Enthalpy:

$$E + P/\rho = \text{const}$$

<u>Steady state shock profile</u> (6)

$$\left(\frac{1}{\gamma-1} + \frac{\gamma-1}{\gamma-1}\right) P/\rho + \frac{1}{2}u^2 = \text{Const}$$

$$\frac{\gamma}{\gamma-1} P/\rho + \frac{1}{2}u^2 = C_3$$

$$P/\rho = \frac{T}{\gamma M_\infty^2} \qquad \boxed{\left(\frac{1}{\gamma-1}\right)\frac{T}{M_\infty^2} + \frac{1}{2}u^2 = \frac{1}{\gamma-1}\frac{T_L}{M_\infty^2} + \frac{1}{2}u_L^2}$$

$$T = T_L + \frac{\gamma-1}{2} M_\infty^2 (u_L^2 - u^2)$$

$$T_L = a_L^2 M_\infty^2 = u_L^2$$

$$T = u_L^2 + \frac{\gamma-1}{2} M_\infty^2 (u_L^2 - u^2)$$

$$P = \frac{\dot m}{\gamma M_\infty^2}\frac{T}{u} = \frac{\dot m}{\gamma M_\infty^2}\left(\frac{u_L^2 + \frac{\gamma-1}{2}M_\infty^2(u_L^2-u^2)}{u}\right) \qquad 1)$$

$$P_L = \frac{\dot m}{\gamma M_\infty^2}\frac{u_L^2}{u_L} = \frac{\dot m u_L}{\gamma M_\infty^2}$$

$$Pu = \frac{\dot m}{\gamma M_\infty^2}\left[u_L^2 + \frac{\gamma-1}{2}M_\infty^2(u_L^2-u^2)\right]$$

$$\dot m u + P - \dot m u_L - P_L = \mu' u_x \qquad \mu' = \frac{1}{Re}(\lambda + 2\mu)$$

$$\dot m(u^2 - u u_L) + (P - P_L)u = \mu' u u_x$$

$$\dot m(u^2 - u u_L) + \frac{\dot m}{\gamma M_\infty^2}\left[u_L^2 + \frac{\gamma-1}{2}M_\infty^2(u_L^2-u^2)\right] - \frac{\dot m}{\gamma M_\infty^2}u_L \cdot u = \mu' u u_x$$

$$\dot m\left[u^2 - u u_L + \frac{u_L^2}{\gamma M_\infty^2} + \frac{\gamma-1}{2\gamma}(u_L^2 - u^2) - \frac{1}{\gamma M_\infty^2}u_L u\right] = \mu' u u_x$$

Divide by u_L^2 & Define $\frac{u}{u_L} = V$

$$\dot{m}\left[V^2 - V + \frac{1}{\gamma M_\infty^2} + \frac{\gamma-1}{2\gamma}(1 - V^2) - \frac{1}{\gamma M_\infty^2} V\right] = \mu' V V_x \quad 2)$$

From RH ~~Tables~~ Relations

$$V_f = \frac{u_R}{u_L} = \frac{\left(\gamma - 1 + \frac{2}{M_\infty^2}\right)}{\gamma+1}$$

$$\boxed{\frac{(\gamma+1)V_f - (\gamma-1)}{2\gamma} = \frac{1}{\gamma M_\infty^2}}$$

subst into 2)

$$\dot{m}\left[V^2 - \tilde{V} + \frac{(\gamma+1)V_f}{2\gamma} - \frac{(\gamma-1)}{2\gamma} + \frac{\gamma-1}{2\gamma}(1 - V^2) - \frac{(\gamma+1)V_f - (\gamma-1)}{2\gamma} V\right] = \mu' V V_x$$

$$\dot{m}\left[V^2\left(1 - \frac{\gamma-1}{2\gamma}\right) + V\left(-1 + \frac{(\gamma-1)}{2\gamma}\right) + V\left(\frac{-(\gamma+1)}{2\gamma}\right)V_f + \frac{\gamma+1}{2\gamma} V_f\right] = \mu' V V_x$$

$$\dot{m}\left[V^2\left(\frac{\gamma+1}{2\gamma}\right) - V\left(\frac{\gamma+1}{2\gamma}\right) - \frac{\gamma+1}{2\gamma} V V_f + \frac{\gamma+1}{2\gamma} V_f\right] = \mu' V V_x$$

$$\dot{m}\,\frac{\gamma+1}{2\gamma}\left[V^2 - V - V V_f + V_f\right] = \mu' V V_x$$

$$\boxed{\dot{m}\,\frac{\gamma+1}{2\gamma}\left\{(V - 1)(V - V_f)\right] = \mu' V V_x}$$

