A Novel Image Interpolation Technique Based on Fractal Theory

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Abstract

Fractal interpolation can be used for scaling-up natural images, and it can retain the texture characters of the image well. But this approach must calculate the Hausdorff dimension first which is difficult to calculate by old ways, like Box-Counting approach and others. This paper proposes a new K dimension which is approximated to Hausdorff dimension and proposes an efficient way to calculate this dimension. Then we use the fractal Brownian motion (FBM) approach in the process of interpolation and get the post-processing images. In the end, the hardware architecture for fast implementation is proposed. Comparing to the other two traditional interpolation approaches, this approach can dramatically reduce the loss of the image's high-frequency components in scaling-up processing and easily applied in hardware design.

Keywords: *Fractal interpolation, Image scaling, Fractal dimension*

1. Introduction

In Recent years image scaling-up becomes more and more important [1][2]. By now, the traditional interpolation algorithms for scaling-up, such as bi-linear, nearest-neighbor interpolation and etc.. are widely used in TV chipset or other filed[1][2]. Nearest-neighbor interpolation just calculates the added pixel's gray value using the gray value of the original pixeles nearest it, sometimes just copy one of them. The arithmetic is very simple and can be easily applied in hardware design. But there are some problems which is hard to avoid. If the original picture has texture, the approach may loss the texture characters, produce some square areas which have the same gray value and produce saw teeth at the edge of the objects in the post-interpolated picture [3]. Bi-linear interpolation has some benefits than nearest-neighbor interpolation. It uses four original pixels which surround the added pixel to calculate the gray value of the added pixel [3]. Bi-linear interpolation avoids some disadvantage of gray value's discontinuity which caused by nearest-neighbor interpolation. Although gray value is continued at added pixel, but the differential coefficient of the gray value is discontinued. So compared to the original picture, the post-scaled picture also loss lots of high frequency components and the image's edge is not very clear. These approaches above all have weakness of lossing image detail and texture imformation in scaling-up processing. It is essential to find a new way to scale up nature image and reduce the loss of high-frequency components.

The nature fractal morphology theory is proposed by B.B Mandelbrot in 1970[4], which had been used in many science fields. And all are being actively studied, such as computer grapics, remote sensing, image compressing and etc.. The term has its roots in the Latin word fractus which is related to the verb fangere (meaning:to break)[10]. In the theory, B.B Mandelbrot pointed out that most texture in nature share the same basic random fractal characters. A fractal dimension which characterized the fractalness is strongly correlated with a sense of roughness. Therefore, applying fractal theory in video image interpolation for nature image has its advantage in keeping target object's texture characters, especially perfect in high definition image. This paper focuses on a new scaling-up interpolation algorithm, which first calculates the selected region's fractal dimension and average variance, and then calculates the interpolated pixel's gray value by its surrounding neighbor pixels and Gauss random numbers.

2.Fractal theory

The basic fractal theory is iterated function systems. It represents an extremely versatile method for conveniently generating a wide variety of useful fractal structures. This reflects the fractal's self-similarity character. We use a series of affine transformations to apply these iterated function systems. Supposing a parameter K:

$$k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \tag{1}$$

or also defined:

$$k(x,y) = (ax+by+e, cx+dy+f) \tag{2}$$

where a, b, c, d, e and f are real numbers. Hence, the affine transformation, k, is represented by six parameters. a, b, c, and d control rotation and scaling, while e and f control linear translation.

Now suppose k1, k2, k3, … , kN as a set of affine linear transformations, and let C be the initial geometry. Then a new geometry, produced by applying the set of transformations to the original geometry, C, and collecting the results from k1(C), k2(C), … , kn(C), can be represented by

$$k(C) = \bigcup_{n=1}^{N} k_n(C) \tag{3}$$

where k is known as the Hutchinson operator. A fractal geometry can be obtained by repeatedly applying k to the previous geometry[11]. For example, we could get the Koch Curve from this iteration function system. Let us suppose that the initiator is place along the x axis, with its left end at the origin. The transformations to obtain the curve of the generator are

$$K_1\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/3 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \tag{4}$$

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$$K_2\begin{pmatrix}x_2\\ y_2\end{pmatrix}=\begin{bmatrix}(1/3)\cos(\pi/3) & -(1/3)\sin(\pi/3)\\ (1/3)\sin(\pi/3) & (1/3)\cos(\pi/3)\end{bmatrix}\begin{pmatrix}x\\ y\end{pmatrix}+\begin{pmatrix}1/3\\ 0\end{pmatrix} \quad (5)$$

Then we get the Koch Cruve using the iteration function system shown in Fig.1 .

