MASTER'S DIAGNOSTIC EXAMINATION - August 15, 2013

Student's Name ____

INSTRUCTIONS FOR STUDENTS:

1. The exam is to be started at 1 pm (CDT) and completed by 5 pm (CDT) on August 15 2013.
2. Put your name above but DO NOT put your NAME on the **SOLUTIONS** to the exam.
3. Place the NUMBER assigned to you on the

   UPPER RIGHT HAND CORNER of EACH PAGE of your SOLUTIONS.
4. Please start your answer to EACH QUESTION on a SEPARATE sheet of paper.
5. Use only one side of each sheet of paper.
6. You must answer all four questions: Questions I, II, III and IV.
7. Be sure to attempt all parts of the four questions. It may be possible to answer a later part of a question without having solved the earlier parts.
8. Be sure to hand in all of your exam. No additional material will be accepted once the exam has ended and you have left the exam room.
9. You may use the following:
   - Calculator which does not have capability to phone, text, or access the Web
   - Pencil or pen
   - Blank paper for the solutions for this examination
   - No other materials are allowed

- I attest that I spent no more than 4 hours to complete the exam.
- I used only the materials described above.
- I did not receive assistance from anyone during the taking of this exam.

Student's Signature ____________________________________

INSTRUCTIONS FOR PROCTOR:

Immediately after the student completes the exam, **fax** the student's solutions to **979-845-6060** or

Scan the solutions into a **single** pdf file and **email to longneck@stat.tamu.edu**

Do not send the questions, just send the student's solutions.

(1) I certify that the time at which the student started the exam was ____________________

   and the time at which the student completed the exam was ____________________

(2) I certify that the student has followed all the **INSTRUCTIONS FOR STUDENTS** listed above.

(3) I certify that the student's solutions were faxed to **979-845-6060** or

   emailed to **longneck@stat.tamu.edu**.

Proctor's Signature____

QUESTION I.

Sleep-disordered breathing is common among adults. To estimate the prevalence of this disorder, a questionnaire concerning sleep habits was mailed to 3514 individuals from thirty to sixty years of age who worked for three large state agencies in Wisconsin. Subjects were classified as habitual snorers if they reported snoring, snorting, or breathing pauses every night or almost every night.

Use the attached SAS output in answering the following questions.

1. Are either of the two potential explanatory variables, gender and agegroup associated with being a snorer? Explain your reasoning.

2. Carry out a test that the odds ratio for being a snorer between women and men differs for the three age groups.

3. Carry out a test for conditional dependence of snoring and gender, controlling for age group.

4. Carry out a test for dependence of snoring and gender, ignoring the effect of age group.

5. Discuss which test is more appropriate, the one in part (3.) or the one in part (4.).

6. Construct and interpret a confidence interval for an assumed common odds ratio. Is it appropriate to use for these data.

7. What is Simpson's paradox? Does Simpson's paradox occur for this data set? Explain.

SAS OUTPUT FOR QUESTION I

The FREQ Procedure
Table of agegroup by gender

agegroup / Frequency	gender: female	male	Total
30-39	799	536	1335
40-49	709	696	1405
50-59	336	438	774
Total	1844	1670	3514

Statistics for Table of agegroup by gender

Statistic	DF	Value	Prob
Chi-Square	2	56.8978	<.0001
Likelihood Ratio Chi-Square	2	57.1344	<.0001

Sample Size = 3514

The FREQ Procedure
Table of agegroup by snorer

agegroup / Frequency	snorer: yes	no	Total
30-39	384	951	1335
40-49	536	869	1405
50-59	335	439	774
Total	1255	2259	3514

Statistics for Table of agegroup by snorer

Statistic	DF	Value	Prob
Chi-Square	2	51.0224	<.0001
Likelihood Ratio Chi-Square	2	51.4389	<.0001

Sample Size = 3514

The FREQ Procedure
Table of gender by snorer

gender / Frequency	snorer: yes	no	Total
female	522	1322	1844
male	733	937	1670
Total	1255	2259	3514

Statistics for Table of gender by snorer

Statistic	DF	Value	Prob
Chi-Square	1	92.7017	<.0001
Likelihood Ratio Chi-Square	1	92.9613	<.0001

Estimates of the Relative Risk (Row1/Row2)

Type of Study	Value	95% Confidence Limits	
Case-Control (Odds Ratio)	0.5047	0.4388	0.5806
Cohort (Col1 Risk)	0.6449	0.5891	0.7061
Cohort (Col2 Risk)	1.2778	1.2140	1.3449

Table 1 of gender by snorer
Controlling for agegroup=30-39

gender / Frequency	snorer: yes	no	Total
female	196	603	799
male	188	348	536
Total	384	951	1335

Statistics for Table 1 of gender by snorer
Controlling for agegroup=30-39

Statistic	DF	Value	Prob
Chi-Square	1	17.4056	<.0001
Likelihood Ratio Chi-Square	1	17.2335	<.0001
Continuity Adj. Chi-Square	1	16.8948	<.0001

Estimates of the Relative Risk (Row1/Row2)

Type of Study	Value	95% Confidence Limits	
Case-Control (Odds Ratio)	0.6017	0.4734	0.7646
Cohort (Col1 Risk)	0.6994	0.5915	0.8269
Cohort (Col2 Risk)	1.1624	1.0798	1.2513

Sample Size = 1335

Table 2 of gender by snorer
Controlling for agegroup=40-49

gender / Frequency	snorer: yes	no	Total
female	223	486	709
male	313	383	696
Total	536	869	1405

Statistics for Table 2 of gender by snorer
Controlling for agegroup=40-49

Statistic	DF	Value	Prob
Chi-Square	1	27.2023	<.0001
Likelihood Ratio Chi-Square	1	27.3005	<.0001

Estimates of the Relative Risk (Row1/Row2)

Type of Study	Value	95% Confidence Limits	
Case-Control (Odds Ratio)	0.5615	0.4516	0.6981
Cohort (Col1 Risk)	0.6994	0.6103	0.8015
Cohort (Col2 Risk)	1.2457	1.1457	1.3543

Sample Size = 1405

Table 3 of gender by snorer
Controlling for agegroup=50-59

gender / Frequency	snorer: yes	no	Total
female	103	233	336
male	232	206	438
Total	335	439	774

Statistics for Table 3 of gender by snorer
Controlling for agegroup=50-59

Statistic	DF	Value	Prob
Chi-Square	1	38.5631	<.0001
Likelihood Ratio Chi-Square	1	39.1621	<.0001

Estimates of the Relative Risk (Row1/Row2)

Type of Study	Value	95% Confidence Limits	
Case-Control (Odds Ratio)	0.3925	0.2913	0.529
Cohort (Col1 Risk)	0.5787	0.4817	0.6953
Cohort (Col2 Risk)	1.4744	1.3048	1.6661

Sample Size = 774

Summary Statistics for gender by snorer
Controlling for agegroup

Cochran-Mantel-Haenszel Statistics (Based on Table Scores)

Statistic	Alternative Hypothesis	DF	Value	Prob
1	Nonzero Correlation	1	78.3284	<.0001
2	Row Mean Scores Differ	1	78.3284	<.0001
3	General Association	1	78.3284	<.0001

Estimates of the Common Relative Risk (Row1/Row2)

Type of Study	Method	Value	95% Confidence Limits	
Case-Control	Mantel-Haenszel	0.5296	0.4598	0.6100
(Odds Ratio)	Logit	0.5305	0.4604	0.6114
Cohort	Mantel-Haenszel	0.6667	0.6083	0.7306
(Col1 Risk)	Logit	0.6672	0.6088	0.7311
Cohort	Mantel-Haenszel	1.2520	1.1901	1.3171
(Col2 Risk)	Logit	1.2411	1.1801	1.3053

Breslow-Day Test for
Homogeneity of the Odds Ratios

Chi-Square	5.2478
DF	2
Pr > ChiSq	0.0725

Total Sample Size = 3514

QUESTION II.

