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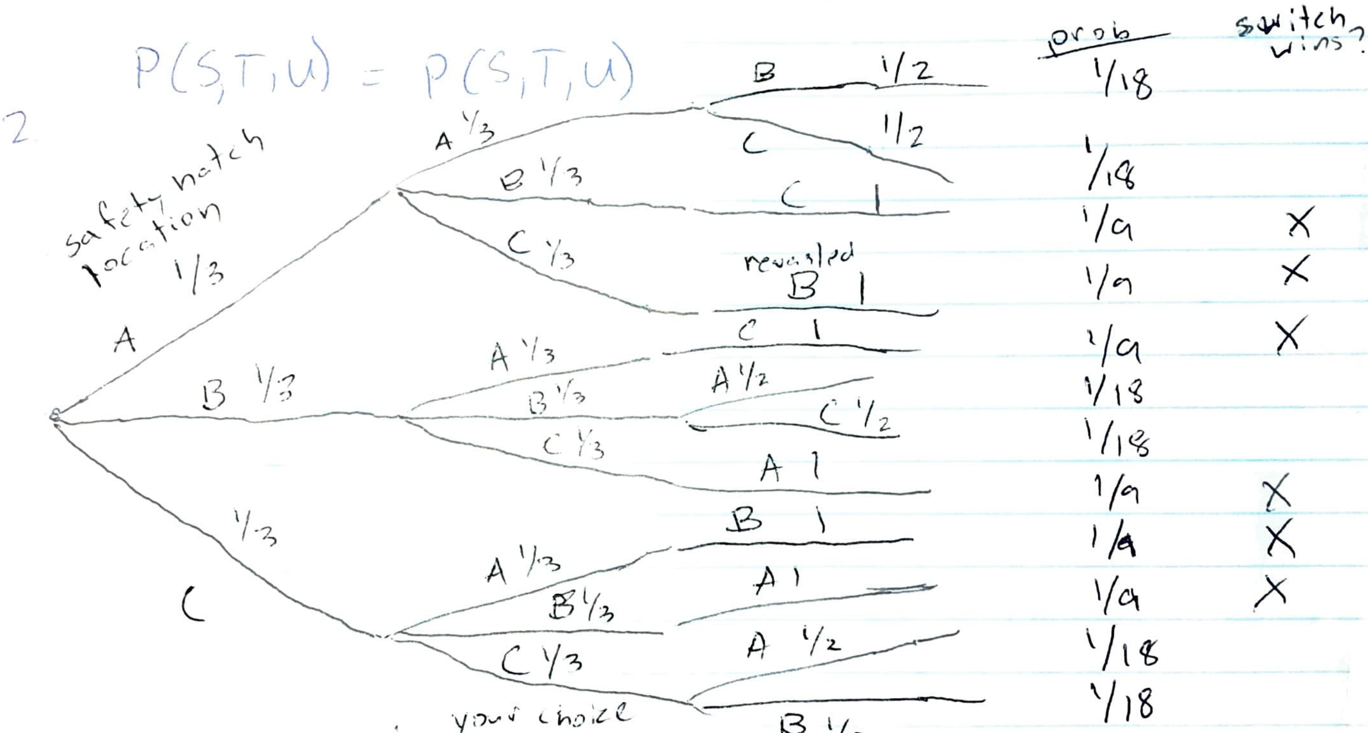
1. $P(S|T) = \frac{P(S,T)}{P(T)}$ Conditional probability

$$P(S,T,U) = P(S|T,U)\,P(T|U)\,P(U)$$

$$\frac{P(S,(T,U))}{\cancel{P(T,U)}} \times \frac{\cancel{P(T,U)}}{\cancel{P(U)}} \times \frac{\cancel{P(U)}}{1}$$

$$P(S,T,U) = P(S,T,U)$$

2.



~~safety guess~~ | ~~door guessed~~ | ~~door revealed~~ | ~~probability~~

$P(\text{Safety}) = 1/3$ $\quad P(\text{HAL reveals}) = 1/2$

$P(\text{safety } 1 \cap \text{HAL reveals } 3) = 1/3 \times 1/2 = 1/6$

$P(\text{safety } 2 \cap \text{HAL reveals } 3) = 1/3 \times 1 = 1/3$

$P(\text{safety } 3 \cap \text{HAL reveals } 2) = 1/3 \times 1 = 1/3$

win $P(\text{stay}) = P(\text{safety } 1 \mid \text{HAL reveals } 3) \rightarrow P(\text{safety } 1 \cap \text{HAL}3) / P(\text{HAL } 3)$

lose $P(\text{stay}) = P(\text{safety } 2 \mid \text{HAL reveals } 3) \rightarrow P(\text{safety} 2 \cap \text{HAL}3) / P(\text{HAL } 3)$

$P(\text{stay} \cap \text{win}) = 1/3$ $\quad P(\text{stay} \cap \text{lose}) = 2/3$

It is always smarter to switch given the possible events.

