What would you do with more free time?

Introduction to Basic Reliability Statistics

James Wheeler, CMRP

Allied®

Objectives

- Arithmetic Mean
- Standard Deviation
- Correlation Coefficient
- Estimating MTBF
  - Type I Censoring
  - Type II Censoring
- Exponential Distribution
- Reliability Predictions
- Weibull Curves and Intro to Weibull Analysis
- Basic System Reliability
  - Series System
  - Active Parallel Systems

Arithmetic Mean

- The arithmetic mean or simply “mean” is the sum of a group of numbers divided by the number of items in the group.
- In statistics, this is denoted by $\bar{x}$ (pronounced “x bar”)
- Example: What is the arithmetic mean of 24,37,16 and 21?

$$\bar{x} = (24 + 37 + 16 + 21) = 98 \div 4 = 24.5$$

Arithmetic Mean

Example

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$x_1 = 24$$

$$x_2 = 37$$

$$x_3 = 16$$

$$x_4 = 21$$

$$\bar{x} = \frac{1}{4}\sum_{i=1}^{4} x_i$$

$$\bar{x} = \frac{1}{4}(24 + 37 + 16 + 21)$$

$$\bar{x} = \frac{1}{4}(98)$$

$$\bar{x} = \frac{98}{4}$$

$$\bar{x} = 24.5$$

Standard Deviation

- Standard deviation is the measure of statistical dispersion in a set of numbers.
- It is the Root Mean Square (RMS) of the deviation from the arithmetic mean of a group of numbers.
- If the data points are all close to the mean then the Standard Deviation is close to zero.
- If the data points are far from the mean then the standard deviation is far from zero.
- Standard deviation is noted by the lower case Greek letter Sigma (σ)

Standard Deviation
Example

$x_1 = 24$
$x_2 = 37$
$x_3 = 16$
$x_4 = 21$

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

For known population size

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Estimate for unknown population size

$$\sigma = \sqrt{\frac{1}{4}[(24 - 24.5)^2 + (37 - 24.5)^2 + (16 - 24.5)^2 + (21 - 24.5)^2]}$$

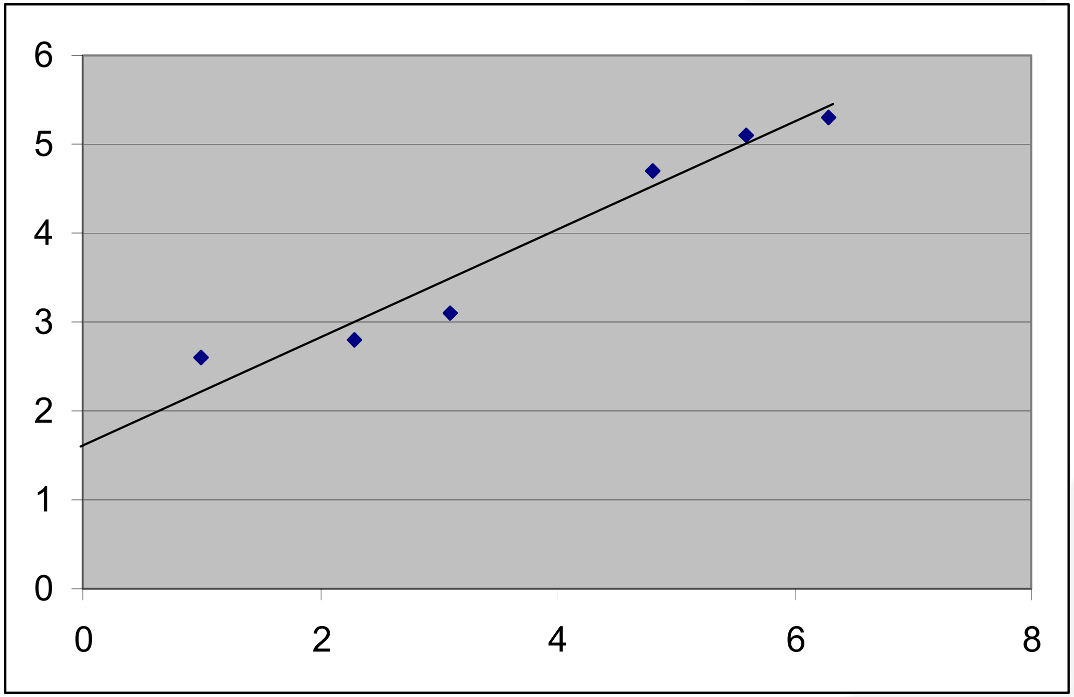
$$\sigma = 7.76208$$

Correlation Coefficient

- Is the likelihood that 2 sets of numbers are related
  - The closer the correlation value gets to 1.0, the more linear the relationship between the 2 sets of numbers.
- It is based on calculations of slope (m), y-intercept (b) and correlation (r)

x	y
1	2.6
2.3	2.8
3.1	3.1
4.8	4.7
5.6	5.1
6.3	5.3

Slope	0.584
Y-Intercept	1.684
Correlation	0.974



Correlation Coefficient

- Using the X & Y values, there is a non-graphical method for calculating slope (m), y intercept (b) and correlation (r).

