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MADRID 2018 IROS

Tutorial

Screw Theory for Robotics

A practical approach for modern Robot Mechanics

(5th October - Room 1.L3 BERLIN)

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Tutorial
Screw Theory for Robotics
A practical approach for modern Robot Mechanics

CASE STUDY: A compelling computational approach of Screw theory for robotics

ABSTRACT:

Many say that kinematics for robots of many degrees of freedom, can only be addressed in a practical way with numeric algorithms. However, this approach is not very suitable for real time applications. Screw Theory overcomes this problem, and paves the way for solving kinematics with very compelling computational geometric algorithms truly effective and efficient.

Join us and take a modern stance towards robot mechanics with Screw Theory!

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

- Take a sneak peak at the FUTURE of Robotics.
- Kinematics approach NUMERIC vs. GEOMETRIC
- The cornerstone of the CANONICAL Subproblems
- Inverse Kinematics Geometric Solutions EXAMPLES: PUMA, GANTRY & REDUNDANT robots
- Inverse Kinematics SIMULATIONS: ABB IRB120, ABB IRB6620LX, KUKA IIWA robots
- DIFFERENTIAL Kinematics & GEOMETRIC JACOBIAN
- Screw Theory for Robotics BENEFITS summary

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CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

Who thinks we will see millions of commercial humanoid robots working in our society before 2050?

CASE Study: A compelling Computational Approach of Screw Theory for Robotics

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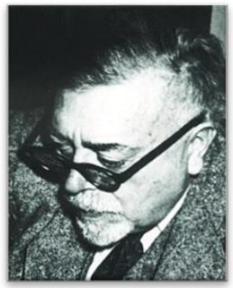
"I believe computers/robots will match and quickly exceed human capabilities in the areas where humans are still superior today by 2029" *- R. Kurzweil*

…"Not in our lifetimes. Not even in Ray Kurzweil's lifetime. It will be well over 100 years before we see this level in our machines. Maybe many hundred years" *- R. Brooks*

"Theoretically, if we could build a machine whose mechanical structure duplicated human physiology, then we could have a machine whose intellectual capacities would duplicate those of human beings" *- N. Wiener*



Then, let's focus on building a robot whose MECHANICS replicate those of a human body!

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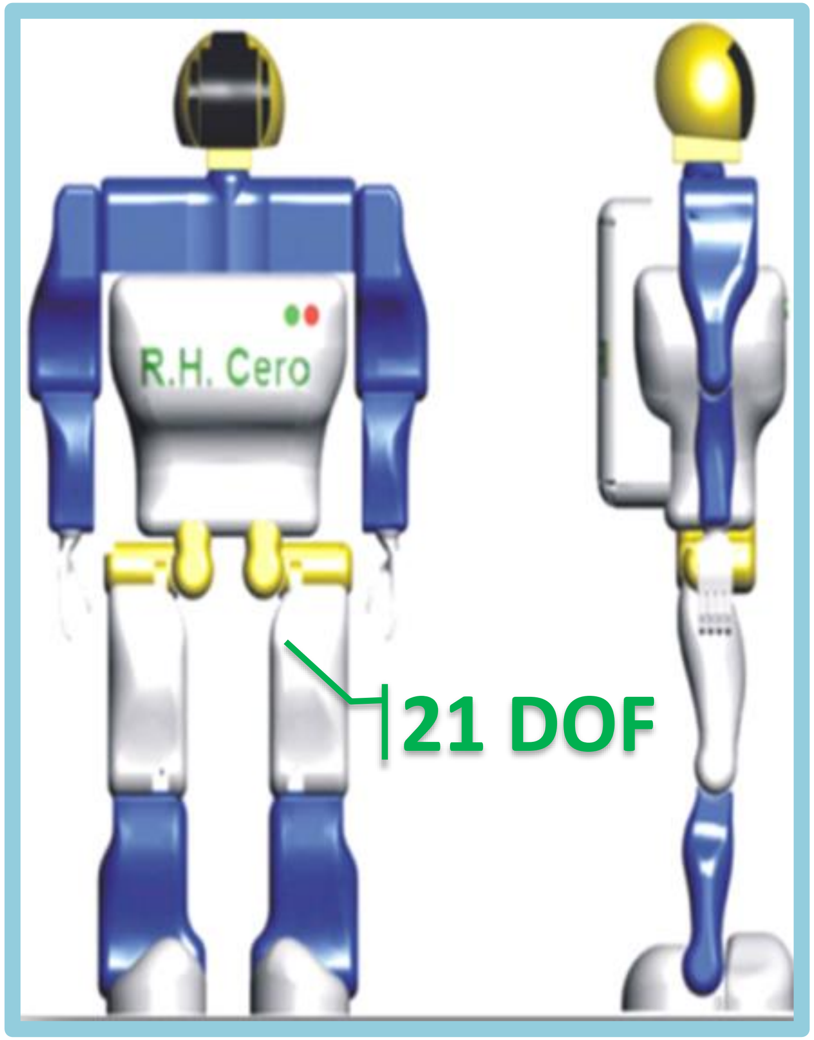

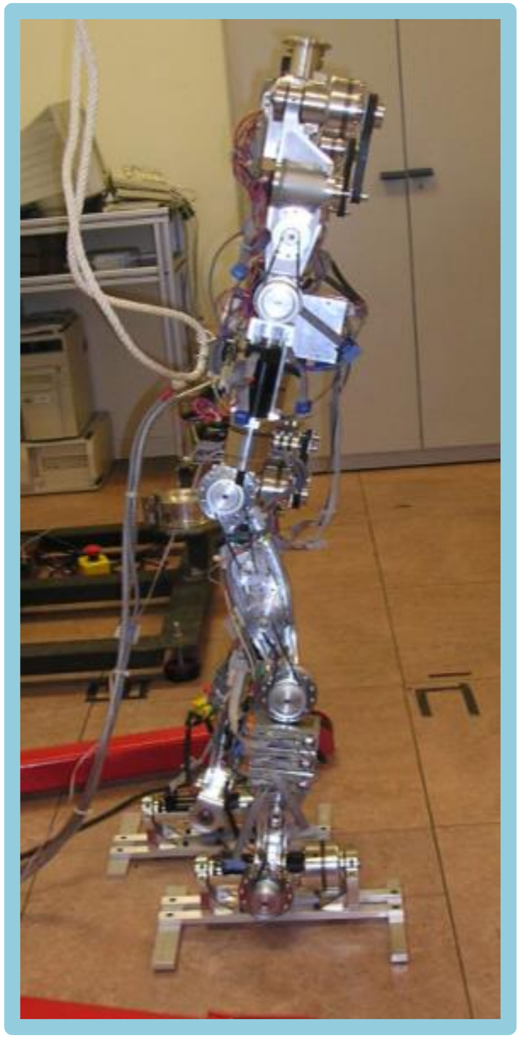

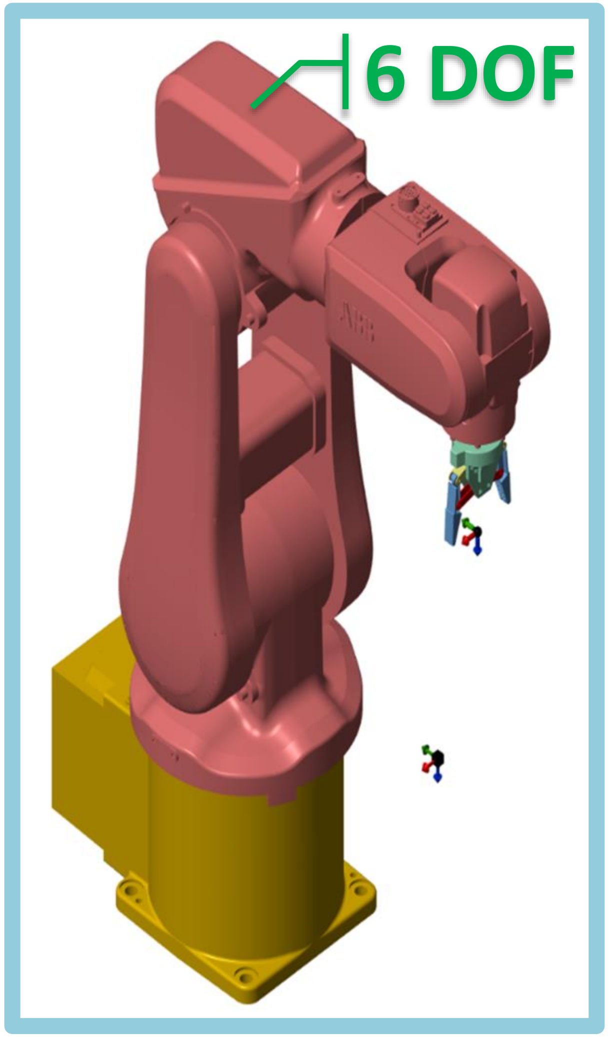

Who has ever been frustrated by the performance of the available solutions to your robot KINEMATICS applications?

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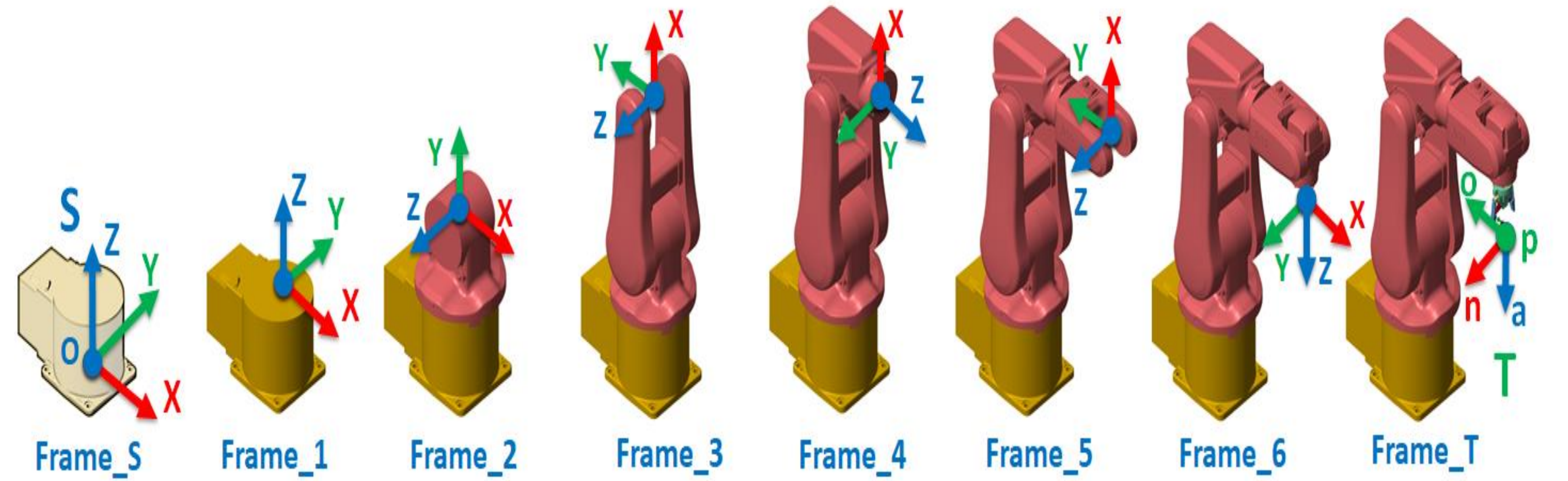

$$H_{ST}(\theta) = \prod_{i=1}^{n} {}^{n-1}H_n = {}^{0}H_1\,{}^{1}H_2\,{}^{2}H_3\,{}^{3}H_4\,{}^{4}H_5\,{}^{5}H_6\,{}^{6}H_T = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Denavit-Hartenberg continues to be the quasi standard approach used in the treatment of ROBOT KINEMATICS

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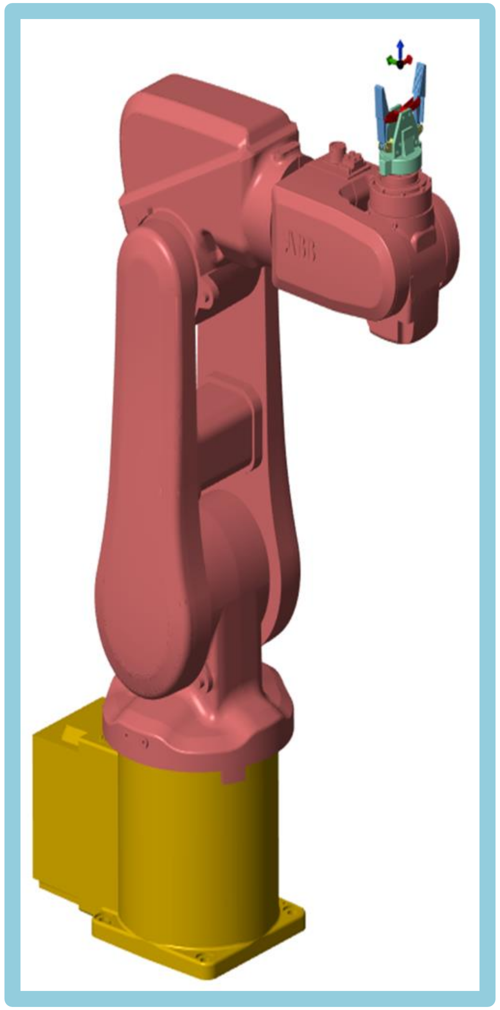


The joint MAGNITUDES to solve (θ1,θ2,θ3,θ4,θ5,θ6)

The TARGET Pose

$$H_{ST}(\theta) = \boxed{{}^{0}H_1\,{}^{1}H_2\,{}^{2}H_3\,{}^{3}H_4\,{}^{4}H_5\,{}^{5}H_6} = \begin{bmatrix} n_X & o_X & a_X & p_X \\ n_Y & o_Y & a_Y & p_X \\ n_Z & o_Z & a_Z & p_X \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

?

