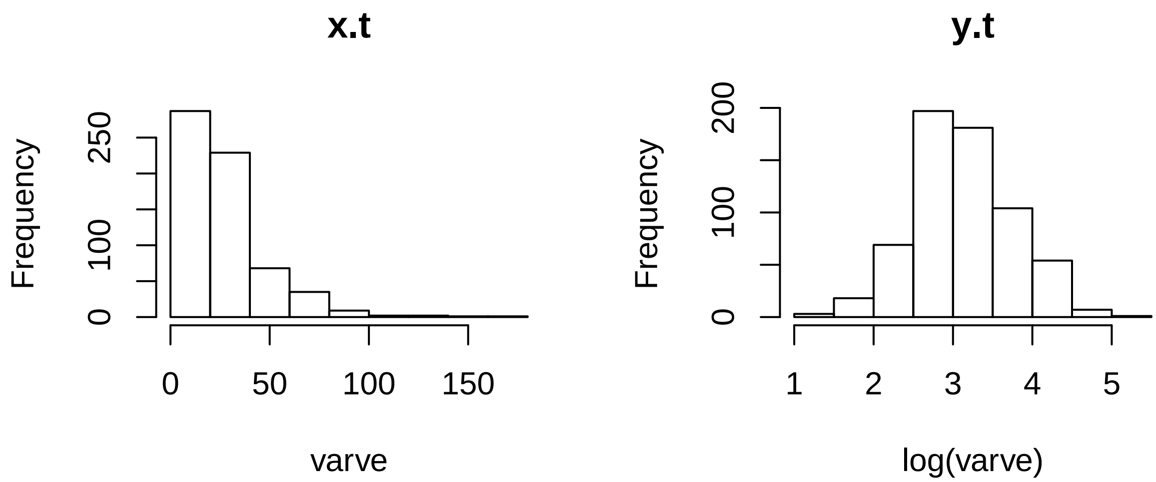
STAT 626 HW06 BLUBAUGH

I

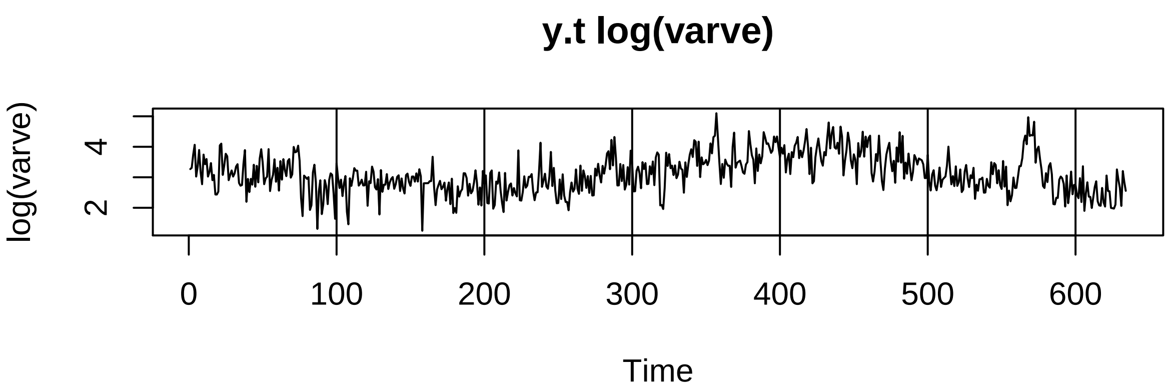
2.8

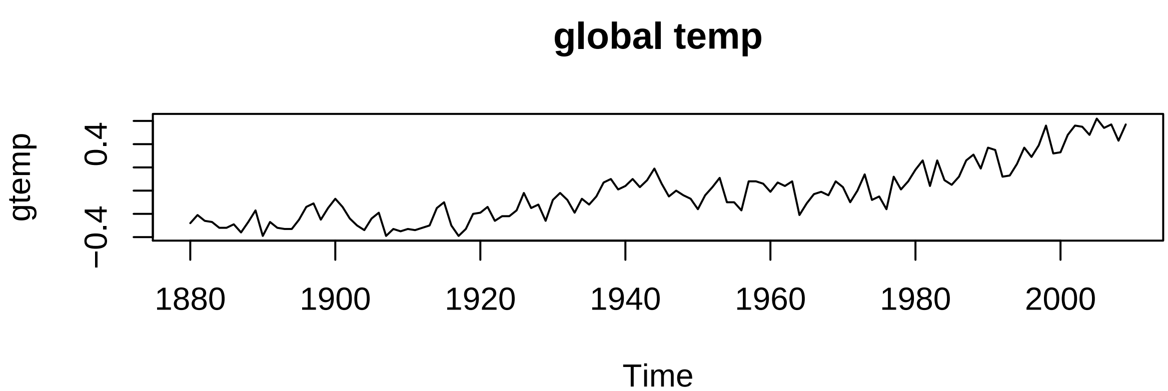
a) There are 634 observations in the series. In the first half (1:317), $\sigma^2 = 113$, in the second half (318:634) $\sigma^2 = 594$ The difference between the two makes it clear that variance is not constant across the series. Applying a log transformation to the series and calculating variance across the same intervals returns $\sigma^2 = [.27, .45]$ respectively. Although the variance is not the same for the two segments, it is much closer than before the transformation. A histogram of varve before the transformation shows a heavily right skewed, non normal distribution where as the log transformation of varve shows a much more normal distribution although slightly right skewed.



b)

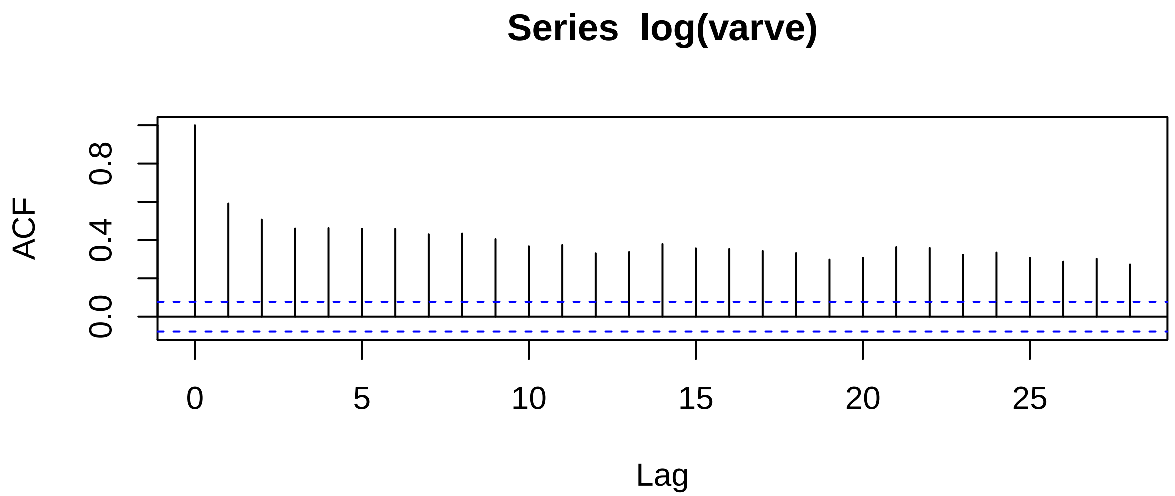
There doesn't appear to be a noticeable similarity between the two series when breaking up the varve set into 100 year intervals other than the linear trend seems to stay relatively stable for long periods of time. The two datasets are related to different periods in time so it may not be appropriate to compare the two.





