STATISTICS 630 - Test II

July 10, 2012

Name _____ Email Address _____

INSTRUCTIONS FOR STUDENTS:

(1) There are six pages including this cover page and four formula sheets. Each of the five numbered problems is weighted equally.

(2) You have exactly 70 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper. Put the worked problems in numerical order for scanning.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc., unless otherwise specified.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 70 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature_____

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: _____

(2) Record the time at which the student ends the exam: _____

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until July 16, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature_____

1. A regional jet that can seat 36 passengers has a weight capacity of 7500 pounds. Suppose that the passengers that use this plane come from a population with a mean weight of 200 pounds and a standard deviation of 40 pounds. (Assume that this weight includes any baggage they may bring on board the plane.) Obtain the approximate probability that the weight of 36 randomly selected passengers from this population boarding the plane will exceed the weight capacity of the regional jet.

2. Suppose that (X, Y) have the joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} x+y & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise,} \end{cases}$$

and marginal probability density functions

$$f_X(x) = \begin{cases} \frac{1}{2} + x & 0 \le x \le 1, \\ 0 & \text{otherwise,} \end{cases} \qquad f_Y(y) = \begin{cases} \frac{1}{2} + y & 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find the values of $E(X),\ E(Y),\ \mathrm{Var}(X),\ \mathrm{Var}(Y)$, and $\mathrm{Cov}(X, Y)$.

3. The moment generating function of the Gamma(α, λ) distribution is

$$M(s) = \frac{\lambda^\alpha}{(\lambda - s)^\alpha}, s < \lambda.$$

Suppose that X and Y are independent random variables with the Gamma(α, λ) distribution. Obtain the moment generating functions of (i) $W = X + Y$ and (ii) $V = 5X$. Then use the moment generating function to identify the distributions of W and V.

4. Suppose that Z_1, Z_2, Z_3, Z_4 are independent standard normal random variables. Let $U = Z_1^2,\ V = Z_2^2 + Z_3^2 + Z_4^2$, and $W = 3U/V$. Identify the distributions of $U,\ V$, and W. Then derive $E(W)$.

5. Suppose that $X_1, \ldots, X_n$ are a random sample from a distribution with pdf

$$f(x|\theta) = \begin{cases} \frac{1}{\theta} x^{(1/\theta)-1}, & 0 < x < 1,\ \theta > 0 \\ 0 & \text{otherwise,} \end{cases}$$

and mean $E(X_i) = 1/(1+\theta)$. Find the maximum likelihood estimator estimator of θ and also the method of moments estimator of θ. Are they the same?