$$n_x = (C_1C_2S_3 + S_1C_3)(C_4C_5C_6 - S_4S_6) + C_1S_2(S_4C_5C_6 + C_4S_6) + (-C_1C_2C_3 + S_1S_3)S_5C_6$$

$$n_y = (-S_1C_2S_3 + S_1C_3)(C_4C_5C_6 - S_4S_6) + S_1S_2(S_4C_5C_6 + C_4S_6) + (-S_1C_2C_3 - C_1S_3)S_5C_6$$

$$n_z = (-S_2S_3)(C_4C_5C_6 - S_4S_6) + C_2(S_4C_5C_6 + C_4S_6) + S_2C_3S_5C_6$$

$$o_x = (C_1C_2S_3 + S_1C_3)(-C_4C_5C_6 - S_4S_6) + C_1S_2(-S_4C_5C_6 + C_4S_6) + (-C_1C_2C_3 + S_1S_3)(-S_5C_6)$$

$$o_y = (-S_1C_2S_3 + S_1C_3)(-C_4C_5C_6 - S_4S_6) + S_1S_2(-S_4C_5C_6 + C_4S_6) + (-S_1C_2C_3 - C_1S_3)(-S_5C_6)$$

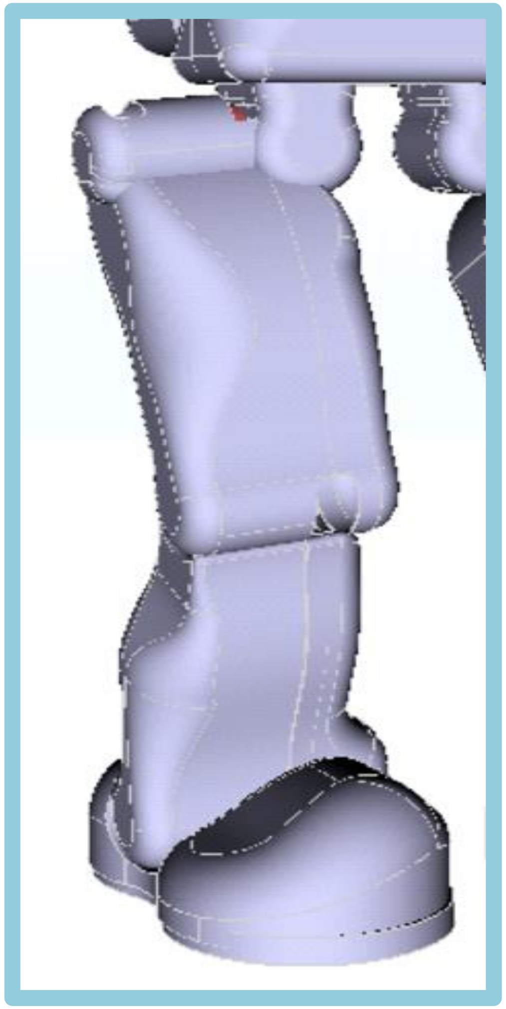
$$o_z = (-S_2S_3)(-C_4C_5C_6 - S_4S_6) + C_2(-S_4C_5C_6 + C_4S_6) + S_2C_3(-S_5C_6)$$

$$a_{xyz} = n_{xyz} \times o_{xyz}$$

$$p_x = (C_1C_2S_3 + S_1C_3)(l_4C_4S_5) + C_1S_2(l_4S_4S_5) + (C_1C_2C_3 + S_1S_3)(-l_4C_5 + l_3) + (-l_2C_1C_2S_3 - l_2S_1C_3 - l_1S_1)$$

$$p_y = (-S_1C_2S_3 - C_1C_3)(l_4C_4S_5) + S_1S_2(l_4S_4S_5) + (-C_1C_2C_3 - C_1S_3)(-l_4C_5 + l_3) + (-l_2S_1C_2S_3 - l_2C_1C_3 + l_1C_1)$$

$$p_z = (-S_2S_3)(l_4C_4C_5) + C_2(l_4S_4S_5) + S_2C_3(-l_4C_5 + l_3) + l_2S_2S_3$$



The INVERSE KINEMATICS problem of these mechanisms has 12 nonlinear coupled equations with multiple solutions of 6 unknowns!

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CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

D-H leads to NUMERIC SOLUTIONS for solving Robot INVERSE KINEMATICS (they are a kind of BRUTE FORCE APPROACH)

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e.g. Numeric
INVERSE KINEMATICS

The Broyden-Fletcher-Goldfarb-Shanno (BFGS) gradient projection algorithm: iterative, gradient-based optimization methods.

[configSol, solInfo] = ik(endeffector, pose, weights, initialguess)

Theta Solution (Optimal – Not Exact)

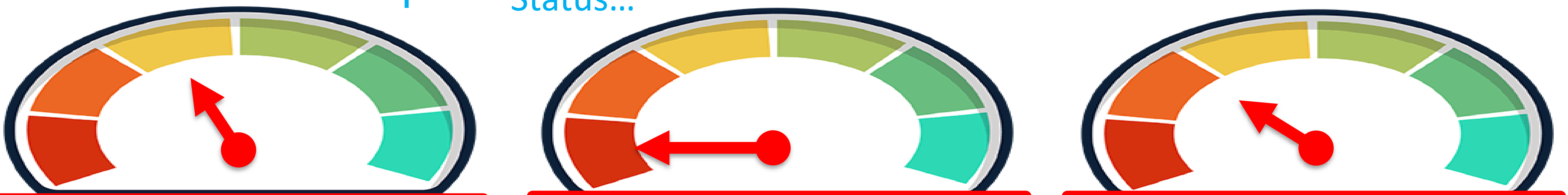
Solution info: Iterations PoseErrorNorm Status…

Body in the Robot

POSE TARGET (noap)

Error Tolerance

Theta guess (sometimes difficult)



EFFICIENCY | **EFFECTIVENESS** | **# SOLUTIONS**

NUMERIC algorithms are NON SUITABLE for robotics REAL TIME APPLICATIONS because they lack certainty for their convergence

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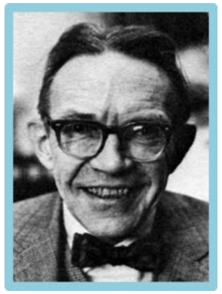
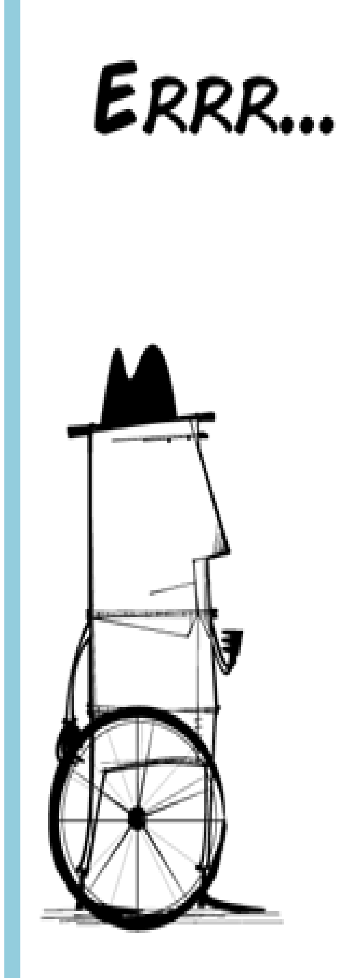
D-H





Screw Theory

It is difficult the trade-off between working with the STANDARD (D-H) and the INNOVATION (Screw Theory)!

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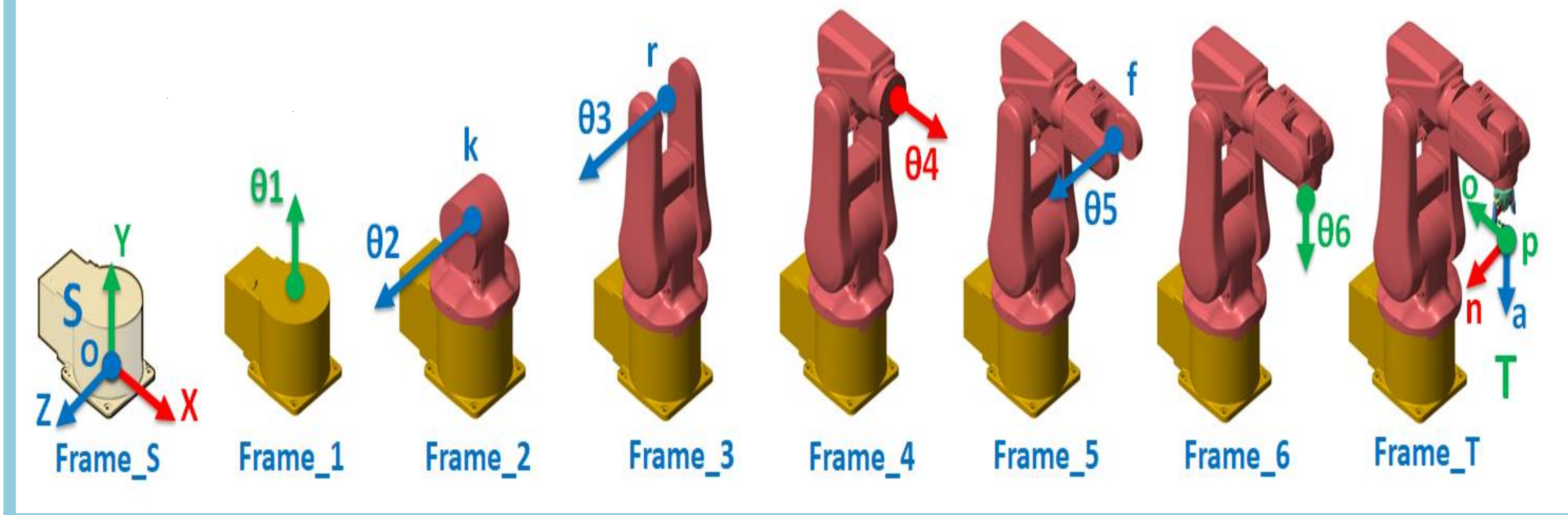

$$H_{ST}(\theta) = \prod_{i=1}^{n} e^{\hat{\xi}_i \theta_i} H_{ST}(0) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} e^{\hat{\xi}_3 \theta_3} e^{\hat{\xi}_4 \theta_4} e^{\hat{\xi}_5 \theta_5} e^{\hat{\xi}_6 \theta_6} H_{ST}(0) = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Screw Theory provides the Product Of Exponentials (POE) to generalize the KINEMATICS map for any open-chain robot

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e.g. Geometric INVERSE KINEMATICS

INVERSE KINEMATICS Algorithm based on SCREW THEORY for ROBOTICS with CLOSED-FORM Solutions (ST24R toolbox)

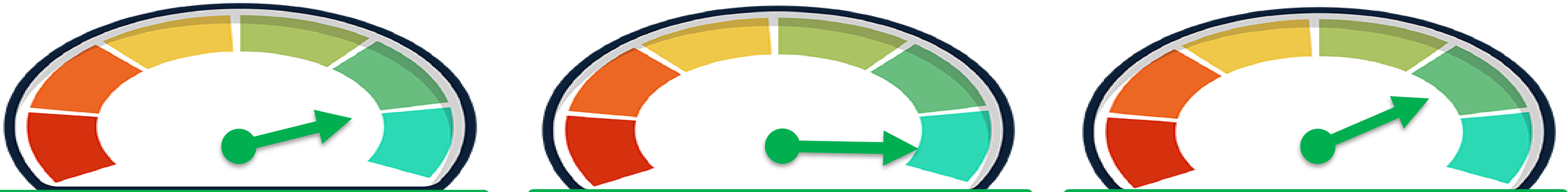
ThetaIK = PUMAtype_ikinePOE(TcpGoal, TcpInit, Dimensi)

Theta Solutions (Geometric - Exact)

Tool POSE TARGET (noap)

Tool Pose at Reference

Robot Dimensions



EFFICIENCY | **EFFECTIVENESS** | **# SOLUTIONS**

GEOMETRIC algorithms are suitable for REAL TIME APPLICATIONS and "Screw Theory" provides a useful & ELEGANT approach for them

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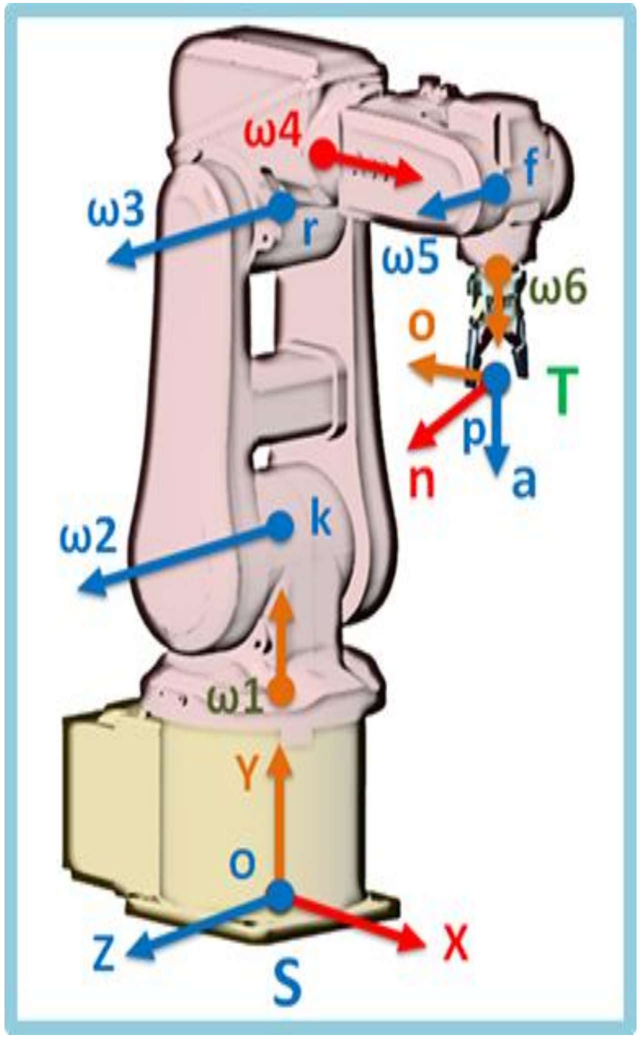

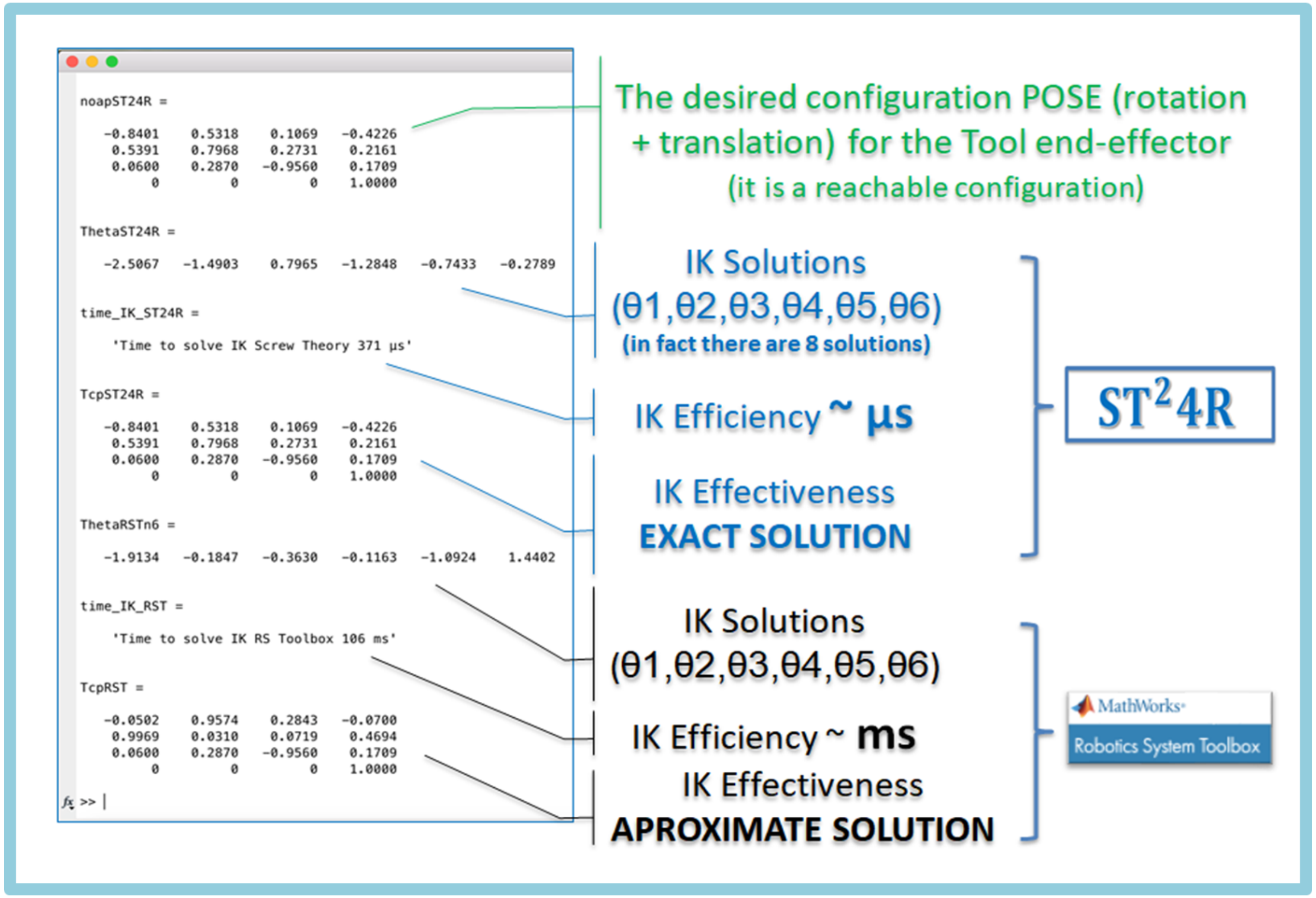

e.g. "ST24R" (Screw Theory Toolbox for Robotics) is faster, exact & gives 8 solutions, vs. "RST" (Robotics System Toolbox - MATLAB)

