MASTER'S DIAGNOSTIC EXAMINATION
January 2011

Student's Name ____

INSTRUCTIONS FOR STUDENTS:

1. DO NOT put your NAME on the exam. Place the NUMBER assigned to you on the UPPER RIGHT HAND CORNER of EACH PAGE of your solutions.
2. Please start your answer to EACH QUESTION on a SEPARATE sheet of paper.
3. Use only one side of each sheet of paper.
4. You must answer all four questions: Questions I, II, III and IV.
5. Be sure to attempt all parts of the four questions. It may be possible to answer a later part of a question without having solved the earlier parts.
6. Be sure to hand in all of your exam. No additional material will be accepted once the exam has ended and you have left the exam room.
7. You may use only a calculator, pencil or pen, and blank paper for this examination. No other materials are allowed.

I attest that I spent no more than 4 hours to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature________________________________

INSTRUCTIONS FOR PROCTOR:

Immediately after the student completes the exam, **fax** the student's solutions to **979-845-6060** or email to **longneck@stat.tamu.edu** Do not send the questions, just send the student's solutions.

(1) I certify that the time at which the student started the exam was ____________________

and the time at which the student completed the exam was ____________________

(2) I certify that the student has followed all the **INSTRUCTIONS FOR STUDENTS** listed above.

(3) I certify that the student's solutions were faxed to **979-845-6060** or

emailed to **longneck@stat.tamu.edu**.

Proctor's Signature________________________________

PROBLEM I.

A computer programmer claims that $U_1, \ldots, U_{30}$ is a random sample of size 30 from a uniform (0,1) distribution.

1. Describe a graphical method to evaluate the programmer's claim. Be sure to label your axes.

2. Describe a test of hypotheses to evaluate the programmer's claim.

3. If $U_1, \ldots, U_{30}$ are determined to be in fact a random sample from a uniform on (0,1) distribution, show how $U_1, \ldots, U_{30}$ could be used to generate a random sample of 30 observations, $Y_1, \ldots, Y_{30}$ from a distribution having cdf given by

$$F(y) = 1 - exp(-\alpha(y - \beta)) \quad if \quad y \geq \beta,$$

   where α and β are known constants.

4. Suppose we have n independent observations, $Y_1, \ldots, Y_n$ on a random variable having cdf, F(y). Describe a graphical procedure to evaluate whether F(y) has the form:

$$F(y) = 1 - exp(-\alpha(y - \beta)) \quad if \quad y \geq \beta,$$

   where α and β are **unknown** constants.

5. How would you modify your graphical procedure in Part (4.) if you were told that the data was Type I censored data with $m < n$ of the observations censored?

PROBLEM II:

There are two experimental situations described below. For each of the experiments provide the following information:

1. Type of Randomization, for example, CRD, RCBD, LSD, BIBD, SPLIT-PLOT, Crossover, etc.;
2. Type of Treatment Structure, for example, Single Factor, Crossed, Nested, Fractional, etc.;
3. Identify each of the factors as being Fixed or Random;
4. Describe the Experimental Units and Measurement Units;
5. Describe the Measurement Process: Response Variable, Covariates, SubSampling, Repeated Measures;
6. A partial ANOVA Table containing just Sources of Variation (SV) and Degrees of Freedom (DF);

Experiment A: A research specialist for a large seafood company investigated bacterial growth on oysters and mussels subjected to three different storage temperatures. Nine cold storage units were available. Three storage units were randomly assigned to be used for each of the storage temperatures: 0, 5, and 10 degrees C. Oysters and mussels were stored for two weeks in each of the cold storage units. A bacterial count was made from a sample of oysters and a sample of mussels from each storage unit at the end of two weeks so that for each storage unit there is a bacterial value for oysters and a bacterial value for mussels, yielding a total of 18 observations.

Experiment B: A study was designed to compare the effect of a vitamin E supplement on the growth of guinea pigs. There are 15 guinea pigs available for the study. The guinea pigs are randomly assigned to one of the three dose levels of vitamin E with 5 animals per level. For each animal the body weight was recorded at the end of weeks 1, 3, 4, 5, 6, and 7. All 15 animals were given a growth-inhibiting substance during week 1 and given identical diets during the first four weeks of the study. At the beginning of week 5, the vitamin E treatments were implemented. The three treatment levels (doses of vitamin E) were 0, L (low), and H (high). The data include the response variable WEIGHT for each of the 15 animals for each of the 6 weekly weighings (total of 90 measurements). The other information available for each observation are the levels of DOSE (0, L, H) and the WEEK (1, 3, 4, 5, 6, 7). The animals are numbered 1 through 15. In addition, a variable called BEFAFT is created which has the following values:
BEFAFT = B for weeks 1, 3, and 4, that is, before the start of the vitamin E doses
BEFAFT = A for weeks 5, 6, and 7, that is, after starting the vitamin E doses

PROBLEM III:

1. A researcher, who has run a simple linear regression through the origin, states “My residuals do not sum to 0. Thus my model assumptions have been violated. I must transform Y or X.” Is this a correct statement? Must the residuals sum to 0? Please explain your answer.

2. A researcher believes that the log of the odds is a linear combination of x, x*x and log x. Write out the logit function.

