CS 132 Assignment1

Gordon Ng

TOTAL POINTS

25.75 / 32

QUESTION 1

4 pts

1.1 Question 1.a 2 / 2

✓ **- 0 pts** **Correct work and answer**

- **1 pts** Correct work, but incorrect final answer
- **1 pts** Correct answer, but no work
- **1.25 pts** Incorrect work and answer
- **2 pts** No answer / incorrect answer

1.2 Question 1.b 0.75 / 2

- **0 pts** Correct work and answer
- **1 pts** Correct work, but incorrect final answer
- **1 pts** Correct answer, but no work

✓ **- 1.25 pts** **Incorrect work and answer**

- **2 pts** No answer / incorrect answer

QUESTION 2

4 pts

2.1 Question 2.a 2 / 2

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- **2 pts** No solution provided.

2.2 Question 2.b 2 / 2

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- **2 pts** No solution provided.

QUESTION 3

4 pts

3.1 Question 3.a 2 / 2

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- **2 pts** No solution provided.

3.2 Question 3.b 2 / 2

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- **2 pts** No solution provided.

QUESTION 4

4 Question 4 2 / 4

- **0 pts** Correct answer and correct explanation

✓ **- 2 pts** **Correct answer, no explanation or incorrect explanation**

- **4 pts** No answer or incorrect answer

The solution will be unique because a1 and a2 are not co-linear. This means that there is only one combination of the two vectors to reach every point in R2.

QUESTION 5

5 Question 5 3 / 4

- **0 pts** Correct explanation

✓ **- 1 pts** **Did not mention "no free variables"**

- **1 pts** Did not mention "every column is a pivot column"
- **1 pts** Did not conclude matrix A spans R3
- **4 pts** No answer or incorrect answer

QUESTION 6

6 Question 6 4 / 4

✓ **- 0 pts** **Correct answer and explanation**

- **2 pts** Correct answer, no explanation
- **4 pts** No answer or incorrect answer

QUESTION 7

4 pts

7.1 Question 7.a **2 / 2**

✓ **- 0 pts** **Correct work and correct conclusion**

- 1 pts Correct work, incorrect conclusion

- 2 pts No answer or incorrect answer

7.2 Question 7.b **2 / 2**

✓ **- 0 pts** **Correct work and correct conclusion**

- 1 pts Correct work, incorrect conclusion

- 2 pts No answer or incorrect answer

QUESTION 8

4 pts

8.1 Question 8.a **2 / 2**

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- 2 pts No solution provided.

8.2 Question 8.b **0 / 1**

- 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

✓ **- 2 pts** **No solution provided.**

8.3 Question 8.c **0 / 1**

- 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

✓ **- 2 pts** **No solution provided.**

consistent with all
values of h.

$$\left[\begin{array}{cc:c} 1 & 2 & -4 \\ 0 & h-6 & 0 \end{array}\right]$$

Gordon Ng

1a $\begin{bmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{bmatrix}$ $2h=7$

$h=3.5$

1b.

$-2, 0, 5, 10, 20$

2 a) $v_1 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ $v_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$ weight

-2

$= 2 \cdot v_2$ $\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 0 \\ -6 \end{bmatrix}$

$\begin{bmatrix} 15 \\ 0 \\ 10 \end{bmatrix}$ $\begin{bmatrix} -10 \\ 0 \\ 15 \end{bmatrix}$ | $c_1 = 10$ $c_2 = 10$ $\begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix}$

$c_1 = 0$ $c_2 = 0$ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

1.1 Question 1.a 2 / 2

✓ **- 0 pts** **Correct work and answer**

- 1 pts Correct work, but incorrect final answer

- 1 pts Correct answer, but no work

- 1.25 pts Incorrect work and answer

- 2 pts No answer / incorrect answer consistent with all values of h.

$$\begin{bmatrix} 1 & 2 & -4 \\ 0 & h-6 & 0 \end{bmatrix}$$

$2h = 7$
$h = 3.5$

16.