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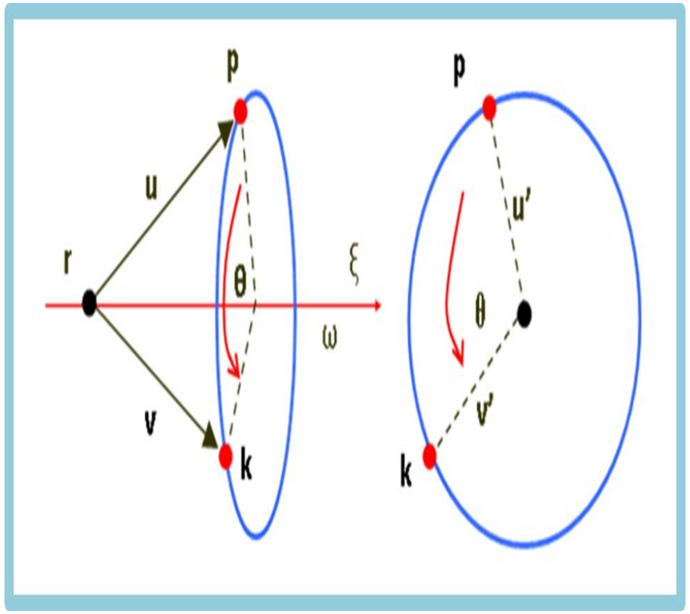
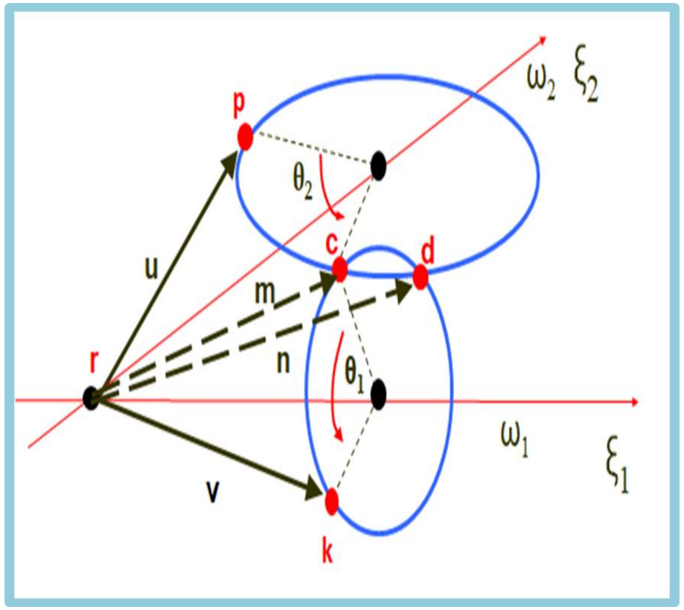
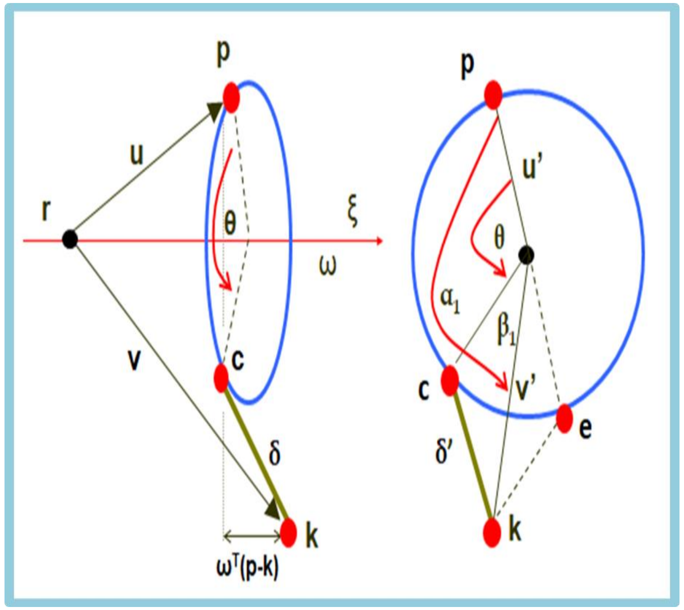
Paden-Kahan Subproblems One, Two & Three (PK1-2-3)







PK1 Definition

$$e^{\hat{\xi}_1\theta_1}\cdot p = k$$

PK2 Definition

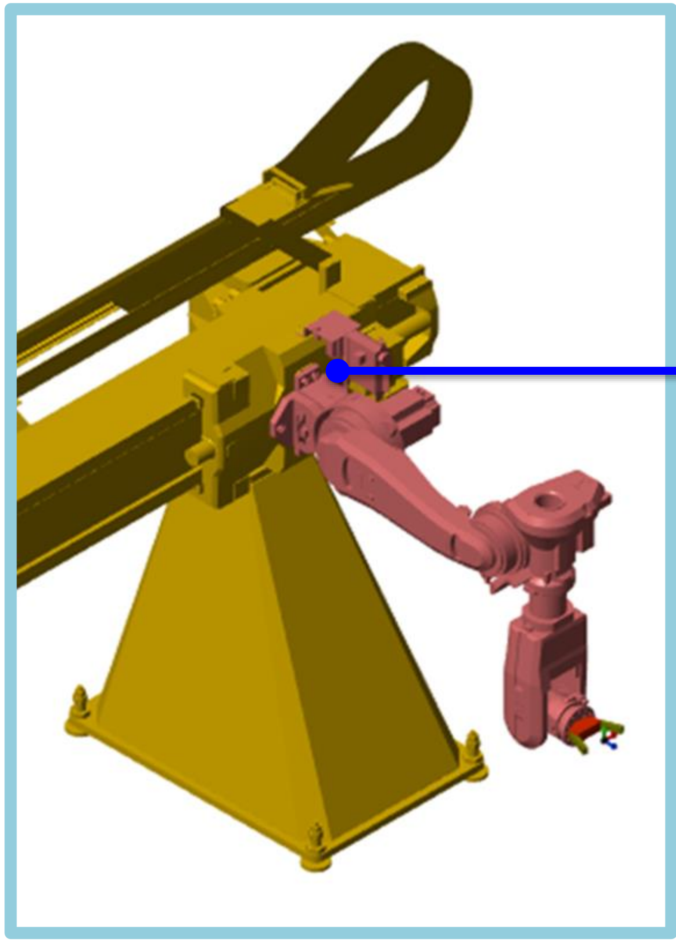
$$e^{\hat{\xi}_1\theta_1}\cdot e^{\hat{\xi}_2\theta_2}\cdot p = k$$

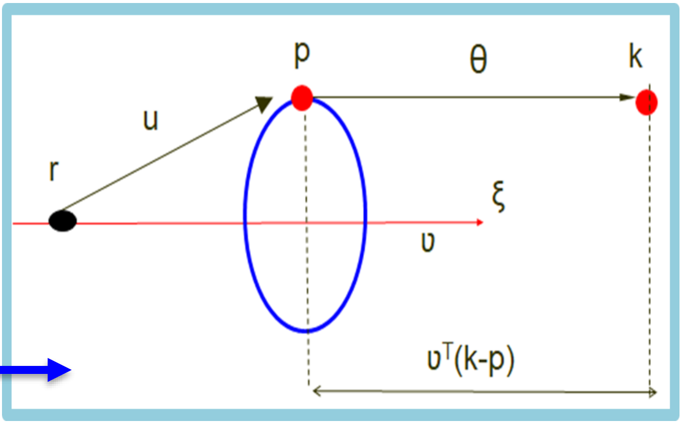
PK3 Definition

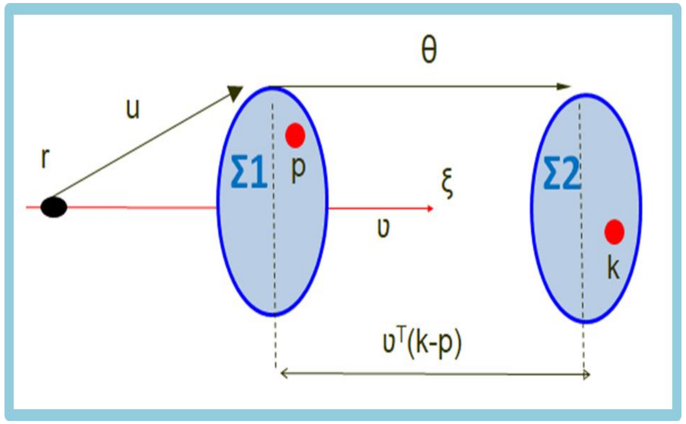
$$\left\| e^{\hat{\xi}_1\theta_1}\cdot p - k \right\| = \delta$$

The POE grants geometric solutions for the inverse kinematics problem by breaking it down into simpler CANONICAL SUBPROBLEMS

TRANSLATION ALONG A SINGLE AXIS







PG1 Definition

$$e^{\hat{\xi}_1\theta_1} \cdot p = k$$

PG1 Solution

$$\theta_1^{01} = \upsilon^T (k - p)$$

PG1 Definition

$$e^{\hat{\xi}_1\theta_1} \cdot \Sigma_1 = \Sigma_2$$

PG1 Solution

$$\theta_1^{01} = \upsilon^T (k - p)$$

Pardos-Gotor Subproblem ONE (PG1)

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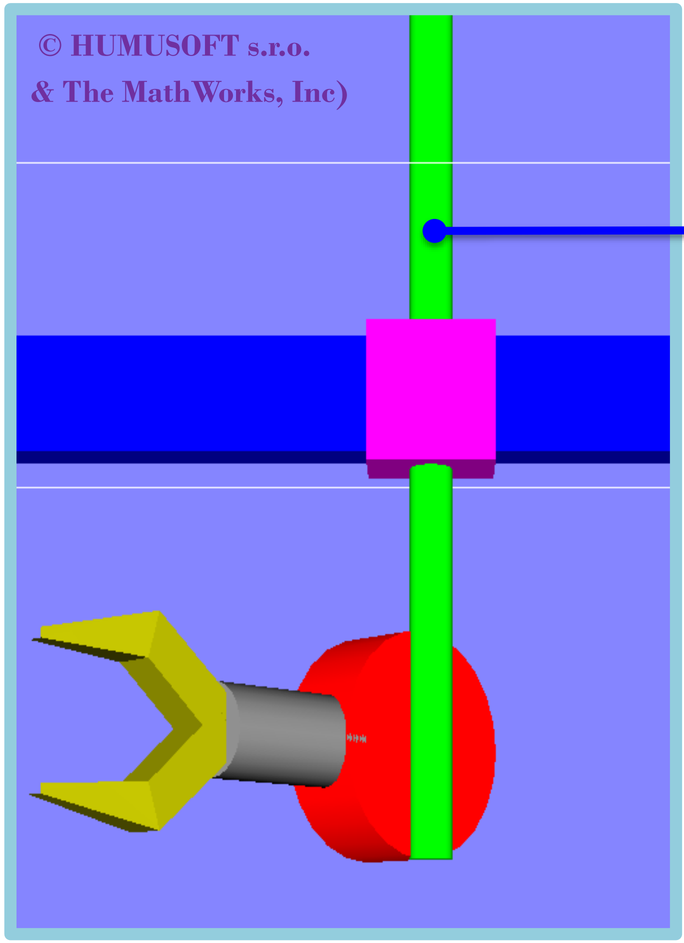
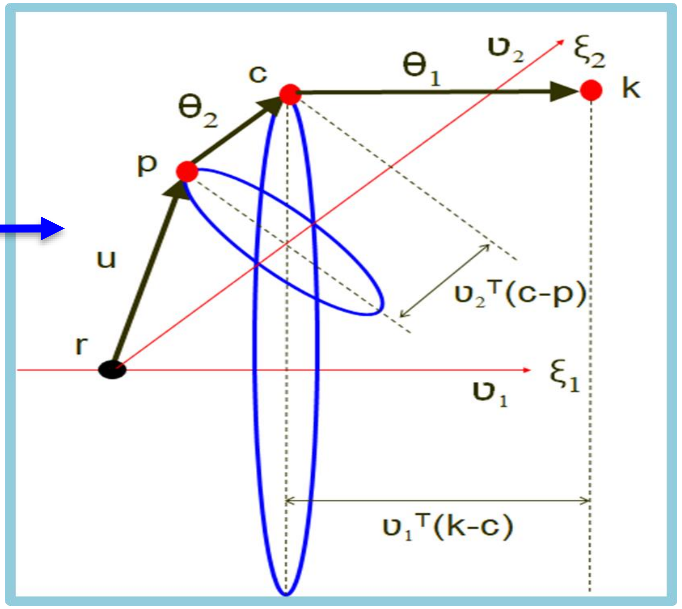
TRANSLATION ALONG TWO SUBSEQUENT CROSSING AXES





$$c = k + \frac{\left\| \upsilon_2 x (p - k) \right\|}{\left\| \upsilon_2 x \upsilon_1 \right\|} \upsilon_1$$

$$\forall \left\| \upsilon_2 x (p - k) \right\| * \left\| \upsilon_2 x \upsilon_1 \right\| \geq 0$$

or

$$c = k - \frac{\left\| \upsilon_2 x (p - k) \right\|}{\left\| \upsilon_2 x \upsilon_1 \right\|} \upsilon_1$$

$$\forall \left\| \upsilon_2 x (p - k) \right\| * \left\| \upsilon_2 x \upsilon_1 \right\| < 0$$

PG2 Definition

$$e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} p = k$$

PG2 Solution

$$\theta_1^{01} = \upsilon_1^{T} \left(k - c \right)$$

$$\theta_2^{01} = \upsilon_2^{T} \left(c - p \right)$$

Pardos-Gotor Subproblem TWO (PG2)

