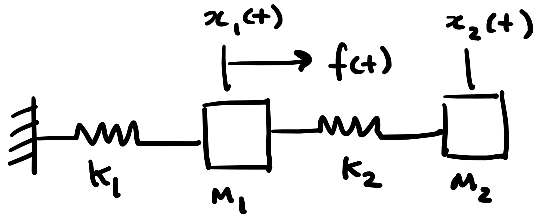
Example



Here we 2DOF translational system. Mass 1, M_1 is connected to a fixed frame of reference by a spring, k_1. Mass 2, M_2 is connected to M_1 by a spring, k_2. A force, $f(t)$ is applied to M_1. We want to find the transfer function that describes M_2's displacement as a function of $f(t)$.

$$\frac{X_2(s)}{F(s)} = ?$$

Using mesh analysis, we can write the following equations:

$$(M_1 s^2 + k_1 + k_2)X_1(s) - (k_2)X_2(s) = F(s)$$

$$-(k_2)X_1(s) + (M_2 s^2 + k_2)X_2(s) = 0$$

In matrix form:

$$\begin{bmatrix} F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} M_1 s^2 + k_1 + k_2 & -k_2 \\ -k_2 & M_2 s^2 + k_2 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

Using Cramer's rule to solve for $X_2(s)$:

$$X_2(s) = \frac{\left| \begin{bmatrix} M_1 s^2 + k_1 + k_2 & F(s) \\ -k_2 & 0 \end{bmatrix} \right|}{\left| \begin{bmatrix} M_1 s^2 + k_1 + k_2 & -k_2 \\ -k_2 & M_2 s^2 + k_2 \end{bmatrix} \right|}$$

$$= \frac{k_2 F(s)}{(M_1 s^2 + k_1 + k_2)(M_2 s^2 + k_2) - k_2^2}$$

Multiplying the polynomials:

$$\begin{array}{ccccc} & & M_1 s^2 & + (k_1 + k_2) & \\ \times & & M_2 s^2 & + k_2 & \\ \hline s^4 & s^3 & s^2 & s^1 & s^0 \\ & & k_2 M_1 & 0 & k_1 k_2 + k_2^2 \\ M_1 M_2 & 0 & (k_1 + k_2) M_2 & 0 & 0 \end{array}$$

$$= M_1 M_2 s^4 + k_2 M_1 s^2 + (k_1 + k_2) M_2 s^2 + k_1 k_2 + k_2^2$$

$$\frac{X_2(s)}{F(s)} \rightarrow \frac{k_2}{M_1 M_2 s^4 + k_2 M_1 s^2 + (k_1 + k_2) M_2 s^2 + k_1 k_2}$$

This is the transfer function that describes M_2's displacement as a function of time.

Substituting for some values:

If $M_1 = M_2 = k_1 = k_2 = 1$,

$$\frac{X_2(s)}{F(s)} = \frac{1}{s^4 + 3s^2 + 1}$$