Homework 07
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STAT 642-720

1.

1) Design - 2^{6-2}

   a) Fraction of Full Design: $\frac{1}{2}^2 = \frac{1}{4}$
   b) 2 Generators Required
   c) Generalized Interactions: $2^6 - 6 - 1 = 57$
   d) Alias for each effect: $2^2 = 4$
   e) Number of Runs: $2^{6-2} = 16$

2) Design - 2^{7-3}

   a) Fraction of Full Design: $\frac{1}{2}^3 = \frac{1}{8}$
   b) 3 Generators Required
   c) Generalized Interactions: $2^7 - 7 - 1 = 120$
   d) Alias for each effect: $2^3 = 8$
   e) Number of Runs: $2^{7-3} = 16$

3) Design - 2^{7-4}

   a) Fraction of Full Design: $\frac{1}{2}^4 = \frac{1}{16}$
   b) 4 Generators Required
   c) Generalized Interactions: $2^7 - 7 - 1 = 120$
   d) Alias for each effect: $2^4 = 16$
   e) Number of Runs: $2^{7-4} = 8$

2. 2^{7-3}

```
suppressPackageStartupMessages(library(FrF2))

dsgn = FrF2(nruns = 16, nfactors = 7, generators = c('ABC', 'BCD', 'ACD'))
dsgn = cbind(y = 1:nrow(dsgn), dsgn)
alias.sets = aliases(lm(y ~ (.)^4, data = dsgn))

cat('Alias Sets \n'); alias.sets
```

```
Alias Sets


 A = B:C:E = B:F:G = C:D:G = D:E:F
 B = A:C:E = A:F:G = C:D:F = D:E:G
 C = A:B:E = A:D:G = B:D:F = E:F:G
 D = A:C:G = A:E:F = B:C:F = B:E:G
 E = A:D:F = B:D:G = C:F:G = A:B:C
 F = A:B:G = A:D:E = B:C:D = C:E:G
```

```
G = A:B:F = A:C:D = B:D:E = C:E:F
A:B = A:C:D:F = A:D:E:G = B:C:D:G = B:D:E:F = C:E = F:G
A:C = A:B:D:F = A:E:F:G = B:C:F:G = C:D:E:F = B:E = D:G
A:D = A:B:C:F = A:B:E:G = B:C:D:E = B:D:F:G = C:G = E:F
A:E = A:B:D:G = A:C:F:G = B:E:F:G = C:D:E:G = B:C = D:F
A:F = A:B:C:D = A:C:E:G = B:C:E:F = C:D:F:G = B:G = D:E
A:G = A:B:D:E = A:C:E:F = B:C:E:G = D:E:F:G = B:F = C:D
B:D = A:B:C:G = A:B:E:F = A:C:D:E = A:D:F:G = C:F = E:G
A:B:D = A:C:F = A:E:G = B:C:G = B:E:F = C:D:E = D:F:G
```

```
cat('Treatment Design \n'); dsgn
```

```
Treatment Design

    y  A  B  C  D  E  F  G
1   1  1 -1  1 -1 -1  1 -1
2   2  1  1 -1  1 -1 -1 -1
3   3  1 -1  1  1 -1 -1  1
4   4  1  1  1  1  1  1  1
5   5 -1 -1  1  1  1 -1 -1
6   6 -1  1  1  1 -1  1 -1
7   7 -1  1  1 -1 -1 -1  1
8   8  1 -1 -1 -1  1 -1  1
9   9  1  1 -1 -1 -1  1  1
10 10 -1  1 -1  1  1 -1  1
11 11 -1 -1 -1  1 -1  1  1
12 12 -1 -1 -1 -1 -1 -1 -1
13 13  1  1  1 -1  1 -1 -1
14 14  1 -1 -1  1  1  1 -1
15 15 -1  1 -1 -1  1  1 -1
16 16 -1 -1  1 -1  1  1  1
```

3. Fractional Factorial 2^{5-2}

i. $\frac{2^3}{2^5} = \frac{1}{4}$
ii. $I = AE$ is the preferred generating relationship although neither design is good because the max resolution for AE is 2 which means some main effects will be confounding.

$$I = ABCD = BCE = AB^2C^2E = AE$$
$$I = ABCDE = ABCD = A^2B^2C^2D^2E = E$$

4.

