Assigmnent 06 solution   Summer 2015

Problem 1. (35 points) Traffic Engineering Study:

a. Model : $y_{ijk\ell} = \mu+\alpha_i+\beta_j+I_{k(i)}+(\alpha\beta)_{ij}+(\beta I)_{jk(i)}+\gamma_\ell+(\alpha\gamma)_{i\ell}+(\beta\gamma)_{j\ell}+(\gamma I)_{\ell k(i)}+(\alpha\beta\gamma)_{ij\ell}+e_{ijk\ell},\ i = 1,2,3,\ j = 1,2,\ k = 1,2,\ \ell = 1,2$, where

- α_i is the fixed effect for signal type
- β_j is the fixed effect for level of traffic
- γ_ℓ is the fixed effect for method of measuring
- $(\alpha\gamma)_{i\ell}$ is the fixed interaction effect between signal type and method of measuring
- $(\beta\gamma)_{j\ell}$ is the fixed interaction effect between level of traffic and method of measuring
- $(\alpha\beta\gamma)_{ij\ell}$ is the fixed interaction effect between signal type, level of traffic and method of measuring
- $I_{k(i)}$ is the random effect due to intersection nested within signal type
- $(\beta I)_{jk(i)}$ is the random effect due to interaction between intersection nested within signal type and traffic level
- $(\gamma I)_{\ell k(i)}$ is the random effect due to interaction between intersection nested within signal type and method of measurement
- $I_{k(i)},\ (\beta I)_{jk(i)},\ (\gamma I)_{\ell k(i)}$ and $e_{ijk\ell}$ are independent
- $I_{k(i)} \sim iid\ N(0,\sigma^2_{I(S)}),\ (\beta I)_{jk(i)} \sim iid\ N(0,\sigma^2_{T*I(S)}),\ (\gamma I)_{\ell k(i)} \sim iid\ N(0,\sigma^2_{M*I(S)})$ and $e_{ijk\ell} \sim iid\ N(0,\sigma^2_e)$

b. AOV Table - From SAS output: S=Signal Type, T=Traffic level, I=Intersection, M=Method

Source	df	SS	MS	F	$Pr > F$	Significant at $\alpha = 0.05$?
S	2	3143.02	1517.51	2.30	0.2484	Not significant
T	1	236.88	236.88	7.37	0.0728	Not significant
I(S)	3	2053.05	684.35	11.4134*	0.0101*	Significant
S*T	2	275.77	137.89	4.2924	0.1318	Not significant
T*I(S)	3	96.37	32.12	7.7065	0.0638	Not significant
M	1	96.00	96.00	2.9995	0.1817	Not significant
S*M	2	51.43	25.72	0.8035	0.5255	Not significant
T*M	1	31.74	31.74	7.6146	0.0702	Not significant
M*I(S)	3	96.02	32.01	7.6781	0.0641	Not significant
S*T*M	2	11.97	5.99	1.4358	0.3652	Not significant
Error	3	12.51	4.17			
Total	23	6104.76				

$$* \begin{cases} F = \frac{MS_{I(S)}}{M},\ \text{where } M = MS_{T*I(S)} + MS_{M*I(S)} - MSE, \\ F \dot{\sim} F_{3,\upsilon},\ \text{where } \upsilon = \frac{M^2}{\frac{MS^2_{T*I(S)}}{df_{T*I(S)}} + \frac{MS^2_{M*I(S)}}{df_{T*I(S)}} + \frac{MSE^2}{df_E}}. \end{cases}$$

c. AOV ($a = 3,\ b = 2,\ c = 2,\ d = 2$)

