MFCS

Unit 1 : Statement & Notation

* A statement is a declarative sentence to which it is possible to assign a truth value true or false but not both.

Ex:

$2+2=4$ (T)

$2+5=7$ (T)

$2+2=7$ (F)

* The statements are denoted by distinct alphabetical letters A, B, C . . . P, Q, R . . . X, Y, Z.

LA-3

* [Connectives:

A simple statement may be called as a primary statement or atomic statements. To make a simple statement into compound stmts connectives are used. These connectives are and, or, conditional, bi-conditional & negation.

) Negation:

The negation of a stmt is formed by introducing the word 'not' at a proper place in the stmt or by prefixing the statement with the phrase "It is not the case that" or "It is not true that".

If P denotes a stmt then the negation of P is denoted by $\sim P$ (or) $\neg P$

Truth table:

P	~P/¬P
T	F
F	T

Ex: ① P: Hyd is a city

~P: Hyd is not a city

② P: $2+2=4$

~P: $2+2 \neq 4$ / It is not true that $2+2=4$

ii) Conjunction (∧):

Let P, Q are two stmts then the compound stmt made with the phrase 'and' in b/w the two stmts is called as conjunction i.e P and Q it is denoted as $P \wedge Q$

Truth table:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex: ① P: It is raining today

Q: $2+6=8$

$P \wedge Q$: It is raining today and $2+6=8$

Ex ②: Write the symbolic form for the compound stmt : Raju and Raghu went to the school

P : Raju went to the school

Q : Raghu went to the school

$P \wedge Q$

iii) Disjunction ($\vee$):

Let P, Q are two stmts then the compound stmt made by using the phrase 'or' is called disjunction i.e P or Q it is denoted by $P \vee Q$

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Ex: P : I will buy a computer

Q : I will buy a car

$P \vee Q$: I will buy a computer/or car

SA-5

v) Conditional stmts:

If P, Q are any two stmts then the stmt "$P \rightarrow Q$" read as "if P then Q" ~~the~~ is called a conditional stmt.

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

The stmt P is called the anticident and Q is the consequent in $P \to Q$. $P \to Q$ can be read as

i) P only if Q

ii) Q if P

iii) Q provided that P

iv) P is sufficient condition for Q

v) Q is necessary condition for P

vi) P implies Q

vii) Q is implied by P

Ex: P : Amulya ~~works~~ works hard

Q : Amulya will pass the exam

$P \to Q$: If Amulya works hard then she will pass the exam)

SA-6

v) Bi-conditional stmts:

If P, Q are any two stmts then $P \Leftrightarrow Q$ is read as P if and only if Q. is called a bi-conditional stmt. It is abrievated as 'P iff Q

P	Q	$P \rightleftarrows Q$
T	T	T
T	F	F
F	T	F
F	F	T

$P \rightleftarrows Q : (P \to Q) \wedge (Q \to P)$

The bi-conditional stmt can be read as follows:

i) P if and only if Q

ii) P is equivalent to Q

iii) P is necessary and sufficient condition for Q

iv) Q is necessary and sufficient condition for P

Ex: P : Two lines are parallel

Q : Two lines have the same slope

$P \rightleftarrows Q$: Two lines are parallel if and only if they have same slope.)]

Q. Write the following stmt into a symbolic form.

If either Mr. Seenu takes calculus or Mr. Swami takes graph theory then Mr. Mahesh will take computer programming.

Sol: $(P \vee Q) \to R$

* Statement formula:

Stmts which do not contain any connectives are called atomic / simple / primary stmts.

The stmts which contain one or more primary stmts and atleast one connective are called molecular / composite / compound stmts.

Ex: $\sim P$, $P \vee Q$, $\sim P \vee Q$, $\sim P \wedge Q$, $P \wedge \sim Q$, $\sim P \vee \sim Q$, $\sim P \to Q$

The above compound stmts are called stmt formulas derived from the stmt variables P & Q . P & Q are called as components of the stmt formulas.

* Truth table:

A table showing all the possible truth values of a stmt formula for each possible combinations of the truth values of the component stmts is called the truth table of the formula.

In general if there are 'n' distinct components in a stmt formula then we'll have 2^n possible combinations of truth values.

Ex: construct a truth table for $P \wedge \sim P$, $P \vee \sim P$, $P \to (Q \to$

Sol:

P	$\sim P$	$P \wedge \sim P$
T	F	F
F	T	F

P	$\sim P$	$P \vee \sim P$
T	F	T
F	T	T

P	Q	R	$Q \to R$	$P \to (Q \to R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

Ex: $(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \vee \sim Q)$

P	$\sim P$	Q	$\sim Q$	$P \wedge Q$	$\sim P \wedge Q$	$P \wedge \sim Q$	$\sim P \vee \sim Q$
T	F	T	F	T	F	F	F
F	T	F	T	F	F	F	T
T	F	F	T	F	F	T	T
F	T	T	F	F	T	F	T

$(P \wedge Q) \vee (\sim P \wedge Q)$	$(P \wedge \sim Q) \vee (\sim P \vee \sim Q)$	$\vee$
T	F	T
F	T	T
F	T	T
T	T	T

Q. Write the following stmts in a symbolic form.

1. P : Pavan is rich

Q : Raghav is happy

a) Pavan is rich and Raghav is not happy

$P \wedge (\sim Q)$

b) Pavan is not rich and Raghav is happy

$\sim P \wedge Q$

2. P : Naveen is rich

Q : Naveen is happy

a) Naveen is poor but happy $\sim P \wedge Q$

b) Naveen is neither rich nor happy $\sim P \wedge \sim Q$

c) Naveen is rich or unhappy $P \vee \sim Q$

d) Naveen is poor or he is both rich and unhappy

$\sim P \vee (P \wedge \sim Q)$

3. P: Naveen is smart

Q : Amal is smart

a) Naveen is smart and Amal is not smart $P \wedge \sim Q$

b) Naveen and Amal are both smart $P \wedge Q$

c) Neither Naveen nor Amal is smart $\sim P \wedge \sim Q$

d) It is not true that Naveen and Amal are smart

$\sim (P \wedge Q)$

Q. Let P, Q, R denote the following stmts.

P : Triangle ABC is isosceles

Q : Triangle ABC is equilateral

R : Triangle ABC is equiangular

Translate each of the following into english stmt.

a) $Q \rightarrow P$

If triangle ABC is equilateral then it is isosceles

b) $\sim P \rightarrow \sim Q$

If Δ^{le} ABC is not isosceles then Δ^{le} ABC is not equilateral

c) $Q \rightleftarrows R$

Δ^{le} ABC is equilateral if and only if Δ^{le} ABC is equiangular

d) $P \wedge \sim Q$

Δ^{le} ABC is isosceles but not equilateral

e) $R \rightarrow P$

If Δ^{le} ABC is equiangular then Δ^{le} ABC is isosceles.

Q. If P, Q, R are the 3 stmts with truth values True, True, False resp. Find the truth values of the following

1. $P \vee Q$ (T) 2. $P \wedge R$ (F) 3. $(P \vee Q) \wedge R$ (F)

4. $P \wedge (\sim R)$ (T) 5. $P \wedge \sim Q$ (F) 6. $P \rightarrow R$ (F)

7. $P \rightarrow Q$ (T) 8. $R \rightarrow P$ (T) 9. $(R \wedge P) \rightarrow Q$ (T)

10. $(P \wedge \sim Q) \rightarrow R$ (T) 11. $(P \vee Q) \rightleftarrows (P \rightarrow \sim R)$ (T) 12. $(P \rightleftarrows R) \rightarrow R$ (T)

$P \longrightarrow T$ $\quad Q \longrightarrow T$ $\quad R \longrightarrow F$

Q. If P, Q are the stmts with truth value True and R, S are stmts with truth value False.

1. $(R \wedge P) \rightarrow S$ (T)

2. $(P \wedge Q) \wedge R$ (F)

3. $(P \rightleftarrows Q) \rightarrow (S \rightleftarrows R)$ (T)

4. $P \vee (Q \wedge S)$ (T)

5. $(P \rightarrow \sim Q) \rightarrow (S \rightleftarrows R)$ (T)

6. $(P \rightarrow \sim Q) \rightarrow (P \vee Q)$ (T)

7. $P \rightarrow (Q \rightleftarrows (R \rightarrow S))$ (T)

8. $S \rightarrow P$ (T)

$P \longrightarrow T$

$Q \longrightarrow T$

$R \longrightarrow F$

$S \longrightarrow F$

Q. Construct the truth tables for the following formulas.

a) $\sim(\sim P \wedge \sim Q)$

b) $(\sim P \vee Q) \wedge (\sim Q \vee P)$

c) $(P \wedge Q) \rightarrow (P \vee Q)$

d) $P \vee (Q \wedge R)$

e) $(P \wedge (Q \wedge R)) \wedge \sim((P \vee Q) \wedge (P \wedge R))$

a)

P	Q	$\sim P$	$\sim Q$	$x = \sim P \wedge \sim Q$	$\sim(x)$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

b)

P	Q	$\sim P$	$\sim Q$	$\sim P \vee Q$	$\sim Q \vee P$	$\wedge$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

c)

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \rightarrow (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

d)

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	F	T
F	F	T	F	F
F	T	F	F	F
T	F	F	F	T

e)

P	Q	R	$Q \wedge R$	$P \wedge (Q \wedge R)$	$P \vee Q$	$P \wedge R$	$(P \vee Q) \wedge (P \wedge R)$	$\sim$	$\wedge$
T	T	T	T	T	T	T	T	F	F
T	T	F	F	F	T	F	F	T	F
T	F	T	F	F	T	T	T	F	F
T	F	F	F	F	T	F	F	T	F
F	T	T	T	F	T	F	F	T	F
F	T	F	F	F	T	F	F	T	F
F	F	T	F	F	F	F	F	T	F
F	F	F	F	F	F	F	F	T	F

Q. Given that truth values of P, Q as T and R, S as F then find the truth values of following stmt formula

a) $P \wedge (Q \wedge R) \wedge \sim((P \vee Q) \wedge (R \vee S))$

$(T \wedge (T \wedge F)) \wedge \sim((T \vee T) \wedge (F \vee F))$

$(T \wedge F) \wedge \sim(T \wedge F)$

$F \wedge \sim F$

$F \wedge T$

F

* <u>Well formed formula</u>:

A stmt formula is an expression which is a string consisting of variables, parenthesis & connective symbols, which produces a stmt. when the variables are replaced by stmts.

