



# Statistics for the Australian Grains Industry Technical Report Series

## Efficient linkage map construction using R/ASMap

Julian Taylor  
Research Scientist, SAGI  
School of Food, Agriculture & Wine  
University of Adelaide, Waite Campus  
PMB 1, Glen Osmond, SA, 5064  
email: [julian.taylor@adelaide.edu.au](mailto:julian.taylor@adelaide.edu.au)



February 3, 2023

---

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Background . . . . .	1
1.2	MSTmap . . . . .	3
1.2.1	Clustering . . . . .	3
1.2.2	Marker Ordering . . . . .	4
<b>2</b>	<b>A closer look at R/ASMap functions</b>	<b>7</b>
2.1	Map construction functions . . . . .	8
2.1.1	mstmap.data.frame() . . . . .	8
2.1.2	mstmap.cross() . . . . .	9
2.2	Pulling and pushing markers . . . . .	13
2.3	Improved heat map . . . . .	17
2.4	Genotype and marker/interval profiling . . . . .	19
2.5	Miscellaneous additional R/qtl functions . . . . .	22
2.5.1	Genetic clones . . . . .	22
2.5.2	Breaking and merging linkage groups . . . . .	23
2.5.3	Rapid genetic distance estimation . . . . .	25
2.5.4	Subsetting in R/ASMap . . . . .	26
2.5.5	Combining maps . . . . .	27
<b>3</b>	<b>Worked example</b>	<b>29</b>
3.1	Pre-construction . . . . .	29
3.2	MSTmap construction . . . . .	33
3.3	Pushing back markers . . . . .	36
3.4	Post-construction linkage map development . . . . .	41
3.4.1	Combining two linkage maps of the same population . . . . .	42
3.4.2	Fine mapping . . . . .	42
3.4.3	Unknown linkage groups . . . . .	43
<b>4</b>	<b>Aspects of the MSTmap algorithm</b>	<b>44</b>

4.1	MSTmap and distance calculations . . . . .	44
4.2	Use of the argument <code>mvest.bc</code> . . . . .	47
4.3	Use of the argument <code>detectBadData</code> . . . . .	48
	<b>Bibliography</b>	<b>49</b>

# 1 Introduction

## 1.1 Background

Genetic linkage maps are widely used in the biological research community to explore the underlying DNA of populations. They generally consist of a set of polymorphic genetic markers spanning the entire genome of a population generated from a specific cross of parental lines. This exploration may involve the dissection of the linkage map itself to understand the genetic landscape of the population or, more commonly, it is used to conduct gene-trait associations such as quantitative trait loci (QTL) analysis or genomic selection (GS). For QTL analysis, the interpretation of significant genomic locations is enhanced if the linkage map contains markers that have been assigned and optimally ordered within chromosomal groups. This can be achieved algorithmically by using linkage map construction techniques that utilise assumptions of Mendelian genetics.

In the R statistical computing environment ([R Development Core Team, 2014](#)) there is only a handful of packages that can perform linkage map construction. A popular package is the linkage map construction and QTL analysis package R/qtl ([Broman & Wu, 2014](#)). Since its inception in late 2001 it has grown considerably and incorporates functionality for a wide set of populations. These include a simple Backcross (BC), Doubled Haploid (DH), intercrossed F2 (F2) and 4-way crosses. More recently, the authors have added functionality for additional populations generated from advanced Recombinant Inbred Lines (RIL) as well as populations generated from multiple backcrossing and selfing processes. The package offers an almost complete set of tools to construct, explore and manipulate genetic linkage maps for each population. To complement the package the authors have also published a comprehensive book ([Broman & Sen, 2009](#)). A more recent available addition to the contributed package list in R is R/onemap ([Margarido & Mollinari, 2014](#)). This package provides a suite of tools for genetic linkage map construction for BC, DH and RIL inbred populations and also has functionality for outcrossing populations.

Both packages contain functions that perform moderately well at clustering markers into homogeneous linkage groups. Unfortunately, the functions in R/qtl and R/onemap used to perform optimal marker ordering within linkage groups are tediously slow. In both packages there are functions that compute initial orders using non-exhaustive methods such as SERIATION ([Buetow & Chakravarti, 1987](#)) and RECORD ([Van Os et al., 2005](#)). As these methods may not return an optimal order, a second stage antiquated brute force

## 1.1 Background

---

approach is used that checks all order combinations within a marker window. This is known throughout the linkage map construction community as “rippling”. For linkage groups with large numbers of markers and a moderate sized window (eight markers), rippling can be very computationally cumbersome and also may need to be performed several times before an optimal order is reached.

In an attempt to circumvent these computational issues the R/ASMap package was developed (Taylor & Butler, 2014). The package contains linkage map construction functions that utilise the MSTmap algorithm derived in Wu et al. (2008) and computationally implemented as C++ source code freely available at <http://alumni.cs.ucr.edu/~yonghui/mstmap.html>. The algorithm uses the minimum spanning tree of a graph (Cheriton & Tarjan, 1976) to cluster markers into linkage groups as well as find the optimal marker order within each linkage group in a very computationally efficient manner. In contrast to R/qtl and R/onemap, genetic linkage maps are constructed using a one-stage approach. The algorithm is restricted to linkage map construction with Backcross (BC), Doubled Haploid (DH) and Recombinant Inbred (RIL) populations. For RIL populations, the level of self pollination can also be given and consequently the algorithm can handle selfed F2, F3, . . . , F $n$  populations where  $n$  is the level of selfing. Advanced RIL populations are also allowed as they are treated like a bi-parental for the purpose of linkage map construction.

The R/ASMap package uses an R/qtl format for the structure of its genetic objects. Once the class of the object is appropriately set, both R/ASMap and R/qtl functions can be used synergistically to construct, explore and manipulate the object. To complement the efficient MSTmap linkage map construction functions the R/ASMap package also contains a function that “pulls” markers of different types from the linkage map and places them aside. A complementary “push” function also exists to push the markers back to linkage groups ready for construction or reconstruction. There are also several numerical and graphical diagnostic tools to efficiently check the quality of the constructed map. This includes the ability to simultaneously graphically display multiple panel profiles of linkage map statistics for the set of genotypes used. Additionally, profile marker/interval statistics can be simultaneously displayed for the entire genome or subsetted to pre-defined linkage groups. In tandem, these fast graphical diagnostic tools and efficient linkage map construction assist in providing a rapid turnaround time in linkage map construction and diagnosis.

The vignette is set out in chapters. The final section of the introduction provides a brief technical explanation of the components of the MSTmap algorithm. In the second chapter the functions of R/ASMap are discussed in detail and examples are provided, where appropriate, using a data set integrated into the package. Chapter 3 contains a completely worked example of a barley Backcross population including pre-construction diagnostics, linkage map construction, and post-construction diagnostics using the available R/ASMap functions. The chapter also discusses how R/ASMap can be used for post construction linkage map improvement through techniques such as fine mapping or combining linkage maps. The last chapter presents some additional useful information on aspects of the MSTmap algorithm learnt through detailed exploration of the linkage map construction

## 1.2 MSTmap

---

functions in this package.

## 1.2 MSTmap

It is important to understand some of the technical features of the MSTmap algorithm as they are contained in the two linkage map construction functions that are available with the R/ASMap package.

Following the notation of [Wu et al. \(2008\)](#) consider a Doubled Haploid population of  $n$  individuals genotyped across a set of  $t$  markers where each  $(i, j)$ th entry of the  $n \times t$  matrix  $\mathbf{M}$  is either an  $A$  or a  $B$  representing the two parental homozygotes in the population. Let  $\mathbf{P}_{jk}$  be the probability of a recombination event between the markers  $(\mathbf{m}_j, \mathbf{m}_k)$  where  $0 \leq \mathbf{P}_{jk} < 0.5$ . MSTmap uses two possible weight objective functions based on recombination probabilities between the markers

$$w_p(j, k) = \mathbf{P}_{jk} \quad (1.1)$$

$$w_{ml}(j, k) = -(\mathbf{P}_{jk} \log \mathbf{P}_{jk} + (1 - \mathbf{P}_{jk}) \log(1 - \mathbf{P}_{jk})) \quad (1.2)$$

In general  $\mathbf{P}_{jk}$  is not known and so it is replaced by an estimate,  $d_{jk}/n$  where  $d_{jk}$  corresponds to the hamming distance between  $\mathbf{m}_j$  and  $\mathbf{m}_k$  (the number of non-matching alleles between the two markers). This estimate,  $d_{jk}/n$ , is also the maximum likelihood estimate for  $\mathbf{P}_{jk}$  for the two weight functions defined above.

### 1.2.1 Clustering

If markers  $\mathbf{m}_j$  and  $\mathbf{m}_k$  belong to two different linkage groups then  $\mathbf{P}_{jk} = 0.5$  and the hamming distance between them has the property  $E(d_{jk}) = n/2$ . A simple thresholding mechanism to determine whether markers belong to the same linkage group can be calculated using Hoeffdings inequality,

$$P(d_{jk} < \delta) \leq \exp(-2(n/2 - \delta)^2/n) \quad (1.3)$$

for  $\delta < 0.5$ . For a given  $P(d_{jk} < \delta) = \epsilon$  and  $n$ , the equation  $-2(n/2 - \delta)^2/n = \log \epsilon$  is solved to determine an appropriate hamming distance threshold,  $\hat{\delta}$ . [Wu et al. \(2008\)](#) indicate that the choice of  $\epsilon$  is not crucial when attempting to form linkage groups. However, the equation that requires solving is highly dependent on the number of individuals in the population. For example, for a DH population, Figure 1.1 shows the profiles of the  $-\log 10\epsilon$  against the number of individuals in the population for four threshold minimum cM distances (25, 30, 35, 40). MSTmap uses a default of  $\epsilon = 0.00001$  which would work universally well for population sizes of  $n \sim 150$  to 200. For larger numbers of individuals, for example 350, the plot indicates an  $\epsilon = 1.0e^{-12}$  to  $1.0e^{-15}$  would use a conservative minimum threshold of 30-35 cM before linking markers between clusters. If the default  $\epsilon = 0.00001$  is given in this instance this threshold is dropped to  $\sim 45$  cM and consequently distinct clusters of markers will appear linked. For this reason, Figure 1.1 should always initially be checked before linkage map construction to ensure an appropriate p-value is given to the MSTmap algorithm.

## 1.2 MSTmap

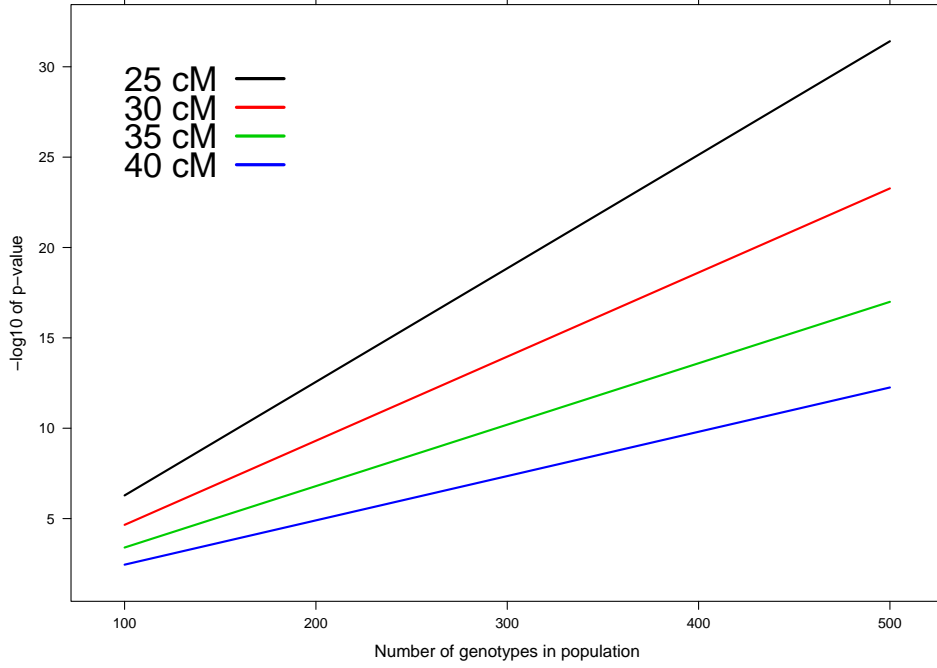


Figure 1.1: Negative log10  $\epsilon$  versus the number of genotypes in the population for four threshold cM distances.

To cluster the markers MSTmap uses an edge-weighted undirected complete graph, for  $\mathbf{M}$  where the individual markers are vertices and the edges between any two markers  $\mathbf{m}_j$  and  $\mathbf{m}_k$  is the pairwise hamming distance  $d_{jk}$ . Edges with weights greater than  $\hat{\delta}$  are then removed. The remaining connected components allow the marker set  $\mathbf{M}$  to be partitioned into  $r$  linkage groups,  $\mathbf{M} = [\mathbf{M}_1, \dots, \mathbf{M}_r]$ .

### 1.2.2 Marker Ordering

For simplicity, consider the  $n \times t$  matrix of markers  $\mathbf{M}$  belongs to the same linkage group. Preceding marker ordering, the markers are “binned” into groups where, within each group, the pairwise distance between any two markers is zero. The markers within each group have no recombinations between them and are said to be co-locating at the same genomic location for the  $n$  genotypes used to construct the linkage map. A representative marker is then chosen from each of the bins and used to form the reduced  $n \times t^*$  marker set  $\mathbf{M}^*$ .

For the reduced matrix  $\mathbf{M}^*$ , consider the complete set of entries  $(j, k) \in (1, \dots, t^*)$  for either weight function (1.1) or (1.2). These complete set of entries can be viewed as the upper triangle of a symmetric weight matrix  $\mathbf{W}$ . MSTmap views all these entries as being connected edges in an undirected graph where the individual markers are vertices. A marker order for the set  $\mathbf{M}^*$ , also known as a travelling salesman path (TSP), can be determined by visiting each marker once and summing the weights from the connected edges. To find a minimum weight ( $\text{TSP}_{\min}$ ), MSTmap uses a minimum spanning tree

## 1.2 MSTmap

(MST) algorithm (Cheriton & Tarjan, 1976), such as Prim's algorithm (Prim, 1957). If the  $TSP_{min}$  is unique then the MST is the correct order for the markers. For cases where the data contains genotyping errors or lower numbers of individuals the MST may not be a complete path and contain markers or small sets of markers as individual nodes connected to the path. In these cases, MSTmap uses the longest path in the MST as the backbone and employs several efficient local optimization techniques such as K-opt, node-relocation and block-optimize (see Wu et al., 2008) to improve the current minimum TSP. By integrating these local optimization techniques into the algorithm, MSTmap provides users with a true one stage marker ordering algorithm.

One excellent feature of the MSTmap algorithm is the utilisation of an EM type algorithm for the imputation of missing allele scores that is tightly integrated with the ordering algorithm for the markers. To achieve this the marker matrix  $\mathbf{M}^*$  is converted to a matrix,  $\mathbf{A}$ , where the entries represent the probabilistic certainty of the allele being A. For the  $j$ th marker and  $i$ th individual then

$$\mathbf{A}(i, j) = \begin{cases} 1 & \text{if } \mathbf{M}^*(i, j) \text{ is the A allele} \\ 0 & \text{if } \mathbf{M}^*(i, j) \text{ is the B allele} \\ \frac{(1-\hat{\mathbf{P}}_{j-1,j})(1-\hat{\mathbf{P}}_{j,j+1})}{(1-\hat{\mathbf{P}}_{j,j+1})(1-\hat{\mathbf{P}}_{j-1,j})+\hat{\mathbf{P}}_{j-1,j}\hat{\mathbf{P}}_{j,j+1}} & \text{if } \mathbf{M}^*(i, j) \text{ is missing} \end{cases} \quad (1.4)$$

where  $\hat{\mathbf{P}}_{j,j-1}$  and  $\hat{\mathbf{P}}_{j,j+1}$  are estimated recombination fractions between the  $(j-1)$ th and  $j$ th marker and  $j$ th and  $(j+1)$ th marker respectively. The equation on the right hand side is the posterior probability of the missing value in marker  $j$  being the A allele for genotype  $i$  given the current estimate. The ordering algorithm begins by initially calculating pairwise normalized distances between all markers in  $\mathbf{M}^*$  and deriving an initial weight matrix,  $\mathbf{W}$ . An undirected graph is formed using the markers as vertices and the upper triangular entries of  $\mathbf{W}$  as connected edges. An MST of the undirected graph is then found to establish an initial order for the markers of the linkage group. For the current order at the  $(j-1, j, j+1)$ th markers the E-step of algorithm requires updating the missing observation at marker  $j$  by updating the estimates  $\hat{\mathbf{P}}_{j-1,j} = \hat{d}_{j-1,j}/n$  and  $\hat{\mathbf{P}}_{j,j+1} = \hat{d}_{j,j+1}/n$  in (1.4). The M-step then re-estimates the pairwise distances between all markers in  $\mathbf{M}^*$  where, for the  $j$ th and  $k$ th marker, is

$$\hat{d}_{jk} = \sum_{i=1}^{t^*} \mathbf{A}(i, j)(1 - \mathbf{A}(i, k)) + \mathbf{A}(i, k)(1 - \mathbf{A}(i, j)) \quad (1.5)$$

and the weight matrix  $\mathbf{W}$  is recalculated. An undirected graph is formed with the markers as vertices and the upper triangular entries of  $\mathbf{W}$  as connected edges. A new order of the markers is derived by obtaining an MST of the undirected graph and the algorithm is repeated to convergence. Although this requires several iterations to converge, the computational time for the ordering algorithm remains expedient. However, an increase in the number of missing values will increase computation time.

