



Level 3 Mathematics and Statistics (Statistics), 2013

9.30 am Wednesday 20 November 2013

FORMULAE AND TABLES BOOKLET for 91584, 91585 and 91586

Refer to this booklet to answer the questions in your Question and Answer booklets.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

STATISTICS AND MODELLING - USEFUL FORMULAE AND TABLES

Permutations and Combinations

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

Expectation Algebra

$$E[aX + b] = aE[X] + b$$

$$Var[aX + b] = a^2 Var[X]$$

$$E[aX + bY] = aE[X] + bE[Y]$$

$$Var[aX + bY] = a^2 Var[X] + b^2 Var[Y]$$

if X, Y are independent

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Mean and Variance of a Discrete Random Variable

$$\mu = E(X)$$

$$= \sum x.P(X = x)$$

$$\sigma^{2} = \operatorname{Var}(X)$$

$$\sigma = \operatorname{SD}(X)$$

$$= \sqrt{\sum (x - \mu)^{2}.P(X = x)}$$

$$= \sqrt{E(X^{2}) - [E(X)]^{2}}$$

Continuous Uniform Distribution

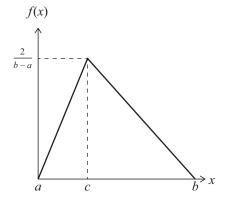
The probability density function, f(x), for a continuous uniform distribution is defined as:

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{for } a \le x \le b \\ 0, & \text{elsewhere} \end{cases}$$

Triangular Distribution

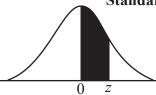
The probability density function, f(x), for a triangular distribution is defined as:

$$f(x) = \begin{cases} 0, & x < a \\ \frac{2(x-a)}{(b-a)(c-a)}, & a \le x \le c \\ \frac{2(b-x)}{(b-a)(b-c)}, & c \le x \le b \\ 0, & x > b \end{cases}$$



Area of a triangle = $\frac{1}{2}$ base × height

_ Standard Normal Distribution



3

aru Normai Distribu

$$\left(Z = \frac{X - \mu}{\sigma}\right)$$

Each entry gives the probability that the standardised normal random variable Z lies between 0 and z.

Differences

										Billerenees									
Z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4	8	12	16	20	24	28	32	36
0.1	1									.0754	4	8	12		20		28	32	36
0.2				.0910							4	8			19			31	
0.3	1			.1293							4		11		19			30	
0.4	1			.1664							4		11		18		-	29	
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3	7	10	14	17	21	24	27	31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3	6	10	13	16	19	23	26	29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3	6	9	12	15	18	21	24	27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3	6	8	11	14	17	19	22	25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3	5	8	10	13	15	18	20	23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2	5	7	9	12	14	16	18	21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2	4	6	8	10	12	14	16	19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2	4	5	7	9	11	13	15	16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2	3	5	6	8	10	11	13	14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1	3	4	6	7	8	10	11	13
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1	2	4	5	6	7	8	10	11
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1	2	3	4	5	6	7	8	9
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1	2	3	3	4	5	6	7	8
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1	1	2	3	4	4	5	6	6
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1	1	2	2	3	4	4	5	5
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	0	1	1	2	2	3	3	4	4
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	0	1	1	2	2	2	3	3	4
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	0	1	1	1	2	2	2	3	3
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	0	0	1	1	1	2	2	2	2
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	0	0	1	1	1	1	1	2	2
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	0	0	0	1	1	1	1	1	1
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	0	0	0	0	1	1	1	1	1
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	0	0	0	0	0	1	1	1	1
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	0	0	0	0	0	0	0	0	1
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	0	0	0	0	0	0	0	0	1
3.0	ł			.4988							0	0	0	0	0	0	0	0	0
3.1				.4991							0	0	0	0	0	0	0	0	0
3.2				.4994							0	0	0	0	0	0	0	0	0
3.3				.4996							0	0	0	0	0	0	0	0	0
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998	0	0	0	0	0	0	0	0	0
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	0	0	0	0	0	0	0	0	0
3.6				.4999							0	0	0	0	0	0	0	0	
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.8				.4999							0	0	0	0	0	0	0	0	0
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0

L3-STATF

Binomial Distribution

Each entry gives the probability that a binomial random variable X, with the parameters n and π , has the value x.

$$\left(\begin{array}{l} P(X=x) = \binom{n}{x} \pi^x \left(1 - \pi \right)^{n-x} \\ \mu = n\pi, & \sigma = \sqrt{n\pi \left(1 - \pi \right)} \end{array} \right)$$

