Math 146 - Calculus II Name________________________

Improper Integrals

1. First, let's remember some important facts about the natural logarithm.

   (a) Sketch a quick graph of $\ln(x)$ for $0 \leq x \leq 5$.

   (b) We should all know the value of $\ln(1)$:

   (c) These two limits both diverge, but are they $+\infty$ or $-\infty$?

   $$\lim_{x \to \infty} \ln(x) = \qquad\qquad \lim_{x \to 0^+} \ln(x) =$$

2. Water is being drained from a 3000 liter pool. The initial rate is 100 liters per hour, but the rate decreases by 5% each hour. The rate r satisfies the differential equation $\frac{dr}{dt} = -.05r$ with $r(0) = 100$.

   (a) It turns out that $r(t) = 100e^{-0.05t}$. (Not a bad idea to try to solve this in your spare time.) Now, the amount of water drained from the pool in the first three hours is given by $\int_0^3 r(t)\, dt$. (Because the integral of a rate of change gives total change.) Calculate how much water was drained from the pool in that 3 hours.

   (b) How much water is left in the pool after 3 hours?

(c) Calculate how much water has drained **and** how much is left after 10 hours.

(d) Complete the following table, starting with your answers from the previous page:

hours	amount drained	amount left
3		
10		
48		
100		
250		

(e) If we let the pool continue to drain, what will happen in the long run? Will the pool ever be empty?

(f) What integral would represent the amount of water drained from the pool if we let it drain indefinitely?