Simulation Parameters and Equations: Geometric Lattice Model for the Cusp-Core Problem

Michael Feldstein

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Simulation Overview

This document summarizes the equations, parameters, and initial conditions used in the 2D particle-on-a-grid simulation exploring galaxy core and cusp formation via a dynamic curvature lattice.

Equations of Motion

Curvature Field Evolution

$$\frac{\partial C}{\partial t} = s\,\rho(x, y) - \lambda C + D_C \nabla^2 C$$

where:

- $C(x, y, t)$: lattice curvature field
- s: source term controlling curvature generation from particle density
- $\rho(x, y)$: particle density via Cloud-in-Cell deposition
- λ: decay rate of curvature
- D_C: diffusion coefficient

Particles experience curvature-driven acceleration:

$$a_{x,i}^{\text{curv}} = \mu \frac{\partial C}{\partial x}\bigg|_{x_i, y_i}, \quad a_{y,i}^{\text{curv}} = \mu \frac{\partial C}{\partial y}\bigg|_{x_i, y_i}$$

Density-Dependent Repulsion

Local density around particle i:

$$\rho_{\text{local}}(x_i, y_i) = \frac{1}{\pi r^2} \sum_{j \neq i} \Theta(r - d_{ij}), \quad d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

Repulsive acceleration:

$$\mathbf{a}_i^{\text{rep}} = K_{\text{rep}} \sum_{j \neq i} \frac{\mathbf{r}_i - \mathbf{r}_j}{d_{ij}} \rho_{\text{local}}(x_j, y_j) \Theta(r - d_{ij})$$

Thermal Motion

$$v_{x,i}^{\text{therm}} \sim \mathcal{N}(0, v_{\text{th}}^2), \quad v_{y,i}^{\text{therm}} \sim \mathcal{N}(0, v_{\text{th}}^2)$$

Total Acceleration

$$\mathbf{a}_i = \mathbf{a}_i^{\text{curv}} + \mathbf{a}_i^{\text{rep}} + \mathbf{a}_i^{\text{therm}}$$

Simulation Parameters

Parameter	Dwarf Analog	Massive Analog
Grid size $N \times N$	128	128
Number of particles N_P	2000	20000
Timestep Δt	0.05	0.05
Steps	1000	1000
Repulsion K_{rep}	1.0	0.2
Thermal velocity v_{th}	0.1	0.05
Curvature source s	2.5	2.5
Curvature decay λ	0.005	0.005
Curvature diffusion D_C	0.08	0.08
Curvature-to-accel factor μ	2.0	2.0
Repulsion radius r_{rep}	3	3

Initial Conditions

Particles are initialized from a 2D Gaussian around the center:

$$x_i, y_i \sim \mathcal{N}\left(\frac{N}{2}, \sigma^2\right), \quad \sigma = 10$$

Velocities are initialized to zero. Periodic boundary conditions are applied in both dimensions.

Diagnostics

- Radial density profiles are computed relative to the lattice center.
- Maximum central curvature ($maxC$) is recorded at each timestep.
- Parameter sweep performed for $K_{\text{rep}} \in [0.01, 2.0]$.