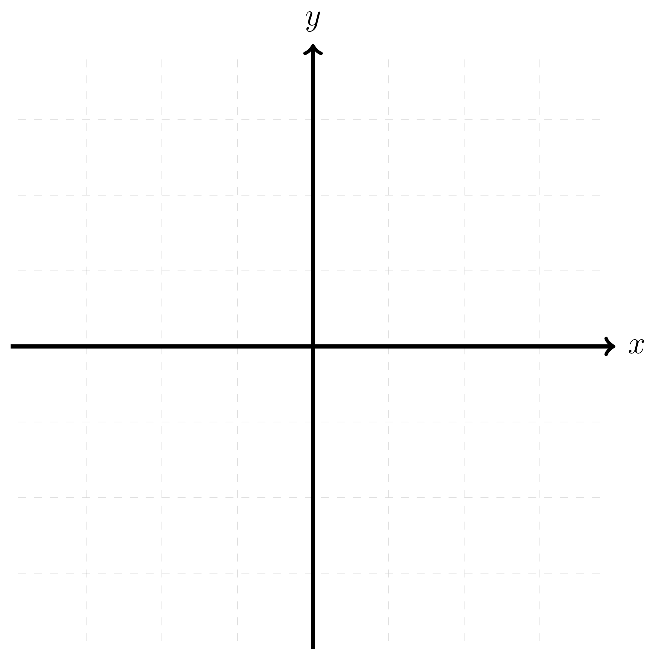
Math 145 - Calculus I                                        Name________________________

**Instantaneous Rate of Change**                                        Day 4

We've looked at the rate of change of a function on an interval—the average rate of change. Today we will shift our focus to the rate of change at a point—the instantaneous rate of change.

1. Let $f(x) = 1 - x^2$.

   (a) Sketch a graph of $f$ on the interval $-2 \leq x \leq 2$.




   (b) We are interested in the instantaneous rate of change at $x = 1$. Draw a straight line through the point $(1, f(1))$ in the direction of the graph of $f$ (i.e., the *tangent* line).

   (c) The instantaneous rate of change is the slope of this line. Estimate the slope from the graph.

   (d) Let's try to calculate the instantaneous rate of change by calculating the average rate of change on smaller and smaller intervals containing the point $x = 1$:

| Interval | Average rate of change |
|---|---|
| $[1, 1.5]$ | |
| $[1, 1.1]$ | |
| $[1, 1.01]$ | |

   (e) Based on any of your work on this page, what is your best guess for the instantaneous rate of change of $f(x) = 1 - x^2$ at $x = 1$?

2. Use a similar procedure as in 1(d) above to estimate the instantaneous rate of change of $y = 3^x$ at $x = 0$. *(For example, your first interval could be* $[0, 0.5]$*.)* Show your work.

3. The other day we talked about the "affection" function, given by $A(d) = 1 - (1 - e^{1-d})^2$ where $d$ is "distance" and $A$ is the amount of affection.

   (a) Use the approach from 1(d) to estimate the instantaneous rate of change of $A$ at $d = 2$.

   (b) Interpret this value in terms of proximity and affection by completing the sentence:

   *When the distance is* $d =$_____ *units, then an increase in distance by* 1 *additional unit would result in an increase/decrease (circle one) in affection by about* __________ *units.*