STATISTICS 630 - Final Exam

December 10, 2013

Name ____________________________________ Email Address ____________________________

INSTRUCTIONS FOR STUDENTS:

(1) There are 8 pages including this cover page and 5 formula sheets. Each of the six numbered problems is weighted equally.

(2) You have exactly 120 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 120 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature__________________________________

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: ______________________

(2) Record the time at which the student ends the exam: ______________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until December 17, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature__________________________________

1. Let X and Y be jointly distributed random variables with means $\mu_X = 0$ and $\mu_Y = -2$, variances $\sigma_X^2 = 2$ and $\sigma_Y^2 = 4$, and covariance $\mathrm{Cov}(X, Y) = -1$. Let $U = X - Y - 3$ and $V = 2X + 3Y + 5$. Find $E(U)$, $\mathrm{Var}(U)$, $E(V)$, $\mathrm{Var}(V)$ and $\mathrm{Cov}(U, V)$.

2. Let $X_1, \ldots, X_n$ be a random sample from the normal$(0, \sigma_1^2)$ distribution for $\sigma_1 > 0$ and $Y_1, \ldots, Y_m$ be a random sample from the normal$(0, \sigma_2^2)$ distribution for $\sigma_2 > 0$. Assume that $X_1, \ldots, X_n, Y_1, \ldots, Y_m$ are mutually independent.

   (a) Let
   $$W = \frac{\sum_{i=1}^n X_i^2/(n\sigma_1^2)}{\sum_{j=1}^m Y_j^2/(m\sigma_2^2)}.$$
   Explain why W has an $F(n, m)$ distribution.

   (b) Using W as a pivot, derive a level γ confidence interval for σ_1^2/σ_2^2.

3. Suppose that T_1 and T_2 are independent random variables such that $E(T_1) = \theta$, $E(T_2) = 2\theta$, $\mathrm{Var}(T_1) = 2\theta^2$ and $\mathrm{Var}(T_2) = 4\theta^2$. Consider the following estimators of θ:
   $$\hat{\theta}_1 = \frac{T_1 + T_2}{3} \qquad \text{and} \qquad \hat{\theta}_2 = \frac{T_1 + T_2}{4}.$$
   Find the bias, variance, and mean squared error of each of these estimators. Then determine which estimator is preferable.

4. Let $X_1, \ldots, X_n$ be a random sample from the normal$(0, 1/\theta)$ distribution for $\theta > 0$ with probability density function
   $$f_\theta(x) = \frac{\theta^{1/2}}{\sqrt{2\pi}} e^{-\theta x^2/2}, \quad -\infty < x < \infty.$$
   Suppose that θ has the prior density
   $$\pi(\theta) = \begin{cases} 4\theta^2 e^{-2\theta}, & \theta > 0 \\ 0 & \text{otherwise.} \end{cases}$$
   Obtain the posterior distribution of θ given $X_1 = x_1, \ldots, X_n = x_n$. Then obtain the mean and variance of the posterior distribution.

5. A statistics professor enjoys playing tennis and needs to practice his serves. Suppose that he attempts three serves and the number of good serves is a random variable Y with moment generating function

$$M_Y(s) = \frac{1}{27} + \frac{6}{27}e^s + \frac{12}{27}e^{2s} + \frac{8}{27}e^{3s}.$$

(a) Use the moment generating function to show that

$$E(Y) = 2 \quad \text{and} \quad \mathrm{Var}(Y) = \frac{2}{3}.$$

(b) Suppose $Z_n = Y_1 + \cdots + Y_n$ where $Y_1, ..., Y_n$ are independent random variables with the above mean and variance. Find a number m (with proof) such that $\frac{1}{n} Z_n \xrightarrow{P} m$.

6. Suppose that $X_1, \ldots, X_n$ are a random sample from a distribution with probability density function

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1 \\ 0 & \text{otherwise}, \end{cases}$$

(a) Obtain the maximum likelihood estimator and Fisher's information for θ.

(b) Write out expressions for the Wald statistic and the score statistic for testing $H_0 : \theta = \theta_0$ versus $H_0 : \theta \neq \theta_0$.

.