STATISTICS 630 - Test I
June 19, 2012

Name _____ Email Address _____

INSTRUCTIONS FOR STUDENTS:

(1) There are five pages including this cover page and three formula sheets. Each of the five numbered problems is weighted equally.

(2) You have exactly 70 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper. Put the worked problems in numerical order for scanning.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc., unless otherwise specified.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 70 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature_____

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: _____

(2) Record the time at which the student ends the exam: _____

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until June 29, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature_____

1. A box in a supply room contains four 60W bulbs, six 75W bulbs, and five 100W bulbs. Suppose that three bulbs are randomly selected (without replacement).

   (a) Find the probability that all three bulbs have the same wattage.
   (b) Conditional on all three bulbs having the same wattage, obtain the probability that all three bulbs are 100W bulbs.

2. Suppose that X is a random variable with cumulative distribution function

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ x^2, & 0 < x < 0.4 \\ x, & 0.4 \le x < 1 \\ 1, & 1 \le x. \end{cases}$$

   Compute each of the following:

   (a) $P(X = 0.5)$
   (b) $P(X = 0.4)$
   (c) $P(0.3 \le X \le 0.6)$
   (d) $P(0.4 \le X \le 0.6)$

3. Suppose that V is a random variable with the cumulative distribution function

$$F_V(v) = \begin{cases} 0 & v \le -1 \\ \frac{(1+v)^2}{4}, & -1 < v < 1 \\ 1, & v \ge 1 \end{cases}$$

   Obtain the probability density function of $W = V^2$.

4. Consider rolling a fair six-sided die and independently tossing two fair two-sided coins. Let Z = the number showing on the die, and let W = the number of heads showing in the two tosses. Find the probability distributions of (i) W and (ii) $U = WZ$.

5. Suppose that (X, Y) have the joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} x + Cy & 0 \le x \le 1, \ 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

   Find the value of C that makes $f_{X,Y}(x, y)$ a valid probability density function. Then determine whether X and Y are independent. Justify your answer.