Math 145 - Calculus I Name________________________

Substitute Teacher - Day 31

Today we're going to learn about the method of finding antiderivatives called *substitution*, which is closely related to the chain rule for derivatives.

1. To warm up, let's do a couple of chain rule problems. Find $f'(x)$ in each case:

   (a) $f(x) = 3e^{-4x}$

   (b) $f(x) = \sin(x^3) - 2$

2. Use the previous results to evaluate the following integrals:

   $\int (-12e^{-4x})\,dx =$ $\qquad\qquad$ $\int 3x^2\cos(x^3)\,dx =$

3. Consider $y = (2x^3 + 3)^4$.

   (a) Calculate $\dfrac{dy}{dx}$.

   (b) Use that answer to evaluate $\int 24x^2(2x^3+3)^3\,dx$.

   (c) One more: what is $\int x^2(2x^3+3)^3\,dx$? *(Use your answer from (b) here.)*

   (d) Check that last answer by calculating a derivative.

4. So, if we can recognize the function that we want to find the antiderivative of as a result of the chain rule, then we can find the answer. But how can we do that if we aren't told the function to begin with? The *method of substitution* gives us a process to follow.

   Let's again consider $\int x^2(2x^3+3)^3\,dx$. The function we want to integrate (called the *integrand*) has an "inside" or "inner" function, so we will set $u = 2x^3 + 3$. Calculate $\dfrac{du}{dx}$.

5. Rewrite the previous answer as $du = 6x^2\,dx$. Our goal is now to rewrite the entire integral in terms of our new variable u and du. Since $du = 6x^2\,dx$ and our integrand includes $x^2\,dx$, we will write $\frac{1}{6}\,du = x^2\,dx$. So we can rewrite the integral as follows:

$$\int x^2(2x^3+3)^3\,dx = \int \frac{1}{6}u^3\,du.$$

   This new integral is one that we can do easily. Complete this integral, getting an answer in terms of u:

   $\int \frac{1}{6}u^3\,du =$

6. Finally substitute back in the expression for u to get an answer in terms of x:

   $\int x^2(2x^3+3)^3\,dx =$

   Does your answer agree with 3(c)?

7. So let's see if we can do this with a brand new problem. Try to evaluate $\int xe^{1-x^2}\,dx$. Start with $u =$ "inner" function.