Polynomial Regression

Overfitting and Underfitting

Further explained from Chapter 2

Underfitting

- Over-simplified
- Not sufficiently expressive
- a.k.a **high bias problem** (assuming it to be linear when it is actually quadratic)

Fixing Underfitting

- Generate **additional feautres (non-linear terms)** from **existing feature set (polynomial transformation)**
- Add more useful features
- Adding new samples won't help

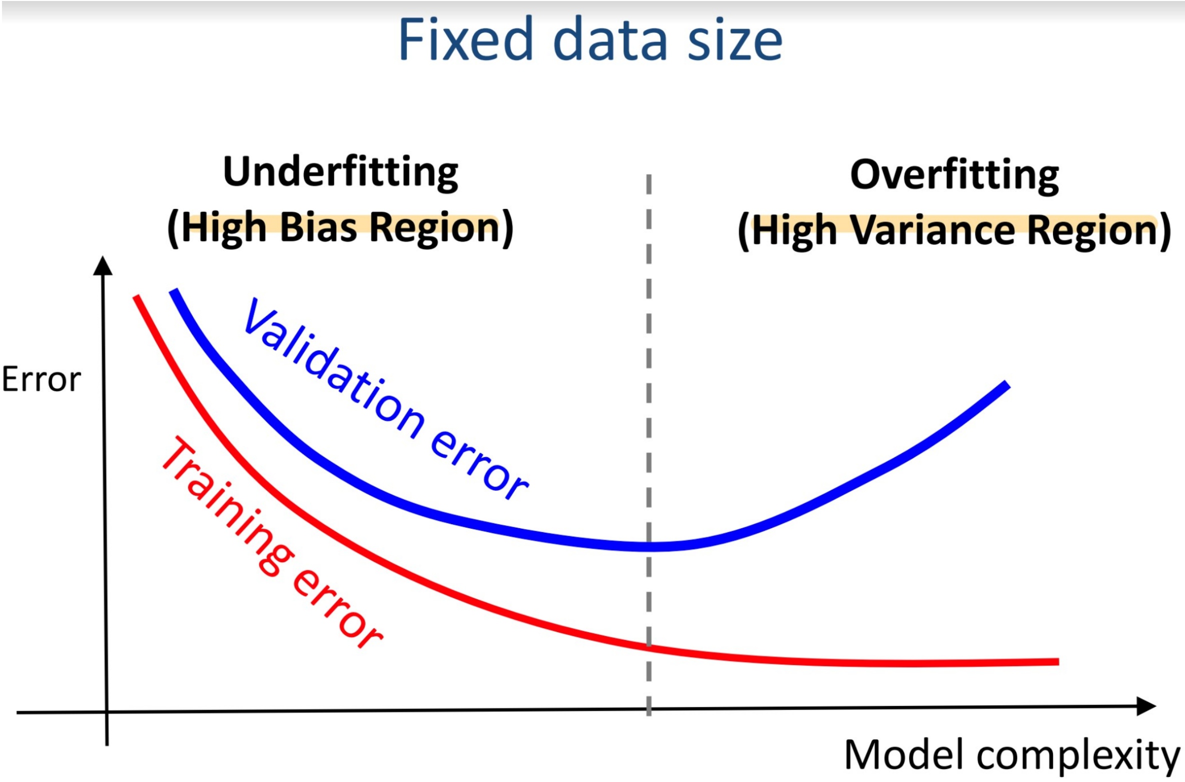
Overfitting

- Model too complex (e.g 3000 degree model)
- a.k.a **high variance problem** (model fits too specifically to traninig set, cannot adapt to unseen data)
- Excessively sensitive to small variations of training data (too many curves and turns)

Fixing Overfitting

- Reduce number of feautres
  - Manually select which features to keep
  - Model selection algorithm
- Regularization
  - Keep all features, but reduce magnitude/values of parameters θ_j
  - Works well when there is alot of features as each feature contributes a bit to predict y
- Adding more samples

Effect of model complexity



Effect of training size

Refer Slide 18 🔍

Regularization

- Impose some **constraints** (e.g make param θ values smaller, not feature values!)
- To simplify model
- **Reduce overfitting**

Ridge (Tikhonov) Regression

> **Strategy :** Enforce parameters θ to **prefer smaller values**

We add a **regularization term** to the cost function to **penalize** parameters with big values

$j(\theta) = MSE(\theta) + \alpha \frac{1}{2} \sum_{i=1}^{n} \theta_i^2$ **(non-vectorized)**

$j(\theta) = MSE(\theta) + \alpha \frac{1}{2} \theta^T \theta$ **(vectorized)**

Note : We don't regularize θ_0

> **Regularization Strength, α :** Hyperparam.
> $\alpha = 0$, no reg.
> $\alpha > 0$, strong reg.

α **Value**	**Errors**
Set Properly	Train Error = Low, Val Error = Low
Too Small (no reg.)	Train Error = Low, Val Error = High (Overfitting)
Too large (strong reg.)	Train Error = High, Val Error = High

Regularized Normal Equation

Original Normal Equation:

$\hat{\theta} = (X^T X)^{-1} X^T \cdot y$

Regularized Normal Equation:

$\hat{\theta} = (X^T X + \alpha A)^{-1} X^T \cdot y$

> n: # features
> m: # samples
> Size of **X**: (m, n+1)
> Size of **y**: (m,)
> Size of θ: (n+1,)

A: an (n+1) $\times$ (n+1) identity matrix except with a 0 on the top-left cell, corresponding to the bias term
α: regularization strength

$$\begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \end{bmatrix}$$

Lasso Regression

Lasso means **L**east **A**bsolute **S**hrinkage and **S**election **O**perator Regression

$J(\theta) = MSE(\theta) + \alpha \sum_{i=1}^{n} |\theta_i|$

- Generate solution that is **sparse** (most of weights are full of zeros)
- Eliminates the weights of the least important features (set them to 0)
  - This is an auto perform **feature selection**

Elastic Net

The **middle ground** between **Ridge Regression** and **Lasso Regression**

$J(\theta) = MSE(\theta) + r\alpha \sum_{i=1}^{n} |\theta_i| + (1-r)\alpha \frac{1}{2} \sum_{i=1}^{n} \theta_i^2$

$r = 0$: Elastic Net becomes Ridge Regression
$r = 1$: Elastic Net becomes Lasso Regression