MASTER'S DIAGNOSTIC EXAMINATION - August 20, 2015

Student's Name ____

INSTRUCTIONS FOR STUDENTS:

1. The exam will start at Noon (CDT) on August 20, 2015 and must be completed by 4 pm (CDT) on August 20, 2015.
2. Put your name above but **DO NOT put your NAME** on the **SOLUTIONS** to the exam.
3. Place the **NUMBER** assigned to you on the

   UPPER RIGHT HAND CORNER of EACH PAGE of your SOLUTIONS.
4. Please start your answer to each of the four questions on a SEPARATE sheet of paper.
5. Use only one side of each sheet of paper.
6. You must answer all four problems: Problems I, II, III and IV.
7. Be sure to attempt all parts of the four problems. It may be possible to answer a later part of a problem without having solved the earlier parts.
8. Be sure to hand in all of your exam. No additional material will be accepted once the exam has ended and you have left the exam room or submitted your solutions.
9. You may use the following:
   - Calculator which does not have capability to phone, text, or access the Web
   - Pencil or pen
   - Blank paper for the solutions for this examination
   - No other materials are allowed

- I attest that I spent no more than 4 hours to complete the exam.
- I used only the materials described above.
- I did not receive assistance from anyone during the taking of this exam.

Student's Signature ____

INSTRUCTIONS FOR PROCTOR:

Immediately after the student completes the exam, **fax** the student's solutions to **979-845-6060** or

Scan the solutions into a **single** pdf file and **email to contact@stat.tamu.edu**

Do not send the questions, just send the student's solutions.

(1) I certify that the time at which the student started the exam was ____________________

and the time at which the student completed the exam was ____________________

(2) I certify that the student has followed all the **INSTRUCTIONS FOR STUDENTS** listed above.

(3) I certify that the student's solutions were faxed to **979-845-6060** or

emailed to **contact@stat.tamu.edu**.

Proctor's Signature____

Problem I.

1) Given below is scatter plot with the least squares regression line marked on it. We shall consider the point in the bottom right hand corner of the plot which is marked as "Point under scrutiny". This point has a Cook's distance equal to 4.1 and a standardized residual equal to -3.4. An analyst added a dummy variable corresponding to "Point under scrutiny" to the model and refitted the model. Thus the fitted model became

   $Y = b_0 + b_1 x_1 + b_2 \text{PointUnderScrutiny} + e.$

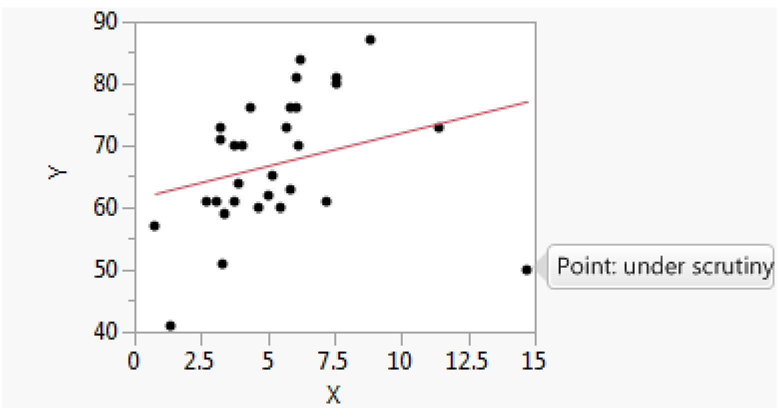
   Comparing this second model to the first one fit

   $Y = b_0 + b_1 x_1 + e.$

   a. The slope of the regression line (i.e. b_1) will be
      i. The same in both models
      ii. Higher in the second model
      iii. Lower in the second model

   b. R^2 will
      i. The same in both models
      ii. Higher in the second model
      iii. Lower in the second model

   c. The regression coefficient of the dummy variable will be
      i. Zero
      ii. Positive
      iii. Negative



2. Explain how to check whether the following modeling assumption is a reasonable for a given multiple regression model

$$Y = g(b_0 + b_1x_1 + b_2x_2 + \ldots + b_kx_k) + e$$

where g is an unspecified function. In particular, what plot should you look at and what should you look for?

3. Consider the situation in which the following regression model has been fit to a set of data

$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + e$$

Suppose that the hypothesis test based on Analysis of Variance leads to a strong rejection of the joint hypothesis that regression coefficients are all zero and that each individual t-tests of the form H_{01}: $b_1 = 0$, H_{02}: $b_2 = 0$ and H_{03}: $b_3 = 0$ are strongly rejected. Suppose that the estimated values of b_1 and b_2 are positive as expected. On the other hand, suppose that the estimated value of b_3 is negative, when it was expected to be positive. Describe why this is likely to have occurred and what plots and which statistics you would look at to diagnose the problem.

Problem II.

An experiment was conducted to assess the effect of a new drug on weight loss. A random sample of 160 overweight people was obtained, consisting of 80 females and 80 males. Within each gender group, half of the people were randomized to receive the drug, with the remaining half receiving a placebo. Let $\mathbf{y}' = [y_1, y_2, \ldots, y_{160}]$ be the vector containing weight loss measurements in pounds for each of the participants, ordered so that the first 40 entries are for females in the placebo group, followed by females in the treatment group, then males in the placebo group, then males in the treatment group. Define the model matrix

$$\mathbf{X}_{160\times 4} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

where $\mathbf{1}$ and $\mathbf{0}$ are column vectors containing 40 ones and zeros, respectively. We have

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 0.025 & -0.025 & -0.025 & 0.025 \\ -0.025 & 0.050 & 0.025 & -0.050 \\ -0.025 & 0.025 & 0.050 & -0.050 \\ 0.025 & -0.050 & -0.050 & 0.100 \end{bmatrix}$$

The regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ was fit, where $\boldsymbol{\epsilon}$ is a vector of random errors. The coefficient estimates are

$\hat{\boldsymbol{\beta}}' = [4.74, 3.13, 7.47, 3.77]$, and the residual standard deviation is $\hat{\sigma} = 4.92$.

(1) What are the assumptions of the model?

(2) Interpret all model coefficients.

(3) Testing at significance level $\alpha = 0.05$, does weight loss in the placebo group differ between males and females? Support your answer. Also (while you do not have to compute it here), carefully interpret what the p-value means.

(4) In terms of the model coefficients, what is the appropriate null hypothesis for testing whether there is an overall treatment effect, and how would you test this?

