

## STATISTICS 630 - Test I

June 21, 2013

Name \_\_\_\_\_ Email Address \_\_\_\_\_

### INSTRUCTIONS FOR STUDENTS:

- (1) There are five pages including this cover page and three formula sheets. Each of the five numbered problems is weighted equally.
- (2) You have exactly 70 minutes to complete the exam.
- (3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper. Put the worked problems in numerical order for scanning.
- (4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as  $\frac{12}{19}$ ,  $\binom{32}{14}$ ,  $e^{-3}$ ,  $\Phi(1.4)$ , etc., unless otherwise specified.
- (5) Show *ALL* your work. Give reasons for your answers.
- (6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.
- (7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 70 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature \_\_\_\_\_

### INSTRUCTIONS FOR PROCTOR:

- (1) Record the time at which the student starts the exam: \_\_\_\_\_
- (2) Record the time at which the student ends the exam: \_\_\_\_\_
- (3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.
- (4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.
- (5) Please keep these materials until June 29, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature \_\_\_\_\_

1. Suppose that we independently toss five fair coins. Let  $X$  = the number of heads on the five tosses.
  - (a) Find the probability distribution of  $X$ .
  - (b) Find the conditional probability of tossing exactly three heads given that you toss an odd number of heads.

2. The joint probability mass function (pmf)  $p_{X,Y}(x, y)$  for  $(X, Y)$  is given in the following table:

		$y$			
		0	1	2	3
$x$	1	1/6	1/6	0	0
	2	1/12	2/12	1/12	0
	3	1/24	3/24	3/24	1/24

We note that  $p_{X,Y}(x, y) = 0$  for all other values of  $(x, y)$ .

- (a) Compute  $P(X = Y)$  and  $P(X > Y)$ .
  - (b) Obtain the marginal pmf of  $X$  and the conditional pmf of  $X$  given  $Y = 2$ . Based on these pmfs, what can you say about the independence of  $X$  and  $Y$ ?
3. A professor carries 12 bills of various denominations in her wallet. Suppose that she has 5 one-dollar bills, 3 five-dollar bills, and 4 ten-dollar bills. She selects three of the bills at random from her wallet.
  - (a) Find the probability that the bills are all of the same denomination.
  - (b) Find the probability that the bills are all of different denominations.
4. Suppose that  $X$  is a random variable with the probability density function

$$f_X(x) = \begin{cases} \frac{3}{8}x^2 & \text{for } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the cumulative distribution function of  $X$ .
  - (b) Let  $Y = X^2$ . Derive the cumulative distribution function of  $Y$ .
5. A professor at a major university uses a plagiarism checker website to check for the originality of the papers turned in for his class. Of papers that have been copied from the web,  $\frac{5}{6}$  will test positive (labelled “copied”). Of papers that are original (not copied from the web),  $\frac{2}{3}$  will test negative (labelled “original”). Suppose that  $\frac{1}{4}$  of all papers in this class have been copied from the web.
  - (a) Obtain the probability that a randomly selected paper from this class will test negative (be identified as “original.”)
  - (b) Given that a randomly selected paper tests positive (is identified as “copied”), obtain the conditional probability that the paper was actually copied from the web.