A well formed formula can be generated by the following rules:

1. The stmt variable standing alone is a well formed formula.

2. If A is a well formed formula then $\sim A$ also a well formed formula.

3. If A and B are well formed formulas then $(A \wedge B)$, $(A \vee B)$, $(\sim A \wedge \sim B)$, $(\sim P \rightleftarrows \sim Q)$ also are well formed formulas.

The following stmt-formulas are not well formed formulas.

$\vee P \rightarrow (\sim P \vee Q)$, $((P \rightarrow Q) \rightarrow (\wedge Q))$, $(P \wedge Q \rightarrow Q))$

SA-2a)b)

* Tautology:

A stmt-formula which is true regardless of the truth values of the stmts which replace the variables in it is called a universally valid formula (or) tautology(or) a logic truth i.e If each entry in the final column of the truth table of a statement formula is true (T) alone then it is called tautology.

* Contradiction:

A stmt formula which is false regardless of the truth values of the stmt which replaces the variables in it is called contradiction i.e if each entry in the final column of the truth table of a stmt. formula is false (F) alone then it is called contradiction.

Q. Prove the following are tautology.

i) $\sim(P \vee Q) \vee (\sim P \wedge Q) \vee P$

ii) $(((P \rightarrow Q) \wedge (R \rightarrow S)) \wedge (P \vee R)) \rightarrow (Q \vee S)$

iii) $((P \rightarrow R) \wedge (Q \rightarrow R)) \rightarrow ((P \vee Q) \rightarrow R)$

iv) $((P \rightarrow (Q \vee R)) \wedge (\sim Q)) \rightarrow (P \rightarrow R)$

v) $(((P \vee Q) \rightarrow R) \wedge (\sim P)) \rightarrow (Q \rightarrow R)$

vi) $(P \rightarrow Q) \rightleftarrows (\sim P \vee Q)$

vii) $Q \vee (P \wedge \sim Q) \vee (\sim P . \wedge \sim Q)$

Sol:

i)

P	Q	$\sim P$	$P \vee Q$	$\sim P \wedge Q$	$\sim(P \vee Q)$	$\vee$	$\vee$
T	T	F	T	T	F	T	T
T	F	F	T	F	F	F	T
F	T	T	T	T	F	T	T
F	F	T	F	F	T	T	T

ii)

P	Q	R	S	$P \to Q$	$R \to S$	$P \vee R$	$Q \vee S$	$(P \to Q) \wedge (R \to S)$	$(\therefore) \wedge (P \vee R)$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	T	T	F	F
T	T	F	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T	T
T	F	T	T	F	T	T	T	F	F
T	F	T	F	F	F	T	F	F	F
T	F	F	T	F	T	T	T	F	F
T	F	F	F	F	T	T	F	F	F
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	T	T	F	F
F	T	F	T	T	T	F	T	T	F
F	T	F	F	T	T	F	T	T	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	T	F	F	F
F	F	F	T	T	T	F	T	T	F
F	F	F	F	T	T	F	F	T	F

(QVS)
T
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iii)

P	Q	R	$P \to R$	$Q \to R$	$P \vee Q$	$(P \to R) \wedge (Q \to R)$	$(P \vee Q) \to R$	$\to$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T	T
T	F	F	F	T	T	F	F	T
F	T	T	~~F~~ T	T	T	T	T	T
F	T	F	T	F	T	F	F	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	T

iv)

P	Q	R	$Q \vee R$	$P \to (Q \vee R)$	$\sim Q$	$(P \to (Q \vee R)) \wedge (\sim Q)$	$P \to R$	$\to$
T	T	T	T	T	F	F	T	T
T	T	F	T	T	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	F	F	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

vi)

P	Q	$\sim P$	$P \to Q$	$\sim P \vee Q$	$(P \to Q) \rightleftarrows (\sim P \vee Q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

v)

P	Q	R	PVQ	(PVQ)→R	(~P)	( )∧(~P)	Q→R	→
T	T	T	T	T	F	F	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	F	F	T	T
T	F	F	T	F	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

vii)

P	Q	~P	~Q	P∧~Q	~P∧~Q	( )∨( )	Q∨(
T	T	F	F	F	F	F	T
T	F	F	T	T	F	T	T
F	T	T	F	F	F	F	T
F	F	T	T	F	T	T	T

Q. Construct the truth table:

i) P∧(Q∨R)

ii) (P∧Q)∨(P∧R)

iii) P∨~(P∧Q)

iv) (P∧Q)∧~(P∨Q)

v) (P→Q)→(P∧Q)

vi) ~P→(Q→P)

Sol: i)

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

ii)

P	Q	R	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

iii)

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$P \vee \sim(P \wedge Q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

iv)

P	Q	$P\wedge Q$	$P\vee Q$	$\sim(P\vee Q)$	$(P\wedge Q)\wedge\sim(P\vee Q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

v)

P	Q	$P\rightarrow Q$	$P\wedge Q$	$(P\rightarrow Q)\rightarrow(P\wedge Q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

vi)

P	Q	$\sim P$	$Q\rightarrow P$	$\sim P\rightarrow(Q\rightarrow P)$
T	T	F	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

* **Equivalence formula:**

Two formulas A and B are said to be $\Rightarrow$ equivalent to each other if and only if $A\rightleftarrows B$ is a tautology. It is denoted as $A\Leftrightarrow B$ i.e. the truth values of A and B are same.

Q. 1. $P \vee Q \Leftrightarrow \sim(\sim P \wedge \sim Q)$

2. $P \rightarrow Q \Leftrightarrow \sim P \vee Q$

Sol: 1.

P	Q	$\sim P$	$\sim Q$	$P \vee Q$	$\sim P \wedge \sim Q$	$\sim(\ )$	$\Leftrightarrow$
T	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	F	T

2.

P	Q	$\sim P$	$P \rightarrow Q$	$\sim P \vee Q$	$\Leftrightarrow$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

* Equivalence formulae:

$a+b = b+a$

$a(b+c) = ab+ac$

1. $P \vee P \Leftrightarrow P$, $P \wedge P \Leftrightarrow P$ — Idempotent law

2. $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ — Associative law

3. $P \vee Q \Leftrightarrow Q \vee P$, $P \wedge Q \Leftrightarrow Q \wedge P$ — Commutative law

4. $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ — Distributive law

$P \vee F \Leftrightarrow P$, $P \wedge T \Leftrightarrow P$

$P \wedge F \Leftrightarrow F$ $\quad P \vee T \Leftrightarrow T$

$P \vee \sim P \Leftrightarrow T$; $P \wedge \sim P \Leftrightarrow F$

$$\left.\begin{array}{l} {}^{*}P \vee (P \wedge Q) \Leftrightarrow P \\ P \wedge (P \vee Q) \Leftrightarrow P \end{array}\right\} \text{Absorption law}$$

$$\left.\begin{array}{l} {}^{*}\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q \\ \sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q \end{array}\right\} \text{Demorgan's law}$$

$P \vee T \Rightarrow T$

$P \wedge T \Rightarrow P$

$P \vee \sim P \rightarrow T$

<u>Absorption law</u>:

P	Q	$P \wedge Q$	$P \vee (P \wedge Q)$	$\Leftrightarrow P$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	F	F	F

✓

P	Q	$P \vee Q$	$P \wedge (P \vee Q)$	$\Leftrightarrow P$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	F	T

✓

$F \wedge Q$ $\quad$ F . T

F F

Demorgan's law:

P	Q	P∨Q	∼	∼P	∼Q	∼P∧∼Q	⇔
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

P	Q	P∧Q	∼(P∧Q)	∼P	∼Q	∼P∨∼Q	⇔
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

Distributive law:

P	Q	R	Q∧R	P∨(Q∧R)	P∨Q	P∨R	( )∧( )	( )⇔(
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

P	Q	R	$Q\vee R$	$P\wedge(Q\vee R)$	$P\wedge Q$	$P\wedge R$	$(\ )\vee(\ )$	$\Leftrightarrow$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	R	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

P	F	$P\vee F$	$P\vee F\Leftrightarrow P$	$P\wedge F$	$P\wedge F\Leftrightarrow F$
T	F	T	T	F	T
F	F	F	T	F	T

P	T	$P\wedge T$	$P\wedge T\Leftrightarrow P$	$P\vee T$	$P\vee T\Leftrightarrow T$
T	T	T	T	T	T
F	T	F	T	T	T

Q:

* Replacement process:

~~$\sim(P\wedge Q)$~~ ~~$\sim P\vee Q$~~

1. $P\rightarrow Q \Leftrightarrow \sim P\vee Q$
2. $P\rightleftarrows Q \Leftrightarrow (P\rightarrow Q)\wedge(Q\rightarrow P)$

   $(\sim P\vee Q)\wedge(\sim Q\vee P)$

Q. Prove that $P \to (Q \to R) \Leftrightarrow P \to (\sim Q \vee R) \Leftrightarrow (P \wedge Q) \to R$

Sol: $P \to (Q \to R)$

$\Leftrightarrow \underbrace{P}_{A} \to \underbrace{(\sim Q \vee R)}_{B}$ $\quad (\because A \to B \Leftrightarrow \sim A \vee B)$

$\sim A \vee B$

$\Leftrightarrow \sim P \vee (\sim Q \vee R)$

$\Leftrightarrow (\sim P \vee \sim Q) \vee R$

$\Leftrightarrow \sim \underbrace{(P \wedge Q)}_{A} \vee \underbrace{R}_{B}$

$A \to B$

$\Leftrightarrow (P \wedge Q) \to R$

$P \to (Q \to R)$

$P \to (\sim Q \vee R)$

$\sim P \vee (\sim Q \vee R) \Rightarrow (\sim P \vee \sim Q) \vee R$

$\sim (P \wedge Q) \vee R$

$(P \wedge Q) \to R$

~~Q. Prove that $(P \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$~~

~~Sol: $(P \to Q) \wedge (R \to Q)$~~

~~$(\sim P \vee Q) \wedge (\sim R \vee Q) \Leftrightarrow (\sim P \wedge \sim R) \vee Q$~~

~~$\sim (P \vee R) \vee Q$~~

~~$(P \vee R) \to Q$~~

Q. Prove that $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

$(Q \vee P) \wedge (R \vee R)$

Sol: $(\sim P \wedge (\sim Q \wedge R)) \vee ((Q \vee P) \wedge R)$

$((\sim P \wedge \sim Q) \wedge R) \vee ((P \vee Q) \wedge R)$

$(\sim \underline{(P \vee Q)} \wedge \underline{R}) \vee ((P \vee Q) \wedge R)$ $\quad a(b+c)$ $\quad ab+ac$

$(\sim (P \vee Q) \vee (P \vee Q)) \wedge R$

$\boxed{\sim P \vee P \Leftrightarrow T}$

$R \wedge T$

R

$\boxed{\sim P \vee P \Leftrightarrow T}$

$(\sim\underbrace{(P \vee Q)}_{\sim P} \vee \underbrace{(P \vee Q)}_{P}) \wedge R$

$\because \boxed{\sim P \vee P \Leftrightarrow T}$

$T \wedge R \quad (\because P \wedge T \Leftrightarrow P)$

R

Q. Prove that $P \to (Q \to R) \Leftrightarrow \sim P \to (P \to Q)$

Sol:

$\underbrace{P}_{A} \to \underbrace{(Q \to R)}_{B}$

$\sim P \vee (\underbrace{Q}_{A} \to \underbrace{R}_{B})$

$\sim P \vee (\sim Q \vee R)$

$(\sim P \vee P) \vee (\sim Q)$

$T \vee \sim Q$

$T \quad (\because P \vee T \Leftrightarrow T)$

$\sim P \to (P \to Q)$

$\sim(\sim P) \vee (P \to Q)$

$P \vee (\sim P \vee Q)$

$\underline{P \vee \sim P} \vee Q$

$T \vee Q$

T

Q. Prove that $(P \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R) \to Q$

Sol: $(P \to Q) \wedge (R \to Q)$

$(\sim P \vee Q) \wedge (\sim R \vee Q)$

$(\sim P \wedge \sim R) \vee (Q \wedge Q)$

$\sim (P \vee R) \vee Q$

$(P \vee R) \to Q$

Q. Prove that $\sim(P \rightleftarrows Q) \Leftrightarrow (P \vee Q) \wedge \sim(P \wedge Q)$

Sol: $\sim((\sim P \vee Q) \wedge (\sim Q \vee P))$

$\sim(\sim(P \vee Q) \vee (Q \wedge P))$

$(P \vee Q) \wedge \sim(P \wedge Q)$

$\sim((P \to Q) \wedge (Q \to P))$

$\sim((\sim P \vee Q) \wedge (\sim Q \vee P))$

$\sim((\sim P \wedge \sim Q) \vee (Q \wedge P))$

$\sim(\sim(P \vee Q) \vee (Q \wedge P))$

$(P \vee Q) \wedge \sim(Q \wedge P)$

Q. Prove that $\sim(P \rightleftarrows Q) \Leftrightarrow (P \wedge \sim Q) \vee (\sim P \wedge Q)$

Sol: $\sim((P \to Q) \wedge (Q \to P))$

$\sim((\sim P \vee Q) \wedge (\sim Q \vee P))$

$\cancel{\sim((\sim P \wedge \sim Q) \vee (Q \wedge P))}$

$(P \wedge \sim Q) \vee (Q \wedge \sim P)$

$\cancel{\sim(\sim(P \vee Q)) \vee (Q \wedge P))}$

$(P \wedge \sim Q) \vee (\sim P \wedge Q)$

Q. Prove that $((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee ((\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)) \Leftrightarrow (P \vee Q)$

Sol: $(P \vee Q) \wedge \sim((\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)) \vee (\sim P \wedge (\sim Q \vee \sim R))$

$(P \vee Q) \wedge ((P \vee Q) \wedge (P \vee R)) \vee \sim(P \vee (Q \wedge R))$

$(P \cap P)$

$(P \vee Q) \wedge \underbrace{(P \vee (Q \wedge R))}_{P} \vee \underbrace{\sim(P \vee (Q \wedge R))}_{\sim P}$

$(P \vee Q) \wedge T$

$\Leftrightarrow$

$(P \vee Q)$

Q. Show that P is equivalent to the following formulas. by using truth tables

i) $\sim(\sim P)$ ii) $P \wedge P$ iii) $P \vee P$ iv) $P \vee (P \wedge Q)$ × Absorption law

v) $P \wedge (P \vee Q)$ × Absorption law vi) $(P \wedge Q) \vee (P \wedge \sim Q)$ vii) $(P \vee Q) \wedge (P \vee \sim Q)$

Sol:

i)

P	$\sim P$	$\sim(\sim P)$
T	F	T
F	T	F

ii)

P	P	$P \wedge P$
T	T	T
F	F	F

iii)

P	P	$P \vee P$
T	T	T
F	F	F

vi)

P	Q	$P \wedge Q$	$\sim Q$	$P \wedge \sim Q$	V
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	F	F
F	F	F	T	F	F

vii)

P	Q	$P \vee Q$	$\sim Q$	$P \vee \sim Q$	$\wedge$
T	T	T	F	T	T
T	F	T	T	T	T
F	T	T	F	F	F
F	F	F	T	T	F

vi) $(P \wedge Q) \vee (P \wedge \sim Q)$

$P \wedge (Q \vee \sim Q)$

$P \wedge T$

P

vii) $(P \vee Q) \wedge (P \vee \sim Q)$

$P \vee (Q \wedge \sim Q)$

$P \vee F$

P

Q. Solve by using equivalence formulas/replacement method.

i) $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

Sol: RHS: $(P \to Q) \vee (P \to R)$

$(\sim P \vee Q) \vee (\sim P \vee R)$

$\sim P \vee (Q \vee R)$

$P \to (Q \vee R)$

ii) $\sim(P \to Q) \Leftrightarrow P \wedge \sim Q$

Sol: $\sim(\sim P \vee Q)$

$P \wedge \sim Q$

iii) $(P \rightleftarrows Q) \Leftrightarrow (P \to Q) \wedge (Q \to P)$

Sol: RHS: $(P \to Q) \wedge (Q \to P)$

$(\sim P \vee Q) \wedge (\sim Q \vee P)$

P	Q	$\sim P$	$\sim Q$	$\sim P \vee Q$	$\sim Q \vee P$	$\wedge$

can be solved only by T.T

$P \vee (P \wedge Q) \Leftrightarrow P$

$\sim P \wedge (\sim P \vee Q) \Leftrightarrow \sim P$

iv) $\sim (P \wedge Q) \to (\sim P \vee (\sim P \wedge Q)) \Leftrightarrow (\sim P \vee Q)$

Sol: $\sim(P \wedge Q) \to (\sim P)$ (Absorption law)

$\sim(\sim(P \wedge Q)) \vee \sim P$

$(P \wedge Q) \vee \sim P$

$(P \vee \sim P) \wedge (\sim P \vee Q)$

$T \wedge (\sim P \vee Q)$

$(\sim P \vee Q)$

v) $(P \vee Q) \wedge (\sim P \wedge (\sim P \vee Q)) \Leftrightarrow (\sim P \wedge \sim Q)$

Sol: $(P \vee Q) \wedge (\sim P)$

~~$(P \vee \sim P) \wedge (\sim P \vee Q)$~~

~~$T \wedge (\sim P \vee Q)$~~

~~Ans. $\sim(P \wedge \sim Q)$~~

$\sim(P \vee Q)$

$(P \wedge \sim P) \vee (Q \wedge \sim P)$

$F \vee (Q \wedge \sim P)$

$Q \wedge \sim P$

vi) $P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$

Sol: $\sim P \vee Q \; \cancel{\Leftrightarrow Q \vee \sim R}$

$Q \vee \sim P$

$\sim Q \rightarrow \sim P$

* **Law of duality:**

Two formulas A and A^* are said to be duals to each other if either one can be obtained from the other by replacing $\wedge$ by $\vee$ (or) $\vee$ by $\wedge$. The connectives $\wedge$ and $\vee$ are also called duals to each other.

Ex: $A = (P \wedge Q) \vee R \wedge (P \wedge \sim P)$

$A^* = (P \vee Q) \wedge R \vee (P \vee \sim P)$

If the formula A contains the special variables T or F then A^* is obtained by replacing T by F (or) F by T.

