Math 145 - Calculus I Name____________________________

Parametric Curves - Day 24

1. Open the following Desmos notebook: `https://www.desmos.com/calculator/cyzdr1tzku` and answer the questions below regarding the motion of the planet and the moon.

   (a) Why does this animated plot make sense in terms of what is orbiting around what?

   (b) If we denote time by t, then the coordinates (x, y) of the two moving objects change over time and are therefore functions of t. From the notebook you see that the functions are

planet:	moon:
$x = 3\cos(t)$	$x = 3\cos(t) + \cos(4t)$
$y = 3\sin(t)$	$y = 3\sin(t) + \sin(4t)$

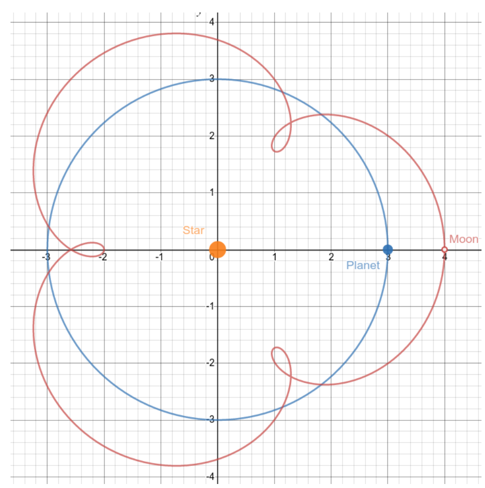
   They are called *parametric equations* and t is called a *parameter*. Use them to calculate the positions (as coordinate pairs (x, y)) for both the planet and its moon at times $t = 1$ and $t = 4$ in addition to the locations already provided. Mark them in the plot below.

   Positions of the planet at various times:

t	0	1	4	2π
x	3			3
y	0			0

   Positions of the moon at various times:

t	0	1	4	2π
x	4			4
y	0			0



2. Use a separate Desmos window to sketch below the curves corresponding to parametric equations

$$x = 6\cos(3t), \quad y = 3\sin(4t),$$

   one for the interval $0 \le t \le \pi$ and one for the interval $0 \le t \le 2\pi$. These are called *parametric curves*. In each plot, indicate the locations of the *initial point* (where $t = 0$) and the *terminal point* (where $t = \pi$ or $t = 2\pi$) and also indicate with arrows the direction of increasing t.

3. In addition to parametric equations, we can also use *implicit equations* to create cool curves in the plane. The most familiar example of this is the equation of a circle,

$$(x-h)^2 + (y-k)^2 = r^2.$$

   (a) Open a new window in Desmos and type the equation $(x-1)^2 + (y+1)^2 = 2^2$. For the general equation, what are the meanings of the constants h, k, and r?

   (b) A simpler example is $x^2 + y^2 = 9$. What is the radius and center of this circle?

   (c) If $x = 1$, what y values are on the circle in (b)? *(Solve an equation here.)*

4. A couple of general notes.

   (a) Note that an implicitly defined curve doesn't have a direction like a parametric curve does.

   (b) Also note that with both implicit and parametric curves we can get graphs that are not the graph of a function. Explain why none of your graphs in this activity is the graph of a function.

   (c) What happens if you solve the equation from 3(b) for y? Show that you don't just get the equation of a (single) function $y = f(x)$. How many functions are needed to describe the graph?