STATISTICS
TEXAS A&M UNIVERSITY

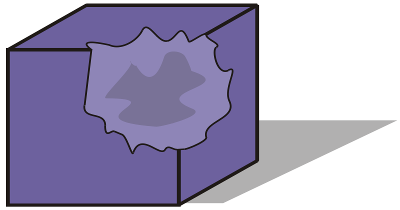
Handout 05

Multi Factor Designs and Blocking

Randomized Incomplete Block Design

Incomplete Block Design

Incomplete block:



Incomplete Block Design: Each treatment cannot be applied in each block an equal number of times

The size of convenient blocks will not accommodate all the treatments of interest.

Suppose you wanted to test four types of automobile tires for wear. An obvious choice for a block would be an automobile. You might select ten automobiles for the study. Assuming that the tires were rotated among the four positions, this experiment would control for differences in tire wear due to the type of automobile and the terrain that each traveled.

However, blocking difficulties arise if you want to test six types of tires. You could redesign the automobile , or you could adopt a balanced incomplete block design.

In a balanced incomplete block design, the treatments are assigned to the blocks so that every pair of treatments occurs together in a block the same number of times. This achieves the balance (all differences between treatments are measured with equal precision).

Incomplete Block Design

Plant Example: You are working in a greenhouse, and want to compare 9 varieties of flowers. The benches in this greenhouse are only large enough to hold 6 pots, and you know benches differ and so must be blocked on. Here the block size of 6 is too small to hold all 9 treatments, making the blocks "incomplete".

Animal Example: You are testing 4 meat storage methods, which must be applied to entire cuts of meat. One animal can only produce 2 such cuts, one from each side of the animal. Individual animals differ in meat quality, so must be blocked on. An incomplete design is produced by having blocks of size 2 that can not hold all 4 treatments.

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Balanced Incomplete Block Design

Rules:

(1) Randomly assign the numbers to the blocks

(2) Randomly assign the letters to the treatments.

(3) Randomly assign the treatments within the blocks.

(4) Randomly group blocks as replicates. A replicate is a complete set of all treatments

Properties of BIBD (balanced Incomplete Block Design):

- Each block has the same number of experimental units.
- Each treatment occurs the same number of times in the experiment.
- The number of times any two treatments occur together in the same block is the same for all pairs of treatments.

Block	Treatment
I	1 ,2, 3
II	1, 2, 4
III	1, 3, 4
IV	2, 3, 4

If the design is an incomplete block design and any one of these properties is not met, the design is an unbalanced incomplete block design.

Balanced Incomplete Block Design

$y_{ij} = \mu + \alpha_i + b_j + \varepsilon_{ij}$, i=1,…,t, j=1,…,b

Not all y_{ij} exists because of incompleteness

Additive effect due to block (no interaction)

This design should only be used if the experimental situation forces blocks to be too small, but also is recommended if more than 10 treatments are being tested.

In the latter case, experience suggests that complete blocks of 10 or more experimental units generally are so large that they will contain experimental units that are not similar.

Such blocks should be divided into more homogeneous groups, producing an incomplete block design, because the divided complete block will now be too small to hold all treatments.

Balanced Incomplete Block Design (BIBD)

t: number of treatments
b: number of blocks
r: number of blocks in which each treatment appears (number of replications per treatment)
k: number of EUs per block where k<t

λ: number of blocks in which each pair of treatments appear together

n: number of EUs in the experiment.

– BIBD do not exist for every t, b and k<t

– First Order Balance: r=n/t=bk/t where n=tr=bk is the BIBD restriction

– Second Order Balance: number of blocks in which each pair of treatments appear together, $\lambda = \frac{r(k-1)}{t-1}$ must be an integer

Balanced Incomplete Block Design (BIBD)

<u>Easy Case if you only know t and k and need to identify the rest:</u>

Any k<t, one block for each unique subset of k treatments

$$b = \binom{t}{k}$$

$$r = \binom{t-1}{k-1}$$

$$\lambda = \binom{t-2}{k-2}$$

Hardness Example

t=4 tips,   k=3 EUs per block and n=12 EUs

12=4r then r=3

12=3b then b=4

$$\lambda = \frac{3(2)}{3} = 2$$

Is it a BIBD?

Example

The four blocks of design with six treatments, each treatment appears in two blocks and three EUs per block:

{1,2,3}

{1,4,5}

{2,4,6}

{3,5,6}

$t=\quad, b=\quad, r=\quad, k=\quad, \lambda =$

Is it a BIBD?