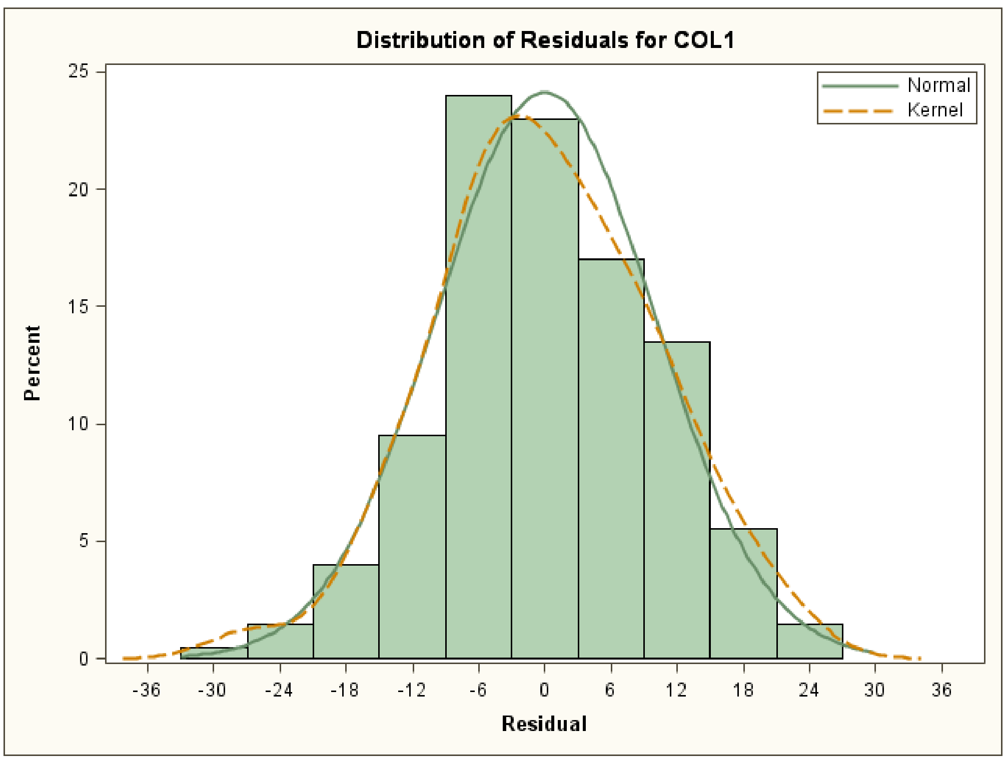
3. Carefully study the results for the three models given below. What model would you recommend to your client? Explain why. Discuss the issues with validity of the models. Is there anything you can tell the client about his predictor variables?

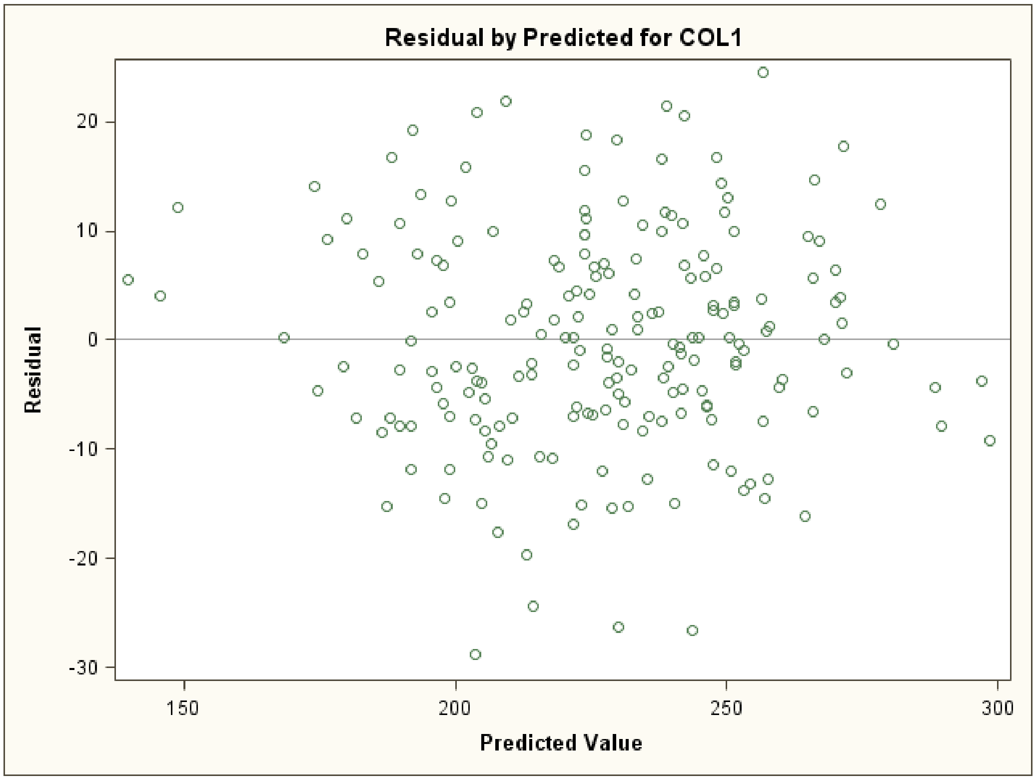
Model 1:

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	9	154584	17176	166.62	<.0001
Error	190	19586	103.08628		
Corrected Total	199	174171			