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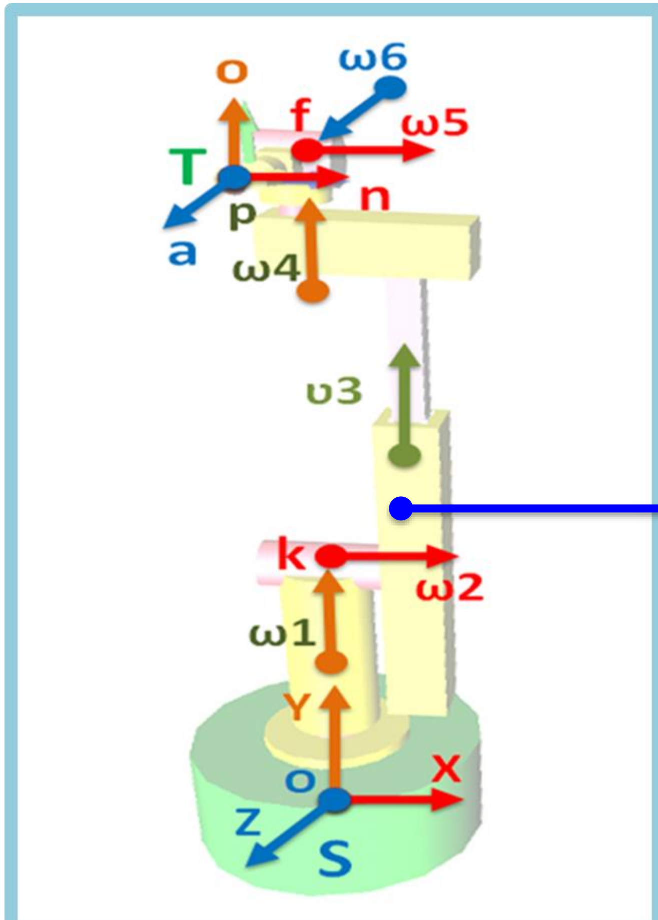
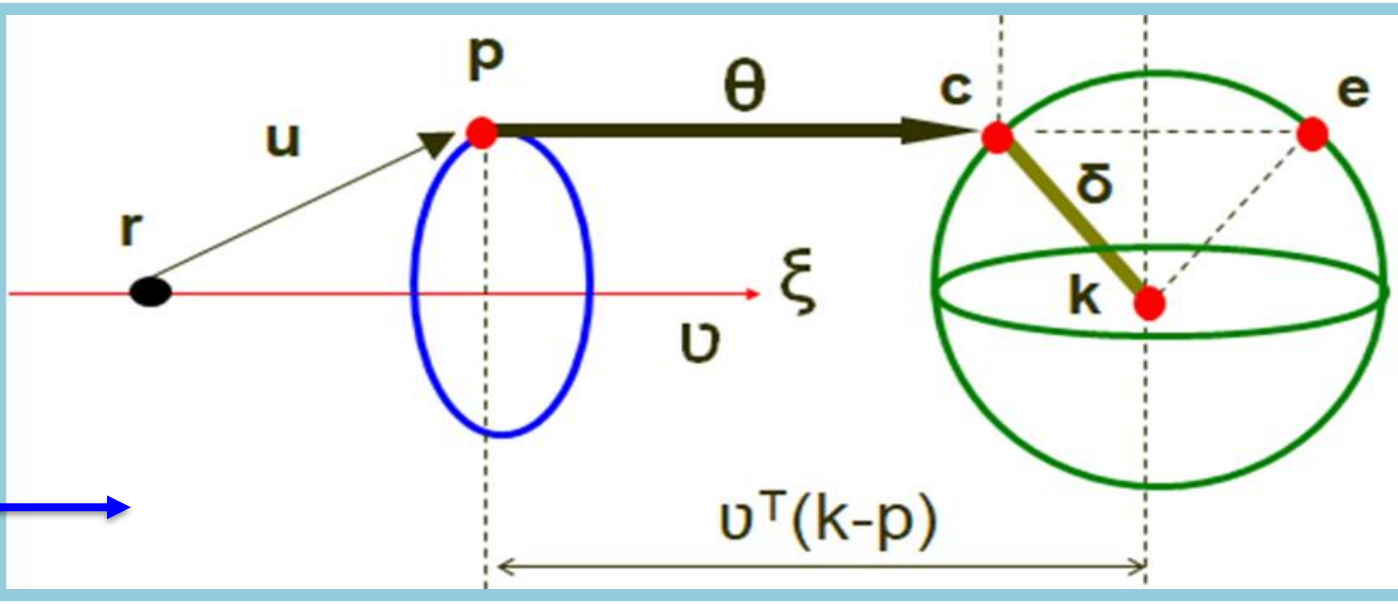
TRANSLATION TO A GIVEN DISTANCE





PG3 Definition

$$\left\| e^{\hat{\xi}_1 \theta_1} p - k \right\| = \delta$$

PG3 Solution

$$\theta_1^{01} = \upsilon^T (k-p) + \sqrt{\left[\upsilon^T (k-p)\right]^2 - \|k-p\|^2 + \delta^2}$$

$$\theta_1^{02} = \upsilon^T (k-p) - \sqrt{\left[\upsilon^T (k-p)\right]^2 - \|k-p\|^2 + \delta^2}$$

Pardos-Gotor Subproblem THREE (PG3)

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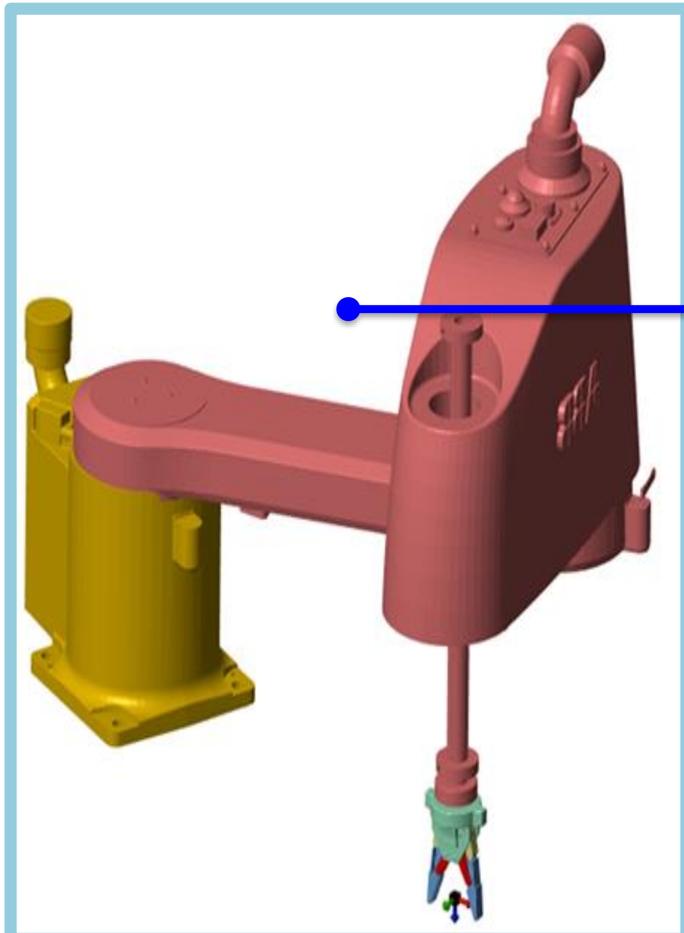
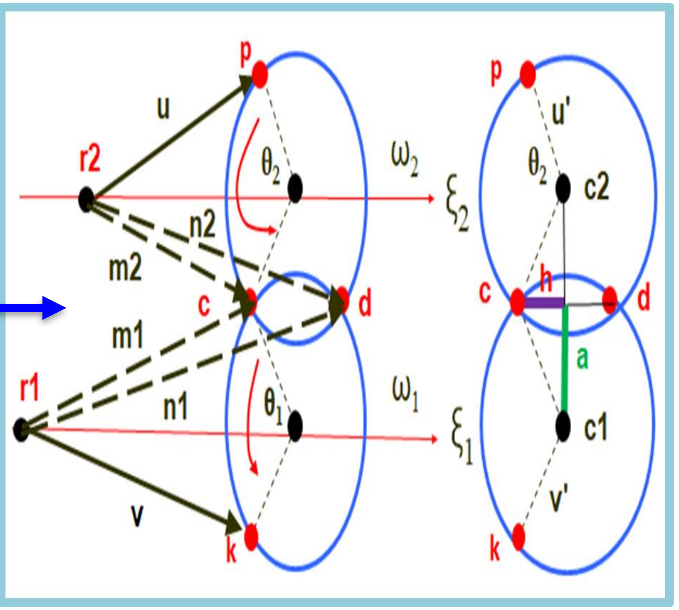
ROTATION ABOUT TWO SUBSEQUENT PARALLEL AXES





$$c = c_1 + a\omega_a + h\omega_h$$

$$d = c_1 + a\omega_a - h\omega_h$$

$$\omega_a = c_2 - c_1 / \|c_2 - c_1\|$$

$$\omega_h = \omega_1 x \omega_a$$

PG4 Definition

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} p = k$$

PG4 Solution

$$\theta_2^{01} = \arctan 2\left[\omega_2^T (u' \times m_2'), u'^T \cdot m_2'\right]$$

$$\theta_1^{01} = \arctan 2\left[\omega_1^T (m_1' \times v'), m_1'^T \cdot v'\right]$$

$$\theta_2^{02} = \arctan 2\left[\omega_2^T (u' \times n_2'), u'^T \cdot n_2'\right]$$

$$\theta_1^{02} = \arctan 2\left[\omega_1^T (n_1' \times v'), n_1'^T \cdot v'\right]$$

Pardos-Gotor Subproblem FOUR (PG4)

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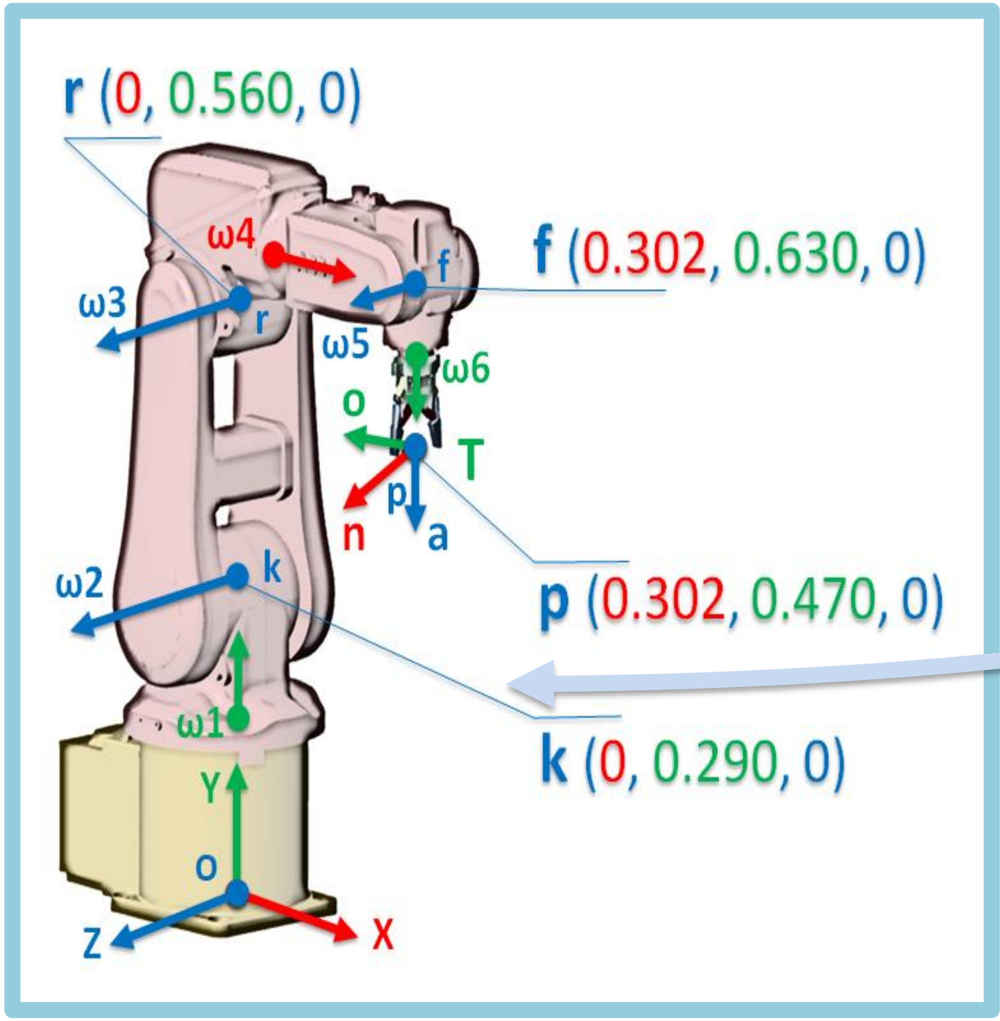
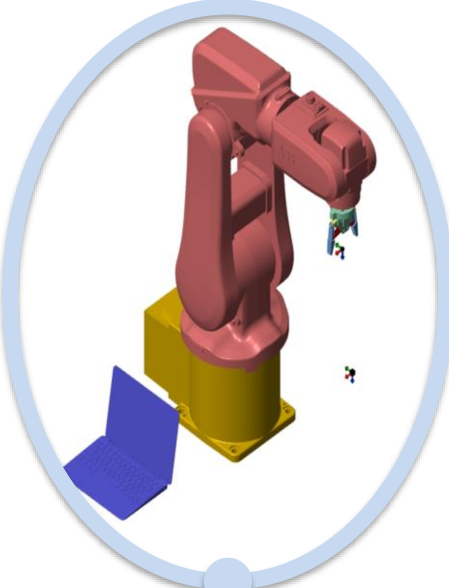




$$
\begin{array}{l}
\theta_1^{01}, \theta_2^{01}, \theta_3^{01}, \theta_4^{01}, \theta_5^{01}, \theta_6^{01} \\
\theta_1^{01}, \theta_2^{01}, \theta_3^{01}, \theta_4^{02}, \theta_5^{02}, \theta_6^{02} \\
\theta_1^{02}, \theta_2^{02}, \theta_3^{01}, \theta_4^{03}, \theta_5^{03}, \theta_6^{03} \\
\theta_1^{02}, \theta_2^{02}, \theta_3^{01}, \theta_4^{04}, \theta_5^{04}, \theta_6^{04} \\
\theta_1^{03}, \theta_2^{03}, \theta_3^{02}, \theta_4^{05}, \theta_5^{05}, \theta_6^{05} \\
\theta_1^{03}, \theta_2^{03}, \theta_3^{02}, \theta_4^{06}, \theta_5^{06}, \theta_6^{06} \\
\theta_1^{04}, \theta_2^{04}, \theta_3^{02}, \theta_4^{07}, \theta_5^{07}, \theta_6^{07} \\
\theta_1^{04}, \theta_2^{04}, \theta_3^{02}, \theta_4^{08}, \theta_5^{08}, \theta_6^{08}
\end{array}
$$

"Puma" Robot INVERSE KINEMATICS

ABB IRB 120

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

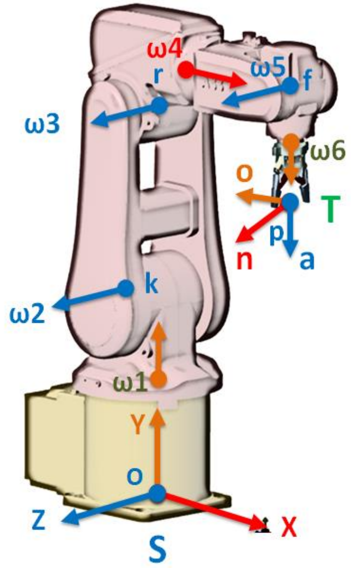
The desired Tool POSE TARGET (it is a reachable configuration)

