Math 145 - Calculus I   Name_____
Predictions - Day 18

Suppose that $C(v)$ is a function that gives the cost of a speeding ticket if you are driving v miles per hour **above** the speed limit.

1. What is the value of $C(0)$ and why?

2. Suppose you only know that $C(10) = 70$. Based on this, what would you guess for the cost of a speeding ticket for going 15 miles above the limit? How confident are you in that answer?

3. Suppose that you know $C(10) = 70$ and $C'(10) = 3$. Determine the units for these two numbers and use this new information to make a new guess for the cost of a speeding ticket for going 15 miles above the limit.

4. To be more formal about this, assuming again that we know $C(10) = 70$ and $C'(10) = 3$, determine the equation of the tangent line to $C(v)$ at $v = 10$. Write your answer in the form $L(v) = a + b(v - 10)$ with a and b replaced by the appropriate numbers. *(L for linearization.)*

5. Using your equation for $L(v)$, determine $L'(10)$. This should look familiar, if not, ask your professor. Why should this result not be surprising?

Better approximations to $C(v)$ at $v = 10$ might be obtained by using a quadratic function instead of a linear one. The quadratic function would mimic the linear function with an extra squared term:

$$Q(v) = a + b(v - 10) + c(v - 10)^2.$$

In problem 5. you should have observed that $L'(10) = C'(10)$, and we of course also have that $L(10) = C(10)$. So for our quadratic approximation we should require its value and its derivatives at $v = 10$ to match those of $C(v)$, that is, we should eventually want:

$$Q(10) = C(10), \qquad Q'(10) = C'(10), \qquad Q''(10) = C''(10).$$

6. Using $Q(v)$ defined above, evaluate the quantities below (answers will have a, b, or c in them):

$$Q(10) \qquad\qquad Q'(10) \qquad\qquad Q''(10)$$

7. Suppose now we know that $C(10) = 70$, $C'(10) = 3$ (as before), but we also know that $C''(10) = 1$. Use this information, your work in 6., and the matching conditions above to determine the coefficients a, b, and c in $Q(v)$, and write down a formula for $Q(v)$.

8. Use $Q(v)$ to estimate the cost of a speeding ticket for going 15 miles above the limit.

9. As a final step, write down both your approximations, $L(v)$ from problem 4. and $Q(v)$ from problem 7. What similarities do you notice in the two formulas?