STATISTICS 641 - ASSIGNMENT 2

DUE DATE: Noon (CDT), Friday, February 06, 2015

Name ____

Email Address ____

Please TYPE your name and email address. Often we have difficulty in reading the handwritten names and email addresses. Make this cover sheet the first page of your Solutions.

STATISTICS 641 - ASSIGNMENT #2 - Due Noon, Friday - 02/06/2015

- Read Handout 3
- Read Chapter 2 in the Textbook
- Hand in the following Problems:

(1.) (10 points) Assume that the random variable Y has pmf with parameter p, $0 < p < 1$:

$$f(y) = \begin{cases} p(1-p)^y & \text{for } \ y = 0, 1, 2, 3, \ldots \\ \\ 0 & \text{otherwise} \end{cases}$$

(a.) Find the cdf, $F(y)$ for Y

(b.) Find the quantile function, $Q(u)$ for Y

(2.) (20 points) Let Y have a 3-parameter Weibull distribution, that is, Y has pdf and cdf in the following form with $\alpha > 0$, $\gamma > 0$, $\theta > 0$:

$$f(y) = \begin{cases} \frac{\gamma}{\alpha^\gamma}(y-\theta)^{\gamma-1} e^{-\left(\frac{y-\theta}{\alpha}\right)^\gamma} & \text{for} \quad y \geq \theta \\ \\ 0 & \text{for} \quad y < \theta \end{cases} \qquad F(y) = \begin{cases} 1 - e^{-\left(\frac{y-\theta}{\alpha}\right)^\gamma} & \text{for} \quad y \geq \theta \\ \\ 0 & \text{for} \quad y < \theta \end{cases}$$

(a.) Verify that the pair $(\theta,\ \alpha)$ are location-scale parameters for this family of distributions.

(b.) Derive the quantile function for the three parameter Weibull family of distributions.

(c.) What is the probability that a random selected value from a Weibull distribution with $\theta = 10$, $\gamma = 2$ and $\alpha = 25$ has value greater than 30?

(d.) Compute the 40th percentile from a Weibull distribution with with $\theta = 10$, $\gamma = 2$ and $\alpha = 25$.

(3.) (10 points) An alternative form of the 2-parameter Weibull distribution is given as follows with $\beta > 0$, $\gamma > 0$

$$f(y) = \begin{cases} \frac{\gamma}{\beta} y^{\gamma-1} e^{-y^\gamma/\beta} & \text{for} \quad y \geq 0 \\ \\ 0 & \text{for} \quad y < 0 \end{cases} \qquad F(y) = \begin{cases} 1 - e^{-y^\gamma/\beta} & \text{for} \quad y \geq 0 \\ \\ 0 & \text{for} \quad y < 0 \end{cases}$$

(a.) Show that β is not a scale parameter for this family of distributions?

(b.) Suggest a function of γ and β which would be a scale parameter for this family of distributions.

(4.) (10 points) An experiment measures the number of particle emissions from a radioactive substance. The number of emissions has a Poisson distribution with rate $\lambda = .25$ particles per week.

(a.) What is the probability of at least 1 emission occurring in a randomly selected week?

(b.) What is the probability of at least 1 emission occurring in a randomly selected year?

(5.) (10 points) Let $Z_1,\ Z_2,\ Z_3,\ Z_4,\ Z_5,\ Z_6,\ Z_7,\ Z_8$ be independent N(0,1) r.v.'s. Identify the distributions of the following random variables.

(a.) $R = Z_1^2 + Z_2^2 + Z_5^2 + Z_6^2$.

(b.) $W = Z_7/\sqrt{[Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2 + Z_6^2]/6}$.

(c.) $Y = 7Z_2^2/[Z_1^2 + Z_3^2 + Z_4^2 + Z_5^2 + Z_6^2 + Z_7^2 + Z_8^2]$.

(d.) $T = Z_1/Z_4$.

(e.) $S = 3(Z_2^2 + Z_4^2)/[2(Z_1^2 + Z_3^2 + Z_5^2)]$.

(6.) (10 points) Let $U = .38$ be a realization from a Uniform on (0,1) distribution.

Express a single realization from each of the following random variables using just the fact $U = .38$.

(a.) W = Weibull(γ=4, α=1.5)

(b.) N = NegBin(r=8, p=.7)

(c.) B = Bin(20, .4)

(d.) P = Poisson(λ=3)

(e.) U = Uniform on (0.3, 2.5)

(7.) (30 points) For each of the following situations described below, select the distribution which best models the given situation. Provide a short justification for your answer.

