Homework 02
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STAT 641-720

1. a)

$$
\begin{aligned}
f(y) &= P(Y = y) = p(1-p)^y \\
F(y) &= \sum_{k=0}^{y} P(Y = k) \\
&= P\sum_{k=0}^{y}(1-p)^k \\
&= P\frac{1-(1-p)^{y+1}}{1-(1-p)} \\
&= 1-(1-p)^{y+1}
\end{aligned}
$$

b)

$$
\begin{aligned}
F(y) = 1-(1-p)^{y+1} &\geq q \\
1-q &\geq (1-p)^{y+1} \\
log(1-q) &\geq (y+1)log(1-p) \\
\frac{log(1-q)}{log(1-p)} &\geq (y+1) \\
\frac{log(1-q)}{log(1-p)} - 1 &\geq y
\end{aligned}
$$

2. a) The cdf of z is not affected by α and θ

$$
\begin{aligned}
F_Z(z) = P(Z \leq z) &= P(\frac{y-\theta}{\alpha} \leq z) \\
&= P(y \leq \alpha z + \theta) \\
&= Fy(\alpha z + \theta) \\
&= 1 - exp\Big[-\Big(\frac{\alpha z + \theta - \theta}{\alpha}\Big)^{\gamma}\Big] \\
&= 1 - exp\Big[-z^{\gamma}\Big]
\end{aligned}
$$

b)

$$
\begin{aligned}
F_Z = 1 - exp\Big(-\frac{y-\theta}{\alpha})^{\gamma} \geq q \\
1 - q \geq exp\Big(-\frac{y-\theta}{\alpha})^{\gamma} \\
log(1-q) \geq (-\frac{y-\theta}{\alpha})^{\gamma} \\
log(1-q)^{\frac{1}{\gamma}} \geq -\frac{y-\theta}{\alpha} \\
-a(-log(1-q))^{\frac{1}{\gamma}} \geq y - \theta \\
\frac{a(-log(1-q))^{\frac{1}{\gamma}}}{\theta} \geq= y
\end{aligned}
$$

c)

```
y = 30; theta = 10; gamma = 2; alpha = 25
1 - exp(-(y - theta)/alpha)^gamma
```

```
## [1] 0.7981035
```

d)

```
q = .4; theta = 10; gamma = 2; alpha = 25
(alpha * (-log(1 - q))^(1/gamma)) / theta
```

```
## [1] 1.786802
```

3. a)

$$
\begin{aligned}
F_Z(z) = P(Z \leq z) &= P(\frac{Y}{B} \leq z) \\
&= P(y \leq z\beta) \\
&= F_y(z\beta) \\
&= 1 - exp\Big(-\frac{(z\beta)^{\gamma}}{\beta}\Big) \\
&= 1 - exp\Big(-z^{\gamma}\beta^{\gamma-1}\Big)
\end{aligned}
$$

b)

$$\begin{aligned} Z &= \frac{y}{\beta^{y\gamma}} \\ F_Z(z) &= P(\frac{Y}{\beta^{y\gamma}}) \\ &= P(y \leq z\beta^{y\gamma}) \\ &= 1 - exp\Big[- \Big(\frac{z\beta^{y\gamma}}{\beta}\Big)^{\gamma}\Big] \\ &= 1 - exp\Big[- z\frac{\beta}{beta}\Big] \\ &= 1 - e^{-z} \end{aligned}$$

4. a) Expected value of the probability function, λ = .25
   b) 52 weeks in a year with a λ = .25, so it is certain at least one event will occur in any give year.

```r
ppois(52, .25)
```

```
## [1] 1
```

5. a) Chi-square(6)
   b) t(6)
   c) F(1,7)
   d) Cauchy(0,1)
   e) F(2,3)

6. a) W = Weibull(γ = 4, α = 1.5)

$$\begin{aligned} 38 &= 1 - exp - \Big(\Big(\frac{y}{1.5}\Big)^4\Big) \\ 1 - .38 &= e^{\left(\frac{y}{1.5}\right)^4} \\ -\Big(\frac{y}{1.5}\Big)^4 &= log(.62) \\ \frac{y}{1.5} &= \Big(- log(.62)\Big)^{\frac{1}{4}} \\ y &= 1.5\Big[- log(.62)\Big]^{.25} \\ y &= 1.24 \end{aligned}$$

b) N = NegBin(r = 8, p = .7), C = 2

```r
pnbinom(1, 8, .7); pnbinom(2, 8, .7)
```

```
## [1] 0.1960032

## [1] 0.3827828
```

c) B = Bin(20, .4), C = 7

```r
pbinom(6, 20, .4); pbinom(7, 20, .4)
```

```
## [1] 0.2500107

## [1] 0.4158929
```

d) P = Poisson(λ = 3), C = 2

```r
ppois(1, 3); ppois(2, 3)
```

```
## [1] 0.1991483

## [1] 0.4231901
```

e) U = Uniform(.3, 2.5)

$$
\begin{aligned}
.38 &= \frac{1}{b-a} \\
Q &= a + p(b-a) \\
&= .3 + .38(2.5 - .3) \\
&= 1.36
\end{aligned}
$$

7.
   a) Cauchy, because its the ratio of 2 standard normal distributions
   b) Gamma, because T is the length of time between events
   c) Uniform, the probability is equally likely
   d) Weibull, because we are measuring the time it takes for an event to occur
   e) Bernoulli, the possible values are 1/0, pass/fail
   f) Poisson, because the interval of time is fixed
   g) Double Exponential, a cubic function would have a very steep incline
   h) Binomial, because known probability of failure with fixed n
   i) Hypergeometric, sampling without replacement would make most sense
   j) Normal, 68 & 95 percent are the 1st and 2nd standard deviations of the normal distribution
   k) Negative Binomial, because the interviews will continue until the 50th success
   l) Poisson, because the space of the wing is fixed and we are counting events
   m) Exponential, because we are measuring the time till an event occurs based on a poisson distribution
   n) Chi-square, because of the small sample size, it should be normal as the samples increase
   o) F, right skewed distribution