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STAT 630-720
HW 05

1) 3.1.1

a) E(x) = -4(1/7) + 0(2/7) + 3(4/7) = 1.142
b) E(x) = 2x^-x-1 / log(2)

2) 3.1.2

a) 5(3/7) + 8(4/7)
b) 3(1/7) + 4(2/7)
c) E(3x + 7y) = 3(5(3/7) + 8(4/7)) + 7(3(1/7) + 4(2/7))
d) E(x^2) = 5^2(3/7) + 8^2(4/7)
e) (5 (3/7) + 8 (4/7)) * (3 (1/7) + 4 (2/7))

3) 3.1.6 E(Y ~ Binomial(100, .3)) = 30, E(Z ~ Poisson(7)) = 7, E(Y + Z) = 37

4) 3.1.7 E(X ~ Binomial(80, 1/4)) = 20, E(Y ~ Poisson(3/2)) = 3/2, (EXY) = 20 * 3/2 = 30

5) 3.2.1

a) E(x) = 8, C = 1/4
b) E(x) = 512/3 + 18 = 188.6, C = 16

6) 3.2.2

a) 2
b) 1
c) 4
d) 1

7) 3.2.5 E(x) = 5, E(y) = 1/9, E(-5x - 6y) = -25.666

8) 3.2.12

a) 11
b) 30
c) 270
d) 273

9) 3.2.18

$$\begin{aligned} E(x) &= \int x f(x) dx \\ &= \int x a x^{(a-1)} e^{(-x^a)} \\ &= \frac{x^2}{2} e^{-x^a} \end{aligned}$$

10) 3.2.22

$$\begin{aligned} E(x) &= \int x x^{a-1} (1-x)^{b-1} \\ &= \frac{x^2}{2}(a-1)x^{a-2}(b-1)(1-\frac{x^2}{2})^{(b-2)} \\ &= \frac{a}{a+b} \end{aligned}$$

11) 3.3.3

b) E(x) = 5(3/7) + 8(4/7) = 47/7, E(y) = 3(1/7) + 4(2/7) = 11/7, E((x - 47/7)(y - 11/7)) = -2.759
c) var(x) = (5 - 6.71)^2 3/7 + (8 - 6.71)^2 4/7 = 2.2, var(y) = (0 - 1.57)^2 4/7 + (3 - 1.57)^2 1/7 + (4 - 1.57)^2 2/7 = .57
d) corr(x,y) = -2.7/1.11

12) 3.3.7 E(x) = 2/5, E(y) = 46/63, E(xy) = 34/21, cov(x,y) = 1.132, E(x,y) = .476, Cov(x,y) = -.010, var(x^2) = 19/30, var(y^2) = 7/12, var(x) = .188, var(y) = .118, corr(x,y) = -.010/.148 = -.06

13) 3.3.14 var(z) = 17/36, var(w) = 1/36, E(z,w) = 5/36, cov(z,w) = .125, corr(z,w) = 1.09

14) 3.3.21

$$\begin{aligned} E(x) &= \int x a x^{a-1} e^{-x^a} \\ set(t) &= a x^a \\ &= \frac{t^{\frac{-1}{a}}}{a} \\ &= a^{\frac{-1}{1}} \int t^{\frac{1}{a}} e^{-t} \\ E(x) &= a^{\frac{-1}{a}} \rho(1+\frac{1}{a}) \\ Var(x) &= E(x^2) - E(x)^2 \\ &= a^{\frac{-1}{a}} \rho(1+\frac{1}{a}) - (\rho(1+\frac{1}{a})^2) \end{aligned}$$

15) 3.3.24

$$\begin{aligned}
E(x) &= \int x x^{a-1}(1-x)^{b-1} \\
&= \frac{x^2}{2}(a-1)x^{a-2}(b-1)(1-x)^{b-2} \\
&= \frac{a}{a+b} \\
E(x^2) &= \int x^2 x^{a-1}(1-x)^{b-2} \\
&= \frac{x^3}{3}(a-1)x^{a-2}(b-1)(1-x)^{b-2} \\
Var(x) &= E(x^2) - E(x)^2 \\
&= \frac{ab}{(a+b)^2(a+b+1)}
\end{aligned}$$

16) 3.4.5 My(s) = eˆ(4 s) Mx(3 s)

17) 3.4.8

c) M'x(0) = 17/6
d) M"x(0) = 18.5

18) 3.4.12

a)

$$mx(s) = e^{sx}\theta(1-\theta)^x$$

b)

$$\begin{aligned}
m'x(s) &= xe^{sx}\theta(1-\theta)^x + e^{sx}x\theta(1-\theta)^{x-1} \\
(1-\theta)/\theta
\end{aligned}$$

c)

$$\begin{aligned}
m''x(s) &= x^2e^{sx}\theta(1-\theta)^x + e^{sx}(x-1)x\theta(1-\theta)^{x-2} \\
&= (1-\theta)/\theta^2
\end{aligned}$$

19) 3.4.16

$$\begin{aligned} mx(s) &= xe^{sx}e^{-|x|}/2x \\ &= 2xe^{sx}e^{-|x|}/2x \end{aligned}$$

$$\begin{aligned} m'x(s) &= 2x^2e^{sx}(-xe^{-|x|}/2x) \\ &= 0 \end{aligned}$$

$$\begin{aligned} m''x(s) &= 2x^3e^{sx}e^{-|x|}/2x \\ &= 0 \end{aligned}$$

20) 3.4.20

$$\begin{aligned} Mx(s) &= \int \frac{e^{sx}\lambda x^{a-1}}{\rho(a)}e^{-2x} \\ &= \frac{\lambda}{\rho(a)}\int e^{sx}\lambda x^{a-1}/\rho(a)e^{-2x} \\ &= x^{a-1}e^{-(2-5)x} \end{aligned}$$

$$Let b = (\lambda - t)x$$

$$\begin{aligned} &= \frac{\lambda}{\rho(a)}\int \frac{b}{\lambda - t}^{a-1} e^{-b}\frac{db}{\lambda - t} \\ &= \frac{\lambda^a}{\rho(a)(\lambda - t)^a}\int b^{a-1}e^{-b}db \\ &= \frac{\lambda}{\lambda - t}^{a} \end{aligned}$$

21) 3.4.22 $E(e^{sx}) = \sum e^{sx}\binom{r-1+k}{k}\theta^r(1-\theta)^k$

22) additional question uncorrelated variables that are independent will have a covariance of 0 and a correlation of 0