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Hw06, STAT 630

1) 3.5.4

a) -4(1/11 / 2/11) + 6(1/11 / 2/11) = -2 + 3 = 1
b) -4(2/11 / 3/11) + 6(1/11 / 3/11) = -8/3 + 2 = -2/3
c) -4(4/11 / 5/11) + 6(1/11 / 5/11) = -16/5 + 6/5 = -2
d) -4(0) + 6(1/11 / 1/11) = 6
e) 1 + -2/3 - 2 + 6 = 4.333

2) 3.5.11ace

a)

$$\begin{aligned}
E(x) &= \frac{6}{19}\int_0^2\int_0^1 x^2 + y^3 \, dydx \\
&= \frac{6}{19}\int_0^2 x^2y + \frac{y^4}{4}|_0^1 \, dx \\
&= \frac{6}{19}\int_0^2 x(x^2 + \frac{1}{4}) \, dx \\
&= \frac{6}{19}(\frac{x^4}{4} + \frac{x^2}{8}|_0^2) \\
&= 4.5
\end{aligned}$$

b)

$$\begin{aligned}
E(x|y) &= \frac{57}{96}\int_0^2 x^2y^{-3} \, dx \\
&= \frac{57}{96}(\frac{x^3}{3}y^-3|_0^2) \\
&= \frac{1.58}{y^3}
\end{aligned}$$

c)

3) 3.5.16

$$E(x) = \int xf(x) \, dx$$

$$\begin{aligned}
F_{x|y}(x|y) &= \frac{\lambda^a x^{a-1}/\gamma(a)}{e^{-x/\lambda}/\lambda} \\
&= \frac{\lambda\gamma(a)}{e^{-x/\lambda}\lambda^a x^{a-1}}
\end{aligned}$$

4) 3.6.3

a) P(x > 9) = E(x) = 1, 1/9 = .111
b) P(x > 2) = 1/2
c) P(|x - 1| > 1) = 2 / 1^2 = 2, but this extends outside of the probability bounds and doesn't tell us anything
d) b is equal to the mean of the distribution and c is equal to the variance
e) P(x > 9) = (1 - .5)^9 * .5 = .0009, P(x > 2) = (1 - .5)^2 * .5 = .25, P(|x - 1| > 1) = .5

5) 3.6.11

a) $E(z) = \int_0^2 z\frac{z^3}{4}\, dz = \frac{z^5}{20}|_0^2 = 1.6$

b)

$$\begin{aligned} E(z^2) &= \int_0^2 z^2 f(x)\, dx \\ &= int_0^2 \frac{z^5}{4}\, dz \\ &= \frac{z^6}{24}|_0^2 \\ &= 2.66 \end{aligned}$$

$$\begin{aligned} Var(z) &= 2.66 - 1.6 \\ &= 1.06 \\ &= \frac{1}{1.06} \\ &= .94 \end{aligned}$$

6) additional a $\lambda\theta$ is a poisson distribution because the following holds true when using the mgf to find the mean and variance

$$\begin{aligned} withmgf & \\ f(t) &= e^{\lambda\theta(e^t-1)} \\ f'(t) &= \lambda\theta e^{\lambda\theta(e^t-1)+t} \\ f'(0) &= \lambda\theta \\ \\ f''(t) &= \lambda\theta(e\lambda\theta+1)^{e\lambda\theta(ex-1)+x} f''(0) & &= \lambda\theta \end{aligned}$$

7) additional b

$$T\ exponential(\lambda), conditional on T$$
$$U\ uniform[0,T]$$

$$E[E(U|T)] = E(U)$$
$$mean = \frac{T}{2}$$
$$variance = \frac{T^2}{12}$$

```r
# a
max.norm = as.numeric()
for (i in 1:10){max.norm = c(max.norm, max(rnorm(10^6, mean = 0, sd = 1)))}
max.norm
```

8) 4.1.11

```
##  [1] 4.992599 5.625752 4.768809 4.751971 4.964059 4.742862 4.986028
##  [8] 4.916164 4.950232 4.808086
```

