Math 145 - Calculus I Name________________________

Something's Fishy - Day 28 [1]

Most fish can be classified as *indeterminate* growers, meaning that their length, $L(t)$, increases with their age t throughout their lifetime. The growth rate slows down as they age, giving us the differential equation

$$\frac{dL}{dt} = Ae^{-kt},$$

where A and k are positive constants that vary with the species or habitat of the fish.

1. To warm up, let's suppose $A = 2$ and $k = 1$. So we have $\frac{dL}{dt} = 2e^{-t}$. To **solve** this differential equation, we want a function $L(t)$ whose derivative is $2e^{-t}$. Try to come up with one such function. *(Check your work by computing the derivative of your answer!)*

2. There are actually lots of functions that work (but they are all pretty similar). Can you come up with at least one more function?

3. Okay, so the **general solution** to $\frac{dL}{dt} = 2e^{-t}$ is $L(t) = -2e^{-t} + C$, where C is an arbitrary constant. Right? Right.

   Suppose that the fish start out at 3 centimeters long (at time $t = 0$). What is the value of C in this case and what is $L(t)$ equal to?

4. Continuing with the specific case from the previous problem, graph $L(t)$ for $0 \le t \le 8$ and sketch below. What happens to the length of the fish in the long run?

[1]This example is based on problem 5.10.80 in Calculus for Biology and Medicine by Neuhauser and Roper

5. What is the equation for the general solution if we change to $A = 4$ but keep $k = 1$?

6. Finally, what is the general solution if $A = 4$ and $k = 2$?

So, one kind of differential equation is solved by finding an *antiderivative*; in this case $\frac{dy}{dx}$ is equal to a function of just x. The answer always includes a "$+C$". Find y in each example below. *(Check all your answers by differentiating.)*

1. $\frac{dy}{dx} = x^2$

2. $\frac{dy}{dx} = 4\cos(x)$

3. $\frac{dy}{dx} = 1 + e^x$

4. $\frac{dy}{dx} = \frac{1}{x}$ (assuming $x > 0$)

5. **(Optional challenge:)** $\frac{dy}{dx} = \frac{1}{x}$ for $x < 0$