PO91Q: Fundamentals in Quantitative Research Methods
Week 5 - Solutions

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Effect size, sample size, and power

1. Same effect size, different n (mapping t, n, and $\hat{d}$)
   a. With $t = 1.6$ and $n = 16$, $\hat{d} = t/\sqrt{n} = 1.6/4 = 0.40$. The power at $n = 16$ for $\hat{d} \approx 0.40$ and $\alpha = 0.05$ will be modest (typically well below 0.80).
   b. Keeping $\hat{d}$ fixed at 0.40 and raising n to 64 requires $t = \hat{d}\sqrt{n} = 0.40 \times 8 = 3.2$. The app will show the same $\hat{d}$ but a higher power at $n = 64$.
   c. As n increases while the underlying effect stays fixed, the standard error shrinks: $\text{SE} = s/\sqrt{n}$. The test statistic is $t = \frac{\bar{x} - \mu_0}{\text{SE}}$, so a smaller SE makes $|t|$ larger on average. For a fixed α, the critical cutoff (e.g., $t_{\alpha/2,df}$ for a two-sided test) is essentially fixed, so larger typical $|t|$ increases the chance that $|t| > t_{\alpha/2,df}$. Therefore, bigger $n \Rightarrow$ less sampling noise $\Rightarrow$ tighter estimates $\Rightarrow$ higher power.

2. Planning with a SESOI and α sensitivity
   a. For $t = 2.0$ and $n = 30$, $\hat{d} = t/\sqrt{n} \approx 2.0/\sqrt{30} \approx 0.37$. The app will report the power at $n = 30$ using this $\hat{d}$ (typically moderate at $\alpha = 0.05$).
   b. Turning on the SESOI with $d = 0.5$ switches the curve to a fixed target effect. The orange marker shows the n giving 80% power at $\alpha = 0.05$; for $d = 0.5$ this is typically in the few–dozen range for a one-sample t-test (on the order of the 30s).
   c. Increasing α to 0.10 lowers the critical threshold and reduces the required n for a given power; decreasing α to 0.01 raises the threshold and increases the required n. Formally, stricter α increases the critical t value, so a larger n is needed for the same probability of exceeding it under the alternative.

3. Was the study well powered? Post-hoc check and replication planning
   a. With $t = 2.1$ and $n = 25$, $\hat{d} = 2.1/\sqrt{25} = 2.1/5 = 0.42$. The app will show the power at $n = 25$ for $\hat{d} \approx 0.42$ and $\alpha = 0.05$; this is typically only moderate, not comfortably high.
   b. For replication planning, turn on SESOI and choose the closest option to $\hat{d}$ (e.g., $d = 0.5$). The orange marker will give the n needed for 80% power at the chosen α; expect a sample size in the few–dozen range for $d = 0.5$ at $\alpha = 0.05$.
   c. A significant result with low power can be fragile because small shifts in sampling may miss the effect and estimates are noisier. Picking a SESOI (a target d tied to practical importance) and planning for 80% power helps ensure a replication has adequate sensitivity to detect a meaningfully sized effect.

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Summarising Conclusions

- Power is the probability your test will detect a real effect (reject H_0 when the effect truly exists).
- The curve fixes an effect size d and shows how power increases with n.
- If you base d on your observed test ($\hat{d} = t/\sqrt{n}$), the curve answers: ”If the effect really was the size we observed, how likely would the test be to detect that effect at other sample sizes n?”
- If you base d on a meaningful minimum (SESOI), the curve answers: ”How large should n be to reliably detect an effect that actually matters?”
- The 0.80 (80%) line is a convention (not a law). Many fields use it as a planning minimum; higher power (e.g., 90%) is desirable when stakes are high.
- Rule of thumb: because $t = d\sqrt{n}$, halving d requires about 4× the sample size to keep power roughly the same.

Why do many plots show an 80% power line?

- 80% power means choosing $\beta = 0.20$ (so power $= 1 - \beta = 0.80$). This is a planning convention, not a mathematical law.
- The convention became common through methodological guidance (e.g., Cohen’s power analysis texts) and is echoed in many applied fields.
- In higher-stakes settings (e.g., confirmatory clinical trials), targets of 80–90% power are typical; 90

Case Study: Alcohol-Related Hospital Admissions and Mortality in Scotland

Data Exploration

Before starting, we need to load libraries and install packages if not already installed. In these exercises we will be using the `tidyverse` package.

1. Set your working directory, place the data set in it, and load it into R.
2. Create a new RScript for this case study and annotate it as you go through the exercises presented here.
3. Load the `tidyverse` package.

