Homework 01
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2.1

Rows: Fathers Profession, Cols: Sons Profession

	farm	operative	craftsman	sales	professional
farm	0.018	0.035	0.031	0.008	0.018
operative	0.002	0.112	0.064	0.032	0.069
craftsman	0.001	0.066	0.094	0.032	0.084
sales	0.001	0.018	0.019	0.010	0.051
professional	0.001	0.029	0.032	0.043	0.130

a) Fathers marginal density

```
rowSums(jobs)
```

```
       farm    operative    craftsman        sales professional
      0.110        0.279        0.277        0.099        0.235
```

b) Sons marginal density

```
colSums(jobs)
```

```
       farm    operative    craftsman        sales professional
      0.023        0.260        0.240        0.125        0.352
```

c) P(Son Profession | Father = Farmer)

```
jobs[1, ] / rowSums(jobs)[1]
```

```
          farm operative craftsman      sales professional
farm 0.1636364 0.3181818 0.2818182 0.07272727    0.1636364
```

d) P(Father Profession | Son = Farmer)

```
data.frame(farm = jobs[, 1] / colSums(jobs)[1],
           row.names = c('farm', 'operative', 'craftsman', 'sales', 'professional'))
```

```
                farm
farm         0.78260870
operative    0.08695652
craftsman    0.04347826
sales        0.04347826
professional 0.04347826
```

2.2

a) $E[a_1Y_1 + a_2Y_2] = a_1E[Y_1] + a_2E[Y_2]$
$Var[a_1Y_1 + a_2Y_2] = a_1^2Var[Y_1] + a_2^2Var[Y_2]$

b) $E[a_1Y_1 - a_2Y_2] = a_1E[Y_1] - a_2E[Y_2]$
$Var[a_1Y_1 - a_2Y_2] = a_1^2Var[Y_1] + a_2^2Var[Y_2]$

2.3

a) $P(X|Y) = P(X)$; $P(X|Y,Z) = P(X,Z) \approx f(X,Z)$

b) $P(Y|X) = P(Y)$; $P(Y|X,Z) = P(Y,Z) \approx f(Y,Z)$

2.5

a)

Rows: X, Cols: Y

	Green	Red	Marginal.X
Head	0.2	0.3	0.5
Tail	0.3	0.2	0.5
Marginal.Y	0.5	0.5	1.0

b) $E[Y] = .5$
$P[Y = G|X = H] = .2$
$P[Y = G|X = T] = .3$
$P[Y = G] = .5$

c) $Var[Y|X = 0] = 0^2(.2) + 1^2(.3) = .3$
$Var[Y|X = 1] = 0^2(.3) + 1^2(.2) = .2$

The difference in variance is because we are essentially comparing the following conditional probabilities: $P(Y = 1|X = 0) = .3$ and $P(Y = 1|X = 1) = .2$. The smaller probability will have smaller variation than the larger probability because we are more certain that the smaller probability will occur less often than the larger probability.

d) $P(X = 0|Y = 1) = .2/.5 = .4$

2.6

Under this scenario the following are conditionally dependent. $P(A \perp B|C); P(A^c \perp B|C); P(A \perp B^c|C)$ and A^c, B^c are the same area. $P(A \perp B|C^c)$ are not conditionally independent because A and B intersect outside of C.