

# CS 132 Assignment1

Gordon Ng

TOTAL POINTS

**25.75 / 32**

## QUESTION 1

4 pts

### 1.1 Question 1.a 2 / 2

- ✓ - **0 pts** Correct work and answer
- **1 pts** Correct work, but incorrect final answer
- **1 pts** Correct answer, but no work
- **1.25 pts** Incorrect work and answer
- **2 pts** No answer / incorrect answer

### 1.2 Question 1.b 0.75 / 2

- **0 pts** Correct work and answer
- **1 pts** Correct work, but incorrect final answer
- **1 pts** Correct answer, but no work
- ✓ - **1.25 pts** Incorrect work and answer
- **2 pts** No answer / incorrect answer

## QUESTION 2

4 pts

### 2.1 Question 2.a 2 / 2

- ✓ - **0 pts** We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.
- **2 pts** No solution provided.

### 2.2 Question 2.b 2 / 2

- ✓ - **0 pts** We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.
- **2 pts** No solution provided.

## QUESTION 3

4 pts

### 3.1 Question 3.a 2 / 2

- ✓ - **0 pts** We are not grading this question for

correctness. Please check the solutions if you want to see whether you have the correct answers.

- **2 pts** No solution provided.

### 3.2 Question 3.b 2 / 2

- ✓ - **0 pts** We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.
- **2 pts** No solution provided.

## QUESTION 4

### 4 Question 4 2 / 4

- **0 pts** Correct answer and correct explanation
- ✓ - **2 pts** Correct answer, no explanation or incorrect explanation
- **4 pts** No answer or incorrect answer

💬 The solution will be unique because  $a_1$  and  $a_2$  are not co-linear. This means that there is only one combination of the two vectors to reach every point in  $R^2$ .

## QUESTION 5

### 5 Question 5 3 / 4

- **0 pts** Correct explanation
- ✓ - **1 pts** Did not mention "no free variables"
- **1 pts** Did not mention "every column is a pivot column"
- **1 pts** Did not conclude matrix A spans  $R^3$
- **4 pts** No answer or incorrect answer

## QUESTION 6

### 6 Question 6 4 / 4

- ✓ - **0 pts** Correct answer and explanation
- **2 pts** Correct answer, no explanation
- **4 pts** No answer or incorrect answer

QUESTION 7

4 pts

7.1 Question 7.a 2 / 2

- ✓ - 0 pts Correct work and correct conclusion
- 1 pts Correct work, incorrect conclusion
- 2 pts No answer or incorrect answer

7.2 Question 7.b 2 / 2

- ✓ - 0 pts Correct work and correct conclusion
- 1 pts Correct work, incorrect conclusion
- 2 pts No answer or incorrect answer

QUESTION 8

4 pts

8.1 Question 8.a 2 / 2

- ✓ - 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.
- 2 pts No solution provided.

8.2 Question 8.b 0 / 1

- 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.
- ✓ - 2 pts No solution provided.

8.3 Question 8.c 0 / 1

- 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.
- ✓ - 2 pts No solution provided.

Consistent with all values of  $h$ .

$$\begin{bmatrix} 1 & 2 & -4 \\ 0 & h-6 & 0 \end{bmatrix} \rightarrow$$

Gordon Ng

$$1a \begin{bmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{bmatrix}$$

$$2h=7$$

$$h=3.5$$

1b.

$$2a) \begin{matrix} -2, 2, 5, 10, 20 \\ v_1 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} \end{matrix}$$

$$v_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} \text{ weight } -2, 20$$

$$= 2 \cdot v_2 \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ -6 \end{bmatrix} \text{ Error}$$

$$5: \begin{bmatrix} 15 \\ 0 \\ 10 \end{bmatrix} \begin{bmatrix} -10 \\ 0 \\ 15 \end{bmatrix} \quad \begin{matrix} C_1=10 \\ C_2=10 \end{matrix} \begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix}$$

$$\begin{matrix} C_1=0 \\ C_2=0 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

## 1.1 Question 1.a 2 / 2

- ✓ - **0 pts** Correct work and answer
- **1 pts** Correct work, but incorrect final answer
- **1 pts** Correct answer, but no work
- **1.25 pts** Incorrect work and answer
- **2 pts** No answer / incorrect answer



Consistent with all values of  $h$ .

$$\begin{bmatrix} 1 & 2 & -4 \\ 0 & h-6 & 0 \end{bmatrix} \downarrow$$

$$2h=7 \\ h=3.5$$

1b.

$$\text{Ex } \begin{bmatrix} 10 & -30 & -20 \\ 10 & 2h & -14 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2h-30 & 6 \\ 10 & 2h & -14 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} \text{ weight } \begin{bmatrix} -6 \\ 0 \\ -4 \end{bmatrix} - 2 \cdot V_1$$

$$\boxed{C_1 = -2} \\ \boxed{C_2 = -2}$$

$$\begin{bmatrix} 4 \\ 0 \\ -6 \end{bmatrix}$$

$$\text{Ex } \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} -6 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 10 \end{bmatrix} \quad \boxed{\text{K.O.}}$$

$$\left[ \begin{array}{c|c} C_1=10 & \begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix} \\ C_2=10 & \begin{bmatrix} -20 \\ 0 \\ 30 \end{bmatrix} \end{array} \right] \left[ \begin{array}{c|c} C_1=20 & \begin{bmatrix} 60 \\ 0 \\ 40 \end{bmatrix} \\ C_2=20 & \begin{bmatrix} -40 \\ 0 \\ -60 \end{bmatrix} \end{array} \right]$$

$$\left[ \begin{array}{c|c} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} & \begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix} + \begin{bmatrix} -20 \\ 0 \\ 30 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 50 \end{bmatrix} \end{array} \right]$$

$$1, -6, -5, 0$$

$$\left[ \begin{array}{c|c} C_1=7, C_2=7 & \begin{bmatrix} 49 \\ 7 \\ 40 \end{bmatrix} \\ & \begin{bmatrix} -35 \\ 21 \\ 0 \end{bmatrix} \end{array} \right] \left[ \begin{array}{c|c} C_1=1 & \begin{bmatrix} 7 \\ 1 \\ 6 \end{bmatrix} \\ C_2=1 & \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} \end{array} \right]$$

## 1.2 Question 1.b 0.75 / 2

- **0 pts** Correct work and answer
- **1 pts** Correct work, but incorrect final answer
- **1 pts** Correct answer, but no work
- ✓ - **1.25 pts** Incorrect work and answer
- **2 pts** No answer / incorrect answer



$$2 \begin{pmatrix} 1 & -4 & 2 \end{pmatrix}$$

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ -1 & 2 & 5 \end{bmatrix} \xrightarrow{+R_1} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \xrightarrow{\div R_3} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\div R_2} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+7R_2} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+2R_2} \begin{bmatrix} 1 & 0 & -10/3 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+10/3 R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  4 line-c combination is formed.

6.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  at most  $n$  vectors can't span  $\mathbb{R}^n$ ,  $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^4$ .  
 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  < nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , linear it will reach  
 b The solution isn't unique! B is the span of A.

Solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of A  
 uniqueness  $\leftrightarrow A$  is linearly independent (not all zero)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 we dimensional ~~linearly~~ which will span  $\mathbb{R}^3$ .  $\forall b \in \mathbb{R}^3, Ax=b$  has solution  
 REF of  $[A|b]$  cannot have  $[0 \dots 0 | n] \ n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a.  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 0 & 9 & 2 & 15 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_1 \div 9} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 - 7R_1, R_3 - 6R_1, R_4 + 5R_1} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \div 7, R_3 \div 6, R_4 \div -5} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 2/7 & -5/7 & 8/7 \\ 1 & 5/3 & -1/3 & 7/6 \\ -1 & -3/5 & 4/5 & -9/5 \end{bmatrix} \xrightarrow{R_2 - R_1, R_3 - R_1, R_4 + R_1} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 1 & 5/3 & -1/3 & 7/6 \\ -1 & -3/5 & 4/5 & -9/5 \end{bmatrix} \xrightarrow{R_3 - R_2, R_4 + R_2} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 0 & 5/3 & 2/3 & 1/2 \\ 0 & -3/5 & 11/5 & -8/5 \end{bmatrix} \xrightarrow{R_3 \div 5/3, R_4 \div -3/5} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 0 & 1 & 2/3 & 1/2 \\ 0 & 1 & 11/5 & -8/5 \end{bmatrix} \xrightarrow{R_3 - R_2, R_4 - R_2} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 0 & 0 & 1/3 & -1/4 \\ 0 & 0 & 0 & -14/5 \end{bmatrix} \xrightarrow{R_3 \div 1/3, R_4 \div -14/5} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 + 11/7 R_3, R_1 - 2/9 R_3} \begin{bmatrix} 0 & 1 & 0 & 11/12 \\ 1 & 0 & 0 & -1/2 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 + 1/2 R_4, R_1 - 11/12 R_4} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

