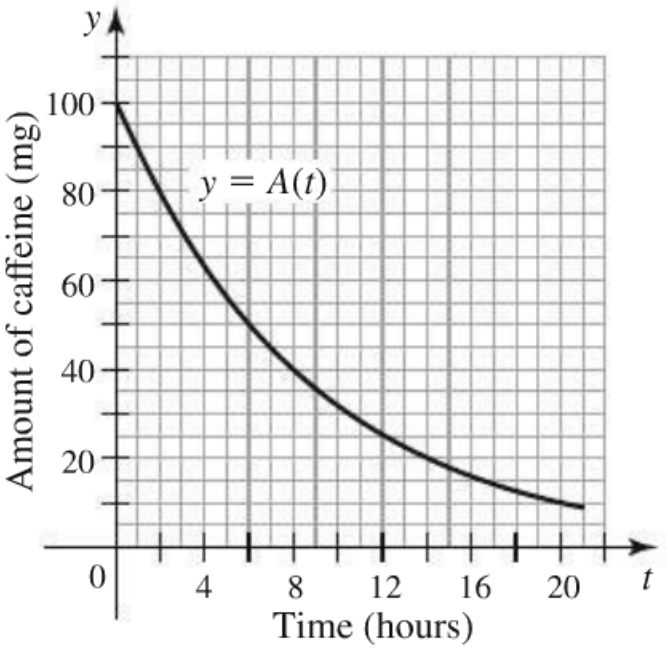
Math 145 - Calculus I Name_____

Derivative as a Function - Day 6 [1]

Caffeine is a chemical found in coffee, tea, cola, guarana, mate, and other products. It is most commonly used to improve mental alertness.[2] According to the Mayo Clinic, it is safe for most healthy adults to consume up to 400 milligrams (mg) per day.[3] An 8-oz cup of coffee contains between 80 and 200 mg of caffeine, depending on the varietal and brew method, while a 12-oz can of Red Bull contains 111 mg. In about 45 minutes, 99 % of the caffeine from your coffee, tea or energy drink has entered the bloodstream fully.[4]

The graph on the right shows the amount $A(t)$ of caffeine (in mg) in the bloodstream t hours after 100 mg have been fully absorbed.



1. What is the amount of caffeine in the bloodstream after 5 hours? 10, 15, 20 hours? Make a small table with the relevant values of t and $A(t)$.

2. What is the sign of the derivative, $A'(t)$, for any t between 0 and 21 hours? How do we know?

3. Draw the graph of the tangent line to A at $t = 5$. Plot your tangent line for t between 0 and 10. Remember that its slope is $A'(5)$. Determine $A'(5)$ including its units.

4. Repeat the process with $t = 10$ to find $A'(10)$ with units.

5. Think about how the values of the derivative at other points (e.g., $A'(0)$, $A'(15)$) would compare to $A'(5)$ and $A'(10)$ and sketch the derivative $y = A'(t)$ for $0 \leq t \leq 21$ hours by smoothly connecting the computed values of the derivative.

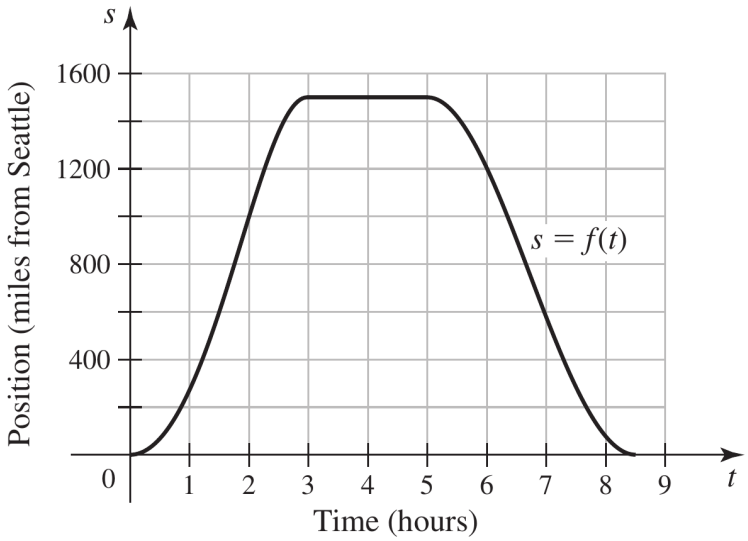
[1] Examples adapted from our textbook.

[2] https://www.webmd.com/vitamins/ai/ingredientmono-979/caffeine

[3] https://www.mayoclinic.org/healthy-lifestyle/nutrition-and-healthy-eating/in-depth/caffeine/art-20045678

[4] https://www.goviter.com/blogs/viter-energy-blog/caffeine-kick-in

The figure shows the position function of an airplane on an out-and-back trip from Seattle to Minneapolis-St. Paul, where $s = f(t)$ is the number of ground miles from Seattle t hours after take-off at 6:00 AM. The plane returns to Seattle 8.5 hrs later at 2:30 PM. So, the flight consists of two legs, one from Seattle to MSP and one from MSP back to Seattle, with a break between the two legs.



1. Based on the graph, what is the distance between Seattle and Minneapolis-St. Paul? Why?

2. Estimate the duration of each of the two legs of the flight (to the nearest half hour) from the graph.

3. The derivative of the position function with respect to time is called *velocity*. What are its units?

4. What is the **average** velocity for each of the two legs of the flight? Include units.

5. Use the graph to estimate the velocity of the airplane at $t = 0$, 2, 4, 6, and 8 hours. Put your answers in a table. *(Some of them will be positive, some negative, some zero.)*

6. As in the previous part, plot the graph of the derivative $\dfrac{ds}{dt}$ for $0 \leq t \leq 8.5$ hours. Connect your points with a smooth curve.

   Where is the derivative largest/smallest? What do the largest/smallest values represent? How do they compare?