Define $\alpha = \frac{\mu'}{\dot{m}}\frac{2\gamma}{\gamma+1} = \frac{1}{Re}\frac{(\lambda + 2\mu)}{\dot{m}}\frac{2\gamma}{\gamma+1}$

$$\boxed{(V - 1)(V - V_f) = \alpha V V_x}$$

7a

where

$$A=\frac{\varphi(a)}{f'(a)}\qquad B=\frac{\varphi(b)}{f'(b)},\ \ldots,\ M=\frac{\varphi(m)}{f'(m)}.$$

If some of the roots of the equation $f(x)=0$ are imaginary, we group together the fractions that represent conjugate roots of the equation. Then, after certain manipulations, we represent the corresponding pairs of fractions in the form of real fractions of the form

$$\frac{M_1x+N_1}{x^2+2Bx+C}+\frac{M_2x+N_2}{(x^2+2Bx+C)^2}+\ldots+\frac{M_px+N_p}{(x^2+2Bx+C)^p}.$$

2.103 Thus, the integration of a proper rational fraction $\frac{\varphi(x)}{f(x)}$ reduces to integrals of the form $\int\frac{g\,dx}{(x-a)^{\alpha}}$ or $\int\frac{Mx+N}{(A+2Bx+Cx^2)^p}dx$. Fractions of the first form yield rational functions for $\alpha>1$ and logarithms for $\alpha=1$. Fractions of the second form yield rational functions and logarithms or arctangents:

1. $$\int\frac{g\,dx}{(x-a)^{\alpha}}=g\int\frac{d(x-a)}{(x-a)^{\alpha}}=-\frac{g}{(\alpha-1)(x-a)^{\alpha-1}}.$$

2. $$\int\frac{g\,dx}{x-a}=g\int\frac{d(x-a)}{x-a}=g\ln|x-a|.$$

3. $$\int\frac{Mx+N}{(A+2Bx+Cx^2)^p}dx=\frac{NB-MA+(NC-MB)x}{2(p-1)(AC-B^2)(A+2Bx+Cx^2)^{p-1}}+$$
$$+\frac{(2p-3)(NC-MB)}{2(p-1)(AC-B^2)}\int\frac{dx}{(A+2Bx+Cx^2)^{p-1}}.$$

4. $$\int\frac{dx}{A+2Bx+Cx^2}=\frac{1}{\sqrt{AC-B^2}}\operatorname{arctg}\frac{Cx+B}{\sqrt{AC-B^2}}\qquad [AC>B^2];$$
$$=\frac{1}{2\sqrt{B^2-AC}}\ln\left|\frac{Cx+B-\sqrt{B^2-AC}}{Cx+B+\sqrt{B^2-AC}}\right|\qquad [AC<B^2].$$

5. $$\int\frac{(Mx+N)\,dx}{A+2Bx+Cx^2}=\frac{M}{2C}\ln|A+2Bx+Cx^2|+$$
$$+\frac{NC-MB}{C\sqrt{AC-B^2}}\operatorname{arctg}\frac{Cx+B}{\sqrt{AC-B^2}}\qquad [AC>B^2];$$
$$=\frac{M}{2C}\ln|A+2Bx+Cx^2|+$$
$$+\frac{NC-MB}{2C\sqrt{B^2-AC}}\ln\left|\frac{Cx+B-\sqrt{B^2-AC}}{Cx+B+\sqrt{B^2-AC}}\right|\qquad [AC<B^2].$$

