PO91Q: Fundamentals in Quantitative Research Methods

Worksheet Week 8 - Solutions

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# 1 | Core - Applied

1. Assuming a regression model of the type $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$, calculate the estimators for $\beta_0$ and $\beta_1$. Use Table 1 as a guide to the required intermediary calculations.

| i | age (x) | income (y) | $y - \bar{y}$ | $x - \bar{x}$ | $(x - \bar{x})^2$ | $(x - \bar{x})(y - \bar{y})$ |
|---|---|---|---|---|---|---|
| 1 | 22 | 700 | -795 | -17 | 289 | 13515 |
| 2 | 19 | 650 | -845 | -20 | 400 | 16900 |
| 3 | 56 | 2300 | 805 | 17 | 289 | 13685 |
| 4 | 45 | 1900 | 405 | 6 | 36 | 2430 |
| 5 | 37 | 2000 | 505 | -2 | 4 | -1010 |
| 6 | 23 | 900 | -595 | -16 | 256 | 9520 |
| 7 | 32 | 1000 | -495 | -7 | 49 | 3465 |
| 8 | 65 | 2500 | 1005 | 26 | 676 | 26130 |
| 9 | 43 | 1800 | 305 | 4 | 16 | 1220 |
| 10 | 48 | 1200 | -295 | 9 | 81 | -2655 |
| MEAN | 39 | 1495 | | | | |
| SUM | | | | | 2096 | 83200 |

Table 1: Intermediary Regression Calculations

$$\begin{aligned}\hat{\beta_0} &= \bar{Y} - \hat{\beta_1}\bar{X} \\ &= 1495 - 39.69 \times 39 \\ &= -53.09\end{aligned}$$

$$\hat{\beta}_1 = \frac{\Sigma_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma_{i=1}^{N}(X_i - \bar{X})^2}$$
$$= \frac{83200}{2096}$$
$$= 39.69$$

2. Specify the SRF and interpret the estimators of $\beta_0$ and $\beta_1$.

$$\hat{\text{income}} = -53.09 + 39.7 \text{ age}$$

- At age zero, a person would have an income of -53.09, on average.
- For every additional year of age, a person;s income increases by 39.7 units, on average.

3. Calculate the coefficient of determination, $r^2$, with $\hat{Y}_i = -5\hat{3}.1 + 3\hat{9}.7X_i$.

| i | age (x) | income (y) | $(y - \bar{y})^2$ | $\hat{y}$ | $(\hat{y} - \bar{y})$ | $(\hat{y} - \bar{y})^2$ | $(y - \hat{y})$ | $(y - \hat{y})^2$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 22 | 700 | 632025 | 820.3 | -674.7 | 455220.09 | -120.3 | 14472.09 |
| 2 | 19 | 650 | 714025 | 701.2 | -793.8 | 630118.44 | -51.2 | 2621.44 |
| 3 | 56 | 2300 | 648025 | 2170.1 | 675.1 | 455760.01 | 129.9 | 16874.01 |
| 4 | 45 | 1900 | 164025 | 1733.4 | 238.4 | 56834.56 | 166.6 | 27755.56 |
| 5 | 37 | 2000 | 255025 | 1415.8 | -79.2 | 6272.64 | 584.2 | 341289.64 |
| 6 | 23 | 900 | 354025 | 860.0 | -635.0 | 403225.00 | 40.0 | 1600.00 |
| 7 | 32 | 1000 | 245025 | 1217.3 | -277.7 | 77117.29 | -217.3 | 47219.29 |
| 8 | 65 | 2500 | 1010025 | 2527.4 | 1032.4 | 1065849.76 | -27.4 | 750.76 |
| 9 | 43 | 1800 | 93025 | 1654.0 | 159.0 | 25281.00 | 146.0 | 21316.00 |
| 10 | 48 | 1200 | 87025 | 1852.5 | 357.5 | 127806.25 | -652.5 | 425756.25 |
| **MEAN** | 39 | 1495 | | | | | | |
| **SUM** | | | 4202250 | | | 3303485 | | 899655 |

Table 3: Intermediary Regression Calculations

$$R^2 = \frac{\text{ESS}}{\text{TSS}}$$
$$= \frac{3303485}{4202250}$$
$$= 0.79$$

or

$$R^2 = 1 - \frac{\text{RSS}}{\text{TSS}}$$
$$= \frac{899655}{4202250}$$
$$= 0.79$$