Math 145 - Calculus I Name________________________

F is for Factorials and Fitting - Day 19

Part I: Factorials

You might have seen factorials before. For example "5 factorial" is $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

1. Calculate the following factorials:
   (a) $3!$
   (b) $4!$
   (c) $6!$

2. Consider the fraction $\frac{6!}{4!}$.
   (a) Use your answers from problem 1. to evaluate $\frac{6!}{4!}$.
   (b) Expand the numerator and the denominator of the fraction as in the example above for $5!$, cancel terms that can be canceled, evaluate, and compare to your answer in (a).

3. One interpretation of factorials is that $n!$ gives the number of different ways to arrange (or order) n objects one after another. Use this idea to explain why $3! = 6$ by writing down different possible orders of the letters A, B, and C (two of which are ABC and ACB).

4. From the definition at the top of the page, it should be clear that $1! = 1$. In how many ways can you arrange 1 object "one after another"? Is this answer consistent with $1! = 1$?

(One can also think that there is 1 *way to order* 0 *items, so in mathematics we define* $0! = 1$.*)*

(Optional:) How could you compute $\frac{1001!}{999!}$? Try this expression in Desmos. Why do you think it fails?

Part II: Fitting Functions

We've been talking about linear approximations all semester, and we began talking about quadratic approximations in the last class. Today we'll see that you can use higher-degree polynomials to get an even closer match.

5. Go to `https://www.desmos.com/calculator/tlzypi43az`. Your first job is to "play with the sliders" and choose values of a, b, c, d, f (in this order) to best fit the function $g(x) = \ln(1+x)$ near $x = 0$. Write down your chosen values of a, b, c, d, f below.

6. Today we'll learn how to use derivatives to calculate the "best" values for these constants. In this case the answer is $p(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$. Add this function to your graph as a third curve. Describe how this function fits the original, and how the values differ from your choices.

7. If we call the function above $p_4(x)$ (since it is a polynomial of degree 4), what do you think the formula for $p_5(x)$ will be? *(Hint: Look at the pattern in the first* 4 *terms to guess the* 5^{th}*.)*

   Add this to your graph to see how it looks. Better?

8. Modify the function in the Desmos notebook to $g(x) = \cos(x)$, zoom out and pan to see it well, and choose values of a, b, c, d, e, f to approximate $g(x)$ near $x = 0$. Record your choices below.

9. This time the "best" answer is $p(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$. Again add this to your graph and compare and describe the result.