Stat 642 Summer 2015
Solutions for Homework 5

1. (10points) In the Cell Means model: $Y_{ijk} = \mu_{ij} + e_{ijk}$

   a. H_o : No interaction $\Rightarrow$ $H_o : \mu_{ij} - \mu_{kj} = \mu_{ih} - \mu_{kh}$ for all (i, k, j, h) $\Rightarrow$
   For all $j = 1, \ldots, b : \mu_{2j} - \mu_{1j} = C_1;\ \mu_{3j} - \mu_{2j} = C_2;\ \cdots; \mu_{aj} - \mu_{a-1j} = C_{a-1}$ $\Rightarrow$
   Under H_o only b + (a-1) parameters are needed to express μ_{ij}, namely,
   $(\mu_{11}, \mu_{12}, \cdots, \mu_{1b}, C_1, C_2, \cdots, C_{a-1})$, that is, given these parameters we have
   $\mu_{ij} = \mu_{1j} + (\mu_{2j} - \mu_{1j}) + (\mu_{3j} - \mu_{2j}) + \cdots + (\mu_{ij} - \mu_{i-1j}) = \mu_{1j} + C_1 + C_2 + \cdots + C_{i-1}$
   Thus, $df_{A*B} = (\#\text{ parameters under } H_1) - (\#\text{ parameters under } H_o) = ab - (b + a - 1) = (a-1)(b-1)$

   b. H_o : No main effect Factor A $\Rightarrow$ $H_o : \bar{\mu}_{1.} = \bar{\mu}_{2.} = \cdots = \bar{\mu}_{a.} = C$ $\Rightarrow$
   $\bar{\mu}_{i.} = C \ \Rightarrow \frac{1}{b}\sum_{j=1}^{b} \mu_{ij} = C \ \Rightarrow \mu_{ib} = bC - \sum_{j=1}^{b-1} \mu_{ij}$ $\Rightarrow$
   Under H_o only 1 + a(b-1) parameters are needed to express μ_{ij}, namely,
   C plus, for each i, need b-1 parameters $\mu_{i1}, \mu_{i2}, \cdots, \mu_{ib-1}$ to express μ_{ij} under H_o
   Thus, $df_A = (\#\text{ parameters under } H_1) - (\#\text{ parameters under } H_o) = ab - (1 + a(b-1)) = a - 1$

2. (25 points) This is a CRD experiment involving three treatments with fixed levels, Soil Type, and 10 replications/treatment. The EU's are the fields and there is subsampling with the MU's being the locations within the fields.

   a. $Y_{ijk} = \mu + \tau_i + e_{ij} + d_{ijk},\ i = 1, \cdots, 15,\ j = 1, 2,\ k = 1, \cdots, n_{ij}$, where μ is overall mean, τ_i is the fixed effect due to the Soil Type, e_{ij} is the random effect due to the selected fields, d_{ijk} is the random effect due to variation in Soil subsamples within the same Soil Type-Field and all other sources.
   $\tau_3 = 0,\ e_{ij} \sim iid\ N(0, \sigma_e^2),\ d_{ijk} \sim iid\ N(0, \sigma_d^2)$, and e_{ij} and d_{ijk} are independent

   b. $t = 3,\ n_i = r = 10,\ m_{ij} = 1$ or 2:

Source	df	SS	MS	EMS	F	$Pr > F$
SoilType	2	36.67107	18.33554	$\sigma_d^2 + 1.6342\sigma_e^2 + 14.66\theta_\tau$	29.92	$< .0001$
Field(SoilType)	27	15.67045	0.58039	$\sigma_d^2 + 1.448\sigma_e^2$	1.769	0.1318
Location(Field)	14	4.59362	0.32812	σ_d^2		
Total	43	56.93514				

