

Table 2.3 Important Laplace Transform Pairs

| $f(t)$ | $F(s)$ |
|---|--|
| Step function, $u(t)$ | $\frac{1}{s}$ |
| e^{-at} | $\frac{1}{s+a}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2 + \omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2 + \omega^2}$ |
| t^n | $\frac{n!}{s^{n+1}}$ |
| $f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$ | $s^k F(s) - s^{k-1}f(0^-) - s^{k-2}f'(0^-) - \dots - f^{(k-1)}(0^-)$ |
| $\int_{-\infty}^t f(t) dt$ | $\frac{F(s)}{s} + \frac{1}{s} \int_{-\infty}^0 f(t) dt$ |
| Impulse function $\delta(t)$ | 1 |
| $e^{-at} \sin \omega t$ | $\frac{\omega}{(s+a)^2 + \omega^2}$ |
| $e^{-at} \cos \omega t$ | $\frac{s+a}{(s+a)^2 + \omega^2}$ |
| $\frac{1}{\omega} [(\alpha - a)^2 + \omega^2]^{1/2} e^{-at} \sin(\omega t + \phi),$ $\phi = \tan^{-1} \frac{\omega}{\alpha - a}$ | $\frac{s+\alpha}{(s+a)^2 + \omega^2}$ |
| $\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t, \zeta < 1$ $\frac{1}{a^2 + \omega^2} + \frac{1}{\omega \sqrt{a^2 + \omega^2}} e^{-at} \sin(\omega t - \phi),$ $\phi = \tan^{-1} \frac{\omega}{-a}$ | $\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$ |
| $1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \phi),$ $\phi = \cos^{-1} \zeta, \zeta < 1$ | $\frac{1}{s[(s+a)^2 + \omega^2]}$ |
| $\frac{\alpha}{a^2 + \omega^2} + \frac{1}{\omega} \left[\frac{(\alpha - a)^2 + \omega^2}{a^2 + \omega^2} \right]^{1/2} e^{-at} \sin(\omega t + \phi),$ $\phi = \tan^{-1} \frac{\omega}{\alpha - a} - \tan^{-1} \frac{\omega}{-a}$ | $\frac{\omega_n^2}{s(s^2 + 2\zeta \omega_n s + \omega_n^2)}$ |
| | $\frac{s+\alpha}{s[(s+a)^2 + \omega^2]}$ |