Homework 07
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8.1)

a) I would expect $Var[Y_{i,j}|\theta_i, \sigma^2]$ to be less than $Var[Y_{i,j}|\mu, \tau^2]$ because the former can be considered a sub grouping of the latter and should have less spread than the overall data set.

b) I would expect $Cov[Y_{i1,j}, Y_{i2,j}|\theta_j, \sigma^2]$ to be positive meaning that the samples within the group are not independent when the mean is known. Likewise $Cov[Y_{i1,j}, Y_{i2,j}|\mu, \tau^2]$ would be zero because the mean is unknown and the samples are assumed independent.

c)

$$
\begin{aligned}
var[y_{i,j}|\theta, \sigma^2] =& var[y_{i,j}] + var[E(y_{i,j}|\theta, \sigma^2)] \\
=& \tilde{\sigma}^2 + var[\theta] \\
var[y_{i,j}|\mu, \tau^2] =& var[y_{i,j}] + var[E(y_{i,j}|\mu, \tau^2)] \\
=& \tilde{\sigma}^2 + \sigma^2 \\
var[\hat{y}_{.,j}|\theta, \sigma^2] =& var[\hat{y}_{.,j}] + var[E(\hat{y}_{.,j}|\theta, \sigma^2)] \\
=& \tilde{\sigma}^2 + var[\theta]/n \\
var[\hat{y}_{.,j}|\mu, \tau^2] =& var[\hat{y}_{.,j}] + var[E(\hat{y}_{.,j}|\mu, \tau^2)] \\
=& \tilde{\sigma}^2 + \sigma^2/n \\
cov[y_{i,j}|\theta, \sigma^2] =& var[y_{1i,j}] + var[y_{2i,j}] - var[E(y_{i,j}, y_{2i,j}|\theta, \sigma^2)] \\
=& var[\theta] + var[\mu] - \sigma^2 \\
cov[y_{i,j}|\mu, \tau^2] =& var[y_{1i,j}] + var[y_{2i,j}] - var[E(y_{i,j}, y_{2i,j}|\mu, \tau^2)] \\
=& var[\theta] + var[\mu] - \tau^2
\end{aligned}
$$

d)

$$
\begin{aligned}
&p(\mu|\theta_1 ... \theta_m, \sigma^2, \tau^2, y) \\
&\qquad \propto p(\mu, \theta_1 ... \theta_m, \sigma^2, \tau^2 | y) \\
&\qquad = \tau^{-m} exp[-\frac{1}{2\tau^2}\sum_{j=1}^{m}(\theta_j - \mu)^2] exp[\frac{1}{2\gamma_0^2}(\mu - \mu_0)^2]
\end{aligned}
$$

