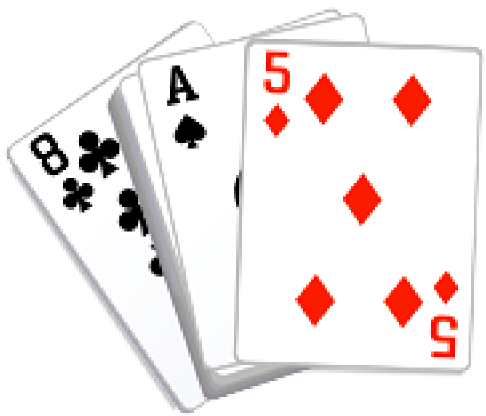
Basic Principles of Probability and Statistics



Lecture notes for PET 472

Spring 2010

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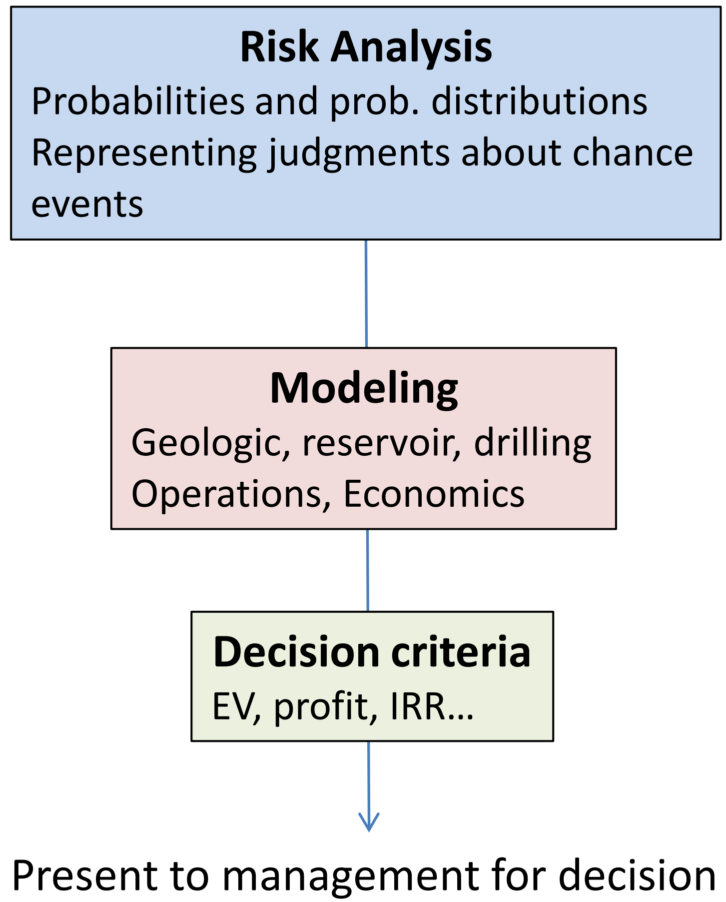
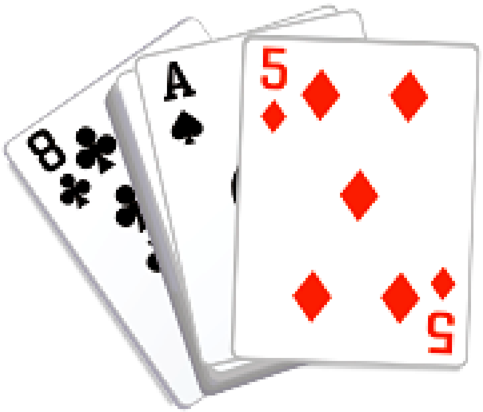
Definitions

- Risk Analysis
  - Assessing probabilities of occurrence for each possible outcome

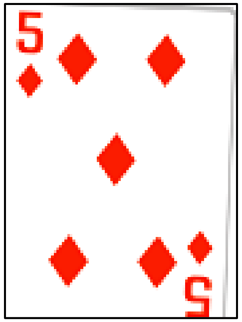


Definitions

- Sample Space
  - Complete set of outcomes (52 cards)



- Outcome
  - Subset of the sample space (drawing a “5” of any suit)



- Probability
  - Likelihood of drawing a “5”

    P(A) = 4/52

Definitions

- Equally likely outcomes
  - Have same probability to occur

- Mutually exclusive outcomes
  - The occurrence of any given outcome excludes the occurrence of other outcomes
- Independent events
  - The occurrence of one outcome does not influence the occurrence of another
- Conditional probability
  - The probability of an outcome is dependent upon one or more events that have previously occurred.

Rules of Operation

Symbol	Definition	Expression
P(A)	Probability of outcome A occurring	
P(A+B)	Probability of outcome A and/or B occurring	P(A+B)=P(A)+P(B)-P(AB)
P(AB)	Probability of A and B occurring	P(AB) = P(A) P(B|A)
P(A|B)	Probability of A given B has occurred.	

