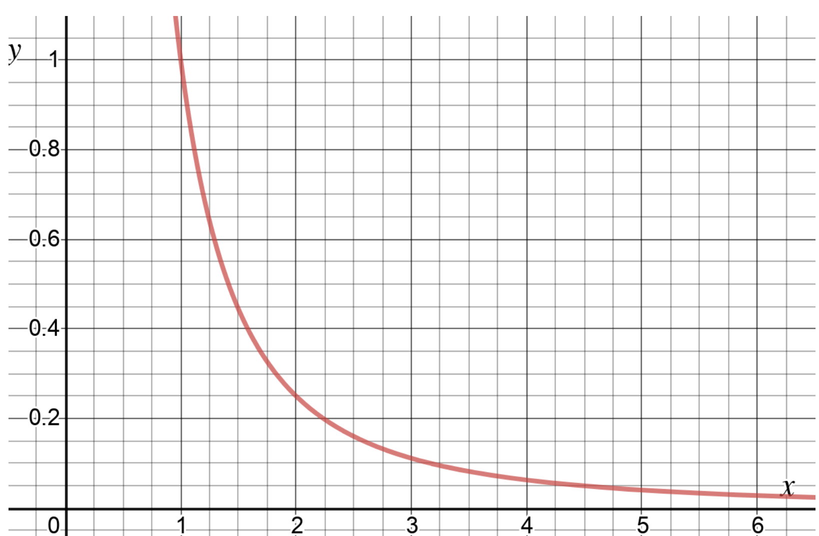
Math 146 - Calculus II Name_______________________

Integrals vs. Series

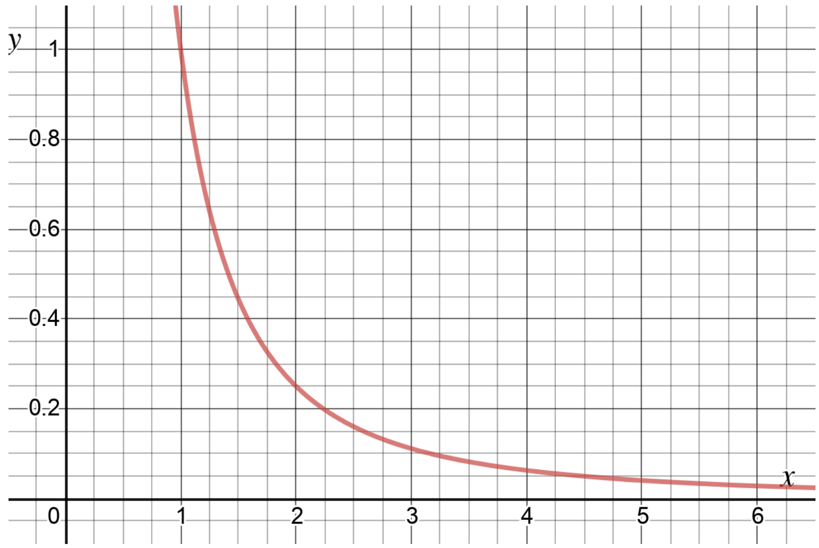
1. (a) Evaluate the improper integral $\displaystyle\int_1^\infty \frac{dx}{x^2}$.

(b) What does the value of that integral tell you about the area under the curve $y = \dfrac{1}{x^2}$ over the interval $[1, \infty)$?

(c) Shade in the region whose area you found on the graph drawn below.



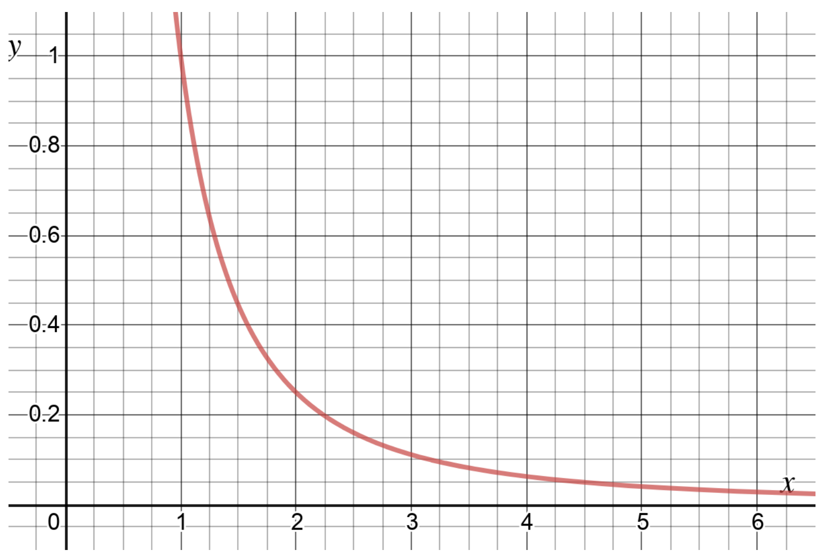
(d) Draw a left endpoint Riemann sum with $\Delta x = 1$ for $\displaystyle\int_1^\infty \frac{dx}{x^2}$.



(e) Write the sum of the areas of those rectangles as an infinite series.

(f) Which is larger: the value of the integral or the series?

(g) Now draw a right endpoint Riemann sum with $\Delta x = 1$ for $\displaystyle\int_1^\infty \frac{dx}{x^2}$. Write the sum of the areas of those rectangles as an infinite series. Which is larger: the value of the integral or the series?



(h) Use your results to give upper and lower bounds for the value of the series $\sum_{k=1}^{\infty} \frac{1}{k^2}$.

Hint: $\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \sum_{k=2}^{\infty} \frac{1}{k^2}$

(i) If I told you the value of that series was $\pi^2/6$ would that agree with your bounds?

2. Evaluate the improper integral $\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$.