$$m = \frac{n\Sigma(xy) - \Sigma x \Sigma y}{n\Sigma(x^2) - (\Sigma x)^2}$$

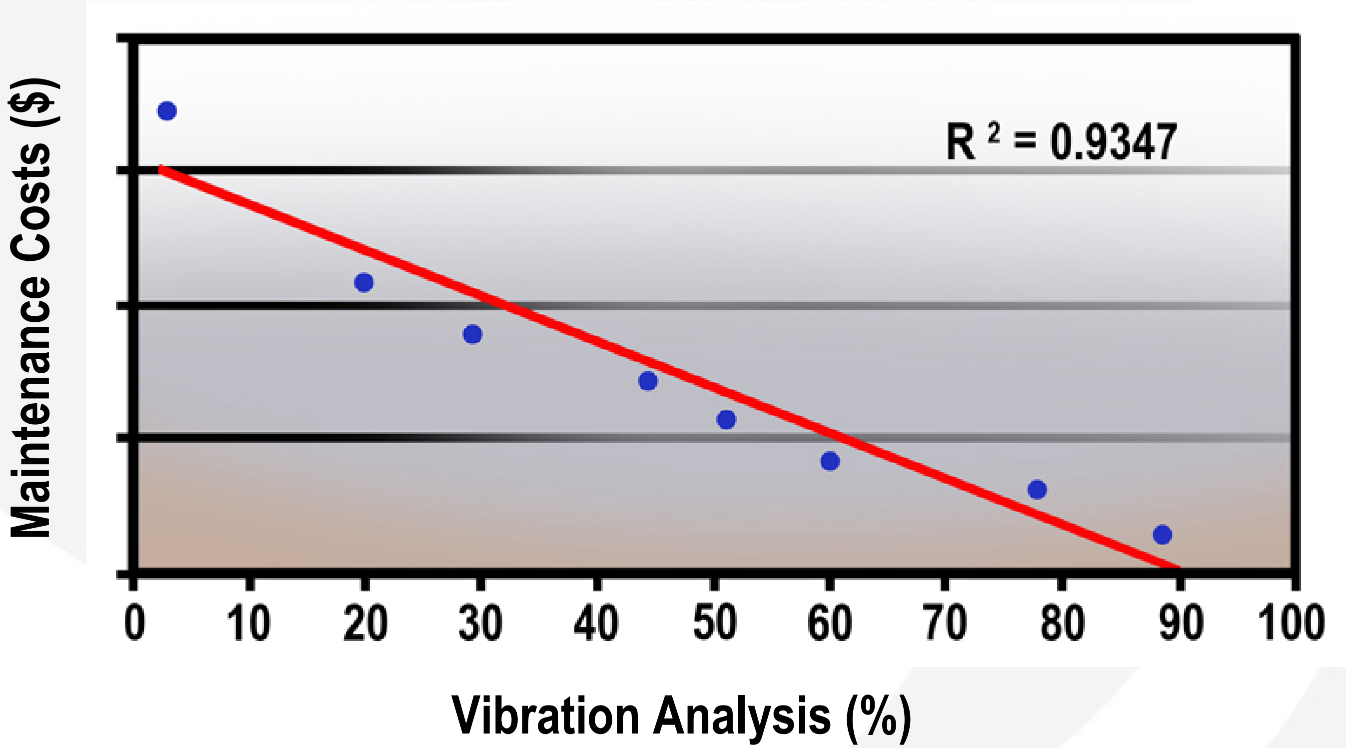
$$b = \frac{\Sigma y - m\Sigma x}{n}$$

$$r = \frac{n\Sigma(xy) - \Sigma x \Sigma y}{\sqrt{\left[n\Sigma(x^2) - (\Sigma x)^2\right]\left[n\Sigma(y^2) - (\Sigma y)^2\right]}}$$

