Analysis of BOD data to Determine the Kinetic Coefficients

The first phase of the BOD reaction involves the oxidation of the substance, carbonaceous organic material. The reaction is approximated by 1st-order kinetics, i.e., the rate of oxygen consumption is assumed to be proportional to the amount of oxygen demanding material, either substrate or cells:

$$\frac{dy}{dt} = K_1(L_o - y) \tag{1}$$

In this equation both K_1 and L_o are **unknown**. Integration of Eq. 1 yields:

$$y = L_o(1 - e^{-K_1 t}) \tag{2}$$

If it is assumed that *dy/dt* represents the value of the slope of the BOD curve to be fitted through all the data points for a given K_1 and L_u value, then because of experimental error, the two sides of Eq.1 will not be equal but will differ by an amount *R*. Rewriting Eq.1 in terms of *R* yields:

$$R = K_1(L_o - y) - y' \tag{3}$$

where $y' = dy/dt$. Simplifying Eq.3 and substituting *a* for K_1L_o and $-b$ for K_1 gives

$$R = a + by - y' \tag{4}$$

Now, if the sum of the squares of the residuals *R* is to be minimum, the following equations must hold:

$$\frac{\partial}{\partial a}\Sigma R^2 = \Sigma 2R\frac{\partial R}{\partial a} = 0 \tag{5}$$

$$\frac{\partial}{\partial b}\Sigma R^2 = \Sigma 2R\frac{\partial R}{\partial b} = 0 \tag{6}$$

If the indicated operations in Eqs. 5 and 6 are carried out using the value of the residual R defined by Eq. 4, the following set of equation result:

$$\begin{aligned} na + b\Sigma y - \Sigma y' &= 0 \\ a\Sigma y + b\Sigma y^2 - \Sigma yy' &= 0 \end{aligned} \tag{7}$$

where n = number of data points

$a = -bL_o$

$b = -K_1$ (base e)

$L_o = -a/b$

y = any given BOD value

$y' = (y_{n+1} - y_{n-1})/2\Delta t$

Ex. Calculation of BOD kinetic coefficients K and L_o using the lease squares for the following BOD data reported for a stream receiving some waste effluent:

t (day)	2	4	6	8	20
y (mg/L)	11	18	22	24	26

Solution

1. Set up a computation table and perform the indicated steps.

Time	y	y^2	y'	yy'
2	11	121	4.50	49.5
4	18	324	2.75	49.5
6	22	484	1.50	33.0
8	24	576	1.00	24.0
	75	1,505	9.75	156.0

2. Substituting the values from the above table to Eq. 6 and solve for a and b.

$$4a + 75b - 9.75 = 0$$

$$75a + 1505b - 156.0 = 0$$

$$a = 7.5 \quad \text{and} \quad b = -0.271 \text{(base e)}$$

3. Determine the values of K_1 and L_o

$K_1 = -b = 0.271$ (base e)

$L_o = a/b = 7.5/0.271 = 27.7$ mg/L