STATISTICS 642 - ASSIGNMENT #3 Solution

1. 16 points

   (a) (i) We have relatively homogeneous experimental units which are randomly assigned to the treatments in order to avoid any subjective assignment of treatments.

   (ii) A probability sample of units should be selected from available members of each treatment population. Units are selected from within each population such that each unit has an equal chance of entering sample. Note that each population represents a separate treatment classification, and random sampling is maintained only within the population.

   (b) The author on page 47 states "If the experimental errors are independent with a mean of zero and have homogeneous variances the least squares estimators are unbiased with minimum variance". This statement is somewhat incomplete in that what should be stated is " If the e_{ij}s are uncorrelated with mean 0 and the same variance then the least squares estimators are Best Linear Unbiased Estimators (BLUE) of the population parameters. If we further include the condition that the e_{ij}s are independent, normally distributed then the least squares estimators are Uniformly Minimum Variance Unbiased Estimators (UMVUE). Note the difference in these two statements. BLUEs have smallest variance amongst all unbiased estimators which are linear functions of the data and UMVUEs have smallest variance amongst all unbiased estimators no matter the form of the estimator.

   (c) y_{ij} have independent $N(\mu_i, \sigma_e^2)$ distributions for all i and j.

   (d) No test, that we have studied, would be valid. The permutation and rank based procedures would all require at least the condition "e_{ij}s are independently distributed with mean 0 and the same variance".

   We will discuss alternative procedures in Handout 4 for when this condition does not hold.

2. 14 points

a. To calculate the power of the $\alpha = .05$ test of

$H_o : \mu_{Premolt} = \mu_{Fasting} = \mu_{60g} = \mu_{80g} = 110 = \mu_{Mash}$;

when $\mu_{Premolt} = 90$, $\mu_{Fasting} = 100$, $\mu_{60g} = 120$, $\mu_{80g} = 110$, $\mu_{Mash} = 80$; and $\sigma_e^2 = 150$,

First compute the noncentrality parameter (ncp): $\lambda = \frac{\sum_{i=1}^{t} n_i(\mu_i - \bar{\mu}_.)^2}{\sigma_e^2}$

$\bar{\mu}_. = \frac{1}{n}\sum_{i=1}^{5} n_i\mu_i = \frac{1}{21}(4)(90) + (5)(100) + (3)(120) + (5)(110) + (4)(80) = 99.5238$

$\lambda = \frac{(4)(90-99.5238)^2+(5)(100-99.5238)^2+(3)(120-99.5238)^2+(5)(110-99.5238)^2+(4)(80-99.5238)^2}{150} = 24.635$

Power at $\lambda = 24.635$ is

$$\gamma(24.635) = P[F_{4,16} \geq qf(1 - .05, 4, 16)] = P[F_{4,16} \geq 3.007] = 1 - pf(3.007, 4, 16, 24.635) = .9463$$

using the noncentral F cdf function from R: $pf(x, df_1, df_2, ncp)$

- Alternatively, you could use the graphs in Table IX on page 607 in Kuehl's book with $\alpha = .05$, $\nu_1 = 4$, $\Phi = \sqrt{\frac{L}{t}} = \sqrt{\frac{24.635}{5}} = 2.22$, $\nu_2 = 16$, to obtain Power $\approx .94$

b. $\alpha = 0.01, \gamma_0 = 0.90, t = 5, \hat{\sigma}^2 = MSE = 150$, $D = 20$. By using the SAS code for Approach 5:

`repsize_Approach5.sas`,

alpha	gamma	t	r	u1	u2	L	phi	Power
0.01	0.9	5	12	4	55	16.0000	1.78885	0.70266
0.01	0.9	5	13	4	60	17.3333	1.86190	0.75675
0.01	0.9	5	14	4	65	18.6667	1.93218	0.80326
0.01	0.9	5	15	4	70	20.0000	2.00000	0.84256
0.01	0.9	5	16	4	75	21.3333	2.06559	0.87527
0.01	0.9	5	17	4	80	22.6667	2.12916	0.90211

$\lambda \geq \frac{rD^2}{2\hat{\sigma_e}^2} = L$. Find smallest r such that $\gamma(L) \geq \gamma_0 = 0.90 \Rightarrow r = 17$

