4c. iff $x^2+2x-3<0$ then $-3<x<1$

$x=0$: $0^2+0-3<0$ then $-3<0<1$ ✓

$x>0$: $-5^2+2(-5)-3<0 \Rightarrow 12<0$ (False) $-3<-5<0$ (False) ✓

$x<0$: $3^2+2(3)-3<0$ $12<0$ (False) $-3<3<1$ (Also False) ✓

$x\geq 0$: $1^2+2-3<0 \Rightarrow 0<0$ (False) $-3<1<1$ (also False) ✓

Valid Proof with Proof by Cases

4d. $x^2-3x-10<0$, then $-2<x<5$

$x<0$: $(-1)^2-3(-1)-10<0 \Rightarrow -6<0$ $-2<-1<5$ ✓

$x=0$: $0^2-3(0)-10<0 \Rightarrow -10<0$ $-2<0<5$ ✓

$x>0$: $3^2-3(3)-10<0 \Rightarrow -10<0$ $-2<-3<5$ ✓

Valid Proof with Proof by Cases.

(2) $\frac{b_1}{a} = ?$ Remainder r_1 $\frac{b_2}{a} = ?$ Remainder r_2 | $\frac{b_1+b_2}{a} = ?$ remainder r_1+r_2 or r_1+r_2-a

Sol x   Sol y   Sol 2

Let r1 be the reminder of b1 when divided by a, meaning $0<=r1<a$ and there exists a q1 such that $b1=aq1+r1$

Let r2 be the reminder of b2 when divided by a, meaning $0<=r2<a$ and there exists a q2 such that $b2=aq2+r2$.

So that means $b1+b2=(aq1+r1)+(aq2+r2)$,

by using communatie laws $b1+b2=(aq1+aq2)+(r1+r2)$

factoring a out gives us $b1+b2=2a(q1+q2)+(r1+r2)$

q1 and q2 are both quotients, so $b1+b2=2aq+(r1+r2)$ fits the definition of a reminder

(3) a) For every pair of integers a, b and $b\neq 0$, $a=q_1b+r_1$ and $a=q_2b+r_2$

$(a=q_2b+r_2)-(a=q_1b+r_1) = (q_1-q_2)b+(r_1-r_2)=0$

The reminder after mod b is $(r_1-r_2)=0 \pmod b$, which gives us an integer k where $bk=r_1-r_2$. $0\leq r_1<|b|$ and $0\leq r_2<|b|$, where $0\leq r_1-r_2<|b|$, r_1-r_2 is shown to be less than $|b|$, so bk must also be less than $|b|$.

The only possible integer for k is 0. So, $r_1-r_2=0$, there is at most one remainder

b) Since the set of integers of progession $b-qa$ for $q=1,2\ldots$ has a least element r, there is a least element remainder.

Therefore it is unique since there is both at most one remainder and at least one remainder

5) ① $\forall n \in \mathbb{N}\ \sum_{i=0}^{n} 2^i = 2^{n+1}-1$ Premise

② ~~P(n) is true and~~ Prove that P(k-1) is true, $\exists n \forall n \in \mathbb{N} \sum_{i=0}^{k}$

③ $\sum_{i=1}^{k-1} 2^{n+1}-1$

④ $\sum_{i=1}^{k-1} 2^{n+1}-1 = 2^{k+1}-1 + 2^{k}-1$

⑤ $2^{k}-1 = 2^{(k-1)+1}-1$, contradiction is true.

⑥ By having a smaller number outside the well ordering principle be true, having the equation not true for all numbers satisfies the contradiction.

3) b) In any non-empty set, there must be a least element.
ex. set x: $1, 2, 10, 8, -1 \ldots$ -1 is the least.
This least element would have to range from 0 to b, if it wasn't if you subtract b, it would be smaller than the original and replace the smallest number. So, there is at least one remainder after the division of b by a.

c) Since $r \in S$ by definition there exists q such that $r = b - aq$
Let $r \geq a$ then $r - a \geq 0$, so $b - (q+1)a \geq 0$. This would be in set S. $b - (q+1)a \in S$, but $b(q+1)a = r - a < r$ contradicts that fact which makes r not the smallest number. So with proof by contradiction $b = aq + r$ where $0 \leq r < a$.

d) If there are two remainders r_1 and r_2, $b = aq_1 + r_1$ and $b = aq_2 + r_2$ Set them equal in respects to b, $aq_1 + r = aq_2 + r$. The quotients would be te same, leaving us with $r_1 = r_2$. The division of b by a leads to the same remainder.

