INF 4300 – Hough transform

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•This lecture goes more in detail than Gonzales and Woods 10.2.

•Introduction to Hough transform

• Using gradient information to detect lines

• Representing a line

• The [a,b]-representation

• The [ρ,θ]-representation

• Algorithm for detection of lines

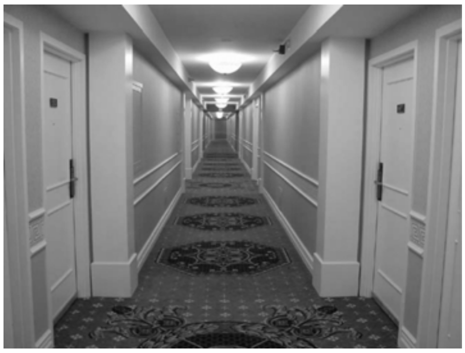
• Detecting circles

Introduction to Hough transform

- The Hough transform (HT) can be used to detect lines, circles or other parametric curves.
- It was introduced in 1962 (Hough 1962) and first used to find lines in images a decade later (Duda 1972).
- The goal is to find the location of lines in images.
- This problem could be solved by e.g. Morphology and a linear structuring element, or by correlation.
  - Then we would need to handle rotation, zoom, distortions etc.
- Hough transform can detect lines, circles and other structures if their parametric equation is known.
- It can give robust detection under noise and partial occlusion.

An image with linear structures

- Borders between the regions are straight lines.
- These lines separate regions with different grey levels.
- Edge detection is often used as preprocessing to Hough transform.



Remember edge types?

a b c

FIGURE 10.8 From left to right, models (ideal representations) of a step, a ramp, and a roof edge, and their corresponding intensity profiles.

Step edge Ramp edge Roof edge

Hough-transform – the input

- The input image must be a thresholded edge image.
- The magnitude results computed by the Sobel operator can be thresholded and used as input.

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Repetition - Basic edge detection

- A thresholded edge image is the starting point for Hough transform.
- What does a Sobel filter produce?
- Approximation to the image gradient:

$$\nabla f(x)$$

- ...which is a vector quantity given by:

$$\nabla \mathbf{f}(x,y) = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Repetition – Edge magnitude

- The gradient is a measure of how the function f(x,y) changes as a function of changes in the arguments x and y.
- The gradient vector points in the direction of maximum change.
- The length of this vector indicates the size of the gradient:

$$\nabla f = |\nabla \mathbf{f}| = \sqrt{G_x^2 + G_y^2}$$

G_x, G_y and the gradient operator

- Horisontal edges:
  - Compute $g_x(x,y)=H_x*f(x,y)$
  - Convolve with the horisontal filter kernel H_x
- Vertical edges:
  - Compute $g_t(x,y)=H_y*f(x,y)$
- Compute the gradient operator as:

$$g(x,y) = \sqrt{g_x^2(x,y) + g_y^2(x,y)}$$ Gradient-magnitude (kant-styrke)

$$\theta(x,y) = \tan^{-1}\left(\frac{g_y(x,y)}{g_x(x,y)}\right)$$ Gradient-retning

Repetition – Edge direction

- The direction of this vector is also an important quantity.
- If α(x,y) is the direction of f in the point (x,y) then:

$$\alpha(x,y) = \tan^{-1}(\frac{G_y}{G_x})$$

- Remember that α(x,y) will be the angle with respect to the x-axis
- Remember also that the direction of an edge will be perpendicular to the gradient in any given point

Input to Hough – thresholded edge image

Prior to applying Hough transform:

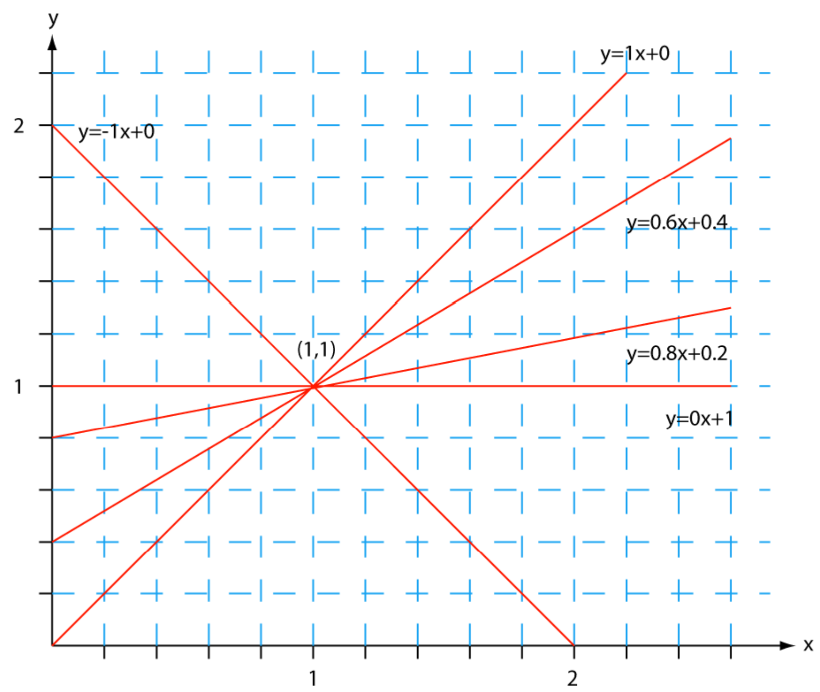
- Compute edge magnitude from input image.
- As always with edge detection, simple lowpass filtering can be applied first.
- Threshold the gradient magnitude image.

Hough-transform

- Assume that we have performed some edge detection, and a thresholding of the edge magnitude image.
- Thus, we have *n* pixels that may partially describe the boundary of some objects.
- We wish to find sets of pixels that make up straight lines.
- Regard a point $(x_i; y_i)$

  and a straight line $y_i = ax_i + b$
  - There are many lines passing through the point (x_i, y_i).
  - Common to them is that they satisfy the equation for some set of parameters (a,b).