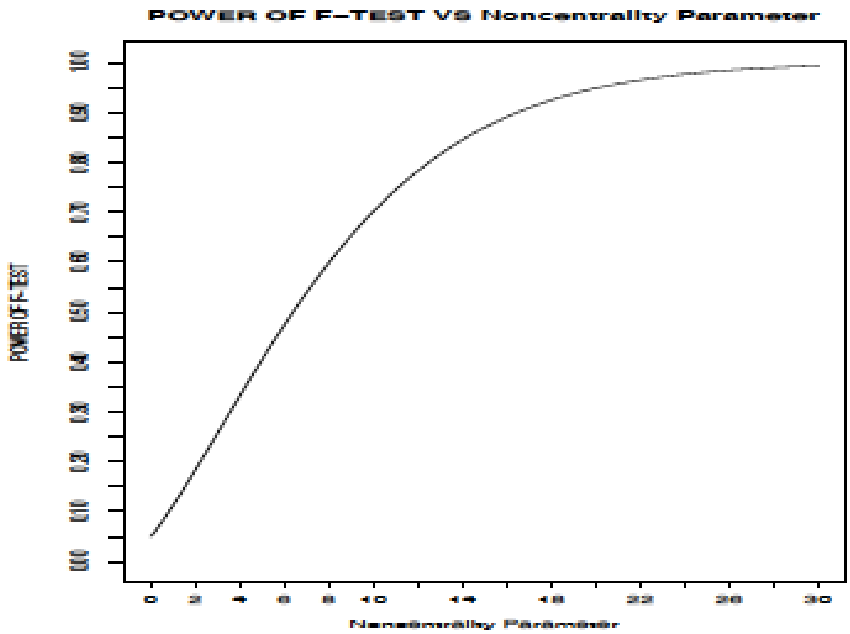
- Using the graph on page 607 in Kuehl's book, $\nu_1 = t - 1 = 4$, with $\alpha = .01, t = 5, D = 20, \hat{\sigma_e}^2 = 150$,

$\Phi = \sqrt{\frac{rD^2}{2t\hat{\sigma_e}^2}} = \sqrt{\frac{r(20)^2}{2(5)(150)}} = .5164\sqrt{r}$, $\nu_2 = t(r-1) = 5(r-1)$

For $r = 12, 13, 14, 15, 16, 17$ the above values of power are obtained from the graphs using $\Phi = .5164\sqrt{r}$ and $\nu_2 = 5(r-1)$. Thus, r must be at least 17 to have power of at least .90.

3. 13 points

a. The power curve can be obtained from the R program in Dostat/Files/Rcode: **powerf2.R** with $t = 3,\ n = 15,\ \alpha = .05,\ \sigma_\varepsilon^2 = 12\ \lambda = seq(0, 30.01)$



b. The μ_i are specified as $\mu_1 = 16, \mu_2 = 17, \mu_3 = 19$ and $\alpha = 0.05$.

From data in Problem 1, $\gamma_0 = 1 - 0.20 = 0.80,\ t = 3,\ \hat{\sigma_e}^2 = MSE = 12$.

noncentrality parameter $= \lambda = \frac{r}{\sigma^2}\sum_{i=1}^{t}(\mu_i - \bar{\mu}.)^2 = \frac{r}{\hat{\sigma_e}^2}\{(16 - \frac{52}{3})^2 + (17 - \frac{52}{3})^2 + ((19 - \frac{52}{3})^2\} = \frac{4.6667r}{12}$.

By using the SAS code for Approach 3:

`repsize_Approach3.sas`,

alpha	gamma	t	r	u1	u2	L	Phi	Power
0.05	0.8	3	21	2	60	8.1667	2.72222	0.70387
0.05	0.8	3	22	2	63	8.5556	2.85185	0.72641
0.05	0.8	3	23	2	66	8.9444	2.98148	0.74759
0.05	0.8	3	24	2	69	9.3333	3.11111	0.76745
0.05	0.8	3	25	2	72	9.7222	3.24074	0.78603
0.05	0.8	3	26	2	75	10.1111	3.37037	0.80337

Find smallest r s.t. $\gamma(L) \geq 0.80$. Then, $r = 26$.

