STATISTICS 630 - Test II

November 6, 2013

Name ______________________________________ Email Address ______________________________

INSTRUCTIONS FOR STUDENTS:

(1) There are seven pages including this cover page and four formula sheets. Each of the five numbered problems is weighted equally.

(2) You have exactly 70 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper. Put the worked problems in numerical order for scanning.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc., unless otherwise specified.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test.

(8) Do not use your textbook, class notes, or any other written material except for the formula sheets. Do not use a computer, cell phone, or any other electronic device.

I attest that I spent no more than 70 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature__________________________________

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: _______________________

(2) Record the time at which the student ends the exam: _______________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until November 13, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature__________________________________

1. Suppose that the random variables (X, Y) have joint probability density function (pdf)

$$f(x,y) = \begin{cases} 15x^2y, & 0 < x < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

You may use without proof the results from the first exam that the marginal pdfs of X and Y are

$$f_X(x) = \begin{cases} \frac{15}{2}x^2(1-x^2), & 0 < x < 1, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad f_Y(y) = \begin{cases} 5y^4, & 0 < y < 1, \\ 0, & \text{otherwise,} \end{cases}$$

and also that the conditional pdf of X given $Y = y$ for $0 < y < 1$ is given by

$$f_{X|Y}(x|y) = \frac{3x^2}{y^3}, \ 0 < x < y.$$

(a) Obtain $E(X)$.

(b) Compute $E[X|Y = y]$. Then compute $E[E[X|Y]]$ to verify directly that $E(X) = E[E(X|Y)]$.

2. Suppose that $X_1, \ldots, X_n$ is a random sample from a distribution with probability density function

$$f_X(x) = \begin{cases} 2\theta x e^{-\theta x^2}, & 0 < x < \infty, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$.

(a) Find the maximum likelihood estimator of θ.

(b) Find the method of moments estimator based on the mean of the distribution,

$$E(X) = \sqrt{\pi}/(2\sqrt{\theta}).$$

3. The statistical software package SAS is used in an applied statistics course where many of the students are not experienced programmers. Suppose that the number of errors X made by a randomly chosen student for a given SAS program has the following probability mass function:

x	0	1	2	3
$p_X(x)$	0.4	0.3	0.2	0.1

(a) Verify that $E(X) = 1$ and $\text{Var(X)} = 1$ (be sure to show your calculations).

(b) Suppose that a class of 64 students is taking this course. Suppose that the numbers of errors committed by the students in the class on the given SAS program can be considered to be a random sample from the above distribution. Obtain an expression for the approximate probability that the total number of errors committed by the 64 students in the class exceeds 60.

4. Suppose that $X_1 \sim N(2, 2^2)$, $X_2 \sim N(0, 3^2)$, and $X_3 \sim N(-1, 1^2)$ are independent random variables.

   (a) Let $U = X_1 - 2X_2 + 5X_3 + 7$. Find the distribution of U.

   (b) Find values of $C_1,\ C_2,\ C_3,\ C_4,\ C_5$, and C_6, (where $C_2 \neq 0$ and $C_4 \neq 0$) so that

$$\frac{X_1 + C_1}{\sqrt{C_2(X_2 + C_3)^2 + C_4(X_3 + C_5)^2}} \sim t(C_6).$$

5. Let T_1 and T_2 be unbiased estimators of μ. Consider the estimator $T = \frac{1}{2}T_1 + \frac{1}{2}T_2$ of μ. Suppose that $\mathrm{Var(T_1)} = 2$ and $\mathrm{Var(T_2)} = 1$.

   (a) Show that T is an unbiased estimator of μ.

   (b) Obtain mean squared error of T under each of the following scenarios. Use this information to determine which scenario would lead to the best estimation for μ. Explain why.

      i. $\mathrm{Cov(T_1, T_2)} = 0$

      ii. $\mathrm{Cov(T_1, T_2)} = 1$

      iii. $\mathrm{Cov(T_1, T_2)} = -1$