Math 145 - Calculus I Name________________________

The Wheels on the Bus - Day 36

The class is going on a field trip, but the odometer on the school bus (which tracks distance) is broken. The speedometer works, so you begin keeping track of velocity data on the trip:

Time, t (min)	0	15	30	45	60
Velocity, v (mph)	0.0	42.9	50.0	52.9	54.5

We will be estimating the distance traveled using Riemann sums of the form

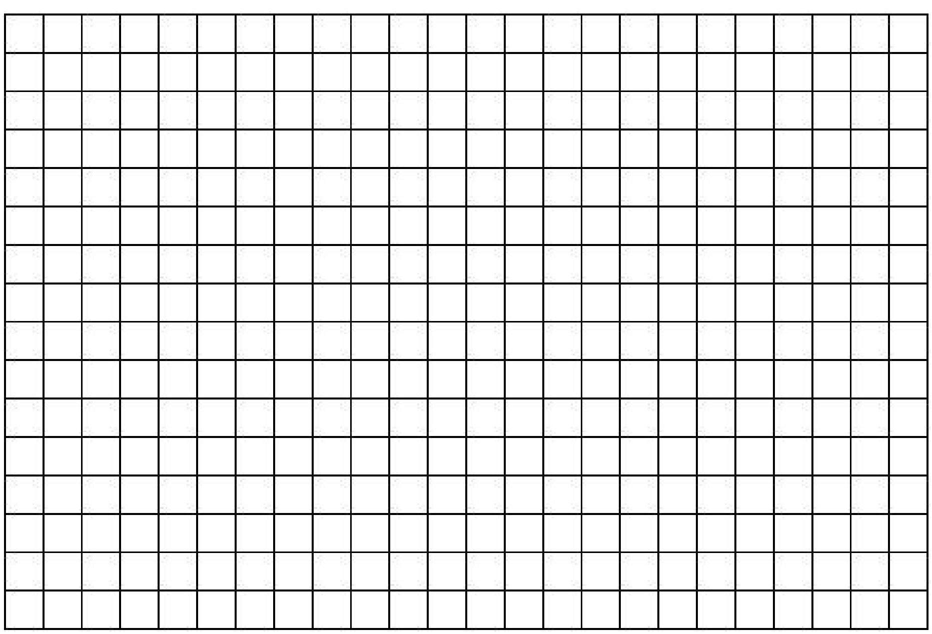
$$\sum_{k=1}^{n} v(t_k)\Delta t = (v(t_1) + v(t_2) + \cdots + v(t_n))\,\Delta t.$$

Recall that "velocity $\times$ time" gives "displacement," but we need to make sure the units are consistent.

1. We will keep the velocities in miles per hour and convert time to hours. What is Δt in hours?

2. Compute the *left* Riemann sum with $n = 4$ subintervals to estimate the distance traveled. Assuming the velocity is increasing during the 60 minutes, is this result an *overestimate* or an *underestimate* of the distance traveled over the hour?

3. Let's make this visual. Plot the velocity data on the grid provided. Then draw appropriate rectangles to represent your calculation from 2. in the grid below.



4. Now compute the *right* Riemann sum with $n = 4$ subintervals to estimate the distance traveled. With the same assumption as in 2., is this result an *overestimate* or an *underestimate* of the distance traveled over the hour? Also represent your calculation in the grid on the previous page, using different color rectangles.

5. Compute the average of the left Riemann sum and the right Riemann sum to produce a single estimate of the distance traveled. What is the answer?

6. Suppose we had velocity measurements not every 15 minutes, but every 7.5 minutes instead. We would expect our approximations to the distance traveled to be *more/less accurate* then before (circle the correct answer).

7. Consider now the updated data in the table below.

Time, t (min)	0	7.5	15	22.5	30	37.5	45	52.5	60
Velocity, v (mph)	0.0	33.3	42.9	47.4	50.0	51.7	52.9	53.8	54.5

Redo the work from parts 2., 4., and 5. to produce a new estimate of the distance traveled. Note that in this case $n = 8$.

8. Our intuition tells us that in order to produce better estimates of the distance traveled, we would like to have more data points, or more intervals to work with. Use this idea to replace the question marks in the (very important) statement below:

$$\text{distance traveled by the bus} = \lim_{???} \sum_{k=1}^{n} v(t_k)\Delta t.$$

(Hint: We used $n = 4$ and $n = 8$ in the previous parts.)

9. In the previous parts, the velocity function $v(t) = \dfrac{60t}{t+0.1}$ (with time t in hours) was used to generate the data (but you didn't know it). A friend comes to you and tells you that the position function for the bus is

$$s(t) = 60t - 6\ln(t + 0.1).$$

How can we check that this is a possible position function? Do the work below and ask if unsure.

10. Using the position function $s(t)$ from 9., how far exactly does the bus travel in 1 hour? How does the answer compare to your estimates from 5. and 7.?