					4.								
n	π	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5
4	0 1 2 3 4	0.8145 0.1715 0.0135 0.0005	0.6561 0.2916 0.0486 0.0036 0.0001	0.5220 0.3685 0.0975 0.0115 0.0005	0.4823 0.3858 0.1157 0.0154 0.0008	0.4096 0.4096 0.1536 0.0256 0.0016	0.3164 0.4219 0.2109 0.0469 0.0039	0.2401 0.4116 0.2646 0.0756 0.0081	0.1975 0.3951 0.2963 0.0988 0.0123	0.1785 0.3845 0.3105 0.1115 0.0150	0.1296 0.3456 0.3456 0.1536 0.0256	0.0915 0.2995 0.3675 0.2005 0.0410	0.0625 0.2500 0.3750 0.2500 0.0625
5	0 1 2 3 4	0.7738 0.2036 0.0214 0.0011	0.5905 0.3281 0.0729 0.0081 0.0005	0.4437 0.3915 0.1382 0.0244 0.0022	0.4019 0.4019 0.1608 0.0322 0.0032	0.3277 0.4096 0.2048 0.0512 0.0064	0.2373 0.3955 0.2637 0.0879 0.0146	0.1681 0.3602 0.3087 0.1323 0.0284	0.1317 0.3292 0.3292 0.1646 0.0412	0.1160 0.3124 0.3364 0.1811 0.0488	0.0778 0.2592 0.3456 0.2304 0.0768	0.0503 0.2059 0.3369 0.2757 0.1128	0.0313 0.1563 0.3125 0.3125 0.1563
	5			0.0001	0.0001	0.0003	0.0010	0.0024	0.0041	0.0053	0.0102	0.0185	0.0313
6	0 1 2 3 4	0.7351 0.2321 0.0305 0.0021 0.0001	0.5314 0.3543 0.0984 0.0146 0.0012	0.3771 0.3993 0.1762 0.0415 0.0055	0.3349 0.4019 0.2009 0.0536 0.0080	0.2621 0.3932 0.2458 0.0819 0.0154	0.1780 0.3560 0.2966 0.1318 0.0330	0.1176 0.3025 0.3241 0.1852 0.0595	0.0878 0.2634 0.3292 0.2195 0.0823	0.0754 0.2437 0.3280 0.2355 0.0951	0.0467 0.1866 0.3110 0.2765 0.1382	0.0277 0.1359 0.2780 0.3032 0.1861	0.0156 0.0938 0.2344 0.3125 0.2344
	5 6		0.0001	0.0004	0.0006	$0.0015 \\ 0.0001$	$0.0044 \\ 0.0002$	$0.0102 \\ 0.0007$	$0.0165 \\ 0.0014$	$0.0205 \\ 0.0018$	$0.0369 \\ 0.0041$	$0.0609 \\ 0.0083$	0.0938 0.0156
7	0 1 2 3 4	0.6983 0.2573 0.0406 0.0036 0.0002	0.4783 0.3720 0.1240 0.0230 0.0026	0.3206 0.3960 0.2097 0.0617 0.0109	0.2791 0.3907 0.2344 0.0781 0.0156	0.2097 0.3670 0.2753 0.1147 0.0287	0.1335 0.3115 0.3115 0.1730 0.0577	0.0824 0.2471 0.3177 0.2269 0.0972	0.0585 0.2048 0.3073 0.2561 0.1280	0.0490 0.1848 0.2985 0.2679 0.1442	0.0280 0.1306 0.2613 0.2903 0.1935	0.0152 0.0872 0.2140 0.2918 0.2388	0.0078 0.0547 0.1641 0.2734 0.2734
	5 6 7		0.0002	0.0012 0.0001	0.0019 0.0001	0.0043 0.0004	0.0115 0.0013 0.0001	$\begin{array}{c} 0.0250 \\ 0.0036 \\ 0.0002 \end{array}$	0.0384 0.0064 0.0005	0.0466 0.0084 0.0006	0.0774 0.0172 0.0016	0.1172 0.0320 0.0037	0.1641 0.0547 0.0078