If required, the MSTmap algorithm also detects and removes genotyping errors as well as integrates this process into the ordering algorithm. The technique involves using a



## 1.2 MSTmap

---

weighted average of nearby markers to determine the expected state of the allele. For individual  $i$  and marker  $j$  the expected value of the allele is calculated using

$$E[\mathbf{A}(i, j)] = \sum_{j \neq k} d_{j,k}^{-2} \mathbf{A}(i, k) \Big/ \sum_{j \neq k} d_{j,k}^{-2} \quad (1.6)$$

In this equation the weights are the inverse square of the distance from marker  $j$  to its nearby markers. MSTmap only uses a small set of nearby markers during each iteration and the observed allele is considered suspicious if  $|E[\mathbf{A}(i, j)] - \mathbf{A}(i, j)| > 0.75$ . If an observation is detected as suspicious it is treated as missing and imputed using the EM algorithm discussed previously. The removal of the suspicious allele has the effect of reducing the number of recombinations between the marker containing the suspicious observation and the neighbouring markers. This has an influential effect on the genetic distance between markers and the overall length of the linkage group.

The complete algorithm used to initially cluster the markers into linkage groups and optimally order markers within each linkage group, including imputing missing alleles and error detection, is known as the MSTmap algorithm.

## 2 A closer look at R/ASMap functions

This chapter explores the R/ASMap functions in greater depth and shows how they can be used to efficiently explore, manipulate and construct genetic linkage maps. It will also showcase the graphical tools that will allow efficient diagnosis of linkage map problems post construction.

The package contains multiple data sets listed as follows

**mapDH** : A constructed genetic linkage map for a Doubled Haploid population in the form of an R/qlt object. The genetic linkage map contains a total of 599 markers spanning 23 linkage groups genotyped across 218 individuals. The linkage map contains a small set of co-located markers and a small set of markers with excessive segregation distortion

**mapDHf** : An unconstructed version of **mapDH** in the form a data frame. The data frame has dimensions  $599 \times 218$  and the rows (markers) have been randomized.

**mapBCu** : An unconstructed set of markers for a backcross population in the form of an R/qlt object. The marker set contains a total of 3023 markers genotyped on 326 individuals. This marker set can be used in conjunction with the detailed process presented in chapter 3 to construct a genetic linkage map.

**mapBC** : A constructed linkage map of **mapBCu** in the form of an R/qlt object. The linkage map contains a total of 3019 markers genotyped on 300 individuals.

**mapF2** : A simulated linkage map for a self-pollinated F2 population consisting of 700 markers spanning 7 linkage groups genotyped across 250 individuals.

Each of these data sets is accessible using commands similar to

```
R> data(mapDHf, package = "ASMap")
R> data(mapDH, package = "ASMap")
R> data(mapBCu, package = "ASMap")
```

## 2.1 Map construction functions

---

## 2.1 Map construction functions

The R/ASMap package contains two linkage map construction functions that allow users to fully utilize the MSTmap parameters listed at <http://alumni.cs.ucr.edu/~yonghui/mstmap.html>. Some additional information on aspects of the MSTmap algorithm and the appropriate use of the arguments `mvest.bc` and `detectBadData` is given in Chapter 4.

### 2.1.1 `mstmap.data.frame()`

The first of these functions allows users to input a data frame of genetic markers ready for construction. For a more detailed explanation of the arguments users should consult the help documentation found by typing `?mstmap.data.frame` in R.

```
mstmap.data.frame(object, pop.type = "DH", dist.fun = "kosambi", objective.fun  
  = "COUNT", p.value = 1e-06, noMap.dist = 15, noMap.size = 0, miss.thresh = 1,  
  mvest.bc = FALSE, detectBadData = FALSE, as.cross = TRUE, return.imputed =  
  TRUE, trace = FALSE, ...)
```

The explicit form of the data frame `object` is borne from the syntax of the marker file required for using the stand alone MSTmap software. It must have markers in rows and genotypes in columns. Marker names are required to be in the `rownames` component of the object with genotype names residing in the `names`. Spaces in any of the marker or genotype names should be avoided but will be replaced with a “-” if found. Each of the columns of the data frame must be of class “character” (not factors). If converting from a matrix, this can easily be achieved by using the `stringAsFactors = FALSE` argument for any `data.frame` method.

The available populations that can be passed to `pop.type` are “BC” Backcross, “DH” Doubled Haploid, “ARIL” Advanced Recombinant Inbred and “RILn” Recombinant Inbred with n levels of selfing. The allelic content of the markers in the `object` must be explicitly adhered to. For `pop.type` “BC”, “DH” or “ARIL” the two allele types should be represented as (“A” or “a”) and (“B” or “b”). Thus for `pop.type = “ARIL”` it is assumed the minimal number of heterozygotes have been set to missing. For non-advanced RIL populations (`pop.type = “RILn”`) phase unknown heterozygotes should be represented as “X”. For all populations, missing marker scores should be represented as (“U” or “-”).

Users need to be aware that the `p.value` argument plays an important role in determining the clustering of markers to distinct linkage groups. Section 1.2.1 shows the separation of marker groups is highly dependent on the the number of individuals in the population. For this reason, some trial and error may be required to determine an appropriate `p.value` for the linkage map being constructed.

Although this function contains arguments that utilize the complete set of available MSTmap parameters it is less flexible than its sister function `mstmap.cross()` (see section 2.1.2) that uses the flexible structure of an R/qlt “cross” object. For this reason,

## 2.1 Map construction functions

---

it is recommended that users set `as.cross = TRUE` to ensure the constructed object is returned as a R/qrtl cross object with an appropriate class structure. For population types "BC" and "DH" the class of the constructed object is given "bc" and "dh" respectively. For "RILn" the `qrtl` package conversion function `convert2bcsft` is used to ensure the class of the object is assigned "bcsft" with arguments `F.gen = n` and `BC.gen = 0`. For "ARIL" populations the constructed object is given the class "riself". The correct assignation of these classes ensures the objects can be used synergistically with the suite of functions available in the R/qrtl package as well as other functions in the R/ASMap package.

The R/ASMap package contains an unconstructed Doubled Haploid marker set `mapDHf` with 599 markers genotyped across 218 individuals. The marker set is formatted correctly for input into the `mstmap.data.frame()` function.

```
R> testd <- mstmap(mapDHf, dist.fun = "kosambi", trace = TRUE, as.cross = TRUE)
R> nmar(testd)
```

```
  L1  L2  L3  L4  L5  L6  L7  L8  L9 L10 L11 L12 L13 L14 L15 L16 L17 L18 L19 L20
56  54  35  41   4  37   6  30  40  27  37  13  15  10  21   6  32  33  33  41
L21 L22 L23 L24
   6   9   5   8
```

```
R> chrrlen(testd)
```

```
      L1      L2      L3      L4      L5      L6      L7
142.806369 181.935011 97.376797 115.405242 21.660902 108.162200 13.256931
      L8      L9      L10     L11     L12     L13     L14
133.885978 153.235513 98.705941 145.839837 60.441336 72.211612 81.224002
      L15     L16     L17     L18     L19     L20     L21
98.211685   8.738474 109.472813 60.913640 134.491476 103.954898 18.019891
      L22     L23     L24
7.801089   7.810474 19.196583
```

As `as.cross = TRUE` the usual functions available in the R/qrtl package are available for use on the returned object.

### 2.1.2 mstmap.cross()

The second linkage map construction function allows users to input an unconstructed or constructed linkage map in the form of an R/qrtl cross object. See `?mstmap.cross` for a more detailed description.

```
mstmap.cross(object, chr, id = "Genotype", bychr = TRUE, suffix = "numeric",
             anchor = FALSE, dist.fun = "kosambi", objective.fun = "COUNT", p.value = 1e-06,
```

## 2.1 Map construction functions

---

```
noMap.dist = 15, noMap.size = 0, miss.thresh = 1, mvest.bc = FALSE,  
detectBadData = FALSE, return.imputed = FALSE, trace = FALSE, ...)
```

The cross `object` needs to inherit from one of the allowable classes available in the R/qtl package, namely "bc", "dh", "riself", "bcsft" where "bc" is a Backcross "dh" is Doubled Haploid, "riself" is an advanced Recombinant Inbred and "bcsft" is a Backcross/Self.

It is important to understand how these classes are encoded into the object for the specific populations. If `read.cross()` is used to read in any bi-parental populations it will be given the class "bc". Doubled Haploid populations can be changed to "dh" just by changing the class. For the purpose of linkage map construction, both classes "bc" and "dh" will produce equivalent results. For non-advanced Recombinant Inbred populations markers are required to be fully informative (i.e. contain 3 distinct allele types such as AA, BB for parental homozygotes and AB for phase unknown heterozygotes) and the use of `read.cross()` will result in the cross object being given a class "f2". The level of selfing is required to be encoded into the object by applying one of the two conversion functions available in the R/qtl package. For a population that has been generated by selfing  $n$  times, the conversion function `convertbcsft` can be used by setting the arguments `F.gen = n` and `BC.gen = 0`. This will attach a class "bcsft" to the object. Populations that are genuine advanced RILs can be converted using the `convert2riself` function. This function will replace any remaining heterozygosity, if it exists, with missing values and attach the class "riself" to the object.

Similar to the `mstmap.data.frame()` function, users need to be aware that the `p.value` argument is highly dependent on the number of individuals in the population and may require some trial and error to ascertain an appropriate value. After construction the cross object is returned with an identical class structure as the inputted object. All R/qtl and R/ASMap package functions can be used synergistically with this object.

### Examples

The constructed linkage map `mapDH` available in the R/ASMap package will be used to showcase the flexibility of this function. Before attempting re-construction, some preliminary output of `mapDH` is presented.

```
R> nmar(mapDH)
```

```
1A 1B1 1B2 1D 2A 2B 2D1 2D2 3A 3B 3D 4A 4B 4D 5A 5B 5D 6A 6B 6D  
41 5 33 10 40 35 8 13 37 30 6 30 32 6 54 56 6 27 41 15  
7A 7B 7D  
33 37 4
```

```
R> pull.map(mapDH)[[4]]
```

## 2.1 Map construction functions

---

```
1D.m.1 1D.m.2 1D.m.3 1D.m.4 1D.m.5 1D.m.6 1D.m.7 1D.m.8
0.000000 2.285816 2.742698 2.742698 21.465574 25.652893 57.200980 58.104090
1D.m.9 1D.m.10
58.568087 81.061896
attr(,"class")
[1] "A"
```

The output shows that there are 23 groups that have been appropriately been assigned linkage group or chromosome names. The markers within each linkage group have been named according to the order of the markers.

### Example 1: Completely construct or reconstruct a linkage map.

To completely re-construct this map set the argument `bychr = FALSE`. This will bulk the genetic data from all linkage groups, re-cluster the markers into groups and then optimally order the markers within each linkage group. This linkage map is small so these two tasks happen almost instantaneously.

```
R> mapDHa <- mstmap(mapDH, bychr = FALSE, dist.fun = "kosambi", trace = TRUE)
R> nmar(mapDHa)
```

```
L.1 L.10 L.11 L.12 L.13 L.14 L.15 L.16 L.17 L.18 L.19 L.2 L.20 L.21 L.22 L.23
41 30 6 9 21 32 6 54 56 6 27 5 41 15 33 37
L.24 L.3 L.4 L.5 L.6 L.7 L.8 L.9
4 33 10 40 35 8 13 37
```

```
R> pull.map(mapDHa)[[4]]
```

```
4A.m.9 4A.m.7 4A.m.8 4A.m.6 4A.m.5 4A.m.3 4A.m.4 4A.m.2
0.000000 1.835687 1.835687 2.294415 2.753144 3.670678 4.129406 6.424595
4A.m.1
7.801089
attr(,"class")
[1] "A"
```

The reconstructed map contains 24 linkage groups with the extra linkage group coming from a minor split in 4A. As the linkage map is constructed from scratch, it assumed that former linkage group names are no longer required. A standard “L.” prefix is provided for the new linkage group names. This example also indicates that MSTmap by default does not respect the inputted marker order.

### Example 2: Optimally order markers within linkage groups of an established map.

## 2.1 Map construction functions

---

It may be necessary to only perform marker ordering within already established linkage groups. This can be achieved by setting `bychr = TRUE`. In some cases it also be preferable to ensure the marker orders of the linkage groups are respected and this can be achieved by setting `anchor = TRUE`.

```
R> mapDHb <- mstmap(mapDH, bychr = TRUE, dist.fun = "kosambi", anchor = TRUE,
  trace = TRUE)
R> nmar(mapDHb)
```

1A	1B1	1B2	1D	2A	2B	2D1	2D2	3A	3B	3D	4A.1	4A.2	4B	4D	5A
41	5	33	10	40	35	8	13	37	30	6	9	21	32	6	54
5B	5D	6A	6B	6D	7A	7B	7D								
56	6	27	41	15	33	37	4								

This map is identical to `mapDH` with the exception that chromosome 4A has been split into two linkage groups. As `bychr = TRUE` the function understands the origin of the linkage group was 4A and consequently uses it as a prefix in the naming of the two new linkage groups.

### Example 3: Optimally order markers within linkage groups of an established map without breaking linkage groups.

The splitting of the linkage groups in the last example only occurred due to choice of default `p.value = 1e-06` set in the function. A slight change to this `p.value` will ensure that 4A remains linked during the algorithm.

```
R> mapDHc <- mstmap(mapDH, bychr = TRUE, dist.fun = "kosambi", anchor = TRUE,
  trace = TRUE, p.value = 1e-04)
R> nmar(mapDHc)
```

1A	1B1	1B2	1D	2A	2B	2D1	2D2	3A	3B	3D	4A	4B	4D	5A	5B	5D	6A	6B	6D
41	5	33	10	40	35	8	13	37	30	6	30	32	6	54	56	6	27	41	15
7A	7B	7D																	
33	37	4																	

An identical result can be achieved by setting `p.value = 2` or any number greater than 1. Doing this instructs the MSTmap algorithm to not split linkage groups regardless of how weak the linkages are between markers within any group. Users should be aware that if this latter method is used then linkage groups that contain groups of markers separated by a substantial distance (i.e. very weak linkages) may suffer from local orientation problems.

### Example 4: Reconstruct map within predefined linkage groups of an established map.

## 2.2 Pulling and pushing markers

---

There may only be a need to reconstruct a predefined set of linkage groups. By setting the `chr` argument and `bychr = FALSE` users can determine which linkage groups require the complete reconstruction using `MSTmap`.

```
R> mapDHd <- mstmap(mapDH, chr = names(mapDH$geno)[1:3], bychr = FALSE, dist.fun =  
  "kosambi", trace = TRUE, p.value = 1e-04)  
R> nmar(mapDHd)
```

```
1D  2A  2B 2D1 2D2  3A  3B  3D  4A  4B  4D  5A  5B  5D  6A  6B  6D  7A  7B  7D  
10  40  35   8  13  37  30   6  30  32   6  54  56   6  27  41  15  33  37   4  
L.1 L.2 L.3  
41   5  33
```

Again, the algorithm assumes that the original linkage group names are no longer necessary and defines new ones. An obvious extension of this example is to set `bychr = TRUE` and then the algorithm will order markers within the predefined linkage groups stipulated by `chr`. Similar to the previous example, an appropriate choice of `p.value` will ensure that linkage groups remain unbroken.

## 2.2 Pulling and pushing markers

Often in linkage map construction, some pruning of the markers occurs before initial construction. This may be the removal of markers with a proportion of missing values higher than some desired threshold as well as markers that are significantly distorted from their expected Mendelian segregation patterns. Often the removal is permanent and the possible importance of some of these markers is overlooked. A preferable system would be to identify and place the problematic markers aside with the intention of checking their usefulness at a later stage of the construction process.

The `R/ASMap` package contains two functions that allow you to do this. The first is the function `pullCross()` which provide users with a mechanism to “pull” markers of certain types from a linkage map and place them aside. The complementary function `pushCross()` then allows users to “push” markers back into the linkage map at any stage of the construction process. These can be seen as helper functions for more efficient construction of linkage maps (see `?pullCross` and `?pushCross` for complete details).

```
pullCross(object, chr, type = c("co.located", "seg.distortion", "missing"),  
  pars = NULL, replace = FALSE, ...)  
pushCross(object, chr, type = c("co.located", "seg.distortion", "missing",  
  "unlinked"), unlinked.chr = NULL, pars = NULL, replace = FALSE, ...)  
pp.init(seg.thresh = 0.05, seg.ratio = NULL, miss.thresh = 0.1, max.rf = 0.25,  
  min.lod = 3)
```



## 2.2 Pulling and pushing markers

---

In the current version of the package, the helper functions share three types of markers that can be “pulled/pushed” from linkage maps. These include markers that are co-located with other markers, markers that have some defined segregation distortion and markers with a defined proportion of missing values. If the argument `type` is `"seg.distortion"` or `"missing"` then the initialization function `pp.init` is used to determine the appropriate threshold parameter setting (`seg.thresh`, `seg.ratio`, `miss.thresh`) that will be used to pull/push markers from the linkage map. Users can set their own parameters by setting the `pars` argument (see examples below). For each of the different types, `pullCross()` will pull markers from the map and place them in separate elements of the cross object. Within the elements, vital information is kept that can be accessed by `pushCross()` to push the markers back at a later stage of linkage map construction.