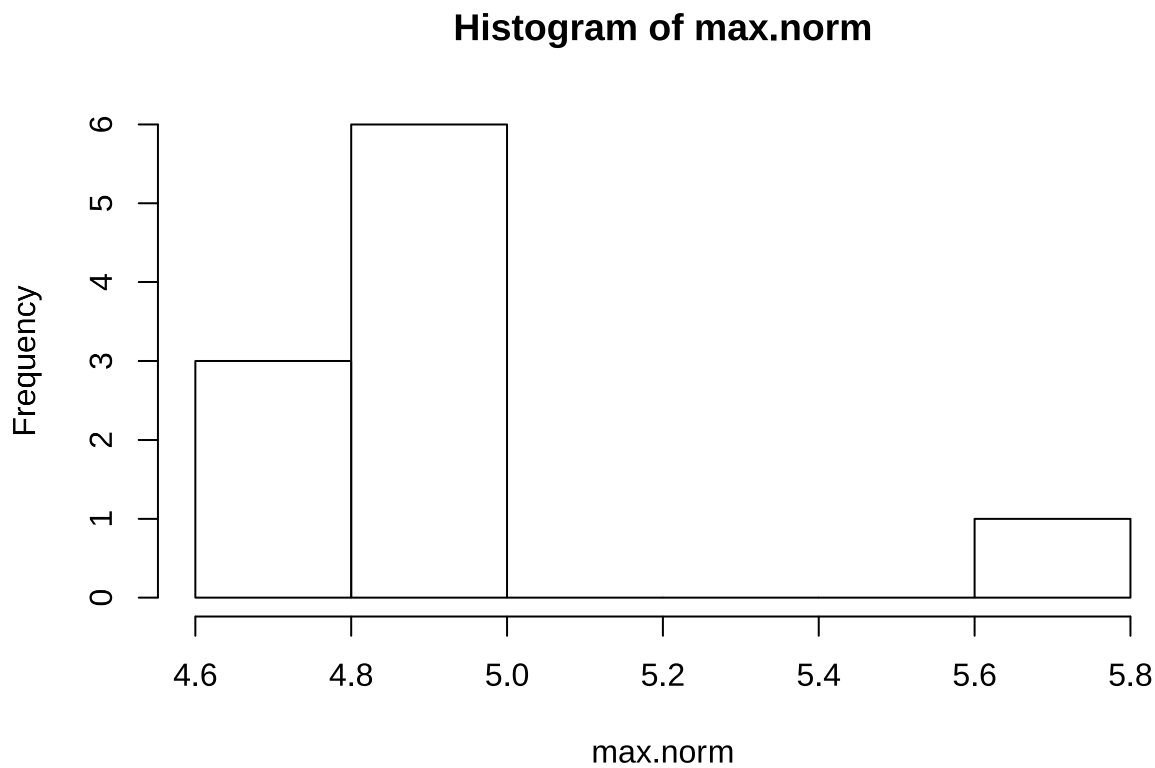
```r
mean(max.norm)
```

```
## [1] 4.950656
```

```r
sd(max.norm)
```

```
## [1] 0.2573765
```

```r
# b) The histogram changes dramatically with only 10 total samples.
# It looks as though it will be normally distributed given more samples
hist(max.norm)
```




```
# c
max.norm.20 = as.numeric()
for (i in 1:20){max.norm.20 = c(max.norm.20, max(rnorm(10^6, mean = 0, sd = 1)))}
max.norm.20
```

```
##  [1] 4.902280 4.581705 4.860167 4.789735 5.110561 4.774266 4.496828
##  [8] 4.532096 5.063028 4.586803 4.621516 4.735194 4.846381 5.078942
## [15] 5.064213 5.018005 4.622862 4.847498 5.243185 5.655335
```

