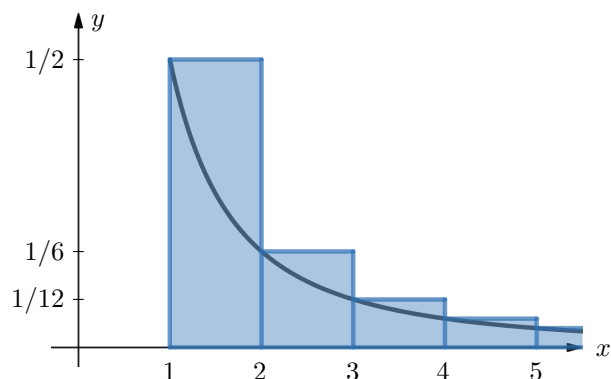


1. Consider the function defined by $f(x) = \frac{1}{x(x+1)}$ for $x \geq 1$. We would like to know if the area under this curve is finite or infinite.



- (a) Show that the sum of the areas of the first three rectangles drawn in the picture is $\frac{3}{4}$.
- (b) Fill in the missing values in the table:

n	Area under first n rectangles
1	
2	
3	$\frac{3}{4}$
4	

- (c) If we continue to draw rectangles in this manner, what is the area of the first n rectangles? Write your answer using “...” and in Sigma (summation) notation:

(d) Using the values from your table, make a conjecture about the explicit form of the sum $\sum_{k=1}^n \frac{1}{k(k+1)}$. Pause here and check your answer with someone outside your group (because we will use the answer in the rest of this activity.)

(e) If we continue to draw rectangles in this manner extending to the right indefinitely, what is the total area of those rectangles? Write your answer using \dots and also using Sigma (summation) notation.

(f) Calculate $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k(k+1)}$. What does that tell you about the infinite series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$?

(g) Explain why $\int_1^{\infty} \frac{dx}{x(x+1)} < \sum_{k=1}^{\infty} \frac{1}{k(k+1)}$.

(h) What can you conclude about the area under the curve $y = \frac{1}{x(x+1)}$ for $x \geq 1$? Is it finite or infinite?

2. (a) Evaluate $\sum_{k=1}^4 \left(\frac{1}{k} - \frac{1}{k+1} \right)$

(b) Evaluate $\sum_{k=1}^{99} \left(\frac{1}{k} - \frac{1}{k+1} \right)$

(c) Make a conjecture about the explicit form of $\sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right)$.

(d) Calculate $\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right)$.

(e) Use algebra to show that $\frac{1}{k} - \frac{1}{k+1} = \frac{1}{k(k+1)}$.

(f) It follows that $\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right)$. Did you get the same answers in 1)f) and 2)d)?