# Neural Dynamics

Martin Giese, Ahmed Abdelrazik, Albert Mukovskiy

Email: ahmed.abdelrazik@student.uni-tuebingen.de

Exercise Sheet 1: Due 11.11.24 10:00 Am

**Disclaimer:** *The full number of points can only be assigned if the way how the results were derived is understandable for us. Just "seeing" the solution and writing it down is not sufficient to obtain the full number of points!*

## Exercise 1. Nernst and GHK Equation. Credits: 4

### 1.1

$\frac{\mathrm{d}Q}{\mathrm{d}t}$ Derive the Nernst equation (2), from the Nernst-Planck equation (1).
*Hints: In equilibrium, it holds $j_A = 0$. Integrate the equation spatially across the membrane.*

$$J_\mathrm{A} = J_{\mathrm{A,diff}} + J_{\mathrm{A,drift}} = -D_A\left(\frac{\mathrm{d}\,[\mathrm{A}]}{\mathrm{d}\,x} + \frac{z_\mathrm{A}F}{RT}\,[\mathrm{A}]\,\frac{\mathrm{d}\,V}{\mathrm{d}\,x}\right), \tag{1}$$

$$E_m = \frac{RT}{z_\mathrm{A}F}\log\left(\frac{[\mathrm{A}]_\mathrm{out}}{[\mathrm{A}]_\mathrm{in}}\right). \tag{2}$$

Variables:

- $J_\mathrm{A}$ = The flux of ion A within the electric field, sum of diffusion and drift flux.
- $D_\mathrm{A}$ = The diffusion constant of ion A.
- $[\mathrm{A}]$ = The concentration of ion A at current location.
- $x$ = The current position, from $x_\mathrm{inside}$ =0 to $x_\mathrm{outside}$ =thickness of membrane.
- $z_\mathrm{A}$ = The signed valency of ion A.
- $F$ = Faraday's constant (Coulombs per mole).
- $R$ = The ideal gas constant (Joules per Kelvin per mole).
- $T$ = The temperature (in Kelvin).
- $V$ = The membrane potential (in Volt).
- $E_m$ = The membrane equilibrium potential (in Volt).
- $[\mathrm{A}]_\mathrm{out}$ = The extracellular concentration of ion A.
- $[\mathrm{A}]_\mathrm{in}$ = The intracellular concentration of ion A.

### 1.2

Consider the Goldman Hodgkin Katz (GHK) voltage equation (3), and assume that the membrane of a neuron is selectively permeable for only **one monovalent anion** type A with permeability $\mathrm{P_A}$ (all other permeabilities are zero). Derive the Nernst equation (2) from the GHK voltage equation

$$E_m = \frac{RT}{F}\log\left(\frac{\sum_{i=1}^{N} P_{\mathrm{C}_i^+}\left[\mathrm{C}_i^+\right]_\mathrm{out} + \sum_{j=1}^{M} P_{\mathrm{A}_j^-}\left[\mathrm{A}_j^-\right]_\mathrm{in}}{\sum_{i=1}^{N} P_{\mathrm{C}_i^+}\left[\mathrm{C}_i^+\right]_\mathrm{in} + \sum_{j=1}^{M} P_{\mathrm{A}_j^-}\left[\mathrm{A}_j^-\right]_\mathrm{out}}\right). \tag{3}$$

Variables:

- $\mathrm{C}_i^+$ = One type of monovalent cation (positive charge, e.g. $\mathrm{K}^+$).
- $N$ = Number of relevant cations.
- $\mathrm{A}_j^-$ = One type of monovalent anion (negative charge, e.g. $\mathrm{Cl}^-$).
- $M$ = Number of relevant anions.
- $P_\mathrm{ion}$ = The permeability of an ion type (meters per second).

## Exercise 2. Linear Membrane Model. Credits: 4

### 3.1 Equivalent simplified electrical circuit.

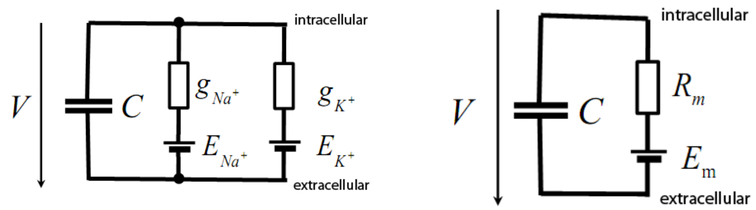


Figure 1: *Left)* Detailed circuit with single channel conductances and reversal potentials. *Right)* Equivalent simplified circuit.

Consider the electrical circuits in Fig. 1. Compute $E_m$ as a function of $g_{\mathrm{K}^+}$, $g_{\mathrm{Na}^+}$, $E_{\mathrm{K}^+}$, $E_{\mathrm{Na}^+}$.

### 3.2

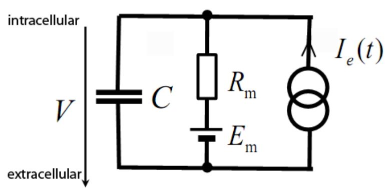


Figure 2: Equivalent circuit with external current $I_e$.

Consider the equivalent circuit of a membrane in Fig. 2. Assuming $V(t) = E_m$ for $t \leq t_0$, derive the response of the membrane potential $V(\cdot)$ to a step input current $I_e(\cdot)$ given by

$$I_e(t) = \begin{cases} 0 & t \leq t_0 \\ I_0 & t > t_0, \end{cases} \tag{4}$$

where $I_0$ is a constant.

### 3.3

Consider the equivalent circuit of a membrane in Fig. 2. Derive the response (of the membrane potential $V(\cdot)$) to a rectangular input current $I_e(t)$ given by

$$I_e(t) = \begin{cases} 0 & t \leq 0 \\ I_0 & 0 < t \leq t_e \\ 0 & t_e < t. \end{cases} \tag{5}$$

*Hint: Use the previous result and exploit that the network is time-invariant.*

## Exercise 3. Programming. Credits: 4

See Jupyter Notebook.