Q. $A = \sim(P \vee Q) \wedge (P \vee \sim(Q \wedge \sim R))$

Sol: $A^* = \sim(P \wedge Q) \vee (P \wedge \sim(Q \vee \sim R))$

Q. $\sim(P \wedge Q) \rightarrow (\sim P \vee (\sim P) \vee Q)) \Leftrightarrow (\sim P \vee Q)$

Sol: $\cancel{(P \wedge Q) \vee (\sim P \vee (\sim P \vee Q))}$

$\cancel{(P \wedge Q) \vee (\sim P \vee \sim (P \wedge Q))}$

$\sim(P \wedge Q) \rightarrow (\sim P \vee Q)$

$(P \wedge Q) \vee (\sim P \vee Q)$

$\cancel{(P \wedge Q) \vee \sim (R \wedge (P \wedge \sim R))}$

$\cancel{(P \wedge Q) \vee \sim (R}$

$(P \vee (\sim P \vee Q) \wedge (Q \vee (\sim P \vee Q))$

$(P \vee \sim P \vee Q) \wedge \cancel{(Q \vee \sim P \vee Q)} \wedge \sim P \vee Q \vee Q$

$T \vee Q$

$T \wedge (\sim P \vee Q)$

$\sim P \vee Q$

Q. $(P \vee Q) \wedge (\sim P \wedge (\sim P \wedge Q)) \Leftrightarrow \sim P \wedge Q$

$((\sim P \wedge \sim P) \wedge Q)$

Sol: $(P \vee Q) \wedge (\sim P \wedge Q)$

$P \wedge (\sim P \wedge Q) \vee (Q \wedge (\sim P \wedge Q))$

$(P \wedge \sim P \wedge Q) \vee (\sim P \wedge (Q \wedge Q))$

$F \wedge Q$

$F \vee (\sim P \wedge Q)$

$\sim P \wedge Q$

* Normal forms:

To transform the stmt. formulas A and B to some standard forms A' and B' such that a simple comparison of A' and B' shows whether $A \Leftrightarrow B'$. The standard forms are called normal forms (or) canonical forms.

Let $A(P_1, P_2 \ldots P_n)$ be a stmt. formula where $P_1, P_2, \ldots P_n$ are primitive variables. If A has the truth value T for atleast one combination of truth values assigned to $P_1, P_2 \ldots P_n$ then A is said to be satisfiable.

Def: A product of variables and their negations is called elementary product.

Ex: $(P \wedge Q), (\sim P \wedge Q), (P \wedge \sim Q), (P \wedge \sim Q)$

$P, Q, \sim P, \sim Q \ldots$

Def: A sum of the variables and their negations is called elementary sums.

Ex: $P, Q, \sim P, \sim Q \ldots$

$P \vee Q,\ P \vee \sim Q,\ \sim P \vee Q,\ \sim P \vee \sim Q, \ldots$

* **Disjunctive normal form (DNF)**

A formula which is equivalent to a given formula and which consist of sum of elementary products (SOP) is called a disjunctive normal form.

* **Procedure to obtain a disjunctive normal form of a given formula:**

Step 1: If the connectives $\rightarrow$ and $\rightleftarrows$ appears in the given formula obtain an equivalence formula in which $\rightarrow$ and $\rightleftarrows$ do not appear i.e an equivalence formula can be obtained in which $\rightarrow$ and $\rightleftarrows$ are replaced by $\vee$, $\wedge$ and $\sim$

Ex: $P \rightarrow Q \Leftrightarrow \sim P \vee Q$

$P \rightleftarrows Q \Leftrightarrow (\sim P \vee Q) \wedge (\sim Q \vee P)$

Step 2: If the negation is applied to the formula and not to the variables appearing in it, then by using demorgan's law, an equivalent formula can be obtained.

Ex: $\sim (P \wedge Q)$

$\rightarrow \sim P \vee \sim Q$

Step 3: Apply the distributive law until a sum of elementary products is obtained

Ex: $P \wedge (Q \vee R)$

$\rightarrow (P \wedge Q) \vee (P \wedge R)$

* Extended distributive law:

$(P \vee Q) \wedge (R \vee S) \Leftrightarrow (P \wedge R) \vee (P \wedge S) \vee (Q \wedge R) \vee (Q \wedge S)$

LHS: $P \wedge (R \vee S) \vee Q \wedge (R \vee S)$

$(P \wedge R) \vee (P \wedge S) \vee (Q \wedge R) \vee (Q \wedge S)$

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Q. Obtain a dnf of $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

Sol: Sum of products $(\wedge) \vee (\wedge) \vee (\wedge)$

$P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

$P \rightarrow ((P \rightarrow Q) \wedge (Q \wedge P))$

$P \rightarrow ((\sim P \vee Q) \wedge (P \wedge Q))$

$P \rightarrow (\sim P \wedge (P \wedge Q) \vee (Q \wedge (P \wedge Q)))$

$P \vee T \rightarrow T$

$P \wedge T \rightarrow P$

$P \rightarrow ((F \wedge Q) \vee (P \wedge Q))$

$P \rightarrow (F \vee (P \wedge Q))$

$P \rightarrow (P \wedge Q)$

$(\sim P) \vee (P \wedge Q)$

~~$(\sim R \vee P) \wedge (\sim P \vee Q) \wedge$~~

$(\sim P \wedge \sim P) \vee (P \wedge Q)$

sum of products

Q. Find dnf for $P \wedge (P \to Q)$

Sol: $P \wedge (P \to Q)$

$P \wedge (\sim P \vee Q) \Rightarrow (P \wedge \sim P) \vee (P \wedge Q)$

$F \vee (P \wedge Q)$

Q. Solve $\sim (P \vee Q) \rightleftarrows (P \wedge Q)$

Sol: $\cancel{\sim(P \vee Q)}$

$\cancel{((\sim(P \vee Q)) \to (P \wedge Q)) \wedge (P \wedge Q) \to (\sim(P \vee Q))}$

$\cancel{\sim(P \wedge \sim Q)}$

$\cancel{((P \vee Q) \vee (P \wedge Q)) \wedge (\sim(P \wedge Q) \vee (\sim(P \vee Q)))}$

$\cancel{((P \vee Q) \vee (P \wedge Q)) \wedge \sim((P \wedge Q) \wedge (P \vee Q))}$

$\cancel{(\sim P \vee Q) \wedge (\sim Q \vee P)}$

$\cancel{P \rightleftarrows Q}$

$\boxed{\therefore P \rightleftarrows Q \; (\equiv) \quad (\sim P \vee Q) \wedge (P \vee \sim Q)}$

$\cancel{(P \vee Q)}$

$\cancel{P \vee (P \wedge Q) \vee Q \wedge (P \wedge Q) \wedge \sim (P \wedge (P \vee Q) \wedge Q \wedge (P \vee Q}$

$P \vee$

$(\sim(\sim(P \vee Q)) \vee (P \wedge Q)) \wedge (\sim(P \vee Q) \vee \sim(P \wedge Q))$

1 $((P \vee Q) \vee (P \wedge Q)) \wedge ((\sim P \wedge \sim Q) \vee (\sim P \vee \sim Q))$

$\cancel{(P \vee Q)}(P \wedge Q) \vee (P \vee Q)$

$(P \vee (P \vee Q)) \wedge Q \vee (P \vee Q) \wedge$

$(P \vee Q) \wedge (P \vee Q) \quad (P \vee Q) \wedge$

(2) $(\sim P \wedge \sim Q) \vee (\sim P \vee \sim Q)$

$\sim P \vee (\sim P \vee \sim Q) \wedge (\sim Q) \vee (\sim P \vee \sim Q)$

$(\sim P \vee \sim Q) \wedge (\sim P \vee \sim Q)$

(1) + (2)

$(P \vee Q) \wedge (\sim P \vee \sim Q)$

$(P \wedge (\sim P \vee \sim Q)) \vee Q \wedge (\sim P \vee \sim Q))$

$(P \wedge \sim P) \vee (P \wedge \sim Q) \vee (Q \wedge \sim P) \vee (Q \wedge \sim Q)$

$\cancel{F} \vee (P \wedge \sim Q) \vee (\sim P \wedge Q) \vee \cancel{F}$

$\boxed{\Leftrightarrow (P \wedge \sim Q) \vee (\sim P \wedge Q)}$ //

$F \vee T \Leftrightarrow T$

$\sim (P \vee Q) \rightleftarrows (P \wedge Q)$

$(\sim (P \vee Q) \rightarrow (P \wedge Q)) \wedge ((P \wedge Q) \rightarrow (\sim (P \vee Q)))$

$((P \vee Q) \vee (P \wedge Q)) \wedge (\sim (P \wedge Q) \vee (\sim (P \vee Q)))$

$((P \vee Q) \vee (P \wedge Q)) \wedge (\sim P \vee \sim Q \vee \sim P \wedge \sim Q$

$(P \vee (P \wedge Q)) \vee (Q \vee (P \wedge Q)) \wedge \sim P \vee (\sim P \wedge \sim Q) \vee \sim Q \vee (\sim P \wedge$

$(P \vee P \wedge Q \vee Q \vee (Q \wedge P) \wedge (\sim P \vee \sim P) \wedge \sim Q \vee (\sim Q \vee (\sim Q) \wedge$

$(P \wedge Q) \vee (Q \wedge P) \wedge (\sim P \wedge \sim Q) \vee (\sim Q \wedge \sim P)$