The function `pushCross()` also contains another marker type called `"unlinked"` which, in conjunction with the argument `unlinked.chr`, allows users to push markers from an unlinked linkage group in the `geno` element of the `object` into established linkage groups. This mechanism becomes vital, for example, when pushing new markers into an established linkage map. An example of this is presented in section [3.4.3](#).

Again, the constructed linkage map `mapDH` will be used to showcase the power of `pullCross()` and `pushCross()`. Markers are pulled from the map that are co-located with other markers, have significant segregation distortion with a p-value less than 0.02 and have a missing value proportion greater than 0.03.

```
R> mapDHs <- pullCross(mapDH, type = "co.located")
R> mapDHs <- pullCross(mapDHs, type = "seg.distortion", pars = list(seg.thresh =
  0.02))
R> mapDHs <- pullCross(mapDHs, type = "missing", pars = list(miss.thresh = 0.03))
R> names(mapDHs)
```

```
[1] "geno"          "pheno"          "co.located"     "seg.distortion"
[5] "missing"
```

```
R> names(mapDHs$co.located)
```

```
[1] "table" "data"
```

The cross object now contains three new elements named by the marker types that are pulled from the map. In each of the elements there are two elements, a table of information for the markers that are pulled and the actual marker data in genotype by marker format (i.e. exactly the same as the data contained in the linkage groups themselves).

```
R> mapDHs$seg.distortion$table
```

## 2.2 Pulling and pushing markers

---

	mark	chr	pos	neglog10P	missing	AA	AB
1	1A.m.34	1A	77.35004	1.830929	0.000000000	0.5825688	0.4174312
2	1A.m.37	1A	91.14416	1.830929	0.000000000	0.5825688	0.4174312
3	3B.m.15	3B	64.19254	1.997307	0.000000000	0.5871560	0.4128440
4	3B.m.16	3B	64.65088	2.170970	0.000000000	0.5917431	0.4082569
5	3B.m.17	3B	65.10962	1.872327	0.027522936	0.5849057	0.4150943
6	3B.m.18	3B	65.10962	1.997307	0.000000000	0.5871560	0.4128440
7	3B.m.19	3B	65.56831	1.830929	0.000000000	0.5825688	0.4174312
8	3B.m.20	3B	66.48546	1.830929	0.000000000	0.5825688	0.4174312
9	6D.m.12	6D	60.69446	1.756872	0.004587156	0.4193548	0.5806452
10	7B.m.6	7B	23.03041	1.769873	0.013761468	0.4186047	0.5813953

The table element of each of the marker types "missing" and "seg.distortion" consists of a summary of positional information as well as information from `geno.table()` in the R/qtl package.

```
R> head(mapDHs$co.located$table)
```

	bins	chr	mark
1	1	1B1	1B1.m.4
2	1	1B1	1B1.m.5
3	2	1D	1D.m.3
4	2	1D	1D.m.4
5	3	2B	2B.m.31
6	3	2B	2B.m.32

The table element for the marker type "co.located" contains information on the markers that are co-located and the group or bin they belong to. In each group the first marker is the reference marker that remains in the linkage map and the remaining markers are pulled from the map and placed aside.

Suppose now the map is undergone a construction or re-construction process so that the linkage groups are artificially renamed. To do this we will re-run MSTmap with `bychr = FALSE`.

```
R> mapDHs <- mstmap(mapDHs, bychr = FALSE, dist.fun = "kosambi", trace = TRUE,
  anchor = TRUE)
R> nmar(mapDHs)
```

L.1	L.10	L.11	L.12	L.13	L.14	L.15	L.16	L.17	L.18	L.19	L.2	L.20	L.21	L.22	L.23
38	23	5	8	21	32	6	52	55	5	27	4	41	13	32	34
L.24	L.3	L.4	L.5	L.6	L.7	L.8	L.9								
3	31	9	37	34	7	13	35								

## 2.2 Pulling and pushing markers

---

The markers can now be pushed back into the linkage map using `pushCross()`. The complete set of co-located markers are pushed back as well as markers that have significant segregation distortion with p-values greater than 0.001 and markers that have a missing value proportion less than 0.05.

```
R> mapDHs <- pushCross(mapDHs, type = "co.located")
R> mapDHs <- pushCross(mapDHs, type = "seg.distortion", pars = list(seg.thresh =
  0.001))
R> mapDHs <- pushCross(mapDHs, type = "missing", pars = list(miss.thresh = 0.05))
R> names(mapDHs)
```

```
[1] "geno" "pheno"
```

With the above parameter settings all markers from each marker type are pushed back into the map and the marker type elements are removed from the `object`.

```
R> pull.map(mapDHs)[[4]]
```

```
  4A.m.1  4A.m.2  4A.m.4  4A.m.3  4A.m.5  4A.m.6  4A.m.7  4A.m.8
0.000000 1.376494 3.671683 4.130412 5.047946 5.506674 5.965403 5.965403
  4A.m.9
7.801089
attr(,"class")
[1] "A"
```

```
R> pull.map(mapDHs)[[21]]
```

```
  2B.m.1  2B.m.2  2B.m.3  2B.m.4  2B.m.5  2B.m.6  2B.m.7  2B.m.8
0.000000 2.779284 25.847324 26.306053 31.833042 41.112443 49.445632 51.281319
  2B.m.9  2B.m.10  2B.m.11  2B.m.12  2B.m.13  2B.m.14  2B.m.15  2B.m.16
51.740047 52.198776 61.004291 61.921825 63.757512 65.134006 65.592735 66.051463
  2B.m.17  2B.m.18  2B.m.19  2B.m.20  2B.m.21  2B.m.22  2B.m.23  2B.m.24
66.510191 67.427726 68.345260 69.721754 70.180483 71.556977 72.933472 73.392200
  2B.m.25  2B.m.26  2B.m.27  2B.m.28  2B.m.29  2B.m.30  2B.m.31  2B.m.32
75.227887 75.686615 76.604149 77.521684 77.980412 78.439141 78.897869 78.897869
  2B.m.34  2B.m.33  2B.m.35
80.274363 80.733092 97.376797
attr(,"class")
[1] "A"
```

For co-located markers the reference marker for each group is used as a guide to place the set of co-locating markers back into the linkage map adjacent to the reference marker. For

## 2.3 Improved heat map

---

example, the co-locating marker 1D.m.4 on 1D appears adjacent to its reference marker 1D.m.3. Markers from the "seg.distortion" and "missing" elements are pushed back to the end of the linkage groups ready for the map to be re-constructed by chromosome. For example the distorted marker 6D.m.12 is pushed to the end of 6D. A final run of MSTmap within each linkage group will produce the desired map with all markers in their optimal position.

```
R> mapDHs <- mstmap(mapDHs, bychr = TRUE, dist.fun = "kosambi", trace = TRUE,  
  anchor = TRUE, p.value = 2)
```

## 2.3 Improved heat map

A visual diagnostic that is very useful for checking how well a linkage map is constructed is a heat map that combines the estimates of the pairwise recombination fraction (RF) between markers as well as LOD scores reflecting the strength of linkage between each pair. Let the estimate of RF between marker  $i$  and  $j$  be  $r_{ij}$  then the LOD score is a test of no linkage ( $r_{ij} = 0.5$ ). Higher LOD scores indicate that the hypothesis of no linkage is rejected and the strength of the connection between the pair of markers is strong.

In R/qtl users can plot the heat map of pairwise linkage between markers using `plot.rf()`. The `plot.rf()` version of `mapDH` is given in Figure 2.1. In this plot strong pairwise linkages between markers are represented as “hot” or red areas and weak pairwise linkages between markers are displayed as “cold” or blue areas.

Unfortunately the plot contains some inadequacies that are mostly pointed out in the documentation (see `?plot.rf`). “Recombination fractions are transformed by  $-4(\log_2(r)+1)$  to make them on the same sort of scale as LOD scores. Values of LOD or the transformed recombination fraction that are above 12 are set to 12.” This transformation and the arbitrary threshold induce multiple restrictions on the visualization of the heat map. Firstly, the LOD score is highly dependent on the number of individuals in the population and the upper limit of 12 may be a vastly inadequate representation of the pairwise linkage between some markers. In Figure 2.1 this latter inadequacy is displayed as red or hot areas of linkage between markers that should otherwise be cooler colours. Additionally, the actual estimated recombination fractions are not displayed in the upper triangle of the heat map and colour legends matching the numerical LOD and RF’s are not present. From a visual standpoint, it is well known the rainbow colour spectrum is perceptually inadequate for representing diverging numerical data.

R/ASMap contains an improved version of the heat map called `heatMap()` that rectifies the numerical and visual issues of the `qtl` heat map.

```
heatMap(x, chr, mark, what = c("both", "lod", "rf"), lmax = 12, rmin = 0,  
  markDiagonal = FALSE, color = rev(rainbow(256, start = 0, end = 2/3)), ...)
```

## 2.3 Improved heat map

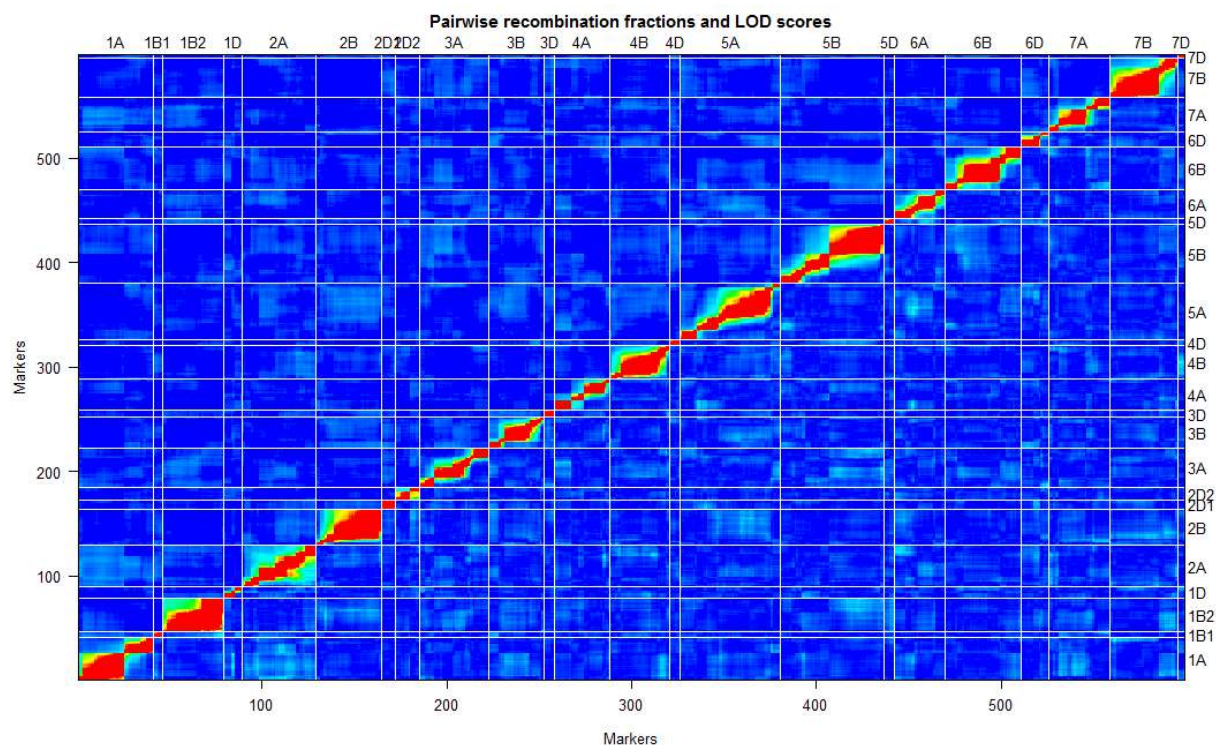


Figure 2.1: Heat map of the constructed linkage map `mapDH` using `plot.rf()`.

The function independently plots the LOD score on the bottom triangle of the heatmap as well as the actual estimated recombination fractions on the upper triangle. The function also releases the arbitrary threshold on the LOD score and recombination fractions by providing a user defined `lmax` and `rmin` argument that is adjustable to suit the population size and any requirements of the plot required. For example, Figure 2.2 shows the `heatMap()` equivalent of `plot.rf()` for the linkage map `mapDH` made with the call

```
R> heatMap(mapDH, lmax = 50)
```

The instantly visual difference between the figures is the chosen colour palette. R/ASMap uses a diverging colour palette "Spectral" from the default color palettes of **RColorBrewer**. The palette softens the plot and provides a gentle distinction between strongly linked and weakly linked markers. The plot also includes a legend for the LOD score and recombination fractions on either side of Figure 2.2. As the estimated recombination fractions are being freely plotted without transformation, the complete scale is included in the heat map. This scale also includes estimated recombination fractions that are above the theoretical threshold of 0.5. By increasing this scale beyond 0.5, potential regions where markers out of phase with other markers can be recognised. The key to obtaining an "accurate" heat map is to match the heat of the LOD scores to the heat of the estimated recombination fractions and setting `lmax = 50` achieves this goal. Similar to `plot.rf()`, the `heatMap()` function allows subsetting of the linkage map by `chr`. In ad-

## 2.4 Genotype and marker/interval profiling

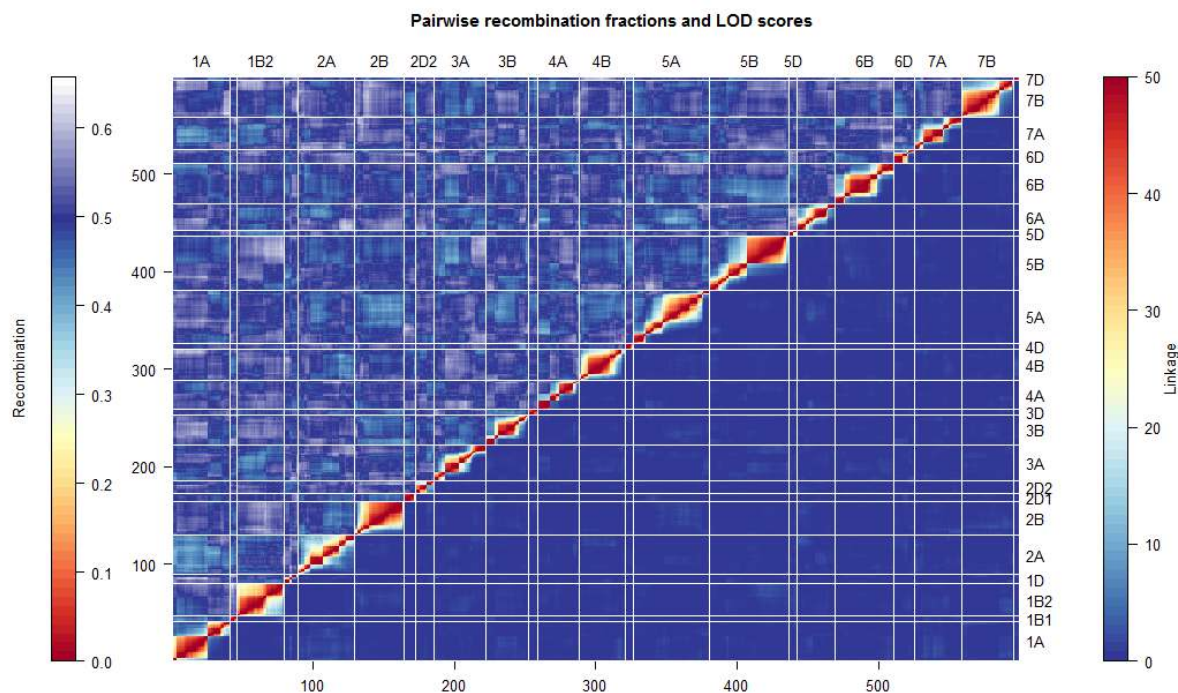


Figure 2.2: Heat map of the constructed linkage map mapDH using `heatMap()`.

dition, users can further subset the linkage groups using the argument `mark` by indexing a set of markers within linkage groups defined by `chr`.

