Liquidity Constraints, Precautionary Saving, and Counterclockwise Concavification

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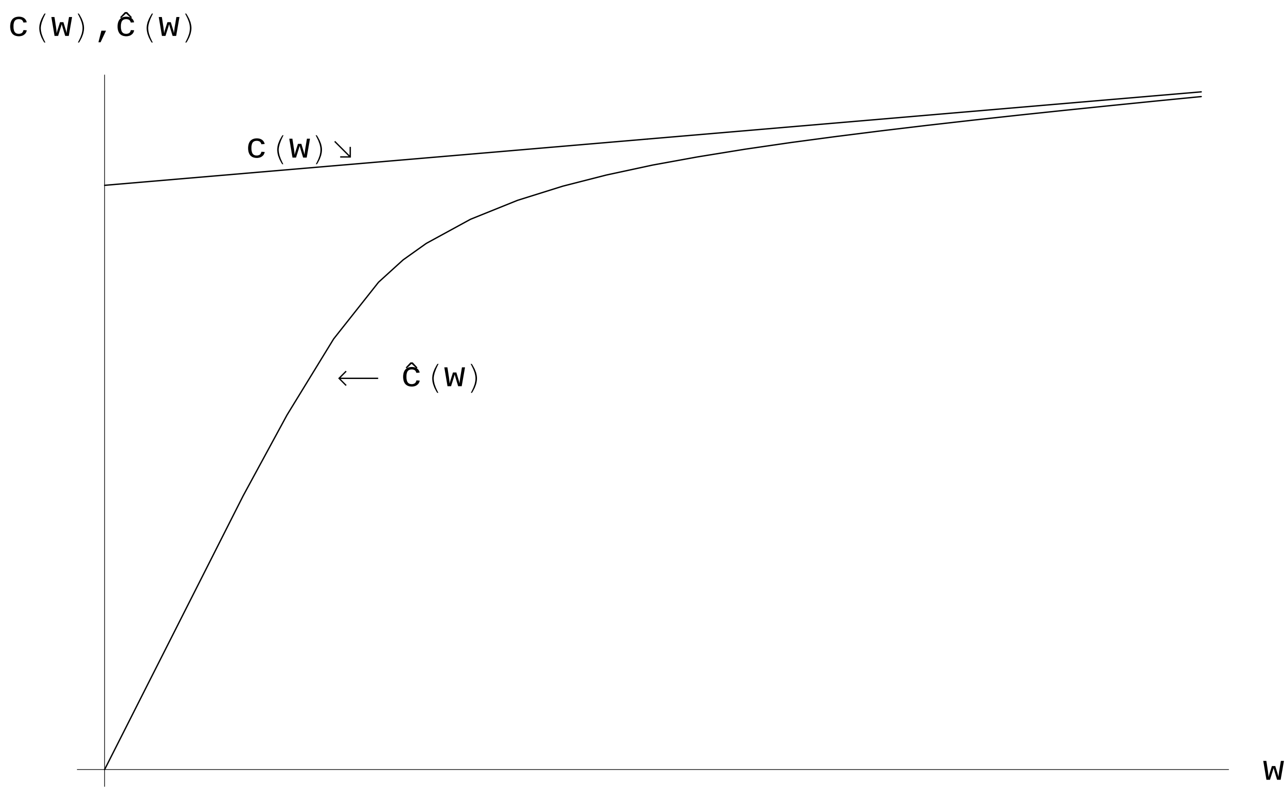
  - Knowing you might face risk intensifies LC effect .

Answer: It's All About Concavity



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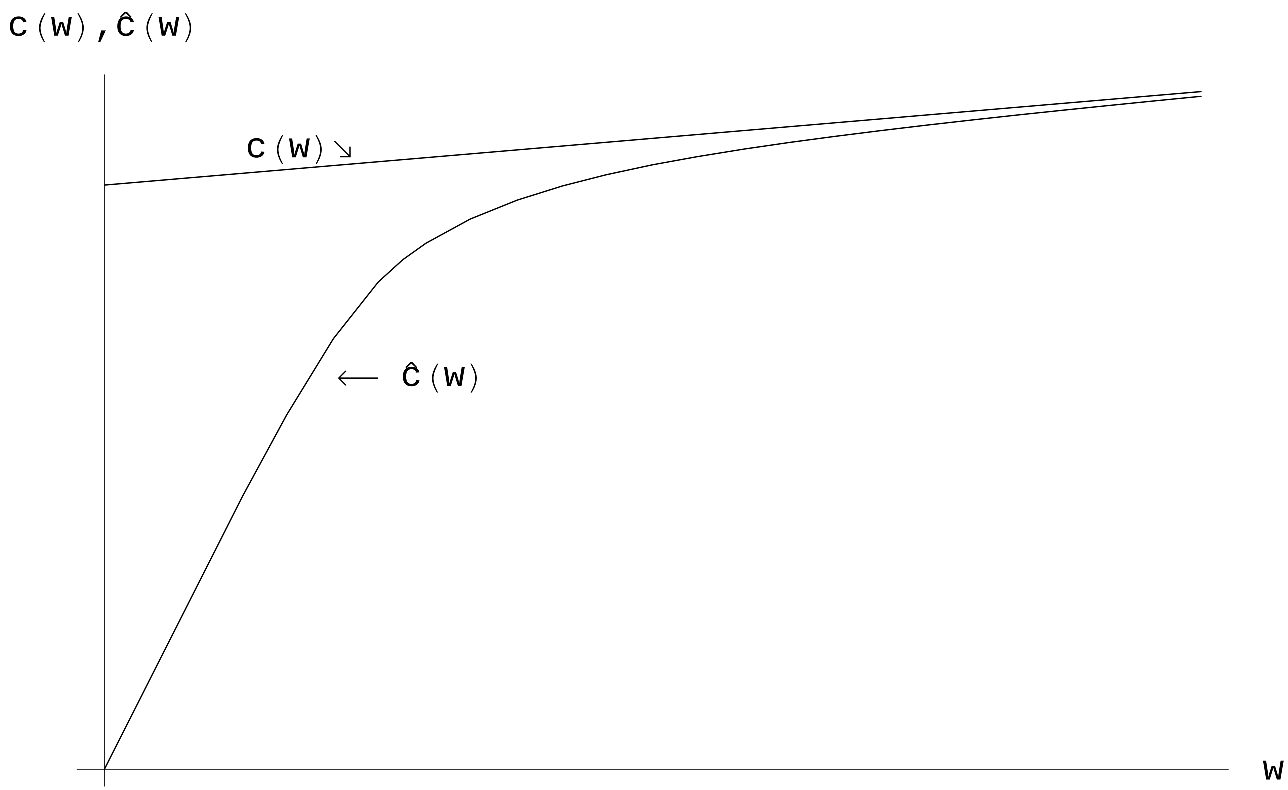

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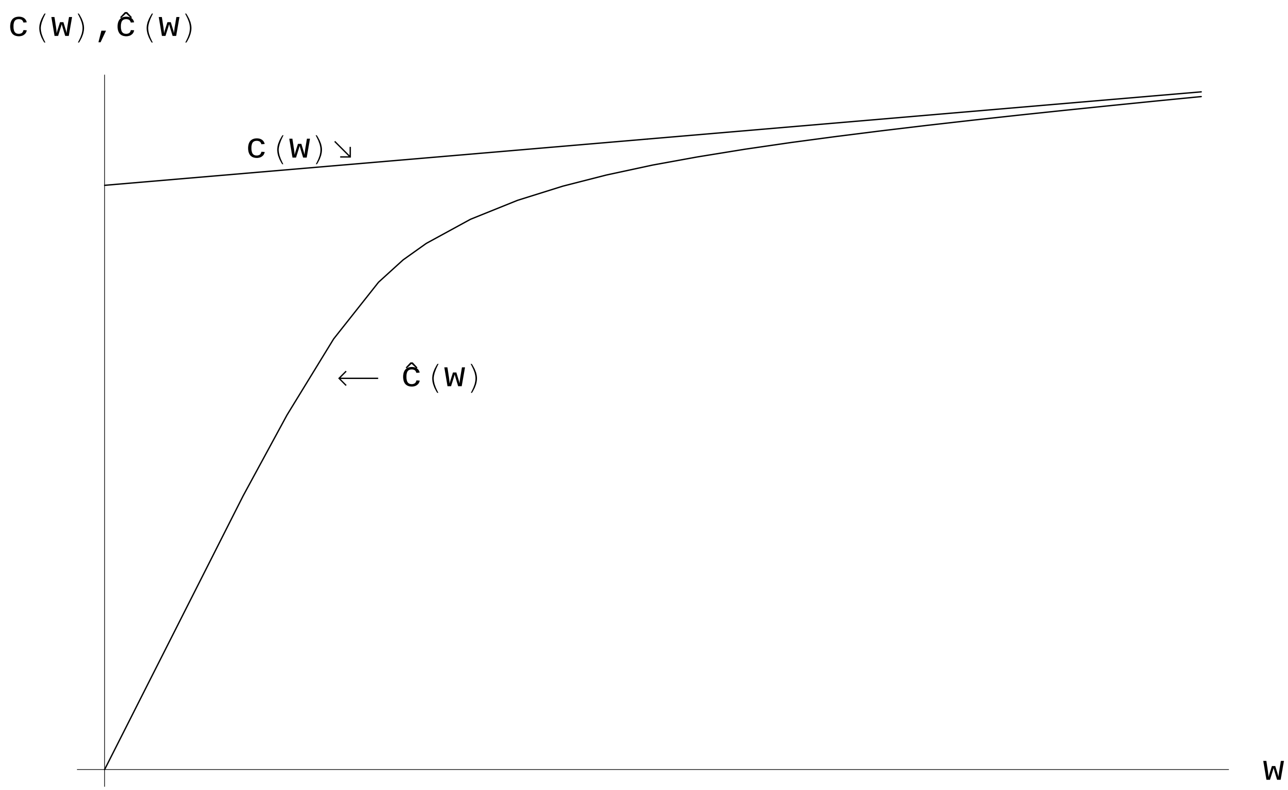

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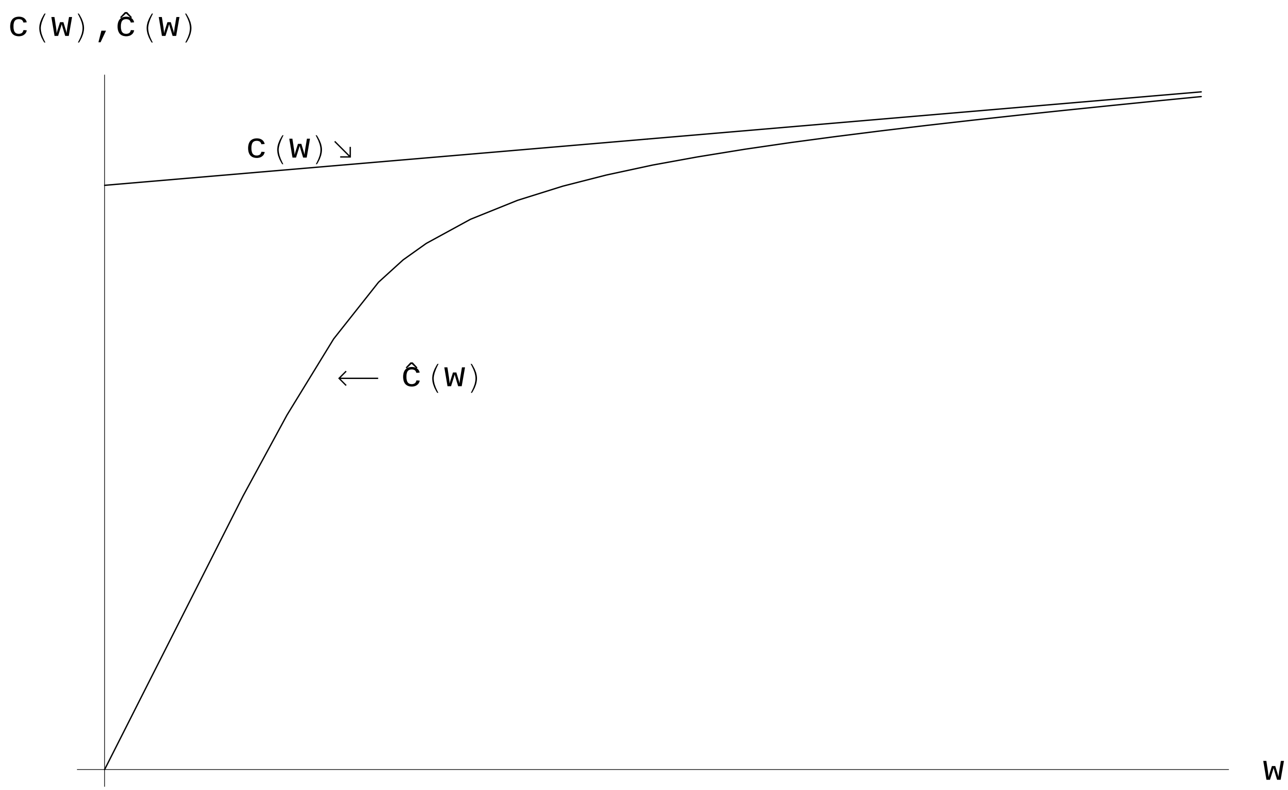

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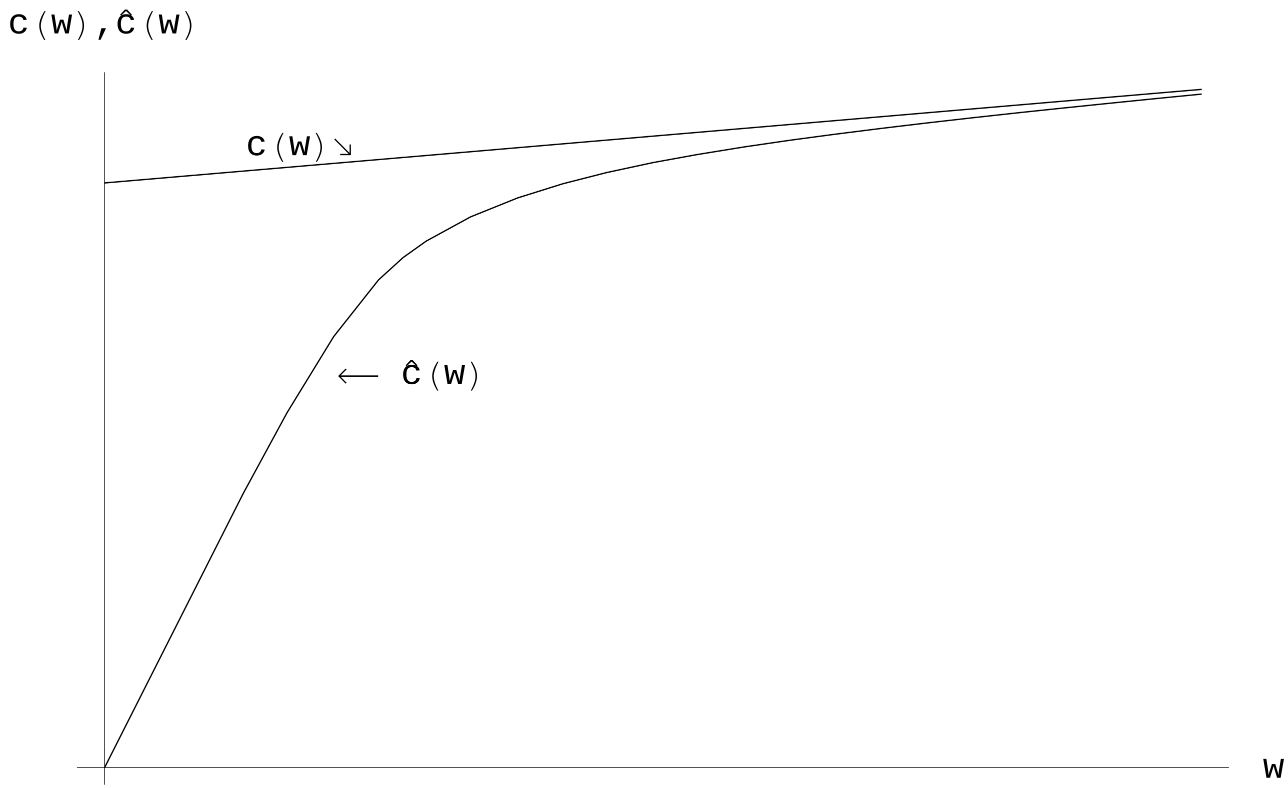

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- Browning, others: "Red Herring!"

