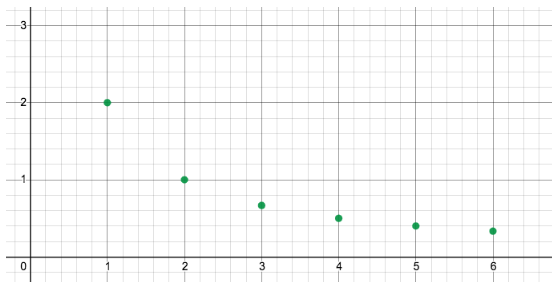
Math 146 - Calculus II Name________________________

**Limits of Sequences**

One method for making a conjecture about the limit of a sequence is to draw its graph. The horizontal axis marks the values of the index $n$ and the vertical axis marks the values of the terms $a_n$. Notice that the graph is just dots (no connecting curve) so we refer to this image as the "dot graph" of the sequence.

For example, here is the dot graph of the sequence $a_n = \dfrac{2}{n}$ for $n \geq 1$. It appears that $a_n$ is




- decreasing
- bounded (above by 2, below by 0)
- limit = 0

1. For each of the sequences listed,
   - Create a dot graph including points for $n = 0$ to $n = 4$.
   - Determine if the sequence is increasing, decreasing, or neither.
   - Determine if the sequence is bounded above, bounded below, bounded (both), or neither.
   - Make a conjecture about the limit of the sequence, or explain why the limit does not exist.

   (a) $a_n = 2^n$

(b) $b_n = \left(\frac{1}{2}\right)^n$

(c) $c_n = 2 - \frac{1}{n+1}$

(d) $d_n = 2 + \frac{(-1)^n}{n+1}$

(e) $e_n = 2 + (-1)^n$

(f) $f_n = \dfrac{2^n}{2^n + 1}$

2. (a) Each of the following sequences is of the form $a_n = r^n$ for $n \geq 0$ where $r$ is a constant. State the value of $r$ and make a conjecture about the limit of each sequence (or say that it does not exist (dne)). Hint: we did the first two sequences in problem 1. For the remaining sequences you may want to calculate the first few terms.

| | | |
|---|---|---|
| $a_n = 2^n$ | $r =$ | $\lim(a_n) =$ |
| $a_n = \left(\frac{1}{2}\right)^n$ | $r =$ | $\lim(a_n) =$ |
| $a_n = (-2)^n$ | $r =$ | $\lim(a_n) =$ |
| $a_n = \left(-\frac{1}{2}\right)^n$ | $r =$ | $\lim(a_n) =$ |
| $a_n = 1$ | $r =$ | $\lim(a_n) =$ |
| $a_n = 0$ | $r =$ | $\lim(a_n) =$ |
| $a_n = (-1)^n$ | $r =$ | $\lim(a_n) =$ |

(b) What is $\lim\limits_{n \to \infty} r^n$ ? Your answer will depend on the value of $r$.