

Statistics 630 - Assignment 10
(due Wednesday, December 3, 2014, 11:59 pm)

Instructions:

- The textbook exercises are in the book by Evans and Rosenthal. This assignment covers the material on Bayes analysis from Chapter 7 and hypothesis testing from Chapters 6 and 8 discussed in Lectures 33–36.
- Whether you write out the solutions by hand or in a text document, be sure that they are *neat, legible and in order* (even if you choose to solve them in a different order).
- **Type** your name, email address, course number, section number and assignment number at the top of the first page (or cover page).
- Either scan or print your solutions to a **PDF** file under 15MB in size. It must be in a *single* file, not separate files for separate pages. Name the file using your name (for example, I could use twehrly630hw01.pdf) to avoid confusion with other students and/or assignments. *Do not* take a photo of each page and then paste them into a document – this will make your file too big and the results will generally not be very readable anyway.
- Login to your WebAssign account to upload your file. You must do this by **11:59 pm U.S. Central time**, according to the WebAssign server, on the due date. We highly recommend that you start the upload at least 15 minutes earlier. You can make multiple submissions, but *only the last submission will be graded*.

Answer the following problems from Chapter 7:

7.1.3

7.1.4

7.1.4 (b) Find the posterior mean, posterior mode, and posterior variance.

7.1.9, 7.1.14

7.2.1, 7.2.2

7.2.10

7.2.10 (b) Consider now a sample of size 20 with a sample mean of 5.1. Consider two cases for the prior:

- (i) the mean of the gamma is 0.5 and the standard deviation is 1
- (ii) the mean of the gamma is 10 and the standard deviation is 20.

Plot the two posterior distributions and compare them. Find the two posterior means and compare them.

Answer the following problem from Chapter 6:

6.3.27 (This corresponds to a test that rejects for $Z \geq Z_{1-\alpha}$ where Z is defined on the bottom of slide 11.)

Answer the following problem from Chapter 8:

8.2.5

Additional Problems:

A. Suppose that X binomial $(100, \theta)$. Consider the test that rejects $H_0 : \theta = 0.5$ in favor of $H_a : \theta \neq 0.5$ for $|X - 50| > 10$. Use the normal approximation to answer the following:

- (a) What is the level of significance, α ?
- (b) Derive the power function and graph it as a function of θ .

B. Let X have one of the following distributions:

| x | 1 | 2 | 3 | 4 |
|----------|-----|-----|-----|-----|
| $f_0(x)$ | 0.2 | 0.3 | 0.3 | 0.2 |
| $f_1(x)$ | 0.1 | 0.4 | 0.1 | 0.4 |

- (a) Compute the likelihood ratio for each possible value of X and order the values of x according to the LR.
- (b) What is the likelihood ratio test of $H_0 : \theta = 0$ versus $H_a : \theta = 1$ at level $\alpha = 0.2$?
What is the likelihood ratio test at level $\alpha = 0.5$?
- (c) Find the power for each of the tests found in part b.
- (d) If the prior probabilities are $P(H_0) = P(H_a)$, which outcomes favor H_0 ?