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  - Concavity might go up at some m, down at others

Digression on Herrings

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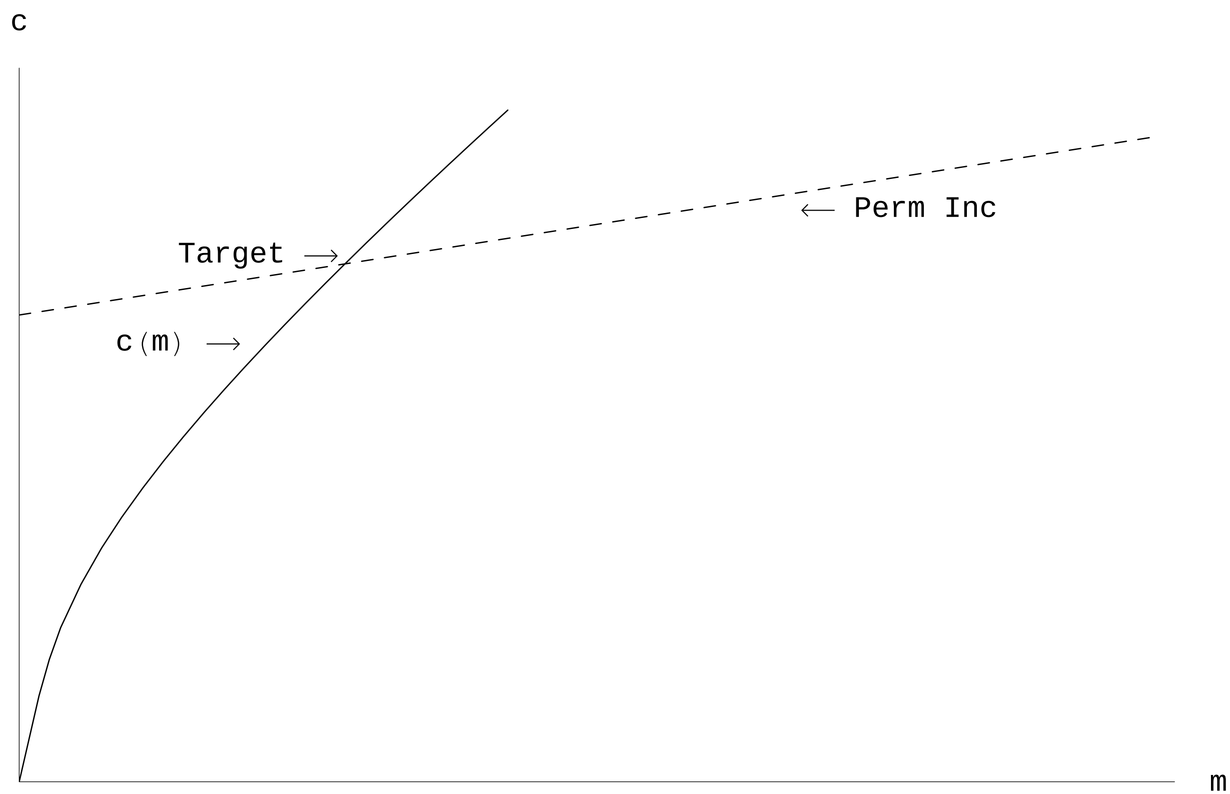

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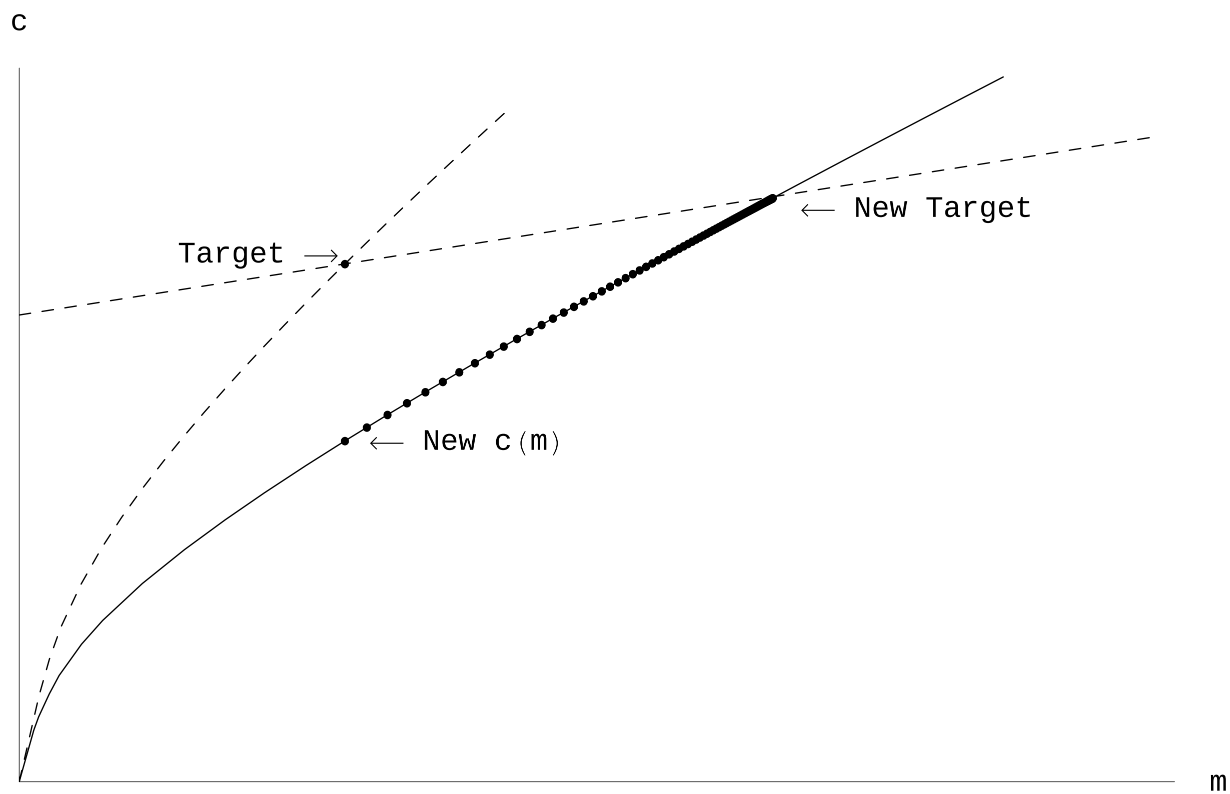

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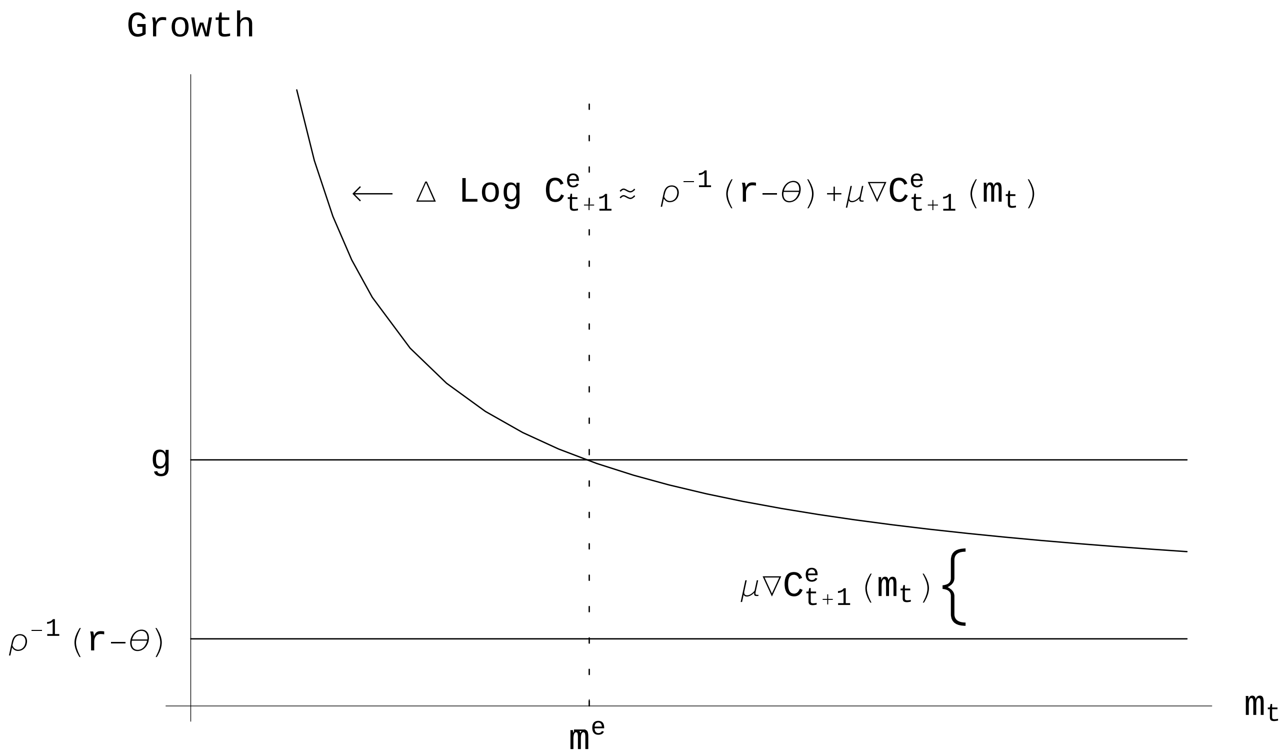
Digression: Target Wealth Ratio



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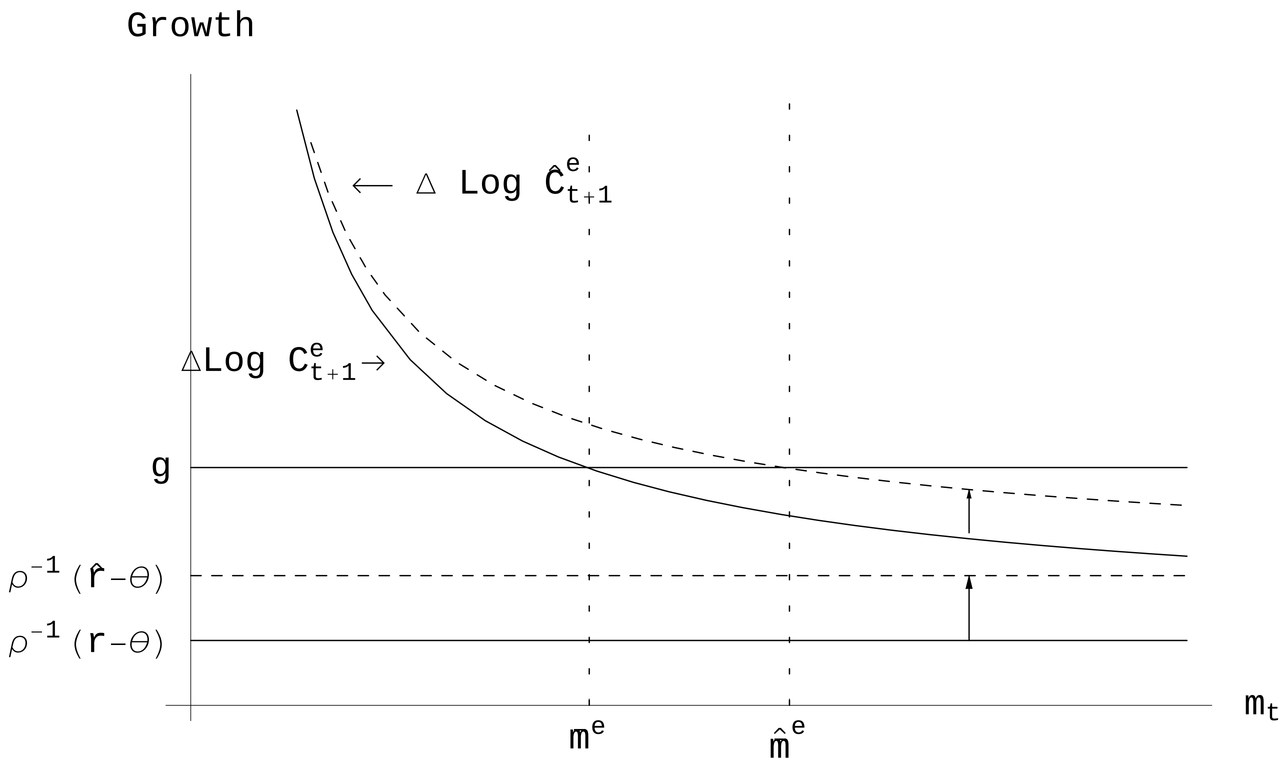

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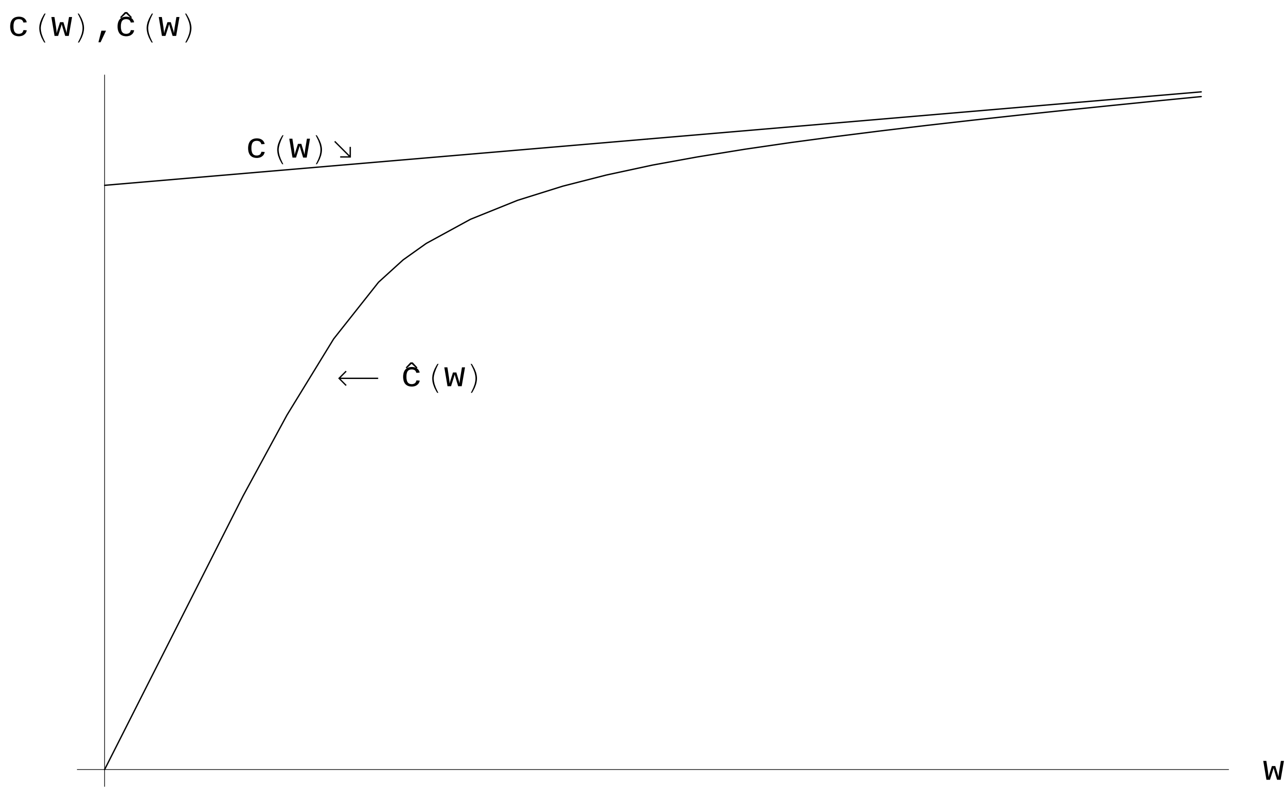
- Prudence of $\hat{V}(m)$ exceeds that of $V(m)$ at m if

$$\mathcal{P}(\hat{c}(m))\hat{c}'(m) \;>\; \mathcal{P}(c(m))c'(m)$$

.

$\mathcal{P}(\hat{c}(m))\hat{c}'(m) > \mathcal{P}(c(m))c'(m)$?



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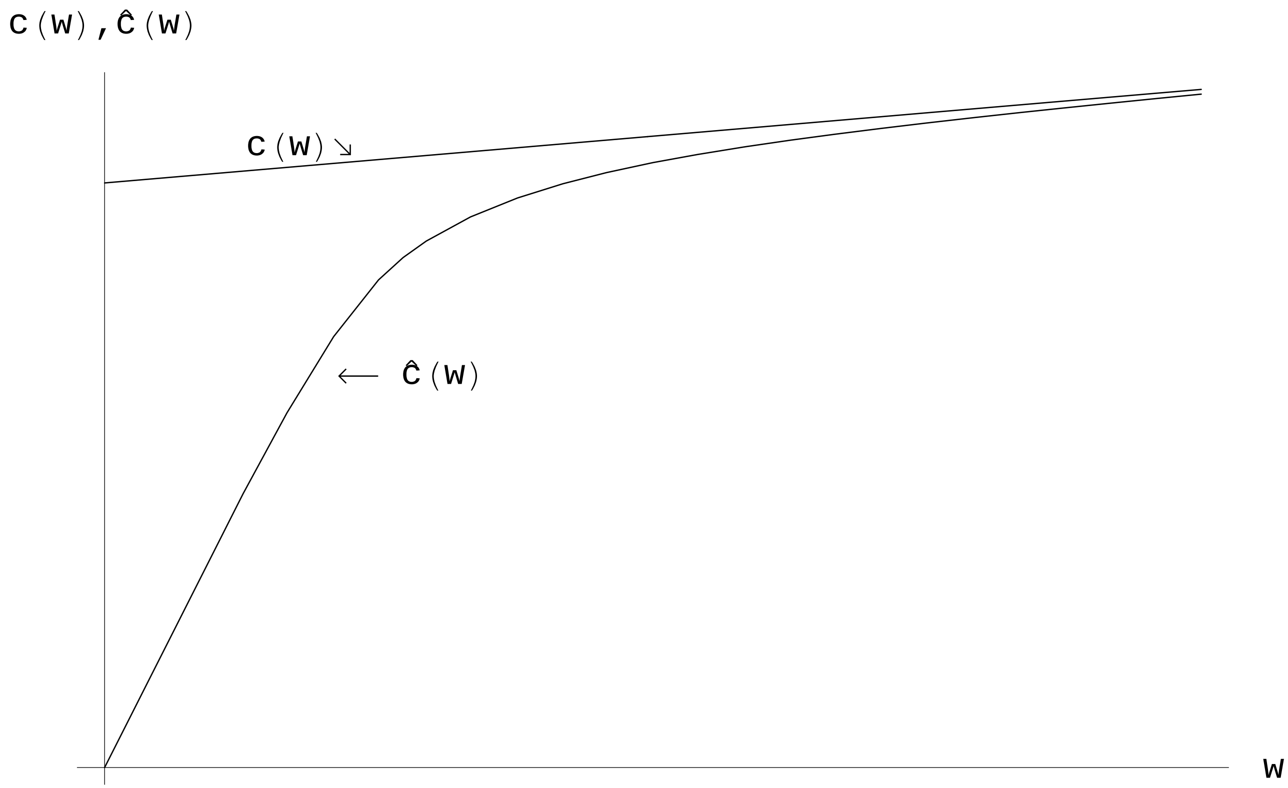