Source	df	EMS
S	2	$\sigma_e^2 + 2\sigma^2_{MI(S)} + 2\sigma^2_{TI(S)} + 4\sigma^2_{I(S)} + 8Q_S$
T	1	$\sigma_e^2 + 2\sigma^2_{TI(S)} + 12Q_T$
I(S)	3	$\sigma_e^2 + 2\sigma^2_{MI(S)} + 2\sigma^2_{TI(S)} + 4\sigma^2_{I(S)}$
S*T	2	$\sigma_e^2 + 2\sigma^2_{TI(S)} + 4Q_{ST}$
T*I(S)	3	$\sigma_e^2 + 2\sigma^2_{TI(S)}$
M	1	$\sigma_e^2 + 2\sigma^2_{MI(S)} + 12Q_M$
S*M	2	$\sigma_e^2 + 2\sigma^2_{MI(S)} + 4Q_{SM}$
T*M	1	$\sigma_e^2 + 6Q_{TM}$
M*I(S)	3	$\sigma_e^2 + 2\sigma^2_{MI(S)}$
S*T*M	2	$\sigma_e^2 + 2Q_{STM}$
Error	3	σ_e^2
Total	23	

d. From AOV table above, we conclude there is significant evidence of an effect due to intersections nested within signal type but all other effects are not significant.

e. $\hat{\sigma}^2_{I(S)} = 156.10 - 83.0\%$; $\hat{\sigma}^2_{T*I(S)} = 13.98 - 7.4\%$; $\hat{\sigma}^2_{M*I(S)} = 13.92 - 7.4\%$; $\hat{\sigma}^2_e = 4.17 - 2.2\%$,

Problem 2: (41 points) Factor A has 4 randomly selected levels, Factor B has 5 fixed levels, Factor C has 3 randomly selected levels at each level of Factor B, and there are 6 EU's at each of the t=60 treatments:

a.

Source	DF	**MS**	**Expected Mean Squares**
A	3	24.5	$\sigma_e^2 + 6\sigma^2_{A*C(B)} + 18\sigma^2_{A*B} + 90\sigma^2_A$
B	4	19.7	$\sigma_e^2 + 6\sigma^2_{A*C(B)} + 18\sigma^2_{A*B} + 24\sigma^2_{C(B)} + 72Q_B$
$A \times B$	12	8.9	$\sigma_e^2 + 6\sigma^2_{A*C(B)} + 18\sigma^2_{A*B}$
$C(B)$	10	7.5	$\sigma_e^2 + 6\sigma^2_{A*C(B)} + 24\sigma^2_{C(B)}$
$A \times C(B)$	30	6.8	$\sigma_e^2 + 6\sigma^2_{A*C(B)}$
Error	300	5.8	σ_e^2

b. Test for a significant AB interaction ($\alpha = 0.05$). Note that the AOV table is providing the MS, not SS for each source of variation.

$H_o : \sigma^2_{AB} = 0$ vs $H_1 : \sigma^2_{AB} > 0$ $F = \frac{MS_{AB}}{MS_{A*C(B)}} = \frac{8.9}{6.8} = 1.309 < 2.09 = F_{.05,12,30}$ and $p-value = 1 - pf(1.309, 12, 30) = .2644 > .05 \Rightarrow$

There is not significant evidence that $\sigma_{AB} > 0$

c. Test for a significant B main effect ($\alpha = 0.05$): $H_o : Q_B = 0$ vs $H_1 : Q_B \neq 0$

Let $M = MS_{AB} + MS_{C(B)} - MS_{AC(B)} = 9.6$. When $Q_B = 0, E[M] = E[MS_B]$, thus the appropriate test statistic is

$F = \frac{MS_B}{M} = \frac{19.7}{9.6} = 2.052$ with $p-value = pf(2.052, 4, 6.6942) = .1952 > \alpha = .05 \Rightarrow$ There is not significant evidence that $Q_B \neq 0$, that is, there is not significant evidence of a difference in the 5 treatment means associated with the levels of Factor B.

