STATISTICS 630 - Final Exam

December 11, 2012

Name ____________________________________ Email Address ____________________________

INSTRUCTIONS FOR STUDENTS:

(1) There are 8 pages including this cover page and 5 formula sheets. Each of the six numbered problems is weighted equally.

(2) You have exactly 120 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 120 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature__________________________________

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: ______________________

(2) Record the time at which the student ends the exam: ______________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until December 18, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature__________________________________

1. Suppose that X and Y are independent random variables with respective moment generating functions

$$M_X(s) = \exp(2s + 8s^2) \quad \text{and} \quad M_Y(s) = \exp(-s + s^2).$$

   (a) Obtain the moment generating functions of (i) $W = X + Y$ and (ii) $V = 5X$.

   (b) Obtain $E(W)$ and $E(V)$.

2. Let $X_1, \ldots, X_n$ be a random sample from the Weibull distribution with density

$$f(x|\theta) = 3\theta x^2 e^{-\theta x^3}, \qquad x > 0, \quad 0 < \theta < \infty.$$

   Suppose that θ has the prior density

$$\pi(\theta) = \theta e^{-\theta}, \quad \theta > 0.$$

   Obtain the posterior distribution of θ given $X_1 = x_1, \ldots, X_n = x_n$. Then obtain the mean of the posterior distribution.

3. The weekly CPU time used by an accounting firm has probability density function (measured in hours) given by

$$f(x) = \begin{cases} \frac{3}{64}x^2(4-x), & 0 < x < 4, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Obtain $E(X)$ and $\mathrm{Var}(X)$. You may use without proof the fact that $E(X^2) = 32/5$.

   (b) Suppose that the weekly CPU times are recorded for 50 weeks during the year. Assuming that the times are independent and all have the above distribution, obtain an expression in terms of $E(X)$ and $\mathrm{Var}(X)$ that approximates the probability that the total CPU time used during the 50 weeks exceeds 130 hours.

4. Let X be a single observation from one of the two following distributions:

x	0	1	2	3	4	5	6	7
$p_0(x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$p_1(x)$	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{5}{32}$	$\frac{8}{32}$	$\frac{8}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

   Compute the likelihood ratio for testing $H_0 : \theta = 0$ versus $H_a : \theta = 1$. Then obtain the most powerful test with size $\alpha = \frac{1}{4}$ and compute its power.

5. The reading on a voltage meter connected to a test circuit is uniformly distributed over the interval $[\theta, \theta + 1]$ where $\theta > 0$ is the true but unknown voltage of the circuit. Suppose that $Y_1, \ldots, Y_n$ is a random sample of such readings.

   (a) Obtain the method of moments estimator $\hat{\theta}$ of θ based on $E(Y_i)$.

   (b) Obtain the mean squared errors of the two estimators, $\hat{\theta}$ and the sample mean, $\bar{Y}$. Which is the better estimator?

6. Suppose that $X_1, \ldots, X_n$ are a random sample from a distribution with probability mass function

$$p_\theta(x) = \begin{cases} (x+1)\theta^2(1-\theta)^x, & x = 0, 1, 2, 3, \ldots, \ (0 \le \theta \le 1) \\ 0 & \text{otherwise,} \end{cases}$$

   and mean $E(X_i) = 2(1-\theta)/\theta$. In Test 2, you found that the maximum likelihood estimator is $\hat{\theta} = \frac{2n}{2n + \sum_{i=1}^n X_i}$.

   (a) Obtain Fisher's information for θ.

   (b) Use Fisher's information to construct an approximate level γ confidence interval for θ based on the maximum likelihood estimator.

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