Differentialkvotient for $f(x) = \sqrt{x}$

(1) Opskriv sekanthældningen:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

(2) Reducér udtrykket:

$$= \frac{\left(\sqrt{x+h} - \sqrt{x}\right) \cdot \left(\sqrt{x+h} + \sqrt{x}\right)}{h \cdot \left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{\left(\sqrt{x+h}\right)^2 - \left(\sqrt{x}\right)^2}{h \cdot \left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{x + h - x}{h \cdot \left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{h}{h \cdot \left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

(3) Lad $h \to 0$:

$$f'(x) = \lim_{h \to 0} \left(\frac{1}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$