Math 145 - Calculus I Name_____

Up and Down - Day 13

Part I: (From Section 3.7) Yancey, Uri, and Xan work at an apple farm. Yancey picks apples three times as fast as Uri, and Uri picks apples twice as fast as Xan. We could write the first fact as $\frac{dy}{du} = 3$; it is the rate at which Yancey picks with respect to Uri.

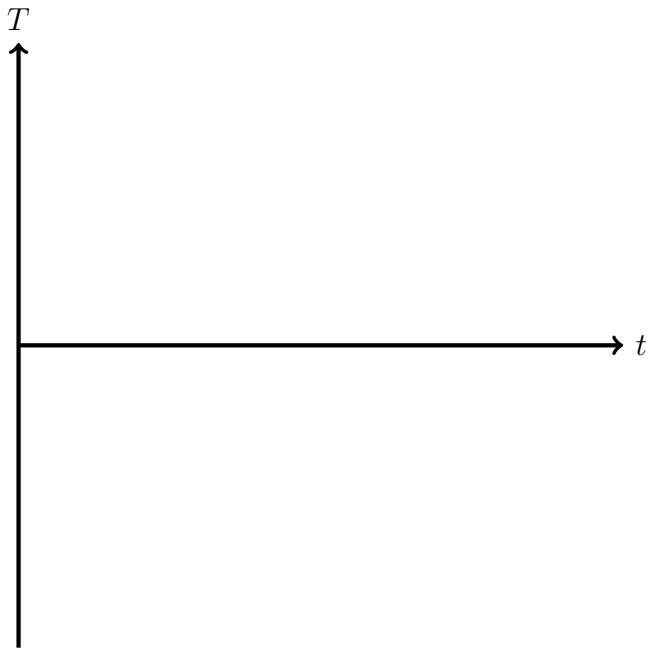
1. Express the fact that Uri picks apples twice as fast as Xan as a derivative.

2. How fast does Yancey pick apples with respect to Xan?

3. Write your answer to 2. in terms of derivatives. This is one important form of today's topic, the chain rule.

Part II: The temperature T in degrees Fahrenheit on a typical day in July can be modeled by the function $T(t) = 78 - 12\cos(\frac{\pi t}{12})$, where t is the number of hours after midnight.

1. Sketch a graph of $T(t)$ for $0 \leq t \leq 24$.



2. Describe the temperature variation over the course of the day.

3. Recall that $T'(x) \approx \dfrac{T(x+h) - T(x)}{h}$ for small h and estimate $T'(6)$ and $T'(10)$.

4. Let $f(x) = 78 - 12\cos(x)$ and let $g(x) = \dfrac{\pi x}{12}$. Compute $f(g(x))$ and $g(f(x))$. Does one of them look familiar?

5. Compute $f'(x)$ and $g'(x)$ for the functions in 4.

6. Calculate these quantities (note that $\dfrac{\pi}{2} = \dfrac{\pi \cdot 6}{12}$):
   - $f(6) =$
   - $g(6) =$
   - $f'(6) =$
   - $g'(6) =$
   - $f(\frac{\pi}{2}) =$
   - $f'(\frac{\pi}{2}) =$

7. Can you find a combination of some of your answers in 6. that agrees with your estimate for $T'(6)$? Write out the formula using a product of two of the values.

8. Using 7., write the chain rule, which shows how to take the derivative of $f(g(x))$.

$$\frac{d}{dx}f(g(x)) =$$

9. Use the chain rule to find the derivative of $T(t) = 78 - 12\cos(\frac{\pi t}{12})$.

10. Use the chain rule to find the derivative of $y = \cos(x^5)$.