7b.  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 5 & 4 & 1 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 5 & 4 & 1 \\ 0 & -11 & 3 & -23 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_2 \div 5, R_3 + 11R_2, R_4 - 11R_2} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 39 & -11 \\ 0 & 0 & -47 & 15 \end{bmatrix} \xrightarrow{R_2 \div 5, R_3 \div 39, R_4 + 47R_2} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 4/5 & 1/5 \\ 0 & 0 & 1 & -11/39 \\ 0 & 0 & 0 & -14/13 \end{bmatrix} \xrightarrow{R_2 - 4/5 R_3, R_4 \div -14/13} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 0 & 11/13 \\ 0 & 0 & 1 & -11/39 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 - 11/13 R_4, R_1 + 5R_4} \begin{bmatrix} 7 & 2 & 0 & 18 \\ 0 & 1 & 0 & 11/13 \\ 0 & 0 & 1 & -11/39 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \div 7, R_2 \div 11/13} \begin{bmatrix} 1 & 2/7 & 0 & 18/7 \\ 0 & 1 & 0 & 11/13 \\ 0 & 0 & 1 & -11/39 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 - 2/7 R_2} \begin{bmatrix} 1 & 0 & 0 & 10/13 \\ 0 & 1 & 0 & 11/13 \\ 0 & 0 & 1 & -11/39 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

7c.  $\begin{bmatrix} 1 & 1 & 7 & -7 & -9 & -6 \\ 0 & -3 & 11 & 6 & 11 & 11 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \xrightarrow{R_2 \div -3, R_3 - 8R_1, R_4 + 3R_1} \begin{bmatrix} 1 & 1 & 7 & -7 & -9 & -6 \\ 0 & -3 & 11 & 6 & 11 & 11 \\ 0 & 19 & -54 & 55 & 85 & 55 \\ 0 & 5 & 17 & -19 & -41 & -25 \end{bmatrix} \xrightarrow{R_2 \div -3, R_3 \div 19, R_4 \div 5} \begin{bmatrix} 1 & 1 & 7 & -7 & -9 & -6 \\ 0 & -3 & 11 & 6 & 11 & 11 \\ 0 & 1 & -28/19 & 275/19 & 425/19 & 275/19 \\ 0 & 1 & 17/5 & -19/5 & -41/5 & -25/5 \end{bmatrix} \xrightarrow{R_2 + 3R_3, R_4 - R_3} \begin{bmatrix} 1 & 1 & 7 & -7 & -9 & -6 \\ 0 & -3 & 11 & 6 & 11 & 11 \\ 0 & 1 & -28/19 & 275/19 & 425/19 & 275/19 \\ 0 & 0 & 45/19 & -10/19 & 6/19 & 5/19 \end{bmatrix} \xrightarrow{R_2 \div -3, R_3 \div 19, R_4 \div 45/19} \begin{bmatrix} 1 & 1 & 7 & -7 & -9 & -6 \\ 0 & -3 & 11 & 6 & 11 & 11 \\ 0 & 1 & -28/19 & 275/19 & 425/19 & 275/19 \\ 0 & 0 & 1 & -2/9 & 2/15 \end{bmatrix} \xrightarrow{R_2 + 3R_3, R_1 - 7R_3} \begin{bmatrix} 1 & 0 & 0 & -11/9 & -19/15 & -12/5 \\ 0 & 0 & 0 & -2/9 & 2/15 & -12/5 \\ 0 & 1 & -28/19 & 275/19 & 425/19 & 275/19 \\ 0 & 0 & 1 & -2/9 & 2/15 \end{bmatrix} \xrightarrow{R_2 \div -2/9, R_1 + 11/9 R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & -12/5 \\ 0 & 0 & 0 & 1 & -2/15 \\ 0 & 1 & -28/19 & 275/19 & 425/19 & 275/19 \\ 0 & 0 & 1 & -2/9 & 2/15 \end{bmatrix} \xrightarrow{R_1 + 12/5 R_2, R_3 + 28/19 R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2/15 \\ 0 & 1 & 0 & 275/19 & 425/19 \\ 0 & 0 & 1 & -2/9 & 2/15 \end{bmatrix} \xrightarrow{R_3 \div 275/19, R_1 + 2/15 R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2/15 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -2/9 & 2/15 \end{bmatrix} \xrightarrow{R_1 + 2/15 R_2, R_3 - R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2/15 \\ 0 & 1 & 0 & 0 & 13/15 \\ 0 & 0 & 1 & -2/9 & 2/15 \end{bmatrix} \xrightarrow{R_1 + 2/15 R_2, R_3 \div 13/15} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2/15 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2/9 & 2/15 \end{bmatrix} \xrightarrow{R_1 + 2/15 R_2, R_3 \div 13/15} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2/15 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2/9 & 2/15 \end{bmatrix}$

Spans  $\mathbb{R}^4$ , 4 pivots

## 2.1 Question 2.a 2 / 2

✓ - 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

- 2 pts No solution provided.



