Joseph Blubaugh
Stat 630
Hw08

1) 6.1.7

$$P(\theta) = \frac{e^{-\theta}\theta^x}{x!}$$

$$a)X = Poisson(\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\begin{aligned} L(\theta|x_1,...,x_n) &= \prod_{i=1}^{n}\frac{e^{-\theta}\theta^{x_i}}{x_i!} \\ &= \frac{e^{-\theta}\theta^x}{x!}, where, x = \sum_{i=1}^{n} x_i \end{aligned}$$

$$\begin{aligned} b)f_\theta &= h(s)g_\theta(T(s)) \\ f_\theta(x_1,...,x_n) &= \prod_{i=1}^{n}\frac{e^{-\theta}\theta^{x_i}}{x!} \\ &= \frac{e^{-\theta}\theta^{T(x_1,...,x_n)}}{T(x_1,...,x_n)}1 \\ where, T(x_1,...,x_n) &= \sum_{i=1}^{n} x_1, and \\ g_\theta(T) &= \frac{e^{-\theta}\theta^T}{T!}, and \\ h(x_1,...,h_n) &= 1 \end{aligned}$$

2) 6.1.19

$$\begin{aligned} f_\theta &= \frac{\theta^{a_0}}{\gamma(a_0)}x^{a_0-1}e^{-\theta x} \\ f_\theta &= h(x)g_\theta(T(x)) \\ &= \prod_{i=1}^{n}\frac{\theta^{a_0}}{\gamma(a_0)}x^{a_0-1}e^{-\theta x}1 \\ where, h &= 1 \\ T(x_1,...,x_n) &= \sum_{i=1}^{m} x_i \\ g_\theta(T) &= \frac{\theta^(a_0)}{\gamma(a_0)}\gamma^{a_0-1}e^{-\theta\gamma} \end{aligned}$$

3) 6.2.4

$$Y = Poisson(\theta) = \frac{e^{-\theta}\theta^x}{n!}$$
$$l(\theta|x_1, ..., x_n) = \sum_{i=1}^{n} log(f_\theta(x_i))$$
$$log(f_\theta(x) = -\theta + xlog(\theta) + log(\frac{1}{n}))$$
$$\frac{dl(\theta|x_1, ..., x_n)}{d\theta} = \frac{x}{\theta} - n = 0$$
$$\theta = \frac{x}{n}$$

$$b)Bias_\theta(T) = E(T) - \psi(\theta)$$
$$= \theta - \frac{x}{n}$$
$$variance = \theta$$
$$MSE = \theta - \theta - \frac{x}{n} = \frac{x}{n}$$

4) 6.2.5

$$f = a_0 log(\theta) + (a_0 - 1)log(x) - \theta x + log(\frac{1}{\gamma a_0})$$
$$f' = \frac{a_0}{\theta} - x = 0$$
$$\theta = \frac{a_0}{x}$$

$$b)bias = E_\theta(T) - \psi(\theta)$$
$$= \frac{a_0}{\theta} - \frac{a_0}{\theta} = 0$$

$$c)E(x) = a_0\theta, \theta = \frac{a_0}{E(x)}$$
$$\hat{\theta} = \frac{a_0}{\bar{x}}$$

5) 6.2.6

a) These are independent evnts of successes or failurs so we can use the negative binomial distribution

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\begin{aligned}
P_x(x) &= \binom{r-1-x}{x}\theta^r(1-\theta)^x where, r=1 \\
&= \binom{x}{x}\theta(\theta^x) = \theta^{x+1} \\
log(\theta^{x+1}) &= (x+1)log(\theta) \\
\frac{dl(\theta^{x+1})}{d\theta} &= \frac{x+1}{\theta} = 0
\end{aligned}
$$

$$
b)E(x) = \frac{(1-\theta)}{\theta}, \neq \frac{x+1}{\theta}
$$

6) 6.2.8

$$
\begin{aligned}
Weibull(\beta) &= \beta(1+x)^{-\beta-1} \\
l(\beta|x) &= log(\beta) + (-\beta-1)log(x+1) \\
\frac{dl(\beta|x)}{x\beta} &= \frac{1}{\beta} + \frac{(-\beta-1)}{1+x}
\end{aligned}
$$

7) 6.2.12

a) $mle = \hat{\sigma}^2 = \frac{n-1}{n}s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ any distribution with a mean μ and variance σ^2... They are the same

$$
\begin{aligned}
L(\mu, \sigma|x_1, ...x_n) &= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma}} e^{.5(\frac{x_i-\mu}{\sigma})^2} \\
l(\mu, \sigma) &= log(L(\mu, \sigma|x_1, ..., x_n)) \\
&= \frac{-n}{2}log(2\pi) - nlog(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2 \\
\frac{dl(\mu, \sigma)}{d\sigma} &= \frac{-n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(x_i - \mu)^2 = 0 \\
\sigma^2 &= \frac{1}{n}\sum i = 1^n(x_i - \hat{x})^2
\end{aligned}
$$

8) 6.2.19

$$AA = \theta, Aa = 2\theta(1-\theta), aa = (1-\theta)^2$$

$$a) Bernoulli Distribution$$

$$\begin{aligned} b) L(\theta|x_1, x_2, x_3) &= \theta^{x_1} 2\theta(1-\theta)^{x_2}(1-\theta)^{2x_3} \\ l(\theta|x_1, x_2, x_3) &= x_1 log(\theta) + x_2 log(2\theta - \theta^2) + 2x_3 log(1-\theta) \\ score &= \frac{x_1}{\theta} + \frac{x_2}{2\theta - \theta^2} + \frac{2x_3}{1-\theta} = 0 \end{aligned}$$

9) 6.3.15

a) yes
b) no

10) 6.3.24

$$\begin{aligned} \psi(\theta) &= aT_1 + (1-a)T_2 \\ Bias_\theta(T) &= E_\theta(T) - \psi(\theta) \\ 0 &= E_\theta(T) - \psi(\theta) \\ \psi(\theta) &= E_\theta(T) \end{aligned}$$

$$b) var_\theta(T_1) = aT_1, var_\theta(T_2) = T_2(1-a)$$

c) when a = .5, both T1 and T2 have the same weight and will minimize $T_1 + (1-a)T_2$ when T_1 and T_2 are unknown

10) additional

a) $x_1 ... x_n, exponential(\lambda) = \lambda e^{-\lambda x}$ $T_c = \frac{c}{\sum_{i=1}^n x_i}$, when C = 1 the estimator will have the smallest MSE because the expected mean $exponential(\lambda) = \frac{1}{\lambda}$