$(P \wedge Q) \wedge (\sim P \wedge \sim Q)$

Q. Find the dnf of $P\wedge(\sim(Q\vee R))\vee(((P\wedge Q)\vee\sim R)\wedge P)$

Sol: $P\wedge(\sim Q\wedge\sim R)\vee((P\wedge Q)\vee\sim R)\wedge P)$

$\cancel{P\wedge((\sim Q\wedge\sim R)\vee((P\wedge Q)\vee\sim R))}$

$P\wedge(\sim Q\wedge\sim R)\vee((P\wedge Q)\wedge P)\vee(\sim R\wedge P)$

$(P\wedge\sim Q\wedge\sim R)\vee(P\wedge Q)\vee(P\wedge\sim R)$

Q. Find the dnf of $(Q\vee(P\wedge R))\wedge\sim((P\vee R)\wedge Q)$

Sol: $(\underbrace{Q}_{P}\vee\underbrace{(P\wedge R)}_{Q})\wedge(\underbrace{\sim(P\vee R)}_{R}\vee\underbrace{\sim Q}_{S})$

$\boxed{(P\wedge R)\vee(P\wedge S)\vee(Q\wedge R)\vee(Q\wedge S)}$

$Q\wedge(\sim(P\vee R))\vee(Q\wedge(\sim Q))\vee((P\wedge R)\wedge(\sim(P\vee R)))\vee((P\wedge R)\wedge\sim$

$Q\wedge(\sim P\wedge\sim R)\vee F\vee((P\wedge R)\wedge(\sim P\wedge\sim R))\vee(P\wedge R\wedge\sim Q)$

$(\sim P\wedge Q\wedge\sim R)\vee F\vee(P\wedge R\wedge\sim P\wedge\sim R)\vee(P\wedge\sim Q\wedge R)$

$(\sim P\wedge Q\wedge\sim R)\vee(P\wedge\sim Q\wedge R)$

Q. Find the dnf of $[\underbrace{(P\wedge Q)}_{P}\vee\underbrace{(P\wedge\sim Q)}_{Q}]\wedge[\underbrace{(P\wedge\sim Q)}_{R}\vee\underbrace{(\sim P\wedge\sim Q)}_{S}]$

Sol: $((P\wedge Q)\wedge(P\wedge\sim Q))\vee((P\wedge Q)\wedge(\sim P\wedge\sim Q))\vee((P\wedge\sim Q)\wedge(P\wedge\sim$

$\vee((P\wedge\sim Q)\wedge(\sim P\wedge\sim Q))$

$(P\wedge F)\vee(F\wedge F)\vee(P\wedge\sim Q)\vee(F\wedge\sim Q)$

$F\vee F\vee(P\wedge\sim Q)\vee F$

$F\vee(P\wedge\sim Q)$

$P\wedge\sim Q$

Q. Find the dnf of $\sim(P \to (Q \wedge R))$

Sol: $\sim(\sim P \vee (Q \wedge R))$

$P \wedge \sim(Q \wedge R)$

$P \wedge \sim Q \vee \sim R$

Q. Find the dnf of $(P \wedge \sim(Q \wedge R)) \vee \sim(P \to Q)$

Sol: $(P \wedge (\sim Q \vee \sim R)) \vee \sim(\sim P \vee Q)$

$((P \wedge \sim Q) \vee (P \wedge \sim R)) \vee (P \wedge \sim Q)$

$(P \wedge \sim Q) \vee (P \wedge \sim R)$

Q. Find the dnf of $P \vee (\sim P \to (Q \vee (Q \to R)))$

Sol: $P \vee (\sim P \to (Q \vee (\sim Q \vee R)))$

$P \vee (\sim P \to (T \vee R))$

$P \vee (\sim P \to T)$

$P \vee (P \vee T)$

$P \vee T$

T

Q. Find the dnf of $(\sim P \vee \sim Q) \to (\sim P \wedge R)$

Sol: $(\sim(P \wedge Q) \to (\sim P \wedge R))$

$(P \wedge Q) \vee (\sim P \wedge R)$

Q. Find the dnf of $P \vee (\sim P \wedge (\sim Q \vee R))$

Sol: $P \vee ((\sim P \wedge \sim Q) \vee (\sim P \wedge R))$

$(\sim P \wedge \sim Q) \vee (\sim P \wedge R) \vee P$

Q. Find the dnf of $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

Sol: $(P \rightarrow ((\sim P \vee Q)) \wedge (Q \wedge P))$

$\cancel{\sim P \vee (\sim P \vee Q) \wedge (Q \wedge P)}$ $\cancel{P \rightarrow (\sim P \wedge Q) \vee}$

$\cancel{(\sim P \vee Q) \wedge (Q \wedge P)}$

$\cancel{F \wedge Q}$

$\cancel{F}$

$P \rightarrow ((\sim P \vee Q) \wedge (P \wedge Q))$

$P \rightarrow \sim P \wedge (P \wedge Q) \vee Q \wedge (P \wedge Q)$

$P \rightarrow (F \wedge Q) \vee (P \wedge Q)$

$P \rightarrow F \vee (P \wedge Q)$

$P \rightarrow P \wedge Q$

$\sim P \vee (P \wedge Q)$

Q. Find the dnf using truth tables $(\sim P \rightarrow R) \wedge (P \rightleftarrows Q)$

Sol:

P	Q	R	$\sim P \rightarrow R$	$P \rightleftarrows Q$	$\wedge$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	F	T	F

$(T,T,T), (T,T,F), (F,F,T)$ are having 'T'

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$

* Conjuctive Normal form (cnf):

A formula which is equivalent to a given formula and which consists of a product of elementary sums is called a conjuctive normal form of the given formula

Ex: $(P \vee Q) \wedge (\sim P \vee Q) \wedge (\sim P \vee \sim Q)$

Q. $P \wedge (P \rightarrow Q)$

Sol: $P \wedge (\sim P \vee Q) \Leftrightarrow (P \wedge \sim P) \vee (P \wedge Q) = F \vee (P \wedge Q) \Leftrightarrow P \wedge Q$

Q. $\sim(P \vee Q) \rightleftarrows (P \wedge Q)$

Sol: $((P \vee Q) \vee (P \wedge Q)) \wedge (\sim(P \wedge Q) \vee \sim(P \vee Q))$

$((P \vee Q) \vee (P \wedge Q)) \wedge ((\sim P \vee \sim Q) \vee (\sim P \wedge \sim Q))$

$((P \vee Q) \vee P) \wedge ((P \vee Q) \vee Q) \wedge ((\sim P \vee \sim Q) \vee \sim P) \wedge ((\sim P \vee \sim Q) \vee \sim Q)$

$((P \vee Q) \wedge (P \vee Q)) \wedge ((\sim P \vee \sim Q) \wedge (\sim P \vee \sim Q))$

$(P \vee Q) \wedge (\sim P \vee \sim Q)$

Q. $(Q \vee (P \wedge R)) \wedge \sim[(P \vee R) \wedge Q]$

Sol: $(Q \vee (P \wedge R)) \wedge ((\sim P \wedge \sim R) \vee \sim Q)$

$\cancel{(Q \vee (P \wedge R)) \wedge (\sim P \vee \sim Q \wedge \sim Q))}$

$\cancel{(Q \vee (R \wedge R))} \wedge ((Q \vee P) \wedge (Q \vee R)) \wedge ((\sim P \wedge \sim R) \vee \sim Q)$

$(Q \vee P) \wedge (Q \vee R) \wedge (\sim P \vee \sim Q) \wedge (\sim R \vee \sim Q)$

Q. $(Q \vee (P \wedge \sim Q)) \vee (\sim P \wedge \sim Q)$

$(Q \vee P) \wedge (Q \vee \sim Q) \vee (\sim P \wedge \sim Q)$

$(Q \vee P) \wedge T \vee \sim (P \vee Q)$

$\cancel{(R \vee Q) \wedge T} \; ((Q \vee P) \vee \sim (Q \vee P)) \wedge T$

$\cancel{(R \vee Q) \wedge}$ $\quad T \wedge T$

T

Q. $(P \rightarrow [(P \rightarrow Q) \wedge \sim (\sim Q \vee \sim P)]$

Sol: $P \rightarrow [(P \rightarrow Q) \wedge \sim (\sim (Q \wedge P))$

$P \rightarrow [(P \rightarrow Q) \wedge (Q \wedge P)]$

$P \rightarrow [(\sim P \vee Q) \wedge (Q \wedge P)]$

$P \rightarrow [\sim P \wedge (Q \wedge P) \vee Q \wedge (Q \wedge P)]$

$P \rightarrow [(F \wedge Q) \vee (Q \wedge P)]$

$P \rightarrow [(F) \vee (Q \wedge P)]$

$P \rightarrow (Q \wedge P)$

$\sim P \vee (Q \wedge P)$

dnf $\sim P \vee (P \wedge Q) \rightarrow$ cnf $(\sim P \vee P) \wedge (\sim P \vee Q)$

$T \wedge (\sim P \vee Q)$

$\sim P \vee Q$

$Q \vee \quad P \vee \sim P$

$Q \vee \{ \quad P$

* **Principal disjunctive Normal form (PDNF):**

• **Min term:**

A min term consisting of conjunctions in which each stmt. variable or its negation but not both appears only once

Ex: $P \wedge Q$, $\sim P \wedge Q$, $\sim P \wedge \sim Q$, $P \wedge \sim Q$

Min terms for the three variables P, Q & R are

$P \wedge Q \wedge R$, $\sim P \wedge Q \wedge R$, $P \wedge \sim Q \wedge R$, $P \wedge Q \wedge \sim R$, $\sim P \wedge \sim Q \wedge R$,

$\sim P \wedge Q \wedge \sim R$, $P \wedge \sim Q \wedge \sim R$, $\sim P \wedge \sim Q \wedge \sim R$

* It is clear that no two min terms are equivalent

* Each min term has the truth value 'T' for exactly one combination of the truth values of the variables

* **PDNF:**

An equivalent formula consisting of disjunctions of min terms only is known as its principal disjunctive normal form. It is sum of products canonical form.

