Math 146 - Calculus II Name_____

Probability

Today we're going to learn about the idea of a *probability density function* or pdf, which is used in Statistics to calculate the odds of an event occurring. Given a pdf $p(t)$, the probability of t being between a and b is given by $\int_a^b p(t)\ dt$.

1. As a first example, consider a large class of students taking a 2 hour final exam. The pdf for completing the exam at time t is given by $p(t) = \dfrac{3t^2}{8}$, where t is time in hours and $0 \leq t \leq 2$.

   (a) Sketch a graph of $p(t)$ for $0 \leq t \leq 2$.

   (b) Verify that $\int_0^2 p(t)\ dt = 1$. This means that $1 = 100\%$ of the students finish in two hours or less.

   (c) What percentage of students finish in under 1.5 hours? Set up and evaluate an integral.

(d) What percentage of students finish in exactly 1.5 hours? (exactly 1 hour, 30 minutes, 0 seconds)...

(e) What percentage of students need over 1.8 hours to finish?

2. The pdf for the life of a light bulb is $p(t) = 0.5e^{-0.5t}$ where t is time in years. This is an example of an exponential probability density function.

(a) This pdf is defined for all $0 \leq t < \infty$. But sketch a graph of this pdf for $0 \leq t \leq 4$.

(b) What percentage of light bulbs last less than 1 year?

(c) Using your answer to (b), what percentage of light bulbs last more than 1 year?

(d) Verify your answer to (c) by setting up and evaluating an improper integral to answer the question.

(e) Verify that the 'total area' under this pdf is equal to $1 = 100\%$.