Homework 2: Written Section

1. Show that $Var(Y_i) = Var(e_i)$ in the simple linear regression model. (Yes, this should be that simple.) What did you assume?

2. Define in words only the least squares criterion.

3. Show that the least squares criterion applied to the "intercept-only" model, i.e.

$$y_i = \beta_0 + e_i, i = 1, 2, \ldots, n$$

   results in the least squares estimator of $\beta_0 : \hat{\beta}_0 = \bar{y}$ by following these steps:

   (a) Write down your design matrix, $\mathbf{X}$. (It won't be the same as any we've used in class.) Double check: does $\mathbf{y} = \mathbf{x}\boldsymbol{\beta} + \mathbf{e}$ give the set of equations listed above? Notice this model has no predictor variable.

   (b) Use the previously derived formula $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ to get the least squares estimator.

4. Question 4, page 40 in our textbook, except do:

   (a) Setup:

      i. Write down your design matrix, $\mathbf{X}$.

      ii. Show, using matrix notation and starting with the principle of least squares, that the least squares estimate of β is given by

$$\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}$$

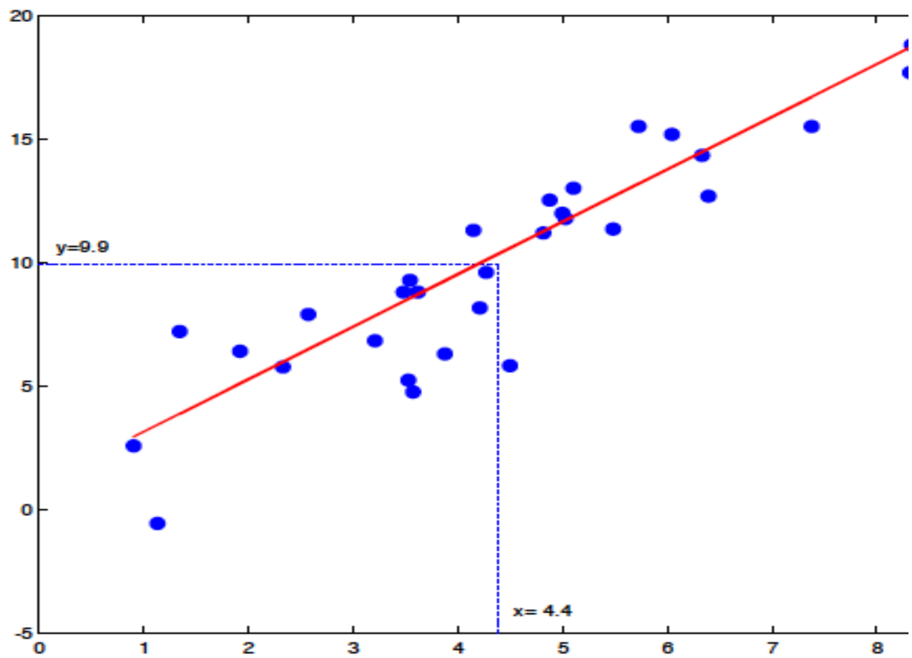
   (b) As in text

5. Using $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$, finish our algebra from class and show that $\hat{\beta}_0 = \bar{y} - \frac{SXY}{SXX}\bar{x}$ for the simple linear regression case. Give a few more algebraic details than are on page 133.

6. Show that for the usual regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, where the usual regression assumptions from question 4 apply, $\text{Var}(\mathbf{a}'\hat{\boldsymbol{\beta}}|\mathbf{X}) = \sigma^2\mathbf{a}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{a}$, where $\mathbf{a}$ is a constant vector. (We'll use this fact later.)

7. Question 7, page 42 in our textbook.

8. The figure below shows a scatterplot of some data together with a line that purports to have been fitted by least squares. The averages of the x and y values are 4.4 and 9.9 respectively. The line in the figure cannot be the least squares line. Say why not AND provide a justification for your answer.



y=9.9
x= 4.4
20
15
10
5
0
-5
0
1
2
3
4
5
6
7
8