* **Methods to obtain PDNF of a given formula:**

1. In order to obtain PDNF of a given formula

Step 1: Replace the conditionals & biconditionals by their equivalent formulas containing $\vee$, $\wedge$ and $\sim$

Step 2: Negations are applied to the variables by using demorgan's laws followed by the applications of distributive laws.

Step 3: Any elementary product which is a contradiction is dropped, min terms are obtained in the disjunctions by introducing the missing variables.

Q. Obtain the PDNF

1. $P \rightleftarrows Q$

Sol: $(\sim P \vee Q) \wedge (P \vee \sim Q)$

$\sim P \wedge (P \vee \sim Q) \vee Q \wedge (P \vee \sim Q)$

$(\sim P \wedge P) \vee (\sim P \wedge \sim Q) \vee (Q \wedge P) \vee (Q \wedge \sim P)$

$F \vee (\sim P \wedge \sim Q) \vee (P \wedge Q) \vee F$

$(\sim P \wedge \sim Q) \vee (P \wedge Q)$

2. $\sim P \vee Q$

Sol: $\sim P \wedge (Q \vee \sim Q) \vee Q \wedge (P \vee \sim P)$

$\underline{(\sim P \wedge Q)} \vee (\sim P \wedge \sim Q) \vee (Q \wedge P) \vee \underline{(Q \wedge \sim P)}$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$

Q: a) $P \vee (P \wedge Q)$ b) $P \vee (\sim P \wedge Q)$

Sol: a) $P \wedge (Q \vee \sim Q) \vee (P \wedge Q)$

$\underline{(P \wedge Q)} \vee (P \wedge \sim Q) \vee \underline{(P \wedge Q)}$

$(P \wedge Q) \vee (P \wedge \sim Q)$

b) $P \vee (\sim P \wedge Q)$

$P \wedge (Q \vee \sim Q) \vee (\sim P \wedge Q)$

$(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

Q. $P \to ((P \to Q) \wedge (\sim Q \vee \sim P))$

Sol: $P \to ((\sim P \vee Q) \wedge (Q \wedge P))$

$P \to ((\sim P \wedge (Q \wedge P) \vee Q \wedge (Q \wedge P))$

$P \to (((F \wedge Q) \vee (Q \wedge P))$

$P \to (F \vee (Q \wedge P))$

$P \to (Q \wedge P)$

$\sim P \vee (Q \wedge P)$

$\sim P \wedge (Q \vee \sim Q) \vee (Q \wedge P)$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$

Q. $P \vee (\sim P \wedge Q \wedge R)$

Sol: $(P \wedge (Q \vee \sim Q) \wedge (R \vee \sim R)) \vee (\sim P \wedge Q \wedge R)$

$((P \wedge Q \vee P \wedge \sim Q) \wedge (R \vee \sim R)) \vee (\sim P \wedge Q \wedge R)$

$((P \wedge Q) \wedge (R \vee \sim R) \vee (P \wedge \sim Q) \wedge (R \vee \sim R)) \vee (\sim P \wedge Q \wedge R)$

$\cancel{P \wedge (R \vee \sim R) \wedge Q \wedge (R \vee \sim R) \vee (P \wedge (R \vee \sim R) \wedge \sim Q \wedge (R \vee \sim R)) \vee (\sim P \wedge Q \wedge R)}$

$\cancel{(P \wedge R) \wedge (P \vee \sim R) \wedge (Q \wedge R) \wedge (Q \vee \sim R)}$

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R)$

$\vee (\sim P \wedge Q \wedge R)$

Q. $(Q \wedge \sim R \wedge \sim S) \vee (R \wedge S)$

Sol: $(Q \wedge \sim R \wedge \sim S) \vee (R \wedge S) \wedge (Q \vee \sim Q)$

$(Q \wedge \sim R \wedge \sim S) \vee (R \wedge S \wedge Q) \vee (R \wedge S \wedge \sim Q)$

* Principal Conjunctive Normal Form (PCNF):

· Max terms:

A max term consists of disjunctions in which each variable or its negation but not both appears only once.

Ex: $P \vee Q$, $P \vee \sim Q$, $\sim P \vee Q$, $\sim P \vee \sim Q$

Max terms for the three variables P, Q and R are $(P \vee Q \vee R)$, $(P \vee Q \vee \sim R)$, $(P \vee \sim Q \vee R)$, $(P \vee \sim Q \vee \sim R)$, $(\sim P \vee Q \vee R)$, $(\sim P \vee Q \vee \sim R)$, $(\sim P \vee \sim Q \vee R)$, $(\sim P \vee \sim Q \vee \sim R)$

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Clearly the max terms are the duals of mins terms.

* PCNF:

An equivalent formula consisting of conjunctions of max terms only is known as PCNF i.e product of sums canonical form

* Methods to obtain PCNF:

· The method for obtaining the PCNF for a given formula is similar to the PDNF.

· If the PDNF of a given formula 'A' contains 'n' variables then the PDNF of '$\neg A$' consists of the disjunction of the remaining min terms i.e which do not appear in the PDNF of A. Then $\sim(\neg A)$ can obtain the PCNF of 'A'.

Q. Obtain the PCNF $(\sim P \to R) \wedge (Q \rightleftarrows P)$

Sol: $(P \vee R) \wedge ((Q \to P) \wedge (P \to Q))$

$(P \vee R) \wedge ((\sim Q \vee P) \wedge (\sim P \vee Q))$

$\uparrow Q \qquad \uparrow R \qquad \uparrow R$

$(P \vee R) \vee (Q \wedge \sim Q) \wedge ((\sim Q \vee P) \vee (R \wedge \sim R) \wedge (\sim P \vee Q) \vee (R \wedge \sim R)$

$\cancel{P \vee (Q \wedge \sim Q) \vee R \vee (Q \wedge \sim Q) \wedge ((\sim Q \vee (R \wedge \sim R) \vee P \vee (R \wedge \sim R)}$

$\cancel{\wedge (\sim P \vee (R \wedge \sim R) \vee Q \vee (R \wedge \sim R)}$

$(P \vee R \vee Q) \wedge \underset{P \vee R \vee}{\sim Q} \wedge ((\sim Q \vee P \vee R) \wedge (\sim Q \vee P \vee \sim R) \wedge$

$(\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R))$

$(P \vee R \vee Q) \wedge (P \vee R \vee \sim Q) \wedge (\sim Q \vee P \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge$

$(\sim P \vee Q \vee \sim R)$

but it is DNF as all terms are not there

Q. Given the PDNF of A: $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$. find PCNF. PDNF

Sol: $A : (P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

$A : (P \wedge Q) \wedge (R \vee \sim R) \vee (\sim P \wedge R) \wedge (Q \vee \sim Q) \vee (Q \wedge R) \wedge (P \vee$

$A : (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge R \wedge Q) \vee (\sim P \wedge R \wedge \sim Q)$

$\vee (Q \wedge R \wedge P) \vee (Q \wedge R \wedge \sim P)$

$A : \overset{T\,T\,T}{(P \wedge Q \wedge R)} \vee \overset{T\,T\,F}{(P \wedge Q \wedge \sim R)} \vee \overset{F\,T\,T}{(\sim P \wedge R \wedge Q)} \vee \overset{F\,T\,F}{(\sim P \wedge R \wedge \sim Q)}$

$\neg A : (P \wedge \sim Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge \sim R)$

$\sim\Gamma A : (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R) \wedge (P \vee \sim Q \vee R) \wedge$
$(P \vee Q \vee R)$

Q. Find the PDNF and PCNF $A : (P \wedge Q) \vee (\sim P \wedge Q) \vee (Q \wedge R)$

Sol: $A : (P \wedge Q) \vee (\sim P \wedge Q) \vee (Q \wedge R)$

$(P \wedge Q) \wedge (R \vee \sim R) \vee (\sim P \wedge Q) \wedge (R \vee \sim R) \vee (Q \wedge R) \wedge (P \vee \sim$

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge \sim R)$
$\vee (Q \wedge R \wedge P) \vee (Q \wedge R \wedge \sim P)$

PDNF
$A : (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge \sim R)$

$\Gamma A : (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R) \vee (\sim P \wedge \sim Q \wedge \sim R)$

PCNF
$\sim(\Gamma A) : (\sim P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee Q \vee \sim R) \wedge (P \vee Q \vee R)$

Q. Find PDNF from PCNF of $S : \{ P \vee (\sim P \rightarrow (Q \vee (\sim Q \rightarrow R)))$

Sol: $S : P \vee (\sim P \rightarrow (Q \vee (\sim Q \rightarrow R)))$

$S : P \vee (\sim P \rightarrow (Q \vee Q \vee R))$

$S : P \vee (\sim P \rightarrow (Q \vee R))$

$S : P \vee (P \vee Q \vee R)$

P)

$S : P \vee Q \vee R$

$\Gamma S : (P \vee Q \vee \sim R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge$
$(\sim P \vee Q \vee \sim R) \wedge (\sim P \vee \sim Q \vee R) \wedge (\sim P \vee \sim Q \vee \sim R)$

$\sim(\Gamma S) : (\sim P \wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R)$
$\vee (P \wedge \sim Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge Q \wedge R)$

* Statement Calculus:

Theory of inference:

- Def: The main aim of logic is to provide rules of inference to infer a conclusion from certain premises. The theory associated with rules of inference is known as inference theory.

