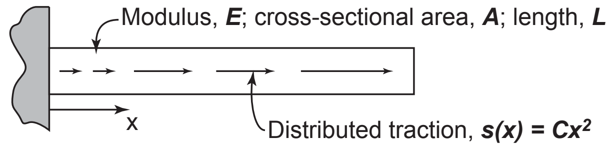
MAE671/CE671: Introduction to Finite Element Analysis

Midterm Instructions

- The goal of this exam is to evaluate your understanding of the basic concepts and mathematical framework utilized in the finite element method. If you do not understand the question, but can illustrate your understanding of course material by working through a related problem, then do that (as opposed to struggling to answer something you really do not understand.)
- Work out enough detail to be convincing that you fully understand the concepts and manipulations involved, but you do not need to provide every single detail.
- Clearly indicate the "end result" for each part by labeling it accordingly (e.g. "**Ans. 1(a) =** ").
- ***Write only on the sheet of paper provided for that problem (including the back):*** the problems will be graded independently (and in different physical locations). That is, do not continue one problem on a different sheet (used to answer a different problem). You may utilize blank scratch paper, but only turn in a single sheet of paper - the sheet with the exam question.
- ***Put your name on every page:*** each problem will be graded independently. Only one page per problem will be graded.
- ***DO NOT PANIC, OR WORRY ABOUT EARNING POINTS.*** (Note that more difficult questions are weighted less!). Simply use this framework of this exam to tell me what you know.

Problem 1:

A one-dimensional bar has an applied traction (force per unit length) that varies quadratically from one end to the other. The only relevant deformation is the displacement in the x-direction.



a) State one kinematic relationship, and one constitutive relationship relevant to the problem illustrated above. Don't make this hard: it is as simple as it gets. (10 pts)

b) Use these relationships with the principle of virtual work to derive the equations governing the displacements that are generated by the distributed traction; illustrate the implications of PVW (and associated variational calculus!) with respect to the relevant boundary conditions. (35 pts)

c) If one wanted to develop a single finite element that would be able to recover an exact solution for the given loading, how many nodes would it have? Fully justify your answer, but do not derive anything. (10 pts)

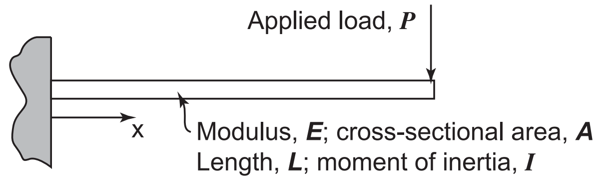
d) Write the expression that one would evaluate to determine the equivalent nodal forces that would be applied at the nodes (to part c) above; you do not have to fill it in or derive any of the expressions, rather explain how that nodal force vector is obtained. (5 pts)

Problem 2:

Illustrate how one can obtain a cubic displacement variation in an element with just two nodes. Clearly state the required number of different interpolation functions, their mathematical form (i.e. cubic, quadratic, exponential), and the equations that are used to explicitly define their form. (60 pts)

Problem 3:

Consider the cantilever beam problem shown: assume Bernoulli-Euler kinematics.



a) Develop a two-term solution for the displacement distribution using a Raleigh-Ritz approach. That is, assume a form of the solution with two unknown constants, and then derive the equations that determine the "best" constants. YOU DO NOT NEED TO SOLVE THE EQUATIONS, IF IT OBVIOUS WHAT THE LINEAR EQNS ARE THAT SHOULD BE SOLVED TO GET THE CONSTANTS. (50 pts)

b) Determine whether or not you have derived a weak solution (generally one does via this approach) or a strong solution (which one can, if assumption is chosen wisely): obviously, you'll need to illustrate you know the difference between the "strong" and "weak" formulations. (10 pts.)

Problem 4:

Describe how the stiffness matrix is derived for a two-dimensional, plane-strain element with three nodes; clearly identify the concepts that you are applying, as well as the key relationships that are utilized in the derivation. If you use matrix notation (a good idea), be sure to identify the dimensions of each matrix, as well as how the entries are computed. NOTE: you do not need to fill every thing in, just make it clear conceptually how entries depend on position (if they do) and how they are calculated.