8.3)

```r
school1 = read.table("../Data/school1.dat", col.names = "S1")[,1]
school2 = read.table("../Data/school2.dat", col.names = "S2")[,1]
school3 = read.table("../Data/school3.dat", col.names = "S3")[,1]
school4 = read.table("../Data/school4.dat", col.names = "S4")[,1]
school5 = read.table("../Data/school5.dat", col.names = "S5")[,1]
school6 = read.table("../Data/school6.dat", col.names = "S6")[,1]
school7 = read.table("../Data/school7.dat", col.names = "S7")[,1]
school8 = read.table("../Data/school8.dat", col.names = "S8")[,1]

dta = list(s1 = c(length(school1), mean(school1), var(school1)),
           s2 = c(length(school2), mean(school2), var(school2)),
           s3 = c(length(school3), mean(school3), var(school3)),
           s4 = c(length(school4), mean(school4), var(school4)),
           s5 = c(length(school5), mean(school5), var(school5)),
           s6 = c(length(school6), mean(school6), var(school6)),
           s7 = c(length(school7), mean(school7), var(school7)),
           s8 = c(length(school8), mean(school8), var(school8)))

gibbs = normal.hierarchy.suff(Y = dta, nreps = 100000, mu0 = 7, gamma0 = 5,
                              eta0 = 2, tau0 = 10, nu0 = 2, sigma0 = 15)


#####
##### A)
#####

## Mean Estimate of Thetas
colMeans(gibbs[[1]])
```

```
[1]  9.413984  7.047829  7.936284  6.266357 10.673994  6.249389  6.176489
[8]  7.384697
```

```r
## Effective Sample Size for Sigma
var(gibbs[[2]]) / apply(X = gibbs[[1]], MARGIN = 2, FUN = var)
```

```
[1] 5.069583 4.647337 4.055971 4.850340 4.822647 4.427428 4.466980 4.073121
```

```r
## Effective Sample Size for Mu
var(gibbs[[3]]) / apply(X = gibbs[[1]], MARGIN = 2, FUN = var)
```

```
[1] 5.063824 4.642058 4.051364 4.844830 4.817168 4.422398 4.461905 4.068494
```

```r
## Effective Sample Size for Tau
var(gibbs[[4]]) / apply(X = gibbs[[1]], MARGIN = 2, FUN = var)
```

```
[1] 579.6894 531.4071 463.7864 554.6198 551.4531 506.2612 510.7838 465.7474
```

```r
#####
##### B)
```

```r
#####

## Posterior Mean and 95% Confidence for Sigma
mean(gibbs[[2]]); quantile(gibbs[[2]], c(.025, .975))
```

```
[1] 16.92979
```

```
    2.5%    97.5%
13.70228 20.86746
```

```r
## Posterior Mean and 95% Confidence for Mu
mean(gibbs[[3]]); quantile(gibbs[[3]], c(.025, .975))
```

```
[1] 7.55428
```

```
     2.5%     97.5%
 3.899602 11.159852
```

```r
## Posterior Mean and 95% Confidence for Tau
mean(gibbs[[4]]); quantile(gibbs[[4]], c(.025, .975))
```

