MASTER'S DIAGNOSTIC EXAMINATION

August 2011

Student's Name ____

INSTRUCTIONS FOR STUDENTS:

1. DO NOT put your NAME on the exam. Place the NUMBER assigned to you on the UPPER RIGHT HAND CORNER of EACH PAGE of your solutions.
2. Please start your answer to EACH QUESTION on a SEPARATE sheet of paper.
3. Use only one side of each sheet of paper.
4. You must answer all four questions: Questions I, II, III and IV.
5. Be sure to attempt all parts of the four questions. It may be possible to answer a later part of a question without having solved the earlier parts.
6. Be sure to hand in all of your exam. No additional material will be accepted once the exam has ended and you have left the exam room.
7. You may use only a calculator, pencil or pen, and blank paper for this examination. No other materials are allowed.

I attest that I spent no more than 4 hours to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature__________________________________

INSTRUCTIONS FOR PROCTOR:

Immediately after the student completes the exam, the student's solutions should be

faxed to 979-845-6060 or **emailed to longneck@stat.tamu.edu**

Do not send the questions, just send the student's solutions.

(1) I certify that the time at which the student started the exam was ______________________

and the time at which the student completed the exam was ______________________

(2) I certify that the student has followed all the **INSTRUCTIONS FOR STUDENTS** listed above.

(3) I certify that the student's solutions were faxed to **979-845-6060** or emailed to **longneck@stat.tamu.edu**.

Proctor's Signature__________________________________

PROBLEM I. In a genetics experiment, investigators examined 300 chromosomes of a particular type and counted the number of sister-chromatid exchanges on each chromosome. The following table displays the number of exchanges for each of the 300 chromosomes and information about model fit.

Number of Exchanges	0	1	2	3	4	5	6	7	8	9	Total
Number of Chromosomes	5	21	40	59	62	44	42	17	7	3	300
$\hat{p}_i$	.018	.074	.146	.195	.196	.156	.104	.060	.030	.021	1.00
$\hat{E}_i$	5.4	22.2	43.8	58.5	58.8	46.8	31.2	18.0	9.0	6.3	300
O_i	5	21	40	59	62	44	42	17	7	3	300
$\frac{(O_i-\hat{E}_i)^2}{\hat{E}_i}$	.030	.065	.330	.004	.174	.168	3.738	.056	.444	1.729	6.74

Summary Statistics for the number of exchanges for the 300 chromosomes were

$N = 300$ $\bar{X} = 4.0$ $S^2 = 3.48$

1. Using the information given above, does a Poisson model appear to be an appropriate model for the data?

2. Estimate, using an appropriate 95% confidence interval, the average number of exchanges per chromosome.

3. Assume that the Poisson model is the correct model. Using an α=0.05 test, test the hypothesis that the number of exchanges per chromosome is less than 4. What is the p-value of your test?

4. What is the power of the test developed in Question c) if the true mean number of exchanges is 3.8 per chromosome?

5. A new study is being designed. Determine the minimum sample size which would yield an $\alpha = .05$ test of the research hypothesis that the mean number of exchanges is less than 4 with the test having power of at least 0.85 whenever the true value of the mean number of exchanges is 3.5 or smaller.

- Hint: You may assume that the sample size is large enough for the central limit theorem to be applicable.

PROBLEM II. Greens for golf courses present a unique problem for use of fertilizers in maintaining turf growth. The soil is usually very sandy and as a consequence has little capacity to retain nitrogen in the root zone after irrigation. Large initial doses of nitrogen are harmful to the grass, but slow release forms may leach out of the soil and be ineffective. A second factor that may affect nitrogen retention is the build-up of thatch, or dead grass.

A soil scientist wanted to investigate the effects of nitrogen supplied in different chemical forms and evaluate those effects combined with the effects of thatch accumulation on the quality of established turf. The first factor in his experiment is N, the form of nitrogen. He will use four forms of nitrogen, two fast release fertilizers: U = urea and AS = Ammonium Sulphate; and two slow release fertilizers: ID = Isobutylidene Diurea and SC = Sulphur Coated urea. Each fertilizer will be applied to the turf at the rate of one pound nitrogen per 1000 square feet of turf. The fertilizer will be applied each year of the experiment. The second factor is T = thatch. The thatch will be allowed to accumulate on an experimental plot for 2, 4, or 6 years. One variable of interest will be the chlorophyll content of grass clippings from the experimental plots.

The soil scientist has two choices for the experimental design:

> **Design A**. This design would use a Randomized Complete Block Design with 12 treatment combinations (N at 4 levels and T at 3 levels). There would be two complete blocks (total of 24 observations).

