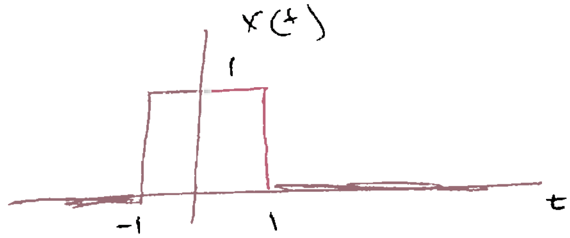
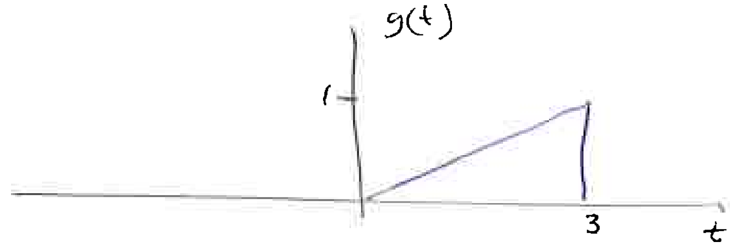
Ex 3 Graphical convolution

Find $c(t) = x(t) * g(t)$.

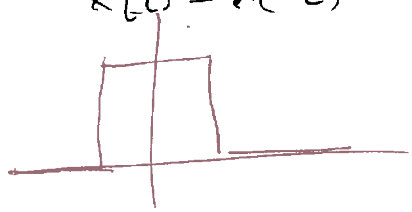




SOLN: (1) Keep $g(\tau)$ fixed

(2) Flip $x(\tau)$ (more simple)

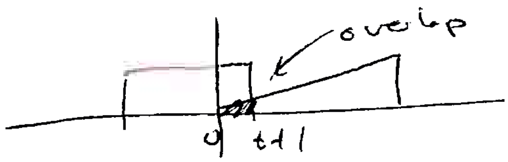
$x(\tau) = x(-\tau)$



(3), (4), (5) shift t, compute area for all cases

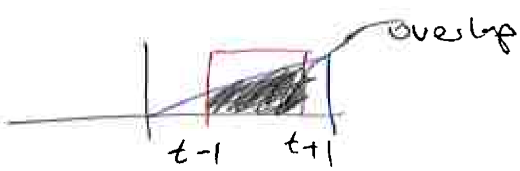
Case 1 $\quad t < -1 \quad c(t) = 0$

Case 2 $\quad -1 \le t \le 1$



$$c(t) = \int_0^{t+1} (1)\left(\frac{1}{3}\tau\right) d\tau = \frac{1}{6}\tau^2 \Big|_0^{t+1} = \underline{\frac{1}{6}(t+1)^2}$$

Case 3 $\quad 1 \le t \le 2$

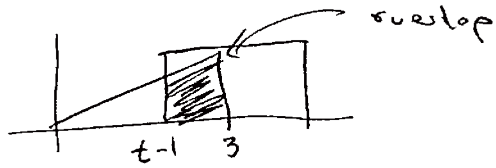


$$\int_{t-1}^{t+1} (1)\left(\frac{1}{3}\tau\right) d\tau = \frac{1}{6}\tau^2 \Big|_{t-1}^{t+1} = \frac{1}{6}(t+1)^2 - \frac{1}{6}(t-1)^2$$

$$= \frac{1}{6}t^2 + \frac{2}{6}t + \frac{1}{6} - \frac{1}{6}t^2 + \frac{2}{6}t - \frac{1}{6}$$

$$= \underline{\frac{2}{3}t}$$

case 4 $2 \le t \le 4$



$$\int_{t-1}^{3} (1)\left(\tfrac{1}{3}\tau\right) d\tau = \frac{1}{6}\tau^2 \Big|_{t-1}^{3} = \frac{9}{6} - \frac{1}{6}(t-1)^2$$

$$= -\frac{1}{6}\left(-9 + t^2 - 2t + 1\right)$$

$$= \underline{-\frac{1}{6}\left(t^2 - 2t - 8\right)}$$

case 5 $t > 4$ $c(t) = 0$

Final

$$\boxed{c(t) = \begin{cases} \frac{1}{6}(t+1)^2 & t \in [-1, 1] \\ \frac{2}{3}t & t \in [1, 2] \\ -\frac{1}{6}(t^2 - 2t - 8) & t \in [2, 4] \\ 0 & \text{o.w.} \end{cases}}$$