Hypergeometric	Equally Likely	Poisson	Binomial
Geometric	Negative Binomial	Normal	Uniform
Gamma	Exponential	Chi-square	Lognormal
Cauchy	Double Exponential	Weibull	F
t	Logistic	Beta	

(a.) A wildlife biologist is studying if there is a difference between ducks in Texas and Michigan. She measures the wing span of each duck and then computes the difference between this wing span and a standard value for a large population of ducks. These differences are known to have a standard normal distribution. A sample of 100 ducks from Texas yield deviations, $T_1, \ldots, T_{100}$. The total squared deviation from the standard value, i.e., $TD = \sum_{n=1}^{100} T_i^2$, is then computed. A similar statistic is computed for Michigan: $MD = \sum_{n=1}^{100} M_i^2$. She now wants to compare the ratio $R = \frac{TD}{MD}$ to 1.0. The distribution of R is ___?

(b.) The Geoscience Department at Stanford monitors the occurrences of earthquakes in the Northern Region of California. One of the variables of interest to the researchers is the length of time T between the occurrence of major earthquakes. The distribution of T is _____?

(c.) A quality control engineer measures the difference D between the nominal diameter of a 5 cm ball bearing and the true bearing diameter. He finds that the bearings are equally likely to have a diameter larger than or smaller than 5 cm. Furthermore, 10% of the bearings have diameters which deviate more than 6 times their scale parameter from 5 cm. The distribution of D is _____?

(d.) In the development of a new treatment for kidney disease in domestic cats, 100 cats with kidney problems are placed on the new treatment. The time T until the cat no longer has kidney disease is recorded for each of the 100 cats. A plot of the hazard rate function yields $h(t) = 3.5t^{.8}$. The distribution of T is _____?

(e.) A manufacturer of computer hard drives ships the drives in boxes containing 30 drives. A box of hard drives is inspected by randomly selecting 6 hard drives from each box and testing the 6 drives for defectiveness. Let D be the number of defective hard drives found in a randomly selected box containing 30 hard drives. The distribution of D is _____?

(f.) For each day during a six month period in Stamford, Connecticut, the maximum daily ozone reading R was recorded. The distribution of R is _____?

(g.) A new type of transistor is in development. Using the data from an accelerated life test of the transistor, the failure rate function is found to be approximately a cubic function. Let T be the time to failure of the transistor. The distribution of T is _____?

(h.) In proof testing of circuit boards, the probability that any particular diode will fail is known to be .001. Suppose a particular type of circuit board contains 200 diodes. Circuit boards are tested and the number N of failed diodes are recorded for each circuit board. The distribution of N is _____?

(i.) A manufacture of spark plugs ships the plugs in packages of 100 plugs. A package is inspected by randomly selecting 5 plugs and testing whether or not the plugs are defective. Let N be the number of defective plugs in the sample of 5 plugs. The distribution of N is _____?

(j.) The distribution of resistance for resistors having a nominal value of 10 ohms is under investigation. An electrical engineer randomly selects 73 resistors and measures their resistance. Based on these 73 values, she determines that the resistance R of the resistors has the following behavior: approximately 70% of resistors have resistance within one standard deviation of 10 ohms, 95% are within two standard deviations, and none of the resistors have resistance greater than three standard deviations from 10 ohms. The distribution of R is _____?

(k.) A veterinarian is trying to recruit people to place their dogs in a study of the effectiveness of a new drug to control ticks on dogs. He needs 50 dogs in order for the study to meet professional standards of significance. Let M be the number of people the veterinarian interviews until he obtains the required 50 dogs for the study. The distribution of M is _____?

(l.) The wings on an airplane are subject to stresses which cause cracks in the surface of the wing. After 1000 hours of flight the wing is inspected with an x-ray machine and the number of cracks N are recorded. The distribution of N is _____?

(m.) Suppose small aircraft arrive at a certain airport according to a Poisson process with rate 8 aircraft per hour. For the next 100 days, the length of time, T, until the 15th aircraft arrives each day is recorded. The distribution of T is _____?

(n.) A manufacturer of piston rings measures the deviation of the true diameter from the nominal value. This measurement is known to have a standard normal distribution. A sample of 10 rings yield deviations, $X_1, \ldots, X_{10}$. The total squared deviation from the nominal value, i.e., W = $\sum_{n=1}^{10} X_i^2$, is then computed. The distribution of W is _____?

(o.) A large corporation has thousands of small suppliers of its raw materials. Let D be the proportion of parts in a randomly selected shipment that are defective. The vast majority of suppliers have small values of D but a few suppliers have large values of D. A possible distribution for D is ____?