Hough transform – basic idea



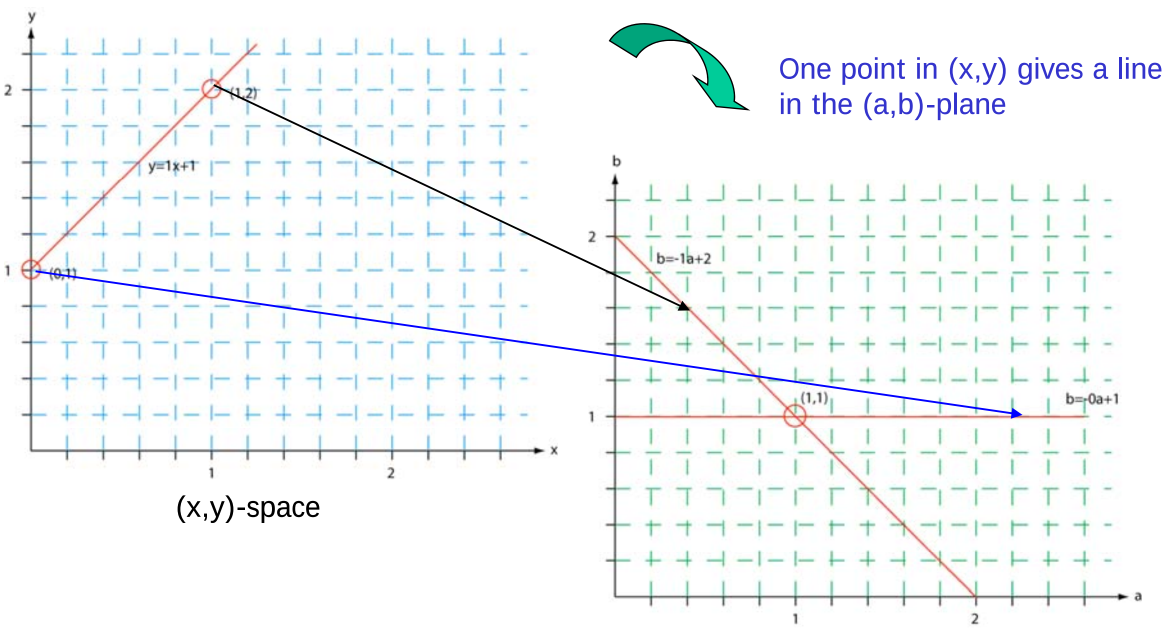
Hough transform – basic idea

- This equation can obviously be rewritten as follows:

$$b = -xa + y$$

- We now consider *x* and *y* as parameters and *a* and *b* as variables.
- This is a line in (*a,b*) space parameterized by *x* and *y*.
  - So: a single point in xy-space gives a line in (a,b) space.
- Another point (*x,y*) will give rise to another line in (*a,b*) space.

Hough transform – basic idea



(x,y)-space

(a,b)-space

Hough transform – basic idea

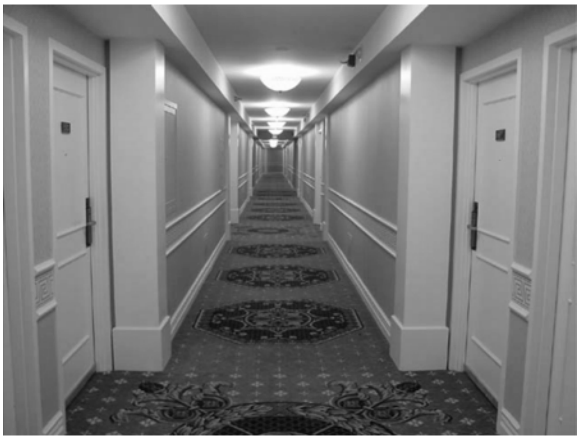
- Two points (x,y) and(z,k) define a line in the (x,y) plane.
- These two points give rise to two different lines in (a,b) space.
- In (a,b) space these lines will intersect in a point (a',b') where a' is the rise and b' the intersect of the line defined by (x,y) and (z,k) in (x,y) space.
- The fact is that all points on the line defined by (x,y) and (z,k) in (x,y) space will parameterize lines that intersect in (a',b') in (a,b) space.
- Points that lie on a line will form a "cluster of crossings" in the (a,b) space.

Hough transform – algorithm

- Quantize the parameter space (a,b), that is, divide it into cells.
- This quantized space is often referred to as the accumulator cells.
- In the figure in the next slide a_{min} is the minimal value of a etc.
- Count the number of times a line intersects a given cell.
  - For each point (x,y) with value 1 in the binary image, find the values of (a,b) in the range $[[a_{min},a_{max}],[b_{min},b_{max}]]$ defining the line corresponding to this point.
  - Increase the value of the accumulator for these [a',b'] point.
  - Then proceed with the next point in the image.
- Cells receiving a minimum number of "votes" are assumed to correspond to lines in (x,y) space.
  - Lines can be found as peaks in this accumulator space.

