STATISTICS 642 - EXAM I - Partial Solutions - Summer 2014

Problem I. (30 points) Provide the details for each of the following items:

1. Type of Randomization: RCBD with sub-sampling
2. Type of Treatment Structure: 2x2 Factorial Treatment Structure with cross of method and gender
3. Identify each of the factors as being Fixed or Random:

- Method: Fixed effect with two levels (M1, M2), Gender: Fixed
- Blocking Factor: hospital (already given)

4. Describe the experimental units.

- Ward is the experimental unit.

5. Measurement units: Patients
6. There are 2 replications since there are 2 EUs measured under the same experimental condition (method) in each block (hospital)

Problem II. (30 points)

1. Using the results of Tukey's HSD procedure from the SAS output, the groups of cotton percentages are

   $G_1 = \{15\%, 35\%\}$, $G_2 = \{20\%, 35\%\}$, $G_3 = \{20\%, 25\%\}$, $G_4 = \{25\%, 30\%\}$.

2. Using $\alpha_{PC} = 1 - (1 - .05)^{1/4} = .01274$. The trend contrasts having p-values less than α_{PC} are the Quadratic(p-value$<$ .0001) and Cubic(p-value = .0124) trends.

3. Using the contrasts given in the SAS code, the hypotheses matrix is given by

$$H = \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 2 & -1 & -2 & -1 & 2 \\ -1 & 2 & 0 & -2 & 1 \\ 1 & -4 & 6 & -4 & 1 \end{pmatrix}$$

4. With $\alpha = .05, t = 5, \sigma_e^2 = 8, r = 13, D = 4; \Rightarrow \quad \lambda = \frac{rD^2}{2\sigma_e^2} = \frac{13\times 4^2}{2\times 8} = 13; \Phi = \sqrt{\lambda/t} = \sqrt{13/5} = 1.6$

- From Table IX on Page 607 with $\nu_1 = t-1 = 4, \nu_2 = t(r-1) = 60, \alpha = .05, \Phi = 1.6$, power is approximately .80 which is less than .90. Thus, r=13 reps is too small to achieve the power specification.

  From R, $\gamma(13) = P[F_{4,60} \geq F_{.05,4,60} | \lambda = 13] = 1 - pf(2.525215, 4, 60, 13) = .803$

Problem III. (40 points) Answer each of the following questions using at most **15 words**.

1. $\alpha_{PC} = 1 - (1 - .01)^{1/8} = .001256$
2. With $n_1 = 2, n_2 = 3, n_3 = 2, n_4 = 4$ and $C_1 = -3\mu_1 - \mu_2 + \mu_3 + 3\mu_4$ and $C_2 = -\mu_1 + 3\mu_2 - 3\mu_3 + \mu_4$ we have

- $\sum_{i=1}^4 c_i a_i = (-3)(-1) + (-1)(3) + (1)(-3) + (3)(1) = 0$ Thus the contrasts are orthogonal
- $\sum_{i=1}^4 c_i a_i / n_i = (-3)(-1)/2 + (-1)(3)/3 + (1)(-3)/2 + (3)(1)/4 = -1/4 \neq 0$. Thus, the contrasts' sum of squares are not independent.

3. The split-plot experimental units are not **randomly assigned** to the 10 levels of the Split-Plot Factor, Time.
4. The experimentwise Type I error rate for Scheffe is over **ALL** contrasts whereas the experimentwise Type I error rate for HSD is just for the $t(t-1)/2$ contrasts which are the pairwise differences of the t treatment means.

5. The effects models with $\tau_4 = 0$:

$$X = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \beta = \begin{pmatrix} \mu_3 \\ \mu_1 - \mu_3 \\ \mu_2 - \mu_3 \end{pmatrix}$$

6. LSD and SNK do not have either exact values for α_F nor upper bounds on α_F

7. $SS_{TREATMENT}$

8. $\tau_5 = 0 \Rightarrow$
   $\hat{\mu_1} = 3 + 1 = 4$
   $\hat{\mu_2} = 3 + 2 = 5$
   $\hat{\mu_3} = 3 + 4 = 7$
   $\hat{\mu}_4 = 3 - 8 = -5$
   $\hat{\mu}_5 = 3 + 0 = 3$