STATISTICS
TEXAS A&M UNIVERSITY

Handout 7

Multi Factor Designs and Blocking

Split Plot Design

Split Plot Design

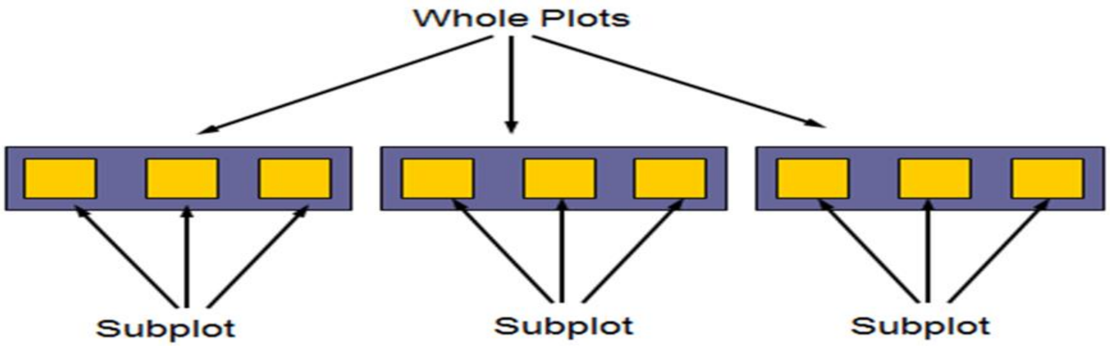
<u>Objectives</u>

– Define a split-plot design.
– Define random and fixed effects.
– Generate and analyze a split-plot design

<u>Properties:</u>

– Split-plot designs are used when factors are impractical, inconvenient, or costly to change.
– The hard-to-change factors are assigned to whole plots and easy-to-change factors are assigned to subplots.
– Randomization is restricted in split-plot designs.
– The estimates for subplot treatments are more precise than the estimates for whole plot treatments.

A Split-Plot Type of Design



- There are two factors: A and B.
- Factor A is applied to the large experimental units (whole unit).
- The large experimental unit (whole unit) is divided into smaller experimental units (sub-units).
- Factor B is applied to the smaller experimental units (sub-units).
- Each whole unit is a complete replicate of all the levels of factor B.
- The whole unit design may be a CRD, RCBD, or LS design.

Split-Plot Designs

Advantages: A split-plot design

– provides greater power for testing the sub-unit treatment factor and the interaction

– allows for different sized experimental units in the same experiment

– allows for including a second factor at very little cost

– involves repeated measures on the same experimental unit (whole unit) in the design.

Disadvantages:

– Analysis is complicated by the presence of two experimental error variances, which leads to several different standard errors for comparisons.

– High variance and few replications of whole units frequently lead to poor sensitivity on the whole-unit factor.

Split Plot Designs versus Blocking Factors

Blocking factors are used in designed experiments to account for variation in the response that might be wrongly attributed to other factors or to experimental error.

The whole plot factor in a split-plot design is of interest to the experimenter but the levels of the whole plot factor are hard-to-change from run to run if a completely randomized design is used.

Blocks are almost always treated as a random effect in a mixed model while the hard-to-change factors in a split-plot design are treated as fixed effects.

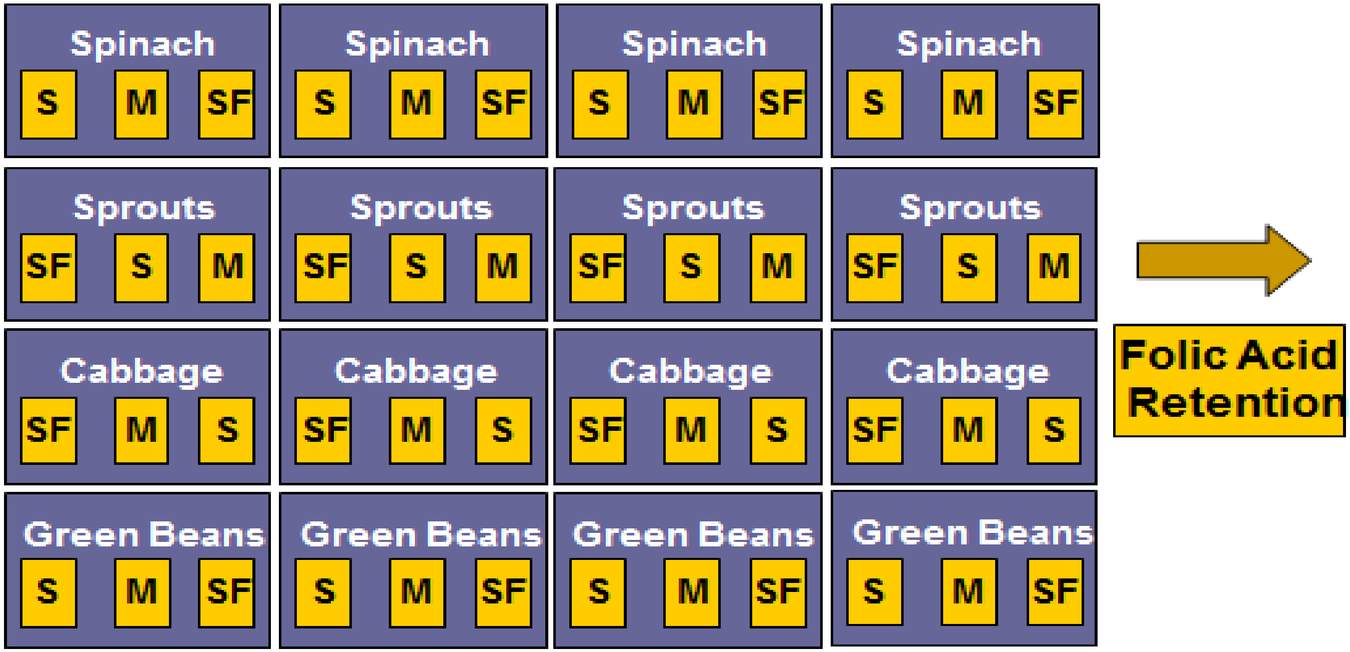
Folic Acid Retention Experiment

An experiment is being conducted to examine the effect of different cooking methods (Microwave, Stir Fry, Steaming) on the folic acid retention of four different vegetables (Spinach, Sprouts, Cabbage and Green Beans).

During the experiment a batch of vegetables, Spinach for example, is prepared and divided into three portions. One portion is randomly assigned each of the three cooking methods. After cooking, the Folic Acid Retention is measured for each portion.

