STATISTICS 630 - Final Exam
July 26, 2013

Name ____________________________________ Email Address ____________________________

INSTRUCTIONS FOR STUDENTS:

(1) There are 8 pages including this cover page and 5 formula sheets. Each of the six numbered problems is weighted equally.

(2) You have exactly 120 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 120 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature__________________________________

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: ______________________

(2) Record the time at which the student ends the exam: ______________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until August 7, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature__________________________________

1. A manufacturer of booklets packages them in boxes of 100. Suppose that the weight X (in ounces) of a single booklet has a distribution with moment generating function

$$M_X(s) = \exp\left(s + \frac{s^2}{800}\right).$$

   (a) Use the moment generating function to show that

$$E(X) = 1 \quad \text{and} \quad \mathrm{Var}(X) = \frac{1}{400}.$$

   (b) Suppose that $X_1, \ldots, X_{100}$ are independent random variables representing the weights of 100 books with the above mean and variance. Obtain an expression for the approximate probability that the total weight of the 100 booklets exceeds 100.5 ounces.

2. Let $X_1, \ldots, X_n$ be a random sample from the Poisson(θ) distribution for $\theta > 0$ with probability mass function

$$f(x|\theta) = \begin{cases} \frac{e^{-\theta}\theta^x}{x!}, & x = 0, 1, 2, ..., \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that θ has the prior density

$$\pi(\theta) = \begin{cases} 4\theta^2 e^{-2\theta}, & \theta > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Obtain the posterior distribution of θ given $X_1 = x_1, \ldots, X_n = x_n$. Then obtain the mean and variance of the posterior distribution.

3. Let $Z_1, \ldots, Z_7$ be independent standard normal random variables. Identify the distribution completely of each of the following random variables. Be sure to explain your reasoning.

   (a) $U = 5Z_1 - 6Z_2 + 3$.
   (b) $V = Z_1^2 + Z_2^2 + Z_4^2 + Z_7^2$.
   (c) $Y = Z_7/\sqrt{[Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2 + Z_6^2]/6}$.
   (d) $W = (Z_1^2 + Z_2^2 + Z_3^2)/(Z_4^2 + Z_5^2 + Z_6^2)$.

4. Suppose that the number of students (Y) enrolling in STAT 630 in a semester is a negative binomial random variable with parameters $r = 20$ and $\theta = 1/4$. Each student that enrolls passes STAT 630 with probability $\pi = 0.8$. Assume that the students' performances are independent and hence, assume that conditional on $Y = y$, the number of students X passing STAT 630 in a semester has a binomial(y, π) distribution. Find the unconditional mean and variance of X, the number of students passing STAT 630 in a semester.

5. Suppose that $X_1, \ldots, X_n$ are a random sample from a distribution with probability mass function

$$f(x|\theta) = \begin{cases} \frac{e^{-\theta^2}\theta^{2x}}{x!}, & x = 0, 1, 2, \ldots, \ 0 < \theta < \infty \\ 0 & \text{otherwise,} \end{cases}$$

and mean $E(X_i) = \theta^2$. In Test 2, you found that the maximum likelihood estimator is $\hat{\theta} = \sqrt{\sum_{i=1}^{n} X_i/n}$.

(a) Obtain Fisher's information for θ.

(b) Use Fisher's information to construct an approximate level γ confidence interval for θ based on the maximum likelihood estimator.

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6. Suppose that you have a single observation V from the beta(θ,1) distribution with probability density function

$$f(v|\theta) = \begin{cases} \theta v^{\theta-1}, & 0 < v < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) You are interested in testing $H_0 : \theta = 1$ versus $H_a : \theta = 2$. Suppose that you reject H_0 whenever $V > 0.9$. Determine the size (or level) of this test. Then determine the power of this test.

(b) Is the test in part (a) most powerful of its size for testing $H_0 : \theta = 1$ versus $H_a : \theta = 2$? Is this test uniformly most powerful for testing $H_0 : \theta = 1$ versus $H_a : \theta > 1$? Be sure to justify your answer.