$A \to B$
$A \to C$
$\therefore A \to B \wedge C$
Conditional Conjunction

$(x \in P(A)) \leftrightarrow (x \subseteq A)$

$P(A \cup B) = P(A) \cup P(B)$

$P(A \cap B) = P(A) \cap P(B)$

2. $P \subseteq Q$ $(x \in P) \to x \in Q$ def of subset
$(x \in A) \to x \in P(A)$ def of powerset

$A = \{1, 2\}$
$B = \{2, 5\}$
$P(A) = \{\{1,2\}, \{2\}, \{1\}, \{\emptyset\}\}$
$P(B) = \{\{2,5\}, \{2\}, \{5\}, \{\emptyset\}\}$

1. $P(A \cup B) = P(A) \cup P(B)$ is wrong because $P(A \cup B)$ includes $\{1,2,5\}$ which is not equal to $P(A) \cup P(B)$

~~$P(A \cap B) = P(A) \cap P(B)$~~

$x \in P(A \cap B) \Rightarrow x \subseteq A \cap B$ . def of power set on ①

~~$x \in P(A) \cap P(B)$~~
~~$y \in P(A) \cap P(B)$~~
~~$y \in P(A)$~~
~~$y \in P(B)$~~
~~$y \in P(A \cap B)$~~

2. $P(A \cap B) = P(A) \cap P(B)$ is right because

| | |
|---|---|
| $x \in P(A \cap B)$ | def of powerset |
| $x \subset (A \cap B)$ | " " |
| $x \subset A$ and $x \subset B$ | " " |
| $x \in P(A)$ and $x \in P(B)$ | " " |
| $x \in P(A) \cap P(B)$ | " " |
| $P(A) \cap P(B)$ | solution |

vice versa

$P(A) \cap P(B) = P(A \cap B)$
$x \in P(A) \cap P(B)$
$x \in P(A)$ and $x \in P(B)$
$x \subset A$ and $x \subset B$
$x \subset A \cap B$
$P(A \cap B)$

3. A is an empty set

$\exists x \in A, x = \emptyset$ $\quad \neg\exists x\,(x \in A)$

$\forall x\,(x \notin A)$

4. $J(A,B) = \dfrac{|A \cap B|}{|A \cup B|}$, $J(\emptyset,\emptyset) = 1$ $\quad d_J(A,B) = 1 - J(A,B)$

a) $J(\{1,3,5\},\{2,4,6\}) = \dfrac{0}{6} = 0$ $\quad d_J(A,B) = 1$

a2) $J(\{1,2,3,4,5,6\}, B = \{1,2,3,4,5,7\}) = \dfrac{5}{7}$ $\quad d_J(A,B) = \dfrac{3}{7}$

4b) A has cardinality of 5

$J(A,A) = \dfrac{5}{5} = 1$ $\quad d_J(A,B) = 1 - 1 = 0$ $\quad$ TRUE

b2) A has cardinality 10, B's cardinality 5. $\quad$ FALSE

$J(A,B) = \dfrac{10}{5} = 2$ $\quad J(B,A) = \dfrac{5}{10} = .5$ $\quad 2 \neq .5$

$d_J(A,B) = 1 - 2 = -1$ $\quad d_J(A,B) = 1 - .5 = 1.5$

b3) $J(A,B) = 1$ iff $A = B$ $\quad A = 0, B = 0$

$J(0,0) = \dfrac{0}{0} = 1$ $\quad$ TRUE ✓

$J(1,1) = \dfrac{1}{1} = 1$ $\quad J(-1,-1) = \dfrac{-1}{-1} = 1$

① c) ① $c \in \underline{P} \cup Q$ hypothesis

② $c \in \overline{Q} \cap R$ hypothesis

~~double negation~~

~~de morgan's law ③~~

6 3 $c \in \underline{P} \vee c \in Q$ Union ①

7 4 $c \in \overline{Q} \cap \cancel{c \in R}$ Intersection ②

8 5 $c \in \overline{Q}$ simplification ④

⑥ $c \in P$ disjunctive syllogism ③

⑦ $c \in R$ disjunctive syllogism ④

⑧ $c \in P \cap R$ solution

4b1 $J(A,A)=1$ $\quad d_j(A,A)=0$

① $\dfrac{|A \cap A|}{|A \cup A|}$

② $\dfrac{|A|}{|A|}=1$ Idempotent laws

$d_j = 1 - J(A,A) = 0$

4b2) $J(A,B) = J(B,A)$ $\quad d_j(A,B) = d_j(B,A)$

$\dfrac{|A \cap B|}{|A \cup B|} = \dfrac{|B \cap A|}{|B \cup A|}$ Def of Jaccard, comm. law