Correlation Coefficient

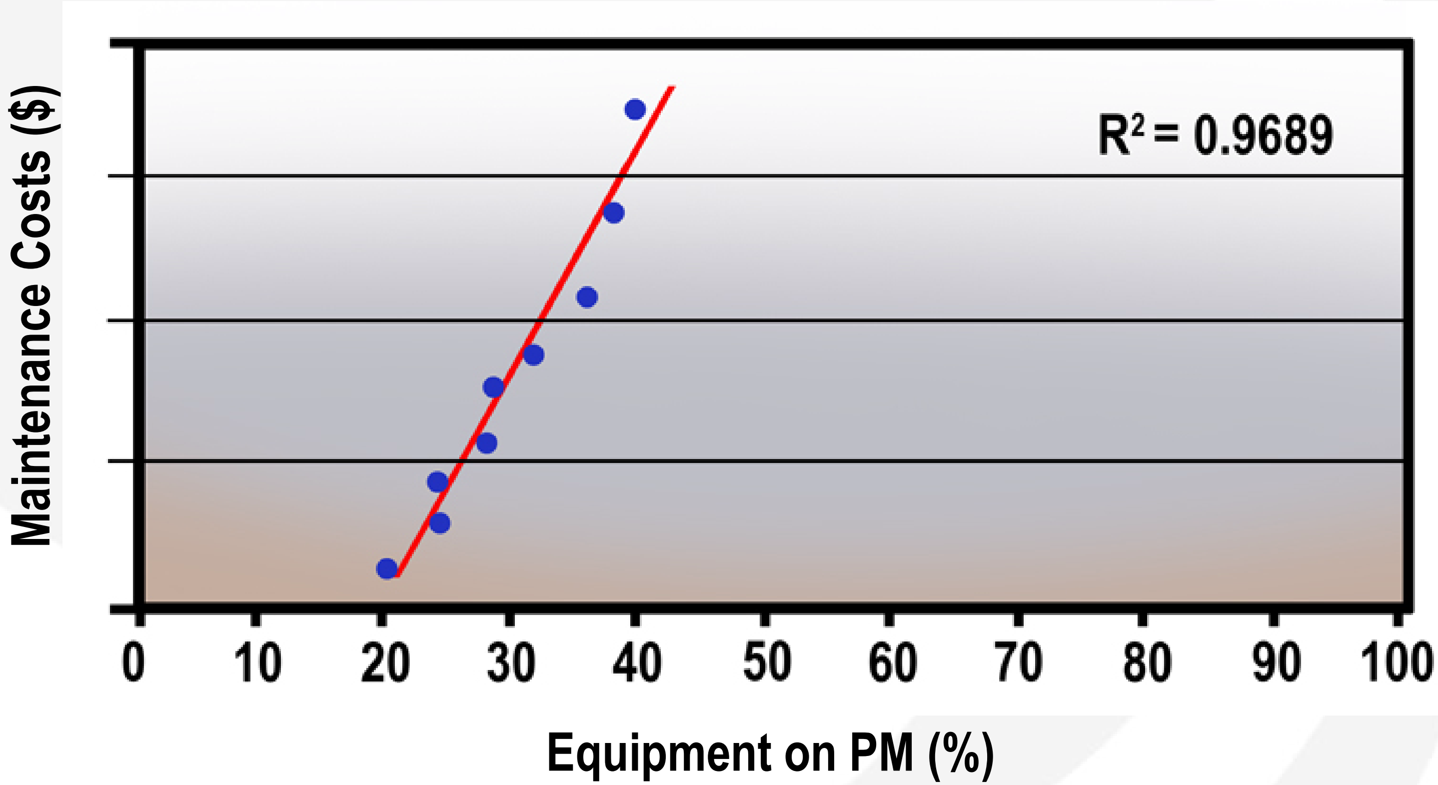
- Why do I need this? How can I use it?
- Does equipment become more prone to failure or more expensive failures as it ages?
  - Collect some ages and failure rate data and find out?
  - Collect some ages and MTBF and find out?
- Other examples:
  - For a pump, are motor amps and gallons per minute perfectly linear?

Maintenance Costs versus Vibration Analysis (PdM)



Source: 1997 Benchmarking Study in Chemical Processing industry, John Schultz to be featured in Ron Moore's new book *What Tool? When? Selecting the Right Manufacturing Improvement Strategies and Tools*

Maintenance Costs versus Equipment on PM



Source: 1997 Benchmarking Study in Chemical Processing industry, John Schultz to be featured in Ron Moore's new book *What Tool? When? Selecting the Right Manufacturing Improvement Strategies and Tools*

Mean Time Between Failures

- MTBF is supposed to be calculated for each individual asset

- Do you calculate it at your plant?