Diets on the weight Gain

A study of the difference 6 Diets on the weight gain of rabbits is proposed. Because weight varies amongst rabbits, it is proposed to block the experiment based on litters. There are 10 litters of rabbits available varying sizes. The minimum litter size is 3. Therefore, only 3 of the 6 diets can be observed in any particular litter.

Is BIBD possible?

t=6, k=3, b=10 then n=tr=30=bk so r=5

$$\lambda = \frac{r(k-1)}{t-1} = \frac{5(3-1)}{(6-1)} = 2$$

For the complete balance, we would need $\binom{6}{3}$=20 litters

Breaking Strength Example

t=5 alloys, k=3 EUs per block, n=15 EUs

15=5r then r=3 is less than unreasonable $\binom{4}{2}$ =6

15=3b then b=5 is less than unreasonable $\binom{5}{3} = 10$

$\lambda = \frac{r(k-1)}{t-1} = \frac{3(2)}{4}$ = 1.5

Is it BIBD?

Differences between Balanced and Unbalanced Designs

- A balanced design can require too many runs to be practical.
- All estimates of treatment means and all comparisons between pairs of treatments have equal precision in a balanced design.
  - Unequal observations then SE of treatment estimates are different
  - Unequal observations then C.Is involving treatments will have different widths
- A balanced design is more statistically efficient than an unbalanced design (i.e., achieves a smaller SE with the same number of runs)

Which of the following is NOT a property of the BIBD?

(a) Each Block contains the same number of EUs

(b) Each treatment occurs the same number of times in the experiment

(c) Each treatment is present in every block

(d) Each pair of treatments occurs together in the same block the same number of times.

Water Quality Example

Water quality monitoring studies often take the form of incomplete block designs. For example, the following data represents TSS (Total Suspended Solids) in water samples taken upstream of a development (the reference sample), at the development (the mid-stream sample), or downstream of the development (the DS sample) are taken during storm events when water quality may be compromised by the development. Here is a small set of data:

Location	Storm 1	Storm 2	Storm 3	Storm 4
Ref	.	.	25	20
Mid	51	.	100	.
DS	173	137	170	110

n= t= k= b= r=

λ=

Waterquality.jmp Design and Analysis

When the data has a wide range and where the ratio among values is of interest, log tranformation may be appropriate.

Response logTSS

Whole Model

Actual by Predicted Plot

Summary of Fit

RSquare	0.968623
RSquare Adj	0.890179
Root Mean Square Error	0.281795
Mean of Response	4.332644
Observations (or Sum Wgts)	8

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	4.9027092	0.980542	12.3481
Error	2	0.1588170	0.079409	Prob > F
C. Total	7	5.0615262		0.0766

Parameter Estimates

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Location	2	2	3.9995791	25.1836	0.0382*
Event	3	3	0.2287853	0.9604	0.5465

Response logTSS

Whole Model

Actual by Predicted Plot

Summary of Fit

RSquare	0.951939
RSquare Adj	0.932715
Root Mean Square Error	0.243934
Mean of Response	4.332644
Observations (or Sum Wgts)	8

Parameter Estimates

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Event	0.2735382	0.0162766	0.0383146	0.0020646	3439932.2	21.479
Residual		0.0595039	0.048186	0.0192219	0.799593	78.521
Total		0.0757805	0.0484095	0.0293159	0.467774	100.000

-2 LogLikelihood = 6.157316192

Note: Total is the sum of the positive variance components.

