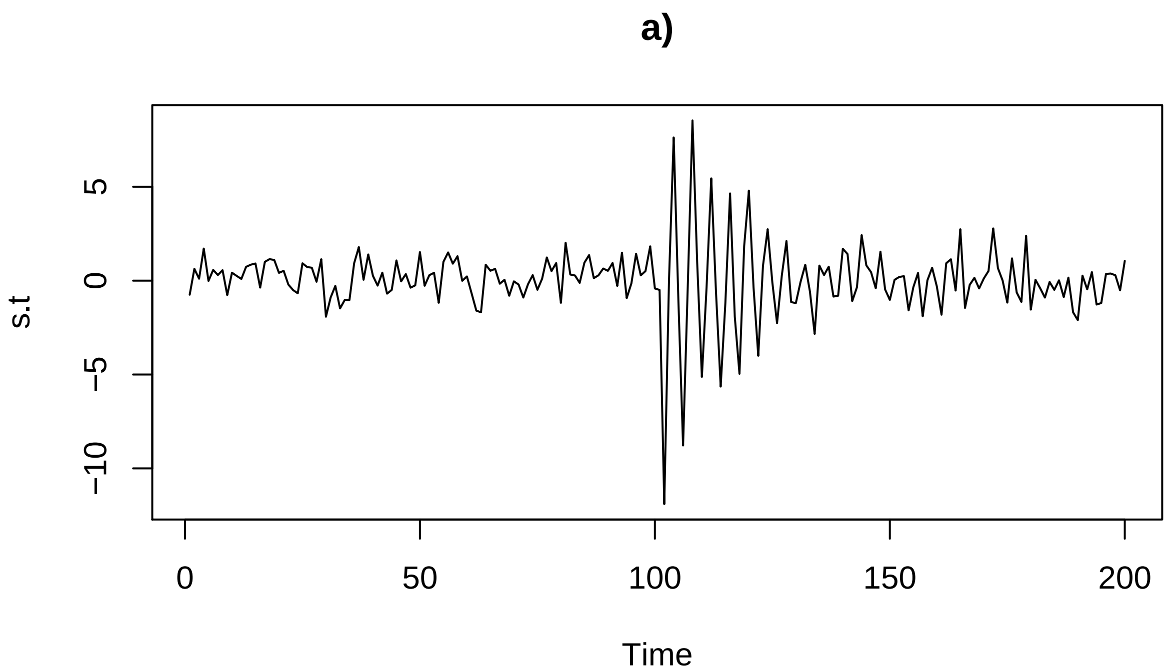
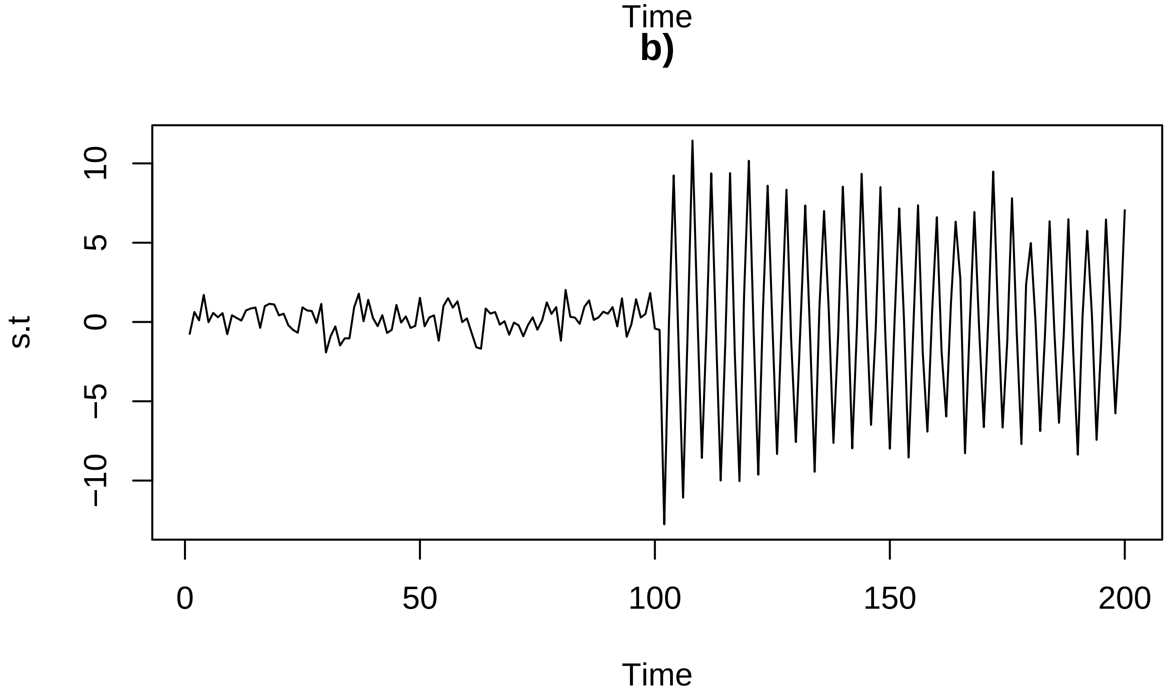
STAT626 HW01 BLUBAUGH