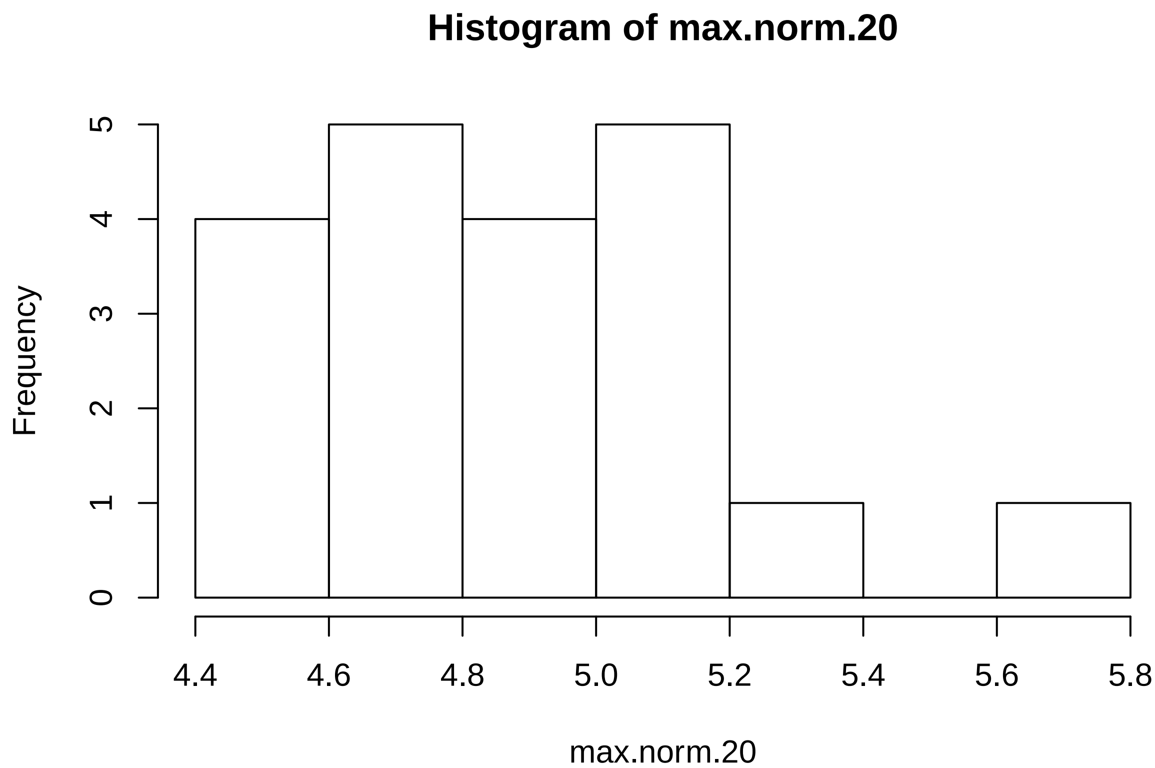
```
mean(max.norm.20)
```

```
## [1] 4.87153
```

```
sd(max.norm.20)
```

```
## [1] 0.2837657
```

```
# d) The histogram is starting to take the shape of a normal distribution
hist(max.norm.20)
```