## 2.4 Genotype and marker/interval profiling

To provide a complete system for efficient linkage map construction and diagnosis the R/ASMap package contains functions that calculate linkage map statistics across the markers for every genotype as well as across the markers/intervals of the genome. These statistics can then be profiled across simultaneous R/lattice panels for quick diagnosis of linkage map attributes.

```
statGen(cross, chr, bychr = TRUE, stat.type = c("xo", "dxo", "miss"), id =  
  "Genotype")  
profileGen(cross, chr, bychr = TRUE, stat.type = c("xo", "dxo", "miss"), id =  
  "Genotype", xo.lambda = NULL, ...)  
statMark(cross, chr, stat.type = c("marker", "interval"), map.function =  
  "kosambi")  
profileMark(cross, chr, stat.type = "marker", use.dist = TRUE, map.function =  
  "kosambi", crit.val = NULL, display.markers = FALSE, mark.line = FALSE, ...)
```

The `statGen()` and `profileGen()` are functions for calculation and profiling of the statistics for the genotypes across the chosen marker set defined by `chr`. Specifically,



## 2.4 Genotype and marker/interval profiling

---

`profileGen()` actually calls `statGen()` to obtain the statistics for profiling and also returns the statistics invisibly after plotting. Setting `bychr = TRUE` will also ensure that the genotype profiles are plotted individually for each linkage groups given `chr`. The current statistics that can be calculated for each genotype include

- `"xo"` : number of crossovers
- `"dxo"` : number of double crossovers
- `"miss"` : number of missing values

The two statistics `"xo"` and `"dxo"` are obviously only useful for constructed linkage maps. However, from my experience, they represent the most vital two statistics for determining a linkage maps quality. Inflated crossover or double crossover rates of any genotypes indicate a lack of adherence to Mendelian genetics. For example, it can be quickly calculated for a Doubled Haploid wheat population of size 100, one recombination is approximately 1cM. Consequently, under Mendelian genetics, a chromosome of length 200 cM would expect to have 200 random crossovers with each genotype having an expected recombination rate of 2. Wheat is a hexaploid, so for excellent coverage over the whole genome the expected recombination rate of any genotype is  $\sim 42$ . This number will obviously be reduced if the coverage of the genome is incomplete. Significant recombination rates can be checked by manually inputting a median recombination rate in the argument `xo.lambda` of `profileGen()`. As an example, the genotype profiles of the `mapDH` can be displayed using the command below and the resultant plot is given in Figure 2.3.

```
R> profileGen(mapDH, bychr = FALSE, stat.type = c("xo", "dxo", "miss"), id =  
  "Genotype", xo.lambda = 25, layout = c(1, 3), lty = 2)
```

The plot neatly displays, the number of missing values, double crossovers and crossovers for each genotype in the order represented in `mapDH`. The plot was aesthetically enhanced by including graphical arguments `layout = c(1,3)` and `lty = 2` which are passed to the high level lattice function `xyplot()`. Plotting these statistics simultaneously allows the users to quickly recognize genotypes that may be problematic and worth further investigation. For example, in Figure 2.3 the line DH218 was identified as having an inflated recombination rate and removing it may improve the quality of the linkage map.

The `statMark()` and `profileMark()` functions are commands for the calculation and profiling of statistics associated with the markers or intervals of the linkage map. `profileMark()` calls `statMark()` and graphically profiles user given statistics and will also return them invisibly after plotting. The current marker statistics that can be profiled are

- `"seg.dist"`:  $-\log_{10}$  p-value from a test of segregation distortion
- `"miss"`: proportion of missing values
- `"prop"`: allele proportions

## 2.4 Genotype and marker/interval profiling

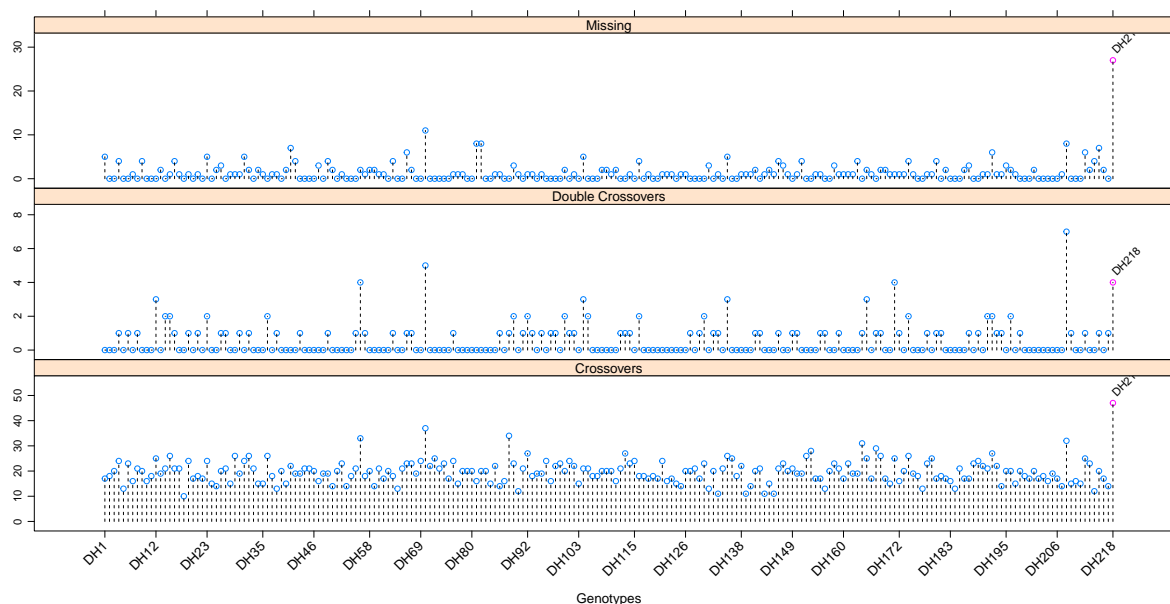


Figure 2.3: Genotype profiles of missing values, double recombinations and recombinations for mapDH.

- "dxo": number of double crossovers

The current interval statistics that can be profiled are

- "erf": estimated recombination fractions
- "lod": LOD score for the test of no linkage
- "dist": interval map distance
- "mrf": map recombination fraction
- "recomb": number of recombinations.

The function allows any or all of the statistics to be plotted simultaneously on a multi-panel lattice display. This includes combinations of marker and interval statistics. There is a `chr` argument to subset the linkage map to user defined linkage groups. If `crit.val = "bonf"` then markers that have significant segregation distortion greater than the family wide alpha level of  $0.05/\text{no.of.markers}$  will be annotated in marker panels. Similarly, intervals that have a significantly weak linkage from a test of the recombination fraction of  $r = 0.5$  will also be annotated in the interval panels. Similar to `profileGen()`, graphical arguments can be used in `profileMark()` and are passed to the high level lattice function `xyplot()`. A plot of the segregation distortion, double crossovers, estimated recombination fractions and LOD scores for the whole genome of mapDH is given in Figure 2.4 and created with the command

```
R> profileMark(mapDH, stat.type = c("seg.dist", "dxo", "erf", "lod"), id =
```



## 2.5 Miscellaneous additional R/qtl functions

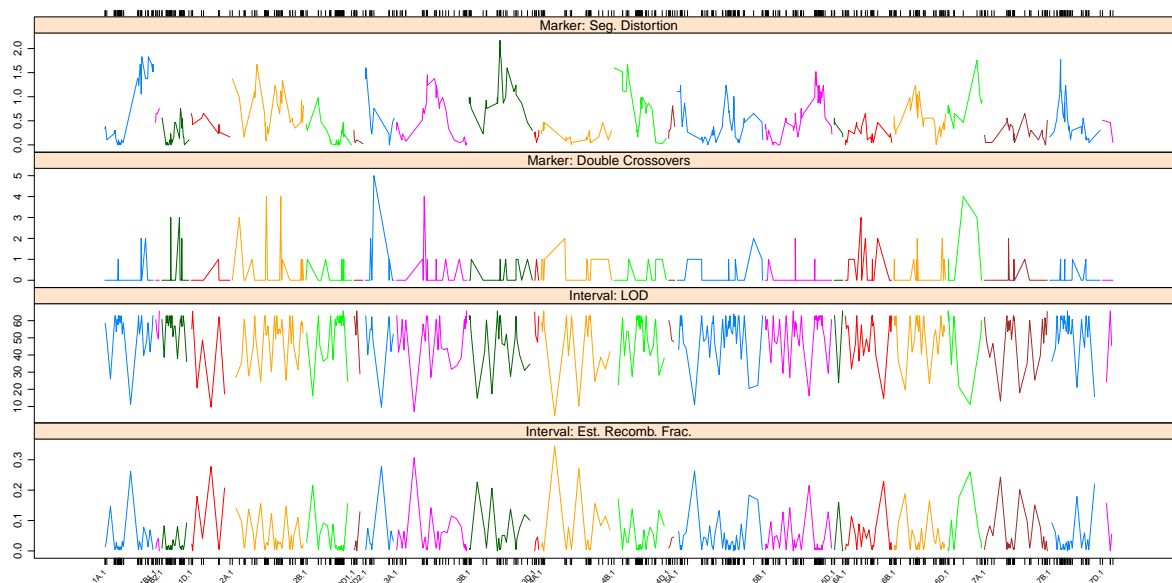


Figure 2.4: Marker and interval profiles of segregation distortion, double crossovers, estimated recombination fractions and LOD scores for mapDH.

```
"Genotype", layout = c(1, 4), type = "l")
```

All linkage groups are highlighted in a different colours to ensure they can be identified clearly. The lattice panels ensure that marker and interval statistics are seamlessly plotted together so problematic regions or markers can be identified efficiently. In this command, the `layout = c(1,4)` and `type = "l"` arguments are passed directly to the high level `xypplot()` lattice function to ensure a more aesthetically pleasing graphic.

## 2.5 Miscellaneous additional R/qtl functions

### 2.5.1 Genetic clones

In my experience, the assumptions of how the individuals of the population are genetically related is rarely checked throughout the construction process. Too often unconstructed or constructed linkage maps contain individuals that are far too closely related beyond the simple assumptions of the population. For example, in a DH population, the assumption of independence would indicate that any two individuals will, by chance, share half of their alleles. Any pairs of individuals that significantly breach this assumption should be deemed suspicious and queried.

The R/ASMap package contains a function for the detection and reporting of the relatedness between individuals as well as a function for forming consensus genotypes if genuine clones are found.

## 2.5 Miscellaneous additional R/qlt functions

---

```
R> genClones(object, chr, tol = 0.9, id = "Genotype")
R> fixClones(object, gc, id = "Genotype", consensus = TRUE)
```

The `genClones()` function uses the power of `comparegeno()` from the R/qlt package to perform the relatedness calculations. It then provides a numerical breakdown of the relatedness between pairs of individuals that share a proportion of alleles greater than `tol`. This breakdown also includes the clonal group the pairs of individuals belong to. The complete set of statistics allows users to make an informed decision about the connectedness of the pairs of individuals. For example, using the constructed map `mapDH` the list of clones can be found using

```
R> gc <- genClones(mapDH, tol = 0.9)
R> gc$cgd
```

	G1	G2	coef	match	diff	na.both	na.one	group
1	DH40	DH24	1	597	0	0	2	1
2	DH59	DH53	1	597	0	0	2	2
3	DH65	DH60	1	598	0	0	1	3
4	DH143	DH139	1	597	0	0	2	4
5	DH186	DH169	1	595	0	0	4	5

The reported information shows five pairs of clones that are, with the exception of missing values, identical.

If clones are found then `fixClones()` can be used to form consensus genotypes for each of the clonal groups. By default it will intelligently collapse the allelic information of the clones within each group (see Table 2.1) to obtain a single consensus genotype. Setting `consensus = FALSE` will choose a genotype with the smallest proportion of missing values as the representative genotype for the clonal group. In both cases genotypes names are given an elongated name containing all genotype names of the clonal group separated by an underscore.

```
R> mapDHg <- fixClones(mapDH, gc$cgd, consensus = TRUE)
R> levels(mapDHg$pheno[[1]])[grep("_", levels(mapDHg$pheno[[1]]))]
```

```
[1] "DH139_DH143" "DH169_DH186" "DH24_DH40" "DH53_DH59" "DH60_DH65"
```

### 2.5.2 Breaking and merging linkage groups

During the linkage map construction process there may be a requirement to break or merge linkage groups. R/ASMap provides two functions to achieve this.

## 2.5 Miscellaneous additional R/qtl functions

Table 2.1: Consensus genotype outcomes for 3 clones across 8 markers in a DH population.

Genotype	M1	M2	M3	M4	M5	M6	M7	M8
G1	AA	BB	AA	BB	AA	BB	NA	AA
G2	AA	BB	NA	NA	NA	NA	NA	BB
G3	AA	BB	AA	BB	NA	NA	NA	AA
Cons.	AA	BB	AA	BB	AA	BB	NA	NA

```
breakCross(cross, split = NULL, suffix = "numeric", sep = ".")
mergeCross(cross, merge = NULL, gap = 5)
```

The **breakCross()** function allows users to break linkage groups in a variety of ways. The **split** argument takes a list with elements named by the linkage group names that require splitting and containing the markers that immediately proceed where the splits are to be made. For example, a split of the linkage group 3B and 6A linkage map **mapDH** after the seventh and fifteenth marker respectively can be easily made using

```
R> mapDHb1 <- breakCross(mapDH, split = list(`3B` = "3B.m.7", `6A` = "6A.m.15"))
R> nmar(mapDHb1)
```

1A	1B1	1B2	1D	2A	2B	2D1	2D2	3A	3B.1	3B.2	3D	4A	4B	4D	5A
41	5	33	10	40	35	8	13	37	7	23	6	30	32	6	54
5B	5D	6A.1	6A.2	6B	6D	7A	7B	7D							
56	6	15	12	41	15	33	37	4							

The **split** argument is flexible and can handle multiple linkage groups as well as multiple markers within linkage groups. The default use of the **suffix** argument produces a numerical suffix attachment to the original linkage groups being split with a separator **sep**. Users can also provide their own complete names for the new split linkage groups by explicitly naming them in the **suffix** argument.

```
R> mapDHb2 <- breakCross(mapDH, split = list(`3B` = "3B.m.7"), suffix = list(`3B`
= c("3B1", "3B2")))
R> nmar(mapDHb2)
```

1A	1B1	1B2	1D	2A	2B	2D1	2D2	3A	3B1	3B2	3D	4A	4B	4D	5A	5B	5D	6A	6B
41	5	33	10	40	35	8	13	37	7	23	6	30	32	6	54	56	6	27	41
6D	7A	7B	7D																
15	33	37	4																

The **mergeCross()** function provides a method for merging linkage groups. Its argument **merge** requires a list with elements named by the proposed linkage group names required

## 2.5 Miscellaneous additional R/qtl functions

---

and containing the linkage groups to be merged. For the linkage map `mapDHb1` containing split linkage groups 3B and 6A created by a call to `breakCross()` the call to `mergeCross()` would be

```
R> mapDHm <- mergeCross(mapDHb1, merge = list(`3B` = c("3B.1", "3B.2"), `6A` =  
      c("6A.1", "6A.2")))  
R> nmar(mapDHm)
```

1A	1B1	1B2	1D	2A	2B	2D1	2D2	3A	3B	3D	4A	4B	4D	5A	5B	5D	6A	6B	6D
41	5	33	10	40	35	8	13	37	30	6	30	32	6	54	56	6	27	41	15
7A	7B	7D																	
33	37	4																	

It should be noted that this function places an artificial genetic distance gap between the merged linkage groups set by the `gap` argument. Accurate distance estimation would require a separate map estimation procedure after the merging has taken place.

### 2.5.3 Rapid genetic distance estimation

The linkage map estimation function in R/qtl called `est.map()` can be invoked individually or can be applied through `read.cross()` when setting the argument `estimate.map = TRUE`. The function applies the multi-locus hidden Markov model technology of [Lander & Green \(1987\)](#) to perform its calculations. Unfortunately this technology is computationally cumbersome if there are many markers on a linkage group and becomes more so if there are many missing allele calls and genotyping errors present.