$5 \; -15 \; -10$

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & 7 \end{bmatrix}$$

$3 \; -9 \; -6$ $\quad 5 \; h-15 \; 7$
$1 \; -3 \; -2$

$$\begin{bmatrix} 10 & -30 & -20 \\ 10 & 2h & -14 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2h-30 & 6 \\ 10 & 2h & -14 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} \quad \text{weight } -2 \quad \begin{bmatrix} -6 \\ 0 \\ -4 \end{bmatrix} -2 \cdot v_1$$

$$\boxed{\begin{matrix} C_1 = -2 \\ C_2 = -2 \end{matrix}}$$

$$\begin{bmatrix} 4 \\ 0 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} -6 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 10 \end{bmatrix}$$

$C_1 = 10$
$C_2 = 10$

$$\begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix} \begin{bmatrix} -20 \\ 0 \\ 30 \end{bmatrix}$$

$C_1 = 20$
$C_2 = 20$

$$\begin{bmatrix} 60 \\ 0 \\ 40 \end{bmatrix} \begin{bmatrix} -40 \\ 0 \\ -60 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix} + \begin{bmatrix} -20 \\ 0 \\ 30 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 50 \end{bmatrix}$$

1, -6, -5, 0

$$\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$$

$C_1 = 7 \; C_2 = 7$

$$\begin{bmatrix} 49 \\ 7 \\ 42 \end{bmatrix} \begin{bmatrix} -35 \\ 21 \\ 0 \end{bmatrix}$$

$C_1 = 1$
$C_2 = 1$

$$\begin{bmatrix} 7 \\ 1 \\ 6 \end{bmatrix} \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$$

1.2 Question 1.b **0.75 / 2**

- **- 0 pts** Correct work and answer
- **- 1 pts** Correct work, but incorrect final answer
- **- 1 pts** Correct answer, but no work
- ✓ **- 1.25 pts** Incorrect work and answer
- **- 2 pts** No answer / incorrect answer

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n \left[\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n} \right]$ ← nope! not enough pivots

at most n pivots, n vectors can't span $\mathbb{R}^m$, $m > n$, three vectors can't span $\mathbb{R}^4$

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existance ⟺ b is the span of A
→ Uniqueness ⟺ A is linearly independent (not ~~l~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least two dimensional ~~mana~~ which will span $\mathbb{R}^3$. $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution $\forall$ REF of $[A|B]$ cannot have $[0 \ldots 0 | n]$ $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ 5/7

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span R4, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 0 & 63/11 & -10/11 & -5/11 & 191/11 \\ 0 & 65/11 & -10/11 & 61/11 & 59/11 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/11 & -10/11 & -5/11 & 191/11 \\ 0 & 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 0 & -12/5 \end{bmatrix}$

Spans $\mathbb{R}^4$, 4 pivots

2.1 Question 2.a **2 / 2**

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- 2 pts No solution provided.

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n}$ ← nope! not enough pivots

at most n pivots, $m > n$, n vectors can't span $\mathbb{R}^m$, three vectors can't span $\mathbb{R}^4$

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existance ⟺ b is the span of A
→ Uniqueness ⟺ A is linearly independent (not linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least two dimensional ~~plane~~ which will span $\mathbb{R}^3$. $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution. ~~V~~ RREF of $[A|B]$ cannot have $[0 \ldots 0|n]$ $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $5/7$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span $\mathbb{R}^4$, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 0 & 65/11 & -10/11 & -5/11 & 191/11 \\ 0 & 65/11 & -10/11 & 61/11 & 59/11 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/11 & -10/11 & -5/11 & 191/11 \\ 0 & 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 0 & -12/5 \end{bmatrix}$

Spans $\mathbb{R}^4$, 4 pivots

2.2 Question 2.b **2 / 2**

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- 2 pts No solution provided.