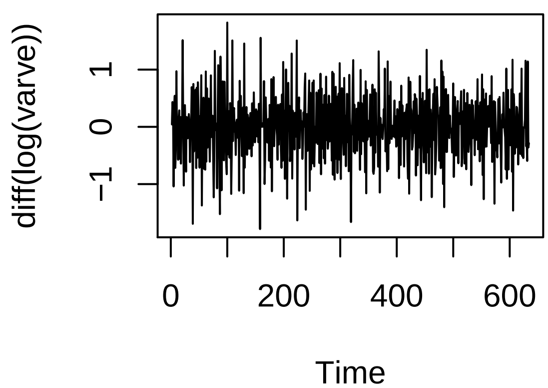
c)

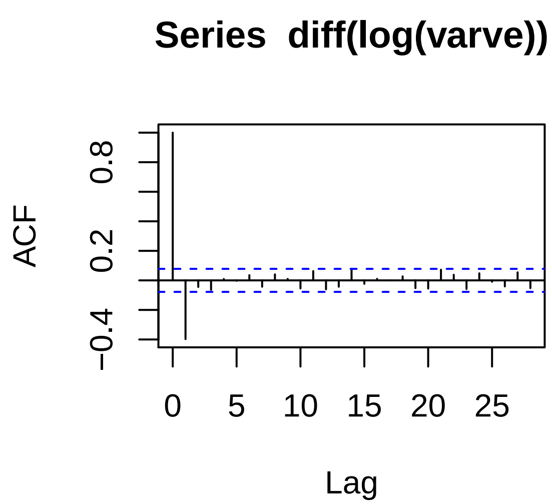
The long tail of the ACF indicates the possibility that the data are not stationary. Differencing would be a suggested method for making the series stationary



d)

The data appears to be stationary with $E(u_t) = 0$. The ACF plot shows significance at lag 1 indicating that an MA(1) model may be an appropriate fit. P can be interpreted as the percent change in the series between y_t and y_{t-1}





e)

$$
\begin{aligned}
u_t =& \mu + w_t - \theta w_{t-1} \\
\gamma(h=0) =& var(\mu + w_t - \theta w_{t-1}) \\
=& (1+\theta^2)\sigma^2 \\
\gamma(h=1) =& cov(w_{t-1} - \theta w_{t-2}, w_t - \theta w_{t-1}) \\
=& -\theta\sigma^2 \\
\gamma(h=1) =& cov(w_{t-2} - \theta w_{t-3}, w_t - \theta w_{t-1}) \\
=& 0
\end{aligned}
$$

f)

4.16ab

a) Since each series is independently stationary, they are also jointly stationary with an expected covariance of 0. $cov(w_t - w_{t-1}, \frac{1}{2}(w_t + w_{t-1})) = \frac{w_t}{2} - \frac{w_{t-1}}{2} = \frac{1}{2} - \frac{1}{2} = 0$

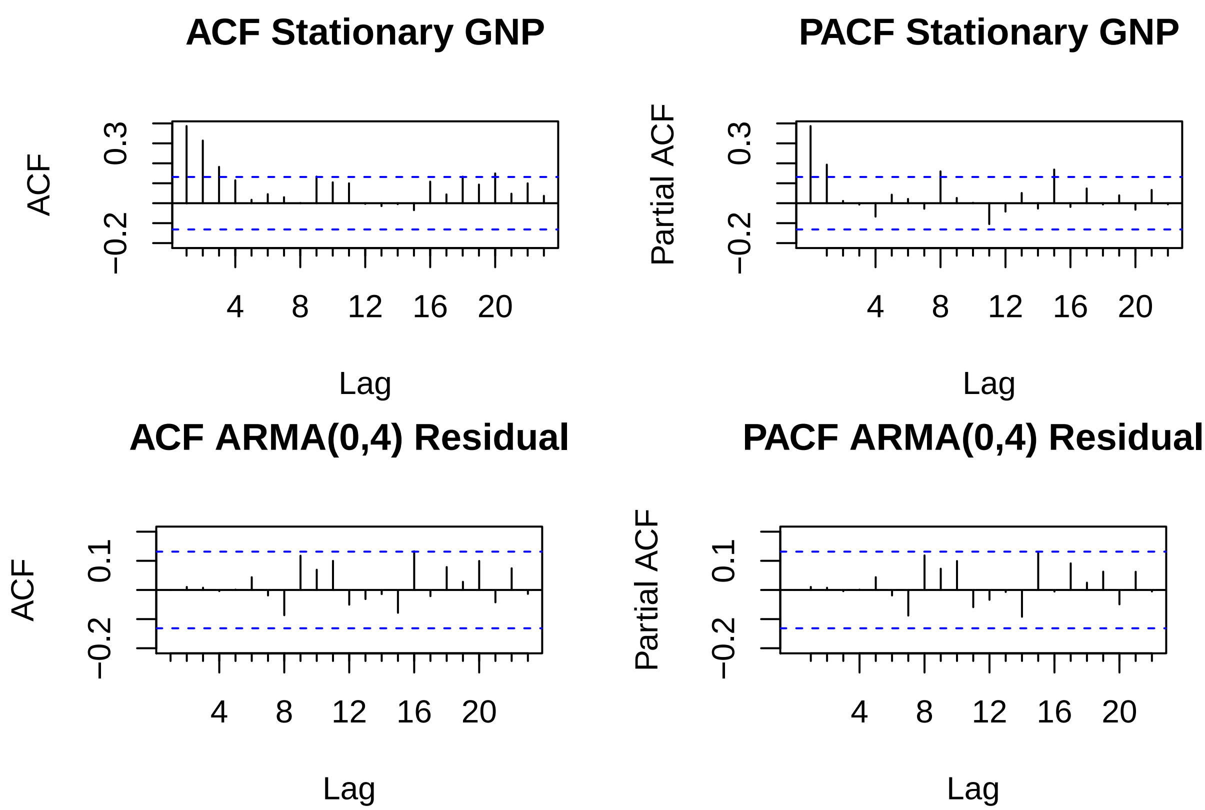
b)

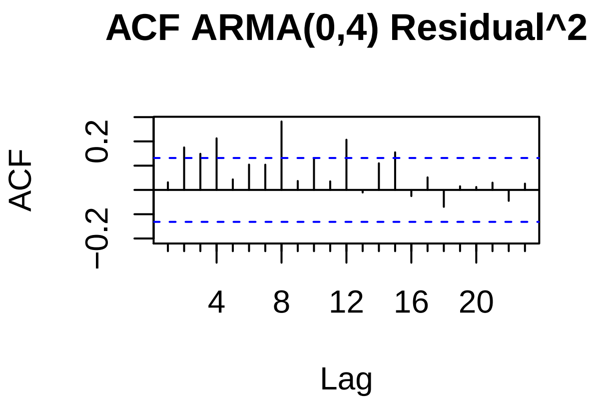
$$\begin{aligned}
\gamma_x(0) =& 1 + 1^2 = 2\sigma^2 \\
\gamma_x(1) =& (1)(-1) = -\sigma^2 \\
f_x(w) =& \sum_{-\infty}^{\infty} = \gamma(h)e^{-2\pi iwh} = \sigma_w^2[2 - e^{-2\pi iw} + e^{2\pi iw}]
\end{aligned}$$

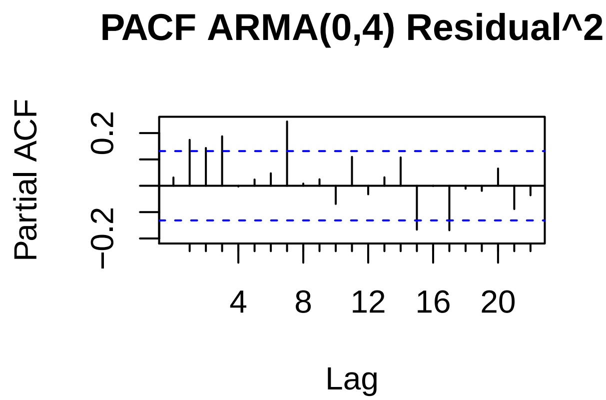
$$\begin{aligned}
\gamma_y(0) =& .5(1^2 + 1^2) = \sigma^2 \\
\gamma_y(1) =& .5\sigma^2 \\
f_y(w) =& \sum_{-\infty}^{\infty} = \gamma(h)e^{-2\pi iwh} = \sigma_w^2[1 + .5(e^{-2\pi iwh} + e^{2piiwh})]
\end{aligned}$$

5.6

Plotting the first difference of gnp to eliminate the trend shows how the variance of the series is not constant. For example the sample variance in the first half of the series is 934 and in the second half 2350. A preliminary model of ARMA(0,4), based on the assessment of acf/pacf plots also shows nonconstant variance in the residuauls. Since there is a case for non-constant variance, a GARCH model is an appropriate modeling choice. An ARCH(3,0) is chosen based on the pacf plot of the squared residuals showing the first 3 lags as being significant.



