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Due: Monday 05/07/2017 by 4 PM

Problem 53: ***Design Problem-2 DOF Helicopter Control by Pole Placement*** [10 pts]

Shown below are the free-body diagram and kinematics of the 2 DOF helicopter used in Project 1.

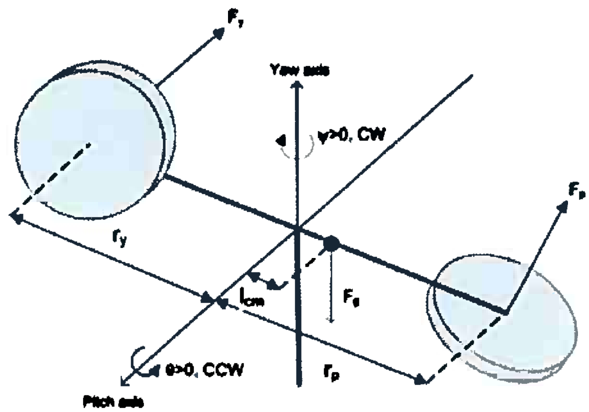

Figure 1: 2 DOF Helicopter free-body diagram.

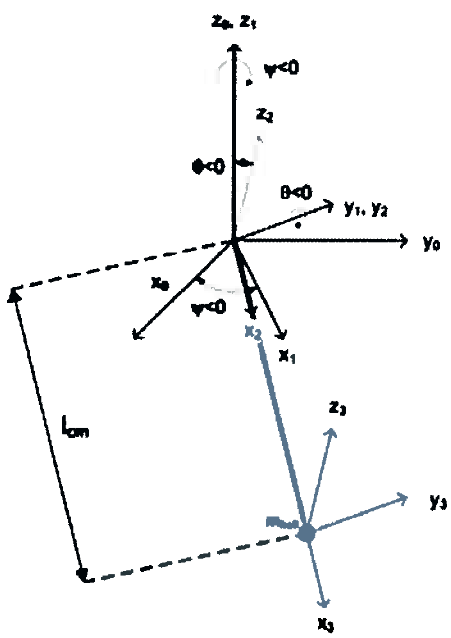

Figure 2: 2 DOF Helicopter kinematics diagram.

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The helicopter can be represented as a two input-two output state space model, where the state vector is defined as

$$x = \begin{bmatrix} \theta(t) \\ \psi(t) \\ \dot{\theta}(t) \\ \dot{\psi}(t) \end{bmatrix}$$

and the output vector is defined as

$$y = \begin{bmatrix} \theta(t) \\ \psi(t) \end{bmatrix}$$

where θ is the pitch angle and ψ is the yaw angle. The inputs are the commanded pitch and commanded yaw. The state space model of this system is represented by the matrices

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{B_p}{J_{T_p}} & 0 \\ 0 & 0 & 0 & -\frac{B_y}{J_{T_y}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{T_p}} & \frac{K_{py}}{J_{T_p}} \\ \frac{K_{yp}}{J_{T_y}} & \frac{K_{yy}}{J_{T_y}} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

where

$$J_{T_p} = J_{eq\_p} + m_{heli} l_{cm}^2$$
$$J_{T_y} = J_{eq\_y} + m_{heli} l_{cm}^2$$

for the parameters in the table below.

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Table 1: Model parameters for the 2 DOF Helicopter

Parameter	Value	Units and Description
K_{pp}	0.204	[Nm/V] Thrust force constant on pitch axis from pitch propeller
K_{yy}	0.072	[Nm/V] Thrust force constant on yaw axis from yaw propeller
K_{py}	0.0068	[Nm/V] Thrust force constant on pitch axis from yaw propeller
K_{yp}	0.0219	[Nm/V] Thrust force constant on yaw axis from pitch propeller
B_p	0.800	[N/V] Equivalent viscous damping about pitch axis
B_y	0.318	[N/V] Equivalent viscous damping about yaw axis
m_{heli}	1.3872	[kg] Total moving mass of the helicopter
l_{cm}	0.186	[m] Center of mass length from pitch axis
J_{eq_p}	0.0384	[kg·m^2] Total moment of inertia about pitch axis
J_{eq_y}	0.0432	[kg·m^2] Total moment of inertia about yaw axis