EFFECTIVENESS the algorithm gives EIGHT EXACT GEOMETRIC (closed-form) SOLUTIONS for the set $(\theta 1,\theta 2,\theta 3,\theta 4,\theta 5,\theta 6)$

EFICIENCY ≈ ms (MATLAB)

Command Window

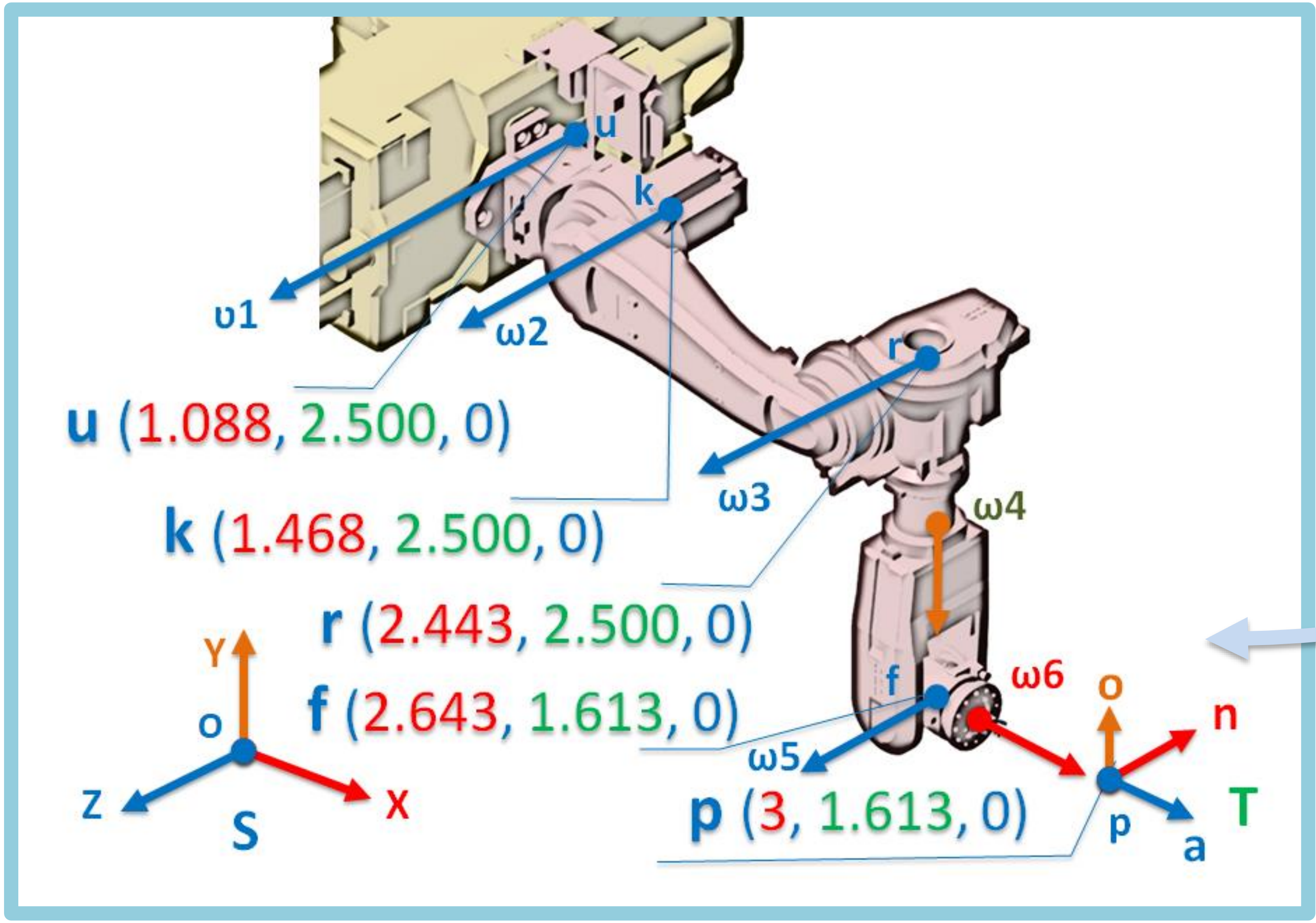
```
noap =

    0.6273   -0.7690    0.1233    0.0793
   -0.5413   -0.5443   -0.6409    0.7614
    0.5600    0.3352   -0.7577   -0.0718
         0         0         0    1.0000


Theta =

   -0.6932   -0.1915   -4.8328    1.8278   -0.8175    0.3857
   -0.6932   -0.1915   -4.8328   -1.3138   -2.3241   -2.7559
    2.4484    0.0767   -4.8328   -1.0085   -0.6729    0.8308
    2.4484    0.0767   -4.8328    2.1331   -2.4687   -2.3108
   -0.6932   -0.0767    1.2356    1.7211   -0.8377    0.2409
   -0.6932   -0.0767    1.2356   -1.4205   -2.3039   -2.9007
    2.4484    0.1915    1.2356   -0.9461   -0.6169    0.9345
    2.4484    0.1915    1.2356    2.1955   -2.5247   -2.2071


time_IK =

    'Time to solve IK Screw Theory 5.5 ms'
```

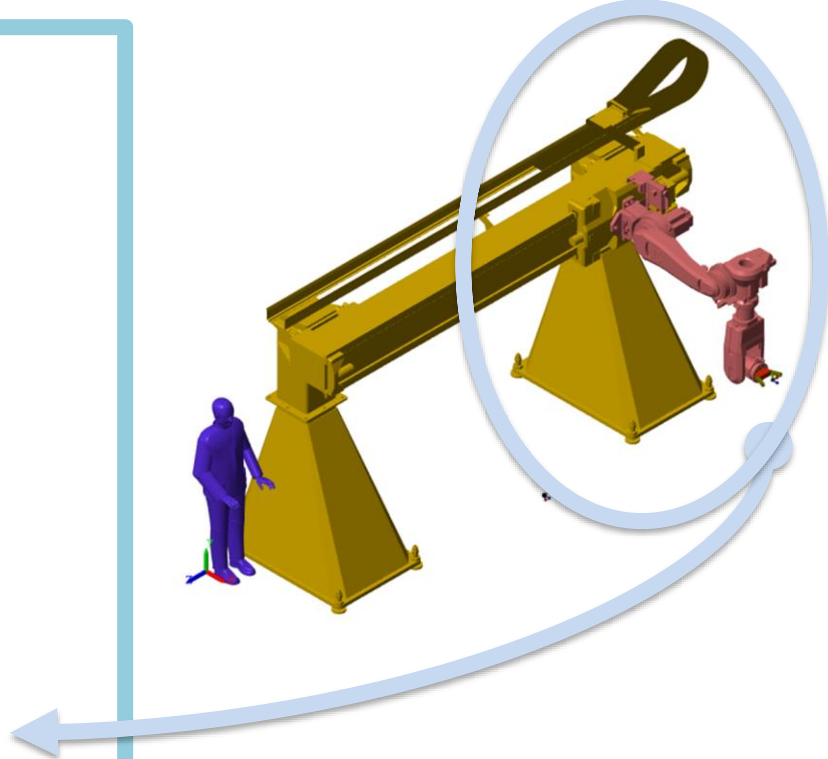


"Puma" Robot INVERSE KINEMATICS

ABB IRB 120 – Algorithm implementation

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

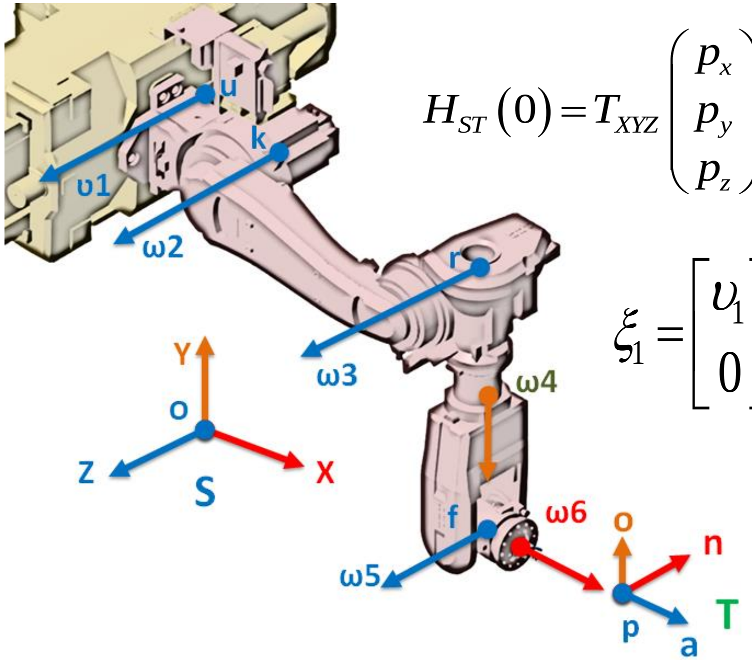




"Gantry" Robot INVERSE KINEMATICS

ABB IRB 6620LX (1/7) - Geometry

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



$$H_{ST}(0) = T_{XYZ}\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} R_o(\frac{\pi}{2}) = \begin{bmatrix} 0 & 0 & 1 & p_x \\ 0 & 1 & 0 & p_y \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \ \upsilon_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}; \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_6 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\xi_1 = \begin{bmatrix} \upsilon_1 \\ 0 \end{bmatrix}; \xi_2 = \begin{bmatrix} -\omega_2 \times k \\ \omega_2 \end{bmatrix}; \xi_3 = \begin{bmatrix} -\omega_3 \times r \\ \omega_3 \end{bmatrix}; \xi_4 = \begin{bmatrix} -\omega_4 \times f \\ \omega_4 \end{bmatrix}; \xi_5 = \begin{bmatrix} -\omega_5 \times f \\ \omega_5 \end{bmatrix}; \xi_6 = \begin{bmatrix} -\omega_6 \times p \\ \omega_6 \end{bmatrix}$$

12 nonlinear equations with four solutions of 6 unknowns (θ1,θ2,θ3,θ4,θ5,θ6)

The desired TARGET Pose

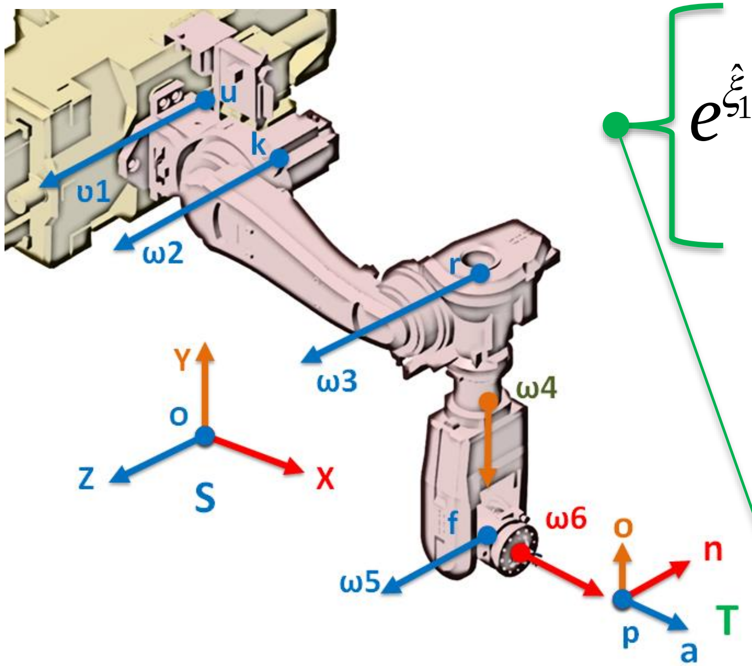
?

$$H_{ST}(\theta) = \boxed{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} H_{ST}(0)} = \boxed{\begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

“Gantry” Robot INVERSE KINEMATICS

ABB IRB 6620LX (2/7) – Forward Kinematics map

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



$$e^{\hat{\xi}_1\theta_1}\underbrace{e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}}\underbrace{e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6}} f = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} f$$

The screw Rotations θ2 and θ3, DO NOT CHAGE the plane where "f" moves, which is perpendicular to the axis of υ1 and so do not affect θ1

The screw Rotations θ4, θ5, θ6, have NO EFFECT when applied to any point "f" on their axis

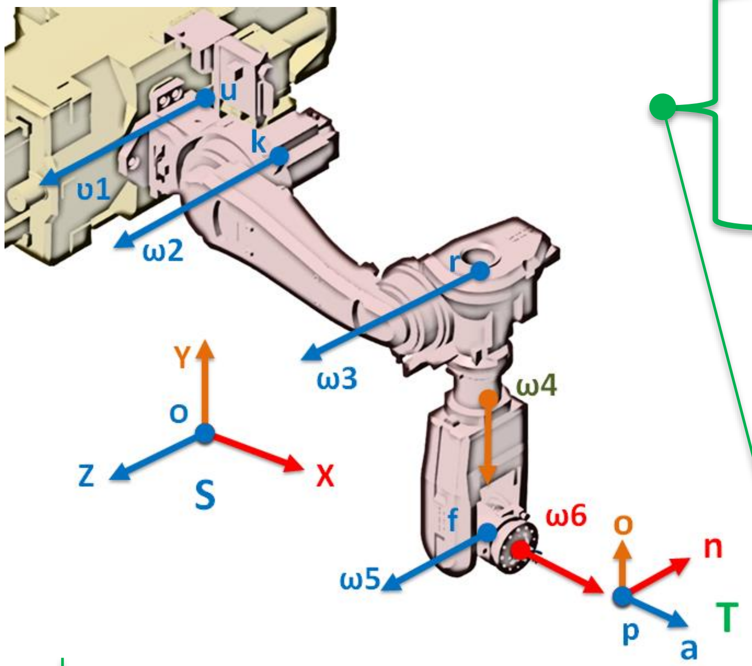
Canonical subproblem Pardos-Gotor ONE

$$\boxed{e^{\hat{\xi}_1\theta_1} f = k_1} \quad \left\{ \boxed{\theta_1^{01}} \right.$$

"Gantry" Robot INVERSE KINEMATICS

ABB IRB 6620LX (3/7) - θ1 Solution

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



$$e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} \underbrace{e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6}} f = e^{-\hat{\xi}_1\theta_1} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} f$$

The screw Rotations θ4, θ5, θ6, have NO EFFECT when applied to any point "f" which is on their axis