4) a) For any c, $\lceil c \rceil$ is unique.
For any c, $\lceil c \rceil$ is not unique

~~① $0 <= z1 + z2 < 1$~~
~~② $c1 + c2 = \lceil c1 \rceil - z1 + \lceil c2 \rceil - z2$~~
~~③ $c1 + c2 = \lceil c1 \rceil + \lceil c2 \rceil - (z1 + z2)$~~
~~④ $z1 + z2$~~

① $d_1 \neq d_2$, $\lceil c \rceil = d_1$, $\lceil c \rceil = d_2$
~~② $d_1 - d_2 = z_1 - z_2 \geq 0$~~
② $c = d_1 - d_2$ and $c = z_1 - z_2$, $0 \leq z_1\ z_2 < 1$
③ $d_1 - d_2 = z_1 - z_2 \geq 0$
④ $|d_1 - z_1| < 1$
⑤ d_1 and z_1 are two integers where $|d_1 - z_1|$ is not distanceable and must be greater than, which is a contradiction. $\lceil c \rceil$ is unique.

4b. $\forall a,b: \lceil a+b \rceil$ is equal to $\lceil a \rceil + \lceil b \rceil$ or $\lceil a \rceil + \lceil b \rceil - 1$

1. if $\lceil a+b \rceil = d$, then $\lceil a+b \rceil = d - z$, $0 \leq z < 1$
2. $\lceil a \rceil = d - z_1$, $0 \leq z_1 < 1$
3. $\lceil b \rceil = d - z_2$, $0 \leq z_2 < 1$
4. $\lceil a \rceil + \lceil b \rceil = \lceil a \rceil - z_1 + \lceil b \rceil - z_2$, $0 \leq z1 + z2 < 1$
5. $(z1 + z2)$ would have to equal zero, 4, inference
6. $\lceil a+b \rceil$ is equal to $\lceil a \rceil + \lceil b \rceil$

~~5) 0 (n=0)~~ $2^0 = 2^{1} - 1$ is true.

$S(k) \rightarrow S(k+1)$ $\quad S(k)\ \sum_{i=1}^{k} 2^i = 2^{k+1} - 1$

$S(k+1)\ \sum_{i=1}^{k+1} 2^i = 2^{k+2} - 1$

$$\sum_{i=1}^{k+1} 2^i = \sum_{i=1}^{k} 2^i + 2^{k+1}$$

$$= 2 \cdot 2^{k+1} - 1 = 2^{k+2} - 1$$

plug in 1 for k

$2^{1-2} - 1 = -\frac{1}{2}$

(1) Gordon Ng CS131 H.W

1a. The theorem you are trying to prove is only proven for $m=7$ and $n=9$, and not all other integers. Skips essential steps

1b. This is a valid proof, it should clarify if n or m is greater/equal or less than 0. ~~[illegible]~~ Skips steps

1c. This uses the variables odd equals to $2k+1$ but never uses it in the proof. ~~[illegible]~~ Invalid Reasoning

1d. This skips the step of distribution of $(2k+1)^2 + (2j+1)^2$, it is unclear if it is actually $2\times$ an integer. Skips steps

1e. This plugs in one variable in the place of two, only tests for same two odd number variables. Invalid reasoning

2a Even numbers $= 2k$. $x+y = 2n + 2m$

$2(n+m)$ is also an even integer as $n+m$ is an integer and it is $2\times$ of it.

This Theorem is true.

2b. False $3+5=8$, 3 and 5 are not even.

2i. A perfect square is defined as n^2, $x \cdot y$ would be n^2m^2

~~n^2m^2 [illegible]~~ False 16 is a perfect square but $2 \cdot 8$, both are not perfect squares.

2k. If you have 5 and 4 as x and y, $x \cdot y = 20$. Twenty is divisible by 10, but you do not get an integer when $5/10$ or $4 \div 10$.

2l. Since y is a divisor for x, then, $y = kx$ for some integer k.

$x \mid y$ would be $y = jx$ for some integer j. This is true as there is a j for when you move x to get $y = \frac{k}{2}x$. True.

3a. Iff n^3 is even, n is even. Iff n^3 is odd, n is odd.

~~[illegible]~~ Since n is odd, then $n = 2k+1$

$(2k+1)^3 = 8k^3 + 12k^2 + 6k + 1 = 2(4k^3 + 6k^2 + 3k) + 1$

$4k^3 + 6k^2 + 3k$ is also an integer

n^3 is odd, proof by contrapositive.

3b. $\sqrt[3]{2}$ is rational, so $\sqrt[3]{2} = \frac{p}{q}$ $p^3 = 2q^3$ and $(p,q) = 1$

Since p^3 is even $p = 2m$

$2q^3 = (2m)^3 = 8m^3$

$q^3 = 4m^3$, q^3 is even and q is even

then, $(p,q) \geq 2$ on p, q are both even, which contradicts $(p,q) = 1$

So, $\sqrt[3]{2}$ is not rational.