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STAT 630
HW10

7.1.3 ...

```
location_normal = 1 - pnorm(0, 0, sqrt(1))
prior = 1 - pnorm(0, 1, sqrt(10))
location_normal * prior
```

```
## [1] 0.3120426
```

7.1.4 ...

$$
\begin{aligned}
Poisson(\lambda) &= \frac{\lambda^x}{x!}e^{-\lambda} \\
Gamma(a,b) &= \frac{b^a x^{a-1}}{\gamma(a)}e^{-bx}
\end{aligned}
$$

$$
\begin{aligned}
f(x,\theta) &= Poisson(\lambda) * Gamma(a,b) \\
&= \frac{b^a x^{a-1}}{\gamma(a)}e^{-bx}\frac{\lambda^x}{x!}e^{-\lambda}
\end{aligned}
$$

$$
m(x) = \frac{b^a x^{a-1}}{\gamma(a)}\frac{1}{x!e^x}e^{-bx}\int_0^1 \frac{x!e^x}{\lambda^{x-1}}\frac{\lambda^x}{x!}e^{-\lambda}d\lambda
$$

$$
\begin{aligned}
Posterior &= f(x,\lambda) \\
&= \frac{\frac{b^a x^{a-1}}{\gamma(a)}e^{-bx}\frac{\lambda^x e^{-\lambda}}{x!}}{\frac{b^a x^{a-1}}{\gamma(a)}e^{-bx}\frac{\lambda^{x-1}}{x!e^x}} \\
&= \frac{x!\lambda^x e^{-\lambda x}}{\lambda^{x-1}x!} \\
&= \lambda e^{-\lambda x}
\end{aligned}
$$

(b) Find the posterior mean, posterior mode, and posterior variance.
PosteriorMean $= 1/\lambda$
PosteriorMode $= 0$
PosteriorVariance $= 1/\lambda^2$

7.1.9 ...

$$Uniform[.4, .6] = 1/(.6 - .4)$$
$$Binomial(n, \theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$$
$$f(x, \theta) = Binomial(n, \theta) * Uniform[.4, .6]$$

$$m(x) = \frac{1}{.6 - .4}(n+1)\theta^n \int_0^1 \frac{\binom{n}{x}\theta^n(1-\theta)^{n-x}}{(n+1)\theta^n}$$

$$posterior = \frac{(n+1)\theta^n I[.4, .6]\theta}{(.6^{n+1} - .4^{n-1})}$$

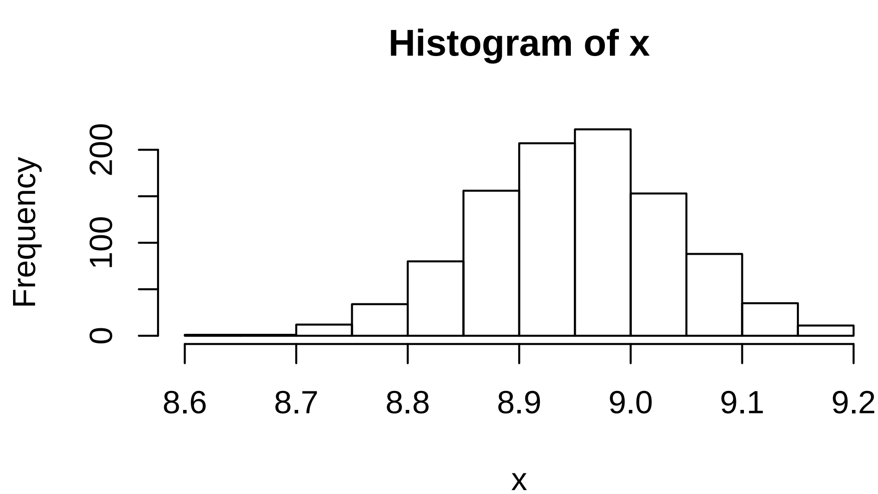
7.1.14 ...

```
mu = function(x, gam, var, n, mu0) ((1 / gam) + (n / var))^-1 * ((mu0/gam) + (n/var)) * x
sd0 = function(n, var, gam) ((1 / gam) + (n / var))^-1

x = rnorm(n = 1000,
          mean = mu(x = 8.2, gam = 1, var = 1, n = 10, mu0 = 2),
          sd = sd0(n = 10, var = 1, gam = 1))

error = sd(x) / sqrt(length(x))
mean(x) - error; mean(x) + error
```

```
## [1] 8.947598
```

```
## [1] 8.953201
```

```
hist(x)
```



7.2.1 ...