The Ostrogradskiy-Hermite method

2.104 By means of the Ostrogradskiy-Hermite method, we can find the rational part of $\int\frac{\varphi(x)}{f(x)}dx$ without finding the roots of the equation $f(x)=0$ and without decomposing the integrand into partial fractions:

$$\int\frac{\varphi(x)}{f(x)}dx=\frac{M}{D}+\int\frac{N\,dx}{Q}.$$ FI II 49

Here, M, N, D, and Q are rational functions of x. Specifically, D is the greatest common divisor of the function $f(x)$ and its derivative $f'(x)$; $Q=\frac{f(x)}{D}$; M is a

$$\int \frac{(Mx+N)\,dx}{A+2Bx+Cx^2} = \text{———} \cdot \text{——}$$

$$\int \frac{\alpha V\,dV}{(V-1)(V-V_f)} = \int \frac{V\,dV}{V^2-(1+V_f)V+V_f}$$

$A = V_f$ $M = 1$

$B = -\frac{(1+V_f)}{2}$ $N = 0$

$C = 1$

$$B^2 - AC = \frac{(1+V_f)^2}{4} - \frac{4V_f}{4} = \frac{1+2V_f+V_f^2-4V_f}{4} = \left[\frac{(1-V_f)}{2}\right]^2$$

$B^2 - AC \geqslant 0$

$$\alpha\int \frac{\alpha V\,dV}{(V-1)(V-V_f)} = \frac{1}{2}\ln\left|(V-1)(V-V_f)\right| +$$

$$\frac{\frac{(1+V_f)}{2}}{\cancel{2}\cdot\frac{(1-V_f)}{\cancel{2}}} \ln\left| \frac{V - \frac{(1+V_f)}{2} - \frac{(1-V_f)}{2}}{V - \frac{1+V_f}{2} + \frac{(1-V_f)}{2}} \right|$$

$$X = \frac{\alpha}{2}\left[\ln\left|(V-1)(V-V_f)\right| + \frac{(1+V_f)}{(1-V_f)}\ln\left|\frac{V-V_f}{V-V_f}\right|\right]$$

NS Equations

$$\rho_t + (\rho u)_x = 0$$

$$(\rho u)_t + (\rho u^2 + P)_x = \frac{1}{Re}\left[(\lambda + 2\mu) u_x\right]_x$$

$$(\rho E)_t + \left[(\rho E + P)u\right]_x = \frac{1}{Re}\left[\frac{K}{Pr}\left[\frac{1}{\rho u}(\rho E + P)u\right]_x + \left[(\lambda + 2\mu) - \frac{K}{Pr}\right]\left(\frac{u^2}{2}\right)_x\right]_x$$

assume $\lambda + 2\mu = \text{const}$

$\frac{K}{Pr} = \text{const}$ The eqns can be cast as

$$\rho_t + (\rho u)_x = 0$$

$$(\rho u)_t + (\rho u^2 + P)_x = \frac{1}{Re}(\lambda + 2\mu) u_{xx}$$

$$(\rho E)_t + \left[(\rho E + P)u\right]_x = \frac{1}{Re}\left[\frac{K}{Pr}\left[\frac{1}{\rho u}(\rho E + P)u\right]_{xx} + \left(\lambda + 2\mu - \frac{K}{Pr}\right)\left(\frac{u^2}{2}\right)_{xx}\right]$$

We have solved in the stationary frame $u = u(\xi)$

in this frame (τ, ξ) The "time" derivatives are zero.

our task is to transform the Data from $(\tau, \xi) \Rightarrow (x, t)$

We Have

$$\rho(\xi)\, u(\xi) = C_1 \qquad 1] \quad \text{mass}$$

$$(V-1)(V-V_f) = \alpha V V_\xi \qquad 2] \quad \text{momentum}$$

$$E + P/\rho = \frac{\gamma}{\gamma-1} P/\rho + \frac{u^2}{2} = C_3 \qquad 3] \quad \text{"Enthalpy" energy}$$

Multiplying 2 by u_L^2

$$(u-u_L)(u-u_R) = \alpha u u_\xi$$

and make the Definition of variables in the ~~stat~~ shock frame are ~ quantities

$$\tilde\rho\, \tilde u = \tilde C_1 \qquad 1)$$

$$(\tilde u - \tilde u_L)(\tilde u - \tilde u_R) = \alpha \tilde u \tilde u_\xi \qquad 2)$$

$$\frac{\gamma}{\gamma-1} \tilde P/\tilde\rho + \frac{\tilde u^2}{2} = \tilde C_3 \qquad 3)$$

