STATISTICS 641 - Spring 2015 - ASSIGNMENT 8

DUE DATE: NOON, Friday, April 17

Name ____

Email Address ____

Make this cover sheet the first page of your Solutions.

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- Read Handout 12 and Chapters 6.3, 7, 9.1, 14.1 in the Textbook
- Solve and hand in the following problems:

1. (10 points) A company purchases connectors in shipments of size 100,000. The shipment is considered acceptable if the mean strength, μ of the connectors exceeds 10,000 psi. The company cannot test all 100,000 connectors so they take a random sample of 10 connectors and test just these 10 connectors. If mean strength, $\bar{Y}$, of the 10 connectors exceeds 10500 psi, the shipment is accepted. Calculated the probability of a Type I error and the probability of a Type II error at $\mu = 10,700$. From many previous studies, a value of $\sigma = 1000$ psi, may be used in the probability calculations.

2. (10 points) Refer to the previous question. Suppose the company wants the probability of Type I error to be at most .01.

   (a.) What is an appropriate decision rule in terms of $\bar{Y}$?

   (b.) Calculate the probability of a Type II error at each of the following values of the true mean strength of the shipment:

   10,600 psi; 10,800 psi; 11,000 psi; 11,500 psi

3. (10 points) A new additive has been formulated to reduce the reaction time in a chemical process. With the previously used additive, the average reaction time was 10 minutes. In order to evaluate the effectiveness of the new additive, 15 batches of the material are formulated and the new additive is placed in the batches. From previous studies, reaction times appear to have a normal distribution.

   (a.) The mean reaction time from the 15 batches was 8.7 minutes with a standard deviation of 2 minutes. Is there significant evidence using an $\alpha = .01$ test that the average reaction time has been reduced? Include the p-value with your decision.

   (b.) The process engineer had claimed that the new additive will reduce the reaction time by at least 1.5 minutes. What is the probability that the experiment will be able to detect a reduction of the average reaction to 8.5 minutes or smaller using $\alpha = .01$?

   (c.) A new study is to be designed. What sample size is needed for an $\alpha = .05$ test to have at least an 80% chance to detect that the average reaction time is 9 minutes or less?

4. (10 points) A new device has been developed which allows patients to evaluate their blood sugar levels. The most widely device currently on the market yields widely variable results. The new device is evaluated by 25 patients having nearly the same distribution of blood sugar levels yielding the following data:

125	123	117	123	115	112	128	118	124
111	116	109	125	120	113	123	112	118
121	118	122	115	105	118	131		

   (a.) Is there significant evidence ($\alpha = .10$) that the standard deviation in the readings from the new device is less than 10?

   (b.) Compute the probability of a Type II error in using your test from part (a.) for the following values of σ: 5, 6, 7, 8, 9, 10

   (c.) Construct an upper 90% confidence bound on the standard deviation of the new device. Is this bound consistent with your answer to the question in part (a.)?

5. (10 points) Refer to the blood sugar device data in Problem 4.

   (a.) Is there significant ($\alpha = .05$) evidence that median blood sugar readings was less than 120 in the population from which the 25 patients were selected? Use the sign test and report the p-value.

   (b.) Is there significant ($\alpha = .05$) evidence that median blood sugar readings was less than 120 in the population from which the 25 patients were selected? Use the Wilcoxon signed rank test and report the p-value.

   (c.) Place a 95% lower bound on the median blood sugar reading.

6. (10 points) The current method of identifying patients at risk of sudden cardiac death can be identified with 80% accuracy. A change in the method has hopefully improved the accuracy. To evaluate the new method, 50 people are tested and the new method produced the result on 46 of the 50 people.

   (a.) Place a 95% confidence interval on the accuracy of the device.

   (b.) Is there substantial evidence ($\alpha = .05$) that the improved method has increased the accuracy over the current method?

   (c.) Compute the power of the test in part (b.) to detect that the accuracy of the improved method is 75%, 80%, 85%, 90%, 95%.

   (d.) How many patients would need to be included in a new study in order to have a power value of 80% if the new method had an accuracy of 90%.

Multiple Choice (40 points) SELECT ONE of the following letters **(A, B, C, D, or E)** corresponding to the **BEST** answer. Show details for partial credit.