- Alternatively using the table on page 605 in the textbook, with $\nu_1 = t - 1 = 2$

  $\nu_2 = n - t = 3(r-1),\ \alpha = .01,\ \Phi = \sqrt{\lambda/t} = \sqrt{\frac{4.6667r}{(3)(12)}} = 0.36\sqrt{r}$,

  you could compute the values of power for r = 21 to 26 as given above and hence conclude that r must be at least 26 to achieve a power of at least .8.

4. 10 points

a. $y_{ij} = \mu_i + e_{ij},\ i = 1,2,3;\ j = 1,\cdots,n_i\ n_1 = 12,\ n_2 = 14,\ n_3 = 11$ where y_{ij} is the response from the jth employee from the ith division, μ_i is the mean response in ith division and e_{ij} is the experiment error with $e_{ij} \overset{iid}{\sim} N(0,\sigma_e^2)$.

b. $\bar{y}_{..} = \frac{1}{n}\sum_{i=1}^{3} n_i\bar{y}_{i.} = \frac{1}{37}[12 * 25.2 + 14 * 32.6 + 11 * 28.1] = 28.86216$

$SS_{TRT} = \sum_{i=1}^{3} n_i(\bar{y}_{i.} - \bar{y}_{..})^2 = 12(25.2 - 28.86216)^2 + 14(32.6 - 28.86216)^2 + 11(28.1 - 28.86216)^2 = 362.927$

$SSE = \sum_{i=1}^{3}(n_i - 1)S_i^2 = (12 - 1)3.6 + (14 - 1)4.8 + (11 - 1)5.3 = 155$

$F = \frac{SS_{TRT}/(t-1)}{SSE/(n-t)} = \frac{362.927/(3-1)}{155/(37-3)} = 39.8049 \Rightarrow p - value = P[F \geq 39.8049] = 1 - pf(39.8049, 2, 34) = 1.239 \times 10^{-9}$ There is significant evidence (p-value $<$ .0001) that the mean acceptance of the plan is different for the three divisions of the company.

5. 20 points

(A) The answer to this question depends on what constraint is placed on the τ_is:

i. For $\sum_{i=1}^{t} n_i\tau_i = 0$, then $\hat{\tau}_4 = -\frac{1}{n_4}\sum_{i=1}^{3} n_i\tau_i = -\frac{1}{3}[(6)(-2.3) + (3)(-1.7) + (5)(1.8)] = 3.3$

ii. For $\tau_t = 0$, then $\tau_4 = 0$.

(B) The randomization provides a reference distribution for statistical inference. Also, the randomization will help to avoid having biases in the experiment due to the assignment of the best experimental units to "preferred" treatments.

(C) Because each manufacturer only supplied one container of paint, there is only a single replication of the four treatments. Therefore, it would not be possible to estimate the variation in the effectiveness of the four paints.

(D) The restriction used in SAS is to set the effect of the last treatment, P5, equal to 0. Thus we have $\hat{\tau}_1 = -\hat{\mu}_1 - \hat{\mu}_5 = -2.3$ which can be interpreted as Program P1 has a mean response which is 2.3 units less than the mean response of program P5.

(E) $\alpha = .01$, $\nu_1 = t - 1 = 4$, $\nu_2 = t(r-1)$, $\hat{\sigma}_e = 2$, $D = 5.1$, $\gamma_o = .95$.

By using the SAS code for Approach 5:

`repsize_Approach5.sas,`

we have the following results:

alpha	gamma	t	r	u1	u2	L	phi	p
0.01	0.95	5	6	4	25	19.5075	1.97522	0.73483
0.01	0.95	5	7	4	30	22.7588	2.13348	0.84315
0.01	0.95	5	8	4	35	26.0100	2.28079	0.91263
0.01	0.95	5	9	4	40	29.2613	2.41914	0.95378

Thus, we need r=9 reps to achieve a power greater than or equal to 0.95.

- Using the graph on page 607 in Kuehl's book, $\nu_1 = t - 1 = 4$, with $\alpha = .01, t = 5, D = 5.1, \hat{\sigma_e} = 2$,

$\Phi = \sqrt{\frac{rD^2}{2t\hat{\sigma_e}^2}} = \sqrt{\frac{r(5.1)^2}{2(5)(2)^2}} = .8064\sqrt{r},\ \ \nu_2 = t(r-1) = 5(r-1)$

For $r = 6,7,8,9$ the above values of power are obtained from the graphs using

$\Phi = .5164\sqrt{r}$ and $\nu_2 = 5(r-1)$. Thus, r must be at least 9 to have power of at least .95.