The treatments are replicated 3 times. There are a total of 16 batches; 4 batches of each vegetable. There are 48 portions.

Folic Acid Retention Experiment



Model: $y_{ijk}=\mu+\alpha_i+\omega_{ij}+\beta_k+(\alpha\,\beta)_{ik}+\varepsilon_{ijk}$,
i=cabbage,greenbeans,spinach,sprouts, j=1,2,3,4,k=M,S,SF,

Y: folic Acid mcg, μ:intercept, α:vegetables effect,
ω :whole plot error, β :cooking method effect,
$(\alpha\,\beta)$: interaction effect, ε:subplot error

Fixed and Random Effects

Fixed Effects

- Factors and levels are chosen by experimenter.
- Inferences apply only to the factor levels in the experiment.

Random Effects

- Levels are randomly selected from a larger population of possible levels.
- Inferences extend to all population levels.

In a split-plot design, the whole plots are chosen at random from a large population of plots.

Match the term on the left with the appropriate description on the right

1. Fixed effect	A. receives the easy-to-change factor
2. Random effect	B. extends inferences to all population levels
3. Whole plot	C. receives the hard-to-change factor
4. Subplot	D. extends inferences only to the levels in the experiment

Generate this Design in JMP

Create a split-plot design with one hard to change factor and one easy to change factor.

Select DOE – Custom Design

Under responses, change Y to Folic Acid Retention and leave the goal as Maximize

Under Factors,

Select Add Factor-Categorical-4 level

Change X1 to vegetable and type the levels Spinach, Sprouts, Cabbage, Green Beans

To make this whole plot factor, select easy to Hard

Select Add Factor-Categorical-3 level

Change X2 to cooking method and type the levels Microwave, stir Fry, steaming

To make this sub plot factor, leave easy as it is.

Select Continue

Under Factors, select vegetable. Under model, select Cooking method. Select Cross to see vegetable, cooking method, vegetable*cooking method

