Math 146 - Calculus II Name________________________

Predictions

Suppose that $c(x)$ is a function that gives the shipping cost when you order x items from an online store.

1. What is the value of $c(0)$? Why?

2. Suppose you only know that $c(4) = 10$. What would you guess for the shipping cost for 6 items? How confident are you in that answer?

3. Suppose that you know $c(4) = 10$ and $c'(4) = 2$. Use this information to make a guess for the shipping cost for 6 items.

4. Let's be more formal about this: determine the equation of the tangent line to $c(x)$ at $x = 4$. The tangent line equation is sometimes called a *linear approximation*, or *local linearization*, of $c(x)$. Write your answer in the form $p_1(x) = a + b(x - 4)$, which should be a natural thing to do.

5. How would you use the tangent line to answer question 3? How accurate do you think your tangent line estimate is? Explain why you believe the tangent line *overestimates* or *underestimates* the shipping cost.

We are going to revisit this problem with more information. Now we know that $c(4) = 10$, $c'(4) = 2$ (as before), but also we know that $c''(4) = -0.5$. One thing we could do here is to determine a *quadratic approximation* of $c(x)$.

6. Let $p_2(x) = a + b(x-4) + c(x-4)^2$. Find values of a, b, c such that $p_2(x)$ fits $c(x)$ at $x = 4$. Specifically we want

$$p_2(4) = c(4) \qquad\qquad p_2'(4) = c'(4) \qquad\qquad p_2''(4) = c''(4)$$

7. Use $p_2(x)$ to estimate the shipping cost for 6 items.

8. What do you notice comparing $p_2(x)$ to the equation of the tangent line?

9. Graph the quadratic approximation $p_2(x)$ and the linear approximation $p_1(x)$ on the same graph.

10. New problem: Determine the quadratic approximation of $f(x) = e^{2x}$ at $x = 0$. This function will have the form $p_2(x) = a + bx + cx^2$.

11. Create a graph of $f(x) = e^{2x}$ along with its quadratic approximation for $-2 \leq x \leq 2$.