(5) What is an approximate 95% confidence interval for mean weight loss among males in the treatment group? `Hint`: If $\mathbf{v}$ is a vector of constants and $\mathbf{Z}$ is a random vector of the same dimension as $\mathbf{v}$, then $\mathrm{Var}(\mathbf{v}'\mathbf{Z}) = \mathbf{v}'\mathrm{Var}(\mathbf{Z})\mathbf{v}$. Carefully interpret what your confidence interval means.

(6) Let $[\mathrm{lo}_{\mathrm{ind}}, \mathrm{hi}_{\mathrm{ind}}]$ be the 95% confidence interval for the mean difference in weight loss comparing treated females to placebo females, based on the above model. Now consider an alternative design for the experiment, in which there were only 40 females and 40 males, each assessed both with and without the drug. Suppose we fit the same model as above to the resulting 160 weight loss observations from the alternative design, this time incorporating correlations when fitting the model. Let $[\mathrm{lo}_{\mathrm{cor}}, \mathrm{hi}_{\mathrm{cor}}]$ be the 95% confidence interval for mean difference in weight loss comparing treated females to placebo females, according to the correlation-based model. How would $[\mathrm{lo}_{\mathrm{cor}}, \mathrm{hi}_{\mathrm{cor}}]$ compare to $[\mathrm{lo}_{\mathrm{ind}}, \mathrm{hi}_{\mathrm{ind}}]$, and why?

Problem III.

1. An experiment was conducted to evaluate the effectiveness of a treatment for tapeworm in the stomachs of sheep. A random sample of 24 worm-infected lambs of approximately the same age and health was randomly divided into two groups of 12 lambs. One group of 12 lambs were injected with the drug, and the remaining 12 were left untreated. After a six-month period, the lambs were slaughtered and the worm counts are given in the following table:

Subject	1	2	3	4	5	6	7	8	9	10	11	12
Drug-treated sheep (x_1)	18	43	28	50	16	32	13	35	38	33	6	7
Untreated sheep (x_2)	40	54	26	63	21	37	39	23	48	58	28	39

Some possibly useful statistics follow:

$$\bar{x}_1 = 26.583,\ s_1 = 14.362,\ \bar{x}_2 = 39.667,\ s_2 = 13.859,\ d = x_2 - x_1,\ \bar{d} = -13.083,\ s_d = 12.887$$

(a) What is the design of the experiment? Discuss its appropriateness for this experiment.

(b) Obtain a 95% confidence interval for the difference in mean tapeworm counts for the two treatments. Interpret your results.

(c) The researcher states that she is certain that the data does not follow a normal distribution. Describe a procedure to determine whether the treatment was effective, that is, the worm counts for the treated sheep tend to be less than worm counts for the untreated sheep. Compute the value of the test statistic. It is not necessary to compute the p-value nor to compare the computed value of the test statistic to a critical value.

2. We wish to assess the efficacy of a psychiatric treatment for depression. Each patient is assessed as to whether they are depressed at the beginning of the study (Year 0) and then assessed as to whether they are depressed after a year of treatment.

(a) The data can be recorded into either of two tables:

Table A

Year	Outcome: Depress Yes	Outcome: Depress No	Total
0	317	164	481
1	285	196	481

Table B

Diagnosis at Year 0	Diagnosis at Year 1: Depress Yes	Diagnosis at Year 1: Depress No	Total
Depress–Yes	276	41	317
Depress–No	9	155	164
Total	285	196	481

Discuss which table is more appropriate for presenting these data. Explain why.

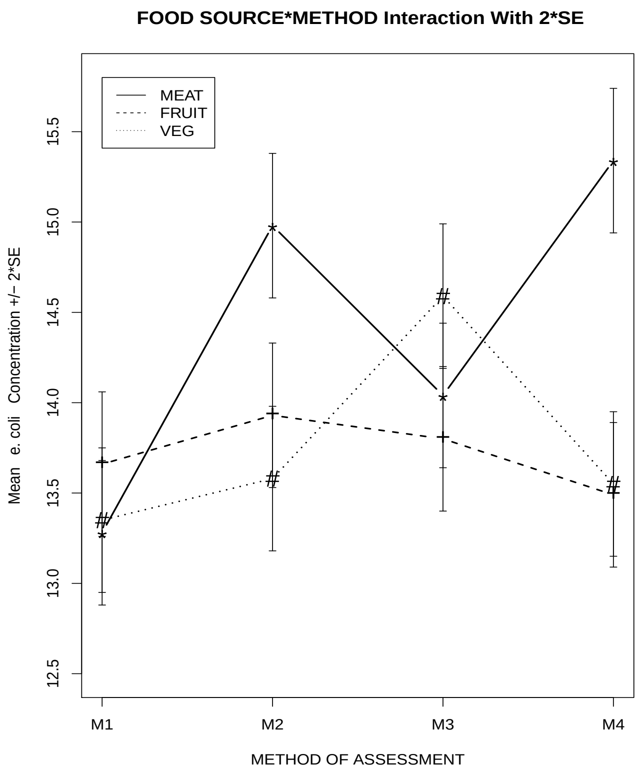
(b) Describe (name) a procedure for testing for a difference in the proportion of depressed patients between Year 0 and Year 1. At the $\alpha = .05$ level, conduct this test to determine if there is significant evidence of a difference in the proportions?

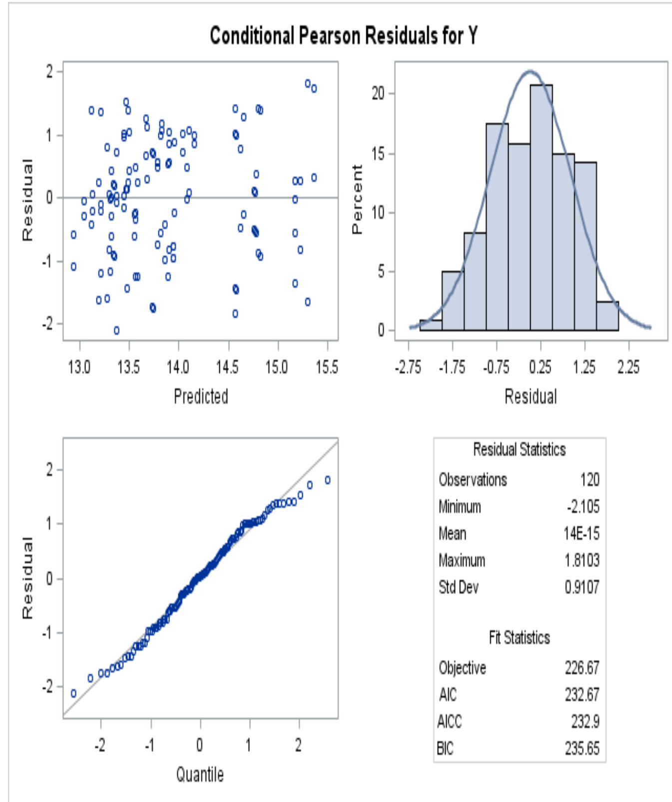
Problem IV.