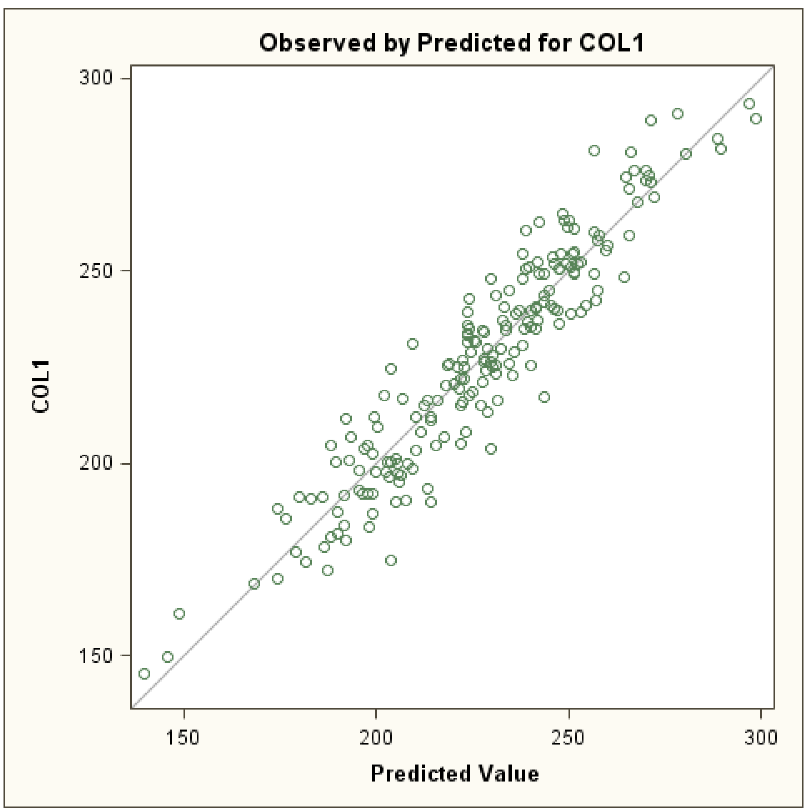
Root MSE	10.15314	R-Square	0.8875
Dependent Mean	227.04725	Adj R-Sq	0.8822
Coeff Var	4.47182		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	3.81085	6.13929	0.62	0.5355	0
COL2	1	0.93090	1.53281	0.61	0.5444	12.23132
COL3	1	2.46945	0.90784	2.72	0.0071	4.27408
COL4	1	3.41763	1.17315	2.91	0.0040	8.77643
COL5	1	3.68754	0.42996	8.58	<.0001	1.05793
COL6	1	4.71660	0.41765	11.29	<.0001	1.06188
COL7	1	6.02569	0.42912	14.04	<.0001	1.03296
COL8	1	7.19138	0.45166	15.92	<.0001	1.06395
COL9	1	7.95603	0.44949	17.70	<.0001	1.05266
COL10	1	4.39536	1.49354	2.94	0.0037	19.06114





Residual by Predicted for COL1
Residual
Predicted Value
20
10
0
-10
-20
-30
150
200
250
300



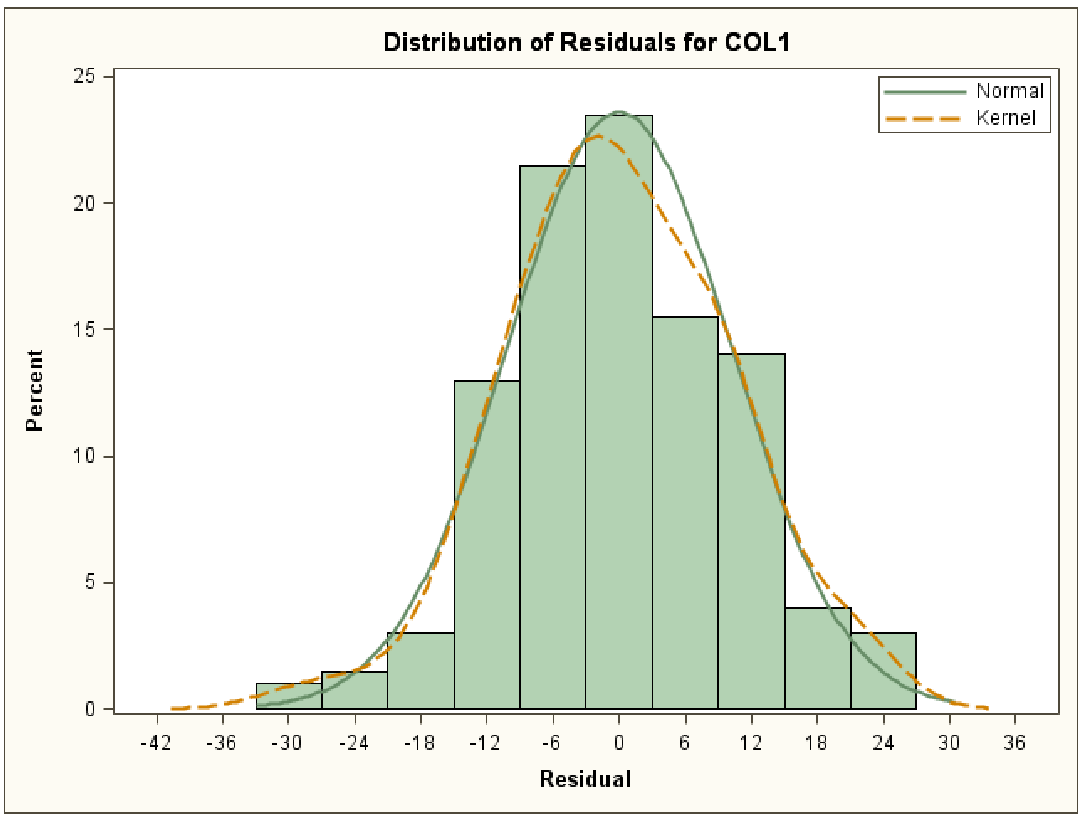
Observed by Predicted for COL1
COL1
Predicted Value
300
250
200
150
150
200
250
300

