Intro to Social Science Data Analysis

Lecture 7: Overview of Statistical Inference

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Assignment 2

Due: Friday 19 October

Describe at least **3** variables in a data set.

You need to select a **range of descriptive statistical tools**. The tools should include both **numerical descriptive statistics** and **graphics**.

These tools should describe the variables':

- ▶ central tendency,
- ▶ variation,
- ▶ their relationships with the other variables.

The descriptions need to be discussed **in paragraph form**.

The description must be **reproducible**. So you should email me the link to a Dropbox folder with:

- ▶ the .csv data set,
- ▶ the .Rmd R markdown file,
- ▶ the final .html file.

What are some of the goals that quantitative graphics should try to acheive?
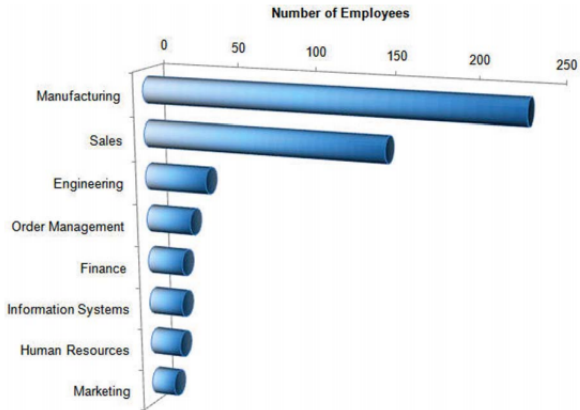
What are some things you should not do in quantitative graphics?

In particular, how does the **data-ink ratio** help us create effective graphics?

Remember:

$$\text{Data Ink Ratio} = \frac{\text{data} - \text{ink}}{\text{total ink}}$$
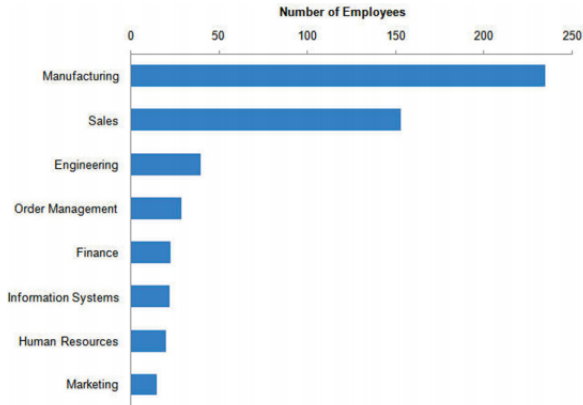
What is wrong with this graph?

Source:

http://www.perceptualedge.com/articles/visual_business_intelligence/rules_for_using_color.pdf

Improved

Source:

http://www.perceptualedge.com/articles/visual_business_intelligence/rules_for_using_color.pdf

Today's Data Download

```r
# Load package RCurl
library(RCurl)

# Create URL object for OpenIntro data
url <- paste("https://raw.github.com/christopher",
             "gandrud/Introduction_to_Statistics_",
             "and_Data_Analysis",
             "_Yonsei/master/Lectures/Lecture7/",
             "OpenIntroData/run10.txt", sep = "")

# Download data
Run10 <- getURL(url, ssl.verifypeer = FALSE)

# Convert Run10 to data.frame
Run10 <- read.table(textConnection(Run10),
                    sep = "\t", header = TRUE)
```

Today's Variables

```
names(Run10)

## [1] "place"    "time"     "pace"     "age"
## [5] "gender"   "location" "state"    "divPlace"
## [9] "divTot"
```

Today we will use these variables (from Diaz et al. (2011, Ch. 4)):

Variable	Description
time	ten mile run time, minutes
age	age of runner in years

Random Sample

Our data contains the **entire population** of 16,974 runners who finished the 2009 Cherry Blosom 10 Mile Run in Washington, DC.

But imagine, however, that we only have a **simple random sample** of 100 runners.