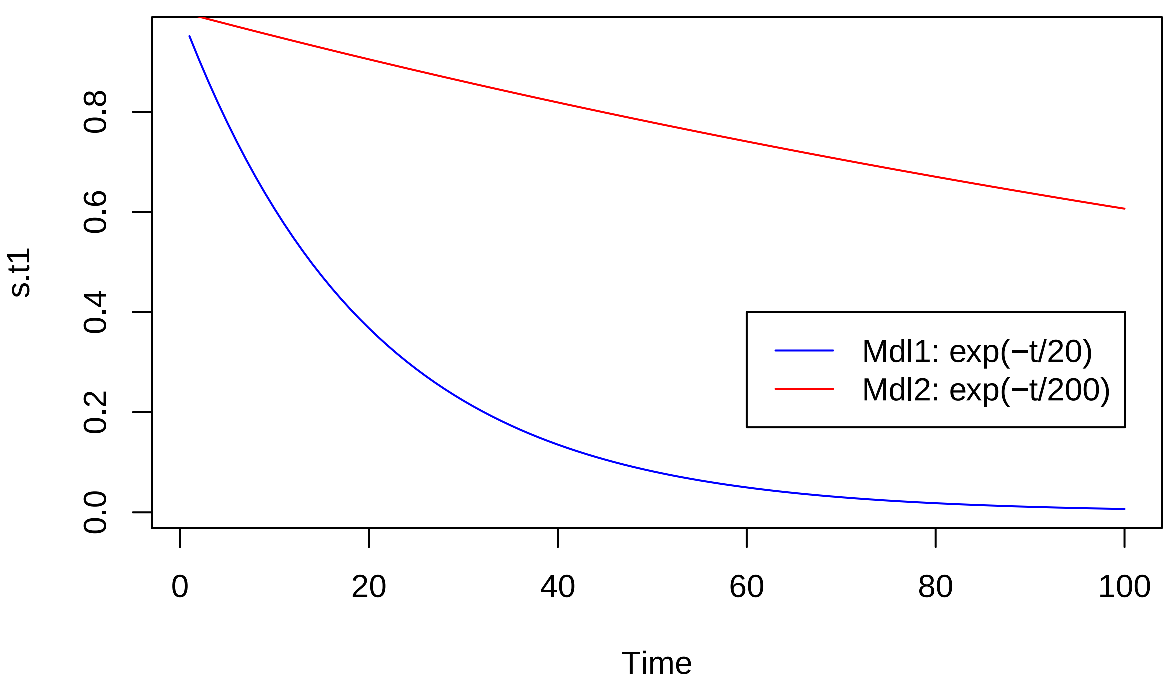
1.2





c) Both models are made up of white noise for the first 100 observations. The second model has a larger amplitude than the first model, but the first model has a much faster rate of decay than the second model and shown by looking at a plot of the signal modulators below.

