Homework 06
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STAT 642-720

1.

a)

$$Y_{ijkl} = \tau_i + \gamma_j + (\tau\gamma)_{ij} + \lambda_l + (\tau\lambda)_{ik} + (\gamma\lambda)_{jk} + (\tau\gamma\lambda)_{ijk} + a_{l(i)} + (\gamma a)_{j,l(i)} + (\lambda a)_{k,l(i)}$$

$$\tau_i = Signal$$
$$\gamma_j = Traffic$$
$$\lambda_k = MeasuredDevice$$
$$a_l = Intersection$$

b) Model AOV

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	20	6092.253	304.612	73.08	0.0023
Error	3	12.505	4.168	.	.
Corrected Total	23	6104.758	.	.	.

Source	DF	Type I SS	Mean Square	F Value	Pr > F
S	2	3143.023	1571.511	377.01	0.0002
T	1	236.881	236.881	56.83	0.0048
I(S)	3	2053.050	684.350	164.18	0.0008
S T	2	275.773	137.886	33.08	0.0090
T I(S)	3	96.370	32.123	7.71	0.0638
M	1	96.000	96.000	23.03	0.0172
S M	2	51.430	25.715	6.17	0.0865
M T	1	31.740	31.740	7.61	0.0702
M I(S)	3	96.015	32.005	7.68	0.0641
S M T	2	11.970	5.985	1.44	0.3652

Source	DF	Sum of Squares	Mean Square	Expected Mean Square
S	2	3143.023	1571.511	(2053.05 + 3143.0233) / 5 = 1039.215
T	1	236.881	236.881	(236.88 + (32.12 * 0.46)) / 4 = 62.913
S T	2	275.773	137.886	(137.88 + (32.12 * 0.54)) / 5 = 31.04
M	1	96.000	96.000	(96 + (96.015 * .65)) / 4 = 39.60
S M	2	51.430	25.715	(51.43 + (96.015 * .35)) / 5 = 17.00
M T	1	31.740	31.740	(31.74 + (12.505 * .134)) / 4 = 8.35
S M T	2	11.970	5.985	(11.97 + (12.505 * .05)) / 5 = 2.51
I(S)	3	2053.050	684.350	(2053.05 + 311.87) / 8.2014 = 288.35

Source	DF	Sum of Squares	Mean Square	Expected Mean Square
T I(S)	3	96.370	32.123	(96.37 + (12.505 * .408)) / 6 = 16.91
M I(S)	3	96.015	32.005	(96.015 + (12.505 * .406)) / 6 = 16.84
Residual	3	12.505	4.168	.

c)

d) With an $\alpha = .05$ the only significant interaction is between the type of signal and the type of traffic. The main effects of Method, Type of Signal, and Type of traffic are all individually significant, as is the specific intersection. Increasing α slightly would make several of the interactions significant.

e)

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    Source       SS Var.Prop
1       S 3143.023    51.6%
2       T  236.881     3.9%
3    I(S) 2053.050    33.7%
4      ST  275.773     4.5%
5   TI(S)   96.370     1.6%
6       M   96.000     1.6%
7      SM   51.430     0.8%
8      MT   31.740     0.5%
9   MI(S)   96.015     1.6%
10    SMT   11.970     0.2%
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2.

Source	DF	MS	Expected Mean Squares
A	3	24.5	$90\sigma_A^2 + 18\sigma_{A*B}^2 + 6\sigma_{A*C(B)}^2 + \sigma_e^2$
B	4	19.7	$72Q_B + 18\sigma_{A*B}^2 + 24\sigma_{C(B)}^2 + 6\sigma_{A*C(B)}^2 + \sigma_e^2$
A B	12	8.9	$18\sigma_{A*B}^2 + 6\sigma_{A*C(B)}^2 + \sigma_e^2$
C(B)	12	7.5	$24\sigma_{C(B)}^2 + 6\sigma_{A*C(B)}^2 + \sigma_e^2$
A C(B)	36	6.8	$6\sigma_{A*C(B)}^2 + \sigma_e^2$
Error	300	5.8	σ_e^2

a)

Source	DF	MS	F	P-value
A B	12	8.9	8.9 / 6.8 = 1.308	1 - pf(q = 1.308, df1 = 12, df2 = 36) = .256

b)

c) Using the Satterthwaite approximation

Source	DF	MS	F	P-value
B	4	19.7	19.7 / (8.9 + 7.5 - 6.8) = 2.052	1 - pf(q = 2.052, df1 = 4, df2 = 11.45332) = .153

d)

$$y_{.1..} = \mu + a_{.} + \beta_1 + c_{.(1)} + (a\beta)_{.1} + (ac)_{..(1)} + e_{.1..}$$
$$y_{.2..} = \mu + a_{.} + \beta_2 + c_{.(2)} + (a\beta)_{.2} + (ac)_{..(2)} + e_{.2..}$$
$$Var[\bar{y}_{.1..} - \bar{y}_{.2..}] = Var[\bar{c}_{.(1)} - \bar{c}_{.(2)}] + Var[\bar{a\beta}_{.1} - \bar{a\beta}_{.2}] + Var[\bar{ac}_{..(1)} - \bar{ac}_{..(2)}] + Var[\bar{e}_{.1..} - \bar{e}_{.2..}]$$
$$= \frac{2\sigma_a^2}{4} + \frac{2\sigma_{a\beta}^2}{4} + \frac{2\sigma_{ac(\beta)}^2}{12} + \frac{2\sigma_e^2}{72}$$
$$= \frac{36\sigma_a^2}{72} + \frac{36\sigma_{a\beta}^2}{72} + \frac{12\sigma_{ac(\beta)}^2}{72} + \frac{2\sigma_e^2}{72}$$
$$= \frac{1}{18}[18\sigma_a^2 + 18\sigma_{a\beta}^2 + 6\sigma_{ac(\beta)}^2 + \sigma_e^2]$$
$$= \frac{1}{18}19.7$$
$$Var = 1.094$$
$$DF = 4$$

e)

$$TukeyHSD = \frac{qtukey(p = .95, nmeans = 6, df = 4)(\sqrt{1.094})}{\sqrt{2}}$$
$$= \frac{(6.7064)(1.0459)}{\sqrt{2}}$$
$$= 4.959$$

3.

a) i. Interaction - F1*F2
    ii. Yes

b) i. Main Effect - F2
    ii. No - missing values

c) i. Main Effect - F2
    ii. No - missing values