PO91Q: Fundamentals in Quantitative Research Methods

Worksheet Week 3 - Solutions

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Exercises – Calculations

1.

a. Billy is looking for the heaviest bag possible and finds one that is 1082 g. What is the probability of finding a heavier bag?

$\mu = 1000$
$\sigma = 50$
$x = 1082$

Normally distributed, so find a z-score for the observed value. Heavier means right tail.

$Z = (x - \mu)/\sigma$
$Z = (1082 - 1000)/50$
$Z = 1.64$

Consult tables area under right tail, close to 0.05. Therefore, probability is 5%.

b. What is the probability that Billy will find a bag lighter than 870g?

$\mu = 1000$
$\sigma = 50$
$x = 870$

Normally distributed so find a z-score for the observed value.

$Z = (x - \mu)/\sigma$
$Z = (870 - 1000)/50$
$Z = -2.6$

Consult table's area under right tail, probability is equal to 0.0047. For a positive z-score this would indicate the probability of a heavier bag, but because our z score is negative, it shows the probability of a lighter bag. This probability is less than 0.5%.

c. How would the results of a. and b. change if the standard deviation was only 40g?

For a.
$\mu = 1000$
$\sigma = 40$
$x = 1082$
$Z = (x - \mu)/\sigma$
$Z = (1082 - 1000)/40$
$Z = 2.05$
Probability is 2% now.

For b.
$\mu = 1000$
$\sigma = 40$
$x = 870$
$Z = (x - \mu)/\sigma$
$Z = (870 - 1000)/40$
$Z = -3.25$
Probability is now about 0.1%

Both of these probabilities are smaller and are a direct reflection of a more narrow distribution.

2. 1.96
3. 12.92%
4. $\frac{50-62.3}{8.5} = -1.447059 \rightarrow 7.35\%$

Exercises – The t-distribution

1. How df changes the shape and critical values
   Using two-tailed, $\alpha = 0.05$ (so $t_{\text{crit}} = t_{0.025,df}$ with $df = n - 1$):
   - $n = 3$ ($df = 2$): $t_{\text{crit}} \approx 4.303$
   - $n = 5$ ($df = 4$): $t_{\text{crit}} \approx 2.776$
   - $n = 10$ ($df = 9$): $t_{\text{crit}} \approx 2.262$
   - $n = 20$ ($df = 19$): $t_{\text{crit}} \approx 2.093$
   - $n = 50$ ($df = 49$): $t_{\text{crit}} \approx 2.009$

   The normal reference is $z_{0.025} \approx 1.960$.
   Within 0.05 of 1.96 occurs by $n \approx 50$ ($df = 49$, 2.009 is 0.049 above 1.96).

   Explanation: as $df \rightarrow \infty$, the t distribution $\rightarrow N(0, 1)$, so tails thin and t_{crit} decreases toward 1.96.

2. One-tailed vs two-tailed critical regions
   With $df = 14$ and $\alpha = 0.05$:

   - Two-tailed: $t_{0.025,14} \approx 2.145$, cut points at ± 2.145
   - One-tailed (upper): $t_{0.05,14} \approx 1.761$, single cut point at $+1.761$

   For $H_0 : \mu = \mu_0$ vs $H_1 : \mu > \mu_0$, the one-tailed threshold $t_{0.05,14}$ is relevant because only the upper tail provides evidence against H_0.

   In both cases, the shaded area equals α:

   - two-tailed shades two symmetric regions each of area $\alpha/2$;
   - one-tailed shades a single upper tail of area α.

3. Relating t statistics to sample size
   Observed $t = 2.10$, two-tailed $\alpha = 0.05$.

   a. $df = 9$: $t_{0.025,9} \approx 2.262$. Since $|2.10| < 2.262$, do not reject H_0.
   b. $df = 29$: $t_{0.025,29} \approx 2.045$. Since $|2.10| > 2.045$, reject H_0.
   c. As n increases ($df \uparrow$), t_{crit} decreases toward 1.96; with the same observed t, rejection becomes more likely.

R Exercises

See RScript in the Online Companion