3. Given $E[f(A)] = E[E(f(A)|B)]$

A & B are random varibles

$= \sum_{b} f(B) P_b(B)$ expected value rule

$= \sum_{b} E[f(a) | B=b] P_b(B)$ total expected theorm

Law of iterated expectations

$= E[X]$ $\quad E[E[X|Y]] = E[X]$

4. $P(S|B) = \prod_{i=1}^{n} \beta e^{-\beta i}$, $P(S|B) = X$

$X = \beta^n e^{-\beta \sum_{i=1}^{n} si}$

$\text{Log} X = n \log \beta - \beta \sum_{i=1}^{n} si$

$\frac{\partial \log L}{\partial B} = \frac{n}{\beta} - \sum_{i=1}^{n} si(1)$

$\frac{n}{\beta} - \sum_{i=1}^{n} si = 0$

$\frac{n}{\beta} = \sum_{i=1}^{n} si$

maximum likelihood

$\hat{\beta} = \frac{n}{\sum_{i=1}^{n} si} = \frac{\partial^2 \log L}{\partial \beta^2}\Big|_{\hat{\beta}} < 0$ $\quad \boxed{\hat{\beta} = \frac{n}{\sum_{i=1}^{n} Si}}$

5. $P(U|S) = 0.9$ $\quad P(U|T) = 0.6$ $\quad P(U|S \cap T)$

$\frac{P(U \cap S)}{P(S)}$ $\quad \frac{P(U \cap T)}{P(T)}$ $\quad \frac{P(U \cap S \cap T)}{P(S \cap T)}$

$\frac{P(S|U \cap T)\, P(U \cap T)}{P(S|T)}$

$P(U|S,T) = \frac{P(S|U,T) \cdot P(S|T)}{P(S|T)}$

Not enough info.

We need $P(U|T)$, $P(S|T)$ and $P(S|U,T)$ but we are only given $P(U|T) = 0.6$ and $P(S|T)$ and $P(S|U,T)$ aren't given.

6. $P(U|S) = 0.9$ $\quad P(S) = .4$ $\quad P(U|T) = 0.6$ $\quad P(T) = 0.5$ $\quad P(U|S \cap T)$

$P(S|T) = \frac{P(S \cap T)}{P(T)} = \frac{.4 \cdot .5}{.5} = .4$

$P(U \cap S) = .9 \times .4$ $\quad P(U \cap T) = .6 \times .5$

$P(U|S,T) = \frac{P(U,S,T)}{P(S,T)}$ not enough information

7. $P(U \cap S) = .3$ $\quad P(S) = .5$ $\quad P(T) = 1$ $\quad P(U|S \cap T) = \frac{P(U,S,T)}{P(S,T)}$

$\frac{P(U,S)}{P(S)} = \frac{.3}{.5} = \boxed{\frac{3}{5}}$

8) $P(G \cap L) = 2/3$ $\quad P(G) = 1/10$ $\quad P(\neg G \cap L) = 1/5$ $\quad P(G \cap L) = ?$

$1/10 \times \frac{2}{3} = \boxed{\frac{1}{15}}$

9. $P(\text{fluffy}) = 2/3$ $P(\text{cute}|\text{fluffy}) = .8$ $P(\text{cute}|\neg\text{fluffy}) = .1$

$$P(B|A') = 0.1 = \frac{P(A' \cap B)}{P(A')} = 0.1$$

$$P(A' \cap B) = .1 \times 1/3 = 1/30$$

$$P(A' \cap B) = P(B) - P(A \cap B)$$

$$P(B) = P(A' \cap B) + P(A \cap B) = 17/30$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{8}{15} \Big/ \frac{17}{30} = \frac{16}{17}$$

$$P(A|B) = 0.9412$$

$$\boxed{0.9412}$$

10.

hungry $1/3$
- cranky 0.7
  - scary $= 0.9$
  - ns $= 0.1$
- notcranky 0.3
  - s $= 0.05$
  - ns $= 0.95$

nothungry $2/3$
- cranky $= 0.1$
  - scary $= 0.9$
  - ns $= 0.1$
- notcranky 0.9
  - scary $= 0.05$
  - ns $= 0.95$

$$\frac{\frac{1}{3} \times 0.7 \times 0.9}{\left(\frac{1}{3} \times 0.7 \times 0.9\right) + \left(1/3 \times 0.3 \times 0.05\right)} = \frac{0.21}{0.21 + 0.005}$$

$$= \boxed{0.9767}$$

10. $P(C|H,S) = P(C,H,S) / P(C,H,S) + P(\neg C,H,S)$

$P(C|H,S) = P(S|CH)\ P(C|H)\ P(H)$

$P(S|CH) = P(S|C)$

$P(S|\neg CH) = P(S|\neg C)$

11. $P(CD) = P(CDA) + P(CD\neg A)$

$P(CD) = 0 + P(CD\neg A)$

$P(C|A) = 0, P(CD) = 0 + P(CD\neg A)$

$P(CD\neg A) = P(C|D\neg A)\ P(D\neg A)$

$= P(C|\neg A)\ P(D\neg A)$

$P(D\neg A) = P(D\neg AB) + P(D\neg A\neg B)$ Knowing B

$P(D\neg AB) = P(D|\neg AB)\ P(B|\neg A)\ P(\neg A)$

$= P(D|B)\ P(B|\neg A)\ P(\neg A)$

$P(D\neg AB) = P(D|\neg AB)\ P(\neg B|\neg A)\ P(\neg A)$

$= P(D|\neg B)\ P(\neg B|\neg A)\ P(\neg A)$

$P(C,D,\neg A) = 1/2.$

d. $E(w) = E(E(w|M))$

e. $E(w^2) = E(E(w^2|M))$

$= E(Var(W|M)) + E(W|M)^2)$ $= E(Var(W|M)) + E(E(W|M)^2)$

12, vanilla $= 1/6$ $Var = E(Var)$

AI prob = given

$E(W) = \sum P(\text{blank} \mid \text{van})m + \sum P(\text{blank} \mid AI)(1-m)$

$Var(w) = \text{Expected}(w^2) - \text{Expected}(w)^2$

$Var(W|M) = E(W^2|M) - E(E(W|M))^2$