MASTER'S DIAGNOSTIC EXAMINATION - JANUARY 8, 2015

Student's Name _____

INSTRUCTIONS FOR STUDENTS:

1. The exam is to be started at Noon (CST) and completed by 4 pm (CST) on January 8, 2015.
2. Put your name above but DO NOT put your NAME on the **SOLUTIONS** to the exam.
3. Place the NUMBER assigned to you on the

   UPPER RIGHT HAND CORNER of EACH PAGE of your SOLUTIONS.
4. Please start your answer to EACH QUESTION on a SEPARATE sheet of paper.
5. Use only one side of each sheet of paper.
6. You must answer all four questions: Questions I, II, III and IV.
7. Be sure to attempt all parts of the four questions. It may be possible to answer a later part of a question without having solved the earlier parts.
8. Be sure to hand in all of your exam. No additional material will be accepted once the exam has ended and you have left the exam room.
9. You may use the following:
   - Calculator which does not have capability to phone, text, or access the Web
   - Pencil or pen
   - Blank paper for the solutions for this examination
   - No other materials are allowed

- I attest that I spent no more than 4 hours to complete the exam.
- I used only the materials described above.
- I did not receive assistance from anyone during the taking of this exam.

Student's Signature_____

INSTRUCTIONS FOR PROCTOR:

Immediately after the student completes the exam, **fax** the student's solutions to **979-845-6060** or

Scan the solutions into a **single** pdf file and **email to longneck@stat.tamu.edu**

Do not send the questions, just send the student's solutions.

(1) I certify that the time at which the student started the exam was _____

and the time at which the student completed the exam was _____

(2) I certify that the student has followed all the **INSTRUCTIONS FOR STUDENTS** listed above.

(3) I certify that the student's solutions were faxed to **979-845-6060** or

emailed to **longneck@stat.tamu.edu**.

Proctor's Signature_____

QUESTION I.

A company is interested in the ability of a machine to consistently place electrical wire on a coil. There are three types of machines available: hand operated(HO), partially computer operated(PCO), and completely automated(CA). Three machines of each type are randomly selected from their suppliers for use in the study. The wire placed on the coils comes in one of three thicknesses: .02mm, .04mm, or .06mm. Each of the machines assembles two coils of each of the three wire thicknesses. Each wound coil is then measured for the uniformity of windings at a middle position on the coil. These measurements are given in the following table.

		TYPE OF MACHINE									
		HO			PCO			CA			
MACHINE ID		1	2	3	4	5	6	7	8	9	THICK MEANS
THICKNESS	.02mm	12.30	13.46	12.35	13.01	13.46	13.15	5.47	5.75	6.24	
		12.59	14.00	12.06	12.63	13.92	13.20	5.96	5.68	6.15	10.63
			(12.79)			(13.23)			(5.87)		
	.04mm	13.16	13.29	12.50	12.74	13.84	13.46	5.73	5.60	5.92	
		13.00	13.62	12.39	12.68	13.75	13.57	5.64	5.65	5.64	10.68
			(12.99)			(13.34)			(5.70)		
	.06mm	12.87	13.46	12.73	12.47	13.62	13.36	5.01	5.80	6.19	
		12.92	13.82	12.15	12.15	13.28	13.42	5.62	5.71	6.23	10.60
			(12.99)			(13.05)			(5.76)		
TYPE MEANS			12.93			13.21			5.78		10.64

A partial ANOVA table for the experiment is given below.

SOURCE	DF	MS	EMS
THICKNESS		0.0263	
TYPE		319.1202	
THICKNESS*TYPE		0.1152	
MACHINE(TYPE)		1.4935	
THICKNESS*MACHINE(TYPE)		0.0878	
ERROR		0.0445	

1. Complete the ANOVA table by determining the values of Degrees of Freedom and Expected Mean Squares for each of the sources of variation.

   Use the information in the ANOVA table to answer the questions on the next page. Use $\alpha = .05$ in reaching your answers to Questions 3 and 4.

2. Write a linear model for the above experiment. Make sure to identify the terms in your model with respect to distributional properties or restrictions on population parameters.

