STATISTICS 642 - ASSIGNMENT #1 Solution

1. Chapter 1 - Problem 5 - 10 points

   a. To reduce experimental error, the selection of uniform experimental units is required. Thus, the 19 volunteers are divided into four groups of individuals constructing of individuals of similar age and same gender. Treatments A and B are then randomly assigned within each group.

   b. By constructing groups of EU's as described above and randomly assigning the treatments within the groups, we can reduce the amount of difference in the individuals assigned to Treatment A and B within a group prior to applying the treatments. Thus, we can reduce the experimental error variance. When modeling the data, we would take into account the actual age (covariate) and not just the two groups of ages.

   c. Each number indicates Individual number in the table.

	Method A	Method B
Group 1 (Male, Age ≤ 35)	10, 11	17, 18
Group 2 (Male, Age > 35)	1, 2, 14	7, 9, 13
Group 3 (Female,Age ≤ 35)	4, 16	5, 12
Group 4 (Female,Age > 35)	3, 19	6, 8, 15

2. Chapter 1 - Problem 6 - 10 points

   a. Method II

   b. Method II has the treatments applied independently to the shirts (EU's) which allows an assessment of the effect of the application of the treatments to the shirts on the measured responses. Also, each run of the simulation machine has a shirt from each of the 3 treatments. Therefore, if there is a large effect due to the particular run all 3 treatments will be equally affected.

   c. Method I applies the treatment to groups of shirts not to the individual shirts and hence does not allow an assessment of the effect of the application of the treatments. Also, with each run of the simulation machine having all 4 shirts from the same treatment, if there is a large effect due to the particular run, the treatment in that run will have a big advantage (disadvantage) over the other two treatment.

   Method III applies the treatments independently to each treatment as was done in Method II but has the same problem as Method I with respect to having all 4 shirts in a given run being associated with the same treatment.

3. Chapter 1 - Problem 8 - 15 points

   a. There is actually only one replication of the experiment. The effect of instructor is confounded with the effect of method of instruction.

   b. There are numerous ways that this experiment could be designed. I will list two:

      i. Design I: Randomized Block Design: Have three or more instructors, with each instructor randomly assigned to three classrooms. The three classrooms are then randomly assigned to a single instruction method. Thus, each instructor applies all three instruction methods but each classroom receives a single instruction method. The blocks are the instructors, the EU's are the classrooms, and the individual students are the measurement units.

      ii. Design II: Latin Square Design: Have three instructors and three classrooms. The three instruction methods would be applied by each instructor in each classroom. The order in which the classrooms receive the instruction would be one of three sequences. This design could be replicated for multiple sets of instructors and classrooms. The blocks are instructors and classrooms. The EU's are an instructor-classroom combination. The MU's are the individual students.

4. Chapter 1 - Problem 12 - 15 points

Problem 1.12 (15 points) We have 20 possible arrangements. See the table below: Consider the alternative hypothesis $H_a : \mu_B - \mu_A > 0$ to the null null hypothesis $H_0 : \mu_B - \mu_A \leq 0$. The arrangement of the actual experiment is arrangement 9 with a mean difference $\bar{y}_B - \bar{y}_A = 3.00$. A difference of 3.00 or larger occurs with 3 arrangements. Under the null hypothesis a mean difference of 3.00 or larger occurs with a frequency of $\frac{3}{20}$. It yields a significance level of 0.15. On the basis of the observed results of the experiment, arrangement 9, there is not significant evidence that μ_B is greater than μ_A.

	Unit:	1	2	3	4	5	6	
Arrangement	Response:	7	10	9	5	10	12	$\bar{y}_B - \bar{y}_A$
1		A	A	A	B	B	B	0.33
2		A	A	B	A	B	B	3.00
3		A	A	B	B	A	B	-0.33
4		A	A	B	B	B	A	-1.67
5		A	B	A	A	B	B	3.67
6		A	B	A	B	A	B	0.33
7		A	B	A	B	B	A	-1.00
8		A	B	B	A	B	A	1.67
9		A	B	B	A	A	B	3.00
10		A	B	B	B	A	A	-1.67
11		B	B	B	A	A	A	-0.33
12		B	B	A	B	A	A	-3.00
13		B	B	A	A	B	A	0.33
14		B	B	A	A	A	B	1.67
15		B	A	B	B	A	A	-3.67
16		B	A	B	A	B	A	-0.33
17		B	A	B	A	A	B	1.00
18		B	A	A	B	A	B	-1.67
19		B	A	A	B	B	A	-3.00
20		B	A	A	A	B	B	1.67

5. (20 points)

a.

$$\binom{21}{7} \times \binom{14}{7} \times \binom{7}{7} = 399,072,960$$

b.

$$\binom{21}{6} \times \binom{15}{6} \times \binom{9}{9} = 271,591,320$$

6. (30 points)

1. Factors: F_1 : Brand F_2 : Thickness of Covers F_3 : Type of Cover Materials
2. Levels: F_1 : 4 Brands F_2 : 3 Thicknesses of Covers F_3 : 2 Types of Cover Materials
3. Treatments: The $(4) \times (3) \times (2) = 24$ combinations of the three factor levels
4. Response: Compressive strength of golf ball cover
5. EU's: A golf ball
6. MU's: A randomly selected spot on a golf ball
7. Replications: There are 3 replications with a given replication consisting of 3 golf balls of a given brand with a given Thickness and Type of Cover evaluated at a given testing facility
8. Subsampling: 5 measurements at random spots on a golf ball
9. Covariates: None in this experiment. However, if the testing would have been done outside at a golf course, temperature and humidity would be possible covariates.
10. Blocking: Two testing facilities
11. Confounding: None in this experiment.