STATISTICS 630 - Test II

July 12, 2013

Name ____________________________________ Email Address ____________________________

INSTRUCTIONS FOR STUDENTS:

(1) There are six pages including this cover page and four formula sheets. Each of the five numbered problems is weighted equally.

(2) You have exactly 70 minutes to complete the exam.

(3) You should write out the answers to the exam questions on blank sheets of paper. Please start each question on a separate sheet of paper. Put the worked problems in numerical order for scanning.

(4) Do not use a calculator. You may leave answers in forms that can easily be put into a calculator such as $\frac{12}{19}$, $\binom{32}{14}$, e^{-3}, $\Phi(1.4)$, etc., unless otherwise specified.

(5) Show *ALL* your work. Give reasons for your answers.

(6) Do not discuss or provide any information to any one concerning any of the questions on this exam or your solutions until I post the solutions next week.

(7) You may use the formula sheets accompanying this test. Do not use your textbook or class notes.

I attest that I spent no more than 70 minutes to complete the exam. I used only the materials described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature__________________________________

INSTRUCTIONS FOR PROCTOR:

(1) Record the time at which the student starts the exam: ______________________

(2) Record the time at which the student ends the exam: ______________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to Webassign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion with him or her.

(5) Please keep these materials until July 20, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to webassign in my presence:

Proctor's Signature__________________________________

1. Suppose that the random variable V has probability density function

$$f_V(v) = \begin{cases} 2v & 0 \le v \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find $E(V)$ and $\mathrm{Var}(V)$.

   (b) Use Chebychev's inequality to give a bound on $P[|V - E(V)| \ge 1/3]$. Compare this bound to the actual probability.

2. We now consider inference for the exponential distribution using a mean parameter instead of a rate parameter. Let $X_1, \dots, X_n$ be a random sample from the exponential distribution with mean parameter $\theta > 0$ and probability density function,

$$f_\theta(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & 0 < x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

   One can show that $E(X_i) = \theta$, $\mathrm{Var}(X_i) = \theta^2$, and that the maximum likelihood estimator of θ is given by $\hat{\theta} = \frac{1}{n} \sum_{i=1}^n X_i$. (You do not have to derive any of these results.) Consider also the alternative estimator

$$W = \frac{1}{n+1} \sum_{i=1}^n X_i.$$

   Obtain the bias, variance, and mean squared error for each estimator, $\hat{\theta}$ and W. Which estimator has smaller mean squared error as an estimator of θ, $\hat{\theta}$ or W?

3. Let X and Y be independent random variables where X has a normal distribution with mean 0 and variance 4 and Y has a Poisson distribution with $\lambda = 3$. Define $Z = X + Y$ and $W = X - Y$. Compute $E(Z)$, $E(W)$, $\mathrm{Var}(Z)$, $\mathrm{Var}(W)$, and $\mathrm{Cov}(W, Z)$.

4. Suppose that $X_1, \dots, X_n$ are a random sample from a distribution with probability mass function,

$$f(x|\theta) = \begin{cases} \frac{e^{-\theta^2} \theta^{2x}}{x!}, & x = 0, 1, 2, \dots, \ 0 < \theta < \infty \\ 0 & \text{otherwise,} \end{cases}$$

   and mean $E(X_i) = \theta^2$. Find the maximum likelihood estimator estimator of θ and also the method of moments estimator of θ. Are they the same?

5. Suppose that the weights $W_1, \dots, W_{10}$ of checked luggage for 10 first class customers on a certain airline are normally distributed with a mean of 40 pounds and a standard deviation of 10 pounds. Also, suppose that the weights of checked luggage $V_1, \dots, V_{25}$ for 25 coach customers on this airline are normally distributed with a mean of 30 pounds and a standard deviation of 5 pounds. You may assume that all the weights are independent.

   Identify the distributions (including the values of the parameters) of

$$\bar{W} = (W_1 + \cdots + W_{10})/10 \qquad \text{and} \qquad \bar{V} = (V_1 + \cdots + V_{25})/25.$$

   Then express $P[\bar{W} > \bar{V}]$ in terms of the cdf of the standard normal distribution.