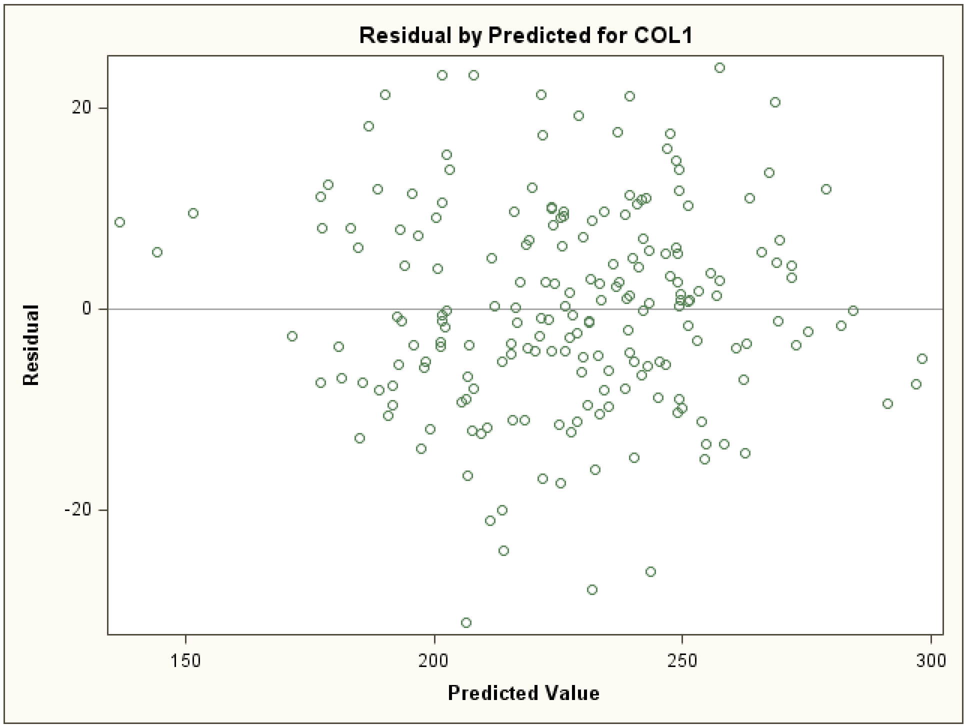
Model 2

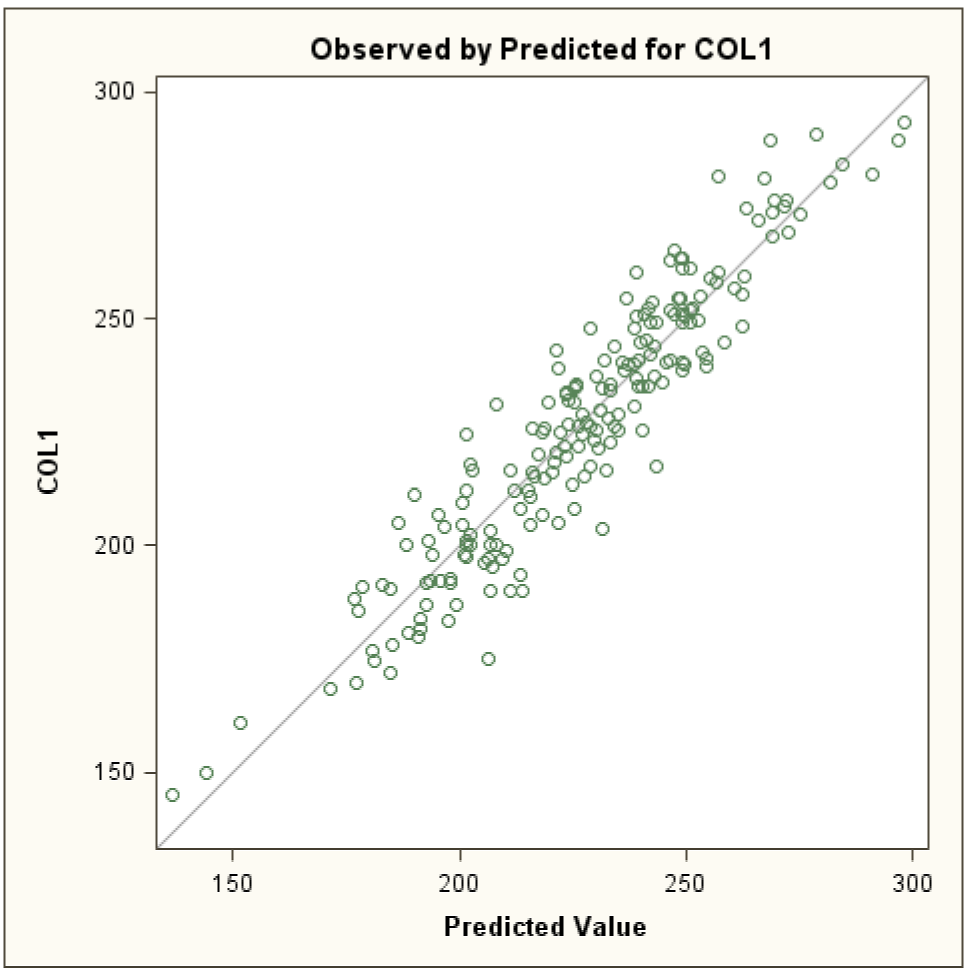
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	153692	19211	179.18	<.0001
Error	191	20479	107.22094		
Corrected Total	199	174171			