- Def: If a conclusion is derived from a set of premises by using the accepted rules of reasoning then such a process of derivation is called a deduction or a formal proof and the argument or conclusion is called a valid argument or valid conclusion. ~~Let A~~

Def: Let A and B be two stmt. formulas and say that B logically follows from A or B is a valid conclusion of the premises A if and only if $A \to B$ is a tautology and it is denoted as $A \Rightarrow B$

A set of premises $\{H_1, H_2, \ldots H_m)$ a conclusion 'C' follows logically if and only if

$$H_1 \wedge H_2 \ldots H_m \Rightarrow C$$

Implications:

$I_1 : P \wedge Q \Rightarrow P$ } simplification

$I_2 : P \wedge Q \Rightarrow Q$

$I_3 : P \Rightarrow P \vee Q$ } addition

$I_4 : Q \Rightarrow P \vee Q$

$I_5 : \sim P \Rightarrow P \rightarrow Q$

$I_6 : Q \Rightarrow P \rightarrow Q$

$I_7 : \sim(P \rightarrow Q) \Rightarrow P$

$I_8 : \sim(P \rightarrow Q) \Rightarrow \sim Q$

$I_9 : P, Q \Rightarrow P \wedge Q$

$I_{10} : \sim P, P \vee Q \Rightarrow Q$

$I_{11} : P, P \rightarrow Q \Rightarrow Q$

$I_{12} : \sim Q, P \rightarrow Q \Rightarrow \sim P \; ; \; Q, P \rightarrow \sim Q \Rightarrow \sim P$

$I_{13} : P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$

$I_{14} : P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$

Q. Determine whether the conclusion C follows logically from the hypothesis H_1 and H_2 by using truth tables.

(1) $H_1 : P \rightarrow Q$, $H_2 : P$, $C : Q$

$H_1 \wedge H_2$

$(P \rightarrow Q) \wedge P$

$(\sim P \vee Q) \wedge P$

$(\sim P \wedge P) \vee (Q \wedge P)$

$F \vee (Q \wedge P)$

$Q \wedge P$ (simplification)

Q

(2) $H_1 : P \rightarrow Q$, $H_2 : Q$, $C : P$

(3) $H_1 : \sim P$, $H_2 : P \rightleftarrows Q$, $C : (P \wedge Q)$

(4) $H_1 : P \rightarrow Q$, $H_2 : \sim P$, $C : Q$

(5) $H_1 : \sim Q$, $H_2 : P \rightarrow Q$, $C : \sim P$

(6) $H_1 : Q$

$H_2 : P \vee \sim Q$

$C : Q$

Soln: ①

P	Q	$H_1: P \to Q$	$H_2: P$	$H_1 \wedge H_2$	$C: Q$	$(H_1 \wedge H_2) \to C$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	F	F	F	T

valid conclusion

②

P	Q	$H_1: P \to Q$	$H_2: Q$	$H_1 \wedge H_2$	$C: P$	$(H_1 \wedge H_2) \to C$
T	T	T	T	T	T	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	F	F	F	T

③

P	Q	$H_1: \sim P$	$H_2: P \rightleftarrows Q$	$H_1 \wedge H_2$	$C: (P \wedge Q)$	$(H_1 \wedge H_2$
T	T	F	T	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	F	F

④

$H_1: P \to Q$	$H_2: \sim P$	$H_1 \wedge H_2$	$C: Q$	$(H_1 \wedge H_2) \to C$
T	F	F	T	T
F	F	F	F	T
T	T	T	T	T
T	T	T	F	F

(5)

$H_1 : \sim Q$	$H_2 : P \to Q$	$H_1 \wedge H_2$	$C : \sim P$	$(H_1 \wedge H_2) \wedge C$
F	T	F	F	T
T	F	F	F	T
F	T	F	T	T
T	T	T	T	T

valid

(6)

$H_1 : Q$	$H_2 : P \vee \sim Q$	$H_1 \wedge H_2$	$C : Q$	$(H_1 \wedge H_2) \to C$
T	T	T	T	T
F	T	F	F	T
T	F	F	T	T
F	F	F	F	T

valid

* **Equivalences:**

$E_1 : \sim(\sim P) \Leftrightarrow P$

$E_2 : P \wedge Q \Leftrightarrow Q \wedge P$

$E_3 : (P \vee Q) \Leftrightarrow Q \vee P$

$E_4 : P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

$E_5 : (P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$

$E_6 : P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

$E_7 : P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

$E_8 : \sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$

$E_9 : \neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

$E_{10} : P \wedge P \Leftrightarrow P$

$E_{11} : P \vee P \Leftrightarrow P$

$E_{12} : R \vee (P \wedge \sim P) \Leftrightarrow R$

$E_{13} : R \wedge (P \vee \sim P) \Leftrightarrow R$

$E_{14}: R \vee (P \vee \sim P) \Leftrightarrow T$

$E_{15}: R \wedge (P \wedge \sim P) \Leftrightarrow F$

$E_{16}: P \rightarrow Q \Leftrightarrow \sim P \vee Q$

$E_{17}: \sim (P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$

$E_{18}: P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$

$E_{19}: P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$

$E_{20}: \sim (P \rightleftarrows Q) \Leftrightarrow P \rightleftarrows \sim Q$

$E_{21}: P \rightleftarrows Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

$E_{22}: P \rightleftarrows Q \Leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$

* **Rule P:**

We may introduce a premises at any point in the derivation.

* **Rule T:**

We may introduce a formula 'S' in a derivation if S is tautologically implied by any one or more of the preceding formulae in the derivation.

Q. Determine that 'S' is a valid influence from the premises

$P \rightarrow \sim Q,\ Q \vee R,\ \sim S \rightarrow P$ and $\sim R$

Sol:

$\left.\begin{array}{l} Q \vee R \\ \sim R \end{array}\right\} I_{10}$ Rule P, Rule P

Q

$\left.\begin{array}{l} P \rightarrow \sim Q \\ Q \end{array}\right\} I_{12}$ Rule P, Rule P

$\sim P$

$\left.\begin{array}{l}\sim S \to P \\ \sim P\end{array}\right\} I_{12}$ $\begin{array}{l}\text{Rule P} \\ \text{Rule P}\end{array}$

$\sim(\sim S)$

$\Rightarrow S$

Q. Show that "$R \vee S$" follows logically from the premises

$C \vee D,\ C \vee D \to \sim H,\ \sim H \to (A \wedge \sim B),\ (A \wedge \sim B) \to (R \vee S)$

Sol:

$\left.\begin{array}{l}C \vee D \to \sim H \\ \sim H \to (A \wedge \sim B)\end{array}\right\} I_{13}$ $\begin{array}{l}\text{Rule P} \\ \text{Rule P}\end{array}$

$\left.\begin{array}{l}C \vee D \to (A \wedge \sim B) \\ C \vee D\end{array}\right\} I_{11}$

$\left.\begin{array}{l}A \wedge \sim B \\ A \wedge \sim B \to (R \vee S)\end{array}\right\} I_{11}$

"$R \vee S$"

LA-2

Q. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises

$P \vee Q,\ \underbrace{Q \to R,\ \overset{P}{\cancel{Q}} \to M}$, and $\sim M$

$\sim Q,\ P \to Q \Rightarrow \sim$
$\sim P,\ P \vee Q \Rightarrow Q$
$P,\ P \to Q \Rightarrow Q$
$P, Q \Rightarrow P \wedge Q$

Sol:

$\cancel{Q \to R}$

$\cancel{R \to M}$

$\cancel{Q \to M}$

$\cancel{\sim M}$

$\cap(Q \vee \sim Q)$

$\left.\begin{array}{l}\sim M \\ P \to M\end{array}\right\} I_{12}$

$\left.\begin{array}{l}\sim P \\ P \vee Q\end{array}\right\} I_{10}$

$\left.\begin{array}{l}Q \\ Q \to R\end{array}\right\} I_{11}$

$\left.\begin{array}{l}R \\ P \vee Q\end{array}\right\} I_9$ Rule T

$R \wedge (P \vee Q)$

* **Rule CP**: If we can derive 'S' from 'R' and a set of premises then we can derive $R \rightarrow S$ from the set of premises alone. This can be written as

$(P \wedge R) \rightarrow S \Leftrightarrow P \rightarrow (R \rightarrow S)$ where P denotes the conjunction of the set of premises say $P_1, P_2 \ldots P_n$

Q. Show that $R \rightarrow S$ can be derived from the premises

$P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$ and Q

Sol:

1: $\sim R \vee P$ Rule P

2: R (additional premises) Rule P

[1,2] 3: P Rule P

4: $P \rightarrow (Q \rightarrow S)$ Rule P

[3,4] 5: $Q \rightarrow S$ Rule P

6: Q Rule P

[5,6] 7: S Rule P

[2,7] 8: $R \rightarrow S$ Rule CP

Q. Show that $P \rightarrow S$ can be derived from

$\sim P \vee Q$, $\sim Q \vee R$, $R \rightarrow S$

Sol:

1: $\sim P \vee Q$ Rule P

2: P (additional)

3: Q

4: $\sim Q \vee R$

5: R

6: $R \to S$

7: S

8: P (additional)

9: $P \to S$ (Rule CP)

** Q. Derive $P \to (Q \to S)$ using rule CP if necessary from $P \to (Q \to R)$, $Q \to (R \to S)$

Sol:

1: $P \to (Q \to R)$

2: P (additional)

3: $Q \to R$

$\Rightarrow \sim Q \vee R$

4: $Q \to (R \to S)$

$\Rightarrow \sim Q \vee (\sim R \vee S)$

$(\sim Q \vee R) \wedge (\sim Q \vee (\sim R \vee S))$

$\sim Q \vee (R \wedge (\sim R \vee S))$

$\sim Q \vee (R, (\sim R \vee S))$

$\sim Q \vee S$

$\Rightarrow Q \to S$

P

$P \to (Q \to S)$ Rule CP

* Indirect Method of proof:

The method of using the rule of conditional proof and the notion of an inconsistent set of premises is called the indirect method of proof or proof by contradiction. The technique of indirect method of proof is as follows:

1. Introduce the negation of the desired conclusion as a new premise

2. From the additional or new premise together with the given premises derive a contradiction.

3. Assert the desired conclusion as a logical inference from the premises.

Q. (LA-4) Using indirect method of proof, derive $P \to \sim S$ from $P \to (Q \vee R)$, $Q \to \sim P$, $S \to \sim R$, P.