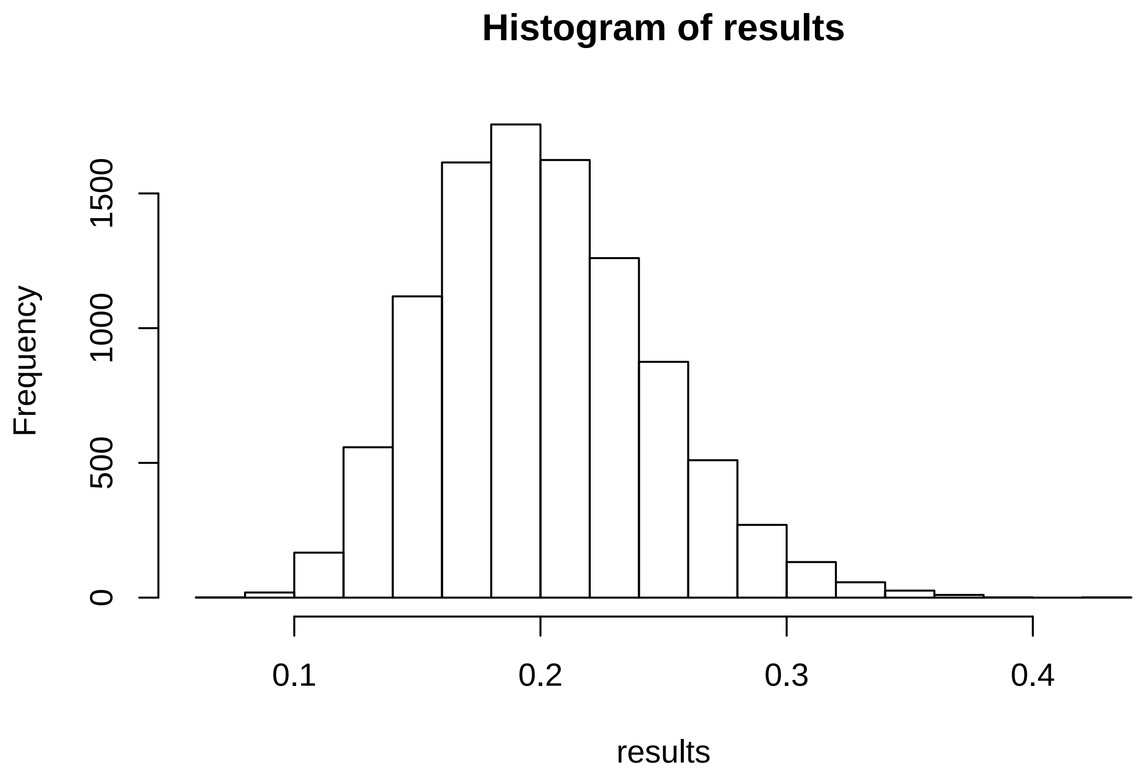


9) 4.2.2

$$ln(X_n) = n$$
$$ln(1) = 0$$
$$lim_{n\to\infty} X_n = 0$$

10) 4.2.10 $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = \frac{91}{6} = 15.1$
For large n, the average is very close to 15.1

```
results = as.numeric()
for (i in 1:10^4) results = c(results, mean(rexp(20,5)))
hist(results)
```

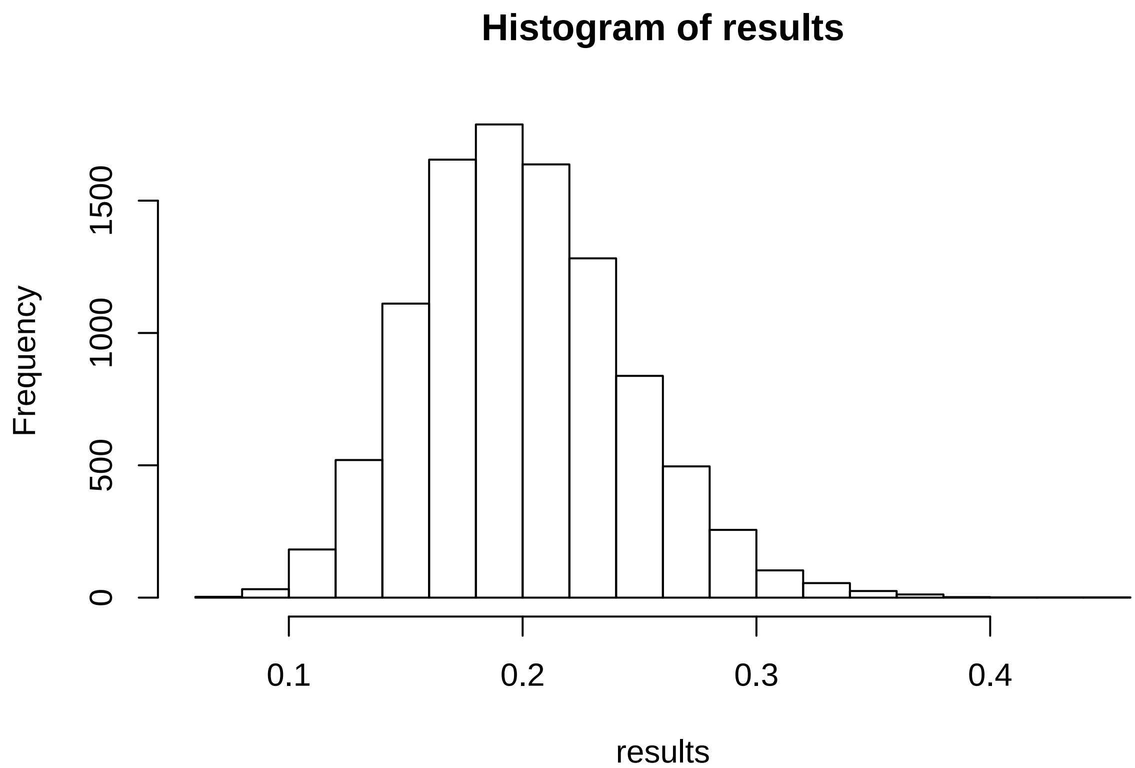


11) 4.2.12

```
n20 = length(which(results >= .19 & results <= .21)) / length(results)
# The proportion of results that are between .19 and .21 are:
n20
```

```
## [1] 0.1741
```

```
results = as.numeric()
for (i in 1:10^4) results = c(results, mean(rexp(20,5)))
hist(results)
```




```
n50 = length(which(results >= .19 & results <= .21)) / length(results)
# Repeated with n = 50, the proportion of results that are between .19 and .21 are:
n50
```