The research department at a major food processor designed a study to assess the amount of salt (NaCl) in three processed foods: Meat, Fruit, and Vegetables. The accuracy of the determination of the amount of salt in foods varies considerably. The researchers wanted to evaluate the four major assessment methods Meth1, Meth2, Meth3, Meth4 and also the variation in the hundreds of laboratories which use one or more of these four methods in their determination of salt content in processed foods. Five laboratories were randomly selected from a long list of labs which used one of the four salt content methods to participate in the study. The researchers then randomly selected 120 containers of food product from the warehouse of the food processor, with 40 containers of each the 3 food sources. Six containers, two from each of the three food sources, are then sent to the 20 laboratories selected for the study. The salt content, Y_{ijkl}, is then determined by the kth Lab using Assessment Method j for the ℓth Container of Food Source i is recorded for the 120 containers. The researchers are interested in comparing the mean salt content of the four assessment methods and their differences across the food sources. Also, the researchers want to determine if there are major differences in determination of the mean salt content across the hundreds of laboratories in the USA.

		Food Source		
AssessMethod	Lab	Meat	Fruit	Vegetable
	L1	12.3, 12.6	13.2, 13.4	13.1, 12.5
	L2	13.2, 13.0	14.4, 14.5	13.4, 14.0
Meth1	L3	12.9, 13.0	12.9, 13.7	13.6, 13.0
	L4	13.2, 14.0	14.1, 14.1	13.8, 14.1
	L5	12.9, 13.9	12.8, 13.4	12.8, 13.3
	L6	14.5, 15.0	13.2, 14.2	13.5, 14.0
	L7	14.3, 15.6	14.2, 13.4	13.3, 12.6
Meth2	L8	14.5, 14.8	14.4, 14.6	13.5, 12.8
	L9	14.3, 15.6	13.3, 13.6	14.3, 13.6
	L10	14.5, 14.8	14.5, 13.8	13.5, 12.8
	L11	14.4, 14.1	13.4, 14.1	14.3, 15.1
	L12	13.5, 13.4	14.5, 14.4	13.5, 15.4
Meth3	L13	14.7, 14.6	12.7, 14.2	13.7, 15.2
	L14	13.5, 13.4	13.5, 14.4	14.5, 15.4
	L15	14.7, 14.2	12.7, 14.2	13.7, 15.2
	L16	14.8, 15.3	12.2, 13.3	12.2, 13.3
	L17	16.4, 14.3	14.4, 13.6	14.3, 13.9
Meth4	L18	15.6, 16.4	12.4, 13.8	13.6, 12.7
	L19	14.8, 15.4	13.4, 13.8	13.4, 13.9
	L20	15.2, 14.4	13.2, 13.3	14.4, 14.1

Use the following plots, the attached tables, and SAS output to answer the following questions. Pages 1-7 in the SAS output are from PROC GLM and pages 8-12 are from PROC MIXED.





2. Do the necessary conditions for testing hypotheses and constructing confidence intervals appear to be satisfied? Justify your answers.

   C_1 Normality:

   C_2 Equal Variance:

   C_3 Independence:

3. Construct a partial ANOVA table for this experiment.

   Include only the following: Source of Variation, df, and Expected Mean Squares.

4. At the $\alpha = .05$ level, which Main effects and Interaction effects are significant? Justify your answer by including the relevant p-values along with their pair of degrees of freedom $(df_{NUM.}, df_{DEN.})$. Test for Main effects even if an interaction is significant. Also, provide a test for all random effects.

5. Separate the four Assessment Methods into groups such that all four Assessment Methods in a group are not significantly different from any other member of the group with respect to their mean salt content. Use an experimentwise error rate of $\alpha = .05$.

6. Use the following estimates of the variance components from PROC MIXED to answer the following two questions:

$$\sigma^2_{LAB(METH)} = .008708 \qquad \sigma^2_{FOOD*LAB(METH)} = .03236 \qquad \sigma^2_{RESIDUAL} = .3357$$

   a. Justify that the estimated standard error of the estimated mean salt content of a container of Meat which was measured using Method M1 is .2044. Hint, compute the variance of the estimated mean and use the estimated variance components to obtain the requested quantity.

   b. Justify that the estimated standard error of the estimated difference between the mean salt content of a container of Meat and a container of Fruit is .1415. Hint, compute the variance of the estimated mean and use the estimated variance components to obtain the requested quantity.

7. The researchers state that from a health point of view two measurements of salt content have a practical difference only if the two means differ by 1.5 units. Using 95% confidence interval, is there a practical difference in the level of salt between a container of Meat and a container of Fruit using measurement Method M1?

