

Differentialkvotient for $f(x) + g(x)$

(1) Opskriv sekanthældningen:

$$\frac{\Delta y}{\Delta x} = \frac{(f(x+h) + g(x+h)) - (f(x) + g(x))}{h}$$

(2) Reducér udtrykket:

$$\begin{aligned} &= \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h} \\ &= \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h} \\ &= \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \end{aligned}$$

(3) Lad $h \rightarrow 0$:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) + \lim_{h \rightarrow 0} \left(\frac{g(x+h) - g(x)}{h} \right) \\ &= f'(x) + g'(x) \end{aligned}$$