3. Is the differences in the mean uniformity of the windings produced by the three types of Machines consistent across the three thicknesses?

4. Is there significant difference in the mean uniformity of the windings produced by the three types of Machines?

5. Using the numeric values of the MS's given above and your EMS's, provide the following information:

    a. Proportionally allocate the variance of the uniformity in windings of a randomly selected coil to the various variance components.

    b. An estimate of the standard error of the the estimated mean uniformity of windings from a CA winding machine.

    c. An estimate of the standard error of the estimated difference in the mean uniformity of windings between type HO and CA winding machines.

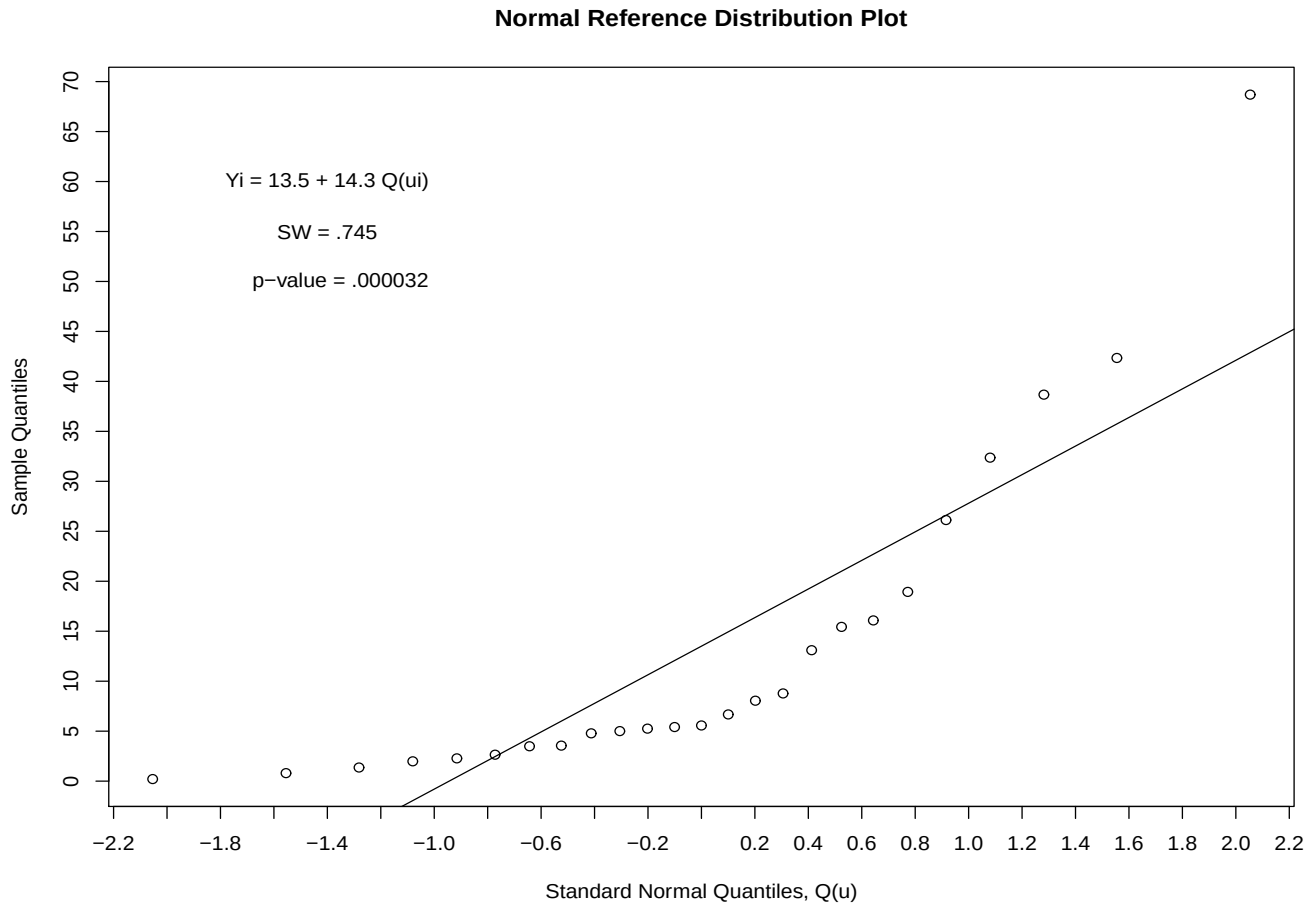
    d. An estimate of the standard error of the estimated difference in the mean uniformity of windings of coils using wire of thickness .02mm and .06mm assembled using a CA winding machine.