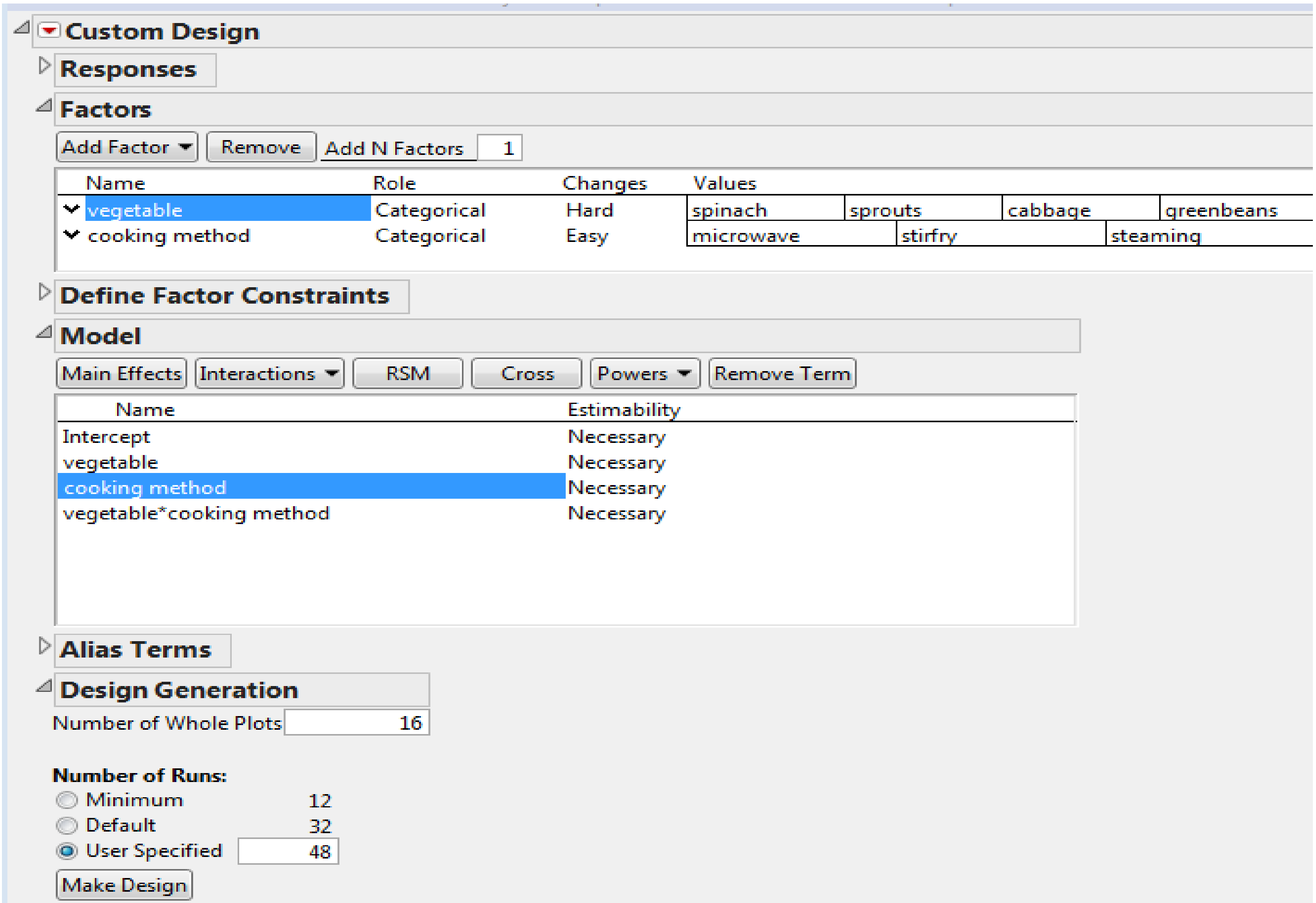
Type 16 for the number of whole plots

Select user specified and type 48

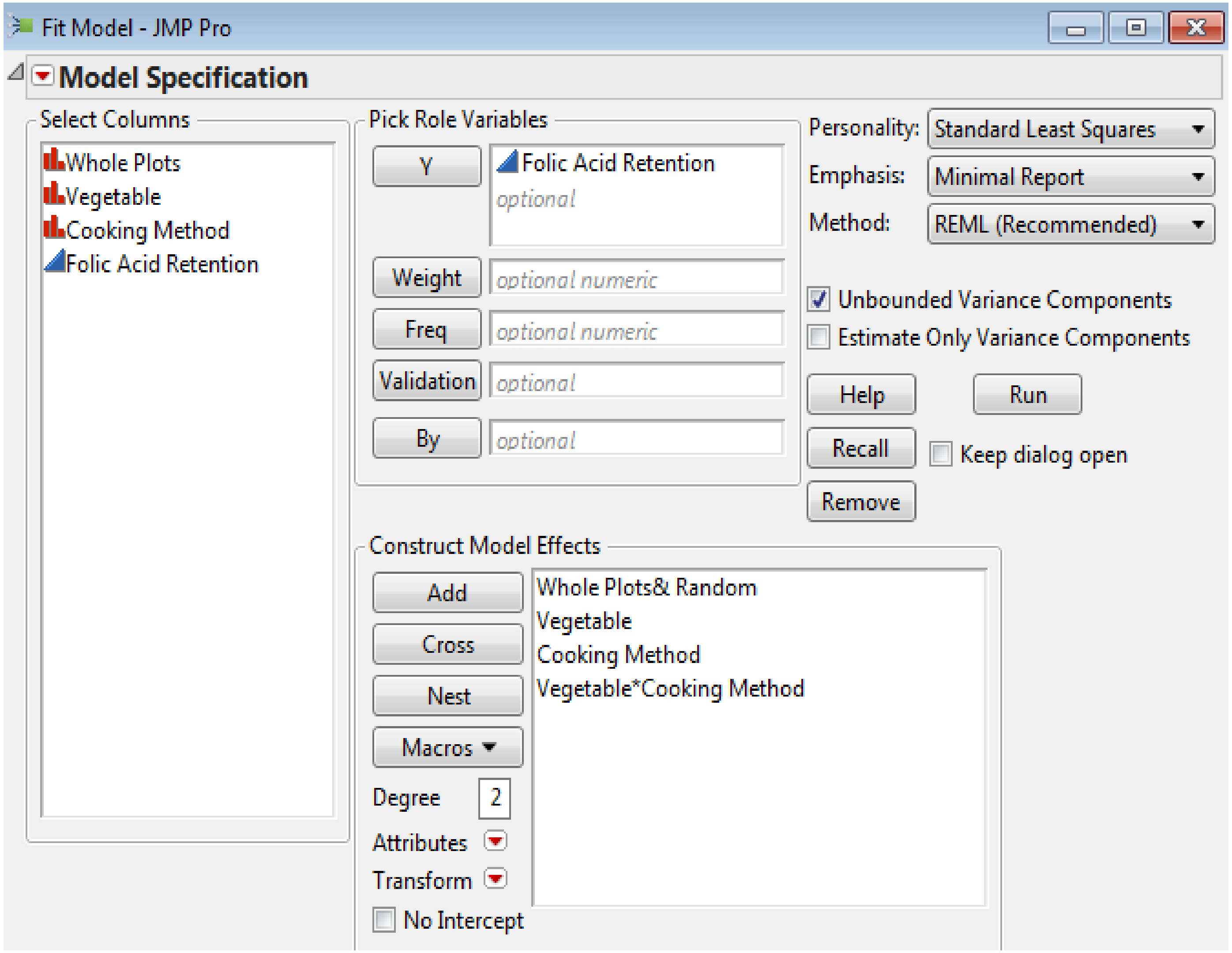
Select Make Design

STATISTICS
TEXAS A&M UNIVERSITY

Analyze vegetables.JMP



Analyze vegetables.JMP



Analyze vegetables.JMP

Vegetables - Fit Least Squares - JMP Pro

Response Folic Acid Retention

Summary of Fit

RSquare	0.979877
RSquare Adj	0.973728
Root Mean Square Error	2.928585
Mean of Response	43.66958
Observations (or Sum Wgts)	48

Parameter Estimates

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Whole Plots	2.3579983	20.223628	9.4594595	9.6133381	66.591533	70.220
Residual		8.5766083	2.4758536	5.2290976	16.598347	29.780
Total		28.800236	9.566852	16.4709	62.780267	100.000

-2 LogLikelihood = 238.33493915

Note: Total is the sum of the positive variance components.

Total including negative estimates = 28.800236

Covariance Matrix of Variance Component Estimates

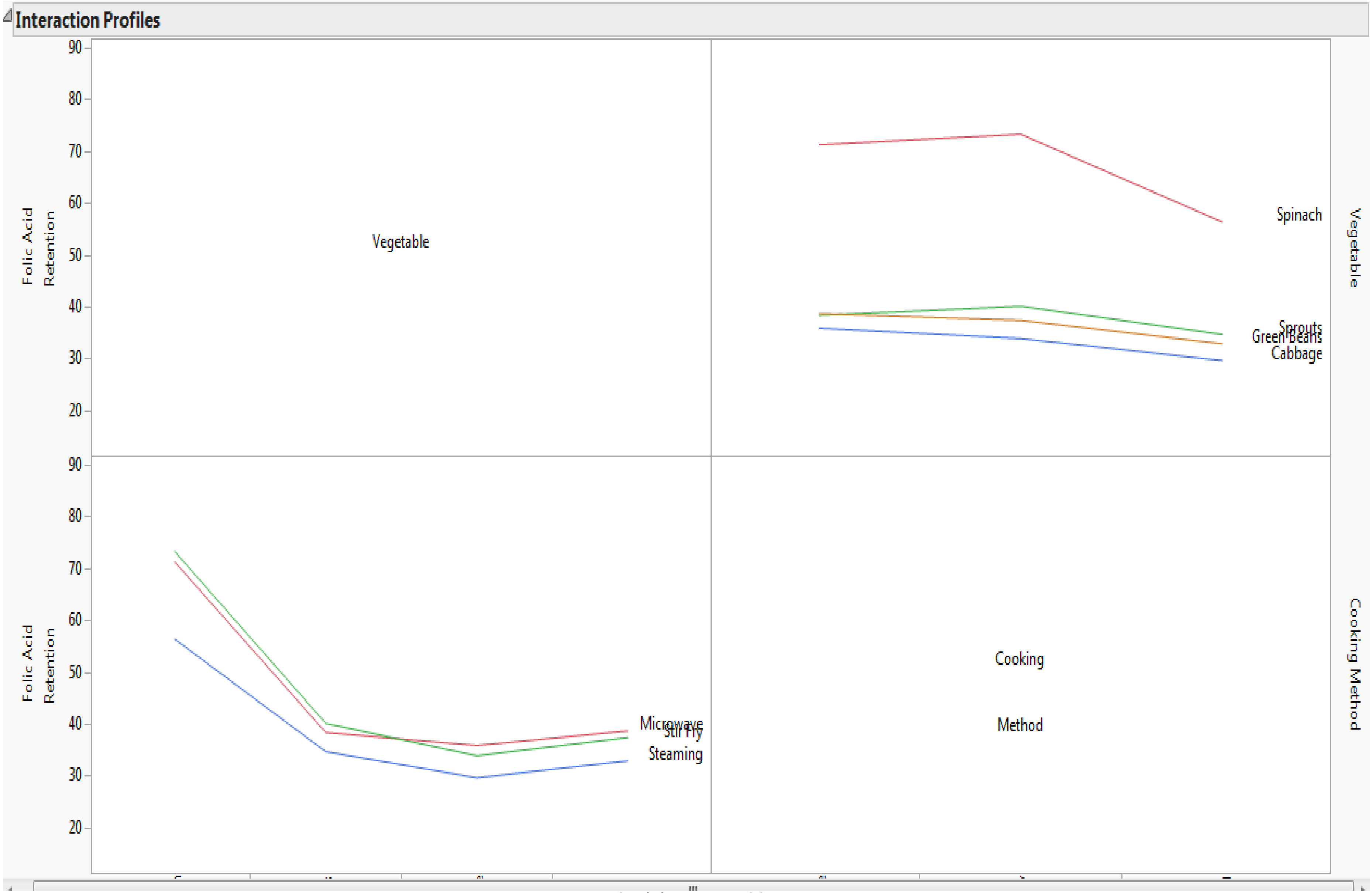
Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Vegetable	3	3	12	42.9685	<.0001*
Cooking Method	2	2	24	36.9850	<.0001*
Vegetable*Cooking Method	6	6	24	5.1469	0.0016*