Ba.  $[2-84] \rightarrow [1-427]$

36. 1-2-67

17

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. Yes the form does have a  $A \neq b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , then it will resolve.

b. The solution is not unique. B is the span of  $A$ .

existence  $\Leftarrow \Rightarrow$  is the span of  $A$

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 dimensional ~~image~~ which will span  $\mathbb{R}^3$ .  $\forall b \in \mathbb{R}^3, Ax=b$  has solution

$$72 - 58 \frac{5}{7}$$

7	2	-3	8	No. It doesn't
0	11	-3	23	spec. R <sub>4</sub> missing

$$76 \quad \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix}$$

Spans  $\mathbb{R}^1$ , 4 pivots.

## 2.2 Question 2.b 2 / 2

✓ - **0 pts** We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

- **2 pts** No solution provided.



$$2 \begin{pmatrix} 1 & -4 & 2 \end{pmatrix}$$

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ -1 & 2 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \quad b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix} \quad \begin{bmatrix} 11 & 10 & -5 & 4 \\ 0 & -15 & 6 & 3 \\ 0 & 0 & 9 & 9 \end{bmatrix}$

4 linear combination is formed.

6.  $n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$  at most  $n$  vectors can't span  $\mathbb{R}^m$   
 $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^4$   
 $\leftarrow$  nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , ~~vector~~ it will reach  $b$ . The solution is not unique,  $B$  is the span of  $A$ .

solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of  $A$   
 $\rightarrow$  uniqueness  $\leftrightarrow A$  is linearly independent (not linearly not coplanar)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 two dimensional ~~image~~ which will span  $\mathbb{R}^3 \quad \forall b \in \mathbb{R}^3, Ax=b$  has solution  
 $\forall b \in \mathbb{R}^3$  of  $[A|b]$  cannot have  $[0 \dots 0 | n] \quad n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a)  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{5/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix} \xrightarrow{-6/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & 10 & -2 & 7 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{1/9} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 2/9 & 5/3 \\ 0 & 10 & -2 & 7 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{-10/9} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 2/9 & 5/3 \\ 0 & 0 & -20/9 & 13/3 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{-11/9} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 2/9 & 5/3 \\ 0 & 0 & -20/9 & 13/3 \\ 0 & 0 & -22/9 & 16/3 \end{bmatrix}$

7b.  $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39 & 6 & 3 & 141 \end{bmatrix} \xrightarrow{-1/39} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 1 & -2/13 & -1/13 & 37/13 \end{bmatrix} \xrightarrow{-65/13} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 1 & -2/13 & -1/13 & 37/13 \end{bmatrix}$

No, it doesn't span  $\mathbb{R}^4$ , missing a pivot. No solution

### 3.1 Question 3.a 2 / 2

✓ - 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

- 2 pts No solution provided.



$$2 \begin{pmatrix} 1 & -4 & 2 \end{pmatrix}$$

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \quad b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix} \quad \begin{bmatrix} 11 & 10 & -5 & 4 \\ 0 & -15 & 6 & 3 \\ 0 & 0 & 9 & 9 \end{bmatrix}$

4 linear combination is formed.

6.  $n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$  at most  $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^3$   
 $\leftarrow$  nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , ~~vector~~ it will reach  $b$ . The solution is not unique,  $B$  is the span of  $A$ .

solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of  $A$   
 $\rightarrow$  uniqueness  $\leftrightarrow A$  is linearly independent (not linearly not coplanar)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 two dimensional ~~image~~ which will span  $\mathbb{R}^3 \quad \forall b \in \mathbb{R}^3, Ax=b$  has solution  
 $\forall b \in \mathbb{R}^3$  of  $[A|b]$  cannot have  $[0 \dots 0 | n] \quad n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a)  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{5/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix} \xrightarrow{-6/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & 10 & -2 & 7 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{1/9} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 2/9 & 5/3 \\ 0 & 10 & -2 & 7 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{10/9} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 2/9 & 5/3 \\ 0 & 0 & -20/9 & 13/3 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{11/9} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 2/9 & 5/3 \\ 0 & 0 & -20/9 & 13/3 \\ 0 & 0 & -20/9 & 13/3 \end{bmatrix}$

No, it doesn't span  $\mathbb{R}^4$ , missing a pivot. No solution

7b.  $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39 & 6 & 3 & 141 \end{bmatrix} \xrightarrow{1/11} \begin{bmatrix} 1 & 7/11 & -7/11 & -9/11 & -6/11 \\ 0 & -39 & 6 & 3 & 141 \end{bmatrix} \xrightarrow{1/39} \begin{bmatrix} 1 & 7/11 & -7/11 & -9/11 & -6/11 \\ 0 & 1 & -2/13 & 1/13 & 37/13 \end{bmatrix}$

### 3.2 Question 3.b 2 / 2

✓ - 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

- 2 pts No solution provided.