(MC1.) Suppose that $X_1, \cdots, X_n$ are to be used to construct a 95% prediction interval for a normal population. The researcher notes that the data was collected by an automatic sampler which may result in $X_1, \cdots, X_n$ having a high positive correlation. If the prediction interval was computed using the formula: $\bar{X} \pm t_{.025,n-1} S\sqrt{1 + \frac{1}{n}}$, the resulting interval

A. will be too wide.

B. will have a level of confidence greater than 95%.

C. will have a level of confidence less than 95%.

D. will have a level of confidence equal to 95%.

E. none of the above

(MC2.) Suppose a normal population has a standard deviation of $\sigma = 9$. In order to be 95% confident that the difference between the sample estimator of the population mean $\hat{\mu}$ and the true value μ is at most 1.5 units, the sample size n must be at least

A. 140

B. 100

C. 98

D. 35

E. cannot be determined without further information

(MC3.) In a level $\alpha = .05$ test of $H_o : \mu \leq 17$ versus $H_1 : \mu > 17$, where μ is the mean of a normally distributed population, the sample size needed to have a Type II error rate of at most 0.10 whenever $\mu > 17 + .5 * \sigma$ is

A. 36
B. 22
C. 13
D. 70
E. need the non-central t cdf in order to determine sample size

(MC4.) The power of a test of the hypothesis: $H_1 : \mu < \mu_o$

A. is not a function of the value of α
B. is the probability of a Type II error
C. is one minus the probability of a Type II error at μ_o
D. varies depending on the value of μ
E. none of the above

(MC5.) In testing the hypotheses $H_o : \sigma \leq 23.8$ versus $H_1 : \sigma > 23.8$, where σ is the standard deviation of a normally distributed population, an $\alpha = .05$ test was run using a independent random sample of size $n = 10$. The probability of a Type II error when $\sigma = 47.9$ is

A. .05
B. .95
C. .10
D. .90
E. need noncentral Chi-squared tables to compute power

(MC6.) A researcher wants to determine if there is an increase in the likelihood that people will purchase a product after a redesign of the product. The current market share is 20%. Initially, the researcher was planning on using a random sample of n=20 persons with an $\alpha = .05$ test to evaluate the product. He wants you to calculate the chance that the study will fail to detect that preference for the product has been increased if in fact the preference for the new product is 40%. This chance is

A. .316
B. .596
C. .416
D. .950
E. cannot be determined with the given information

(MC7.) A random sample of n=15 from a normally distributed population is used to construct a level $\alpha = .01$ test of $H_o : \mu \leq 20$ versus $H_1 : \mu > 20$, where μ is the mean of the population. The probability of a Type II error for $\mu > 20 + .8\sigma$ is at most

A. .05
B. .55
C. .22
D. .32
E. cannot be determined from the given information

(MC8.) A psychologist is investigating the IQ level of young children who have been in a head start program. She wants to determine if the variation in IQ scores for the population of head start students is smaller than the variation in the general population of children under the age of 6 which has a variation of $\sigma = 10.2$. She also informs you that the distribution of IQ scores is highly right skewed. Suppose she uses the test: reject H_o is $\frac{(n-1)S^2}{(10.2)^2} < \chi^2_{.95,n-1}$, where S is the standard deviation from a random sample of n head start students, to test whether σ is less than 10.2 with an α value of 0.05.

A. the actual level of significance will be greater than 0.05.

B. the actual level of significance will be less than 0.05.

C. the actual level of significance will be very close to 0.05.

D. the actual level of significance will be exactly 0.05.

E. it is impossible to determine the effect of skewness on the actual level of significance.

Use the following information for Problems MC9-MC10.

A process engineer samples a continuous flow of the company's product 200 times per day and obtains the following pH levels in the product : $X_1, \cdots, X_{200}$. He determines that the daily pH levels are related by $X_t = \theta + \rho X_{t-1} + e_t$, where the e_ts have independent $N(0, \sigma_e^2)$ distributions and $\rho \approx .92$.

(MC9.) The engineer constructs a nominal 95% confidence interval for the average daily pH level, μ, using the formula $\bar{X} \pm t_{.025,199}(s/\sqrt{200})$, where $\bar{X}$ and s are the sample mean and standard deviation for a given days pH levels. The true coverage probability of this confidence interval

A. is 0.95.

B. is much less than 0.95.

C. is very close to 0.95.

D. is much greater than 0.95.

E. may be greater than 0.95 or less than 0.95 depending on the distribution of the X_t's.

(MC10.) Refer to Problem MC9. The nominal pH level of the product is 5.3. The process engineer wants to test if the pH on a given day is different from 5.3. He uses $t = \frac{\bar{X}-5.3}{s/\sqrt{200}}$ as his test statistic. Next, he uses the t-distribution with df=199 to compute the p-value of the observed data. The computed p-value will be

A. correct because the sample size is large.

B. much smaller than the correct p-value.

C. much larger than the correct p-value.

D. very close to the correct p-value because the sample size is large.

E. may be greater or less than the correct value depending on the size of σ.