SAS Code:

```
ods html;ods graphics on;

OPTIONS LS=90 PS=55 nocenter nodate;
TITLE 'SAS OUTPUT FOR PROBLEM IV';
DATA SALT;
INPUT FOOD $ METH $ LAB  $ Y @@;
TRT=COMPRESS (METH) || COMPRESS (FOOD);
CARDS;

 MEAT M1 L1 12.30  MEAT  M2 L6  14.46  MEAT M3 L11  14.35 MEAT M4 L16  14.85
 MEAT M1 L1 12.59  MEAT  M2 L6  15.00  MEAT M3 L11  14.06 MEAT M4 L16  15.32
 MEAT M1 L2 13.15  MEAT  M2 L7  14.29  MEAT M3 L12  13.50 MEAT M4 L17  16.35
 MEAT M1 L2 13.00  MEAT  M2 L7  15.62  MEAT M3 L12  13.39 MEAT M4 L17  14.34
 MEAT M1 L3 12.87  MEAT  M2 L8  14.46  MEAT M3 L13  14.73 MEAT M4 L18  15.55
 MEAT M1 L3 13.02  MEAT  M2 L8  14.82  MEAT M3 L13  14.65 MEAT M4 L18  16.36
 MEAT M1 L4 13.15  MEAT  M2 L9  14.29  MEAT M3 L14  13.50 MEAT M4 L19  14.75
 MEAT M1 L4 14.00  MEAT  M2 L9  15.62  MEAT M3 L14  13.39 MEAT M4 L19  15.38
 MEAT M1 L5 12.87  MEAT  M2 L10 14.46  MEAT M3 L15  14.73 MEAT M4 L20  15.15
 MEAT M1 L5 13.92  MEAT  M2 L10 14.82  MEAT M3 L15  14.15 MEAT M4 L20  14.39
 FRUIT M1 L1 13.20  FRUIT  M2 L6  13.16  FRUIT M3 L11  13.35 FRUIT M4 L16  12.25
 FRUIT M1 L1 13.39  FRUIT  M2 L6  14.20  FRUIT M3 L11  14.06 FRUIT M4 L16  13.33
 FRUIT M1 L2 14.45  FRUIT  M2 L7  14.23  FRUIT M3 L12  14.50 FRUIT M4 L17  14.35
 FRUIT M1 L2 14.50  FRUIT  M2 L7  13.42  FRUIT M3 L12  14.39 FRUIT M4 L17  13.55
 FRUIT M1 L3 12.86  FRUIT  M2 L8  14.45  FRUIT M3 L13  12.73 FRUIT M4 L18  12.36
 FRUIT M1 L3 13.72  FRUIT  M2 L8  14.62  FRUIT M3 L13  14.15 FRUIT M4 L18  13.75
 FRUIT M1 L4 14.11  FRUIT  M2 L9  13.29  FRUIT M3 L14  13.50 FRUIT M4 L19  13.38
 FRUIT M1 L4 14.10  FRUIT  M2 L9  13.62  FRUIT M3 L14  14.39 FRUIT M4 L19  13.79
 FRUIT M1 L5 12.83  FRUIT  M2 L10 14.46  FRUIT M3 L15  12.73 FRUIT M4 L20  13.15
 FRUIT M1 L5 13.42  FRUIT  M2 L10 13.82  FRUIT M3 L15  14.15 FRUIT M4 L20  13.32
 VEG M1 L1 13.10  VEG  M2 L6  13.46  VEG M3 L11  14.35 VEG M4 L16  12.15
 VEG M1 L1 12.52  VEG  M2 L6  14.00  VEG M3 L11  15.06 VEG M4 L16  13.32
 VEG M1 L2 13.35  VEG  M2 L7  13.29  VEG M3 L12  13.50 VEG M4 L17  14.32
 VEG M1 L2 14.04  VEG  M2 L7  12.62  VEG M3 L12  15.39 VEG M4 L17  13.85
 VEG M1 L3 13.57  VEG  M2 L8  13.46  VEG M3 L13  13.73 VEG M4 L18  13.55
 VEG M1 L3 12.96  VEG  M2 L8  12.82  VEG M3 L13  15.15 VEG M4 L18  12.65
 VEG M1 L4 13.75  VEG  M2 L9  14.29  VEG M3 L14  14.50 VEG M4 L19  13.37
 VEG M1 L4 14.10  VEG  M2 L9  13.62  VEG M3 L14  15.39 VEG M4 L19  13.85
 VEG M1 L5 12.82  VEG  M2 L10 13.46  VEG M3 L15  13.73 VEG M4 L20  14.39
 VEG M1 L5 13.32  VEG  M2 L10 12.82  VEG M3 L15  15.15 VEG M4 L20  14.05
RUN;

PROC GLM ORDER=DATA;
CLASS FOOD METH LAB;
MODEL  Y = FOOD METH FOOD*METH LAB(METH) FOOD*LAB(METH);
RANDOM LAB(METH) FOOD*LAB(METH)/TEST;
LSMEANS FOOD METH FOOD*METH/STDERR PDIFF ADJUST=TUKEY;

PROC MIXED ORDER=DATA;
CLASS FOOD METH LAB;
MODEL  Y = FOOD METH FOOD*METH/RESIDUAL ;
RANDOM LAB(METH) FOOD*LAB(METH);
LSMEANS FOOD METH FOOD*METH/ADJUST=TUKEY;

ods graphics off;ods html close;
```

SAS OUTPUT FOR PROBLEM IV

The GLM Procedure

Class Level Information

Class	Levels	Values
FOOD	3	MEAT FRUIT VEG
METH	4	M1 M2 M3 M4
LAB	20	L1 L6 L11 L16 L2 L7 L12 L17 L3 L8 L13 L18 L4 L9 L14 L19 L5 L10 L15 L20

Number of Observations Read	120
Number of Observations Used	120

SAS OUTPUT FOR PROBLEM IV

The GLM Procedure
Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	59	68.49452917	1.16092422	3.46	<.0001
Error	60	20.14305000	0.33571750		
Corrected Total	119	88.63757917			

R-Square	Coeff Var	Root MSE	Y Mean
0.772748	4.169802	0.579411	13.89542

Source	DF	Type I SS	Mean Square	F Value	Pr > F
FOOD	2	9.38181167	4.69090583	13.97	<.0001
METH	3	11.45262250	3.81754083	11.37	<.0001
FOOD*METH	6	27.60327500	4.60054583	13.70	<.0001
LAB(METH)	16	7.24294000	0.45268375	1.35	0.1995
FOOD*LAB(METH)	32	12.81388000	0.40043375	1.19	0.2734

Source	DF	Type III SS	Mean Square	F Value	Pr > F
FOOD	2	9.38181167	4.69090583	13.97	<.0001
METH	3	11.45262250	3.81754083	11.37	<.0001
FOOD*METH	6	27.60327500	4.60054583	13.70	<.0001
LAB(METH)	16	7.24294000	0.45268375	1.35	0.1995
FOOD*LAB(METH)	32	12.81388000	0.40043375	1.19	0.2734

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance
Dependent Variable: Y

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	FOOD	2	9.381812	4.690906	11.71	0.0002
	FOOD*METH	6	27.603275	4.600546	11.49	<.0001
	LAB(METH)	16	7.242940	0.452684	1.13	0.3705
	Error	32	12.813880	0.400434		

Error: MS(FOOD*LAB(METH))

*** This test assumes one or more other fixed effects are zero.**

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	METH	3	11.452623	3.817541	8.43	0.0014
	Error: MS(LAB(METH))	16	7.242940	0.452684		

*** This test assumes one or more other fixed effects are zero.**

Source	DF	Type III SS	Mean Square	F Value	Pr > F
FOOD*LAB(METH)	32	12.813880	0.400434	1.19	0.2734
Error: MS(Error)	60	20.143050	0.335717		

SAS OUTPUT FOR PROBLEM IV

Least Squares Means
Adjustment for Multiple Comparisons: Tukey

FOOD	Y LSMEAN	Standard Error	Pr > |t|	LSMEAN Number
MEAT	14.2900000	0.0916130	<.0001	1
FRUIT	13.6757500	0.0916130	<.0001	2
VEG	13.7205000	0.0916130	<.0001	3

