STATISTICS
TEXAS A&M UNIVERSITY

Beta Distribution

Function and properties

Beta pdf is

$$f(x;a,b,\alpha,\beta)=\frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}},$$

$$a\le x\le b,\qquad \alpha,\beta>0$$

where $B(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx$

$$\Gamma(\alpha)=\int_0^\infty t^{\alpha-1}e^{-t}\,dt$$

α and β are shape parameters.

Standard Beta (a=0, b=1) pdf is

$$f(x;a,b,\alpha,\beta) = \frac{(x)^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)},\ 0 \leq x \leq 1,\quad \alpha,\beta > 0$$

In SAS, Proc Univariate-Beta Fit :

theta=a,

sigma=b-a,

alpha=α,

beta=β

μ=mean=$\frac{\alpha}{\alpha+\beta}$, scale parameter=ϕ=α+β

Var(y)=$\frac{a(\mu)}{1+\phi}$ where a(μ)=μ(1-μ)

$\alpha=\beta=1$ then Uniform distribution

$\alpha=\beta$ then symmetric

$\alpha<1$, $\beta<1$ then U shaped

$\alpha>1$, $\beta=1$ then strictly increasing

$\alpha=1$, $\beta>1$ then strictly decreasing

$\alpha>1$, $\beta>1$ then unimodal

In SAS, Proc Univariate-Beta Fit, food example :

theta=a=0,

sigma=b-a=1-0=1,

alpha=α=2.464531,

beta=β=4.897577

μ=mean=$\frac{\alpha}{\alpha+\beta} = \frac{2.464531}{2.464531+4.897577}$ =0.334759,

scale parameter=ϕ=α+β= $2.464531 + 4.897577$=7.3621

Var(y)=$\frac{\mu(1-\mu)}{1+\phi} = \frac{0.334759(1-0.334759)}{1+7.3621} = 0.026632$

Std dev.=$\sqrt{Var(y)}$=0.163192

Fitting Proc GLIMMIX without any predictors with the response and beta distribution:

- Logit is estimated by the intercept. Food example has the estimated value, -0.6867

  mean =μ=$\frac{1}{1+e^{-logit}} = \frac{1}{1+e^{0.6867}} = 0.33476$

- Output has the estimated scale, 7.3621

  $\phi=\alpha+\beta=2.464531+4.897577$