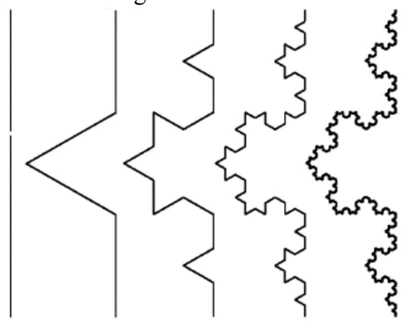

Fig. 1. Koch Curve

Fractal theory proposes us a new way to process and compress image. Because the nature scene has the basic fractal feature like self-similarity and also has the commonest character: random distributing. The conventional math theory has much difficult in describing the complex and anomaly surface of nature objects. In order to simulate the natural processing, it is essential to generate random number to describe the roughness of the nature image. Fractal theory provides the best way to generate the random number, and it is divided into two main directions. One is regulation fractal theories; it is based on some fixed iterative functions. In this approach, we calculated over and over again and get the approximated result ignoring the errors finally. It is usually used in generation the nature scene by computer graphic. The other is random fractal theory. it is based on randomization and describes the nature object well. The latter theory is usually used in image scaling or image compressing. This paper also uses this theory to interpolation nature image, and proposes an efficient way to apply it.

Mandelbrot uses fractal dimension to describe the roughness and self-similarity of the nature image, and proposed fractal Brownian motion (FBM) theory. The track of Brownian motion (BM) is an anomaly fractal curve. it also has the feature of self-similarity and statistic character. FBM is the extension of Brownian motion and satisfied the BM character. Mid-point theory can derived from the FBM theory. There are two simple equations to describe this theory:

$$x_{added} = (x_i + x_{i+1})/2 + s \cdot w \cdot rand(sed1)$$

$$y_{added} = (y_i + y_{i+1})/2 + s \cdot w \cdot rand(sed2) \quad (6)$$

Where xadded, yadded are the pixels interpolated .xi, yi are the pixels already known. s, w are the control dimension, s control the moving direction, w control the moving distance. rand ( ) is a rand number.

According to the mid-point theory, s * w * rand( ) can be replaced by std*N(0,1). And then using a new H parameter to denote the stander variance between the old area and the new one, and the ratio of the two is 2-H.In this way, lots of FBM curves can be drawn. The H parameter is a fractal dimension. If the H equals 0.5, the curve can stand for pure Brownian motion, if H is larger than 0.5, the curve displays more smooth. The approach can also be extended into 2-D space, just like a nature image. This is why people always use it to generate nature objects like mountain or other. We can control the H parameter to control how rough the generated image is.

3. Fractal dimension

The fractal dimension is an important feature of textural images. There are many dimensions to describe this feature like Hausdorff dimension and Minkowski-Bouligand dimension .They are all related to the roughness of the nature image.

The Hausdorff dimension is defined in any subsets of metric space, and the dimension is generalization of the fractal dimensions which includes similarity dimension. It is derived from Lebesgue measure.

We suppose that $F \subset R^n$ is a sets that is not empty, δ is a real number above zero, Ui is a limit subset of Rn .If $F \subset \bigcup U_i$ and $\forall i \quad 0 < |U_i| \le \delta$,then we said that {Ui} is a δ-cover of F. |Ui| is the diameter of Ui, |Ui| is defined as $|U_i| = \sup\{|x-y| : x, y \in U_i\}$,for every δ>0 , define a real number s as:

$$H_\delta^s(F) = \inf\{\sum_{i=1}^{\infty} |U_i|^s : \{U_i\} is\ \delta - cover\ of\ F\} \quad (7)$$

For a fixed F and s, Hsδ(F) is a decreasing function of δ, when δ becoming less , Hsδ(F) becoming more. When $\delta -> 0$, Hs(F) should be described as :

$$H^s(F) = \lim_{\delta \to 0} H_\delta^s(F) \quad (8)$$

Hs(F) is a real number and called Hausdorff dimension.

