STAT 626, Summer 2016: Homeworks

NOTE: (i) The bonus problems get deeper into the concepts, will encourage you to study the relevant sections of of the textbook more systematically.
(ii) To get the bonus points for a problem, a complete solution should be given. No partial credits for the Bonus problems.

6. HW#6, Due Tuesday July 19, 2:15 PM CST,
   I. Do Problems 2.8, 4.16 (a)-(b), 5.6, 5.11, 5.13 (a), 5.15 from the text.

   II. Let $\{w_{t1}\}, \{w_{t2}\}, \{w_t\}$ be three independent WN$(0, 1)$ series and define

   $$y_{t1} = x_{t1} + \sum_{j=1}^{t} w_j, \quad y_{t2} = x_{t2} + 5\sum_{j=1}^{t} w_j,$$

   where

   $$x_{t1} = 0.5x_{t-1,1} + w_{t1}, \quad x_{t2} = 0.9x_{t-12,2} + w_{t2},$$

   are two causal AR time series.
   (a) Simulate and plot $n = 100$ values of the three time series $\{y_{t1}\}, \{y_{t2}\}$ and $z_t = 5y_{t1} - y_{t2}$. Do they appear to be stationary?
   (b) Compute the autocovariance function of $\{y_{t1}\}$. Is $\{y_{t1}\}$ stationary?
   (c) Compute the autocovariance function of the time series $z_t = 5y_{t1} - y_{t2}$. Is it stationary?
   (d) Compare and explain your findings in parts (a)-(c) regarding (non)stationarity of the three time series involved.
   (e) Compute the cross-covariance function and cross-correlation function (CCF) between $\{y_{t1}\}$ and $\{y_{t2}\}$, see Definition 1.5, p.21. Are the two time series $\{y_{t1}\}$ and $\{y_{t2}\}$ jointly stationary? (See Definition 1.10, p. 25.)

5. HW# 5, Due Wed. July 13, 2:15 PM CST,
   I. Do Problems 3.27, 3.31, 3.32, 3.35, 3.36, 3.39 (skip the forecast part), and 4.10.

4. HW#4, Due Wed. July 6, 2:15 PM CST,
   I. Do Problems 3.10, 3.15, 3.17, 3.18 and 3.20.

   II. (Bonus) Problem 3.42

3. HW# 3, Due **Tuesday** June 21, 2:15 PM CST,
   I. (a) Simulate two random walks $y_t, x_t, t = 1, \ldots, 100$, with initial values $x_0 = y_0 = 0$,

using two independent $N(0,1)$ white noises.

(i) Plot y_t vs x_t in the (x, y)-plane. Describe the pattern of dependence (if any) you notice in the scatterplot.

(ii) Consider the linear regression model $y_t = \beta_0 + \beta_1 x_t + w_t$, and the null hypothesis $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$ at $\alpha = 0.05$. Do you expect H_0 would be rejected? Why?

(iii) Perform the test and state your conclusion.

(b) Repeat the above experiment 1000 times and count the number of times H_0 is rejected in (iii). Does it support your expectation in (ii)? If not, find an explanation for this phenomenon based on possible violations of assumptions of inference in regression models. (Hint: Recall the three assumptions of inference for linear regression models: (a) Independence, (b) Homogeniety of variances, (c) Normality. Check whether they hold for the data here.)

II. Do Problems 2.9, 2.11, 3.4, 3.6 and 3.7 from the textbook.

2. HW# 2, Due June 15, 2:15 PM CST,
   Do Problems 1.8, 1.9,1.10, 1.15 and 2.3 from the textbook.

1. HW# 1, Due June 8, 2:15 PM CST,

   I. Do Problems 1.2, 1.4, 1.6 and 1.7 from the textbook.

   II. For regression problems one usually deals with pairs of observations $(x_i, y_i), i = 1, \cdots, n$. Show that

   1. $\sum_{i=1}^n (x_i - \bar{x}) = 0$, and $\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^n (x_i - \bar{x}) y_i$.

   2. $S_x^{-2} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^n c_i y_i$,

   where $c_i = \frac{x_i - \bar{x}}{S_x^2}$, $S_x^2 = \sum_{i=1}^n (x_i - \bar{x})^2$.

   3. (Bonus) For the simple linear regression model, $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$,

(a) derive the equations for minimizing the residual sum of squares (RSS) and give the formulae for the least squares estimates (LSE) $\hat{\beta}_0, \hat{\beta}_1$ of the regression coefficients.

(b) Show that $\hat{\beta}_1 = \sum_{i=1}^n c_i y_i$, with c_i's as above.

(c) Let $e_i = y_i - \hat{y}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i,\ i = 1, 2, \ldots, n$, be the estimated regression residuals. Show that

$$\sum_{i=1}^n e_i = 0, \text{ and } \sum_{i=1}^n e_i x_i = 0.$$

What is the (geometrical) interpretation of the above identities?