Math 146 - Calculus II Name________________________

Inverse Trig

Today we're going to work on derivatives and antiderivatives involving inverse trigonometric functions. Doesn't that sound impressive? Inverse trig is important for some applications, and also gives us some important integration formulas. Note that all angles are measured in radians!

1. First some basic trig values, so we have this all in one place.

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
0			
$\pi/6$			
$\pi/4$			
$\pi/3$			
$\pi/2$			
$3\pi/4$			
π			

2. Let's start with the inverse sine function, which we call $\arcsin(x)$ or $\sin^{-1}(x)$. This is defined as follows:

$$y = \arcsin(x) \text{ if } \sin(y) = x \text{ and} \frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$$

   Use your table to find these values exactly:

   (a) $\arcsin(0)$.

   (b) $\arcsin(1)$.

   (c) $\arcsin(0.5)$.

   Check these answers using your calculator.

3. Sketch a graph of the inverse sine function for $-2 \leq x \leq 2$.

4. Verify this derivative formula:

$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

This derivative formula automatically gives us an integration formula. Write down this integration formula.

5. The other important function for Calc 2 is the inverse tangent function, which is defined as

$$y = \arctan(x) \text{ if } \tan(y) = x \text{and} \frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$$

Find exactly the values below using your knowledge of trigonometry:

   (a) $\arctan(0)$.
   (b) $\arctan(1)$.
   (c) $\arctan(\sqrt{3})$.

   Again check these answers using your calculator

6. Sketch a graph of the inverse tangent function for $-2 \leq x \leq 2$ on your paper.

7. Use the same technique (including the trig identity) that we used to differentiate $\arcsin(x)$ to find the derivative of the inverse tangent function. You should end up finding that:

$$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$$

8. This derivative formula automatically gives us an integration formula. Write this down.

9. Use your new formulas to evaluate following integrals:

(a) $\displaystyle\int \frac{1}{\sqrt{1-4x^2}}\,dx = \int \frac{1}{\sqrt{1-(2x)^2}}\,dx$

(b) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx$