PO12Q: Introduction to Quantitative Political Analysis II
Week 3 - Worksheet Solutions

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Regression

1. a. There is a positive relationship between life expectancy and per capita GDO, meaning that longer life expectancies (higher levels of health) are expected to lead to higher levels of per capita GDP (wealth)
   b. It is a sample, as there are more than those countries represented in most of the regions.
   c. The regional regression line would adapt to that particular subset of observations, so change both intercept and slope
   d. There is a curve suggested in this plot, so a quadratic regression line (second order polynomial) would be better (we will cover this in Week 9)

Matrices

Core Solutions

1. Entries.
   a. $\boldsymbol{A}_{22} = 8$
   b. $\boldsymbol{A}_{31} = 14$
   c. $\boldsymbol{B}_{13} = 12$
   d. $\boldsymbol{B}_{24} = 3$
   e. $\boldsymbol{C}_{12} = 22$
   f. $\boldsymbol{C}_{21} = 31$

2. Dimensions.
   - $\boldsymbol{A} \in \mathbb{R}^{3\times3}$
   - $\boldsymbol{B} \in \mathbb{R}^{2\times4}$
   - $\boldsymbol{C} \in \mathbb{R}^{2\times2}$
   - $\boldsymbol{B}' \in \mathbb{R}^{4\times2}$
   - $\boldsymbol{CB} \in \mathbb{R}^{2\times4}$.

3. Transpose $\boldsymbol{B}$.

$$\boldsymbol{B} = \begin{bmatrix} 13 & 8 & 12 & 2 \\ 1 & 5 & 15 & 3 \end{bmatrix} \implies \boldsymbol{B}' = \begin{bmatrix} 13 & 1 \\ 8 & 5 \\ 12 & 15 \\ 2 & 3 \end{bmatrix}$$

4. Product defined? (and dimensions if defined)
    a. ***AB***: not defined (3×3 with 2×4)
    b. ***BA***: not defined (2×4 with 3×3)
    c. ***AC***: not defined (3×3 with 2×2)
    d. ***CA***: not defined (2×2 with 3×3)
    e. ***BC***: not defined (2×4 with 2×2)

5. Calculate ***D*** where ***CB*** = ***D***.

$$\boldsymbol{D} = \boldsymbol{CB} = \begin{bmatrix} 25 & 22 \\ 31 & 19 \end{bmatrix} \begin{bmatrix} 13 & 8 & 12 & 2 \\ 1 & 5 & 15 & 3 \end{bmatrix} = \begin{bmatrix} 347 & 310 & 630 & 116 \\ 422 & 343 & 657 & 119 \end{bmatrix}$$

6. Determinant

    Generally we calculate the determinant with

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc,$$

    Here, we have $a = 25$, $b = 22$, $c = 31$, $d = 19$ for ***C***. Therefore

$$\det(\boldsymbol{C}) = 25 \cdot 19 - 22 \cdot 31 = 475 - 682 = -207.$$

7. Inverse of ***C***.

    For $\boldsymbol{C} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\boldsymbol{C}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

    Here $a = 25$, $b = 22$, $c = 31$, $d = 19$, so $ad - bc = -207$ and

$$\boldsymbol{C}^{-1} = \frac{1}{-207} \begin{bmatrix} 19 & -22 \\ -31 & 25 \end{bmatrix} = \begin{bmatrix} -\frac{19}{207} & \frac{22}{207} \\ \frac{31}{207} & -\frac{25}{207} \end{bmatrix}.$$

Advanced Solutions

8. Solve $\boldsymbol{Cx} = \boldsymbol{b}$ using $\boldsymbol{C}^{-1}$.

   $\boldsymbol{x} = \boldsymbol{C}^{-1}\boldsymbol{b}$.

- For $\boldsymbol{b} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$:

$$\boldsymbol{x} = \begin{bmatrix} -\frac{19}{207} & \frac{22}{207} \\ \frac{31}{207} & -\frac{25}{207} \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -\frac{85}{207} \\ \frac{106}{207} \end{bmatrix}.$$

- For $\boldsymbol{b} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$:

$$\boldsymbol{x} = \begin{bmatrix} -\frac{19}{207} & \frac{22}{207} \\ \frac{31}{207} & -\frac{25}{207} \end{bmatrix} \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{26}{69} \\ -\frac{17}{69} \end{bmatrix}.$$

9. Vector–matrix products.

   With $\boldsymbol{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\boldsymbol{v} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$:

$$\boldsymbol{Au} = \begin{bmatrix} 9 & 4 & 11 \\ 6 & 8 & 3 \\ 14 & 7 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 27 \\ 4 \\ 25 \end{bmatrix}, \qquad \boldsymbol{v}'\boldsymbol{A} = \begin{bmatrix} 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 9 & 4 & 11 \\ 6 & 8 & 3 \\ 14 & 7 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 13 \end{bmatrix}.$$

10. Linear combination with identity.

$$3\boldsymbol{A} - 2\boldsymbol{I}_3 = 3\begin{bmatrix} 9 & 4 & 11 \\ 6 & 8 & 3 \\ 14 & 7 & 9 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 25 & 12 & 33 \\ 18 & 22 & 9 \\ 42 & 21 & 25 \end{bmatrix}.$$