Root MSE	10.35475	R-Square	0.8824
Dependent Mean	227.04725	Adj R-Sq	0.8775
Coeff Var	4.56062		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	4.08374	6.26048	0.65	0.5150	0
COL2	1	5.24085	0.46140	11.36	<.0001	1.06556
COL3	1	4.79871	0.45351	10.58	<.0001	1.02544
COL4	1	6.65044	0.41997	15.84	<.0001	1.08134
COL5	1	3.60344	0.43753	8.24	<.0001	1.05326
COL6	1	4.57544	0.42313	10.81	<.0001	1.04787
COL7	1	6.08528	0.43715	13.92	<.0001	1.03066
COL8	1	7.25810	0.46005	15.78	<.0001	1.06127
COL9	1	7.96753	0.45840	17.38	<.0001	1.05259




Residual by Predicted for COL1
20
0
-20
Residual
150
200
250
300
Predicted Value

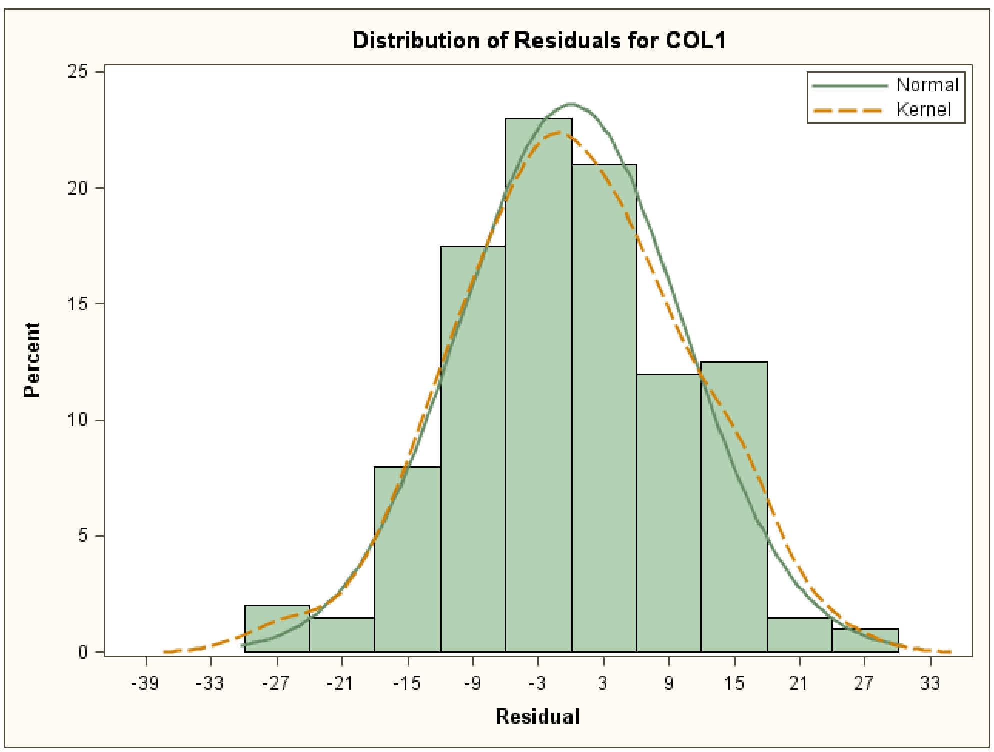

Observed by Predicted for COL1
300
250
200
150
COL1
150
200
250
300
Predicted Value

