

# Homework 1

Instructions: On this and all homeworks and exams, please be sure your file has a cover page with your name, email address, course and section number, and homework or exam number **typed**.

## I. Matrix Algebra Review.

Define matrices **A**, **B**, and **C** as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 2 & 0 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & -1 \\ 3 & 0 \\ 2 & 1 \\ 0 & -2 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

1. Calculate  $\mathbf{A}'$ , the transpose of matrix **A**.
2. Calculate  $\mathbf{A}' + \mathbf{B}$ .
3. Calculate  $\mathbf{AB}$ , where  $\mathbf{AB}$  is the matrix product, or matrix multiplication.
4. Calculate  $\mathbf{BA}$ . Is  $\mathbf{AB} = \mathbf{BA}$ ?
5. Is the matrix  $\mathbf{AB}$  singular? Why or why not?
6. Calculate the trace of  $\mathbf{AB}$ .
7. Write  $(\mathbf{BA})'$  in another form (algebraically).
8. Calculate  $(\mathbf{AB})^{-1}$ .
9. Write  $\mathbf{I}_2$ , the  $2 \times 2$  identity matrix.
10. What is  $\mathbf{I}_2 \mathbf{A}$ ? Why?
11. Describe geometrically the space spanned by **C**. That is, the space spanned by the two vertical vectors in the matrix **C**. Assume we're working in three dimensional space defined by axes  $xyz$ .
12. Calculate the projection matrix for **C**.
13. Project the vector  $\mathbf{d} = [2 \ 2 \ 2]'$  onto the space spanned by **C**.
14. Describe geometrically what you did in the previous step.
15. Are the vectors  $\mathbf{d}$  defined above and  $\mathbf{f} = [1 \ 0 \ 0]'$  orthogonal? Why or why not? (Talk about a dot product in your answer.)
16. Calculate the dot product  $\mathbf{1} \cdot \mathbf{1}$ , where the vector  $\mathbf{1} = [1 \ 1 \ \dots \ 1]'$  is of length  $n$ .
17. Calculate the dot product  $\mathbf{1} \cdot \mathbf{x}$ , where  $\mathbf{1}$  is defined as above and the vector  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]'$ .
18. Calculate the dot product  $\mathbf{x} \cdot \mathbf{x}$ , where  $\mathbf{x}$  is defined as above. Memorize these last three answers (15, 16, and 17) - it's good for you!

## II. Calculus Review.

Define

$$f(x, y) = 3x^2 + 2xy^2 - y.$$

19. Calculate  $\frac{\partial}{\partial x} f(x, y)$ .

20. Calculate  $\frac{\partial}{\partial y} f(x, y)$ .

## III. Log Review.

21. Calculate  $\log(e)$ . (*Remember statisticians tend to write  $\log$  instead of  $\ln$  when they really mean  $\log$  base  $e$ !*)

22. Rewrite  $\log\left(\frac{x}{y}\right)$  in another way.

23. Rewrite  $\log(x^n)$  in another way.

24. Solve  $\log(x) = y$  for  $x$ .

## IV. Statistics and Linear Regression Review.

After regressing eight patients' weights (in kg) on their height (in cm), a doctor found the following output.

Coefficient	Estimate	Std. Error	t-value	$Pr(>  t )$
(Intercept)	-129.1667	24.3610	-5.302	0.001826
Height	1.1667	0.1521	?????	0.000257

25. Write down the least squares regression line using  $\hat{y}$  = predicted weight and  $x$  = height.

26. What weight does the model predict for someone who is 160 cm tall?

27. Interpret the slope of the line in the context of this model.

28. Interpret the standard error of the slope in the context of this model.

29. Calculate the t-statistic for testing whether the slope is significant.

30. Are height and weight linearly associated? Explain. (Assume assumptions are met.)

31. A journal article might report that height is a *significant* predictor of weight. Explain what this means in context, as if to someone with no statistical background.

32. Calculate a 95% confidence interval for the slope.

33. Interpret your interval above in context.