Rules of Operation

Addition Theorem

P(A+B)=P(A)+P(B)-P(AB)

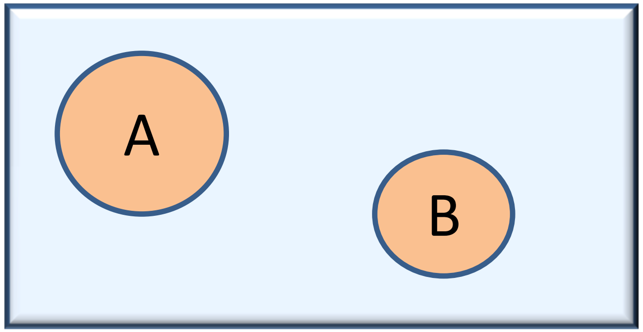
<u>Example</u>

outcome A – drawing 4, 5, 6 of any suit $P(A) = \frac{12}{52}$

outcome B – J or Q of any suit $P(B) = \frac{8}{52}$

$P(A+B) = \frac{20}{52}$

$P(AB) = 0$



Mutually Exclusive events

Venn Diagram

Rules of Operation

Addition Theorem

P(A+B)=P(A)+P(B)-P(AB)

<u>Example</u>

outcome A – drawing 4, 5, 6 of any suit

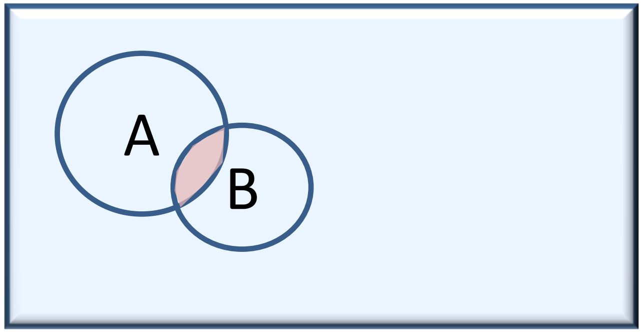
$$P(A) = \frac{12}{52}$$

outcome B – drawing a diamond

$$P(B) = \frac{13}{52}$$

$$P(A + B) = \frac{22}{52}$$

$$P(AB) = \frac{3}{52}$$



Venn Diagram

Rules of Operation | Multiplication Theorem

P(AB)=P(A)P(A|B)

Example

outcome A – drawing any jack

$$P(A) = \frac{4}{52}$$

outcome B – drawing a four of hearts on the second draw

$$P(B \mid A) = \frac{1}{51}$$

conditional

$$P(AB) = \left(\frac{4}{52}\right)\left(\frac{1}{51}\right) = \frac{1}{663}$$

Sampling without replacement

- observed outcome is not returned
- series of dependent events

Rules of Operation | Multiplication Theorem

P(AB)=P(A)P(B)

<u>Example</u>

outcome A – drawing any jack, return $\quad P(A) = \frac{4}{52}$

outcome B – drawing a four of hearts on the second draw $\quad P(B) = \frac{1}{52}$

$$P(AB) = \left(\frac{4}{52}\right)\left(\frac{1}{52}\right) = \frac{1}{676}$$

Sampling with replacement

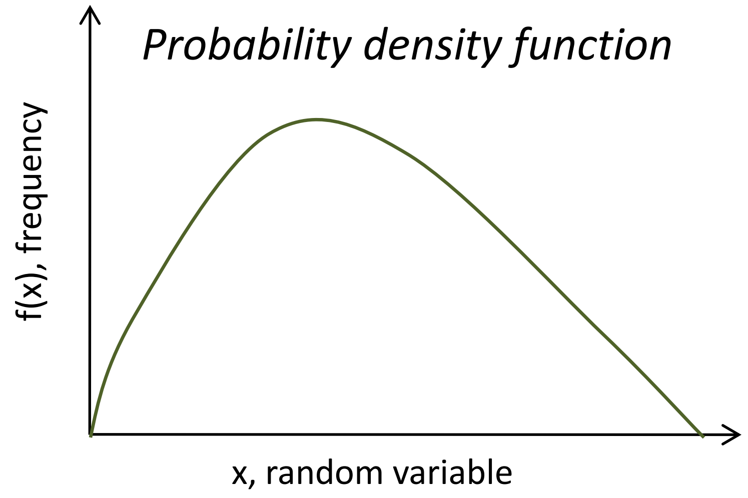
- observed outcome is returned to sample space
- series of independent events

Probability Distributions

- A graphical representation of the range and likelihoods of possible values of a random variable

- Random variable

  a variable that can have more than one possible value, also known as stochastic or deterministic



- Useful method to describe a range of possible values. Basis for **Monte Carlo Simulation**.

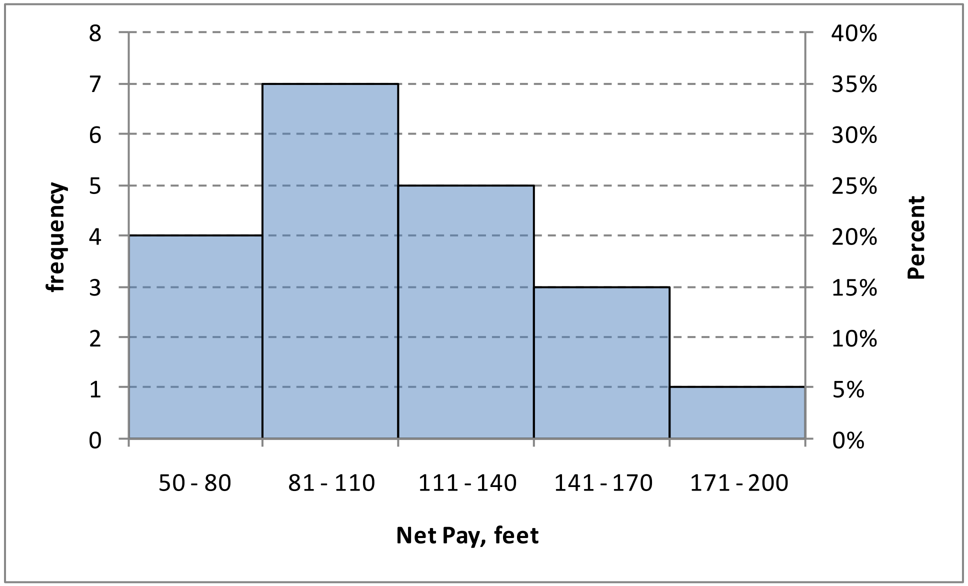
Probability Distributions | Frequency distributions

Data	
Well No	Net pay, ft
1	111
2	81
3	142
4	59
5	109
6	96
7	124
8	139
9	89
10	129
11	104
12	186
13	65
14	95
15	54
16	72
17	167
18	135
19	84
20	154

Divide into intervals
Or bins

Range	frequency	Percent
50 - 80	4	20%
81 - 110	7	35%
111 - 140	5	25%
141 - 170	3	15%
171 - 200	1	5%
	20	100%

Histogram representation
Of statistical data



Probability Distributions

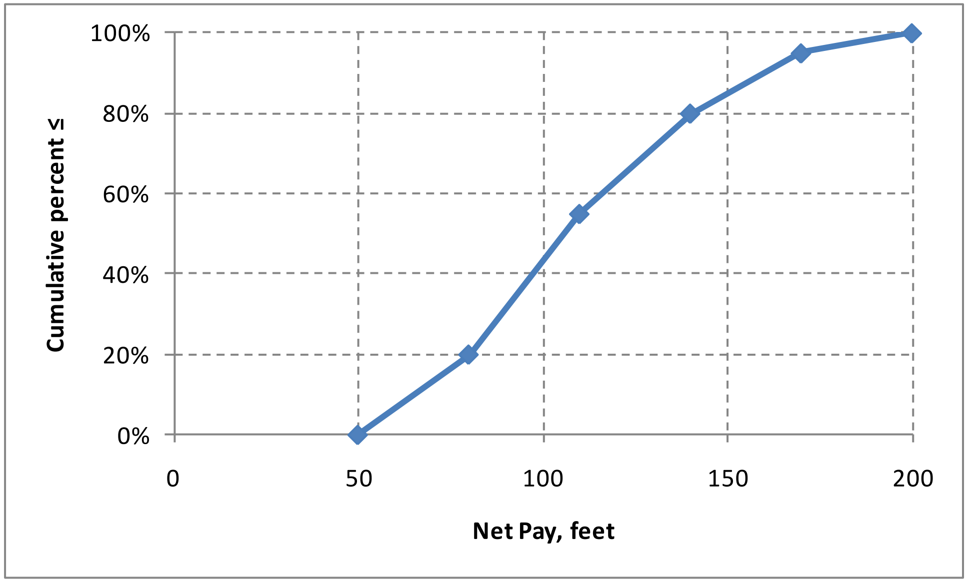
Cumulative frequency distributions

Range	frequency	Percent
50 - 80	4	20%
81 - 110	7	35%
111 - 140	5	25%
141 - 170	3	15%
171 - 200	1	5%
	20	100%

minimum

Range	Cumulative Percent ≤
50	0%
80	20%
110	55%
140	80%
170	95%
200	100%

maximum



<u>Benefits</u>

1. *Can easily read probabilities*
2. *Necessary for Monte Carlo Simulation*

Parameters of distributions

- A parameter that describes central tendency or average of the distribution

  Mean, μ – weighted average value of the random variable

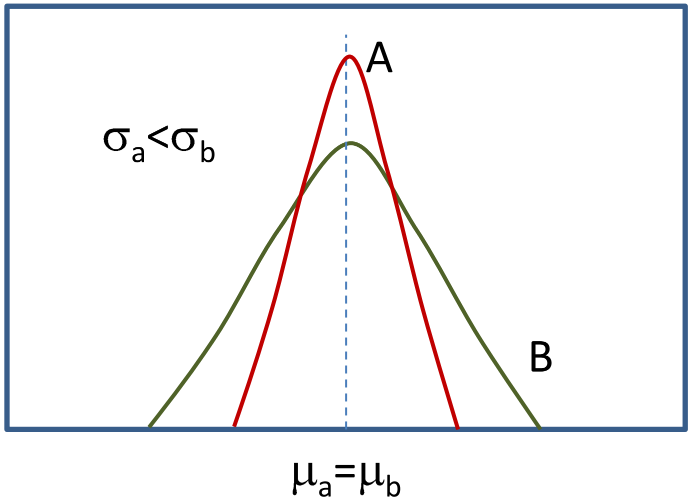
  Median – value of the random variable with equal likelihood above or below

  Mode – value most likely to occur

- A parameter that describes the variability of the distribution

  Variance, σ^2 – mean of the squared deviations about the mean

  Standard deviation, σ – square root of variance…degree of dispersion of distribution abut the mean



Parameters of distributions

Computing mean and standard deviation

1. Arithmetic average of discrete sample data set

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

N – number of equally-probable values

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}}$$

$\mu =$	**17.6**
$\sigma =$	**2.87**

Depth	k,md	ϕ, %
4807.5	2.5	17.0
4808.5	59	20.7
4809.5	221	19.1
4810.5	211	20.4
4811.5	275	23.3
4812.5	384	24.0
4813.5	108	23.3
4814.5	147	16.1
4815.5	290	17.2
4816.5	170	15.3
4817.5	278	15.9
4818.5	238	18.6
4819.5	167	16.2
4820.5	304	20.0
4821.5	98	16.9
4822.5	191	18.1
4823.5	266	20.3
4824.5	40	15.3
4825.5	260	15.1
4826.5	179	14.0
4827.5	312	15.6
4828.5	272	15.5
4829.5	395	19.4
4830.5	405	17.5
4831.5	275	16.4
4832.5	852	17.2
4833.5	610	15.5
4834.5	406	20.2
4835.5	535	18.3
4836.5	663	19.6
4837.5	597	17.7
4838.5	434	20.0
4839.5	339	16.8
4840.5	216	13.3
4841.5	332	18.0
4842.5	295	16.1
4843.5	882	15.1
4844.5	600	18.0
4845.5	407	15.7
4847.5	479	17.8
4847.5	139	20.5
4847.5	135	8.4
	$\mu =$	**17.6**
	$\sigma =$	**2.87**