R/ASMap contains a small map estimation function that circumvents this computational burden.

```
quickEst(object, chr, map.function = "kosambi", ...)
```

The function makes use of another function in R/qtl called `argmax.geno()`. This function is also a multi-locus hidden Markov algorithm that uses the observed markers present in a linkage group to impute pseudo-markers at any chosen cM genetic distance. In this case, we only require a reconstruction or imputation at the markers themselves. For the most accurate imputation to occur there needs to be an estimate of genetic distance already in place. To obtain an initial estimate of distance `est.rf()` is called within each linkage group defined by `chr` and recombination fractions are converted to genetic distances based on `map.function`. Unlike `est.map()` the `quickEst()` function lives up to its namesake by providing the quickest genetic distance calculations for large linkage maps.

```
R> map1 <- est.map(mapDH, map.function = "kosambi")  
R> map1 <- subset(map1, chr = names(nmar(map1))[6:15])
```

## 2.5 Miscellaneous additional R/qtl functions

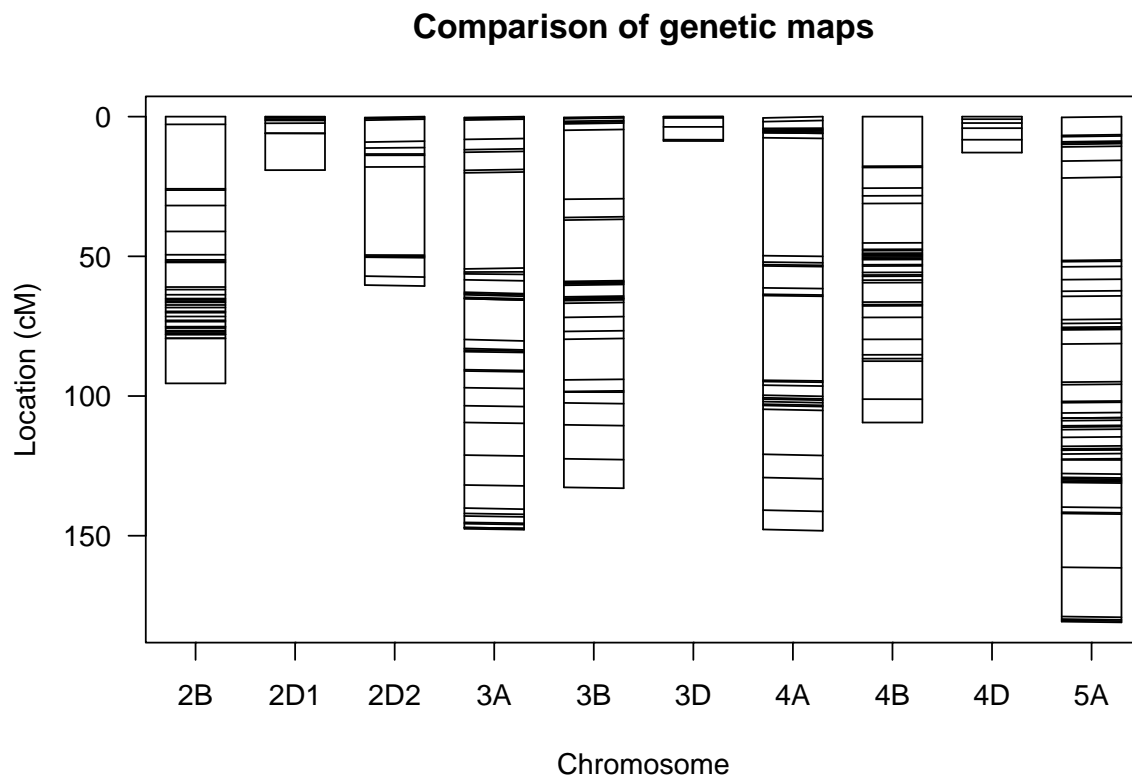


Figure 2.5: Comparison of mapDH using `est.map` and `quickEst`.

```
R> map2 <- quickEst(mapDH, map.function = "kosambi")
R> map2 <- subset(map2, chr = names(nmar(map2))[6:15])
R> plot.map(map1, map2)
```

The linkage map `mapDH` was re-estimated using `est.map()` and `quickEst()` and a comparison of the resulting maps are given in Figure 2.5. The graphic indicates that there negligible changes in marker placement and overall linkage group distances between the two linkage maps.

### 2.5.4 Subsetting in R/ASMap

The functions `pullCross()` and `pushCross()` described in section 2.2 are used to create and manipulate extra list elements `"co.located"`, `"seg.distortion"` and `"missing"` associated with different marker types. Each element contains a data element consisting of a marker matrix equivalent in row dimension to the marker elements of the linkage map they were pulled from. Unfortunately, these list elements are not recognized by the native R/qtl functions. If the R/qtl function `subset.cross()` is used to subset the object to a reduced number of individuals then the data component of each of these elements will not be subsetting accordingly. In addition, the statistics in the table component of the elements `"seg.distortion"` and `"missing"` will be incorrect for the newly subsetting

## 2.5 Miscellaneous additional R/qtl functions

---

linkage map.

The `subsetCross()` function available in the R/ASMap package contains identical functionality to `subset.cross` but also ensures the data components of the extra list elements "co.located", "seg.distortion" and "missing" are subsetted to match the linkage map. In addition, for elements "seg.distortion" and "missing" it also updates the table components to reflect the newly subsetted map. This update ensures that `pushCross()` uses the most accurate information when deciding which markers to push back into the linkage map.

Using the default `seg.thresh = 0.05` for the linkage map `mapDH`, distorted markers are pulled from the map

```
R> mapDH.s <- pullCross(mapDH, type = "seg.distortion")
R> mapDH.s <- subsetCross(mapDH.s, ind = 3:218)
R> dim(mapDH.s$seg.distortion$data)[1]
```

```
[1] 216
```

In this example the use of `subsetCross()` ensures that the data component of the "seg.distortion" element is the same dimension as the map. The table element is also updated to ensure the statistics are correct for the reduced subset of lines.

### 2.5.5 Combining maps

Over the period of time that I have been involved in linkage map construction there has been many occasions where I have required a function that could merge R/qtl cross objects together in an intelligent manner. For example, the merging of two linkage maps from the same population that were independently built with markers from two different platforms. This idea was the motivation behind the `combineMap()` function in the R/ASMap package. The aim of the function was to merge linkage maps based on map information, readying the combined linkage groups for reconstruction through an efficient linkage map construction process such as `mstmap.cross()`.

```
combineMap(..., id = "Genotype", keep.all = TRUE)
```

The function takes an unlimited number of maps through the `...` argument. The linkage maps must all have the same cross class structure and contain the same genotype identifier `id`. At the current stage of writing this vignette the function required unique maker names across all linkage maps. This is expected to be relaxed at a later date so linkage maps that share markers, such as nested association mapping populations, can be merged effectively.

The merging of the maps happens intelligently using several components of the map. Firstly the linkage maps are merged based on commonality between the genotypes. If

## 2.5 Miscellaneous additional R/qtl functions

---

`keep.all = TRUE` the new combined linkage map is “padded out” with missing values where genotypes are not shared. If `keep.all = FALSE` the combined map is reduced to genotypes that are shared among all linkage maps. Secondly, if linkage group names are shared between maps then the markers from the shared linkage groups are merged. To exemplify its use a duplicate of `mapDH` is made and 10 linkage group names have been altered, with the marker names inside each of the linkage groups also altered to ensure they are unique.

```
R> mapDH1 <- mapDH
R> names(mapDH1$geno)[5:14] <- paste("L", 1:10, sep = "")
R> mapDH1$geno <- lapply(mapDH1$geno, function(el) { names(el$map) <-
  dimnames(el$data)[[2]] <- paste(names(el$map), "A", sep = "") el })
R> mapDHc <- combineMap(mapDH, mapDH1)
R> nmar(mapDHc)
```

1A	1B1	1B2	1D	2A	2B	2D1	2D2	3A	3B	3D	4A	4B	4D	5A	5B	5D	6A	6B	6D
82	10	66	20	40	35	8	13	37	30	6	30	32	6	108	112	12	54	82	30
7A	7B	7D	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10							
66	74	8	40	35	8	13	37	30	6	30	32	6							

The resulting combined map includes a combined marker set for the linkage groups that shared the same name and distinct linkage groups for unshared names. Again, note the function only merges the linkage maps and does not reconstruct the final combined linkage map.

The advantages of this function may not be obvious at a first glance. If an attempt is made to completely reconstruct the super set of linkage maps, rather than combine them first, the identification of linkage groups is lost. This function serves to preserve the important identity of linkage groups. More examples of its use in common map construction procedures will be explored in section chapter 3.4 of the next chapter.

## 3 Worked example

This chapter involves the complete linkage map construction process for a barley Backcross population that contains 3024 markers genotyped on 326 individuals in an unconstructed marker set formatted as an R/qlt object with class "bc". The data is available from the R/ASMap package by typing

```
data(mapBCu, package = "ASMap")
```

### 3.1 Pre-construction

The construction of a linkage map does not usually just involve applying a construction algorithm to a supplied set of genetic marker data. It is always prudent to go through a pre-construction checklist to ensure that the best quality genotypes/markers are being used to construct the linkage map.

A non-exhaustive ordered checklist for an unconstructed marker set could be

1. Check missing allele scores across markers for each genotype as well as across genotypes for each marker. Markers or genotypes with a high proportion of missing information could indicate problems with the physical genotyping.
2. Check for genetic clones or individuals that have a high proportion of matching allelic information between them.
3. Check markers for excessive segregation distortion. Highly distorted markers may not map to unique locations.
4. Check markers for switched alleles. These markers will not cluster or link well with other markers during the construction process and it is therefore preferred to repair their alignment before proceeding.
5. Check for co-locating markers. For large linkage maps it would be more computationally efficient from a construction standpoint to temporarily omit markers that are co-located with other markers.



### 3.1 Pre-construction

---

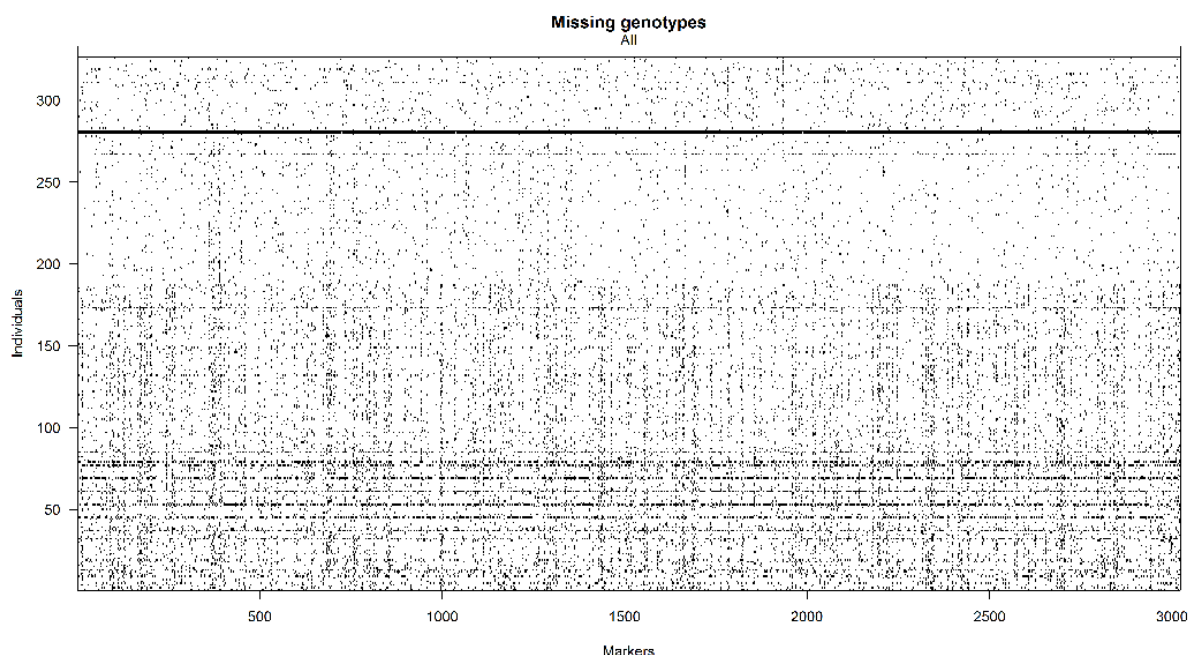


Figure 3.1: Plot of the missing allele scores for the unconstructed map `mapBCu`

R/qtl provides a very simple graphical tool for checking the structure of missing allele score across the genotypes and the markers.

```
R> plot.missing(mapBCu)
```

Figure 3.1 shows the resulting plot for this command. The darkest lines on the plot indicate there are some genotypes with large amounts of missing data. This could indicate poor physical genotyping of these lines and they should be removed before proceeding. The plot also reveals the markers have a large number of typed allele values across the range of genotypes. The R/ASMap function `statGen()` can be used to identify the genotypes with a certain number of missing values. These genotypes are then omitted using the usual functions available in R/qtl.

```
R> sg <- statGen(mapBCu, bychr = FALSE, stat.type = "miss")
R> mapBC1 <- subset(mapBCu, ind = sg$miss < 1600)
```

From a map construction point of view, highly related individuals can enhance segregation distortion of markers. It is therefore wise to determine a course of action such as removal of individuals or the creation of consensus genotypes before proceeding with any further pre-construction diagnostics. The R/ASMap function `genClones()` discussed in section 2.5.1 can be used to identify and report genetic clones.

### 3.1 Pre-construction

---

```
R> gc <- genClones(mapBC1, tol = 0.95)
R> gc$cgd
```

	G1	G2	coef	match	diff	na.both	na.one	group
1	BC045	BC039	0.9919	1466	12	97	1448	1
2	BC052	BC039	1.0000	2423	0	98	502	1
3	BC168	BC039	1.0000	2572	0	47	404	1
4	BC052	BC045	0.9899	1476	15	94	1438	1
5	BC168	BC045	0.9872	1620	21	44	1338	1
6	BC168	BC052	1.0000	2577	0	36	410	1
7	BC067	BC060	1.0000	2759	0	8	256	2
8	BC135	BC086	1.0000	2743	0	17	263	3
9	BC099	BC093	1.0000	2737	0	19	267	4
10	BC120	BC117	1.0000	2699	0	22	302	5
11	BC129	BC126	1.0000	2678	0	35	310	6
12	BC204	BC138	1.0000	2691	0	6	326	7
13	BC147	BC141	0.9996	2753	1	22	247	8
14	BC193	BC144	1.0000	2771	0	7	245	9
15	BC162	BC161	1.0000	2686	0	20	317	10
16	BC205	BC190	1.0000	2886	0	7	130	11
17	BC286	BC285	1.0000	2920	0	1	102	12
18	BC325	BC314	1.0000	2911	0	4	108	13

The table shows 13 groups of genotypes that share a proportion of their alleles greater than 0.95. The supplied additional statistics show that the first group contains three pairs of genotypes that had matched pairs of alleles from 1620 markers or less. These pairs also 1400 markers where an allele was present for one genotype and missing for another. Based on this, there is not enough evidence to suspect these pairs may be clones and they are removed from the table. The `fixClones()` function can then be used to form consensus genotypes for the remaining groups of clones in the table.

```
R> cgd <- gc$cgd[-c(1, 4, 5), ]
R> mapBC2 <- fixClones(mapBC1, cgd, consensus = TRUE)
R> levels(mapBC2$pheno[[1]])[grep("-", levels(mapBC2$pheno[[1]]))]
```

```
[1] "BC039_BC052_BC168" "BC060_BC067"      "BC086_BC135"
[4] "BC093_BC099"        "BC117_BC120"      "BC126_BC129"
[7] "BC138_BC204"        "BC141_BC147"      "BC144_BC193"
[10] "BC161_BC162"        "BC190_BC205"      "BC285_BC286"
[13] "BC314_BC325"
```

```
R> profileMark(mapBC2, stat.type = c("seg.dist", "prop", "miss"), crit.val =
  "bonf", layout = c(1, 4), type = "l", cex = 0.5)
```

### 3.1 Pre-construction

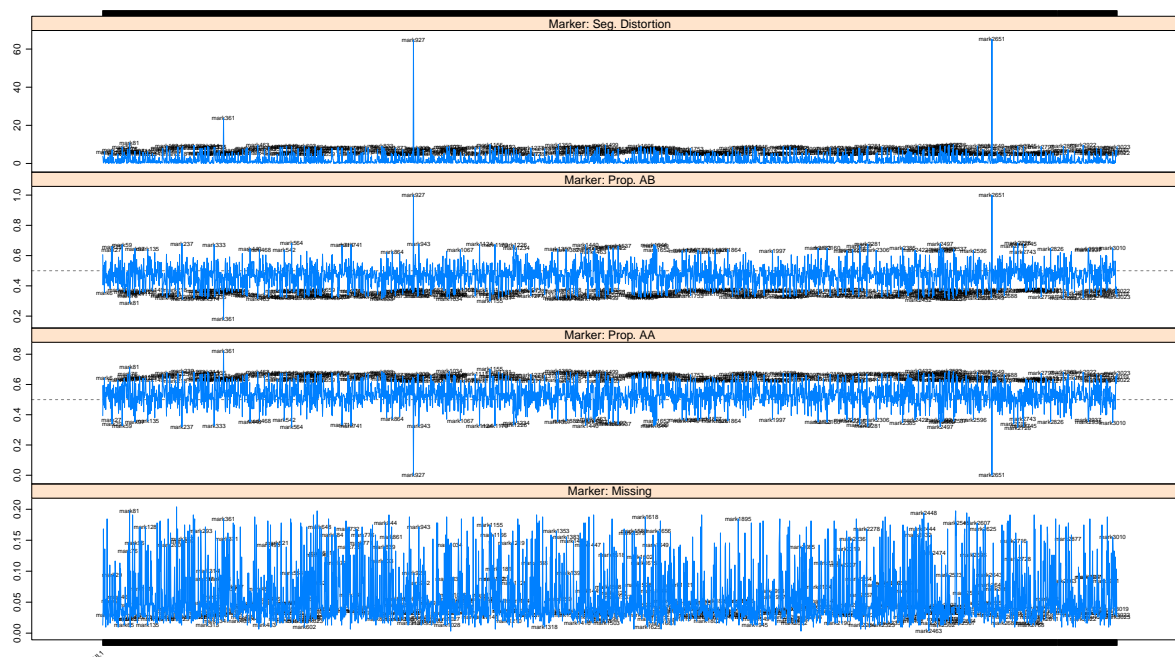


Figure 3.2: For individual markers, the negative log<sub>10</sub> p-value for the test of segregation distortion, the proportion of each contributing allele and the proportion of missing values.

At this juncture it is wise to check the segregation distortion statistics of the markers. Segregation distortion is phenomenon where the observed allelic frequencies at a specific locus deviate from expected allelic frequencies due to Mendelian genetics. It is well known that this distortion can occur from physical laboratorial processes or it may also occur in local genomic regions from underlying biological and genetic mechanisms (Lyttle, 1991).

