Math 146 - Calculus II Name________________________

Ratio Test Prep

Our topic for today is the Ratio Test, our final test for series convergence. This test is inspired by geometric series, so let's start there (yet again).

1. Consider the series $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^k$.

   (a) Does this series converge or diverge? If it converges, what is its sum?

   (b) For the Ratio Test we will be looking at the ratio of successive terms. Here our general term is $a_k = 5\left(\frac{2}{3}\right)^k$. Simplify the ratio $\rho = \frac{a_{k+1}}{a_k}$. (That's the Greek letter rho, btw.)

   Because this ratio is less than 1, the series converges.

2. Now let's consider the series $\sum_{k=1}^{\infty} \frac{k}{3^k}$.

   (a) Finish writing out the first four terms below:

   $$\frac{1}{3} + \qquad + \qquad + \qquad + \ldots$$

   (b) Is this a geometric series? Explain.

   (c) Explain why doing the Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{3^k}$ is not helpful.

(d) This time $a_k = \dfrac{k}{3^k}$. We will again consider the ratio of successive terms, $\rho = \dfrac{a_{k+1}}{a_k}$. Write out this ratio.

(e) This time we don't get a constant; the ratio depends on k. We are interested in what happen in the long term, so calculate $\displaystyle\lim_{k\to\infty} \frac{a_{k+1}}{a_k}$.

Again, because this limiting ratio is less than 1 we can conclude that the series converges.

3. Series involving factorials will come up quite a bit in future weeks, and the ratio test turns out to be a great tool in dealing with them. For now let's just do some practice with factorials. Recall that $4! = 4 \cdot 3 \cdot 2 \cdot 1$. Calculate the following:

   (a) $4!$

   (b) $5!$

   (c) $6!$

   (d) $6 \cdot 5!$

   (e) $(k+1) \cdot k!$

   (f) $\dfrac{5!}{4!}$

   (g) $\dfrac{(k+1)!}{k!}$