Math 145 - Calculus I Name________________________

Multiply and Divide - Day 12

We now know how to compute the derivatives of quite a few functions, and we know how to deal with sums and constant multiples of those functions. But what if we have a product, like $x^2 \sin(x)$, or a quotient, like $\frac{e^x}{x^3}$? We'll learn rules for computing derivatives of functions like this today.

1. Let's start with a practical example. Chen is a runner with a stride length of 3 feet per stride. He takes 2 strides per second.

   (a) What is Chen's speed in feet per second?

   (b) What if Chen's stride length increases by 0.5 feet per stride? How much does Chen's speed increase? *(Write your answer as a product using this number.)*

   (c) What if Chen's pace increases by 0.25 strides per second? How much does Chen's speed increase? *(Again, write your answer as a product using this number.)*

   (d) Now what if Chen has trained very hard and manages to make both of those increases? Explain why it makes sense to say that his speed increases by 1.75 feet per second overall.

   (e) After the class discussion, write this in terms of the product rule.

2. Before we do some geometry, let's look at an example with functions. Consider the function $p(x) = x \sin(x)$.

   (a) Sketch $p(x)$ for $-5 \leq x \leq 5$.

   (b) One might expect that the derivative of $p(x)$ will be simply the derivative of x times the derivative of $\sin(x)$. What would that answer be?

   (c) Sketch a graph of your answer from (b). Explain, based on the two graphs, why this is **not** the correct formula for the derivative of $p(x)$.

3. Here's a geometric view of the product rule:

   (a) Draw a rectangle and label the sides p and q.

   (b) What is the area A of this rectangle in terms of p and q?

   (c) Now extend each side of the rectangle by a little bit, so that the new sides are $p + \Delta p$ and $q + \Delta q$. What is the area of this new rectangle? Write your formula in terms of p, Δp, q, and Δq.

   (d) What is ΔA, the change in area, in terms of $p, q, \Delta p$, and Δq? Simplify your answer.

   (e) Again, after the class discussion, explain how this examples motivates the product rule.