Liquidity Constraints and Precautionary Saving

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Abstract

We provide the analytical explanation of strong interactions between precautionary saving and liquidity constraints that are regularly observed in numerical solutions to consumption/saving models. The effects of constraints and of uncertainty spring from the same cause: counterclockwise concavification of the consumption function, which can be induced either by constraints or by uncertainty. Such concavification propagates back to consumption functions in prior periods. But, surprisingly, once a linear consumption function has been concavified by the presence of either risks or constraints, the introduction of *additional* concavifiers in a given period can *reduce* the precautionary motive in earlier periods at some levels of wealth.

Keywords liquidity constraints, uncertainty, precautionary saving

JEL codes C6, D91, E21

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1 Introduction

A large literature has shown that numerical models that take constraints and uncertainty seriously can yield different conclusions than those that characterize traditional models. For example, Kaplan, Moll, and Violante (2018) show that when sufficiently many households have high marginal propensities to consume (MPC's), a major transmission channel of monetary policy is the 'indirect income effect' – a channel of minimal importance in traditional macro models. Similarly, Guerrieri and Lorenzoni (2017) and Bayer, Lütticke, Pham-Dao, and Tjaden (2019) show that tightened borrowing conditions and heightened income risk can help explain the consumption decline during the great recession.

A drawback to numerical solutions is that it is often difficult to know why results come out the way they do. A leading example is in the complex relationship between precautionary saving behavior and liquidity constraints.[1] At least since Zeldes (1984), economists working with numerical solutions have known that liquidity constraints can strictly increase precautionary saving under very general circumstances. On the other hand, simulations have sometimes found circumstances under which liquidity constraints and precautionary saving are substitutes. In an early example, Samwick (1995) showed that unconstrained consumers with a precautionary saving motive in a retirement saving model behave in ways qualitatively and quantitatively similar to the behavior of liquidity constrained consumers facing no uncertainty.

This paper provides the theoretical tools to make sense of the interactions between liquidity constraints and precautionary saving. The main theoretical innovation is to conceptualize the effects of either constraints or risks in terms of consumption concavity. The advantage of understanding the effects in terms of consumption concavity is that there is a link between more consumption concavity (concavification) and prudence, and therefore also precautionary saving (Kimball, 1990). In particular, we show that prudence of the *value* function is increased by any concavification of the consumption function regardless of its cause.

Our first main result is to show that the introduction of a constraint at the end of period t causes consumption concavity around the point where the constraint binds.[2] Furthermore, once consumption concavity is created, it propagates back to periods before

[1] For the seminal numerical examination of some of the interactions between precautionary saving and liquidity constraints, see Deaton (1991).

[2] The connection between constraints and consumption concavity has been explored in more specific settings. See e.g. Park (2006) for CRRA utility, Seater (1997) for the case where time-discounting equals the interest rate, Nishiyama and Kato (2012) for quadratic utility, and Holm (2018) for the case with infinitely-lived households with HARA utility.

our references (2018); for reference to the toolkit itself see Acknowleding Econ-ARK. Thanks to the Consumer Financial Protection Bureau for funding the original creation of the Econ-ARK toolkit; and to the Sloan Foundation for funding Econ-ARK's extensive further development that brought it to the point where it could be used for this project. The toolkit can be cited with its digital object identifier, 10.5281/zenodo.1001067, as is done in the paper's own references as 2018. This paper supercedes NBER working paper no. 8496 from 2001. We are grateful to Mark Huggett for suggesting the current proof of one of our lemmas, Luigi Pistaferri, Misuka Otsuka, and to conference participants in the conferences "Macroeconomics and Household Borrowing" sponsored by the Finance and Consumption program and the European University in May 2005 and "Household Choice of Consumption, Housing, and Portfolio" at CAM in Copenhagen in June 2005. Kimball is grateful to the National Institute on Aging for research support via grant P01-AG10179 to the University of Michigan.

t. Carroll and Kimball (1996) showed similar results for the effects of risks on consumption concavity. Hence, the two papers establish rigorously that both constraints and risks create a form of consumption concavity that propagates backward.

Since prudence is heightened when the consumption function is more concave, it follows immediately that when a liquidity constraint is added to a standard consumption problem, the resulting value function exhibits increased prudence around the level of wealth where the constraint becomes binding.[3] Constraints induce precaution because constrained agents have less flexibility in responding to shocks when the effects of the shocks cannot be spread out over time. The precautionary motive is heightened by the desire (in the face of risk) to make future constraints less likely to bind.[4] This can explain why such a high percentage of households cite precautionary motives as the most important reason for saving (Kennickell and Lusardi, 1999) even though the fraction of households who report actually having been constrained in the past is relatively low (Jappelli, 1990).

After establishing that the introduction of a constraint increases the precautionary saving motive, we show that the introduction of a *further* future constraint may actually *reduce* the precautionary saving motive by 'hiding' the effects of pre-existing constraints or risks. An existing constraint may be rendered irrelevant at levels of wealth where the new constraint forces more saving than the existing constraint would induce. Identical logic implies that uncertainty can 'hide' the effects of a constraint because the consumer may save so much for precautionary reasons that the constraint becomes irrelevant. Thus, the introduction of a new constraint or risk does not generally strengthen the precautionary motive.

