STATISTICS 630 - Test II
November 7, 2012

Name ____________________________________ Email Address ____________________________

INSTRUCTIONS FOR STUDENTS:

(1) There are six pages including this cover page and four formula sheets. Each of the five numbered problems is weighted equally.

(2) You have exactly 70 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper. Put the worked problems in numerical order for scanning.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc., unless otherwise specified.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 70 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature__________________________________

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: ______________________

(2) Record the time at which the student ends the exam: ______________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until November 16, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature__________________________________

1. Suppose that $X_1 \sim N(2, 2^2)$ and $X_2 \sim N(-1, 3^2)$ are independent random variables.
   (a) Let $U = 4X_1 - X_2$. Find the distribution of U.
   (b) Find values of $C_1,\ C_2,\ C_3,\ C_4$, and C_5 (where $C_1 \neq 0$ and $C_3 \neq 0$) so that
$$C_1(X_1 + C_2)^2 + C_3(X_2 + C_4)^2 \sim \chi^2(C_5).$$

2. Suppose that $X_1, \ldots, X_n$ are a random sample from a distribution with probability mass function
$$p_\theta(x) = \begin{cases} (x+1)\theta^2(1-\theta)^x, & x = 0, 1, 2, 3, \ldots, \ (0 \leq \theta \leq 1) \\ 0 & \text{otherwise,} \end{cases}$$
and mean $E(X_i) = 2(1-\theta)/\theta$. Find the maximum likelihood estimator estimator of θ and also the method of moments estimator of θ. Are they the same?

3. Suppose that X and Y are jointly distributed random variables with means, $E(X) = 0,\ E(Y) = 0$, variances, $\mathrm{Var}(X) = 6,\ \mathrm{Var}(Y) = 5$, and covariance, $\mathrm{Cov}(X, Y) = 2$. Let $U = 3X - 2Y$ and $W = 2X + Y$. Obtain the following expectations:
   (a) $E(U)$
   (b) $\mathrm{Var}(U)$
   (c) $E(W)$
   (d) $\mathrm{Var}(W)$
   (e) $\mathrm{Cov}(U, W)$.

4. Suppose that undergraduate statistics students take a multiple choice exam with 20 questions and that each question has 5 possible answers. Since all the students neglected to study, each student guesses at random on each question. We assume that all the students take the test independently.
   (a) Let X_i be the score of the i^{th} student taking the exam. Find $E(X_i)$ and $\mathrm{Var}(X_i)$.
   (b) Suppose that a class of n students take the exam independently, and let their scores be $X_1, \ldots, X_n$. Find with a proof a number m such that the average score of the class converges in probability to that number as $n \longrightarrow \infty$; i.e., find a number m such that $\frac{1}{n}(X_1 + \cdots + X_n) \xrightarrow{P} m$.

5. Suppose that T is a random variable such that $E(T) = 4\theta$ and $\mathrm{Var}(T) = 8\theta^2$. Consider the following estimators of θ:
$$\hat{\theta}_1 = \frac{T}{4}, \qquad \hat{\theta}_2 = \frac{T}{5}.$$
Find the mean, variance, and mean squared error of each of these estimators. Then determine which one has smaller mean squared error.