

Toppunktsformel for andengradspolynomiet $f(x) = ax^2 + bx + c$

- ① Find den afledte af andengradspolynomiet:

$$f'(x) = 2ax + b$$

- ② Opskriv ligningen for maximum af en funktion:

$$f'(x) = 0$$

$$2ax + b = 0$$

- ③ Isolér x i ligningen:

$$2ax = -b$$

$$x = \frac{-b}{2a}$$

- ④ Indsæt denne værdi for x i f og reducer:

$$\begin{aligned} y = f\left(\frac{-b}{2a}\right) &= a\left(\frac{-b}{2a}\right)^2 + b \cdot \left(\frac{-b}{2a}\right) + c \\ &= a\left(\frac{(-b)^2}{(2a)^2}\right) - \left(\frac{b^2}{2a}\right) + c \\ &= \frac{ab^2}{4a^2} - \frac{b^2}{2a} + c \\ &= \frac{b^2}{4a} - \frac{2 \cdot b^2}{4a} + \frac{4a \cdot c}{4a} \\ &= \frac{-b^2 + 4ac}{4a} \\ &= \frac{-(b^2 - 4ac)}{4a} \\ &= \frac{-d}{4a} \end{aligned}$$