4. ① $A \cap B = \emptyset$ Premise

② $D = C \cap A$, $E = C \cap B$ Premise

③ $\forall_x (x \in D \Leftrightarrow (x \in C \wedge x \in A))$ biconditional implication ②

④ $\forall_x (x \in E \Leftrightarrow (x \in C \wedge x \in B))$ biconditional implication ②

⑤ $\exists_x (x \in D \cap E)$ hypothesis introduction

⑥ $x \in D \cap E$ $\exists$ instatiation

⑦ $(x \in C \wedge x \in A) \wedge (x \in C \wedge x \in B)$ $\exists$ instantiation

⑧ $(x \in A \wedge x \in B) \wedge (x \in C \wedge x \in C)$ associative laws on sets.

⑨ $(x \in A \wedge x \in B)$ hypothesis elimination, idempotent laws

⑩ $A \cap B \neq \emptyset$ False, def of sets, hypothesis elimination

⑪ $(x \notin D \wedge x \notin E)$

5 d) $B(x)$ = x has permission slip

$P(x)$ = x goes to field trip.

$\forall x\,(B(x) \to P(x))$

$\forall x\ B(x)$

$\therefore$ ~~∀x B(x)~~ $\forall x\ P(x)$

Proof

1. $\forall x\ B(x)$ Hypothesis
2. $B(c)$ Universal Distantiation, 2
3. $\forall x\,(B(x) \to P(x))$ hyp.
4. $\forall x\,(\neg B(x) \vee P(x))$ Conditional identity 3
5. $\neg B(c) \vee P(c)$ Universal Instatiation 4
6. $P(c)$ Disjunctive syllogism 5, 2
7. $\forall x\ P(x)$ Universal Generalization

e) $U(x)$ student at university

$B(x)$ taking Boolean logic

$A(x)$ can take algorithms

$U(\text{Larry})$

$U(\text{Hubert})$

$B(\text{Larry}) \wedge B(\text{Hubert})$

$\forall x\,(B(x) \to A(x))$

$\therefore\ A(\text{Larry}) \wedge A(\text{Hubert})$

Proof

1. $U(\text{Larry})$, $U(\text{Hubert})$, $B(\text{Larry}) \wedge B(\text{Hubert})$ Hypothesis
2. $B(\text{Larry})$, $B(\text{Hubert})$ simplification 1
3. $\forall x\,(B(x) \to A(x))$ Hypothesis
4. $B(\text{Larry}) \to A(\text{Larry})$ Universal Instutistion 6, 1
5. $A(\text{Larry})$ Modus Ponens ~~9~~, 4
6. $B(\text{Hubert}) \to A(\text{Hubert})$ Univeral Instatiation 6, 2
7. $A(\text{Hubert})$ Modus Ponen 9, 5
8. $A(\text{Larry}) \wedge A(\text{Hubert})$ 8, 10 Conjunction

4 b) $\exists x \forall y(\neg P(x,y) \vee \neg Q(x,y))$

c) $\forall x \exists y(P(x,y) \wedge \neg Q(x,y))$

d) $\exists x \forall y(P(x,y) \leftrightarrow P(y,x))$

$\neg[\exists x \forall y((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y))$

$\forall x \exists y(\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y)))$

$\forall x \exists y((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))$

5) f) $\neg(\exists x \forall y (x < y))$

g) $\forall x((x \neq 0) \rightarrow (\exists y (xy = 1)))$

h) $\forall x((x \neq 0) \rightarrow (\exists! y(xy = 1)))$

6. f) $\forall x\ B(\text{Josephine}, x)$

g) $\exists x \forall y (B(\text{Nancy}, x) \wedge (x \neq y) \rightarrow \neg B(\text{Nancy}, y)))$

h) $\exists x \exists y(\forall z(\neg B(z,x) \wedge \neg B(z,y))$

2. Def of subset $X \subseteq Y = \forall b \in X\ b \in Y$

① $B \subseteq A$ is $\forall b \in B, b \in A$.

② $\forall a \in A, Q(a)$ is true.

③ $\forall b \in B, Q(b)$ is true. 2, 1

④ $C \subseteq B$ is $\forall b \in C, b \in B$.

⑤ $\forall c \in C, Q(c)$ is true 2, 4.

$B \subseteq A \wedge C \subseteq B \rightarrow C \subseteq B \subseteq A$

$\forall c \in C, c \in B, c \in A$. def of subset

$\forall c \in C\ Q(c)$ holds.

