Math 146 - Calculus II Name________________________

Systems of Differential Equations [1]

This problem considers an insect and its predator, for example we could think about mosquitoes and bats. Let $N(t)$ be the density of insects and let $P(t)$ be the density of predators, noting that both depend on time t. Since we are talking about populations, both $N \geq 0$ and $P \geq 0$. The quantities are related by the **system** of differential equations:

$$\frac{dN}{dt} = 5N - 3PN$$

$$\frac{dP}{dt} = 2PN - P$$

1. We haven't seen two linked differential equations before, but there are things we can determine without much struggle. For example, if $P = 0$, what is the equation for $\frac{dN}{dt}$?

2. What does this suggest about the growth of the insect population when there are no predators?

3. Perform a similar analysis for $\frac{dP}{dt}$ when $N = 0$ and explain what this tells us.

4. Next let's think about equilibrium solutions. For a system, we want values of N and P for which **both** derivatives are zero. Clearly $(N, P) = (0, 0)$ is an equilibrium solution here. Set $\frac{dN}{dt} = 0$ and $\frac{dP}{dt} = 0$ to find any other values.

[1]This example is based on 11.4)15 in Calculus for Biology and Medicine by Neuhauser and Roper

5. There are two equilibrium solutions, $(0,0)$ and what other point?

6. Explain why $(0.5,0)$ is **not** an equilibrium solution.

7. Visualizations are often helpful in complex problems like this. Since N and P are always positive, let's draw a set of axes, just the first quadrant, and label the horizontal axis N and the vertical axis P. Then plot your two equilibrium solutions on this plane.

8. Next sketch the vertical line $N = \dfrac{1}{2}$ and the horizontal line $P = \dfrac{5}{3}$. This divides the first quadrant into 4 sections.

9. The point $(0.1, 0.1)$ is in the lower left section. Determine the signs (positive or negative) of $\dfrac{dN}{dt}$ and $\dfrac{dP}{dt}$ at this point.