

Differentialkvotient for $f(x) = \frac{1}{x}$

(1) Opskriv sekanthældningen:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

(2) Reducér udtrykket:

$$\begin{aligned} &= \frac{\frac{x}{(x+h)\cdot x} - \frac{x+h}{x\cdot(x+h)}}{h} \\ &= \frac{\frac{-h}{(x+h)\cdot x}}{h} \\ &= \frac{-h}{(x+h)\cdot x \cdot h} \\ &= \frac{-1}{x^2 + hx} \end{aligned}$$

(3) Lad $h \rightarrow 0$:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left(\frac{-1}{x^2 + hx} \right) \\ &= \frac{-1}{x^2} \end{aligned}$$