   Variance for Fields=σ_e^2, Variance for Locations=σ_d^2:
   $$A = \tfrac{25}{15} + \tfrac{22}{14} + \tfrac{25}{15} = 4.904762, \quad B = 16(1)^2 + 14(2)^2 = 72, \quad D = (15)^2 + (14)^2 + (15)^2 = 646 \ \Rightarrow$$
   $$C_1 = \tfrac{1}{3-1}\left(4.904762 - \tfrac{72}{44}\right) = 1.6342, \quad C_2 = \tfrac{1}{3-1}\left(44 - \tfrac{646}{44}\right) = 14.66, \quad C_3 = \tfrac{1}{30-3}\left(44 - 4.904762\right) = 1.448$$

   c. The variance components are computed as follows:
   $$E(MS_{SUB}) = \sigma_d^2 \qquad \Rightarrow \quad \hat{\sigma}_d^2 = MS_{SUB} = 0.32812$$
   $$E(MSE) = \sigma_d^2 + c_3\sigma_e^2 \quad \Rightarrow \quad \hat{\sigma}_e^2 = \tfrac{MSE - MS_{SUB}}{c_3} = \tfrac{.58039 - .32812}{1.448} = .17422$$

   Using the more accurate REML (Restricted Maximum Likelihood) estimators from the SAS function PROC MIXED, we obtain $\hat{\sigma}_d^2 = 0.3244$, and $\hat{\sigma}_e^2 = 0.1780$

   Because of the unequal number of subsamples, REML produces estimates which are a little different from the AOV-MOM estimates.

From the model, the variance in the individual porosity readings, $\sigma_y^2 = \sigma_e^2 + \sigma_d^2$. Therefore,

Proportion of variation due to Fields is $\frac{\sigma_e^2}{\sigma_e^2+\sigma_d^2} \approx \frac{.3244}{.3244+.1780} = 64.6\%$

Proportion of variation due to Locations in the field is $\frac{\sigma_d^2}{\sigma_e^2+\sigma_d^2} \approx \frac{.1780}{.3244+.1780} = 35.4\%$

d. Using the results from SAS, PROC MIXED, we have the following groupings, where Soil Types within a group are not significantly different:

$G_1 = \{CLAY, LOAM\}, \ \ G_2 = \{SANDY\}$

Note that if you use PROC GLM to do the grouping, the Standard Errors of the estimates of the Treatment means $\widehat{SE}(\hat{\mu}_i)$ are smaller than the values from PROC MIXED:

PROC GLM - $\widehat{SE}(\hat{\mu}_i)$ = .15687, .16202, .15687; which are incorrect estimates because they ignore the value of σ_e^2

PROC MIXED - $\widehat{SE}(\hat{\mu}_i)$ = .2014, .2054, .2014; which are larger because they include $\hat{\sigma}_e^2$ in the estimation

This results in GLM having different groupings: $G_1 = \{CLAY\}, \ \ G_2 = \{LOAM\}, \ \ G_3 = \{SANDY\}$

e. For testing $H_0 : \sigma_e^2 = 0$ vs $H_1 : \sigma_e^2 > 0$, test statistic is $F = \frac{MSE}{MS_{SUB}} = \frac{.58039}{.32812} = 1.7688$. Because $F = 1.7688 < 2.326 = F_{0.05,27,14}$ and $p - value = 1 - pf(1.7688, 27, 14) = 0.1318 > 0.05$, we fail to reject H_0 and conclude that there is not significant evidence of a difference in porosity in fields of the same soil type.

3. (24 points) This is a 4×3 CRD with two crossed factors, Type of Crop and Nitrogen, having fixed levels. There are four reps/treatment with EU=MU=Growth Chamber.

a. Cell Mean Model: $Y_{ijk} = \mu_{ij} + e_{ijk}; \ i = 1, 2, 3; \ j = 1, 2, 3, 4; \ k = 1, 2, 3, 4$, where Y_{ijk} is acetylene reduction from the kth growth chamber receiving ith Nitrogen level with the jth Crop, μ_{ij} is the mean response of ith Nitrogen level, jth Crop, and $e_{ijk} \sim iid\ N(0, \sigma_e^2)$.
Effects Model: $Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + e_{ijk}$, where τ_i is the fixed effect of ith Nitrogen level, β_j is the fixed effect of jth Crop, $(\tau\beta)_{ij}$ are interaction effects between Nitrogen and Crop, with $\tau_3 = 0, \ \beta_4 = 0, \ (\tau\beta)_{3j} = (\tau\beta)_{i4} = 0$ for $i = 1, 2, 3; j = 1, 2, 3, 4$.

b. Based on the B-F-L test which has p-value = 0.0389 and the plot of the residuals, there is an indication that the assumption of equality of variance is invalid.