a) Resolution 4

Run	A	B	C	D	ABCD = E
1	-1	-1	-1	1	ABCD = -1, E = 1
2	1	-1	1	1	ABCD = -1, E = 1
3	1	1	-1	-1	ABCD = 1, E = -1
4	1	-1	1	-1	ABCD = 1, E = -1
5	-1	1	-1	-1	ABCD = -1, E = 1
6	-1	-1	1	-1	ABCD = -1, E = 1
7	-1	1	-1	1	ABCD = 1, E = -1
8	1	1	1	1	ABCD = 1, E = -1
9	-1	1	1	1	ABCD = -1, E = 1
10	1	1	-1	1	ABCD = -1, E = 1
11	-1	-1	1	1	ABCD = 1, E = -1
12	-1	-1	-1	-1	ABCD = 1, E = -1
13	1	-1	-1	-1	ABCD = -1, E = 1
14	1	1	1	-1	ABCD = -1, E = 1
15	1	-1	-1	1	ABCD = 1, E = -1
16	-1	1	1	-1	ABCD = 1, E = -1

b)

c) Alias Sets

```
mdl = FrF2(nruns = 16, nfactors = 5)
mdl$y = 1:nrow(mdl)
aliases(lm(y ~ (.)^4, data = mdl))
```

```
 A = B:C:D:E
 B = A:C:D:E
 C = A:B:D:E
 D = A:B:C:E
 E = A:B:C:D
 A:B = C:D:E
 A:C = B:D:E
 A:D = B:C:E
 A:E = B:C:D
 B:C = A:D:E
 B:D = A:C:E
 B:E = A:C:D
 C:D = A:B:E
 C:E = A:B:D
 D:E = A:B:C
```

d) All higher order effects are equal to 0 e)

```
   Variation  SS.Sq Var.Prop
1          A 0.0676  0.02801
2          B 0.0361  0.01496
3          C 0.0870  0.03604
4          D 0.0144  0.00597
5          E 0.4761  0.19724
6        A:B 0.0012  0.00050
7        A:C 0.8100  0.33557
8        A:D 0.0156  0.00646
9        A:E 0.0020  0.00083
10       B:C 0.0036  0.00149
11       B:D 0.2652  0.10987
12       B:E 0.6006  0.24882
13       C:D 0.0001  0.00004
14       C:E 0.0001  0.00004
15       D:E 0.0342  0.01417
```

f) Effects E, A:C, D:B, B:E make up 88% of the total variaion on the model. Significance cannot be tested because we are using up all of the degrees of freedom in the model by testing every combination of main effect and 2-way treatment.

g)

```r
mdl = FrF2(nruns = 8, nfactors = 5)
mdl$y = 1:nrow(mdl)
summary(mdl)
```

```
Call:
FrF2(nruns = 8, nfactors = 5)

Experimental design of type  FrF2
8  runs

Factor settings (scale ends):
   A  B  C  D  E
1 -1 -1 -1 -1 -1
2  1  1  1  1  1

Design generating information:
$legend
[1] A=A B=B C=C D=D E=E
```

```
$generators
[1] D=AB E=AC


Alias structure:
$main
[1] A=BD=CE B=AD    C=AE    D=AB    E=AC

$fi2
[1] BC=DE BE=CD


The design itself:
   A  B  C  D  E y
1 -1 -1 -1  1  1 1
2 -1 -1  1  1 -1 2
3 -1  1  1 -1 -1 3
4  1 -1 -1 -1 -1 4
5  1 -1  1 -1  1 5
6  1  1 -1  1 -1 6
7 -1  1 -1 -1  1 7
8  1  1  1  1  1 8
class=design, type= FrF2
```

```r
aliases(lm(y ~ (.)^4, data = mdl))
```

```
 A = C:E = B:D
 B = C:D:E = A:B:C:E = A:D
 C = B:D:E = A:B:C:D = A:E
 D = B:C:E = A:C:D:E = A:B
 E = B:C:D = A:B:D:E = A:C
 B:C = D:E = A:B:E = A:C:D
 B:E = C:D = A:B:C = A:D:E
```

5.

a) Hypothesis testing shows that the Carry Over term is not significant

```r
head(tbl, 3)
```

```
  Sequence Subject Drug     Period Blood.Pressure Carry.Over
1      ABC       1    A   Period.I            174          N
2      ABC       1    B  Period.II            146          A
3      ABC       1    C Period.III            164          B
```

```
mdl = lmer(Blood.Pressure ~ Sequence + Drug + Period + Carry.Over + (1 | Sequence:Subjec
           data = tbl)
```

```
fixed-effect model matrix is rank deficient so dropping 1 column / coefficient
```

```
anova(mdl)
```

```
fixed-effect model matrix is rank deficient so dropping 1 column / coefficient
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```

