

STATISTICS 630 - Test I

October 2, 2013

Name _____ Email Address _____

INSTRUCTIONS FOR STUDENTS:

- (1) There are five pages including this cover page and three formula sheets. Each of the five numbered problems is weighted equally.
- (2) You have exactly 70 minutes to complete the exam.
- (3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper. Put the worked problems in numerical order for scanning.
- (4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3} , $\Phi(1.4)$, etc., unless otherwise specified.
- (5) Show *ALL* your work. Give reasons for your answers.
- (6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.
- (7) You may use the formula sheets accompanying this test.
- (8) Do not use your textbook, class notes, or any other written material except for the formula sheets. Do not use a computer, cell phone, or any other electronic device.

I attest that I spent no more than 70 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature _____

INSTRUCTIONS FOR PROCTOR:

- (1) Record the time at which the student starts the exam: _____
- (2) Record the time at which the student ends the exam: _____
- (3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.
- (4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.
- (5) Please keep these materials until October 11, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature _____

1. After travelling to the Joint Statistical Meetings in Montreal, I arrived back in College Station with 3 U. S. quarters and 2 Canadian quarters in my pocket. I randomly chose two quarters without replacement from my pocket.
 - (a) Obtain the probability mass function of the number of U. S. quarters among the two that I chose from my pocket. Evaluate this probability mass function using fractions (e.g., $P(X=1)=3/4$).
 - (b) Obtain the conditional probability that both the quarters that I randomly selected were U. S. quarters given that at least one of the two quarters was a U. S. quarter.

2. A continuous random variable X has the probability density function (pdf)

$$f_X(x) = \begin{cases} cx^4 & \text{for } 0 < x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Obtain the value of c that makes $f_X(x)$ a valid pdf for a continuous rv.
 - (b) Find the 50th percentile of the distribution of X .
3. Suppose that the random variables (X, Y) have joint probability density function

$$f(x, y) = \begin{cases} 15x^2y, & 0 \leq x \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Obtain the marginal probability density functions of X and Y .
 - (b) Obtain the conditional probability density function of X given $Y = y$ and use this function to determine whether X and Y are independent.
4. A tennis player with a two-handed backhand often has elbow problems. Let A be the event that her right elbow is sore on a given morning and B be the event that her left elbow is sore on that morning. Suppose that $P(A) = 0.3$ and $P(B) = 0.2$. What the probability that at least one of her elbows is sore on the given morning under each of the following assumptions?
 - (a) A and B are mutually exclusive.
 - (b) A and B are independent.
 - (c) $B \subset A$.
 - (d) The probability that both her elbows are sore on the given morning equals 0.1.

5. Suppose that X is an exponential random variable with probability density function

$$f(x) = \begin{cases} 2e^{-2x}, & 0 < x < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Obtain the cumulative distribution function of X .
 - (b) Obtain the probability density function of $Y = e^X$.