Differentialkvotient for $f(x) = x^2$

(1) Opskriv sekanthældningen:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

(2) Reducér udtrykket:

$$\begin{aligned} &= \frac{x^2 + h^2 + 2xh - x^2}{h} \\ &= \frac{h^2 + 2xh}{h} \\ &= \frac{h^2}{h} + \frac{2xh}{h} \\ &= h + 2x \end{aligned}$$

(3) Lad $h \to 0$:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} (h + 2x) \\ &= 2x \end{aligned}$$