STATISTICS 642 - Final Exam Solution

I 24 points

1. Complete the following AOV table: G=Grade, M=Manufacturer, R=Run

SV	DF	Mean Squares	EMS
G	2	44.24	$\sigma_e^2 + 2\sigma_{G*R(M)}^2 + 18Q_G$
M	2	12.63	$\sigma_e^2 + 2\sigma_{G*R(M)}^2 + 6\sigma_{R(M)}^2 + 18Q_M$
G*M	4	5.94	$\sigma_e^2 + 2\sigma_{G*R(M)}^2 + 6Q_{G*M}$
R(M)	6	67.02	$\sigma_e^2 + 2\sigma_{G*R(M)}^2 + 6\sigma_{R(M)}^2$
G*R(M)	12	1.36	$\sigma_e^2 + 2\sigma_{G*R(M)}^2$
Error	27	4.04	σ_e^2

	3	3		3		2
SV	Q_G	Q_M	Q_{G*M}	$\sigma_{R(M)}^2$	$\sigma_{G*R(M)}^2$	σ_e^2
G	18	0	0	0	2	1
M	0	18	0	6	2	1
G*M	0	0	6	0	2	1
R(M)	0	0	0	6	2	1
G*R(M)	0	0	0	0	2	1
Error	0	0	0	0	0	1

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2. At the $\alpha = 0.05$ level, do the controllers from the three Manufacturers have a difference in their mean reliability?

Test for interaction: $H_o : Q_{G*M} = 0$ vs $H_o : Q_{G*M} \neq 0$: $F = \frac{MS_{G*M}}{MS_{G*R(M)}} = \frac{5.94}{1.36} = 4.37 > 3.26 = F_{.05,4,12}$

Test for Main effect of M: $H_o : Q_M = 0$ vs $H_o : Q_M \neq 0$: $F = \frac{MS_M}{MS_{R(M)}} = \frac{12.63}{67.02} = 0.19 < 5.14 = F_{.05,2,6}$

There is significant evidence of an interaction between Grade and Manufacturer but not significant evidence of a Main effect due to Manufacturer but this main effect is not interpretable due to the significant interaction. Thus, to determine if the mean reliability differs between the three Manufacturers it would be necessary to compare the mean reliability of the three manufacturers separately for each Grade of quality.

3. Let y_{ijkl} be the reliability of Controller l, from Run k from Manufacturer i of Grade j, the model is $y_{ijkl} = \mu + \tau_i + \beta_j + (\tau * \beta)_{ij} + c_{k(i)} + d_{jk(i)} + e_{ijkl}; \quad i, j, k = 1, 2, 3; \quad l = 1, 2$

Using the numeric values of the MS's given in the AOV table, compute the estimated standard error of the estimated mean difference in reliability of Grades G1 and G2 controllers from Manufacturer I:

$$\bar{y}_{11..} = \mu + \tau_1 + \beta_1 + (\tau * \beta)_{11} + \bar{c}_{.(1)} + \bar{d}_{1.(1)} + \bar{e}_{11..}$$
$$\bar{y}_{12..} = \mu + \tau_1 + \beta_2 + (\tau * \beta)_{12} + \bar{c}_{.(1)} + \bar{d}_{2.(1)} + \bar{e}_{12..}$$

$$\begin{aligned} Var(\bar{y}_{11..} - \bar{y}_{12..}) &= Var(\bar{d}_{1.(1)} - \bar{d}_{2.(1)}) + Var(\bar{e}_{11..} - \bar{e}_{12..}) \\ &= \frac{2\sigma^2_{G*R(M)}}{3} + \frac{2\sigma^2_e}{6} \\ &= \frac{2}{6}\left(2\sigma^2_{G*R(M)} + \sigma^2_e\right) \\ &= \frac{EMS_{G*R(M)}}{3} \quad \Rightarrow \widehat{SE}(\bar{y}_{11..} - \bar{y}_{12..}) = \sqrt{1.36/3} = .6733 \end{aligned}$$

4. The value of HSD needed to group the mean reliability of the three Grades of controllers at the $\alpha = .05$ level is computed as follows:

Because there is a significant interaction between Manufacturer and Grades, the grouping of the Grades will be done separately for each Manufacturer with a Bonferroni correction to HSD, ie, .05/3.

$$HSD = q(.05/3, 3, \nu)\sqrt{\tfrac{1}{2}\left(\widehat{SE}(\bar{y}_{ij..} - \bar{y}_{ik..})\right)^2} = q(.0167, 3, 12)\sqrt{\tfrac{1}{2}(.6733)^2} = (5.04)(.4761) = 2.40$$

II 18 points

The study design was a fractional factorial with 16 runs using the generators

$I_1 = ABDE = +$ and $I_2 = ACE = +$ to generate the 16 treatments to be used in the study.

1. For each of the following treatments, check YES if the treatment will appear in the experiment, otherwise check NO.

   i. $(A, B, C, D, E, F) = (+, -, +, +, -, +) \Rightarrow$ $ABDE = +$ and $ACE = - \Rightarrow$ No
   ii. $(A, B, C, D, E, F) = (+, -, +, -, +, +) \Rightarrow$ $ABDE = +$ and $ACE = + \Rightarrow$ Yes

2. What is the resolution of this design? Justify your answer.

- (ABDE)(ACE)=BCD therefore the length of the shortest generator and implicit generator is 3. Thus, Resolution = III

3. What effects which must be assumed to be negligible in order that the data from the experiment will provide an estimate of the interaction between Factors C and F.