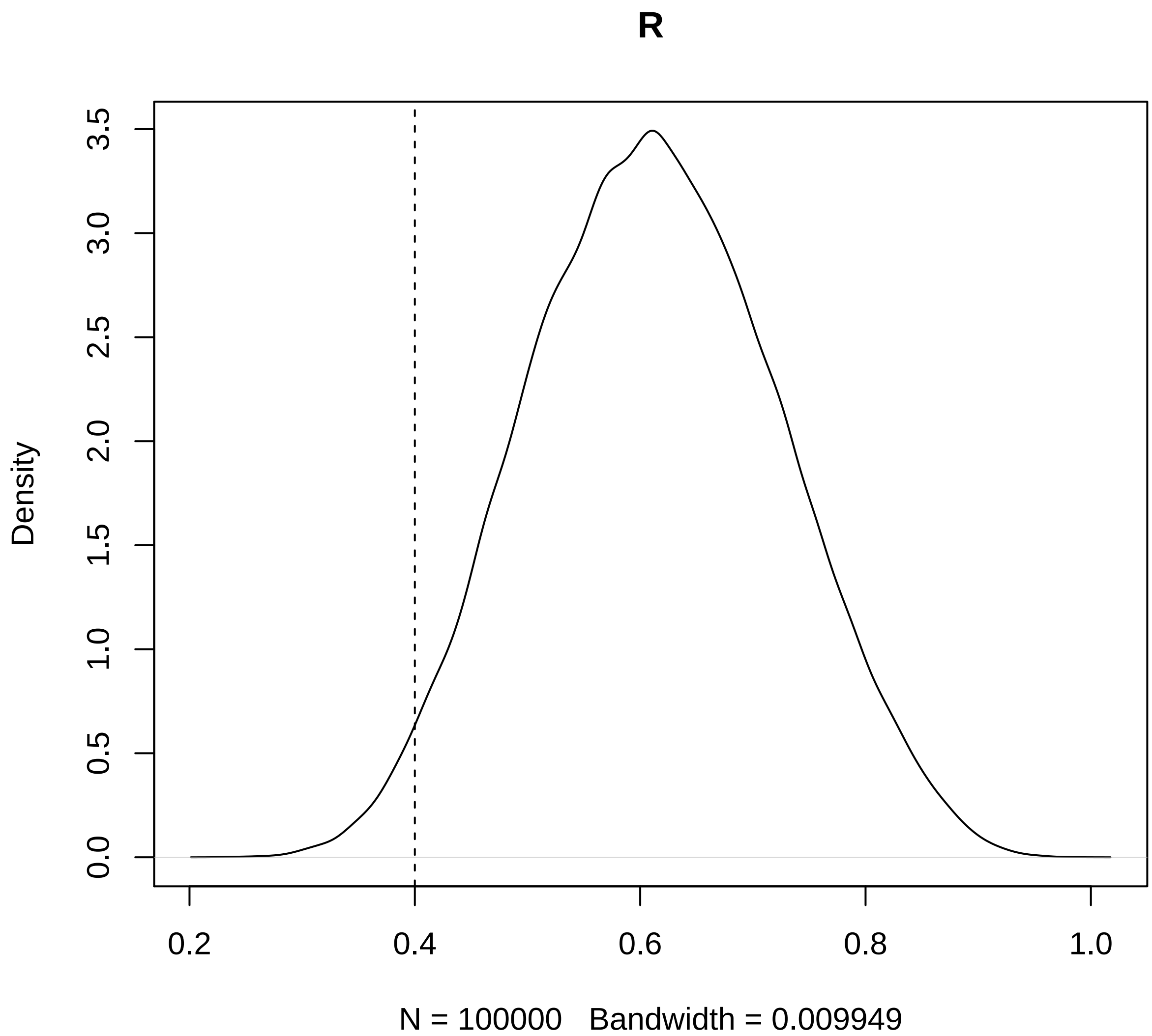
```
[1] 31.20825
```

```
    2.5%    97.5%
11.58911 80.53067
```

```r
##
##

#####
##### C)
#####

plot(density(gibbs[[4]] / (gibbs[[2]] + gibbs[[4]])), main = "R")
abline(v = .4, lty = 2)
```




```
## There is noticeable change in variance between Tau and Sigma which indicates that
## there are differences between the schools. Variance has also increased vs the prior estimat


#####
##### D)
#####

## Probability that Theta.7 is less than theta.8
length(which(gibbs[[1]][,7] < gibbs[[1]][,8])) / 100000
```