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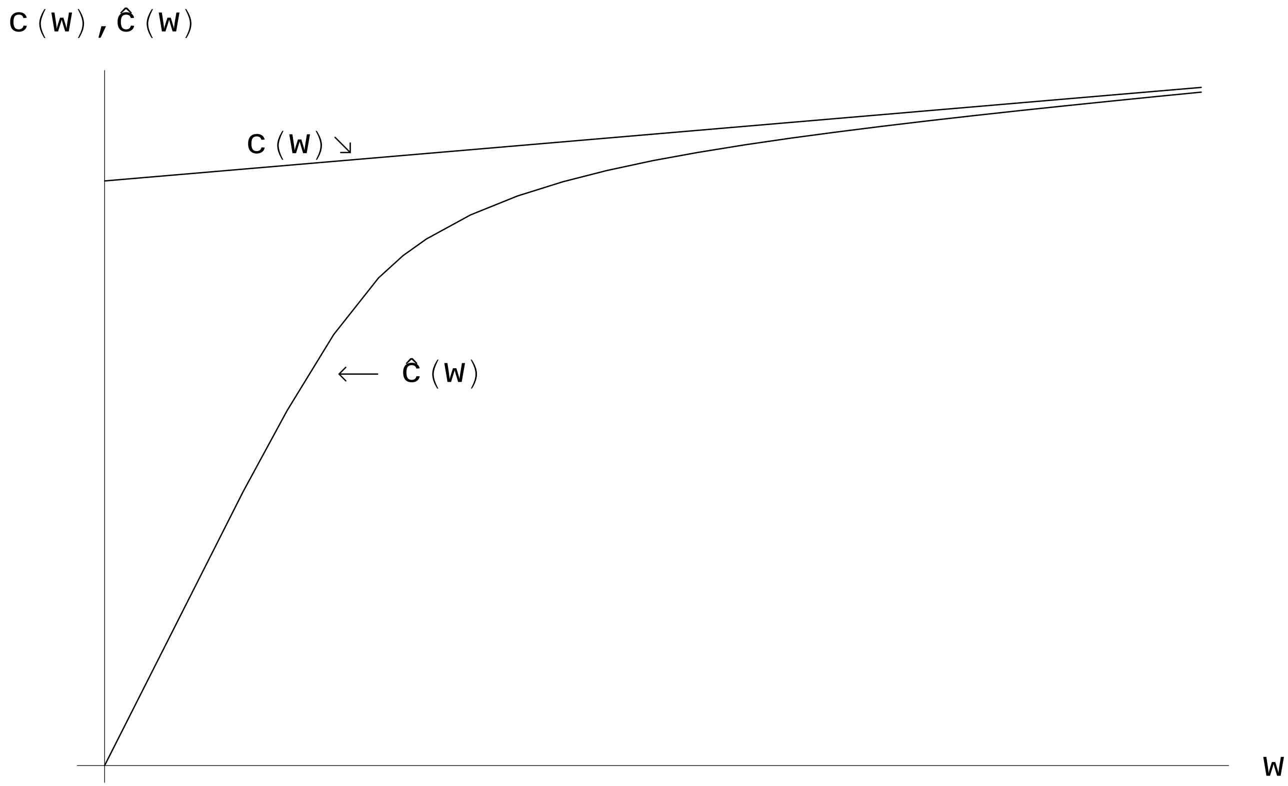

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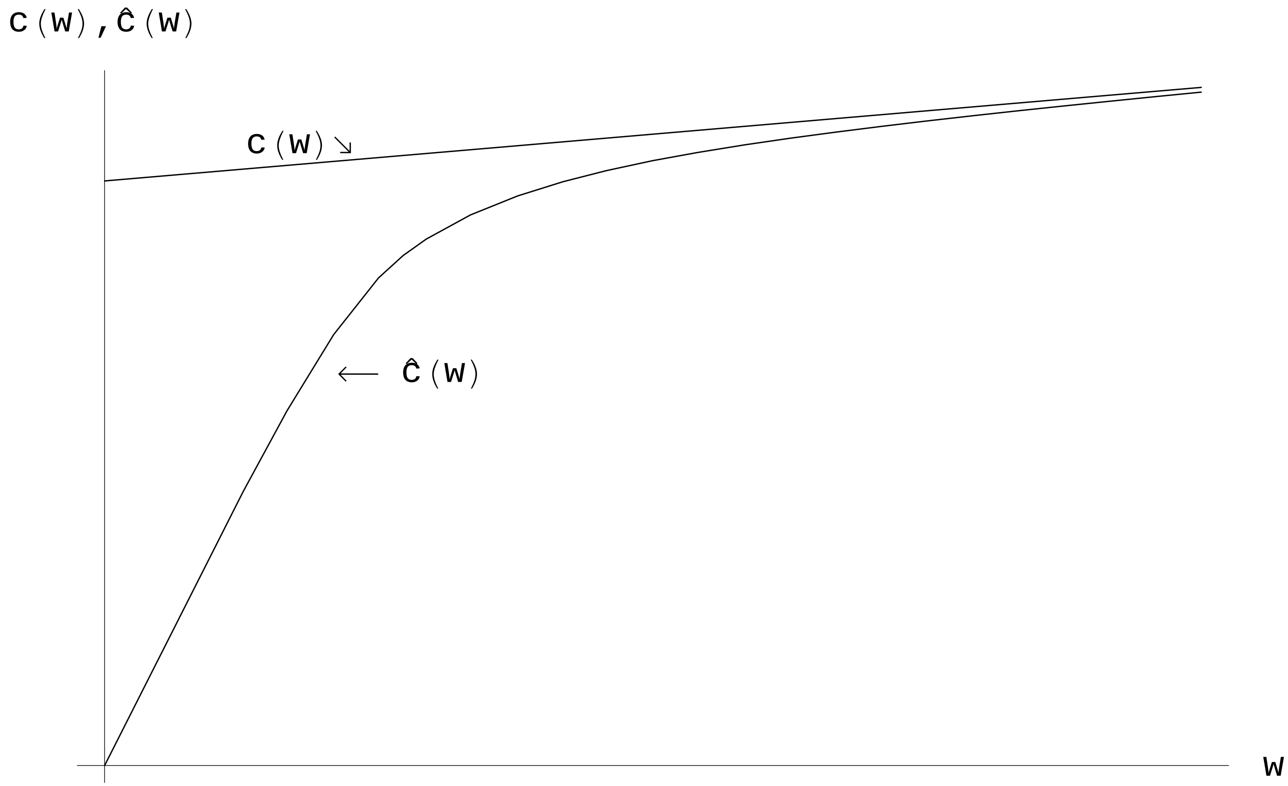

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Consider the consumption function in period $T - 2$

Suppose:

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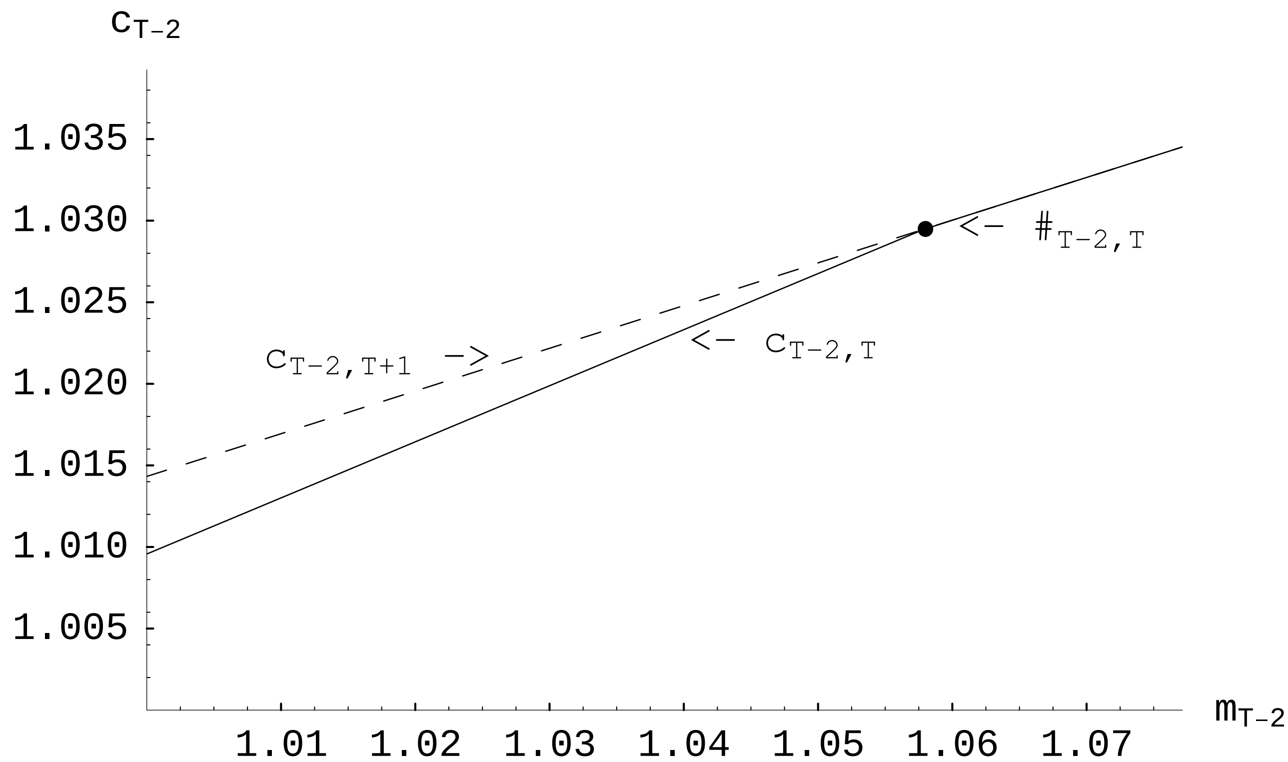

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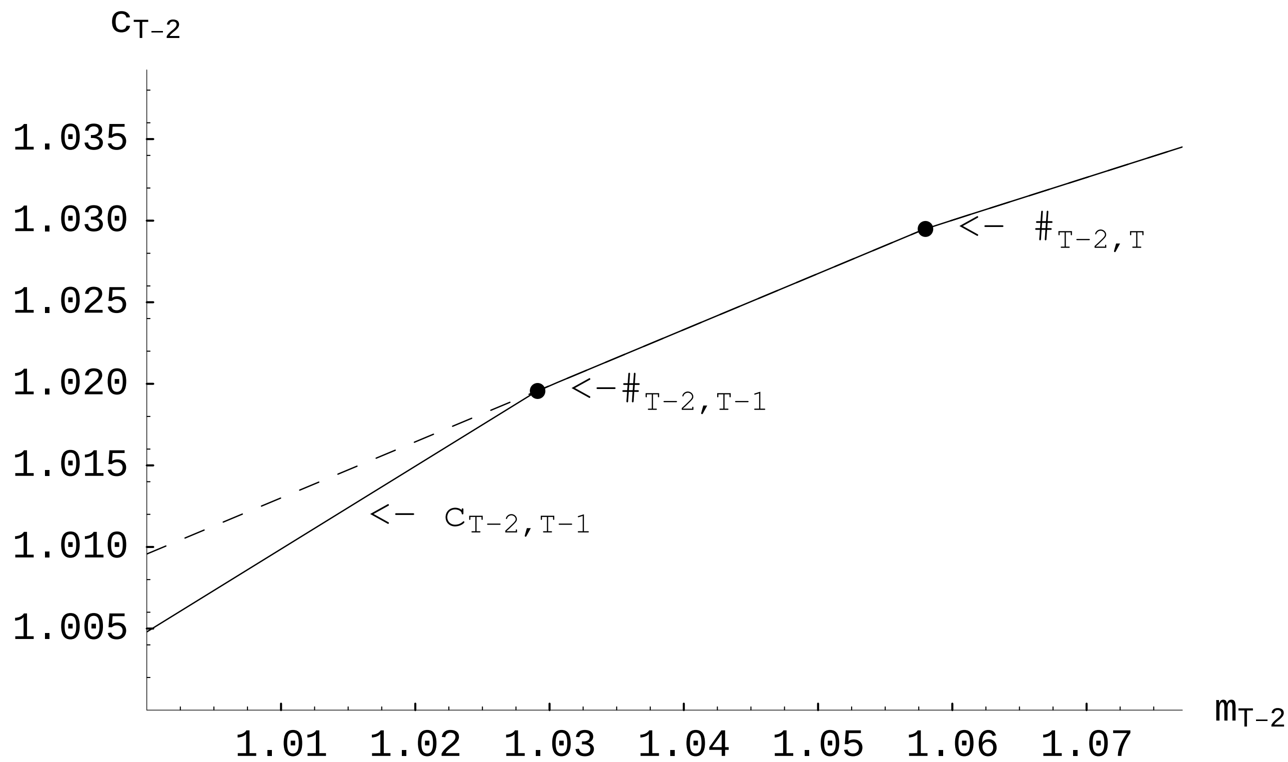

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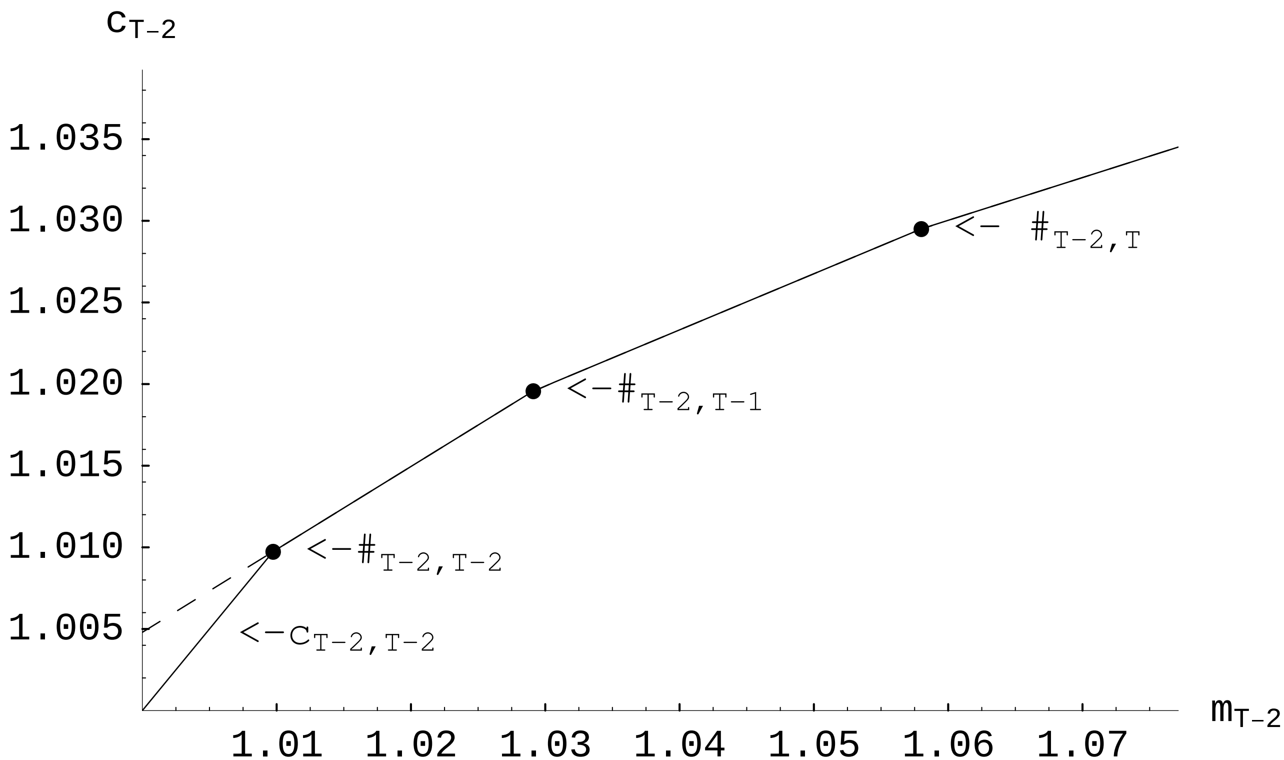

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$\Rightarrow$ Think of prudence as infinite at kink points .

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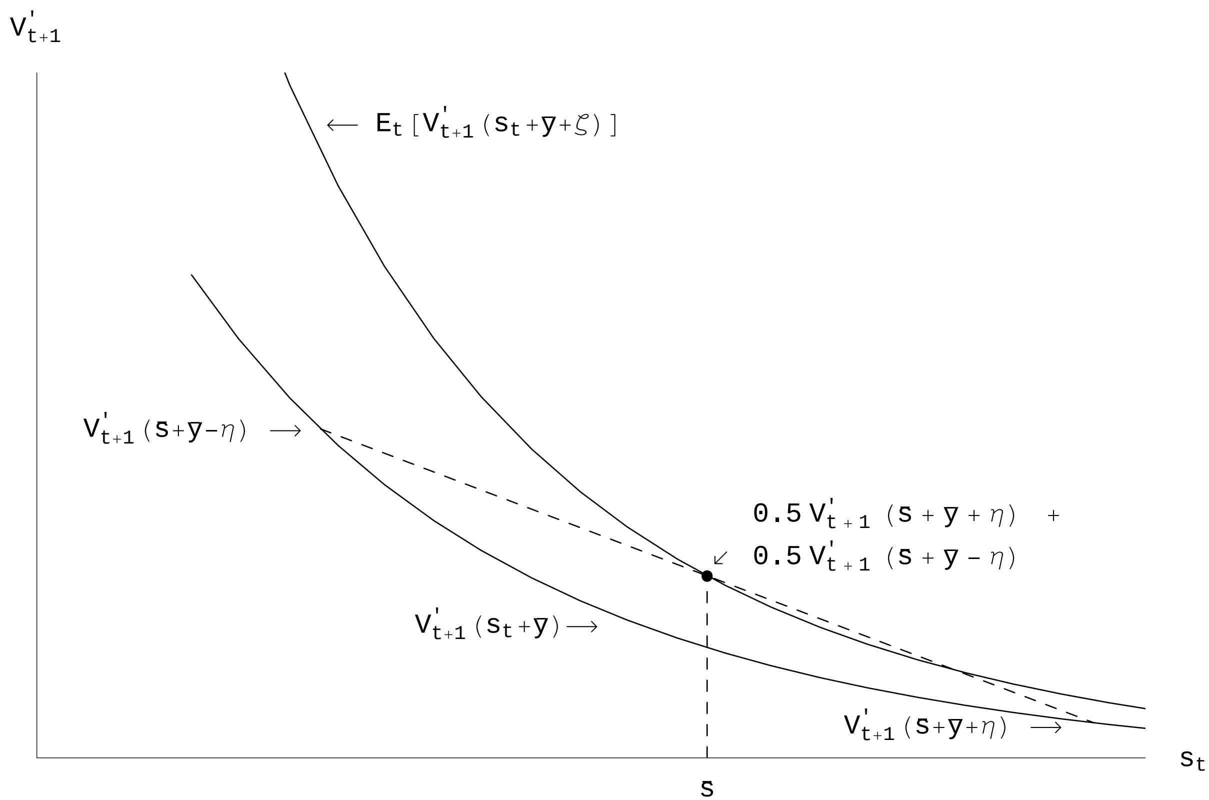
That is, imposing each earlier constraint increases the prudence of the consumption function

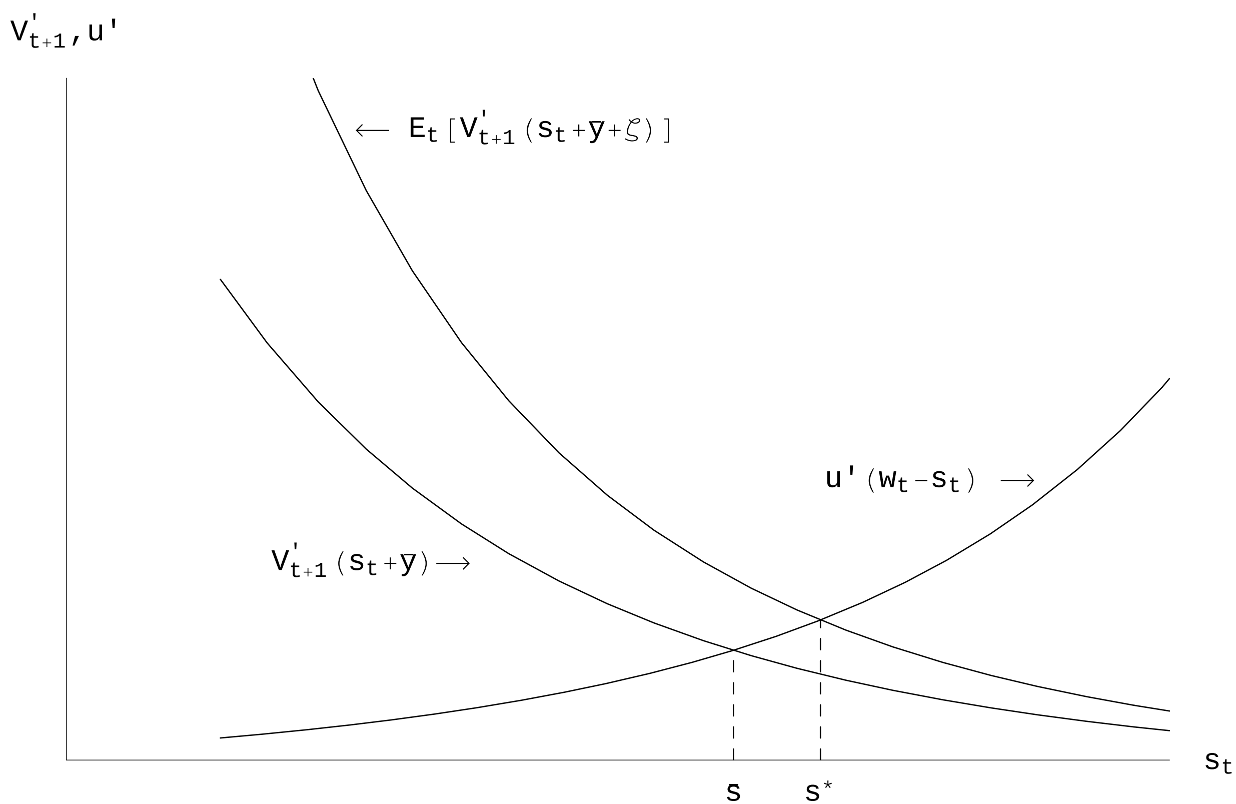
.