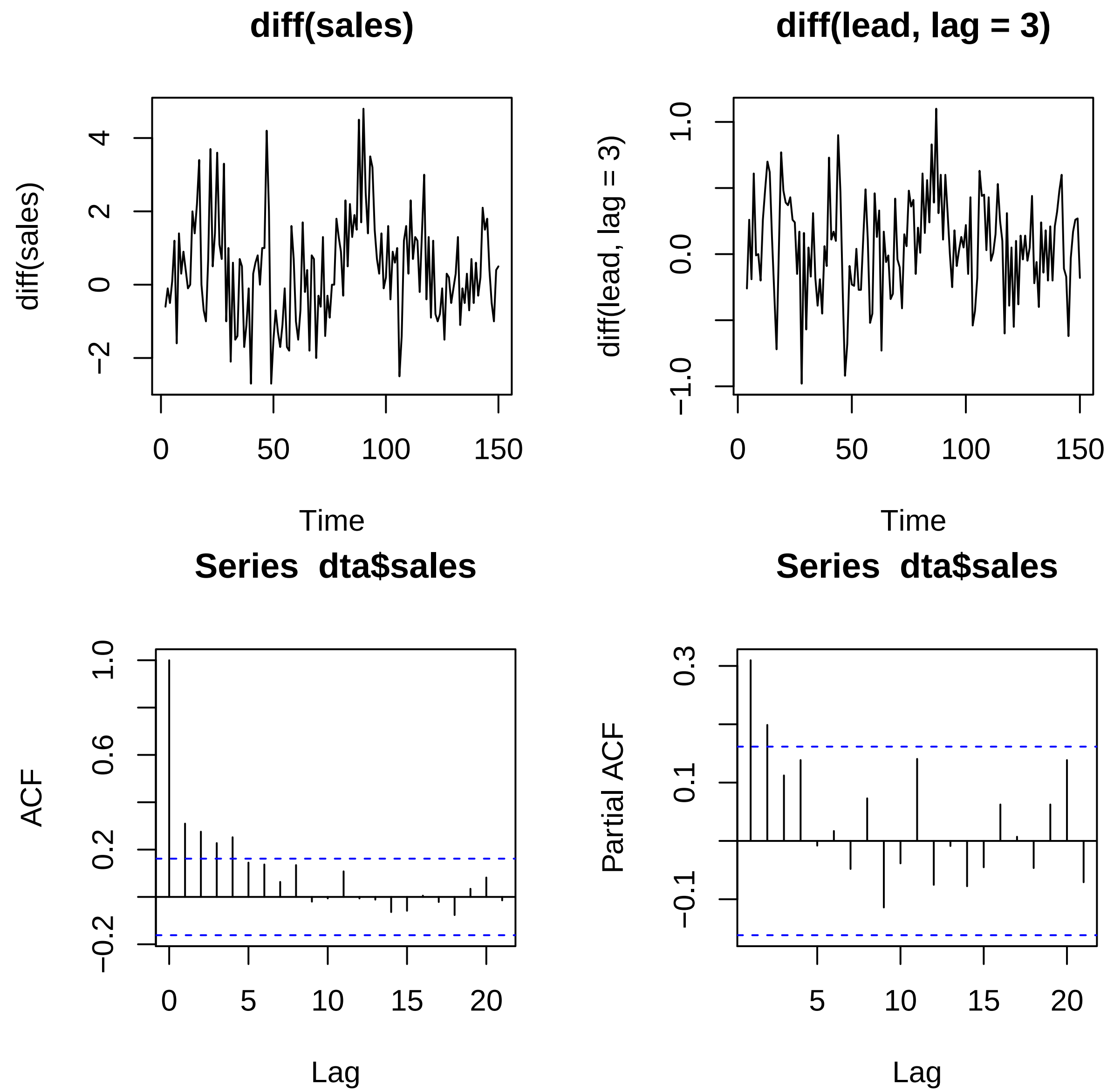


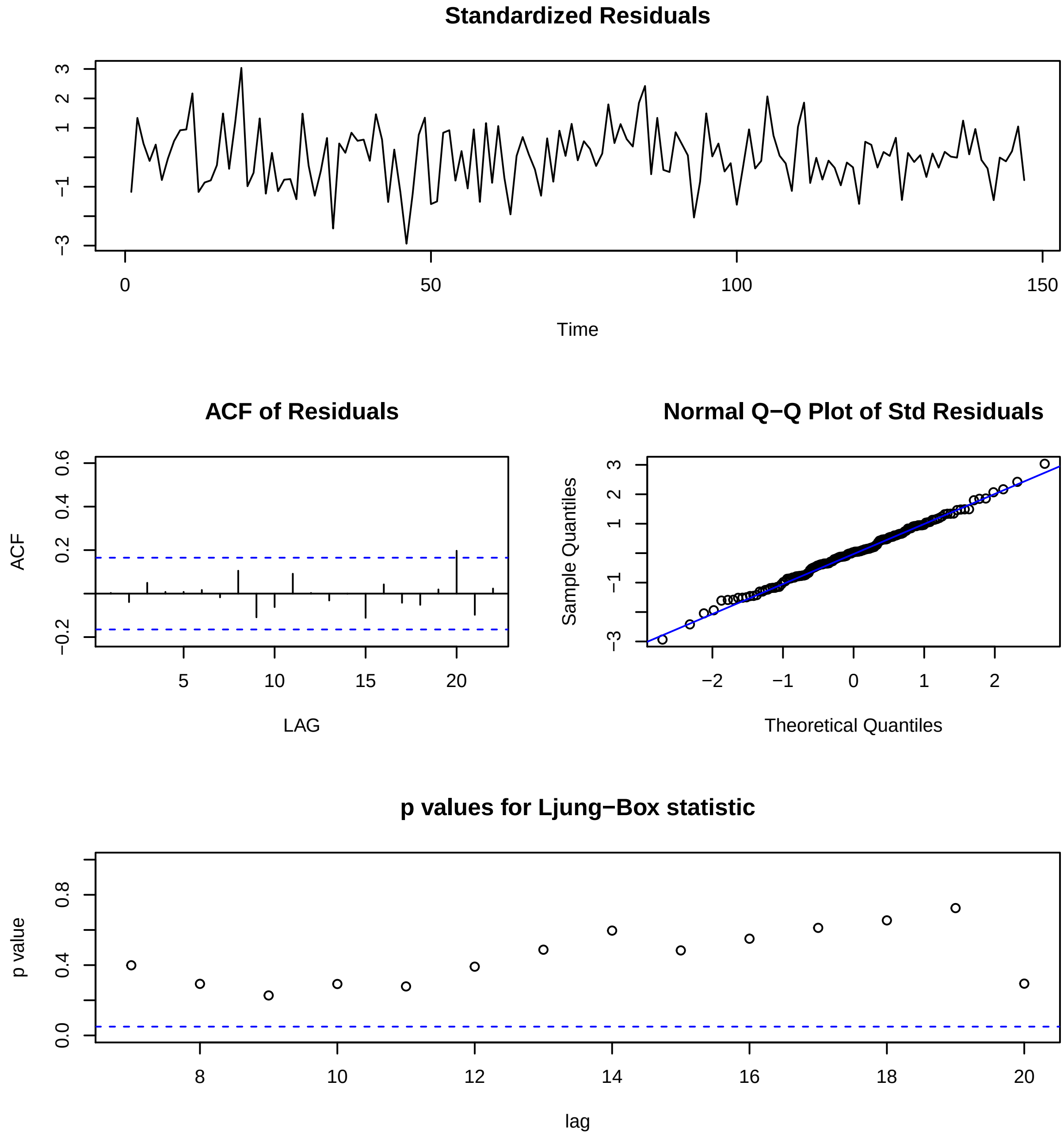



```
FALSE
FALSE Title:
FALSE  GARCH Modelling
FALSE
FALSE Call:
FALSE  garchFit(formula = ~arma(0, 4) + garch(3, 0), data = diff(gnp),
FALSE     trace = F)
FALSE
FALSE Mean and Variance Equation:
FALSE  data ~ arma(0, 4) + garch(3, 0)
FALSE <environment: 0xeb1fb8>
FALSE  [data = diff(gnp)]
FALSE
FALSE Conditional Distribution:
FALSE  norm
FALSE
FALSE Coefficient(s):
FALSE         mu         ma1         ma2         ma3         ma4       omega
FALSE  36.445703    0.400898    0.389817    0.183208    0.214149  670.166959
FALSE     alpha1      alpha2      alpha3
FALSE   0.078343    0.276873    0.234494
FALSE
FALSE Std. Errors:
FALSE  based on Hessian
FALSE
FALSE Error Analysis:
FALSE         Estimate  Std. Error  t value Pr(>|t|)
FALSE mu      36.44570     4.73041    7.705 1.31e-14 ***
FALSE ma1      0.40090     0.06341    6.323 2.57e-10 ***
FALSE ma2      0.38982     0.07707    5.058 4.24e-07 ***
FALSE ma3      0.18321     0.07481    2.449 0.014324 *
FALSE ma4      0.21415     0.06630    3.230 0.001237 **
FALSE omega  670.16696   189.75841    3.532 0.000413 ***
FALSE alpha1   0.07834     0.08711    0.899 0.368439
FALSE alpha2   0.27687     0.10133    2.732 0.006286 **
FALSE alpha3   0.23449     0.11436    2.051 0.040310 *
FALSE ---
FALSE Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
FALSE
FALSE Log Likelihood:
FALSE  -1113.04    normalized:  -5.013695
FALSE
FALSE Description:
FALSE  Sun Jul 17 16:43:37 2016 by user:
```

