STATISTICS 630 - Final Exam
July 26, 2013

Name ____________________________________ Email Address ____________________________

INSTRUCTIONS FOR STUDENTS:

(1) There are 8 pages including this cover page and 5 formula sheets. Each of the six numbered problems is weighted equally.

(2) You have exactly 120 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 120 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature__________________________________

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: ______________________

(2) Record the time at which the student ends the exam: ______________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until August 13, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature__________________________________

1. Let the independent random variables X and Y both be unbiased measurements of a quantity μ; that is, $E(X) = E(Y) = \mu$. Suppose we combine the two measurements using the weighted average
$$T_\alpha = \alpha X + (1-\alpha)Y,$$
where $0 \le \alpha \le 1$. Suppose that the variances of X and Y are $\sigma_X^2 = 2$ and $\sigma_Y^2 = 1$, respectively. First show that T_α is unbiased estimator of μ. Then find the mean squared error of T_α as an estimator of μ and the value of α that minimizes $MSE(T_\alpha)$.

2. Let $X_1, \ldots, X_n$ be a random sample from the Weibull distribution with density
$$f(x|\theta) = 3\theta x^2 e^{-\theta x^3}, \qquad x > 0, \quad 0 < \theta < \infty.$$
Obtain the maximum likelihood estimator of θ and Fisher's information for θ. Use these to construct an approximate level γ confidence interval for θ.

3. Let $X_1, \ldots, X_{16}$ be a random sample from a normal distribution with unknown mean μ and known variance $\sigma^2 = 4$. It is of interest to test the hypotheses
$$H_0 : \mu = 10 \quad \text{vs.} \quad H_a : \mu > 10$$
at level of significance α. Define $\bar{X} = \sum_{i=1}^n X_i/n$. Find the critical value c_α for a level α test of the form:
$$\text{“Reject } H_0 \text{ if } \bar{X} \ge c_\alpha.\text{”}$$
Then obtain an expression in terms of Φ (the standard normal cdf) for the power curve associated with your test. (You will get full credit for a correct expression in terms of Φ and $Z_{1-\alpha}$.)

4. Suppose that (X, Y) have the joint probability density function
$$f_{X,Y}(x,y) = \begin{cases} x + y & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise,} \end{cases}$$
and marginal probability density functions
$$f_X(x) = \begin{cases} \frac{1}{2} + x & 0 \le x \le 1, \\ 0 & \text{otherwise,} \end{cases} \qquad f_Y(y) = \begin{cases} \frac{1}{2} + y & 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$
Find $E[X|Y=y]$ and $\text{Var}[X|Y=y]$.

5. Let $X_1, \ldots, X_n$ be a random sample from the geometric distribution with probability mass function
$$p_\theta(x) = \theta(1-\theta)^x, x = 0, 1, 2, \ldots, \quad 0 < \theta < 1.$$
Suppose that θ has the prior density
$$\pi(\theta) = 6\theta(1-\theta), \quad 0 < \theta < 1.$$
Obtain the posterior distribution of θ given $X = x$. Obtain the mean of the posterior distribution and compare this to the mean of the prior distribution.

6. Let $X \sim N(2, 4)$ and $Y \sim N(-3, 5)$ be independent normal random variables. (Note: The notation $N(a, b)$ indicates a normal distribution with mean a and variance b.)

   (a) Let $U = 2X + 3Y - 1$ and $V = X - CY$ where C is a constant. Identify the distributions of U and V.

   (b) For U and V defined in part (a), what is the value of C that makes U and V independent?

   (c) Let $W = C_1(X + C_2)^2 + C_3(Y + C_4)^2$. Find values of $C_1,\ C_2,\ C_3$, C_4, and C_5 (with $C_1 \neq 0$ and $C_3 \neq 0$) so that W has a chi-squared distribution with C_5 degrees of freedom.