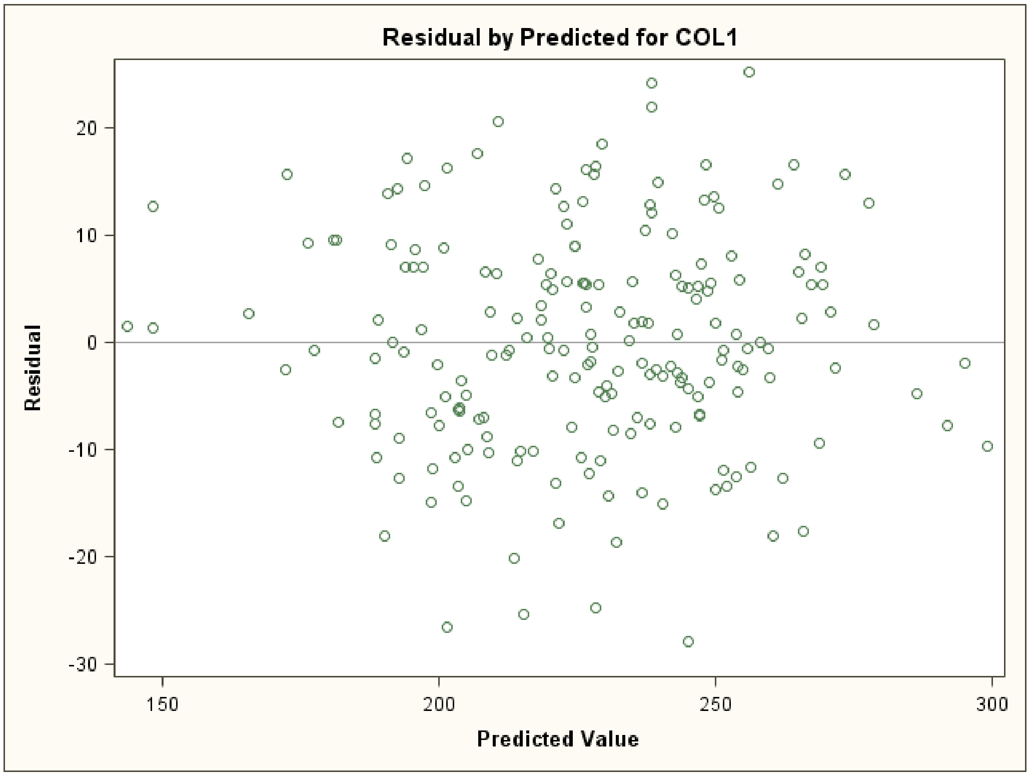
Model 3 – Stepwise

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	153676	21954	205.66	<.0001
Error	192	20495	106.74650		
Corrected Total	199	174171			

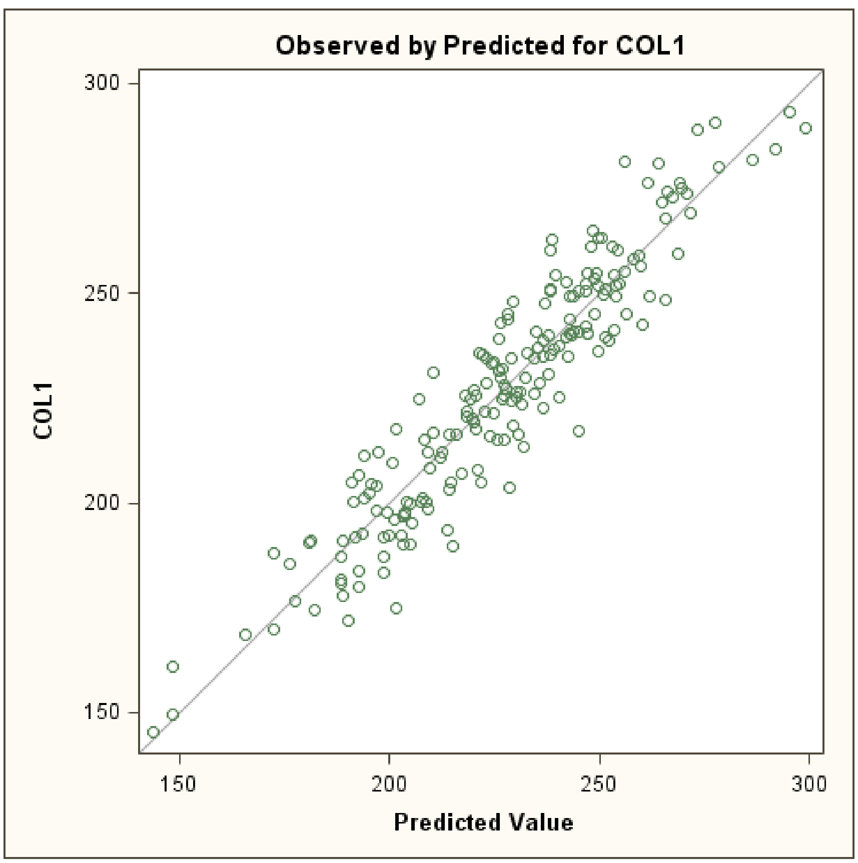
Root MSE	10.33182	R-Square	0.8823
Dependent Mean	227.04725	Adj R-Sq	0.8780
Coeff Var	4.55052		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	5.19190	6.22902	0.83	0.4056	0
COL2	1	-3.29178	0.58148	-5.66	<.0001	1.69988
COL5	1	3.73159	0.43698	8.54	<.0001	1.05532
COL6	1	4.88690	0.41914	11.66	<.0001	1.03276
COL7	1	5.97025	0.43561	13.71	<.0001	1.02796
COL8	1	7.05863	0.45679	15.45	<.0001	1.05090
COL9	1	7.98222	0.45577	17.51	<.0001	1.04517
COL10	1	8.62440	0.45761	18.85	<.0001	1.72803





Residual by Predicted for COL1
Residual
Predicted Value
20
10
0
-10
-20
-30
150
200
250
300



Observed by Predicted for COL1
COL1
Predicted Value
300
250
200
150
150
200
250
300

PROBLEM IV: Provide complete solutions to the following two problems:

Part 1. A study was carried out to examine the effect of injecting Botox on eye pain. Fifteen Patients received a high-dose injection in one eye (experimental treatment E) and a low-dose injection in the other eye (control treatment C). Patients were asked to rate the level of pain in each eye on a 1 to 10 scale, with higher values indicating more pain. Which eye received which treatment was randomized. The pain scores for the two eyes recorded on the last of several visits are given in the table:

Subject	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
E eye (x_1)	1.3	7.3	0	0	3	0	3.5	0	0	2.0	0	3.0	5.0	0.3	0
C eye (x_2)	8.8	1.3	0.8	9.5	7.8	9.0	5.0	2.3	2.5	8.0	4.5	4.5	9.0	7.5	0.5

Some possibly useful statistics follow:

$$\bar{x}_1 = 1.693,\ s_1 = 2.2569,\ \bar{x}_2 = 5.400,\ s_2 = 3.3058,\ d = x_2 - x_1,\ \bar{d} = 3.707,\ s_d = 3.9773$$

1. What is the design of the experiment? Discuss its appropriateness for this experiment.

2. Obtain a 95% confidence interval for the mean difference pain scores between the two eyes. Interpret your results.

3. Another way to look at these data is to consider the percentage of subjects who have less pain with the E eye versus the C eye. Carry out a test of whether the proportion of subjects who have less pain with the E eye differs from 0.5. Interpret your results.

