STAT 630 Fall 2014
Homework 1 Solution

1.2.3

$P(\{2\}) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$
$P(\{3\}) = 1 - \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

1.2.6

$a : A \cap B^c \cap C^c$; $b : A \cap B \cap C^c$;$c : A^c \cap B \cap C^c$;
$d : A \cap B^c \cap C$;$e : A \cap B \cap C$;$f : A^c \cap B \cap C$;$g : A^c \cap B^c \cap C$;

1.2.12

We have $P(\{1\}) - \frac{1}{8} = P(\{2\}) = 3P(\{3\}) = 4P(\{4\})$. Hence, $1 = P(S) = P(\{1\}) + P(\{2\}) + P(\{3\}) + P(\{4\}) = \frac{31}{12}P(\{2\}) + \frac{1}{8}$. This gives us $P(\{2\}) = \frac{21}{62}, P(\{1\}) = P(\{2\}) + \frac{1}{8} = \frac{115}{248}, P(\{3\}) = \frac{1}{3}P(\{2\}) = \frac{7}{62}$, and $P(\{4\}) = \frac{1}{4}P(\{2\}) = \frac{21}{248}$.

1.3.2

Let A be the event "Al watches the six o'clock news" and B be the event "Al watches the eleven o'clock news". Then $P(A) = \frac{2}{3}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{3}$. Therefore, the probability that Al only watches the six o'clock news is $P(A \backslash (A \cap B)) = P(A) - P(A \cap B) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$. The probability that Al watches neither news is given by $P((A \cup B)^c) = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) = 1 - \frac{2}{3} - \frac{1}{2} + \frac{1}{3} = \frac{1}{6}$.

1.3.8

A student is chosen at random. The probability of being a female is 55%, the probability of having long hair is 44%+15%=59%, and the probability that the student is a long haired female is 44%. By Theorem 1.3.3, the probability of either being female or having long hair is 55%+59%-44%=70%.

1.3.10a

$$
\begin{aligned}
P(A \cup B \cup C) &= P(A \cup B) + P(C) - P((A \cup B) \cap C) \\
&= P(A \cup B) + P(C) - P((A \cap C) \cup (B \cap C)) \\
&= P(A) + P(B) - P(A \cap B) + P(C) - P((A \cap C) \cup (B \cap C)) \\
&= P(A) + P(B) - P(A \cap B) + P(C) - (P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C))) \\
&= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)
\end{aligned}
$$

1.4.1

(a) For each roll, the probability to get a six is $\frac{1}{6}$, because the probability should be equal for showing a certain side for a dice. Also, all the eight rolls are independent, thus the probability for the event that all eight dice show a six is $(\frac{1}{6})^8 = \frac{1}{1679616}$.

(b) $P(\text{all eight roll show the same}) = \sum_{i=1}^{6} P(\text{all eight show the number i}) = 6*(\frac{1}{6})^8 = \frac{1}{279936}$.

(c) First for each roll, the smallest number is 1. Thus we need 7 dice showing 1 and one 1 dice showing 2 to make the sum of the eight dice equal to 9. There are 8 ways to let it happen(The number of ways to choose one out of eight dice with number 2 is 8) and for each way the probability is $(\frac{1}{6})^8$. Therefore, the probability that the sum of the eight dice is equal to 9 is $8 * (\frac{1}{6})^8 = \frac{1}{209952}$.

1.4.4

(a) There is only one way this can happen, so the probability is $\frac{1}{\binom{52}{5}} = \frac{1}{2598960}$.

(b) There are $\binom{13}{5}$ ways this can happen, so the probability is $\frac{\binom{13}{5}}{\binom{52}{5}} = \frac{33}{66640}$.

(c) There are totally 13 different numbers. We pick 5 from them and then assign each of them with one of the suits(i.e. spade, heart, diamond and club). So the number of ways this can happen is $\binom{13}{5} \times 4^5 = 1317888$ and the corresponding frobability is $\frac{1317888}{\binom{52}{5}}$.

(d) First we pick 3 cards of a kind, that is pick a number from 1-13 and pick three of four suits. Then we pick a number from the left 12 numbers and make it a pair (pick two of four suits for this number). So the number of ways this can happen is $13 \times \binom{4}{3} \times 12 \times \binom{4}{2} = 3744$ and the corresponding probability is $\frac{3744}{\binom{52}{5}} = \frac{6}{4165}$.

1.4.11

There are two sub-events for this event: all six balls are red or all six balls are blue. $P(\text{all red}) = \frac{\binom{5}{3}\binom{6}{3}}{\binom{12}{3}\binom{18}{3}}$, $P(\text{all blue}) = \frac{\binom{7}{3}\binom{12}{3}}{\binom{12}{3}\binom{18}{3}}$ Then the probability for all six balls having same color is the sum of the above two probabilities, which is 0.044.

