Math 145 - Calculus I Name________________________

Bikes and Curves - Day 25

Part I: Bike Wheels

The path of a point on the wheel of a bike is given by a curve called a *cycloid*. The parametric equations for this curve for a bike with 70 cm wheels are

$$x(t) = 35(t - \sin(t)), \qquad y(t) = 35(1 - \cos(t)),$$

where t is time in seconds, and x and y are in centimeters.

1. Recall how you plot parametric curves in Desmos and use the notebook linked here, `https://www.desmos.com/calculator/rdo2ewnksx`, to re-sketch the cycloid for $0 \leq t \leq 4\pi$.

2. Compute the coordinates of the point on the curve where $t = 4$ and add it to your sketch.

3. Calculate $\left.\dfrac{dx}{dt}\right|_{t=4}$ including units. What does this derivative mean?

4. Calculate $\left.\dfrac{dy}{dt}\right|_{t=4}$ including units. What does this derivative mean?

5. Usually with the graph of a function we are interested in calculating $\dfrac{dy}{dx}$.

   (a) Before we calculate, what is the meaning of $\left.\dfrac{dy}{dx}\right|_{t=4}$?

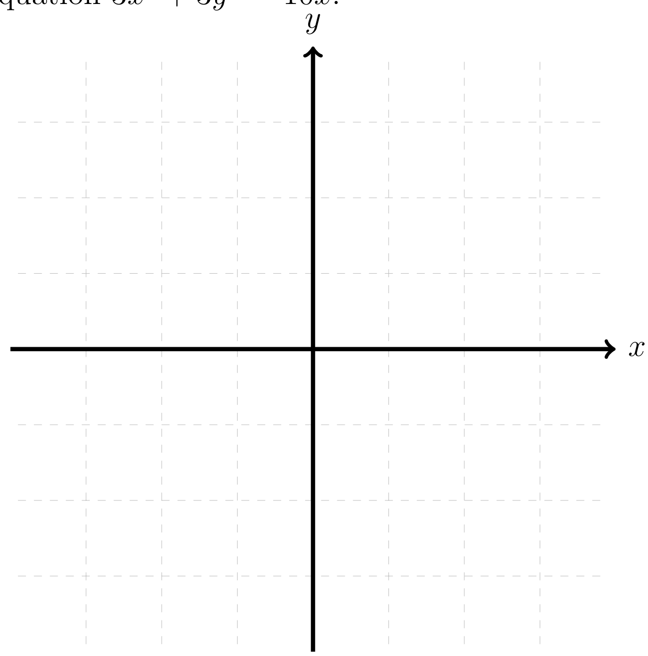
   (b) Roughly estimate $\left.\dfrac{dy}{dx}\right|_{t=4}$ from the plot in the Desmos notebook.

6. Recall from earlier that there is a relationship between $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dy}{dx}$. Write down the relationship and using your answers from 3. and 4. calculate $\frac{dy}{dx}$ at $t = 4$. Address how your computed value makes sense given your plot in problem 1.

Part II: Sharp Curves

As an example of an implicit curve, consider the equation $3x^3 + 3y^3 = 10x$.

1. Open a new window in Desmos, type in this equation, and sketch the graph into the provided axes, using a viewing window $-4 \leq x \leq 4$ and $-4 \leq y \leq 4$.
2. If $y = 0$, what are all three possible values of x on this curve? (Solve for x exactly and visually check with your plot.)



We are again interested in determining the slope of this curve, represented by $\frac{dy}{dx}$. In this case, we will take the **derivative of both sides of this equation**. Since this is an implicit equation, we think of y **as being a function of** x (think $y(x)$ in place of every y). So, when we compute the derivative of $y^3 = (y(x))^3$, we first get $3y(x)^2$ or simply $3y^2$ (*derivative of the outer function,* $(\dots)^3$), but then, using the chain rule, we multiply by the derivative of the inner function (y or $y(x)$) with respect to x, i.e., $\frac{dy}{dx}$. So the derivative of y^3 is $3y^2 \frac{dy}{dx}$.

Thinking of y as a function of x, what would be the derivatives with respect to x of the following expressions?

- y^2
- y^4
- $\sin(y)$

3. Use these ideas to take the derivative of the equation $3x^3 + 3y^3 = 10x$.

4. Once you've taken the derivative of both sides, complete the calculation by solving for $\frac{dy}{dx}$. Your answer will depend on both x and y.

5. One point on the curve is at about $(0.6, 1.2)$. Evaluate $\frac{dy}{dx}$ at this point. Does the value fit with the graph? How do you know?