$$PosteriorMean = \frac{\theta^{(n\bar{x}+\alpha-1)}(1-\theta)^{n(1-\bar{x})+\beta-1}}{n}$$

$$n = 40, \bar{x} = .25, \alpha = 1, \beta = 1$$

$$\begin{aligned} PosteriorMean &= \frac{\theta^{(40*.25+1-1)}(1-\theta)^{40(1-.25)+1-1}}{40} \\ &= \frac{\theta^{10}(1-\theta)^{30}}{40} \end{aligned}$$

7.2.2 since the distribution is symmetrical the mode and the mean are equal

$$\begin{aligned} \psi &= \mu + \sigma_0 z_.75 \\ &= \mu + 1(.67) \end{aligned}$$

$$\psi = (\frac{1}{\gamma_0^2} + \frac{n}{\sigma_0^2})^{-1})(\frac{\mu_0}{\gamma_0^2} + \frac{n}{\sigma_0^2}) + .67$$

$$\gamma_0^2 = 2, \sigma_0^2 = 1, \mu_0 = 0, n = 10$$

$$\begin{aligned} \psi &= (\frac{1}{2} + \frac{n}{1})^{-1})(\frac{0}{2} + \frac{10}{1}) + .67 \\ &= .67 \end{aligned}$$

7.2.10

$$X = Exponential(\lambda) = \lambda e^{-\lambda x}$$
$$\lambda = Gamma(\alpha, \beta) = \frac{\beta^{\alpha} x^{\alpha - 1}}{\gamma(\alpha)} e^{-\beta x}$$

$$f(x|\lambda) = Exponential(\lambda) * Gamma(\alpha, \beta)$$
$$= \frac{\beta^{\lambda} x^{\alpha - 1}}{\gamma(\alpha)} \lambda e^{\lambda \beta x^2}$$

$$m(x) = \frac{\beta^{\alpha} x^{\alpha - 1}}{\gamma(\alpha)} e^{-\beta x} \int_0^1 \frac{1}{e^{-\beta x}} \lambda e^{\lambda \beta x^2}, d\lambda$$

$$Posterior = \lambda e^{-\lambda \beta^2 x^3}$$
$$Expectation = \int_0^1 \lambda^2 e^{-\lambda \beta^2 x^3}$$
$$= -(\frac{1}{\beta^2 x^3} + \frac{2}{\beta^4 x^6} + \frac{2}{\beta^6 x^9}) e^{-\beta x^3}$$

6.3.27 $1 - 2\phi(\frac{\bar{x} - \mu}{\sigma - \sqrt{n}})$

8.2.5

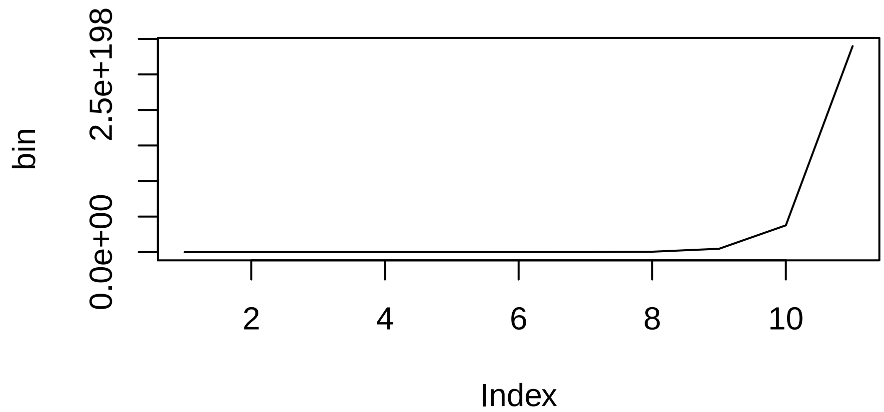
a) 0
b) $1 - (1/\theta)$

Additional A

$$binomial(100, \theta) = \binom{100}{x} \theta^x (1 - \theta)^(n - x)$$

$$\beta(\mu') = \frac{.1 - \mu}{.02/\sqrt{100}} \leq -Z_{1-\alpha}$$
$$\mu = .1$$
$$\alpha = 0$$

```
theta = 40:50
bin = choose(100, 50) * theta^50 * (1 - theta)^(100 - 50)
abs.bin = abs(bin - 50)
plot(bin, type = "l")
```



Additional B

a) best order x = {4, 1, 2, 3}

x	1	2	3	4
f0(x)	0.2	0.3	0.3	0.2
f1(x)	0.1	0.4	0.1	0.4
f1(x)/f0(x)	.5	1.3	0.3	2.0

b) at a = .2 -> x = 1,2,3,4 at a = .5 -> x = 1,2,4

c) at a = .2 -> power = .2 + .3 + .3 + .2 = 1 at a = .5 -> power = .2 + .3 + .2 = .8

d) outcomes x = 1 and x = 3