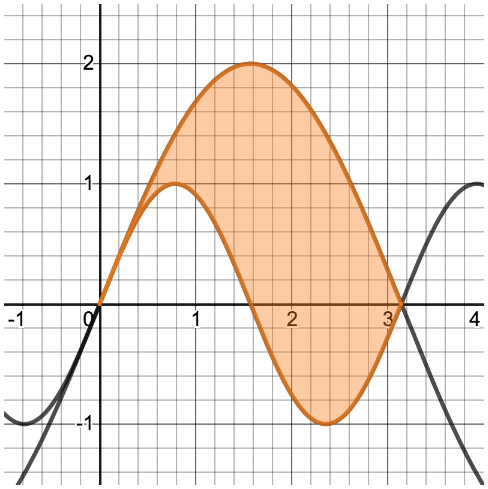
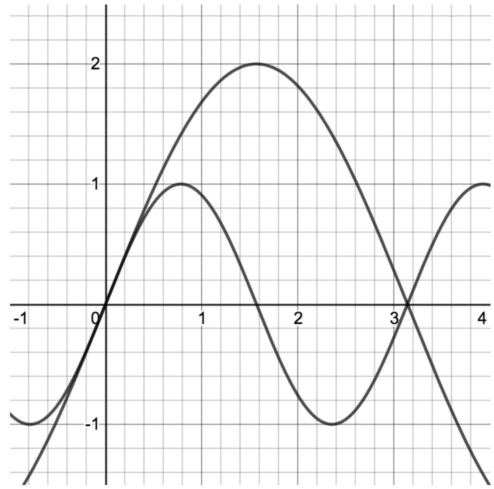
Math 146 - Calculus II Name________________________

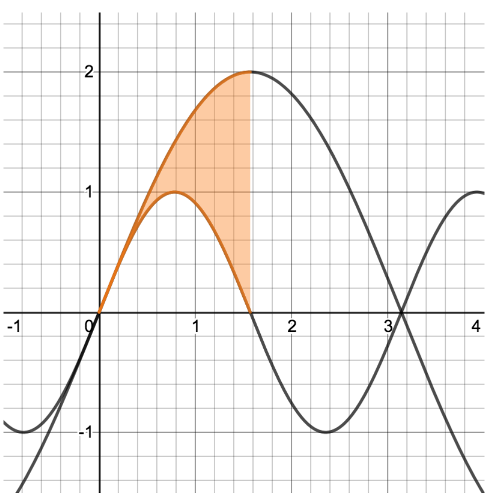
The Space Between

Consider the area between the functions $f(x) = \sin(2x)$ and $g(x) = 2\sin(x)$ as shown on the left below.





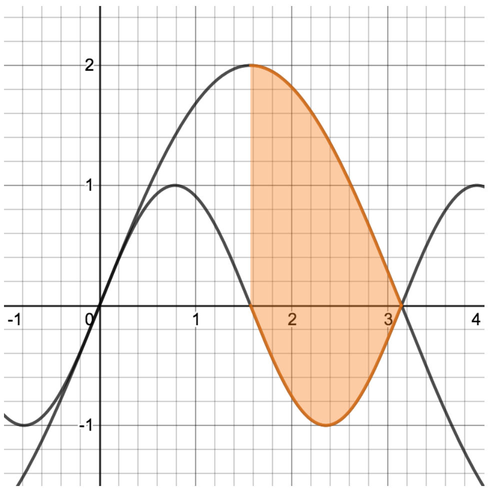
1. Label each curve with the appropriate function.
2. Shade the region corresponding to $\int_0^{\pi} 2\sin(x)\, dx$ on the graph on the right above.
3. Calculate the area of the shaded region below as the difference between two integrals. Find the answer exactly.



4. What is the value of $\int_0^{\pi} \sin(2x)\ dx$? This should be a quick answer.

5. Explain why the area of the shaded region in this image is $\int_{\frac{\pi}{2}}^{\pi} 2\sin(x)\ dx - \int_{\frac{\pi}{2}}^{\pi} \sin(2x)\ dx$.

   No need to do the calculation yet.



6. Now explain why the total area between the curves (the area in the first picture) is given by $\int_0^{\pi} 2\sin(x)\ dx - \int_0^{\pi} \sin(2x)\ dx (= \int_0^{\pi} 2\sin(x) - \sin(2x)\ dx)$.

7. Do the calculations to show that the area is equal to exactly 4.

8. The other topic we're going to discuss today is arclength. This term represents the length of a curve and can be thought of as the length of rope you'd need to cover the curve. We'll discuss the arclength of $y = x^2$ for $0 \leq x \leq 2$ as an example.

   (a) Sketch a graph of this curve.

   (b) Explain why we know the arclength is more than $2\sqrt{5}$. Hint: hypotenuse.