STAT 630–Formulas for Test 1

Axioms of Probability

(i) $P(A) \geq 0$ for any event A. (ii) $P(\mathcal{S}) = 1$

(iii) For mutually exclusive events $A_1, A_2, \ldots$, $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$.

Probability of a Union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Permutations: $P_{k,n} = n(n-1)\cdots(n-k+1)$

Combinations: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Conditional Probability: $P(A|B) = \frac{P(A\cap B)}{P(B)}$ when P(B) > 0

Independent Events: Events A and B are independent if $P(A \cap B) = P(A) \times P(B)$

Law of Total Probability and Bayes Theorem: For mutually exclusive and exhaustive events $B_1, \ldots, B_n$ and any event A with $P(A) > 0$,

$$P(A) = \sum_{j=1}^{n} P(A|B_j)P(B_j) \quad \text{and} \quad P(B_i|A) = \frac{P(B_i)P(A|B_i)}{\sum_{j=1}^{n} P(A|B_j)P(B_j)}.$$

Probability Mass Function of a Discrete Random Variable

The probability mass function p of a discrete random variable X is $p_X(x) = P(X = x)$ for all x.

Discrete Uniform PMF: $p_X(x) = \frac{1}{N},\ x = 1, 2, \ldots N$.

Binomial PMF: $p_X(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x},\ x = 0, 1, 2, \ldots, n$

Negative Binomial PMF: $p_X(x) = \binom{r+x-1}{r-1}\theta^r(1-\theta)^x,\ x = 0, 1, 2, \ldots,$ Special case: geometric distribution when $r = 1$

Geometric Sum: $\sum_{x=0}^{\infty} \alpha^x = \frac{1}{1-\alpha},$ for $0 < \alpha < 1$

Hypergeometric PMF:

$$p_X(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}},\ x = M_1, \ldots, M_2,$$

where $M_1 = \max(0, n - (N - M))$, and $M_2 = \min(n, M)$.

Poisson PMF: $p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots$

PDF of a Continuous Random Variable:

The pdf f_X of a continuous random variable X is a function $f_X(x) \geq 0$ such that

$$P(a < X < b) = \int_a^b f_X(x)\,dx \quad \text{for all } a \leq b.$$

Uniform PDF: $f_X(x; L, R) = \frac{1}{R-L},$ for $L \leq x \leq R$

Normal PDF: $f_X(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \ -\infty < x < \infty.$

Gamma PDF:

$$f_X(x|\alpha,\lambda) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}\, I_{(0,\infty)}(x).$$

Special cases: exponential distribution when $\alpha = 1$, Erlang distribution when $\alpha = r$, a positive integer.

Beta PDF:

$$f_X(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}, \ 0 \le x \le 1, \ a > 0, \ b > 0.$$

Cumulative Distribution Function

The cdf of a random variable X is a function F_X with domain the entire real number line. It is defined as

$$F_X(x) = P(X \le x) \quad \text{for each } x.$$

Relationship of CDF and PDF for a Continuous RV

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt, \qquad f_X(x) = \frac{d}{dx} F_X(x).$$

Quantile Function

For a continuous random variable X with strictly increasing cdf F_X, the quantile function of X is $Q_X(u) = F_X^{-1}(u)$ for each $0 < u < 1$.

Function of a Discrete RV

Let $Y = h(X)$ where X is a discrete rv with pmf $p_X(x)$. Then the pmf of Y is

$$p_Y(y) = \sum_{\{x:h(x)=y\}} p_X(x).$$

Function of a Continuous RV

Let $Y = h(X)$ where X is a continuous rv with pdf $f_X(x)$. Then the cdf of Y is

$$F_Y(y) = P[h(X) \le y] = \int_{\{x:h(x)\le y\}} f_X(x)dx$$

If h is differentiable and strictly monotonic on some interval I which includes the range of X, the pdf of Y equals

$$f_Y(y) = f_X(h^{-1}(y)) \left| \frac{d}{dy} h^{-1}(y) \right|.$$

Joint Probability Mass Function $p_{X,Y}(x,y) = P(X = x, Y = y)$

Joint Probability Density Function A joint pdf f is a nonnegative function such that

$$P\left((X,Y) \in A\right) = \int_A \int f(x,y)\, dxdy.$$

Bivariate Distribution Function This is the function F such that

$$F(x, y) = P(X \leq x, Y \leq y).$$

Obtaining Joint PDF from CDF If X and Y are continuous rvs,

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$$

Marginal Distributions When X and Y have joint pmf p or joint pdf f, the marginal pmf or pdf of X is

$$p_X(x) = \sum_y f(x, y), \text{ when } X \text{ and } Y \text{ are discrete}$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy, \text{when } X \text{ and } Y \text{ are continuous.}$$

Independent RVs When X and Y have joint pmf p or pdf f, they are independent iff

$$p(x, y) = p_X(x) p_Y(y) \qquad \text{or} \qquad f(x, y) = f_X(x) f_Y(y), \text{ all } x, y.$$

Conditional PMF or PDF When X and Y have joint pmf p or pdf f, the conditional pmf or pdf of Y given that $X = x$ is

$$p_{Y|X}(y|x) = \frac{p(x, y)}{p_X(x)}, \qquad \text{or} \qquad f_{Y|X}(y|x) = \frac{f(x, y)}{f_X(x)}$$

where p_X or f_X is the marginal pmf or pdf of X.

Maxima and Minima When $X_1, \ldots, X_n$ are continuous rvs with the same cdf F_X, the cdfs of the maximum U and minimum V are, respectively,

$$F_U(u) = [F_X(u)]^n \qquad \text{and} \qquad F_V(v) = 1 - (1 - F_X(v))^n.$$

Convolutions When X and Y are independent continuous rvs with pdfs f_X and f_Y, the cdf and pdf of $Z = X + Y$ are, respectively,

$$F_Z(z) = \int_{-\infty}^{\infty} f_X(x) F_Y(z - x) dx, \qquad f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z - x) dx.$$

When X and Y are independent discrete rvs with pmfs p_X and p_Y, the pmf of $Z = X + Y$ is

$$p_Z(z) = \sum_{\{(x,y): x+y=z\}} p_X(x) p_Y(y) = \sum_w p_X(z - w) p_Y(w).$$

A Few Indefinite Integrals and One Definite Integral

$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad \text{except for } n = -1$$

$$\int \frac{1}{x} dx = \log_e(x)$$

$$\int u dv = uv - \int v du \qquad \text{integration by parts}$$

$$\Gamma(a) = \int_0^{\infty} x^{a-1} e^{-x} dx, \qquad a > 0$$