5.11

I included the differecend variable lead with lag = 3 as the xregressor in the arima model. β_0 is represented by intercept, β_1 is represented by xreg. x_t is the ARMA process that includes 2 ar terms and 4 ma terms. The ARMA process was determined by looking at the differenced sales variable that shows 4 significant lags on the acf plot and 2 significant lags on the pacf plot. As a result an ARMA(2,4) process was chosen.

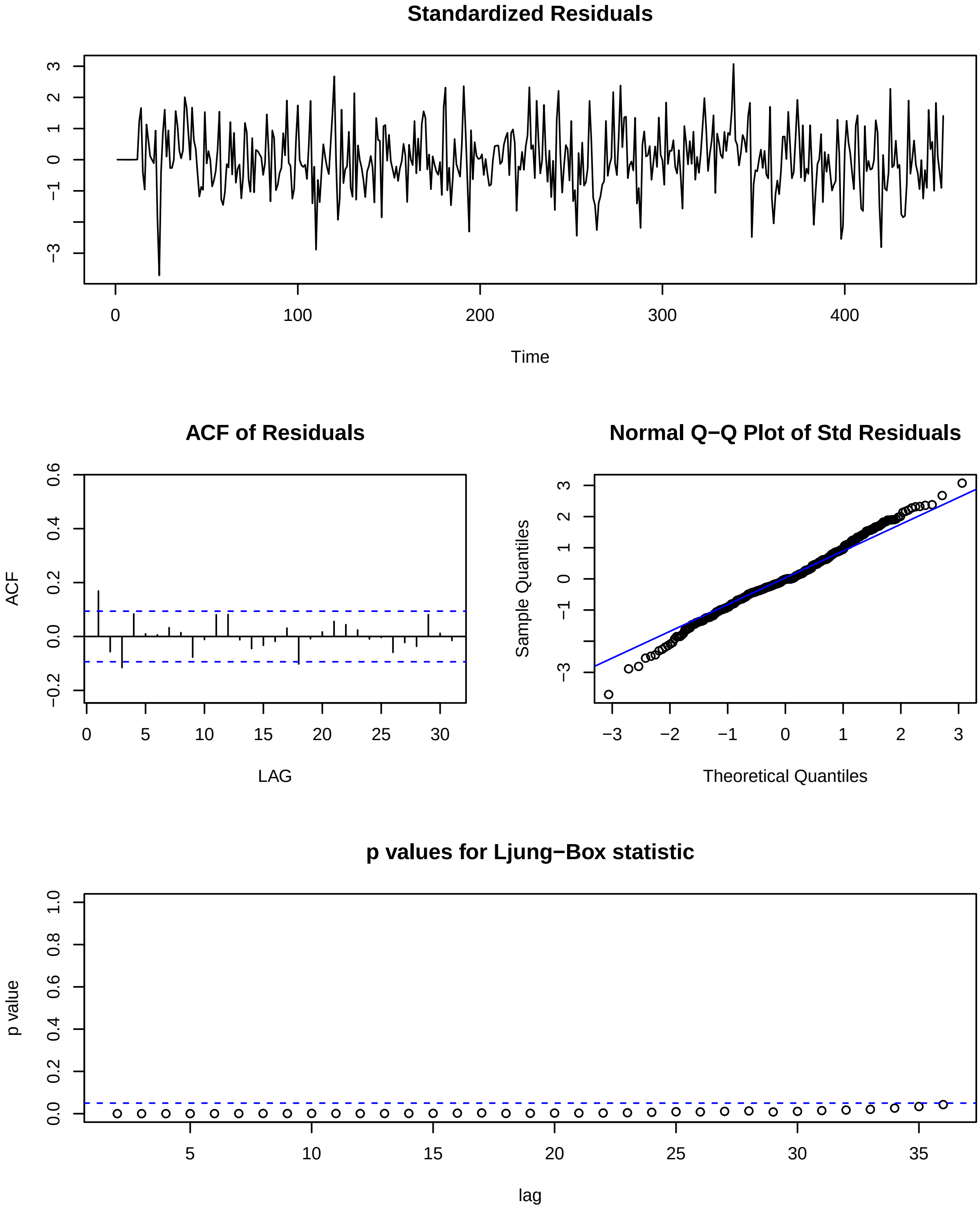




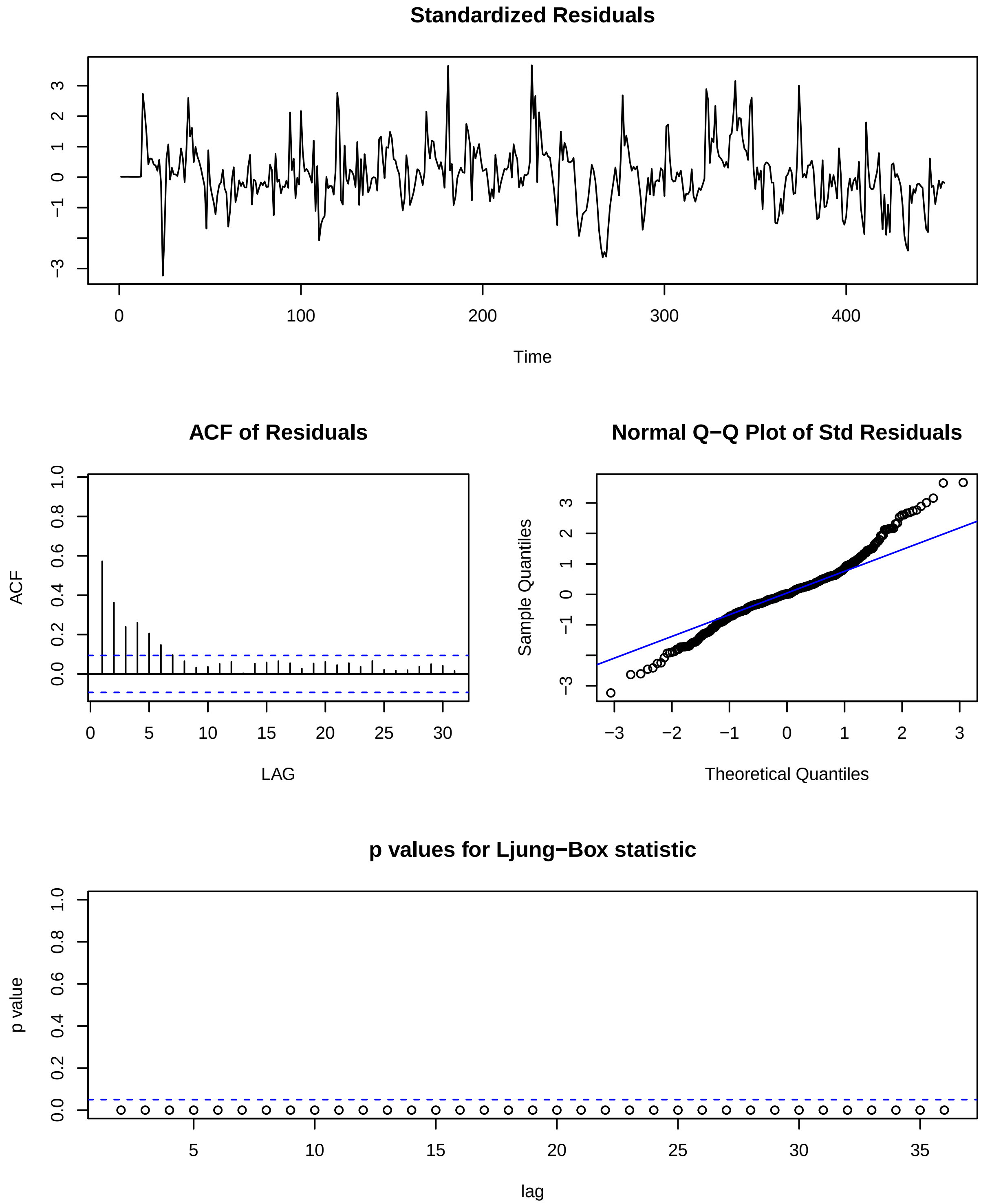

```
FALSE
FALSE Call:
FALSE stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
FALSE     Q), period = S), xreg = xreg, optim.control = list(trace = trc, REPORT = 1,
FALSE     reltol = tol))
FALSE
FALSE Coefficients:
FALSE          ar1      ar2      ma1     ma2     ma3      ma4  intercept
FALSE       0.9797  -0.1291  -0.4996  0.1250  0.5236  -0.3878     0.5704
FALSE s.e.  0.3027   0.2049   0.2875  0.0794  0.0806   0.1772     0.3655
FALSE          xreg
FALSE       -2.6623
FALSE s.e.   0.1341
FALSE
FALSE sigma^2 estimated as 0.8009:  log likelihood = -193.48,  aic = 404.96
```

