BOD/DO MODELING USING STREAM

BOD/DO modeling of rivers and streams has become an integral component of water pollution control and management during the past 35 years. The concentration of dissolved oxygen has been one of the most significant criteria in stream sanitation. The discharge of organic impurities, such as municipal sewage and industrial wastes, into a body of water presents a problem of primary importance in this regard (Eckenfelder and O'Connor, 1961). Bacteria consume oxygen to break down the organic matters. Oxygen is resupplied via the reaeration process. The analysis of such a natural purification capacity of a stream, therefore, is of fundamental and practical value. The first attempt to quantify the oxygen balance in a stream was by Streeter and Phelps (1925) for the Ohio River. O'Connor (1967) expanded this analysis by including other processes such as photosynthesis, respiration, and sediment oxygen demand in a classic paper to present a comprehensive modeling analysis of oxygen balance in streams. His work represents the foundation of modern water quality modeling of natural waters.

Three decades have passed since O'Connor's classic work on stream BOD/DO modeling. The 1-D stream BOD/DO modeling has become a standard practice in the water quality modeling profession. Most 1-D models are run on personal computers at a great speed, making efficient calculations and proving that running these models is not difficult. Yet, how to perform correct analyses by assigning reasonable model coefficients is the key to success of simple 1-D BOD/DO stream modeling.

Heavy use of 1-D BOD/DO stream modeling took place in the 1970's when the NPDES permit process, construction program, and the 208 program were beginning. In an engineering practice article, Lung and Sobeck (1999) demonstrated the renewed use of these models in water quality management. More importantly, additional information on wastewater characteristics and new protocol on long-term BOD measurements have reopened this once thought "straightforward" modeling analysis. The purpose of this technical note is to present 1-D BOD/DO stream modeling with up-to-date information concerning this modeling technology.

The case studies presented focus on the STREAM model, which is based on the original work by O'Connor and his colleagues at Manhattan College. This modeling program is not unique, as many other workers have since written their own code to implement the governing equations for dissolved oxygen balance in streams under steady state conditions.

Equations behind the STREAM Model

The basic principle to formulate a stream water quality model is mass balance. For a given segment of the stream, the accumulation of a water quality constituent over a finite period of time is equal to the mass entering the segment plus the mass added to the segment, less the mass leaving the segment and the mass lost within the segment.

Accumulation = Mass In – Mass Out + Source - Sink

Applying the mass balance principle and considering a small segment of a stream, one may develop:

$$V\Delta C = QC\Delta t - [Q + \Delta Q][C + \frac{\partial C}{\partial x}\Delta x]\Delta t + W\Delta t - KCV\Delta t \quad (1)$$

where:

V = volume of the segment
ΔC = change of concentration
Q = advective flow rate
C = concentration
Δt = small increment of time
ΔQ = change of advective flow rate over the length of the segment, Δx
$\partial C/\partial x$ = concentration gradient with respect to x
W = direct loading rate of constituent
K = first-order reaction rate of constituent

Dividing Eq. 1 by $V\Delta t$ results in

$$\frac{\partial C}{\partial t} = -\frac{Q}{A}\frac{\partial C}{\partial x} - \frac{C}{A}\frac{\partial Q}{\partial x} + \frac{W}{V} - KC \quad (2)$$

Assuming steady-state conditions ($\partial C / \partial t = 0$) and neglecting the flow gradient ($\partial Q / \partial x = 0$), Eq. 2 becomes

$$0 = -\frac{Q}{A}\frac{\partial C}{\partial x} + \frac{W}{V} - KC \quad (3)$$

Q/A in Eq. 3 is the average velocity in the segment of the river, i.e., 1-D advective transport.

STREAM is a steady state, one-dimensional BOD and DO model for streams and rivers. Three water quality parameters are modeled: $CBOD_u$, NBOD, and DO. For $CBOD_u$, Eq. 3 takes the form:

$$0 = -\frac{Q}{A}\frac{dL}{dx} - K_r L \tag{4}$$

where L is the $CBOD_u$ concentration and K_r is the $CBOD_u$ removal rate. The wasteload, W is included at the beginning of the river reach and is therefore not shown in Eq. 4.

The equivalent equation for NBOD is:

$$0 = -\frac{Q}{A}\frac{dN}{dx} - K_n N \tag{5}$$

where N is the NBOD concentration.

The distribution of dissolved oxygen may be formulated in a similar manner by including all dissolved oxygen sources and sinks described earlier:

$$0 = -\frac{Q}{A}\frac{dC}{dx} + K_a(C_s - C) - K_d L - K_n N + P - R - \frac{SOD}{H} \tag{6}$$

If $CBOD_u$ is removed only by direct oxidation, the deoxygenation rate coefficient, K_d, reflecting actual oxygen reduction in the system, is equal to the $CBOD_u$ removal rate coefficient, K_r. The terms on the right side of Eq. 6 represent, respectively: the downstream transport of oxygen with the stream flow; atmospheric reaeration; biological oxidation of $CBOD_u$; biological oxidation of NBOD; algal photosynthesis; algal respiration; and the biological oxidation of sediment materials. The $CBOD_u$ concentration, L, and NBOD, N, in Eq. 6 may be replaced by the functional forms of Eqs. 4 and 5, respectively. The resulting expression may be integrated with given boundary conditions and expressed in terms of dissolved oxygen.