1. Suppose we have n independent observations of the pair (x_i, y_i), $i = 1, 2, \ldots, n$, where y_i is binary, taking only the values 0 or 1, and x_i is continuous. Provide two reasons why linear regression of y on x, with the usual linear regression assumptions, would not be appropriate as an analysis method.

2. In a study of a particular disease in humans, researchers observed the following data.

	Female	Male	Total
Diseased	20	19	39
Healthy	20	41	61
Total	40	60	100

Let y_i be disease status (1 if diseased, 0 if healthy) and x_i the gender (1 if female, 0 if male) of the ith individual, $i = 1, 2, \ldots, 100$. Let $\pi(x)$ be the conditional probability of an individual being diseased, given its gender. Consider the logistic regression model:

$$\log\left(\frac{\pi(x_i)}{1-\pi(x_i)}\right) = \beta_0 + \beta_1 x_i.$$

(a.) Write down the likelihood function $L(\boldsymbol{\beta})$, where $\boldsymbol{\beta}' = (\beta_0, \beta_1)$.

(b.) Report the maximum likelihood estimates $\hat{\beta}_0$ and $\hat{\beta}_1$.

(c.) One way to test $H_0 : \beta_1 = 0$ is with a likelihood ratio test. The test statistic equals $-2\log\left[\frac{L_0(\hat{\beta}_0^0)}{L(\hat{\boldsymbol{\beta}})}\right]$, where L_0 is the likelihood function for the null model, the model with only the intercept, β_0^0, included:

$$\log\left(\frac{\pi_0(x_i)}{1-\pi_0(x_i)}\right) = \beta_0^0,$$

and where $\hat{\beta}_0^0$ and $\hat{\boldsymbol{\beta}}$ are the MLEs under the null and unrestricted models, respectively. Under H_0, the statistic is approximately chi-square distributed with 1 degree of freedom. Test $H_0 : \beta_1 = 0$. Some chi-square cumulative probabilities are provided.

3. A similar study to the one described above was conducted, in which the independent variable was race:

	White	Black	Hispanic	Other	Total
Diseased	5	20	15	10	50
Healthy	20	10	10	10	50
Total	25	30	25	20	100

Consider the logistic regression model

$$\log\left(\frac{\pi(r_i)}{1-\pi(r_i)}\right) = \beta_0 + \beta_1 r_{1i} + \beta_2 r_{2i} + \beta_3 r_{3i},$$

where the r_1, r_2, r_3 independent variables code for race as follows:

	r_1	r_2	r_3
White	0	0	0
Black	1	0	0
Hispanic	0	1	0
Other	0	0	1

Here is partial output from the model fit using `R` code:

```
             Estimate  Std. Error  z value    Pr(>|z|)
(Intercept)  -1.3863     0.5000    -2.773     0.00556 **
  r1          2.0794     0.6325     3.288     0.00101 **
  r2          1.7918     0.6455     2.776     0.00551 **
  r3          1.3863     0.6708     2.067     0.03878 *
```

(a.) According to the model, what is the estimated probability of a white individual being diseased? Show your calculations, in terms of the model coefficients.

(b.) Report a 95% confidence interval for the odds ratio (not *log* odds ratio) comparing Hispanic to White.

QUESTION III.

An analyst for a grocery store chain was interested in the effect of product placement on shelves (Knee, Waist, and Eye levels) and facings (or the amount of shelf space required by the products: Half or Full) on sales, the number of products sold. Three products were placed at each shelf / facing combination, for a total of 18 products. The first model fit to the data was:

$$Sales = \alpha_i + \beta_j + e_{ij}; \quad i = 1, 2, 3; \quad j = 1, 2 \qquad \textbf{(Model 1)}$$

1. Interpret the parameter estimate $\hat{\beta}_1$ from Model 1 in context.

2. If the variable Sales had been an indicator variable for whether or not a product had sold rather than the number of each product sold, would the model above have been a valid model? Why or why not?

It was also suggested that the sugar content of the products may play a part in the number sold. The second fitted model (this time with an intercept) was:

$$Sales = \beta_0 + \beta_1 Sugars + \beta_{2i} + \beta_{3j}; \quad i = 1, 2, 3; \quad j = 1, 2 \qquad \textbf{(Model 2)}$$

Output from this second model is found below and on the following page.

Model 2:

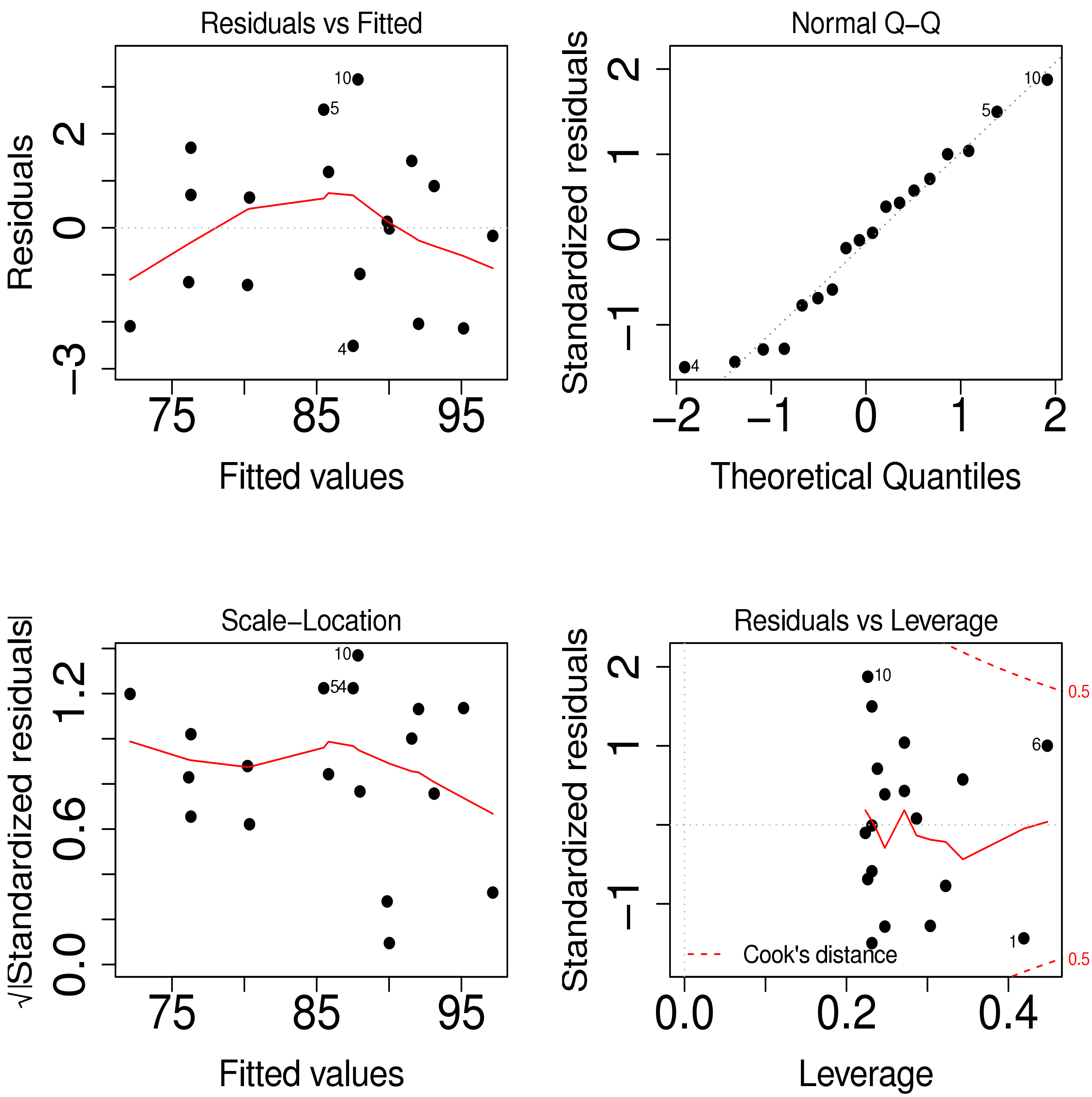
```
Coefficients:
             Estimate   Std. Error  t value      Pr(>|t|)
(Intercept)   73.774      2.875      25.661      1.60e-12 ***
FacingHalf    -7.627      1.025      -7.443      4.88e-06 ***
HeightKnee    -2.172      1.107      -1.962      0.0715
HeightWaist    3.096      1.394       2.222      0.0447 *
Sugars         2.030      0.364       5.578      8.95e-05 ***
```

3. Is Model 2 a valid model? Discuss why or why not.

4. Regardless of your answer above, use Model 2 to test whether Sugars have an effect on Sales, after controlling for shelf placement and facing. Be sure to include the hypotheses used and your conclusion in the context of the problem.

5. Suppose that a third model is fit, and an interaction between shelf placement and facing is found to be significant. How would you explain the interaction to others in the company without using statistical jargon?

Model 2: Graphical Displays

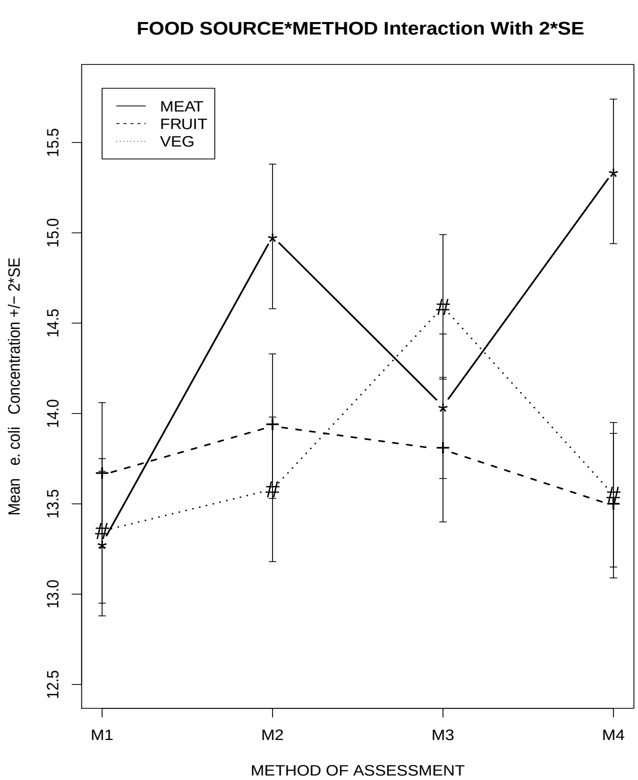


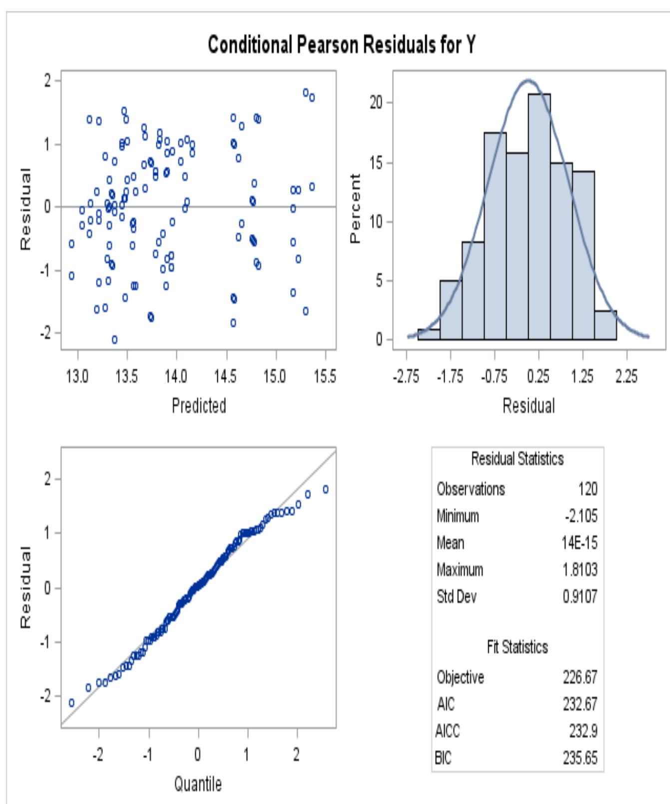
QUESTION IV.

The Center for Disease Control (CDC) conducted an experiment to evaluate the reliability of assessing the level of contamination of *e. coli* in three food sources, Meat, Fruit, and Vegetables. There are four unique methods for assessing *e. coli* : M1, M2, M3, M4 and hundreds of laboratories which use one or more of theses methods in the USA. For each of the methods of assessment, five laboratories are randomly selected to participate in the study. Forty containers are prepared for each food source by spiking the container with a known level of contamination of *e. coli* and then placing the container in a controlled climate for three weeks to allow the *e. coli* level to stabilize. Six containers, two of each of the three food sources, are then sent to each of the 20 laboratories selected for the study. The *e. coli* level (cfu/g), Y_{ijkl}, determined by the kth Lab using Assessment Method j for the ℓth Container of Food Source i is recorded for the 120 containers. The CDC wants to compare the mean *e. coli* level of the four assessment methods and their differences across the food sources. Also, CDC wants to determine if there are major differences in the mean *e. coli* determinations across the many laboratories in the USA.