5.13a

```
## i)
mdl = sarima(xdata = sqrt(climhyd$Precip), 0, 0, 0, 0, 1, 1, 12, details = FALSE)
```



```r
## ii)
mdl = sarima(xdata = log(climhyd$Inflow), 0, 0, 0, 0, 1, 1, 12, details = FALSE)
```

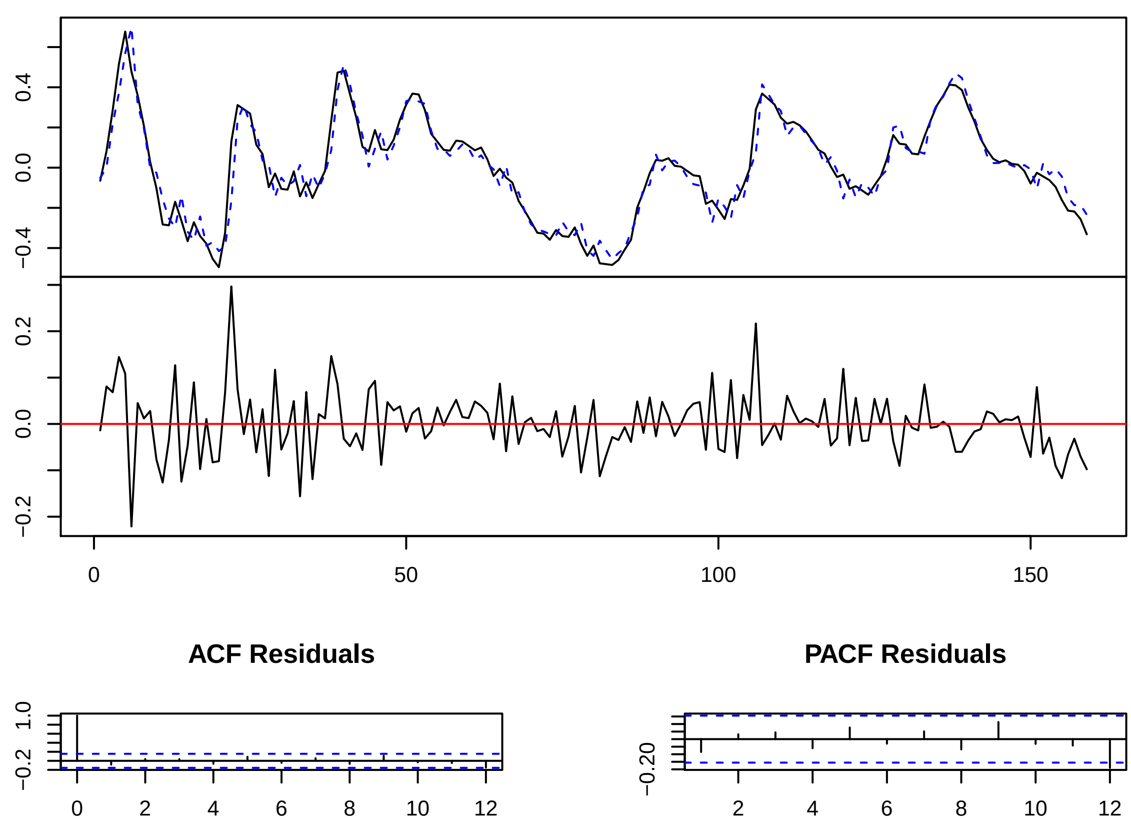


5.15

```
FALSE
FALSE VAR Estimation Results:
FALSE ========================
FALSE
FALSE Estimated coefficients for equation unemp:
FALSE ============================================
FALSE Call:
FALSE unemp = unemp.l1 + gnp.l1 + consum.l1 + govinv.l1 + prinv.l1 + unemp.l2 + gnp.l2 + consum.l2 + govinv.l2
FALSE
FALSE      unemp.l1        gnp.l1     consum.l1     govinv.l1      prinv.l1
FALSE  1.0084922076 -1.7140507670 -2.0699480543 -0.5971166088 -0.3580450855
FALSE      unemp.l2        gnp.l2     consum.l2     govinv.l2      prinv.l2
FALSE -0.2047038595  1.3295385948  2.4740423948  0.5365033451  0.0550888276
FALSE         const
FALSE  0.0006639402
FALSE
FALSE
FALSE Estimated coefficients for equation gnp:
FALSE ==========================================
FALSE Call:
FALSE gnp = unemp.l1 + gnp.l1 + consum.l1 + govinv.l1 + prinv.l1 + unemp.l2 + gnp.l2 + consum.l2 + govinv.l2 +
FALSE
FALSE      unemp.l1        gnp.l1     consum.l1     govinv.l1      prinv.l1
FALSE -0.0112010839  1.2968694627  0.1199108653  0.1285933069 -0.0355635368
FALSE      unemp.l2        gnp.l2     consum.l2     govinv.l2      prinv.l2
FALSE  0.0289604078 -0.4039982370  0.0192795653 -0.1061141302  0.0395245785
FALSE         const
FALSE -0.0002283406
FALSE
FALSE
FALSE Estimated coefficients for equation consum:
FALSE =============================================
FALSE Call:
FALSE consum = unemp.l1 + gnp.l1 + consum.l1 + govinv.l1 + prinv.l1 + unemp.l2 + gnp.l2 + consum.l2 + govinv.l
FALSE
FALSE     unemp.l1       gnp.l1    consum.l1    govinv.l1     prinv.l1
FALSE  0.004884473  0.031627374  0.860454305  0.097045569  0.031190575
FALSE     unemp.l2       gnp.l2    consum.l2    govinv.l2     prinv.l2
FALSE  0.006306513 -0.050741629  0.138843090 -0.078801367 -0.025678355
FALSE        const
FALSE -0.000414320
FALSE
FALSE
FALSE Estimated coefficients for equation govinv:
FALSE =============================================
FALSE Call:
FALSE govinv = unemp.l1 + gnp.l1 + consum.l1 + govinv.l1 + prinv.l1 + unemp.l2 + gnp.l2 + consum.l2 + govinv.l
FALSE
FALSE     unemp.l1       gnp.l1    consum.l1    govinv.l1     prinv.l1
FALSE  0.013758557  0.238572685 -0.012978845  1.301077620 -0.003122029
FALSE     unemp.l2       gnp.l2    consum.l2    govinv.l2     prinv.l2
FALSE -0.034119657  0.261190185 -0.346366379 -0.424658072 -0.075235232
FALSE        const
FALSE  0.001755712
FALSE
FALSE
FALSE Estimated coefficients for equation prinv:
FALSE ============================================
```

```
FALSE Call:
FALSE prinv = unemp.l1 + gnp.l1 + consum.l1 + govinv.l1 + prinv.l1 + unemp.l2 + gnp.l2 + consum.l2 + govinv.l2
FALSE
FALSE      unemp.l1       gnp.l1    consum.l1    govinv.l1     prinv.l1
FALSE -0.017154009  0.028167309  2.882837331  0.326154913  0.914622515
FALSE      unemp.l2       gnp.l2    consum.l2    govinv.l2     prinv.l2
FALSE  0.076178299 -0.281062959 -2.025613415 -0.261500447 -0.115256833
FALSE        const
FALSE -0.001373826
```