Further, dissolved oxygen deficit, D (= $C - C_s$), instead of dissolved oxygen is used to formulate the dissolved oxygen profile, expressing Eq. 6 as:

$$0 = -\frac{Q}{A}\frac{dD}{dx} + K_a D + K_d L + K_n N - P + R + \frac{SOD}{H} \tag{7}$$

The solution to Eq. 7 is:

$$D = \frac{K_d L_0}{K_a - K_r}\left(e^{-K_r \frac{x}{U}} - e^{-K_a \frac{x}{U}}\right) \qquad CBOD_u \qquad (8a)$$

$$+ \quad \frac{K_n N_0}{K_a - K_n}\left(e^{-K_n \frac{x}{U}} - e^{-K_a \frac{x}{U}}\right) \qquad NBOD \qquad (8b)$$

$$+ D_0 e^{-K_a \frac{x}{U}} \qquad Initial\ \ Deficit \qquad (8c)$$

$$- \frac{P}{K_a}\left(1 - e^{-K_n \frac{x}{U}}\right) \qquad Algal\ \ Photosysthesis \qquad (8d)$$

$$+ \frac{R}{K_a}\left(1 - e^{-K_n \frac{x}{U}}\right) \qquad Algal\ \ Respiration \qquad (8e)$$

$$+ \frac{SOD}{H\ K_a}\left(1 - e^{-K_n \frac{x}{U}}\right) \qquad Sediment\ \ Oxygen\ \ Demand \qquad (8f)$$

Note that $U = Q/A$ and is the average stream velocity. D_0 represents an initial dissolved oxygen deficit existing at the origin, if any. The initial $CBOD_u$ concentrations, L_0, in Eq. 8a must be expressed in terms of the ultimate oxygen demand. The dissolved oxygen concentration may then be determined from the computed deficit using the equation

$$C = C_S - D$$

Because of zero-order and first-order kinetics formulated in the model, the dissolved oxygen deficit terms due to varying sources and sinks may be added (i.e., superimposed).

Implementing the Governing Equation in STREAM Code

Equations 8a through 8f are developed for steady-state conditions using constant parameters and represent the analytical solution to the BOD/DO equation (Eq. 7) with spatially constant hydraulic geometry, kinetic coefficients, and wasteloads. In practice, however, the hydraulic geometry varies along the river. Implementing Eqs. 8a-8f for a real stream/river would require dividing the river into a number reaches, each of which has spatially constant parameters. Mass balance at the end of each reach must be maintained prior to starting a new reach. The STREAM model code is designed to accommodate numerous reaches in the one-dimensional configuration.

One of the key parameters of STREAM is the reaeration coefficient, K_a. In the original version of the STREAM model, the O'Connor-Dobbins (1958) Equation was used to quantify K_a:

$$K_a(20^oC) = \frac{12.9U^{0.5}}{H^{1.5}} \qquad (8)$$

where U is average stream velocity (ft/s) and H is average stream depth (ft).

The Tsivoglou Equation (Tsivoglou and Neal, 1976) has now been added to quantify reaeration coefficients for small rivers and streams:

$$K_a(20^oC) = C\,V\,S \tag{9}$$

where

V = stream velocity (ft/s)

S = stream slope (ft/mile)

C is a proportionality constant:

= 1.8 for stream flow rate between 1 cfs and 10 cfs

= 1.3 for stream flow rate between 10 cfs and 25 cfs

= 0.88 for stream flow rate between 25 cfs and 300 cfs

Saturated dissolved oxygen concentrations in each reach are calculated using the following equation (EPA, 1995):

$$C_s = \frac{468}{31.6 + T} \tag{10}$$

where T is water temperature in °C and C_s is saturated DO concentration in mg/L. This equation is accurate to within 0.03 mg/L compared with the Benson-Krause equation on which the Standard Methods tables are based (EPA, 1995).

STREAM Model Input Structure

The STREAM model input structure indicates such a simple design (Table 1). Following the entry of the number of reaches and the starting mile point, each reach is characterized by two lines of input. Note that the tributary flows and wastewater treatment plant flows are entered as incremental flows to the total stream flow. Boundary conditions for CBOD, NBOD, and DO deficit concentrations are also needed. Water column kinetic coefficients, K_r, K_d, K_n, sediment oxygen demand, and net algal photosynthetic oxygen production are entered for each reach. Because all of these coefficients are temperature dependent, average water temperature for each reach is also required. Note that the user can choose either the O'Connor-Dobbins or Tsivoglou Equation for the reaeration coefficient.

Application of STREAM to Rock Creek

Rock Creek (Figure 1) in Gettysburg, Pennsylvania was experiencing poor water quality in the late 1970's resulting from the Gettysburg wastewater treatment plant discharge which failed to meet the NPDES permit limits. Water quality and aquatic biology surveys in 1970's indicated that severe degradation in water quality was found below the treatment discharge in terms of key water quality parameters. During a 1969 water survey, dissolved oxygen concentration was down to 2 mg/L near the outfall of the plant discharge. Nutrient concentrations also were high in the Creek.

