STATISTICS 630 - Test I

October 3, 2012

Name ____________________________________ Email Address ____________________________

INSTRUCTIONS FOR STUDENTS:

(1) There are five pages including this cover page and three formula sheets. Each of the five numbered problems is weighted equally.

(2) You have exactly 70 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper. Put the worked problems in numerical order for scanning.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc., unless otherwise specified.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 70 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature________________________________

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: ______________________

(2) Record the time at which the student ends the exam: ______________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until October 12, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature________________________________

1. Suppose that a random variable X has the probability density function (pdf)

$$f_X(x) = \begin{cases} Cx(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the value of C that makes $f_X(x)$ a valid pdf for a continuous rv.

   (b) Suppose $V = 1/X$. Obtain the pdf of V.

2. Two litters of pet rats have been born, one with two brown-haired and one gray-haired rat (Litter 1 with 3 rats), and the other with three brown-haired and two gray-haired (Litter 2 with 5 rats). We select a litter at random (each litter with probability 1/2) and then select an offspring at random from the selected litter (each offspring in the litter has the same probability of being chosen).

   (a) What is the probability that the chosen animal is brown-haired?

   (b) Given that a brown-haired offspring was selected, what is the probability that the rat was from Litter 1?

3. An absent-minded professor has five similar looking keys in his pocket. Two of the keys will unlock his office door while three of the keys will not. Find the probability that exactly one of the two keys tried will succeed in unlocking the office door in each of the following circumstances:

   (a) He selects two of the keys at random from his pocket. He then tries both of the keys on his office door and records how many of them can unlock the door.

   (b) He selects one of the keys at random. He tries the selected key on his office door and records whether it can unlock the door. He then places the key back in his pocket with the other keys. He again selects a key at random from his pocket. He tries it on his office door and records whether it can unlock the door.

4. Suppose that (X, Y) have the joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} 6xy^2 & \text{for } 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the marginal probability density functions of X and Y. Determine whether X and Y are independent.

   (b) Compute $P(X > Y)$.

5. Let the sample space $\mathcal{S}$ consist of the 6 permutations of the letters a, b, and c along with the three triples of each letter. Thus, $\mathcal{S} = \{aaa, bbb, ccc, abc, bca, cba, acb, bac, cab\}$. Suppose that each element of $\mathcal{S}$ is equally likely. Define the events $A_1 = \{aaa, abc, acb\}$, $A_2 = \{aaa, bac, cab\}$, and $A_3 = \{aaa, bca, cba\}$ with $P(A_1) = P(A_2) = P(A_3) = 1/3$.

   (a) Show that the events A_1, A_2, and A_3 are pairwise independent.

   (b) Show that the events A_1, A_2, and A_3 are not mutually independent.