Probability 1

MML 6.1

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Probability Space

- **Outcome space**: Set Ω
- **Event space** (sigma algebra): $\mathcal{F} \subset 2^{\Omega}$
- **Probability function**: $P \colon \mathcal{F} \to [0, 1]$

Probability Space: Properties

- **Outcome space**: Set Ω
  - Finite or infinite
- **Event space** (sigma algebra): $\mathcal{F} \subset 2^{\Omega}$
  - Contains sample space:
  - Closed under complements:
  - Closed under countable unions:
  - Closed under countable intersections:

- **Probability measure**: $P : \mathcal{F} \to [0, 1]$
  - Mesure of sample space equals 1:
  - Countably additive:

Random Variable

A $\mathcal{T}$-valued random variable X (upper case!) is a function:

$$X : \Omega \to \mathcal{T}$$

Examples $\Omega = \{J, E, M\}$:

Random Variable: Common Types

1. **Continuous (real-valued)**

2. **Discrete**

Random Variable: Pre-image (inverse)

Pre-image $X^{-1}: \mathcal{T} \to 2^{\Omega}$ defined as

$$X^{-1}(x) = \{\omega \in \Omega \mid X(\omega) = x\}$$

Examples $\Omega = \{J, E, M\}$:

Probability Distribution

Always associated with a random variable for some $X : \Omega \to \mathcal{T}$

$$\mathbb{P}[X = x] = P\left(X^{-1}(x)\right) = P\left(\{\omega \in \Omega \mid X(\omega) = x\}\right)$$

Probability Distributions

Wikipedia is a good reference for their properties

Discrete random variable:

- Bernoulli: Heads or tails
- Binomial: Number of heads
- Geometric: Coin flips until heads
- Poisson: Number of customers

Continuous random variable:

- Normal: Central limit theorem
- Multivariate normal: Height and weight
- Laplace: Extreme weather events

Regression vs Classification

Predicting target: $Y : \Omega \to \mathcal{T}$

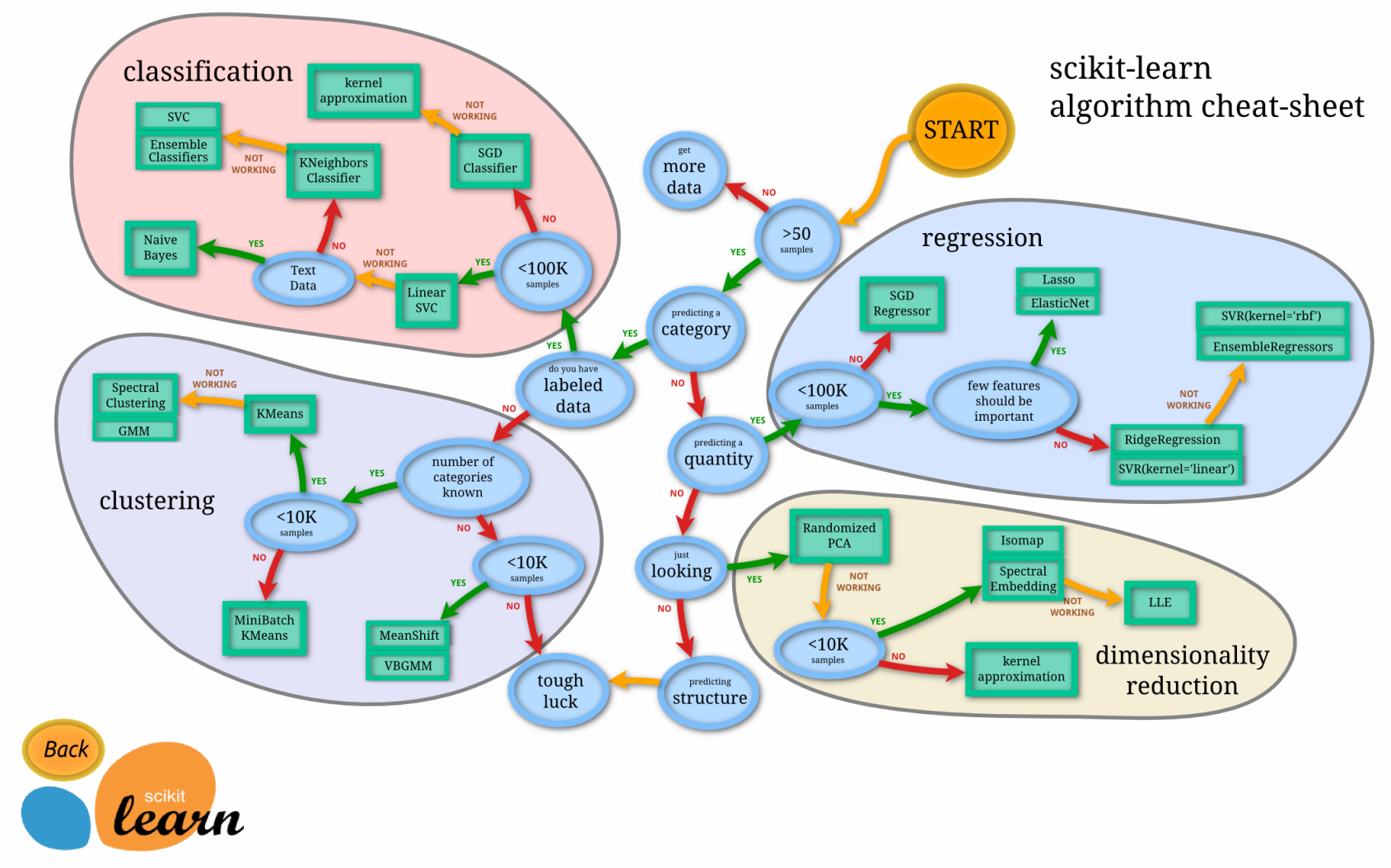
Regression: continuous target $\mathcal{T} = \mathbb{R}$

- Profits
- Probability of survival

Classification: discrete target: $\mathcal{T}$ is finite

- Color
- State
- Year (could be either)

Machine Learning Choices ...



Source: `http://scikit-learn.org/stable/tutorial/machinelearningmap/index.html`