The Mystifying Constants of Proportionality

The definition of the posterior distribution is

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{m(y)},$$

where

$$m(y) = \int_{\Theta} p(y|\tau)p(\tau)\,d\tau.$$

Suppose that

$$p(y|\theta) = h(y)f(y,\theta)$$

and

$$p(\theta) = Cg(\theta),$$

where *neither $h(y)$ nor C depends on θ.*

It follows that

$$\begin{aligned} m(y) &= \int_{\Theta} h(y)f(y,\tau)Cg(\tau)\,d\tau \\ &= Ch(y)\int_{\Theta} f(y,\tau)g(\tau)\,d\tau \\ &= Ch(y)m^*(y). \end{aligned}$$

Therefore

$$
\begin{aligned}
p(\theta|y) &= \frac{Ch(y)f(y,\theta)g(\theta)}{Ch(y)m^*(y)} \\
&= \frac{f(y,\theta)g(\theta)}{m^*(y)}.
\end{aligned}
$$

So, "ignoring" the constant term $Ch(y)$ and treating $f(y,\theta)g(\theta)$ as if it were $p(y|\theta)p(\theta)$ **leads to the correct posterior.**