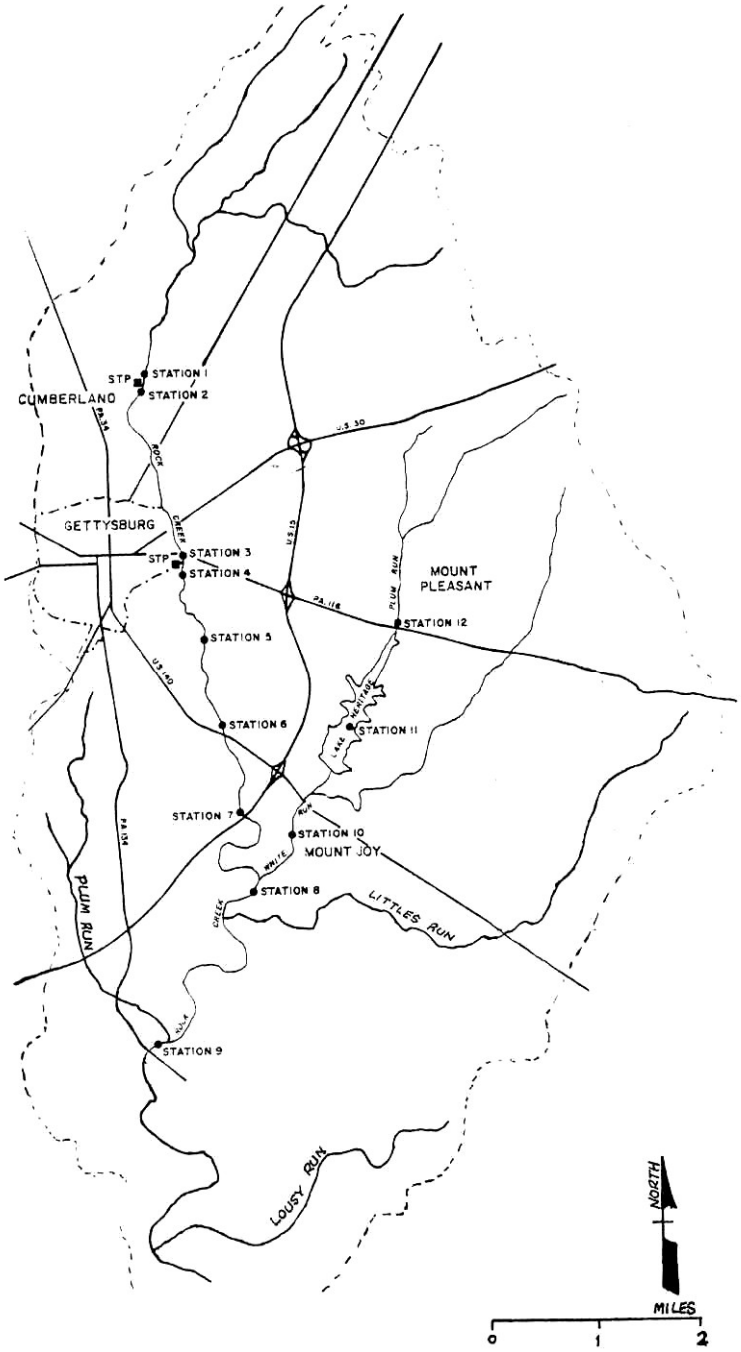

Figure 1. Rock Creek in Gettysburg, Pennsylvania (showing the sampling stations)

Under the Construction Program of the Clean Water Act of 1977, federal grants and state support were secured by the local government to upgrade the Gettysburg treatment plant. As part of the facility planning process, a water quality model was developed in 1979 to evaluate various wastewater management alternatives designed to alleviate these water quality problems. The STREAM BOD/DO model was chosen for this study. To support the modeling work, a water quality survey was conducted in August 1979 to collect data for the

model calibration analysis. Historical data from surveys in 1973 and 1969 were also used for model verifications.

