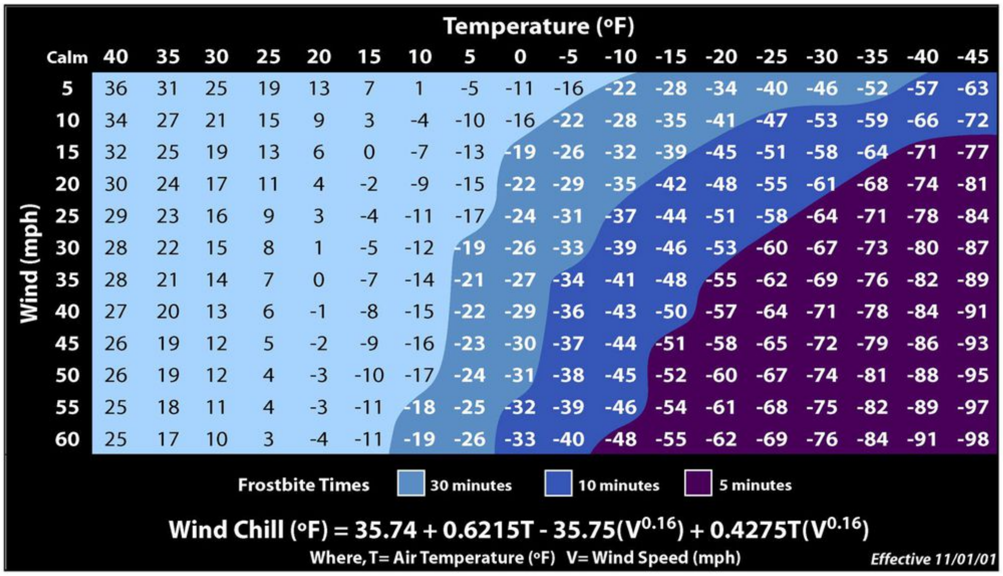
MAT 145 Calculus 1 Name________________________

It Depends - Day 20

The chart below, taken from the National Weather Service, is used to calculate wind chill (in °F) based on the air temperature (in °F) and wind speed (in mph).

Wind Chill Chart



Temperature (°F)

Wind (mph) \ Calm	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45
5	36	31	25	19	13	7	1	-5	-11	-16	-22	-28	-34	-40	-46	-52	-57	-63
10	34	27	21	15	9	3	-4	-10	-16	-22	-28	-35	-41	-47	-53	-59	-66	-72
15	32	25	19	13	6	0	-7	-13	-19	-26	-32	-39	-45	-51	-58	-64	-71	-77
20	30	24	17	11	4	-2	-9	-15	-22	-29	-35	-42	-48	-55	-61	-68	-74	-81
25	29	23	16	9	3	-4	-11	-17	-24	-31	-37	-44	-51	-58	-64	-71	-78	-84
30	28	22	15	8	1	-5	-12	-19	-26	-33	-39	-46	-53	-60	-67	-73	-80	-87
35	28	21	14	7	0	-7	-14	-21	-27	-34	-41	-48	-55	-62	-69	-76	-82	-89
40	27	20	13	6	-1	-8	-15	-22	-29	-36	-43	-50	-57	-64	-71	-78	-84	-91
45	26	19	12	5	-2	-9	-16	-23	-30	-37	-44	-51	-58	-65	-72	-79	-86	-93
50	26	19	12	4	-3	-10	-17	-24	-31	-38	-45	-52	-60	-67	-74	-81	-88	-95
55	25	18	11	4	-3	-11	-18	-25	-32	-39	-46	-54	-61	-68	-75	-82	-89	-97
60	25	17	10	3	-4	-11	-19	-26	-33	-40	-48	-55	-62	-69	-76	-84	-91	-98

Frostbite Times: 30 minutes, 10 minutes, 5 minutes

Wind Chill (°F) = $35.74 + 0.6215T - 35.75(V^{0.16}) + 0.4275T(V^{0.16})$

Where, T= Air Temperature (°F) V= Wind Speed (mph)

Effective 11/01/01

1. Let's get acquainted with this chart. For now, ignore the formula on the bottom.

   (a) What is the wind chill at a temperature of $-5\,°F$ and a wind speed of 30 mph?

   (b) What is the wind chill at a temperature of $-5\,°F$ and a wind speed of 35 mph?

2. Using the data in the table, estimate the rate of change of **wind chill** with respect to **wind speed** when temperature is $-5\,°F$ and the wind speed is 30 mph. Show your work. What are the units of your answer? Explain why the sign of your answer make sense (*e.g., how do we expect the wind chill to change when the wind speed increases?*).

3. Using the data in the table, estimate the rate of change of **wind chill** with respect to **temperature** when the temperature is $-5\,°\mathrm{F}$ and the wind speed is 30 mph. Show your work. What are the units of your answer? Explain why the sign of your answer make sense.

4. Wind chill W depends on two variables, temperature T and wind speed V. So it is a *multi-variable* function. We could say $W = f(T, V)$. The rates of change we are calculating are then called *partial derivatives*, and we use a delta, ∂, rather than a d to describe them. Question 2. asked for the rate of change of **wind chill** with respect to **wind speed**. Specifically you estimated $\left.\dfrac{\partial W}{\partial V}\right|_{(-5,30)}$. Using this notation, what did you estimate in question 3.?

5. The formula used for wind chill is $W = 35.74 + 0.6215T - 35.75V^{0.16} + 0.4275T \cdot V^{0.16}$. To calculate $\dfrac{\partial W}{\partial T}$ we simply take the derivative, considering the other variable, V, to be a constant. Apply this process to show that $\dfrac{\partial W}{\partial T} = 0.6215 + 0.4275V^{0.16}$.

6. Evaluate this partial derivative at the point $(T, V) = (-5, 30)$ and compare this to your answer in 3. *(Is your answer close? Does it have the same sign?)*

7. Now calculate $\dfrac{\partial W}{\partial V}$ and then $\left.\dfrac{\partial W}{\partial V}\right|_{(-5,30)}$ and compare to your answer in 2. *(Is your answer close? Does it have the same sign?)*

8. Here's a whole new example function: $f(x, y) = x^2 + 2xy + y^3$.

   (a) Calculate $\frac{\partial f}{\partial x}$.

   (b) Calculate $\frac{\partial f}{\partial y}$.

   (c) Use your answers in (a) and (b) and solve the system of equations $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$.
   (Points where all partial derivatives of a function are zero are again called critical points of the function and can correspond to local minima or local maxima of the function.)

   (Did you find two critical points, $(x, y) = (0, 0)$ and $(x, y) = (-\frac{2}{3}, \frac{2}{3})$?)