Estimating MTBF
Type I Censoring

- a.k.a. Time/Cycle Truncated Censoring
- Test is halted at a given number of hours.
- Failures during the test are immediately repaired and the test continues

$$\hat{\Theta} = \frac{nt}{r}$$

Where: $\hat{\Theta}$ **= estimate of MTBF**

n = number of items on test

t = total test time per unit

r = # of failures occurring during the test

Estimating MTBF
Type II Censoring

- a.k.a. Failure Truncated Censoring
- Test is halted at a given number of failures
- Failures during the test are immediately repaired and the test continues

$$\hat{\Theta} = \frac{\sum_{i=1}^{r} y_i + (n-r) y_r}{r}$$

$\hat{\Theta}$ = estimate of MTBF

y_i = time to failure i_{th} item

y_r = time to failure of the unit at which time is truncated

n = Total number of assets in test

r = Total number of failures

When would I use MTBF?

- Good question!

- MTBF can be used to help determine maintenance intervals.
- There is a significant flaw with this.

- What does the M in MTBF stand for?
- What does this implicitly tell you?

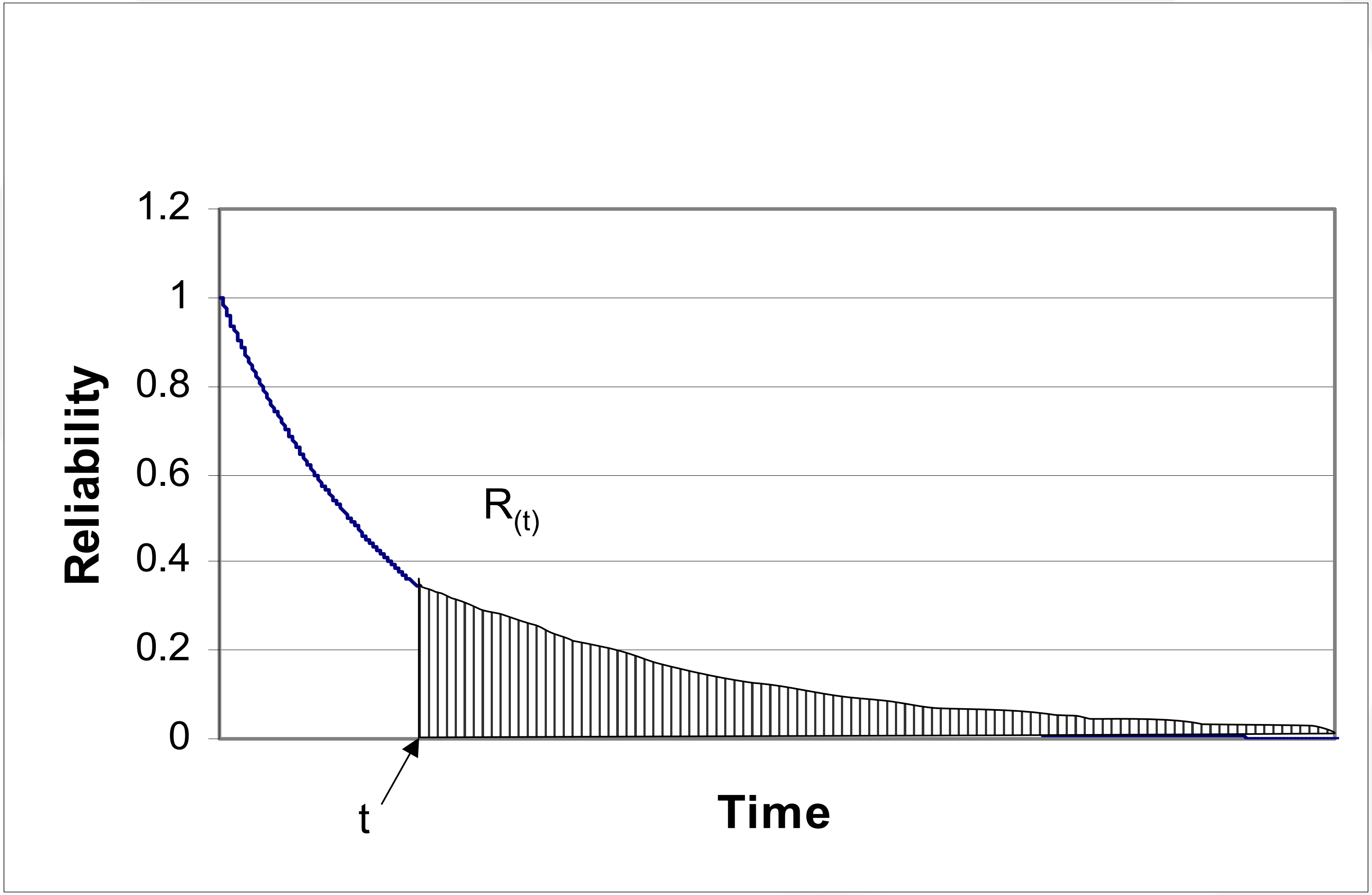
Reliability Predictions

- If I know a little bit about the MTBF for a particular asset…
- I can make some predictions about the life of that asset.