6. 3 points each

(C_1) $30°C$ vs average of 4 other Temperatures: $C_1 = \mu_5 - \frac{1}{4}(\mu_1 + \mu_2 + \mu_3 + \mu_4)$ or

$$C_1 = -\mu_1 - \mu_2 - \mu_3 - \mu_4 + 4\mu_5$$

Using the coefficients in Table XI on page 623 in Kuehl's book with t=5:

(C_2) Linear Trend: $C_2 = -2\mu_1 - \mu_2 + \mu_4 + 2\mu_5$

(C_3) Quadratic Trend: $C_3 = 2\mu_1 - \mu_2 - 2\mu_3 - \mu_4 + 2\mu_5$

(C_4) Cubic Trend: $C_4 = -\mu_1 + 2\mu_2 - 2\mu_4 + \mu_5$

a. There is equal replication so we only have to evaluate $\sum_{i=1}^{5} k_i d_i = 0$ for the 6 pairs of contrasts:

- C_1 and C_2 are not orthogonal: $\sum_{i=1}^{5} k_i d_i = (-1)(-2) + (-1)(-1) + (-1)(0) + (-1)(1) + (4)(2) = 10 \neq 0$
- Similarly, we can show that C_1 and C_3 ($\sum_{i=1}^{5} k_i d_i = 10$) are not orthogonal and C_1 and C_4 ($\sum_{i=1}^{5} k_i d_i = 5$) are not orthogonal
- However, $\sum_{i=1}^{5} k_i d_i = 0$ for the pairs (C_2, C_3), (C_2, C_4), (C_3, C_4)

b. SAS output needed for this problem:

Dependent Variable: C   Ecoli Concentration

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	39.77880000	9.94470000	18.59	<.0001
Error	45	24.07400000	0.53497778		
Corrected Total	49	63.85280000			

Least Squares Means

T	C LSMEAN	Standard Error	Pr > |t|	LSMEAN Number
10	8.3200000	0.2312959	<.0001	1
15	8.6200000	0.2312959	<.0001	2
20	9.8500000	0.2312959	<.0001	3
25	9.9200000	0.2312959	<.0001	4
30	10.7300000	0.2312959	<.0001	5

From the above, we have $\hat{\sigma}_e^2 = \sqrt{MSE} = \sqrt{.53498} = .731422$

The estimates of the four contrasts are computed as $\hat{C}_h = \sum_{i=1}^{5} c_{ih}\bar{y}_{i.}$

with estimated standard errors of the estimates given by $\hat{SE}(\hat{C}_h) = \hat{\sigma}_e \sqrt{c_{ih}^2/n_i}$

For example, $\hat{C}_1 = [(-1)(8.32) + (-1)(8.62) + (-1)(9.85) + (-1)(9.92) + (4)(10.73)] = 6.21$

$\hat{SE}(\hat{C}_1) = (.731422)\sqrt{\frac{1}{10}[(-1)^2 + (-1)^2 + (-1)^2 + (-1)^2 + (4)^2]} = 1.034$ The least square estimates of the contrasts, $\hat{C}_h$, and estimated Standard Error,$\hat{SE}(\hat{C}_h)$ from SAS:

Contrast	Estimate	Error	t Value	p-value
C1: 30 VS AVE REST	6.21000000	1.03438656	6.00	<.0001
C2: LINEAR TREND	6.12000000	0.73142175	8.37	<.0001
C3: QUADRATIC TREND	-0.14000000	0.86542989	-0.16	0.8722
C4: CUBIC TREND	-0.19000000	0.73142175	-0.26	0.7962

c. 95% SCI for the four contrasts as given by Scheffé Procedure:

$\hat{C}_1 \pm \widehat{SE}(\widehat{C_1})\sqrt{(t-1)F_{.05,t-1,n-t}} = 6.21 \pm (1.03438656)\sqrt{4(2.579)} = 6.21 \pm 3.322 = (2.89, 9.53)$

Similarly, we would have the following for the other three contrasts:

