Exercise 1

The density of a colony of microorganisms in a culture (in number of microorganisms/cm³) is given by the relation $D(t) = \frac{a}{3+\cos{(\pi t)}}, t \geq 0$, where t is the time since the start of the culture (in hours) and 'a' is a positive constant. It is known that the maximum value of D(t) is 6 microorganisms/mm³ and that the culture lasts 2 hours.

- Determine the value of a.
- How many microorganisms will there be in 10 cm³ of culture?
- What are the time intervals during which the density of microorganisms increases? When is it decreasing?
- What is the minimum value of the microorganism density in the culture?

Exercise 2

The pain intensity experienced by a patient is given by the amount of two analgesics X and Y administered according to the relation: $S(x,y) = x^2 + y^2 - \frac{x^4+y^4}{4} - 3x^2y^2 + 100$, where x and y are the amounts of analgesics administered in mg. If initially the amounts of analgesics administered are (1, 1.5):

- How should the amounts of analgesics vary to achieve the greatest possible decrease in pain intensity under these conditions?
- What is the maximum possible variation in pain intensity under these conditions?
- What would be the variation in pain intensity if we increase analgesic 'x' twice as much as analgesic 'y'?

Exercise 3

The rate at which a quantity of a radioactive isotope disintegrates is proportional to the amount of isotope present. The radioactive isotope iodine-131 (I-131) is used in the treatment of hyperthyroidism. When administered to a patient, I-131 accumulates in the thyroid gland, where it decomposes and destroys part of the gland. If the half-life of I-131 is 8 days (the time it takes for a quantity of I-131 to decay to half its original amount):

- Set up the equation that defines the amount of active I-131 at each moment.
- Solve the equation.
- If 4 days pass from the shipment of I-131 from the producer until its use in the patient, what active amount will remain at the time of use?
- How much time will pass until only 5% of the original amount supplied by the producer remains?