The df for the F-test are obtained from the Satterthwaite approximation are obtained as follows:

$df_M = \frac{(9.6)^2}{\frac{(8.9)^2}{12}+\frac{(7.5)^2}{10}+\frac{(6.8)^2}{30}} = 6.6942$

d. Compute the variance of the difference in treatment means for levels 1 and 2 of Factor B:

$y_{ijkl} = \mu + a_i + \beta_j + c_{k(j)} + (a\beta)_{ij} + (ac)_{ik(j)} + e_{ijkl} \Rightarrow$

$\bar{y}_{.1..} = \mu + \bar{a}_. + \beta_1 + \bar{c}_{.(1)} + (\bar{a\beta})_{.1} + (\bar{ac})_{..(1)} + \bar{e}_{.1..}$

$\bar{y}_{.2..} = \mu + \bar{a}_. + \beta_2 + \bar{c}_{.(2)} + (\bar{a\beta})_{.2} + (\bar{ac})_{..(2)} + \bar{e}_{.2..} \Rightarrow$

$Var[\bar{y}_{.1..} - \bar{y}_{.2..}] = Var(\bar{c}_{.(1)} - \bar{c}_{.(2)}) + Var((\bar{a\beta})_{.1} - (\bar{a\beta})_{.2}) + Var((\bar{ac})_{..(1)} - (\bar{ac})_{..(2)}) + Var(\bar{e}_{.1..} - \bar{e}_{.2..})$

Therefore, we have

$$\begin{aligned} Var[\bar{y}_{.1..} - \bar{y}_{.2..}] &= \frac{2\sigma^2_{C(B)}}{3} + \frac{2\sigma^2_{AB}}{4} + \frac{2\sigma^2_{AC(B)}}{12} + \frac{2\sigma^2_e}{72} \\ &= 2\left[\frac{24\sigma^2_{C(B)} + 18\sigma^2_{AB} + 6\sigma^2_{AC(B)} + \sigma^2_e}{72}\right] \\ &= \frac{2[EMS_{AB} + EMS_{C(B)} - EMS_{AC(B)}]}{72} \end{aligned}$$

Provide an estimate of this variance and the degrees of freedom of the estimate.

$\widehat{Var}[\bar{y}_{.1..} - \bar{y}_{.2..}] = \frac{2[MS_{AB}+MS_{C(B)}-MS_{AC(B)}]}{72} = 0.2667.$

Using the Sattherwaite approximation: $df_M \approx \frac{(8.9+7.5-6.8)^2}{\frac{(8.9)^2}{12}+\frac{(7.5)^2}{10}+\frac{(6.8)^2}{30}} = 6.6942.$

e. Compute the value of Tukey's HSD with $\alpha = .05$ that would be used to determine which pairs of means across the levels of Factor B are different:

$HSD = q_{.05,5,\nu}\sqrt{\frac{M}{(4)(3)(6)}} = q_{(.05,5,6.6942)}\sqrt{\frac{9.6}{72}} = 5.1257\sqrt{\frac{9.6}{72}} = 1.87$

where $q_{(.05,5,6.6942)} = qtukey(.95, 5, 6.6942) = 5.1257$ using the r-function qtukey.

Problem 3. (24 points)

a. $\mathbf{C}_1 = \mu_{11} - \mu_{14} - \mu_{31} + \mu_{34} = (\mu_{11} - \mu_{14}) - (\mu_{31} - \mu_{34})$

   i. The contrast, C_1, consists of the difference between the first and third levels of F_1 of a contrast in the levels of F_2 and hence is an **interaction** contrast.
   ii. The contrast is estimable because all four μ_{ij}'s in the contrast have estimates from the data, $\hat{\mu}_{ij} = \bar{y}_{ij}$.