Effect Details

Analyze vegetables.JMP



Yield of soybean Experiment

Soybeans are an important crop throughout the world. They are planted for both their use as an oil and as a source for protein. The vast majority of the crop is extracted for vegetable oil or as defatted soy meal, which is then used for feed for various farm animals. To a much lesser extent soybeans are consumed directly as food by humans. However, soybean products are an ingredient in a wide variety of processed foods.

A study was designed to determine if additional phosphorus applied to the soil would increase the yield of soybean. There are three major varieties of soybeans of interest (V1, V2, V3) and four levels of phosphorus (0, 30, 60, 120 pounds per acre).

The researchers have nine plots of land available for the study which are grouped into blocks of three plots each based on the soil characteristics of the plots. Because of the complexities of planting the soybeans on plots of the given size, it was decided to plant a single variety of soybeans on each plot and then divide each plot into four subplots.

The researchers randomly assigned a variety to one plot within each block of three plots and then randomly assigned the levels of phosphorus to the four subplots within each plot. The yields (bushels/acre) from the 36 plots are recorded.

Yield of soybean Experiment(variety.jmp)

	Block								
	B1			B2			B3		
Phosphorus	V_1	V_2	V_3	V_1	V_2	V_3	V_1	V_2	V_3
0	53.5	44.8	50.7	62.2	52.5	61.4	53.4	43.1	50.6
30	60.6	51.0	54.9	68.8	58.7	64.9	59.5	49.6	54.8
60	60.8	51.5	59.4	70.9	59.4	70.0	61.0	49.7	60.5
120	59.6	49.9	64.7	67.8	58.1	74.4	60.3	49.5	65.0

Model: $y_{ijk}=\mu+\alpha_i+b_j+\omega_{ij}+\beta_k+(\alpha\beta)_{ik}+\varepsilon_{ijk}$, i=V1,V2,V3, j=B1,B2,B3, k=0,30,60,120

Y: yield, μ: intercept, α: variety effect, b: block effect
ω : whole plot error (variety*block), β : phosphorus effect,
$(\alpha\beta)$: interaction effect, ε:subplot error

STATISTICS
TEXAS A&M UNIVERSITY

Generate this Design in JMP

Create a split-plot design with one hard to change factor, block and one easy to change factor.

Select DOE – Custom Design

Under responses, change Y to Yield of soybean and leave the goal as Maximize

Under Factors,

- Select Add Factor-Categorical-3 level
  - Change X1 to variety and type the levels V1, V2, V3
  - To make this whole plot factor, select easy to Hard
- Select Add Factor –Categorical-3 level
  - Change X2 to block and type the levels 1, 2, 3
  - To make this whole plot factor, select easy to Hard
- Select Add Factor-Categorical-4 level
  - Change X2 to phosphorus and type the levels 0, 30, 60, 120
  - To make this sub plot factor, leave easy as it is.