QUESTION II.

Twenty five mice are exposed to a high level of radiation and then provided with treatment. The researchers are interested in the survival in days of these mice. The survival times in days for the random sample of 25 mice are given below along with summary statistics and a normal reference plot. The value of the Shapiro-Wilk test was .745 with a p-value of .000032.

0.2	0.8	1.4	2.0	2.3	2.6	3.5	3.6	4.8	5.0	5.3	5.4	5.6
6.7	8.0	8.8	13.1	15.4	16.1	18.9	26.1	32.4	38.7	42.3	68.7	

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	St.Dev.	MAD
.2	3.49	5.57	13.50	16.80	69	16.55	5.32



Use the above information to answer the questions on the next page:

1. Provide a 95% confidence interval for the mean survival of all mice subjected to the treatment using the assumption that the pivot

$$t = \frac{\sqrt{n}\ (\overline{X} - \mu)}{S}$$

has a t-distribution. A t-table is provided with the exam.

2. The researchers feel the interval in part 1. is too imprecise, approximately how many mice would they need in a new experiment to obtain a 95% confidence interval for the mean having a width of 4 days?

3. Explain why the confidence interval derived in part 1. may not be an appropriate confidence interval for the mean.

4. Suppose the distribution of survival times is gamma, that is, has pdf

$$f(x) = \frac{1}{\Gamma(\alpha)}\beta^{-\alpha}x^{\alpha-1}exp(-x/\beta) \quad \text{for some pair } (\alpha,\ \beta)$$

Explain how to use a simulation to determine the sampling distribution of the pivot $t = \sqrt{n}\ (\overline{X} - \mu)\ /\ S$. In particular, explain precisely how to determine t_α, the αth percentile of the distribution of t for any $\alpha \in (0, 1)$?

5. Suppose the simulation yielded $t_{.025} = -3.75$, $t_{.05} = -2.05$, $t_{.95} = 1.45$ and $t_{.975} = 1.75$. Based on these percentiles, display a 95% confidence interval for mean survival?

6. The treatment used previously for the applied level of radiation had a median survival time of 3 days. Is there significant ($\alpha = .05$) evidence that the new treatment has increased the survival time in comparison to the previous treatment? What is the p-value of your test statistic?

QUESTION III.

Consider a regression model for an experiment with a continuous response Y, a treatment factor, A, with three levels, A_1, A_2, and A_3, and a continuous explanatory variable, X. Define the dummy variables,

$D_1 = 1$ if $A = A_1$ and $D_1 = 0$, otherwise,

$D_2 = 1$ if $A = A_2$ and $D_2 = 0$, otherwise.

Consider the regression model

$$E(Y) = \beta_0 + \beta_1 D_1 + \beta_2 D_2 + \beta_3 X + \beta_4 D_1 X + \beta_5 D_2 X$$

1. Obtain expressions for the mean response for each of the three treatments:
   a. A_1
   b. A_2
   c. A_3

2. Write out interpretations in terms of the mean response and its relationship to the treatments and explanatory variable for each of the parameters:
   a. β_0
   b. β_1
   c. β_2
   d. β_3
   e. β_4
   f. β_5

3. Formulate the hypotheses for a test of equal slopes for the three treatments. Explain how to carry out the test if you were provided statistical software that could fit a regression model and provide the usual analysis of variance table for the fitted regression model.

4. Does testing the hypothesis $H_0 : \beta_1 = \beta_2 = 0$ provide a test of equal effects for the three treatments in the above model? Justify your answer.