Diagram of fit and residuals for unemp



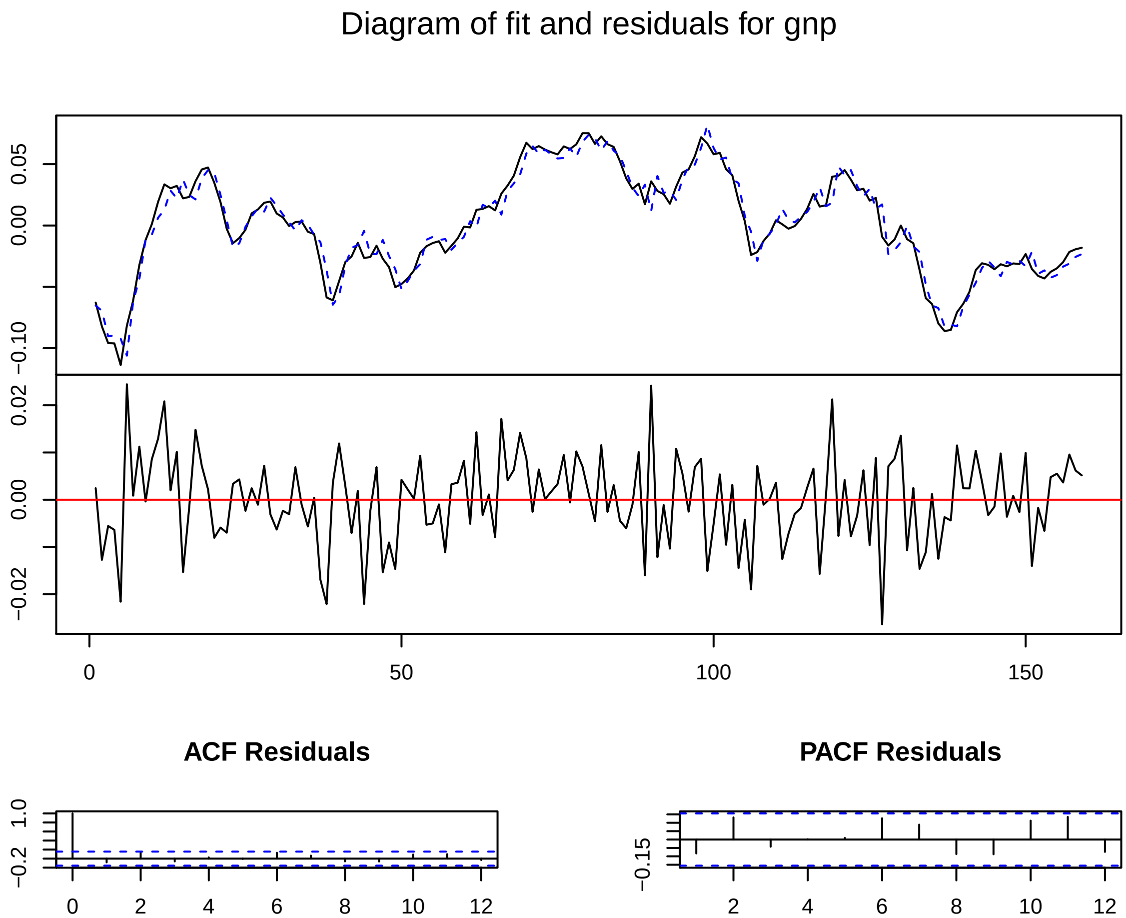


Diagram of fit and residuals for gnp
0.05
0.00
−0.10
0.02
0.00
−0.02
0
50
100
150
ACF Residuals
1.0
−0.2
0
2
4
6
8
10
12
PACF Residuals
−0.15
2
4
6
8
10
12

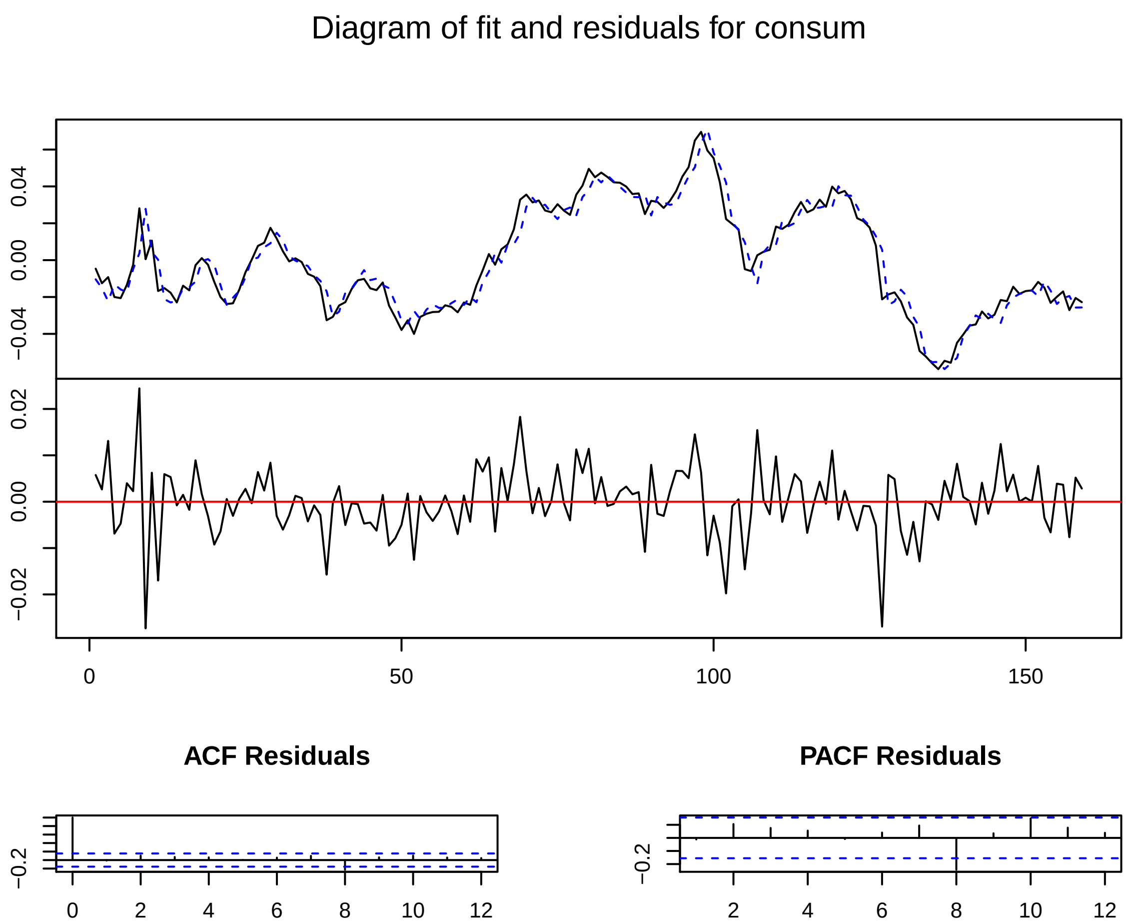


Diagram of fit and residuals for consum
0.04
0.00
−0.04
0.02
0.00
−0.02
0
50
100
150
ACF Residuals
−0.2
0
2
4
6
8
10
12
PACF Residuals
−0.2
2
4
6
8
10
12