Descriptive Statistics

1. Produce descriptive statistics for all three numerical variables.

```
summary(simd$alc16)
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##   48.15   66.83   99.03   98.71  116.07  205.29
```

```
summary(simd$mortality16)
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##   79.04   89.49   94.86   96.72  103.17  126.68
```

```
summary(simd$mortality20)
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's 
##   79.09   89.82   94.26   95.58  100.79  113.59       1
```

Visualisation

Let's visualise the distribution of the variable `alc16`.

```
ggplot(simd, aes(x=alc16)) +
  geom_density(aes(y=..density..)) +
  theme_classic() +
  scale_x_continuous(name="Alcohol-Related Hospital Admissions (2011-2014)")+
  ylab('Density') +
  theme(axis.text=element_text(size=12),
        axis.title=element_text(size=13))
```

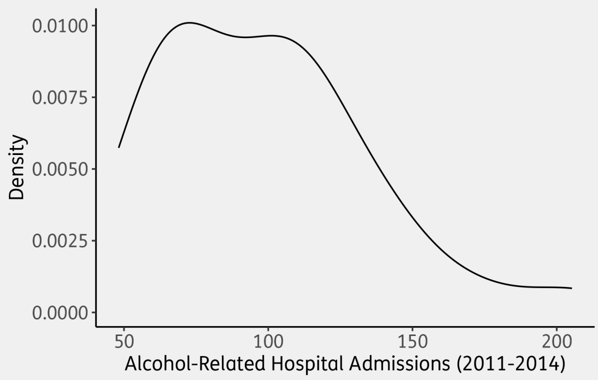

Figure 1: Distribution of Alcohol-Related Hospital Admissions (2011-2014)

1. Reproduce Figure 1.
2. What does the distribution tell us about alcohol-related admissions to hospital?
    - Not perfectly normally distributed, with a positive skew.
3. How does the shape of the distribution in Figure 1 relate to the descriptive statistics calculated in Section ?
    - In a positvely skewed distribution the median is larger than the mean which is the case here. The maximum is also well above the third quartile.
4. What would happen to the shape of the distribution if the median was smaller than the mean?
    - If they were identical, then this would be a normal distribution. If the median was smaller than the mean then we would be dealing with a negatively skewed distribution.

Hypothesis

We are interested in how alcohol-related admissions to hospital have affected mortality rates in Scotland. The following scatter plot uses the variables `alc16` and `mortality20`.

```
ggplot(simd, aes(alc16, mortality20)) +
  geom_point() +
  xlab('Alcohol-Related Hospital Admissions (2011-2014)') +
  ylab('Standardised Mortality Ratio (2014-2018)') +
  theme_classic() +
  theme(axis.text=element_text(size=12),
        axis.title=element_text(size=13))
```

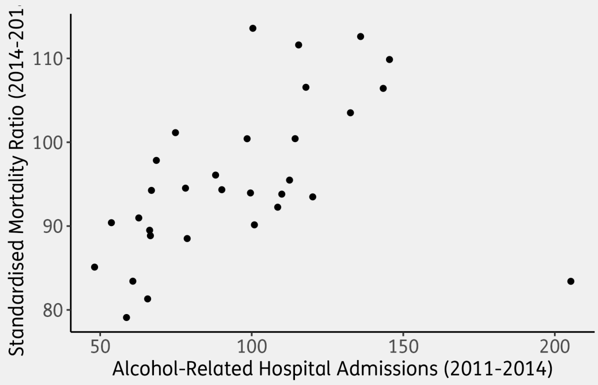

Figure 2: Alcohol-Related Hospital Admissions and Mortality

1. Reproduce Figure 2.
2. Based on this scatter plot, formulate the alternative and the null hypotheses:

H_A: The higher the rate of alcohol-related admissions to hospital between 2011 and 2014, the higher the standardised mortality ratio in 2014-2018.

H_0: The rate of alcohol-related admissions to hospital between 2011 and 2014 and the standardised mortality ratio in 2014-2018 are unrelated.

Sampling

The data frame `simd` which we have been using so far represents the population. Let us now draw a random sample of 15 councils as follows:

```
set.seed(6)
sample <- sample_n(simd, 15)
```

1. Explain the purpose of the `set.seed` function.
    - It creates a pseudo-random number.

Inferential Statistics

Let us now see if the mortality ratio has changed between the two waves of 2016 and 2020. This is the worked example from the lecture, but I am repeating it here deliberately, so that you can carry out the example yourself.

1. As a first step, create a new variable measuring the difference between `mortality20` and `mortality16`. Make sure that increases are positive and decreases negative.

```
sample$diff <- with(sample, mortality20-mortality16)
```

2. What is the sample mean of the differences in mortality rates, variable `diff`?

```
summary(sample$diff)
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -2.9276 -0.1155  1.5762  0.9063  2.2114  2.8747
```

3. The sample size of 15 is small. Will it be appropriate to conduct a t-test? Why? Why not?
    - Yes, as the population distribution is likely to be normal.
4. Find out whether the difference in mortality rates is significantly different from zero.

```
t.test(sample$diff, mu=0,
       data=sample)
##
##  One Sample t-test
##
## data:  sample$diff
## t = 1.9689, df = 14, p-value = 0.06909
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.08096415  1.89353389
## sample estimates:
## mean of x
## 0.9062849
```

- There is no statistically significant difference in mortality ratios between the two waves.

5. Draw a graph which depicts the direction of the alternative hypothesis and the p-value. Try not to look at the lecture slides.

