PO91Q - Fundamentals in Quantitative Research Methods:

Worksheet Week 8 - Solutions

Dr Florian Reiche
F.Reiche@warwick.ac.uk

1 | Core - Applied

1. Assuming a regression model of the type $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$, calculate the estimators for β_0 and β_1. Use Table 2 as a guide to the required intermediary calculations.

i	age (x)	income (y)	$y - \bar{y}$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
1	22	700	-795	-17	289	13515
2	19	650	-845	-20	400	16900
3	56	2300	805	17	289	13685
4	45	1900	405	6	36	2430
5	37	2000	505	-2	4	-1010
6	23	900	-595	-16	256	9520
7	32	1000	-495	-7	49	3465
8	65	2500	1005	26	676	26130
9	43	1800	305	4	16	1220
10	48	1200	-295	9	81	-2655
MEAN	39	1495				
SUM					2096	83200

Table 1: Intermediary Regression Calculations

$$\begin{aligned}\hat{\beta_0} &= \bar{Y} - \hat{\beta_1}\bar{X} \\ &= 1495 - 39.69 \times 39 \\ &= -53.09\end{aligned}$$

$$\begin{aligned}\hat{\beta_1} &= \frac{\Sigma_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma_{i=1}^{N}(X_i - \bar{X})^2} \\ &= \frac{83200}{2096} \\ &= 39.69\end{aligned}$$

2. Specify the SRF and interpret the estimators of β_0 and β_1.

$$\hat{\text{income}} = -53.09 + 39.7 \text{ age}$$

- At age zero, a person would have an income of -53.09, on average.
- For every additional year of age, a person;s income increases by 39.7 units, on average.

3. Calculate the coefficient of determination, r^2, with $\hat{Y}_i = -5\hat{3}.1 + 3\hat{9}.7X_i$.

i	age (x)	income (y)	$(y-\bar{y})^2$	$(\hat{y})$	$(\hat{y}-\bar{y})$	$(\hat{y}-\bar{y})^2$	$(y-\hat{y})$	$(y-\hat{y})^2$
1	22	700	632025	820.3	-674.7	455220.09	-120.3	14472.09
2	19	650	714025	701.2	-793.8	630118.44	-51.2	2621.44
3	56	2300	648025	2170.1	675.1	455760.01	129.9	16874.01
4	45	1900	164025	1733.4	238.4	56834.56	166.6	27755.56
5	37	2000	255025	1415.8	-79.2	6272.64	584.2	341289.64
6	23	900	354025	860	-635	403225	40	1600
7	32	1000	245025	1217.3	-277.7	77117.29	-217.3	47219.29
8	65	2500	1010025	2527.4	1032.4	1065849.76	-27.4	750.76
9	43	1800	93025	1654	159	25281	146	21316
10	48	1200	87025	1852.5	357.5	127806.25	-652.5	425756.25
MEAN	39	1495						
SUM			4202250			3303485		899655

Table 2: Intermediary Regression Calculations

$$R^2 = \frac{\text{ESS}}{\text{TSS}} = \frac{3303485}{4202250} = 0.79$$

or

$$R^2 = 1 - \frac{\text{RSS}}{\text{TSS}} = \frac{899655}{4202250} = 0.79$$