In this problem, you will write MATLAB code to design a controller and simulate the system response. Start the MATLAB code with the system parameters shown in the table, and by defining the state matrices.
(a) Calculate the eigenvalues of the open loop system using the MATLAB command *eig* [1 pt].
(b) Determine if the system is controllable [1 pt].
(c) Choose pole locations to meet the following requirements:

- $M_p(\%) \leq 5\%$
- $T_s < 3$ s

You will want to choose the same pole locations twice so that pitch and yaw both meet the performance criteria. In other words: *p_cl = [p_secorder,p_secorder]* [2 pts].
(d) Use the *place* command to find the K matrix that locates closed loop poles where you want them [1 pt].
(e) Collect the closed loop system using the *ss* command, making certain to be careful with dimensions for B and D [1 pt].
(f) Find the eigenvalues of the closed loop system using the MATLAB command *eig.* Do these match the poles you chose in part (e) [1 pt]?
(g) In the closed loop system, the command *initial* is the same as an input, as $r = -Kx$. Find the system response by using the *initial* command for an initial state of $x^T = [-10, -10, 0, 0]$ and a final time of 5 seconds. This corresponds to a 10 degree step in the pitch and yaw axes. Plot the resulting output in an appropriately labeled figure. Note that the output will be a 2715x2 matrix, so you want to plot each column separately and label them correctly [2 pts].
(h) Does this output meet the requirements we set? Can you think of some reasons why or why not [1 pt]?

SOLN: MATLAB Attached

(f) yes, they match exactly!

(h) Response meets requirements.

used full-state feedback!

```
% Helicopter Design Problem
close all
clear all
clc

%pitch and yaw are both second order

% Start by defining the system parameters
K_pp = 0.204; % Thrust force constant on pitch axis from pitch↙
prop [Nm/V]
K_yy = 0.072; % Thrust torque constant of yaw axis from yaw prop↙
[Nm/V]
K_py = 0.0068; % Thrust torque constant on pitch axis from yaw↙
prop [Nm/V]
K_yp = 0.0219; % Thrust torque constant on yaw axis from pitch↙
prop [Nm/V]
B_p = 0.8; % Equivalent viscous damping about pitch axis [N/V]
B_y = 0.318; % Equivalent viscous damping about yaw axis [N/V]
m_heli = 1.3872; % Total moving mass of the helicopter [kg]
l_cm = 0.186; % Center of mass length from pitch axis [m]
J_eqp = 0.0384; % Total moment of inertia about pitch axis [kg.↙
m^2]
J_eqy = 0.0432;% Total moment of inertia about yaw axis [kg.m^2]

% Calculate the values of J_Tp and J_Ty
J_Tp = J_eqp+m_heli*l_cm^2;
J_Ty = J_eqy+m_heli*l_cm^2;

% Define the state matrices
A = [0,0,1,0;0,0,0,1;0,0,-B_p/J_Tp,0;0,0,0,-B_y/J_Ty];
B = [0,0;0,0;K_pp/J_Tp,K_py/J_Tp;K_yp/J_Ty,K_yy/J_Ty];
C = [1,0,0,0;0,1,0,0];
D = [0,0;0,0];

% (a) Calculate the eigenvalues of A
eig_A = eig(A)

% (b) Check controllability
Pc = ctrb(A,B);
if rank(Pc)==length(eig_A)
    disp('System is Controllable')
end

% (c) Design for pole locations
% Design objectives: MP(%) less than 5%, Ts<3 seconds.  Desire↙
both pitch
% and yaw to meet these requirements.
```