$C_2 : (3.77, 8.47)$; $\quad C_3 : (-2.92, 2.64)$; $\quad C_4 : (-2.54, 2.16)$

Using the above Scheffé confidence intervals, we can conclude that there is significant evidence that contrasts C_1 and C_2 are different from 0 because 0 is not contained in its C.I. However, there is not significant evidence that contrasts C_3 and C_4 are different from 0 because 0 is contained in there C.I.'s.

d. We can conduct the Bonferroni comparisons by comparing the p-values from the four t-tests listed in the SAS output above with the familywise error rate $\alpha_F = .05/4 = .0125$. The p-values for contrasts C_1 and C_2 are less than .0125 so we would conclude that there is significant evidence that these two contrasts are different from 0. However, the p-values for contrasts C_3 and C_4 are greater than .0125 so we would conclude that there is not significant evidence that these two contrasts are different from 0.

When the number of contrasts being compared is relatively small, it is probably advisable to use the Bonferroni procedure instead of Scheffé, although, in this example, the the Bonferroni and Scheffé procedures agreed because two of the contrasts had extreme evidence (very small p-values) that they were different from 0 and the other two contrasts had extreme evidence (very large p-values) that they were not different from 0.

e. The test of $H_o : C_2 = 0, C_3 = 0, C_4 = 0$ versus H_1 : At least one $C_i \neq 0$

is obtained by formulating the Hypothesis matrix:

Test $H_o : \mathbf{H}\boldsymbol{\mu} = 0$ vs $H_o : \mathbf{H}\boldsymbol{\mu} \neq 0$ with

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{pmatrix}; \quad \mathbf{H} = \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 2 & -1 & -2 & -1 & 2 \\ -1 & 2 & 0 & -2 & 1 \end{pmatrix}; \quad \hat{\boldsymbol{\mu}} = \begin{pmatrix} \bar{y}_{1.} \\ \bar{y}_{2.} \\ \bar{y}_{3.} \\ \bar{y}_{4.} \\ \bar{y}_{5.} \end{pmatrix} = \begin{pmatrix} 8.32 \\ 8.62 \\ 9.85 \\ 9.92 \\ 10.73 \end{pmatrix};$$

$$(\mathbf{X}^T\mathbf{X}) = Diag(n_1, n_2, n_3, n_4, n_5) = \begin{pmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{pmatrix}$$

The sum of squares associated $\mathbf{H}$ is given by

$$\begin{aligned} SS_{\mathbf{H}} &= (\mathbf{H}\hat{\boldsymbol{\mu}} - \mathbf{0})^T \left(\mathbf{H}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{H}^T\right)^{-1} (\mathbf{H}\hat{\boldsymbol{\mu}} - \mathbf{0}) \\ &= \begin{pmatrix} 6.12 \\ -0.14 \\ -0.19 \end{pmatrix}^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5/7 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 6.12 \\ -0.14 \\ -0.19 \end{pmatrix} \\ &= 37.5045 \end{aligned}$$

The test statistic for testing $H_o : \mathbf{H}\boldsymbol{\mu} = 0$ vs $H_o : \mathbf{H}\boldsymbol{\mu} \neq 0$ is given by

$$F = \frac{SS_H/k}{MSE} = \frac{37.5045/3}{.53498} = 23.37 \Rightarrow \text{ p-value} = P[F_{3,45} \geq 23.37] = 1 - pf(23.37, 3, 45) = 2.9 \times 10^{-9}$$

Thus, there is significant evidence (p-value $= 2.9 \times 10^{-9}$) that at least one of the contrasts C_2, C_3, C_4 is not different from 0.