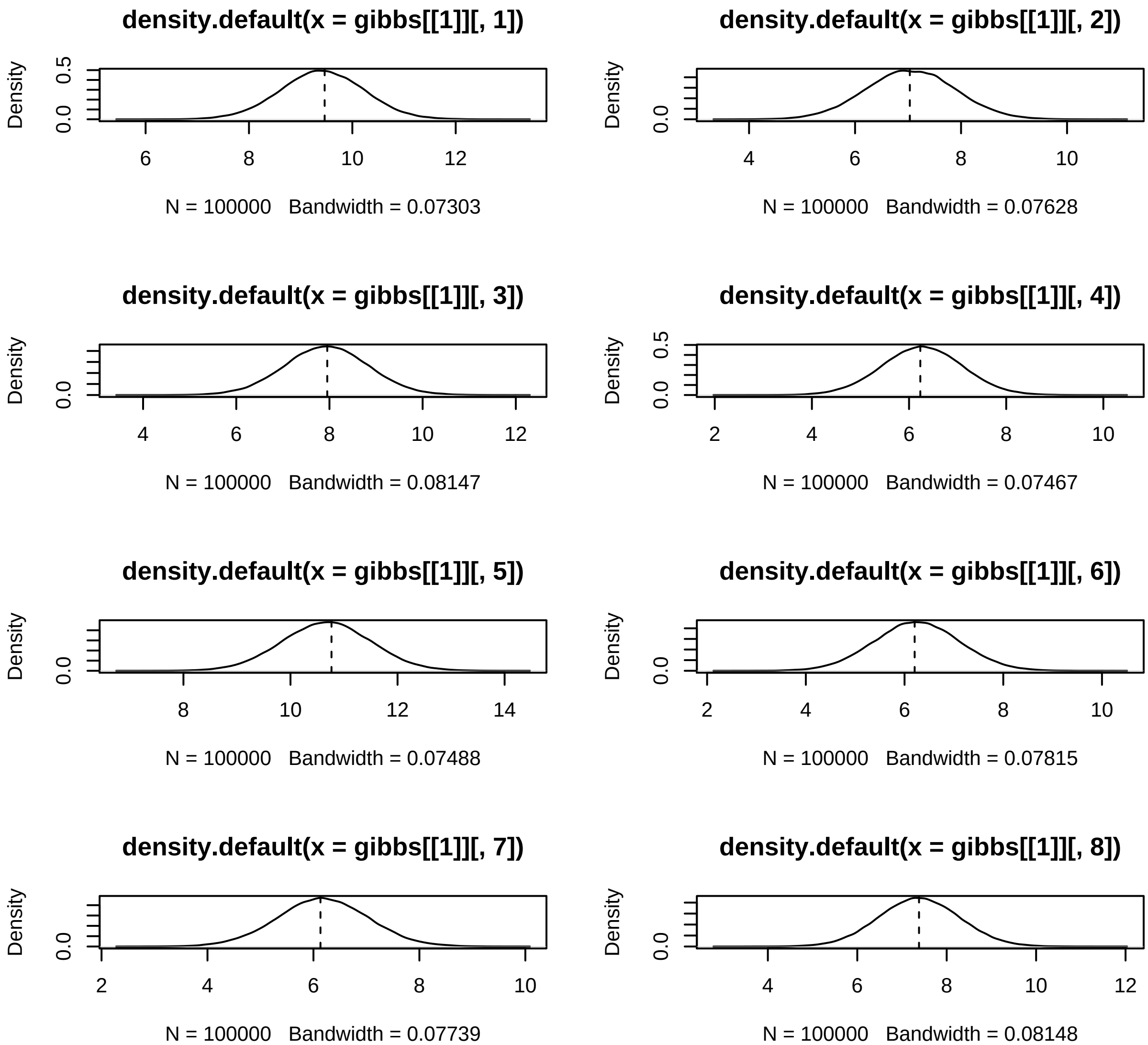
```
[1] 0.83379
```

```
## Probability that Theta.7 is the smallest value
length(which(gibbs[[1]][,7] < gibbs[[1]][,8] & gibbs[[1]][,7] < gibbs[[1]][,6] &
               gibbs[[1]][,7] < gibbs[[1]][,5] & gibbs[[1]][,7] < gibbs[[1]][,4] &
               gibbs[[1]][,7] < gibbs[[1]][,3] & gibbs[[1]][,7] < gibbs[[1]][,2] &
               gibbs[[1]][,7] < gibbs[[1]][,1])) / 100000
```

```
[1] 0.32184
```

```
#####
##### E)
#####

par(mfrow = c(4, 2))
plot(density(gibbs[[1]][,1])); abline(v = mean(school1), lty = 2)
plot(density(gibbs[[1]][,2])); abline(v = mean(school2), lty = 2)
plot(density(gibbs[[1]][,3])); abline(v = mean(school3), lty = 2)
plot(density(gibbs[[1]][,4])); abline(v = mean(school4), lty = 2)
plot(density(gibbs[[1]][,5])); abline(v = mean(school5), lty = 2)
plot(density(gibbs[[1]][,6])); abline(v = mean(school6), lty = 2)
plot(density(gibbs[[1]][,7])); abline(v = mean(school7), lty = 2)
plot(density(gibbs[[1]][,8])); abline(v = mean(school8), lty = 2)
```




```
## All of the sample means are close to the peak of the posterior distributions
## The posterior mu of 7.55 is close to the original prior mu of 7
```