Math 145 - Calculus I Name________________________

Euler's Method and Slope Fields - Day 27

Solving differential equations can be challenging. Sometimes we use tools to *approximate* solutions instead. We'll be learning two such tools today.

We will start with an example of model of language adoption. Here t is time in months and W is the percentage of the population familiar with a new word, such as *cryptocurrency* or *glamping*. The differential equation for this situation is given by:

$$\frac{dW}{dt} = 0.02W(100 - W).$$

1. What does it mean to say $W(0) = 20$?

2. What is the value of $\frac{dW}{dt}$ at the point $t = 0$, $W = 20$? What is the meaning of this answer?

3. Starting with $W(0) = 20$ and using $\frac{dW}{dt}$ from 2., what would you predict $W(1)$ to be?

4. So now we're at a new point, $t = 1$ and $W =$____________. What is $\frac{dW}{dt}$ at this new point?

5. Use the information from 4. to predict the value of $W(2)$.

6. Hmmm, that answer is a bit problematic. Why?

This technique of approximating solutions to differential equations is called **Euler's method** *(pronounced "oiler's method").*

Another idea we'll make use of is *slope fields.* A slope field for a differential equation is an image of the xy-plane with small lines showing the slope at a number of points. Let's look at the example

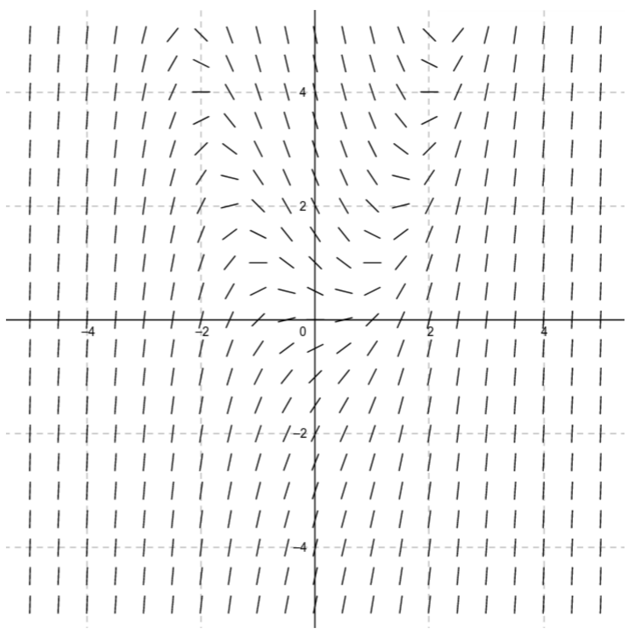
$$\frac{dy}{dx} = x^2 + y.$$

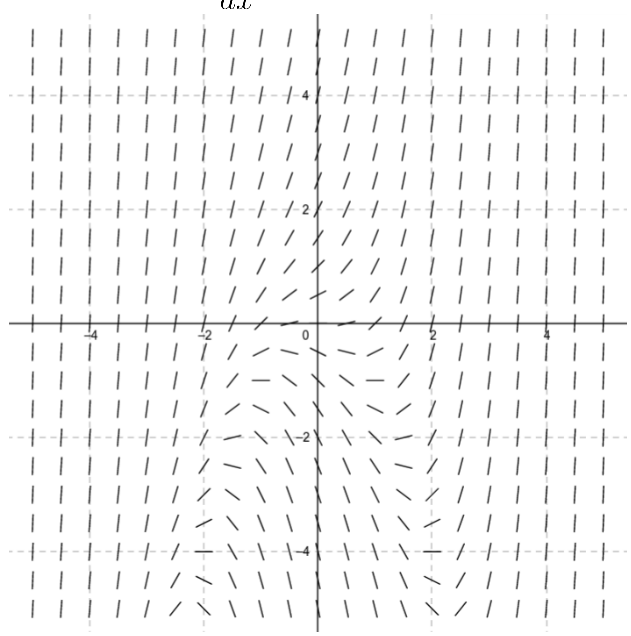
So for example at the point $(x, y) = (-1, 1)$, the slope is $\dfrac{dy}{dx} = (-1)^2 + 1 = 2$. Thus we'd draw a small line with a slope of 2 at the point $(-1, 1)$.

1. Calculate the slope at the following points:

x	-1	0	2	-1
y	1	1	0	-1
slope	2			

2. Below you see two slope fields. Which one is the slope field of $\dfrac{dy}{dx} = x^2 + y$ and why?





3. On the correct graph mark the point satisfying $y(0) = 1$ (which you read as "y of 0 is 1"). Then sketch the solution going through that point as a smooth curve, following the directions suggested by the slope field lines. Be sure to move both forward and backward from your starting point.

4. On the same graph draw the solution curve satisfying $y(2) = 0$.