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n\begin{bmatrix} \overbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix}}^{n} \end{bmatrix}$ at most n pivots, n vectors can't span $\mathbb{R}^m$, $m > n$, three vectors can't span $\mathbb{R}^4$

← nope! not enough pivots

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existance ⟷ b is the span of A

→ Uniqueness ⟷ A is linearly independent (not ~~l~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least two dimensional ~~mana~~ which will span $\mathbb{R}^3$ $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution $\forall b$ REF of [A|B] cannot have $[0 \ldots 0 | n]$ $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $5/7$ $1/3$ ~~0 2 3 v 2/3 5~~ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span R4, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix}$

3.1 Question 3.a 2 / 2

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- 2 pts No solution provided.

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $\begin{array}{cc} & n \\ n & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ at most n pivots, n vectors can't span $\mathbb{R}^m$ $m > n$, three vectors can't span $\mathbb{R}^4$

← nope! not enough pivots

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existance ⟷ b is the span of A

→ Uniqueness ⟷ A is linearly independent (not ~~l~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least two dimensional ~~mana~~ which will span $\mathbb{R}^3$ $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution

$\forall b$ REF of [A|B] cannot have [0 .. 0|n] $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $5/7$ $\frac{1}{2}$ ~~023~~ $\frac{2}{3}$ 5 $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span R4, missing a pivot. No solution

7b $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/ & 6/ & 3/ & 141 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 39/ & 6/ & 3/ & -141 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/ & 10/5 & & 191 \end{bmatrix}$

3.2 Question 3.b 2 / 2

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- 2 pts No solution provided.

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $\begin{array}{cc} & \overbrace{}^{n} \\ n & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ at most n pivots, n vectors can't span $\mathbb{R}^m$, $m > n$, three vectors can't span $\mathbb{R}$

← nope! not enough pivots

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ ↗ existance ⟷ b is the span of A
→ Uniqueness ⟷ A is linearly independent (not ~~l~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least ~~f~~ two dimensional ~~mana~~ which will span $\mathbb{R}^3$ $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution $\forall$ b REF of $[A|B]$ cannot have $[0 \ldots 0|n]$ $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ 5/7 ~~1/3~~ ~~0 2 3 ~~2/3~~ 5~~ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span R4, missing a pivot. No solution

7b $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/ & 6/ & 3/ & -141 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 39/ & 6/ & 3/ & -141 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/ & -10/ & 5/ & 191 \end{bmatrix}$

4 Question 4 2 / 4

- **- 0 pts** Correct answer and correct explanation
- ✓ **- 2 pts Correct answer, no explanation or incorrect explanation**
- **- 4 pts** No answer or incorrect answer

The solution will be unique because a1 and a2 are not co-linear. This means that there is only one combination of the two vectors to reach every point in R2.

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n}$ at most n pivots, n vectors can't span $\mathbb{R}^m$, $m > n$, three vectors can't span $\mathbb{R}$

← nope! not enough pivots

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~[illegible]~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existance ⟷ b is the span of A
→ Uniqueness ⟷ A is linearly independent (not ~~l~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least ~~t~~ two dimensional ~~[illegible]~~ which will span $\mathbb{R}^3$ $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution
~~∀b~~ REF of [A|B] cannot have [0 .. 0|n] $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $\frac{5}{7}$ ~~0 2 3 ~ 2 3 5~~ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -\frac{11}{7} & \frac{3}{7} & -\frac{23}{7} \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-\frac{6}{7}$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -\frac{11}{7} & \frac{3}{7} & -\frac{23}{7} \\ 0 & \frac{58}{7} & \frac{16}{7} & \frac{1}{7} \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & \frac{53}{11} & -\frac{189}{11} \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span R4, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \end{bmatrix}$

5 Question 5 **3 / 4**

- **- 0 pts** Correct explanation
- ✓ **- 1 pts** **Did not mention "no free variables"**
- **- 1 pts** Did not mention "every column is a pivot column"
- **- 1 pts** Did not conclude matrix A spans R3
- **- 4 pts** No answer or incorrect answer

gradescope

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n}$ ← nope! not enough pivots

at most n pivots, n vectors can't span $\mathbb{R}^m$, $m > n$, three vectors can't span $\mathbb{R}^4$