> **Design B**. This design would use a Split Plot Design with the 4 N levels as the whole plot treatments and the 3 T levels as the subplot treatments. There would be two complete blocks for the whole plot design (total of 24 observations).

1. Discuss the conditions under which each of the designs would be favored.

2. Describe the randomization procedure to be followed if the soil scientist chooses to use Design B.

3. The soil scientist uses Design B. A partial AOV table and Means table are given on the next page with notation Blocks (B), Nitrogen (N), Thatch (T). Provide the values for degree of freedom.

AOV Table

Source	DF	Mean Square
B		0.51
N		12.44
N*B		0.42
T		1.91
N*T		0.69
Error		0.21

Table of Treatment Means

		Thatch			
		2	4	6	Nitrogen Mean
	U	3.85	5.35	5.10	4.77
Nitrogen	AS	5.60	5.85	5.80	5.75
	ID	6.50	6.00	7.80	6.77
	SC	7.35	8.60	8.45	8.13
	Thatch Mean	5.82	6.45	6.79	6.35

(a) Using α=0.05 for any tests that you may want to conduct, what is the effect of Nitrogen and Thatch on the average chlorophyll content of the grass clippings.

(b) Are there any additional analyses you would suggest? If so, give a brief outline of these analyses. (You **do not** need to actually perform the analyses.)

PROBLEM III.

Two treatments, A and B, showed promise for treating a potentially fatal disease. A randomized experiment was conducted to determine whether there is a significant difference in the survival rate between patients who receive treatment A and those who receive treatment B. Of 154 patients who received treatment A, 38 survived for at least 15 years, whereas 16 of the 164 patients who received treatment B survived at least 15 years.

1. A double-blind experiment is one in which neither the patient nor the doctor can identify the treatment given to the patient. Treatment A can be administered only as a pill, and treatment B can be administered only as an injection. Can this randomized experiment be performed as a double-blind experiment? Why or why not?

2. Construct and interpret a 95 percent confidence interval for the difference between the proportion of the population who would survive at least 15 years if given treatment A and the proportion of the population who would survive at least 15 years if given treatment B.

   In many of these types of studies, physicians are interested in the ratio of survival probabilities, $\frac{p_A}{p_B}$, where p_A represents the true 15-year survival rate for all patients who receive treatment A and p_B represents the true 15-year survival rate for all patients who receive treatment B. This ratio is usually referred to as the *relative risk* of the two treatments. For example, a relative risk of 1 indicates the survival rates for patients receiving the two treatments are equal, whereas a relative risk of 1.5 indicates that the survival rate for patients receiving treatment A is 50 percent higher than the survival rate for patients receiving treatment B. An estimator of the relative risk is the ratio of estimated probabilities, $\frac{\hat{p}_A}{\hat{p}_B}$

3. Using the data from the randomized experiment described above, compute the estimate of the relative risk.

   The sampling distribution of $\frac{\hat{p}_A}{\hat{p}_B}$ is skewed. However, when both sample sizes, n_A and n_B, are relatively large, the distribution of $\log_e\left(\frac{\hat{p}_A}{\hat{p}_B}\right)$, the natural logarithm of relative risk, is approximately normal with a mean of $\log_e\left(\frac{p_A}{p_B}\right)$ and a standard deviation of $\sqrt{\frac{1-p_A}{n_A p_A}+\frac{1-p_B}{n_B p_B}}$ where p_A and p_B can be estimated by using $\hat{p}_A$ and $\hat{p}_B$. When a 95 percent confidence interval for $\log_e\left(\frac{p_A}{p_B}\right)$ is known, an approximate 95 percent confidence interval for $\frac{p_A}{p_B}$, the relative risk of the two treatments, can be constructed by exponentiating the endpoints of the confidence interval for $\log_e\left(\frac{p_A}{p_B}\right)$.

4. Construct and interpret a 95 percent confidence interval for the relative risk, $\frac{p_A}{p_B}$, of the two treatments.

The *odds ratio*, $OR = \dfrac{p_A/(1-p_A)}{p_B/(1-p_B)} = \dfrac{p_A(1-p_B)}{p_B(1-p_A)}$ is another quantity that can be used to compare proportions. An estimator of the odds ratio is the sample odds ratio, $\widehat{OR} = \dfrac{\hat{p}_A(1-\hat{p}_B)}{\hat{p}_B(1-\hat{p}_A)}$.

5. Using the data from the randomized experiment above, compute the estimate of the odds ratio.

   We will now look at a setting where we can further explore the three measures used to compare proportions. In a long-term study of British male physicians, the sample proportion who died from heart disease was 0.00140 for smokers and 0.00669 for nonsmokers.

