Math 146 - Calculus II Name________________________

Substitution

The integration technique known as *substitution* is the integration version of the chain rule for derivatives. We'll start by practicing some chain rule.

1. Find the derivatives of the following functions:

   (a) $f(x) = 3e^{x^2}$

   (b) $g(t) = \sqrt{1 + 3\sin(t)}$

   (c) $h(a) = 2\ln(e^a + 2)$

2. So the chain rule involves taking the derivative of the "inside" function. We need to keep that in mind when doing antiderivatives. For example consider $\int 3x^2 \cos(x^3)\, dx$.

   (a) Why might someone guess that the antiderivative is $x^3 \sin(x^3)$?

   (b) Explain why the suggestion in (a) is incorrect.

(c) If you think about $3x^2\cos(x^3)$ being the result of the chain rule, what would be the inside function? And what's the derivative of that function?

(d) Explain why $\int 3x^2\cos(x^3)\,dx = \sin(x^3) + C$ is the correct antiderivative.

(e) Finally, what is $\int x^2\cos(x^3)\,dx$?

3. In substitution, we'll be trying to evaluate integrals by letting u be the inside function. Make guess for u for the examples below, and then calculate $\frac{du}{dx}$ for your guess.

(a) $\displaystyle\int xe^{x^2+1}\,dx$

(b) $\displaystyle\int \sin(x)\cos^3(x)\,dx = \int \sin(x)(\cos(x))^3\,dx$

(c) $\displaystyle\int \frac{x^3}{2+3x^4}\,dx$