Math 145 - Calculus I Name________________________

I'm in love with the shape of u - Day 14

1. Let's start with a straightforward mathematical example. Let $y = 3x - x^3$.

   (a) Sketch a quick graph of this function for $-3 \leq x \leq 3$.

   (b) At exactly what x-values is $y = 0$? *(These x-values are called the* roots *of the function.)*

   (c) By looking at the graph, on approximately which interval(s) is this function increasing?

   (d) Use your formula for $f'(x)$ to find exactly all critical points. *(Recall that these are the x-values where $f'(x) = 0$ or $f'(x)$ does not exist.)*

   (e) By looking at the graph of the original function, identify any local maxima and minima.

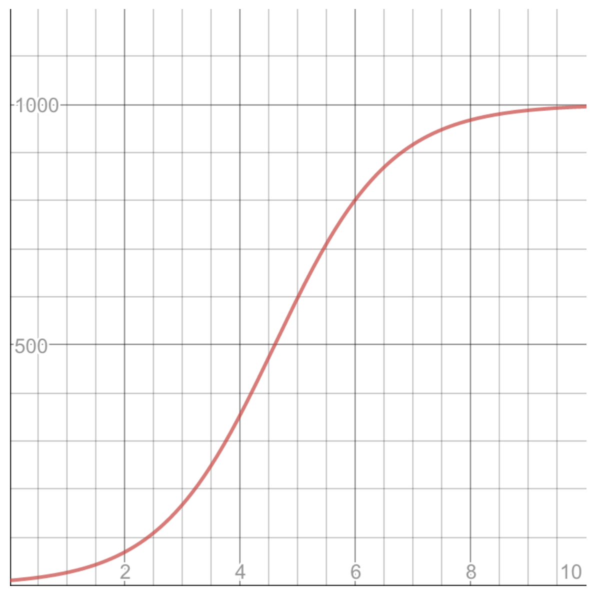
   (f) By looking at the graph of the original function, on what interval(s) is this function concave down?

   (g) Find $\dfrac{d^2y}{dx^2}$.

(h) An *inflection point* is a point where concavity changes from up to down or vice versa, and it is often found by finding where the second derivative of a function changes sign from positive to negative or vice versa. Find any inflection points for this function.

(i) Summarize the shape of the original graph. Where is it increasing/decreasing? Where is it concave up/down?

2. Remember when we did the SIR model of the spread of an infectious disease?



(a) For which population is the graph above the best fit: Susceptible, Infected, or Recovered? Explain your reasoning.

(b) Let's call this function $f(t)$. Based on the graph above, sketch a quick graph showing the general shape of $f'(t)$ for $0 \leq t \leq 10$.

(c) Describe the concavity of $f(t)$ in a sentence.

(d) **(Extra credit:)** Suppose that $f(t) = \dfrac{1000}{1+100e^{-t}}$. Do the analysis to find any inflection points and to determine the intervals on which f is concave up and concave down.

i. Apply the chain rule to $f(t) = 1000(1+100e^{-t})^{-1}$ to get $f'(t) = \dfrac{100000e^{-t}}{(1+100e^{-t})^2}$.

ii. Apply the quotient rule and factor to get $f''(t) = -\dfrac{100000e^{-t}(1-100e^{-t})}{(1+100e^{-t})^3}$.

iii. Now study concavity and find any inflection points. Do they agree with a plot of $f(t)$?