1.1 $\quad g(x) = x^3 - 2x^2 + 1$

$$g'(x) = \frac{\partial}{\partial x} g(x) = 3x^2 - 4x$$

$$g'(1) = 3(1)^2 - 4\cdot(1) = 3-4 = -1$$

$$g'(0) = 3\cdot(0)^2 - 4\cdot(0) = 0$$

$$g'(-1) = 3\cdot(-1)^2 - 4(-1) = 3+4 = 7$$

Tangent line : $y = ax+b$

- $x=1 \Rightarrow g(1) = 1^3 - 2\cdot 1^2 + 1 = 1-2+1 = -1 = y$

  $a = g'(1) = -1$

  $y = ax+b \Rightarrow -1 = (-1)\cdot 1 + b \Rightarrow b = -1+1 = 0$

  $$\boxed{y = -1x+0}$$

- $x=0 \Rightarrow g(0) = 0^3 - 2\cdot 0^2 + 1 = 1 = y$

  $a = g'(0) = 0$

  $y = ax+b \Rightarrow 1 = 0\cdot 0 + b \Rightarrow b = 1$

  $$\boxed{y = 0\cdot x + 1}$$

- $x=-1 \Rightarrow g(-1) = (-1)^3 - 2\cdot(-1)^2 + 1 = -1-2+1 = -2 = y$

  $a = g'(-1) = 7$

  $y = ax+b \Rightarrow -2 = 7\cdot(-1) + b \Rightarrow b = -2+7 = 5$

  $$\boxed{y = 7x+5}$$

1.3

$$m(t) = 10 + \frac{\sqrt{t}}{e^t} \qquad t > 0$$

a) $$\lim_{t\to\infty} m(t) = \lim_{t\to\infty} 10 + \cancel{\frac{\sqrt{\infty}}{e^{\infty}}}^{\to 0} = 10$$

b) maximum $\frac{d}{dt} m(t) = 0$

$$\frac{d}{dt} m(t) = \frac{(\sqrt{t})' \cdot e^t - \sqrt{t} \cdot (e^t)'}{(e^t)^2} =$$

$$= \frac{\frac{1}{2} t^{-1/2} \cdot \cancel{e^t} - t^{1/2} \cdot \cancel{e^t}}{e^t \cdot \cancel{e^t}} = 0$$

maximum or minim

$$\frac{1}{2} t^{-1/2} - t^{1/2} = 0$$

$$\frac{1}{2\sqrt{t}} = \sqrt{t}$$

$$\frac{1}{4t} = t$$

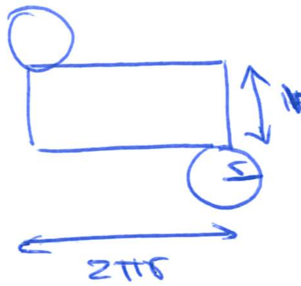
$$\frac{1}{4} = t^2$$

$$\sqrt{\frac{1}{4}} = t$$

$$\frac{1}{2} = t$$

$$m(t = 1/2) = 10 + \frac{\sqrt{1/2}}{e^{1/2}} = 10{,}428 \text{ Mm}^3$$

1.4

$S =$ 

$= 2\pi r \cdot h + \pi r^2 \cdot 2$

$V = \pi r^2 \cdot h = \cancel{0,15}\ 0,15$

$\Downarrow$

$h = \dfrac{0,15}{\pi r^2}$

$$\frac{dS}{dr} = \left(2\cancel{\pi}\cancel{r}\cdot\left(\frac{0,15}{\cancel{\pi}\, r^{\cancel{2}}}\right) + \pi r^2 \cdot 2\right)' =$$

$$= \left(2\cancel{\pi}\cdot 0,15 \cdot (-1)\cdot r^{-2} + \pi \cdot 2 \cdot r \cdot 2\right)$$

max or min $\Rightarrow$ $\dfrac{dS}{dr} = 0 \Rightarrow -2\cdot 0,15 \cdot r^{-2} + 4\pi r = 0$