Reliability Predictions

- Q is the probability of failure.
- $Q = 1 - R$
- So then R is the probability of <u>not</u> failing

Exponential Distribution



Reliability Predictions

The reliability for a given time (t) during the random failure period can be calculated with the formula:

$$R_{(t)} = e^{-\lambda t}$$

<u>Where:</u>
e = base of the natural logarithms which is 2.718281828…
λ = failure rate (1/MTBF)
t = time

e - the base of natural logarithms

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$

$$1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \cdots$$

1	1.00000000000000
+ 1/1!	1.00000000000000
+ 1/2!	0.50000000000000
+ 1/3!	0.16666666666667
+ 1/4!	0.04166666666667
+ 1/5!	0.00833333333333
+ 1/6!	0.00138888888889
+ 1/7!	0.00019841269841
+ 1/8!	0.00002480158730
+ 1/9!	0.00000275573192
+ 1/10!	0.00000027557319
e	2.71828180114638

Reliability Predictions
Example

A particular pump has a MTBF of 4,000 hours. What is the probability of operating for a period of 1,500 hours without a failure?

λ = 0.00025 or 1/4,000
t = 1,500

$$e^{-\lambda t} = e^{-(0.00025)(1,500)} = e^{-0.375} = 0.68728$$

68.73% Probability exists of operating 1,500 hours without a failure exists when the MTBF = 4,000 hours.

31.27% Probability exists of a failure before operating 1,500 hours.

Reliability Predictions

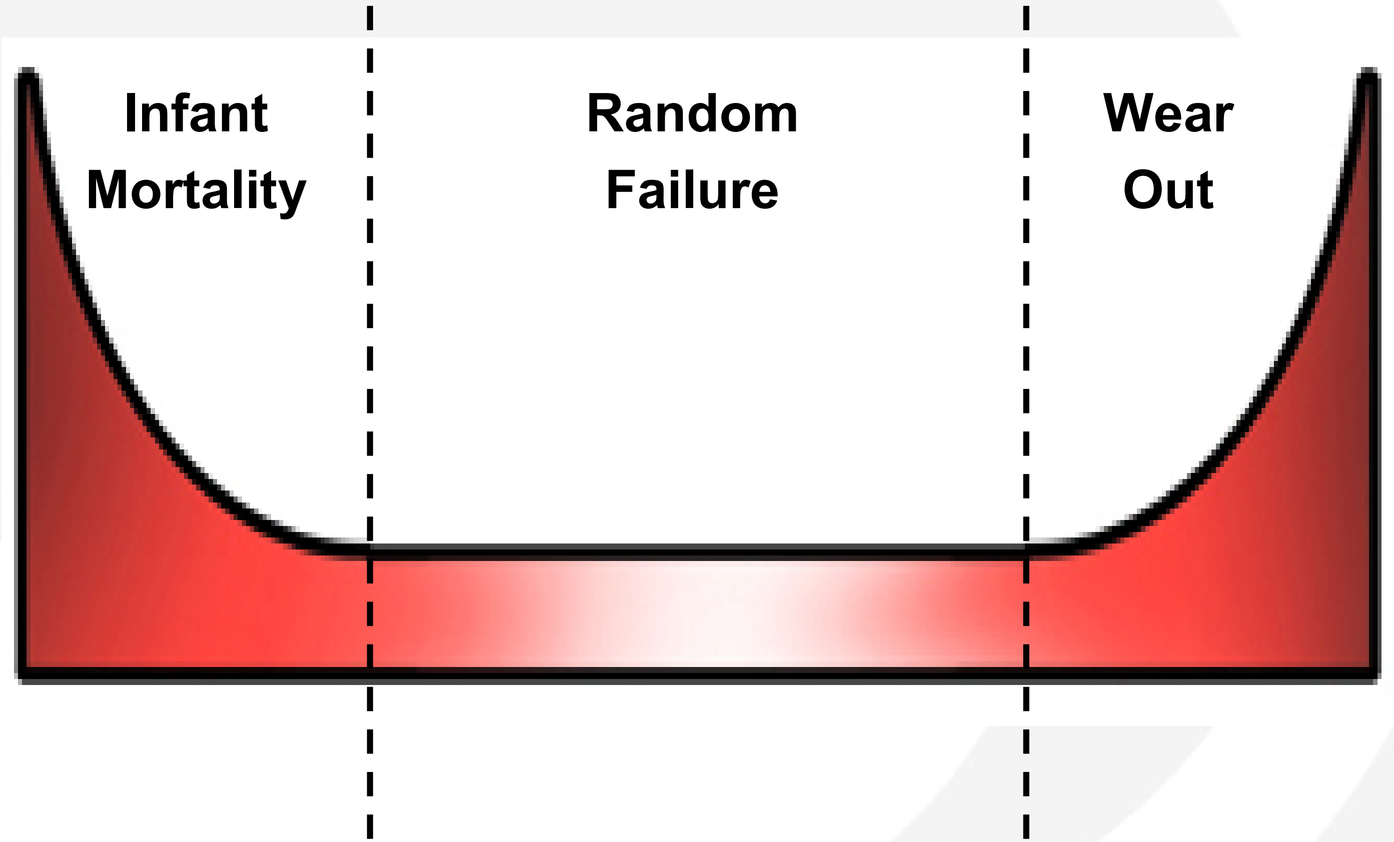
If the reliability for a given time (t) during the random failure period can be calculated with the formula:

$$R_{(t)} = e^{-\lambda t}$$

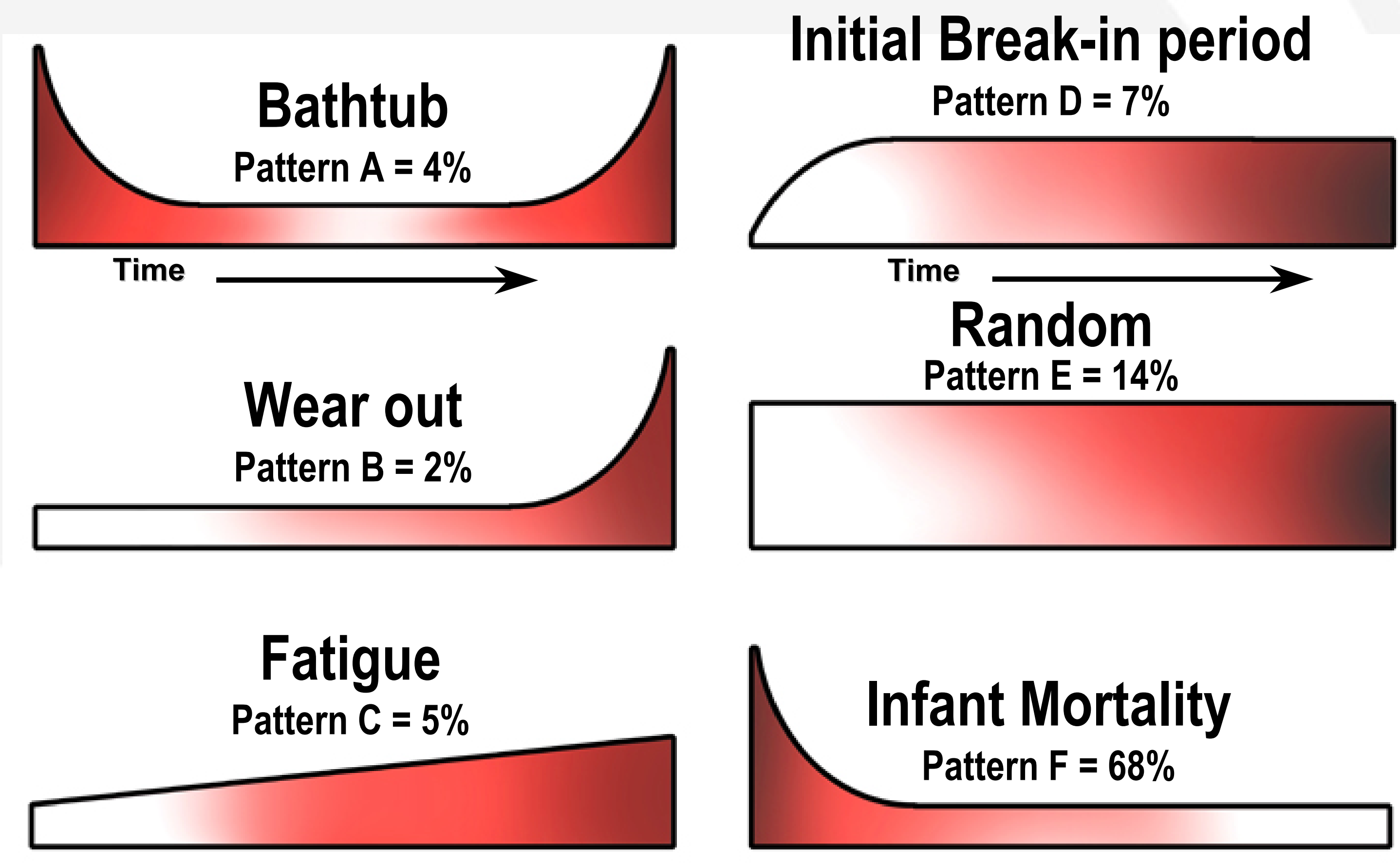
Then what is the equation when I am not in the random failure period? What if I in the infant mortality period or wear-out period?

