Math 146 - Calculus II Name________________________

Comparing Series

Today we're going to learn some comparison tests for infinite series. For a comparison test to work, you need to know some series to serve as a basis for comparison. So let's start there.

1. Geometric Series: A geometric series looks like $a + ar + ar^2 + ar^3 + \dots$.
   (a) Write this series using Σ notation.

   (b) This series diverges if:

   (c) This series converges if:

   (d) If the series converges, it converges to:

   (e) Give an example of a divergent geometric series.

2. p-Series: Last class we used the integral test to classify p-series, which have the form $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^p}$.
   (a) A p-series converges if:

   (b) A p-series diverges if:

   (c) Give an example of a convergent p-series.

3. Okay, so consider the series $\sum_{k=1}^{\infty} \frac{1}{k^2+3}$.

   (a) Explain why this is **not** a p-series.

   (b) What p-series would you choose to compare this with? Does that series converge or diverge?

   (c) Which of these is true? Why?

$$\frac{1}{k^2+3} < \frac{1}{k^2} \qquad\qquad \frac{1}{k^2+3} > \frac{1}{k^2}$$

   (d) So, we have a series of **positive** terms, and each term is less than the corresponding term of a series that we know converges. So we can conclude that our new series converges. Right?

4. Suppose we are given Series A: $\displaystyle\sum_{k=2}^{\infty} \frac{1}{\ln(k)}$.

(a) Sketch a graph to show that $\ln(x) < x$ for $x \geq 2$.

(b) We will compare Series A with Series B: $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k}$. Does this Series B converge or diverge? How do you know?

(c) In order to draw a similar conclusion about Series A, what will we want to show about the relationship between the terms, $a_k = \dfrac{1}{\ln(k)}$ and $b_k = \dfrac{1}{k}$?

(d) Complete the argument to explain why Series A diverges.