Canonical subproblem Pardos-Gotor FOUR

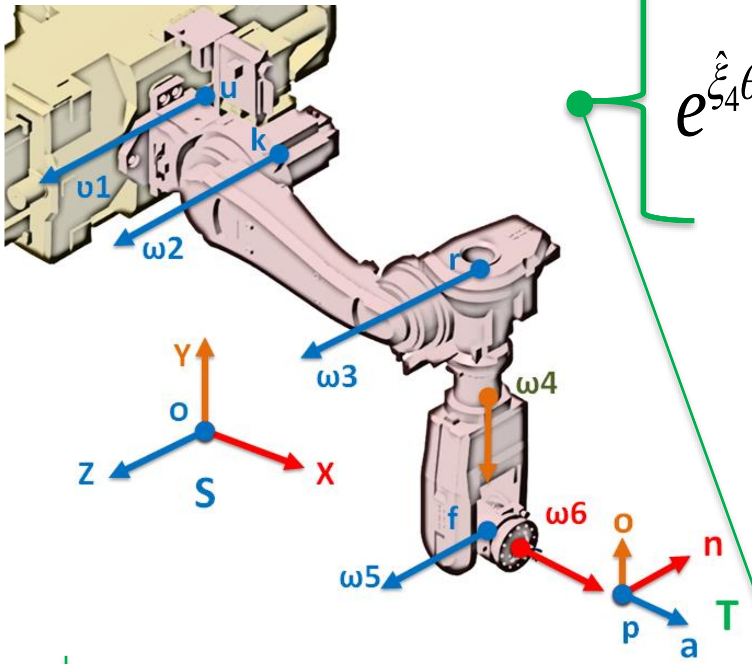
$$\boxed{e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} f = k_2}$$

$$\boxed{\theta_1^{01} \Rightarrow \theta_2^{01}, \theta_3^{01} \;\&\; \theta_2^{02}, \theta_3^{02}}$$

"Gantry" Robot INVERSE KINEMATICS

ABB IRB 6620LX (4/7) - θ2 & θ3 Solutions

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



$$\left\{ e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} \underbrace{e^{\hat{\xi}_6\theta_6} p}_{} = \underbrace{e^{-\hat{\xi}_3\theta_3} e^{-\hat{\xi}_2\theta_2} e^{-\hat{\xi}_1\theta_1}}_{} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} p \right.$$

The screw rotations θ6, has NO EFFECT when applied to any point "p" which is on its axis

θ3, θ2, θ1 are already known from the previous calculation

Canonical subproblem Paden-Kahan TWO

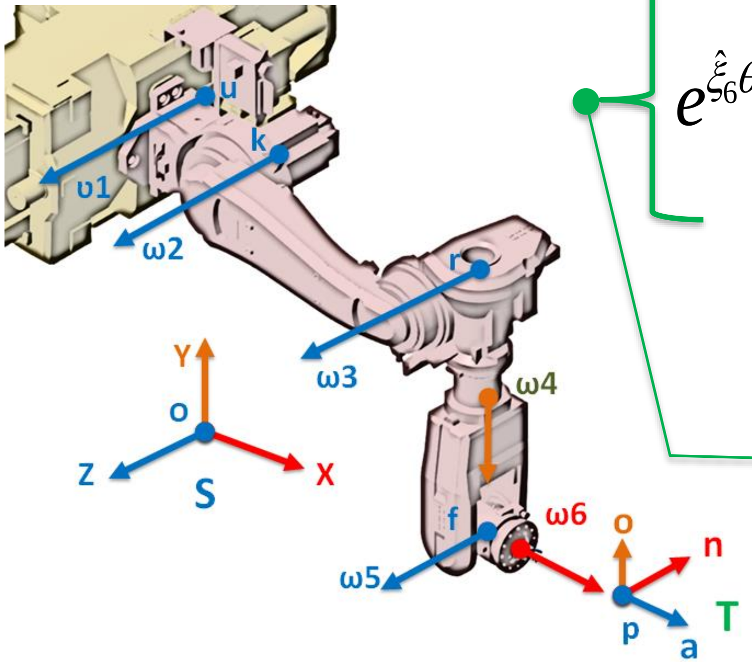
$$\boxed{e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} p = k_3}$$

$$\left\{ \boxed{\begin{array}{l} \theta_1^{01}, \theta_2^{01}, \theta_3^{01} \Rightarrow \theta_4^{01}, \theta_5^{01} \,\&\, \theta_4^{02}, \theta_5^{02} \\ \theta_1^{01}, \theta_2^{02}, \theta_3^{02} \Rightarrow \theta_4^{03}, \theta_5^{03} \,\&\, \theta_4^{04}, \theta_5^{04} \end{array}} \right.$$

"Gantry" Robot INVERSE KINEMATICS

ABB IRB 6620LX (5/7) - θ4 & θ5 Solutions

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



$$e^{\hat{\xi}_6\theta_6} o = \underbrace{e^{-\hat{\xi}_5\theta_5} e^{-\hat{\xi}_4\theta_4} e^{-\hat{\xi}_3\theta_3} e^{-\hat{\xi}_2\theta_2} e^{-\hat{\xi}_1\theta_1}} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} o$$

θ5, θ4, θ3, θ2, θ1 are already known from the previous calculation (4 Solutions)

Subproblem Paden-Kahan ONE

$$\boxed{e^{\hat{\xi}_6\theta_6} o = k_4}$$

$$\begin{aligned}
&\theta_1^{01}, \theta_2^{01}, \theta_3^{01}, \theta_4^{01}, \theta_5^{01} \Rightarrow \theta_6^{01} \\
&\theta_1^{01}, \theta_2^{01}, \theta_3^{01}, \theta_4^{02}, \theta_5^{02} \Rightarrow \theta_6^{02} \\
&\theta_1^{01}, \theta_2^{02}, \theta_3^{02}, \theta_4^{03}, \theta_5^{03} \Rightarrow \theta_6^{03} \\
&\theta_1^{01}, \theta_2^{02}, \theta_3^{02}, \theta_4^{04}, \theta_5^{04} \Rightarrow \theta_6^{04}
\end{aligned}$$

"Gantry" Robot INVERSE KINEMATICS

ABB IRB 6620LX (6/7) - θ6 Solutions

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

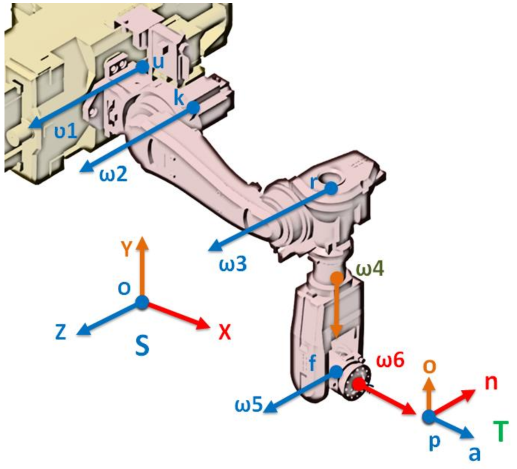
The desired Tool POSE TARGET (it is a reachable configuration)

EFFECTIVENESS the algorithm gives FOUR EXACT GEOMETRIC (closed-form) SOLUTIONS for the set $(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6)$

EFICIENCY ≈ ms (MATLAB)

Command Window

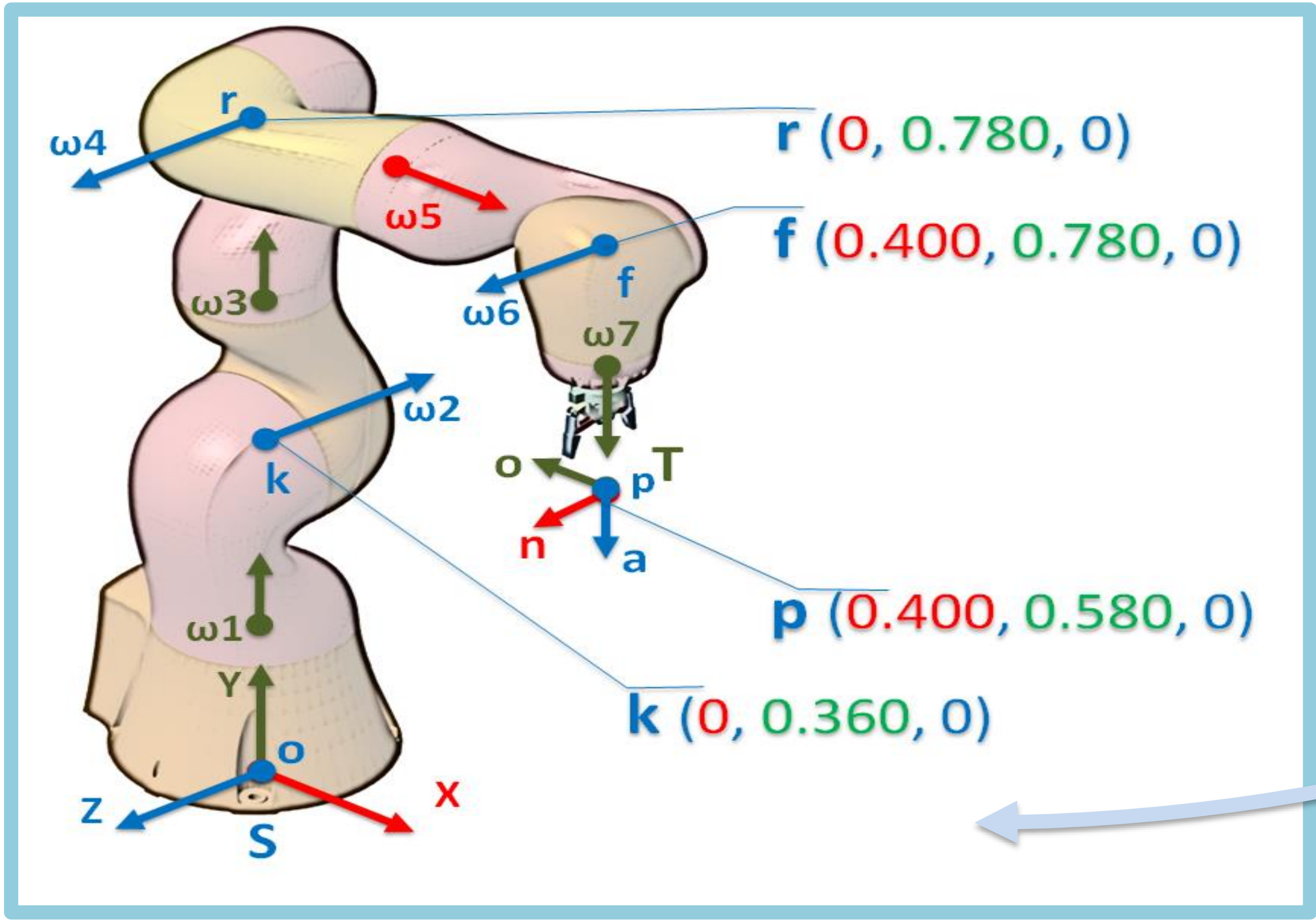
```
noap =

   -0.0390   -0.4913   -0.8701    0.9168
   -0.1617    0.8624   -0.4797    2.5706
    0.9861    0.1220   -0.1131   -0.2255
         0         0         0    1.0000


Theta =

   -0.1852    1.1521   -2.1500    1.0259   -1.7034    1.9965
   -0.1852   -2.7285   -1.4352    1.9464    1.6926   -1.3157
   -0.1852    1.1521   -2.1500   -2.1157   -1.4382   -1.1451
   -0.1852   -2.7285   -1.4352   -1.1952    1.4490    1.8259


time_IK =

    'Time to solve IK Screw Theory 3.5 ms'
```

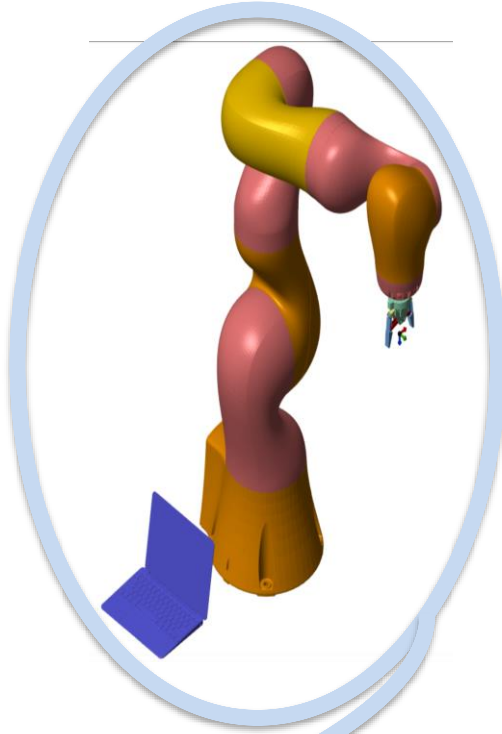


"Gantry" Robot INVERSE KINEMATICS

ABB IRB 6620LX (7/7) – Algorithm implementation

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics





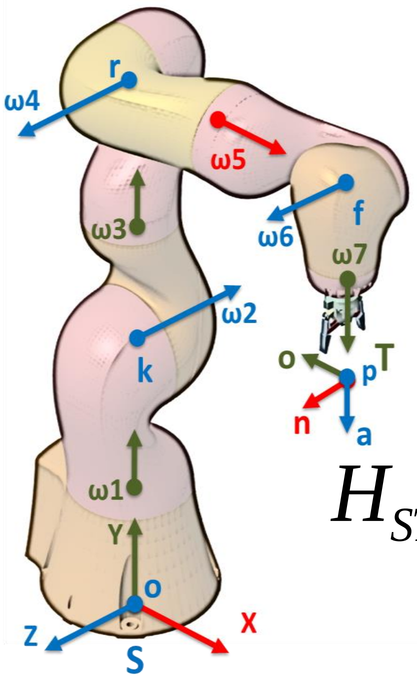
Collaborative Redundant Robot INVERSE KINEMATICS

KUKA IIWA R820 (1/9) - Geometry

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

$$H_{ST}(0) = T_{XYZ}\begin{pmatrix} p_X \\ p_Y \\ p_Z \end{pmatrix} R_n(\frac{\pi}{2}) R_a(\frac{\pi}{2}) = \begin{bmatrix} 0 & -1 & 0 & p_X \\ 0 & 0 & -1 & p_Y \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad \omega_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \omega_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}; \omega_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \omega_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \omega_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_7 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\xi_1 = \begin{bmatrix} -\omega_1 \times k \\ \omega_1 \end{bmatrix}; \xi_2 = \begin{bmatrix} -\omega_2 \times k \\ \omega_2 \end{bmatrix}; \xi_3 = \begin{bmatrix} -\omega_3 \times k \\ \omega_3 \end{bmatrix}; \xi_4 = \begin{bmatrix} -\omega_4 \times r \\ \omega_4 \end{bmatrix}; \xi_5 = \begin{bmatrix} -\omega_5 \times f \\ \omega_5 \end{bmatrix}; \xi_6 = \begin{bmatrix} -\omega_6 \times f \\ \omega_6 \end{bmatrix}; \xi_7 = \begin{bmatrix} -\omega_7 \times f \\ \omega_7 \end{bmatrix}$$



12 nonlinear equations with ∞ solutions of 7 unknowns (θ1,θ2,θ3,θ4,θ5,θ6,θ7)

The desired TARGET Pose

$$H_{ST}(\theta) = \boxed{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} e^{\hat{\xi}_7\theta_7} H_{ST}(0)} = \boxed{\begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

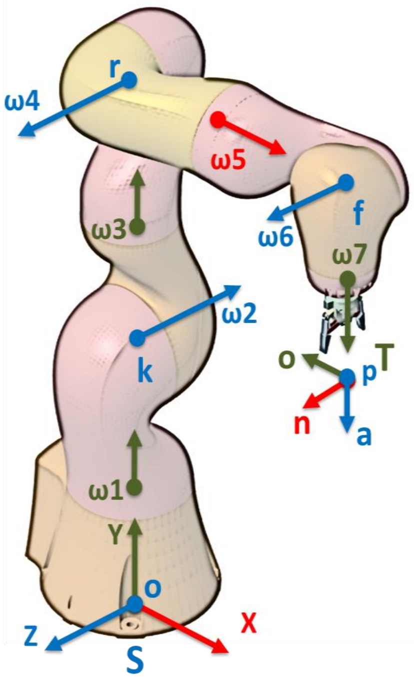
?