2 (1 -4 2)

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \quad b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix} \quad \begin{bmatrix} 11 & 10 & -5 & 4 \\ 0 & -15 & 6 & 3 \\ 0 & 0 & 9 & 9 \end{bmatrix}$

4 linear combination is formed.

6.  $n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$  at most  $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^4$   
 $\leftarrow$  nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , ~~vector~~ it will reach  $b$ . The solution is not unique,  $B$  is the span of  $A$ .

solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of  $A$   
 $\rightarrow$  uniqueness  $\leftrightarrow A$  is linearly independent (not linearly dependent)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 dimensional ~~image~~ which will span  $\mathbb{R}^3 \quad \forall b \in \mathbb{R}^3, Ax=b$  has solution  
 $\forall b \in \mathbb{R}^3$  of  $[A|b]$  cannot have  $[0 \dots 0 | n] \quad n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a)  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{5/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix} \xrightarrow{-6/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & -10/7 & 32/7 & -23/7 \\ -7 & 9 & 2 & 15 \end{bmatrix} \xrightarrow{-6/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & -10/7 & 32/7 & -23/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & -10/7 & 32/7 & -23/7 \\ 0 & 11 & -3 & 23 \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 50/7 & -189/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  No, it doesn't span  $\mathbb{R}^4$ , missing a pivot. No solution

7b.  $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/2 & 6/2 & 3/2 & 141/2 \end{bmatrix} \rightarrow \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/2 & 6/2 & 3/2 & 141/2 \end{bmatrix} \rightarrow \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/2 & -10/2 & 5/2 & 191/2 \end{bmatrix}$

#### 4 Question 4 2 / 4

- 0 pts Correct answer and correct explanation

✓ - 2 pts Correct answer, no explanation or incorrect explanation

- 4 pts No answer or incorrect answer

💬 The solution will be unique because  $a_1$  and  $a_2$  are not co-linear. This means that there is only one combination of the two vectors to reach every point in  $\mathbb{R}^2$ .



2 (1 -4 2)

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \quad b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix} \quad \begin{bmatrix} 11 & 10 & -5 & 4 \\ 0 & -15 & 6 & 3 \\ 0 & 0 & 9 & 9 \end{bmatrix}$

4 linear combination is formed.

6.  $n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$  at most  $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^4$   
 $\leftarrow$  nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , ~~vector~~ it will reach  $b$ . The solution is not unique,  $B$  is the span of  $A$ .

solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of  $A$   
 $\rightarrow$  uniqueness  $\leftrightarrow A$  is linearly independent (not linearly not coplanar)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 two dimensional ~~image~~ which will span  $\mathbb{R}^3 \quad \forall b \in \mathbb{R}^3, Ax=b$  has solution  
 $\forall b \in \mathbb{R}^3$  of  $[A|b]$  cannot have  $[0 \dots 0 | n] \quad n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a)  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{5/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix} \xrightarrow{-6/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & -10/7 & 32/7 & -23/7 \\ -7 & 9 & 2 & 15 \end{bmatrix} \xrightarrow{-6/7} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & -10/7 & 32/7 & -23/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 0 & -10/7 & 32/7 & -23/7 \\ 0 & 11 & -3 & 23 \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 50/7 & -189/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  No, it doesn't span  $\mathbb{R}^4$ , missing a pivot. No solution

7b.  $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/2 & 6/2 & 3/2 & 141/2 \end{bmatrix} \rightarrow \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/2 & 6/2 & 3/2 & 141/2 \end{bmatrix} \rightarrow \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/2 & -10/2 & 5/2 & 191/2 \end{bmatrix}$

## 5 Question 5 3 / 4

- 0 pts Correct explanation
- ✓ - 1 pts Did not mention "no free variables"
- 1 pts Did not mention "every column is a pivot column"
- 1 pts Did not conclude matrix A spans  $\mathbb{R}^3$
- 4 pts No answer or incorrect answer





## 6 Question 6 4 / 4

- ✓ - **0 pts** Correct answer and explanation
- **2 pts** Correct answer, no explanation
- **4 pts** No answer or incorrect answer



$$2 \begin{pmatrix} 1 & -4 & 2 \end{pmatrix}$$

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ -1 & 2 & 5 \end{bmatrix} \xrightarrow{+R_1} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \xrightarrow{\div R_3} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\div R_2} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+7R_2} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+2R_2} \begin{bmatrix} 1 & 0 & -10/3 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+10R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  4 line-c combination is formed.

6.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  at most  $n$  vectors can't span  $\mathbb{R}^n$ ,  $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^4$ .  
 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  < nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , linear it will reach  
 b The solution isn't unique! B is the span of A.

Solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of A  
 uniqueness  $\leftrightarrow A$  is linearly independent (not all linearly dependent)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 we dimensional ~~linearly~~ which will span  $\mathbb{R}^3$ .  $\forall b \in \mathbb{R}^3, Ax=b$  has solution  
 REF of  $[A|b]$  cannot have  $[0 \dots 0 | n] \ n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a.  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 0 & 9 & 2 & 15 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_1 \div 9} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 - 7R_1, R_3 - 6R_1, R_4 + 5R_1} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \div 7, R_3 \div 6, R_4 \div -5} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 2/7 & -5/7 & 8/7 \\ 1 & 5/3 & -1/3 & 7/6 \\ -1 & -3/5 & 4/5 & -9/5 \end{bmatrix} \xrightarrow{R_2 - R_1, R_3 - R_1, R_4 + R_1} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 1 & 5/3 & -1/3 & 7/6 \\ -1 & -3/5 & 4/5 & -9/5 \end{bmatrix} \xrightarrow{R_3 - R_2, R_4 + R_2} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 0 & 5/3 & 2/3 & 1/2 \\ 0 & -3/5 & 7/5 & -8/5 \end{bmatrix} \xrightarrow{R_3 \div 5/3, R_4 \div -3/5} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 0 & 1 & 2/3 & 1/2 \\ 0 & 1 & 7/5 & -8/5 \end{bmatrix} \xrightarrow{R_3 - R_2, R_4 - R_2} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 0 & 0 & 1/3 & -1/4 \\ 0 & 0 & 0 & -15/4 \end{bmatrix} \xrightarrow{R_3 \div 1/3, R_4 \div -15/4} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 0 & -11/7 & 1/7 \\ 0 & 0 & 1 & -1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 - 2/9 R_4, R_2 + 11/7 R_4} \begin{bmatrix} 0 & 1 & 0 & 11/12 \\ 1 & 0 & 0 & 1/7 \\ 0 & 0 & 1 & -1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \div 1/7, R_1 - 11/12 R_4} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

7b.  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 5 & 4 & 1 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 5 & 4 & 1 \\ 0 & -11 & 3 & -23 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_2 \div 5, R_3 + 11R_2, R_4 - 11R_2} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 39 & 11 \\ 0 & 0 & -47 & 15 \end{bmatrix} \xrightarrow{R_2 \div 5, R_3 \div 39, R_4 + 47R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 1 & 11/39 \\ 0 & 0 & 0 & 15/13 \end{bmatrix} \xrightarrow{R_2 \div 5, R_3 \div 11/39, R_4 \div 15/13} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 - 4R_3, R_1 + 5R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 5 & 0 & 1/3 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \div 5, R_1 - 2R_2} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 0 & 1/15 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 - 2R_2} \begin{bmatrix} 7 & 0 & -5 & 14/5 \\ 0 & 1 & 0 & 1/15 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 + 5R_2} \begin{bmatrix} 7 & 0 & -5 & 14/5 \\ 0 & 1 & 0 & 1/15 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 + 5R_2} \begin{bmatrix} 7 & 0 & -5 & 14/5 \\ 0 & 1 & 0 & 1/15 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 + 5R_2} \begin{bmatrix} 7 & 0 & -5 & 14/5 \\ 0 & 1 & 0 & 1/15 \\ 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

7c.  $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 6 & 5 & -10 & 1 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \xrightarrow{R_2 \div -3, R_3 \div 6, R_4 \div 8, R_5 \div -3} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 1 & 5/3 & -5/3 & 1/6 \\ 1 & 11/8 & -3/4 & -7/8 & 13/8 \\ 1 & 4/3 & -1/3 & 8/3 & 7/3 \end{bmatrix} \xrightarrow{R_1 - 11R_4, R_2 + 3R_4, R_3 - 5R_4, R_5 - R_4} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 1 & 5/3 & -5/3 & 1/6 \\ 1 & 11/8 & -3/4 & -7/8 & 13/8 \\ 0 & 1 & -1/3 & 5/3 & 1/3 \end{bmatrix} \xrightarrow{R_1 - 11R_4, R_2 + 3R_4, R_3 - 5R_4, R_5 - R_4} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 1 & 5/3 & -5/3 & 1/6 \\ 1 & 11/8 & -3/4 & -7/8 & 13/8 \\ 0 & 1 & -1/3 & 5/3 & 1/3 \end{bmatrix} \xrightarrow{R_1 - 11R_4, R_2 + 3R_4, R_3 - 5R_4, R_5 - R_4} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 1 & 5/3 & -5/3 & 1/6 \\ 1 & 11/8 & -3/4 & -7/8 & 13/8 \\ 0 & 1 & -1/3 & 5/3 & 1/3 \end{bmatrix}$

Spans  $\mathbb{R}^4$ , 4 pivots

## 7.1 Question 7.a 2 / 2

- ✓ - **0 pts** Correct work and correct conclusion
- **1 pts** Correct work, incorrect conclusion
- **2 pts** No answer or incorrect answer



$$2 \begin{pmatrix} 1 & -4 & 2 \end{pmatrix}$$

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ -1 & 2 & 5 \end{bmatrix} \xrightarrow{+R_1} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \xrightarrow{\frac{1}{11}R_3} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 1 \end{bmatrix}$   
 $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$   
 4 line-c combination is formed.

