Math 145 - Calculus I Name________________________

Part of Your World - Day 32

Last time we learned about substitution, which is the integration version of the chain rule. Today we'll learn another technique called *integration by parts*, which is related to the product rule for derivatives. We'll be able to integrate functions like $\int te^{2t}\,dt$ or $\int 3\ln(2x)\,dx$. These integrals can occur in applications from probability in statistics to heat transfer equations in physics.

1. Let's warm up with some product rule problems. Find $f'(x)$ for each function below:

   (a) $f(x) = 3x\sin(x)$

   (b) $f(x) = \sqrt{x}\ln(x)$

   (c) $f(x) = x^3e^{3x}$

2. Consider the integral $\int x\cos(x)\,dx$. We've talked about how you can't just integrate each part. Demonstrate by differentiating that $\dfrac{x^2}{2}\sin(x) + C$ is **not** the antiderivative here.

3. The integration by parts formula is

$$\int u\,dv = uv - \int v\,du.$$

You'll see where this formula comes from in class. We use this formula to rewrite a given integral as a new integral, which is hopefully easier to find. Let's walk through this for $\int x\cos(x)\,dx$. We will choose

$$u = x \qquad\qquad dv = \cos(x)\,dx$$

Note that we always include the dx from the integrand with the dv term. To use the formula, we need to know du and v. We find these by doing a derivative and an antiderivative, respectively. So

$$du = \qquad\qquad v =$$

Be sure to include a dx with your du. No need for a $+C$ with v (as we will see in class).

4. So now plug the pieces into the integration by parts formula:

$$\int x\cos(x)\;dx =$$

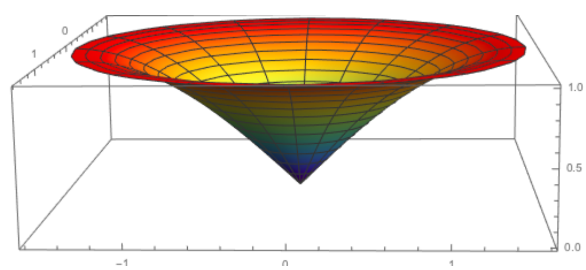
We get two terms, the second of which is a new integral. Is this new integral one that we can easily evaluate?

5. Evaluate the new integral, and complete the problem with a $+C$.

6. Check your answer by taking a derivative. You should see how this process gives you the terms you need to compensate for the product rule.

7. Use integration by parts to evaluate $\int 2xe^x\,dx$. *(Hint: Start with $u = 2x$. What must dv be?)*

8. In Calc II you'll learn how to find the volume of certain objects.



The integral $\int 2\pi x \sin(x)\,dx$ can help to calculate the volume of the bowl pictured. Use integration by parts to evaluate this integral.