

Tabulate Equations of Common Ellipse Parameters

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Introduction

This document tabulates the equations needed to deduce any of the six common parameters of an ellipse given two of its parameters.

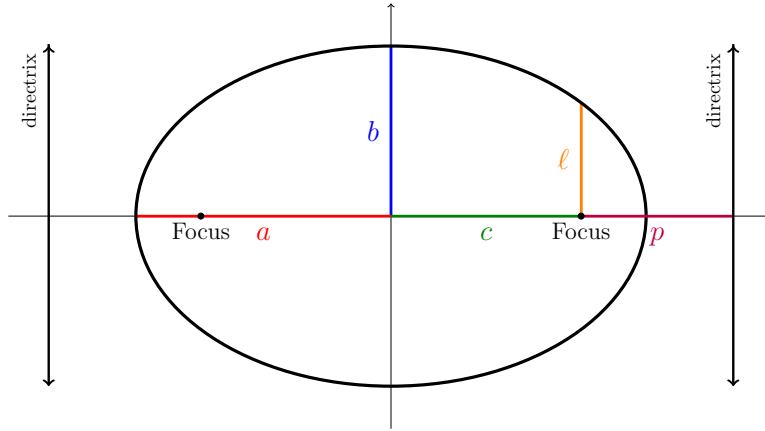


Figure 1: Common parameters labeled on an example ellipse. Eccentricity, e , is not depicted.

Parameters

a Semi-Major Axis.

b Semi-Minor Axis.

c Linear eccentricity.

e Eccentricity. Measurement of deviation from being circular.

ℓ Semi-Latus Rectum. The length of a line segment that begins at the focus and makes contact with the ellipse. It is perpendicular to the major axis.

p Focal parameter. The length from one of the two foci to the nearest directrix.

Other Useful Relations & Terminology

Major Axis Double the length of the semi-major axis ($2a$). The length of the ellipse at its widest point.

Minor Axis Double the length of the semi-minor axis ($2b$). The length of the ellipse at its thinnest point.

Focal Length Double the length of the linear eccentricity ($2e$). The length between the ellipse's two foci.

Flattening A rarer type of measurement for the deviation from being circular. Flattening is given usually in terms of a and b as $f = \frac{a-b}{a}$ or e as $f = 1 - \sqrt{1 - e^2}$.

Latus Rectum Double the length of the semi-latus rectum (2ℓ). The chord that passes through a focus and is perpendicular to the major axis.

Directrix Focal parameter. The length from one of the two foci to the nearest directrix.

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Semi-Major Axis

a	a	b	c	e	ℓ	p
a	—	—	—	—	—	—
b	—	—	$\sqrt{b^2 + c^2}$	$\frac{b}{\sqrt{1 - e^2}}$	$\frac{b^2}{\ell}$	$b\sqrt{1 + \frac{b^2}{p^2}}$
c	—	$\sqrt{b^2 + c^2}$	—	$\frac{c}{e}$	$\frac{\ell + \sqrt{4c^2 + \ell^2}}{2}$	$\sqrt{cp + c^2}$
e	—	$\frac{b}{\sqrt{1 - e^2}}$	$\frac{c}{e}$	—	$\frac{\ell}{1 - e^2}$	$\frac{ep}{1 - e^2}$
ℓ	—	$\frac{b^2}{\ell}$	$\frac{\ell + \sqrt{4c^2 + \ell^2}}{2}$	$\frac{\ell}{1 - e^2}$	—	$\frac{\ell}{1 - \frac{\ell^2}{p^2}}$
p	—	$b\sqrt{1 + \frac{b^2}{p^2}}$	$\sqrt{cp + c^2}$	$\frac{ep}{1 - e^2}$	$\frac{\ell}{1 - \frac{\ell^2}{p^2}}$	—

Semi-Minor Axis

b	a	b	c	e	ℓ	p
a	—	—	$\sqrt{a^2 - c^2}$	$a\sqrt{1 - e^2}$	$\sqrt{a\ell}$	$\sqrt{\frac{\sqrt{4a^2p^2 + p^4} - p^2}{2}}$
b	—	—	—	—	—	—
c	$\sqrt{a^2 - c^2}$	—	—	$\frac{c}{e}\sqrt{1 - e^2}$	$\sqrt{\frac{\sqrt{4c^2\ell^2 + \ell^4} + \ell^2}{2}}$	\sqrt{cp}
e	$a\sqrt{1 - e^2}$	—	$\frac{c}{e}\sqrt{1 - e^2}$	—	$\frac{\ell}{\sqrt{1 - e^2}}$	$\frac{ep}{\sqrt{1 - e^2}}$
ℓ	$\sqrt{a\ell}$	—	$\sqrt{\frac{\sqrt{4c^2\ell^2 + \ell^4} + \ell^2}{2}}$	$\frac{\ell}{\sqrt{1 - e^2}}$	—	$\frac{\ell}{1 - \frac{\ell^2}{p^2}}$
p	$\sqrt{\frac{\sqrt{4a^2p^2 + p^4} - p^2}{2}}$	—	\sqrt{cp}	$\frac{ep}{\sqrt{1 - e^2}}$	$\frac{\ell}{1 - \frac{\ell^2}{p^2}}$	—

