Math 146 - Calculus II   Name_____
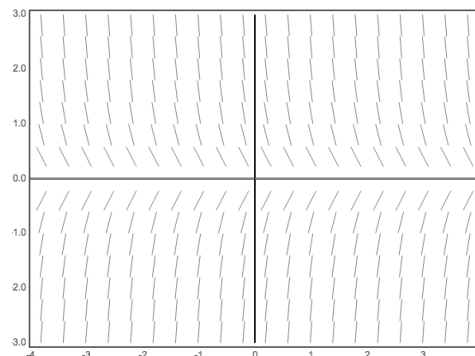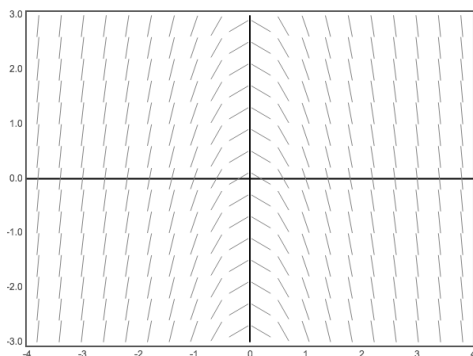**Series and Differential Equations**

1. Consider the differential equation $y' = -5y$ with initial condition $y(0) = 1$.

   (a) Which of these slope fields fits this differential equation? Why?

   (b) Sketch a solution with $y(0) = 1$ on the correct slope field.

   (c) Let $y(x) = c_0 + c_1 x + c_2 x^2 + \cdots = \sum_{k=0}^{\infty} c_k x^k$ be a series solution to this equation. Find c_0 through c_3.

(d) Do you see a pattern in the c_k? If so write down the series as a single sum. If not, find c_4, and beyond if necessary.

(e) Do you recognize this answer as the Taylor Series for a function that you know? Identify the function, and verify that it is a solution to the differential equation.

(f) Graph your solution to check that it fits with your answer to (b). Yes?

2. Let's try this process on a second order differential equation, $y'' = -y$ with initial conditions $y(0) = 1$ and $y'(0) = 0$.

(a) Start with the same general form of a series solution $y(x) = c_0 + c_1 x + c_2 x^2 + \ldots$ and find c_0 through c_4 using whatever method makes sense to you.

(b) Use the pattern you've found to write your answer as a single sum.

(c) You should recognize this series as belonging to a known function. Write down that function and verify that it satisfies $y'' = -y$.