	Assessment Methods																			
	M1					M2					M3					M4				
	Lab					Lab					Lab					Lab				
Source	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20
Meat	12.3	13.2	12.9	13.2	12.9	14.5	14.3	14.5	14.3	14.5	14.4	13.5	14.7	13.5	14.7	14.8	16.4	15.6	14.8	15.2
	12.6	13.0	13.0	14.0	13.9	15.0	15.6	14.8	15.6	14.8	14.1	13.4	14.6	13.4	14.2	15.3	14.3	16.4	15.4	14.4
Fruit	13.2	14.4	12.9	14.1	12.8	13.2	14.2	14.4	13.3	14.5	13.4	14.5	12.7	13.5	12.7	12.2	14.4	12.4	13.4	13.2
	13.4	14.5	13.7	14.1	13.4	14.2	13.4	14.6	13.6	13.8	14.1	14.4	14.2	14.4	14.2	13.3	13.6	13.8	13.8	13.3
Veg	13.1	13.4	13.6	13.8	12.8	13.5	13.3	13.5	14.3	13.5	14.3	13.5	13.7	14.5	13.7	12.2	14.3	13.6	13.4	14.4
	12.5	14.0	13.0	14.1	13.3	14.0	12.6	12.8	13.6	12.8	15.1	15.4	15.2	15.4	15.2	13.3	13.9	12.7	13.9	14.1

Use the following plots, the attached tables, and SAS output to answer the following questions.





1. Write a model which displays an appropriate relationship between level of contamination, Y_{ijkl}, and its possible sources of variation. Include any restrictions on the parameters in your model and any distributional properties on the random variables in your model.

2. Do the necessary conditions for testing hypotheses and constructing confidence intervals appear to be satisfied? Justify your answers.

   C_1 Normality:

   C_2 Equal Variance:

   C_3 Independence:

3. Construct a partial ANOVA table for this experiment.

   Include only the following: Source of Variation, df, and Expected Mean Squares.

4. At the $\alpha = .05$ level, which Main effects and Interaction effects are significant? Justify your answer by including the relevant p-values along with their pair of degrees of freedom $(df_{NUM.}, df_{DEN.})$. Test for Main effects even if an interaction is significant. Also, provide a test for all random effects.

5. Separate the four Assessment Methods into groups such that all Assessment Methods in a group are not significantly different from any other member of the group with respect to their mean *e. coli* level. Use an experimentwise error rate of $\alpha = .05$.

6. Provide justification for the following standard errors listed in the PROC MIXED output using the following estimates of the variance components:

   $$\sigma^2_{LAB(METH)} = .008708 \qquad \sigma^2_{FOOD*LAB(METH)} = .03236 \qquad \sigma^2_{RESIDUAL} = .3357$$

   a. The estimated standard error of the estimated mean *e. coli* level of a container of Meat which was measured by Method M1 is .2044

   b. The estimated standard error of the estimated difference between the mean *e. coli* level of a container of Meat and a container of Fruit is .1415

7. Provide a 95% confidence on the mean *e. coli* level of a container of Meat which was measured by Method M1.

SAS Code:

```
ods html;ods graphics on;

OPTIONS LS=90 PS=55 nocenter nodate;
TITLE 'SAS OUTPUT FOR PROBLEM IV';
DATA ECOLI;
INPUT FOOD $ METH $ LAB  $ Y @@;
TRT=COMPRESS (METH) || COMPRESS (FOOD);
CARDS;

 MEAT M1 L1 12.30  MEAT  M2 L6  14.46  MEAT M3 L11  14.35 MEAT M4 L16  14.85
 MEAT M1 L1 12.59  MEAT  M2 L6  15.00  MEAT M3 L11  14.06 MEAT M4 L16  15.32
 MEAT M1 L2 13.15  MEAT  M2 L7  14.29  MEAT M3 L12  13.50 MEAT M4 L17  16.35
 MEAT M1 L2 13.00  MEAT  M2 L7  15.62  MEAT M3 L12  13.39 MEAT M4 L17  14.34
 MEAT M1 L3 12.87  MEAT  M2 L8  14.46  MEAT M3 L13  14.73 MEAT M4 L18  15.55
 MEAT M1 L3 13.02  MEAT  M2 L8  14.82  MEAT M3 L13  14.65 MEAT M4 L18  16.36
 MEAT M1 L4 13.15  MEAT  M2 L9  14.29  MEAT M3 L14  13.50 MEAT M4 L19  14.75
 MEAT M1 L4 14.00  MEAT  M2 L9  15.62  MEAT M3 L14  13.39 MEAT M4 L19  15.38
 MEAT M1 L5 12.87  MEAT  M2 L10 14.46  MEAT M3 L15  14.73 MEAT M4 L20  15.15
 MEAT M1 L5 13.92  MEAT  M2 L10 14.82  MEAT M3 L15  14.15 MEAT M4 L20  14.39
 FRUIT M1 L1 13.20  FRUIT  M2 L6  13.16  FRUIT M3 L11  13.35 FRUIT M4 L16  12.25
 FRUIT M1 L1 13.39  FRUIT  M2 L6  14.20  FRUIT M3 L11  14.06 FRUIT M4 L16  13.33
 FRUIT M1 L2 14.45  FRUIT  M2 L7  14.23  FRUIT M3 L12  14.50 FRUIT M4 L17  14.35
 FRUIT M1 L2 14.50  FRUIT  M2 L7  13.42  FRUIT M3 L12  14.39 FRUIT M4 L17  13.55
 FRUIT M1 L3 12.86  FRUIT  M2 L8  14.45  FRUIT M3 L13  12.73 FRUIT M4 L18  12.36
 FRUIT M1 L3 13.72  FRUIT  M2 L8  14.62  FRUIT M3 L13  14.15 FRUIT M4 L18  13.75
 FRUIT M1 L4 14.11  FRUIT  M2 L9  13.29  FRUIT M3 L14  13.50 FRUIT M4 L19  13.38
 FRUIT M1 L4 14.10  FRUIT  M2 L9  13.62  FRUIT M3 L14  14.39 FRUIT M4 L19  13.79
 FRUIT M1 L5 12.83  FRUIT  M2 L10 14.46  FRUIT M3 L15  12.73 FRUIT M4 L20  13.15
 FRUIT M1 L5 13.42  FRUIT  M2 L10 13.82  FRUIT M3 L15  14.15 FRUIT M4 L20  13.32
 VEG M1 L1 13.10  VEG  M2 L6  13.46  VEG M3 L11  14.35 VEG M4 L16  12.15
 VEG M1 L1 12.52  VEG  M2 L6  14.00  VEG M3 L11  15.06 VEG M4 L16  13.32
 VEG M1 L2 13.35  VEG  M2 L7  13.29  VEG M3 L12  13.50 VEG M4 L17  14.32
 VEG M1 L2 14.04  VEG  M2 L7  12.62  VEG M3 L12  15.39 VEG M4 L17  13.85
 VEG M1 L3 13.57  VEG  M2 L8  13.46  VEG M3 L13  13.73 VEG M4 L18  13.55
 VEG M1 L3 12.96  VEG  M2 L8  12.82  VEG M3 L13  15.15 VEG M4 L18  12.65
 VEG M1 L4 13.75  VEG  M2 L9  14.29  VEG M3 L14  14.50 VEG M4 L19  13.37
 VEG M1 L4 14.10  VEG  M2 L9  13.62  VEG M3 L14  15.39 VEG M4 L19  13.85
 VEG M1 L5 12.82  VEG  M2 L10 13.46  VEG M3 L15  13.73 VEG M4 L20  14.39
 VEG M1 L5 13.32  VEG  M2 L10 12.82  VEG M3 L15  15.15 VEG M4 L20  14.05
RUN;

PROC GLM ORDER=DATA;
CLASS FOOD METH LAB;
MODEL  Y = FOOD METH FOOD*METH LAB(METH) FOOD*LAB(METH);
RANDOM LAB(METH) FOOD*LAB(METH)/TEST;
LSMEANS FOOD METH FOOD*METH/STDERR PDIFF ADJUST=TUKEY;

PROC MIXED ORDER=DATA;
CLASS FOOD METH LAB;
MODEL  Y = FOOD METH FOOD*METH/RESIDUAL ;
RANDOM LAB(METH) FOOD*LAB(METH);
LSMEANS FOOD METH FOOD*METH/ADJUST=TUKEY;

ods graphics off;ods html close;
```

