EJEMPLO:

REPAIR SHOP: X_n = # machines in the shop on day n

$$X_{n+1} = \underbrace{(X_n - 1)^+}_{\text{c/ día 1 se repara}} + \underbrace{Z_{n+1}}_{\text{entran a ser reparadas}}$$

$$P(Z = k) = a_k \quad ; \quad k \geq 0$$

$$P = \begin{pmatrix} a_0 & a_1 & \dots \\ a_0 & a_1 & \dots \\ 0 & a_0 & \dots \\ 0 & 0 & a_0 \dots \end{pmatrix}$$

$$P(X_{n+1} = j \mid X_n = i) = P(j = (i-1)^+ + Z_{n+1})$$

DISTRIBUCIÓN INVARIANTE

μ dist sobre S

$$P_\mu(X_1 = x) = \mu(x) \quad \forall x \in S$$

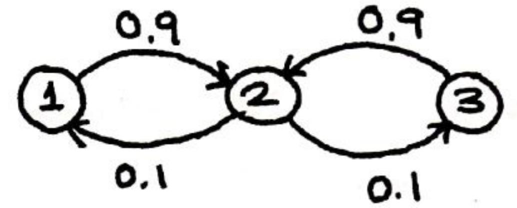
ie; $$\sum_y \mu(y) P(X_1 = x \mid X_0 = y) = \mu(x)$$

$$\boxed{\sum_y \mu(y)\, p(y,x) = \mu(x)}$$

interpretación con harina!

$$\boxed{\mu P = \mu}$$

Para generar intuición:



$$\mu(3) = \mu(1)$$

$$\underbrace{\mu(2) << \mu(1)}_{\text{pues recibe mucho! (2)}}$$

Obs: $$\sum_x \mu(x)\, p(x,z) = \sum_x \left(\sum_y \mu(y)\, p(y,x) \right) p(x,z) = \mu(z)$$

$$\Rightarrow \sum_y \mu(y) \left(\sum_x p(y,x)\, p(x,z) \right) = \mu(z)$$

$$\Rightarrow \sum_y \mu(y)\, p^2(y,z) = \mu(z)$$

$$\boxed{\sum_y \mu(y)\, p^n(y,z) = \mu(z) \quad \forall n}$$

$$\Rightarrow \sum_{n=1}^{k} \left(\sum_y \mu(y)\, p^n(y,z) \right) = k\, \mu(z) \quad \forall k, \forall z$$

$$\Rightarrow \sum_y \mu(y) \frac{\left(\sum_{n=1}^{k} p^n(y,z) \right)}{k} = \mu(z)$$

$$\Rightarrow \sum_y \mu(y)\, \mathbb{E}_y \Bigg[\underbrace{\frac{\sum_{n=1}^{k} \mathbb{1}_{\{X_n = z\}}}{k}}_{F_k(z) = g(X_1, \dots X_k)} \Bigg] = \mu(z)$$

REGENERATIVE CYCLES

$$N_i := \sum_{n\geq 1} \mathbb{1}_{\{X_n = i\}} = \text{\# visits to state } i$$

$$T_i = \inf\{n \geq 1 ; X_n = i\}$$

THM 7.2:

ir de j a i, volver a i r-1 veces, no volver más

$$P_j(N_i = r) = \begin{cases} f_{ji} \cdot f_{ii}^{r-1}(1-f_{ii}) & \text{for } r \geq 1 \\ 1 - f_{ji} & \text{for } r = 0 \end{cases}$$

$$f_{ji} = P_j(T_i < \infty)$$

$$\Rightarrow N_i \mid X_0 = j, N_i \geq 1 \sim G\left(\frac{f_{ji}(1-f_{ii})}{f_{ji}}\right)$$

- $P_i(T_i < \infty) = 1 \iff P_i(N_i = \infty) = 1$
- $E_i[N_i] = \dfrac{f_{ii}}{1-f_{ii}} < \infty \iff P_i(T_i < \infty) < 1$

<u>DEF</u>: $i \in E$ recurrent $P_i(T_i < \infty) = 1$

positive recurrent $E_i[T_i] < \infty$

transient $P_i(T_i < \infty) < 1$

Communication:

- $i \to j$ (j accessible from i) if $\exists\, k \,/\, p_{ij}^{k} > 0$.
- $i \leftrightarrow j$ (both are accessible from both)

DEF: P is irreducible if $i \leftrightarrow j \ \forall\, i,j \in S$

THM 2.1: Regenentive form of Invariant Mesures

$$\mu_j(i) = E_j\left[N_i^{T_j}\right] \quad \forall j$$ es invariante (MEDIDA)

$$= E_j\left[\sum_{n\geq 1} 1_{\{X_n=i\}} \, 1_{\{T_j\geq n\}}\right]$$

Obs. $\sum_i \mu_j(i) = E_j[T_j] \quad \forall j$

x uniqueness $\nu_j(i) = \nu_i(i)$

$$\Rightarrow \nu_j(i) = \frac{\mu_j(i)}{E_j[T_j]} = \nu_i(i) = \frac{\mu_i(i)}{E_i[T_i]} = \frac{1}{E_i[T_i]}$$

ie; $\boxed{\nu(j) = \frac{1}{E_0[T_j]}}$

THM 4.1 ERGODIC THM

$\{X_n\}_n$ irreducible, positive recurrent MC

$\forall \gamma$ initial dist, P_γ - as

$$\Rightarrow \lim_{k\to\infty} \frac{\sum_{n=1}^{k} f(X_n)}{k} = E_\nu[f(X)]$$

$$\parallel$$

$$\sum_i \nu(i) f(i)$$

NOTA PROMEDIOS TEMPORALES = PROMEDIOS ESPACIALES