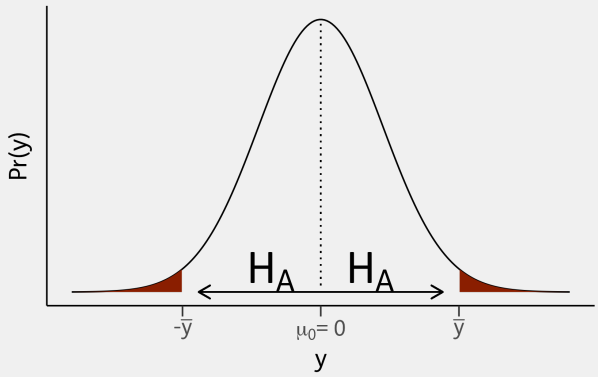

Figure 3: Two-Sided Significance Test

6. Suppose the Scottish Government claims that mortality rates have decreased. Test this claim.

```
t.test(sample$diff, mu=0,
       data=sample,
       alternative = "less")
##
##  One Sample t-test
##
## data:  sample$diff
## t = 1.9689, df = 14, p-value = 0.9655
## alternative hypothesis: true mean is less than 0
## 95 percent confidence interval:
##      -Inf 1.717019
## sample estimates:
## mean of x
## 0.9062849
```

   – Absolutely not!

7. Again, draw a graph which depicts the direction of the alternative hypothesis and the p-value. Try not to look at the lecture slides.

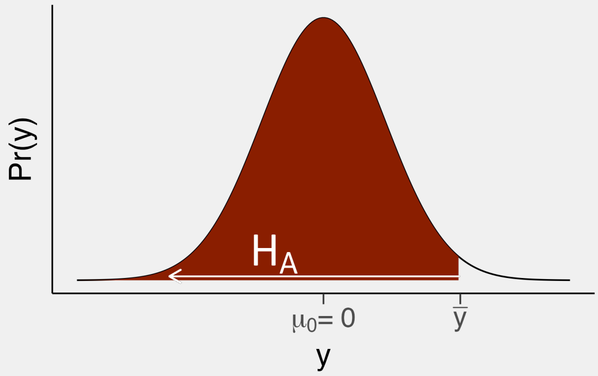

Figure 4: Left-Sided Significance Test

8. Drawing on the results from Exercises 5 and 7, reason about the p-value you would obtain if you tested the hypothesis that mortality rates have increased between the waves of 2016 and 2020.
    - Using Figure 4, the p-value for a right-sided test is indicated by the remaining white area. This area must be half of the blue area in Figure 3. $\frac{0.06909}{2} = 0.03454$. You can confirm this with:

```
t.test(sample$diff, mu=0,
       data=sample,
       alternative = "greater")
##
##  One Sample t-test
##
## data:  sample$diff
## t = 1.9689, df = 14, p-value = 0.03454
## alternative hypothesis: true mean is greater than 0
## 95 percent confidence interval:
##  0.09555077        Inf
## sample estimates:
## mean of x
## 0.9062849
```

- This is significant. Mortality rates have indeed increased.

Causality

1. Identify the elements of symmetry and asymmetry in the setup of this case study.
    - Symmetry: Alcohol abuse leads to health issues and possibly death. It's not really a theory, but medical reasoning.
    - Asymmetry: I have taken the mortality from a later wave than the alcohol-related admissions to hospital. So, rverse causality is not possible, but bear in mind that a time-lag might be insufficient to justify asymmetry.
2. Consider again Figure 5 from the lecture. Which aspects of establishing causality has the case study addressed? What is missing?
    - Practically everything is still missing, bar the theory and historical context, and perhaps asymmetry. We have not touched anything else in this graph, yet.

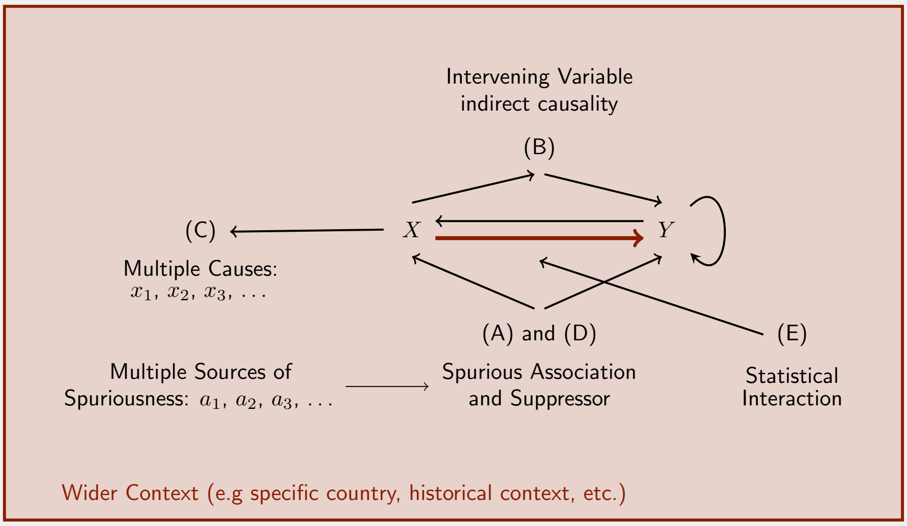

Figure 5: Causality Framework