b. $\mathbf{C}_2 = \mu_{11} + \mu_{21} + \mu_{31} - \mu_{13} - \mu_{23} - \mu_{33} = (\mu_{11} + \mu_{21} + \mu_{31}) - (\mu_{13} + \mu_{23} + \mu_{33}) = \mu^*_{.1} - \mu^*_{.3}$

   i. The contrast is the difference in the first and third levels of F_2 "averaged" over the levels of F_1 (with mean associated with fourth level of F_1 missing from both averages) and hence the contrast is a **Main Effect** contrast for F_2.
   ii. The contrast is not estimable because μ_{13} is not estimable, $\bar{y}_{13}$ was not observed in the data.

c. $\mathbf{C}_5 = \mu_{11} - \mu_{14} + \mu_{21} - \mu_{24} + \mu_{31} - \mu_{34} = (\mu_{11} + \mu_{21} + \mu_{31}) - (\mu_{14} + \mu_{24} + \mu_{34}) = \mu^*_{.1} - \mu^*_{.4}$

   i. The contrast is the difference in the first and fourth levels of F_2 "averaged" over the levels of F_1 (with mean associated with fourth level of F_1 missing from both averages) and hence the contrast is a **Main Effect** contrast for F_2.

   ii. The contrast is not estimable because μ_{24} is not estimable, $\bar{y}_{24}$ was not observed in the data..

Problem 4:

a. i. $H_0 : C_1\boldsymbol{\mu} = \mathbf{0}$ vs $H_1 : C_1\boldsymbol{\mu} \neq \mathbf{0}$,

where $C_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 & -1 \end{pmatrix}$, $\boldsymbol{\mu} = (\mu_{11}, \mu_{12}, \mu_{21}, \mu_{23}, \mu_{31}, \mu_{32}, \mu_{33})^T$.

$\hat{\boldsymbol{\mu}} = (90.3, 90.65, 90.2, 90.0, 90.6, 90.85, 92.25)^T,\ D = diag\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)_{7\times 7},$

$$Q_1 = (C_1\hat{\boldsymbol{\mu}})^T(C_1DC_1^T)^{-1}(C_1\hat{\boldsymbol{\mu}}) = (-0.50, -2.65)\begin{pmatrix} 2 & 1/2 \\ 1/2 & 2 \end{pmatrix}^{-1}\begin{pmatrix} -0.50 \\ -2.65 \end{pmatrix} = 3.5253$$

Then, test statistic is

$$F = \frac{Q_1/df_1}{MSE} = \frac{3.5253/2}{0.0193} = 91.33,$$

where $df_1 = rank(C_1) = 2$. $p - value = Pr(F_{2,7} \geq 91.33) < 0.0001 < \alpha = 0.05$ and so reject H_0. Thus, we conclude that there are significant Temperature effects.

ii. $H_0 : C_2\boldsymbol{\mu} = \mathbf{0}$ vs $H_1 : C_2\boldsymbol{\mu} \neq \mathbf{0}$,

where $C_2 = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 & -1 \end{pmatrix}$.

$$Q_2 = (C_2\hat{\boldsymbol{\mu}})^T(C_2DC_2^T)^{-1}(C_2\hat{\boldsymbol{\mu}}) = (-0.6, -1.45)\begin{pmatrix} 2 & 1/2 \\ 1/2 & 2 \end{pmatrix}^{-1}\begin{pmatrix} -0.6 \\ -1.45 \end{pmatrix} = 1.0813$$

Then, test statistic is

$$F = \frac{Q_2/df_2}{MSE} = \frac{1.0813/2}{0.0193} = 28.03,$$

where $df_2 = rank(C_2) = 2$. $p - value = Pr(F_{2,7} \geq 28.03) = 0.0005 < \alpha = 0.05$ and so reject H_0. Thus, we conclude that there are significant Pressure effects.

