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STAT 630-720
HW 04

1) 2.7.3

a) Compute P_x: P(x) = 1/5 for Px(3, -2, 2, 3, 17), Px(x) = 0 otherwise
b) Compute P_Y: P(y) = 1/5 for Py(-3, -2, 2, 3, 19), Py(y) = 0 otherwise
c) Compute $P(Y > X)$: 3/5
d) Compute $P(Y = X)$: 0
e) Compute $P(XY < 0)$: 0

2) 2.7.4: (change the range to $0 \leq x \leq 2, 0 \leq y \leq 1$ in part (d))

a) $f_{X,Y}(x, y) = 2x^2y + Cy^5, for 0 \leq x \leq 1c, 0 \leq y \leq 1$, otherwise 0

$$f_X(x) = x^2 + \frac{2}{3}$$

$$f_Y(y) = \frac{2y}{3}$$

$$C = 4$$

$$\int_{.6}^{0}\int_{.8}^{0} 2x^2y + 4y^5\, dxdy =$$
$$= \int_{.6}^{0} \frac{x^3y}{1.5} + 4y^5|_0^{.8}\, dy$$
$$= \frac{.512y^2}{3} + \frac{4y^6}{6}|_0^{.6}$$
$$= .088$$

d) $f_{X,Y}(x, y) = Cx^5y^5, for 0 \leq x \leq 2, 0 \leq y \leq 1$

$$\int_0^1 \int_0^2 Cx^5y^5\,dxdy =$$

$$\int_0^1 Cx^5y^5\,dy = C\frac{32}{3}\frac{y^6}{6}|_0^1$$

$$C = \frac{9}{16}$$

$$f_X(x) = \frac{9}{16}x^5$$

$$f_Y(y) = 6y^5$$

$$\int_0^1 \int_0^2 \frac{9}{16}x^5y^5\,dxdy = \int_0^1 3y^6\,dy$$

$$= 3/7$$

3) 2.7.9

a) $f_X(x) = \frac{4+3x^2-2x^3}{8}$
b) $f_Y(x) = \frac{6y+8}{12}$
c) $P(Y < 1) = \frac{1}{3}$

4) 2.7.10 (apply the result in problem 2.7.13 without proof)

a) $f_x(x) = \frac{1}{2\sqrt{2Pi}}e^{\frac{-(x-3)^2}{8}}$

b) $f_y(y) = \frac{1}{4\sqrt{2pi}}e^{\frac{-(y-5)^2}{32}}$

c) not dependent because p != 0

5) 2.7.16

a)

$$C\int_0^y -e^{-(x+y)|_0^y}\,dy = 1 - e^{-2y} = 1$$

$$= \lim_{t\to\infty} y + \frac{e^{-2y}}{2}|_x^t$$

$$C = 1$$

b) $f_X(x) = \lim_{t\to\infty} -e^{-2t} + e^{-2x}$
$f_Y(y) = 1 - e^{-2y}$

6) 2.8.1

a) px(-2) = 3/12, px(9) = 3/12, px(13) = 6/12
b) py(3) = 8/12, py(5) = 4/12
c) X and Y are all independent because in all cases Px(x)Py(y) = Pxy(x,y)

7) 2.8.2

a) Px(-2) = 5/16, Px(9) = 9/16, Px(13) = 2/16
b) Py(3) = 10/16, Py(5) = 6/16
c) No, Px(x)Py(y) != Pxy(x,y)

8) 2.8.5

a) P(y=4 | x = 9) = 2/27
b) P(y=-2 | x = 9) = 2/9
c) P(y = 0 | x = -4) = 0

9) 2.8.7: (change range to $0 \leq x \leq 2, 0 \leq y \leq 1$ in part d)

a) $F_{Y|X}(y|x) = \frac{2x^2y+4y^5}{x^2+2/3}$, not independent
b) $F_{Y|X}(y|x) = 6y^5$, not independent

10) 2.8.10 $P(x = 1, y = 1) = P(x = 1)P(y = 1) = \frac{P(x=1)P(y=1)}{P(Y=1)} = \frac{P(y=1)P(x=1)}{P(x=1)} = 1$

11) 2.8.15

a) $f_{X|Y}(x|y) = \frac{3x^2+3y}{y^3+y}$
b) $f_{Y|X}(y|x) = \frac{x^2+y}{-4x^3+6x^2+8}$
c) x and y are not indepenent because $f_{x|y}(x|y) = \frac{f_{x,y}(x,y)}{f_y(y)} \neq f_x(x)$

12) 2.8.23: (use result of 2.8.22 without proof) $\frac{\binom{3}{2}\binom{2}{1}\theta 1\theta 2}{\binom{3}{1}} = \frac{6\theta 1\theta 2}{3} = 2\theta 1\theta 2$

13) 2.8.24 $f_x1 = \int_0^{x1} \alpha e^{-\alpha y}\, dy$
$f_xn = \int_0^{xn} \alpha e^{-\alpha y}\, dy$

14) 2.9.7 z(2) = 0, z(4) = 1/2, z(5) = 1/12, z(7) = 1/12, z(8) = 1/24, z(9) = 0, z(11) = 3/4, z(12) = 3/8

15) 2.9.14

$$f_Z(z) = P(x + y \leq z)$$

$$f_Z(z) = \int f_x(z - w) f_y(w)\, dw$$

$$\frac{e^{\frac{-y^2}{2}}}{\sqrt{2pi}} \int \frac{\frac{e^{-(z-y)^2/2}}{\sqrt{2pi}}}{2} \sqrt{2pi}$$