1. **(15 points) Analysis of Convolutional Codes:** In this problem, we consider a convolutional encoder with two finite impulse responses (FIR)

$$\mathbf{g}^{(1)} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \qquad \mathbf{g}^{(2)} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}. \tag{1}$$

Do the following problems by your hand to make sure that you understand the mechanism of convolutional codes completely.

(a) Calculate the code rate R.

(b) Draw the shift register structure.

(c) Draw the state transition diagram.

(d) Draw the trellis diagram.

(e) Illustrate the path on the trellis diagram for the received sequence

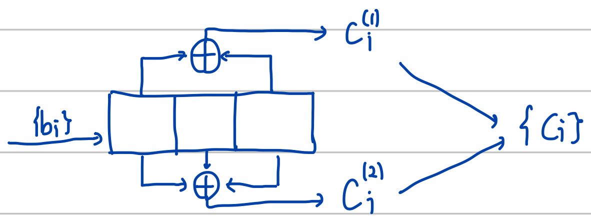
$$\boldsymbol{y} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Also list the decoded codewords $\widehat{\boldsymbol{c}} = \{\widehat{c}_i\}$ and the decoded bits $\widehat{\boldsymbol{b}} = \{\widehat{b}_i\}$.

(f) Based on your results in Problem 1e, calculate the Hamming distance between $\boldsymbol{y}$ and $\widehat{\boldsymbol{c}}$.

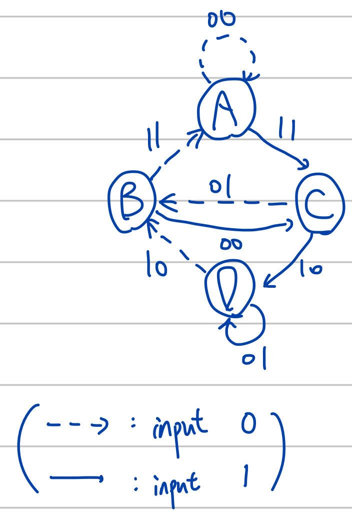
(a) $k=1,\ n=2 \Rightarrow R=\frac{1}{2}$ #

(b)



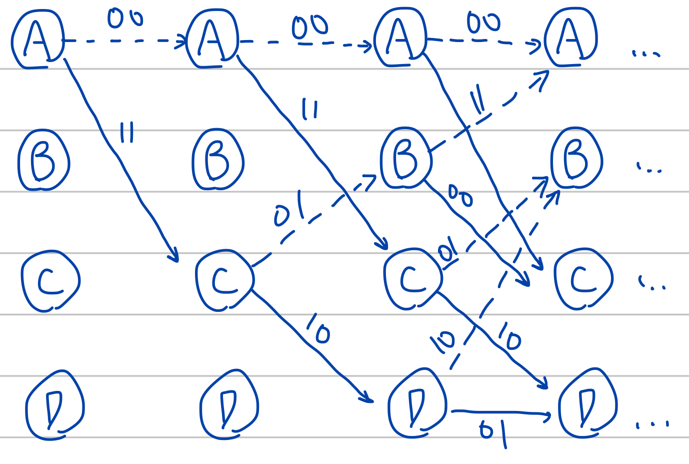
(c)

b_i	before b_{i-1}	before b_{i-2}	$C_i^{(1)}$	$C_i^{(2)}$	after b_{i-1}	after b_{i-2}	
0	0	0	0	0	0	0	(A)
0	0	1	1	1	0	0	
0	1	0	0	1	0	1	(B)
0	1	1	1	0	0	1	
1	0	0	1	1	1	0	(C)
1	0	1	0	0	1	0	
1	1	0	1	0	1	1	(D)
1	1	1	0	1	1	1	



#

(d)

