CS 132 Assignment6

Gordon Ng

TOTAL POINTS

28 / 28

QUESTION 1

1 3 / 3

✓ **- 0 pts** Correct

- 1 pts Incorrect eigenvalues

- 1 pts Incorrect eigenvectors

- 1 pts Incorrect derivation of Ak

QUESTION 2

2 5 / 5

✓ **- 0 pts** Correct

- 5 pts Incorrect

QUESTION 3

3 5 / 5

✓ **- 0 pts** Correct

- 1 pts Incorrect T(b1)

- 1 pts Incorrect T(b2)

- 2 pts Incorrect method of operations (multiplication and subtraction)

- 1 pts Incorrect final answer

QUESTION 4

9 pts

4.1 3 / 3

✓ **- 0 pts** Correct

- 3 pts Missing or incorrect

4.2 3 / 3

✓ **- 0 pts** Correct

- 1 pts Fail to show T(p+q) = T(p)+T(q)

- 1 pts Fail to show T(c.p) = c.T(p)

- 3 pts Missing or wrong answer

4.3 3 / 3

✓ **- 0 pts** Correct

- 3 pts Missing or incorrect

QUESTION 5

6 pts

5.1 3 / 3

✓ **- 0 pts** Correct

- 3 pts Incorrect

- 2.4 pts Pages Not Selected Correctly

- 3 pts Missing

5.2 3 / 3

✓ **- 0 pts** Correct

- 3 pts Incorrect

- 3 pts Missing

1. $A = \begin{bmatrix} .4 & -.3 \\ .4 & 1.2 \end{bmatrix}$ why? $A^k = \begin{bmatrix} -.5 & -.75 \\ 1.0 & 1.5 \end{bmatrix}$ $k \to \infty$.

$A = PDP^{-1}$ $A^k = (PDP^{-1})^k$ $\cancel{P P^{-1}} = 1$

3 matrixes multiplied by itself over and over again makes $A^k = D^k$

$\cancel{PDP^{-1}}\ \cancel{P}DP^{-1}\ PDP^{-1} = A^3 = D^3$

determinant (A) : det(A), and so on . . . to get A^k

$\begin{matrix} .4-\lambda & -.3 \\ .4 & 1.2-\lambda \end{matrix}$ $(.4-\lambda)(1.2-\lambda)$ $.48 + \lambda^2 - 1.6\lambda + (3 \cdot .4)$ $\lambda^2 - 1.6\lambda + .6$ $(\lambda - 1)(\lambda - .6)$

$[A - 1I]X = 0$ $\begin{bmatrix} .4-1 & -.3 \\ .4 & 1.2-1 \end{bmatrix}$ $[A - .6I]X = 0$ $\begin{bmatrix} -.5-.6 & -.75 \\ 1.0 & \cancel{1.5-.6} \end{bmatrix}$

$D = \begin{bmatrix} 1 & 0 \\ 0 & .6 \end{bmatrix}^k$ $\begin{bmatrix} 1 & 0 \\ 0 & .6 \end{bmatrix}^k$ . . .

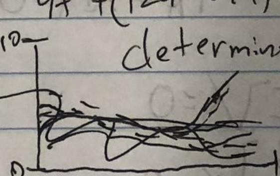
2. Characteristic polynomials → eigen values

$A = \begin{bmatrix} -6 & 28 & 21 \\ 4 & -15 & -12 \\ -8 & a & 25 \end{bmatrix}$ a in set {32, 31.9, 31.8, 32.1, 32.2}

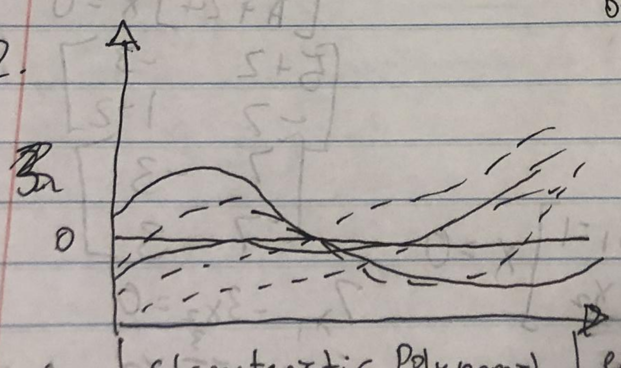
$\Delta(t) = \det(tI - A)$

$\begin{bmatrix} t+6 & t-28 & -21 \\ -4 & t+15 & 12 \\ 8 & -a & t-25 \end{bmatrix}$ $\Delta(t) = t^3 - 4t^2 + (12a - 379)t - 12a + 382 = 0$

graph determinants.