Least Squares Means for effect FOOD
Pr > |t| for H0: LSMean(i)=LSMean(j)
Dependent Variable: Y

i/j	1	2	3
1		<.0001	0.0001
2	<.0001		0.9364
3	0.0001	0.9364	

SAS OUTPUT FOR PROBLEM IV

Least Squares Means
Adjustment for Multiple Comparisons: Tukey

METH	Y LSMEAN	Standard Error	Pr > |t|	LSMEAN Number
M1	13.3660000	0.1057856	<.0001	1
M2	14.0316667	0.1057856	<.0001	2
M3	14.1450000	0.1057856	<.0001	3
M4	14.0390000	0.1057856	<.0001	4

Least Squares Means for effect METH
Pr > |t| for H0: LSMean(i)=LSMean(j)
Dependent Variable: Y

i/j	1	2	3	4
1		0.0002	<.0001	0.0002
2	0.0002		0.8731	1.0000
3	<.0001	0.8731		0.8933
4	0.0002	1.0000	0.8933	

SAS OUTPUT FOR PROBLEM IV

Least Squares Means
Adjustment for Multiple Comparisons: Tukey

FOOD	METH	Y LSMEAN	Standard Error	Pr > |t|	LSMEAN Number
MEAT	M1	13.0870000	0.1832260	<.0001	1
MEAT	M2	14.7840000	0.1832260	<.0001	2
MEAT	M3	14.0450000	0.1832260	<.0001	3
MEAT	M4	15.2440000	0.1832260	<.0001	4
FRUIT	M1	13.6580000	0.1832260	<.0001	5
FRUIT	M2	13.9270000	0.1832260	<.0001	6
FRUIT	M3	13.7950000	0.1832260	<.0001	7
FRUIT	M4	13.3230000	0.1832260	<.0001	8
VEG	M1	13.3530000	0.1832260	<.0001	9
VEG	M2	13.3840000	0.1832260	<.0001	10
VEG	M3	14.5950000	0.1832260	<.0001	11
VEG	M4	13.5500000	0.1832260	<.0001	12

Least Squares Means for effect FOOD*METH
Pr > |t| for H0: LSMean(i)=LSMean(j)
Dependent Variable: Y

i/j	1	2	3	4	5	6	7	8	9	10	11	12
1		<.0001	0.0219	<.0001	0.5536	0.0753	0.2363	0.9988	0.9964	0.9910	<.0001	0.8185
2	<.0001		0.1854	0.8245	0.0029	0.0637	0.0154	<.0001	<.0001	<.0001	0.9998	0.0007
3	0.0219	0.1854		0.0011	0.9367	1.0000	0.9979	0.2122	0.2661	0.3302	0.6093	0.7482
4	<.0001	0.8245	0.0011		<.0001	0.0002	<.0001	<.0001	<.0001	<.0001	0.3572	<.0001
5	0.5536	0.0029	0.9367	<.0001		0.9961	1.0000	0.9770	0.9888	0.9954	0.0276	1.0000
6	0.0753	0.0637	1.0000	0.0002	0.9961		1.0000	0.4672	0.5457	0.6277	0.3150	0.9467
7	0.2363	0.0154	0.9979	<.0001	1.0000	1.0000		0.7998	0.8585	0.9074	0.1096	0.9983
8	0.9988	<.0001	0.2122	<.0001	0.9770	0.4672	0.7998		1.0000	1.0000	0.0004	0.9991
9	0.9964	<.0001	0.2661	<.0001	0.9888	0.5457	0.8585	1.0000		1.0000	0.0006	0.9998
10	0.9910	<.0001	0.3302	<.0001	0.9954	0.6277	0.9074	1.0000	1.0000		0.0010	1.0000
11	<.0001	0.9998	0.6093	0.3572	0.0276	0.3150	0.1096	0.0004	0.0006	0.0010		0.0080
12	0.8185	0.0007	0.7482	<.0001	1.0000	0.9467	0.9983	0.9991	0.9998	1.0000	0.0080	

SAS OUTPUT FOR PROBLEM IV

The Mixed Procedure

Model Information	
Data Set	WORK.SALT
Dependent Variable	Y
Covariance Structure	Variance Components
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Satterthwaite

Class Level Information

Class	Levels	Values
FOOD	3	MEAT FRUIT VEG
METH	4	M1 M2 M3 M4
LAB	20	L1 L6 L11 L16 L2 L7 L12 L17 L3 L8 L13 L18 L4 L9 L14 L19 L5 L10 L15 L20

Number of Observations	
Number of Observations Read	120
Number of Observations Used	120
Number of Observations Not Used	0

Covariance Parameter Estimates

Cov Parm	Estimate
LAB(METH)	0.008708
FOOD*LAB(METH)	0.03236
Residual	0.3357

Type 3 Tests of Fixed Effects				
Effect	**Num DF**	**Den DF**	**F Value**	**Pr > F**
FOOD	2	32	11.71	0.0002
METH	3	16	8.43	0.0014
FOOD*METH	6	32	11.49	<.0001

Least Squares Means							
Effect	**FOOD**	**METH**	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**
FOOD	**MEAT**		14.2900	0.1022	47.8	139.81	<.0001
FOOD	**FRUIT**		13.6758	0.1022	47.8	133.80	<.0001
FOOD	**VEG**		13.7205	0.1022	47.8	134.24	<.0001
METH		**M1**	13.3660	0.1228	16	108.81	<.0001
METH		**M2**	14.0317	0.1228	16	114.23	<.0001
METH		**M3**	14.1450	0.1228	16	115.15	<.0001
METH		**M4**	14.0390	0.1228	16	114.29	<.0001
FOOD*METH	**MEAT**	**M1**	13.0870	0.2044	47.8	64.02	<.0001
FOOD*METH	**MEAT**	**M2**	14.7840	0.2044	47.8	72.32	<.0001
FOOD*METH	**MEAT**	**M3**	14.0450	0.2044	47.8	68.71	<.0001
FOOD*METH	**MEAT**	**M4**	15.2440	0.2044	47.8	74.57	<.0001
FOOD*METH	**FRUIT**	**M1**	13.6580	0.2044	47.8	66.82	<.0001
FOOD*METH	**FRUIT**	**M2**	13.9270	0.2044	47.8	68.13	<.0001
FOOD*METH	**FRUIT**	**M3**	13.7950	0.2044	47.8	67.49	<.0001
FOOD*METH	**FRUIT**	**M4**	13.3230	0.2044	47.8	65.18	<.0001
FOOD*METH	**VEG**	**M1**	13.3530	0.2044	47.8	65.32	<.0001
FOOD*METH	**VEG**	**M2**	13.3840	0.2044	47.8	65.48	<.0001
FOOD*METH	**VEG**	**M3**	14.5950	0.2044	47.8	71.40	<.0001
FOOD*METH	**VEG**	**M4**	13.5500	0.2044	47.8	66.29	<.0001

