# METHODS QUALIFYING EXAM

## AUGUST 2004

**INSTRUCTIONS:**

1. DO NOT put your NAME on the exam. Place the NUMBER assigned to you on the UPPER LEFT HAND CORNER of EACH PAGE of your exam.
2. Please start your answer to EACH QUESTION on a SEPARATE sheet of paper.
3. Answer all the questions.
4. Be sure to attempt all parts of every question. It may be possible to answer a later part of a question without having solved the earlier parts.
5. Be sure to hand in all of your exam. No additional material will be accepted once the exam has ended and you have left the exam room.

## PROBLEM I.

The tensile strength of a material is the ability that the material possesses to resist deformation when a force or a load is applied to it. A metallurgist conducts a study to evaluate the tensile strength of ductile iron strengthened at two different temperatures. She thinks that the lower temperature will yield the higher mean tensile strength. At each of the two temperatures, $800°C$ and $1000°C$, 300 specimens of ductile iron were heat treated. The data consists of the tensile strengths from 300 specimens heated to $800°C$: $X_1, \ldots, X_{300}$ which are iid with mean $\mu_1$ and standard deviation $\sigma_1$ and the tensile strengths from 300 specimens heated to $1000°C$: $Y_1, \ldots, Y_{300}$ which are iid with mean $\mu_2$ and standard deviation $\sigma_2$. Furthermore, the $X's$ and $Y's$ are independent.

(1) The metallurgist is interested in the null hypothesis $\mathrm{H}_0 : \mu_1 \leq \mu_2$ versus the alternative hypothesis $\mathrm{H}_1 : \mu_1 > \mu_2$ Use the following steps to present the customary $t$-test of this null hypothesis based on $X_1, \ldots, X_{300}$ and $Y_1, \ldots, Y_{300}$.

i. Write down a general formula for the $t$ test statistic commonly used for this hypothesis test.

ii. Write down the decision rule for this hypothesis test. Use $\alpha = 0.05$.

iii. State the necessary conditions needed for your procedure to be valid and how you would verify whether the conditions in are satisfied in this experimental setting.

For parts (2), (3), and (4) of this question, you may assume that $\sigma_1 = \sigma_2 = 1$ and that the sample sizes are large enough to invoke the central limit theorem if necessary.

(2) Calculate the power of your test for the following six values of the parameter:

$$\Delta = \frac{\mu_1 - \mu_2}{\sqrt{1/300 + 1/300}} = \quad .5, \quad 1.0, \quad 1.5, \quad 2.0, \quad 2.5, \quad 3$$

(3) Use your power calculations in (2) to sketch a power curve for your test. Be sure to label your axes clearly.

(4) The metallurgist in discussing your results from parts (1) through (3) states, "The power of the test when $\Delta = 1.0$ is not large enough to meet industry standards. What needs to be done to increase it?" Answer the metallurgist's question, paying careful attention to: (i) your specific recommendation on how to increase the power; and (ii) explanation (based on the ideas from parts (1) through (3)) of why your recommendation will result in an increase in power.

(5) The 600 observations considered above represent the tensile strength obtained from the two levels of heat treatment. However, after the experiments were conducted, the metallurgist informs you that the heat treatment for the 300 specimens for each heat level were conducted in the following manner. The furnace used to heat treat the specimens could hold only 5 specimens at a time. Thus, a tray containing 5 randomly selected specimens was heated to the specified temperature for the prescribed length of time and then the tensile strength measurements were taken on the 5 specimens. The metallurgist states that there is some variation in the temperature from one experimental run to the next. Thus, there may be a strong *positive* correlation between tensile strength readings for specimens on the same tray. Given this additional information, answer the following questions without carrying out any additional calculations.

i. How will this positive correlation within specimens affect the expectation of the variance estimator you used in part (1)?

ii. Suppose you did not adjust for the positive correlation within specimens and proceeded to use the ordinary $t$-test you proposed in part (1). Will the positive correlation in the data increase or decrease the numerical values of power you calculated for the test statistic in part (2)? Explain.

(6) In light of your answer to (5), the metallurgist states, "Using the $t$-test from (1) to test the research hypothesis is obviously flawed. What is an alternative approach to testing the research hypothesis?" Answer the metallurgist's question by presenting a standard testing method that will account appropriately for the sampling design described in (5). Be sure to give clear, explicit statements of both your test statistic formula and your decision rule.