Then the equation is slightly more difficult…

Overall (Bathtub) Curve



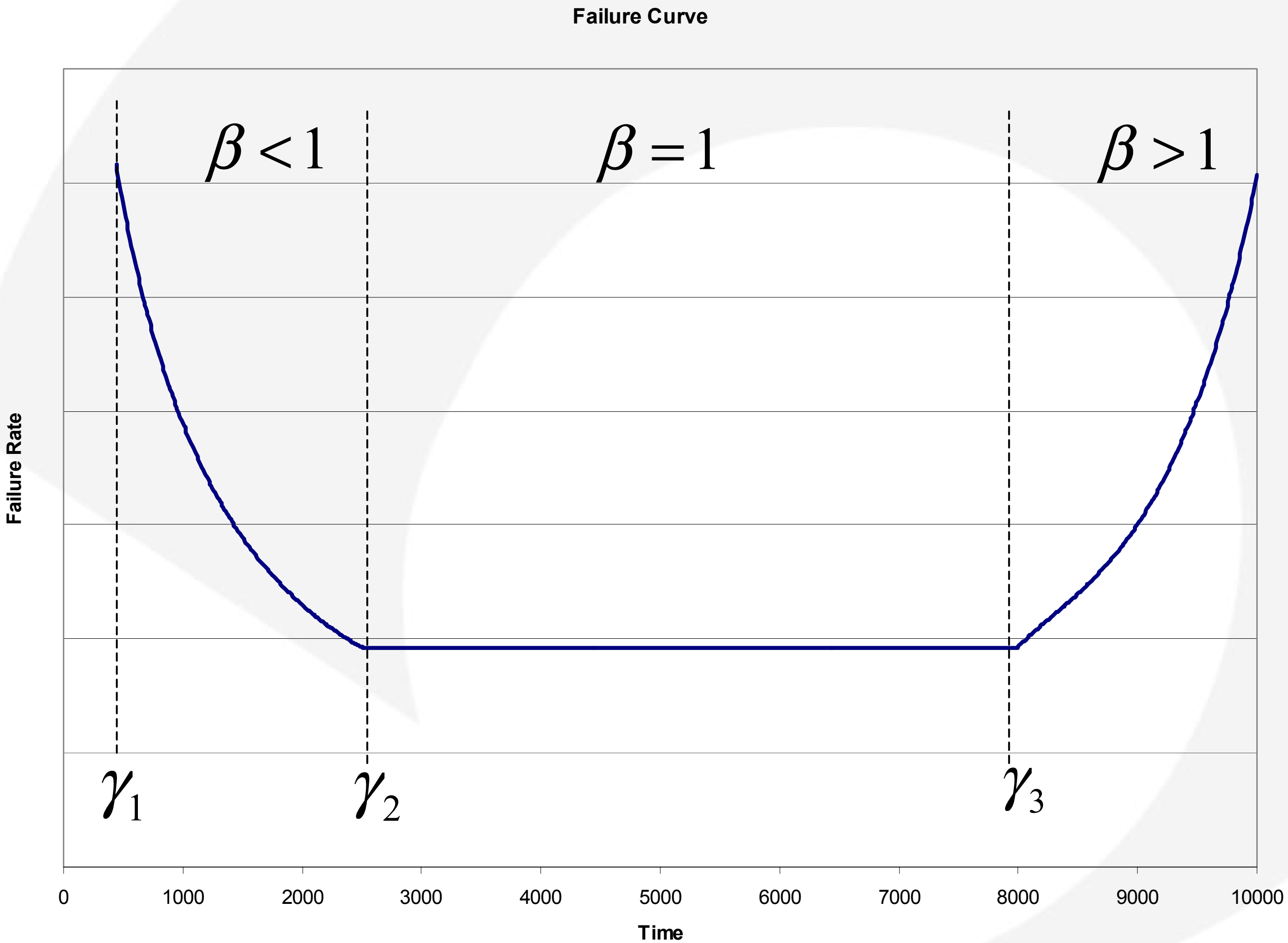
Weibull Shapes
Individual Curves



Age Related = 11%

Random = 89%

Weibull Analysis



$$R_{(t)} = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

System Reliability

- Rarely do assets work alone
- Typically they are a part of a system
- Systems can many different configurations
  - Series
  - Active Parallel
  - “Hot” Standby Parallel
  - “Warm” Standby Parallel
  - “Cold” Standby Parallel
- Reliability calculations for each of these is slightly different