Table 1. Input Data Structure for the STREAM Model

Line	Column	Variable	Description	Units	Format
1	-5	SW1	= 1 for suppressing the listing of input data = 0 for printing the input data		I1
	-10	SW2	= 0 for O'Connor's Reaeration Eq. = 1 for Tsivoglou's Reaeration Eq.		I1
2	(1-80)	Title	Comments to identify the model run		A80
3	(1-10)	NOR	Number of reaches in the river/stream		F10.1
	(11-20)	Starting X	River mile of the location to start the model	mile	F10.0
	(21-30)	PINT	Print interval of model results	mile	F10.0
	(31-40)	Ratio	$CBOD_u$ to $CBOD_5$ ratio[a]		F10.0
4	(1-10)	CBOD	$CBOD_5$ concentration of the tributary inflow	mg/L	F10.0
	(11-20)	WCBOD	Direct $CBOD_5$ load to the head of the reach	lb/day	F10.0
	(21-30)	NBOD	NBOD concentration of the tributary inflow	mg/L	F10.0
	(31-40)	WNBOD	Direct NBOD load to the head of the reach	lb/day	F10.0
	(41-50)	DOD	DO deficit concentration of the tributary inflow	mg/L	F10.0
	(51-60)	WDOD	Direct DO deficit load to the head of the reach	lb/day	F10.0
	(61-70)	BN	Benthic oxygen demand	g O_2 m^{-2} day^{-1}	F10.0
	(71-80)	PNET	Net algal photosynthetic oxygen production	mg O_2 $L^{-1}day^{-1}$	F10.0
	(81-90)	SL	Slope of river channel bottom b	ft/mile	F10.0
5	(1-10)	Q	Incremental flow to the reach c	cfs	F10.0
	(11-20)	DEPTH	Average depth of the reach	ft	F10.0
	(21-30)	AREA	Average cross-sectional area of the reach	ft^2	F10.0
	(31-40)	TEMP	Average water temperature of the reach	C	F10.0
	(41-50)	XR	Mile point that the reach ends	mile	F10.0
	(51-60)	AKR	In stream CBOD removal rate in the reach	day^{-1}	F10.0
	(61-70)	AKD	In stream CBOD deoxygenation rate in the reach	day^{-1}	F10.0
	(71-80)	AKN	In stream NBOD decay rate in the reach	day^{-1}	F10.0
6[d]					
7[e]					

a. This ratio is used if the user prefers to generating results in $CBOD_5$ for comparison with data. However, in many cases, it is desirable to have the results in $CBOD_u$ by simply setting the ratio to 1.0; all model input parameters and results will be in $CBOD_u$.
b. for Tsivoglou Eq. only.
c. positive flow for input and negative flow for withdrawal.
d. repeat lines No. 4 and No. 5 for each subsequent reach.
e. a BLANK line is needed to indicate the end of input.

Stream Hydraulic Geometry and Time of Travel

The physical parameters of hydraulic geometry include width, depth, cross-sectional area, and velocity in different reaches of the stream. In this study, the depth profile across the stream at each sampling station was surveyed. The cross-sectional area was planimetered from the survey data. The average depth was calculated by dividing the cross-sectional area by the width. Finally, the average flow was calculated by multiplying the measured velocity by the cross-sectional area. Figure 2 shows the cross-sectional area, depth, and velocity measured along Rock Creek during the August 1979 survey.

Tributary flows in the study area include the upstream flow in Rock Creek, White Run, Littles Run, Stevens Run, and Plum Run (Figure 1). Since there is no permanent USGS gaging station in the watershed, flows from the nearest gaging station (Evitts Creek near Centerville, PA) with similar watershed characteristics were used to approximate the flows for Rock Creek and its tributaries. Table 2 lists the drainage areas and derived flows at Rock Creek and the tributaries during different times. Additional flows to the stream system are from point source wastewater treatment plants (Cumberland and Gettysburg).

