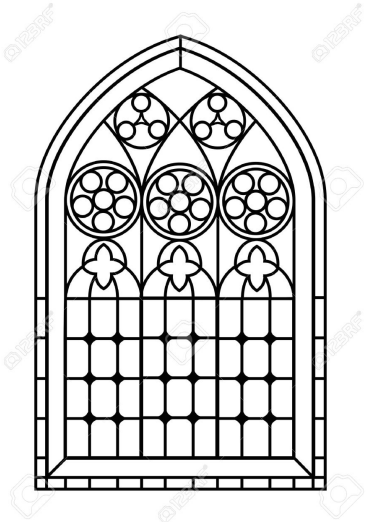
Math 146 - Calculus II Name_____

Gothic Window - Calculating Area

For the next couple of days we're going to be talking about a new mathematical question: how to accurately calculate the area under a curve. Knowing area is important in applications such as architecture, urban planning, and farming. Our first example will be a gothic window something like this.



1. Ignoring the interior details, let's suppose we have a window with this outline and we want to know the area so that we know how much light the window will let in. Suppose the width is 1.4 meters, the height is 2.6 meters, and the vertical sides are straight up to 1.8 meters. Draw a sketch of this window on your graph paper. Make each tick mark represent .1 meters.

2. Explain how we can say for certain the the area of the window is more than $2.52m^2$ and less than $3.64m^2$.

3. One way to get a better approximation of the area is by counting squares. The bottom rectangle has an area of exactly $2.52m^2$, so let's just focus on the top part of the window. First, what is the area of one of your squares?

4. Okay, now how many squares are completely contained in the top part of the window? This gives you what underestimate of the area?

5. Now let's count the squares for which any part of the square is in the window. This gives you what overestimate of the area?

6. By averaging the previous two answers and adding the area of the big rectangle, come up with your best guess of the area of the window.

7. Let's get some functions involved here. On Desmos graph the function $f(x) = 1.76x - 0.88x^2$ for $0 \leq x \leq 0.7$. This should look like half of the top part of our gothic window. What point should indicate the peak of the window? Verify that this function goes through the correct point (approximately).

8. We want to find the area of this part of the window, so we want the area under the curve for $0 \leq x \leq 0.7$. Let's divide that interval into 7 subintervals, each with a length of 0.1. We will then use the right endpoint of each subinterval to determine the height of a rectangle (with a base of 0.1 m), and add up the areas of all of the rectangles to get an approximation of the area. Sketch a picture and then do the calculations below, or on your graph paper.

9. Using the answer from above, what would be your new approximation of the total area of the window?

   The beauty of this method is that it's an algorithm that can be automated and improved by using more rectangles. That's much more difficult to do with counting squares.