SAS OUTPUT FOR PROBLEM IV

The GLM Procedure

Class Level Information		
Class	**Levels**	**Values**
FOOD	3	MEAT FRUIT VEG
METH	4	M1 M2 M3 M4
LAB	20	L1 L6 L11 L16 L2 L7 L12 L17 L3 L8 L13 L18 L4 L9 L14 L19 L5 L10 L15 L20

Number of Observations Read	120
Number of Observations Used	120

SAS OUTPUT FOR PROBLEM IV

The GLM Procedure
Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	59	68.49452917	1.16092422	3.46	<.0001
Error	60	20.14305000	0.33571750		
Corrected Total	119	88.63757917			

R-Square	Coeff Var	Root MSE	Y Mean
0.772748	4.169802	0.579411	13.89542

Source	DF	Type I SS	Mean Square	F Value	Pr > F
FOOD	2	9.38181167	4.69090583	13.97	<.0001
METH	3	11.45262250	3.81754083	11.37	<.0001
FOOD*METH	6	27.60327500	4.60054583	13.70	<.0001
LAB(METH)	16	7.24294000	0.45268375	1.35	0.1995
FOOD*LAB(METH)	32	12.81388000	0.40043375	1.19	0.2734

Source	DF	Type III SS	Mean Square	F Value	Pr > F
FOOD	2	9.38181167	4.69090583	13.97	<.0001
METH	3	11.45262250	3.81754083	11.37	<.0001
FOOD*METH	6	27.60327500	4.60054583	13.70	<.0001
LAB(METH)	16	7.24294000	0.45268375	1.35	0.1995
FOOD*LAB(METH)	32	12.81388000	0.40043375	1.19	0.2734

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance
Dependent Variable: Y

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	**FOOD**	2	9.381812	4.690906	11.71	0.0002
	FOOD*METH	6	27.603275	4.600546	11.49	<.0001
	LAB(METH)	16	7.242940	0.452684	1.13	0.3705
	Error	32	12.813880	0.400434		

Error: MS(FOOD*LAB(METH))

*** This test assumes one or more other fixed effects are zero.**

	Source	DF	Type III SS	Mean Square	F Value	Pr > F
*	**METH**	3	11.452623	3.817541	8.43	0.0014
	Error: MS(LAB(METH))	16	7.242940	0.452684		

*** This test assumes one or more other fixed effects are zero.**

Source	DF	Type III SS	Mean Square	F Value	Pr > F
FOOD*LAB(METH)	32	12.813880	0.400434	1.19	0.2734
Error: MS(Error)	60	20.143050	0.335717		

SAS OUTPUT FOR PROBLEM IV

Least Squares Means
Adjustment for Multiple Comparisons: Tukey

FOOD	Y LSMEAN	Standard Error	Pr > |t|	LSMEAN Number
MEAT	14.2900000	0.0916130	<.0001	1
FRUIT	13.6757500	0.0916130	<.0001	2
VEG	13.7205000	0.0916130	<.0001	3

Least Squares Means for effect FOOD Pr > |t| for H0: LSMean(i)=LSMean(j) Dependent Variable: Y			
i/j	**1**	**2**	**3**
1		<.0001	0.0001
2	<.0001		0.9364
3	0.0001	0.9364	

SAS OUTPUT FOR PROBLEM IV

Least Squares Means
Adjustment for Multiple Comparisons: Tukey

METH	Y LSMEAN	Standard Error	Pr > |t|	LSMEAN Number
M1	13.3660000	0.1057856	<.0001	1
M2	14.0316667	0.1057856	<.0001	2
M3	14.1450000	0.1057856	<.0001	3
M4	14.0390000	0.1057856	<.0001	4

Least Squares Means for effect METH
Pr > |t| for H0: LSMean(i)=LSMean(j)
Dependent Variable: Y

i/j	1	2	3	4
1		0.0002	<.0001	0.0002
2	0.0002		0.8731	1.0000
3	<.0001	0.8731		0.8933
4	0.0002	1.0000	0.8933	

SAS OUTPUT FOR PROBLEM IV

Least Squares Means
Adjustment for Multiple Comparisons: Tukey

FOOD	METH	Y LSMEAN	Standard Error	Pr > |t|	LSMEAN Number
MEAT	**M1**	13.0870000	0.1832260	<.0001	1
MEAT	**M2**	14.7840000	0.1832260	<.0001	2
MEAT	**M3**	14.0450000	0.1832260	<.0001	3
MEAT	**M4**	15.2440000	0.1832260	<.0001	4
FRUIT	**M1**	13.6580000	0.1832260	<.0001	5
FRUIT	**M2**	13.9270000	0.1832260	<.0001	6
FRUIT	**M3**	13.7950000	0.1832260	<.0001	7
FRUIT	**M4**	13.3230000	0.1832260	<.0001	8
VEG	**M1**	13.3530000	0.1832260	<.0001	9
VEG	**M2**	13.3840000	0.1832260	<.0001	10
VEG	**M3**	14.5950000	0.1832260	<.0001	11
VEG	**M4**	13.5500000	0.1832260	<.0001	12