- I = ABDE = ACE = BCD. Therefore, the following effects are confounded with CF:

  (CF)(ABDE) = ABCDEF; (CF)(ACE)= AEF (CF)(BCD) = BDF.

  Thus, in order to estimate CF we would need to be assured that the three effects ABCDEF, AEF, and BDF are negligible.

III 21 points

a. C - Trend analysis in levels of F_1 averaged over the levels of F_2 because F_2 has randomly selected levels

b. P - because the significant interaction involved F_2 which had randomly selected levels

c. F - Comparing one treatment level to all the other levels

d. S - Because of the significant interaction, the comparison should not be done on the marginal means

e. Q - Hsu's procedure determines a group of treatments having a specified probability of containing the "best" treatment

f. T or E - Should not construct contrasts using the observed data, but if the selected contrasts "must" be tested then use the most conservative of all procedures, Scheffe.

g. D - Need to perform trend analyzes across the levels of F_1 separately at each level of F_2 because of the significant interaction

IV 12 points

i. D because $C = \mu_{.1} - 2\mu_{.2} + \mu_{.3}$, a contrast in the levels of $\mu_{.j}$

ii. B because C is a contrast in the levels of factor two at the first level of factor one

iii. E because $C = (\mu_{11} - 2\mu_{12} + \mu_{13}) - (\mu_{21} - 2\mu_{22} + \mu_{23})$, the difference in a quadratic trend in levels of Factor 2 between levels 1 and 2 of Factor 1.

iv. D because L_1 is the difference in the two linear trends and L_2 is the difference in the two quadratic trends.

V 15 points

1. Because we are comparing the levels of factor F_2 three separate times, once for each level of F_1, the value selected for α in obtaining the Tukey HSD coefficient is .05/3.

2. The researcher's analysis was correct because there are 12 distinct treatments with the four treatments consisting of the 0 level of nitrogen combined with each of the four crops being distinctly different treatments due to there being nitrogen naturally occurring in the soil and the crops having different nitrogen fixation rates.

3. Case 1: If the researcher is certain that there is not an $F_1 * F_2$ interaction, then we take $\alpha = .05, \gamma_o = .80, t = 5, \nu_1 = t - 1 = 4, \nu_2 = (5)(2)(r-1) = 10(r-1), D = 6, \sigma_e^2 = 6.5$ and compute $\phi = \sqrt{\frac{rD^2}{2t\sigma_e^2}} = \sqrt{\frac{r36}{(2)(5)(6.5)}} = .7442\sqrt{r}$. From Table IX, $\nu_1 = 4$, we find that $r = 4$ has power approximately .68 and $r = 5$ has power approximately .82. Thus, r=5 yields the desired specification. However, because the analysis is being conducted on the marginal means of factor F_1: $\bar{\mu}_{1.}, \bar{\mu}_{2.}, \ldots, \bar{\mu}_{5.}$, the required number of reps is r/b = 5/2 = 2.5, thus take 3 reps.

- Case 2: If the researcher is uncertain about the $F_1 * F_2$ interaction, then the comparisons across the levels of F_1 must be conducted separately at the two levels of F_2 using first the treatment means $\mu_{11}, \ldots, \mu_{51}$ and then $\mu_{12}, \ldots, \mu_{52}$. This requires using $\alpha = .05/2 = .025, \gamma_o = .80, t = 5, \nu_1 = t - 1 = 4, \nu_2 = (2)(5)(r-1) = 10(r-1), D = 6, \sigma_e^2 = 6.5$ and compute $\phi = \sqrt{\frac{rD^2}{2t\sigma_e^2}} = \sqrt{\frac{r36}{(2)(5)(6.5)}} = .7442\sqrt{r}$. From Table IX with $\nu_1 = 4$ and $\alpha = .05$, we determined in Case 1 that r=5 yields the desired specification. From Table IX with $\alpha = .01$, we find that $r = 6$ has power approximately .72 and $r = 7$ has power approximately .82. Thus, the necessary number of reps is between 5 and 7 to obtain the desired specification. Using R or SAS, with $\alpha = .025$, $r = 6$ meets the desired specifications.

VI 12 points

a. Main Effects (s=1) are not confounded with effects having fewer than 8-1=7 elements, that is, with any main effects, two-way interactions, three-way interactions, four-way interactions, five-way interactions, and six-way interactions.

b. Two-way interactions (s=2) are not confounded with effects having fewer than 8-2=6 elements, that is, with main effects, two-way interactions, three-way interactions, four-way interactions, and five-way interactions.

c. Three-way interactions (s=3) are not confounded with effects having fewer than 8-3=5 elements, that is, with main effects, two-way interactions, three-way interactions, and four-way interactions.

d. Four-way interactions (s=4) are not confounded with effects having fewer than 8-4=4 elements, that is, with main effects, two-way interactions, and three-way interactions elements.