Illustrate Random Sample

Take random sample

```
# Find number of runners in population
nrow(Run10)

## [1] 16924


# Take a random sample of 100 runners
Run10Samp <- Run10[sample(1:nrow(Run10), 100,
                          replace=FALSE),]

# Find number of runners in sample
nrow(Run10Samp)

## [1] 100
```
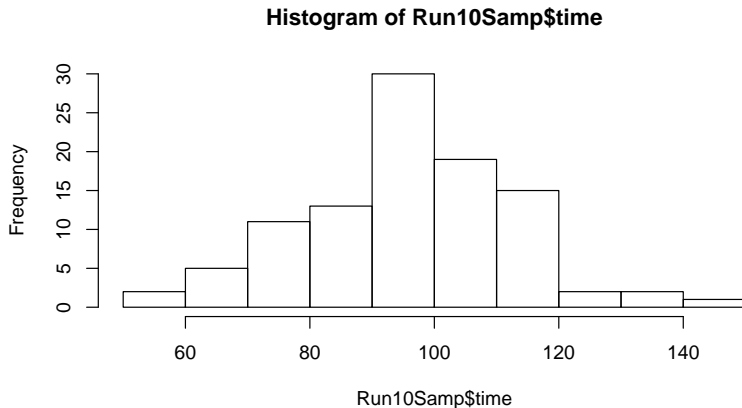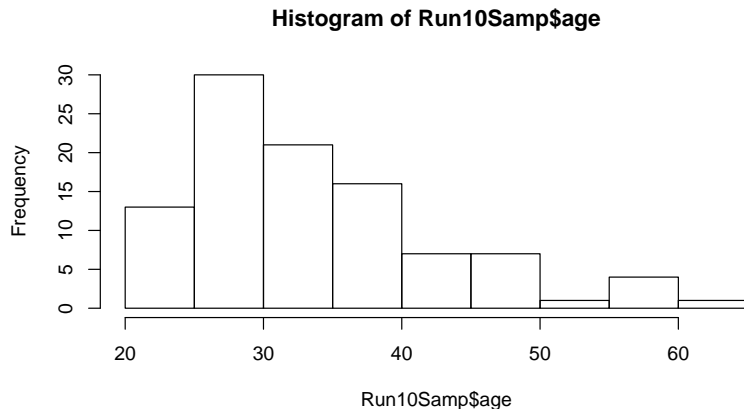
Sample Distribution of `time`

```
hist(Run10Samp$time)
```

Histogram of Run10Samp$time

Sample Distribution of age

```
hist(Run10Samp$age)
```

Histogram of Run10Samp$age

How similar is the sample to the population?

In other words, how **representative** is the sample?

In other other words, what can we **infer** about the population from the sample?

More specifically: Means

Today we are particularly interested in how similar the **sample means** $\bar{x}$ are to the **true population means** μ **for:**

- average time it took to complete the race,
- average age of the runners.

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To answer this question we will need tools of **statistical inference**.

Today we will learn the basics of statistical inference.

We will use these tools throughout the rest of the course and statistics.

Statistical inference is key for linking our (sampled) data to what we care about: questions about populations.

The Sample Means

Lets find our sample means.

Time:

```
mean(Run10Samp$time)
```

```
## [1] 95.78
```

Age:

```
mean(Run10Samp$age)
```

```
## [1] 34.37
```

These are **point estimates** of a **population parameter**, the population mean.

Point Estimates (2)

We can find point estimates of other parameters.

For example, lets find point estimates for the sample **standard deviation of the mean** (s). (The population parameter of the standard deviation is usually written σ).

Time:

```
sd(Run10Samp$time)

## [1] 17.13
```

Age:

```
sd(Run10Samp$age)

## [1] 9.138
```

Summary of our Random Sample

Variable	$\bar{x}$	μ	s	σ
time	95.8	?	17.1	?
age	34.4	?	9.1	?

But, **how do we know** how **close** the point estimates are to the real population parameters?

Especially, since when we take **other samples** we will get **different estimates**.
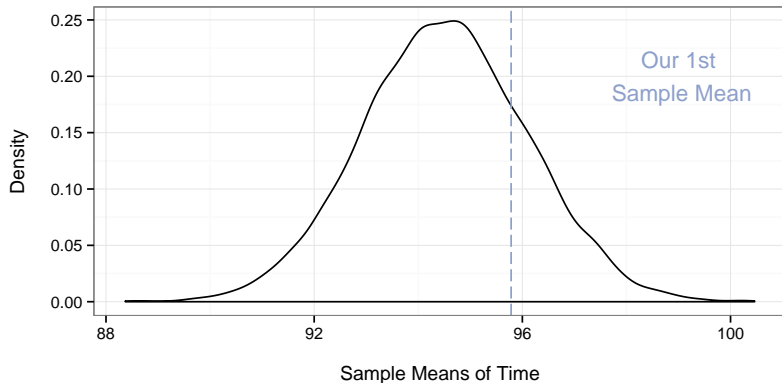
10000 Sample Means!

Lets find the sample mean of `time` ($\bar{x}_{time}$) in 10,000 samples!

```
# Find the sample means from 10,000 samples
TimeSampMeans <- rep(0, 10000)
  for (i in 1:10000) {
    Samp <- sample(Run10$time, 100)
    TimeSampMeans[i] <- mean(Samp)
    }

# Create a data frame object
TimeSampMeans <- data.frame(TimeSampMeans)
```

Distribution of the 10,000 Sample Means!