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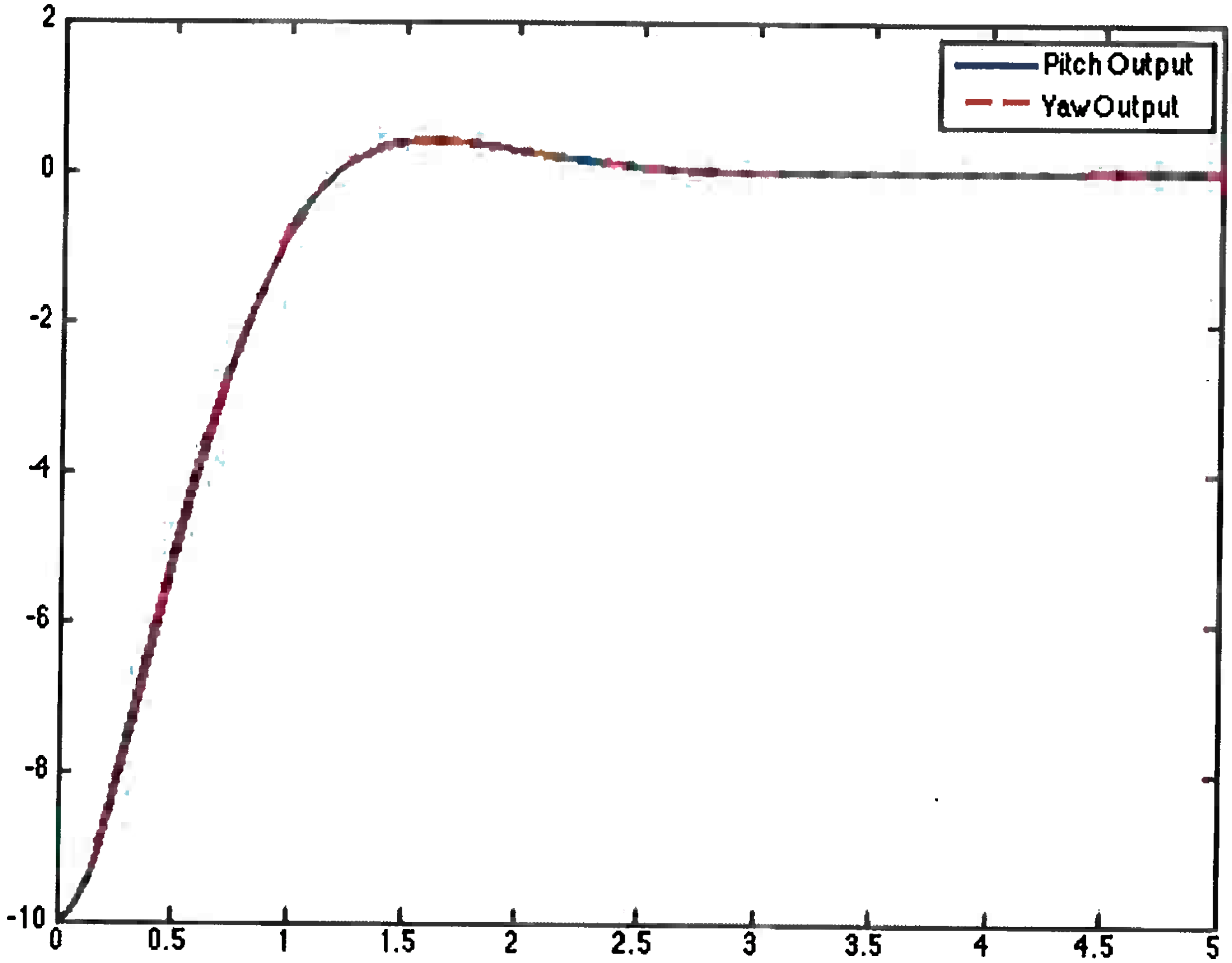
```
% Design so that the MP(%) is less than 5%.  choose 4%
zeta = -log(4/100)/sqrt(pi^2+log(4/100)^2);
% Design so that Ts<3 seconds.  Choose Ts = 2 seconds.
wn = 4/zeta/2;
% Find the second order coefficients for this response
a1 = 2*zeta*wn;
a0 = wn^2;
% Find the corresponding roots for the second order system
sec_poles1 = roots([1,a1,a0]);
% These roots have a real part of -2.

p_cl = [sec_poles1,sec_poles1];

% (d) Place the poles
K = place(A,B,p_cl);

% (e) Full state system
sys_full = ss(A-B*K,[0,0,0,0;0,0,0,0]',C,[0,0;0,0]);

% (f) Find eigenvalues of the full state feedback system
eig_full = eig(sys_full)

% (g) Find results for an initial state of [-10,-10,0,0]'
x0 = [-10,-10,0,0]';
tfinal = 5;
[y_full,t_full,x_full] = initial(sys_full,x0,tfinal);

% (g) cnt'd: Plot closed loop response
figure(1)
plot(t_full,y_full(:,1),'b','Linewidth',2)
hold on
plot(t_full,y_full(:,2),'r--','Linewidth',2)
legend('Pitch Output','Yaw Output')
```

```
eig_A =

         0
         0
   -9.2602
   -3.4872

System is Controllable

eig_full =

  -2.0000 + 1.9520i
  -2.0000 - 1.9520i
  -2.0000 + 1.9520i
  -2.0000 - 1.9520i

EDU>> p_cl

p_cl =

  -2.0000 + 1.9520i  -2.0000 + 1.9520i
  -2.0000 - 1.9520i  -2.0000 - 1.9520i

EDU>>
```


