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Matematika I

Rešena 4. domača naloga

Napisal : Luka Orlić, Vpis. Št.: 28221084
Tutor : Urša Mati Djuraki

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Kazalo

Z oddajo domače naloge potrjujem, da sem domačo nalogo reševal samostojno.

1 Prva naloga

1.1 Navodila

Funkcija ene realne spremenljivke f naj bo dana s predpisom:

$$f(x) = \begin{cases} e^{\frac{1}{1-x}} + a & ; x > 1 \\ \sin(\frac{\pi(4x+9)}{6}) & ; 0 \leq x \leq 1 \\ b - \arctan(\frac{1}{x}) & ; x < 0 \end{cases} \tag{1}$$

Določi konstanti $a,\ b \in \mathbb{R}$ tako, da bo funkcija f zvezna na celi realni osi. Določi tudi zalogo vrednosti $f(R)$ zvezne funkcije f.

1.2 Reševanje naloge

$$L_0 = \lim_{x\to 0}\left[\sin(\frac{\pi(4x+9)}{6})\right] = \sin\left(\lim_{x\to 0}\left[\frac{\pi(4x+9)}{6}\right]\right)$$
$$L_0 = \sin(\frac{9\pi}{6}) = -1$$
$$L_{0-} = \lim_{x\to 0^-}\left[b - \arctan(\frac{1}{x})\right] = \lim_{x\to 0^-}(b) - \lim_{x\to 0^-}\left[\arctan(\frac{1}{x})\right] = b + \frac{\pi}{2}$$
$$L_{0-} = L_0 \implies b + \frac{\pi}{2} = -1 \implies$$
$$b = -(\frac{\pi}{2} + 1)$$

$$L_1 = \lim_{x\to 1}\left[\sin(\frac{\pi(4x+9)}{6})\right] = \sin\left(\lim_{x\to 1}\left[\frac{\pi(4x+9)}{6}\right]\right)$$
$$L_1 = \sin(\frac{13\pi}{6}) = \frac{1}{2}$$
$$L_{1+} = \lim_{x\to 1^+}\left[e^{\frac{1}{1-x}} + a\right] = \lim_{x\to 1^+}\left[e^{\frac{1}{1-x}}\right] + \lim_{x\to 1^+}[a]$$
$$L_{1+} = 0 + a = a$$
$$L_{1+} = L_1 \implies$$
$$a = \frac{1}{2}$$

$$ \tag{2} $$

$$
\begin{aligned}
&\lim_{x\to\infty}\left[e^{\frac{1}{1-x}}+a\right]=1+a=\frac{3}{2}\\
&\lim_{x\to-\infty}\left[b-\arctan(\frac{1}{x})\right]=b-0=-(\frac{\pi}{2}+1)
\end{aligned}
\tag{3}
$$

Funkcija je strogo naraščajoča torej sledi, da je zaloga vrednosti $\left(\frac{-\pi-2}{2},\ \frac{3}{2}\right)$.

2 Druga naloga

2.1 Navodila

zračunaj odvod funkcije ene realne spremenljivke g, podane s predpisom:

$$ g(x)=x^{\sin(x)} \tag{4} $$

in izračunaj limito:

$$ \lim_{x\to 0^+}\left[x^{\sin(x)}\right] \tag{5} $$

2.2 Reševanje naloge

2.2.1 prva podnaloga

$$
\begin{aligned}
&y=x^{\sin(x)}\\
&\ln(y)=\sin(x)\ln(x)\\
&y'\cdot\frac{1}{y}=\cos(x)\ln(x)+\frac{\sin(x)}{x}\\
&g'(x)=x^{\sin(x)}(\cos(x)\ln(x)+\frac{\sin(x)}{x})
\end{aligned}
\tag{6}
$$

2.2.2 druga podnaloga

$$L = \lim_{x \to 0^+} \left[x^{\sin(x)} \right]$$
$$L = \lim_{x \to 0^+} \left[x^{x \cdot \frac{\sin(x)}{x}} \right]$$
$$L = \lim_{x \to 0^+} [x^x] = \lim_{x \to 0^+} \left[e^{x \ln(x)} \right] = \exp\left(\lim_{x \to 0^+} [x \ln(x)] \right)$$

$$\begin{gathered} t = \frac{1}{x} \wedge x \to 0^+ \implies t \to \infty \\ \lim_{x \to 0^+} [x \ln(x)] = \lim_{t \to \infty} \left[-\frac{\ln(t)}{t} \right] = \\ = \lim_{t \to \infty} \left[-\frac{\ln(t)}{t} \right] = \\ = \lim_{t \to \infty} \left[-\frac{1}{t} \right] = 0 \implies \end{gathered} \tag{7}$$

$$L = e^0 = 1$$

$$\exp\lim_{x \to \infty} \left((2x+1) \ln(\frac{x+1}{x-1}) \right) \tag{8}$$