Part I:

Last class we learned the powerful and important Fundamental Theorem of Calculus. In words, this theorem says that the integral of a rate of change gives total change. In symbols, it says that if F'(x) = f(x), then

$$\int_a^b f(x) \ dx = F(b) - F(a).$$

In tropical climates during rainy season, it rains every day in a predictable pattern. Suppose the rain falls at a rate $r(t) = 3 - 3\cos(2\pi t)$ centimeters per day. Here time t is measured in days starting at midnight, so t = 0.5 is noon and t = 1.5 is noon on the next day.

1. Sketch a graph of r(t) for $0 \le t \le 4$.

(Don't forget all the necessary labeling.)

- 2. From the graph, what is the maximum rate of rainfall and approximately what time of each day does it occur?
- 3. Evaluate the indefinite integral $\int r(t) dt$.
- 4. Note that the rate of rainfall function, r(t), is periodic with a period of 1 day. So the amount of rainfall over any 24-hour time period will be the same. Let's specifically focus on the first 24 hours starting with t = 0. Write down the definite integral that represents the amount of rain that falls in the first day.
- 5. Evaluate your answer using the FTC.

¹This example is based on one in Calculus for Biology and Medicine by Neuhauser and Roper, p. 336

6. Let's take this one step further: set up and evaluate an integral to calculate the amount of rain that falls during the first T days, so for $0 \le t \le T$. Your answer will depend on T.

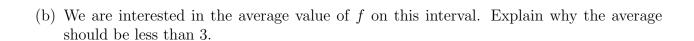
7. Call the function from the previous question R(T). Sketch a graph of R for $0 \le T \le 4$.

8. Describe some of the properties of R, using words like positive, negative, increasing, decreasing, concave up, concave down, etc.

Part II:

Another important use of definite integrals is in calculating the average value of a function.

- 1. Consider the piecewise function $f(x) = \begin{cases} 1 & \text{for } x < 3, \\ 5 & \text{for } x \ge 3. \end{cases}$
 - (a) Sketch a graph of f for $0 \le x \le 4$. (Use filled and empty circles where appropriate.)



(c) It makes sense to consider a weighted average in this case. Show why the average value should actually be 2.

2. Sketch a graph of $f(x) = x^2$ for $0 \le x \le 2$.

(a) Using the graph explain why the average value on this interval would be less than 2.

(b) The formula for the average value \bar{f} of a function f(x) on an interval $a \leq x \leq b$ is $\bar{f} = \frac{1}{b-a} \int_a^b f(x) \, dx$. Verify your explanation by calculating the average value.