Pitch Output
Yaw Output
2
0
-2
-4
-6
-8
-10
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Problem 54: ***Design Problem-Observer Based Design*** [6 pts EC]
You are given the state space system described by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -12 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 65939 & 0 & 0 \end{bmatrix}$$

$$D = 0$$

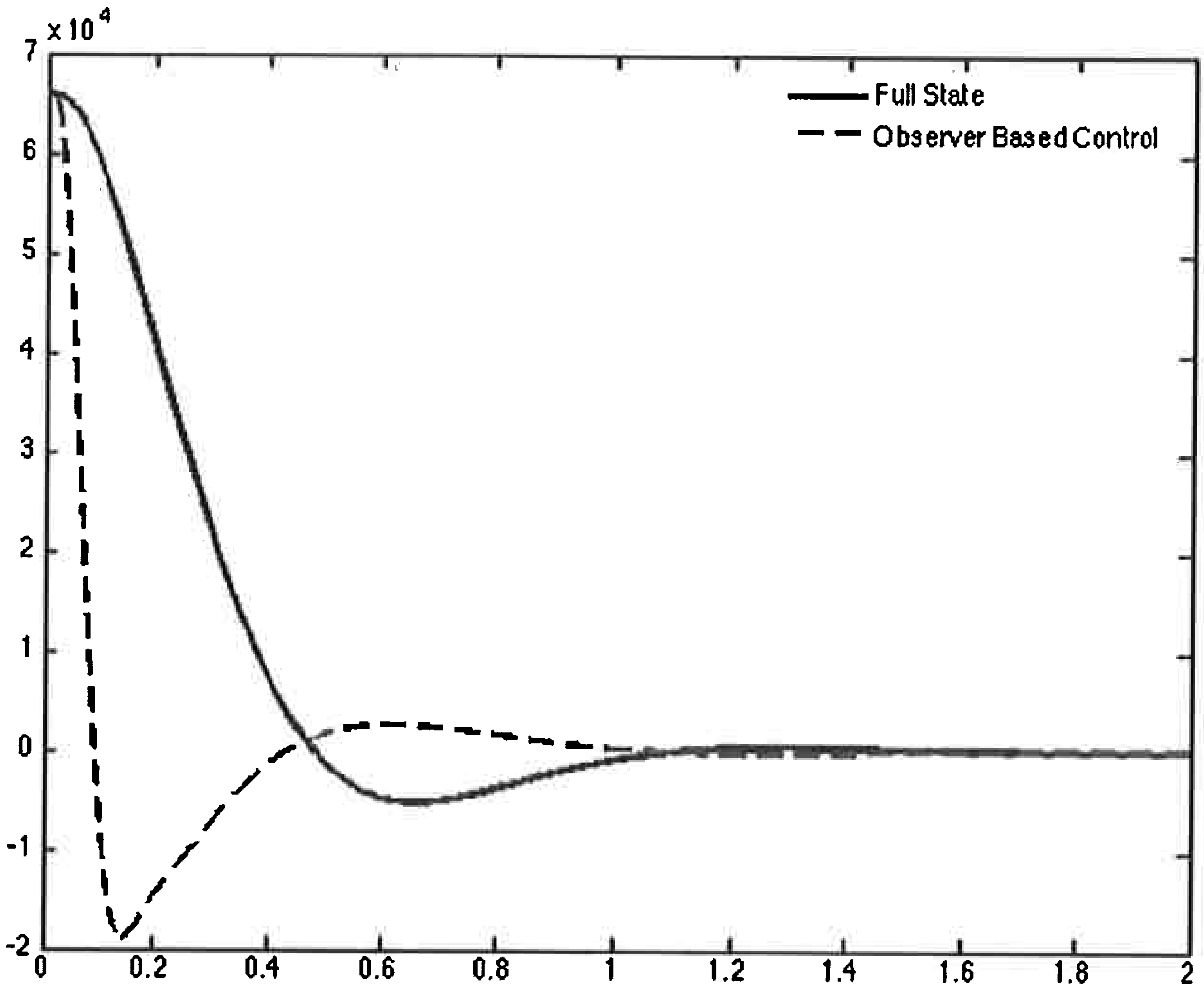
(a) Check the system controllability and observability in MATLAB.
(b) Design a feedback controller that places a dominant complex pole pair for a 1 second settling time and $M_p(\%) < 10\%$. Place the third pole far enough into the left half plane to ensure that the system can be approximated as a second order system. Justify the location of the third pole.
(c) Using MATLAB and the handout from lecture, *place* the poles of the full state feedback system and plot the output y_full for an initial state of x0 = [1; 0; 0], corresponding to an initial disturbance of 1, for a tfinal of 2 seconds. Make sure the plot is labeled appropriately.
(d) Does the output of the full state system meet the settling time requirement? Explain why or why not. If not, change the location of the third pole to meet this requirement.
(e) Design an estimator gain matrix L that satisfies deadbeat response characteristics for a settling time of 0.1 seconds.
(f) Using MATLAB and the handout from Lecture 39, place the poles of the observer. Plot the output for both the full state feedback and the combined systems (y_full and y_reg). Find y_reg for an initial state of x0 = [1; 0; 0; 1; 0; 0], corresponding to an initial disturbance of 1 and an initial estimation error of 1, for a tfinal of 2 seconds. Make sure the plot is labeled appropriately, including using different line type for each output and a legend.
(g) Does the output of the combined system meet the settling time requirement? Explain why it is ok to use a combined system designed in this way (Hint: what is the settling time of the estimator?)