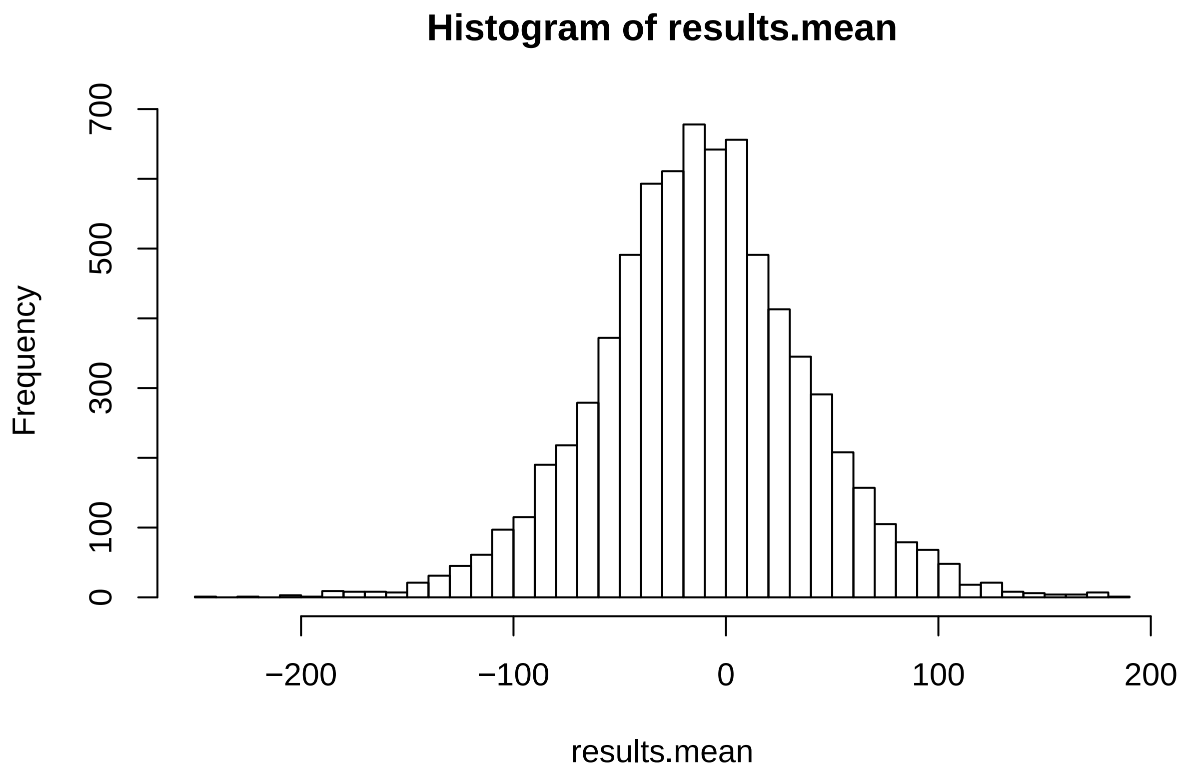
```
## [1] 0.1771
```

12) 4.4.4

$$
\begin{aligned}
W_n(x) &= \frac{1+\frac{x}{n}}{1+\frac{1}{2n}} \\
&= \int_0^\infty && = W_n(x)\,dx \\
&= lim_{n\to\infty}\frac{\frac{x^2}{2n}+x}{\frac{n}{2}+1} \\
&= x = 1
\end{aligned}
$$