Total including negative estimates = 0.0757805

Covariance Matrix of Variance Component Estimates

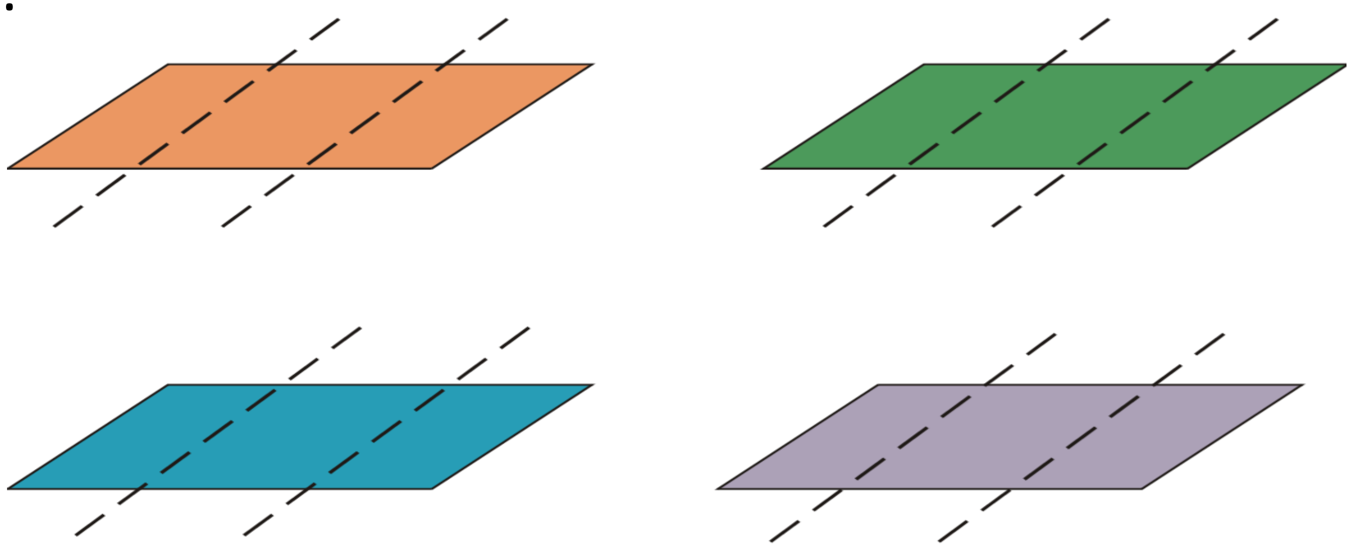
Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Location	2	2	3.712	32.1987	0.0045*

Drill Tip Experiment

Suppose that you determine that only three observations can be obtained from each metal sheet. Because you have four treatments in the experiment, and all four treatments cannot be applied in one block, you would have to use an incomplete block design.