Sol:

$P \to \sim S$

$\sim P \vee \sim S$

$\sim(\sim P \vee \sim S)$

$P \wedge S$ → Negation of desired conclusion

The additional premise is $P \wedge S$

1: $P \to (Q \vee R)$ Rule P

2: P Rule P

3: $Q \vee R$

4: $S \to \sim R$

5: $P \wedge S$

6: S I_1

7: $\sim R$ (I_{11})

8: $(Q \vee R), \sim R$

9: Q

10: $Q \to \sim P$

11: $\sim P$

12: $P \wedge \sim P$ Rule T

F

contradiction

Q. Prove by indirect method

$\sim Q,\ P \rightarrow Q,\ P \vee R \Rightarrow R$

Sol: New premise $\sim R$

1: $P \vee R$

2: $\sim R$

3: P

4: $P \rightarrow Q$

5: Q

6: $\sim Q$

7: $Q \wedge \sim Q$

F

Q. By indirect method

$P \rightarrow Q,\ Q \rightarrow R,\ \cancel{\sim(P \wedge R)},\ (P \vee R) \rightarrow R$

Sol: New premise $\sim R$

1: $Q \rightarrow R$

2: $P \rightarrow Q$

[1,2] 3: $P \rightarrow R$

4: $\sim R$

[3,4] 5: $\sim P$

6: $P \vee R$

[5,6] 7: R

8: $\sim R$ Rule T

[7,8] 9: $R \wedge \sim R$

F

$\boxed{7731011233}$

* Predicate Calculus:

The part of the logic which deals with the content of stmts, based upon the analysis of predicates is called predicate calculus.

Ex: 'x' is a student. This stmt has two parts, the second part 'is a student' is called the predicate.

* Quantifiers:

Certain stmts involve words that indicate quantity such as "all, some, none or one", they answer the question how many. Such words indicate quantity, they are called quantifiers.

Ex: 1. Some men are tall

2. All birds have wings

3. No air balloon is perfectly round

4. There is a real no. less than 11

* Universal quantifier:

The quantifier 'all' is the universal quantifier. It is denoted as "$\forall x$". The symbol ($\forall x$) represents for all x, for every x, for each x, every thing x such that, each thing x such that.

* Existential quantifier:

The quantifier '~~some~~ some' is the existential quantifier. It is denoted as "$\exists x$". The symbol ($\exists x$) represents for some x, some x such that, there exists an x such that, there is an x such that, there is atleast one x such that

Ex: Write each of the following in a symbolic form

① All monkeys have tails.

$M(x)$ = is a monkey $\qquad (\forall x)(M(x) \rightarrow T(x))$

$T(x)$ = has tail

② No monkey has a tail

$$\forall x[M(x) \rightarrow \sim T(x)]$$

③ Some monkeys have tails

$$(\exists x)[M(x) \wedge T(x)]$$

④ Some monkeys have no tails

$$\exists(x)[M(x) \wedge \sim T(x)]$$

Ex: ① All men are good

$M(x)$ = is a man

$G(x)$ = good

$$\forall(x)[M(x) \rightarrow G(x)]$$

② No men are good

$$\forall(x)[M(x) \rightarrow \sim G(x)]$$

③ Some men are good

$$\exists(x)[M(x) \wedge G(x)]$$

④ Some men are not good

$$\exists(x)[M(x) \wedge \sim G(x)]$$

Ex: ① Some people who trust others ~~are~~ one rewarded

$$\exists(x)\,[P(x) \wedge T(x) \wedge R(x)]$$

Ex: It is not true that all roads lead to Rome

$$\sim[\forall(x)\,[R(x) \longrightarrow r(x)]]$$

(or)

$$\exists(x)\,[R(x) \wedge \sim r(x)]$$

* Statement Formulas in Predicate Calculus:

Bound and Free Variable:

Predicate formulas contain a part of the form ~~for~~ $(\forall x)P(x)$ or $\exists(x)P(x)$, such a part is called x-bound part of the formula. Any variable appearing in an x-bound part of the formula is called bound variable, otherwise it is called free variable.

* The smallest formula immediately following $(\forall x)$ or $\exists(x)$ is called "scope of the quantifier".

Ex: ① $(\forall x)\,P(x,y)$

is the scope of the quantifier, and occurance of x is bound occurance, occurance of y is free ~~variable~~ occurance.

② $\exists(x)P(x) \wedge Q(x)$

The scope of the $\exists(x)$ is $P(x)$ and the $Q(x)$ is free

* Predicate Calculus: Theory of inference:

In order to draw the conclusions from quantified premises, a need to know how to remove the quantifiers properly, argue with the resulting stmts and then properly prefix or add the correct the quantifiers. We can use the rules of information given for the stmt calculus.

The rules P & T are used in the introduction of a premise at any stage of the derivation. Also use the rule CP if the conclusion is given in the form a conditional.

The ~~elima~~ elimination of the quantifiers can be done by rules of specification called universal US and existential ES. To prefix the correct quantifier, the rules of generalization called UG and EG are needed.

1. **Rule US: (Universal Specification):**

If a stmt of the form $(\forall x)P(x)$ is assumed to be true then the universal quantifier can be dropped to obtain $P(t)$ is true for an arbitary object "t" in the universe

In symbols, the rule is $\dfrac{(\forall x)P(x)}{\therefore P(t) \text{ for all } t}$

2. **Rule UG: (Universal Generalization)**

If a statement $P(t)$ is true for each element 't' of the universe, then the universal quantifier may be prefixed to obtain $(\forall x)P(x)$

i.e $\dfrac{P(t) \text{ for all } t}{\therefore (\forall x)P(x)}$

3. Rule ES (Existential Specification)

If $\exists(x)P(x)$ is assumed to be true, then there is an element 't' in the universe such that $P(t)$ is true.

i.e $\dfrac{\exists(x)P(x)}{\therefore P(t) \text{ for some } t}$

4. Rule EG (Existential Generalization)

If $P(t)$ is true for some element 't' in the universe then $\exists(x)P(x)$ is true

i.e $\dfrac{P(t) \text{ for some } t}{\therefore \exists(x)P(x)}$

LA-5

Q. Verify the validity of the following argument

Every living thing is a plant or an animal. John's gold fish ~~at~~ is alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart

Sol: The universe consists of all living things

$P(x)$: x is a plant

$A(x)$: x is an animal

$H(x)$: x has a heart

g : John's gold fish

Then the ~~interfe~~ inference is

$\forall(x)(P(x) \vee A(x))$

$\sim P(g)$

$\forall(x)(A(x) \rightarrow H(x))$

$\Rightarrow$ $H(g)$

Arguments:

[1] 1: $\forall(x)\, P(x) \vee A(x)$ rule P

2: $\sim P(g)$ rule P

[1] 3: $P(g) \vee A(g)$ rule US

[2,3] 4: $A(g)$

5: $\forall(x)\, A(x) \rightarrow H(x)$ Rule P

6: $A(g) \rightarrow H(g)$ rule US

[4,6] 7: $H(g)$

Q. Verify the validity of the arguement

Tigers are dangerous animals

These are tigers

Therefore these are dangerous animals.

Sol: $T(x)$: x - is a Tiger

$d(x)$: x - is a dangerous animal

Then the inference is

$$\forall(x)(T(x) \rightarrow d(x))$$
$$\exists(x)\, T(x)$$
$$\overline{\exists(x)\, d(x)}$$

1. $\exists(x)\, T(x)$ rule P

2. $T(b)$ any element from universe rule ES

3. $\forall(x)(T(x) \rightarrow d(x))$ rule P

4. $T(b) \rightarrow d(b)$ rule US

[2,4] 5: $d(b)$

6: $\exists(x)\, d(x)$ rule EG

Q. Verify the argument

All men are mortal

Socrates is a man

Therefore Socrates is mortal

Sol: $H(x)$: x - is a man

$M(x)$: x - is a mortal

s : Socrates

The inference is

$$\forall(x)\left(H(x) \rightarrow M(x)\right)$$

$$\underbrace{H(s)}$$

$$M(s)$$

1. $\forall(x)\left(H(x) \longrightarrow M(x)\right)$ rule P
2. $H(s) \longrightarrow M(s)$ rule US
3. $H(s)$ rule P

[2,3] 4: $M(s)$

Q. All the integers are rational no.s

Some integers are powers of 3

Therefore, some rational no.s are powers of 3

Sol: $P(x)$: x - is an integer

$R(x)$: x - is a rational no.

$S(x)$: x - is a power of 3

Then inference is

$$\forall(x)\left(P(x) \rightarrow R(x)\right)$$

$$\exists(x)\left(P(x) \wedge S(x)\right)$$

$\exists(x)(R(x) \land S(x))$

1: $\exists(x)(P(x) \land S(x))$ rule P

2: $P(t) \land S(t)$ rule ES

3: $P(t)$

4: $S(t)$

5: $\forall(x)(P(x) \rightarrow R(x))$ Rule P

6: $P(t) \rightarrow R(t)$ rule US

[3,6] 7: $R(t)$

[4,7] 8: $R(t) \land S(t)$

9: $\exists(x)(R(x) \land S(x))$ Rule EG