It is very difficult to calculate the Hausdorff dimension from equation (8). Nipon proposed an efficient differential Box-Counting approach to compute fractal dimension of image in 2002.A bounded set A is said to be self-similar if A is the union of a number (Nr) of non-overlapping scaled copies of itself, where r is the scaling factor. The fractal dimension D of A can be calculated by equation (9).

$$D = \frac{\log(N_r)}{\log(1/r)} \quad (9)$$

And the H dimension equals to D-2. There are also many other techniques proposed to estimate the fractal dimension of an image. The techniques proposed by Gangepain and Roques-Garmers and Keller etc.. do not cover much dynamic range of fractal dimension[5-8]. The technique based on the differential box-counting (DBC) proposed in "An Efficient Differential Box-Counting Approach to Compute Fractal Dimension of Image" paper[9] is proved to be the least computational complexity. But the approach is also very complex and also very unpractical for hardware implication in ASIC design.

It is essential to find a new dimension to approximately describe the textural feature of a nature image. In order to easily calculate the fractal dimension in hardware design, this paper proposes a new k dimension to describe the textural feature. It is also a real number form 0 to 1 like Hausdorff dimension in Fig.2.

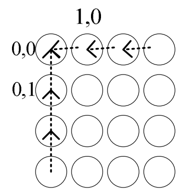

Fig. 2. Hausdorff dimension

K dimension can be calculated as:

$$k=\frac{\sum_{j=0}^{n-1}\sum_{i=0}^{n-2}(|Y_{(i+1,j)}-Y_{(i,j)}|)+\sum_{i=0}^{n-1}\sum_{j=0}^{n-2}(|Y_{(i,j+1)}-Y_{(i,j)}|)}{2\times 255^{n}} \qquad (10)$$

Y(i,j) is the gray value of the pixel (i,j).Because the Y's value is between 0 and 255 ,so the most contrast value of two pixel is 255,it means that the image has the most rough texture. If dividing a picture into 4*4 sub pictures, then n is fixed to 4. Then the k dimension can be easily calculated.

Fig.3 and Fig.4 are two pictures, in which the k dimension value was calculated. The dimensions describe how rough the pictures are.

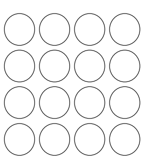

Fig. 3. k=0

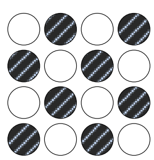

Fig. 4. k=1

4. Fractal interpolation for natural images

A picture can be segmented into M*M pixels subset and the k dimension can be calculated. Fractal interpolation theory means to calculate the gray value of the added pixel from the original ones. The calculate order is shown in figure 5. The circles in gray stand for the original pixels, in white stand for the added pixels .

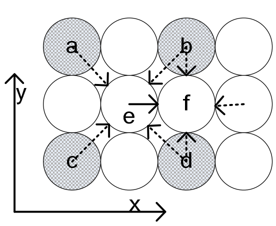

Fig. 5. calculate order

The pixel marked with e should be calculated first, then the one marked with f. In this paper, the random midpoint displacement method is used for generation the added pixels. And the pixel masked with e can be calculated as:

$$Y(x,y)=\frac{1}{4}\{Y(x-1,y-1)+Y(x+1,y-1)+Y(x-1,y+1)+Y(x+1,y+1)\}+\sqrt{1-2^{2k-2}}\cdot\|\Delta X\|\cdot\sigma\cdot Gauss \qquad (11)$$

And the pixel marked with f can be calculated as:

$$Y(x,y)=\frac{1}{4}\{Y(x,y-1)+Y(x-1,y)+Y(x+1,y)+Y(x,y+1)\}+2^{-\frac{k}{2}}\sqrt{1-2^{2k-2}}\cdot\|\Delta X\|\cdot\sigma\cdot Gauss \qquad (12)$$

Where $\Delta X=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ (13)

Gauss is the gauss random number and σ is the average variance of the pixels, ||△X|| is the distance between pixels.

In order to reduce the resource consuming, pictures are divided into 4*4 pixel blocks. This method can be applied by hardware, the diagram is shown below in figure 6.