Transforming to stationary coordinates by $(\tau, \xi) \to (x, t)$

$$\tau = t \quad ; \quad t = \tau$$

$$\xi = x - ct \quad ; \quad x = \xi + c\tau$$

$$\frac{\partial}{\partial \xi} = \frac{\partial}{\partial x} x_\xi + \frac{\partial}{\partial t} t_\xi = \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial x} x_\tau + \frac{\partial}{\partial t} t_\tau = \frac{\partial}{\partial t} + c\frac{\partial}{\partial x}$$

$$\boxed{u = \tilde u + c \quad ; \quad P = \tilde P \quad ; \quad \rho = \tilde\rho \quad ;}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}\xi_x + \frac{\partial}{\partial \tau}\tau_x = \frac{\partial}{\partial \xi}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \xi}\xi_t + \frac{\partial}{\partial \tau}\tau_t = -c\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \tau}$$

Stationary frame coordinates

$$\rho_t + (\rho u)_x = 0$$

$$(\rho u)_t + (\rho u^2 + P)_x = \frac{1}{Re}\left[(\lambda + 2\mu) u_x\right]_x$$

$$(\rho E)_t + \left[(\rho E + P) u\right]_x = \frac{1}{Re}\left[\frac{K}{Pr}\left[\frac{(\rho E + P)u}{\rho u}\right]_x + \left((\lambda + 2\mu) - \frac{K}{Pr}\right)\frac{u^2}{2}\right]_x$$

needed at boundaries are

inviscid fluxes

$$\rho u \qquad \frac{(\lambda+2\mu)}{Re} u_x$$

$$\rho u^2 + P$$

$$(\rho E + P) u \qquad \frac{1}{Re}\frac{K}{Pr}\left[\frac{(\rho E + P)u}{\rho u}\right]_x$$

viscous fluxes

Test 1)

$$\rho_t + (\rho u)_x = 0$$

$$\rho_\tau - C\rho_\xi + \left[\rho(\tilde{u} + C)\right]_\xi = 0$$

$$\rho_\tau - C\rho_\xi + (\rho\tilde{u})_\xi + C\rho_\xi = 0$$

$(\rho\tilde{u})_\xi = 0$ by Definition $\rho_\tau = 0$ by definition

$$0 = 0$$

Test 2)

$$(\rho u)_t + (\rho u^2 + P)_x = \frac{1}{Re}\left[(\lambda + 2\mu) u_x\right]_x$$

$$\cancel{(\rho u)}_\tau - C\cancel{(\rho u)}_\xi + \left[\rho(\tilde{u} + C)^2 + P\right]_\xi = \frac{1}{Re}\left[(\lambda + 2\mu) u_\xi\right]_\xi$$

$$\cancel{(\rho\tilde{u})_\tau}^{0} + \cancel{(\rho C)_\tau}^{0} - \cancel{C(\rho\tilde{u})_\xi}^{0} \left[\rho\tilde{u}^2 + P\right]_\xi + \cancel{\left[2(\rho\tilde{u})\cdot C\right]_\xi}^{0} + \left[\cancel{\rho C^2}\right]_\xi = \frac{1}{Re}\left[(\lambda + 2\mu)\tilde{u}_\xi\right]_\xi$$

$$- C\cancel{(\rho C)}_\xi$$

Test 3] $E = \frac{1}{\gamma-1} P/\rho + \frac{u^2}{2}$; $\tilde{E} = \frac{1}{\gamma-1} P/\rho + \frac{\tilde{u}^2}{2}$

Pr = 1 Energy eqn Becomes $E = \tilde{E} + \frac{1}{2}(2\tilde{u}C + C^2)$

$$(\rho \hat{E})_t + [(\rho E + P)u]_x = \frac{1}{Re}\left\{\frac{K}{Pr}\left[\frac{(\rho E + P)u}{\rho u}\right]_x\right\}_x + \frac{1}{Re}\left\{\underbrace{\left[(\lambda+2\mu) - \frac{K}{Pr}\right]}_{=0}\left(\frac{u^2}{2}\right)_x\right\}_x$$