Select Continue

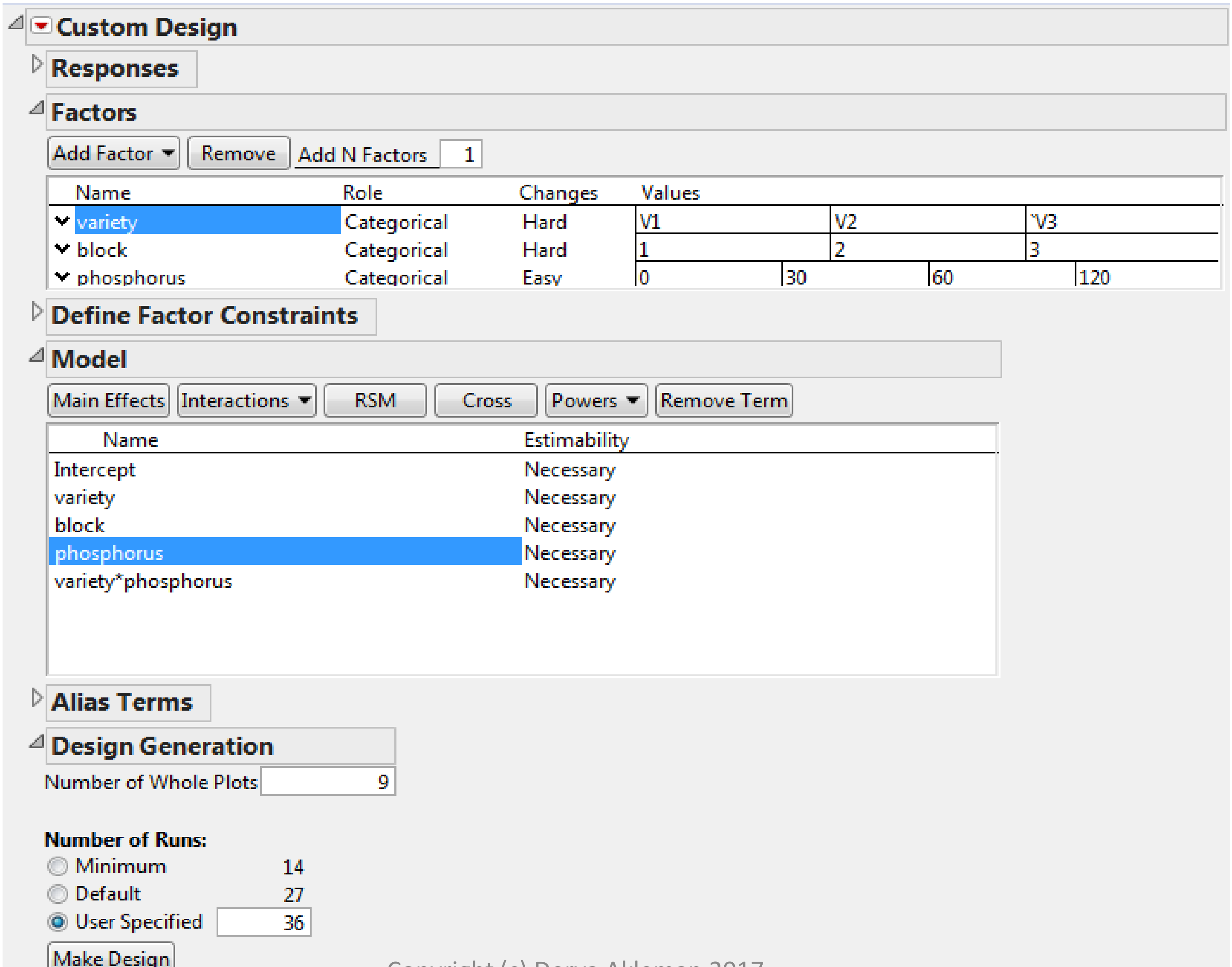
Under Factors, select variety. Under model, select phosphorus. Select Cross to see variety, phosphorus, variety*phosphorus

Type 9 for the number of whole plots

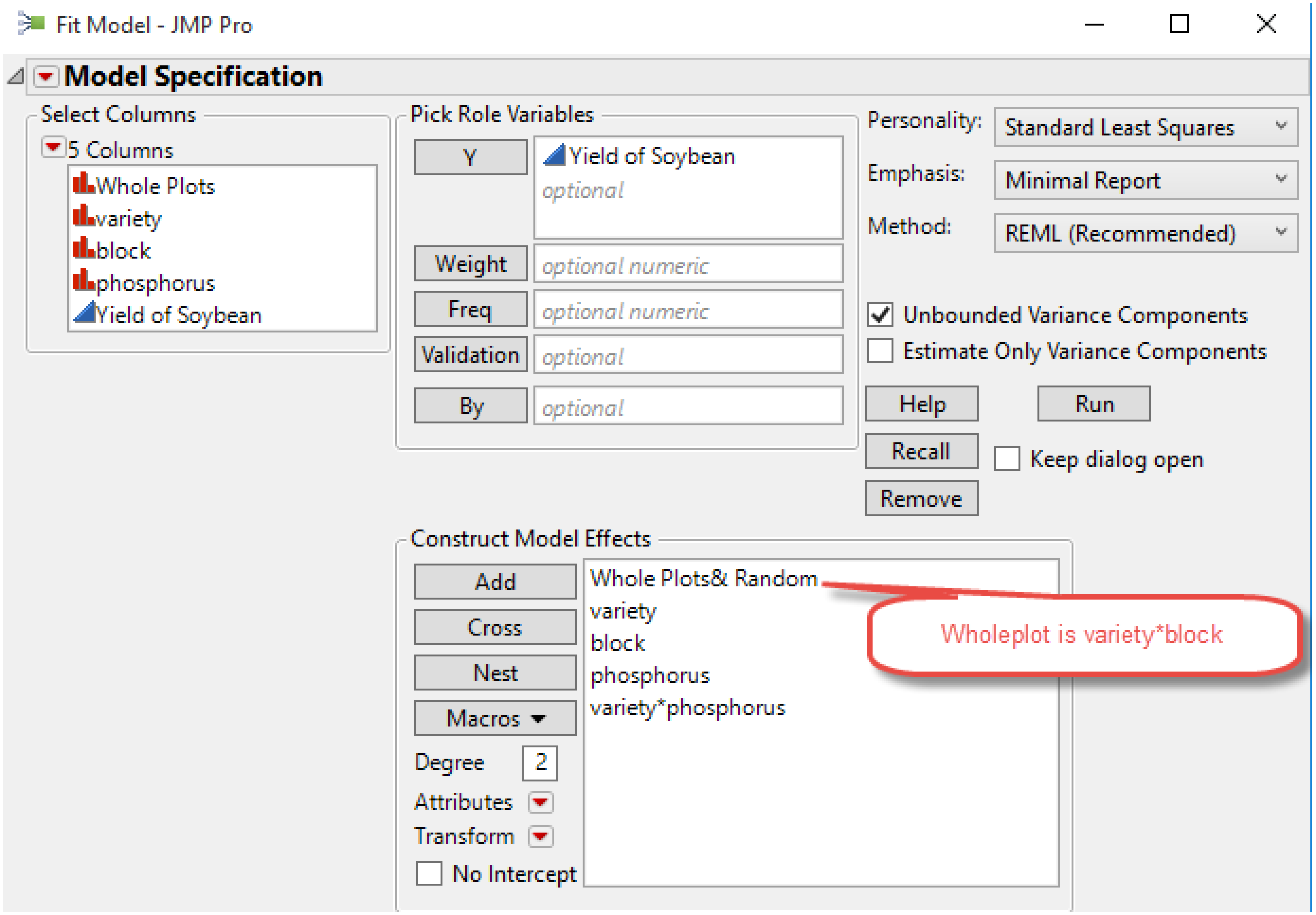
Select user specified and type 36

Select Make Design

Analyze jmpdesign_variety.JMP



Analyze jmpdesign_variety.JMP



Split-split plot designs

Split-split plot designs are a three stratum extension of split plot designs. There are factors that are Very-Hard-to-change, Hard-to-change, and Easy-to-change. In the top stratum, the Very-Hard-to- change factors stay fixed within each whole plot. In the middle stratum the Hard-to-change factors stay fixed within each subplot. Finally, the Easy-to-change factors may vary (and should be reset) between runs within a subplot.

