Math 145 - Calculus I Name______________________

The FTC - Day 37

We have already connected the area between the velocity curve, $v(t)$, and the t-axis to the displacement of the object in motion with velocity $v(t)$. This will lead us to a very important theorem today, the *Fundamental Theorem of Calculus*.

1. If $s(t)$ is the position function at time t of an object moving along a straight line, how can we write the *displacement* of the object over the time interval $a \leq t \leq b$ using $s(t)$? (E.g., if your odometer at the start of a short straight trip reads 120,356.5 mi and at the end 120,363.8 mi, what was the displacement?)

2. If $v(t)$ is the velocity function at time t, what is the relationship between $s(t)$ and $v(t)$?

   - The function $v(t)$ is ______________________________ of the function $s(t)$.
   - The function $s(t)$ is ______________________________ of the function $v(t)$.

3. We know that geometrically $\int_a^b v(t)\,dt$ denotes the "net area" between $v(t)$ and the t-axis. But, one more time, it has another meaning. If the units of t are hours and the units of $v(t)$ are miles per hour, what are the units of the integral and what does the integral represent in the context of the motion?

4. For each of the following velocity functions $v(t)$, determine a possible position function $s(t)$:

   (a) $v(t) = 3$ $\qquad s(t) =$

   (b) $v(t) = 3t$ $\qquad s(t) =$

   (c) $v(t) = 3t^2 - t + 2$ $\qquad s(t) =$

   (d) $v(t) = 8\sqrt{t}$ $\qquad s(t) =$

   (e) $v(t) = \dfrac{1}{t}$ $\qquad s(t) =$

   (f) $v(t) = \sin(2t) + 1$ $\qquad s(t) =$

5. We have now seen two ways to compute the displacement of a moving object: via an integral of the velocity function $v(t)$ and via a difference of values of the position function $s(t)$. This can be summarized by the following statement:

$$\int_a^b v(t)\,dt = s(b) - s(a).$$

Use this idea to compute the integrals of the functions from 4. on the previous page:

(a) $\displaystyle\int_0^3 3\,dt =$

(b) $\displaystyle\int_1^3 3t\,dt =$

(c) $\displaystyle\int_{-1}^1 (3t^2 - t + 2)\,dt =$

(d) $\displaystyle\int_0^1 8\sqrt{t}\,dt =$

(e) $\displaystyle\int_1^e \frac{1}{t}\,dt =$

(f) $\displaystyle\int_0^\pi (\sin(2t) + 1)\,dt =$

6. The following integrals do not have an independent variable t. However, you can still imagine that the integrand is a "velocity" in some context. Use the ideas you discovered above to evaluate the integrals below:

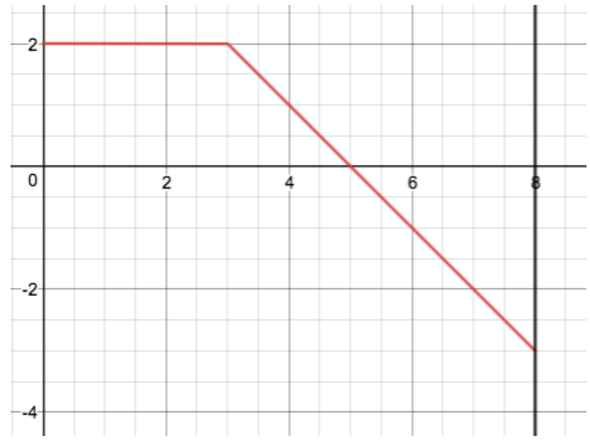
(a) $\displaystyle\int_{-1}^2 (2x - 1)\,dx =$

(b) $\displaystyle\int_0^\pi \cos(z)\,dz =$

(c) $\displaystyle\int_1^3 \frac{1}{w^3}\,dw =$

Extra Credit (up to 5 points)

One other big concept is the idea of the *net area* between a curve and the horizontal axis. *(Net here means overall, as in net profits.)* Let's try to clarify this idea with some examples. Let's call the function graphed on the right $f(x)$.



Below, questions 1.-4. each ask two questions. Make sure to answer both, they may not have the same answer!

1. What is the area between f and the x-axis for $0 \leq x \leq 3$? What is $\int_0^3 f(x)\,dx$ equal to?

2. What is the area between f and the x-axis for $3 \leq x \leq 5$? What is $\int_3^5 f(x)\,dx$ equal to?

3. What is the area between f and the x-axis for $0 \leq x \leq 5$? What is $\int_0^5 f(x)\,dx$ equal to?

4. What is the area between f and the x-axis for $5 \leq x \leq 8$? What is $\int_5^8 f(x)\,dx$ equal to?

5. All of your answers to areas so far should be positive, but only the first three integrals are positive. Integrals corresponds to *net areas*, where areas below the x-axis count as negative. Using this observation, what is $\int_0^8 f(x)\,dx$ equal to? Show your calculation giving the result. (This is quantity is the *net area* between $f(x)$ and the x-axis between $x = 0$ and $x = 8$.)