8	0 1 2 3 4	0.6634 0.2793 0.0515 0.0054 0.0004	0.4305 0.3826 0.1488 0.0331 0.0046	0.2725 0.3847 0.2376 0.0839 0.0185	0.2326 0.3721 0.2605 0.1042 0.0260	0.1678 0.3355 0.2936 0.1468 0.0459	0.1001 0.2670 0.3115 0.2076 0.0865	0.0576 0.1977 0.2965 0.2541 0.1361	0.0390 0.1561 0.2731 0.2731 0.1707	0.0319 0.1373 0.2587 0.2786 0.1875	0.0168 0.0896 0.2090 0.2787 0.2322	0.0084 0.0548 0.1569 0.2568 0.2627	0.0039 0.0313 0.1094 0.2188 0.2734
	5 6 7 8		0.0004	0.0026 0.0002	0.0042 0.0004	0.0092 0.0011 0.0001	0.0231 0.0038 0.0004	0.0467 0.0100 0.0012 0.0001	0.0683 0.0171 0.0024 0.0002	0.0808 0.0217 0.0033 0.0002	0.1239 0.0413 0.0079 0.0007	0.1719 0.0703 0.0164 0.0017	0.2188 0.1094 0.0313 0.0039
9	0 1 2 3 4	0.6302 0.2985 0.0629 0.0077 0.0006	0.3874 0.3874 0.1722 0.0446 0.0074	0.2316 0.3679 0.2597 0.1069 0.0283	0.1938 0.3489 0.2791 0.1302 0.0391	0.1342 0.3020 0.3020 0.1762 0.0661	0.0751 0.2253 0.3003 0.2336 0.1168	0.0404 0.1556 0.2668 0.2668 0.1715	0.0260 0.1171 0.2341 0.2731 0.2048	0.0207 0.1004 0.2162 0.2716 0.2194	0.0101 0.0605 0.1612 0.2508 0.2508	0.0046 0.0339 0.1110 0.2119 0.2600	0.0020 0.0176 0.0703 0.1641 0.2461
	5 6 7 8 9		0.0008 0.0001	0.0050 0.0006	0.0078 0.0010 0.0001	0.0165 0.0028 0.0003	0.0389 0.0087 0.0012 0.0001	0.0735 0.0210 0.0039 0.0004	0.1024 0.0341 0.0073 0.0009 0.0001	0.1181 0.0424 0.0098 0.0013 0.0001	0.1672 0.0743 0.0212 0.0035 0.0003	0.2128 0.1160 0.0407 0.0083 0.0008	0.2461 0.1641 0.0703 0.0176 0.0020
10	0 1 2 3 4	0.5987 0.3151 0.0746 0.0105 0.0010	0.3487 0.3874 0.1937 0.0574 0.0112	0.1969 0.3474 0.2759 0.1298 0.0401	0.1615 0.3230 0.2907 0.1550 0.0543	0.1074 0.2684 0.3020 0.2013 0.0881	0.0563 0.1877 0.2816 0.2503 0.1460	0.0282 0.1211 0.2335 0.2668 0.2001	0.0173 0.0867 0.1951 0.2601 0.2276	0.0135 0.0725 0.1757 0.2522 0.2377	0.0060 0.0403 0.1209 0.2150 0.2508	0.0025 0.0207 0.0763 0.1665 0.2384	0.0010 0.0098 0.0439 0.1172 0.2051
	5 6 7 8 9	0.0001	0.0015 0.0001	0.0085 0.0012 0.0001	0.0130 0.0022 0.0002	0.0264 0.0055 0.0008 0.0001	0.0584 0.0162 0.0031 0.0004	0.1029 0.0368 0.0090 0.0014 0.0001	0.1366 0.0569 0.0163 0.0030 0.0003	0.1536 0.0689 0.0212 0.0043 0.0005	0.2007 0.1115 0.0425 0.0106 0.0016	0.2340 0.1596 0.0746 0.0229 0.0042	0.2461 0.2051 0.1172 0.0439 0.0098
	10	(all other entries < 0.0001) 0.0003 0.0010											0.0010