Drill Tip Experiment

DOE-Custom Design in JMP

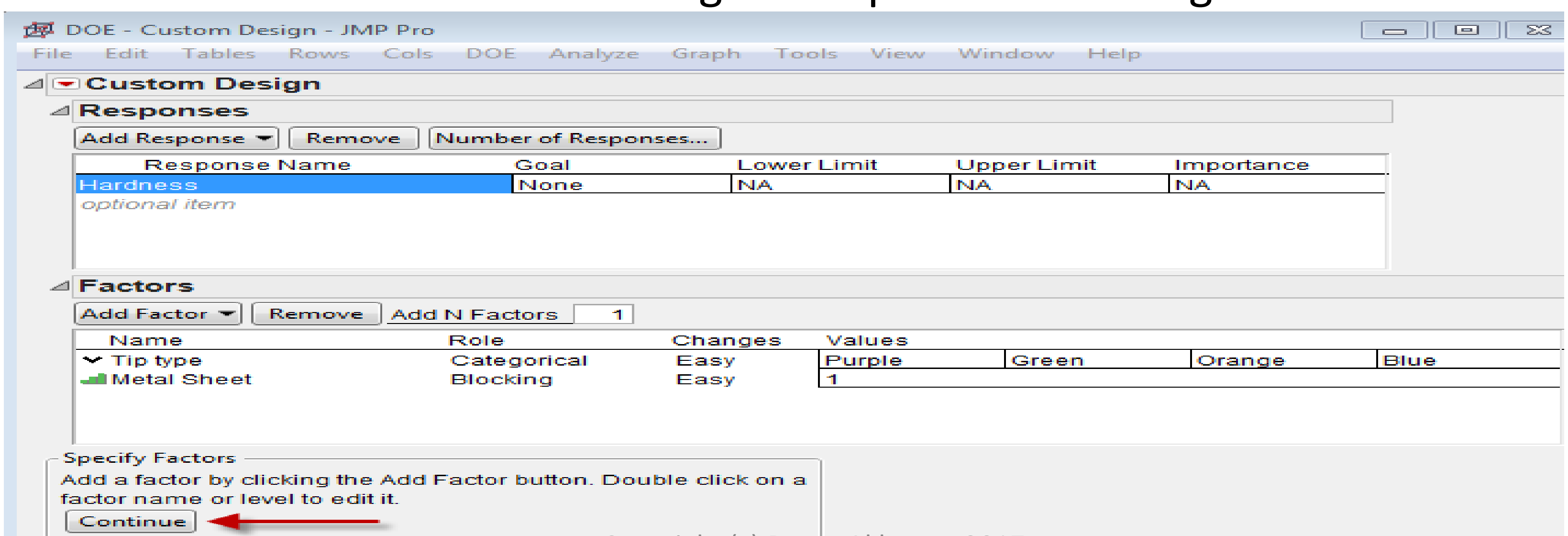
Under Responses,

Change Y to Hardness, Change Goal from Maximize to None

Under Factors,

select add factor-categorical-4level . Change X1 to Tip type then type the levels Purple, Green, Orange, Blue

select add factor-blocking-3runs per block. Change X2 to Metal Sheet

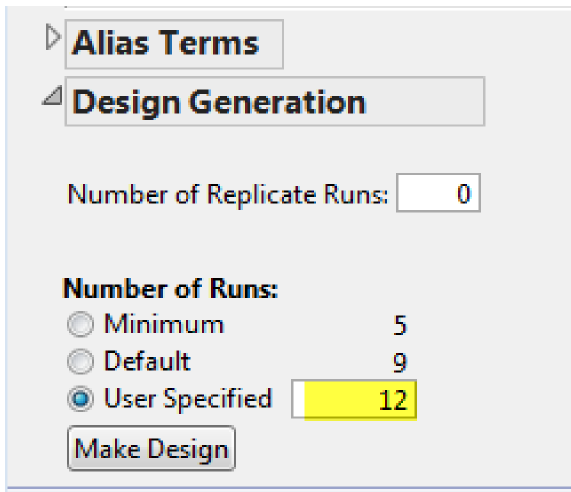


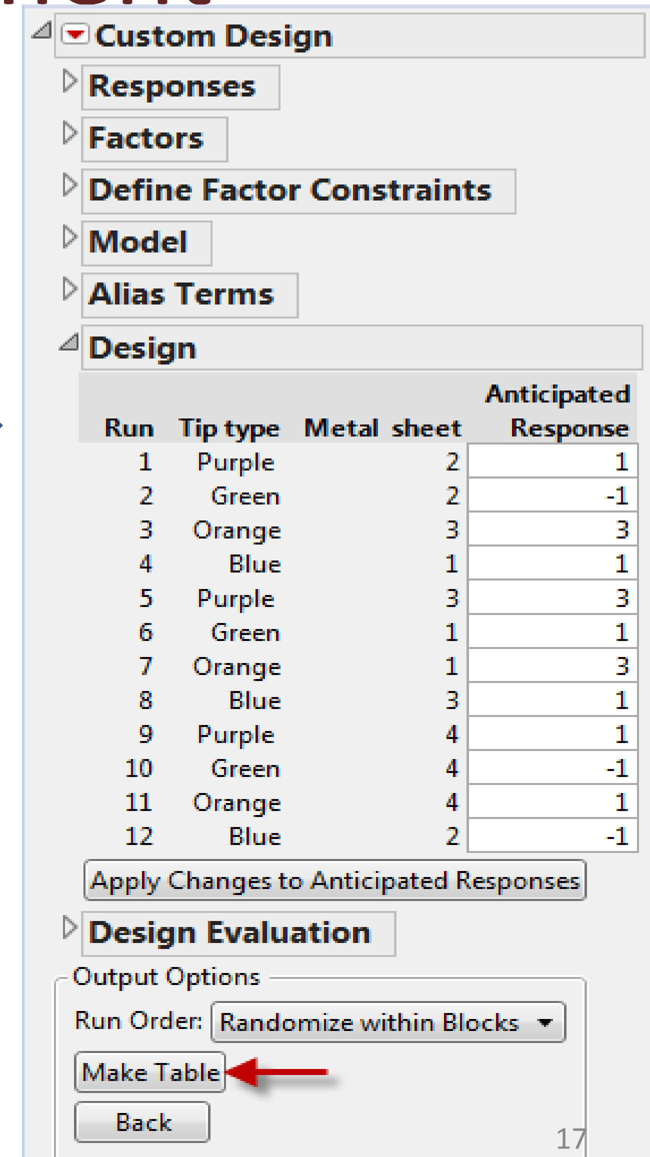
Drill Tip Experiment

Default design has 9 runs but we want 12

Select Make Design then

Make Table.