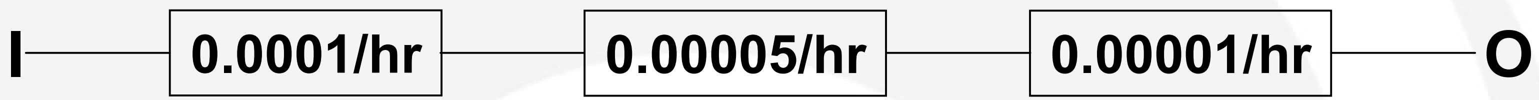
Series Systems - Reliability

- A system whereby the failure of a single machine shuts down the entire system is said to be a “series designed system”

$$R_s = R_1 \times R_2 \times R_3$$

$$R_s = 0.93 \times 0.91 \times 0.80 = 0.677 \text{ or } 67.7\%$$

Series Systems – Failure Probability



$$R_{s(t)} = e^{-\Sigma \lambda_i t_i}$$

$$R_{s(1500)} = e^{-\Sigma(0.0001+0.00005+0.00001)(1500)}$$

$$R_{s(1500)} = e^{-(0.00016)(1500)}$$

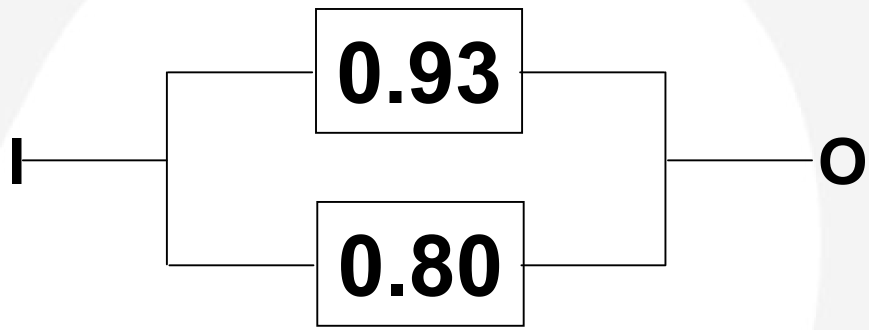
$$R_{s(1500)} = e^{-(0.24)}$$

$$R_{s(1500)} = 78.6\%$$

$$R_{s(1500)} = 0.7866$$

Active Parallel Systems - Reliability

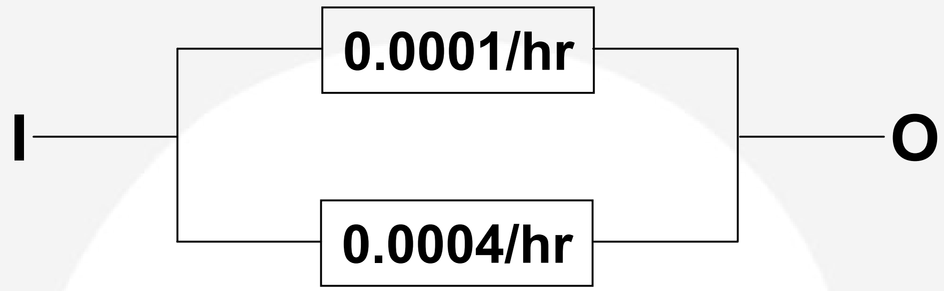
- A system where either machine can carry the full system load and a single failure does not disrupt the system is said to be an "active parallel system"



$$R_s = R_1 + R_2 - R_1R_2$$

$$R_s = 0.93 + 0.80 - 0.93 \times 0.80 = 0.986 \text{ or } 98.6\%$$

Active Parallel Systems – Failure Probability



$$R_{(t)} = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

$$R_{(1500)} = e^{-(0.0001)(1500)} + e^{-(0.0004)(1500)} - e^{-(0.0001+0.0004)(1500)}$$

$$R_{(1500)} = 0.9372$$

$$R_{(1500)} = 93.72\%$$

Questions?

Thanks!

James Wheeler, CMRP
Allied Reliability
wheelerj@alliedreliability.com