CE 691 Computational Methods in Civil Engineering

Total Phosphorus Modeling of Lakes

Develop a phosphorus model for Platte Lake, Michigan to simulate the total phosphorus concentrations in the water column and sediment for an entire year. The basic model governing equations are inspired by Chapra and Canale (1991) as follows:

$$V_1 \frac{dp_1}{dt} = W - Qp_1 - v_s A_s p_1 + v_r A_s p_2$$

$$V_2 \frac{dp_2}{dt} = v_s A_s p_1 - v_r A_s p_2 - v_b A_s p_2$$

where subscripts a1 and 2 designate the water and sediment systems, respectively. In addition, the following model parameters and coefficients are defined:

p_1 = total phosphorus concentration (mg/L) in the lake water column
p_2 = total phosphorus concentration (mg/L) in the sediment
t = time (day)
V_1 = volume of the water column
V_2 = volume of the sediment system (assumed constant with time)
W = total phosphorus loading rate to the lake water column
Q = daily variable flow rate through the water column
v_s = settling velocity of phosphorus from the water column to the sediment (m/yr)
A_s = average area of the lake water column
v_r = recycle mass transfer coefficient from the sediment to the water for phosphorus (m/yr)
v_b = a burial mass transfer coefficient from the sediment system to the deep sediments (m/yr).

Go to the following website to get the daily flow rate data for USGS gaging station 04126740 at Platte River at Honor, MI to retrieve the 2006 flow rate data:

http://waterdata.usgs.gov/nwis/dv?cb_00060=on&format=gif_default&begin_date=2006-01-01&end_date=2006-12-31&site_no=04126740&referred_module=sw

Note that the unit for flow rates is cfs!

The daily total phosphorus loading rates are provided in a separate file called LOAD.DAT. **Note that the unit for these loads is kg/month**.

The following table lists the key model parameters:

V_1 = 78.71 x 10^6 m^3
V_2 = 0.981 x 10^6 m^3
A_s = 10.6 x 10^6 m^2
v_s = 17.6 m/yr
v_b = 6.45 x 10 m/yr
v_r = 0.002 m/yr
p_1 (initial) = 0.0085 mg/L
p_2 (initial) = 300 mg/L

Use the RK 4^{th} order method as the numerical engine for this model. Try the following constant integration time steps for the RK-4 scheme: 0.2 day, 0.5 day, and 1.0 day and see if the results are different.

Plot your results for total phosphorus concentration in water column and sediment vs. time from January 1, 2006 to December 31, 2006.

Implement your work using MATLAB.

References Cited:

Chapra, S.C. and Canale, R.P., 1991. Long-Term Phenomenological Model of Phosphorus and Oxygen for Stratified Lakes. *Water Research*, 35(6):707-715.

Supplemental Reading:

Lung, W.S., 2001. Water Quality Modeling for Wasteload Allocations and TMDLs. John Wiley & Sons, New York, NY.