Differences of Least Squares Means											
Effect	**FOOD**	**METH**	**_FOOD**	**_METH**	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**	**Adjustment**	**Adj P**
FOOD	**MEAT**		**FRUIT**		0.6142	0.1415	32	4.34	0.0001	Tukey-Kramer	0.0004
FOOD	**MEAT**		**VEG**		0.5695	0.1415	32	4.02	0.0003	Tukey-Kramer	0.0009
FOOD	**FRUIT**		**VEG**		-0.04475	0.1415	32	-0.32	0.7539	Tukey-Kramer	0.9465
METH		**M1**		**M2**	-0.6657	0.1737	16	-3.83	0.0015	Tukey	0.0072
METH		**M1**		**M3**	-0.7790	0.1737	16	-4.48	0.0004	Tukey	0.0019
METH		**M1**		**M4**	-0.6730	0.1737	16	-3.87	0.0013	Tukey	0.0066
METH		**M2**		**M3**	-0.1133	0.1737	16	-0.65	0.5234	Tukey	0.9132
METH		**M2**		**M4**	-0.00733	0.1737	16	-0.04	0.9669	Tukey	1.0000
METH		**M3**		**M4**	0.1060	0.1737	16	0.61	0.5503	Tukey	0.9274
FOOD*METH	**MEAT**	**M1**	**MEAT**	**M2**	-1.6970	0.2891	47.8	-5.87	<.0001	Tukey-Kramer	<.0001
FOOD*METH	**MEAT**	**M1**	**MEAT**	**M3**	-0.9580	0.2891	47.8	-3.31	0.0018	Tukey-Kramer	0.0799
FOOD*METH	**MEAT**	**M1**	**MEAT**	**M4**	-2.1570	0.2891	47.8	-7.46	<.0001	Tukey-Kramer	<.0001
FOOD*METH	**MEAT**	**M1**	**FRUIT**	**M1**	-0.5710	0.2830	32	-2.02	0.0521	Tukey-Kramer	0.6790
FOOD*METH	**MEAT**	**M1**	**FRUIT**	**M2**	-0.8400	0.2891	47.8	-2.91	0.0055	Tukey-Kramer	0.1864
FOOD*METH	**MEAT**	**M1**	**FRUIT**	**M3**	-0.7080	0.2891	47.8	-2.45	0.0180	Tukey-Kramer	0.4047
FOOD*METH	**MEAT**	**M1**	**FRUIT**	**M4**	-0.2360	0.2891	47.8	-0.82	0.4183	Tukey-Kramer	0.9994
FOOD*METH	**MEAT**	**M1**	**VEG**	**M1**	-0.2660	0.2830	32	-0.94	0.3543	Tukey-Kramer	0.9980
FOOD*METH	**MEAT**	**M1**	**VEG**	**M2**	-0.2970	0.2891	47.8	-1.03	0.3094	Tukey-Kramer	0.9957
FOOD*METH	**MEAT**	**M1**	**VEG**	**M3**	-1.5080	0.2891	47.8	-5.22	<.0001	Tukey-Kramer	0.0006
FOOD*METH	**MEAT**	**M1**	**VEG**	**M4**	-0.4630	0.2891	47.8	-1.60	0.1158	Tukey-Kramer	0.8960
FOOD*METH	**MEAT**	**M2**	**MEAT**	**M3**	0.7390	0.2891	47.8	2.56	0.0138	Tukey-Kramer	0.3439
FOOD*METH	**MEAT**	**M2**	**MEAT**	**M4**	-0.4600	0.2891	47.8	-1.59	0.1181	Tukey-Kramer	0.8999
FOOD*METH	**MEAT**	**M2**	**FRUIT**	**M1**	1.1260	0.2891	47.8	3.90	0.0003	Tukey-Kramer	0.0200
FOOD*METH	**MEAT**	**M2**	**FRUIT**	**M2**	0.8570	0.2830	32	3.03	0.0048	Tukey-Kramer	0.1465
FOOD*METH	**MEAT**	**M2**	**FRUIT**	**M3**	0.9890	0.2891	47.8	3.42	0.0013	Tukey-Kramer	0.0627
FOOD*METH	**MEAT**	**M2**	**FRUIT**	**M4**	1.4610	0.2891	47.8	5.05	<.0001	Tukey-Kramer	0.0009
FOOD*METH	**MEAT**	**M2**	**VEG**	**M1**	1.4310	0.2891	47.8	4.95	<.0001	Tukey-Kramer	0.0012
FOOD*METH	**MEAT**	**M2**	**VEG**	**M2**	1.4000	0.2830	32	4.95	<.0001	Tukey-Kramer	0.0012
FOOD*METH	**MEAT**	**M2**	**VEG**	**M3**	0.1890	0.2891	47.8	0.65	0.5164	Tukey-Kramer	0.9999
FOOD*METH	**MEAT**	**M2**	**VEG**	**M4**	1.2340	0.2891	47.8	4.27	<.0001	Tukey-Kramer	0.0076

