



## Regression

1.
  - a. There is a positive relationship between life expectancy and per capita GDO, meaning that longer life expectancies (higher levels of health) are expected to lead to higher levels of per capita GDP (wealth)
  - b. It is a sample, as there are more than those countries represented in most of the regions.
  - c. The regional regression line would adapt to that particular subset of observations, so change both intercept and slope
  - d. There is a curve suggested in this plot, so a quadratic regression line (second order polynomial) would be better (we will cover this in Week 9)

## Matrices

### 1. Entries.

- a.  $A_{22} = 8$
- b.  $A_{31} = 14$
- c.  $B_{13} = 12$
- d.  $B_{24} = 3$
- e.  $C_{12} = 22$
- f.  $C_{21} = 31$

### 2. Dimensions.

- $A \in \mathbb{R}^{3 \times 3}$
- $B \in \mathbb{R}^{2 \times 4}$
- $C \in \mathbb{R}^{2 \times 2}$
- $B' \in \mathbb{R}^{4 \times 2}$
- $CB \in \mathbb{R}^{2 \times 4}$ .

### 3. Transpose $B$ .

$$B = \begin{bmatrix} 13 & 8 & 12 & 2 \\ 1 & 5 & 15 & 3 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} 13 & 1 \\ 8 & 5 \\ 12 & 15 \\ 2 & 3 \end{bmatrix}$$

4. Product defined? (and dimensions if defined)

- a. **AB**: not defined ( $3 \times 3$  with  $2 \times 4$ )
- b. **BA**: not defined ( $2 \times 4$  with  $3 \times 3$ )
- c. **AC**: not defined ( $3 \times 3$  with  $2 \times 2$ )
- d. **CA**: not defined ( $2 \times 2$  with  $3 \times 3$ )
- e. **BC**: not defined ( $2 \times 4$  with  $2 \times 2$ )

5. Vector-matrix products.

$$\text{With } \mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} :$$

$$\mathbf{A}\mathbf{u} = \begin{bmatrix} 9 & 4 & 11 \\ 6 & 8 & 3 \\ 14 & 7 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 27 \\ 4 \\ 25 \end{bmatrix}, \quad \mathbf{v}'\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 9 & 4 & 11 \\ 6 & 8 & 3 \\ 14 & 7 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 13 \end{bmatrix}.$$

6. Calculate **D** where **CB = D**.

$$\mathbf{D} = \mathbf{CB} = \begin{bmatrix} 25 & 22 \\ 31 & 19 \end{bmatrix} \begin{bmatrix} 13 & 8 & 12 & 2 \\ 1 & 5 & 15 & 3 \end{bmatrix} = \begin{bmatrix} 347 & 310 & 630 & 116 \\ 422 & 343 & 657 & 119 \end{bmatrix}$$

7. Linear combination with identity.

$$3\mathbf{A} - 2\mathbf{I}_3 = 3 \begin{bmatrix} 9 & 4 & 11 \\ 6 & 8 & 3 \\ 14 & 7 & 9 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 25 & 12 & 33 \\ 18 & 22 & 9 \\ 42 & 21 & 25 \end{bmatrix}.$$

8. Determinant

Generally we calculate the determinant with

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc,$$

Here, we have  $a = 25$ ,  $b = 22$ ,  $c = 31$ ,  $d = 19$  for **C**. Therefore

$$\det(\mathbf{C}) = 25 \cdot 19 - 22 \cdot 31 = 475 - 682 = -207.$$

9. Inverse of  $\mathbf{C}$ .

$$\text{For } \mathbf{C} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \mathbf{C}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Here  $a = 25, b = 22, c = 31, d = 19$ , so  $ad - bc = -207$  and

$$\mathbf{C}^{-1} = \frac{1}{-207} \begin{bmatrix} 19 & -22 \\ -31 & 25 \end{bmatrix} = \begin{bmatrix} -\frac{19}{207} & \frac{22}{207} \\ \frac{31}{207} & -\frac{25}{207} \end{bmatrix}.$$

10. Solve  $\mathbf{C}\mathbf{x} = \mathbf{b}$ .

Using  $\mathbf{C}^{-1}$  we can solve for  $\mathbf{x}$ :  $\mathbf{x} = \mathbf{C}^{-1}\mathbf{b}$ .

- For  $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ :

$$\mathbf{x} = \begin{bmatrix} -\frac{19}{207} & \frac{22}{207} \\ \frac{31}{207} & -\frac{25}{207} \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -\frac{85}{207} \\ \frac{106}{207} \end{bmatrix}.$$

- For  $\mathbf{b}_2 = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$ :

$$\mathbf{x} = \begin{bmatrix} -\frac{19}{207} & \frac{22}{207} \\ \frac{31}{207} & -\frac{25}{207} \end{bmatrix} \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{26}{69} \\ -\frac{17}{69} \end{bmatrix}.$$

11. Difference between  $\mathbf{C}^{-1}\mathbf{b}$  and  $\mathbf{b}\mathbf{C}^{-1}$ .

With  $\mathbf{b} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$  (a  $2 \times 1$  column) and  $\mathbf{C} = \begin{bmatrix} 25 & 22 \\ 31 & 19 \end{bmatrix}$ , we have  $\mathbf{C}^{-1} \in \mathbb{R}^{2 \times 2}$ .

- $\mathbf{C}^{-1}\mathbf{b}$  is defined: ( $2 \times 2$  times  $2 \times 1$  gives a  $2 \times 1$  column vector), and equals the unique solution to  $\mathbf{C}\mathbf{x} = \mathbf{b}$ :

$$\mathbf{C}^{-1} = \frac{1}{-207} \begin{bmatrix} 19 & -22 \\ -31 & 25 \end{bmatrix} = \begin{bmatrix} -\frac{19}{207} & \frac{22}{207} \\ \frac{31}{207} & -\frac{25}{207} \end{bmatrix}, \quad \mathbf{C}^{-1}\mathbf{b} = \begin{bmatrix} -\frac{85}{207} \\ \frac{106}{207} \end{bmatrix}.$$

- $\mathbf{b}\mathbf{C}^{-1}$  is not defined because the inner dimensions do not match:  $(2 \times 1)(2 \times 2)$  has  $1 \neq 2$ .

*Note:* If you instead interpret  $\mathbf{b}$  as a row vector  $\mathbf{b}' = \begin{bmatrix} 1 & -3 \end{bmatrix}$ , then  $\mathbf{b}'\mathbf{C}^{-1}$  is defined and yields a  $1 \times 2$  row:

$$\mathbf{b}'\mathbf{C}^{-1} = \begin{bmatrix} 1 & -3 \end{bmatrix} \begin{bmatrix} -\frac{19}{207} & \frac{22}{207} \\ \frac{31}{207} & -\frac{25}{207} \end{bmatrix} = \begin{bmatrix} -\frac{112}{207} & \frac{97}{207} \end{bmatrix}.$$

So the “difference” is both existence (only the left product is defined for a column  $\mathbf{b}$ ) and shape (column vs row), illustrating that matrix multiplication is order-sensitive.