2.



a_1 one root $0 \le t \le 3$

a_2 three roots $0 \le t \le 3$

a_3 3 roots, one repeated (bounced off 0 zeros) $0 \le t \le 3$

a_4 1 root $0 \le t \le 3$

a_5 one root $0 \le t \le 3$

a	characteristic Polynomial	eigenvalues
31.8	$-4 - 2.6t + 4t^2 - t^3$	3.12, 1, -.12
31.9	$.8 - 3.8t + 4t^2 - t^3$	2.70, 1, 2.98
32.0	$2 - 5t + 4t^2 - t^3$	2, 1, 1
32.1	$3.2 - 6.2t + 4t^2 - t^3$	1.5 ± .97i, 1
32.2	$4.4 - 7.4t + 4t^2 - t^3$	1.5 ± 1.4i, 1

The graph shows the roots are the eigenvalues. As a increases the graph has a lower curve at the end.

1 3 / 3

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- 1 pts Incorrect eigenvalues

- 1 pts Incorrect eigenvectors

- 1 pts Incorrect derivation of Ak

why?

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determinant (A) : det(A), and so on. . . to get A^k

$\begin{matrix} .4-\lambda & -.3 \\ .4 & 1.2-\lambda \end{matrix}$

$(.4-\lambda)(1.2-\lambda)$

$.48+\lambda^2-1.6\lambda + (.3 \cdot .4)$

$\lambda^2 - 1.6\lambda + .6$

$(\lambda-1)(\lambda-.6)$

$[A-1I]X=0$ $\begin{bmatrix} .4-1 & -.3 \\ .4 & 1.2-1 \end{bmatrix}$

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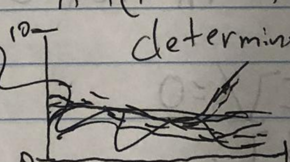
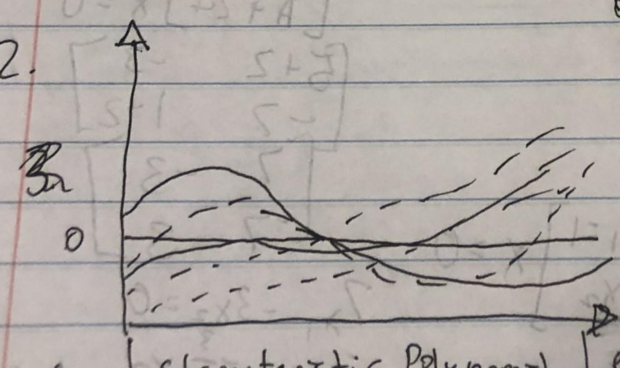
graph



determinants.

2.



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The graph shows the roots are the eigenvalues.
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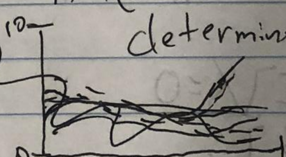

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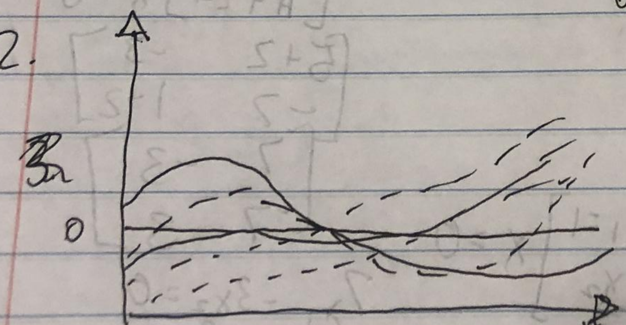

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The graph shows the roots are the eigenvalues. As a increases the graph has a lower curve at the end.