Transform

$$(\rho\tilde{E})_t + \left[\frac{\rho}{2}(2\tilde{u}C + C^2)\right]_t + [(\rho\tilde{E}+P)\tilde{u}]_x + \left\{\left[\frac{\rho}{2}(2\tilde{u}C + C^2)\right]\tilde{u}\right\}_x$$

$$+ [(\rho\hat{E}+P)C]_x + \left(\frac{\rho}{2}(2\tilde{u}C + C^2)C\right)_x = \frac{1}{Re}\left[\frac{K}{Pr}(\hat{E} + P/\rho)_x\right]$$

$$+ \frac{1}{Re}\left[\frac{K}{Pr}\frac{1}{2}(2\tilde{u}C + C^2)_x\right]_x$$

$$\cancel{(\rho\tilde{E})_\tau}^{0} - C(\rho\hat{E})_\xi + \cancel{\left[\frac{\rho}{2}(2\tilde{u}C + C^2)\right]_\tau}^{0} - C\left[\frac{\rho}{2}(2\tilde{u}C + C^2)\right]_\xi$$

$$+ [(\rho E + P)\tilde{u}]_\xi + \left\{\frac{\rho}{2}(2\tilde{u}C + C^2)\tilde{u}\right\}_\xi + [(\rho\hat{E} + P)C]_\xi + \left[\frac{\rho}{2}(2\tilde{u}C + C^2)C\right]_\xi$$

$$= \frac{1}{Re}\left[\frac{K}{Pr}(\hat{E} + P/\rho)_\xi\right]_\xi + \frac{1}{Re}\left[\frac{K}{Pr}\hat{u}_{\xi\xi}C\right]$$

$$[(\rho\tilde{E}+P)\tilde{u}]_\xi + [(\rho u^2 + P)C]_\xi = \frac{1}{Re}\left[\frac{K}{Pr}\left[\frac{(\rho E + P)u}{\rho u}\right]_\xi\right]_\xi + \frac{1}{Re}\left[\frac{K}{Pr}\tilde{u}_{\xi\xi}C\right]$$

But $[\rho u^2 + P]_\xi = \frac{1}{Re}[(\lambda + 2\mu)u_{\xi\xi}]$

$$\boxed{[(\rho\hat{E} + P)\tilde{u}]_\xi = \frac{1}{Re}\left\{\frac{K}{Pr}\left[\frac{(\rho E + P)u}{\rho u}\right]_\xi\right\}_\xi} + \frac{1}{Re}\left(\underbrace{\left(\frac{K}{Pr} - (\lambda+2\mu)\right)}_{0}C\,u_{\xi\xi}\right]$$

Boundary Data: in Shock frame

Fluxes $\rho(\xi)\,u(\xi)$

$(\rho u^2 + P)$

$(\rho E + P)u$

Viscous fluxes

$\frac{(\lambda+2\mu)}{Re} u_\xi$

$\frac{1}{Re}\left[\frac{K}{Pr}\left[\frac{(\rho E+P)u}{\rho u}\right]_\xi\right]$

in Physical frame

$\hat{u} = u_L \tilde{V}$

$x - c\tau = \frac{\alpha}{2}\left[\ln\left|(V-1)(V-V_f)\right| + \frac{(1+V_f)}{(1-V_f)} \ln\left|\frac{V-1}{V-V_f}\right|\right]$

~~$u(x-ct) =$~~

$u(x,\tau) = \tilde{u} + c$

$\rho(\xi) = \frac{\tilde{C}_1}{\tilde{u}(\xi)}$

$\rho u = \rho(\tilde{u}+c) = \rho\tilde{u} + \rho c = \tilde{C}_1 + \rho c = \tilde{C}_1\left(1 + \frac{1}{\tilde{u}}\right)$

$\rho u^2 + P = \rho(\tilde{u}+c)^2 + P$

$(\rho E + P)u = \rho(\tilde{u}+c)(E + P/\rho) = \rho(\tilde{u}+c)\left[\tilde{E} + P/\rho + \frac{1}{2}(2\tilde{u}c + c^2)\right]$

$= \rho\tilde{u}\left[\tilde{E} + P/\rho\right] + \rho c\left[\tilde{E} + P/\rho\right]$

$+ \rho\tilde{u}\left[\frac{1}{2}(2\tilde{u}c + c^2)\right] + \rho c\left[\frac{1}{2}(2\tilde{u}c + c^2)\right]$