Diagram of fit and residuals for govinv

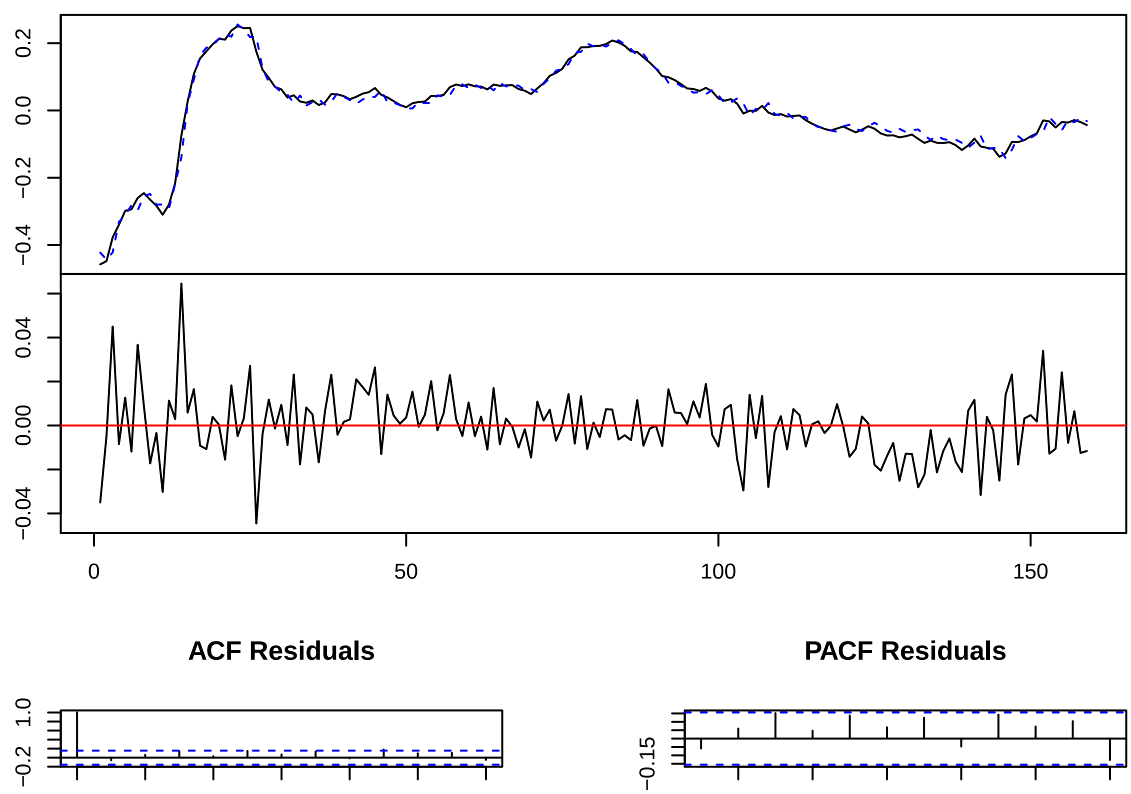
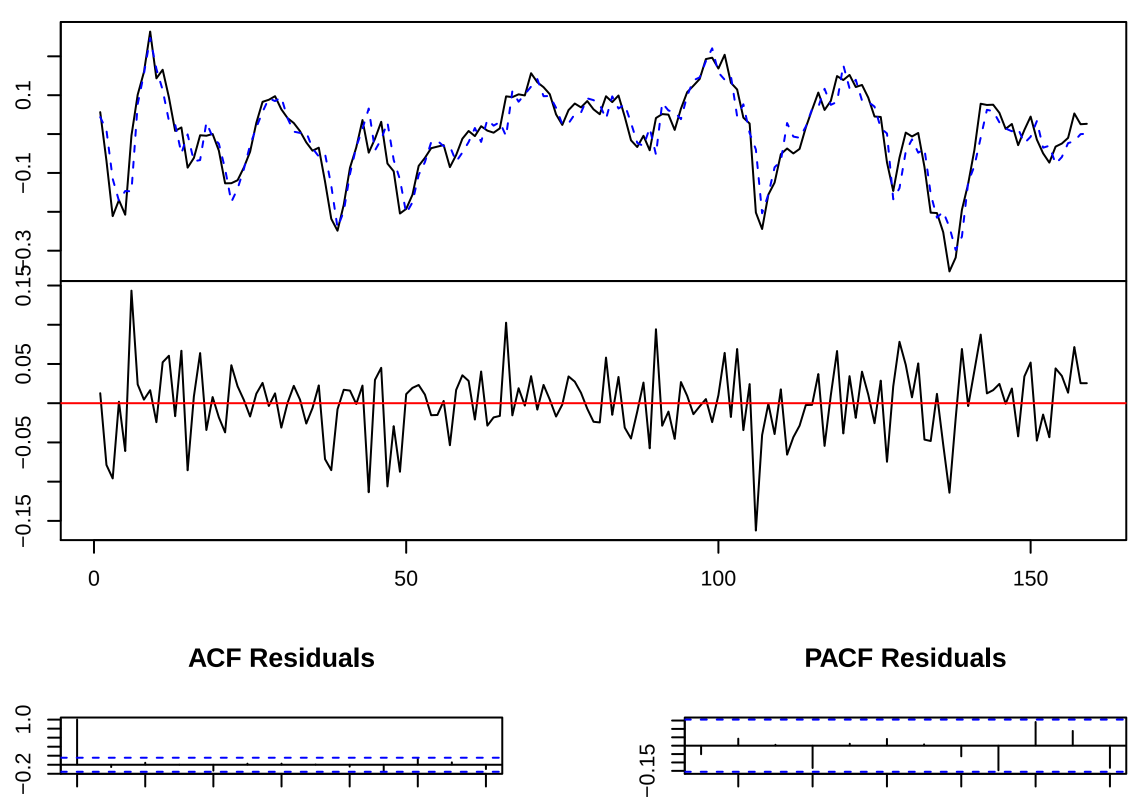


