NAME MT-22 COURSE ENGR 4020

DATE

LESSON State Space Control 1 PAGE 1 of 2

More sophisticated than you will implement in this course, but important to learn

- Let's assume you know how to get state space models (ENGR 3030)
  - ↳ If not, check out videos posted in CMS.

state space model

$\dot{x}(t) = Ax(t) + Bu(t)$

$y(t) = Cx(t) + Du(t)$

$x(t)$: vector of length n, n number of states, called system order

$y(t)$: vector of length m, m number of sensors/outputs

$u(t)$: vector of length r, r number of inputs

$A \in \mathbb{R}^{n \times n}$ $B \in \mathbb{R}^{n \times r}$ } matrizes

$C \in \mathbb{R}^{m \times n}$ $D \in \mathbb{R}^{m \times r}$

will assume Linear, Time Invariant, Dynamic systems for this course

Solution of State Equations

$x(t) = \underset{u=0}{x_{zi}(t)} + \underset{x(0)=0}{x_{zs}(t)}$

$\boxed{x_{zi}(t) = \underbrace{e^{At}}_{\text{matrix exponential}} x(0)}$

matrix exponential $\neq e^{[A_{mx}]t}$

- Let's figure out how to get the matrix exponential

- Let $u=0$ (condition for x_{zi}) then

$\dot{x} = Ax$

take Laplace transform

$\mathcal{L}[\dot{x}] = \mathcal{L}[Ax]$

$sX(s) - x(0) = AX(s)$

$sX(s) - AX(s) = x(0)$

$(sI - A)X(s) = x(0)$

$\underbrace{(sI-A)^{-1}(sI-A)}_{I} X(s) = (sI-A)^{-1}x(0)$

$X(s) = (sI-A)^{-1}x(0)$

$x_{zi}(t) = \mathcal{L}^{-1}[(sI-A)^{-1}]x(0)$

so $\boxed{e^{At} = \mathcal{L}^{-1}[(sI-A)^{-1}]}$

$\boxed{x_{zs}(t) = e^{At}Bu(t)}$

$\Rightarrow x_{zs}(t) = \mathcal{L}^{-1}[(sI-A)^{-1}]Bu(t)$

$y(t) = Cx(t) + Du(t)$

$\boxed{y(t) = Ce^{At}(x(0) + B)u(t) + Du(t)}$

- For a system transfer function, initial conditions are zero and

$Y(s) = [C(sI-A)^{-1}B + D]u(s)$

$\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$

Note that $(sI-A)^{-1}$ has the det $(sI-A)$ in the denominator!

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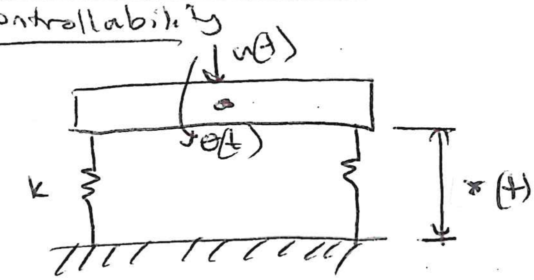
det(sI−A) is characteristic equation
roots(det(sI−A)) are system poles, (eigenvalues)
↳ from these, we know about stability (all in RHP), response characteristics (ζ, ω_n) etc!
↳ maybe this means we can design a control law to get response we want
↳ of course we can: caveat! system must be controllable / observable

controllability



placement of control actuator at center of beam can control vertical displacement but not rotation θ

θ(t) is an uncontrollable state!

Defn An LTIC system is controllable if for every state x^* and time $T>0$ there exists a control actuation u(t) for all time $0 \le t \le T$ such that the state is transfered from x(0) to $x(T) = x^*$

$$\forall x^*, T>0 \ \exists u(t) \ \forall t \in [0,T] \ \text{s.t.} \ x(0) \rightarrow x^* = x(T)$$

∘ How to determine if the system is controllable?
↳ define controllability mx $\mathcal{C}$

$$\mathcal{C} = [B \ \ AB \ \dots \ A^{n-1}B]$$

$$\mathcal{C} \in \mathbb{R}^{n \times nr}$$

Then, a system is controllable if $\text{rank}(\mathcal{C}) = n$

MATLAB
```
>> rank(ctrb(A,B))
```

Try with $A = \begin{bmatrix} 0 & 1 \\ -2 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

then $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

so, we can design state space controllers for full state feedback if a system is controllable
↳ pole placement
↳ put poles where we want