6.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  at most  $n$  vectors can't span  $\mathbb{R}^n$ ,  $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^4$   
 < nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , linear it will reach  
 b The solution isn't unique! B is the span of A.

Solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of A  
 $\rightarrow$  uniqueness  $\leftrightarrow A$  is linearly independent (not all zero)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 we dimensional ~~linear~~ which will span  $\mathbb{R}^3$ .  $\forall b \in \mathbb{R}^3, Ax=b$  has solution  
 REF of  $[A|b]$  cannot have  $[0 \dots 0 | n] \quad n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a.  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 0 & 9 & 2 & 15 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 0 & 9 & 2 & 15 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 58 & 14 & 17 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 58 & 14 & 17 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_3 - 5R_2} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 19 & -109 \\ 0 & 11 & -3 & 23 \end{bmatrix}$

7b.  $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39 & 6 & 11 & -14 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39 & 6 & 11 & -14 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39 & 6 & 11 & -14 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$

Spans  $\mathbb{R}^4$ , 4 pivots

## 7.2 Question 7.b 2 / 2

- ✓ - **0 pts** Correct work and correct conclusion
- **1 pts** Correct work, incorrect conclusion
- **2 pts** No answer or incorrect answer





### 8.1 Question 8.a 2 / 2

✓ - 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

- 2 pts No solution provided.



$$2 \begin{pmatrix} 1 & -4 & 2 \end{pmatrix}$$

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ -1 & 2 & 5 \end{bmatrix} \xrightarrow{+R_1} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \xrightarrow{\div R_3} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\div R_2} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+7R_2} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+2R_2} \begin{bmatrix} 1 & 0 & -10/3 \\ 0 & 1 & 7/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{+10R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  4 line-c combination is formed.

6.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  at most  $n$  vectors can't span  $\mathbb{R}^n$ ,  $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^4$ .  
 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  < nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , linear it will reach  
 b The solution isn't unique! B is the span of A.

Solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of A  
 $\rightarrow$  uniqueness  $\leftrightarrow A$  is linearly independent (not all zero)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 we dimensional ~~linear~~ which will span  $\mathbb{R}^3$ .  $\forall b \in \mathbb{R}^3, Ax=b$  has solution  
 REF of  $[A|b]$  cannot have  $[0 \dots 0 | n] \ n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a.  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 0 & 9 & 2 & 15 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_1 \div 9} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 - 7R_1, R_3 - 6R_1, R_4 + 5R_1} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \div 7, R_3 \div 6, R_4 \div -5} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 1 & 2/7 & -5/7 & 8/7 \\ 1 & 5/3 & -1/3 & 7/6 \\ 1 & -3/5 & 4/5 & -9/5 \end{bmatrix} \xrightarrow{R_2 - R_1, R_3 - R_1, R_4 - R_1} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 0 & -2/7 & 2/7 & -1/7 \\ 0 & 2/3 & -4/3 & 1/6 \\ 0 & -8/5 & 16/5 & -18/5 \end{bmatrix} \xrightarrow{R_2 \div -2/7, R_3 \div 2/3, R_4 \div -8/5} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -2 & 1/3 \\ 0 & 1 & -2 & 9/5 \end{bmatrix} \xrightarrow{R_2 - R_1, R_3 - R_1, R_4 - R_1} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 0 & 0 & -11/9 & -2/3 \\ 0 & 0 & -10/9 & -2/3 \\ 0 & 0 & -10/9 & -2/3 \end{bmatrix} \xrightarrow{R_2 \div -11/9, R_3 \div -10/9, R_4 - R_3} \begin{bmatrix} 0 & 1 & 2/9 & 5/3 \\ 0 & 0 & 1 & 2/11 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

