

# Parameterization of 2D and 3D Cusp–Core Simulations

## 1 Introduction

This document summarizes the mathematical formulation and parameter choices used in the 2D and 3D particle-on-grid simulations designed to study cusp–core formation. Both frameworks share the same physical principles but differ in dimensional scaling, smoothing kernels, and threshold criteria.

## 2 Governing Equations

The general curvature field  $C(\mathbf{r}, t)$  evolves under particle contributions, smoothing, and repulsion terms:

$$C(\mathbf{r}, t + \Delta t) = C(\mathbf{r}, t) + \Delta t \left[ \sum_i K_{\text{well}} f(|\mathbf{r} - \mathbf{r}_i|) - K_{\text{rep}} g(C) - K_{\text{decay}} C \right]. \quad (1)$$

Here:

- $K_{\text{well}}$ : well strength coefficient.
- $K_{\text{rep}}$ : repulsion coefficient.
- $K_{\text{decay}}$ : damping/decay constant.
- $f(|\mathbf{r} - \mathbf{r}_i|)$ : smoothing kernel centered on particle  $i$ .
- $g(C)$ : nonlinear repulsion function.

### 2.1 2D Kernel

In two dimensions, the smoothing kernel is typically Gaussian-like:

$$f_{2D}(r) = \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad (2)$$

with normalization

$$\int f_{2D}(r) d^2r = 1. \quad (3)$$

## 2.2 3D Kernel

In three dimensions, the kernel generalizes to:

$$f_{3D}(r) = \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad (4)$$

with normalization

$$\int f_{3D}(r) d^3r = 1. \quad (5)$$

Although the functional form matches, the scaling of normalization differs between 2D and 3D.

## 3 Thresholds for Well Formation

A central diagnostic of cusp-core behavior is the detection of curvature wells above a threshold  $C_{\text{th}}$ .

### 3.1 2D Criterion

A well is registered when

$$C_{\text{max}, 2D} > C_{\text{th}, 2D} \approx 0.8 C_{\text{sat}, 2D}, \quad (6)$$

where  $C_{\text{sat}, 2D}$  is the saturation curvature amplitude from repeated particle accumulation.

### 3.2 3D Criterion

Similarly, in 3D:

$$C_{\text{max}, 3D} > C_{\text{th}, 3D} \approx 0.8 C_{\text{sat}, 3D}. \quad (7)$$

The threshold value differs numerically due to dimensional scaling.

## 4 Stochastic Smoothing

Both 2D and 3D formulations include a stochastic smoothing term to mimic thermal fluctuations:

$$C_{\text{smoothed}} = (1 - \eta)C + \eta \xi, \quad (8)$$

where  $\eta$  is the noise strength and  $\xi$  is Gaussian white noise. This term prevents artificial sharp wells and encourages realistic diffusion of curvature.

## 5 Parameter Comparison

Parameter	2D Simulation	3D Simulation
Grid size	$256 \times 256$	$128 \times 128 \times 128$
Particles	$N_{2D} \sim 10^4$	$N_{3D} \sim 10^5$
Kernel width $\sigma$	2–3 grid units	1–2 grid units
Well strength $K_{\text{well}}$	0.1–0.5	0.05–0.2
Repulsion $K_{\text{rep}}$	0.8–1.2	0.6–1.0
Decay $K_{\text{decay}}$	0.01–0.05	0.01–0.03
Threshold $C_{\text{th}}$	$0.8 C_{\text{sat}, 2D}$	$0.8 C_{\text{sat}, 3D}$
Noise strength $\eta$	0.02–0.05	0.01–0.03

## 6 Discussion

The 2D and 3D models reproduce the same qualitative cusp–core transition, despite differences in parameter scaling and kernel normalization. Both display saturation at  $\sim 0.8C_{\text{sat}}$  and a flattening of central curvature profiles, consistent with observational interpretations of dwarf galaxy cores.