CS330 Homework 4

1. $L = 12\ 4\ 9\ 6\ 15\ 2\ 7\ 3$

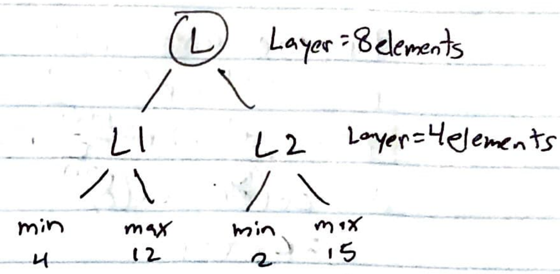
$L1 = 12\ 4\ 9\ 6$ $\quad L2 = 15\ 2\ 7\ 3$

$\min(L1) = 4$ $\quad \min(L2) = 2$

$\max(L1) = 12$ $\quad \max(L2) = 15$

$\min(L1, L2) = 4 > 2$, $\min(L) = 2$

$\max(L1, L2) = 15 > 12$, $\max(L) = 15$




We know that for every min-and-max it recieves $3n/2 - 1$ comparisons for even arrays, in this case we have $n = 4$ elements in each array, giving us $3(4)/2 - 1$ or 4 comparisons per array and we have 2 arrays giving us 8 comparisons. Comparing $L(1,2)$ and $\max L(1,2)$ is 2 more comparisons, which is 10 comparisons

ii) DandC MinMax (L) where $n = |L|$, $M(n) = 2(M(n/2)) + 2$

$m(1) = 0$, $m(2) = 1$, $m(4) = 4$, $m(8) = 10$, $m(16) = 22$

For $n \geq 2$, $3(2^{n-1}) - 2$

2. $T(n) = 3T(n/3) + 2$, with n power of 3 ($n = 3^k$)

base case $T(1) = 0$

$T(3) = 3T(1) + 2 = 2$

$T(9) = 3T(3) + 2 = 8$

$T(27) = 3T(9) + 2 = 26$

$T(81) = 3T(27) + 2 = 80$

$T(243) = 3T(81) + 2 = 242$

T has linear growth. x ~~+~~ y intercept = y

ii) $R_n = a_n = a_{n-2} + a_{n-1} + 3$ base cases are $R(1) = a_1 = 1$ and $R(2) = a_2 = 2$

$a_0 = -2$

$a_3 = a_3 + a_2 + 3 + a_3 + 3$

| | |
|---|---|
| $R(1) = 1$ | 1 |
| $R(2) = 2$ | 2 |
| $R(3) = a_3 = a_2 + a_1 + 3 = 6$ | $(1+2)+3$ |
| $R(4) = a_4 = a_3 + a_2 + 3 = 11$ | $(1+2)+2+3+3$ |
| $R(5) = a_5 = a_4 + a_3 + 3 = 20$ | $(1+2)+(1+2)+2+3+3+3+(1+2)$ |
| $R(6) = a_6 = a_5 + a_4 + 3 = 34$ | $a_0 = -2$ |
| $R(7) = a_7 = a_6 + a_5 + 3 = 57$ | $a_1 = -2+3$ |
| $R(8) = a_8 = a_7 + a_6 + 5 = 94$ | $a_2 = -2+3+1$ |
| | $a_3 = -2+3+2+3$ |
| | $a_4 = -2+3+2+3+2+3$ |

R2 has ~~exponential~~ polynomial growth

Linear recurrence relations produce exponentially growing terms like: $a_n = r^n$.

$r^n = r^{n-1} + r^{n-2} + 3$ divide both sides knowing $r^{n-2}$ is not 0

$r^2 = r + 3 ; x_1 = \frac{1+\sqrt{13}}{2}, x_2 = \frac{1-\sqrt{13}}{2}$ gives...

$a_n = c_0(\cancel{x_1})^n + c_1(\cancel{x_2})^n$ auxiliary/characteristic equation

~~$-2 = c_0 + c_1$~~ Find $c_1$ and $c_0$..

~~$+2 = -c_0 + 2c_1$~~

~~$+2 = 3c_1, c_1 = -1/3$~~

~~$-2 = c_0 - 1/3$~~

~~$c_0 = -1\frac{2}{3}$~~

~~$a_n = -\frac{5}{3}(-1)^n - \frac{1}{3}(2)^n$~~

$\cancel{2} = c_0 + c_1$ make $a_0 = -2$ and $a_1 = 1$

$1 = -c_0 + 3c_1$

$c_0 = -1\frac{3}{4}, c_1 = -1/4$

$\left(-\frac{7}{4}\left(\frac{1+\sqrt{13}}{2}\right)^n + \left(-\frac{1}{4}\left(\frac{1-\sqrt{13}}{2}\right)^n\right)\right)$ Final equation

~~This has~~ 1, 2, 6, 11, 20, 34 ...

~~$Ax = x + 2x^2 + 6x^3$~~

~~$Ax = x + 1x^2(Ax) + 6x^3(Ax)$~~

~~$\frac{x}{1 - x - 5x^2}$~~

~~It is not a pure exponential function~~ Because we have 2 terms for $r_1$ and $r_0$, it is an exponential function with $O\left(\frac{1+\sqrt{13}}{2}^n\right)$ growth.

iii) $S(n) = a_n = 4a_{n-1} - 2$, $S(0) = a_0 = 2$

| | |
|---|---|
| 4 | 2 |
| 16 | 6 |
| 64 | 22 |
| 256 | 86 |

$S(1) = 6$ ~~$4a_{n-1} - 2 = 4(4a_{n-2} - 2) - 2 = 4^2 a_{n-2} + 4 \cdot 2 + 4^0 \cdot 2$~~

$S(2) = 22$ ~~$4^2(4a_{n-3} - 2) + 4 \cdot -2 + 4^0 \cdot -2 = 4^3 a_{n-3} + 4^2 \cdot -2 + 4 \cdot -2 + 4^0 \cdot -2$~~

$S(3) = 86$ ~~$= 4^n a_0 + 4^{n-1} \cdot -2 + 4^{n-2} \cdot -2 \cdots + 4^0 \cdot -2$~~

$S(4) = 342$ ~~$4^n(2) + -2(4^{n-1} + 4^{n-2} + 4^{n-3} + \cdots + 1) =$~~

$S(5) = 1366$ ~~$= \frac{-2(4^n - 1)}{4-1} = -\frac{2}{3}(4^n - 1)$~~

$S(6) = 5462$

$-2(4^n + 4^{n-1} + 4^{n-2} + \ldots + 1)$

~~$S_n = \frac{a - ar^{n+1}}{1-r}$~~

homogeneous equation

$a_n = 4a_{n-1} + -2$ , $r = 4$

$B = 4B + -2$ for particular soln' $b_n = B$

$3B = -2$ , $B = -2/3$

General soln:

$a_n = A4^n - 3$ , $a_0 = 2$

$2 = A - 3$ , $A = 5$

$a_n = 5 \cdot 4^n + -2/3$

This experiences exponential growth