Collaborative Redundant Robot INVERSE KINEMATICS

KUKA IIWA R820 (2/9) – Forward Kinematics map

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

The proposed CRITERIA (to limit the number of solutions) is to orientate the robot TCP in the direction of the TARGET pose, so as to get θ1in & θ3in as inputs.
The result gives a quite natural motion to the robot.



Subproblem
Paden-Kahan
ONE PK1

$$e^{\hat{\xi}_1\theta_1} p_{Hst(0)} = p_{noap} \quad \left\{ \boxed{\theta_1^{in}} \right.$$

$$e^{\hat{\xi}_3\theta_3} p_{Hst(0)} = p_{noap} \quad \left\{ \boxed{\theta_3^{in}} \right.$$

Collaborative Redundant Robot INVERSE KINEMATICS

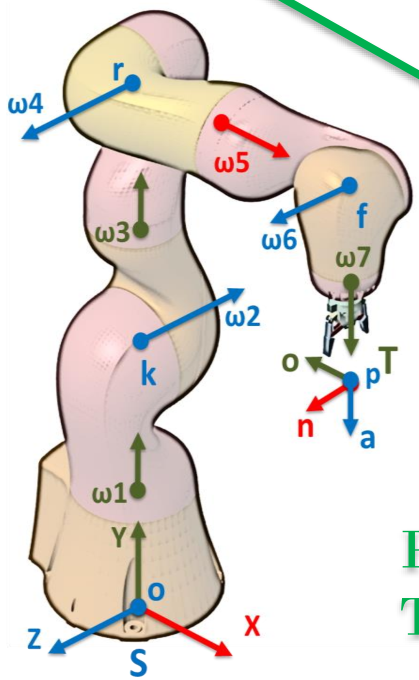
KUKA IIWA R820 (3/9) – θ1in & θ3in Solutions

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

$$\left\| \underbrace{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3}} e^{\hat{\xi}_4\theta_4} \underbrace{e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} e^{\hat{\xi}_7\theta_7}} f - k \right\| = \left\| \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} f - k \right\|$$

The screw Rotations θ5, θ6, θ7, have NO EFFECT when applied to any point "f" which is on its axis

The screw Rotations θ1, θ2, θ3, DO NOT CHAGE the MODULE when applied to any vector with an extreme "k" which is on its axis



Subproblem
Paden-Kahan
THREE PK3

$$\left\| e^{\hat{\xi}_4\theta_4} f - k \right\| = \delta$$

$$\theta_4^{01} \;\&\; \theta_4^{02}$$

Collaborative Redundant Robot INVERSE KINEMATICS

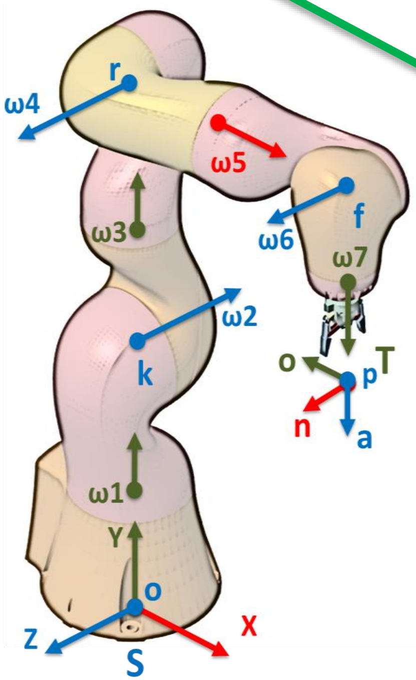
KUKA IIWA R820 (4/9) – θ4 Solutions

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} e^{\hat{\xi}_7\theta_7} f = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} f$$

The screw Rotations θ5, θ6, θ7, have NO EFFECT when applied to any point "f" which is on its axis

θ3in and θ4 are already known from the previous steps



Subproblem
Paden-Kahan TWO PK2

$$\boxed{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} f_1 = k_1}$$

$$\boxed{\begin{matrix} \theta_3^{in}, \theta_4^{01} \Rightarrow \theta_1^{01}, \theta_2^{01} \;\&\; \theta_1^{02}, \theta_2^{02} \\ \theta_3^{in}, \theta_4^{02} \Rightarrow \theta_1^{03}, \theta_2^{03} \;\&\; \theta_1^{04}, \theta_2^{04} \end{matrix}}$$

Collaborative Redundant Robot INVERSE KINEMATICS

KUKA IIWA R820 (5/9) – θ1 & θ2 Solutions (considering θ3in)

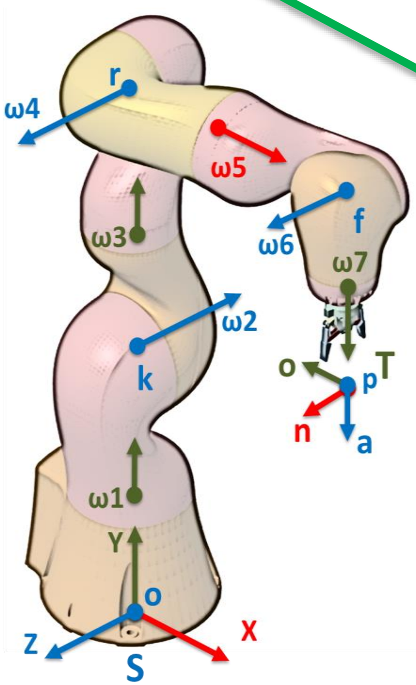
CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

$$e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} \underbrace{e^{\hat{\xi}_4\theta_4}} \underbrace{e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} e^{\hat{\xi}_7\theta_7}} f = \underbrace{e^{-\hat{\xi}_1\theta_1}} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} f$$

θ1in is known from the first step

The screw Rotations θ5, θ6, θ7, have NO EFFECT when applied to any point “f” which is on its axis

θ4 is already known from the previous steps



Subproblem
Paden-Kahan TWO PK2

$$\boxed{e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} f_2 = k_2}$$

$$\left\{ \begin{array}{l} \theta_1^{in}, \theta_4^{01} \Rightarrow \theta_2^{05}, \theta_3^{01} \;\&\; \theta_2^{06}, \theta_3^{02} \\ \theta_1^{in}, \theta_4^{02} \Rightarrow \theta_2^{07}, \theta_3^{03} \;\&\; \theta_2^{08}, \theta_3^{04} \end{array} \right.$$

Collaborative Redundant Robot INVERSE KINEMATICS

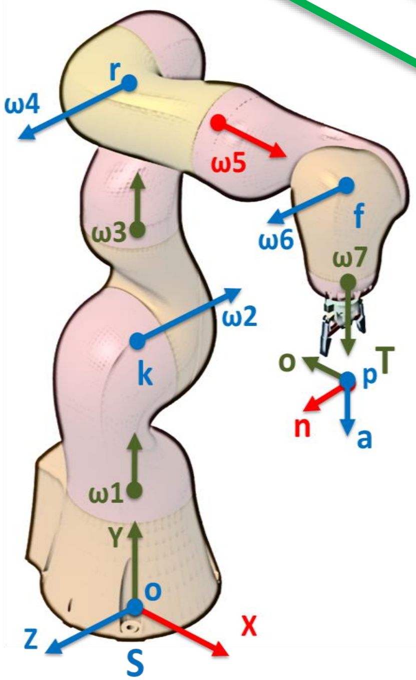
KUKA IIWA R820 (6/9) – θ2 & θ3 Solutions (considering θ1in)

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

$$e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} e^{\hat{\xi}_7\theta_7} p = e^{-\hat{\xi}_4\theta_4} e^{-\hat{\xi}_3\theta_3} e^{-\hat{\xi}_2\theta_2} e^{-\hat{\xi}_1\theta_1} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} p$$

θ4, θ3, θ2 and θ1 are already known

θ7 has NO EFFECT when applied to “p” on its axis



Subproblem Paden-Kahan TWO PK2

$$\boxed{e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} p = k_3}$$

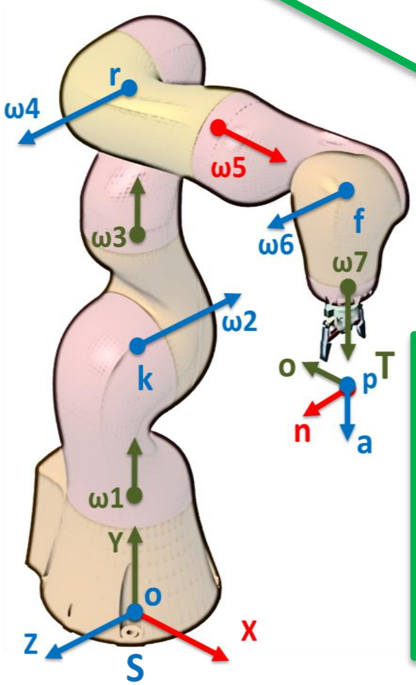
$$\left\{\begin{array}{l}
\theta_1^{01},\theta_2^{01},\theta_3^{in},\theta_4^{01} \Rightarrow \theta_5^{01},\theta_6^{01} \,\&\, \theta_5^{02},\theta_6^{02} \\
\theta_1^{02},\theta_2^{02},\theta_3^{in},\theta_4^{01} \Rightarrow \theta_5^{03},\theta_6^{03} \,\&\, \theta_5^{4},\theta_6^{04} \\
\theta_1^{03},\theta_2^{03},\theta_3^{in},\theta_4^{02} \Rightarrow \theta_5^{05},\theta_6^{05} \,\&\, \theta_5^{06},\theta_6^{06} \\
\theta_1^{04},\theta_2^{04},\theta_3^{in},\theta_4^{02} \Rightarrow \theta_5^{07},\theta_6^{07} \,\&\, \theta_5^{08},\theta_6^{08} \\
\theta_1^{in},\theta_2^{05},\theta_3^{01},\theta_4^{01} \Rightarrow \theta_5^{09},\theta_6^{09} \,\&\, \theta_5^{10},\theta_6^{10} \\
\theta_1^{in},\theta_2^{06},\theta_3^{02},\theta_4^{01} \Rightarrow \theta_5^{11},\theta_6^{11} \,\&\, \theta_5^{12},\theta_6^{12} \\
\theta_1^{in},\theta_2^{07},\theta_3^{03},\theta_4^{02} \Rightarrow \theta_5^{13},\theta_6^{13} \,\&\, \theta_5^{14},\theta_6^{14} \\
\theta_1^{in},\theta_2^{08},\theta_3^{04},\theta_4^{02} \Rightarrow \theta_5^{15},\theta_6^{15} \,\&\, \theta_5^{16},\theta_6^{16}
\end{array}\right.$$

Collaborative Redundant Robot INVERSE KINEMATICS

KUKA IIWA R820 (7/9) – θ5 & θ6 Solutions

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

$$e^{\hat{\xi}_7\theta_7} o = \underbrace{e^{-\hat{\xi}_6\theta_6} e^{-\hat{\xi}_5\theta_5} e^{-\hat{\xi}_4\theta_4} e^{-\hat{\xi}_3\theta_3} e^{-\hat{\xi}_2\theta_2} e^{-\hat{\xi}_1\theta_1}}_{\text{θ6, θ5, θ4, θ3, θ2 and θ1 are already known}} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} o$$



Subproblem Paden-Kahan ONE PK1

$$e^{\hat{\xi}_7\theta_7} o = k_4$$

$\theta_1^{01},\theta_2^{01},\theta_3^{in},\theta_4^{01},\theta_5^{01},\theta_6^{01} \Rightarrow \theta_7^{01}$	$\theta_1^{in},\theta_2^{05},\theta_3^{01},\theta_4^{01},\theta_5^{09},\theta_6^{09} \Rightarrow \theta_7^{09}$
$\theta_1^{01},\theta_2^{01},\theta_3^{in},\theta_4^{01},\theta_5^{02},\theta_6^{02} \Rightarrow \theta_7^{02}$	$\theta_1^{in},\theta_2^{05},\theta_3^{01},\theta_4^{01},\theta_5^{10},\theta_6^{10} \Rightarrow \theta_7^{10}$
$\theta_1^{02},\theta_2^{02},\theta_3^{in},\theta_4^{01},\theta_5^{03},\theta_6^{03} \Rightarrow \theta_7^{03}$	$\theta_1^{in},\theta_2^{06},\theta_3^{02},\theta_4^{01},\theta_5^{11},\theta_6^{11} \Rightarrow \theta_7^{11}$
$\theta_1^{02},\theta_2^{02},\theta_3^{in},\theta_4^{01},\theta_5^{04},\theta_6^{04} \Rightarrow \theta_7^{04}$	$\theta_1^{in},\theta_2^{06},\theta_3^{02},\theta_4^{01},\theta_5^{12},\theta_6^{12} \Rightarrow \theta_7^{12}$
$\theta_1^{03},\theta_2^{03},\theta_3^{in},\theta_4^{02},\theta_5^{05},\theta_6^{05} \Rightarrow \theta_7^{05}$	$\theta_1^{in},\theta_2^{07},\theta_3^{03},\theta_4^{02},\theta_5^{13},\theta_6^{13} \Rightarrow \theta_7^{13}$
$\theta_1^{03},\theta_2^{03},\theta_3^{in},\theta_4^{02},\theta_5^{06},\theta_6^{06} \Rightarrow \theta_7^{06}$	$\theta_1^{in},\theta_2^{07},\theta_3^{03},\theta_4^{02},\theta_5^{14},\theta_6^{14} \Rightarrow \theta_7^{14}$
$\theta_1^{04},\theta_2^{04},\theta_3^{in},\theta_4^{02},\theta_5^{07},\theta_6^{07} \Rightarrow \theta_7^{07}$	$\theta_1^{in},\theta_2^{08},\theta_3^{04},\theta_4^{02},\theta_5^{15},\theta_6^{15} \Rightarrow \theta_7^{15}$
$\theta_1^{04},\theta_2^{04},\theta_3^{in},\theta_4^{02},\theta_5^{08},\theta_6^{08} \Rightarrow \theta_7^{08}$	$\theta_1^{in},\theta_2^{08},\theta_3^{04},\theta_4^{02},\theta_5^{16},\theta_6^{16} \Rightarrow \theta_7^{16}$