6. Compute sample estimates of the difference in proportions, relative risk, and odds ratio of having heart disease for smokers versus nonsmokers.

7. Explain why the estimated relative risk and the estimated odds ratio have similar values.

8. Which measure(s) best indicate the relationship between the probabilities for having heart disease for smokers and nonsmokers? Explain why.

Problem IV:

Part A.

In a study of the percentage of raw material that responds in a reaction, researchers identified the following five factors:

- the feed rate of the chemicals (*FeedRate*), ranging from 10 to 15 liters per minute
- the percentage of the catalyst (*Catalyst*), ranging from 1% to 2%
- the agitation rate of the reactor (*AgitRate*), ranging from 100 to 120 revolutions per minute
- the temperature (*Temperature*), ranging from 140 to 180 degrees Celsius
- the concentration (*Concentration*), ranging from 3% to 6%

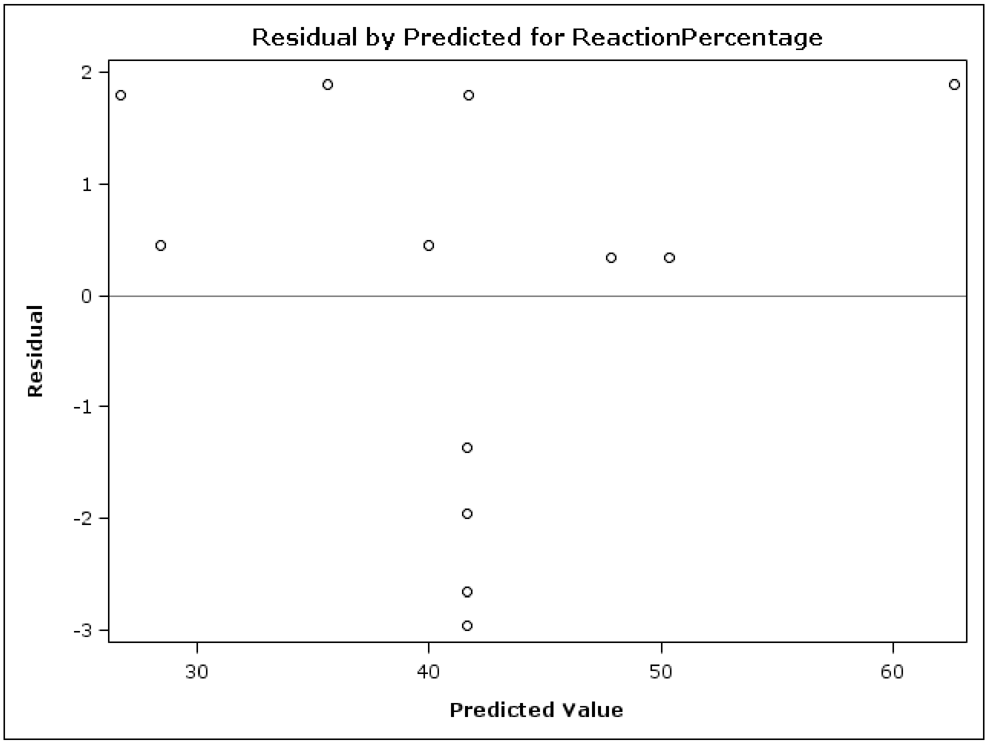
The following data set contains the results of an experiment ***designed*** to estimate main effects for all factors:

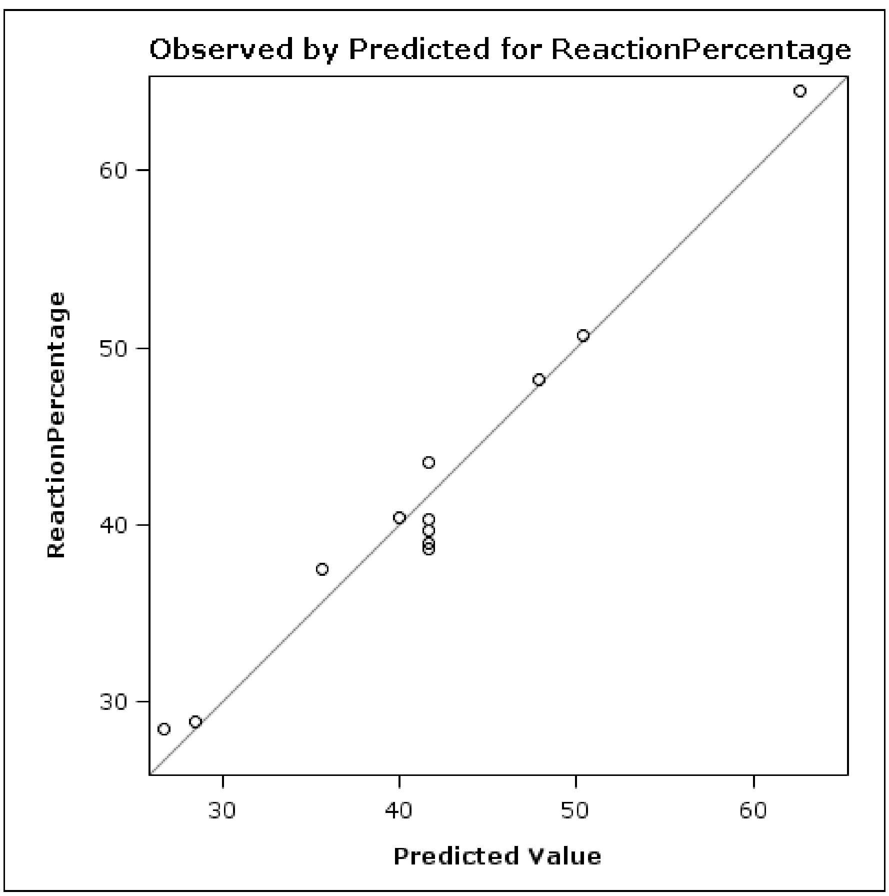
	FeedRate	Catalyst	AgitRate	Temperature	Concentration	ReactionPercentage
1	10	1	100	140	6	37.5
2	10	1	120	180	3	28.5
3	10	2	100	180	3	40.4
4	10	2	120	140	6	48.2
5	15	1	100	180	6	50.7
6	15	1	120	140	3	28.9
7	15	2	100	140	3	43.5
8	15	2	120	180	6	64.5
9	12.5	1.5	110	160	4.5	39
10	12.5	1.5	110	160	4.5	40.3
11	12.5	1.5	110	160	4.5	38.7
12	12.5	1.5	110	160	4.5	39.7

The model is:

$$\text{ReactionPercentage} = \beta_o + \beta_1 \text{*FeedRate} + \beta_2 \text{*Catalyst} + \beta_3 \text{*AgitRate} + \beta_4 \text{*Temperature} + \beta_5 \text{*Concentration} + \varepsilon$$

Some of the results of the regression analyses are given below:


Residual by Predicted for ReactionPercentage
Residual
Predicted Value
2
1
0
-1
-2
-3
30
40
50
60


Observed by Predicted for ReactionPercentage
ReactionPercentage
Predicted Value
60
50
40
30
30
40
50
60

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	990.27000	198.05400	33.29	0.0003
Error	6	35.69917	5.94986		
Corrected Total	11	1025.96917			

Root MSE	2.43923	R-Square	0.9652
Dependent Mean	41.65833	Adj R-Sq	0.9362
Coeff Var	5.85533		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	-43.69167	13.04097	-3.35	0.0154	0
FeedRate	1	1.65000	0.34496	4.78	0.0031	1.00000
Catalyst	1	12.75000	1.72480	7.39	0.0003	1.00000
AgitRate	1	-0.02500	0.08624	-0.29	0.7817	1.00000
Temperature	1	0.16250	0.04312	3.77	0.0093	1.00000
Concentration	1	4.96667	0.57493	8.64	0.0001	1.00000

What would you recommend to your client?

Answer the above question in terms of the following questions:

1) Is this a valid model? Why or why not?

2) If you need to add interactions and squared terms, can you just add them as a group to the model and run the analyses? Why or why not?

3) What is being tested by the F value of 33.29 in the ANOVA table? Answer in terms of the $\beta's$?

Part B.

The usual multiple linear regression model can be written as:

$$Y = X\beta + \varepsilon$$

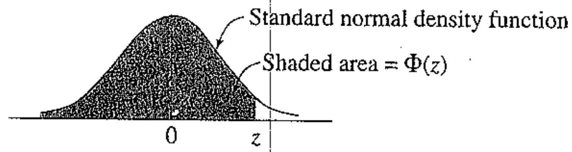
where $V(\varepsilon) = \sigma^2 I$ and I is the (nxn) identity matrix so that $V(Y|X) = \sigma^2 I$.

However, if $V(\varepsilon) = R_{nxn}$, then (Show work)

a) What is the $V(Y|X)$?

