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Matematika I

Rešena 2. domača naloga

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29. september 2024

Kazalo

Z oddajo domače naloge potrjujem, da sem domačo nalogo reševal samostojno.

V prostoru $\mathbb{R}^3$ naj bo dan paralelogram $ABCD$ z oglišči $A(-3,-2,0),\ B(3,-3,1),\ C(5,0,2),\ D(-1,1,1)$.

1 Prva naloga

1.1 Navodila

Določi enačbo ravnine Π, na kateri leži paralelogram $ABCD$.

1.2 Reševanje naloge

$$\vec{a} = \vec{AB} \wedge \vec{b} = \vec{BC}$$

$$\vec{a} = \begin{bmatrix} 3-(-3)=6 \\ -3-(-2)=-1 \\ 1-0=1 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} 5-3=2 \\ 0-(-3)=3 \\ 2-1=1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\vec{a} \times \vec{b} = \vec{n}_1$$

$$\vec{n}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = \begin{bmatrix} -4 \\ -4 \\ 20 \end{bmatrix} \implies \tag{1}$$

$$\vec{n}_1 \| \vec{n}\,;\ \vec{n} = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$$

$$ax + by + cz - d = 0\,;\ \vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \implies$$

$$-x - y + 5z - d = 0$$

$$C(5,0,2) \implies$$

$$-5 - 0 + 10 - d = 0 \implies d = 5$$

$$-x - y + 5z = 5$$

2 Druga naloga

2.1 Navodila

Naj bo K krožnica, ki gre skozi oglišča A, B in C paralelograma $ABCD$. Določi koordinate središča krožnice K.

2.2 Reševanje naloge

$$p = A + \frac{1}{2}\,\vec{a} \implies p(0, -\frac{5}{2}, \frac{1}{2})$$
$$q = B + \frac{1}{2}\,\vec{b} \implies q(4, -\frac{3}{2}, \frac{3}{2})$$
$$\vec{P} = \vec{a} \times \vec{n}$$
$$\vec{Q} = \vec{b} \times \vec{n}$$
$$\vec{P} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -1 & 1 \\ -1 & -1 & 5 \end{vmatrix} = \begin{bmatrix} -4 \\ -31 \\ -7 \end{bmatrix}$$
$$\vec{Q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ -1 & -1 & 5 \end{vmatrix} = \begin{bmatrix} 16 \\ -11 \\ 1 \end{bmatrix}$$

$$\begin{aligned} p + t\vec{p} &= q + s\vec{q} \implies \\ 0 + t(-4) &= 4 + 16s \\ -\frac{5}{2} + t(-31) &= -\frac{3}{2} + t(-11) \\ -t &= 4s + 1 \\ 62t + 10 &= 22s + 3 \end{aligned} \tag{2}$$

$$t = -\frac{1}{9}$$
$$s = -\frac{2}{9}$$
$$q + s\vec{q} = \begin{bmatrix} 4 \\ -\frac{3}{2} \\ \frac{3}{2} \end{bmatrix} + (-\frac{2}{9}) \begin{bmatrix} 16 \\ -11 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{9} \\ \frac{17}{18} \\ \frac{23}{18} \end{bmatrix}$$

$$S\left(\frac{4}{9}, \frac{17}{18}, \frac{23}{18}\right)$$