Differences of Least Squares Means											
Effect	**FOOD**	**METH**	**_FOOD**	**_METH**	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**	**Adjustment**	**Adj P**
FOOD*METH	**MEAT**	**M3**	**MEAT**	**M4**	-1.1990	0.2891	47.8	-4.15	0.0001	Tukey-Kramer	0.0104
FOOD*METH	**MEAT**	**M3**	**FRUIT**	**M1**	0.3870	0.2891	47.8	1.34	0.1870	Tukey-Kramer	0.9670
FOOD*METH	**MEAT**	**M3**	**FRUIT**	**M2**	0.1180	0.2891	47.8	0.41	0.6850	Tukey-Kramer	1.0000
FOOD*METH	**MEAT**	**M3**	**FRUIT**	**M3**	0.2500	0.2830	32	0.88	0.3836	Tukey-Kramer	0.9989
FOOD*METH	**MEAT**	**M3**	**FRUIT**	**M4**	0.7220	0.2891	47.8	2.50	0.0160	Tukey-Kramer	0.3766
FOOD*METH	**MEAT**	**M3**	**VEG**	**M1**	0.6920	0.2891	47.8	2.39	0.0206	Tukey-Kramer	0.4379
FOOD*METH	**MEAT**	**M3**	**VEG**	**M2**	0.6610	0.2891	47.8	2.29	0.0267	Tukey-Kramer	0.5052
FOOD*METH	**MEAT**	**M3**	**VEG**	**M3**	-0.5500	0.2830	32	-1.94	0.0608	Tukey-Kramer	0.7249
FOOD*METH	**MEAT**	**M3**	**VEG**	**M4**	0.4950	0.2891	47.8	1.71	0.0933	Tukey-Kramer	0.8496
FOOD*METH	**MEAT**	**M4**	**FRUIT**	**M1**	1.5860	0.2891	47.8	5.49	<.0001	Tukey-Kramer	0.0003
FOOD*METH	**MEAT**	**M4**	**FRUIT**	**M2**	1.3170	0.2891	47.8	4.56	<.0001	Tukey-Kramer	0.0035
FOOD*METH	**MEAT**	**M4**	**FRUIT**	**M3**	1.4490	0.2891	47.8	5.01	<.0001	Tukey-Kramer	0.0010
FOOD*METH	**MEAT**	**M4**	**FRUIT**	**M4**	1.9210	0.2830	32	6.79	<.0001	Tukey-Kramer	<.0001
FOOD*METH	**MEAT**	**M4**	**VEG**	**M1**	1.8910	0.2891	47.8	6.54	<.0001	Tukey-Kramer	<.0001
FOOD*METH	**MEAT**	**M4**	**VEG**	**M2**	1.8600	0.2891	47.8	6.43	<.0001	Tukey-Kramer	<.0001
FOOD*METH	**MEAT**	**M4**	**VEG**	**M3**	0.6490	0.2891	47.8	2.25	0.0294	Tukey-Kramer	0.5319
FOOD*METH	**MEAT**	**M4**	**VEG**	**M4**	1.6940	0.2830	32	5.99	<.0001	Tukey-Kramer	<.0001
FOOD*METH	**FRUIT**	**M1**	**FRUIT**	**M2**	-0.2690	0.2891	47.8	-0.93	0.3568	Tukey-Kramer	0.9982
FOOD*METH	**FRUIT**	**M1**	**FRUIT**	**M3**	-0.1370	0.2891	47.8	-0.47	0.6377	Tukey-Kramer	1.0000
FOOD*METH	**FRUIT**	**M1**	**FRUIT**	**M4**	0.3350	0.2891	47.8	1.16	0.2523	Tukey-Kramer	0.9887
FOOD*METH	**FRUIT**	**M1**	**VEG**	**M1**	0.3050	0.2830	32	1.08	0.2892	Tukey-Kramer	0.9937
FOOD*METH	**FRUIT**	**M1**	**VEG**	**M2**	0.2740	0.2891	47.8	0.95	0.3480	Tukey-Kramer	0.9979
FOOD*METH	**FRUIT**	**M1**	**VEG**	**M3**	-0.9370	0.2891	47.8	-3.24	0.0022	Tukey-Kramer	0.0937
FOOD*METH	**FRUIT**	**M1**	**VEG**	**M4**	0.1080	0.2891	47.8	0.37	0.7104	Tukey-Kramer	1.0000
FOOD*METH	**FRUIT**	**M2**	**FRUIT**	**M3**	0.1320	0.2891	47.8	0.46	0.6500	Tukey-Kramer	1.0000
FOOD*METH	**FRUIT**	**M2**	**FRUIT**	**M4**	0.6040	0.2891	47.8	2.09	0.0420	Tukey-Kramer	0.6332
FOOD*METH	**FRUIT**	**M2**	**VEG**	**M1**	0.5740	0.2891	47.8	1.99	0.0528	Tukey-Kramer	0.6991
FOOD*METH	**FRUIT**	**M2**	**VEG**	**M2**	0.5430	0.2830	32	1.92	0.0640	Tukey-Kramer	0.7397
FOOD*METH	**FRUIT**	**M2**	**VEG**	**M3**	-0.6680	0.2891	47.8	-2.31	0.0252	Tukey-Kramer	0.4897
FOOD*METH	**FRUIT**	**M2**	**VEG**	**M4**	0.3770	0.2891	47.8	1.30	0.1984	Tukey-Kramer	0.9725

Differences of Least Squares Means											
Effect	**FOOD**	**METH**	**_FOOD**	**_METH**	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**	**Adjustment**	**Adj P**
FOOD*METH	**FRUIT**	**M3**	**FRUIT**	**M4**	0.4720	0.2891	47.8	1.63	0.1091	Tukey-Kramer	0.8840
FOOD*METH	**FRUIT**	**M3**	**VEG**	**M1**	0.4420	0.2891	47.8	1.53	0.1329	Tukey-Kramer	0.9211
FOOD*METH	**FRUIT**	**M3**	**VEG**	**M2**	0.4110	0.2891	47.8	1.42	0.1616	Tukey-Kramer	0.9503
FOOD*METH	**FRUIT**	**M3**	**VEG**	**M3**	-0.8000	0.2830	32	-2.83	0.0080	Tukey-Kramer	0.2162
FOOD*METH	**FRUIT**	**M3**	**VEG**	**M4**	0.2450	0.2891	47.8	0.85	0.4009	Tukey-Kramer	0.9992
FOOD*METH	**FRUIT**	**M4**	**VEG**	**M1**	-0.03000	0.2891	47.8	-0.10	0.9178	Tukey-Kramer	1.0000
FOOD*METH	**FRUIT**	**M4**	**VEG**	**M2**	-0.06100	0.2891	47.8	-0.21	0.8338	Tukey-Kramer	1.0000
FOOD*METH	**FRUIT**	**M4**	**VEG**	**M3**	-1.2720	0.2891	47.8	-4.40	<.0001	Tukey-Kramer	0.0054
FOOD*METH	**FRUIT**	**M4**	**VEG**	**M4**	-0.2270	0.2830	32	-0.80	0.4284	Tukey-Kramer	0.9995
FOOD*METH	**VEG**	**M1**	**VEG**	**M2**	-0.03100	0.2891	47.8	-0.11	0.9151	Tukey-Kramer	1.0000
FOOD*METH	**VEG**	**M1**	**VEG**	**M3**	-1.2420	0.2891	47.8	-4.30	<.0001	Tukey-Kramer	0.0071
FOOD*METH	**VEG**	**M1**	**VEG**	**M4**	-0.1970	0.2891	47.8	-0.68	0.4989	Tukey-Kramer	0.9999
FOOD*METH	**VEG**	**M2**	**VEG**	**M3**	-1.2110	0.2891	47.8	-4.19	0.0001	Tukey-Kramer	0.0094
FOOD*METH	**VEG**	**M2**	**VEG**	**M4**	-0.1660	0.2891	47.8	-0.57	0.5685	Tukey-Kramer	1.0000
FOOD*METH	**VEG**	**M3**	**VEG**	**M4**	1.0450	0.2891	47.8	3.61	0.0007	Tukey-Kramer	0.0398