$(\rho E + P)u = \tilde{C}_1\tilde{C}_3 + \rho c C_3 + \tilde{C}_1\left[\frac{1}{2}(2\tilde{u}c + c^2)\right] + \rho c\left[\frac{1}{2}(2\tilde{u}c + c^2)\right]$

$= \tilde{C}_1\tilde{C}_3 + \frac{\tilde{C}_1 c C_3}{\tilde{u}} + \tilde{C}_1 c\tilde{u} + \tilde{C}_1\frac{c^2}{2} + \tilde{C}_1 c^2 + \frac{1}{2}\frac{\tilde{C}_1}{\tilde{u}}c^3$

$= \tilde{C}_1\left[\tilde{C}_3\left(1 + \frac{c}{\tilde{u}}\right) + \left\{c\left(\tilde{u} + \frac{3}{2}c\right)\right\} + \frac{1}{2}\frac{c^3}{\tilde{u}}\right]$

$$(\rho \tilde{E} + P)\tilde{u} = \tilde{C}_1 \tilde{C}_3$$

$$\tilde{E} = \frac{1}{\gamma-1} P/\rho + \frac{\tilde{u}^2}{2}$$

$$\left(\frac{\gamma}{\gamma-1} P/\rho + \frac{\tilde{u}^2}{2}\right)\cancel{\rho\tilde{u}} = \cancel{\tilde{C}_1}\tilde{C}_3$$

$$P/\rho = \frac{\gamma-1}{\gamma}\left(C_3 - \frac{\tilde{u}^2}{2}\right)$$

$$P = \frac{C_1}{\tilde{u}} \frac{\gamma-1}{\gamma}\left(C_3 - \frac{\tilde{u}^2}{2}\right)$$

$$\rho u^2 + P = \cancel{\text{eq}}\ \rho(\tilde{u}+c)^2 + P$$

$$\rho\tilde{u}^2 + P + 2\rho\tilde{u}c + \rho c^2$$

$$\rho u^2 + P = C_1\tilde{u} + \frac{C_1}{\tilde{u}}\frac{\gamma-1}{\gamma}\left(C_3 - \frac{\tilde{u}^2}{2}\right) + 2C_1 c + \frac{C_1}{\tilde{u}} c^2$$

$$\rho u^2 + P = \tilde{C}_1\left[\tilde{u} + 2c + \frac{c^2}{\tilde{u}} + \frac{\gamma-1}{\gamma}\left(\frac{C_3}{\tilde{u}} - \frac{\tilde{u}}{2}\right)\right]$$

$$\frac{\lambda+2\mu}{Re} u_\xi = \frac{\lambda+2\mu}{Re} u_x = \frac{\lambda+2\mu}{Re} \frac{(V-1)(V-V_f)}{\alpha V} \cdot u_L$$

$$\frac{1}{Re}\left[\frac{K}{Pr}\left[\cancel{E} + P/\rho\right]_{\cancel{x}}\right] = \frac{1}{Re}\left[\frac{K}{Pr}\left[\tilde{E} + P/\rho\right]_\xi\right] + \frac{1}{Re}\left[\frac{K}{Pr}\frac{1}{2}(2\tilde{u}c + c^2)\right]_\xi$$

$$\frac{1}{Re}\left[\frac{K}{Pr}\left[E + P/\rho\right]_x\right] = \frac{1}{Re}\frac{K}{Pr}\tilde{u}_{\cancel{x}} c = \frac{1}{Re}\frac{K}{Pr} c \cdot \frac{(V-1)(V-V_f)}{\alpha V} u_L$$

$$\frac{\lambda+2\mu}{Re}\tilde{u}_x = \cancel{\frac{\lambda+2\mu}{Re}} \frac{\gamma+1}{2\gamma} \frac{\dot{m}\cancel{Re}}{\cancel{(\lambda+2\mu)}} \frac{(V-1)(V-V_f)}{V} u_L$$

$$\frac{1}{Re}\frac{K}{Pr}\hat{u}_x c = \frac{1}{\cancel{Re}}\frac{K}{Pr}\frac{\gamma+1}{2\gamma}\frac{\dot{m}\cancel{Re}}{\lambda+2\mu}\frac{(V-1)(V-V_f)}{V} u_L \cdot c$$