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STAT 630
HW11

6.3.1 When $\alpha = .05$ we do not reject the null hypothesis.

```
data = c(4.7, 5.5, 4.4, 3.3, 4.6, 5.3, 5.2, 4.8, 5.7, 5.3)
x = (mean(data) - 5) / (sqrt(.5 / 10))
2 * pnorm(x)
```

```
## [1] 0.591505
```

6.3.2 When $\alpha = .05$ we do not reject the null hypothesis.

```
data = c(4.7, 5.5, 4.4, 3.3, 4.6, 5.3, 5.2, 4.8, 5.7, 5.3)
x = (mean(data) - 5) / (sqrt(sd(data) / 10))
2 * pnorm(x)
```

```
## [1] 0.6491405
```

6.3.8 Using the $Binomial(250, .62)$ distribution

```
## Wald Test
x = (250  * (.62 - .65)^2) / (.62 * (1 - .62))
1 - pnorm(x)
```

```
## [1] 0.1697867
```

```
## Score Test
y = (250  * (.62 - .65)^2) / (.65 * (1 - .65))
1 - pnorm(y)
```

```
## [1] 0.1613289
```

8.2.16 for $N(0, \sigma_0)$

$$L(\theta) = \frac{1}{2\pi\theta}^{\frac{n}{2}} e^{\frac{-1}{2}} \sum_{i=1}^{n} \frac{X_i^2}{\theta}$$

$$\frac{L(\theta_0|x_1, ..., x_n)}{L(\theta_a|x_1, ..., x_n)} \leq K$$

$$\frac{\theta_a}{\theta_0}^{\frac{n}{2}} e^{\frac{-1}{2}} \sum_{i=1}^{n} X_i^2 \frac{(\theta_a - \theta_0)}{(\theta_0\theta_a)} \leq K$$

$$\alpha = P(\sum_{i=1}^{2} \frac{X_i^2}{\theta_0} \geq \frac{c}{\theta_0}|H_0)$$

8.2.20

$$
\begin{aligned}
P(X = x|\theta) &= \frac{1}{\Pi_{i=1}^{n} x_i!} e^{-n\theta} \theta \sum_{i=1}^{n} x_i \\
&= \frac{1}{\Pi_{i=1}^{n} x_i!} e^{\sum_{i=1}^{n} x_i log(\theta) - n\theta}
\end{aligned}
$$

Additional A Likelihood ratio test

$$
L(\lambda) = -n\lambda + \sum_{i=1}^{n} x_i log(\lambda) - \sum_{i=1}^{n} log(x_i!)
$$

$$
LR = \frac{L(\lambda_1|x_1, ..., x_n)}{L(\lambda_0|x_1, ..., x_n)}
$$

$$
\begin{aligned}
&= \frac{-n\lambda_1 + \sum_{i=1}^{n} x_i log(\lambda_1) - \sum_{i=1}^{n} log(x_i!)}{-n\lambda_0 + \sum_{i=1}^{n} x_i log(\lambda_0) - \sum_{i=1}^{n} log(x_i!)} \\
&= \frac{\lambda_1}{\lambda_0}
\end{aligned}
$$

Substitute the MLE for λ_1

$$
\frac{\sum_{i=1}^{n} x_i}{n\lambda_0}
$$

Wald Statistic

$$
Z = \frac{(\hat{\lambda} - \lambda_0)}{\lambda_0}
$$

Score Statistic

$$
\begin{aligned}
l(\lambda|x) &= \sum_{i=1}^{n} x_i log(\lambda - n\lambda) \\
dl(\lambda|x) &= \frac{1}{\lambda} \sum_{i=1}^{n} x_i - n = 0
\end{aligned}
$$

Additional B

$$\frac{L(\theta_0|x_1,...,x_n)}{L(\theta_a|x_1,...,x_n)} = \frac{\Pi_{i=1}^{n}\frac{1}{\sqrt{2\pi\sigma_0}}e^{\frac{-x_i^2}{2\sigma_0^2}}}{\Pi_{i=1}^{n}\frac{1}{\sqrt{2\pi\sigma_0}}e^{\frac{-x_i^2}{2\sigma_0^2}}}$$

$$= e^{\frac{-1}{2\sigma_0^2}\sum_{i=1}^{n}x_i^2 - \sum_{i=1}^{n}x_i^2}$$

$$LR = e^{\frac{n}{2\sigma_0^2}\hat{x^2}}$$

$$2logLR = \frac{nx^2}{\sigma}$$

Additional C

$$MLE(\hat{\theta}) = \frac{2x_1 + x_2}{2x_1 + 2x_2 + 2x_3}$$

$$\frac{L(\theta_1|y_1,...,y_n)}{L(\theta_0|y_1,...,y_n)} = \frac{(\theta^2)^{x_1}(2\theta(1-\theta))^{x_2}(1-\theta)^{x_3}}{.25^{x_1}(.5)^{x_2}(.5)^{x_3}}$$

$$2logLR = 2log(\frac{(\theta^2)^{x_1}(2\theta(1-\theta))^{x_2}(1-\theta)^{x_3}}{.25^{x_1}(.5)^{x_2}(.5)^{x_3}})$$

$$Reject H_0 for 2logLR > X_{.95}^2(1)$$

Additional D

$$\bar{x} - z_{1-\alpha/2}\frac{2}{\sqrt{16}} < \mu < \bar{x} + z_{1-\alpha/2}\frac{2}{\sqrt{16}}$$

$$\bar{x} - \frac{1.96}{2} < \mu < \bar{x} + \frac{1.96}{2}$$