Core porosity and permeability

Parameters of distributions

Computing mean and standard deviation

2. Values listed as frequencies in groups

$$\mu = \frac{\sum_i n_i x_i}{\sum_i n_i}$$

i – index to denote number of intervals
n – frequency of data points in each interval
x – midpoint value of each interval

$$\sigma = \sqrt{\frac{\sum_i n_i \left[(x_i - \mu)^2 \right]}{\sum_i n_i}}$$

i	Porosity interval	n_i frequency	p_i prob.	x_i midpoint	μ mean	deviation	σ^2 variance
1	$7 \le x < 10$	1	0.024	8.5	0.202	85.342	2.032
2	$10 \le x < 12$	0	0.000	11.0	0.000	45.402	0.000
3	$12 \le x < 14$	1	0.024	13.0	0.310	22.450	0.535
4	$14 \le x < 16$	10	0.238	15.0	3.571	7.497	1.785
5	$16 \le x < 18$	12	0.286	17.0	4.857	0.545	0.156
6	$18 \le x < 20$	8	0.190	19.0	3.619	1.592	0.303
7	$20 \le x < 22$	7	0.167	21.0	3.500	10.640	1.773
8	$22 \le x < 25$	3	0.071	23.5	1.679	33.200	2.371
		42	1.00	$\mu =$	**17.74**	$\sigma^2 =$	8.96
						$\sigma =$	**2.993**

Applicable for large data sets
Results are approximate

Parameters of distributions

Computing mean and standard deviation

3. Discrete probability distributions

$$\mu = \sum_i p_i x_i$$

$$\sigma = \sqrt{\sum_i p_i (x_i - \mu)^2}$$

x drilling costs $M	probability	midpoint of range $M	EV $M	xi*pi $M	$(x_i-\mu)^2$ $(\$M)^2$	$p(x_i)(x_i-\mu)$ $(\$M)^2$
100.0	0					
105.2	0.007	102.6	0.7	0.7	1641.3	10.7
111.5	0.040	108.4	4.3	4.5	1208.5	48.3
130.6	0.229	121.1	27.7	29.9	486.8	111.5
136.3	0.093	133.5	12.4	12.7	93.4	8.7
148.2	0.225	142.3	32.0	33.3	0.7	0.2
165.2	0.278	156.7	43.6	45.9	184.6	51.3
168.7	0.035	167.0	5.8	5.9	568.2	19.9
178.5	0.066	173.6	11.5	11.8	929.5	61.3
183.7	0.021	181.1	3.8	3.9	1443.0	30.3
190.0	0.007	186.9	1.3	1.3	1912.9	13.4
		$\mu =$	**143.1**	**149.9**		355.6
			$\sigma =$	**15.8**	$\sigma =$	**18.9**

p_i is the probability of occurrence of the x_ith value of the random variable

Parameters of distributions

Computing mean and standard deviation

4. Cumulative frequency distribution

x drilling costs $M	probability	midpoint of range $M	EV $M	xi*pi $M	$(x_i-\mu)^2$ $(\$M)^2$	$p(x_i)(x_i-\mu)$ $(\$M)^2$
100.0	0					
105.2	0.007	102.6	0.7	0.7	1641.3	10.7
111.5	0.040	108.4	4.3	4.5	1208.5	48.3
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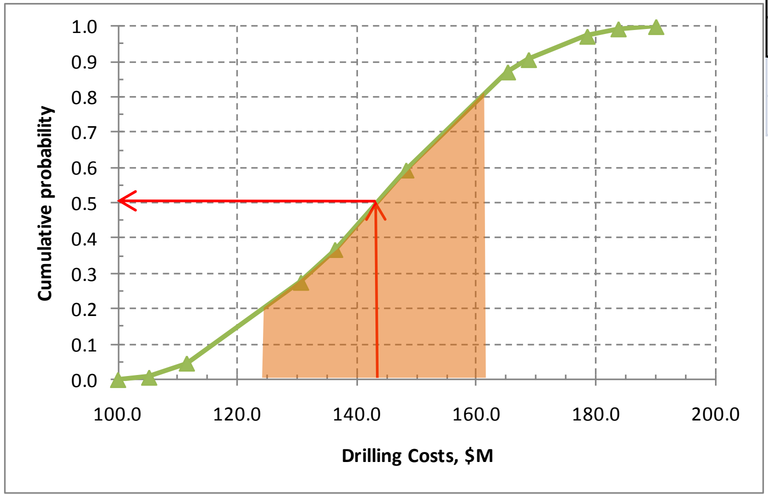
190.0	0.007	186.9	1.3	1.3	1912.9	13.4
		$\mu =$	**143.1**	**149.9**		355.6
			$\sigma =$	**15.8**	$\sigma =$	**18.9**