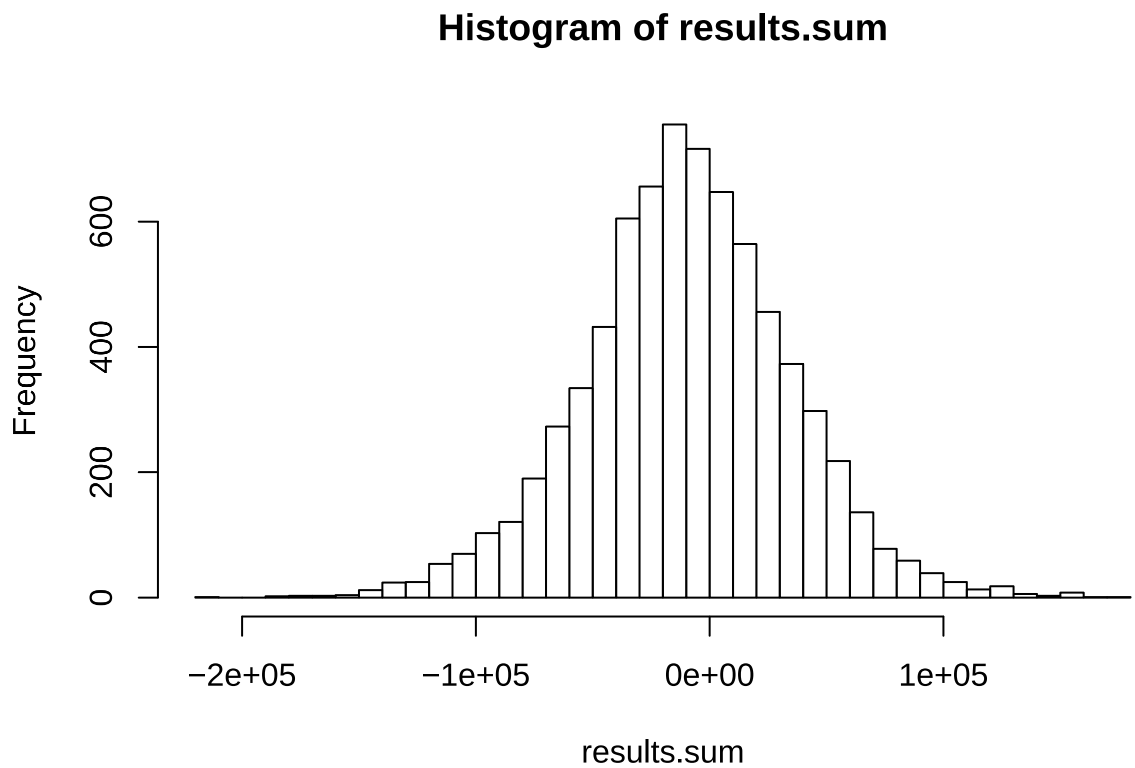
```
func = function(x) 1/x * sum(x/(-10))
results.mean = as.numeric()
results.sum = as.numeric()
```

```
for (i in 1:10000) {
  results.mean[i] = mean(func(sample(seq(-20,10,.01) , 900, replace = T)))
  results.sum[i] = sum(func(sample(seq(-20,10,.01) , 900, replace = T)))
  }
hist(results.mean, breaks = 40)
```



13) 4.4.6

```
hist(results.sum, breaks = 40)
```



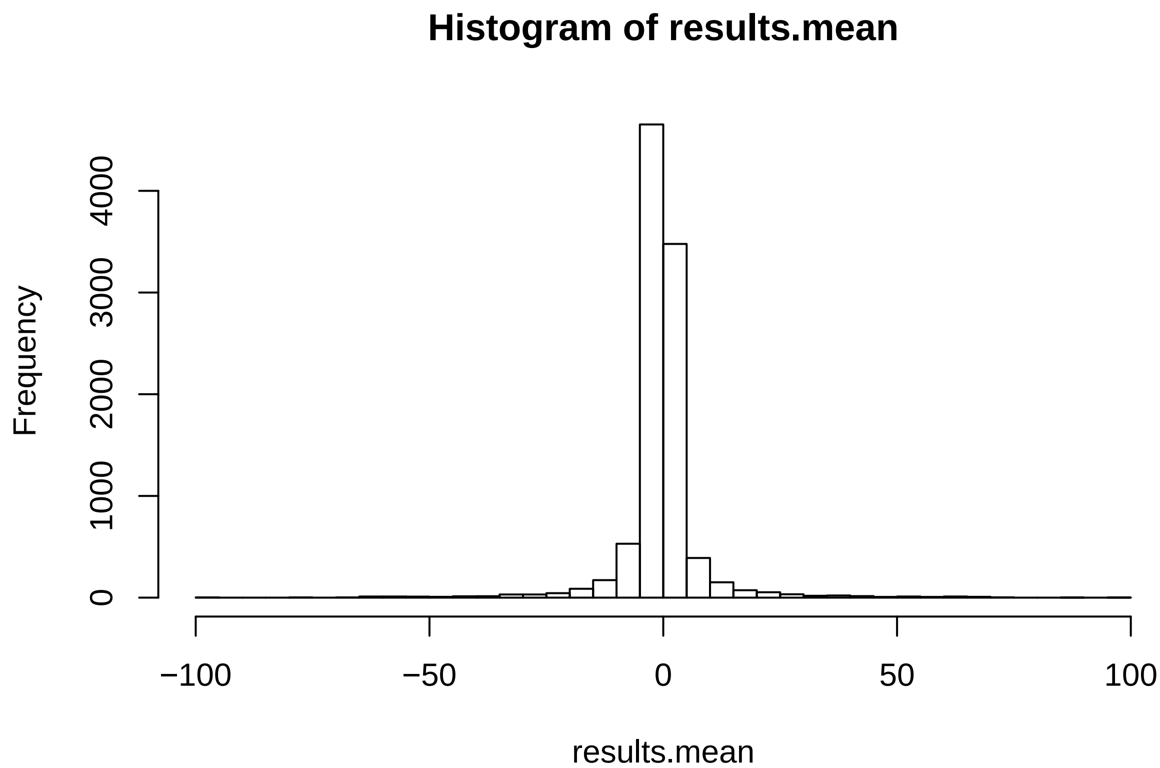

```
length(which(results.sum > -4470))/length(results.sum)
```

```
## [1] 0.5935
```

