Homework 06
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STAT 608-720

1.

$$
\begin{aligned}
E[\hat{\beta}|X] &= Var((X'X)^{-1}X'Y) \\
&= (X'X)^{-1}X'Var(Y)X(X'X) \\
&= (X'X)^{-1}X'Var(X\beta + e)X(X'X)^{-1} \\
&= (X'X)^{-1}X'\Sigma X(X'X)^{-1} \\
&= (X'X)^{-1}X'\sigma^2 IX(X'X)^{-1} \\
&= \sigma^2(X'X)^{-1}X'X(X'X)^{-1} \\
&= \sigma^2(X'X)^{-1}
\end{aligned}
$$

2.

a) Weights might be neccesary if we were looking at variation in weight gain for a single diet, but since we are looking at multiple diets we want to fit multiple slopes so weighting is not neccesary.

b) A polynomial model might be a good fit in this case because there is a curving pattern in the data. I would use a polynomial(3) because higher order polynomials will fit closer to the data points than lower orders

c) The data points seem to spread out more as the dose increases and all of the points seem to be overlapping at dose = 0 so a good idea is to use a polynomial model with a single intercept and different slopes

d) $y_i = \beta_0 + \beta_1 x_i + \beta_2(iSource2)(iSource3)x_i$

3.

a)

$$
\mathbf{Y}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_10 \\ y_11 \\ y_12 \end{bmatrix} = \mathbf{A}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \mathbf{B}\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} + \mathbf{E}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ e_10 \\ e_11 \\ e_12 \end{bmatrix}
$$

$H_0 : \frac{\beta_1+\beta_2}{2} - \beta_3 = 0, H_1 : A\beta \neq h$

$F = \frac{(A\hat{\beta}-h)'(A(X'X)^{-1}A')^{-1}(A\hat{\beta}-h)(n-p-1)}{r(\hat{e}'\hat{e})}$

b) $H_0 : \frac{(\mu_1+\mu_2)}{2} - \mu_3 = \frac{5.6+7.9}{2} - 6.1 = .65 = 0$
$H_1 : .65 \neq 0$

$F = 0.101^2 = .01$

4.

a) 37.5 is the average for treatment group A. [-11.5 1 -27.7] is the difference between groups B, C, D and group A so the average values for B, C, and D are [26 38.5 9.8]

b)
1) X_1, X_2, X_3, X_4 are independent
2) Errors are normally distributed around 0
3) Variance is constant
4) Errors are independent of each other

c) B_2 is the difference between μ_B and μ_A so $-11.5 \pm (1.96 * 3.89) = (-19.1244, -3.8756)$

d) $37.5 - 11.5 = 26$