Types of distributions

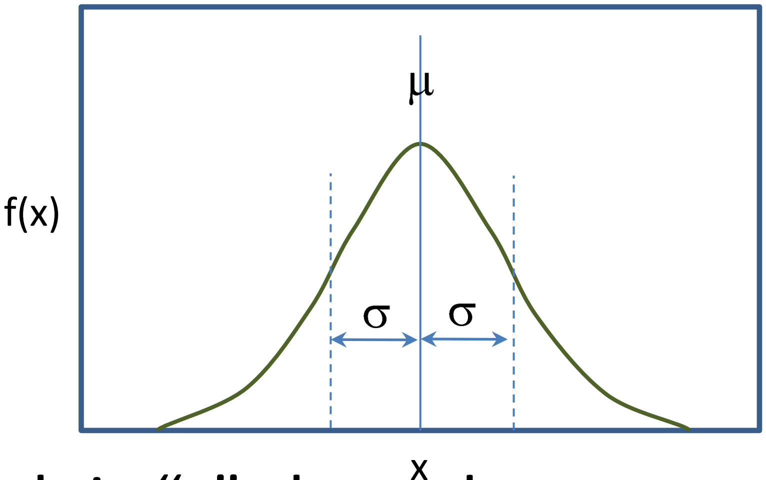
- Normal
- Lognormal
- Uniform
- Triangle
- Binomial
- Multinomial
- hypergeometric

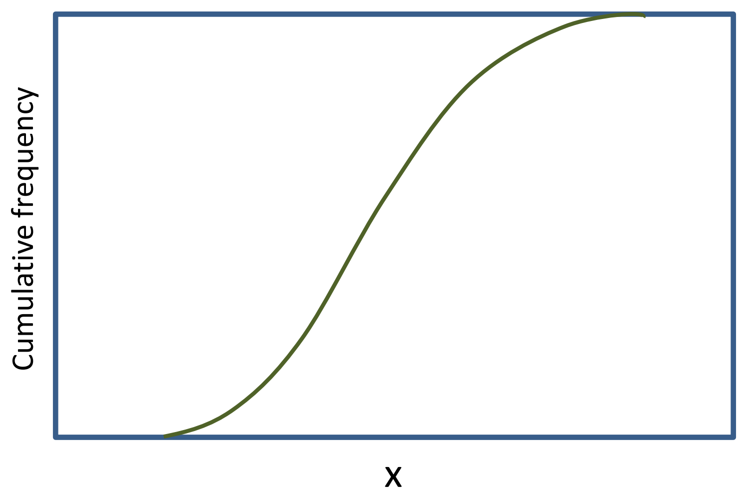
Types of distributions	Normal

Characteristics

- Define by μ and σ
- Mode=mean=median
- Curve is symmetric
- Cumulative frequency graph is "s" shaped
- Can normalize and obtain area (probability) under the curve.

$$t = \frac{x - \mu}{\sigma}$$



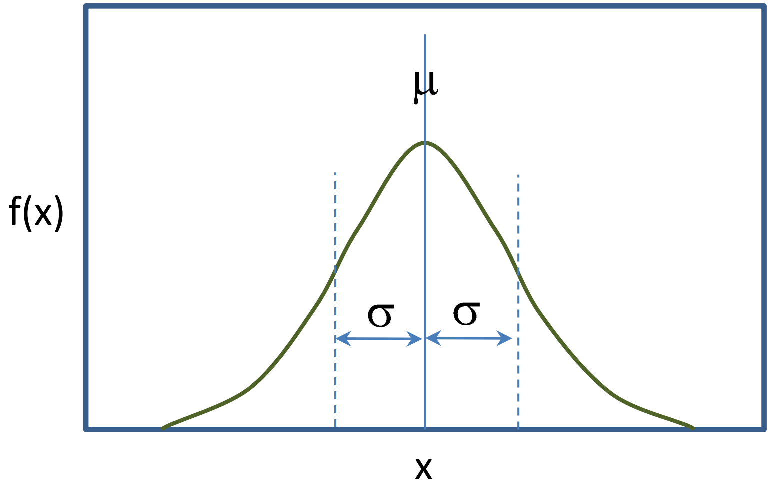


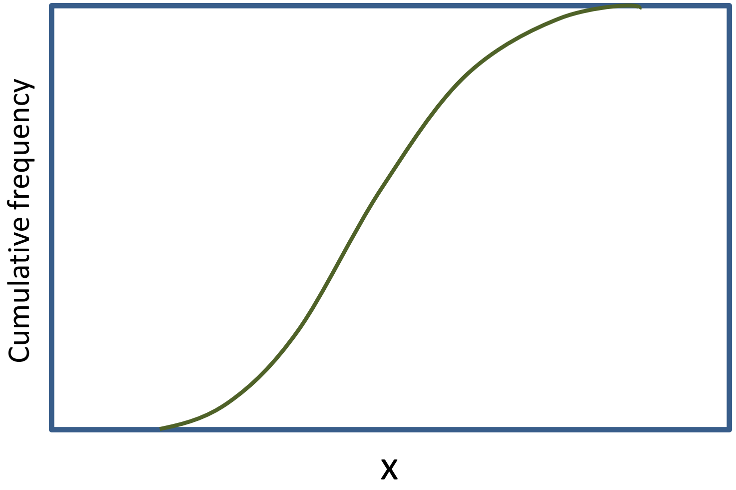
Types of distributions	Normal

Given a set of data how do you know whether it is normally distributed?

- Shape of curves
- median = mean

Examples: porosity, fractional flow

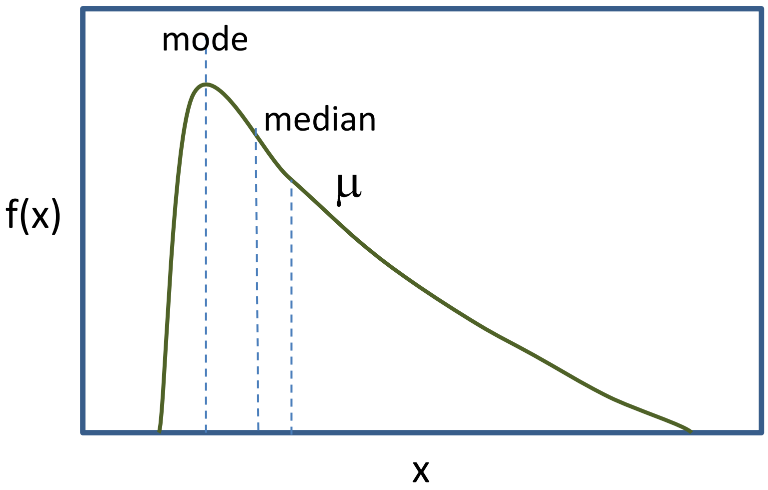


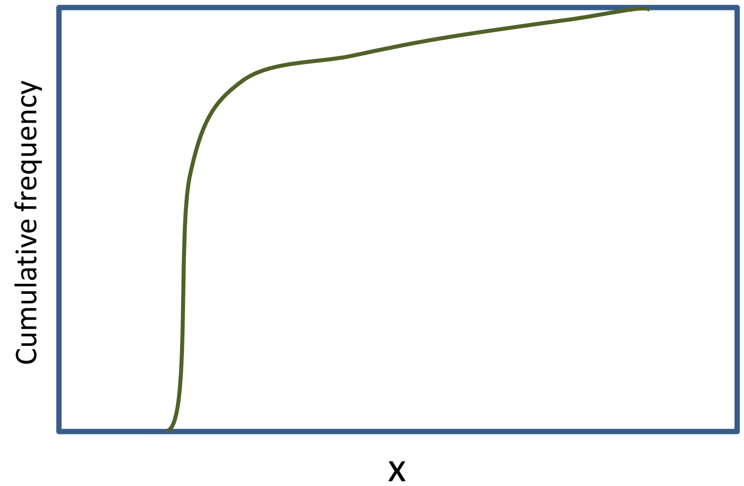


Types of distributions | Lognormal

Characteristics

- Define by μ and σ
- Mode≠mean≠median
- Curve is asymmetric
- Cumulative frequency graph exhibits rapid rise
- Can transform to normal variable by y=ln(x)

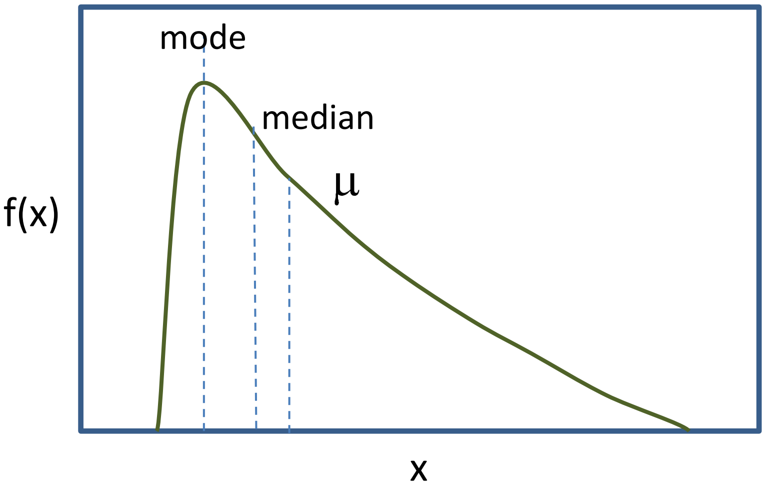




Types of distributions | Lognormal

Examples:

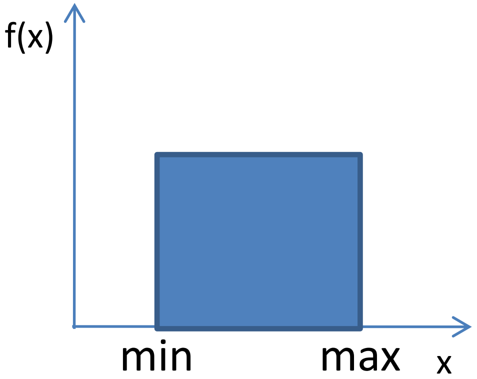
- permeability
- thickness
- oil recovery (bbls/acre-foot)
- field sizes in a play

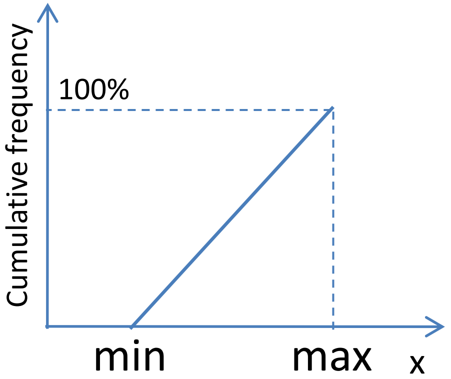


Types of distributions | Uniform

Characteristics:

- all values are equi-probable
- specify min and max
- allows for uncertainty
- used in Monte Carlo simulation