Poisson Distribution

Each entry gives the probability that a Poisson random variable X, with parameter λ , has the value x.

$$\begin{pmatrix} P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!} \\ \mu = \lambda, \quad \sigma = \sqrt{\lambda} \end{pmatrix}$$

$x \lambda$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0 1 2 3 4	0.9048 0.0905 0.0045 0.0002	0.8187 0.1637 0.0164 0.0011 0.0001	0.7408 0.2222 0.0333 0.0033 0.0003	0.6703 0.2681 0.0536 0.0072 0.0007	0.6065 0.3033 0.0758 0.0126 0.0016	0.5488 0.3293 0.0988 0.0198 0.0030	0.4966 0.3476 0.1217 0.0284 0.0050	0.4493 0.3595 0.1438 0.0383 0.0077	0.4066 0.3659 0.1647 0.0494 0.0111	0.3679 0.3679 0.1839 0.0613 0.0153
5 6 7				0.0001	0.0002	0.0004	0.0007 0.0001	0.0012 0.0002	0.0020 0.0003	0.0031 0.0005 0.0001
x λ	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0 1 2 3 4	0.3329 0.3662 0.2014 0.0738 0.0203	0.3012 0.3614 0.2169 0.0867 0.0260	0.2725 0.3543 0.2303 0.0998 0.0324	0.2466 0.3452 0.2417 0.1128 0.0395	0.2231 0.3347 0.2510 0.1255 0.0471	0.2019 0.3230 0.2584 0.1378 0.0551	0.1827 0.3106 0.2640 0.1496 0.0636	0.1653 0.2975 0.2678 0.1607 0.0723	0.1496 0.2842 0.2700 0.1710 0.0812	0.1353 0.2707 0.2707 0.1804 0.0902
5 6 7 8 9	0.0045 0.0008 0.0001	0.0062 0.0012 0.0002	0.0084 0.0018 0.0003 0.0001	0.0111 0.0026 0.0005 0.0001	0.0141 0.0035 0.0008 0.0001	0.0176 0.0047 0.0011 0.0002	0.0216 0.0061 0.0015 0.0003 0.0001	0.0260 0.0078 0.0020 0.0005 0.0001	0.0309 0.0098 0.0027 0.0006 0.0001	0.0361 0.0120 0.0034 0.0009 0.0002
$x \lambda$	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
0 1 2 3 4	0.1108 0.2438 0.2681 0.1966 0.1082	0.0907 0.2177 0.2613 0.2090 0.1254	0.0743 0.1931 0.2510 0.2176 0.1414	0.0608 0.1703 0.2384 0.2225 0.1557	0.0498 0.1494 0.2240 0.2240 0.1680	0.0408 0.1304 0.2087 0.2226 0.1781	0.0334 0.1135 0.1929 0.2186 0.1858	0.0273 0.0984 0.1771 0.2125 0.1912	0.0224 0.0850 0.1615 0.2046 0.1944	0.0183 0.0733 0.1465 0.1954 0.1954
5 6 7 8 9	0.0476 0.0174 0.0055 0.0015 0.0004	0.0602 0.0241 0.0083 0.0025 0.0007	0.0735 0.0319 0.0118 0.0038 0.0011	0.0872 0.0407 0.0163 0.0057 0.0018	0.1008 0.0504 0.0216 0.0081 0.0027	0.1140 0.0608 0.0278 0.0111 0.0040	0.1264 0.0716 0.0348 0.0148 0.0056	0.1377 0.0826 0.0425 0.0191 0.0076	0.1477 0.0936 0.0508 0.0241 0.0102	0.1563 0.1042 0.0595 0.0298 0.0132
10 11 12 13 14	0.0001	0.0002	0.0003 0.0001	0.0005 0.0001	0.0008 0.0002 0.0001	0.0013 0.0004 0.0001	0.0019 0.0006 0.0002	0.0028 0.0009 0.0003 0.0001	0.0039 0.0013 0.0004 0.0001	0.0053 0.0019 0.0006 0.0002 0.0001
$x \lambda$	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0
0 1 2 3 4	0.0150 0.0630 0.1323 0.1852 0.1944	0.0123 0.0540 0.1188 0.1743 0.1917	0.0101 0.0462 0.1063 0.1631 0.1875	0.0082 0.0395 0.0948 0.1517 0.1820	0.0067 0.0337 0.0842 0.1404 0.1755	0.0055 0.0287 0.0746 0.1293 0.1681	0.0045 0.0244 0.0659 0.1185 0.1600	0.0037 0.0207 0.0580 0.1082 0.1515	0.0030 0.0176 0.0509 0.0985 0.1428	0.0025 0.0149 0.0446 0.0892 0.1339
5 6 7 8 9	0.1633 0.1143 0.0686 0.0360 0.0168	0.1687 0.1237 0.0778 0.0428 0.0209	0.1725 0.1323 0.0869 0.0500 0.0255	0.1747 0.1398 0.0959 0.0575 0.0307	0.1755 0.1462 0.1044 0.0653 0.0363	0.1748 0.1515 0.1125 0.0731 0.0423	0.1728 0.1555 0.1200 0.0810 0.0486	0.1697 0.1584 0.1267 0.0887 0.0552	0.1656 0.1601 0.1326 0.0962 0.0620	0.1606 0.1606 0.1377 0.1033 0.0688
10 11 12 13 14	0.0071 0.0027 0.0009 0.0003 0.0001	0.0092 0.0037 0.0013 0.0005 0.0001	0.0118 0.0049 0.0019 0.0007 0.0002	0.0147 0.0064 0.0026 0.0009 0.0003	0.0181 0.0082 0.0034 0.0013 0.0005	0.0220 0.0104 0.0045 0.0018 0.0007	0.0262 0.0129 0.0058 0.0024 0.0009	0.0309 0.0157 0.0073 0.0032 0.0013	0.0359 0.0190 0.0092 0.0041 0.0017	0.0413 0.0225 0.0113 0.0052 0.0022
15 16 17	(all o	other entri	0.0001 es < 0.000	0.0001	0.0002	0.0002 0.0001	0.0003 0.0001	0.0005 0.0002 0.0001	0.0007 0.0002 0.0001	0.0009 0.0003 0.0001