Math 145 - Calculus I Name________________________

Compare and Contrast - Day 15

Our goal is to compare the growth rates of a selection of different basic functions:

- Linear: $f(x) = 3x + 5$
- Quadratic: $g(x) = 5x^2 + 1$
- Quartic: $h(x) = 2x^4 + 3$
- Square Root: $j(x) = \sqrt{x}$
- Exponential: $E(x) = e^x$
- Logarithmic: $L(x) = \ln(x)$

1. What happens to each one of these functions as x gets larger and larger? More specifically, what is $\lim_{x\to\infty} f(x)$? $\lim_{x\to\infty} g(x)$? $\lim_{x\to\infty} h(x)$? Etc.

2. So in some ways all of these functions are similar. But they differ in how fast they grow as x gets larger and larger. We can explore those differences by looking at ratios. Let's start with $\dfrac{f(x)}{g(x)} = \dfrac{3x+5}{5x^2+1}$. We are interested in $\lim_{x\to\infty} \dfrac{f(x)}{g(x)} = \lim_{x\to\infty} \dfrac{3x+5}{5x^2+1}$. In this case we can do some algebra. Multiply the top and bottom of your fraction by $\dfrac{1}{x^2}$ and distribute. Use the facts that $\dfrac{1}{x} \to 0$ and $\dfrac{1}{x^2} \to 0$ as $x \to \infty$ to evaluate the limit. *(The x^2 was chosen because it is the largest power of x in the denominator.)*

3. What if we instead want to calculate $\lim_{x\to\infty} \frac{g(x)}{f(x)} = \lim_{x\to\infty} \frac{5x^2+1}{3x+5}$? What will the answer be now? Show some work using the idea from problem 2. *(What is the largest power in the denominator now?)*

4. Try a similar technique with $\lim_{x\to\infty} \frac{h(x)}{g(x)} = \lim_{x\to\infty} \frac{2x^4+3}{5x^2+1}$.

5. Write at least one generalization you think is true from the previous two examples.

6. Let's mix things up. Consider $\lim_{x\to\infty} \frac{g(x)}{E(x)} = \lim_{x\to\infty} \frac{5x^2+1}{e^x}$. I don't have any algebraic tricks for you this time, so let's investigate this numerically. Calculate this ratio for $x = 5, x = 10$, and $x = 50$.

   What can you conclude about the value of the limit?

7. Look at the graph of $\dfrac{g(x)}{E(x)} = \dfrac{5x^2+1}{e^x}$ to confirm your answer. Sketch the relevant graph and discuss.

8. What if we compare logarithmic and quadratic? Consider $\lim\limits_{x\to\infty} \dfrac{g(x)}{L(x)} = \lim\limits_{x\to\infty} \dfrac{5x^2+1}{\ln(x)}$. Numerically approximate this limit as in 6. showing some evidence.

9. We'd like to end up with an ordering of these six types of functions: Linear, Quadratic, Quartic, Square Root, Exponential, and Logarithmic. Do whatever approximations or graphing you need to to put these in order, with the slowest growing function first and the fastest growing function last.