The Shapiro-Wilk test has p-value< .0001, the Stem-Leaf plot appears heavy tailed with numerous outliers, and the normal probability plot has the residuals both above and below a straight line; therefore, the normal condition appears not to be valid.

There is not an index of time or space relative to the measurements or experimental units so a valid measure of correlation in the residuals is not available.

c. Using the Box-Cox procedure, a transformation X = log(Y) was suggested. An evaluation of the AOV conditions using log(Y) yielded:

• Based on the B-F-L test which has p-value = 0.1940 and the plot of the residuals, the evidence indicates that the assumption of equality of variance is valid.

The Shapiro-Wilk test has p-value= .9289, the Stem-Leaf plot appears symmetric with no outliers, and the normal probability plot has the residuals very close to a straight line; therefore, the normal condition appears to be valid for the transformed data.

The ANOVA table from SAS is given below:

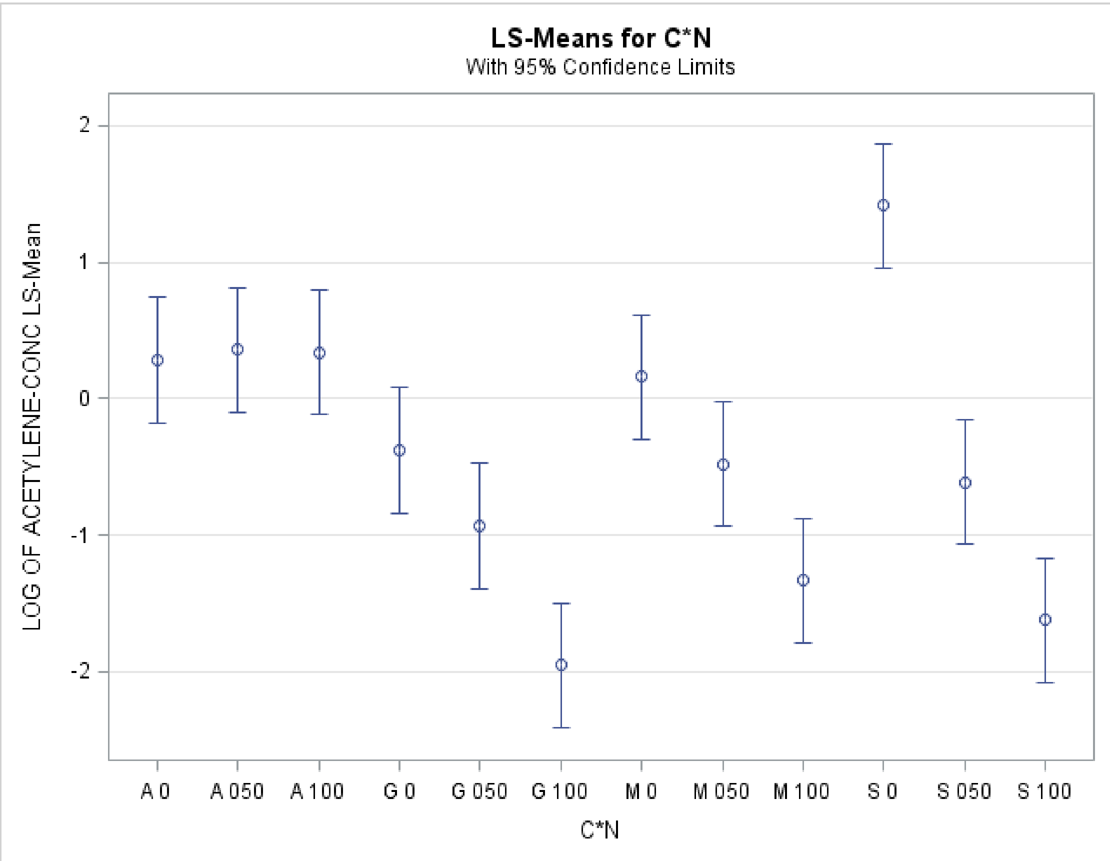
Dependent Variable: X = log(ACE-CONC)

