Math 146 - Calculus II Name________________________

Integration by Parts

The integration technique known as *integration by parts* is based on the product rule for derivatives. So let's start with some product rule practice.

1. Find the derivatives of the following functions:

   (a) $f(x) = x^2 e^x$

   (b) $g(t) = \sin(t)\ln(t)$

   (c) $h(y) = y(1+2y)^4$

2. The product rule can be written a few different ways.

   (a) First write it out using f and g.

   $$(f(x) \cdot g(x))' =$$

   (b) If we call the functions u and v we could write the product rule as $(u \cdot v)' = u \cdot v' + v \cdot u'$. Or if we use a d to indicate a derivative, we could write $d(u \cdot v) = u \cdot dv + v \cdot du$. Solve this last expression for $u \cdot dv$. This will be useful later.

3. Integration by Parts is useful when the integrand is a a product (and substitution isn't helpful), such as the classic example $\int x\cos(x)\ dx$.

   (a) What if we guessed that the antiderivative here was $F(x) = x\sin(x)$? What is $F'(x)$?

   (b) That's close but what's the problem?

   (c) So we could add something to F so that the extra term in the derivative would cancel out. In this case we want to add something whose derivative is $-\sin(x)$. So let's try a new function

   $$G(x) = x\sin(x)+$$

   (d) Show that $G'(x) = x\cos(x)$ as desired.

   This adjustment will be carried out more automatically when we use the technique of Integration by Parts.