**PROBLEM II.**

A colleague in meat science has approached you for help with an experiment she conducted. The experiment consisted of asking a sample of consumers to taste five different recipes for meat loaf. When a consumer tastes a sample, he or she will give scores to several characteristics of the meat loaf. These scores are combined into a single overall rating, called TASTE. Hence, there is one response variable for each consumer for each meat loaf product. The meat tasting literature indicates that in this type of study some consumers tend to give all recipes low scores, while other consumers tend to give all recipes high scores.

(1) There are at least two possible experimental designs:

> Design A: For a random sample of 100 consumers, 20 would be randomly assigned to taste Recipe R1, 20 randomly assigned to taste Recipe R2, and so on.
>
> Design B: A random sample of 20 consumers would taste all five recipes. The recipes would be presented to the 20 consumers in random order.

Both designs result in a total of 100 data values, twenty for each recipe. Which design would you recommend? Provide an explanation of your choice of design.

(2) The meat scientist selects Design B (perhaps contrary to your advice). She runs the experiment and collects the 100 data values. She asks if the correct model statement for **PROC GLM** in SAS is given by

> **MODEL TASTE = CONSUMER RECIPE CONSUMER*RECIPE;**

What would you tell her? Explain your answer.

(3) When asked if there was any problems in running the experiment, she replies that one recipe smelled so bad she had eliminated it from the experiment. Is this a problem for conducting the proper analysis? Why or why not?

(4) She also mentions that several consumers were unable to complete the tasting of all four remaining recipes. For some consumers she has only data for only one, two, or three recipes. She used the same model statement as given above in SAS. Are the results from the SAS output correct? Explain your answer.

(5) Although the subjects use water to rinse their mouths between tasting the different recipes, there is some concern that a lingering flavor from tasting one recipe may influence the response obtained from the tasting of another recipe. Based on this information, would you suggest a different experimental design than the design you selected in (1)? If yes, give a brief description of the design. You do not need to be limited to having twenty observations per recipe, and you may assume that each consumer will complete his or her tasting assignment for your design.

**PROBLEM III.**

This problem is about classical multiple linear regression; i.e. $Y_{nx1} = X_{nxp}\beta_{px1} + e_{nx1}$.
Here p = 5 and miles per gallon is the dependent variable. The following table is part of the data:

| mpg | engine | horse | weight | accel |
|---|---|---|---|---|
| 10 | 360 | 215 | 4615 | 15 |
| 10 | 307 | 200 | 4376 | 15 |
| 11 | 318 | 210 | 4382 | 16 |
| 11 | 429 | 208 | 4633 | 15 |
| 11 | 400 | 150 | 4997 | 15 |
| 11 | 350 | 180 | 3664 | 16 |
| 12 | 383 | 180 | 4955 | 15 |
| 12 | 350 | 160 | 4456 | 15 |
| 12 | 429 | 198 | 4952 | 15 |
| 12 | 455 | 225 | 4951 | 15 |
| 12 | 400 | 167 | 4906 | 15 |
| 12 | 350 | 180 | 4499 | 15 |
| 13 | 400 | 170 | 4746 | 16 |
| 13 | 400 | 175 | 5140 | 15 |
| 13 | 350 | 165 | 4274 | 16 |
| 13 | 350 | 155 | 4502 | 16 |

Engine = Engine Displacement (cu. inches)
Horse = Horsepower
Weight = Vehicle Weight (lbs.)
Accel = Time to Accelerate from 0 to 60 mph (sec)

Assume that there is an intercept in the model. Answer the following questions:

1) Explain, in layperson's terms, what is meant by multicollinearity.
2) What does the determinant of $X'X$ have to do with multicollinearity?
3) Given Tables 1 and 2 below, do you think that there is multicollinearity. Explain your answer?
4) Given Table 2 below, under the Column Dimension – see row 5, both the Constant Term and Time to Accelerate have Variance Proportion = 1 and the Condition Index = 168.934. Explain what that means?
5) Given Table 2 below, under the Column Dimension – see row 4, both the Engine and Weight have high Variance Proportions and the Condition Index = 33.48. Explain what that means?
6) Given Tables 3 and 4 below, do you think that there is multicollinearity. Explain your answer?