The level of segregation distortion, the allelic proportions and the missing value proportion across the genome can be graphically represented using the marker profiling function `profileMark()` and the result is displayed in Figure 3.2.

Setting `crit.val = "bonf"` annotates the markers that have a p-value for the test of segregation distortion lower than the family wide bonferroni adjusted alpha level of  $0.05/\text{no.of.markers}$  on each of the figures in each panel. Additional plotting parameters `layout = c(1,4)`, `type="p"` and `cex = 0.5` are passed to the high level lattice plotting function `xyplot()` to provide a more aesthetically pleasing plot. The plot indicates there are numerous markers that are considered to be significantly distorted with three highly distorted markers. The plot also indicates that the missing value proportion of the markers does not exceed 20%.

The highly distorted markers can easily be dropped using

```
R> mm <- statMark(mapBC2, stat.type = "marker")$marker$AB
```

## 3.2 MSTmap construction

---

```
R> mapBC3 <- drop.markers(mapBC2, c(markernames(mapBC2)[mm > 0.98],
  markernames(mapBC2)[mm < 0.2]))
```

Without a constructed map it is impossible to determine the origin of the segregation distortion. However, the blind use of distorted markers may also create linkage map construction problems. It may be more sensible to place the distorted markers aside and construct the map with less problematic markers. Once the linkage map is constructed the more problematic markers can be introduced to determine whether they have a useful or deleterious effect on the map. The R/ASMap functions `pullCross()` and `pushCross()` are designed to take advantage of this scenario. To showcase their use in this example, they are also used to pull markers with 10-20% missing values as well as co-located markers.

```
R> mapBC3 <- pullCross(mapBC3, type = "missing", pars = list(miss.thresh = 0.1))
R> mapBC3 <- pullCross(mapBC3, type = "seg.distortion", pars = list(seg.thresh =
  "bonf"))
R> mapBC3 <- pullCross(mapBC3, type = "co.located")
R> names(mapBC3)
```

```
[1] "geno"          "pheno"          "missing"         "seg.distortion"
[5] "co.located"
```

```
R> sum(ncol(mapBC3$missing$data), ncol(mapBC3$seg.dist$data),
  ncol(mapBC3$co.located$data))
```

```
[1] 847
```

A total of 847 markers are removed and placed aside in their respective elements and the map is now constructed with the remaining 2173 markers.

## 3.2 MSTmap construction

The curated genetic marker data in `mapBC3` can now be constructed using the `mstmap.cross` function available in R/ASMap.

```
R> mapBC4 <- mstmap(mapBC3, bychr = FALSE, trace = TRUE, dist.fun = "kosambi",
  p.value = 1e-12)
R> chrLen(mapBC4)
```

```
      L.1      L.2      L.3      L.4      L.5      L.6      L.7
304.910957 266.240647 78.982131 252.281760 33.226962 233.485952 153.315888
      L.8      L.9
106.290403   6.657526
```

## 3.2 MSTmap construction

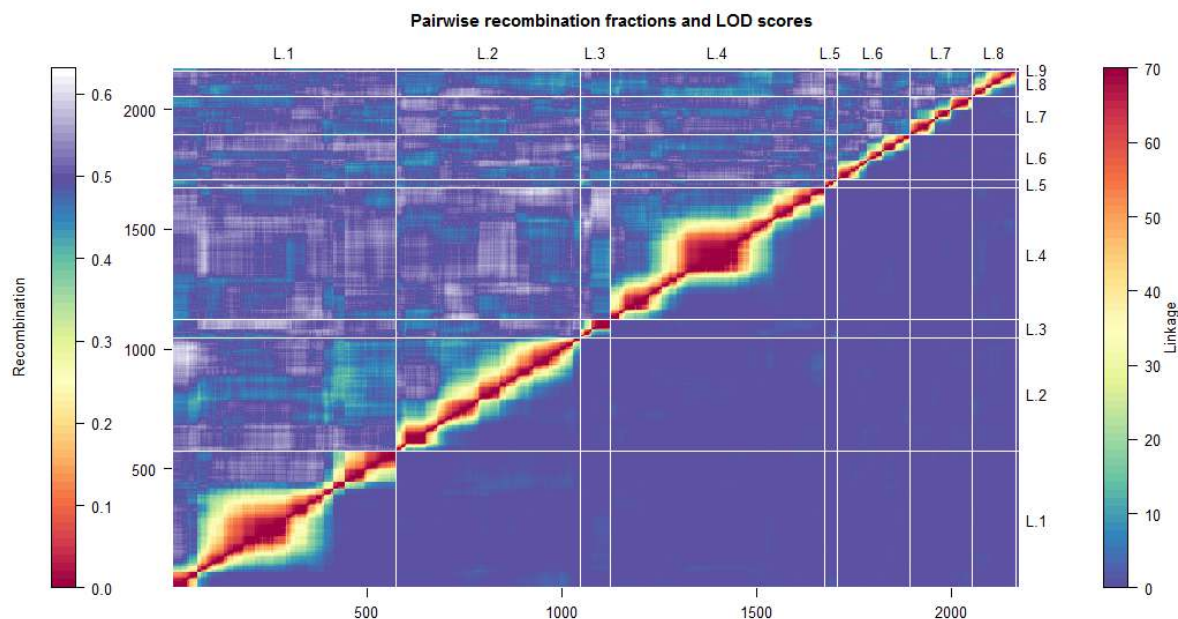


Figure 3.3: Heat map of the constructed linkage map mapBC4.

By setting `bychr = FALSE` the complete set of marker data from `mapBC3` is bulked and constructed from scratch. This construction involves the clustering of markers to linkage groups and then the optimal ordering of markers within each linkage group. An initial check of Figure 1.1, indicates for a population size of 309 the `p.value` should be set to  $1e-12$  to ensure a 30cM threshold when clustering markers to linkage groups. The newly constructed linkage map contains 9 linkage groups each containing markers that are optimally ordered. The performance of the MSTmap construction can be checked by plotting the heat map of pairwise recombination fractions between markers and their pairwise LOD score of linkage using the R/ASMap function `heatMap()`

```
R> heatMap(mapBC4, lmax = 70)
```

As discussed in section 2.3, an aesthetic heat map is attained when the heat on the upper triangle of the plot used for the pairwise estimated recombination fractions matches the heat of the pairwise LOD scores. Figure 3.3 displays the heat map and shows this was achieved by setting `lmax = 70`. The heat map also shows consistent heat across the markers within linkage groups indicating strong linkage between nearby markers. The linkage groups appear to be very distinctly clustered.

Although the heat map is indicating the construction process was successful it does not highlight subtle problems that may be existing in the constructed linkage map. As stated in section 2.4 one of the key quality characteristics of a well constructed linkage map is an appropriate recombination rate of the the genotypes. For this barley Backcross population each line is considered to be independent with an expected recombination

## 3.2 MSTmap construction

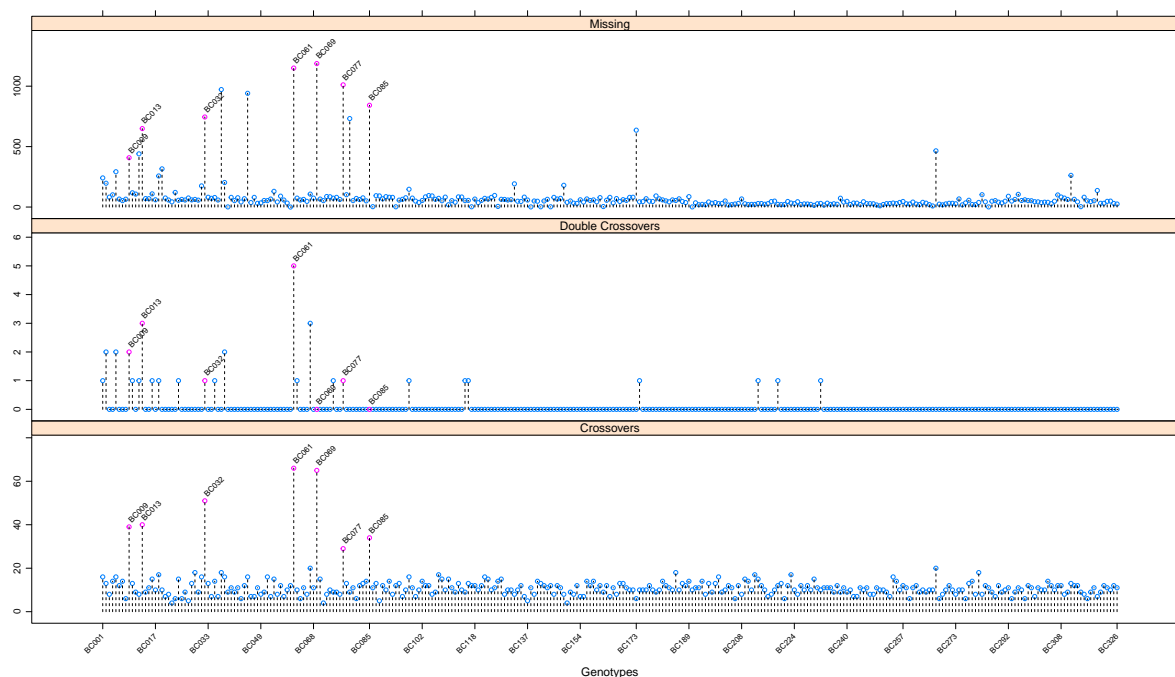


Figure 3.4: For individual genotypes, the number of recombinations, double recombinations and missing values for mapBC4

rate of 14 across the genome. The current linkage map contains linkage groups that exceed the theoretical cutoff of 200cM indicating there may be genotypes with inflated recombination rates. This can easily be ascertained from the `profileGen()` function available in R/ASMap.

```
R> pg <- profileGen(mapBC4, bychr = FALSE, stat.type = c("xo", "dxo", "miss"), id
  = "Genotype", xo.lambda = 14, layout = c(1, 3), lty = 2, cex = 0.7)
```

Figure 3.4 show the number of recombinations, double recombination and missing values for each of 309 genotypes. The plot also annotates the genotypes that have recombination rates significantly above the expected recombination rate of 14. A total of seven lines have recombination rates above 20 and the plots also show that these lines have excessive missing values. To ensure the extra list elements `"co.located"`, `"seg.distortion"` and `"missing"` of the object are subsetted and updated appropriately, the offending genotypes are removed using the R/ASMap function `subsetCross()`. The linkage map is then be reconstructed.

```
R> mapBC5 <- subsetCross(mapBC4, ind = !pg$xo.lambda)
R> mapBC6 <- mstmap(mapBC5, bychr = TRUE, dist.fun = "kosambi", trace = TRUE,
  p.value = 1e-12)
R> chrLen(mapBC6)
```

### 3.3 Pushing back markers

---

L.1	L.2	L.3	L.4	L.5	L.6	L.7
229.578103	223.170605	65.806120	214.380402	27.297642	191.060846	140.114968
L.8	L.9					
88.687053	4.875885					

Users can check the recombination rates of the remaining genotypes in the re-constructed map are now within respectable limits. As a result the lengths of the linkage groups have dropped dramatically.

It is also useful to graphically display statistics of the markers and intervals of the current constructed linkage map. For example, Figure 3.5 shows the marker profiles of the  $-\log_{10}$  p-value for the test of segregation distortion, the allele proportions and the number of double crossovers. It also displays the interval profile of the number of recombinations occurring between adjacent markers. This plot reveals many things that are useful for the next phase of the construction process.

```
R> profileMark(mapBC6, stat.type = c("seg.dist", "prop", "dxo", "recomb"), layout  
  = c(1, 5), type = "l")
```

The plot instantly reveals the success of the map construction process with no more than one double crossover being found at any marker and very few being found in total. The plot also reveals the extent of the biological distortion that can occur within a linkage group. A close look at the segregation distortion and allele proportion plots shows the linkage group L.3 and the short linkage group L.5 have profiles that could be joined if the linkage groups were merged. In addition, L.8 and L.9 also have profiles that could be joined if the linkage groups were combined. This will be discussed in more detail in the next section.

### 3.3 Pushing back markers

In this section the markers that were originally placed aside in the pre-construction of the linkage map will be pushed back into the constructed linkage map and the map carefully re-diagnosed. To begin the 515 external markers that have between 10% and 20% missing values residing in the list element "missing" are pushed back in using `pushCross()`

```
R> mapBC6 <- pushCross(mapBC6, type = "missing", pars = list(miss.thresh = 0.22,  
  max.rf = 0.3))
```

The parameter `miss.thresh = 0.22` is used to ensure that all markers with a threshold less 0.22 are pushed back into the linkage map. Note, this pushing mechanism is not re-constructing the map and only assigns the markers to the most suitable linkage group. At this point it is worth re-plotting the heatmap to check whether the push has been successful. In this vignette we will confine the graphic to linkage groups L.3, L.5, L.8 and

### 3.3 Pushing back markers

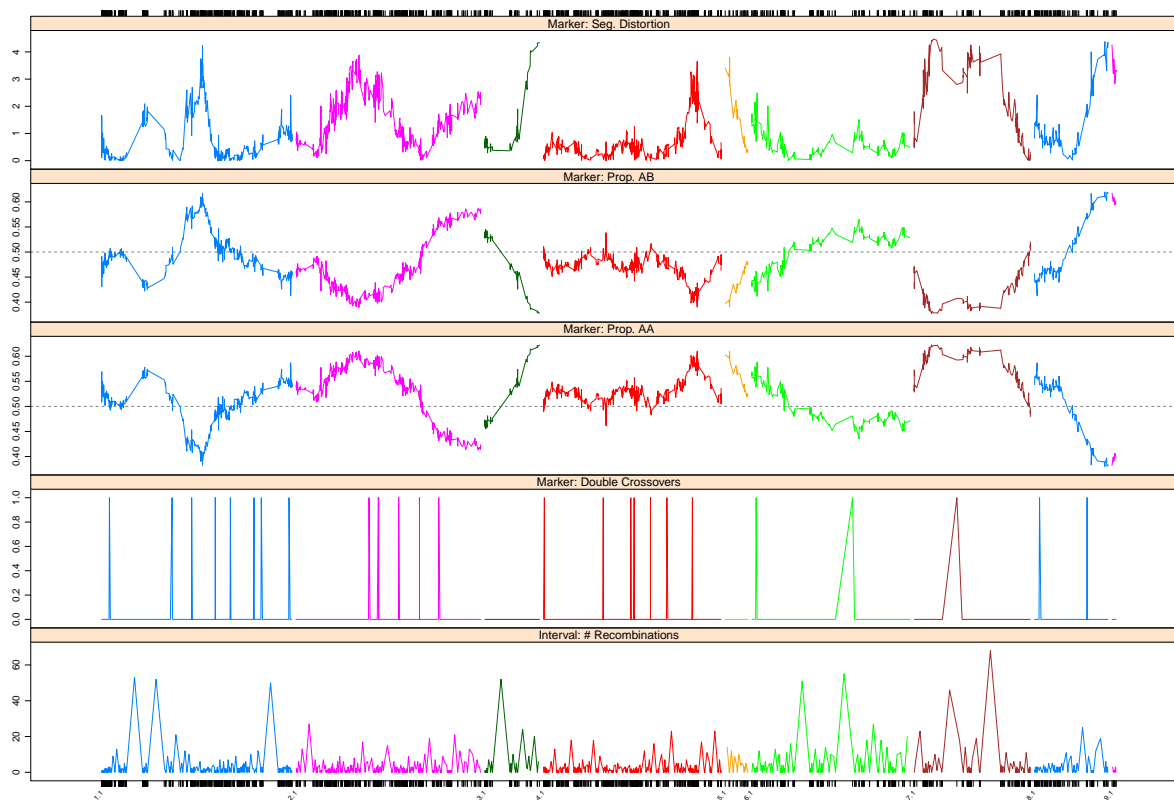


Figure 3.5: Marker profiles of the  $-\log_{10}$  p-value for the test of segregation distortion, allele proportions and the number of double of crossovers as well as the interval profile of the number of recombinations between adjacent markers in mapBC6.