Example: Three-stage processing involving the production of cheese that leads to a split-split plot design.
The first processing step is milk storage. Typically, milk from one storage facility provides the raw material for several curds processing units the second processing stage. Then the curds are further processed to yield individual cheeses.

In a split-split plot design the material from one processing stage passes to the next stage in such a way that nests the subplots within a whole plot.

In this example, milk from a storage facility becomes divided into two curds processing units. Each milk storage tank provided milk to a different set of curds processors. So, the curds processors were nested within the milk storage unit.

Split-split plot designs

An experiment was conducted to measure the effect of three factors,
A = row spacing, B = plant density, and C = date, on a complex response variable y.

The response variable is related to the relationship between a plant's leaf area and the amount of light intercepted at various levels in its canopy. Destruction of several plants is necessary to obtain a single value of y.

A field was divided into r=4 blocks. Each block was divided into two whole plots, and a=2 row spacings (38 and 76 cm) were randomly assigned to the whole plots within each block.

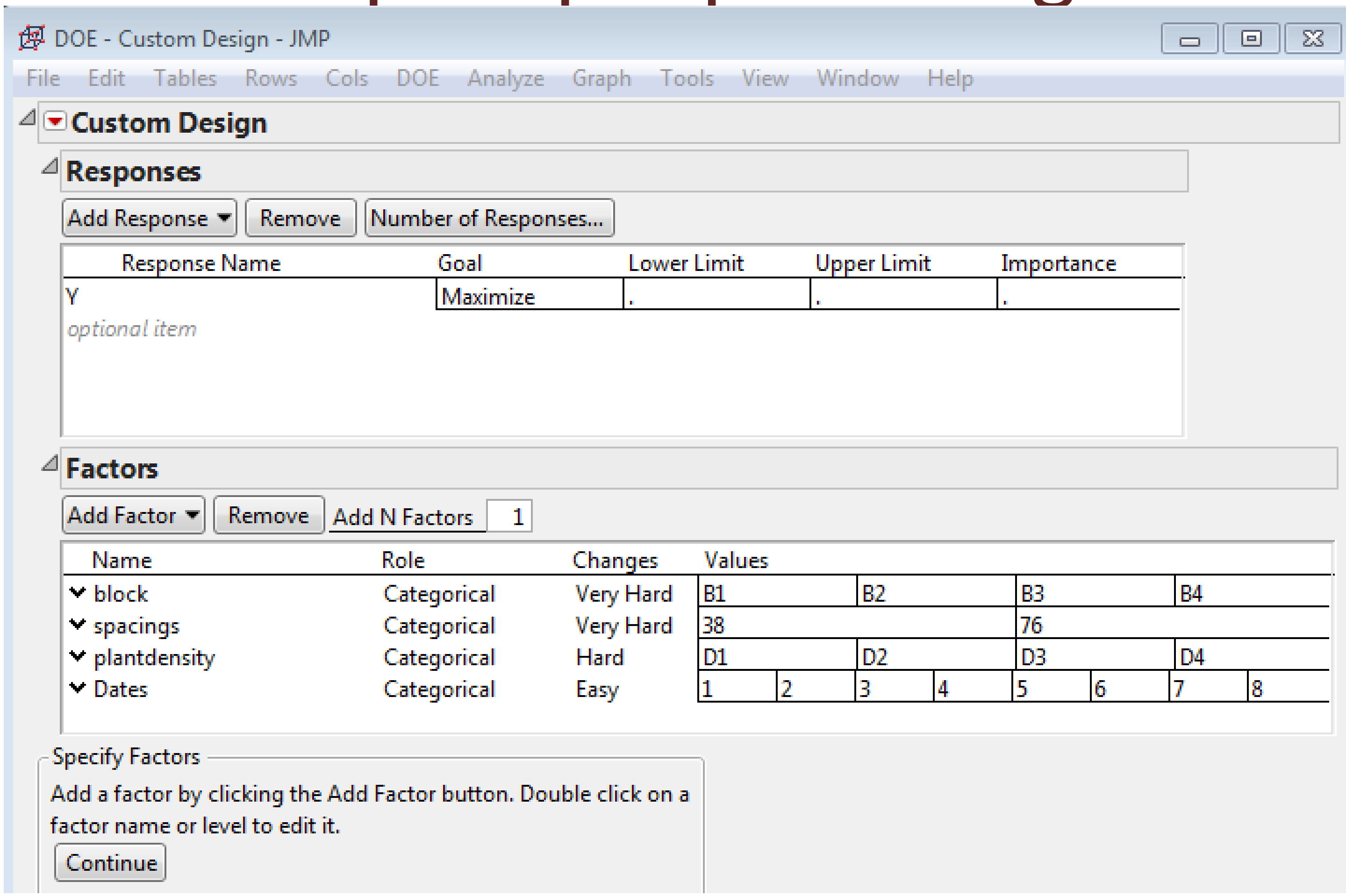
Each whole plot was divided into four split plots, and b=4 plant densities were randomly assigned to the split plots within each whole plot.

Each split plot was divided into eight split-split plots, and c=8 dates were randomly assigned to each split-split plot. At each of the eight dates during the growing season, the appropriate split-split plots were used to obtain (4)(2)(4) = 32 measures of the response variable. A grand total of 256 measurements.

