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Matematika II

Rešena 1. domača naloga za matematiko II

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Kazalo

1 Prva naloga

1.1 Navodila

Komutator kvadratnih matrik A in B iste velikosti je matrika $[A, B] = AB - BA$.

Definirajmo matrike E, F, $H \in \mathbb{R}^{2\times 2}$ s predpisi:

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Izračunaj komutatorje $[E, F]$, $[H, E]$ in$[H, F]$.

1.2 Reševanje naloge

$$[E, F] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = H$$

$$[H, E] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$[H, F] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix}$$

2 Druga naloga

2.1 Navodila

Z uporabo Gauss-Jordanove eliminacije izračunaj inverz matrike:

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 2 & 2 & 2 \\ -1 & 2 & 1 & 1 \end{bmatrix} \tag{1}$$

2.2 Reševanje naloge

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 2 & 0 & 0 & 1 & 0 \\ -1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 & -1 & 1 & 0 \\ 0 & 2 & 1 & 2 & 0 & 1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -2 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & -3 & 2 & -2 & 3 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & -3 & 2 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -2 & -2 & 3 & -2 \\ 0 & 1 & 2 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 2 & 3 & -3 & 2 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -2 & -2 & 3 & -2 \\ 0 & 1 & 0 & 0 & -3 & -3 & 4 & -2 \\ 0 & 0 & 1 & 0 & 2 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 2 & 3 & -3 & 2 \end{array}\right] \Longrightarrow$$

$$A^{-1} = \begin{bmatrix} -2 & -2 & 3 & -2 \\ -3 & -3 & 4 & -2 \\ 2 & 1 & -2 & 1 \\ 2 & 3 & -3 & 2 \end{bmatrix}$$