SOLN: Attached

(a) both observable and controllable

```
% problem 54

close all
clear all
clc

% state matrices
A = [0,1,0;0,0,1;-10,-12,-3];
B = [0,0,1]';
C = [40.4^3,0,0];
D = 0;

% ol system
sys_ol = ss(A,B,C,D);

%% Full State feedback

% check controllability
Pc = ctrb(A,B);
[m,n] = size(A);

if rank(Pc)==n
    disp('System is Controllable')
else
    disp('System is Not Controllable')
end

% desired CL poles
p_cl = [-4+4.9753i;-4-4.9753i;-40]; % set other pole at least 10
times further from jw axis

% place the poles
K = place(A,B,p_cl);

% full state system
sys_full = ss(A-B*K,[0,0,0]',C,0);

% find results for an initial state of [1,0,0]'
x0 = [1,0,0]';
tfinal = 2;
[y_full,t_full,x_full] = initial(sys_full,x0,tfinal);

%% Observer feedback design

% check observability
Po = obsv(A,C);
[m,n] = size(A);
```

```
if rank(Po)==n
    disp('System is Observable')
else
    disp('System is Not Observable')
end

% desired observer poles
% deadbeat response
alpha = 1.9;
beta = 2.2;
wn = 40.4;
den_des = [1,alpha*wn,beta*wn^2,wn^3];
p_ob = roots(den_des);

% observer pole placement
L = acker(A',C',p_ob).';

% combined CL system
A_cl = [A-B*K, B*K; zeros(3,3), A-L*C];
B_cl = [0,0,0,0,0,0]';
C_cl = [C,zeros(1,3)];
D_cl = 0;

sys_obsv = ss(A_cl,B_cl,C_cl,D_cl);

% find the results for an initial disturbance of 1 and an initial
% estimation error of 1
x0 = [1,0,0,1,0,0]';
tfinal = 2;

[y_reg,t_reg,x_reg] = initial(sys_obsv,x0,tfinal);

figure(1)
plot(t_full,y_full,'b','Linewidth',2)
hold on
plot(t_reg,y_reg,'r--','Linewidth',2)

figure(2)
plot(t_full,x_full(:,1),'b','Linewidth',2)
hold on
plot(t_reg,x_reg(:,1),'r--','Linewidth',2)

figure(3)
plot(t_reg,x_reg(:,4),'k-','Linewidth',2)
```



x 10^4
7
6
5
4
3
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Full State
Observer Based Control