Run	Tip type	Metal sheet	Anticipated Response
1	Purple	2	1
2	Green	2	-1
3	Orange	3	3
4	Blue	1	1
5	Purple	3	3
6	Green	1	1
7	Orange	1	3
8	Blue	3	1
9	Purple	4	1
10	Green	4	-1
11	Orange	4	1
12	Blue	2	-1

4 blocks of size 3

The order of trts/block is randomized

Balanced:

(i) 3 runs/block (3 EU/block)

(ii) Each trt occurs 3 times in the experiment

(iii) Any 2 trts appear in the same block twice in the experiment

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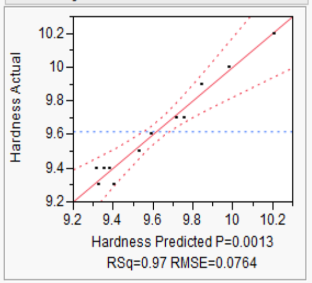
Tips Incomplete Block.JMP

Tips_Incomplete_Block - Fit Least Squares - JMP Pro

Response Hardness

Whole Model

Actual by Predicted Plot



Summary of Fit

RSquare	0.968861
RSquare Adj	0.931495
Root Mean Square Error	0.076376
Mean of Response	9.616667
Observations (or Sum Wgts)	12

Analysis of Variance

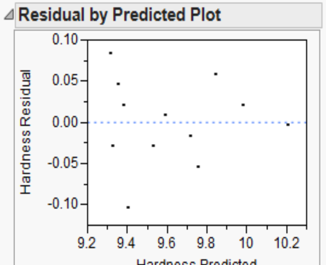
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	0.90750000	0.151250	25.9286
Error	5	0.02916667	0.005833	Prob > F
C. Total	11	0.93666667		0.0013*

Parameter Estimates

Effect Tests

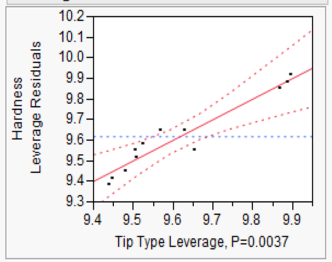
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Tip Type	3	3	0.33083333	18.9048	0.0037*
Metal Sheet	3	3	0.39750000	22.7143	0.0024*

Residual by Predicted Plot



Tip Type

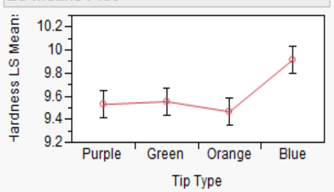
Leverage Plot



Least Squares Means Table

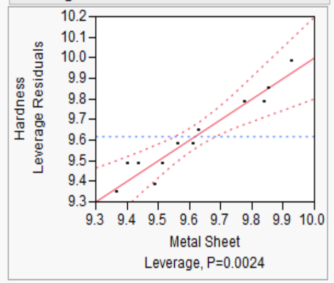
Level	Least Sq Mean	Std Error	Mean
Purple	9.5291667	0.04611655	9.43333
Green	9.5541667	0.04611655	9.53333
Orange	9.4666667	0.04611655	9.53333
Blue	9.9166667	0.04611655	9.96667

LS Means Plot



Metal Sheet

Leverage Plot



Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
1	9.4166667	0.04611655	9.46667
2	9.4666667	0.04611655	9.36667
3	9.6791667	0.04611655	9.70000
4	9.9041667	0.04611655	9.93333

Tips_Incomplete_Block - Fit Least Squares - JMP Pro

Response Hardness

Effect Summary

Summary of Fit

RSquare	0.968039
RSquare Adj	0.956053
Root Mean Square Error	0.076376
Mean of Response	9.616667
Observations (or Sum Wgts)	12

Parameter Estimates

Random Effect Predictions

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Metal Sheet	8.1428571	0.0475	0.0405933	0.0146907	0.8053602	89.063
Residual		0.0058333	0.0036893	0.0022729	0.0350893	10.937
Total		0.0533333	0.0406351	0.0180786	0.5666898	100.000

-2 LogLikelihood = -1.914296926

Note: Total is the sum of the positive variance components.

Total including negative estimates = 0.0533333

Covariance Matrix of Variance Component Estimates

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Tip Type	3	3	5.049	19.0014	0.0035*

Tips Incomplete Block.JMP

In the output window, click the red and save column-prediction formula.