Precautionary Saving



Symmetric Two Point Background Risk

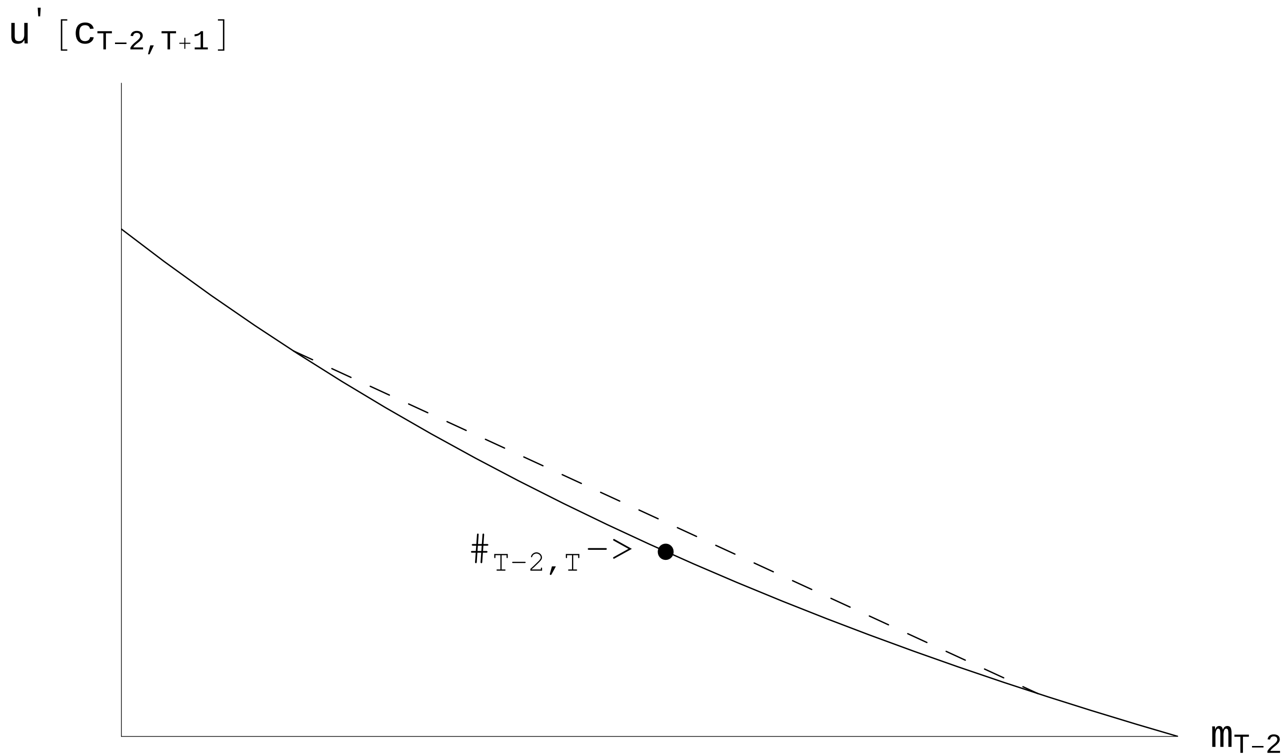
Finding Optimal Saving



$$u'(c_t) = E_t[V'_{t+1}(m_{t+1})]$$

.

C C C Effect



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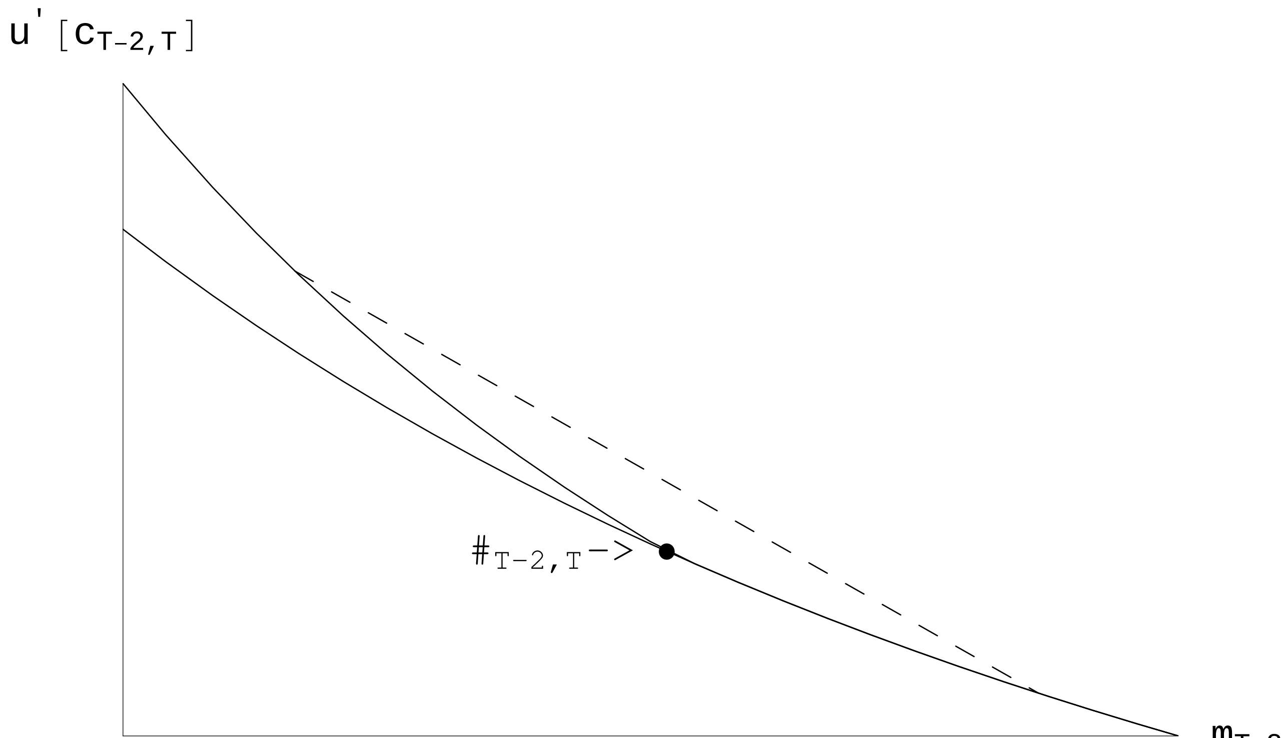

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imposing a *new* constraint that will hold at some date in the future will probably *not* generate a $\hat{c}(m)$ that is a CCC of $c(m)$

.

Example

Setup: Impatient consumer with

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- 3 period life, t to $t+2=T+1$

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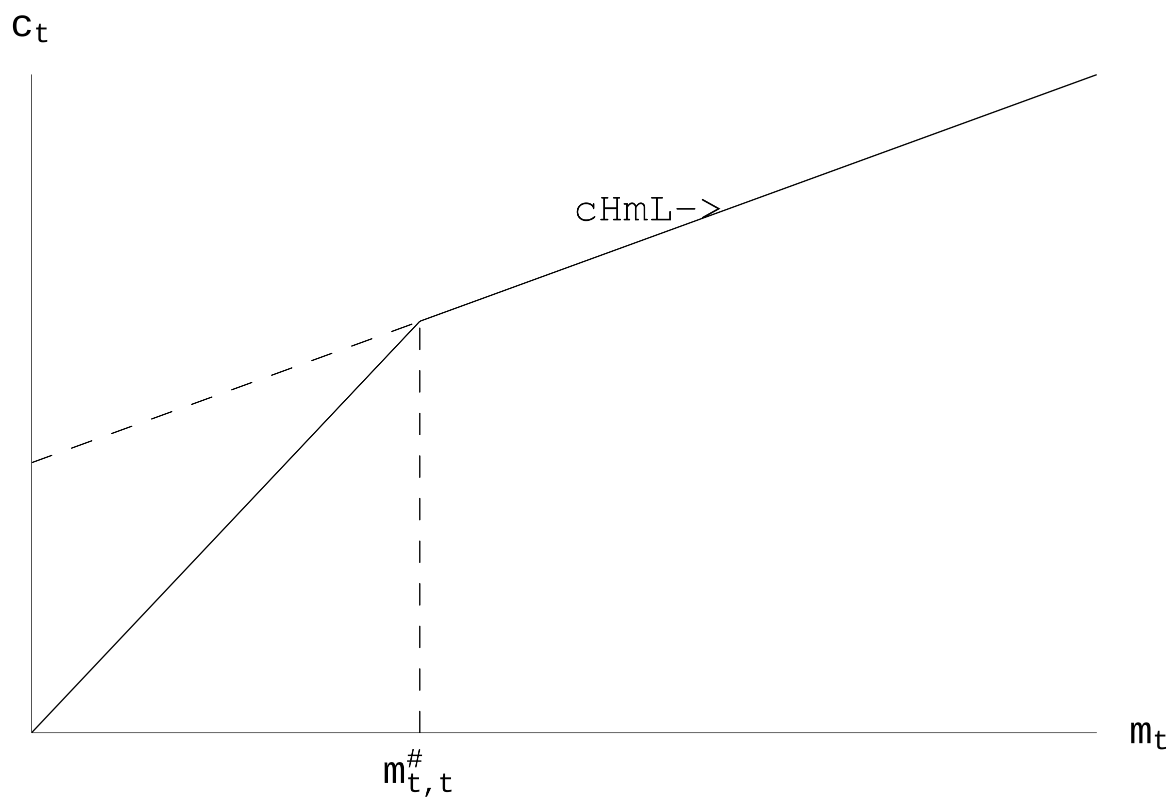
- 3 period life, t to $t + 2 = T + 1$
- Social Security income = 1 in period $t + 2$
- Labor income = 1 in periods t and $t + 1$.

Baseline Constraints: $\mathcal{T}_t$



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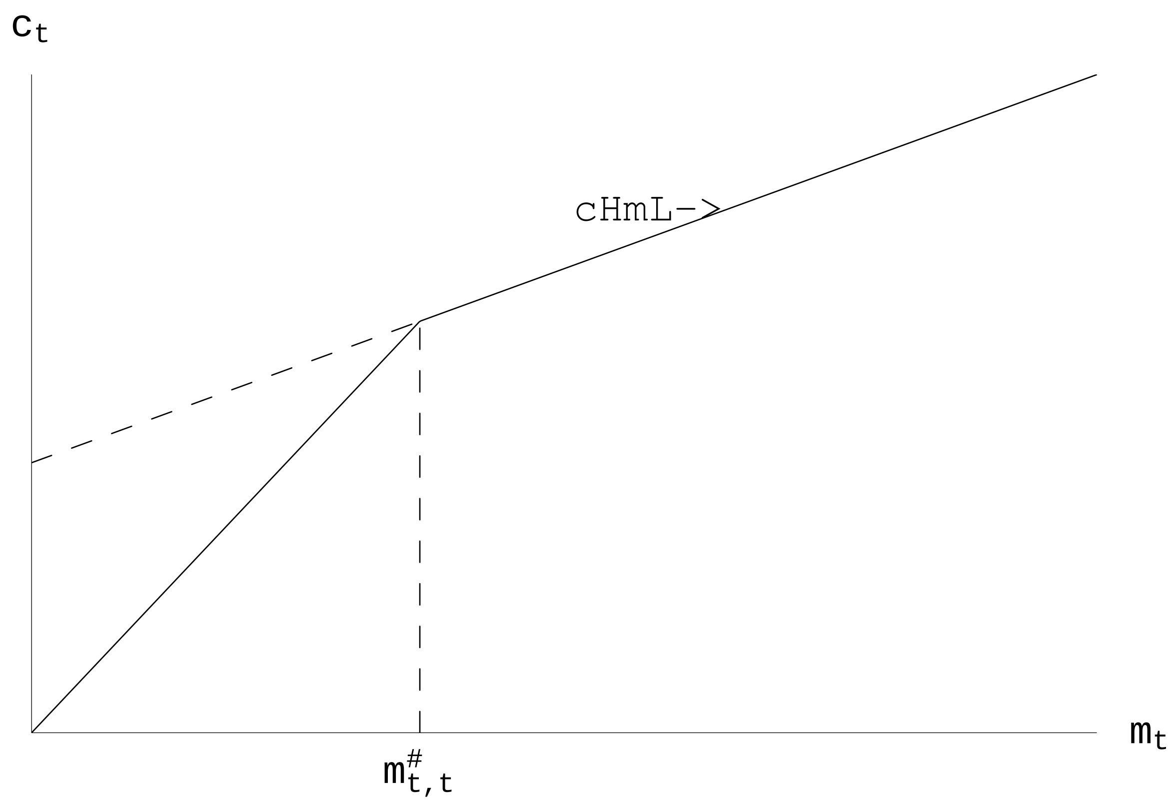

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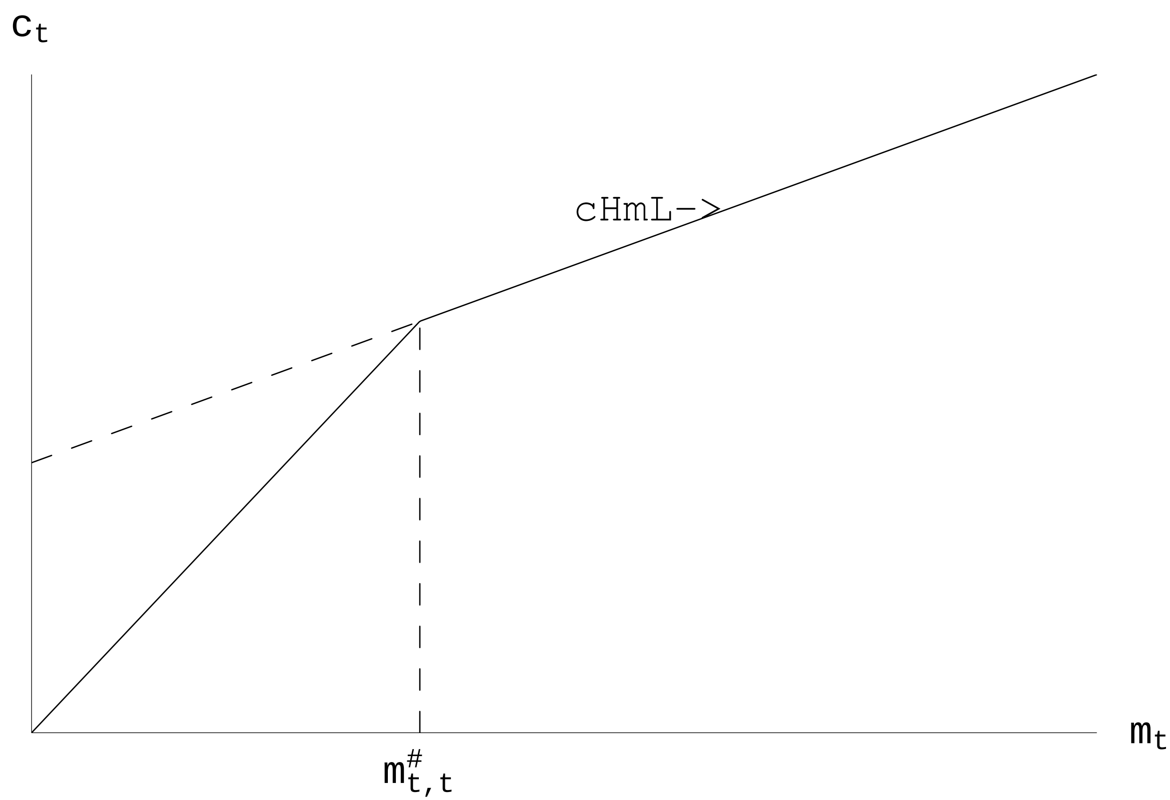

- Baseline: Constraint *only* at date t (constraint set $\mathcal{T}_t$)
- Induces kink in $\mathbf{c}_{t,t}(m)$ at $m^{\#}_{t,1}$

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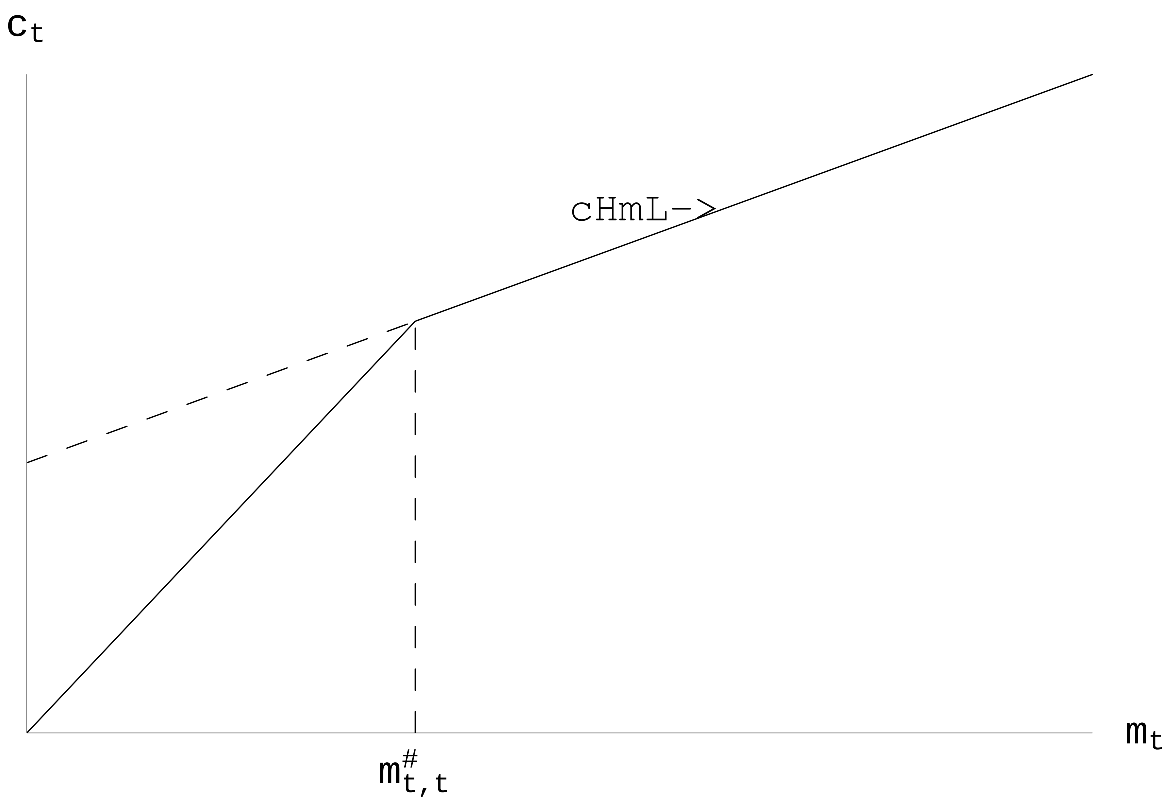
- Can't borrow against SS
- *Want* to *plan* to borrow against SS if $\hat{m}^{\#}_{t,t} < m_t < \hat{m}^{\#}_{t,t+1}$.