Part 2. We wish to compare two different chemotherapy regimens for breast cancer after mastectomy. Patients are placed into pairs based on age and clinical condition. Then a random member of each pair receives treatment A and the other member treatment B. The patients are followed for five years and the number surviving is recorded. The data can be recorded into either of two tables:

Table A

Treatment	Outcome: Survive for 5 years	Outcome: Die within 5 years
A	526	95
B	515	106

Table B

Outcome for A patient	Outcome for B Patient: Survive for 5 years	Outcome for B Patient: Die within 5 years
Survive for 5 years	510	16
Die within 5 years	5	90

1. Discuss which table is more appropriate for presenting these data. Explain why.

2. Carry out a test for association between treatment and survival for 5 years. You may use the SAS output on the next pages in your solution.

The FREQ Procedure

Table of Survive by Treatment

Survive / Treatment

Frequency Percent	A	B	Total
yes	526 42.35	515 41.47	1041 83.82
no	95 7.65	106 8.53	201 16.18
Total	621 50.00	621 50.00	1242 100.00

Statistics for Table of Survive by Treatment

Statistic	DF	Value	Prob
Chi-Square	1	0.7182	0.3967
Likelihood Ratio Chi-Square	1	0.7185	0.3966
Continuity Adj. Chi-Square	1	0.5936	0.4410
Mantel-Haenszel Chi-Square	1	0.7176	0.3969
Phi Coefficient		0.0240	
Contingency Coefficient		0.0240	
Cramer's V		0.0240	

Fisher's Exact Test

Cell (1,1) Frequency (F)	526
Left-sided Pr <= F	0.8224
Right-sided Pr >= F	0.2205
Table Probability (P)	0.0429
Two-sided Pr <= P	0.4411

Estimates of the Relative Risk (Row1/Row2)

Type of Study	Value	95% Confidence Limits	
Case-Control (Odds Ratio)	1.1396	0.8422	1.5420
Cohort (Col1 Risk)	1.0691	0.9129	1.2520
Cohort (Col2 Risk)	0.9381	0.8118	1.0840

Sample Size = 1242

The FREQ Procedure

Table of SurviveA by SurviveB

SurviveA / SurviveB (Frequency, Percent)	yes	no	Total
yes	510 82.13	16 2.58	526 84.70
no	5 0.81	90 14.49	95 15.30
Total	515 82.93	106 17.07	621 100.00

Statistics for Table of SurviveA by SurviveB

McNemar's Test

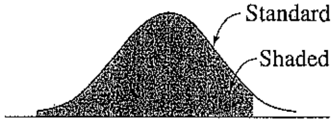
Statistic (S)	5.7619
DF	1
Pr > S	0.0164

Simple Kappa Coefficient

Kappa	0.8754
ASE	0.0266
95% Lower Conf Limit	0.8233
95% Upper Conf Limit	0.9275

Sample Size = 621

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0,0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0394	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0,0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$ (*cont.*)

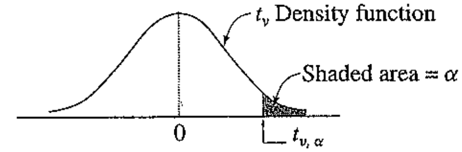
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0,4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1,7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table A.1 Cumulative Binomial Probabilities (*cont.*)

c. $n = 15$

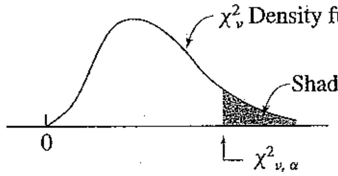
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.860	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.990	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
2	1.000	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000	.000
3	1.000	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000	.000
4	1.000	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000	.000
5	1.000	1.000	.998	.939	.852	.722	.403	.151	.034	.004	.001	.000	.000	.000	.000
6	1.000	1.000	1.000	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000	.000
7	1.000	1.000	1.000	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000	.000
8	1.000	1.000	1.000	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000	.000
9	1.000	1.000	1.000	1.000	.999	.996	.966	.849	.597	.278	.148	.061	.002	.000	.000
10	1.000	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.314	.164	.013	.001	.000
11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.539	.352	.056	.005	.000
12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.764	.602	.184	.036	.000
13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.920	.833	.451	.171	.010
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.987	.965	.794	.537	.140

Table A.4 Critical Values $t_{\nu,\alpha}$ for the t-Distribution



ν	α = .10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	1.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table A.5 Critical Values $\chi^2_{v,\alpha}$ for the Chi-square Distribution



	α									
v	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.420	65.473
40*	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

