Numerical calculation of magnetic fields generated by coils

一、 Governing equation

The governing equation for magnetic fields generated by coils is

$$\nabla \times \boldsymbol{B} = \mu \boldsymbol{J}.$$

Introducing the magnetic vector potential

$$\boldsymbol{B} = \nabla \times \boldsymbol{A},$$

The original formula becomes

$$\nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A} = \mu \boldsymbol{J}$$

In the cylindrical coordinate system, consider the magnetic field generated by the coil, i.e., the axisymmetric structure that produces $\boldsymbol{B} = B_r \boldsymbol{e}_r + B_z \boldsymbol{e}_z$, so the magnetic vector potential contains only polar component, i.e., $\boldsymbol{A} = A_\theta(r,z)\boldsymbol{e}_\theta$, and we have

$$-\left(\frac{\partial^2 A_\theta}{\partial r^2} + \frac{\partial^2 A_\theta}{\partial z^2} + \frac{1}{r}\frac{\partial A_\theta}{\partial r} - \frac{A_\theta}{r^2}\right) = \mu J_\theta.$$

Here the current of the coil $\boldsymbol{J}$ contains only the polar component J_θ. The magnetic vector potential A_θ can be found by the Gaussian iteration method, and we can obtain the magnetic field by

$$\boldsymbol{B} = -\frac{\partial A_\theta}{\partial z}\boldsymbol{e}_r + \left(\frac{\partial A_\theta}{\partial r} + \frac{A_\theta}{r}\right)\boldsymbol{e}_z.$$

The specific iteration format is

$$\begin{aligned}\left(\frac{2}{\Delta r^2} + \frac{2}{\Delta z^2} + \frac{1}{r^2}\right) A_\theta(r,z) \\ &= \mu_0 J_\theta \\ &+ \left(\frac{A_\theta(r+1,z) + A_\theta(r-1,z)}{\Delta r^2} + \frac{A_\theta(r,z+1) + A_\theta(r,z-1)}{\Delta z^2}\right. \\ &\left. + \frac{A_\theta(r+1,z) - A_\theta(r-1,z)}{2r\Delta r}\right).\end{aligned}$$

二、 Example

Here we use four identical and equally spaced coils, each with a current of 500A*100 (100 turns), shown in Figure 1. Figure 2 shows the magnetic field lines. Figure 3 shows the magnitude of the magnetic field at the axis. Note in particular that vacuum magnetic permeability ($\mu = \mu_0$) is used here. If one wants to consider a ferromagnetic medium, changing the magnitude of the permeability is sufficient. If interested, you can also do a comparison with the theoretical solution.

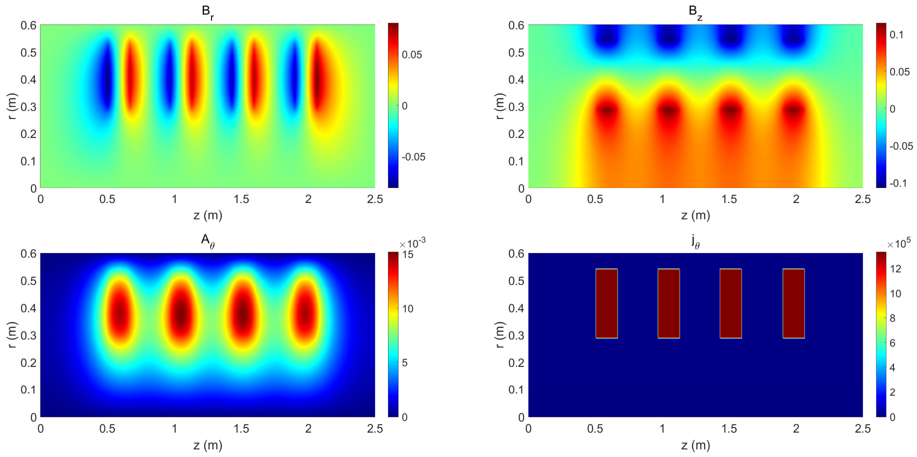

Figure 1. The calculated magnetic field size, magnetic vector potential, and the current density set in this case.

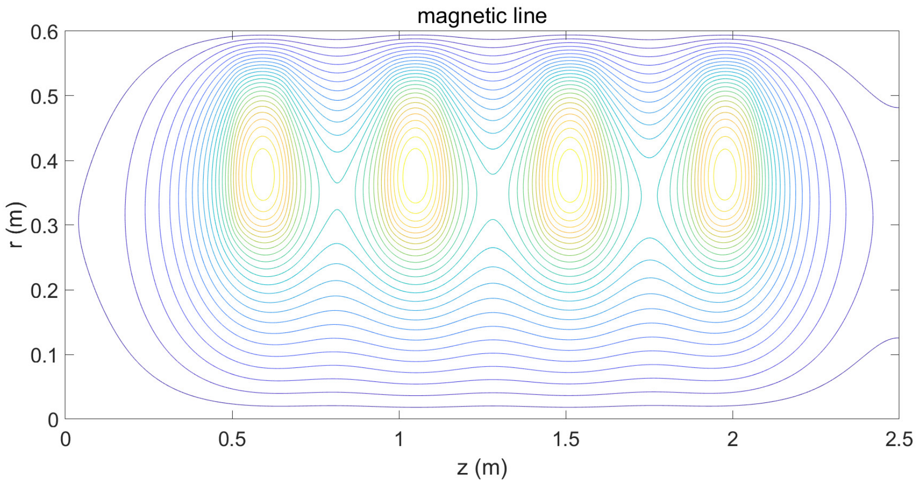

Figure 2. The magnetic field lines.

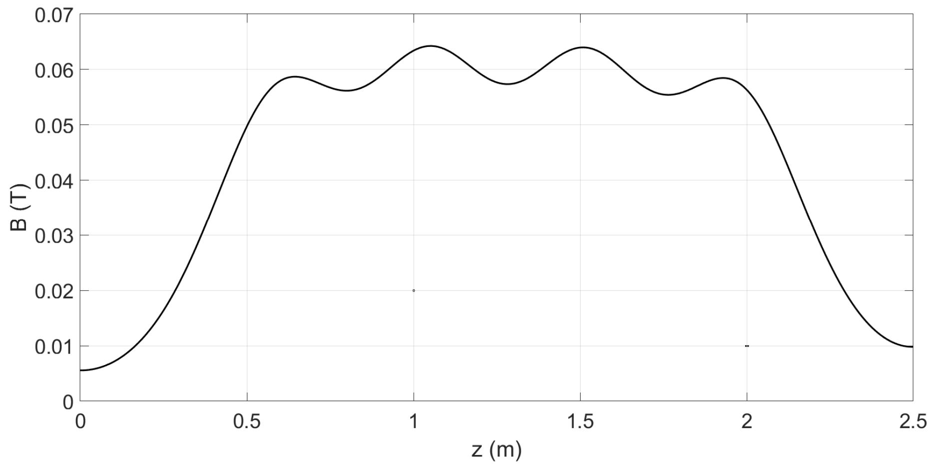

Figure 3. The magnitude of the magnetic field at the axis (r=0).