```
Analysis of Variance Table of type III  with  Satterthwaite
approximation for degrees of freedom
           Sum Sq Mean Sq NumDF  DenDF F.value    Pr(>F)
Sequence    758.0  151.60     5 18.639  0.5466 0.7388586
Drug       6010.6 3005.31     2 42.000 10.8364 0.0001604 ***
Period      609.2  609.19     2 42.000  2.1966 0.1457827
Carry.Over   92.4   46.18     2 42.000  0.1665 0.8471661
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b. to extract the LSE contrasts we have to drop the carry over term from the model and rerun the analysis.

```
                   Estimates Std.Error
Sequence  ABC       171.0833   11.1941
Sequence  ACB       175.6667   11.1941
Sequence  BAC       179.8333   11.1941
Sequence  BCA       167.9167   11.1941
Sequence  CAB       188.5833   11.1941
Sequence  CBA       185.3333   11.1941
Drug   A            191.0000    5.3194
Drug   B            164.8333    5.3194
Drug   C            178.3750    5.3194
Period  Period.I    182.0833    5.3194
Period  Period.II   172.5000    5.3194
Period  Period.III  179.6250    5.3194
```

6.

b) The covariate weld diameter is shown to be very significant as is the interaction between weld diameter and alloy. The adjusted means are significantly different as well.

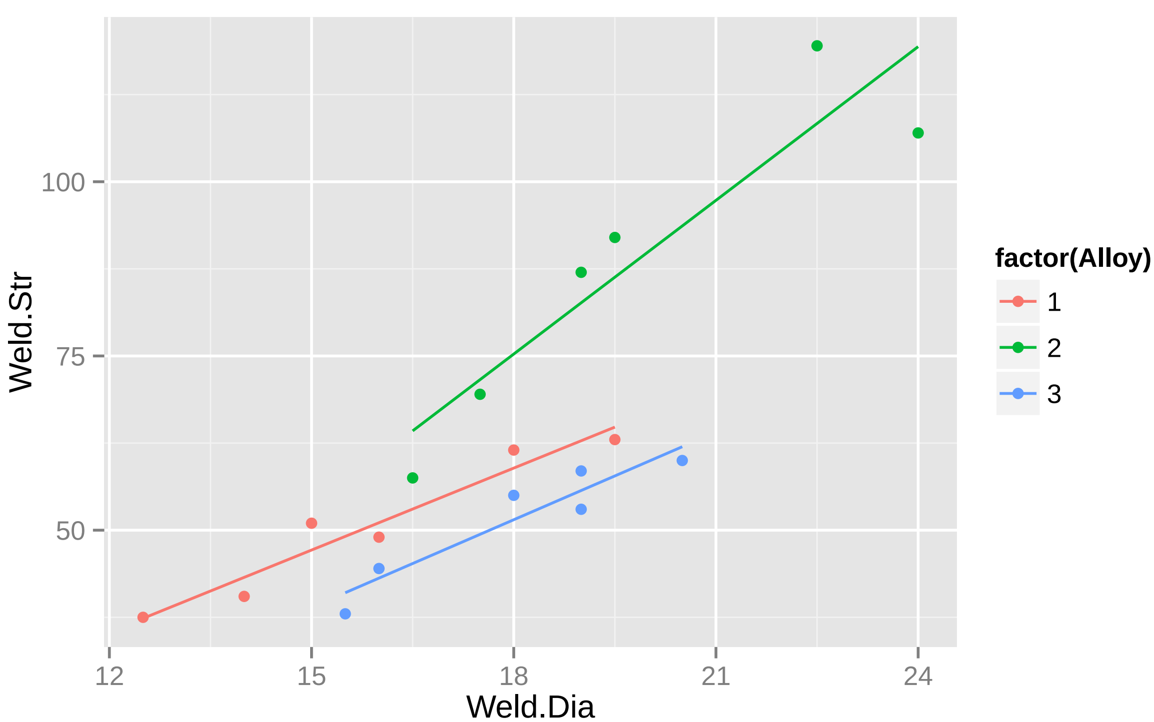
```
suppressPackageStartupMessages(library(ggplot2))

mdl = lm(Weld.Str ~ Weld.Dia * Alloy, data = dt)
anova(mdl)
```

```
Analysis of Variance Table

Response: Weld.Str
               Df Sum Sq Mean Sq  F value    Pr(>F)
Weld.Dia        1 6549.3  6549.3 171.6354 1.808e-08 ***
Alloy           2 2006.0  1003.0  26.2849 4.120e-05 ***
Weld.Dia:Alloy  2  258.3   129.1   3.3841   0.06832 .
Residuals      12  457.9    38.2
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
ggplot(dt, aes(x = Weld.Dia, y = Weld.Str, color = factor(Alloy))) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE)
```



c)

```
      fit   se.fit df
1 58.9100 3.425444 12
2 75.2739 3.070105 12
3 51.5000 2.521846 12
```

```
  Difference Diff.mu  Diff.se
1      T1.T2 16.3639 1.327881
2      T1.T3  7.4100 1.227917
3      T2.T3 23.7739 1.146925
```

