Distribution of the Non-Central F

The non-central F distribution with dfs $\nu_1,\ \nu_2$ and non-centrality parameter λ has pdf

$$g_\lambda(x) = \frac{C(x)}{B(\nu_1/2,\ \nu_2/2)}\ e^{-\lambda/2}\nu_1^{\nu_1/2}\nu_2^{\nu_2/2}(\nu_2+\nu_1 x)^{-(\nu_1+\nu_2)/2}x^{(\nu_1-2)/2}$$

where $B(\nu_1/2,\ \nu_2/2)$ is the beta function and

$$C(x) = 1 + \sum_{j=1}^{\infty}\left(\frac{(\nu_1\lambda x)/2}{\nu_2+\nu_1 x}\right)^j \frac{(\nu_1+\nu_2)(\nu_1+\nu_2+2)(\nu_1+\nu_2+2j-2)}{j!\ \nu_1(\nu_1+2)\cdots(\nu_1+\nu_2+2j-2)}$$

The cdf is then given by

$$G_\lambda(x) = \int_0^x g_\lambda(y)dy$$

Just by examining the function it is somewhat difficult to determine the relationship between $G_\lambda(x)$ and λ. However, after considering numerous combinations values of $\nu_1,\ \nu_2,\ \lambda$, it was always true that for a fixed values of x as λ increases $G_\lambda(x)$ decreases. See plot below as an illustration for $\nu_1 = 3,\ \nu_2 = 56,\ \lambda = .5,\ 2,\ 5.$