Graphically



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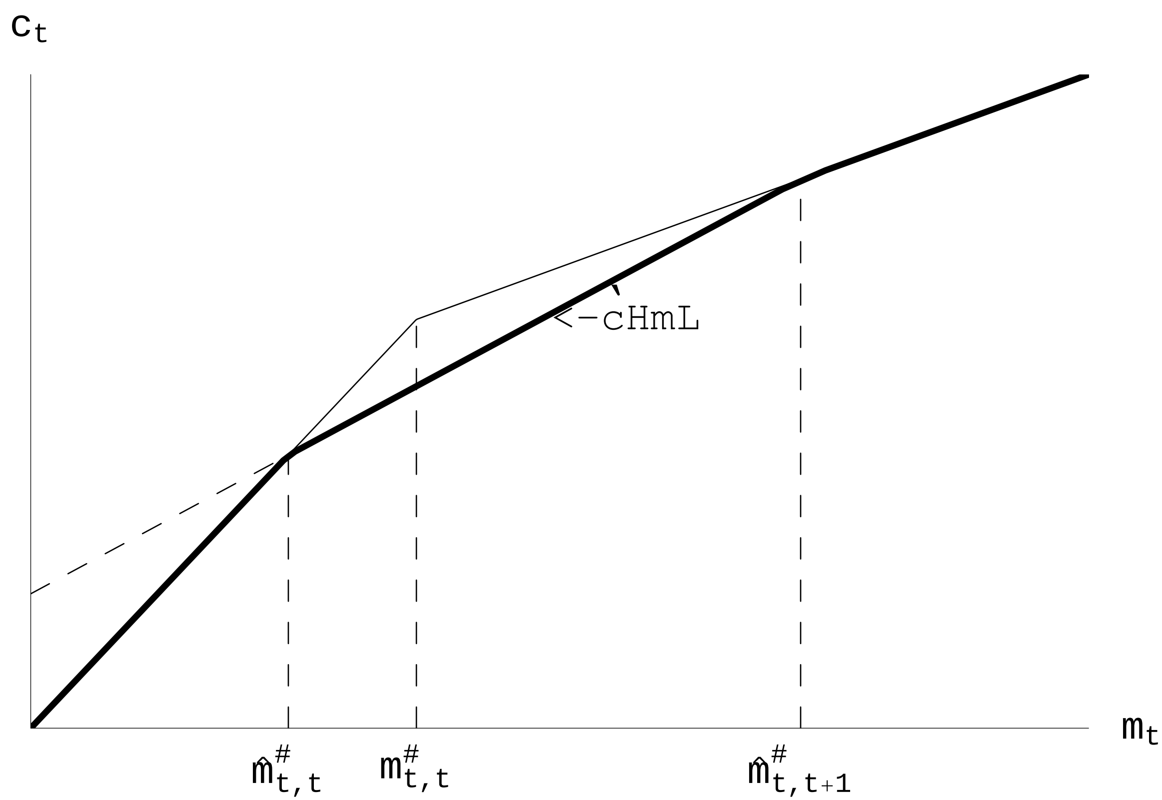

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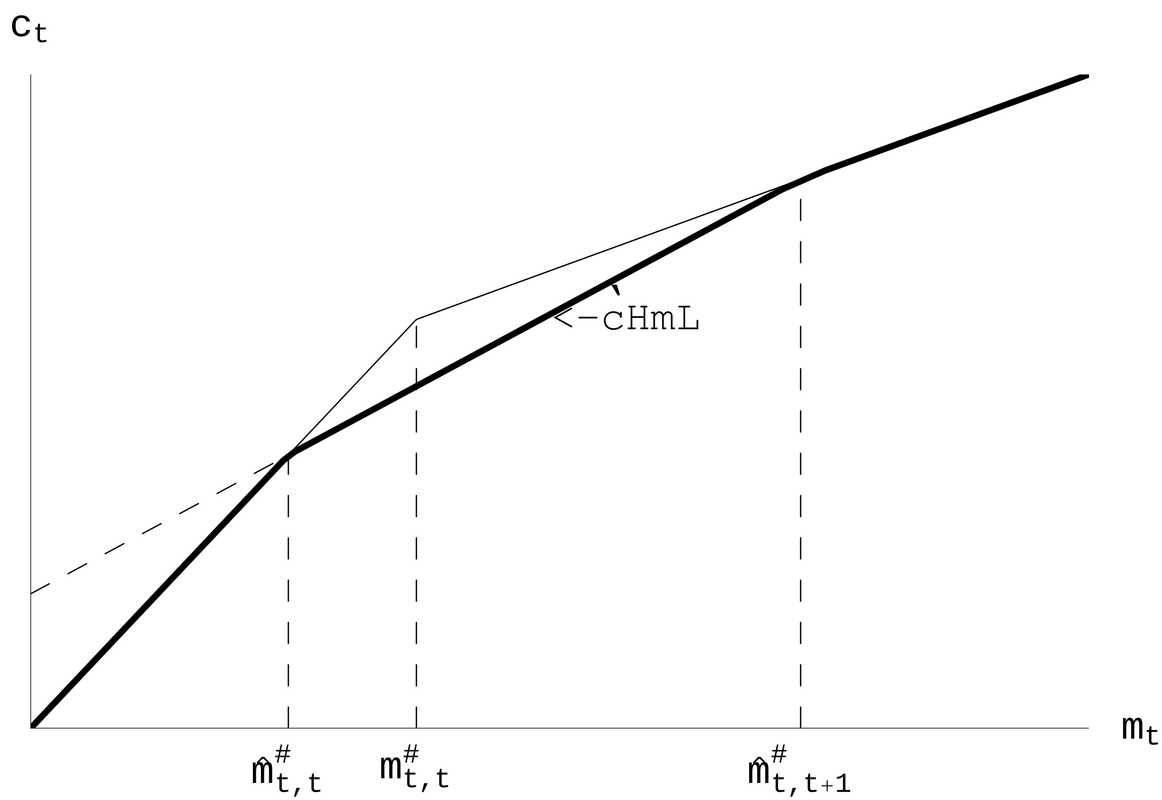

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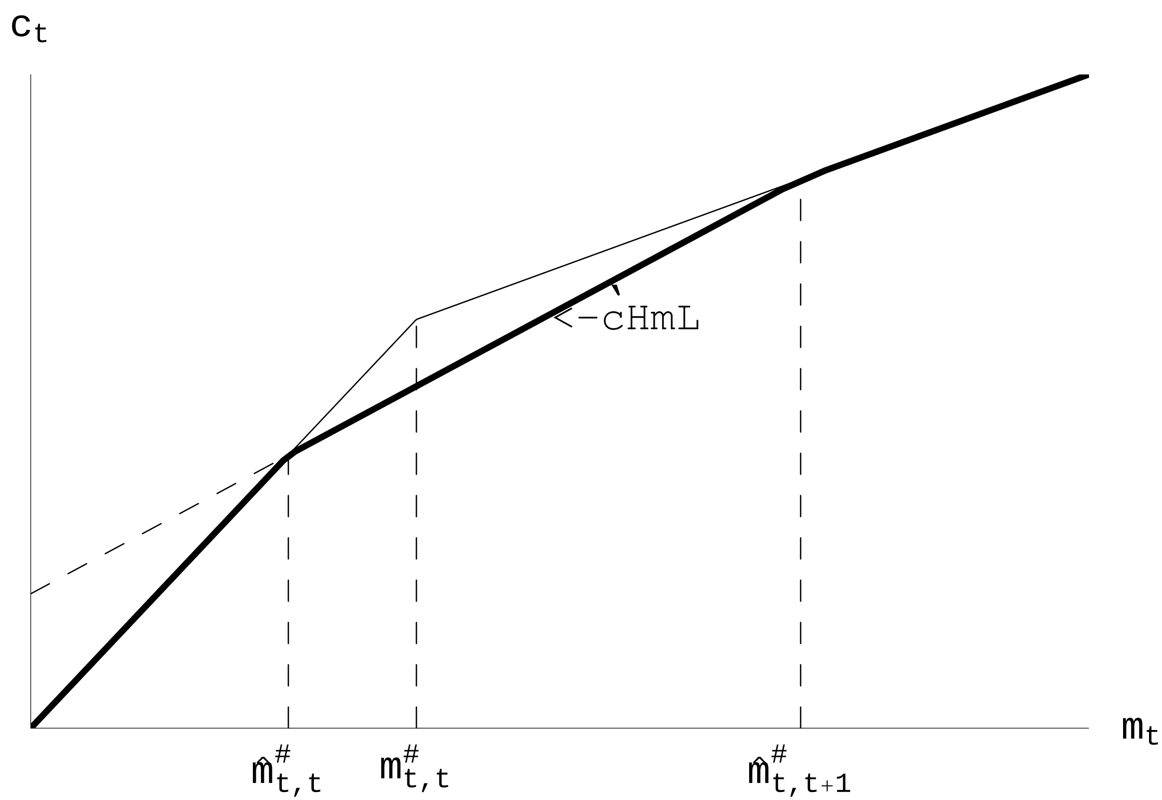

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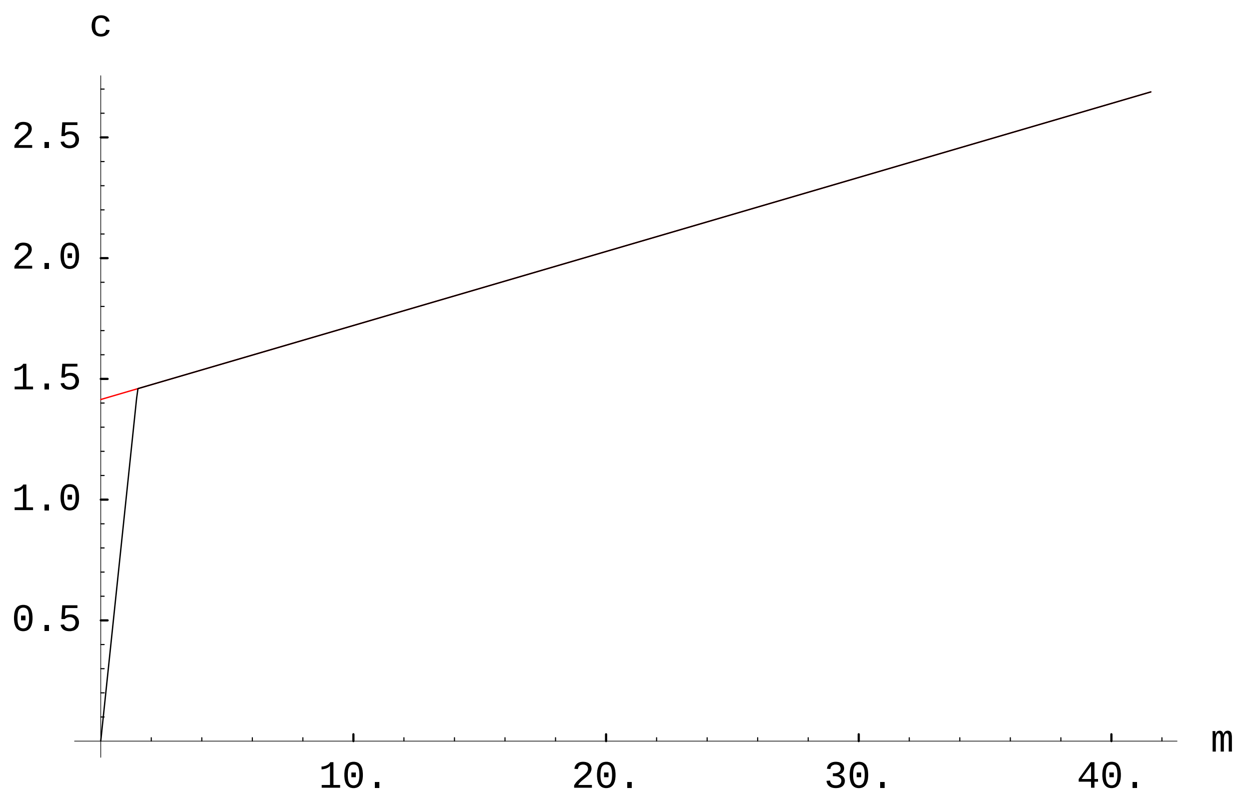
- Modified: $\hat{c}(m)$ prudence finite at $m_{t,t}^{\#}$

Imposing Constraints Forward



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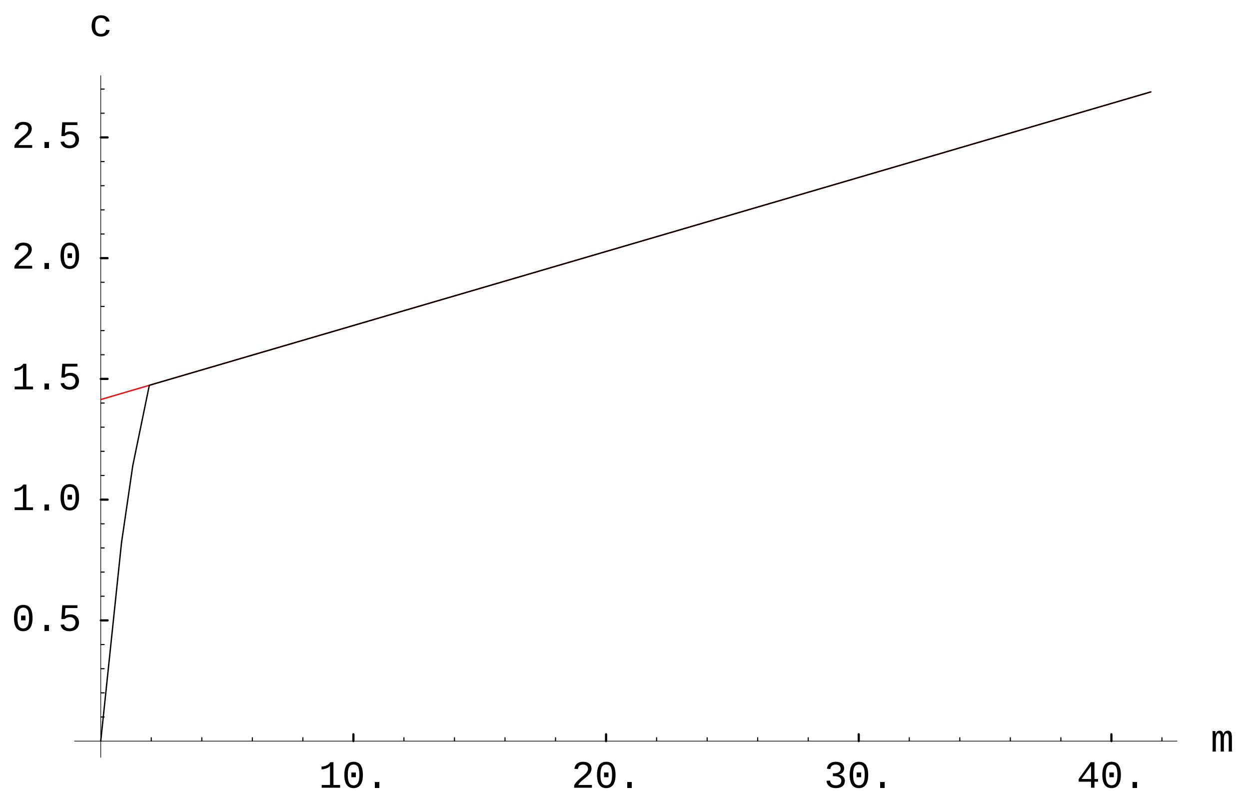

