Math 145 - Calculus I Name________________________

Using the FTC - Day 38 [1]

Part I:

Last class we learned the powerful and important *Fundamental Theorem of Calculus*. In words, this theorem says that the integral of a rate of change gives total change. In symbols, it says that if $F'(x) = f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

In tropical climates during rainy season, it rains every day in a predictable pattern. Suppose the rain falls at a rate $r(t) = 3 - 3\cos(2\pi t)$ centimeters per day. Here time t is measured in days starting at midnight, so $t = 0.5$ is noon and $t = 1.5$ is noon on the next day.

1. Sketch a graph of $r(t)$ for $0 \le t \le 4$. *(Don't forget all the necessary labeling.)*

2. From the graph, what is the maximum rate of rainfall and approximately what time of each day does it occur?

3. Evaluate the indefinite integral $\int r(t)\,dt$.

4. Note that the rate of rainfall function, $r(t)$, is periodic with a period of 1 day. So the amount of rainfall over any 24-hour time period will be the same. Let's specifically focus on the first 24 hours starting with $t = 0$. Write down the definite integral that represents the amount of rain that falls in the first day.

5. Evaluate your answer using the FTC.

[1] This example is based on one in Calculus for Biology and Medicine by Neuhauser and Roper, p. 336

6. Let's take this one step further: set up and evaluate an integral to calculate the amount of rain that falls during the first T days, so for $0 \leq t \leq T$. Your answer will depend on T.

7. Call the function from the previous question $R(T)$. Sketch a graph of R for $0 \leq T \leq 4$.

8. Describe some of the properties of R, using words like positive, negative, increasing, decreasing, concave up, concave down, etc.

Part II:
Another important use of definite integrals is in calculating the average value of a function.

1. Consider the piecewise function $f(x) = \begin{cases} 1 & \text{for } x < 3, \\ 5 & \text{for } x \geq 3. \end{cases}$

   (a) Sketch a graph of f for $0 \leq x \leq 4$. *(Use filled and empty circles where appropriate.)*

(b) We are interested in the average value of f on this interval. Explain why the average should be less than 3.

(c) It makes sense to consider a weighted average in this case. Show why the average value should actually be 2.

2. Sketch a graph of $f(x) = x^2$ for $0 \leq x \leq 2$.

(a) Using the graph explain why the average value on this interval would be less than 2.

(b) The formula for the average value $\bar{f}$ of a function $f(x)$ on an interval $a \leq x \leq b$ is $\bar{f} = \dfrac{1}{b-a} \displaystyle\int_a^b f(x)\,dx$. Verify your explanation by calculating the average value.