A concrete example helps clarify the intuition. A typical perfect foresight model of consumption for a retired consumer with guaranteed income (e.g., 'Social Security') implies that a legal constraint on borrowing can make the consumer run their wealth down to zero (thereafter setting consumption equal to income). Now consider modifying the model to incorporate the possibility of large medical expenses near the end of life (e.g. nursing home fees; see Ameriks, Caplin, Laufer, and Van Nieuwerburgh, 2011). Under reasonable assumptions, a consumer facing such a risk may save enough for precautionary reasons to render the no-borrowing constraint irrelevant.

Although there is no general result for the effects of additional constraints or risks when the consumer already faces existing constraints or risks, we can establish how the introduction of *all* constraints and risks affects the precautionary saving motive. We show that the precautionary saving motive is stronger at every level of wealth[5] in the presence of *all* future risks and constraints than in the case with *no* risks and constraints. This is because the consumption function is concave everywhere in the presence of all future risks and constraints,[6] and since consumption concavity heightens prudence of the value

[3] A relationship between constraints and prudence has also been noted by Lee and Sawada (2007) and documented empirically in Lee and Sawada (2010).

[4] To be clear, the liquidity constraint we analyze here must be satisfied in each period (one-period bonds). This implies that the interactions between constraints and income volatility where some households may prefer to increase (credit card) debt today because they expect tighter credit conditions in the future are ruled out (Fulford, 2015; Druedahl and Jørgensen, 2018).

[5] More precisely, we show that there is no level of wealth at which the motive is weaker, and at least some levels at which it is strictly stronger.

[6] Again, there is no level of wealth at which the consumption function becomes less concave, and at least some levels of wealth at which it becomes strictly more concave.

function, the precautionary saving motive is also stronger in the presence of all risks and constraints than in the case with no risks and constraints.

Hence, we can summarize this paper as follows. The effects of liquidity constraints and risks are similar because both stem from the same source: a concavification of the consumption function. The effects work independently, meaning that neither risks nor constraints are necessary to concavify the consumption function. And since a more concave consumption function exhibits heightened prudence, both constraints and risks strengthen the precautionary saving motive. In addition, we explain the apparently contradictory results that constraints and risks in some cases intensify, but in other cases weaken the precautionary saving motive. The central insight is that the effect of introducing an additional constraint or risk depends on whether it weakens the effects of any pre-existing constraints or risks. If it does not interact with any pre-existing constraints or risks, it intensifies the precautionary saving motive. If it does interact, it may weaken the precautionary saving motive at some levels of wealth.

The rest of the paper is structured as follows. To fix notation and ideas, the next section sets out the general theoretical framework. Section **??** then defines what we mean by consumption concavity and shows how consumption concavity propagates backward and heightens prudence of the value function. In Section **??**, we show how liquidity constraints cause consumption concavity and thereby also prudence. And Section **??** presents our results on the interactions between liquidity constraints and precautionary saving. The final section concludes.

2 The Setup

In this section we present the consumer framework underlying all results. We consider a finitely-lived consumer living from period t to T who faces some future risks and liquidity constraints. The consumer is maximizing the time-additive present discounted value of utility from consumption $u(c)$. With interest and time preference factors $R \in (0, \infty)$ and $\beta \in (0, \infty)$, and labeling consumption c, stochastic labor income y, end-of-period assets a, liquidity constraint ς, and 'market resources' (the sum of current income and spendable wealth from the past) m_t, the consumer's problem can be written as

$$
\begin{aligned}
V_t(m_t) &= \max_c \ \mathbb{E}_t \left[\sum_{k=0}^{T-t} \beta^k u(c_{t+k}) \right] \\
&\quad s.t. \\
m_{t+1} &= (m_t - c_t)R + y_{t+1} \\
a_t &= m_t - c_t \\
a_t &\geq \varsigma_t
\end{aligned}
$$

As usual, the recursive nature of the problem makes this equivalent to the Bellman equation

$$V_t(m_t) = \max_c \ u(c) + \mathbb{E}_t[\beta V_{t+1}((m_t - c)R + y_{t+1})].$$

We define $\Omega_t(a_t) = \mathbb{E}_t[\beta V_{t+1}(Ra_t + y_{t+1})]$ as the end-of-period value function and rewrite the problem as[7]

$$V_t(m_t) = \max_c \ u(c) + \Omega_t(m_t - c).$$

Throughout, what we call 'the consumption function' is the mapping from market resources m_t to consumption. In some of our results we consider utility functions of the HARA class

$$u(c) = \begin{cases} \frac{1}{\alpha_1 - 1} (\alpha_1 c + \alpha_2)^{\frac{\alpha_1 - 1}{\alpha_1}} & \alpha_1 \neq 0, 1 \\ -\alpha_2 e^{-c/\alpha_2} & \alpha_1 = 0 \\ \log(c + \alpha_2) & \alpha_1 = 1 \end{cases} \tag{1}$$

with $\alpha_2 > \max\{-\alpha_1 c, 0\}$. Note that that (1) also covers the case with quadratic utility ($\alpha_1 = -1$).

[7] For notational simplicity we express the value function $V_t(m)$ and the expected discounted value function $\Omega_t(s)$ as functions simply of wealth and savings, but implicitly these functions reflect the entire information set as of time t; if, for example, the income process is not i.i.d., then information on lagged income or income shocks could be important in determining current optimal consumption. In the remainder of the paper the dependence of functions on the entire information set as of time t will be unobtrusively indicated, as here, by the presence of the t subscript. For example, we will call the policy rule in period t which indicates the optimal value of consumption $c_t(m)$. In contrast, because we assume that the utility function is the same from period to period, the utility function has no t subscript.

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