Homework 10 (Written Section)

1. (For review; see the next problem.) Consider the regression model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, where the n errors in the error vector $\mathbf{e}$ are independent and identically distributed.

   (a) Derive the estimate $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ found by minimizing the residual sum of squares.

   (b) Show that $\hat{\beta}$ is unbiased for the parameter vector $\boldsymbol{\beta}$.

   (c) Calculate the variance matrix of $\hat{\beta}$.

2. Consider the model $Y_t = \beta_0 + \beta_1 x_t + e_t$, where $e_t = \rho\, e_{t-1} + \nu_t$ and $\nu_t \sim \text{iid } N(0, \sigma_\nu^2)$.

   (a) Use the log-likelihood function given on page 312 to show (using matrix notation) that

   $$\hat{\beta}_{GLS} = (\mathbf{X}'\Sigma^{-1}\mathbf{X})^{-1}\mathbf{X}'\Sigma^{-1}\mathbf{Y}$$

   (b) Explain why the estimate $\hat{\beta}_{GLS}$ is not defined when $\rho = 1$. Describe the resulting covariance matrix of the errors Σ.

   (c) Show that for $\rho < 1, \text{Corr}(e_t, e_{t-3}) = \rho^3$.

   (d) Show that $\hat{\beta}_{GLS}$ is unbiased for $\boldsymbol{\beta}$.

   (e) Calculate the variance matrix of $\hat{\beta}_{GLS}$.

   (f) How are the answers to 1 (b) and 1(c) similar to or different from the answers to 2(d) and 2(e) above?

3. Question 3, page 329.