13.1.4) need to choose a pencil from case of n.
Cant choose from an empty case ($\binom{n}{0}$ ways)
Choose a pencil from 1 pencil $\binom{n}{1}$ ways
Choose a pencil from 2 pencils $\binom{n}{2}$ 2ways

the total to do so is $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}\dots+\binom{n}{n}$

A pencil is only selected or not selected, so 2^n fits.

b) ~~places~~ n pencils
k places in blue case, m places in red case
$k \leq m \leq n$.

Choose m to be in full blue case $\binom{m}{m}$ ways, then choose out of the m ~~[illegible]~~ $\binom{n}{m}\binom{m}{k}$

Choose pencils to be put in blue case $\binom{n}{m}$ ways, then choose the remaining $\binom{n-k}{m-k}$ ~~[illegible]~~ in the red case $\binom{n}{k}\binom{n-k}{m-k}$

c) $2n$ pencils n ~~[illegible]~~ blue, n red, need to choose to fill pencil case.
All blue pencils and no red is $\binom{n}{0}\binom{n}{n}$.
$(n-1)$ blue pencils, 1 red, $\binom{n}{1}\binom{n}{n-1}$
0 blues, n reds $\binom{n}{n}\binom{n}{0}$

$$\binom{n}{0}\binom{n}{n}+\binom{n}{1}\binom{n}{n-1}+\dots+\binom{n}{n}\binom{n}{0}$$

Choose any n from $2n$ pencils is
$\binom{2n}{n}$

Order doesnt matter

# of comps that doesnt contain files

4. 11.5.6) C(37,2) 2 wouldnot contain file, 40-3=37

11.7.3) 2^8 is how many bit strings there are, 256,

10101010 and 01010101 are not included in this set,

256 - 2 = 254.

11.8.1) SUBSETS = 7 letters

$\frac{7!}{3!}$ # of letters / # of distinct letters — Permutation formula.

Example abc, cba, cab, etc.

11.8.7b) If she wants to cook each meal the same # of times

each meal will be made twice.

We have two 1's to choose from 20 places $\binom{20}{2}$

18 remains which is then $\binom{18}{2}$ ... etc.

$$\binom{20}{2} \times \binom{18}{2} \cdots \binom{4}{2} \cdots \binom{2}{2} = \frac{20!}{2^{10}}$$

11.9.1)

Racer	mins	secs
1	6	1
2	6	10
3	6	19
4	6	28
5	6	37
6	6	46
7	6	55

we can't say there are two runners that are less than 9 seconds apart.

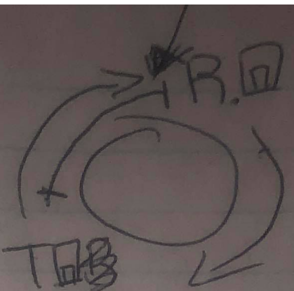
what about 8?

1	6	1
2	6	10
3	6	19
4	6	28
5	6	37
6	6	46
7	6	55
8	[illegible]	04

We only have 6-7 minutes

Gordon Ng CS131

① Base Step:



R = Reward Booth
T = Toll Booth

For Base case $n=1$, we get 1$ from reward booth and then Pay 1$ for the toll, so we can start in Area x.

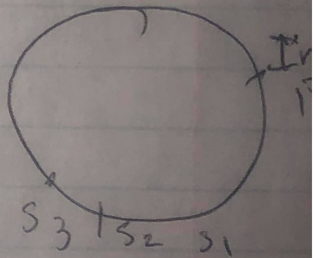
Inductive Step:

Rewards must always be greater than or equal to tolls.

$(n+1)$

You can't go from a toll booth to reward booth. Reward must come after toll, or reward comes after reward, but we must have toll booths.

From the position before R, we start and go around we get that



$\sum_i R_i \geq \sum_j T_j$ (using ①)

At S_2

$\sum_i R_i + 1 \geq \sum_j T_j \equiv$ Possible from ①

At position S_3

$\sum_i R_i + 1 \geq \sum_j T_j + 1 \equiv$ Also possible

$= \sum R_i \geq \sum_j T_j$

After S_3, You will have a Toll booth after a reward booth (n

3. Let player 1 = A and Player 2 = B
1) two piles where $m = n$ initially, and after that, $m \leq n$, where $m, n \in \mathbb{N}$.
2) if B chooses the same number of matches of A, he will always win.
ase: If both piles are 1, ~~and~~ that means A has already lost.

Induction:
~~The~~ Both piles start out at 100,
1. ~~An~~ ~~m~~ 100, ~~m~~ = 100
A takes 40 m
2) $m = 60$, $n = 100$
B takes 40
3) $m = 60$, $n = 60$
A takes 2
4) $m = 58$, $n = 60$
B takes 2
5) $m = 58$, $n = 58$
A takes 59
6) $m = 1$, $n = 58$
B takes 59
7) $m = 1$, $n = 1$
Player A has lost.

(2) Base Case:

P(8) 8 cents = 1 3-cent stamp and 1 5-cent stamp

P(9) 9 cents can be formed with 3 three-cent stamps

P(10) 10 cents: can be formed with two 5-cent stamps

If we give the number of cents ~~variable~~ variable k, we get

$k = \cancel{}\ 5x + 3y$

~~what~~ Whatever doesn't divide 3 gets carried over for 5 cent stamps

$3x$, $3x+1$ and $3x+2$

if we set "$k+1$" to $3x$, then this would be 1 3cent stamp, 0 5-c stamp

if we set "$k+1$" to $3x+1$, then $3(x-3)+9+1 = 3(x-3)+2 \cdot 5$.

if we set "$k+1$" to $3x+2$, then $3(r-1)+3+2 = 3(r-1)+5$.

If cents is greater than 8, then it is true.

and

11.9.3b) For 20 people to share a month for birthdays we would have $20 \times 12 = 240$ → worst case.

11.9.4a) From the numbers ~~1~~-14, ~~each~~ there are 7 pairs of numbers that add to 15. If you choose 8 numbers, there is garunteed a pair.

12.1.3a) Order doesn't matter 20 choose 5 books $C(20,5)$

b) Order does matter 20 choose 5 in permutation $P(20,5)$

c) The total number of ways to distribute with no situation, $20^{\wedge}5$.

12.2.8a) The coefficient would be

$$\frac{25!}{9!\,2!\,5!\,7!\,2!}$$, each degree is treated as a different variable.

b) There are 4 ~~degrees~~ terms in ~~x~~ $v^9 w^2 x^5 y^7 z^2$. w and z have the same degrees.

12.3.6) There are 98 different locations for a neighbor and 50 even numbers. 1 placed at the beginning, 1 placed at end and 1 can be placed in the middle, giving us

$2 \times 50 \times 98!$ + ~~[illegible]~~ $(99! - P(49,2) \times 97!)$

order matters when 1 is placed by odd numbers