c)



1.4

$$\begin{aligned}\gamma(s,t) =& E[(s-\mu_s)(t-\mu_t)]\\ =& E[st-\mu_t s-\mu_s t+\mu_s\mu_t]\\ =& E[st]-\mu_t E[s]-\mu_s E[t]+E[s]E[t]\\ =& E[st]-\mu_t\mu_s\end{aligned}$$

1.6

a) $x_t = \beta_1 + \beta_2 t + w_t$ is stationary because β_1 does not depend on t.

b) $y_t = x_t - x_{t-1} \rightarrow \gamma_x(t-1,t) \rightarrow cov(t-1-t, t-1-t) \rightarrow \sigma_x^2$

c)

$$\begin{aligned}v_t =& \frac{1}{2q+1}\sum_{i=-q}^{q} x_{t-q}\\ =& \frac{x_{t+q}}{2q+1}\\ =& \bar{x}_t\\ \text{ACF Function} =& \sigma_t^2\end{aligned}$$

1.7

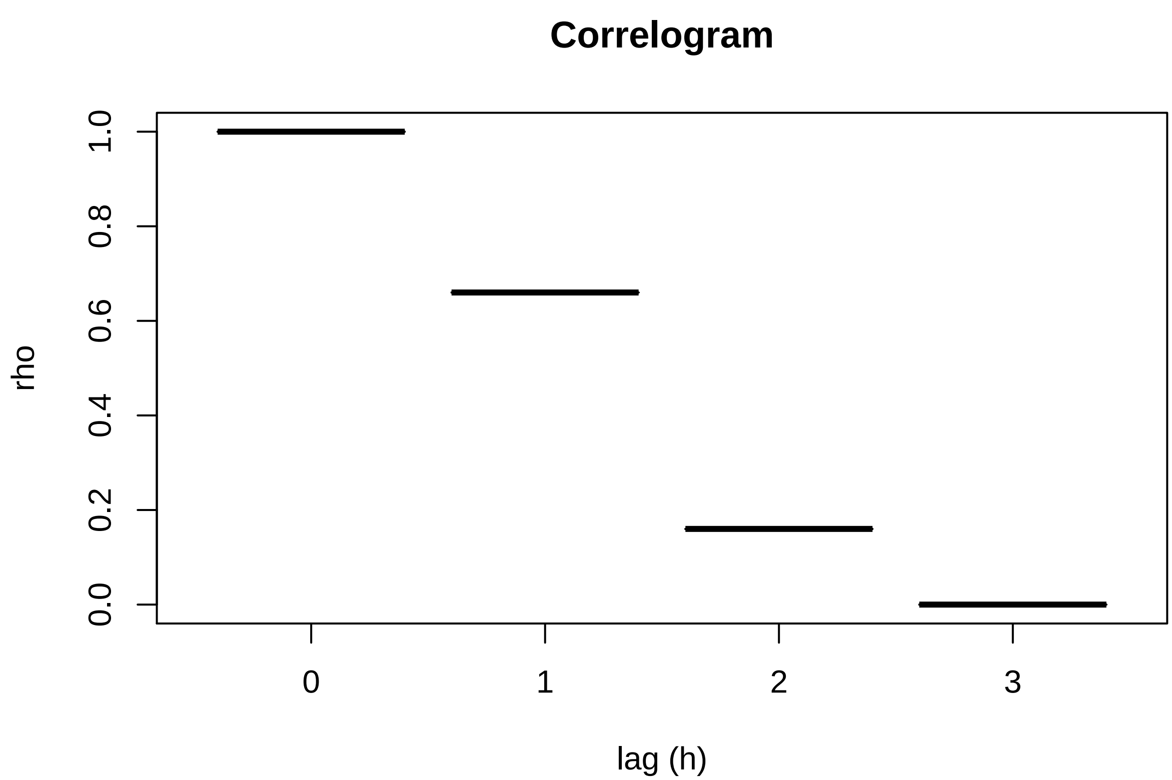
Autocovariance Function

$$\begin{aligned}
x_t =& w_{t-1} + 2w_t + w_{t+1} \\
\gamma_x(t,t) =& var(x_t) = (1 + 2^2 + 1^2)\sigma_w^2 \\
=& 6\sigma_w^2 \\
\gamma_x(s-t,t) =& cov(x_{s-t}, x_t) = cov(x_{s-t-1} + 2x_{s-t} + x_{s-t+1}, x_{t-1} + 2x_t + x_{t+1})
\end{aligned}$$

Autocorrelation Function

$$\begin{aligned}
h =& s - t \\
\rho(h,t) =& \gamma(h,t)/\sqrt{\gamma(h,h)\gamma(t,t)} = \gamma(h,t)/6\sigma_w^2
\end{aligned}$$

$$\begin{aligned}
\gamma_x(h=0,t) =& (1 + 2^2 + 1^2)\sigma_w^2 = 6\sigma_w^2 \\
\gamma_x(h=1,t) =& (2+2)\sigma_w^2 = 4\sigma_w^2 \\
\gamma_x(h=2,t) =& 1\sigma_w^2 \\
\gamma_x(h=3,t) =& 0
\end{aligned}$$



1

$$
\begin{aligned}
\sum_{i=1}^{n}(x_i - \bar{x}) = 0 \\
\sum_{i=1}^{n}x_i - \bar{x}n = 0 \\
\sum_{i=1}^{n}x_i = \bar{x}n \\
\sum_{i=1}^{n}x_i/n = \bar{x}