Least Squares Means for effect FOOD*METH
Pr > |t| for H0: LSMean(i)=LSMean(j)
Dependent Variable: Y

i/j	1	2	3	4	5	6	7	8	9	10	11	12
1		<.0001	0.0219	<.0001	0.5536	0.0753	0.2363	0.9988	0.9964	0.9910	<.0001	0.8185
2	<.0001		0.1854	0.8245	0.0029	0.0637	0.0154	<.0001	<.0001	<.0001	0.9998	0.0007
3	0.0219	0.1854		0.0011	0.9367	1.0000	0.9979	0.2122	0.2661	0.3302	0.6093	0.7482
4	<.0001	0.8245	0.0011		<.0001	0.0002	<.0001	<.0001	<.0001	<.0001	0.3572	<.0001
5	0.5536	0.0029	0.9367	<.0001		0.9961	1.0000	0.9770	0.9888	0.9954	0.0276	1.0000
6	0.0753	0.0637	1.0000	0.0002	0.9961		1.0000	0.4672	0.5457	0.6277	0.3150	0.9467
7	0.2363	0.0154	0.9979	<.0001	1.0000	1.0000		0.7998	0.8585	0.9074	0.1096	0.9983
8	0.9988	<.0001	0.2122	<.0001	0.9770	0.4672	0.7998		1.0000	1.0000	0.0004	0.9991
9	0.9964	<.0001	0.2661	<.0001	0.9888	0.5457	0.8585	1.0000		1.0000	0.0006	0.9998
10	0.9910	<.0001	0.3302	<.0001	0.9954	0.6277	0.9074	1.0000	1.0000		0.0010	1.0000
11	<.0001	0.9998	0.6093	0.3572	0.0276	0.3150	0.1096	0.0004	0.0006	0.0010		0.0080
12	0.8185	0.0007	0.7482	<.0001	1.0000	0.9467	0.9983	0.9991	0.9998	1.0000	0.0080	

SAS OUTPUT FOR PROBLEM IV

The Mixed Procedure

Model Information	
Data Set	WORK.ECOLI
Dependent Variable	Y
Covariance Structure	Variance Components
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Satterthwaite

Class Level Information		
Class	**Levels**	**Values**
FOOD	3	MEAT FRUIT VEG
METH	4	M1 M2 M3 M4
LAB	20	L1 L6 L11 L16 L2 L7 L12 L17 L3 L8 L13 L18 L4 L9 L14 L19 L5 L10 L15 L20

Number of Observations	
Number of Observations Read	120
Number of Observations Used	120
Number of Observations Not Used	0

Covariance Parameter Estimates	
Cov Parm	**Estimate**
LAB(METH)	0.008708
FOOD*LAB(METH)	0.03236
Residual	0.3357

Type 3 Tests of Fixed Effects				
Effect	**Num DF**	**Den DF**	**F Value**	**Pr > F**
FOOD	2	32	11.71	0.0002
METH	3	16	8.43	0.0014
FOOD*METH	6	32	11.49	<.0001

Least Squares Means							
Effect	**FOOD**	**METH**	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**
FOOD	**MEAT**		14.2900	0.1022	47.8	139.81	<.0001
FOOD	**FRUIT**		13.6758	0.1022	47.8	133.80	<.0001
FOOD	**VEG**		13.7205	0.1022	47.8	134.24	<.0001
METH		**M1**	13.3660	0.1228	16	108.81	<.0001
METH		**M2**	14.0317	0.1228	16	114.23	<.0001
METH		**M3**	14.1450	0.1228	16	115.15	<.0001
METH		**M4**	14.0390	0.1228	16	114.29	<.0001
FOOD*METH	**MEAT**	**M1**	13.0870	0.2044	47.8	64.02	<.0001
FOOD*METH	**MEAT**	**M2**	14.7840	0.2044	47.8	72.32	<.0001
FOOD*METH	**MEAT**	**M3**	14.0450	0.2044	47.8	68.71	<.0001
FOOD*METH	**MEAT**	**M4**	15.2440	0.2044	47.8	74.57	<.0001
FOOD*METH	**FRUIT**	**M1**	13.6580	0.2044	47.8	66.82	<.0001
FOOD*METH	**FRUIT**	**M2**	13.9270	0.2044	47.8	68.13	<.0001
FOOD*METH	**FRUIT**	**M3**	13.7950	0.2044	47.8	67.49	<.0001
FOOD*METH	**FRUIT**	**M4**	13.3230	0.2044	47.8	65.18	<.0001
FOOD*METH	**VEG**	**M1**	13.3530	0.2044	47.8	65.32	<.0001
FOOD*METH	**VEG**	**M2**	13.3840	0.2044	47.8	65.48	<.0001
FOOD*METH	**VEG**	**M3**	14.5950	0.2044	47.8	71.40	<.0001
FOOD*METH	**VEG**	**M4**	13.5500	0.2044	47.8	66.29	<.0001

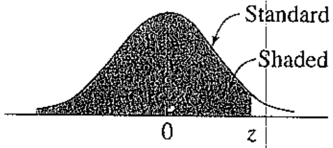
Differences of Least Squares Means											
Effect	**FOOD**	**METH**	**_FOOD**	**_METH**	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**	**Adjustment**	**Adj P**
FOOD	**MEAT**		**FRUIT**		0.6142	0.1415	32	4.34	0.0001	Tukey-Kramer	0.0004
FOOD	**MEAT**		**VEG**		0.5695	0.1415	32	4.02	0.0003	Tukey-Kramer	0.0009
FOOD	**FRUIT**		**VEG**		-0.04475	0.1415	32	-0.32	0.7539	Tukey-Kramer	0.9465
METH		**M1**		**M2**	-0.6657	0.1737	16	-3.83	0.0015	Tukey	0.0072
METH		**M1**		**M3**	-0.7790	0.1737	16	-4.48	0.0004	Tukey	0.0019
METH		**M1**		**M4**	-0.6730	0.1737	16	-3.87	0.0013	Tukey	0.0066
METH		**M2**		**M3**	-0.1133	0.1737	16	-0.65	0.5234	Tukey	0.9132
METH		**M2**		**M4**	-0.00733	0.1737	16	-0.04	0.9669	Tukey	1.0000
METH		**M3**		**M4**	0.1060	0.1737	16	0.61	0.5503	Tukey	0.9274
FOOD*METH	**MEAT**	**M1**	**MEAT**	**M2**	-1.6970	0.2891	47.8	-5.87	<.0001	Tukey-Kramer	<.0001
FOOD*METH	**MEAT**	**M1**	**MEAT**	**M3**	-0.9580	0.2891	47.8	-3.31	0.0018	Tukey-Kramer	0.0799
FOOD*METH	**MEAT**	**M1**	**MEAT**	**M4**	-2.1570	0.2891	47.8	-7.46	<.0001	Tukey-Kramer	<.0001
FOOD*METH	**MEAT**	**M1**	**FRUIT**	**M1**	-0.5710	0.2830	32	-2.02	0.0521	Tukey-Kramer	0.6790
FOOD*METH	**MEAT**	**M1**	**FRUIT**	**M2**	-0.8400	0.2891	47.8	-2.91	0.0055	Tukey-Kramer	0.1864
FOOD*METH	**MEAT**	**M1**	**FRUIT**	**M3**	-0.7080	0.2891	47.8	-2.45	0.0180	Tukey-Kramer	0.4047
FOOD*METH	**MEAT**	**M1**	**FRUIT**	**M4**	-0.2360	0.2891	47.8	-0.82	0.4183	Tukey-Kramer	0.9994
FOOD*METH	**MEAT**	**M1**	**VEG**	**M1**	-0.2660	0.2830	32	-0.94	0.3543	Tukey-Kramer	0.9980
FOOD*METH	**MEAT**	**M1**	**VEG**	**M2**	-0.2970	0.2891	47.8	-1.03	0.3094	Tukey-Kramer	0.9957
FOOD*METH	**MEAT**	**M1**	**VEG**	**M3**	-1.5080	0.2891	47.8	-5.22	<.0001	Tukey-Kramer	0.0006
FOOD*METH	**MEAT**	**M1**	**VEG**	**M4**	-0.4630	0.2891	47.8	-1.60	0.1158	Tukey-Kramer	0.8960
FOOD*METH	**MEAT**	**M2**	**MEAT**	**M3**	0.7390	0.2891	47.8	2.56	0.0138	Tukey-Kramer	0.3439
FOOD*METH	**MEAT**	**M2**	**MEAT**	**M4**	-0.4600	0.2891	47.8	-1.59	0.1181	Tukey-Kramer	0.8999
FOOD*METH	**MEAT**	**M2**	**FRUIT**	**M1**	1.1260	0.2891	47.8	3.90	0.0003	Tukey-Kramer	0.0200
FOOD*METH	**MEAT**	**M2**	**FRUIT**	**M2**	0.8570	0.2830	32	3.03	0.0048	Tukey-Kramer	0.1465
FOOD*METH	**MEAT**	**M2**	**FRUIT**	**M3**	0.9890	0.2891	47.8	3.42	0.0013	Tukey-Kramer	0.0627
FOOD*METH	**MEAT**	**M2**	**FRUIT**	**M4**	1.4610	0.2891	47.8	5.05	<.0001	Tukey-Kramer	0.0009
FOOD*METH	**MEAT**	**M2**	**VEG**	**M1**	1.4310	0.2891	47.8	4.95	<.0001	Tukey-Kramer	0.0012
FOOD*METH	**MEAT**	**M2**	**VEG**	**M2**	1.4000	0.2830	32	4.95	<.0001	Tukey-Kramer	0.0012
FOOD*METH	**MEAT**	**M2**	**VEG**	**M3**	0.1890	0.2891	47.8	0.65	0.5164	Tukey-Kramer	0.9999
FOOD*METH	**MEAT**	**M2**	**VEG**	**M4**	1.2340	0.2891	47.8	4.27	<.0001	Tukey-Kramer	0.0076