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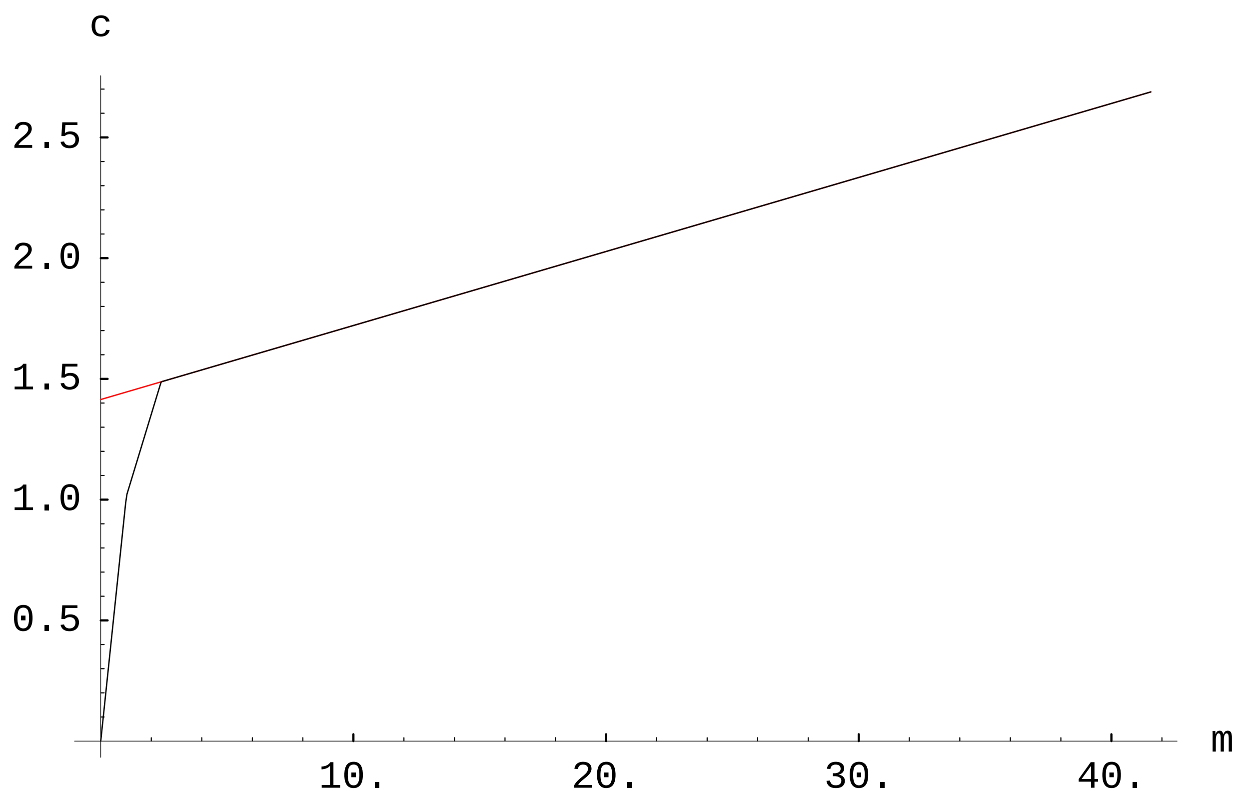

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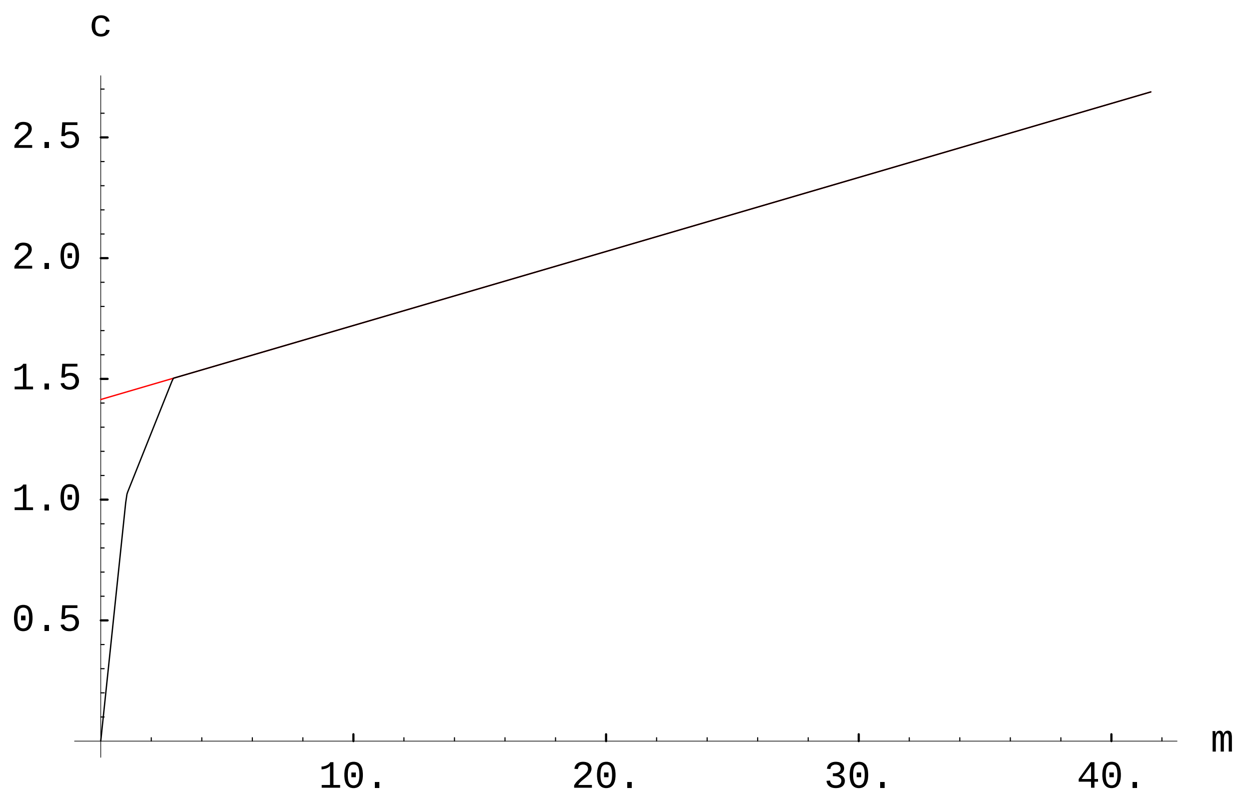

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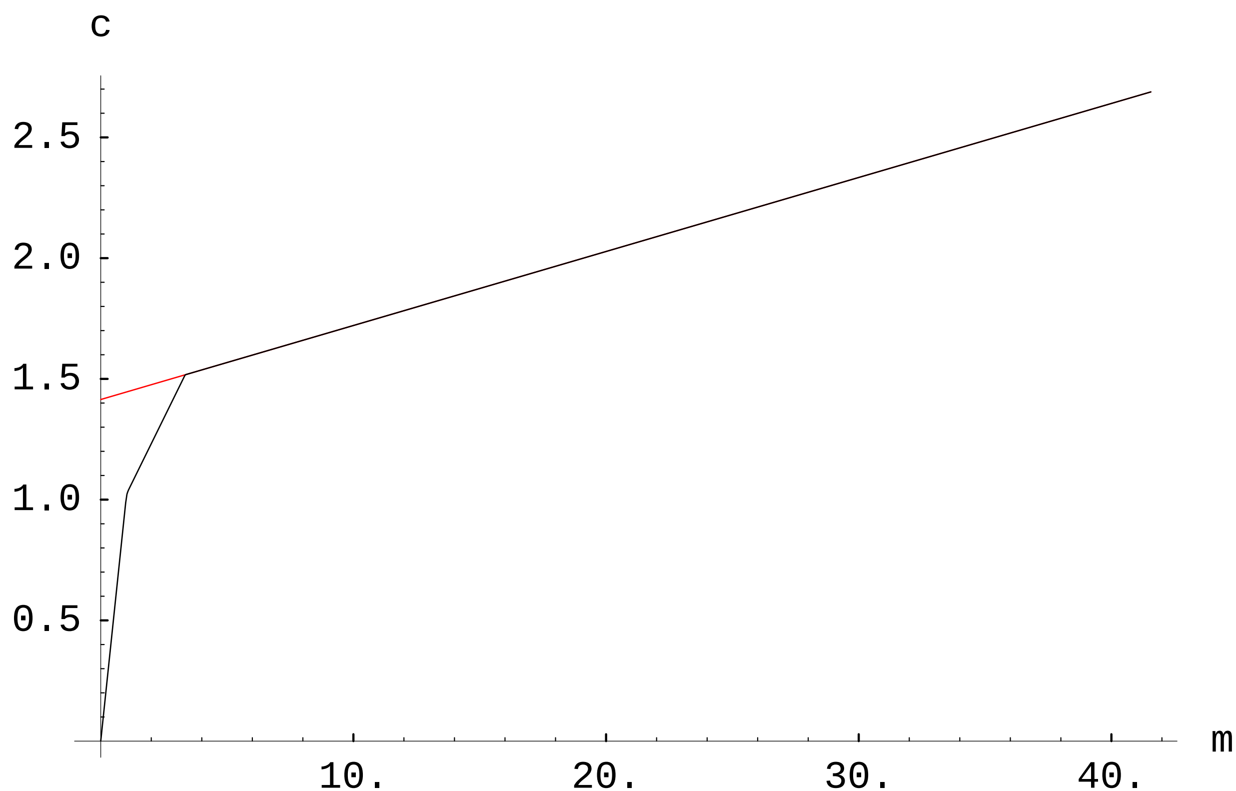

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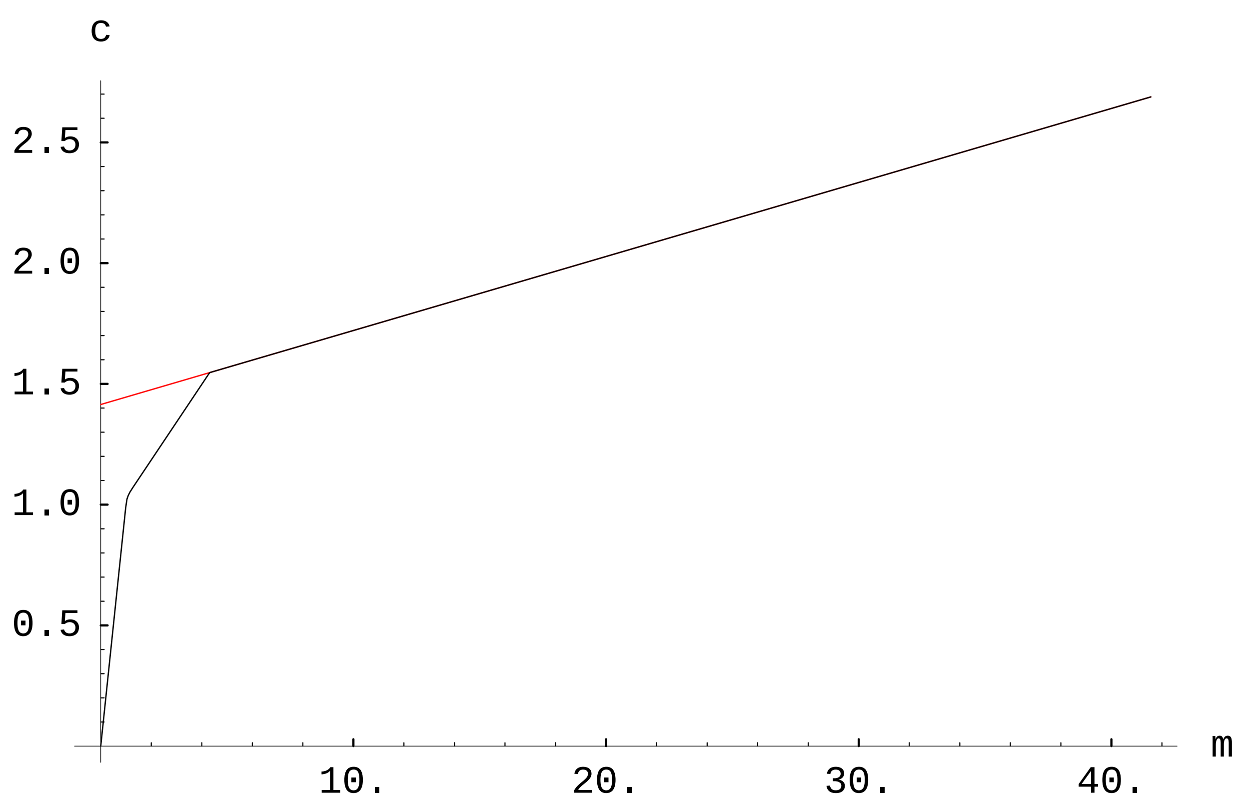

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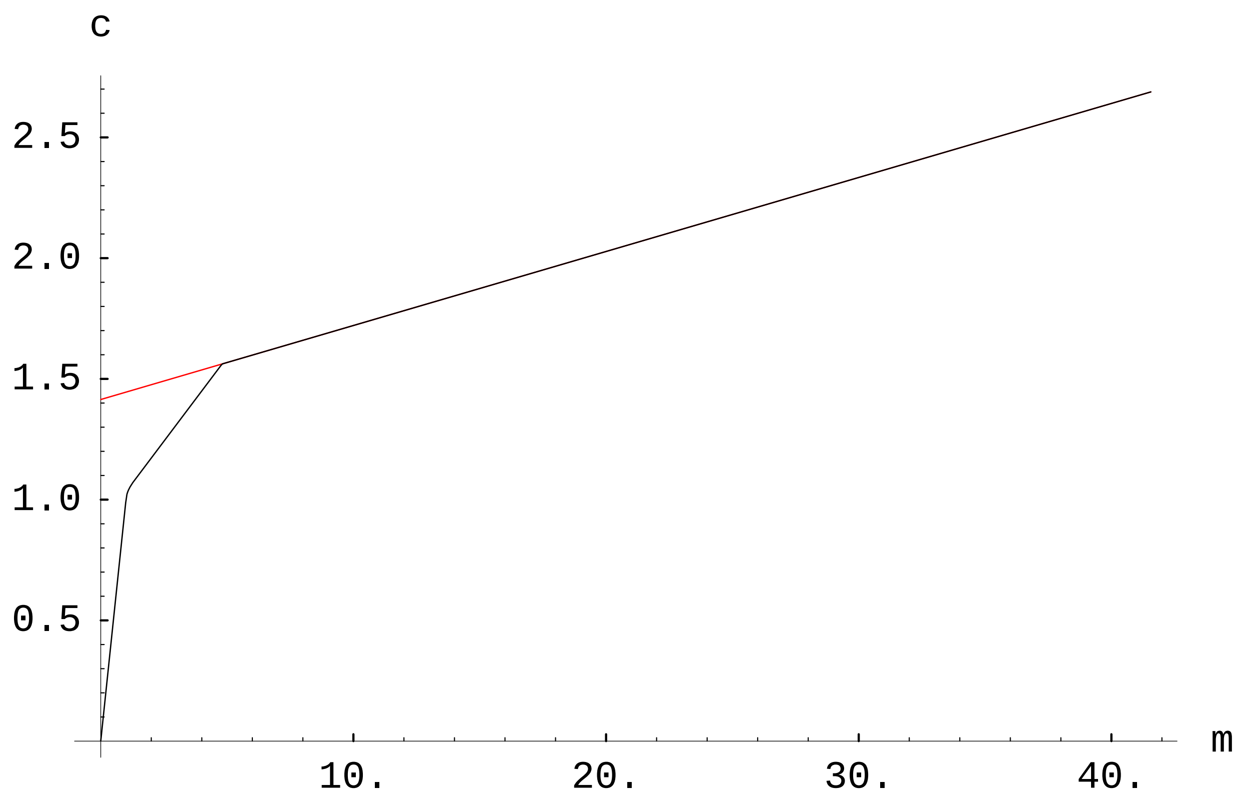

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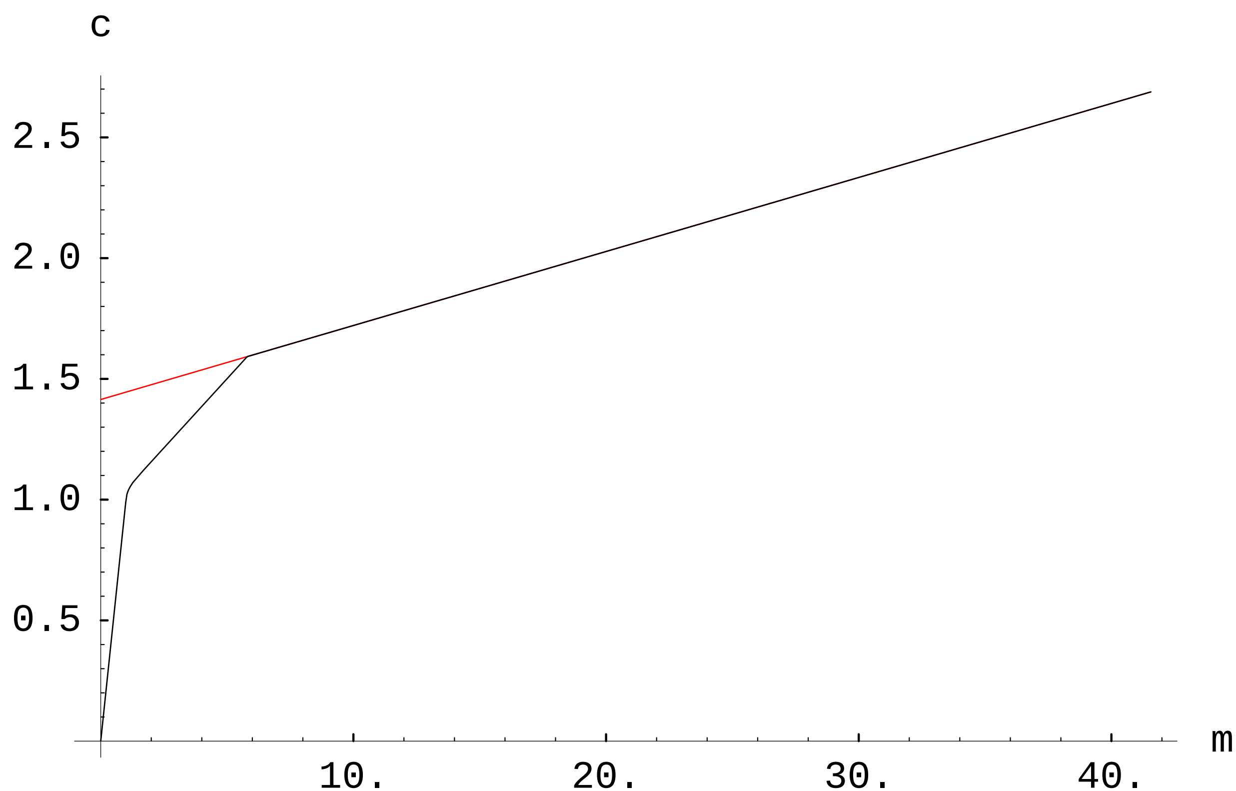