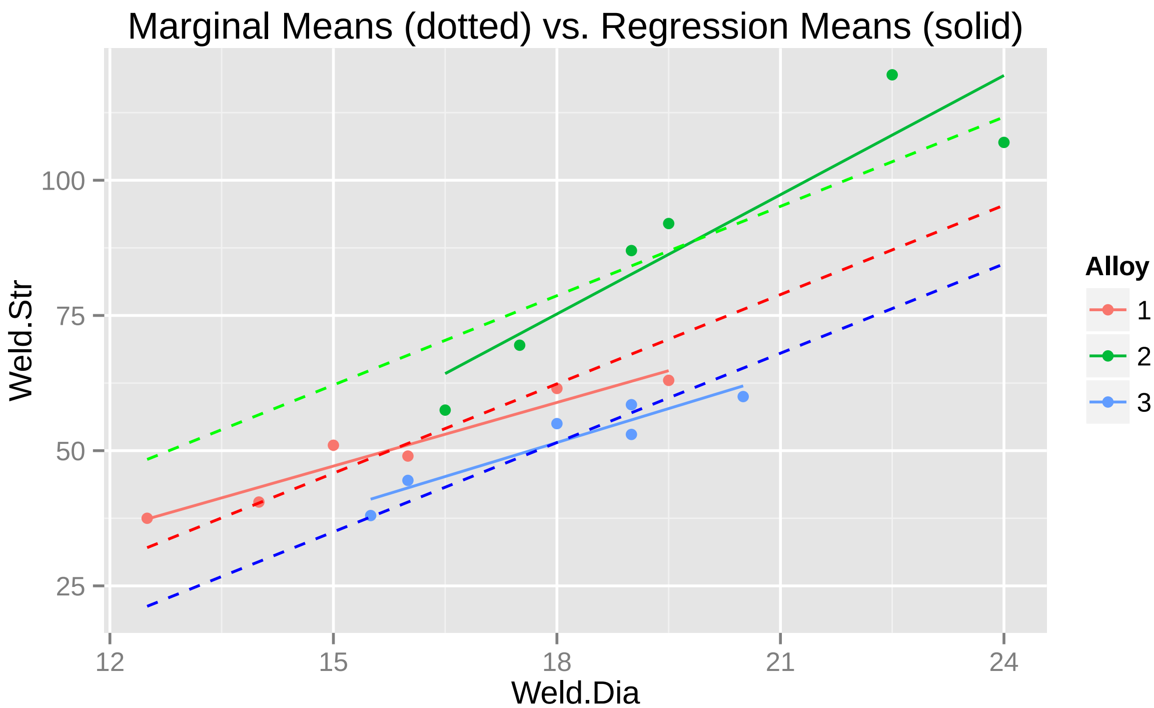
Adjusted Means

T	Y LSMEAN	Standard Error	Pr > t	LSMEAN Number
1	58.4744444	3.3461353	<.0001	1
2	74.4571713	3.1318402	<.0001	2
3	51.0345345	2.5268901	<.0001	3

d)

```
t1 = function(x) {-47.59608541 + 10.84489917 + x*5.50533808}
t2 = function(x) {-47.59608541 + 27.15688019 + x*5.50533808}
t3 = function(x) {-47.59608541 + x*5.50533808}

ggplot(dt, aes(x = Weld.Dia, y = Weld.Str, color = Alloy)) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE) +
  stat_function(fun = t1, color = "red", linetype = 2) +
  stat_function(fun = t2, color = "green", linetype = 2) +
  stat_function(fun = t3, color = "blue", linetype = 2) +
  ggtitle("Marginal Means (dotted) vs. Regression Means (solid)")
```



f) We can conclude from the bartlett test that we have homogeneity of variance with an alpha of .05

```
bartlett.test(Weld.Str ~ Alloy, data = dt)
```

```

	Bartlett test of homogeneity of variances

data:  Weld.Str by Alloy
Bartlett's K-squared = 5.2408, df = 2, p-value = 0.07277
```