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	41.23719964	3.74883633	18.41	<.0001
Error	36	7.32907897	0.20358553		
Corrected Total	47	48.56627862			

Source	DF	Type III SS	Mean Square	F Value	Pr > F
C	3	12.46379492	4.15459831	20.41	<.0001
N	2	18.34306101	9.17153051	45.05	<.0001
C*N	6	10.43034372	1.73839062	8.54	<.0001

(d) Test for Interaction: $H_0 : \mu_{ij} - \mu_{ij'} = \mu_{i'j} - \mu_{i'j'}$ for all (i, i', j, j') vs $H_1 : \mu_{ij} - \mu_{ij'} \neq \mu_{i'j} - \mu_{i'j'}$ for some choice (i, i', j, j'):
Test statistic is $F = \frac{MS_{C\times N}}{MSE} = \frac{1.73839}{0.20358553} = 8.54$ with $F = 8.54 > 2.36 = F_{0.05,6,36}$ and $p - value = 1 - pf(8.54, 6, 36) = 8.494528e - 06 < 0.05$, we reject H_0. Thus, we conclude that there is significant evidence of an interaction between Nitrogen level and Crop. The ANOVA table shows both main effect have very small p-values and hence there is strong evidence of a main effect of both Nitrogen and Crop however, neither of these results have meaningful interpretation.

(e) The profile plot is given here and the graph confirms the conclusion from the test of hypotheses. There is very little change in the mean level of log-acetylene with increasing levels of nitrogen for Alfalfa but there is a strong decrease in log-acetylene with increasing nitrogen for the other three crops.



(f) Because of the significant interaction, the Nitrogen levels will be grouped separately for each of the 4 Types of Crops using the Tukey adjusted p-values:
Crop = Alfalfa: G1 = {0, 50, 100}
Crop = Soybean: G1 = {0} G2 ={ 50, 100}
Crop = Guar: G1 = {0, 50} G2 ={ 50, 100}
Crop = Mungbean: G1 = {0, 50} G2 ={ 50, 100}

- The groupings are a bit surprising after viewing the profile plot. The 0 and 50 level of Nitrogen were not found to be different for Mungbean whereas in the profile plot there appears to be a sizable difference in the two means. This seeming contradiction reflects the simultaneous testing of all pairs which results in fewer pairs being declared different.

4. (20 points)

a. Cell Mean Model: $Y_{ijk} = \mu_{ij} + e_{ijk}$; $i = 1, 2, 3$; $j = 1, 2, 3, 4$; $k = 1, 2, 3, 4$, where Y_{ijk} is acetylene reduction from the kth receiving ith Nitrogen level with the jth Crop, μ_{ij} is the mean response of ith Nitrogen level, jth Crop, and $e_{ijk} \sim iid\ N(0, \sigma_e^2)$.

b. Effects Model: $Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + e_{ijk}$, where τ_i is the fixed effect of ith Nitrogen level, β_j is the fixed effect of jth Crop, $(\tau\beta)_{ij}$ are interaction effects between Nitrogen and Crop, with $\tau_3 = 0,\ \beta_4 = 0,\ (\tau\beta)_{3j} = (\tau\beta)_{i4} = 0$ for $i = 1, 2, 3; j = 1, 2, 3, 4$.