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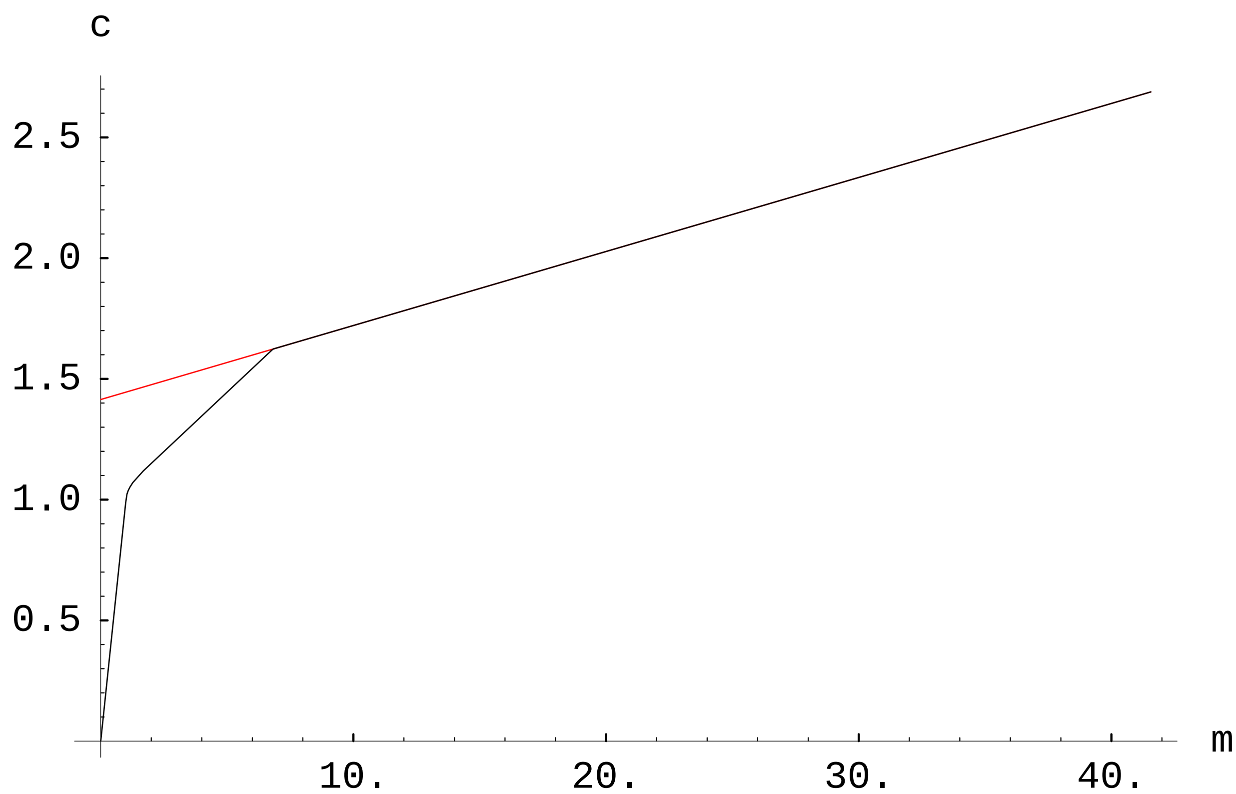

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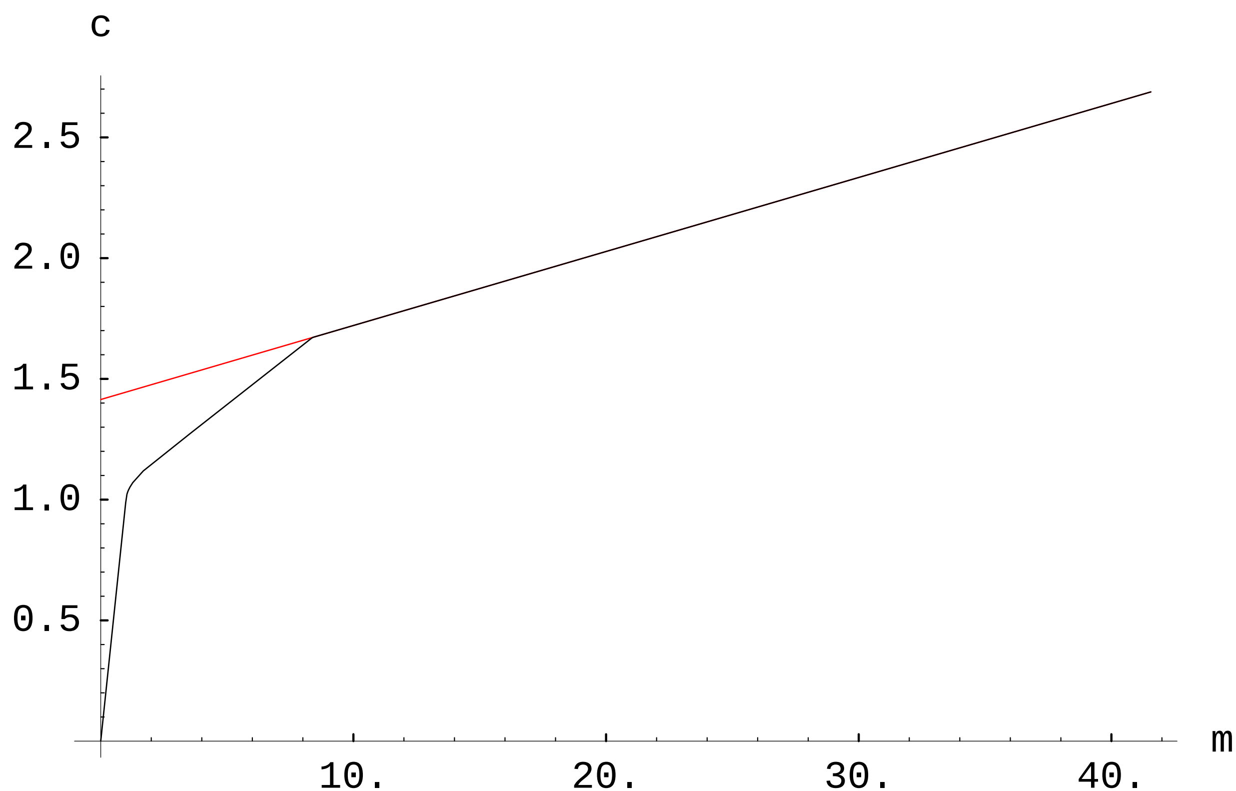

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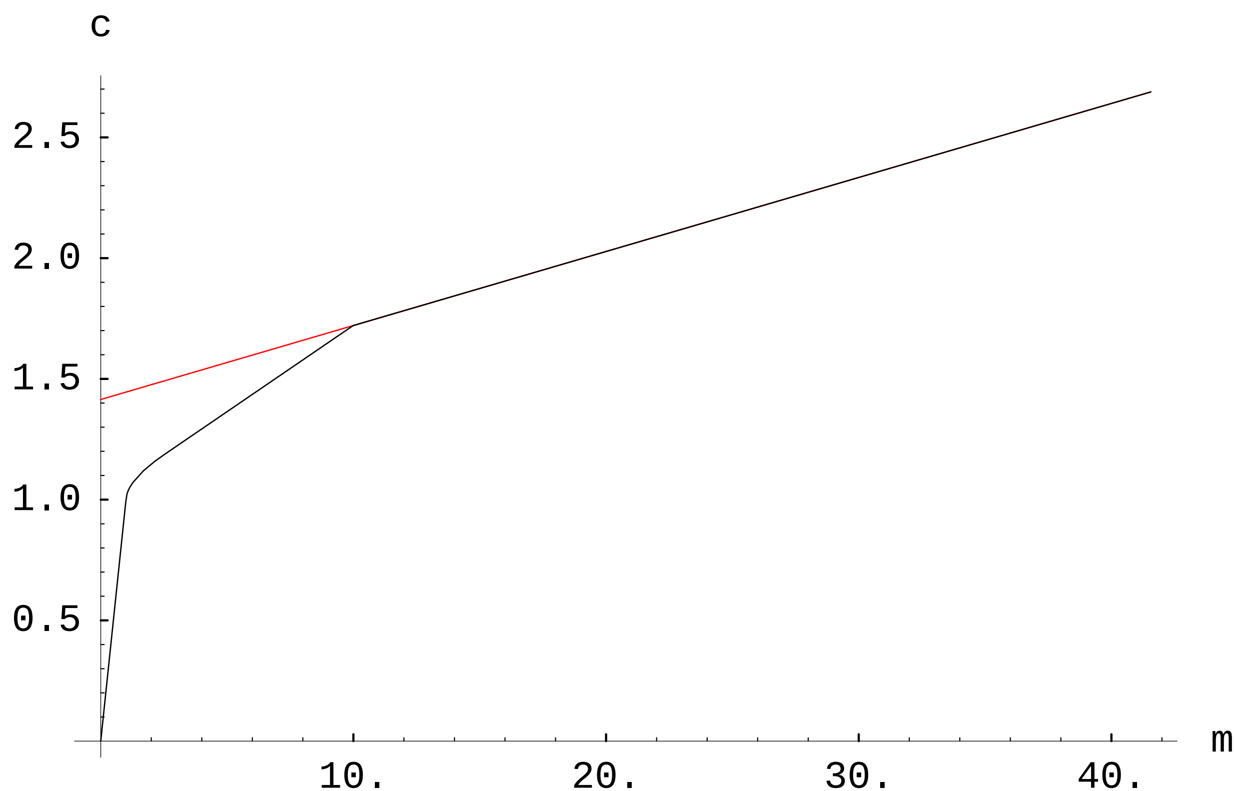

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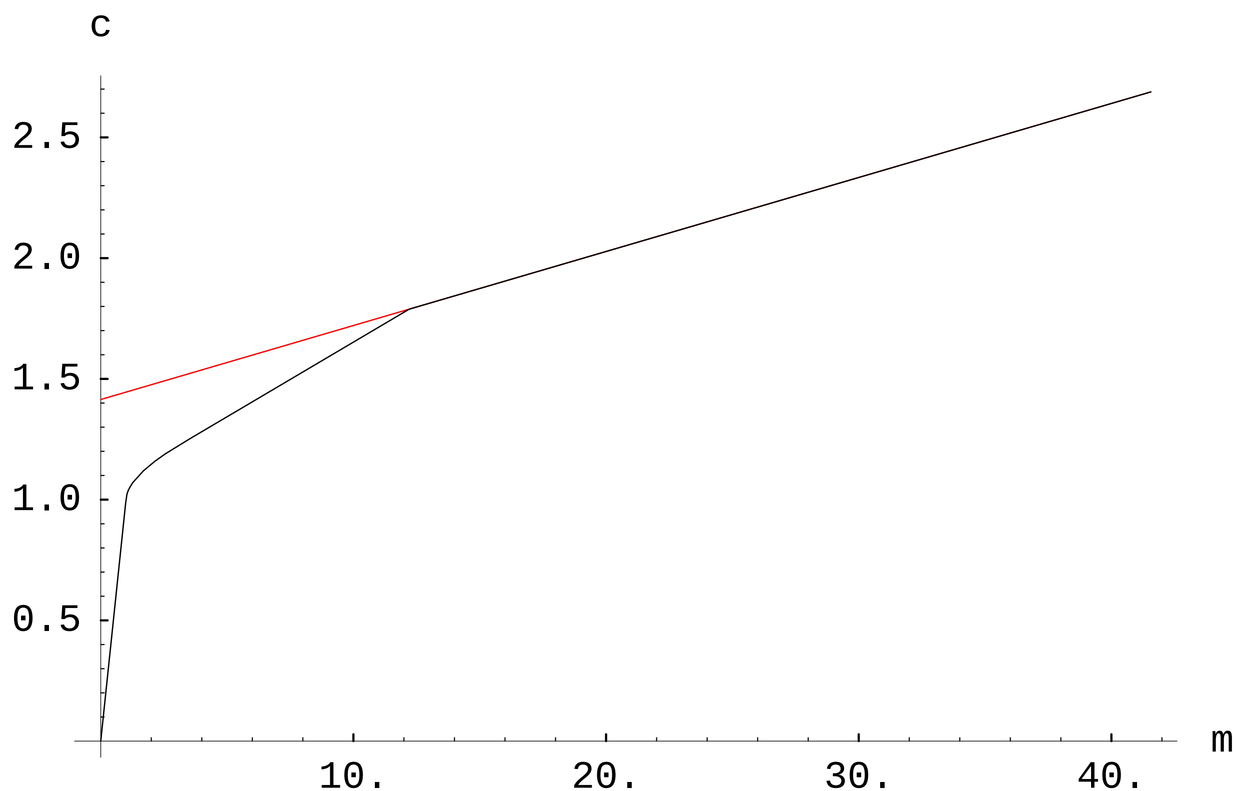

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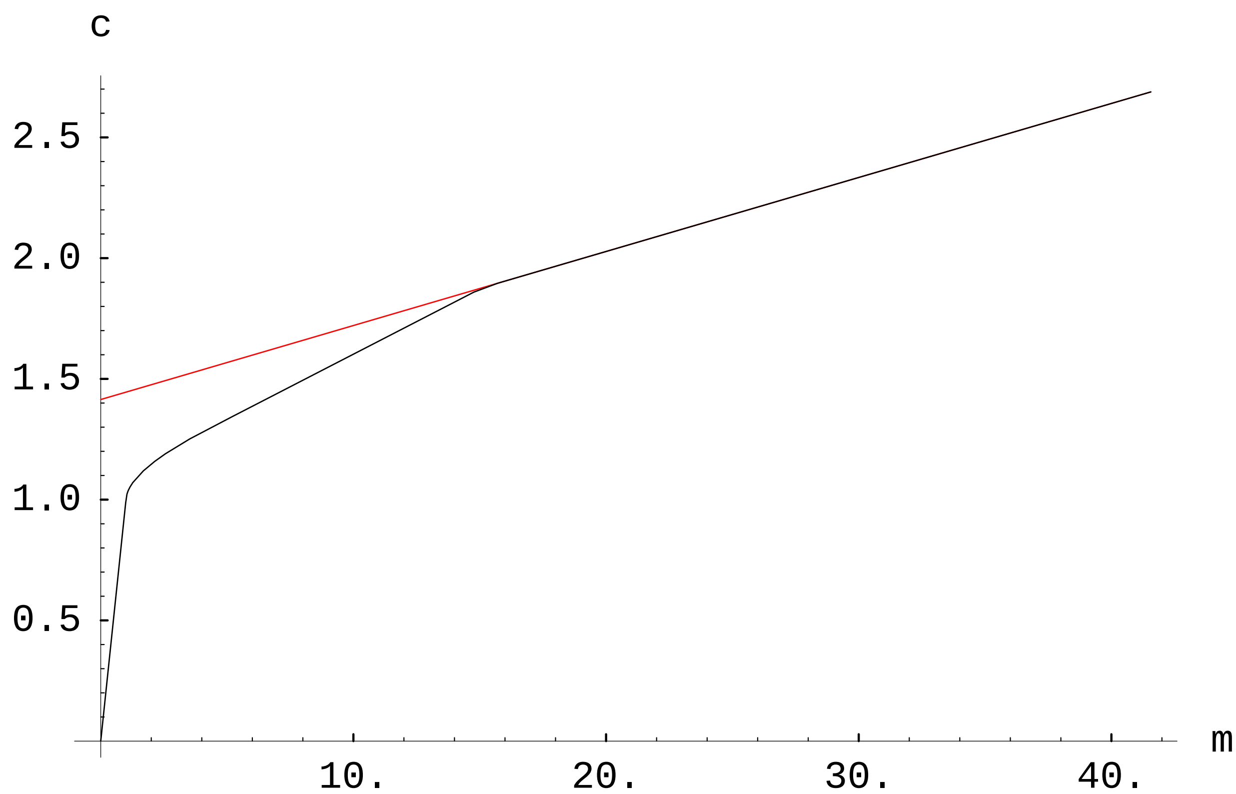

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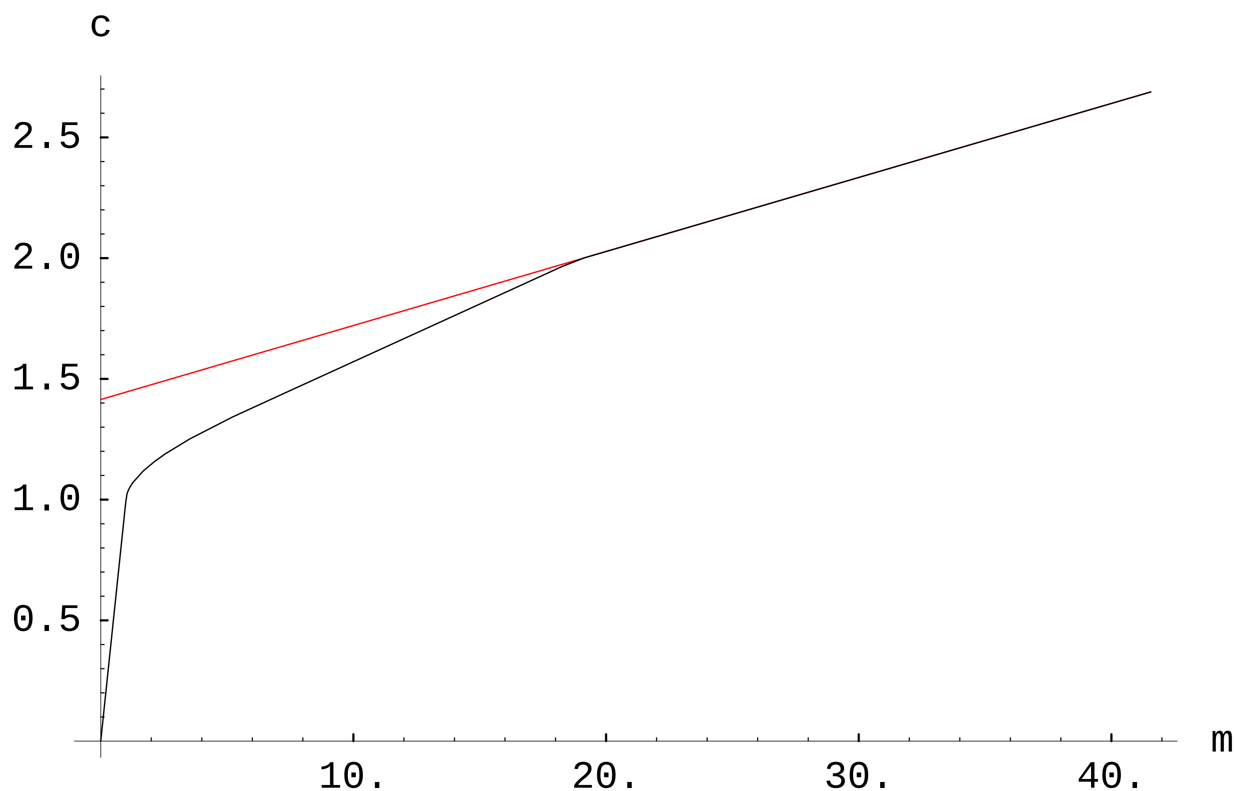

