Problem Set 10 - Gordon Ng gng8@bu.edu

Problem 1)

binomial formula $= p^k(1-p)^{(n-k)}$

$X_A = \Sigma \binom{n}{k}(p^n)(1-p)^{n-k}$

$X_B = \Sigma \binom{m}{k}(p^m)(1-p)^{m-k}$

b) $P(X_A \cap X_B)$ = total obtained heads

$$f(x)^{(x)} = \begin{cases} X_A & 0 < x \leq n \\ X_B & 0 < x \leq m \end{cases}$$ two different coins

PDF of x is $\left(\Sigma \binom{n}{k}(p^n)(1-k)^{n-k}\right) + \left(\Sigma \binom{m}{k}(p^m)(1-p)^{m-k}\right)$

Problem 2)

a) $P(\text{Chocolate}) = 1/3$ $P(\text{Any}) = 2/3$

$E(X) = ((1)(\frac{2}{3})^0(\frac{1}{3}) + ((2)(2/3)^1(1/3)) ((3)(\frac{2}{3})^2(\frac{1}{3}) + \ldots = \frac{1}{3}(1 + 2r + 3r^2 + \ldots)$

$r = 2/3$.

Geometric Sequence with $r = 2/3$.

Geometric formula is $\frac{a}{1-r}$

$E(x) = \frac{1}{1-\frac{2}{3}} = 3$

b) If the expected is 3, and we must stop at chocolate, then we must have 2 not chocolate and 1 chocolate. We can also find $E(Y)$.

$E(Y) = (\frac{1}{3})(\frac{2}{3})(1 + 2r + 3r^2)$, where $r = 2/3$, $= (\frac{2}{9})(9) = 2$

Problem 3)

a) Every ball is independent, so $P(\text{Black}) = \frac{b}{w+b}$ with a geometric ~~sequence~~ sequence

Probability of black $\times$ Probability not black gives us PDF at k.

$$P(X=k) = \left(\frac{b}{w+b}\right)\left(1 - \frac{b}{w+b}\right)^{k-1}$$

b) You want the probability of drawing black at the very end, or ith position.

$i-1$ = white balls before black ball i = black ball

$w+b-i$ is for every other ball

Using choose/counting formula ith position is black ball is $\binom{w+b-i}{b-1}$

The ways we can select the balls is $\binom{w+b}{b}$

$$P(X=i) = \frac{\binom{w+b-i}{b-1}}{\binom{w+b}{b}}$$

4) $n=10$, $p=.75$, if choosing or $n=10$, $p=.5$ if guessing

$P(X\geq 7)=P(7)+P(8)+P(9)+P(10)$

$P(X\geq 7)=\binom{10}{7}(.75^7)(.25^3)+\binom{10}{8}(.75^8)(.25^2)+\binom{10}{9}(.75)^9)(.25^1)+\binom{10}{10}(.75^{10})(.25^0)$

$P(X\geq 7)=\boxed{.77875090}$ if Charles chooses

if he guesses: $n=10$, $p=.5$

$P(X\geq 7)=1-(P(7)+P(8)+P(9)+P(10))$

$P(X\geq 7)=1-\binom{10}{7}(.5)^7(.5)^3+\binom{10}{8}(.5)^8(.5)^2+\binom{10}{9}(.5)^9)(.5^3)+\binom{10}{10}(.5^{10})(.5)^0$

$P(X\leq 7)=\boxed{.828125}$

5) $Pr(1\text{ day})=2/3 \quad P(2\text{day})=1/3$

$E(x)=2/3+2\cdot\frac{1}{3}=2/3+2/3=4/3$

$E(B)=$ pset 1 + problem set 2 + pro set 3 $=4/3+4/3+4/3=\boxed{4}$

6) $E_x(R)=\sum_{x=0}^{\infty}P(R>x)$

$=5/6+(5/6)^2+(5/6)^3$

$=5/6(1/1-5/6)$

$=\boxed{5}$

c) $E(D)=E(\text{Dice}\cdot\text{coin})=E(\text{Dice})\cdot E(\text{coin})$

$E(\text{dice})=1\cdot 1/6+2\cdot\frac{1}{6}+3\cdot\frac{1}{6}+4\cdot\frac{1}{6}+5\cdot\frac{1}{6}+6\cdot\frac{1}{6}=3.5$

$P(1)=1/9 \quad p(3)=1/9 \quad p(5)=1/9$

$P(2\text{day})=2/9 \quad p(4)=2/9 \quad p(6)=2/9$

$p(8\text{day})=1/9 \quad p(10)=1/9 \quad p(12)=1/9$

$1/9+\frac{4}{9}+\frac{3}{9}+\frac{3}{9}+\frac{5}{9}+\frac{12}{9}+\frac{8}{9}+\frac{10}{9}+\frac{12}{9}+\frac{1}{9}+\frac{4}{9}+\frac{3}{9}+\frac{8}{9}+\frac{5}{9}+\frac{12}{9}+\frac{8}{4}+\frac{10}{4}+\frac{12}{9}=\frac{63}{9}$

$63/9=7$

$\boxed{7\text{ days}} \quad E(D)=7$

$P(R)=1/3 \quad P(U)=1/6 \quad P(B)=1/2$

$E(D)=1/2(E(B))+1/6(E(U)+1/3E(R)$

$E(D)=\boxed{29/6}$

6) Since $(4-K)$ figures have not been reached the trials can be computed with

$= 0.50 \times (4-K) \div 4$

$= .125 \times (4-K)$

The expected value is computed with geometric sequences /distribution:

$$\frac{1}{p} = \frac{1}{0.125(4-k)} = \boxed{8\left(\frac{1}{(4-K)}\right)}$$

6) total # of trials expected is

$E(K=0) + E(k=1) + E(k=2) + E(k=3)$

$= \frac{8}{4} + \frac{8}{3} + \frac{8}{2} + \frac{8}{1} = 16.6667.$