The following R-code would yield $SS_{\mathbf{H}}$

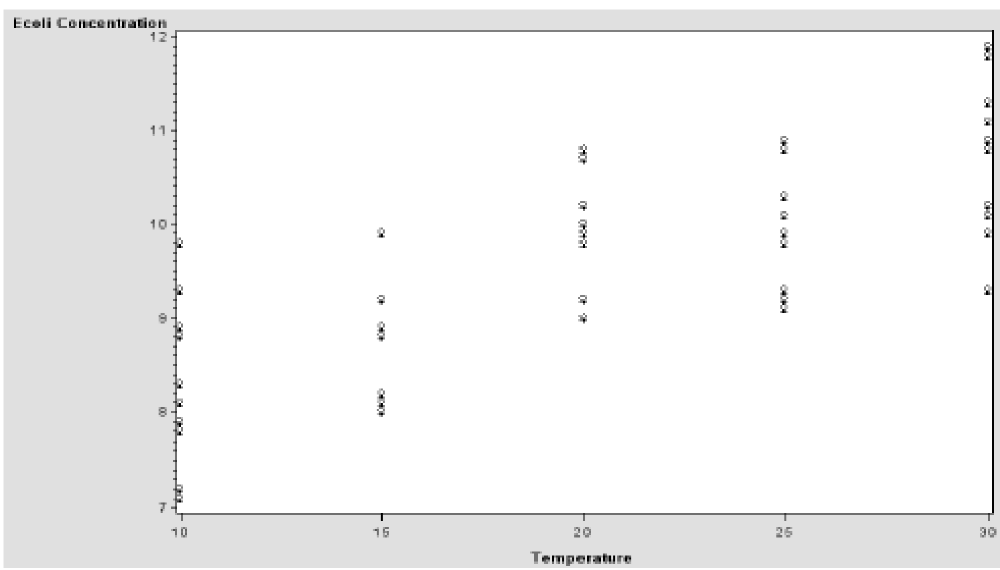
```
library(MASS)
H = matrix(c(-2,-1,0,1,2,2,-1,-2,-1,2,-1,2,0,-2,1),nrow=3,byrow=TRUE)
muhat = matrix(c(8.32,8.62,9.85,9.92,10.73),nrow=5,byrow=TRUE)
h = matrix(c(0,0,0),nrow=3,byrow=TRUE)
D = diag(c(10,10,10,10,10),5,5)
SSH = t(H%*%muhat - h) %*% ginv((H%*%ginv(D)%*%t(H))) %*% (H%*%muhat -  h)
SSH
37.5045
```

From the SAS output we can confirm our calculations by examining the F value and corresponding p-value for the contrast COMBINED TRENDS :

Dependent Variable: C   Ecoli Concentration

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
30 VS REST	1	19.28205000	19.28205000	36.04	<.0001
LINEAR TREND	1	37.45440000	37.45440000	70.01	<.0001
QUADRATIC TREND	1	0.01400000	0.01400000	0.03	0.8722
CUBIC TREND	1	0.03610000	0.03610000	0.07	0.7962
COMBINED TRENDS	3	37.50450000	12.50150000	23.37	<.0001

f. Based on the tests of hypotheses (both the cubic and quadratic trends are not significant but the linear trend is significant), the LSE of the contrasts (the sign of the linear trend is positive), and the scatterplot of the data, I would conclude that there is significant evidence of an increasing trend in the mean **e. coli** concentration with increases in temperature.



7. 3 points each

a. Use the **e. coli** data with Hsu's procedure: "best"=the temperature having the smallest mean **e. coli** concentration.
$m_i = min_{j \neq i}(\bar{y}_{j\cdot}),\ d(0.05, 4, 45) = 2.225,\ k_i = m_i + (2.225)\sqrt{.53498}\sqrt{\frac{2}{10}} = m_i + .7278.$

Treatment	$\bar{y}_{i\cdot}$	m_i	$k_i = m_i + .7278$
30	10.73	8.32	9.0478
25	9.92	8.32	9.0478
20	9.85	8.32	9.0478
15	8.62	8.32	9.0478
10	8.32	8.62	9.3478

With a probability of 0.95, the group of temperatures with smallest mean e. coli concentration are $10°C$ and $15°C$ because both of these temperatures have $\bar{y}_{i.} < k_i$

b. Use a 1-sided Dunnett's procedure. The parameters of interest are $\mu_i - \mu_{30}$, $i = 1, 2, 3, 4$, where μ_{30} is the mean e. coli concentration at $30°C$ and we want to test the research hypotheses: $H_1 : \mu_i < \mu_{30}$.

