1) give the inverse, converse and contrapositive for each of the following statements:

a) If she finished her homework, then she went to the party.

- inverse: If she did not finish her homework, then she did not go to the party.
- contrapositive: If she did not go to the party, then she did not finish her homework.
- converse: If she went to the party, then she finished her homework.

b) If he trained for the race, then he finished the race.

- inverse: If he did not train for the race, then he did not finish the race.
- contrapositive: If he did not finish the race, then he did not train for the race.
- converse: If he finished the race, then he trained for the race.

2 a) $(p \to q) \land (r \to q)$ and $(p \land r) \to q$ not logically equivalent

~~$p \to q$~~	~~$r \to q$~~	~~$(p \to q) \land (r \to q)$~~
~~T~~	~~T~~	~~T~~
~~F~~	~~F~~	~~T~~
~~T~~	~~T~~	~~T~~
~~T~~	~~T~~	~~T~~

~~$(p \land r) \to q$~~
~~F~~
~~F~~
~~F~~
~~T~~

2a)

p	q	$(p \land r) \to q$	$((p \to q) \land (r \to q))$
F	F	T	T
F	F	T	F
F	T	T	T
F	T	T	T
T	F	T	F
T	F	F	F
T	T	T	T
T	T	T	T

These two are not equivalent ~~because~~ because any falses in q lead to a false in $((p \to q) \land (r \to q))$, but if they are both false for $((p \land r) \to q)$, then it may be true.

a) If $(L1 \wedge L2)$ is true, then the treasure is in 2nd path, but this means $(L1 \wedge T2) \vee (L2 \wedge T1)$ also true because of denomination law. $A \vee T = T$, But the seller only said one is true, so $(L1 \wedge T2) \vee (L2 \vee T1)$ Is true.

b) $L1 \wedge T2$ is true and $(L1 \wedge T2) \vee (L2 \wedge T1)$ is false

$(L1 \wedge T2) \wedge (\neg(L1 \wedge T2) \vee (L2 \wedge T1)))$
$= (L1 \wedge T2) \wedge (\neg(L1 \wedge T2) \wedge \neg(L2 \wedge T1))$ Demorgan's Law $\neg(A \vee B) = \neg A \wedge \neg$
$= (L1 \wedge T2) \wedge \neg(L1 \wedge T2) \wedge \neg(L2 \wedge T1)$ Associative Law
$= F \wedge \neg(L2 \wedge T1)$ $A \wedge \neg A = F$
$= F$
$(L1 \wedge T2)$ is false and $(L1 \wedge T2) \vee (L2 \wedge T1)$ is true (check)

$\neg(L1 \wedge T2) \wedge (L1 \wedge T2) \vee (L2 \wedge T2)$ ~~is false~~ distributive law
$(\neg(L1 \wedge T2) \wedge (L1 \wedge T2)) \vee (\neg(L1 \wedge T2) \wedge (L2 \wedge T1))$
$F \vee (\neg(L1 \wedge T2) \wedge (L2 \wedge T1))$ | $F \vee A = A$
$T \wedge (L2 \wedge T1)$ | $T \wedge A = A$
$L2 \wedge T1$
$(L1 \wedge T2)$ is False and $(L1 \wedge T2) \vee (L2 \wedge T1)$ is true, $L2 \wedge T1$ is true

The lying inscription is true, and the other is false, contradiction.

c) ~~it does make~~ $L2 \wedge T1$ is true so the first path has the treasure and the second one ~~has~~ makes you lost forever
proof: part b.
d) It is a correct assumption because both of the paths do not intersect. it can not coincide. Either ~~they~~ go down path 1 for treasure or you go down path two to be lost forever.

4a) $p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$

p	q	$p \oplus q$	$((p \wedge \neg q) \vee (\neg p \wedge q))$	$p \wedge \neg q$	$\neg p \wedge q$
F	F	F	F	F	F
F	T	T	T	F	T
T	F	T	T	T	F
T	T	F	F	F	F

b)

p	q	$p \wedge q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
F	F	F	T	F
F	T	F	T	F
T	F	F	T	F
T	T	T	F	T

$\neg(\neg p \vee \neg q) \equiv p \wedge q$

c)

$p \wedge q$	$p \rightarrow \neg q$	$\neg(p \rightarrow \neg q)$
F	T	F
F	T	F
F	T	F
T	F	T

$\neg(p \rightarrow \neg q \equiv p \wedge q$

d) Since we can keep reordering $p \wedge q$ with 2 operators, there's 4 operators, so my assumption is $4!$, or 24 ways.

e)

p	q	p nand q	$\overline{p \cdot q}$
F	F	T	
T	F	T	
F	T	T	
T	T	F	

f)

p	q	$p \vee q$
F	F	F
T	F	T
F	T	T
T	T	T

p	$\neg p$
F	T
T	F

Nand is equivilent of adding a not to $p \wedge q$, which we have proved can be used with two expressions to become equivalent to it.

5 p you study computer science

q you'll be smart

r you'll be happy

Proof:

$p <= q$ and $p <= r$

~~na~~ $(\neg p \vee q) \wedge (\neg p \vee r)$ conditional identity

$(\neg p$ or $(q \wedge r))$ distributive law

$p <= (q \wedge r)$ Conditional probability