Linear Eccentricity

c	a	b	c	e	ℓ	p
a	—	$\sqrt{a^2 - b^2}$	—	ae	$\sqrt{a^2 - a\ell}$	$\frac{-p + \sqrt{4a^2 + p^2}}{2}$
b	$\sqrt{a^2 - b^2}$	—	—	$\frac{be}{\sqrt{1 - e^2}}$	$b\sqrt{\frac{b^2}{\ell^2} - 1}$	$\frac{b^2}{p}$
c	—	—	—	—	—	—
e	ae	$\frac{be}{\sqrt{1 - e^2}}$	—	—	$\frac{e\ell}{1 - e^2}$	$\frac{pe^2}{1 - e^2}$
ℓ	$\sqrt{a^2 - a\ell}$	$b\sqrt{\frac{b^2}{\ell^2} - 1}$	—	$\frac{e\ell}{1 - e^2}$	—	$\frac{p}{\frac{p^2}{\ell^2} - 1}$
p	$\frac{-p + \sqrt{4a^2 + p^2}}{2}$	$\frac{b^2}{p}$	—	$\frac{pe^2}{1 - e^2}$	$\frac{p}{\frac{p^2}{\ell^2} - 1}$	—

Eccentricity

e	a	b	c	e	ℓ	p
a	—	$\sqrt{1 - \frac{b^2}{a^2}}$	$\frac{c}{a}$	—	$\sqrt{1 - \frac{\ell}{a}}$	$\frac{-p + \sqrt{4a^2 + p^2}}{2a}$
b	$\sqrt{1 - \frac{b^2}{a^2}}$	—	$\frac{c}{\sqrt{b^2 + c^2}}$	—	$\sqrt{1 - \frac{\ell^2}{b^2}}$	$\frac{b}{\sqrt{b^2 + p^2}}$
c	$\frac{c}{a}$	$\frac{c}{\sqrt{b^2 + c^2}}$	—	—	$\frac{-\ell + \sqrt{4c^2 + \ell^2}}{2c}$	$\sqrt{\frac{c}{c + p}}$
e	—	—	—	—	—	—
ℓ	$\sqrt{1 - \frac{\ell}{a}}$	$\sqrt{1 - \frac{\ell^2}{b^2}}$	$\frac{-\ell + \sqrt{4c^2 + \ell^2}}{2c}$	—	—	$\frac{\ell}{p}$
p	$\frac{-p + \sqrt{4a^2 + p^2}}{2a}$	$\frac{b}{\sqrt{b^2 + p^2}}$	$\sqrt{\frac{c}{c + p}}$	—	$\frac{\ell}{p}$	—

Semi-Latus Rectum

ℓ	a	b	c	e	ℓ	p
a	—	$\frac{b^2}{a}$	$\frac{a^2 - c^2}{a}$	$a(1 - e^2)$	—	$\frac{-p^2 + \sqrt{4a^2p^2 + p^4}}{2a}$
b	$\frac{b^2}{a}$	—	$\frac{b^2}{\sqrt{b^2 + c^2}}$	$b\sqrt{1 - e^2}$	—	$\frac{bp}{\sqrt{b^2 + p^2}}$
c	$\frac{a^2 - c^2}{a}$	$\frac{b^2}{\sqrt{b^2 + c^2}}$	—	$\frac{c}{e}(1 - e^2)$	—	$\sqrt{\frac{cp^2}{c + p}}$
e	$a(1 - e^2)$	$b\sqrt{1 - e^2}$	$\frac{c}{e}(1 - e^2)$	—	—	ep
ℓ	—	—	—	—	—	—
p	$\frac{-p^2 + \sqrt{4a^2p^2 + p^4}}{2a}$	$\frac{bp}{\sqrt{b^2 + p^2}}$	$\sqrt{\frac{cp^2}{c + p}}$	ep	—	—

Focal Parameter

p	a	b	c	e	ℓ	p
a	—	$\frac{b^2}{\sqrt{a^2 - b^2}}$	$\frac{a^2 - c^2}{c}$	$\frac{a}{e}(1 - e^2)$	$\sqrt{\frac{a\ell^2}{a - \ell}}$	—
b	$\frac{b^2}{\sqrt{a^2 - b^2}}$	—	$\frac{b^2}{c}$	$\frac{b}{e}\sqrt{1 - e^2}$	$\frac{b\ell}{\sqrt{b^2 - \ell^2}}$	—
c	$\frac{a^2 - c^2}{c}$	$\frac{b^2}{c}$	—	$\frac{c}{e^2}(1 - e^2)$	$\frac{\ell^2 + \sqrt{4c^2\ell^2 + \ell^4}}{2c}$	—
e	$\frac{a}{e}(1 - e^2)$	$\frac{b}{e}\sqrt{1 - e^2}$	$\frac{c}{e^2}(1 - e^2)$	—	$\frac{\ell}{e}$	—
ℓ	$\sqrt{\frac{a\ell^2}{a - \ell}}$	$\frac{b\ell}{\sqrt{b^2 - \ell^2}}$	$\frac{\ell^2 + \sqrt{4c^2\ell^2 + \ell^4}}{2c}$	$\frac{\ell}{e}$	—	—
p	—	—	—	—	—	—