3. $[T]_B = \begin{bmatrix} 0 & -6 & 1 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$ $T(3b_1 - 4b_2) = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & -6 & 1 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+24+0 \\ 0+-20-0 \\ 3+8+0 \end{bmatrix} = \begin{bmatrix} 24 \\ -20 \\ 11 \end{bmatrix}$$

$24b_1 - 20b_2 + 11b_3$

4a. $p(t) = 5+3t$ $p(-1) = 5+3(-1) = 2$ $p(0) = 5+3(0) = 5$ $p(1) = 5+3(1) = 8$

$T(p) = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$ $T(p) = \begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix}$

4b.
$T(cp) = \begin{bmatrix} (cp)(-1) \\ (cp)(0) \\ (cp)(1) \end{bmatrix} = c\begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix}$ $T(p+q) = T(p) + T(q)$
$T(cp) = c \cdot T(p)$

T is linear.
$T(p+q) = \begin{bmatrix} (p+q)(-1) \\ (p+q)(0) \\ (p+q)(1) \end{bmatrix}$ $\begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix} + \begin{bmatrix} q(-1) \\ q(0) \\ q(1) \end{bmatrix}$

4c. $T(1) = T(p(t))$ $1 = p(t)$ $T(t) = T(p(t))$ $T(t^2) = T(p(t))$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

3 5 / 5

- ✓ **- 0 pts** Correct
- **- 1 pts** Incorrect T(b1)
- **- 1 pts** Incorrect T(b2)
- **- 2 pts** Incorrect method of operations (multiplication and subtraction)
- **- 1 pts** Incorrect final answer

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$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

4.1 3 / 3

✓ **- 0 pts** Correct

- 3 pts Missing or incorrect

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$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

4.2 3 / 3

✓ **- 0 pts** Correct

- 1 pts Fail to show T(p+q) = T(p)+T(q)

- 1 pts Fail to show T(c.p) = c.T(p)

- 3 pts Missing or wrong answer

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$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

4.3 3 / 3

✓ **- 0 pts** Correct

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4c) $\cancel{1+4t+8t^2}$

Diagonal matrices = perpendicular

$\cancel{\vec{a}_1 = T(1)}$
$\cancel{\vec{a}_2 = T(t)}$
$\cancel{\vec{a}_3 = T(t^2)}$

$T = \cancel{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}$

5a. $A = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix}$

nullspace find x for eigen vector
$|A - \lambda I| = 0$ iff diagonal, then $A = PDP^{-1}$ $A\vec{x} - \lambda\vec{x} = 0$

$\begin{bmatrix} 0-\lambda & 1 \\ -3 & 4-\lambda \end{bmatrix}$ $-\lambda(4-\lambda)$
$-4\lambda+\lambda^2+3$
$(\lambda-3)(\lambda-1)$

$[A - 3I]$
$\begin{bmatrix} 0-3 & 1 \\ -3 & 4-3 \end{bmatrix}$
$\begin{bmatrix} -3 & 1 \\ -3 & 1 \end{bmatrix}$ $x_1 = \frac{1}{3}x_2$

$[A - 1I]$
$\begin{bmatrix} 0-1 & 1 \\ -3 & 4-1 \end{bmatrix}$
$\begin{bmatrix} -1 & 1 \\ -3 & 3 \end{bmatrix}$ $-1x_1\ 1x_2$

$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $b_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

eigen to standard
P
standard to eigen
P^{-}

5b. $A = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix}$ $|A - \lambda I| = 0$ $\begin{vmatrix} 5-\lambda & -3 \\ -7 & 1-\lambda \end{vmatrix} = 0$ $\lambda^2 - 6\lambda - 16 = 0$
$(\lambda-8)(\lambda+2) = 0$
$\lambda = 8, -2$

eigen vector $[A - 8I]X = 0$
$\begin{bmatrix} 5-8 & -3 \\ -7 & 1-8 \end{bmatrix} X = 0$
$\begin{bmatrix} -3 & -3 \\ -7 & -7 \end{bmatrix} X = 0$
$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} X = 0$ $\begin{bmatrix} x_1 = 1 \\ x_2 \end{bmatrix} X = 0$