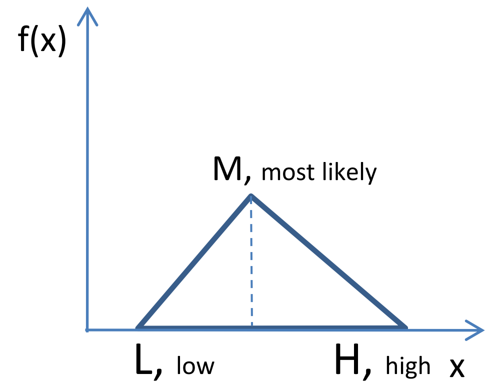


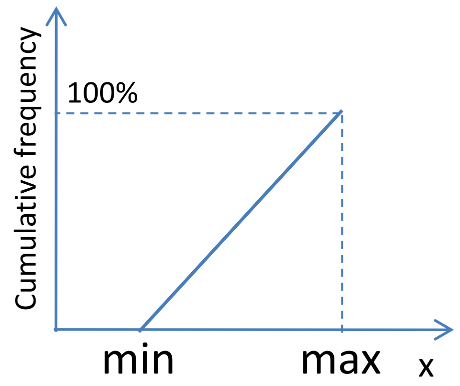


Types of distributions | Triangle

Characteristics:

- all values are equi-probable
- specify min and max
- allows for uncertainty
- used in Monte Carlo simulation

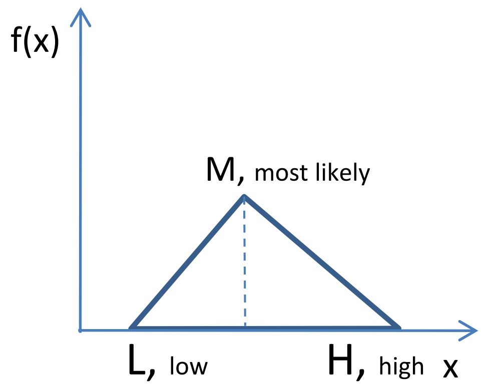




Types of distributions

Triangle

Convert to cumulative frequency plot:



- normalize to a 0 to 1 scale:

$$x' = \frac{x - L}{H - L}$$

- Define m as:

$$m = \frac{M - L}{H - L}$$

- For x’ ≤ m, cumulative probability is given by:

$$P(\leq x) = \frac{(x')^2}{m}$$

- For x’ > m,

$$P(\geq x) = 1 - \left[\frac{(1 - x')^2}{1 - m} \right]$$

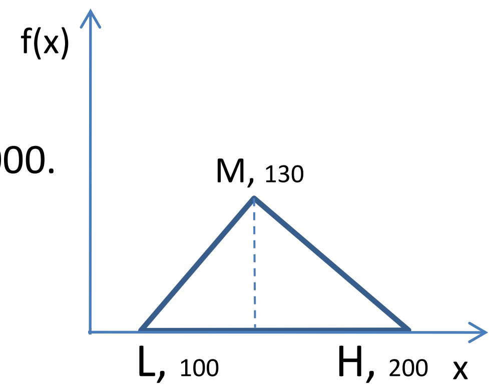
Types of distributions

Triangle

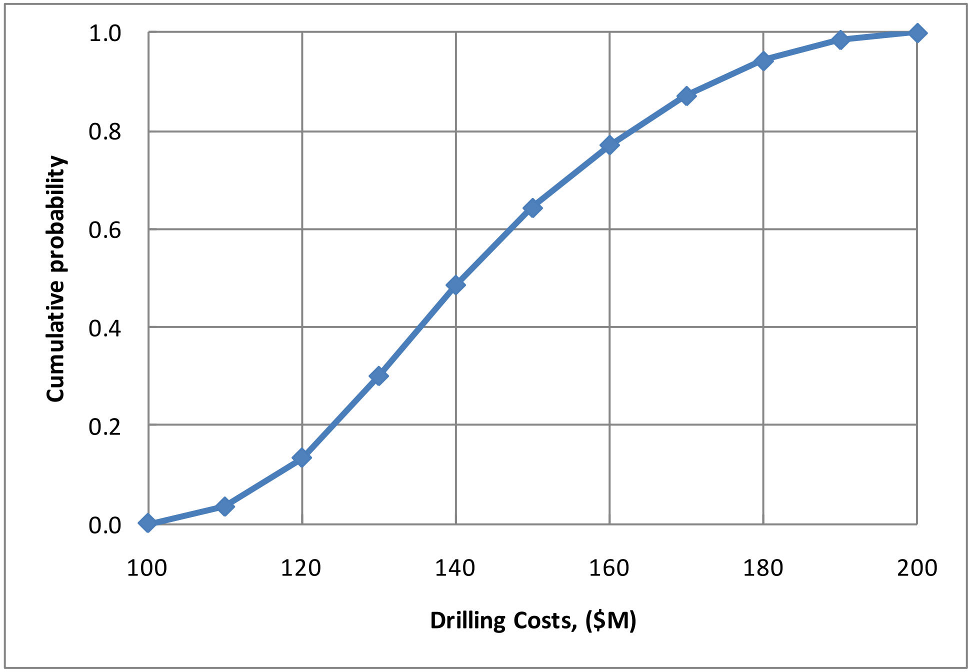
Example

- Estimated costs to drill a well vary from a minimum of $100,000 to a maximum of $200,000,with the most probable value at $130,000.