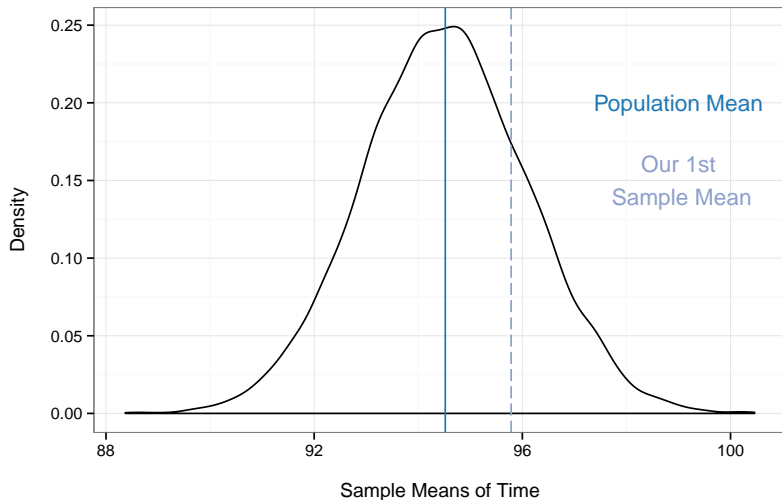The Sampling Distribution of the Mean of Time

The sampling distribution is **centered** on the population mean!

Stated much more confusingly: the mean of the sampling distribution of the sample means is the population mean.

Population Mean vs. Sample Means

The Sampling Distribution of the Mean of Time

Ok, that is nice in theory, but what if we don't
know the sampling distribution?
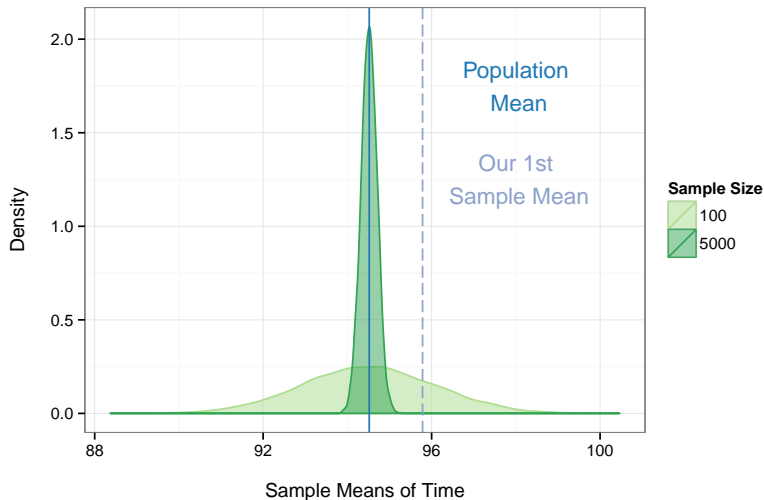
In real life we can't take 10,000 samples!

What would the sampling distribution of the sample mean look like if the sample size was 5,000 rather than 100?

Comparing Sample Sizes

The **likely distance** between the sample mean and
the population mean is related to the **sample size**
(n),

The **bigger** the sample size, the more likely it is
that the point estimate is **close** to the population
parameter.

Also, a **smaller sample standard deviation** (s) indicates less variability $\rightarrow$ more likely that the point estimate is **close** to the population parameter.

So. . .

Finally, the Standar Error!

Standard Error (SE):

$$SE = \frac{s}{\sqrt{n}}$$

Assuming that the observations are *independent*, i.e. the value of one observation does not depend on the value of another observation.

The standard error is a measure of our **uncertainty** about how well the point estimate represents the population parameter.

The Standard Error of the Mean of time

```r
# Load plotrix
library(plotrix)

# Calculate standard error of the mean of time
std.error(Run10Samp$time)

## [1] 1.713
```

So, time in our original sample with 100 observations had a sampleing mean of 95.8 and a standard error of 1.7.

Um . . . a sampleing mean of 95.8 and a standard error of 1.7?

That's not very easy to understand.

We need something more usefull.

We need something that is more **intuitive** to interpret.

Maybe a **range** of likely population parameter values.

The Confidence Interval!

About **95%** of the time the population paramenter will be within **about 2 standard errors** of the point estimate.