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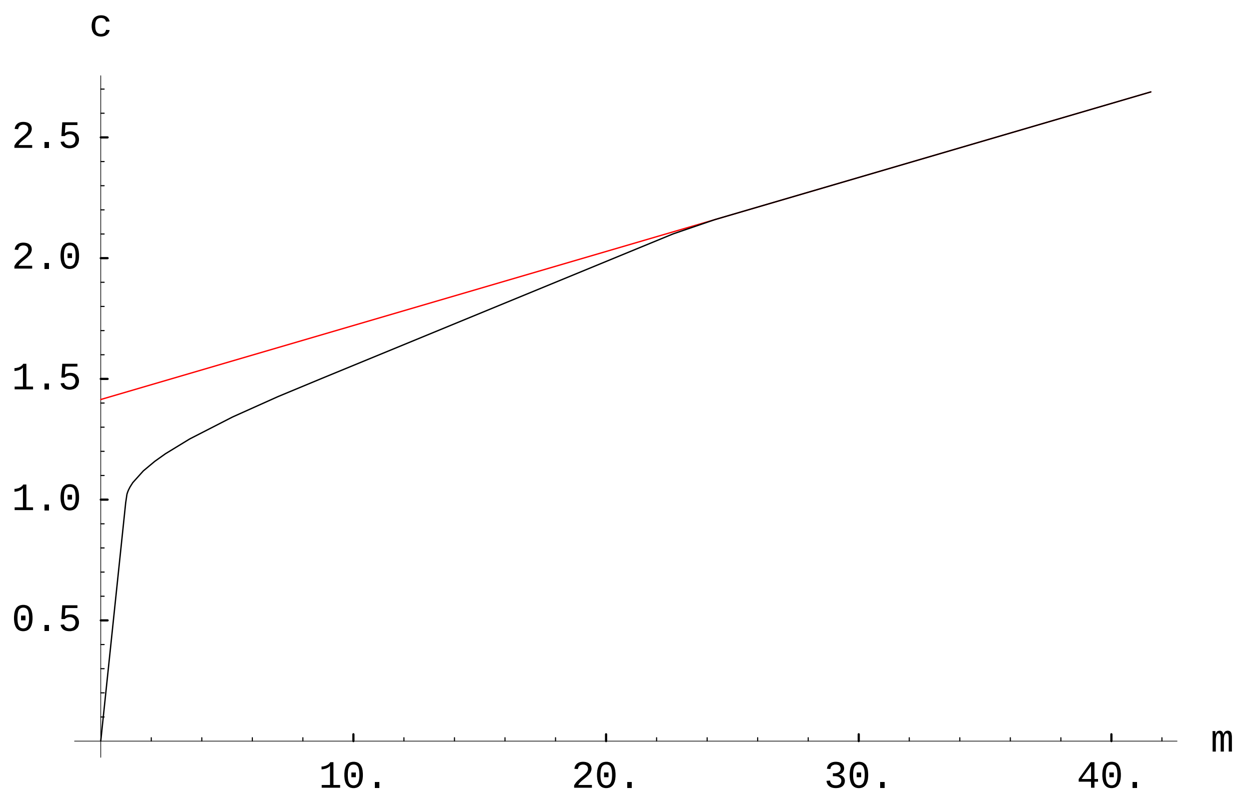

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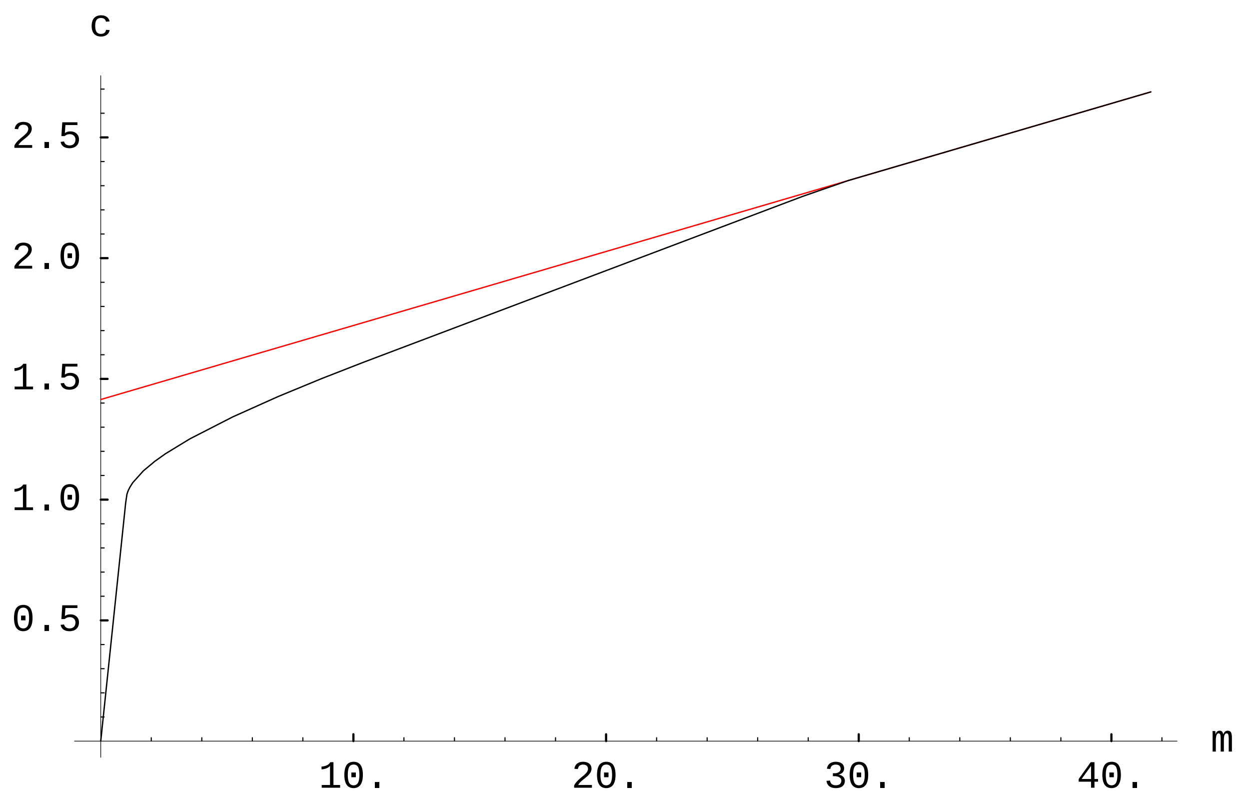

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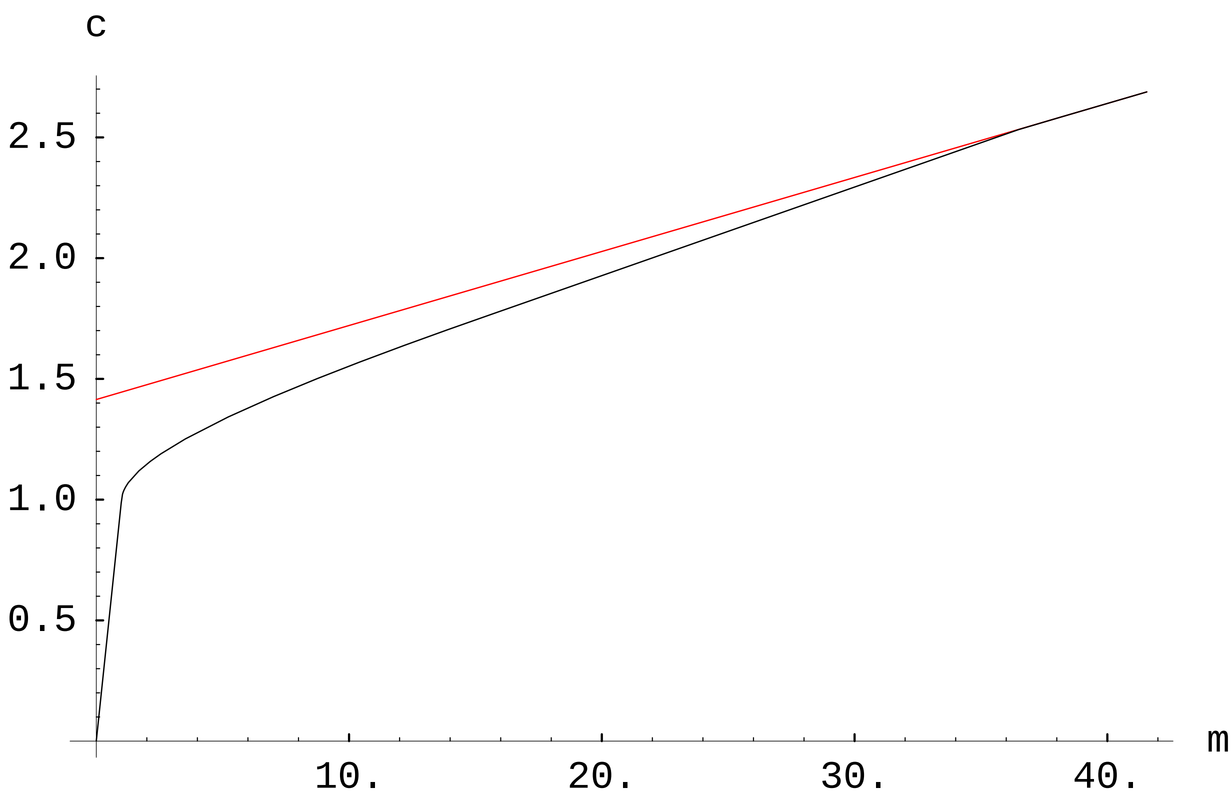

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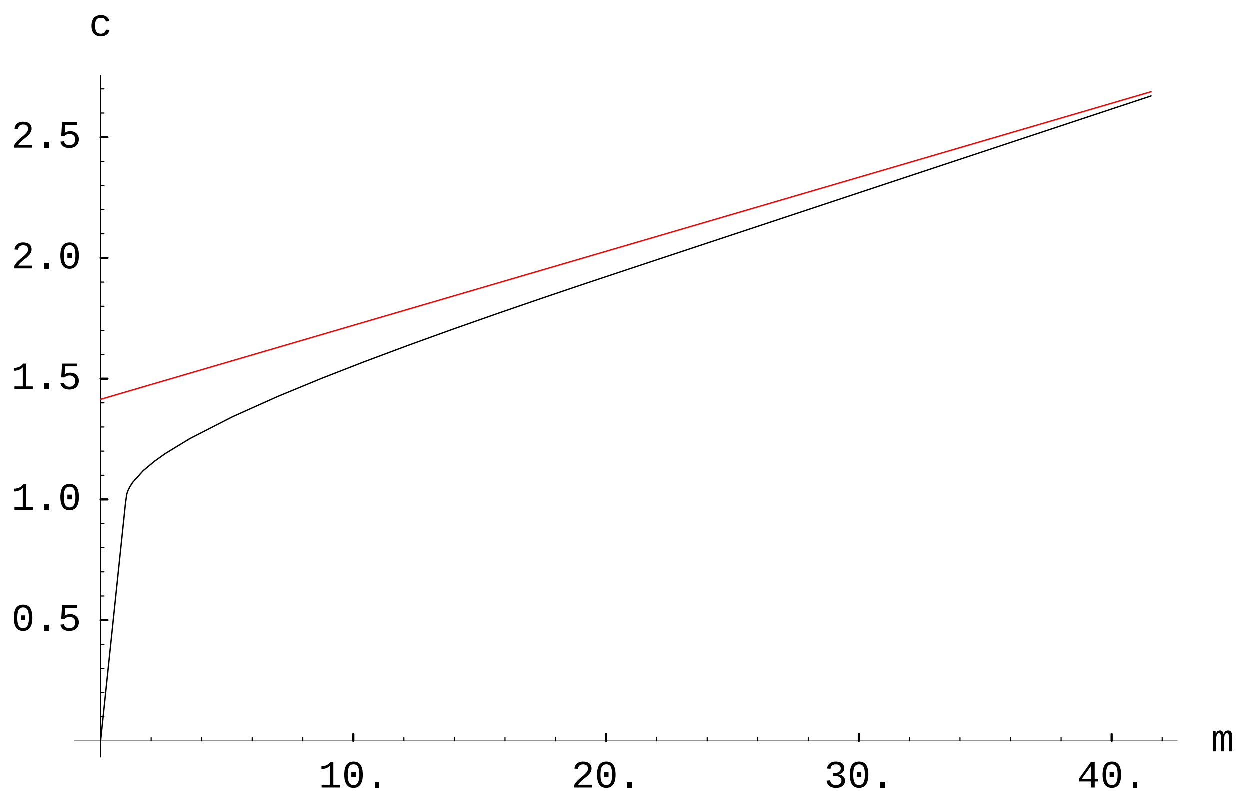

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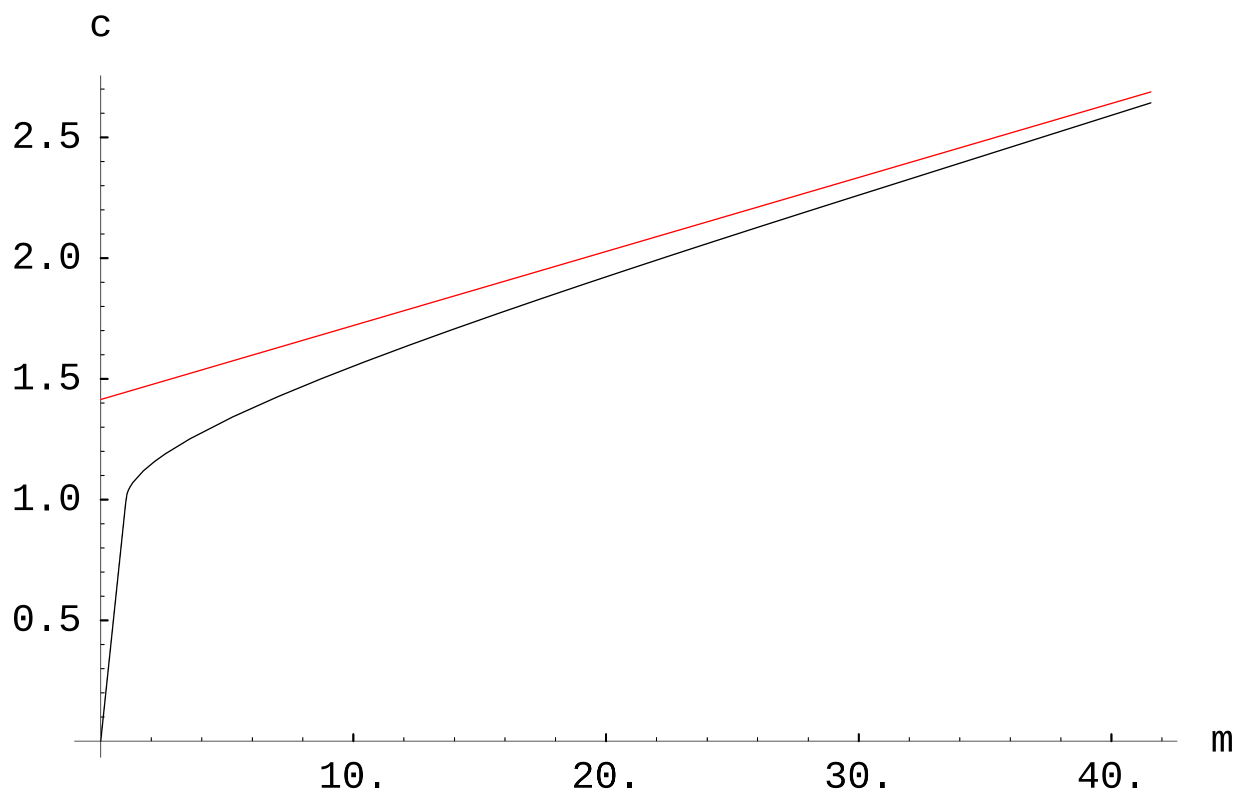

Risks $\approx$ Constraints

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Result:

$$\lim_{p \downarrow 0} \tilde{\mathbf{a}}_{T,T}(m_T) = \mathbf{a}_{T,T}(m_T)$$

.

Positive Result 1

Theorem 3 *Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ increases precautionary saving more for a perfect foresight consumer who faces $n+1$ relevant liquidity constraints in $\mathcal{T}_t$ (counting backwards) than for a perfect foresight consumer who faces only n relevant constraints in $\mathcal{T}_t$. That is,*

$$\mathbf{c}_{t,T-(q+1)}(m) - \tilde{\mathbf{c}}_{t,T-(q+1)}(m) \geq \mathbf{c}_{t,T-q}(m) - \tilde{\mathbf{c}}_{t,T-q}(m)$$

.

Negative Result 1

Consider two different sets of dates at which constraints apply, $\mathcal{T}_t$ and $\hat{\mathcal{T}}_t$, where $\hat{\mathcal{T}}_t$ is a strict superset of $\mathcal{T}_t$. Indicate the consumption function for the consumer who faces the extra constraints by $\hat{\mathbf{c}}_{t,\bullet}$.

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Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ does not *necessarily increase precautionary saving more for the consumer facing a larger number of future constraints. That is,*

$$\mathbf{c}_{t,T-n}(m) - \tilde{\mathbf{c}}_{t,T-n}(m) \lesseqgtr \hat{\mathbf{c}}_{t,T-n}(m) - \tilde{\hat{\mathbf{c}}}_{t,T-n}(m)$$

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Negative Result 2

Consider two different sets of dates at which risks apply, $\mathcal{Q}_t$ and $\hat{\mathcal{Q}}_t$, where $\hat{\mathcal{Q}}_t$ is a strict superset of $\mathcal{Q}_t$. Indicate the consumption function for the consumer who faces the extra risk(s) by $\hat{\mathbf{c}}_{t,\bullet}$.

Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ does not *necessarily increase precautionary saving more, at a given m, than for the consumer facing a larger number of future risks. That is,*

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This can be seen from the previous fact and from the essential equivalence of constraints and risks. .

Positive Result 2

Define as ‘blighted’ a consumer who faces some combination of future risks and future constraints; the unconstrained perfect foresight consumer with the same horizon is unblighted. Indicate the consumption function for the blighted consumer as $\hat{\mathrm{c}}_{t,\bullet}$. Our final result can be stated as

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Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ increases precautionary saving more, at a given m, for the blighted than for the unblighted consumer. That is,

$$\hat{\mathbf{c}}_{t,T-n}(m) - \tilde{\hat{\mathbf{c}}}_{t,T-n}(m) \geq \mathbf{c}_{t,T}(m) - \tilde{\mathbf{c}}_{t,T}(m)$$

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  - Future risks/constraints can ‘hide’ effect of current risks/constraints