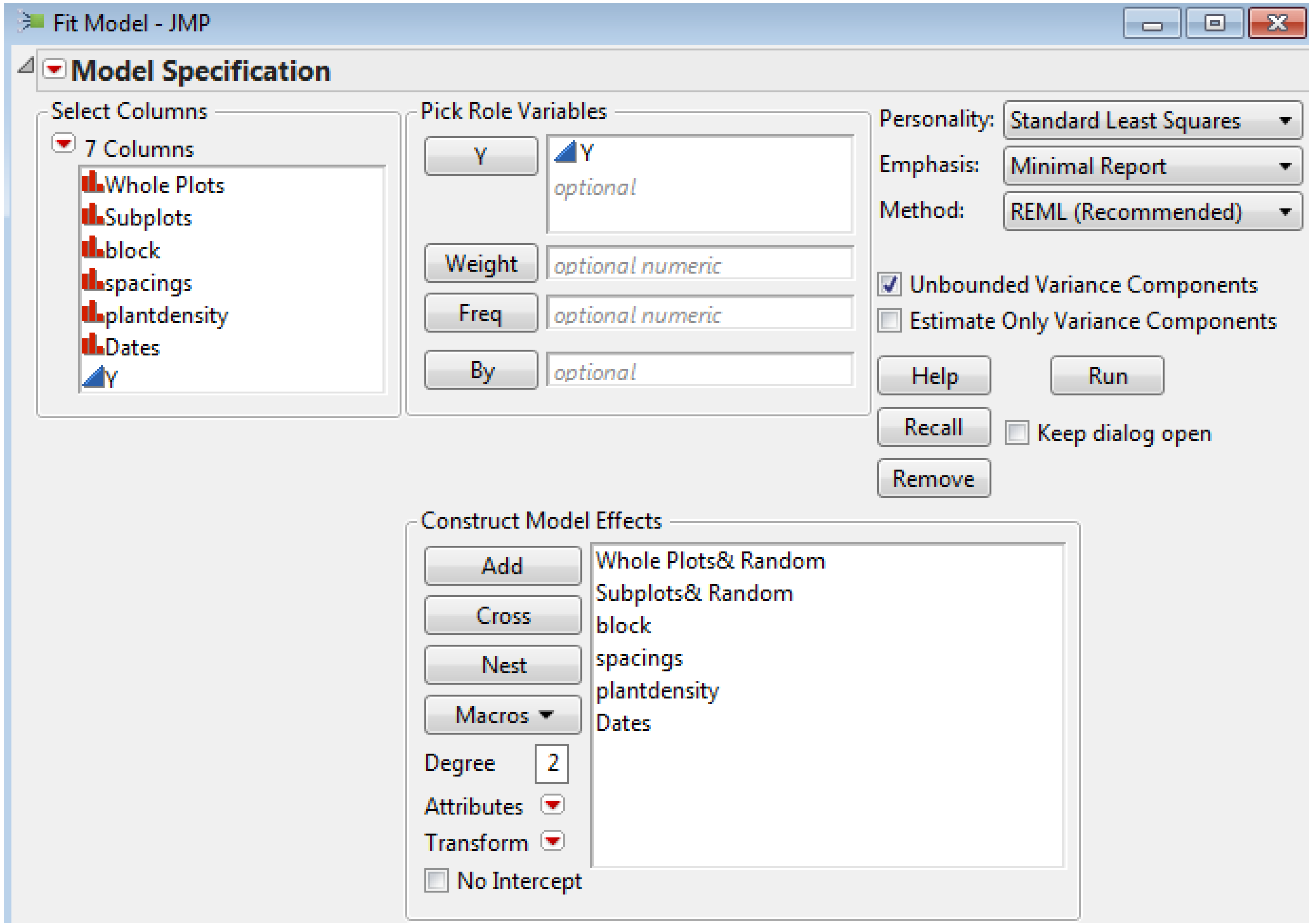
$$
\begin{aligned}
y_{ijkl} &= \mu + \rho_l + \alpha_i + (wp)_{il} && \text{(whole-plot portion)} \\
&\quad + \beta_j + (\alpha\beta)_{ij} + (sp)_{ijl} && \text{(split-plot portion)} \\
&\quad + \gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + (ssp)_{ijkl}, && \text{(split-split-plot portion)} \\
&\quad (i = 1, \ldots, a \quad j = 1, \ldots, b \quad k = 1, \ldots, c \quad l = 1, \ldots, r)
\end{aligned}
$$

where $(wp)_{il} \sim N(0, \sigma^2_{wp})$, $(sp)_{ijl} \sim N(0, \sigma^2_{sp})$, $(ssp)_{ijkl} \sim N(0, \sigma^2_{ssp})$, and all random effects are independent.

Split-split plot designs



Split-split plot designs



STATISTICS TEXAS A&M UNIVERSITY

Water Resistance Example

An experiment was conducted to investigate the effects of different types of pretreatments and stains on the water resistance property of wood.

2 types of pretreatments (A and B) and 4 types of stains (1,2,3,,4) were included in the study.

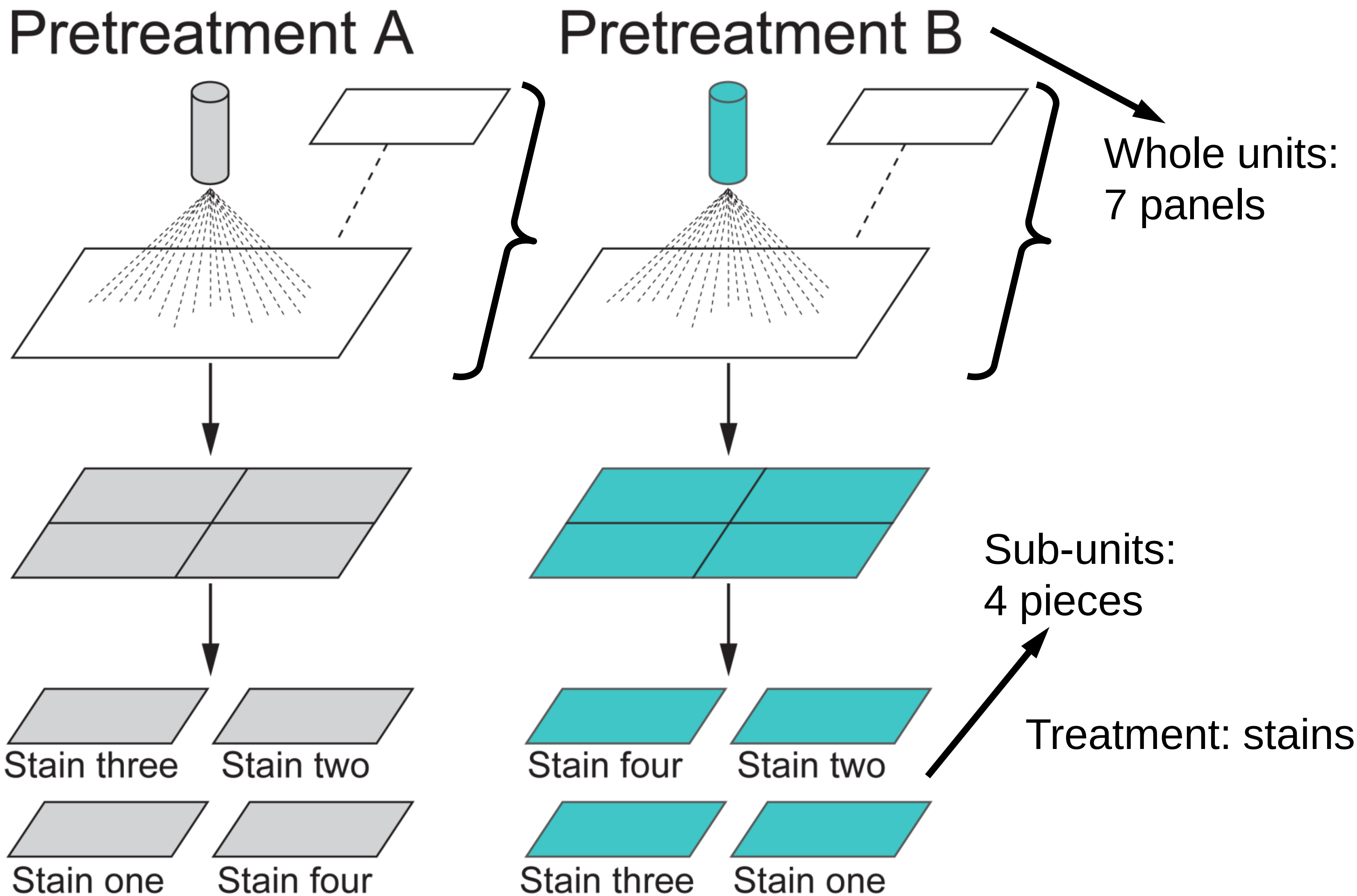
14 wood panels were randomly selected and pretreatment A was applied to 7 of them, pretreatment B was applied to the other 7 wood panels.