$\dfrac{|B \cap A|}{|B \cup A|} \equiv J(B,A)$ Definition

$d_j(A,B) = d_j(B,A)$

$1 - J(A,B) \equiv 1 - (|A \cap B| / |A \cup B|)$ def of jaccard

$1 - (|A \cap B| / |A \cup B|) \equiv 1 - (|B \cap A|)/(|B \cup A|)$ comm law.

$1 - (|B \cap A| / |B \cup A|) \equiv 1 - J(B,A)$ def of Jaccard

$1 - J(B,A) \equiv d_j(B,A)$ Def of jaccard.

4b3) ① $(A = B)$

② $(A \not\subseteq B) \vee (B \not\subseteq A)$

③ $(A \not\subseteq B)$

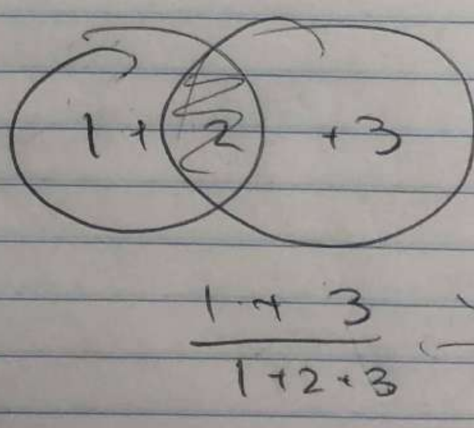
④ $((x \in A) \rightarrow x \in B)$

⑤ $(x \in B \vee \neg x \in A)$

⑥ $\neg x \in B \wedge x \in A$

⑦ $J(A,B) = |A \cap B| / |A \cup B|$

⑧ $\neg(x \in A \cap B)$




$\dfrac{1+3}{1+2+3}$ $\geq$ $\leq 1$

⑨ $|A \cup B| < |A \cap B| \Leftrightarrow A \cap B \not\subseteq A \cup B$

⑩ $\dfrac{|A \cap B|}{|A \cup B|} < 1$ $\quad$ $(A \cap B)$ will always be less than $|A \cup B|$.

(1) 1. $P \subseteq Q$ hypothesis

2. $x \in P \rightarrow x \in Q$ def of subset

~~3. $x \in P \vee x \in Q$~~

~~4. $x \in P \vee \neg\neg x \in P$~~

~~5. $\neg\neg x \in Q \vee$~~

3. $x \in \overline{P} \vee x \in Q$ conditional

4 $x \in \overline{P} \vee \neg\neg x \in Q$ double negation

5 $\neg\neg x \in Q \vee x \in \overline{P}$ commutative law

6 $x \in \neg Q \rightarrow x \in \neg P$ Conditional identity

7 $Q \subseteq P$ solution

(2) (1) $P \subseteq R$ hypothesis

(2) $Q \subseteq \overline{R}$ hypothesis

(3) $x \in P \rightarrow x \in R$ def of subset (1)

(4) $x \in Q \rightarrow x \in \overline{R}$ def of subset (2)

~~(5) [illegible] conditional [illegible]~~

(5) $x \in \overline{P} \vee x \in R$ conditional (3)

(6) $x \in \overline{Q} \vee x \in \overline{R}$ conditional (4)

(7) $x \in R \vee x \in \overline{P}$ comm law (5)

(8) $x \in \overline{R} \rightarrow x \in \overline{P}$ conditional identity (7)

(9) $x \in Q \rightarrow x \in \overline{R}$ conditional identity (6)

(10) $x \in Q \rightarrow x \in \overline{P}$ hypothetical syllogism 8, 9

(11) $Q \subseteq \overline{P}$ def of subset

(3) (1)

(2)