STATISTICS 642 - Exam I Solution

I 30 points

1. Type of Randomization: RCBD with A Split-Plot Treatment Structure
2. Type of Treatment Structure: Alloy crossed with Roller Gap, 5×4 Factorial Treatment Structure
3. Identify each of the factors as being Fixed or Random:

- Whole Plot Treatment Factor: Alloys - Fixed; Split Plot Treatment Factor: Roller Gaps - Fixed;
- Blocking Factor: Days - Random

4. Describe the experimental units and/or measurement units:

- EU for Alloy: Batch of Steel; EU for Roller Bars: Portion of Batch; MU = Portion of Batch

5. Response Variable: Tensile strength; Covariate: Carbon content of Portion of Batch
6. 1 rep per level of Alloy on each Day and 3 reps per level of Roller Gap on each Day

II 20 points

1. Using the results of Tukey's HSD procedure from the SAS output, the groups of cotton percentages are

$$G_1 = \{15\%, 35\%\}, \quad G_2 = \{20\%, 35\%\}, \quad G_3 = \{20\%, 25\%\}, \quad G_4 = \{25\%, 30\%\}.$$

2. Using $\alpha_{PC} = 1 - (1 - 0.05)^{1/4} = 0.01274$, the trend contrasts having p-values less than α_{PC} are: Linear and Quadratic trends.
3. Using the contrasts given in the SAS code, the hypotheses matrix is given by

$$H = \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 2 & -1 & -2 & -1 & 2 \\ -1 & 2 & 0 & -2 & 1 \\ 1 & -4 & 6 & -4 & 1 \end{pmatrix}$$

4. With $\alpha = 0.05$, $t = 5$, $\sigma_2^2 = 8$, $r = 13$, $D = 4$; $\lambda = \frac{rD^2}{2\sigma_e^2} = 13$; $\Phi = \sqrt{\lambda/t} = 1.6$. Then from Tble IX on p.607 with $\nu_1 = t - 1 = 4$, $\nu_2 = t(r-1) = 60$, $\alpha = 0.05$, $\Phi = 1.6$, power is approximately 0.8, which is less than 0.9. Thus $r = 13$ is too small to achieve the power specification.

III 30 points

a. For the Cell Means model: $Y_{ij} = \mu_i + e_{ij}$, $i = 1, 2, 3, 4$, $j = 1, \ldots, n_i$, the Design Matrix is

$$X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b. The effect models with no constraints:

$$X = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

c. The effects model with $\tau_4 = 0$: $Y_{ij} = \mu + \tau_i + e_{ij}$, $i = 1, 2, 3$, $j = 1, \ldots, n_i$. $Y_{ij} = \mu + e_{ij}$, $j = 1, \ldots, n_4$.

$$X = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

d. Without constraints:

$$\tau_i = \mu_i - \mu,\ i = 1, 2, 3, 4 \text{ with } \mu = \frac{1}{4}\sum_{i=1}^{4} \mu_i$$

e. For constraint $\tau_4 = 0$, $\mu = \mu_4$ and

$$\begin{gathered} \tau_i = \mu_i - \mu = \mu_i - \mu_4,\ i = 1, 2, 3, 4 \\ \Rightarrow \tau_1 = \mu_1 - \mu_4;\ \tau_2 = \mu_2 - \mu_4;\ \tau_3 = \mu_3 - \mu_4,\ \tau_4 = 0 \end{gathered}$$

IV 3 points each

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