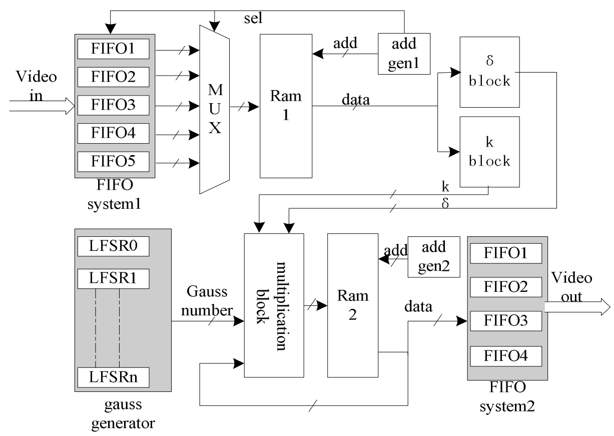

Fig. 6. Hardware Architect diagram

Because there are extraction calculating in δ calculating block, look-up table is adapted to evaluate it. The FIFO system1 contains 5 individual fifos in which 4 fifos used for transfer pixels Y value every line, 1fifo used for transfer UV value. The FIFO system in backside contains 4 fifos. Because they need to wait one line pixels, they depths are 1440. The Ram1 block is the 36*8 bit single port ram, it stores the gray value of the 4*4 pixels blocks' original pixels, the add gen1 block generates the address and control singals of this ram. The ram2 block sis the 100*8 dule port ram, it stores the gray value of the added pixel and the original pixels every 8*8 pixel blocks.

The central limit theorem gives a very simple method to generate a white Gaussian noise. Linear Feedback Shift Registr of length n can be generating a random like binary variable of periodicity2n-1. The concatenation of m individual LFSRs give a m bit value U, if q is big enough. The U is approximate to N(0,1) distribution Gauss random number. So, this paper uses 10 LFSR to generate the Gauss random number.

5. RESULT

In this paper, we proposed a new method to calculate the K dimension, and to application the FBM theory in image interpolation. We did some experiments in two pictures to reveal the relations between the original image and the generated image by fractal theory. The three steps are shown below. First the original image should be divided into 4*4 pixels blocks, the blocks are not overlapped. Second, every block's K dimension should be calculated by the approach mentioned before. Then we use equation (11) and (12) to calculate the gray value of added pixels.

The picture below is tested by this arithmetic for scaling-up twice the size.

Fig. 7. the original one

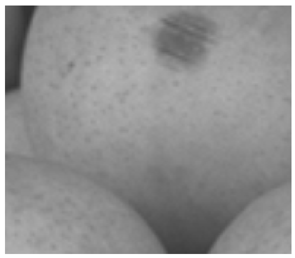

Fig. 8. twice the size

In figure 8, we just displayed a part of the interpolated picture. From it, we saw that the scar and spots on the pears were very clear, but the edge of the pears was not very clear also were the original one. So we changed the picture in order to test more character.

Then we used Lena picture 256*256 pixels to compare the nearest-neighbor, bilinear and fractal interpolation by twice the original picture. And the standard test picture Lena picture has the clear edge and excellent light effect also some texture components.

Fig. 9. original picture

Fig. 10. Nearest-neighbor

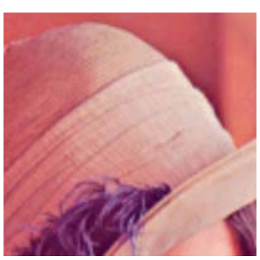

Fig. 11. Bilinear

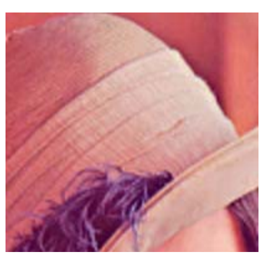

Fig. 12. fractal

Nearest-neighbor interpolation, Bilinear interpolation, fractal interpolation are separated shown in figure 10, 11 and 12. In figure 10, we see that the edge of the hat has lots of saw tooth also the feather on the hat has lots of saw tooth. And the texture on the hat is a little bit more clear than figure 11, the bilinear interpolation.

The bilinear approach didn't have saw tooth at the edge of the hat and feather, but the texture on the hat is very faint and loss lots of high-frequent components. Also the feather on the hat is very faint and dark. Figure 12, the fractal interpolation has the best performance of all the three approaches. The texture on the hat is very clear, also the feather on the hat. The edge of the hat didn't have saw-tooth and was clearer than figure 11.

6. Conclution

This paper proposes an efficient approach to estimate the fractal dimension and uses FBM theory to interpolate nature images. Also this paper experienced two nature image's scaling-up by three different interpolation algorithms. From the compared pictures, we knew that the loss of high-frequency components could be reduced by the proposed interpolation. Therefore, the fractal interpolation of nature picture has higher quality than the conventional interpolation. Because of the new approach to estimate the fractal dimension, fractal interpolation can be easily applied by hardware.

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