In the data file, type

Orange for Tip type and 1 for metal sheet for row 13

Blue for Tip type and 2 for metal sheet for row 14

Green for Tip type and 3 for metal sheet for row 15

Purple for Tip type and 4 for metal sheet for row 16

Select Tables-Summary

Select Tip type-Group

Select Pred Formula Hardness-Statistics-Mean

then OK

	Tip Type	N Rows	Mean(Pred Formula...
1	Purple	4	9.5291666667
2	Green	4	9.5541666667
3	Orange	4	9.4666666667
4	Blue	4	9.9166666667

	Tip Type	Metal Sheet	Hardness	Pred Formula Hardness
1	Green	1	9.4	9.3541666667
2	Blue	1	9.7	9.7166666667
3	Purple	1	9.3	9.3291666667
4	Orange	2	9.4	9.3166666667
5	Purple	2	9.4	9.3791666667
6	Green	2	9.3	9.4041666667
7	Purple	3	9.6	9.5916666667
8	Orange	3	9.5	9.5291666667
9	Blue	3	10	9.9791666667
10	Blue	4	10.2	10.204166667
11	Orange	4	9.7	9.7541666667
12	Green	4	9.9	9.8416666667
13	Orange	1	•	9.2666666667
14	Blue	2	•	9.7666666667
15	Green	3	•	9.6166666667
16	Purple	4	•	9.8166666667

Tips Incomplete Block.JMP

LSMeans Differences Student's t

α= 0.050 t= 2.57058

LSMean[j]

LSMean[i]	Mean[i]-Mean[j] / Std Err Dif / Lower CL Dif / Upper CL Dif	Purple	Green	Orange	Blue
Purple	Mean[i]-Mean[j]	0	-0.025	0.0625	-0.3875
	Std Err Dif	0	0.06614	0.06614	0.06614
	Lower CL Dif	0	-0.195	-0.1075	-0.5575
	Upper CL Dif	0	0.14503	0.23253	-0.2175
Green	Mean[i]-Mean[j]	0.025	0	0.0875	-0.3625
	Std Err Dif	0.06614	0	0.06614	0.06614
	Lower CL Dif	-0.145	0	-0.0825	-0.5325
	Upper CL Dif	0.19503	0	0.25753	-0.1925
Orange	Mean[i]-Mean[j]	-0.0625	-0.0875	0	-0.45
	Std Err Dif	0.06614	0.06614	0	0.06614
	Lower CL Dif	-0.2325	-0.2575	0	-0.62
	Upper CL Dif	0.10753	0.08253	0	-0.28
Blue	Mean[i]-Mean[j]	0.3875	0.3625	0.45	0
	Std Err Dif	0.06614	0.06614	0.06614	0
	Lower CL Dif	0.21747	0.19247	0.27997	0
	Upper CL Dif	0.55753	0.53253	0.62003	0

Level			Least Sq Mean
Blue	A		9.9166667
Green		B	9.5541667
Purple		B	9.5291667
Orange		B	9.4666667

Levels not connected by same letter are significantly different.

Power Details

Test Tip Type

Power

α	σ	δ	Number	Power
0.0500	0.076376	0.16604	12	0.9893

Example

The steel company has determined that only three types of **Alloy** can be used at an one **Mill** because of production constraints; it is not possible to conduct a RCBD.

-Use a Custom Design platform to generate a randomized incomplete block design. Specify 3 runs per block. Choose 15 runs for this experiment. Is the generated design balanced? Why or why not?

-Use Rods Incomplete Block.jmp

Did the blocking factor help the experiment?

Are there differences in the mean strengths, compare all trts using a multiple comparison test that controls the FWER.

Are the SE for differences between the trt means the same? Why or why not?

Are the C.Is for the differences the same width? Why or why not?

Other Incomplete Designs

Youden square

Latin Square with one row (column) deleted

Each trt occurs same number of times in each row (column)

Columns (rows) for BIBD

Combination of Latin Square and BIBD

Partially BIBD (PBIBD)

Does not require each pair to occur together λ times.

Pair in associate class i appears together λ_i times

All trts have same number of i^{th} associates

Cyclic Designs

Includes some BIB and PBIB designs

Square, Cubic and Rectangular Lattices

Square: $t=k^2$

Cubic: $t=k^3$

Rectangular: $t=k*(k+1)$