Differences of Least Squares Means

Effect	FOOD	METH	_FOOD	_METH	Estimate	Standard Error	DF	t Value	Pr > |t|	Adjustment	Adj P
FOOD*METH	MEAT	M3	MEAT	M4	-1.1990	0.2891	47.8	-4.15	0.0001	Tukey-Kramer	0.0104
FOOD*METH	MEAT	M3	FRUIT	M1	0.3870	0.2891	47.8	1.34	0.1870	Tukey-Kramer	0.9670
FOOD*METH	MEAT	M3	FRUIT	M2	0.1180	0.2891	47.8	0.41	0.6850	Tukey-Kramer	1.0000
FOOD*METH	MEAT	M3	FRUIT	M3	0.2500	0.2830	32	0.88	0.3836	Tukey-Kramer	0.9989
FOOD*METH	MEAT	M3	FRUIT	M4	0.7220	0.2891	47.8	2.50	0.0160	Tukey-Kramer	0.3766
FOOD*METH	MEAT	M3	VEG	M1	0.6920	0.2891	47.8	2.39	0.0206	Tukey-Kramer	0.4379
FOOD*METH	MEAT	M3	VEG	M2	0.6610	0.2891	47.8	2.29	0.0267	Tukey-Kramer	0.5052
FOOD*METH	MEAT	M3	VEG	M3	-0.5500	0.2830	32	-1.94	0.0608	Tukey-Kramer	0.7249
FOOD*METH	MEAT	M3	VEG	M4	0.4950	0.2891	47.8	1.71	0.0933	Tukey-Kramer	0.8496
FOOD*METH	MEAT	M4	FRUIT	M1	1.5860	0.2891	47.8	5.49	<.0001	Tukey-Kramer	0.0003
FOOD*METH	MEAT	M4	FRUIT	M2	1.3170	0.2891	47.8	4.56	<.0001	Tukey-Kramer	0.0035
FOOD*METH	MEAT	M4	FRUIT	M3	1.4490	0.2891	47.8	5.01	<.0001	Tukey-Kramer	0.0010
FOOD*METH	MEAT	M4	FRUIT	M4	1.9210	0.2830	32	6.79	<.0001	Tukey-Kramer	<.0001
FOOD*METH	MEAT	M4	VEG	M1	1.8910	0.2891	47.8	6.54	<.0001	Tukey-Kramer	<.0001
FOOD*METH	MEAT	M4	VEG	M2	1.8600	0.2891	47.8	6.43	<.0001	Tukey-Kramer	<.0001
FOOD*METH	MEAT	M4	VEG	M3	0.6490	0.2891	47.8	2.25	0.0294	Tukey-Kramer	0.5319
FOOD*METH	MEAT	M4	VEG	M4	1.6940	0.2830	32	5.99	<.0001	Tukey-Kramer	<.0001
FOOD*METH	FRUIT	M1	FRUIT	M2	-0.2690	0.2891	47.8	-0.93	0.3568	Tukey-Kramer	0.9982
FOOD*METH	FRUIT	M1	FRUIT	M3	-0.1370	0.2891	47.8	-0.47	0.6377	Tukey-Kramer	1.0000
FOOD*METH	FRUIT	M1	FRUIT	M4	0.3350	0.2891	47.8	1.16	0.2523	Tukey-Kramer	0.9887
FOOD*METH	FRUIT	M1	VEG	M1	0.3050	0.2830	32	1.08	0.2892	Tukey-Kramer	0.9937
FOOD*METH	FRUIT	M1	VEG	M2	0.2740	0.2891	47.8	0.95	0.3480	Tukey-Kramer	0.9979
FOOD*METH	FRUIT	M1	VEG	M3	-0.9370	0.2891	47.8	-3.24	0.0022	Tukey-Kramer	0.0937
FOOD*METH	FRUIT	M1	VEG	M4	0.1080	0.2891	47.8	0.37	0.7104	Tukey-Kramer	1.0000
FOOD*METH	FRUIT	M2	FRUIT	M3	0.1320	0.2891	47.8	0.46	0.6500	Tukey-Kramer	1.0000
FOOD*METH	FRUIT	M2	FRUIT	M4	0.6040	0.2891	47.8	2.09	0.0420	Tukey-Kramer	0.6332
FOOD*METH	FRUIT	M2	VEG	M1	0.5740	0.2891	47.8	1.99	0.0528	Tukey-Kramer	0.6991
FOOD*METH	FRUIT	M2	VEG	M2	0.5430	0.2830	32	1.92	0.0640	Tukey-Kramer	0.7397
FOOD*METH	FRUIT	M2	VEG	M3	-0.6680	0.2891	47.8	-2.31	0.0252	Tukey-Kramer	0.4897
FOOD*METH	FRUIT	M2	VEG	M4	0.3770	0.2891	47.8	1.30	0.1984	Tukey-Kramer	0.9725