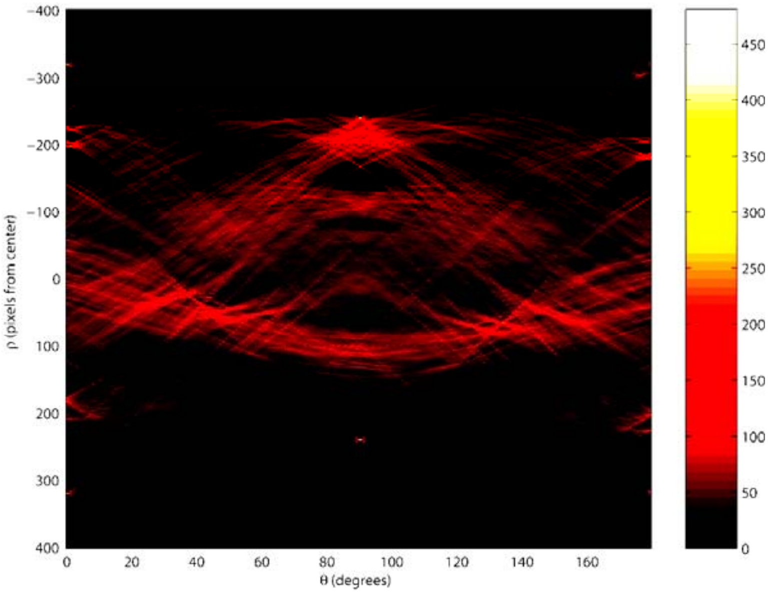
Hough transform – example 3

- Example 3: Natural scene and result of Sobel edge detection followed by thresholding:



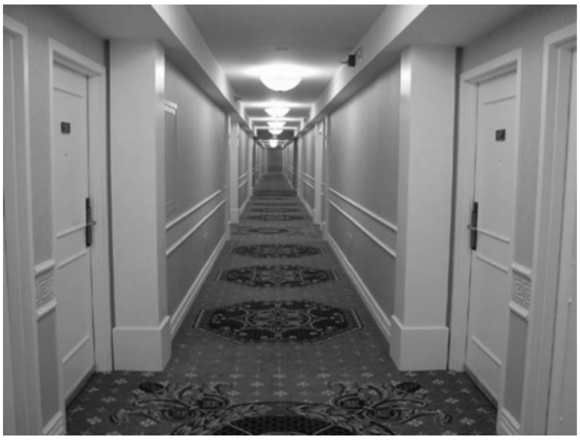
Hough transform – example 3

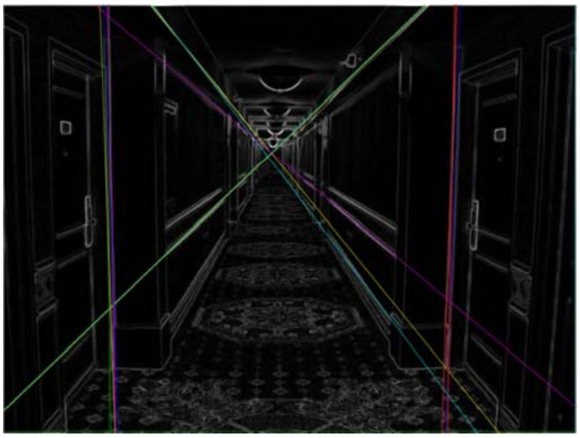
- Example 3: Accumulator matrix:



Hough transform – example 3

- Example 3: Original image and 20 most prominent lines:





Hough transform - advantages

- Advantages:
  - Conceptually simple.
  - Easy implementation.
  - Handles missing and occluded data very gracefully.
  - Can be adapted to many types of forms, not just lines.

Hough transform - disadvantages

- Disadvantages:
  - Computationally complex for objects with many parameters.
  - Looks for only one single type of object.
  - Can be "fooled" by "apparent lines".
  - The length and the position of a line segment cannot be determined.
  - Co-linear line segments cannot be separated.

Hough-transform using the full gradient information – a variant

- Given a gradient magnitude image g(x,y) containing a line.
- Simple algorithm:

  for all $g(x_i,y_i)>T$ do
      for all θ do
          $\rho = x_i \cos\theta + y_i \sin\theta$
          find corresponding indexes (m,n) and increment A(m,n)

- Better algorithm if we have both
  - The gradient magnitude g(x,y)
  - The gradient components g_x and g_y and can compute

$$\phi_g(x,y) = \arctan\left(\frac{g_x}{g_y}\right)$$

- Algorithm:

  for all $g(x_i,y_i)>T$ do
      for all θ do
          $\rho = x_i \cos(\phi_g(x,y)) + y_i \sin(\phi_g(x,y))$
          find index m corresponding to ρ and increment A(m, $\phi_g(x,y)$)