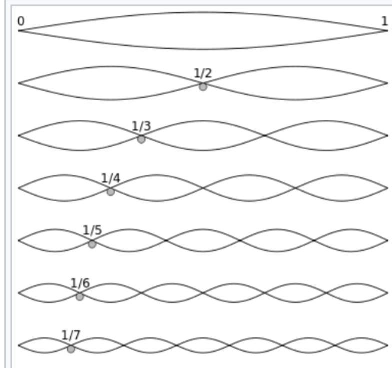
Math 146 - Calculus II Name________________________

Harmonics

Our story of the harmonic series begins with harmonics in music.

> When a string, for example the low, low C on a piano, vibrates, it oscillates up and down over the length of the string. . . . [This] begin[s] to create the air oscillations that our ears perceive as sound. . . . However, this is not all. In addition to the fundamental frequency, the string also vibrates at integer divisions of the string, creating higher pitches . . . based on these ratios.



Harmonics of a string showing the periods of the harmonics (period = 1/frequency)

This phenomena of overtones is illustrated in the diagram on the right. This set of overtones is called the harmonic series

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots$$

SOURCES: Opening quote from *A Feeling for Harmony*, Earlham College of Music, retrievhttps://www.overleaf.com/4269285245mwfchgxthxbned from `http://legacy.earlham.edu/~tobeyfo/musictheory/Book1/FFH1_CH2/2M_HarmonicSeries.html` Image from Wikipedia(Harmonic Series-Music).

1. Consider the harmonic series and the alternating harmonic series

$$\sum_{k=1}^{\infty}\frac{1}{k}=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots \qquad \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\dots$$

(a) Calculate the third partial sum S_3 for each series.

(b) Use appropriate technology to calculate and fill in the indicated partial sums.

n	10	100	1,000
$\sum_{k=1}^{n}\frac{1}{k}$			
$\sum_{k=1}^{n}\frac{(-1)^{k+1}}{k}$			

(c) Make a conjecture about whether each series converges or diverges.

2. Consider the infinite series

$$\sum_{k=1}^{\infty} \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots$$

(a) Compute the first five partial sums S_1, S_2, S_3, S_4, S_5.

n	1	2	3	4	5
$S_n = \sum_{k=1}^{n} \frac{1}{2}$					

(b) Make a conjecture about an explicit formula for S_n.

(c) Find $\lim_{n\to\infty} S_n$.

(d) What can you conclude about this infinite series?

3. Consider the infinite series $\sum_{k=1}^{\infty} \frac{k}{k+1}$. Notice that each term is larger than $\frac{1}{2}$.

(a) What can we conclude about the partial sums of this series?

(b) What does that tell us about the infinite series? Explain.