~~3 $\forall y \in Y\ \exists x \in X\ f(x) = y$~~

~~① $f(x)$ is surjective, $g(x)$ is surjective.~~

~~④ $\forall y \in A\ \exists x \in B\ f(x) = y$~~

~~② $\forall x \in A\ P(x)$ $\forall x(x \in A \rightarrow P(x))$ domain restriction rewriting~~

~~③ $\forall y \in B\ \exists x \in C\ f(x) = y$~~

~~⑥ $g(b) = c$, $f(a) = b$~~

~~⑦ $g(f(a)) = g(b) = c$~~

3. ① $f(x)$ is surjective, $g(x)$ is surjective
② $\forall x \in A\ P(x)$ $\forall x(x \in A \to P(x))$ rewriting domain restriction
③ $\forall y \in Y\ \exists x \in X, f(x) = y$ def of surjection
④ $\forall y \in A\ \exists x \in B, f(x) = y$, $A \to B$ def. ③,
⑤ $\forall y \in B\ \exists x \in C, f(x) = y$, $B \to C$ def ③
⑥ $\exists a \in A\ \exists b \in B\ f, f(a) = b$ existantial instantiation ④
⑦ $\exists b \in B\ \exists c \in C, g(b) = c$ existantial instantiation ⑤
⑧ $g(f(a)) = c$ generalization
⑨ $\forall c \in C\ \exists a \in A, h(a) = c$ def of surjection
⑩ $h: A \to C$ is onto / surjective conclusion

Problem Set 6

1. f) $\forall x((\neg W(x)) \to (S(x) \vee V(x))$
   g) $\exists x((\neg W(x)) \to \neg(S(x) \vee V(x)))$
   h) $\forall x((\neg W(x) \to (S(x) \vee V(x)))$
   i) $\exists x(S(x) \wedge W(x))$
   j) $\exists x((x \neq \text{Ingrid})) \wedge (S(x))$
   k) $\forall x((x \neq \text{Ingrid}) \wedge (S(x)))$

2. e) Proposition. ~~All values are not true according to the table~~. For all x, x had migraines if and only if x had fainting spells (False)
   f) Proposition. ~~Values are not true according to the table~~ (TRUE) For all x, x had migraines and x had fainting spells if x wasn't given medication
   g) Proposition, For some x, it is true. For some x, x was given the medication and x had no fainting spells and x had no migraines (True)
   h) proposition. ~~Values are not true according to the table~~. For all x, x was given the medication if x had fainting spells or x had migraines (False)

3 d) 1 $\forall x(P(x) \to M(x))$ ①
   2 Negation: $\neg \forall x(P(x) \to M(x))$ ②
   3 De Morgan's: $\exists x(P(x) \wedge \neg M(x))$

Some patient who took the placebo didn't have migraines

e) $\exists x(M(x) \wedge P(x))$
   ② $\neg \exists x(M(x) \wedge P(x))$ Negation 1
   ③ $\forall x(\neg M(x) \vee \neg P(x))$ Simplification
   ④ For each patient, they did not recieve medication or the placebo.

Valid Conclusion

Proof

1. $\forall x((M(x) \vee D(x)) \rightarrow \neg(CA(x)))$ Hypothesis
2. Penelope is a particular student Hypothesis
3. $(M(\text{Penelope} \vee D(\text{Penelope}) \rightarrow \neg(A(\text{Penelope}))$ Universal Instantiation
4. $\neg(M(\text{Penelope}) \vee D(\text{Penelope})) \vee \neg(A(\text{Penelope}))$ Cond. identity 3
5. $A(\text{Penelope})$ Hypothesis
6. $\neg(M(\text{Penelope}) \vee D(\text{Penelope}))$ Disjunctive Syllogisms
7. $\neg(M(\text{Penelope}) \wedge \neg D(\text{Penelope})$ De Morgans, 6
8. $\neg D(\text{Penelope})$ Simplification 7.

PR5

7.6.2b) It is not valid because even if she recieved a prize, she could have won it at another competition. This does not imply that she sold 50 Cookies.

$Q(a)$ $P(a)$

3b)

	P	Q
a	F	F
b	F	T

$(P(x) \vee Q(x))$ is True

$Q(a) = F$

$\exists x (\neg Q(x))$

There doesn't exist an x such that $P(x)$ is True, so $\exists x\, P(x)$ doesn't hold for the given hypothesis/inputs, so the conclusion is False.

5d) $D(x)$ has detention

$M(x)$: missed class

$\forall x (M(x) \to D(x))$

Penelope is a particular student

$\neg M(\text{Penelope})$

$\neg D(\text{Penelope})$

Not valid due to other ways of recieving detention like punching your teacher, ~~It could still~~ The hypothesis could still hold true, but it can't be used to imply the conclusion.

5e) $M(x)$ missed class

$D(x)$ recieved detention

$A(x)$ recieved an A.

$\forall x ((M(x) \vee D(x)) \to \neg((A(x))))$

Penelope is a particular student

$A(\text{Penelope})$

$\neg D(\text{Penelope})$