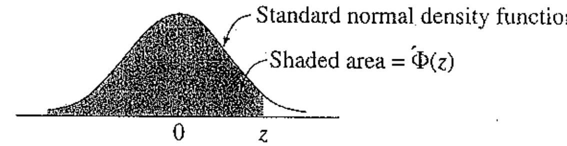
B) If $\hat{Y} = X\hat{\beta} = X(X'X)^{-1}X'Y = HY$ then what is $V(\hat{Y}|X)$?

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$ *(cont.)*



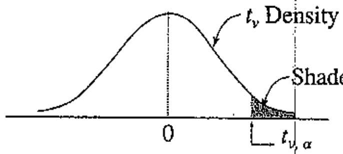
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0,4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1,7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0,0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0394	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0,0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.4 Critical Values $t_{v,\alpha}$ for the t-Distribution



v	α = .10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	1.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

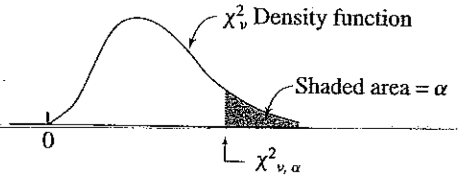
SOURCE: This table is produced with the kind permission of the Trustees of Biometrika from E. S. Pearson and H. O. Hartly (eds.) *The Biometrika Tables for Statisticians*, vol. 1, 3rd ed. (1966) Biometrika.

Table A.2 Cumulative Poisson Probabilities (*cont.*)

	λ = 2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	15.0	20.0
0	.135	.050	.018	.007	.002	.001	.000	.000	.000	.000	.000
1	.406	.199	.092	.040	.017	.007	.003	.001	.000	.000	.000
2	.677	.423	.238	.125	.062	.030	.014	.006	.003	.000	.000
3	.857	.647	.433	.265	.151	.082	.042	.021	.010	.000	.000
4	.947	.815	.629	.440	.285	.173	.100	.055	.029	.001	.000
5	.983	.916	.785	.616	.446	.301	.191	.116	.067	.003	.000
6	.995	.966	.889	.762	.606	.450	.313	.207	.130	.008	.000
7	.999	.988	.949	.867	.744	.599	.453	.324	.220	.018	.001
8	1.000	.996	.979	.932	.847	.729	.593	.456	.333	.037	.002
9		.999	.992	.968	.916	.830	.717	.587	.458	.070	.005
10		1.000	.997	.986	.957	.901	.816	.706	.583	.118	.011
11			.999	.995	.980	.947	.888	.803	.697	.185	.021
12			1.000	.998	.991	.973	.936	.876	.792	.268	.039
13				.999	.996	.987	.966	.926	.864	.363	.066
14				1.000	.999	.994	.983	.959	.917	.466	.105
15					.999	.998	.992	.978	.951	.568	.157
16					1.000	.999	.996	.989	.973	.664	.221
17						.999	.998	.995	.986	.749	.297
18						1.000	.999	.998	.993	.819	.381
19							1.000	.998	.997	.875	.470
20								1.000	.998	.917	.559
21									.999	.947	.644
22									1.000	.967	.721
23										.981	.787
24										.989	.843
25										.994	.888
26										.997	.922
27										.998	.948
28										.999	.966
29										1.000	.978
30											.987
31											.992
32											.995
33											.997
34											.999
35											.999
36											1.000

SOURCE: L. L. Chao (1974), *Statistics: Methods and Analysis*, 2nd ed. New York: McGraw-Hill.

Table A.5 Critical Values $\chi^2_{v,\alpha}$ for the Chi-square Distribution

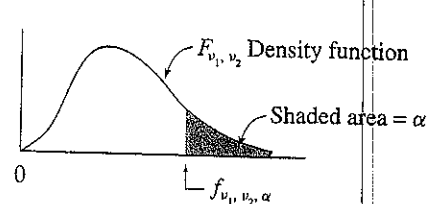


v	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.420	65.473
40*	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

* For $v > 40$, $x^2_{v,\alpha} \simeq v\left(1 - \frac{2}{9v} + z_\alpha\sqrt{\frac{2}{9v}}\right)^3$.

SOURCE: This table is produced with the kind permission of the Trustees of Biometrika from E. S. Pearson and H. O. Hartly (eds.) *The Biometrika Tables for Statisticians*, vol. 1, 3rd ed. (1966) Biometrika.

Table A.6 Critical Values $f_{v_1,v_2,\alpha}$ for the F-Distribution ($\alpha = .05$) (*cont.*)



Degrees of freedom for the denominator (v_2)	Degrees of freedom for the numerator (v_1) 1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.49	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.69	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.81	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.91	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.09	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.59	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.81	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00