- Alternatively, we could use the cell means model in SAS to obtain:

```
option ls=80 ps=50 nocenter nodate;
title 'Problem 4                         ';

data CHEM;
array X X1-X2;
INPUT T $ P $ X1-X2;
TRT=COMPRESS (T) || COMPRESS (P);
do over X;
Y=X;
output; end;
      drop  X1-X2;

cards;
150 LOW   90.4 90.2
150 MED   90.7 90.6
150 HGH   .    .
200 LOW   90.1 90.3
200 MED   .    .
200 HGH   89.9 90.1
250 LOW   90.5 90.7
250 MED   90.8 90.9
250 HGH   92.4 92.1
RUN;
PROC PRINT;
RUN;
proc glm DATA = CHEM ORDER=DATA;
class T P;
model Y = T|P/SS4;
RUN;

TITLE "ANALYSIS OF 7 TREATMENTS";
PROC GLM DATA=CHEM ORDER=DATA;
CLASS TRT;
```

```
MODEL Y=TRT/SS4;
contrast   'C1 AND C2'       TRT   1  1  0  0 -1 -1  0,
                             TRT   0  0  1  1 -1  0 -1;
contrast  'C3 AND C4'        TRT   1 -1  0  0  1 -1  0,
                             TRT   0  0  1 -1  1  0 -1;
RUN;

OUTPUT FROM SAS:

Source      DF   Sum of Squares   Mean Square    F Value   Pr > F
Model       6     6.67428571      1.11238095     57.68     <.0001
Error       7     0.13500000      0.01928571
CTotal     13     6.80928571

Source       DF    Type IV SS     Mean Square    F Value    Pr > F
TRT           6    6.67428571      1.11238095    57.68      <.0001

Contrast     DF   Contrast SS    Mean Square    F Value   Pr > F
C1 AND C2     2    3.52533333    1.76266667      91.40    <.0001
C3 AND C4     2    1.08133333    0.54066667      28.03    0.0005
```

b. i. $H_0 : (\mu_{11} + \mu_{12}) = (\mu_{31} + \mu_{32})$ and $(\mu_{21} + \mu_{23}) = (\mu_{31} + \mu_{33})$.
$\Leftrightarrow H_0 : \frac{1}{2}(\mu_{11} + \mu_{12}) = \frac{1}{2}(\mu_{31} + \mu_{32})$ and $\frac{1}{2}(\mu_{21} + \mu_{23}) = \frac{1}{2}(\mu_{31} + \mu_{33})$.
$\Leftrightarrow H_0 : \bar{\mu}^*_{1.} = \bar{\mu}^*_{3.}, \bar{\mu}^*_{2.} = \bar{\mu}^{**}_{3.}$ vs H_1 : Not all equal.
$\Leftrightarrow$ Tests for Temperature "Main Effect".

ii. $H_0 : (\mu_{11} + \mu_{31}) = (\mu_{12} + \mu_{32})$ and $(\mu_{21} + \mu_{31}) = (\mu_{23} + \mu_{33})$.
$\Leftrightarrow H_0 : \frac{1}{2}(\mu_{11} + \mu_{31}) = \frac{1}{2}(\mu_{12} + \mu_{32})$ and $\frac{1}{2}(\mu_{21} + \mu_{31}) = \frac{1}{2}(\mu_{23} + \mu_{33})$.
$\Leftrightarrow H_0 : \bar{\mu}^*_{.1} = \bar{\mu}^*_{.2}, \bar{\mu}^{**}_{.1} = \bar{\mu}^*_{.3}$ vs H_1 : Not all equal.
$\Leftrightarrow H_0$: No main effect due to Pressure. vs H_1 : There is Main effect due to Pressure.
$\Leftrightarrow$ Tests for Pressure "Main Effect".

c. SAS Output from Effects Model:

```
Source                      DF      Type IV SS     Mean Square    F Value    Pr > F
T                           2*      3.52533333      1.76266667      91.40    <.0001
P                           2*      2.28142857      1.14071429      59.15    <.0001
T*P                         2       1.88000000      0.94000000      48.74    <.0001

* NOTE: Other Type IV Testable Hypotheses exist which may yield different SS.
```

Notice that the Type IV SS for Temperature is the same as the value we obtained in Part (a.) but is different from the value obtained for Pressure in Part (b.). There are many possible pairs of contrasts which can be taken to be like a Pressure main effect. SAS selected a different pair of contrasts than the pair selected for this assignment.