$$4\pi r = \frac{2\cdot 0,15}{r^2}$$

$$r^3 = \frac{2\cdot 0,15}{4\pi}$$

$$r = \sqrt[3]{\frac{2\cdot 0,15}{4\pi}}$$

$r = 0,2879$ cm

$$h = \frac{0,15}{\pi\cdot(0,2879)^2}$$

$h = 0,576$ cm

1.5

$$f(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}+2x-2}$$

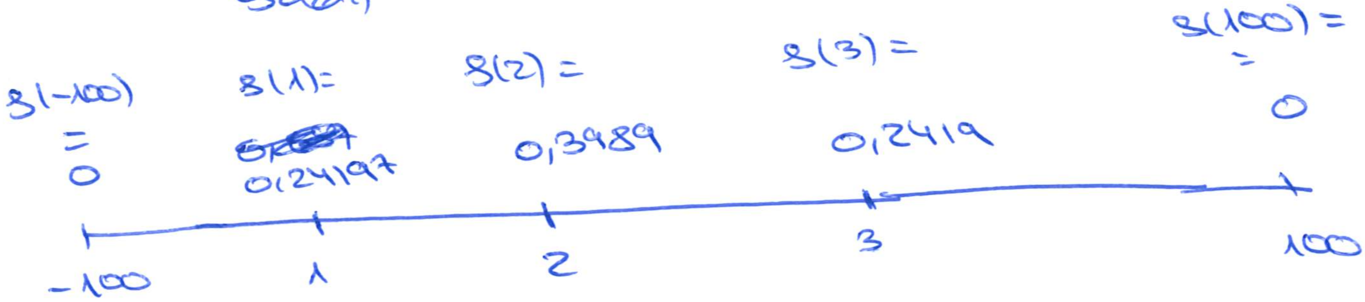
maximum

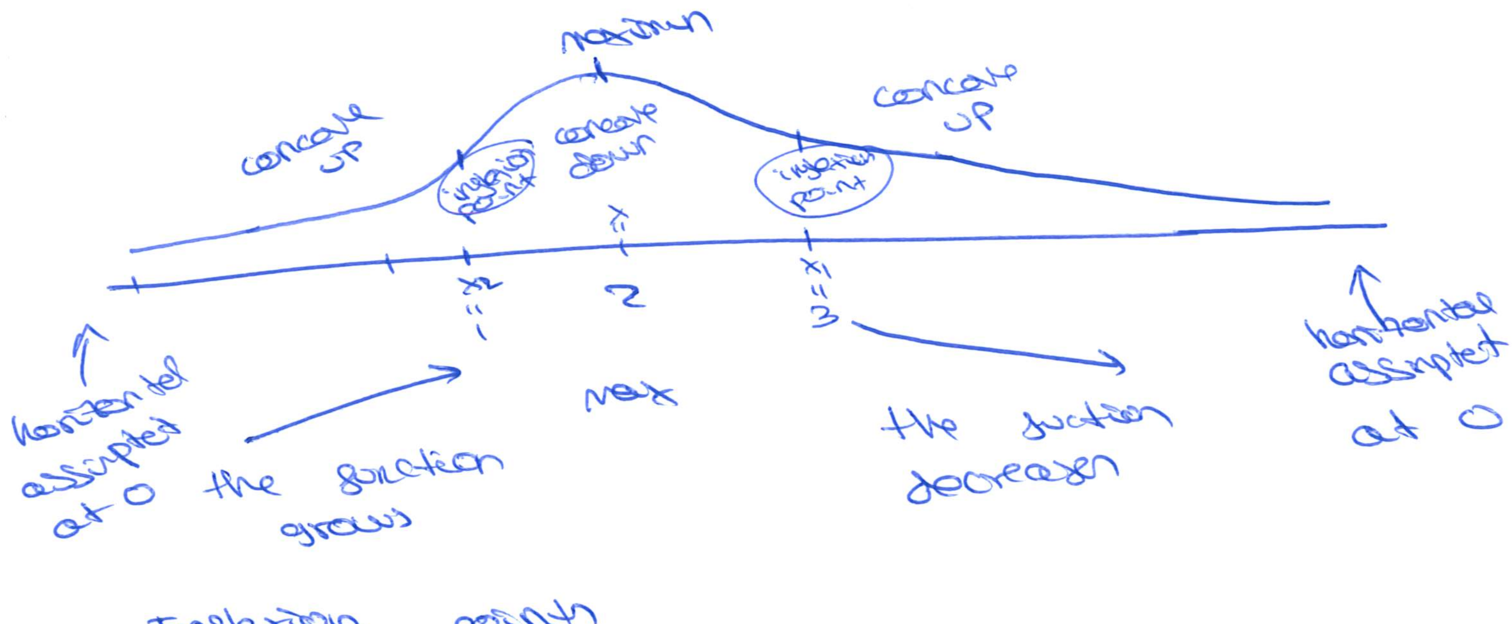
$$f'(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}+2x-2} \cdot \left(-\frac{2x}{2}+2\right)$$

$$\boxed{f'(x)=0} \Rightarrow -\frac{\cancel{2}x}{\cancel{2}}+2=0 \Rightarrow \boxed{x=2}$$

maximum

~~f''(x)~~





Inflexion points

$$\boxed{f''(x)=0}$$

$$f''(x) = \frac{1}{\sqrt{2\pi}}\left[e^{-\frac{x^2}{2}+2x-2}\left(-\frac{2x}{2}+2\right)^2 + e^{-\frac{x^2}{2}+2x-2}(-1)\right]$$

$$e^{-\frac{x^2}{2}+2x-2}$$

$g''(x) = 0 \leftarrow$ inflexion points

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2} + 2x - 2} \left[\left(-\frac{2x}{2} + 2 \right)^2 - 1 \right] = 0$$

$$\left(\frac{-2x}{2} + 2 \right)^2 - 1 = 0$$

$$(2 - x)^2 - 1 = 0$$

$$2^2 - 2 \cdot 2 \cdot x + x^2 - 1 = 0$$

$$x^2 - 4x + 3 = 0 \xrightarrow[\text{solve}]{} \begin{cases} x_1 = 3 \\ x_2 = 1 \end{cases}$$