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## Neural Network

### Project 1: Radial Basis Function

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The purpose of this project is to approximate the function  $h(x) = 0.5 + 0.4\sin(2\pi x)$  using Radial Basis Function (RBF) method. To capture the experimental spirit of data, a random uniform distribution noise in the interval  $[-0.1, 0.1]$  is added to the output function of 75 randomized data set. The code is written in Python (v.3). Initial weights and bias term are generated using the random function in Python. Most of the operations are done using the brilliant “list” feature in Python, however Numpy package is also used occasionally. The function approximation is done by first generating the clusters based on the kmean algorithm discussed in class. Having the clusters determined, we can treat RBF as a MLP with one hidden layer, in which the Gaussian activation function is applied to the data in the first layer and the second layer is the weighted sum of hidden layer output followed by the LMS weight update with 100 epochs.

In this project, there are multiple factors that affect the accuracy and convergence scheme of the model. Number of clusters in the hidden layer (neuron), learning rate parameter and the variance width used in the Gaussian activation function are these factors. In the project instruction, we have been asked to try the number of 2,4,7,11,16 clusters and  $\eta = 0.01, 0.02$  learning rates. To be able to compare the trends,  $\eta = 0.03$  is also evaluated and included in the error figures to better see the effect of learning rates. All the results are shown in the attached figures. The last row shows the absolute error between desired and RBF evaluated values averaged over all the 75 sample data as a function of learning rate and number of clusters. Referring to the attached figures, the results reveal that although increasing the learning rate decreases the absolute error, the change of error is not significant. As far as number of clusters goes, very few number of clusters results in a large error. Increasing the number of clusters up to about 11 clusters found to decrease the error. However, large number of clusters might not decrease the error and in some cases it can increase the error due to the too much spread of data. This shows that the number of clusters should be optimized based on the number of sample data points fed into the model to avoid over-fitting.



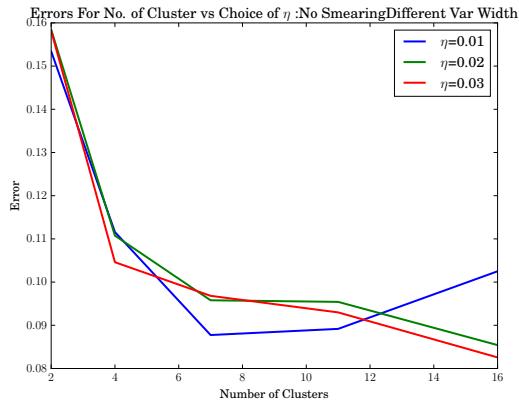
The most interesting feature can be pointed as the effect of cluster variances on the function approximation. The left and right column of figures belong to the use of each cluster variance and the same variance width for all of the cluster width for the Gaussian mapping, respectively. As it can be conceived, different variance for each cluster causes tremendous oscillation over some data points and in some cases, it is very similar to the terrible model in the lecture notes. However, using the same variance width for all clusters results in a very smooth RBF approximation. Referring to the last row of figures, it can be inferred that the scale of error is also decreased in the case of using the same variance width.

One other interesting finding is the clustering method we used in this project and its impact on the reported error. Various methods of k-mean clustering have been reported in the literature, from which we are implementing the unsupervised clustering. In this method, we initially choose the centers randomly from the data and continue the clustering/averaging the process until the centers no longer differ. This method of clustering has some randomization character inherently that produces sharp edges in the error curves for each case (shown below). These sharp edges are the artifact of how the error is quantified and is due to the high sensitivity of error to the unsupervised clustering method. To address this, for each tuple of learning rate/cluster number iteration, RBF algorithm is performed for 10 iterations and the absolute error is averaged through all these iterations. This can help “**smearing**” the artificial sharp edges of error curve produced in a single attempt and evaluates more realistic trend of error over the changes in clusters and learning rate parameter. As a result, the final error curves at the end of this report are the averaged error for 10 smearing iterations in each of the case studies.

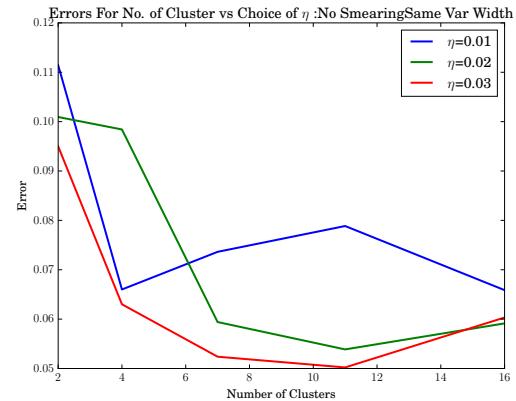


No Smearing

Different Variance Width



Same Variance Width



Smearing:  
10  
iterations

