

Let's learn the T-distribution! Note: Learn about the normal distribution first!

For previous distributions the sample size was assumed large ($N > 30$). For sample sizes that are less than 30, otherwise ($N < 30$). Note: Sometimes the t-distribution is known as the student distribution.

The t-distribution allows for use of small samples, but does so by sacrificing certainty with a margin-of-error trade-off. The t-distribution takes into account the sample size using $n-1$ degrees of freedom, which means there is a different t-distribution for every different sample size. If we see the t-distribution against a normal distribution, you'll notice the tail ends increase as the peak get 'squished' down.

It's important to note, that as n gets larger, the t-distribution converges into a normal distribution.

To further explain degrees of freedom and how it relates to the t-distribution, you can think of degrees of freedom as an adjustment to the sample size, such as $(n-1)$. This is connected to the idea that we are estimating something of a larger population, in practice it gives a slightly larger margin of error in the estimate.

Let's define a new variable called t , where :

$$t = \frac{\bar{X} - \mu}{s} \sqrt{N-1} = \frac{\bar{X} - \mu}{s/\sqrt{N}}$$

which is analogous to the z statistic given by

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{N}}$$

The sampling distribution for t can be obtained:

$$f(t) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\Gamma(\frac{v}{2})} \left(1 + \frac{t^2}{v}\right)^{-\frac{v+1}{2}}$$

Where the gamma function is:

$$\Gamma(n) = (n-1)!$$

And v is the number of degrees of freedom, typically equal to $N-1$.

Similar to a z -score table used with a normal distribution, a t-distribution uses a t-table. Knowing the degrees of freedom and the desired cumulative probability (e.g. $P(T \geq t)$) we can find the

value of t . Here is an example of a lookup table for a t -distribution:

<http://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf>
(<http://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf>)

Now let's see how to get the t -distribution in Python using scipy!

```
In [5]: #Import for plots
import matplotlib.pyplot as plt
%matplotlib inline

#Import the stats library
from scipy.stats import t

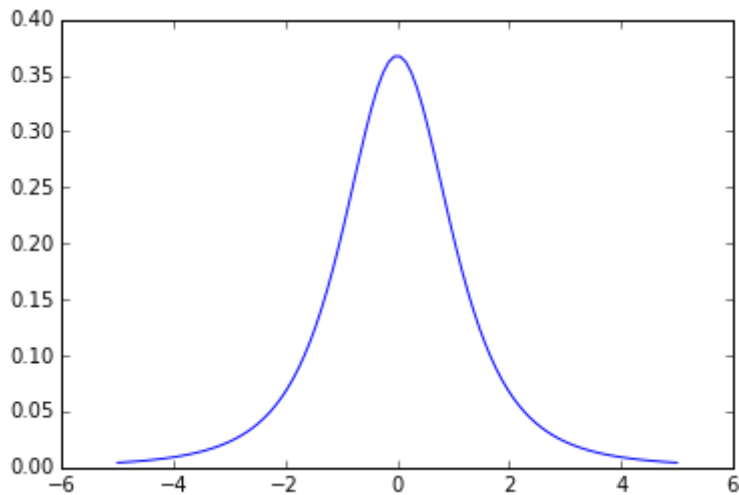
#import numpy
import numpy as np

# Create x range
x = np.linspace(-5,5,100)

# Create the t distribution with scipy
rv = t(3)

# Plot the PDF versus the x range
plt.plot(x, rv.pdf(x))
```

Out[5]: [



Additional resources can be found here:

1.) http://en.wikipedia.org/wiki/Student%27s_t-distribution
(http://en.wikipedia.org/wiki/Student%27s_t-distribution)

2.) <http://mathworld.wolfram.com/Studentst-Distribution.html>
(<http://mathworld.wolfram.com/Studentst-Distribution.html>)

3.) <http://stattrek.com/probability-distributions/t-distribution.aspx> (<http://stattrek.com/probability-distributions/t-distribution.aspx>)

In []: