NAME MT-24 COURSE ENGR 4020

DATE

LESSON SS 3 - observer Design PAGE 1 of 2

Observer Design by Pole Placement ①

- need to estimate state of plant based on available information
- This happens when we don't know all the states.
- Given

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

②

- Let's define a new variable $\hat{x}$, which is the estimated design
- The dynamics that govern $\hat{x}$ are

$$\dot{\hat{x}} = A\hat{x} + Br + L(y - C\hat{x})$$

- L is observer gain matrix
  $n \times m$
  states ↑ measurements

③

$C\hat{x} = \hat{y}$ = measurement

$(y - C\hat{x})$ = residual

set

$e(t) = x(t) - \hat{x}(t)$ to be error

Then

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t)$$

④

so

$$\dot{e}(t) = (Ax + \cancel{Br}) - (A\hat{x} + \cancel{Br} + L(y - C\hat{x}))$$

$$\dot{e}(t) = A(x - \hat{x}) - L(y - C\hat{x})$$

- we know $y = Cx$

$$\dot{e}(t) = A(x - \hat{x}) - LC(x - \hat{x})$$

$$\dot{e} = (A - LC)\, e(t)$$

⑤

Notice:

- Estimation error is not dependent on control $r(t)$
- solution

$$e(t) = e^{(A-LC)t} e(0)$$

- performance of observer is governed by roots of

$$\det\left[(sI - (A - LC)\right] = 0$$

⑥

Ex 1: consider

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$A = \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$C = [1 \quad 0]$$

Find observer gains so that estimation error has $\zeta = 0.8$ and $\omega_n = 2$ rad/s

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SOLN: check observability

$$P_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\det(P_o) = 1$ observable!

Find the roots of

$\det(sI - (A-LC))$

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$$A - LC = \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix} - \begin{bmatrix} L_1 & 0 \\ L_2 & 0 \end{bmatrix}$$

$$A - LC = \begin{bmatrix} -L_1 & 1 \\ 16-L_2 & 0 \end{bmatrix}$$

$\det(sI - (A - LC)) = 0$

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$$\det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -L_1 & 1 \\ 16-L_2 & 0 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} s+L_1 & -1 \\ L_2-16 & s \end{bmatrix}\right) = 0$$

$s^2 + L_1 s + (L_2 - 16) = 0$

compare to $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

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$s^2 + 3.2s + 4 = 0$

$s^2 + L_1 s + (L_2 - 16) = 0$

$\boxed{L_1 = 3.2}$

$L_2 = 4 + 16 = \boxed{20}$