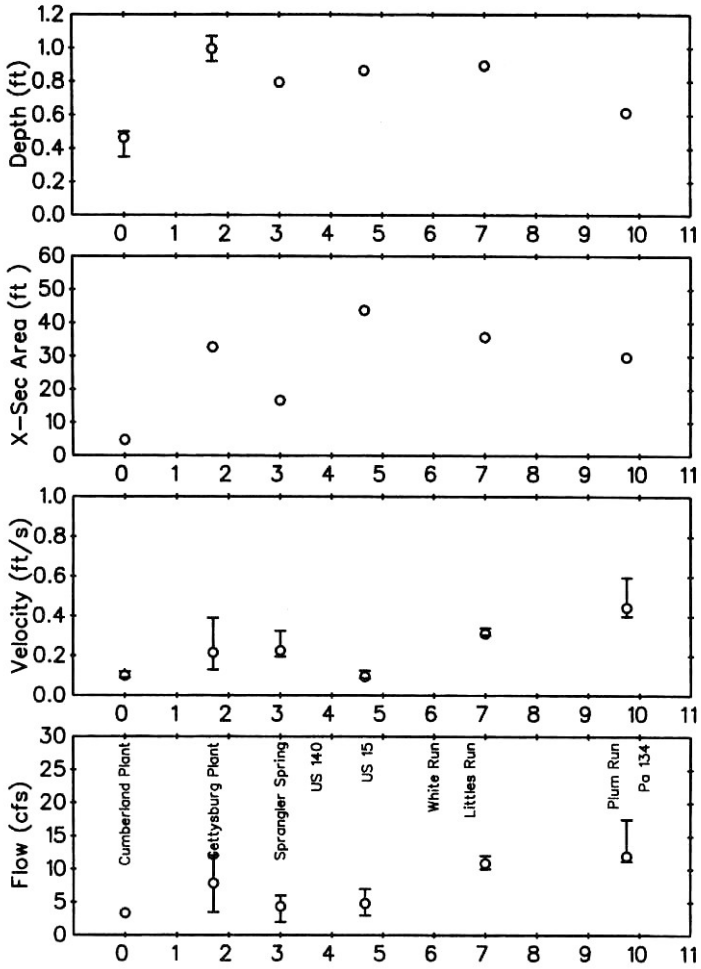

Figure 2. Cross-sectional area, depth, and velocity measured in Rock Creek

Table 2. Drainage Areas and Derived Flows of Rock Creek Watershed

Location/ Tributary	Drainage Area (mi^2)	Aug. 1979 Flow (cfs)	Oct. 1973 Flow (cfs)	Aug. 26 Flow (cfs)	7Q10 Flow (cfs)
Rock Creek above Gettysburg Plant	20	3.8	7.5	6.3	1.13
White Run below Heritage	13	2.5	5.2	4	0.73
Littles Run	7.67	1.5	3.1	2.4	0.42
Plum Run	3.84	0.8	1.5	1.2	0.22
Lousey Run	2.93	0.6	1.2	0.9	0.17
Stevens Run	1.15	0.2	0.5	0.4	0.07

In-stream CBOD Removal Rate, K_r and Nitrification Rate, K_n

The overall decay phenomena as measured in Rock Creek reflects the disappearance of CBOD from the water column due to a combination of settling and biological oxidation. Plotting the CBOD data vs. time-of-travel provides a first estimate of the removal coefficient, K_r (Figure 3). Since the effluent quality from the wastewater treatment plants, particularly the Gettysburg plant, is poor and has a significant concentration of suspended solids, the CBOD removal rate immediately below the treatment plants is much higher than the biological oxidation rate, K_d (Figure 3).

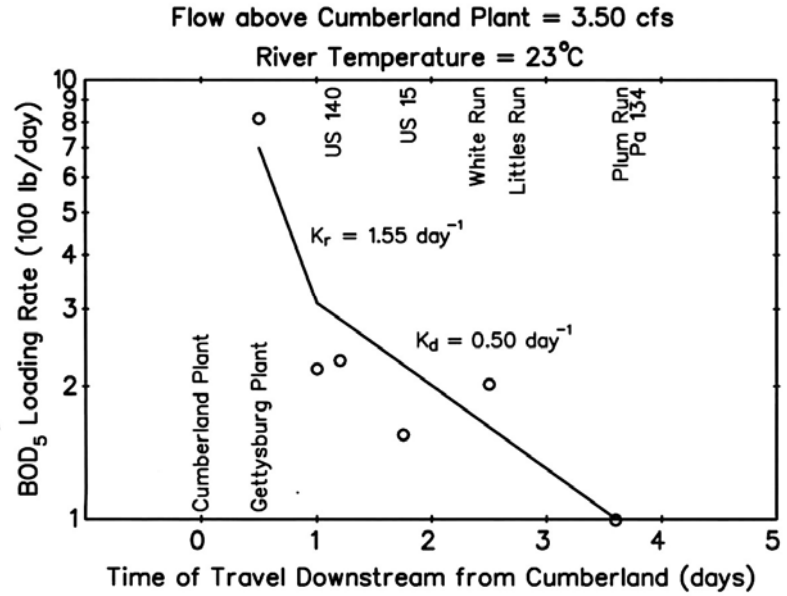

Figure 3. Estimation of CBOD removal and biological oxidation rates in Rock Creek

The in-stream nitrification rate, K_n, is determined by plotting the NBOD concentration vs. time-of-travel along the stream (Figure 4).

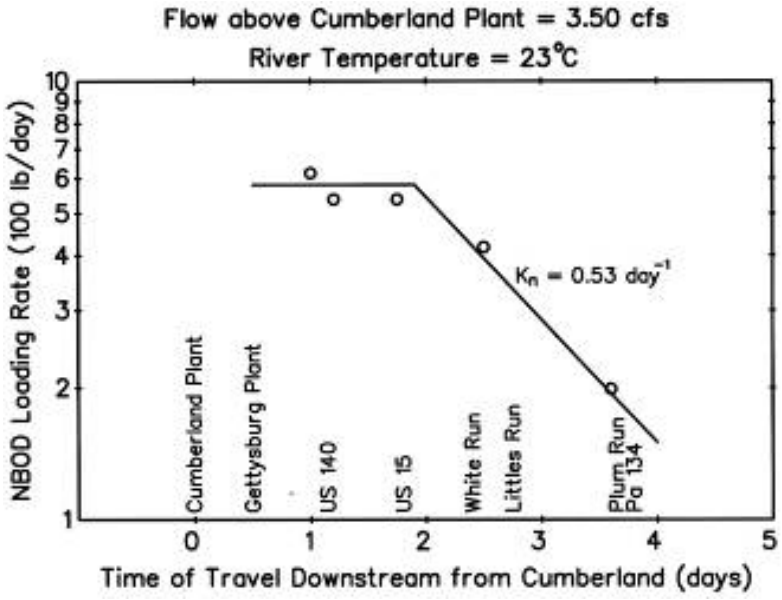

Figure 4. Estimation of in-stream nitrification rate in Rock Creek

Deriving Other Kinetic Coefficients

The Tsivoglou Equation is used to calculate the reaeration coefficients for Rock Creek, as the stream flow rates are small and well within the range of flow rates that the Tsivoglou Equation deems appropriate. Since there was no available data on sediment oxygen demand (SOD) in Rock Creek, literature values were used. They range from 2 gm $O_2/m^{2/}$day to 10 gm O_2/m^2/day with an approximate average of 4 gm O_2/m^2/day for municipal wastewater sludge near the outfall (Thomann and Mueller, 1987). The SOD values decrease to between 1 gm O_2/m^2/day and 2 gm O_2/m^2/day for aged sludge downstream of the outfall. In this study, a value of 2 gm O_2/m^2/day was initially used in the reach from the Gettysburg wastewater treatment plant outfall to 2.3 miles downstream from the plant. Subsequent model calibration and verification runs would eventually fine-tune the SOD values (see Table 3 for the model input data file).