Each wood panel was divided into 4 pieces and one of the four stains was applied to the smaller piece of wood.

The water resistance property was characterized by measuring how long it takes for three drops of water to pass through the treated materials.

This experiment applies each of the pretreatment types to an entire wood panel, then cut each panel into four pieces and apply the four stain types to the smaller pieces.

Water Resistance Example



Water Resistance Data

wood	pretrt	stain	resistance
1	A	1	5.79
1	A	2	5.22
1	A	3	3.80
1	A	4	3.45
2	B	1	6.21
2	B	2	5.94
2	B	3	4.69
2	B	4	4.36
3	B	1	7.61
3	B	2	5.16
3	B	3	6.30
3	B	4	4.90
4	A	1	5.47
4	A	2	5.31
4	A	3	4.18
4	A	4	4.33
...	...	...	...

Model for the Split-Plot Design

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + w_k + e_{ijk}$$

pretrt effect, fixed

stain effect, fixed

wood effect, random

i = 1, 2 (pretrt)

j = 1 to 4 (stain)

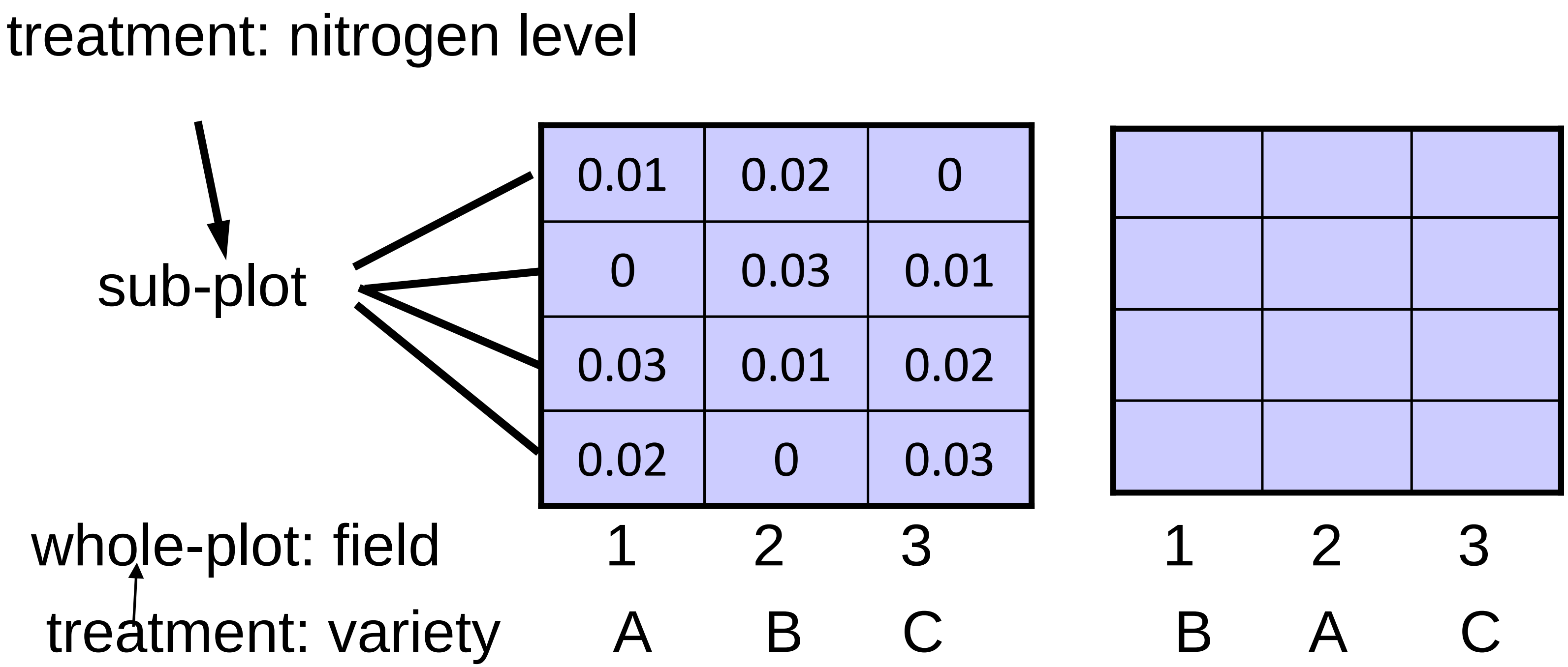
k = 1 to 14 (wood)

$$w_k \sim N(0, \sigma_w^2)$$
$$e_{ijk} \sim N(0, \sigma^2)$$

Exercise

There are two blocks of land.
Each block is divided into three fields, and one of the three varieties (A, B, or C) is assigned to one of the three fields.
Each field is then divided into four smaller fields, and one of the four different nitrogen levels (0, 0.01, 0.02, or 0.03) is applied to one of the smaller fields.
Yield is observed at the end of the season.

Is this a split-plot design?



Fitting a Model for a Split-Plot Design

This demonstration illustrates the concepts discussed previously.

WoodExample.sas