Math 146 - Calculus II Name________________________

I'm Thirsty!

Today we will look at how to approximate solutions to equations that may not be easy to find otherwise. We will work with our hot chocolate problem we have seen before, in which we can actually find the exact solution and check how well our approximation approach is doing.

1. The temperature of a cup of hot chocolate after t minutes is given by $C(t) = 72 + 125e^{-0.2t}$. The temperature is in degrees Fahrenheit.

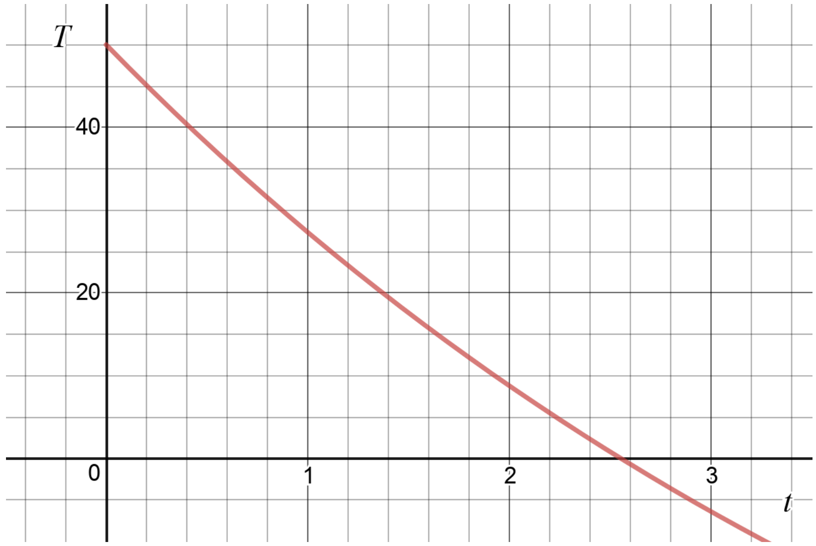
   (a) What is the initial temperature of your chocolate? Do you think it's ready to be drunk?

   (b) At what rate is the temperature changing at $t = 0$? Be sure to include units.

   (c) Using this rate of change, how long would you predict it would take the chocolate to cool to your favorite drinking temperature of 147°F?

2. If we want to determine when the temperature would be equal to 147°F, we set $C(t) = 147$, or alternatively $C(t) - 147 = 0$ and solve for t. Solve this equation and round the answer to three (3) decimal places.

3. We will now work with $T(t) = C(t) - 147 = 125e^{-0.2t} - 75$, whose graph is shown here, and we will attempt to find its zero (our drinking-temperature time) using *Newton's method.*



(a) In parts 1(a) and 1(b), you computed $C(t)$ and $C'(t)$ at $t = 0$. How do they relate to those of $T(t)$?

(b) Sketch the tangent line to $T(t)$ at the point $(0, 50)$ and mark its t-intercept. How does this value relate to your answer in part 1(c)?

(c) Continue this process. Sketch the tangent line at your newly found value of t and mark its t intercept. Repeat at least one more time. What do you observe the t-intercepts are doing?

4. Mathematically, we will see shortly that you are performing the following process. Set $t_0 = 0$, and compute the sequence

$$t_{n+1} = t_n - \frac{T(t_n)}{T'(t_n)} \quad \text{for } n = 0, 1, 2, 3, \ldots$$

This process is called Newton's method. Compute t_1, t_2, and t_3 and record their values below.

5. How does t_3 compare to your "exact" answer from 2. on the previous page?