

Homework 09
Joseph Blubaugh
jblubau1@tamu.edu
STAT 608-720

1)

a)

$$\begin{aligned}\log\left(\frac{\theta(x)}{1-\theta(x)}\right) &= \beta_0 + \beta_1 x \\ \left(\frac{\theta(x)}{1-\theta(x)}\right) &= \exp(\beta_0 + \beta_1 x) \\ \theta(x) &= \frac{\exp(\beta_0 + \beta_1 x)}{1 + \exp(\beta_0 + \beta_1 x)}\end{aligned}$$

b)

$$\begin{aligned}\theta(x) &= \frac{\exp(\beta_0 + \beta_1 x)}{1 + \exp(\beta_0 + \beta_1 x)} \\ \theta(x) &= \frac{1}{1 + \exp(-(\beta_0 + \beta_1 x))}\end{aligned}$$

2) We might be interested in transforming the predictor variables in a logistic regression in order to get a linear relationship between the predictor variables and $\log(\text{odds})$. Using \log on the predictor variables would enable the interpretation of percent changes.

3)

a) & b)

$$Z = \text{Bin}(n, p) = P(Z = z) = \binom{n}{z} p^z (1-p)^{n-z}$$

$$n = 1 : P(Z = z) = p^z (1-p)^{n-z}$$

$$z = 0 : P(Z = 0) = (1-p)$$

$$z = 1 : P(Z = 1) = p(1-p)^{1-z}$$

$$\begin{aligned}\log\left(\frac{\theta(x)}{1-\theta(x)}\right) &= \log\left(\frac{P(Y=1)}{P(Y=0)}\right) + \log\left(\frac{P(X=x|Y=1)}{P(X=x|Y=0)}\right) \\ &= \log\left(\frac{p_1}{p_0}\right) + \log\left(\frac{p_1(1-p_1)^{1-z}}{(1-p_0)}\right) \\ &= \beta_0 + \beta_1 x\end{aligned}$$