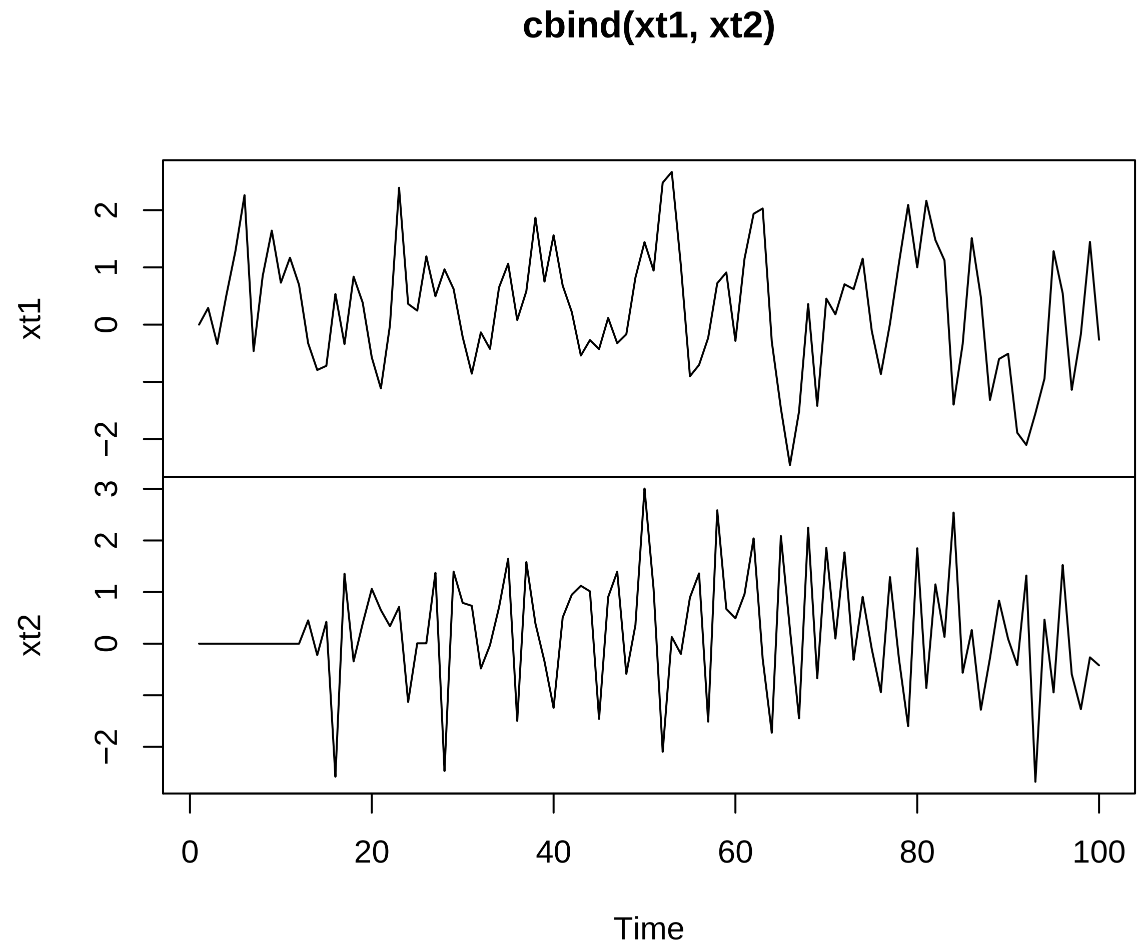
Diagram of fit and residuals for prinv

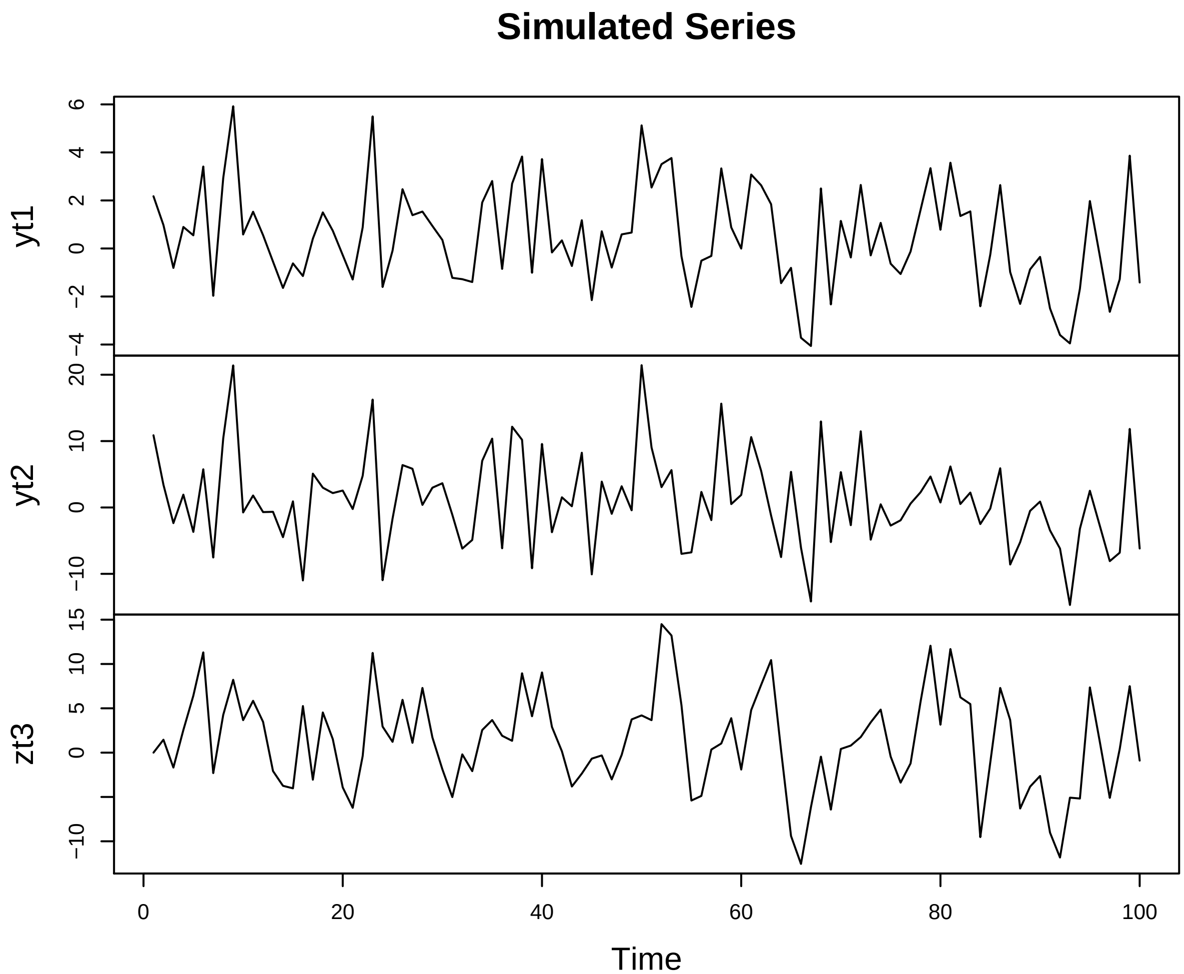


II

a

All 3 series appear to be stationary with an expected mean of 0.





b

$$
\begin{aligned}
\gamma(0) =& var(.5x_{t-1} + w_{t,1} + w_{t,1} + w_{t,2}) \\
=& .5^2 var(x_t) + 1^2 + 1^2 + 1^2 \\
=& \frac{3\sigma^2}{.75} = 4\sigma^2
\end{aligned}
$$

$$
\begin{aligned}
\gamma(1) =& cov[.5x_{t-2} + w_{t-1,1} + w_{t-1,1} + w_{t-1,2}, .5x_{t-1} + w_{t,1} + w_{t,1} + w_{t,2}] \\
=& 0
\end{aligned}
$$

ACF Function: when h = 0: $4\sigma^2$, when h > 0: $0, Y_{t,1}$ is stationary

c

There is a correlation at lag 2 so the series is not stationary

$$\begin{aligned}
z_t =& 5(.25x_{t-1,1} + 2w_{t-1,1} + w_{t,2}) - .9x_{t-12,2} - 2w_{t,2} - w_{t,1} \\
var(z_t) =& 25var[.25x_{t-1,1} + w_{t-1,1} - .9x_{t-12,12} - w_{t,2}] \\
5\begin{bmatrix}(1-.25)x_{t-1,1} \\ (1+.9)x_{t-12,2}\end{bmatrix} =& w_{t-1,1} - w_{t,2} = 2\sigma^2 \\
\gamma(1) =& cov[w_{t-2,1} - w_{t-1,2}, w_{t-1,1} - w_{t-2,2}] = 0 \\
\gamma(2) =& cov[1_{t-3,1} - w_{t-2,2}, w_{t-1,1} - w_{t-2,2}] = 2\sigma^2
\end{aligned}$$

d

y_{t1} and y_{t2} are both stationary, but the linear combination of the 2 are not stationary.

e

$$\begin{aligned}
y_{t1} =& .5x_{t-1,1} + 2w_{t1} + w_{t2} \\
y_{t2} =& .9x_{t-12,1} + 2w_{t2} + w_{t1} \\
cov(y_{t1}, y_{t2}) =& cov[.5x_{t-1,1} + 2w_{t1} + w_{t2}, .9x_{t-12,1} + 2w_{t2} + w_{t1}] \\
=& 2w_{t1} + 2w_{t2} = 4\sigma_w^2 \\
\gamma(1) =& cov[.5x_{t-2,1} + 2w_{t-1,1} + w_{t-1,2}, .9x_{t-12,1} + 2w_{t2} + w_{t1}] \\
=& 0 \\
\gamma(11) =& cov[.5x_{t-12,1} + 2w_{t-11,1} + w_{t-11,2}, .9x_{t-12,1} + 2w_{t2} + w_{t1}] \\
=& .45\sigma_w^2
\end{aligned}$$

Autocovariance Function: when h=0: $4\sigma_2^2$ when h=11.45σ_w^2 else 0
Autocorrelation Function: when h=0: 1 when h=11: .1125 else: 0