- Convert the probability distribution to a cumulative frequency distribution



x, random variable (drilling costs)	x' normalized	cumulative probability ≤ x
100	0.0	0.000
110	0.1	0.033
120	0.2	0.133
130	0.3	0.300
140	0.4	0.486
150	0.5	0.643
160	0.6	0.771
170	0.7	0.871
180	0.8	0.943
190	0.9	0.986
200	1.0	1.000



Types of distributions

Binomial

Describes a stochastic process characterized by:

1. Only two outcomes can occur
2. Each trial is an independent event
3. The probability of each outcomes remains constant over repeated trials
4. Binomial probability equation is given by:

$$P(x) = C_x^n p^x (1-p)^{n-x}$$

where

x = number of successes ($0 \le x \le n$)

n = total number of trials

p = probability of success on any given trial

and "the combination of n things taken x at a time"

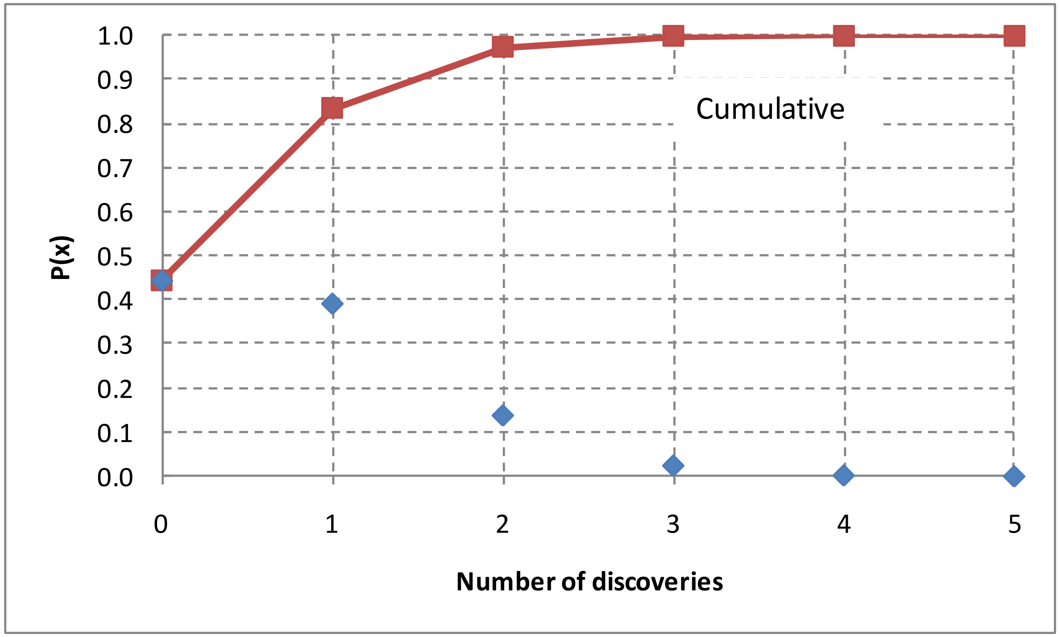
$$C_x^n = \frac{n!}{x!(n-x)!}$$

Types of distributions	Binomial

Example

- Your company proposes to drill 5 wells in a new basin where the chance of success is 0.15 per well
- What is the probability of *only one* discovery in the five wells drilled?
- What is the probability of *at least* one discovery in the 5-well drilling program?

Number of discoveries	P(x)	Cumulative P(x)
0	0.4437	0.4437
1	0.3915	0.8352
2	0.1382	0.9734
3	0.0244	0.9978
4	0.0022	0.9999
5	0.0001	1.0000



Types of distributions

Multinomial

<u>Describes a stochastic process characterized by:</u>

1. Any number of discrete outcomes
2. Each trial is an independent event
3. The probability of each outcomes remains constant over repeated trials
4. Multinomial probability equation is given by:

$$P(x_1, x_2, \ldots, x_r) = \frac{n!}{x_1! x_2! \ldots x_r!} p_1^{x_1} p_2^{x_2} \ldots p_r^{x_r}$$

where

r = number of possible outcomes

x_1 = number of times outcome 1 occurs in *n* trials

x_2 = number of times outcome 2 occurs in *n* trials

x_r = number of times outcome r occurs in *n* trials

n = total number of trials

p_r = probability of outcome *r* on any given trial

Types of distributions	Multinomial

Example

- Your company proposes to drill 10 wells in a new basin where the chance of success is 15% per well
- What is the probability of obtaining 7 dry holes, 2 fields in the 1-2 mmbbl range and 1 field in the 8-12 mmbbl range?

outcome range mmbbl	probability of outcome
1-2	0.08
2-4	0.04
4-8	0.02
8-12	0.01
	0.150
probability of dry hole	0.850

number of trials (wells) in program	n =	10
probability of dry holes	x1 =	7
probability of 1-2 mmbbl	x2 =	2
probability of 2-4 mmbbl	x3 =	0
probability of 4-8 mmbbl	x4 =	0
probability of 8-12 mmbbl	x5 =	1
		0.7%

Types of distributions

Hypergeometric

<u>Describes a stochastic process characterized by:</u>

1. Any number of discrete outcomes
2. Each trial is dependent on the previous event (sampling without replacement)
3. The probability of each outcomes remains constant over repeated trials
4. Hypergeometric probability equation for two possible outcomes:

$$P(x) = \frac{C_x^{d_1} C_{n-x}^{N-d_1}}{C_n^N}$$

where

n=number of trials

d_i = number of successes in the sample space before the n trials

x_i = number of successes in n trials

N = total number of elements in the sample space before the n trials

C^a_b = the number of combinations of a things taken b at a time.

Types of distributions

Hypergeometric

Example

- Our company has identified ten seismic anomalies of about equal size in a new offshore area. In an adjacent area, 30% of the drilled structures were oil productive.
- If we drill 5 wells (test 5 anomalies) what is the probability of two discoveries?

number_sample	n =	5
number_pop	N =	10
population_s	d1 =	3
sample_s	x1 =	2
		42%