$\bar{y}_{1.} = 8.32,\ \bar{y}_{2.} = 8.62,\ \bar{y}_{3.} = 9.85,\ \bar{y}_{4.} = 9.92,\ \bar{y}_{30} = 10.73,\ d1(0.05, 4, 45) = 2.222$

Using the R function in library(mtvnorm),

d1(.05,4,45) = qmvt(.95,tail="lower.tail",df=45,corr=matrix(rep(.5,16),4)+diag(4)*.5)$quantile = 2.221608

$D1(4, 45) = d(0.05, 4, 45)\sqrt{MSE}\sqrt{\frac{2}{r}} = (2.222)\sqrt{.53498}\sqrt{\frac{2}{10}} = .727.$

Thus our decision rule is: State $\mu_i < \mu_{30}$ if $\hat{\mu}_i - \hat{\mu}_{30} < -.727$

```
Comparison    Difference in  Means           Conclusion
 25 - 30          -0.8100            Significant evidence that Mean of 25 is Less than Mean of 30
 20 - 30          -0.8800            Significant evidence that Mean of 20 is Less than Mean of 30
 15 - 30          -2.1100            Significant evidence that Mean of 15 is Less than Mean of 30
 10 - 30          -2.4100            Significant evidence that Mean of 10 is Less than Mean of 30
```

Using the SAS code: **MEANS TRT/DUNNETTL('30');**

```
Dunnett's One-tailed t Tests for L
Alpha                                    0.05
Error Degrees of Freedom                   45
Error Mean Square                    0.534978
Critical Value of Dunnett's t        2.22241
Minimum Significant Difference         0.727
     T                Between      Simultaneous 95%
Comparison            Means        Confidence Limits
 25 - 30             -0.8100      -Infinity  -0.0830  ***
 20 - 30             -0.8800      -Infinity  -0.1530  ***
 15 - 30             -2.1100      -Infinity  -1.3830  ***
 10 - 30             -2.4100      -Infinity  -1.6830  ***
```

There is significant evidence ($\alpha = .05$) that all four temperatures have a mean e. coli concentration that is less than the mean e. coli concentration at $30°C$.

c. Tukey's HSD $= q(\alpha_e, t, \nu_2)\hat{\sigma}_e\sqrt{1/r} = qtukey(1 - .05, 5, 45)\sqrt{MSE}\sqrt{1/10} = 4.018\sqrt{.53498}\sqrt{1/10} = .929$

Declare there is significant evidence that a pair of treatment means ($\mu_i,\ \mu_h$) are different if $|\bar{y}_{i.} - \bar{y}_{h.}| > .929$

```
Comparison  Difference in Means    Conclusion
 30  - 25         0.8100         Not significant evidence of a difference in the pair of means
 30  - 20         0.8800         Not significant evidence of a difference in the pair of means
 30  - 15         2.1100         Significant evidence of a difference in the pair of means
 30  - 10         2.4100         Significant evidence of a difference in the pair of means
 25 - 20          0.0700         Not significant evidence of a difference in the pair of means
 25 - 15          1.3000         Significant evidence of a difference in the pair of means
 25 - 10          1.6000         Significant evidence of a difference in the pair of means
 20 - 15          1.2300         Significant evidence of a difference in the pair of means
 20 - 10          1.5300         Significant evidence of a difference in the pair of means
 15 - 10          0.3000         Not significant evidence of a difference in the pair of means
```

- The above results can be obtained using the SAS code: **MEANS TRT/Tukey;**

```
Tukey's Studentized Range (HSD) Test for L
NOTE: This test controls the Type I experimentwise error rate.
Alpha                                   0.05
Error Degrees of Freedom                  45
Error Mean Square                   0.534978
Critical Value of Studentized Range  4.01842
Minimum Significant Difference        0.9294

               Difference
    T             Between     Simultaneous 95%
Comparison          Means    Confidence Limits
 30  - 25          0.8100     -0.1194   1.7394
 30  - 20          0.8800     -0.0494   1.8094
 30  - 15          2.1100      1.1806   3.0394  ***
 30  - 10          2.4100      1.4806   3.3394  ***
 25 - 20           0.0700     -0.8594   0.9994
 25 - 15           1.3000      0.3706   2.2294  ***
 25 - 10           1.6000      0.6706   2.5294  ***
 20 - 15           1.2300      0.3006   2.1594  ***
 20 - 10           1.5300      0.6006   2.4594  ***
 15 - 10           0.3000     -0.6294   1.2294
```

We thus conclude there is significant evidence ($\alpha_F = .05$) that the following pairs of Thicknesses are different:

(30,15), (30,10), (25,15), (25,10), (20,15), (20,10)

The groupings would be given as

Temperature $^{\circ}C$				
30	25	20	15	10
a	a	a	b	b