7b.  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 5 & 4 & 1 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_2 \div -11, R_4 \div 11} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & -3/11 & 23/11 \\ 0 & 5 & 4 & 1 \\ 0 & 1 & -3 & 23 \end{bmatrix} \xrightarrow{R_4 - R_2} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & -3/11 & 23/11 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & -20/11 & 18/11 \end{bmatrix} \xrightarrow{R_3 \div 5, R_4 \div -20/11} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & -3/11 & 23/11 \\ 0 & 1 & 4/5 & 1/5 \\ 0 & 0 & 1 & 9/5 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & -3/11 & 23/11 \\ 0 & 0 & 17/5 & -14/5 \\ 0 & 0 & 1 & 9/5 \end{bmatrix} \xrightarrow{R_3 \div 17/5, R_2 + 3/11 R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 9/5 \\ 0 & 0 & 1 & 9/5 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 9/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

7c.  $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -3 & 1/11 & 6/11 & 3/11 \\ 0 & 6 & 3/11 & -10/11 & -3/11 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \xrightarrow{R_2 \div -3, R_3 \div 6, R_4 \div 8, R_5 \div -3} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 1 & -1/11 & -2/11 & 1/11 \\ 0 & 1 & 1/22 & -5/22 & -1/22 \\ 1 & 11/8 & -3/4 & -7/8 & 13/8 \\ 1 & 4/3 & 1/3 & 8/3 & 7/3 \end{bmatrix} \xrightarrow{R_1 \div 11, R_4 - R_1, R_5 - R_1} \begin{bmatrix} 1 & 7/11 & -7/11 & -9/11 & -6/11 \\ 0 & 1 & -1/11 & -2/11 & 1/11 \\ 0 & 1 & 1/22 & -5/22 & -1/22 \\ 0 & 0 & 1/2 & -13/8 & 5/8 \\ 0 & 0 & 1/3 & 2/3 & 1/3 \end{bmatrix} \xrightarrow{R_1 - 7R_2, R_3 - R_2, R_4 \div 1/2, R_5 \div 1/3} \begin{bmatrix} 1 & 0 & -5/11 & -13/11 & -12/11 \\ 0 & 1 & -1/11 & -2/11 & 1/11 \\ 0 & 0 & 3/22 & -9/22 & -3/22 \\ 0 & 0 & 1 & -13/4 & 5/4 \\ 0 & 0 & 1 & 2 & 1 \end{bmatrix} \xrightarrow{R_1 + 5/11 R_2, R_3 + 3/22 R_2, R_4 - R_5, R_5 \div 3} \begin{bmatrix} 1 & 0 & 0 & -13/4 & 5/4 \\ 0 & 1 & 0 & -13/4 & 5/4 \\ 0 & 0 & 1 & -13/4 & 5/4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Spans  $\mathbb{R}^5$ , 4 pivots



## 8.2 Question 8.b 0 / 1

- **0 pts** We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

✓ - **2 pts** No solution provided.

$$2 \begin{pmatrix} 1 & -4 & 2 \end{pmatrix}$$

3a.  $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$  Not a linear combination, no solution

3b.  $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ -1 & 2 & 5 \end{bmatrix} \xrightarrow{+R_1} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix} \xrightarrow{\frac{1}{11}R_3} \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 1 \end{bmatrix}$   
 $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$   
 4 line-c combination is formed.

6.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  at most  $n$  vectors can't span  $\mathbb{R}^n$ ,  $n$  pivots,  $m > n$ , three vectors can't span  $\mathbb{R}^4$   
 < nope! not enough pivots

4. Yes the form does have a  $Ax=b$  solution because if you include negative numbers as  $c_1$  and  $c_2$ , linear it will reach  
 b The solution isn't unique! B is the span of A.

Solution  $Ax=b \rightarrow$  existence  $\leftrightarrow b$  is the span of A  
 $\rightarrow$  uniqueness  $\leftrightarrow A$  is linearly independent (not all zero)

5. Since it is a  $3 \times 3$  and it has a solution then it is at least 4 we dimensional ~~linear~~ which will span  $\mathbb{R}^3$ .  $\forall b \in \mathbb{R}^3, Ax=b$  has solution  
 REF of  $[A|b]$  cannot have  $[0 \dots 0 | n] \quad n \neq 0$  3 pivots so spans  $\mathbb{R}^3$ , unique

7a.  $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 0 & 9 & 2 & 15 \\ 7 & 2 & -5 & 8 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 58 & 14 & 17 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 58 & 14 & 17 \\ 0 & -11 & 3 & -23 \\ 0 & 11 & -3 & 23 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 58 & 14 & 17 \\ 0 & 11 & -3 & 23 \end{bmatrix}$

7b.  $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39 & 6 & 11 & -14 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 0 & -39 & 6 & 11 & -14 \\ 11 & 7 & -7 & -9 & -6 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39 & 6 & 11 & -14 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$

Spans  $\mathbb{R}^4$ , 4 pivots

### 8.3 Question 8.c 0 / 1

- **0 pts** We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

✓ - **2 pts** No solution provided.