1.4.12

The possible values for the total number of heads are $\{0, 1, 2, 3\}$ and the possible values for tolling a dice are $\{1, 2, 3, 4, 5, 6\}$. Thus to let these two numbers equal, the possible total numbers are $\{1, 2, 3\}$. Since the tolling a dice and flipping a coin are independent, thus the probability that the total number of heads is equal to the number showing on the die is:

$$P(one\ head\ and\ die = 1) + P(two\ heads\ and\ die = 2) + P(three\ heads\ and\ die = 3)$$
$$= \frac{1}{6}\left(3\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3\right)$$

which is $\frac{7}{48} = 0.1458$.

1.5.3

(a) $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

(b) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{8}/\frac{1}{2} = \frac{1}{4}$

(c) $P(A|B) = \frac{P(A \cap B)}{P(B)} = 0/\frac{1}{2} = 0$

1.5.7

(a) This question also tests on the apply of conditional probability. First, from the information in this question, we can know $P(fast\ ball) = 0.8$ and $P(curve\ ball) = 0.2$. In addition, $P(hit|fast\ ball) = 0.08$ and $P(hit|curve\ ball) = 0.05$. Therefore, $P(hit) = P(hit|fast\ ball)P(fast\ ball) + P(hit|curve\ ball)P(curve\ ball) = 0.8 * 0.08 + 0.2 * 0.05 = 0.074$

(b) $P(curve\ ball|hit) = \frac{P(hit \cap curve\ ball)}{P(hit)} = \frac{P(hit|curve\ ball)P(curve\ ball)}{P(hit)} = \frac{0.05*0.2}{0.074} = 0.135$

(c) $P(curve\ ball|not\ hit) = \frac{P(not\ hit \cap curve\ ball)}{P(not\ hit)} = \frac{P(not\ hit|curve\ ball)P(curve\ ball)}{1-P(hit)} = \frac{0.95*0.2}{1-0.074} = 0.205$

1.5.10

From the previous question 1.4.11, we know $P(all\ same\ color) = 0.044$, $P(all\ red) = 0.0011$. Therefore $P(all\ red|all\ same\ color) = \frac{P(all\ red)}{P(all\ same\ color)} = 0.02532$.

Problem A

(a) S={ (R1,G1),(R1,G2),(R1,G3),(R1,G4),(R1,G5),(R1,G6), (R2,G1),(R2,G2),(R2,G3),(R2,G4), (R2,G5),(R2,G6),(R3,G1),(R3,G2),(R3,G3), (R3,G4),(R3,G5),(R3,G6),(R4,G1),(R4,G2), (R4,G3),(R4,G4),(R4,G5),(R4,G6),(R5,G1), (R5,G2),(R5,G3),(R5,G4),(R5,G5),(R5,G6), (R6,G1),(R6,G2),(R6,G3),(R6,G4),(R6,G5),(R6,G6) }

(b) A={(R3,G6),(R4,G5),(R4,G6),(R5,G4),(R5,G5),(R5,G6),(R6,G3),(R6,G4),(R6,G5),(R6,G6)}
B={(R2,G1),(R3,G1),(R4,G1),(R5,G1),(R6,G1),(R3,G2),(R4,G2),(R5,G2),(R6,G2),(R4,G3), (R5,G3),(R6,G3),(R5,G4),(R6,G4),(R6,G5)}
C={(R1,G4),(R2,G4),(R3,G4),(R4,G4),(R5,G4),(R6,G4)}

(c) $A \cap C = \{(R5, G4), (R6, G4)\}$
$B\cup$ C={(R1,G4),(R2,G1),(R2,G4),(R3,G1),(R3,G2),(R3,G4),(R4,G1),(R4,G2),(R4,G3), (R4,G4),(R5,G1),(R5,G2),(R5,G3),(R5,G4),(R6,G1),(R6,G2),(R6,G3),(R6,G4),(R6,G5)}
$A \cap (B \cup C) = \{(R5, G4), (R6, G3), (R6, G4), (R6, G5)\}$

(d) $P(A \cap C) = \frac{2}{36}$; $P(B \cup C) = \frac{19}{36}$; $P(A \cap (B \cup C)) = \frac{4}{36}$

(e) $P(A \cap C) = \frac{2}{36} \neq P(A)P(C) = \frac{10}{36}\frac{6}{36}$

(f) $P(sum = 7) = \frac{6}{36} = \frac{1}{6}$

Problem B

(a) R code:

```
#B is the number of replications
B=10000
#count record the number of replications when two students have same ID
count=0
for(i in 1:B)
{
ID=sample(0:9999,100,replace=TRUE)
#unique function find the number of distinct elements in ID
uniquenum=length(unique(ID))
if(uniquenum!=100)
count=count+1
}
prob=count/B
```

Simulation result: 0.3861

(b) The true probability is $1 - \frac{10{,}000!/9{,}900!}{10{,}000^{100}} = 0.3914$. The answer is obtained through the following R code:

```
1-exp(lfactorial(10000)-lfactorial(9900)-400*log(10))
```

The reason that we use "lfactorial" is that the computer can't compute such a big number as 10,000!.

(c) You can increase the number of n and see the simulation result whether it is greater than 0.5. The answer is 119.