Model Results

The BOD/DO model of Rock Creek is first calibrated using the August 1979 data. Subsequent model verification runs are made using the data sets of October 1973 and August 1969. CBOD, NBOD, and DO deficit loads from point sources are developed from the data for each data set. In addition, boundary conditions for these three model constituents are also derived from the data. No algae have been observed in the study area and therefore, net production of oxygen due to algal photosynthesis is not considered in the model.

Figure 5 presents the BOD/DO model results compared to the field data. Also shown in Figure 5 are the stream flows along Rock Creek and the average water temperature for these three surveys. In general, the model results match the field data very well throughout these three data sets. In the August 1979 data, the dissolved oxygen standard of 5 mg/L is violated in a portion of the stream. Specifically, significant CBOD load from the Gettysburg plant contributes to the DO sag of 2 mg/L immediately below the outfall. Due to such a low minimum DO, nitrification in the stream is lagging behind, as shown in the horizontal lines between mile point 1.7 and 4. Only after the dissolved oxygen concentration starts to recover, nitrification begins to take place. The October 1973 data were collected during a higher stream flow, resulting in much lower CBOD and NBOD concentrations along Rock Creek. Dissolved oxygen concentrations during this survey are much higher and do not violate the DO standard. No NBOD data are available from the October 1973 and August 1969 surveys for comparison with the model-calculated concentrations. However, nitrification is not lagging during these two model runs.

Table 3. Input Data File for the STREAM Model of Rock Creek

0	1[a]							
Rock Creek in Gettysburg, Pennsylvania (August 1979)								
11.0[b]	-1.0	0.20	1.50[c]					
1.50	0.00	5.00	0.0	1.7	0.0	0.0	0.0	5.435
1.50	0.40	15.0	25.0	0.0	0.50	0.50	0.00	
0.00	20.00	0.00	40.0	0.0	5.60	1.4	0.0	9.235
0.30	0.80	17.0	25.0	1.7	1.00	1.00	0.0	
0.00	400.0	0.00	525.0	0.0	15.5	4.0[d]	0.0	25.689
1.30	0.90	23.08	25.0	3.0	1.00	0.80	0.0	
1.00	0.0	2.00	0.0	4.0	0.0	3.5	0.0	14.345
1.0	1.00	32.31	25.0	4.0	0.90	0.70	0.0	
1.00	0.0	2.0	0.0	3.0	0.0	0.7	0.0	21.789
0.4	1.10	46.00	25.00	5.0	0.5	0.5	0.63	
1.00	0.0	2.00	0.0	3.0	0.0	0.0	0.0	13.981
0.0	1.20	46.0	25.0	6.0	0.5	0.5	0.63	
1.00	0.0	2.0	0.0	3.0	0.0	0.0	0.0	6.678
2.50	1.30	44.38	25.0	6.7	0.5	0.5	0.63	
1.00	0.0	2.0	0.0	3.0	0.0	0.0	0.0	5.234
1.50	1.30	45.26	25.0	8.0	0.5	0.5	0.63	
1.00	0.0	2.00	0.0	3.0	0.0	0.0	0.0	6.129
0.0	1.40	45.26	25.0	9.0	0.5	0.5	0.63	
1.00	0.0	2.00	0.0	3.0	0.0	0.0	0.0	4.978
0.0	1.40	45.26	25.0	9.5	0.5	0.5	0.63	
1.00	0.0	2.00	0.0	3.0	0.0	0.0	0.0	3.457
0.8	1.50	44.76	25.0	11.0	0.5	0.5	0.63	

a. The Tsivoglou's reaeration equation is selected.
b. Rock Creek is divided into 11 reaches.
c. $CBOD_u$ –to-$CBOD_5$ratio is 1.50.
d. SOD values in gm $O_2 m^{-2} day^{-1}$

To better understand the significance of each dissolved oxygen-consuming source, a unit response analysis is conducted using the BOD/DO model of Rock Creek. The four major oxygen-consuming sources are Gettysburg and Cumberland plants, upstream loads, and sediment oxygen demand. Figure 6 shows the DO deficit contributed by these individual sources. It is evident that the Gettysburg plant and the sediment oxygen demand contribute most to the DO deficit while the other sources are insignificant in the dissolved oxygen budget of Rock Creek.

