STAT 638 — Exam 1 **Name:** ______________________________

Fall 2016

INSTRUCTIONS FOR THE STUDENT:

1. You have exactly 1 hour to complete the exam.
2. There are 5 pages including this cover sheet, and 14 questions.
3. Each question is worth 7 points, which means that everyone gets a bonus of two points. **Please circle the letter of the correct answer for each question.**
4. Do not discuss or provide any information to anyone concerning any of the questions on this exam or your solutions until I post the solutions.
5. The only materials you may use are a calculator, an 8 1/2 by 11 inch formula sheet of your own making (with writing on both sides), and a copy of the common distributions on pp. 253-258 of Hoff. **Do not use the textbook or class notes.**

I attest that I spent no more than 1 hour to complete the exam. I used only the allowed materials as described above. I did not receive assistance from anyone during the taking of this exam.

Student's Signature ______________________________

INSTRUCTIONS FOR THE PROCTOR:

(1) Record the time at which the student starts the exam: ____________________

(2) Record the time at which the student ends the exam: ____________________

(3) Immediately after the student completes the exam, please scan the exam to a .pdf file and have the student upload it to WebAssign.

(4) Collect all portions of this exam at its conclusion. Do not allow the student to take any portion of the exam.

(5) Please keep these materials until October 14, 2016, at which time you may either dispose of them or return them to the student.

I attest that the student has followed all the INSTRUCTIONS FOR THE STUDENT listed above and that the exam was scanned into a pdf and uploaded to WebAssign in my presence.

Proctor's Signature ______________________________

1. One percent of a population has malaria. Among people who have malaria, a malaria test correctly diagnoses the disease 98% of the time. Among people who do not have malaria, the same test makes a correct diagnosis 99% of the time. Suppose a person is given the malaria test and is diagnosed as having the disease. The probability that the person really does have malaria is

(a) 0.013.

(b) 0.296.

(c) 0.497.

(d) 0.761.

(e) 0.979.

2. We will observe a single observation Y that has a uniform distribution of the form

$$p(y|\theta) = \frac{1}{\theta} I_{(0,\theta)}(y),$$

where θ is an unknown parameter that could be any positive number. A gamma(2,1) prior will be used for θ. If Y is observed to be 5, then an expression for the posterior probability that θ is larger than 6 is given by

(a) e^{-1}.

(b) $e^{-\theta}$.

(c) $1 - e^{-1}$.

(d) e^{-6}.

(e) $1 - e^{-6}$.

3. Frequentist inference is based on

(a) conditioning on the observed data.

(b) computing posterior probabilities.

(c) repeated sampling from the same population.

(d) repeated sampling from the prior distribution.

(e) all the above.

4. The observations Y and Z are independent given the parameter θ. It follows that

(a) $p(y, z|\theta) = f(y|\theta)g(z|\theta)$, where f and g are the marginals of Y and Z (given θ), respectively.

(b) $p(z|y, \theta) = p(z|\theta)$.

(c) both (a) and (b) are true.

(d) none of the above is true.

(e) Y and Z are not Facebook friends.

5. A binomial experiment is conducted to estimate the unknown success probability θ. The experiment has 20 trials and 7 successes. If a Jeffreys noninformative prior is used, the maximum likelihood estimate and posterior mode

(a) are 13/20 and 12.5/19, respectively.

(b) are 7/20 and 7/20, respectively.

(c) are 7/20 and 6.5/19, respectively.

(d) are the same.

(e) cannot be determined from the information given.

6. Suppose $Y_1, \ldots, Y_n$ is a random sample from a normal distribution with known mean 0 and unknown variance σ^2. The Jeffreys noninformative prior for σ is

(a) proportional to $1/\sigma$ and proper.

(b) proportional to $1/\sigma$ and improper.

(c) constant for all σ.

(d) proportional to $1/\sigma^2$.

(e) not calculable when one of the parameters is known.

7. In the situation of problem 6, let $\theta = 1/\sigma^2$. A conjugate prior for θ is

(a) all gamma(a, b) distributions.

(b) all normal distributions.

(c) all inverse gamma distributions.

(d) all Poisson distributions.

(e) something that eluded Jeffreys his entire career.

8. A certain posterior distribution is normal with mean 10 and standard deviation 2. Denote the $(1-\alpha)$100th percentile of the standard normal distribution by z_α. Which of the following is the best answer?

(a) A 95% credible interval for the unknown parameter is $(10 - 2z_{0.04}, 10 + 2z_{0.01})$.

(b) A 95% HPD region for the unknown parameter is $10 \pm 2z_{0.025}$.

(c) Both (a) and (b) are correct.

(d) Neither (a) nor (b) is correct.

(e) Bayesian probability intervals rule!

9. Hypotheses H_0 and H_1 are to be tested. The prior and posterior probability of H_0 are 0.50 and 0.03, respectively. The posterior odds ratio and Bayes factor

(a) are 3/97 and 1, respectively.

(b) are both 3/50.

(c) are both 3/97.

(d) cannot be determined from the information given.

(e) have feuded for years about the best way to measure evidence in a test of hypotheses.

10. Generally speaking, when the number of observed data increases

(a) the prior distribution becomes less influential.

(b) the effect of the prior remains about the same.

(c) the prior distribution becomes more influential.

(d) the posterior mean becomes closer and closer to the prior mean.

(e) the cost of living for average Americans increases.

11. The Jeffreys prior for a parameter θ is $p(\theta)$. The Jeffreys prior for $\tau = \exp(\theta)$

(a) is $p(\tau)$.

(b) is $p(\tau)/\tau$.

(c) is $p(\log \tau)$.

(d) is $p(\log \tau)/\tau$.

(e) cannot be determined because the likelihood function is not given.

12. Given θ, $Y_1, \ldots, Y_{n+1}$ are independent and each one has density $p(y|\theta)$. Let $\hat{\theta}$ be the maximum likelihood estimate of θ based on the data $Y_1, \ldots, Y_n$. If n is quite large, then the posterior predictive density of Y_{n+1} given $Y_1, \ldots, Y_n$ is approximately equal to

(a) $p(y|\hat{\theta})$.

(b) the posterior.

(c) the prior.

(d) $p(y_1, \ldots, y_n|\hat{\theta})$.

(e) the marginal distribution of $Y_1, \ldots, Y_n$.

13. When testing a point null hypothesis against a two-sided alternative, a frequentist P-value

(a) often overstates the significance of evidence against the alternative hypothesis.

(b) often overstates the significance of evidence against the null hypothesis.

(c) is approximately equal to the posterior probability of the null hypothesis.

(d) is approximately equal to the posterior probability of the alternative hypothesis.

(e) often overstates its importance in the world of statistics.

14. Suppose that, given θ, Y has density $p(y|\theta)$. The prior for θ is $p(\theta)$. Which of the following is a valid way to generate a value from the marginal density of Y?

(a) First generate a value, call it $\tilde{\theta}$, from the posterior. Then generate a value of Y from $p(\,\cdot\,|\tilde{\theta})$.

(b) First generate a value, call it $\tilde{\theta}$, from the prior. Then generate a value of Y from $p(\,\cdot\,|\tilde{\theta})$.

(c) Generate a value of Y directly from $p(\,\cdot\,|\theta)$.

(d) Generating values from the marginal is unnecessary since the marginal can always be computed exactly.

(e) Call Dr. Hart at 2 a.m. and he'll be glad to generate a value of Y for you.