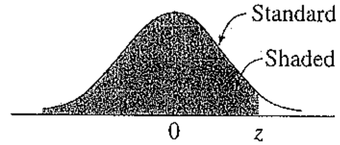
Differences of Least Squares Means											
Effect	**FOOD**	**METH**	**_FOOD**	**_METH**	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**	**Adjustment**	**Adj P**
FOOD*METH	**FRUIT**	**M3**	**FRUIT**	**M4**	0.4720	0.2891	47.8	1.63	0.1091	Tukey-Kramer	0.8840
FOOD*METH	**FRUIT**	**M3**	**VEG**	**M1**	0.4420	0.2891	47.8	1.53	0.1329	Tukey-Kramer	0.9211
FOOD*METH	**FRUIT**	**M3**	**VEG**	**M2**	0.4110	0.2891	47.8	1.42	0.1616	Tukey-Kramer	0.9503
FOOD*METH	**FRUIT**	**M3**	**VEG**	**M3**	-0.8000	0.2830	32	-2.83	0.0080	Tukey-Kramer	0.2162
FOOD*METH	**FRUIT**	**M3**	**VEG**	**M4**	0.2450	0.2891	47.8	0.85	0.4009	Tukey-Kramer	0.9992
FOOD*METH	**FRUIT**	**M4**	**VEG**	**M1**	-0.03000	0.2891	47.8	-0.10	0.9178	Tukey-Kramer	1.0000
FOOD*METH	**FRUIT**	**M4**	**VEG**	**M2**	-0.06100	0.2891	47.8	-0.21	0.8338	Tukey-Kramer	1.0000
FOOD*METH	**FRUIT**	**M4**	**VEG**	**M3**	-1.2720	0.2891	47.8	-4.40	<.0001	Tukey-Kramer	0.0054
FOOD*METH	**FRUIT**	**M4**	**VEG**	**M4**	-0.2270	0.2830	32	-0.80	0.4284	Tukey-Kramer	0.9995
FOOD*METH	**VEG**	**M1**	**VEG**	**M2**	-0.03100	0.2891	47.8	-0.11	0.9151	Tukey-Kramer	1.0000
FOOD*METH	**VEG**	**M1**	**VEG**	**M3**	-1.2420	0.2891	47.8	-4.30	<.0001	Tukey-Kramer	0.0071
FOOD*METH	**VEG**	**M1**	**VEG**	**M4**	-0.1970	0.2891	47.8	-0.68	0.4989	Tukey-Kramer	0.9999
FOOD*METH	**VEG**	**M2**	**VEG**	**M3**	-1.2110	0.2891	47.8	-4.19	0.0001	Tukey-Kramer	0.0094
FOOD*METH	**VEG**	**M2**	**VEG**	**M4**	-0.1660	0.2891	47.8	-0.57	0.5685	Tukey-Kramer	1.0000
FOOD*METH	**VEG**	**M3**	**VEG**	**M4**	1.0450	0.2891	47.8	3.61	0.0007	Tukey-Kramer	0.0398

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$ *(cont.)*



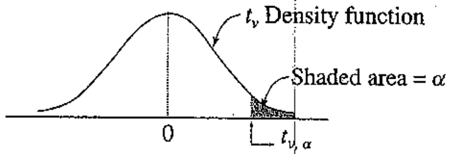
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0394	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.4 Critical Values $t_{\nu,\alpha}$ for the t-Distribution



ν	α = .10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	1.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

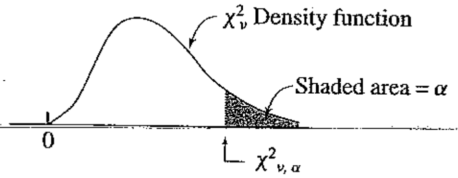
SOURCE: This table is produced with the kind permission of the Trustees of Biometrika from E. S. Pearson and H. O. Hartly (eds.) *The Biometrika Tables for Statisticians*, vol. 1, 3rd ed. (1966) Biometrika.

Table A.2 Cumulative Poisson Probabilities (*cont.*)

	λ = 2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	15.0	20.0
0	.135	.050	.018	.007	.002	.001	.000	.000	.000	.000	.000
1	.406	.199	.092	.040	.017	.007	.003	.001	.000	.000	.000
2	.677	.423	.238	.125	.062	.030	.014	.006	.003	.000	.000
3	.857	.647	.433	.265	.151	.082	.042	.021	.010	.000	.000
4	.947	.815	.629	.440	.285	.173	.100	.055	.029	.001	.000
5	.983	.916	.785	.616	.446	.301	.191	.116	.067	.003	.000
6	.995	.966	.889	.762	.606	.450	.313	.207	.130	.008	.000
7	.999	.988	.949	.867	.744	.599	.453	.324	.220	.018	.001
8	1.000	.996	.979	.932	.847	.729	.593	.456	.333	.037	.002
9		.999	.992	.968	.916	.830	.717	.587	.458	.070	.005
10		1.000	.997	.986	.957	.901	.816	.706	.583	.118	.011
11			.999	.995	.980	.947	.888	.803	.697	.185	.021
12			1.000	.998	.991	.973	.936	.876	.792	.268	.039
13				.999	.996	.987	.966	.926	.864	.363	.066
14				1.000	.999	.994	.983	.959	.917	.466	.105
15					.999	.998	.992	.978	.951	.568	.157
16					1.000	.999	.996	.989	.973	.664	.221
17						.999	.998	.995	.986	.749	.297
18						1.000	.999	.998	.993	.819	.381
19							1.000	.998	.997	.875	.470
20								1.000	.998	.917	.559
21									.999	.947	.644
22									1.000	.967	.721
23										.981	.787
24										.989	.843
25										.994	.888
26										.997	.922
27										.998	.948
28										.999	.966
29										1.000	.978
30											.987
31											.992
32											.995
33											.997
34											.999
35											.999
36											1.000

SOURCE: L. L. Chao (1974), *Statistics: Methods and Analysis*, 2nd ed. New York: McGraw-Hill.

Table A.5 Critical Values $\chi^2_{v,\alpha}$ for the Chi-square Distribution

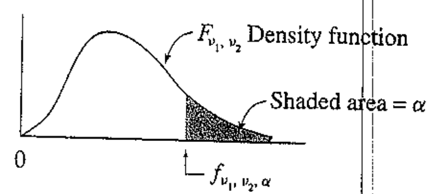


	α									
v	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.420	65.473
40*	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

* For $v > 40$, $\chi^2_{v,\alpha} \simeq v\left(1 - \frac{2}{9v} + z_\alpha\sqrt{\frac{2}{9v}}\right)^3$.

SOURCE: This table is produced with the kind permission of the Trustees of Biometrika from E. S. Pearson and H. O. Hartly (eds.) *The Biometrika Tables for Statisticians*, vol. 1, 3rd ed. (1966) Biometrika.

Table A.6 Critical Values $f_{v_1,v_2,\alpha}$ for the F-Distribution ($\alpha = .05$) (*cont.*)



Degrees of freedom for the denominator (v_2)	Degrees of freedom for the numerator (v_1)																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.49	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.69	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.81	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.91	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.09	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.59	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.81	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00