14) 4.4.12

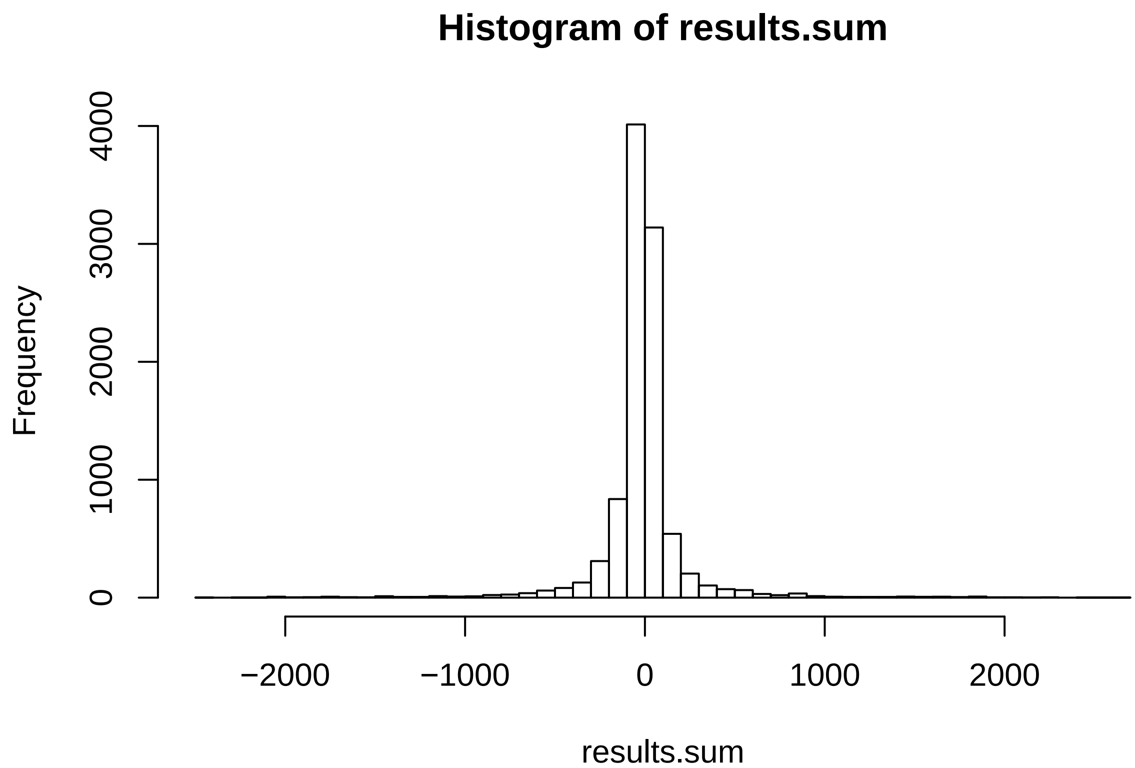
a) for n = 16, $\frac{2.5-2}{2\sqrt{16}} = \frac{.5}{8} = .062z - score, 1 - .4761 = .523$
b) for n = 36, $\frac{2.5-2}{2-sqrt36} = \frac{.5}{12} = .04z - score, 1 - .484 = .516$
c) for n = 100, $\frac{2.5-2}{2-sqrt100} = \frac{.5}{20} = .025z - score = 1 - .492 = .508$

```
func = function(x) 1/x * sum(x/(-10))
results.mean = as.numeric()
results.sum = as.numeric()
for (i in 1:10000) {
  results.mean[i] = mean(func(sample(seq(-20,10,.01) , 30, replace = T)))
  results.sum[i] = sum(func(sample(seq(-20,10,.01) , 30, replace = T)))
  }
hist(results.mean, breaks = 40)
```