$b_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$[A + 2I]X = 0$
$\begin{bmatrix} 5+2 & -3 \\ -7 & 1+2 \end{bmatrix}$
$\begin{bmatrix} 7 & -3 \\ -7 & 3 \end{bmatrix}$
$7x_1 - 3x_2 = 0$
$x_1 = \frac{3}{7}x_2$ $x_2 = x_2$
$b_2 = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$P = \begin{bmatrix} -1 & 3 \\ 1 & 7 \end{bmatrix}$ $P = \begin{bmatrix} -1 & 3 \\ 1 & 7 \end{bmatrix}$ $D = \begin{bmatrix} 8 & 0 \\ 0 & -2 \end{bmatrix}$

B matrix of the transformation $X \mapsto AX$ is a diagonal matrix

5.1 3 / 3

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect
- **- 2.4 pts** Pages Not Selected Correctly
- **- 3 pts** Missing

4c) $\cancel{1+4t+8t^2}$

Diagonal matrices = perpendicular

$\cancel{\vec{a}_1 = T(1)}$

$\cancel{\vec{a}_2 = T(t)}$

$\cancel{\vec{a}_3 = T(t^2)}$

$T = \cancel{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}$

5a. $A = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix}$

nullspace

$|A - \lambda I| = 0$

find x for eigen vector

iff diagonal, then $A = PDP^{-1}$

$A\vec{x} - \lambda\vec{x} = 0$

$\begin{bmatrix} 0-\lambda & 1 \\ -3 & 4-\lambda \end{bmatrix}$

$-\lambda(4-\lambda)$

$-4\lambda + \lambda^2 + 3$

$(\lambda-3)(\lambda-1)$

$[A - 3I]$

$\begin{bmatrix} 0-3 & 1 \\ -3 & 4-3 \end{bmatrix}$

$\begin{bmatrix} -3 & 1 \\ -3 & 1 \end{bmatrix}$ $x_1 = \frac{1}{3}x_2$

$[A - 1I]$

$\begin{bmatrix} 0-1 & 1 \\ -3 & 4-1 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 \\ -3 & 3 \end{bmatrix}$ $-\frac{1}{x_1}\, 1 x_2$

$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $b_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

eigen to standard

P

standard to eigen

P^{-}

5b. $A = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} 5-\lambda & -3 \\ -7 & 1-\lambda \end{vmatrix} = 0$

$\lambda^2 - 6\lambda - 16 = 0$

$(\lambda-8)(\lambda+2) = 0$

$\lambda = 8, -2$

eigen vector $[A - 8I]X = 0$

$\begin{bmatrix} 5-8 & -3 \\ -7 & 1-8 \end{bmatrix} X = 0$

$\begin{bmatrix} -3 & -3 \\ -7 & -7 \end{bmatrix} X = 0$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} X = 0$

$\begin{bmatrix} x_1 = 1 \\ x_2 \end{bmatrix} X = 0$

$b_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$[A + 2I]X = 0$

$\begin{bmatrix} 5+2 & -3 \\ -7 & 1+2 \end{bmatrix}$

$\begin{bmatrix} 7 & -3 \\ -7 & 3 \end{bmatrix}$

$7x_1 - 3x_2 = 0$

$x_1 = \frac{3}{7}x_2$ $x_2 = x_2$

$b_2 = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$P = \begin{bmatrix} -1 & 3 \\ 1 & 7 \end{bmatrix}$ $P = \begin{bmatrix} -1 & 3 \\ 1 & 7 \end{bmatrix}$ $D = \begin{bmatrix} 8 & 0 \\ 0 & -2 \end{bmatrix}$

B matrix of the transformation $X \mapsto AX$ is a diagonal matrix

5.2 3 / 3

✓ **- 0 pts** Correct

- 3 pts Incorrect

- 3 pts Missing