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existence ⟺ b is the span of A
→ Uniqueness ⟺ A is linearly independent (not ~~l~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least ~~f~~ two dimensional ~~plane~~ which will span $\mathbb{R}^3$. $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution ~~V~~ RREF of $[A|B]$ cannot have $[0 \dots 0|n]$ $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $5/7$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span $\mathbb{R}^4$, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 0 & 63/11 & -10/11 & -3/11 & 191/11 \\ 0 & 65/11 & -10/11 & 6/11 & 59/11 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/11 & -10/11 & -5/11 & 191/11 \\ 0 & 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 0 & -12/5 \end{bmatrix}$

Spans $\mathbb{R}^4$, 4 pivots

6 Question 6 4 / 4

✓ - **0 pts** **Correct answer and explanation**

- **2 pts** Correct answer, no explanation
- **4 pts** No answer or incorrect answer

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n}$ ← nope! not enough pivots

at most n pivots, n vectors can't span $\mathbb{R}^m$, $m > n$, three vectors can't span $\mathbb{R}^4$

4. Yes the form does have a $Ax = b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax = b$ → existence ⟺ b is the span of A
→ Uniqueness ⟺ A is linearly independent (not ~~l~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least ~~t~~ two dimensional ~~mana~~ which will span $\mathbb{R}^3$. $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution ~~V~~ REF of $[A|B]$ cannot have $[0 \ldots 0|n]$ $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $5/7$, ~~$1/2$~~

~~$R_1 3 R_2 4 R_3 5$~~ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span $\mathbb{R}^4$, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 8 & 11 & 6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 0 & 63/11 & -10/11 & -3/11 & 191/11 \\ 0 & 65/11 & -10/11 & 61/11 & 59/11 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/11 & -10/11 & -5/11 & 191/11 \\ 0 & 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 0 & -12/5 \end{bmatrix}$

Spans $\mathbb{R}^4$, 4 pivots

7.1 Question 7.a 2 / 2

✓ **- 0 pts** **Correct work and correct conclusion**

- 1 pts Correct work, incorrect conclusion

- 2 pts No answer or incorrect answer

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n}$ ← nope! not enough pivots

at most n pivots, n vectors can't span $\mathbb{R}^m$, $m > n$, three vectors can't span $\mathbb{R}^4$

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existence ⟺ b is the span of A
→ Uniqueness ⟺ A is linearly independent (not ~~l~~ linear, not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least ~~t~~ two dimensional ~~plane~~ which will span $\mathbb{R}^3$. $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution ~~V~~ REF of $[A|B]$ cannot have $[0 \ldots 0|n]$ $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $5/7$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span $\mathbb{R}^4$, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 0 & 65/11 & -10/11 & -5/11 & 191/11 \\ 0 & 65/11 & -10/11 & 61/11 & 59/11 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/11 & -10/11 & -5/11 & 191/11 \\ 0 & 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 0 & -12/5 \end{bmatrix}$

Spans $\mathbb{R}^4$, 4 pivots.

7.2 Question 7.b 2 / 2

✓ **- 0 pts** **Correct work and correct conclusion**

- 1 pts Correct work, incorrect conclusion

- 2 pts No answer or incorrect answer

$2\,(1\ \ -4\ \ 2)$

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n}$ ← nope! not enough pivots

at most n pivots, $m > n$ — n vectors can't span $\mathbb{R}^m$, three vectors can't span $\mathbb{R}^4$

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existence ⟺ b is the span of A
→ Uniqueness ⟺ A is linearly independent (not ~~linear~~ not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least ~~t~~wo dimensional ~~image~~ which will span $\mathbb{R}^3$. $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution
~~V~~ REF of $[A|B]$ cannot have $[0 \ldots 0|n]$ $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $5/7$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span R4, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -\frac{141}{11} \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -\frac{141}{11} \\ 0 & 63/11 & -10/11 & -5/11 & \frac{191}{11} \\ 0 & 65/11 & -10/11 & 61/11 & 59/11 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/11 & -10/11 & -5/11 & \frac{191}{11} \\ 0 & 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 0 & -12/5 \end{bmatrix}$

Spans $\mathbb{R}^4$, 4 pivots

8.1 Question 8.a 2 / 2

✓ **- 0 pts** **We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.**

- 2 pts No solution provided.