Problem 5:

a. Two contrasts which evaluate the Main Effect of F_1, contrasts in levels of F_1 averaged over levels of F_2:

Contrast	μ_{11}	μ_{13}	μ_{14}	μ_{22}	μ_{23}	μ_{24}	μ_{31}	μ_{33}	μ_{34}
C_1	1	1	1	0	0	0	-1	-1	-1
C_2	0	1	1	0	-2	-2	0	1	1

$C_1 = (\mu_{11}+\mu_{13}+\mu_{14})-(\mu_{31}+\mu_{33}+\mu_{34}) = \mu^*_{1.} - \mu^*_{3.}$ is comparing the 1 and 3 levels of factor F_1 averaged over the levels of factor F_2 but with the second level of factor F_2, μ_{12} and μ_{32} missing from the averages.

$C_2 = (\mu_{13}+\mu_{14}) - 2(\mu_{23}+\mu_{24}) + (\mu_{33}+\mu_{34}) = \mu^*_{1.} - 2\mu^*_{2.} + \mu^*_{3.}$ is a contrast in the means of the three levels of F_1 but with the 1 and 2 levels of F_2 missing from the averages.

The two contrasts C_1 and C_2 are orthogonal:

$(1)(0)+(1)(1)+(1)(1)+(0)(0)+(0)(-2)+(0)(-2)+(-1)(0)+(-1)(1)+(-1)(1)=0$

b. Two contrasts which evaluate the Interaction between F_1 and F_2, $F_1 \times F_2$, first comparing contrasts in levels of F_2 at two levels of F_1 and then comparing contrasts in levels of F_1 at two levels of F_2:

Contrast	μ_{11}	μ_{13}	μ_{14}	μ_{22}	μ_{23}	μ_{24}	μ_{31}	μ_{33}	μ_{34}
C_3	1	1	-2	0	0	0	-1	-1	2
C_4	0	1	-1	0	-2	2	0	1	-1

$C_3 = (\mu_{11}+\mu_{13}-2\mu_{14}) - (\mu_{31}+\mu_{33}-2\mu_{34})$ is comparing a contrast with coefficients (1,1,-2) in the 1, 3 and 4 levels of factor F_2 at the 1 and 3 levels of factor F_1

$C_4 = (\mu_{13}-2\mu_{23}+\mu_{33}) - (\mu_{14}-2\mu_{24}+\mu_{34})$ is comparing a contrast (1,-2,1) in the 1, 2, and 3 levels of factor F_1 at the 3 and 4 levels of factor F_2

The two contrasts C_3 and C_4 are orthogonal:
$(1)(0)+(1)(1)+(-2)(-1)+(0)(0)+(0)(-2)+(0)(2)+(-1)(0)+(-1)(1)+(2)(-1)=0$

Problem 6.

a. A-Runs, B-Patients have random effects; model $y_{ijk} = \mu + a_i + b_j + (ab)_{ij} + e_{ijk}$; $a=4, b=5, r=2$:

	4	5		2	
SV	σ^2_A	σ^2_B	σ^2_{AB}	σ^2_e	EMS
A	10	0	2	1	$\sigma^2_e + 2\sigma^2_{AB} + 10\sigma^2_A$
B	0	8	2	1	$\sigma^2_e + 2\sigma^2_{AB} + 8\sigma^2_B$
AB	0	0	2	1	$\sigma^2_e + 2\sigma^2_{AB}$
e(A,B)	0	0	0	1	σ^2_e

b. A-Summer has random effect and B-Water treatment is fixed; model $y_{ijk} = \mu + a_i + \beta_j + (a\beta)_{ij} + e_{ijk}$; $a=2, b=4, r=2$:

	2	4		2	
SV	σ^2_A	Q_B	σ^2_{AB}	σ^2_e	EMS
A	8	0	2	1	$\sigma^2_e + 2\sigma^2_{AB} + 8\sigma^2_A$
B	0	4	2	1	$\sigma^2_e + 2\sigma^2_{AB} + 4Q_B$
AB	0	0	2	1	$\sigma^2_e + 2\sigma^2_{AB}$
e(A,B)	0	0	0	1	σ^2_e

c. A is random, B is random and nested within A; C is random and nested within B; D is random and nested within C; $a = 4, b = 3, c = 2, d = 3$:

Model : $y_{ijk\ell} = \mu + a_i + b_{j(i)} + c_{k(i,j)} + d_{\ell(ijk)},\ i = 1,2,3,4\ j = 1,2,3\ k = 1,2,\ \ell = 1,2,3,$

SV	σ^2_A	$\sigma^2_{B(A)}$	$\sigma^2_{C(AB)}$	σ^2_e	EMS
A	18	6	3	1	$\sigma^2_d + 3\sigma^2_{C(AB)} + 6\sigma^2_{B(A)} + 18\sigma^2_A$
B(A)	0	6	3	1	$\sigma^2_d + 3\sigma^2_{C(AB)} + 6\sigma^2_{B(A)}$
C(AB)	0	0	3	1	$\sigma^2_d + 3\sigma^2_{C(AB)}$
D(A,B,C)	0	0	0	1	σ^2_d

d. A,B,D are fixed, C is random nested within A and B and A,B, and D are crossed:

The model is given by

$$y_{ijkl} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + c_{k(i,j)} + \delta_l + (\alpha\delta)_{il} + (\beta\delta)_{jl} + (\alpha\beta\delta)_{ijl} + (cd)_{lk(i,j)} + e_{m(i,j,k,l)}$$

$$\text{with}\ \ i = 1,2,3;\ \ j = 1,2;\ \ k = 1,2,3,4,5,6;\ \ l = 1,2,3,4,5;\ \ m = 1,2,3,4,5,6$$

SV	Q_A	Q_B	Q_{A*B}	$\sigma^2_{C(A,B)}$	Q_D	Q_{A*D}	Q_{B*D}	Q_{A*B*D}	$\sigma^2_{D*C(A,B)}$	σ^2_e
A	360	0	0	30	0	0	0	0	6	1
B	0	540	0	30	0	0	0	0	6	1
AB	0	0	180	30	0	0	0	0	6	1
C(AB)	0	0	0	30	0	0	0	0	6	1
D	0	0	0	0	216	0	0	0	6	1
AD	0	0	0	0	0	72	0	0	6	1
BD	0	0	0	0	0	0	108	0	6	1
ABD	0	0	0	0	0	0	0	36	6	1
CD(AB)	0	0	0	0	0	0	0	0	6	1
e(A,B,C,D)	0	0	0	0	0	0	0	0	0	1

$$\begin{aligned}
E(MS_A) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} + 30\sigma^2_{C(A,B)} + 360Q_A \\
E(MS_B) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} + 30\sigma^2_{C(A,B)} + 540Q_B \\
E(MS_{A*B}) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} + 30\sigma^2_{C(A,B)} + 180Q_{A*B} \\
E(MS_{C(A,B)}) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} + 30\sigma^2_{C(A,B)} \\
E(MS_D) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} + 216Q_D \\
E(MS_{A*D}) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} + 72Q_{A*D} \\
E(MS_{B*D}) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} + 108Q_{B*D} \\
E(MS_{A*B*D}) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} + 36Q_{A*B*D} \\
E(MS_{C*D(A,B)}) &= \sigma^2_e + 6\sigma^2_{D*C(A,B)} \\
E(MSE) &= \sigma^2_e
\end{aligned}$$