### PO11Q & PO12Q

### Formula Collection and Notation

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## 1 | PO11Q

Statistic	Formula
Confidence Interval	$Pr(\bar{y} - t_{\alpha/2} \cdot se \le \mu \le \bar{y} + t_{\alpha/2} \cdot se) = 1 - \alpha$
Deviation	$d = y_i - \bar{y}$
Mean	$\bar{y} = \frac{\sum y_i}{n}$
	$\mu = \Sigma y P(y) = E[y]$
Position of p <sup>th</sup> percentile	$P=(n+1)\cdot\frac{p}{100}$
Range	$y_{range} = y_{max} - y_{min}$
Standard Deviation	$S = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n - 1}}$
Standard Error	$\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}}$
	$Se = \frac{S}{\sqrt{n}}$
t-test	$t = \frac{\bar{y} - \mu_0}{se}$
Variance	$S^2 = \frac{\Sigma (y_i - \bar{y})^2}{n - 1}$
z-score	$Z = \frac{y - \mu}{\sigma}$

Table 1: Formulae for PO11Q



Statistic	Formula	
Crosstabulations		
$\chi^2$ -Test Statistic	$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$	
Two Sample Tests		
Standard Error of Difference	$se = \sqrt{(se_1)^2 + (se_2)^2}$	
Means		
Standard Error of Difference	$se = \sqrt{\frac{se_1^2}{n_1} + \frac{se_2^2}{n_2}}$	
Standard Error (pooled)	$se_0 = \sqrt{s_0^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$	
Variance (pooled)	$s_0^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	
Proportions		
Standard Error	$se = \sqrt{\frac{\hat{\pi}(1 - \hat{\pi})}{n}}$	
Standard Error of Difference	$se = \sqrt{\frac{\hat{\pi}_1(1 - \hat{\pi}_1)}{n_1} + \frac{\hat{\pi}_2(1 - \hat{\pi}_2)}{n_2}}$ $se_0 = \sqrt{\bar{\pi}(1 - \bar{\pi})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$	
Standard Error (pooled)	$se_0 = \sqrt{\bar{\pi}(1-\bar{\pi})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$	
Regression		
Adjusted R <sup>2</sup>	$\bar{R}^2 = 1 - \frac{\sum \hat{\epsilon}_i^2 / (n - k - 1)}{\sum (y_i - \bar{y})^2 / (n - 1)}$	
Coefficient of Determination	$R^{2} = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum \hat{\epsilon}_{i}^{2}}{\sum (y_{i} - \bar{y})^{2}}$	
Estimated Variance	$\hat{\sigma}^2 = \frac{\sum \hat{\epsilon}_i^2}{n - p}$	
Estimator of $oldsymbol{eta}_0$	$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$	
Estimator of $\beta_1$	$\hat{\beta}_1 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})^2}$	
Explained Sum of Squares	$\sum (\hat{y}_i - \bar{y})^2$	
Residual Sum of Squares	$\sum \hat{\epsilon}_i^2 = \sum (y_i - \hat{y}_i)^2$	
Standard Error of $oldsymbol{eta}_0$	$se(\hat{\beta}_0) = \sqrt{\frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2}} \ \sigma$	$\begin{cases} v\hat{\mathbf{g}}\mathbf{r}(\hat{\mathbf{g}}) = \hat{\sigma}^2 (\mathbf{X}'\mathbf{X})^{-1} \end{cases}$
Standard Error of $oldsymbol{eta}_1$	$se(\hat{\beta}_1) = \frac{\sigma}{\sqrt{\sum (x_i - \bar{x})^2}}$	
Total Sum of Squares	$\sum (y_i - \bar{y})^2$	

Table 2: Formulae for PO12Q

### 2.1 Matrices

$$X'X = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{bmatrix} \times \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} = \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix}$$

$$(\mathbf{X}'\mathbf{X})^{-1} = \frac{1}{n \sum x_i^2 - \sum x_i \sum x_i} \begin{bmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{bmatrix}$$

$$\mathbf{X'Y} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

# 3 | Notation

Symbol	Explanation
PO11Q	·
d	Deviation
n	Sample Size
S	Standard Deviation
$s^2$	Variance
$ar{y}$	Mean
$y_i$	Observation i
f	(Absolute) Frequency
cf	Cumulative (Absolute) Frequency
rf	Relative Frequency
crf	Cumulative Relative Frequency
<i>E</i> [ <i>x</i> ]	The expected value of x
μ	Mean of the Population
se	Standard Error (with s of sample)
σ	Standard Deviation of the Population
$\sigma_{ar{y}}$	Standard Error (with $\sigma$ of population)
t	t-value
Z	z-value
PO12Q	
$\chi^2$	Chi-Squared for test of independence
π	Population Proportion
π̂	Sample Proportion
π	Pooled Proportion
se <sub>0</sub>	Standard Error under the Null Hypothesis
β	Regression Coefficient
β	Estimated Regression Coefficient
€	Error Term
Ê	Estimated Error Term / Residual
$A^{-1}$	Inverse of Matrix A
Α′	Transpose of Matrix A
I	Identity Matrix
$R^2$	R-Squared / Model Fit
$\sigma^2$	Mean Squared Error
k -2	Number of Slope Coefficients
$\bar{R}^2$	Adjusted R-Squared
log(x)	Logarithm of variable x
$\alpha_i$	Regression coefficients of secondary regression models
p V V	Total Number of Regression Coefficients, including the Intercept
VcV	Variance-Covariance Matrix
Ω	Is equal to $\sigma^2 I$ , where $\sigma^2$ represents the mean squared error
t	Time period in time-series data

Table 3: Notation