L.9 to determine whether the extra markers have provided useful additional information about the possible merging of the groups. The resulting heat map is given in Figure 3.6.

```
R> heatMap(mapBC6, chr = c("L.3", "L.5", "L.8", "L.9"), lmax = 70)
```

It is clear from the heat map that there are genuine linkages between L.3 and L.5 as well as L.8 and L.9. These two sets of linkage groups can be merged using `mergeCross()` and the linkage group names are renamed to form the optimal 7 linkage groups that are required for the barley genome.

```
R> mapBC6 <- mergeCross(mapBC6, merge = list(L.3 = c("L.3", "L.5"), L.8 = c("L.8",
  "L.9")))
R> names(mapBC6$geno) <- paste("L.", 1:7, sep = "")
R> mapBC7 <- mstmap(mapBC6, bychr = TRUE, trace = TRUE, dist.fun = "kosambi",
  p.value = 2)
R> chrLen(mapBC7)
```



### 3.3 Pushing back markers

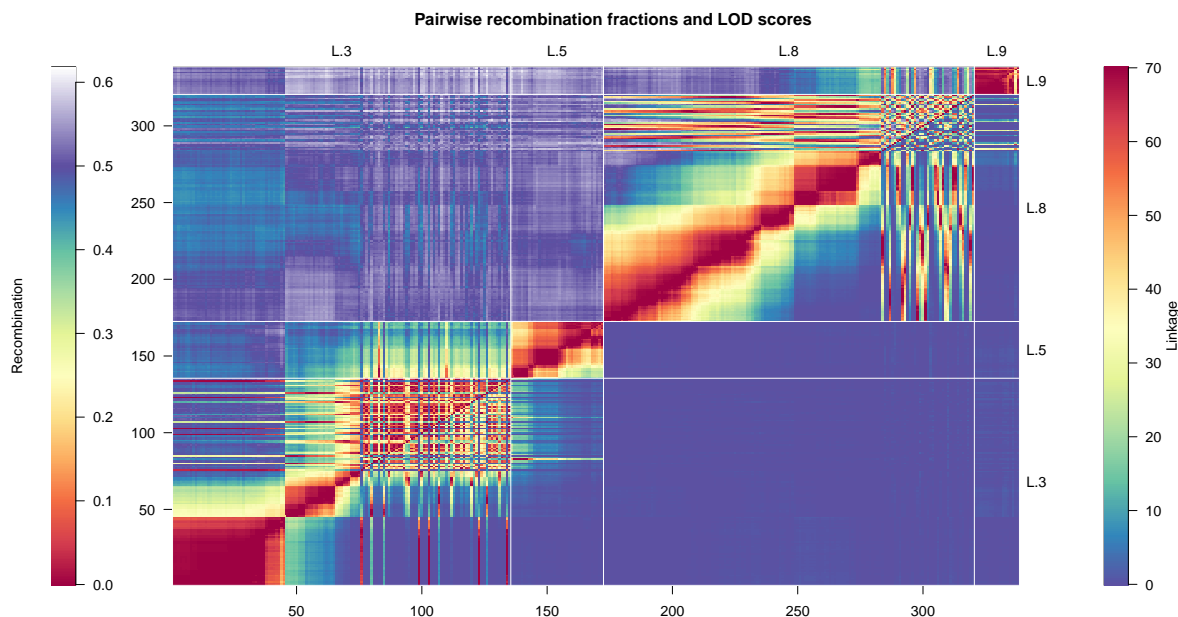


Figure 3.6: Heat map of the constructed linkage map mapBC6.

L.1	L.2	L.3	L.4	L.5	L.6	L.7
242.4104	233.1945	162.6205	224.1773	201.1503	147.7669	159.5308

As the optimal number of linkage groups have been identified the re-construction of the map is performed by linkage group only through setting the `p.value = 2`. At this point there is no need to anchor the map during construction as orientation of linkage groups has not been formally identified. The linkage group lengths of L.1, L.2 and L.4 are slightly elevated indicating excessive recombination across these groups. The reason for this inflation is quickly understood by checking the profile of the genotypes again using `profileGen()`

```
R> pg1 <- profileGen(mapBC7, bychr = FALSE, stat.type = c("xo", "dxo", "miss"), id
  = "Genotype", xo.lambda = 14, layout = c(1, 3), lty = 2, cex = 0.7)
```

Figure 3.7 shows the genotype profiles for the 302 barley lines. The introduction of the markers with missing value proportions between 10% and 20% into the linkage map has highlighted two more problematic lines that have a high proportion of missing values across the genome. Again, these should be removed using `subsetCross()` and the map reconstructed.

```
R> mapBC8 <- subsetCross(mapBC7, ind = !pg1$xo.lambda)
R> mapBC9 <- mstmap(mapBC8, bychr = TRUE, dist.fun = "kosambi", trace = TRUE,
  p.value = 2)
R> chrLen(mapBC9)
```

### 3.3 Pushing back markers

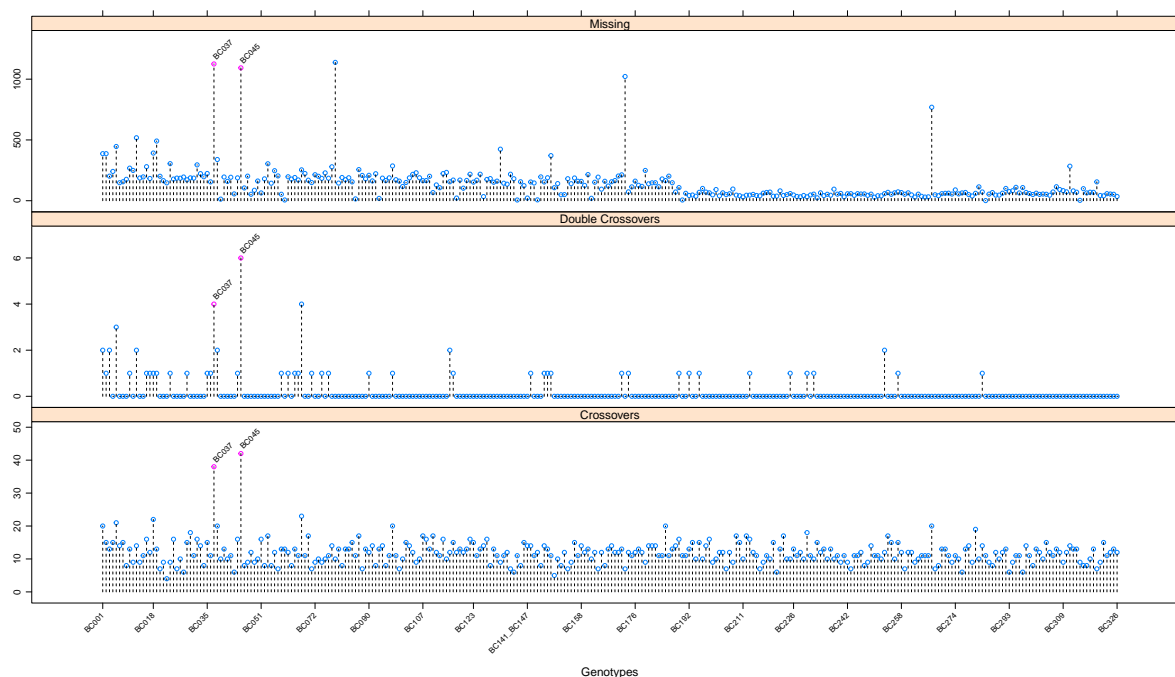


Figure 3.7: For individual genotypes, the number of recombinations, double recombinations and missing values for mapBC7.

L.1	L.2	L.3	L.4	L.5	L.6	L.7
225.7900	223.3063	157.0592	206.4801	194.3003	145.7554	149.2467

The removal of these two lines will not have a deleterious effect on the number of linkage groups and therefore the linkage map should be reconstructed by linkage group only. The length of most linkage groups has now been appreciably reduced. Figure 3.8 displays the segregation distortion and allele proportion profiles for the markers from using `profileMark()` again.

```
R> profileMark(mapBC9, stat.type = c("seg.dist", "prop", "dxo", "recomb"), layout
= c(1, 5), type = "1")
```

The plot indicates a spike of segregation distortion on L.2 that does not appear to be biological and should be removed. The plot also indicates the significant distortion regions on L.3, L.6 and L.7 indicating some of the markers with missing value proportions between 10% and 20% pushed back into the linkage map also had some degree of segregation distortion. The marker positions on the x-axis of the plot suggests these regions are sparse. The 295 external markers in the list element "seg.distortion" may hold the key to this sparsity and are pushed back to determine their effect on the linkage map.

```
R> dm <- markernames(mapBC9, "L.2")[statMark(mapBC9, chr = "L.2", stat.type =
"marker")$marker$neglog10P > 6]
```

### 3.3 Pushing back markers

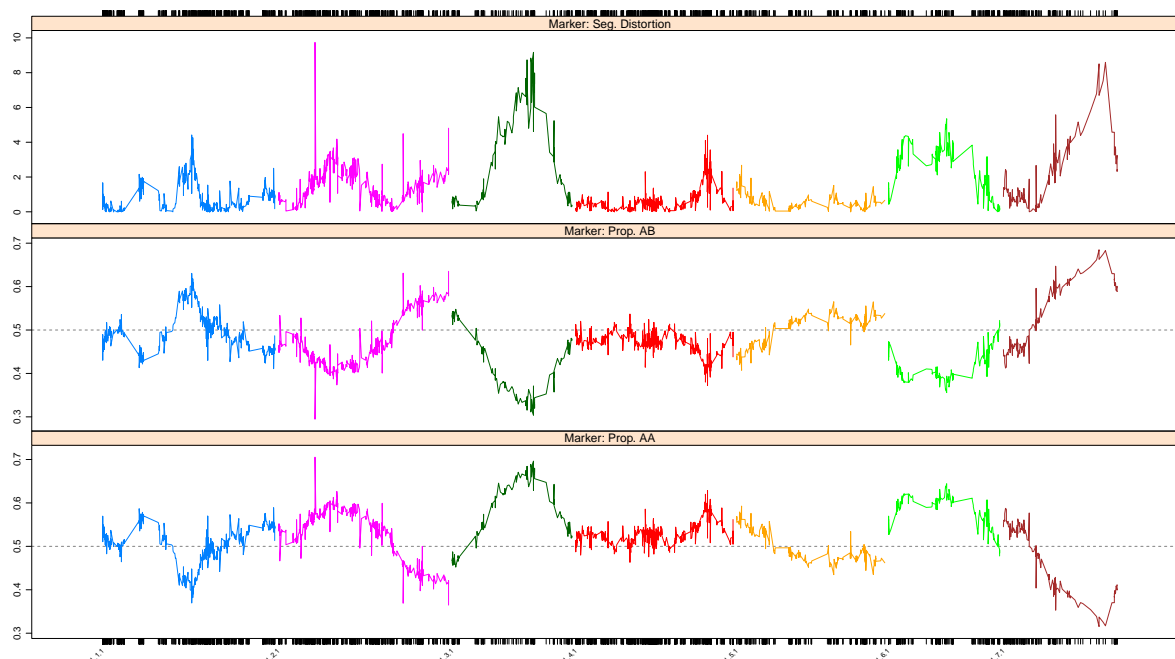


Figure 3.8: Marker profiles of the  $-\log_{10}$  p-value for the test of segregation distortion, allele proportions for mapBC9

```
R> mapBC10 <- drop.markers(mapBC9, dm)
R> mapBC11 <- pushCross(mapBC10, type = "seg.distortion", pars = list(seg.ratio =
  "70:30"))
R> mapBC12 <- mstmap(mapBC11, bychr = TRUE, trace = TRUE, dist.fun = "kosambi",
  p.value = 2)
R> round(chrlen(mapBC12) - chrlen(mapBC9), 5)
```

L.1	L.2	L.3	L.4	L.5	L.6	L.7
0.20398	1.11431	0.60228	-0.92150	0.86448	-0.25043	-6.29943

```
R> nmar(mapBC12) - nmar(mapBC10)
```

L.1	L.2	L.3	L.4	L.5	L.6	L.7
0	1	156	0	0	86	52

After checking the table element of the "seg.distortion" element, a 70:30 distortion ratio was chosen to ensure all the markers were pushed back into the linkage map. After the pushing was complete, the linkage map was reconstructed and linkage group lengths only changed negligibly from the previous version of the map with distorted markers removed. This suggests the extra markers have been inserted successfully with nearly all distorted markers being pushed into L.3, L.6 and L.7.

### 3.4 Post-construction linkage map development

Table 3.1: Table of statistics for the final linkage map, mapBC

	L.1	L.2	L.3	L.4	L.5	L.6	L.7	Total	Ave.
No. of markers	681	593	335	679	233	279	219	3019	431
Lengths	225.9	224.4	157.6	205.5	195.1	145.5	142.9	1297.2	185.3
Ave. interval	0.33	0.39	0.48	0.31	0.84	0.53	0.66		0.51

To finalise the map the 39 co-locating markers residing in the "co.located" list element of the object are pushed back into the linkage map and placed adjacent to the markers they were co-located with. Note that all external co-located markers have an immediate linkage group assignation.

```
R> mapBC <- pushCross(mapBC12, type = "co.located")
R> names(mapBC)
```

```
[1] "geno" "pheno"
```

A check of the final structure of the object shows the extra list elements have been removed and only the "pheno" and "geno" list elements remain. Users can graphically plot the genotypes and marker/interval profiles to diagnostically assess the final linkage map. These plots have been omitted from this report for brevity. The final linkage map has statistics given in Table 3.1.

### 3.4 Post-construction linkage map development

After a linkage map is constructed it is very common to attempt linkage map development through the insertion of additional markers. This may be a simple fine mapping exercise where the linkage group is known in advance for an additional set of markers or it may be a more complex tasks such as the insertion of markers from an older linkage map into a newly constructed map. The procedure to successfully achieve either of these can be problematic. For example, there may need to be external manipulation, such as removal of non-concurrent genotypes, before the additional set of markers is inserted into the map. Additionally, in some cases, the linkage map may need to be partially or completely reconstructed. An unfortunate result of this reconstruction process is the possible loss of known linkage group identities.

R/ASMap provides functionality to insert additional markers into an established linkage map without losing important linkage group identification. The methods applied in this section assume the additional markers, as well as the constructed linkage map, are R/qtl cross objects of the same class. The methods are best described by presenting several examples that mimic common post-construction linkage map development tasks. In these examples additional markers will be obtained by randomly selecting markers from the final linkage map, mapBC.

```
R> set.seed(123)
R> add1 <- drop.markers(mapBC, markernames(mapBC)[sample(1:3019, 2700, replace =
```

## 3.4 Post-construction linkage map development

---

```
FALSE)])
R> mapBCs <- drop.markers(mapBC, markernames(add1))
R> add3 <- add2 <- add1
R> add2 <- subset(add2, chr = "L.1")
R> add3$geno[[1]]$data <- pull.geno(add1)
R> add3$geno[[1]]$map <- 1:ncol(add3$geno[[1]]$data)
R> names(add3$geno[[1]]$map) <- markernames(add1)
R> names(add3$geno)[1] <- "ALL"
R> add3 <- subset(add3, chr = "ALL")
```

### 3.4.1 Combining two linkage maps of the same population

Many populations that are currently being researched have been genotyped on multiple platforms and separate maps constructed for one or both of the populations. For example, a linkage map may have been constructed from markers genotyped on the new Illumina 90K SNP (Single Nucleotide Polymorphism) array but the population may also have a legacy linkage map constructed from markers stemming from SSRs (Single Sequence Repeats) or DaRTs (Diversity Array Technology). The `add1` R/qtl object mimics an older map of `mapBC` with 319 markers spanning the seven linkage groups. As the linkage groups are known between maps there is only a requirement to combine the maps in a sensible manner and reconstruct. This can be done efficiently in R/ASMap without external manipulation and loss of linkage group information.

```
R> add1 <- subset(add1, ind = 2:300)
R> full1 <- combineMap(mapBCs, add1, keep.all = TRUE)
R> full1 <- mstmap(full1, bychr = TRUE, trace = TRUE, anchor = TRUE, p.value = 2)
```

This example is a classic use of the R/ASMap function `combineMap()` described in section 2.5.5. The two maps are first merged on their matching genotypes and, as the first genotype in `add1` has been removed, there are missing cells placed in the first genotype of `mapBCs` for the markers in `add1`. The `combineMap()` function also understands that both linkage maps share common linkage group names and places the markers from shared linkage groups together. The map is then reconstructed by linkage group using `p.value = 2` in the `mstmap.cross()` call, ensuring the important identity of linkage groups are retained. In addition, setting `anchor = TRUE` will ensure that the orientation of the larger linkage map `mapBCs` is preserved.

### 3.4.2 Fine mapping

In marker assisted selection breeding programmes it is common to increase the density of markers in a specific genomic region of a linkage group for the purpose of more accurately identifying the position of quantitative trait loci (QTL). This is known as fine mapping. For example, the `add2` object contains only markers from the L.1 linkage group of `mapBCs`. When the linkage group for the additional markers is known in advance and matches a linkage group in the constructed map, the insertion of the new markers is very similar to the previous section.

```
R> add2 <- subset(add2, ind = 2:300)
R> full2 <- combineMap(mapBCs, add2, keep.all = TRUE)
```

## 3.4 Post-construction linkage map development

---

```
R> full12 <- mstmap(full12, chr = "L.1", bychr = TRUE, trace = TRUE, anchor = TRUE,
  p.value = 2)
```

Again, the removal of the first genotype of `add2` will cause missing values to be added in the first genotype of `full12` for the markers that were in `add2`. The `mstmap.cross()` call is also similar to the previous section with the exception that only the first linkage group `L.1` needs optimal ordering.

### 3.4.3 Unknown linkage groups

There may be occasions when the linkage group identification of the additional markers is not known in advance. For example, an incomplete set of markers was used to construct the map or a secondary set of markers is available that come from an unconstructed linkage map. These additional markers can be pushed into a constructed linkage map efficiently using the functions available in R/ASMap. In this example the `add3` R/qtl object consists of one linkage group called `ALL` that contains all the markers that spanned the seven linkage groups in `add1`.

```
R> add3 <- subset(add3, ind = 2:300)
R> full13 <- combineMap(mapBCs, add3, keep.all = TRUE)
R> full13 <- pushCross(full13, type = "unlinked", unlinked.chr = "ALL")
R> full13 <- mstmap(full13, bychr = TRUE, trace = TRUE, anchor = TRUE, p.value = 2)
```

Again, `combineMap()` is used to merge the linkage maps to avoid the hassle of having to manually match the dimensions of the marker sets. The R/ASMap function `pushCross()` can then be used to push the additional markers into the constructed linkage map. By choosing the marker type argument `type = "unlinked"` and providing the `unlinked.chr = "ALL"` the function recognises that the markers require pushing back into the remaining linkage groups of `full13`. Again, the `mstmap.cross()` call only requires optimal ordering of the markers within linkage groups.