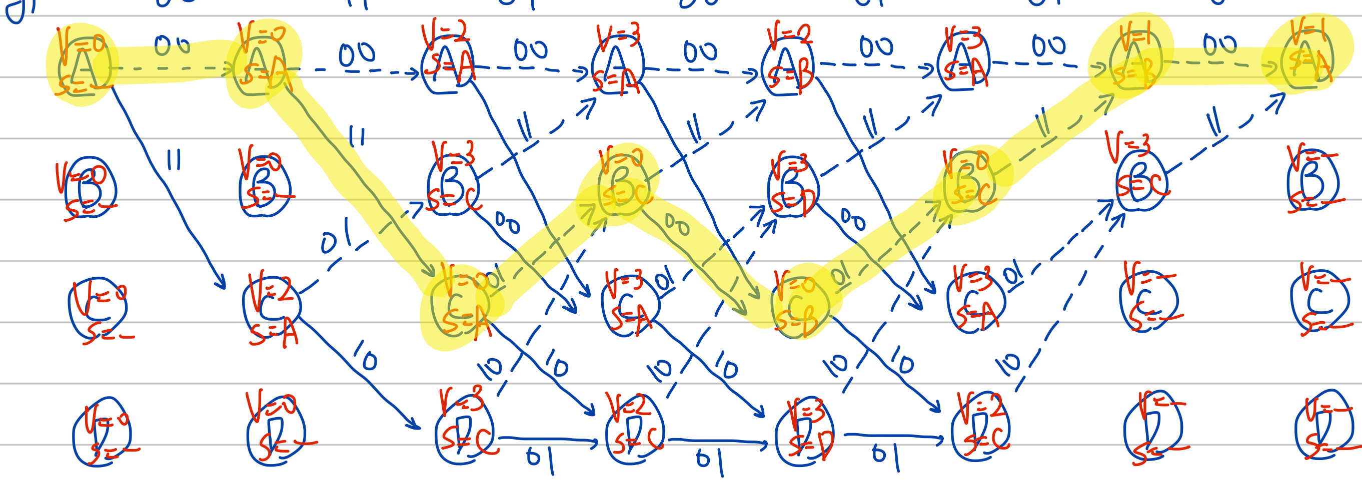


#

(- - -> : input 0
——> : input 1)

(e)

y_i 00 11 01 00 01 01 00



$\hat{C} = [00110100011100]$, $\hat{b} = [0101000]$

(d) hamming weight between y and $\hat{C}$: 1

2.

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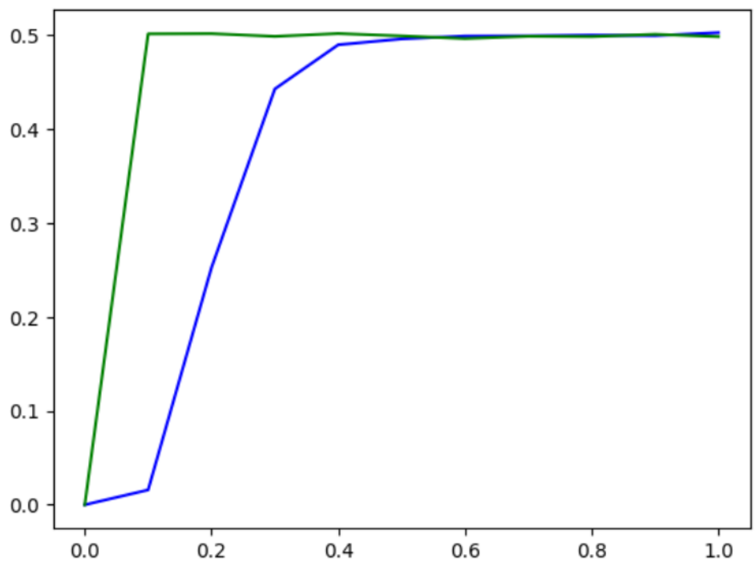
result of 2a:
[1, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 0]
result of 2b:
[0, 0, 0, 1, 0, 0, 0]
```

2.(c): blue line



When the flip probability is higher than 0.5, the error rate is close to 0.5, which symbols an ineffective message transmission.

3: green line

We can see that even p=0.1 brings the bit error rate to near 0.5. This means that the provided FIR is not effective for any p higher than 0.1. Compared to the blue line of 2.c, we can see that the bit error rate of the green line escalates more rapidly, meaning that it is more vulnerable to channel noise.