- What if we don't have perfect information about the states? We need an estimator/observer
  - ↳ Can design as long as system is observable

Defn An LTIC system is observable if $\text{rank}(\mathcal{O}) = n$

where $\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$ is the observability matrix

MATLAB: obsv gives the observability mx

Pole placement using Full state Feedback

- consider an LTIC system

$\dot{x} = Ax + Bu \qquad y = Cx$

- let control rule be

$u = -Kx$

$K = [k_1, k_2, k_3, \dots k_n]$, the gain matrix

let $r = 1$ (single input)

$u = -k_1 x_1 - k_2 x_2 \dots - k_n x_n$

- if we know all states, then

$\dot{x} = Ax - BKx$

$\dot{x} = (A - BK)x$ closed loop response

$x(t) = e^{(A-BK)t} x(0)$

↳ not like xfer function, depends on initial conditions of x

poles of CL system are at

$\det(sI - (A - BK)) = 0$

↳ compare to characteristic eq and get response desired!

EX 1; $\dot{x} = Ax + Bu$

$A = \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$K = [k_1 \ k_2]$

Find K so closed loop poles are placed at $s = -2 \pm j$

- SOLN: First; controllable?

$P_c = [B \ \ AB]$

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \det(P_c) = -1$

Yes!

Poles of CL system are

$\det(sI - (A - BK)) = 0$

$A - BK = \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1 \ \ k_2]$

$= \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 16 - k_1 & k_2 \end{bmatrix}$

$\det(sI - (A - BK)) = 0$

$= \det\left(\begin{bmatrix} s & -1 \\ k_1 - 16 & s + k_2 \end{bmatrix}\right) = 0$

$s(s + k_2) + k_1 - 16 = 0$

$s^2 + k_2 s + (k_1 - 16) = 0$

pick k_1, k_2 s.t.

$(s - (-2 + j))(s - (-2 - j)) = 0$

$s^2 + 4s + 5 = 0$

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yields

$\boxed{k_2 = 4}$

$k_1 - 16 = 5$

$\boxed{k_1 = 21}$

$u = -kx = -21x_1 - 4x_2$

(4)

BEx3: Given

$A = \begin{bmatrix} 0 & 1 \\ 10 & 5 \end{bmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$k = [k_1 \; k_2]$

Find k s.t. $mp(\%) = 10\%$

and $t_s = 2$ s

o SOLN:

find ζ, ω_n

$\zeta = \dfrac{-\ln(mp(\%)/100)}{\sqrt{\pi^2 + \ln^2(mp(\%)/100)}} = 0.32$

$t_s = 2s = \dfrac{4}{\zeta\omega_n}$

$\omega_n = \dfrac{2}{\zeta} = 6.28$ rad/s

closed loop ch. eq.

$s^2 + 4s + 39.44 = 0$

o check controllable:

$P_c = \begin{bmatrix} 0 & 1 \\ 1 & 5 \end{bmatrix} \Rightarrow$ controllable.

(11)

$\det(sI - (A - Bk)) = 0$

$= \det \begin{bmatrix} s & -1 \\ k_1 - 10 & s + k_2 - 5 \end{bmatrix} = 0$

$s(s + k_2 - 5) + k_1 - 10 = 0$

$s^2 + (k_2 - 5)s + (k_1 - 10) = 0$

$k_2 - 5 = 4 \Rightarrow \boxed{k_2 = 9}$

$k_1 - 10 = 39.44 \Rightarrow \boxed{k_1 = 49.44}$