## 4 Aspects of the MSTmap algorithm

This chapter presents miscellaneous additional information associated within the MSTmap algorithm and the arguments supplied to the R/ASMap functions `mstmap.data.frame()` and `mstmap.cross()`.

### 4.1 MSTmap and distance calculations

After close scrutiny and experience with the MSTmap algorithm it appears there may be circumstances where the algorithm produces inflated genetic distances. This is especially prevalent when the linkage map being constructed contains genotypes with many missing values. In these cases the constructed linkage map is likely to contain runs of missing values for these genotypes. For example, this phenomenon can be seen in Figure 4.1 (the dark lines) for the initial constructed linkage map `mapBC4` of the previous chapter. Seven of these genotypes were eventually removed for having excess recombinations across the genome but their missing value structure has also contributed substantially to the inflated distances of the linkage groups.

```
R> plot.missing(mapBC4)
```

This problem is easier to explain by presenting various outputs of the linkage map `mapBC4`. Firstly, the map is reconstructed but with `return.imputed = TRUE` added to the call to ensure the imputed marker probability matrix for the linkage groups is added to the returned cross object.

```
R> mapBC4i <- mstmap(mapBC3, bychr = FALSE, trace = TRUE, dist.fun = "kosambi",  
  p.value = 1e-12, return.imputed = TRUE)  
R> mapBC4i$geno[[1]]$map[1:14]
```

mark246	mark986	mark1452	mark2266	mark391	mark426	mark670	mark1220
0.0000000	0.0000000	0.0000000	0.0000000	0.7641466	0.7641466	0.7641466	0.7641466
mark1789	mark2193	mark2269	mark2382	mark2579	mark2934		
0.7641466	0.7641466	0.7641466	0.7641466	0.7641466	0.7641466		

```
R> mapBC4i$imputed.geno[[1]]$map[1:5]
```

## 4.1 MSTmap and distance calculations

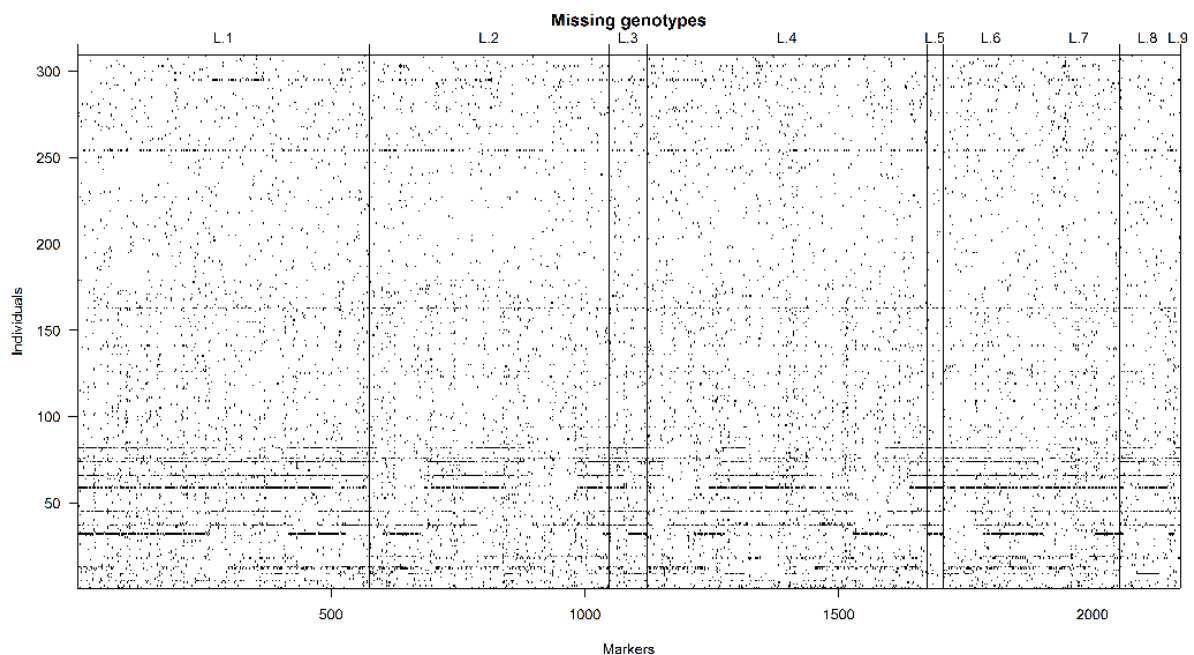


Figure 4.1: Plot of the missing allele scores for the constructed map mapBC4

```
mark246  mark391  mark284  mark157  mark1990
0.0000000 0.7641466 1.5507414 2.3555151 3.4031908
```

To exemplify the problem the markers on L.1 are used. The first 14 markers of L.1 contain the first two co-locating sets of markers and have been calculated to be 0.76 cM apart. The imputed map for L.1 is a reduced version of the linkage map from L.1 that contains all unique markers and one member from each co-locating set. Markers mark246 and mark391 have been chosen as the members for the first two co-locating sets. It can be quickly calculated how many pairs of alleles are observed between the two markers and how many observed recombinations there were across these pairs.

```
R> len <- apply(mapBC4$geno[[1]]$data[, c(1, 5)], 1, function(el)
  length(el[!is.na(el)]))
R> length(len[len > 1])
```

```
[1] 281
```

```
R> bca <- apply(mapBC4i$geno[[1]]$data[, c(1, 5)], 1, function(el) { el <-
  el[!is.na(el)] sum(abs(diff(el))) })
R> bca[bca > 0]
```

```
BC004
```

```
1
```



## 4.1 MSTmap and distance calculations

---

One genuine recombination from 281 observed pairs creates a 0.35 cM distance between the markers, falling short of 0.76 cM. The reason behind this shortfall becomes clear when the imputed marker probability data is printed for the first five markers of the seven genotypes with excessive missing values

```
R> mapBC4i$imputed.geno[[1]]$data[pg$xo.lambda, 1:5]
```

	mark246	mark391	mark284	mark157	mark1990
BC009	1.000000e+00	1.0000000000	1.0000000000	1.0000000000	1.0000000000
BC013	1.026860e-03	0.0000000000	0.002317796	0.0000000000	0.003386561
BC032	1.000000e+00	0.735586234	0.672130843	0.60287457	0.639670176
BC061	5.363308e-01	0.552323021	0.539409197	0.54183522	1.0000000000
BC069	0.000000e+00	0.008689148	0.0000000000	0.02198752	0.035294621
BC077	3.478224e-07	0.0000000000	0.0000000000	0.0000000000	0.0000000000
BC085	9.095952e-01	0.955278155	1.0000000000	0.90765213	0.881923903

Genotype BC061 has an extended set of missing values across the first four co-located sets of markers. Using the distance formula in the M-step of the EM algorithm given in (1.5), the estimates of probability for the two missing allele scores in the first two markers for BC061 adds another 1/2 a recombination or 0.15 cM distance between the markers. Genotype BC061 will add a similar distance between the second and third marker set and the third and fourth marker sets due the imputed probabilities being close to 0.5 for the run of missing values across the four co-located sets. If there are multiple runs of missing values for genotypes, these small distances accumulate quickly and linkage group lengths appear inflated.

There are two solutions to this problem. The obvious first solution is to remove the genotypes regardless of their usefulness in the genetic map. The second less obvious solution is to use the R/ASMap function `quickEst()` to re-estimate the genetic distances (see section 2.5.3). As the function imputes the missing values in the linkage map with actual allele calls it circumvents the uncertain probabilities that MSTmap uses to form its genetic distances.

```
R> mapBC4e <- quickEst(mapBC4)
R> chrLen(mapBC4)
```

	L.1	L.2	L.3	L.4	L.5	L.6	L.7
304.910957	266.240647	78.982131	252.281760	33.226962	233.485952	153.315888	
	L.8	L.9					
106.290403	6.657526						

```
R> chrLen(mapBC4e)
```

	L.1	L.2	L.3	L.4	L.5	L.6	L.7
213.056491	201.340804	63.404825	187.741586	26.574889	189.370258	132.623157	

## 4.2 Use of the argument `mvest.bc`

---

	L.8	L.9
	77.211705	2.912879

The difference in the lengths of the linkage groups for the two linkage maps is dramatic with an 80+ cM reduction in L.1 and 60+ cM reductions in L.2 and L.4.

In summary, if a linkage map is constructed with genotypes containing many missing values then the missing value structure of the constructed object should be checked. If runs of missing values are detected then the lengths of linkage groups are most likely inflated and estimation of genetic distances should be checked using `quickEst()`.

## 4.2 Use of the argument `mvest.bc`

The inflated distances caused by excessive missing values across the marker set for a subset of genotypes can be further exacerbated by an injudicious use of the argument `mvest.bc` in the R/ASMap construction functions `mstmap.data.frame()` and `mstmap.cross()`.

To understand how this is possible, there is a requirement to understand how the MSTmap algorithm determines co-segregating or co-locating markers. In the initial stages of the algorithm two markers are deemed co-located if the pairwise distance is zero with this calculation occurring only from pairs of alleles that are observed in both markers. For each co-locating set a marker is chosen as the representative marker and the remaining markers in the set are placed aside with a knowledge of their direct link to the representative marker. The linkage map is then constructed with only the representative markers. This scenario is equivalent to setting `mvest.bc = FALSE`.

If `mvest.bc = TRUE` then missing allele calls contained in each marker are estimated to have probability 0.5 of being an A allele *before* clustering of the markers has occurred. This has an obvious advantage of involving the complete set of genotypes when calculating pairwise information between markers. However, for any two markers where there is some proportion of unobserved allele pairs, the pairwise distance between the markers becomes non-zero and they are deemed to be not co-located. This results in a larger set of markers being involved in the linkage map construction with the original missing allele calls being continually imputed as part of the marker ordering algorithm (see section 1.2.2). For situations where there is an increased number of missing values across a set of markers for a subset of genotypes, the cumulative distances between adjacent markers quickly increases.

To illustrate this problem, the barley backcross marker set `mapBC3` is constructed using the function `mstmap.cross()` with the argument `mvest.bc = TRUE`.

```
R> mapBC4a <- mstmap(mapBC3, bychr = FALSE, trace = TRUE, dist.fun = "kosambi",
  p.value = 1e-12, mvest.bc = TRUE)
R> nmar(mapBC4)
```

L.1	L.2	L.3	L.4	L.5	L.6	L.7	L.8	L.9
574	473	75	551	32	188	160	110	10

### 4.3 Use of the argument `detectBadData`

---

The clustering algorithm produces an identical number of linkage groups with the same markers within each linkage group. However, the number of uniquely located markers within each linkage group increases dramatically between `mapBC4` and `mapBC4a`. This increase in the number of uniquely located markers also increases the length of each linkage group.

```
R> sapply(mapBC4a$geno, function(el) length(unique(round(el$map, 4)))) -  
      sapply(mapBC4$geno, function(el) length(unique(round(el$map, 4))))
```

```
L.1 L.2 L.3 L.4 L.5 L.6 L.7 L.8 L.9  
384 208 44 305 14 97 62 49 4
```

```
R> chrLen(mapBC4a)
```

```
      L.1      L.2      L.3      L.4      L.5      L.6      L.7  
498.063994 340.716640 96.513164 356.269346 41.016980 275.675999 164.254019  
      L.8      L.9  
123.280455 6.952472
```

Again, the solution to this problem is to use the efficient distance estimation function `quickEst()` to provide an estimate of the genetic distances of the markers in each of the linkage groups.

```
R> mapBC4b <- quickEst(mapBC4a)  
R> chrLen(mapBC4b)
```

```
      L.1      L.2      L.3      L.4      L.5      L.6      L.7      L.8  
207.88659 199.40969 62.42760 184.52014 25.92514 187.42973 132.71937 75.91499  
      L.9  
2.58925
```

### 4.3 Use of the argument `detectBadData`

The last two sections discussed constructed linkage map scenarios where the genetic distance calculations became inflated due to an excessive number of missing values being present for a subset of genotypes in the marker set. If this situation is not present and the linkage group distances are inflated then it is most likely caused by an increased recombination rate in some or all of the genotypes. This is easily checked using the appropriate call to `statGen()` or `profileGen()` after an initial linkage map is constructed. If there is an increased recombination rate in a small subset of genotypes they can be removed before further linkage map construction. If there appears to be a general inflation in the recombination rate across a large set or all of the genotypes then there may be genotyping errors that have occurred in the physical process used to obtain the allele calls for the markers. Consequently, reduction of the linkage group lengths becomes more problematic.

### 4.3 Use of the argument `detectBadData`

---

To circumvent this issue, the construction functions `mstmap.data.frame()` and `mstmap.cross()` in the R/ASMap package contain an argument called `detectBadData` that, if set to `TRUE`, instructs MSTmap to detect suspiciously called alleles (see equation 1.6 of section 1.2.2 for more details). If suspicious allele calls are found they are set to missing and imputed using the EM algorithm detailed in section 1.2.2. This creates a reduction in the number of recombinations between adjacent markers. The consequence of this is a possibly dramatic reduction in the distances between adjacent markers and overall linkage group lengths.

To exemplify the use of `detectBadData`, simulated genotyping errors are added to the final barley backcross linkage map `mapBC` by randomly switching singular allele calls in the data for each linkage group.

```
R> mapBCd <- mapBC
R> mapBCd$geno <- lapply(mapBCd$geno, function(el) { ns <- sample(1:ncol(el$data),
  ncol(el$data)/2, replace = TRUE) ns <- cbind(sample(1:nrow(el$data),
  ncol(el$data)/2, replace = TRUE), ns) el$data[ns] <- abs(1 - el$data[ns])
  el$data[el$data == 0] <- 2 el })
R> mapBCd <- quickEst(mapBCd)
R> chrLen(mapBCd)
```

L.1	L.2	L.3	L.4	L.5	L.6	L.7
281.8210	267.0753	171.9688	252.7070	216.8915	157.7318	153.2963

The function `quickEst()` will not ignore the genotyping errors that have been added to the data and linkage groups will appear inflated.

```
R> mapBCda <- mstmap(mapBCd, bychr = TRUE, trace = TRUE, dist.fun = "kosambi",
  p.value = 1e-12, detectBadData = TRUE)
R> chrLen(mapBCda)
```

L.1	L.2	L.3	L.4	L.5	L.6	L.7
227.1167	220.7915	150.1100	209.0600	187.1651	144.1092	140.4516

A judicious use of the `detectBadData = TRUE` in the `mstmap.cross()` call instructs the algorithm to detect and ignore these errors during the marker ordering algorithm of MSTmap. This deflates the linkage group distances to similar lengths as the linkage groups lengths of the final linkage map `mapBC`.

# References

- BROMAN, K. W. & SEN, S. (2009). *A Guide to QTL Mapping with R/qtl*. Springer-Verlag. ISBN: 978-0-387-92124-2.
- BROMAN, K. W. & WU, H. (2014). *qtl: Tools for Analyzing QTL Experiments*. R package version 1.33-7.
- BUETOW, K. H. & CHAKRAVARTI, A. (1987). Multipoint gene mapping using seriation. 1 Gneeral methods. *American Journal of Human Genetics* **41**, 180–188.
- CHERITON, D. & TARJAN, R. E. (1976). Finding minimum spanning trees. *SIAM Journal on Computing* **5**, 724–742.
- LANDER, E. & GREEN, P. (1987). Construction of multilocus genetic linkage maps in humans. *Proceedings of the National Academy of Science* **84**, 2363–2367.
- LYTTLE, T. W. (1991). Segregation distorters. *Annula Reviews of Genetics* **25**, 511–586.
- MARGARIDO, G. & MOLLINARI, M. (2014). *onemap: Software for constructing genetic maps in experimental crosses: full-sib, RILs, F2 and backcrosses*. R package version 2.0-4.
- PRIM, R. C. (1957). Shortest connection networks and some generalizations. *Bell System Techical. Journal* **36**, 1389–1401.
- R DEVELOPMENT CORE TEAM (2014). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0.
- TAYLOR, J. D. & BUTLER, D. (2014). *ASMap: An (A)ccurate and (S)peedy linkage map construction package for inbred populatons that uses the extremely efficient MSTmap algorithm*. R package version 0.3.
- VAN OS, H., STAM, P., VISSER, R., & VAN ECK, H. (2005). Record: a novel method for ordering loci on a genetic linkage map. *TAG Theoretical and Applied Genetics* **112**, 30–40. 10.1007/s00122-005-0097-x.
- WU, Y., BHAT, P. R., CLOSE, T. J., & LONARDI, S. (2008). Efficient and accurate construction of genetic linkage maps from the minimum spanning tree of a graph. *PLoS Genetics* **4**.