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) \\
\frac{d}{d\bar{x}} =& x_i y_i - \bar{x}y_i - x_i\bar{y} + \bar{x}n\bar{y} \\
=& \sum_{i=1}^{n}(x_i - \bar{x})y_i - \sum_{i=1}^{n}x_i\bar{y} + \bar{x}n\bar{y} \\
=& \sum_{i=1}^{n}(x_i - \bar{x})y_i
\end{aligned}
$$

2

$$
\begin{aligned}
S_x^2 =& SXX \\
c =& \frac{(x_i - \bar{x}}{SXX} \\
\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) =& SXY = \sum_{i=1}^{n}(x_i - \bar{x})y_i \\
\frac{SXY}{SXX} =& \frac{(x_i - \bar{x})y_i}{SXX}
\end{aligned}
$$

3

a)

$$
\begin{aligned}
R(\beta_0,\beta_1) &= \sum (y_i - (\beta_0 + \beta_1 x_i))^2 \\
\frac{dR(\beta_0,\beta_1)}{d\beta_0} &= -\sum 2(y_i - \beta_0 - \beta_x i) \\
0 &= \sum y_i - n\beta_0 - \beta_1 \sum x_i \\
n\beta_0 &= \sum y_i - \beta_1 \sum x_i \\
\beta_0 &= \bar{y} - \beta_1 \bar{x}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dR(\beta_0,\beta_1)}{d\beta_1} &= -\sum 2(y_i - \beta_0 - \beta_x i)x_i \\
0 &= \sum y_i x_i \beta_0 \sum x_i - \beta_1 \sum x_i^2 \\
0 &= \sum y_i x_i - (\bar{y} - \beta_1 \bar{x}) \sum x_i - \beta_1 \sum x_i^2 \\
0 &= y_i x_i - \frac{\sum y_i \sum x_i}{n} + \beta_1 \frac{(\sum x_i)^2}{n} - \beta_1 \sum x_i^2 \\
\beta_1 (\sum x_i^2 - \frac{(\sum x_i)^2}{n}) &= \sum y_i x_i - \frac{\sum y_i \sum x_i}{n} \\
\beta_1 &= \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2} \\
&= \frac{SXY}{SXX}
\end{aligned}
$$