95% Confidence Interval (CI):

$$CI = \text{point estmiate} \pm 2 * SE$$

Central Limit Theorem (informal definition):

If a sample has at least 50 independent observations, and the data is not extremely skewed, the distribution of the sampling mean closely approximates the normal distribution (Diaz et al., 2011, 151))

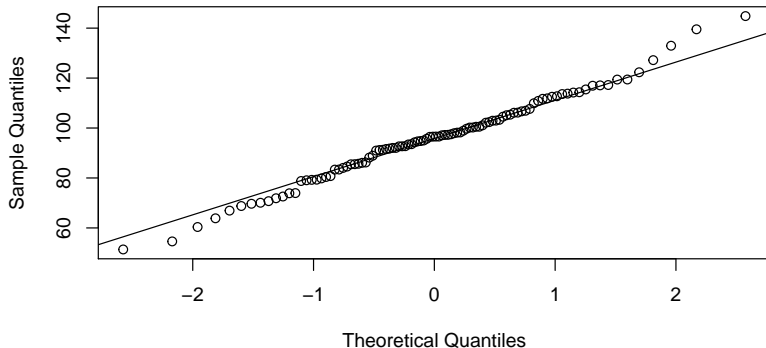So what? This lets our confidence interval be more **precise**.

$$CI = \text{point estmiate} \pm 1.96 * SE$$

Confidence Interval Simulation

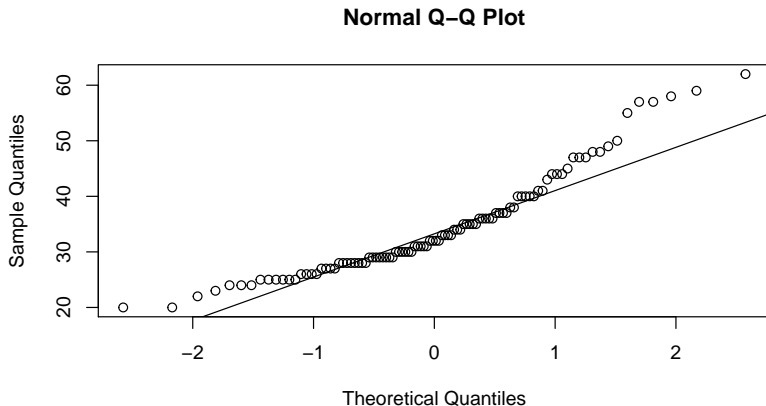# Visually Confirm Normally Distributied Time Data

```
# Create Quantile-Quantile Plots
qqnorm(Run10Samp$time); qqline(Run10Samp$time)
```


Normal Q–Q Plot

Visually Confirm Normally Distributied Age Data

```
# Create Quantile-Quantile Plots
qqnorm(Run10Samp$age); qqline(Run10Samp$age)
```


Normal Q–Q Plot

In general, observations from a simple random sample of **less than 10% of the population** will be independent.

Let's calculate some 95% confidence intervals for `time`

```
# Find Mean of time
MeanTime <- mean(Run10Samp$time)

# Find Standard Error of the mean of time
SETime <- std.error(Run10Samp$time)

# Lower CI
LowerTime <- MeanTime - 1.96 * SETime

# Upper CI
UpperTime <- MeanTime + 1.96 * SETime
```

```
# Show Confidence Intervals
LowerTime

## [1] 92.43


UpperTime

## [1] 99.14
```
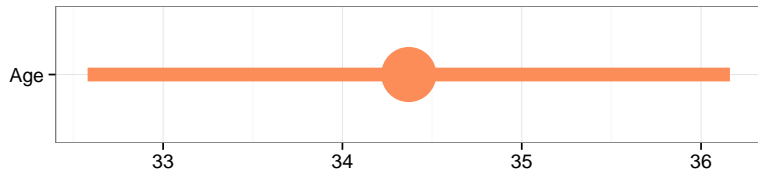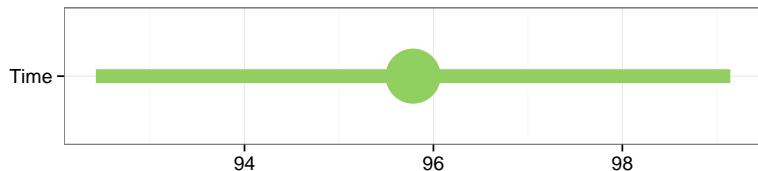
Summary of our Parameter Estimates

Variable	Lower CI	Mean	Upper CI	Population Mean
time	92.4	95.8	99.1	94.5
age	32.6	34.4	36.2	35.5

Alternative

95% Confidence Intervals for the Mean (n = 100)

References I

Diaz, David M., Christopher D. Barr, and Mine Çetinkaya-Rundel. 2011. OpenIntro Statistics. 1st ed. http://www.openintro.org/stat/downloads.php.