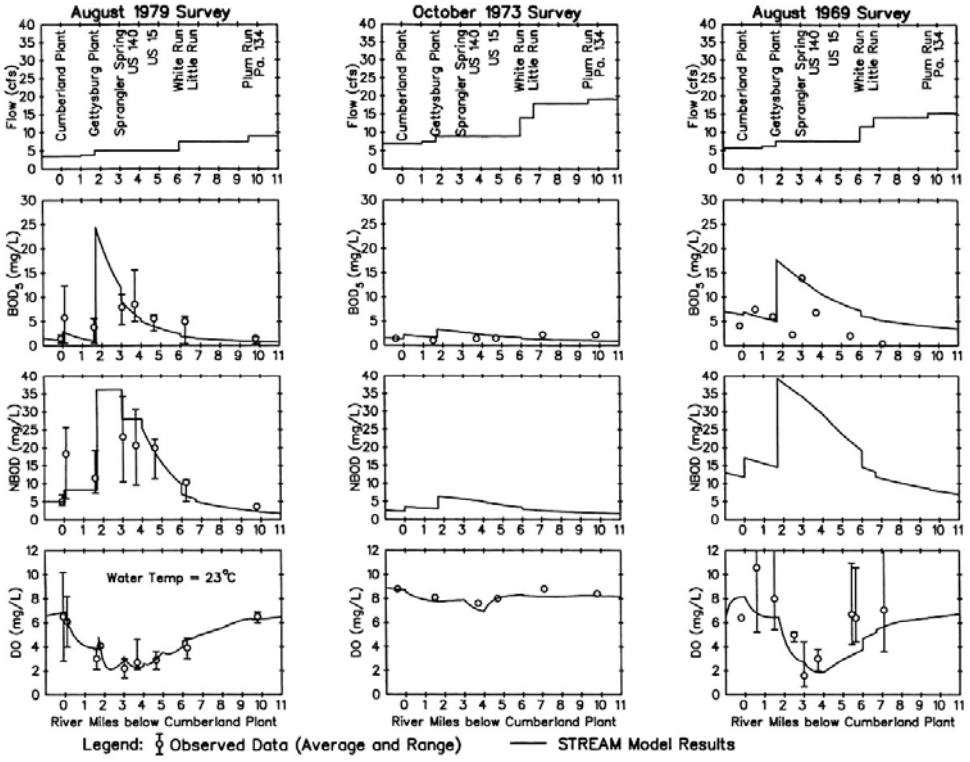

Figure 5. STREAM model calibration results for Rock Creek (1979. 1973, and 1969 data)

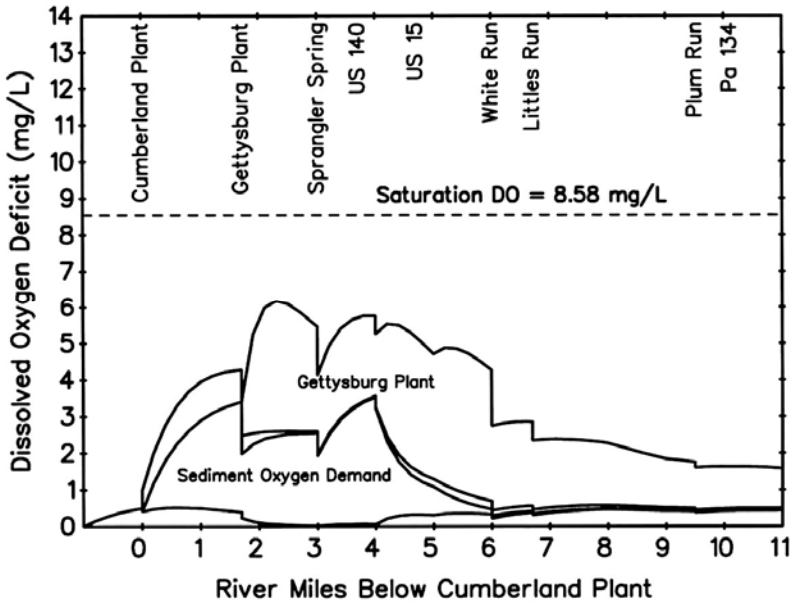

Figure 6. Unit response analysis results of Rock Creek using the STREAM Model

Modeling Total Phosphorus and Fecal Coliform Using STREAM

Phosphorus compounds, like nitrogen compounds, can contribute to aquatic life, but in excessive amounts cause eutrophication of a stream. While phosphorus enters Rock Creek primarily from the Gettysburg plant, a number of processes are responsible for the removal of phosphorus from the water column. Suspended solids in the wastewater eventually settle to the bottom of the stream. Any phosphorus bound to the solids, including the algal biomass, would be incorporated, at least temporarily, into bottom deposits. Soluble phosphorus may combine chemically with metallic cations to form precipitates. Phosphorus sorption by particulate materials plays another role in soluble concentration reduction. Subsequent settling and deposition of this particulate material will reduce the total phosphorus mass in the water column. These processes all contribute to phosphorus removal and can be approximated by a first-order removal process in a similar fashion to the CBOD removal.

