3.1

$$\frac{\partial x}{\partial t} + ax = 0$$

$x(t=0) = x_0$

$x(t=T) = x_0/2$

$T(a) = ?$

$a(T = 4{,}5 \cdot 10^9\,y) = ?$

$$\frac{\partial x}{\partial t} = -ax$$

$$\int_{x_0}^{x_0/2} \frac{\partial x}{x} = \int_{0}^{T} -a\,\partial t$$

$$\left[\ln x\right]_{x_0}^{x_0/2} = \left[-at\right]_0^T$$

$$\ln x_0/2 - \ln x_0 = -aT + a \cdot 0$$

$$\ln \frac{x_0/2}{x_0} = -aT$$

$$\ln 1/2 = -aT$$

$$-\ln 2 = -aT$$

$$\boxed{T = \frac{\ln 2}{a}}$$

$$4{,}5 \cdot 10^9\,y = \frac{\ln 2}{a}$$

$$a = \frac{\ln 2}{4{,}5 \cdot 10^9\,y} = 1{,}54 \cdot 10^{-10}\ 1/y$$

3.2

$$\frac{\partial x}{\partial t} = -\frac{\lambda}{8000} x$$

a) $x(t=0) = x_0$

$x(t_1) = 0{,}0624\, x_0$ $\Big\}$ find t_1

b) find T such as $x(T) = x_0/2$

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a)
$$\int_{x_0}^{0{,}0624 x_0} \frac{\partial x}{x} = \int_0^{t_1} -\frac{\lambda}{8000} \partial t$$

$$\ln\left(\frac{0{,}0624\,\cancel{x_0}}{\cancel{x_0}}\right) = -\frac{\lambda}{8000} t_1 \Rightarrow t_1 = -8000 \ln(0{,}0624)$$

$t_1 = 22193{,}52$ y

b)
$$\int_{x_0}^{x_0/2} \frac{\partial x}{x} = \int_0^{T} -\frac{\lambda}{8000} \partial t$$

$$\ln \frac{\cancel{x_0}/2}{\cancel{x_0}} = -\frac{\lambda}{8000} T \Rightarrow T = +8000 \ln(2)$$

$T = 5545{,}177$ y

4.3

$$S(t=0) = \overset{S_0}{\overset{\shortparallel}{13{,}6}}\ \text{kg}$$

$$S(t=4h) = 4{,}5\ \text{kg}$$

$$\frac{dS}{dt} = aS$$

$S(t_1) = 0{,}05 \cdot S_0$ find t_1

↑

5% of sugar left

95% of sugar dissolved

$$\int_{13{,}6}^{4{,}5} \frac{dS}{S} = \int_0^4 a\,dt$$

$$\ln S \Big]_{13{,}6}^{4{,}5} = at \Big]_0^4$$

$$\ln \frac{4{,}5}{13{,}6} = a\,\cancel{t}(4-0) \Rightarrow a = \frac{1}{4} \ln \frac{4{,}5}{13{,}6} = -0{,}2765$$

$$\int_{13{,}6}^{0{,}05 \cdot 13{,}6} \frac{dS}{S} = \int_0^{t_1} \cancel{a}\,(-0{,}2765) \cdot dt$$

$$\ln \frac{0{,}05 \cdot \cancel{13{,}6}}{\cancel{13{,}6}} = -0{,}2765 \cdot (t_1 - 0)$$

$$t_1 = \frac{\ln 0{,}05}{-0{,}2765} = 10{,}834\ h$$

3.4

$S(t=0) = 5 kg$

$$\frac{dS}{dt} = 0{,}4 \, kg/l \cdot 10 \, l/min - \frac{S}{500 \, l} \cdot 10 \, l/min$$

$$\frac{dS}{dt} = \left(4 - \frac{S}{50}\right)$$

$$\left(\frac{-50}{-50}\right) \cdot \frac{dS}{4 - \frac{S}{50}} = dt$$

$$\int_{5kg}^{S} \frac{-50 \, dS}{S - 200} = \int_{0}^{t} dt$$

$$-50 \ln(S-200) \Big]_{5kg}^{S} = t$$

$$-50 \ln\left(\frac{S-200}{5-200}\right) = t$$

$$e^{\ln \frac{S-200}{-195}} = e^{-t/50}$$

$$\frac{S-200}{-195} = e^{-t/50}$$

$$\boxed{S = -195 \, e^{-t/50} + 200}$$

at $t = 5 min$

$S(5 min) = 23{,}5567 kg$

S at $t = 1h = 60 \cdot$ min

$S(60 min) = 141{,}267 kg$

9.5

$$\frac{dT}{dt} = k\,(T - T_{amb})$$

$T(t = 9h) = 2°C$

$T_{amb} = 22°C$

$T(t = 10h) = 10°C$

$T(t = ?) = 15°C$

$$\int_2^{10} \frac{dT}{T-22} = \int_9^{10} k\, dt$$

$$\left[\ln \frac{T-22}{}\right]_2^{10} = kt\Big]_9^{10}$$

$$\ln \frac{10-22}{2-22} = k\,(10-9)$$

$$\boxed{k} = \frac{\ln \frac{10-22}{2-22}}{10-9} \boxed{= -0{,}510825}$$

$$\int_2^{15} \frac{dT}{T-22} = \int_9^{t} -0{,}510825\, dt$$

$$\ln \frac{15-22}{2-22} = -0{,}510825 \cdot (t-9)$$

$$t = \frac{\ln \frac{15-22}{2-22}}{-0{,}510825} + 9$$

$$\boxed{t = 11{,}05514\ h}$$

3.6

$$\frac{dT}{dt} = k(T - T_A)$$

$$\boxed{\begin{aligned} &T(t=0) = 4°C \\ &T(t=45\,min) = 37°C \\ &T_A = 50°C \end{aligned}}$$

$$\boxed{\begin{aligned} &T(t=0) = 4°C \\ &T(t=?) = 37°C \\ &T_A = 60°C \end{aligned}}$$

first we extract k of plasma in first oven

$$\int_4^{37} \frac{dT}{T-50} = \int_0^{45} k\,dt$$

$$\Big[\ln(T-50)\Big]_4^{37} = \Big[kt\Big]_0^{45}$$

$$\ln\frac{37-50}{4-50} = k(45-0) \rightarrow k = \frac{\ln\frac{13}{46}}{45} = -0{,}02808$$

now we use $k = -0{,}02808$ in the second oven

$$\int_4^{37} \frac{dT}{T-60} = \int_0^{t} -0{,}0281\,dt$$

$$\ln\frac{37-60}{4-60} = -0{,}02808\,t$$

$$t = \ln\frac{23}{56} / (-0{,}02808) = 31{,}688\ min$$