Table A.10 Upper-Tail Probabilities of the Null Distribution of the Wilcoxon Signed Rank Statistic

n	w	$P(W \geq w \mid H_0)$	n	w	$P(W \geq w \mid H_0)$
3	6	0.125		78	0.011
4	9	0.125		79	0.009
	10	0.062		81	0.005
5	13	0.094	14	73	0.108
	14	0.062		74	0.097
	15	0.031		79	0.052
6	17	0.109		84	0.025
	19	0.047		89	0.010
	20	0.031		92	0.005
	21	0.016	15	83	0.104
7	22	0.109		84	0.094
	24	0.055		89	0.053
	26	0.023		90	0.047
	28	0.008		95	0.024
8	28	0.098		100	0.011
	30	0.055		101	0.009
	32	0.027		104	0.005
	34	0.012	16	93	0.106
	35	0.008		94	0.096
	36	0.004		100	0.052
9	34	0.102		106	0.025
	37	0.049		112	0.011
	39	0.027		113	0.009
	42	0.010		116	0.005
	44	0.004	17	104	0.103
10	41	0.097		105	0.095
	44	0.053		112	0.049
	47	0.024		118	0.005
	50	0.010		125	0.010
	52	0.005		129	0.005
11	48	0.103	18	116	0.098
	52	0.051		124	0.049
	55	0.027		131	0.024
	59	0.009		138	0.010
	61	0.005		143	0.005
12	56	0.102	19	128	0.098
	60	0.055		136	0.052
	61	0.046		137	0.048
	64	0.026		144	0.025
	68	0.010		152	0.010
	71	0.005		157	0.005
13	64	0.108	20	140	0.101
	65	0.095		150	0.049
	69	0.055		158	0.024
	70	0.047		167	0.010
	74	0.024		172	0.005

Table A.11 Upper-Tail Probabilities of the Null Distribution of the Wilcoxon-Mann-Whitney Statistic

n_1	n_2	w_1	u_1	$P(W \geq w_1) = P(U \geq u_1)$	n_1	n_2	w_1	u_1	$P(W \geq w_1) = P(U \geq u_1)$	n_1	n_2	w_1	u_1	$P(W \geq w_1) = P(U \geq u_1)$
3	3	15	9	0.050	4	9	44	34	0.006	6	10	66	45	0.059
	4	17	11	0.057		10	42	32	0.053		10	69	48	0.028
	4	18	12	0.029		10	44	34	0.027		10	72	51	0.011
	5	20	14	0.036		10	46	36	0.012		10	74	53	0.005
	5	21	15	0.018		10	47	37	0.007	7	7	66	38	0.049
	6	22	16	0.048	5	5	36	21	0.048		7	68	40	0.027
	6	23	17	0.024		5	37	22	0.028		7	71	43	0.009
	6	24	18	0.012		5	39	24	0.008		7	72	44	0.006
	7	24	18	0.058		5	40	25	0.004		8	71	43	0.047
	7	26	20	0.017		6	40	25	0.041		8	73	45	0.027
	7	27	21	0.008		6	41	26	0.026		8	76	48	0.010
	8	27	21	0.042		6	43	28	0.009		8	78	50	0.005
	8	28	22	0.024		6	44	29	0.004		9	75	47	0.057
	8	29	23	0.012		7	43	28	0.053		9	78	50	0.027
	8	30	24	0.006		7	45	30	0.024		9	81	53	0.011
	9	29	23	0.050		7	47	32	0.009		9	83	55	0.006
	9	31	25	0.018		7	48	33	0.005		10	80	52	0.054
	9	32	26	0.009		8	47	32	0.047		10	83	55	0.028
	9	33	27	0.005		8	49	34	0.023		10	87	59	0.009
	10	31	25	0.056		8	51	36	0.009		10	89	61	0.005
	10	33	27	0.024		8	52	37	0.005	8	8	84	48	0.052
	10	34	28	0.007		9	50	35	0.056		8	87	51	0.025
	10	35	29	0.003		9	53	38	0.021		8	90	54	0.010
4	4	24	14	0.057		9	55	40	0.009		8	92	56	0.005
	4	25	15	0.029		9	56	41	0.006		9	89	53	0.057
	4	26	16	0.014		10	54	39	0.050		9	93	57	0.023
	5	27	17	0.056		10	56	41	0.027		9	96	60	0.010
	5	28	18	0.032		10	59	44	0.010		9	98	62	0.006
	5	29	19	0.016		10	60	45	0.006		10	95	59	0.051
	5	30	20	0.008	6	6	50	29	0.047		10	98	62	0.027
	6	30	20	0.057		6	52	31	0.021		10	102	66	0.010
	6	32	22	0.019		6	54	33	0.008		10	104	68	0.006
	6	33	23	0.010		6	55	34	0.004	9	9	104	59	0.057
	6	34	24	0.005		7	54	33	0.051		9	108	63	0.025
	7	33	23	0.055		7	56	35	0.026		9	111	66	0.012
	7	35	25	0.021		7	58	37	0.011		9	114	69	0.005
	7	36	26	0.012		7	60	39	0.004		10	110	65	0.056
	7	37	27	0.006		8	58	37	0.054		10	114	69	0.027
	8	36	26	0.055		8	61	40	0.021		10	118	73	0.011
	8	38	28	0.024		8	63	42	0.010		10	121	76	0.005
	8	40	30	0.008		8	65	44	0.004	10	10	127	72	0.053
	8	41	31	0.004		9	62	41	0.057		10	131	76	0.026
	9	39	29	0.053		9	65	44	0.025		10	135	80	0.012
	9	41	31	0.025		9	68	47	0.009		10	138	83	0.006
	9	43	33	0.010		9	70	49	0.004					