Two first-order phosphorus removal rates are estimated: 0.44 day^{-1} from mile point 1.7 to 3.0 and 0.24 day^{-1} for the rest of the stream in a manner similar to the spatial variable CBOD removal rate (see Figure 3). These removals rates and the measured total phosphorus loading rates from the Cumberland and Gettysburg plants are incorporated into the STREAM model, using the CBOD slot to form a total phosphorus model of Rock Creek. The model calculated total phosphorus concentrations match the measured data quite closely (Figure 6). Note that the removal rate considered in this analysis represents the net loss of phosphorus in the water column and inherently includes any resuspension and release of phosphorus from the sediment.

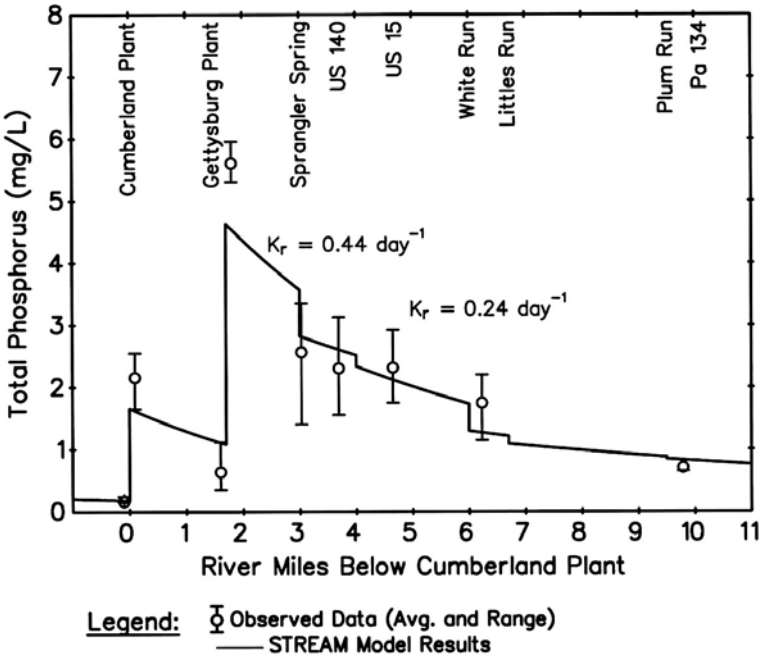

Figure 6. Modeling Total Phosphorus concentrations in Rock Creek using STREAM

For fecal coliform bacteria, a first-order die-off rate is also considered in this analysis. Again, using the CBOD slot in the STREAM model, one can simulate the fecal coliform bacteria concentrations in Rock Creek. For freshwater, the fecal coliform die-off rate has been cited in the range from 0.5 day^{-1} to 3.5 day^{-1}. Plotting the measured fecal coliform bacteria concentration vs. time-of-travel along Rock Creek yields a die-off rate of 2 day^{-1}, consistent with the literature-reported values. Figure 7 shows the comparison between the model calculated fecal coliform levels and the data.

The above applications to total phosphorus and fecal coliform bacteria further demonstrate the versatility of the STREAM model. It not only can be used to model BOD/DO in a 1-D stream, but also is also capable of simulating the fate and transport of other constituents following simple, first-order kinetics. Obviously, the STREAM model can also be used to simulate the fate and transport of any conservative substance in the water column in a 1-D configuration.

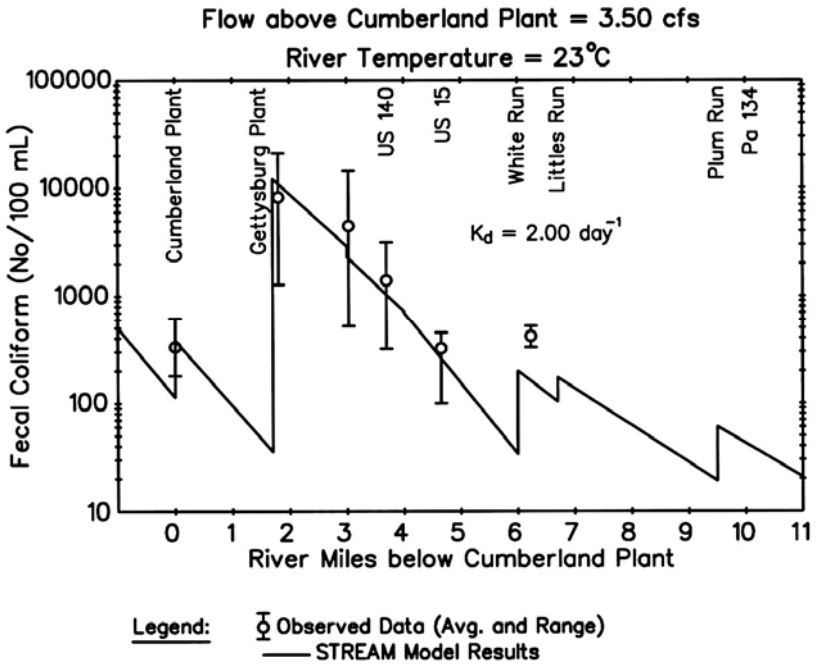

Figure 7. Modeling fecal coliform bacteria levels in Rock Creek using STREAM