c. The matrix form for the cell means model is

$$\mathbf{Y} = X\boldsymbol{\beta} + \mathbf{e},$$

where

$$\begin{aligned}
\mathbf{Y} &= (Y_{111}, Y_{112}, Y_{113}, Y_{114}, Y_{121}, Y_{122}, Y_{123}, Y_{124}, Y_{131}, Y_{132}, Y_{133}, Y_{134}, Y_{141}, Y_{142}, Y_{143}, Y_{144}, \\
&\quad Y_{211}, Y_{212}, Y_{213}, Y_{214}, Y_{221}, Y_{222}, Y_{223}, Y_{224}, Y_{231}, Y_{232}, Y_{233}, Y_{234}, Y_{241}, Y_{242}, Y_{243}, Y_{244}, \\
&\quad Y_{311}, Y_{312}, Y_{313}, Y_{314}, Y_{321}, Y_{322}, Y_{323}, Y_{324}, Y_{331}, Y_{332}, Y_{333}, Y_{334}, Y_{341}, Y_{342}, Y_{343}, Y_{344},)^T, \\
\mathbf{e} &= (e_{111}, e_{112}, e_{113}, e_{114}, e_{121}, e_{122}, e_{123}, e_{124}, e_{131}, e_{132}, e_{133}, e_{134}, e_{141}, e_{142}, e_{143}, e_{144}, \\
&\quad e_{211}, e_{212}, e_{213}, e_{214}, e_{221}, e_{222}, e_{223}, e_{224}, e_{231}, e_{232}, e_{233}, e_{234}, e_{241}, e_{242}, e_{243}, e_{244}, \\
&\quad e_{311}, e_{312}, e_{313}, e_{314}, e_{321}, e_{322}, e_{323}, e_{324}, e_{331}, e_{332}, e_{333}, e_{334}, e_{341}, e_{342}, e_{343}, e_{344},)^T,
\end{aligned}$$

$$X = \begin{pmatrix}
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 \\
\mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4
\end{pmatrix}_{48\times 12} \qquad \boldsymbol{\beta} = \begin{pmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{14} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \\ \mu_{24} \\ \mu_{31} \\ \mu_{32} \\ \mu_{33} \\ \mu_{34} \end{pmatrix}_{12\times 1}$$

d. The matrix form for the effects model with constraints is

$$\mathbf{Y} = X\boldsymbol{\beta} + \mathbf{e},$$

where $\mathbf{Y}$ and $\mathbf{e}$ are the same as those in c,

$$X = \begin{pmatrix}
\mathbf{1}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 \\
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \\
\mathbf{1}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4
\end{pmatrix}_{48\times 12} \qquad \boldsymbol{\beta} = \begin{pmatrix}
\mu \\ \tau_1 \\ \tau_2 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ (\tau\gamma)_{11} \\ (\tau\gamma)_{12} \\ (\tau\gamma)_{13} \\ (\tau\gamma)_{21} \\ (\tau\gamma)_{22} \\ (\tau\gamma)_{23}
\end{pmatrix}_{12\times 1}$$

5.(21 points) 1. Prove that there is an $F_1 * F_2 * F_3$ interaction, that is, show that the $F_2 * F_3$ interaction effect at $F_1 = 1$ is different from the $F_2 * F_3$ interaction effect at $F_1 = 2$.

Verify that $[(\mu_{111} - \mu_{112}) - (\mu_{121} - \mu_{122})] \neq [(\mu_{211} - \mu_{212}) - (\mu_{221} - \mu_{222})]$

$[(\mu_{111} - \mu_{112}) - (\mu_{121} - \mu_{122})] = [(8-4) - (2-4)] = 6$ and

$[(\mu_{211} - \mu_{212}) - (\mu_{221} - \mu_{222})] = [(4-6) - (6-2)] = -6 \neq 6 \quad \Rightarrow$ There is an $F_1 * F_2 * F_3$ interaction.

2. Prove that there is not an $F_2 * F_3$ interaction (ignore the fact that there is an $F_1 * F_2 * F_3$ interaction)..

Verify that $(\bar{\mu}_{.11} - \bar{\mu}_{.12}) = (\bar{\mu}_{.21} - \bar{\mu}_{.22})$

$(\bar{\mu}_{.11} - \bar{\mu}_{.12}) = ((8+4)/2 - (4+6)/2) = 1$ and $(\bar{\mu}_{.21} - \bar{\mu}_{.22}) = ((2+6)/2 - (4+2)/2) = 1 \Rightarrow$ There is not an $F_2 * F_3$ interaction.

3. Prove that there is a main effect due to F_3 (ignore the fact that there is an $F_1 * F_2 * F_3$ interaction).

Verify that $\bar{\mu}_{..1} \neq \bar{\mu}_{..2}$

$\bar{\mu}_{..1} = (8+2+4+6)/4 = 5$ and $\bar{\mu}_{..2} = (4+4+6+2)/4 = 4 \neq 5 \Rightarrow$ There is a F_3 main effect.