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n}$ ← nope! not enough pivots

at most n pivots, $m > n$ n vectors can't span $\mathbb{R}^m$, three vectors can't span $\mathbb{R}^4$

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~vector~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existance ⟺ b is the span of A
→ Uniqueness ⟺ A is linearly independent (not ~~l~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least ~~t~~ two dimensional ~~[illegible]~~ which will span $\mathbb{R}^3$. $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution
~~V~~ REF of [A|B] cannot have [0 ... 0|n] $n \neq 0$ 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ 5/7, 1/3

~~0 2 3 2/3 5~~ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span R4, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -\frac{141}{11} \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -\frac{141}{11} \\ 0 & 63/11 & -10/11 & -5/11 & \frac{191}{11} \\ 0 & 65/11 & -10/11 & 61/11 & 59/11 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/11 & -10/11 & -5/11 & \frac{191}{11} \\ 0 & 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 0 & -12/5 \end{bmatrix}$

Spans $\mathbb{R}^4$, 4 pivots

8.2 Question 8.b **0 / 1**

- 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

✓ **- 2 pts** **No solution provided.**

2 (1 -4 2)

3a. $\begin{bmatrix} 2 & -8 & 4 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ Not a linear combination, no solution

3b. $\begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}$ $a = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 0 & 0 & 11 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$ $\begin{bmatrix} 11 & 10 & -54 \\ 0 & -15 & 63 \\ 0 & 0 & 99 \end{bmatrix}$

A linear combination is formed.

6. $n \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{n}$ at most n pivots, $m > n$ n vectors can't span $\mathbb{R}^m$, three vectors can't span $\mathbb{R}^4$

← nope! not enough pivots

4. Yes the form does have a $Ax=b$ solution because if you include negative numbers as c_1 and c_2, ~~[illegible]~~ it will reach b. The solution is not unique, B is the span of A.

solution $Ax=b$ → existence ⟺ b is the span of A
→ Uniqueness ⟺ A is linearly independent (not ~~[illegible]~~ linear not coplanar)

5. Since it is a 3×3 and it has a solution then it is at least ~~t~~ two dimensional ~~[illegible]~~ which will span $\mathbb{R}^3$. $\forall b \in \mathbb{R}^3$, $Ax=b$ has solution. ~~V~~ RREF of $[A|B]$ cannot have $[0 \ldots 0|n]$ $n \neq 0$. 3 pivots so spans $\mathbb{R}^3$, unique

7a) $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ -5 & -3 & 4 & -9 \end{bmatrix}$ $5/7$ ~~[illegible]~~ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$ $-6/7$

$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11/7 & 3/7 & -23/7 \\ 0 & 58/7 & 16/7 & 1/7 \\ 0 & 11 & -3 & 23 \end{bmatrix}$ $\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & 11 & -3 & 23 \\ 0 & 0 & 53/11 & -189/11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ No. It doesn't span $\mathbb{R}^4$, missing a pivot. No solution

7b. $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 8 & 11 & -6 & -7 & 13 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & -39/11 & 6/11 & 3/11 & -141/11 \\ 0 & 63/11 & -10/11 & -3/11 & 191/11 \\ 0 & 65/11 & -10/11 & 61/11 & 59/11 \end{bmatrix}$ $\begin{bmatrix} 11 & 7 & -7 & -9 & -6 \\ 0 & 65/11 & -10/11 & -5/11 & 191/11 \\ 0 & 0 & 0 & 6 & -12 \\ 0 & 0 & 0 & 0 & -12/5 \end{bmatrix}$

Spans $\mathbb{R}^4$, 4 pivots

8.3 Question 8.c **0 / 1**

- 0 pts We are not grading this question for correctness. Please check the solutions if you want to see whether you have the correct answers.

✓ **- 2 pts** **No solution provided.**