7) Given Table 4 below, under the Column Dimension – see row 5, neither the Constant Term and Time to Accelerate have Variance Proportion = 1 and the Condition Index = 7.235. Explain what that means?
8) Given Table 4 below, under the Column Dimension – see row 5, both the Engine and Weight have high Variance Proportions and the Condition Index =7.235. Explain what that means?
9) Clearly Tables 1 & 2 are different from Tables 3 & 4. Which set of tables would you recommend to your client?

Table 1.

**Coefficients**[a]

| Model | | Unstandardized Coefficients B | Unstandardized Coefficients Std. Error | Standardized Coefficients Beta | t | Sig. | Collinearity Statistics Tolerance | Collinearity Statistics VIF |
|---|---|---|---|---|---|---|---|---|
| 1 | (Constant) | 31.871 | 11.712 | | 2.721 | .007 | | |
| | Engine Displacement (cu. inches) | -.026 | .008 | -.393 | -3.378 | .001 | .083 | 12.008 |
| | Horsepower | .011 | .013 | .069 | .882 | .379 | .183 | 5.452 |
| | Vehicle Weight (lbs.) | -.004 | .001 | -.538 | -5.585 | .000 | .121 | 8.241 |
| | Time to Accelerate from 0 to 60 mph (sec) | .509 | .751 | .023 | .678 | .498 | .986 | 1.015 |

a. Dependent Variable: Miles per Gallon

Table 2

**Collinearity Diagnostics**[a]

| Model | Dimension | Eigenvalue | Condition Index | Variance Proportions (Constant) | Engine Displacement (cu. inches) | Horsepower | Vehicle Weight (lbs.) | Time to Accelerate from 0 to 60 mph (sec) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 4.863 | 1.000 | .00 | .00 | .00 | .00 | .00 |
| | 2 | .119 | 6.386 | .00 | .03 | .02 | .00 | .00 |
| | 3 | .013 | 19.525 | .00 | .16 | .93 | .08 | .00 |
| | 4 | .004 | 33.480 | .00 | .81 | .05 | .92 | .00 |
| | 5 | .000 | 168.934 | 1.00 | .00 | .00 | .00 | 1.00 |

a. Dependent Variable: Miles per Gallon

Table 3. Same data set except that Independent Variables have been Standardized

**Coefficients**[a]

| Model | | Unstandardized Coefficients B | Unstandardized Coefficients Std. Error | Standardized Coefficients Beta | t | Sig. | Collinearity Statistics Tolerance | Collinearity Statistics VIF |
|---|---|---|---|---|---|---|---|---|
| 1 | (Constant) | 20.219 | .215 | | 93.972 | .000 | | |
| | Zscore: Engine Displacement (cu. inches | -2.519 | .746 | -.393 | -3.378 | .001 | .083 | 12.008 |
| | Zscore: Horsepower | .444 | .503 | .069 | .882 | .379 | .183 | 5.452 |
| | Zscore: Vehicle Weight (lbs.) | -3.441 | .616 | -.538 | -5.585 | .000 | .121 | 8.241 |
| | Zscore: Time to Accelerate from 0 to 60 | .148 | .218 | .023 | .678 | .498 | .986 | 1.015 |

a. Dependent Variable: Miles per Gallon

Table 4. Same data set except that Independent Variables have been Standardized

**Collinearity Diagnostics**[a]

| Model | Dimension | Eigenvalue | Condition Index | Variance Proportions (Constant) | Variance Proportions Zscore: Engine Displacement (cu. inches | Variance Proportions Zscore: Horsepower | Variance Proportions Zscore: Vehicle Weight (lbs.) | Variance Proportions Zscore: Time to Accelerate from 0 to 60 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2.815 | 1.000 | .00 | .01 | .02 | .01 | .00 |
| | 2 | 1.005 | 1.674 | .82 | .00 | .00 | .00 | .17 |
| | 3 | .979 | 1.696 | .18 | .00 | .00 | .00 | .82 |
| | 4 | .148 | 4.358 | .00 | .02 | .76 | .30 | .00 |
| | 5 | .054 | 7.235 | .00 | .97 | .22 | .69 | .00 |

a. Dependent Variable: Miles per Gallon