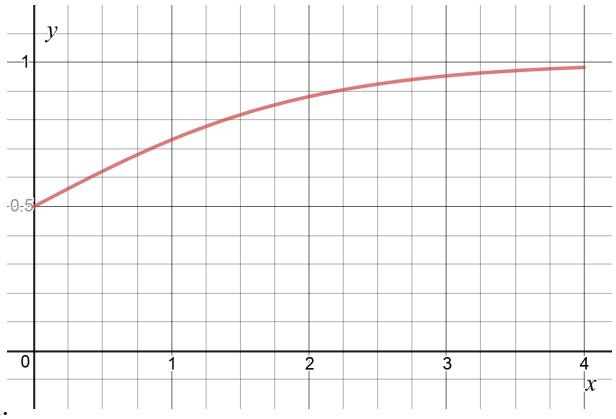
Math 146 - Calculus II Name_____

Approximate the Integral

We have seen approximations of integrals using left, right, and midpoint Riemann sums. Using n subintervals of equal length, we can denote these approximations by L_n, R_n, and M_n, respectively. Today we will look at other approximations, including the **trapezoidal rule** and **Simpson's rule**.

We will work with a form of the logistic function, $f(x) = \dfrac{1}{1+e^{-x}} = \dfrac{e^x}{1+e^x}$, whose graph over the interval $[0, 4]$ is shown here, and we will want to approximate the area under the curve on this interval.



1. Trust me when I say that the exact value of this integral is $I = \ln(1+e^4) - \ln(2) \approx 3.32500275$. Evaluate the left, right, and midpoint approximations: L_4, R_4, and M_4. Compute the errors of each approximation, i.e., compute $I - L_4$, $I - R_4$, and $I - M_4$. Record all of these values in the table below, rounding to 6 decimal places. Include the sign of the error.

Rule	Value of Approximation	Error of the Approximation
L_4		
R_4		
M_4		

2. Notice that L_4 is an underestimate of I and R_4 is an overestimate of I. How could we have known this without computing anything, only based on the graph of f?

3. Since we have an underestimate and an overestimate, it makes sense to do an average. If we average the values of L_n and R_n, we obtain a new approximation, $T_n = \dfrac{L_n + R_n}{2}$, called the **trapezoidal rule**. Compute T_4 and its error, $I - T_4$:

Rule	Value of Approximation	Error of the Approximation
T_4		

4. Look at the errors of the midpoint rule M_4 and the trapezoidal rule, T_4. Explain why it makes sense to compute the **weighted average** $S_{2n} = \dfrac{2}{3}M_n + \dfrac{1}{3}T_n = \dfrac{2M_n + T_n}{3}$. Why is this better than just the average in this case? This is called Simpson's rule.

5. Compute S_8 and its error, $I - S_8$.

Rule	Value of Approximation	Error of the Approximation
S_8		

6. Based on the magnitudes of the errors, order from worst to best the five approximations to I that you computed above.

7. Use the Desmos demo at (`https://www.desmos.com/calculator/qo6i39pkfa`) and fill in the table below. Record all the decimal places Desmos shows you. Briefly comment.

Rule	Approximate Value	Error of the Approximation
L_{20}		
R_{20}		
M_{20}		
T_{20}		
S_{40}		