15) 4.4.16

```
hist(results.sum, breaks = 40)
```



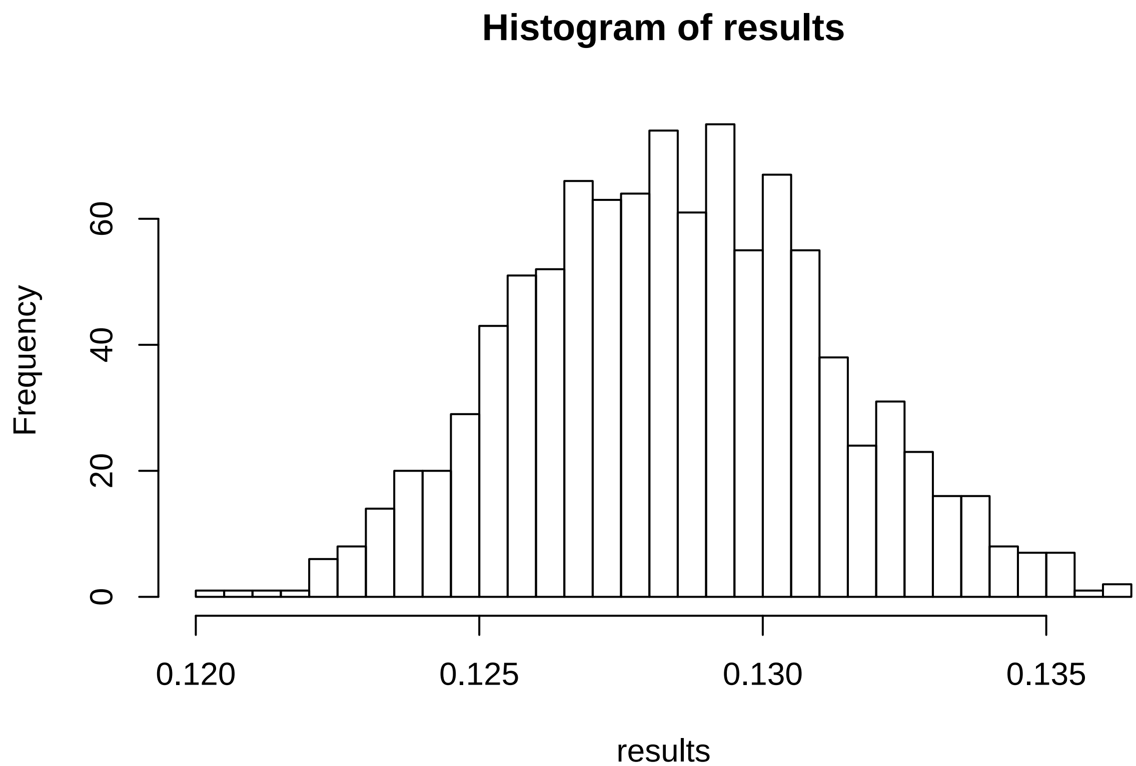

```
length(which(results.sum < -5))/length(results.sum)
```

```
## [1] 0.5264
```

```
results = as.numeric()
n = 10000
t = function(U) cos(U^3) * sin(U^4)

for (i in 1:1000) {
  U = rnorm(n)
  round = mean(t(U))
  results = c(results, round)
  }

hist(results, breaks = 40)
```



16) 4.5.14

```
mean(results)
```

```
## [1] 0.1284558
```

```
mean(results) - sd(results) * 1.96   ## lower 95% confidence
```

```
## [1] 0.1230478
```

```
mean(results) + sd(results) * 1.96   ## upper 95% confidence
```

```
## [1] 0.1338638
```