Table A.1 Cumulative Binomial Probabilities (*cont.*)

e. $n = 25$

	p														
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	.778	.277	.072	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.974	.642	.271	.027	.007	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.998	.873	.537	.098	.032	.009	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	1.000	.966	.764	.234	.096	.033	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	1.000	.993	.902	.421	.214	.090	.009	.000	.000	.000	.000	.000	.000	.000	.000
5	1.000	.999	.967	.617	.378	.193	.029	.002	.000	.000	.000	.000	.000	.000	.000
6	1.000	1.000	.991	.780	.561	.341	.074	.007	.000	.000	.000	.000	.000	.000	.000
7	1.000	1.000	.998	.891	.727	.512	.154	.022	.001	.000	.000	.000	.000	.000	.000
8	1.000	1.000	1.000	.953	.851	.677	.274	.054	.004	.000	.000	.000	.000	.000	.000
9	1.000	1.000	1.000	.983	.929	.811	.425	.115	.013	.000	.000	.000	.000	.000	.000
10	1.000	1.000	1.000	.994	.970	.902	.586	.212	.034	.002	.000	.000	.000	.000	.000
11	1.000	1.000	1.000	.998	.980	.956	.732	.345	.078	.006	.001	.000	.000	.000	.000
12	1.000	1.000	1.000	1.000	.997	.983	.846	.500	.154	.017	.003	.000	.000	.000	.000
13	1.000	1.000	1.000	1.000	.999	.994	.922	.655	.268	.044	.020	.002	.000	.000	.000
14	1.000	1.000	1.000	1.000	1.000	.998	.966	.788	.414	.098	.030	.006	.000	.000	.000
15	1.000	1.000	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.071	.017	.000	.000	.000
16	1.000	1.000	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.149	.047	.000	.000	.000
17	1.000	1.000	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.273	.109	.002	.000	.000
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.439	.220	.009	.000	.000
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.622	.383	.033	.001	.000
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.786	.579	.098	.007	.000
21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.904	.766	.236	.034	.000
22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.968	.902	.463	.127	.002
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.993	.973	.729	.358	.026
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.996	.928	.723	.222

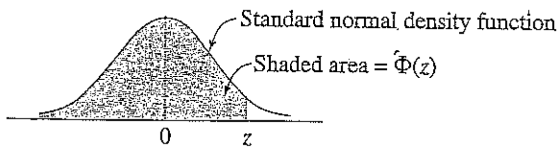
SOURCE: Adapted from L. L. Chao (1980), *Statistics for Management*, Wadsworth, Inc.

Table A.2 Cumulative Poisson Probabilities (*cont.*)

	λ										
	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	15.0	20.0
0	.135	.050	.018	.007	.002	.001	.000	.000	.000	.000	.000
1	.406	.199	.092	.040	.017	.007	.003	.001	.000	.000	.000
2	.677	.423	.238	.125	.062	.030	.014	.006	.003	.000	.000
3	.857	.647	.433	.265	.151	.082	.042	.021	.010	.000	.000
4	.947	.815	.629	.440	.285	.173	.100	.055	.029	.001	.000
5	.983	.916	.785	.616	.446	.301	.191	.116	.067	.003	.000
6	.995	.966	.889	.762	.606	.450	.313	.207	.130	.008	.000
7	.999	.988	.949	.867	.744	.599	.453	.324	.220	.018	.001
8	1.000	.996	.979	.932	.847	.729	.593	.456	.333	.037	.002
9		.999	.992	.968	.916	.830	.717	.587	.458	.070	.005
10		1.000	.997	.986	.957	.901	.816	.706	.583	.118	.011
11			.999	.995	.980	.947	.888	.803	.697	.185	.021
12			1.000	.998	.991	.973	.936	.876	.792	.268	.039
13				.999	.996	.987	.966	.926	.864	.363	.066
14				1.000	.999	.994	.983	.959	.917	.466	.105
15					.999	.998	.992	.978	.951	.568	.157
16					1.000	.999	.996	.989	.973	.664	.221
17						.999	.998	.995	.986	.749	.297
18						1.000	.999	.998	.993	.819	.381
19							1.000	.998	.997	.875	.470
20								1.000	.998	.917	.559
21									.999	.947	.644
22									1.000	.967	.721
23										.981	.787
24										.989	.843
25										.994	.888
26										.997	.922
27										.998	.948
28										.999	.966
29										1.000	.978
30											.987
31											.992
32											.995
33											.997
34											.999
35											.999
36											1.000

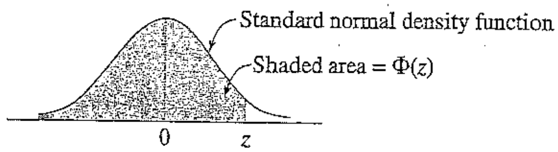
SOURCE: L. L. Chao (1974), *Statistics: Methods and Analysis*, 2nd ed. New York: McGraw-Hill.

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$



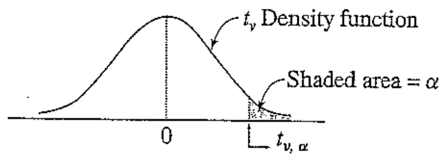
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0,0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0394	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0,0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$ (*cont.*)



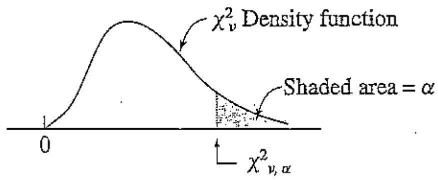
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0,4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1,7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table A.4 Critical Values $t_{\nu,\alpha}$ for the t-Distribution



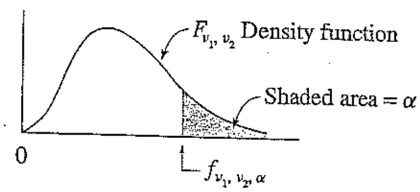
ν	α = .10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	1.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table A.5 Critical Values $\chi^2_{v,\alpha}$ for the Chi-square Distribution



	α									
v	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.420	65.473
40*	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

Table A.6 Critical Values $f_{v_1,v_2,\alpha}$ for the F-Distribution ($\alpha = .05$) (*cont.*)



Degrees of freedom for the denominator (v_2)	Degrees of freedom for the numerator (v_1)																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.49	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.69	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.81	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2 19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.91	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.09	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.59	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.81	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00