Collaborative Redundant Robot INVERSE KINEMATICS

KUKA IIWA R820 (8/9) – θ7 Solutions

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

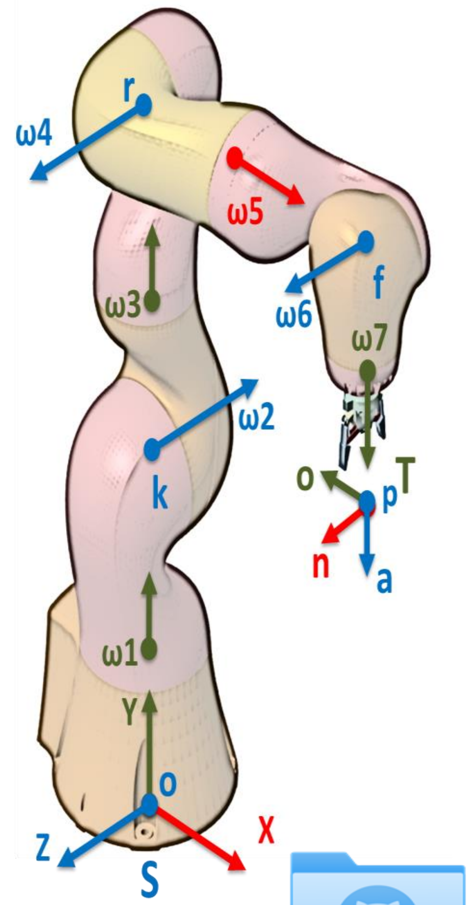
The desired Tool POSE TARGET (it is a reachable configuration)

EFFECTIVENESS the algorithm gives SIXTEEN EXACT GEOMETRIC (closed-form) SOLUTIONS for the set (θ1,θ2,θ3,θ4,θ5,θ6,θ7)

EFICIENCY ≈ ms (MATLAB)

Command Window

```
noap =

   -0.7414    0.6420    0.1953   -0.0930
    0.2781    0.0290    0.9601    0.5790
    0.6107    0.7661   -0.2000   -0.0361
         0         0         0    1.0000


Theta =

   -0.3303   -2.5952    2.7710   -1.8976   -1.6234    1.1815   -2.4421
   -0.3303   -2.5952    2.7710   -1.8976    1.5182    1.9601    0.6995
   -2.7525    0.1162    2.7710   -1.8976    1.5226   -1.3322    0.5938
   -2.7525    0.1162    2.7710   -1.8976   -1.6190   -1.8094   -2.5478
    0.3891   -0.1162    2.7710   -1.2440   -1.6190   -1.3322    0.5938
    0.3891   -0.1162    2.7710   -1.2440    1.5226   -1.8094   -2.5478
    2.8113    2.5952    2.7710   -1.2440    1.5182    1.1815   -2.4421
    2.8113    2.5952    2.7710   -1.2440   -1.6234    1.9601    0.6995
    2.7710    0.1181   -2.7294   -1.8976    1.4913   -1.4231    0.5636
    2.7710    0.1181   -2.7294   -1.8976   -1.6503   -1.7185   -2.5780
    2.7710    2.5865   -0.4121   -1.8976   -1.6201    1.1576   -2.4452
    2.7710    2.5865   -0.4121   -1.8976    1.5215    1.9840    0.6964
    2.7710    0.1181    0.4121   -1.2440   -1.6503   -1.4231    0.5636
    2.7710    0.1181    0.4121   -1.2440    1.4913   -1.7185   -2.5780
    2.7710    2.5865    2.7294   -1.2440    1.5215    1.1576   -2.4452
    2.7710    2.5865    2.7294   -1.2440   -1.6201    1.9840    0.6964


time_IK =

    'Time to solve IK Screw Theory 21.8 ms'
```



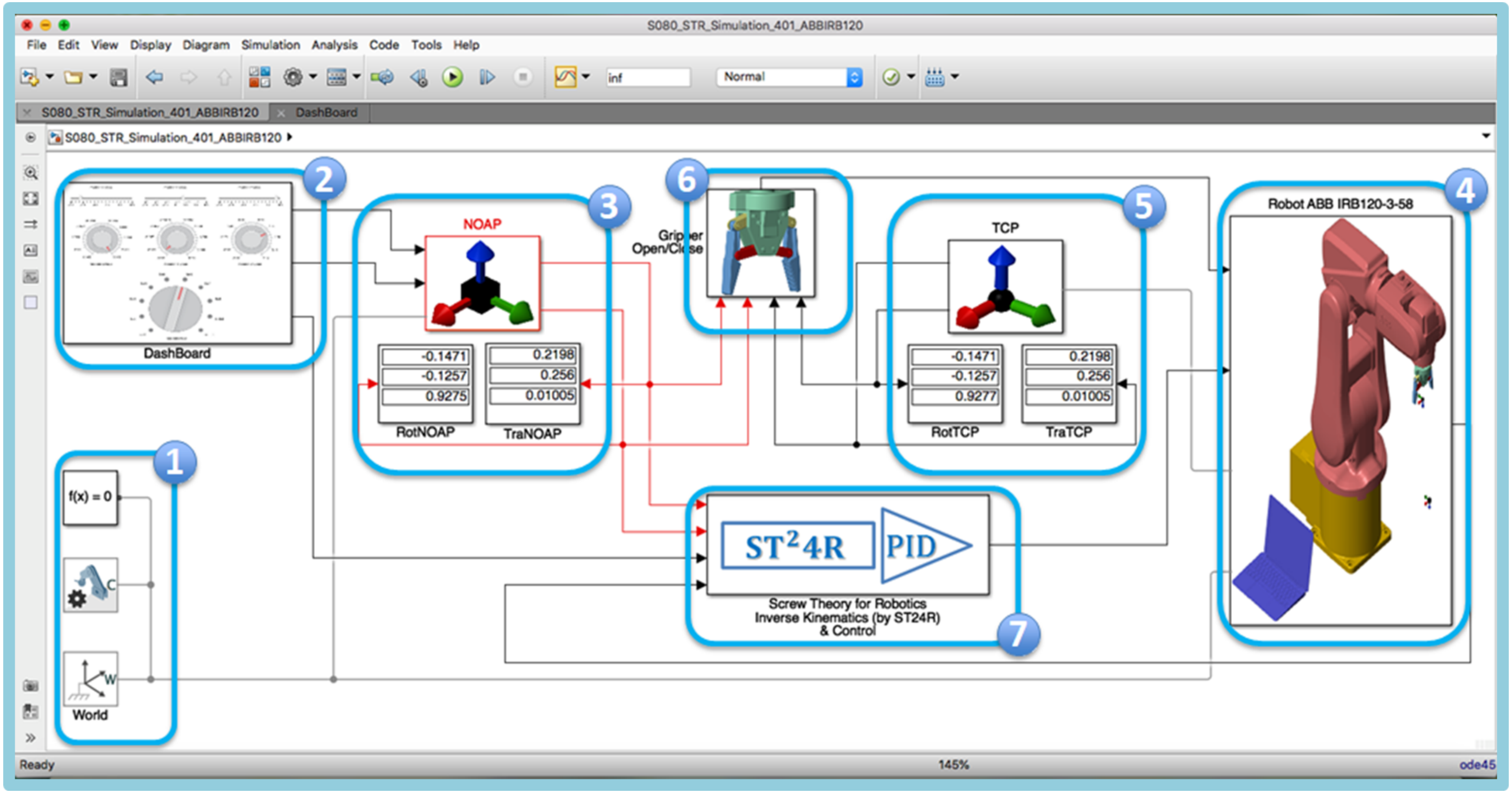
Collaborative Redundant Robot INVERSE KINEMATICS

KUKA IIWA R820 (9/9) – Algorithm implementation

- Take a sneak peak at the FUTURE of Robotics.
- Kinematics approach NUMERIC vs. GEOMETRIC
- The cornerstone of the CANONICAL Subproblems
- Inverse Kinematics Geometric Solutions EXAMPLES: PUMA, GANTRY & REDUNDANT robots
- **Inverse Kinematics SIMULATIONS: ABB IRB120, ABB IRB6620LX, KUKA IIWA robots**
- DIFFERENTIAL Kinematics & GEOMETRIC JACOBIAN
- Screw Theory for Robotics BENEFITS summary

AGENDA

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



SIMULATION: ABB IRB120 Inverse Kinematics -»

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

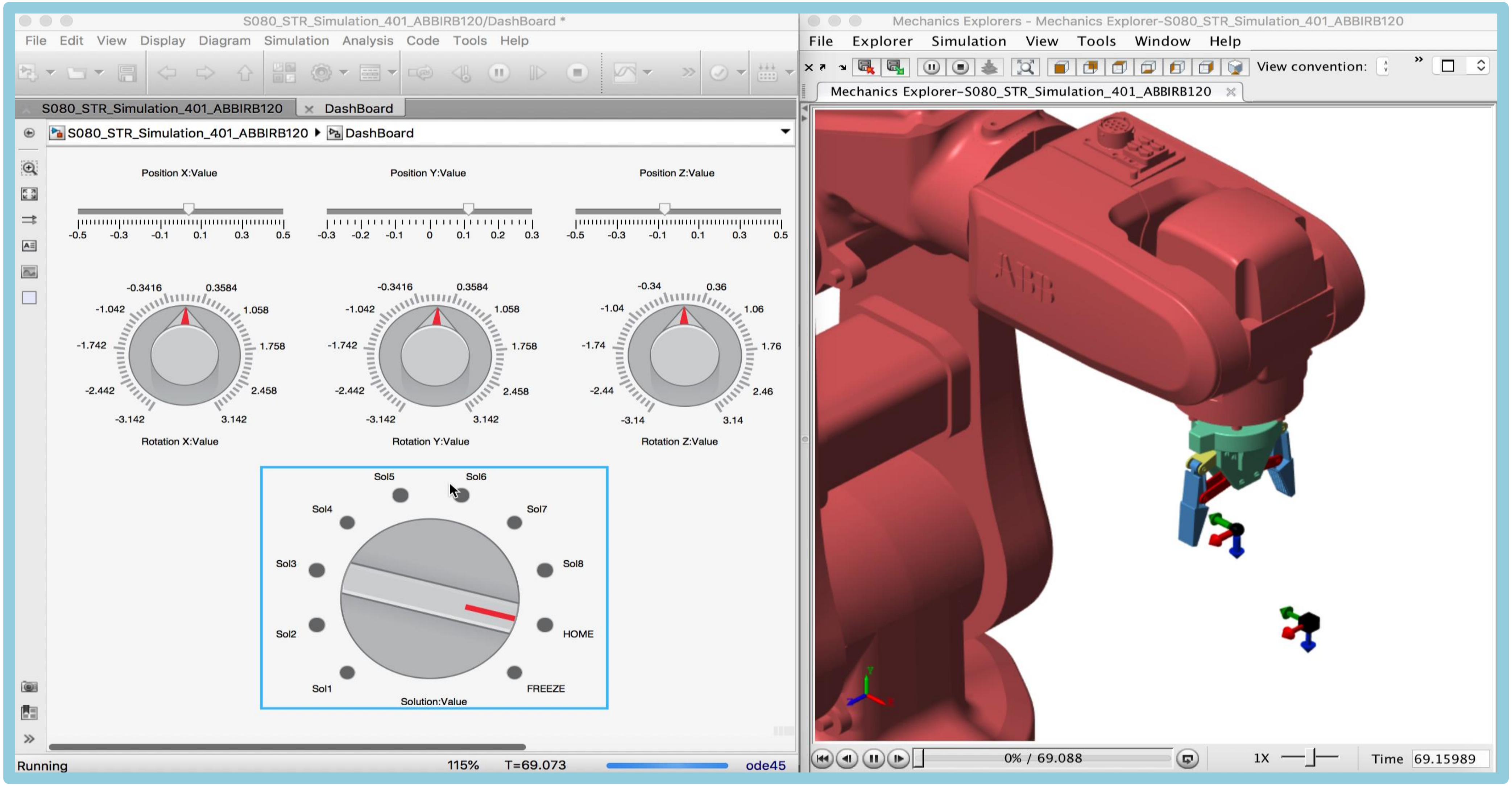
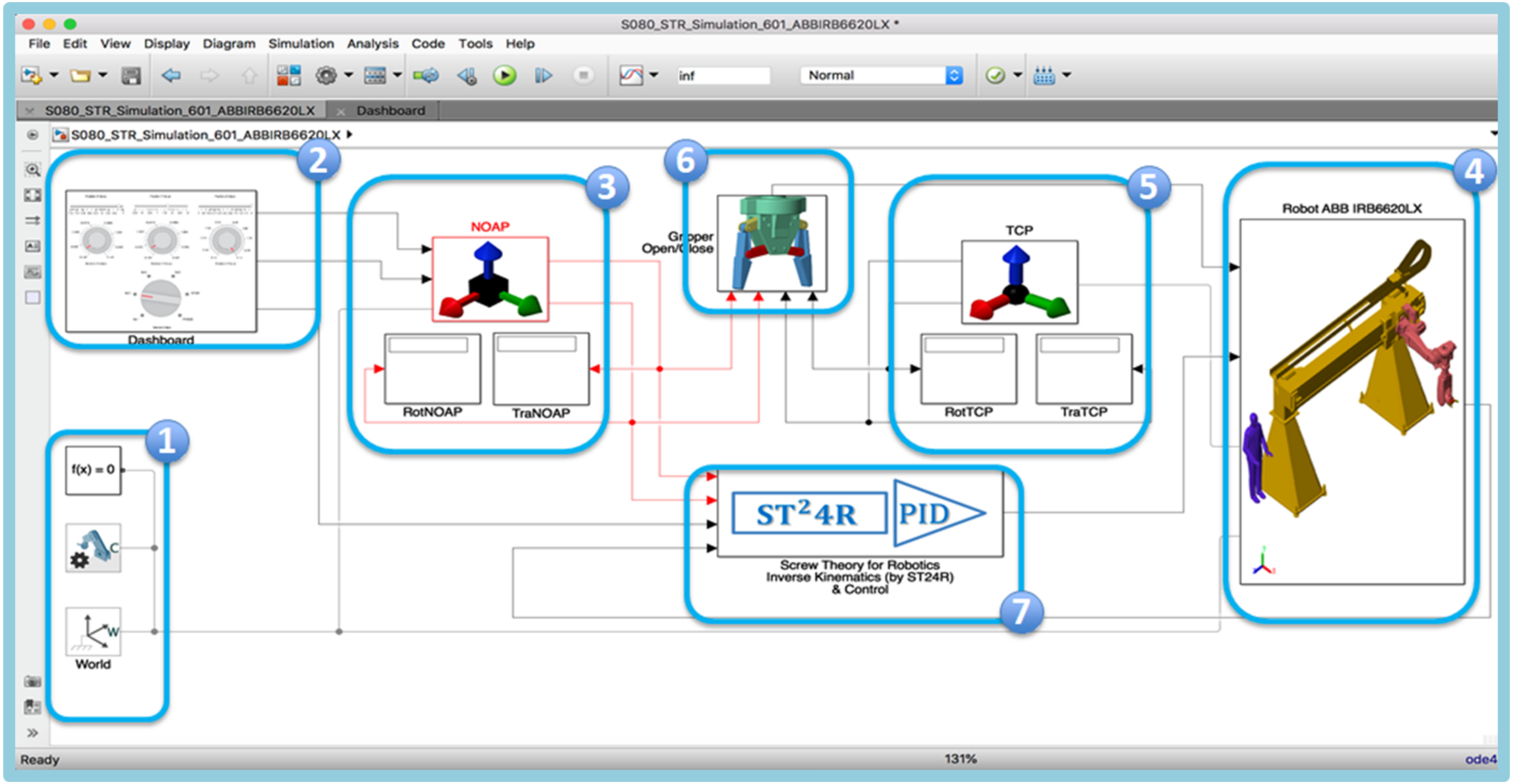


ABB IRB120 Real Time Simulation with EIGHT EXACT Geometric Solutions (θ1,θ2,θ3,θ4,θ5,θ6) -»

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



SIMULATION: ABB IRB6620LX Inverse Kinematics -»

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

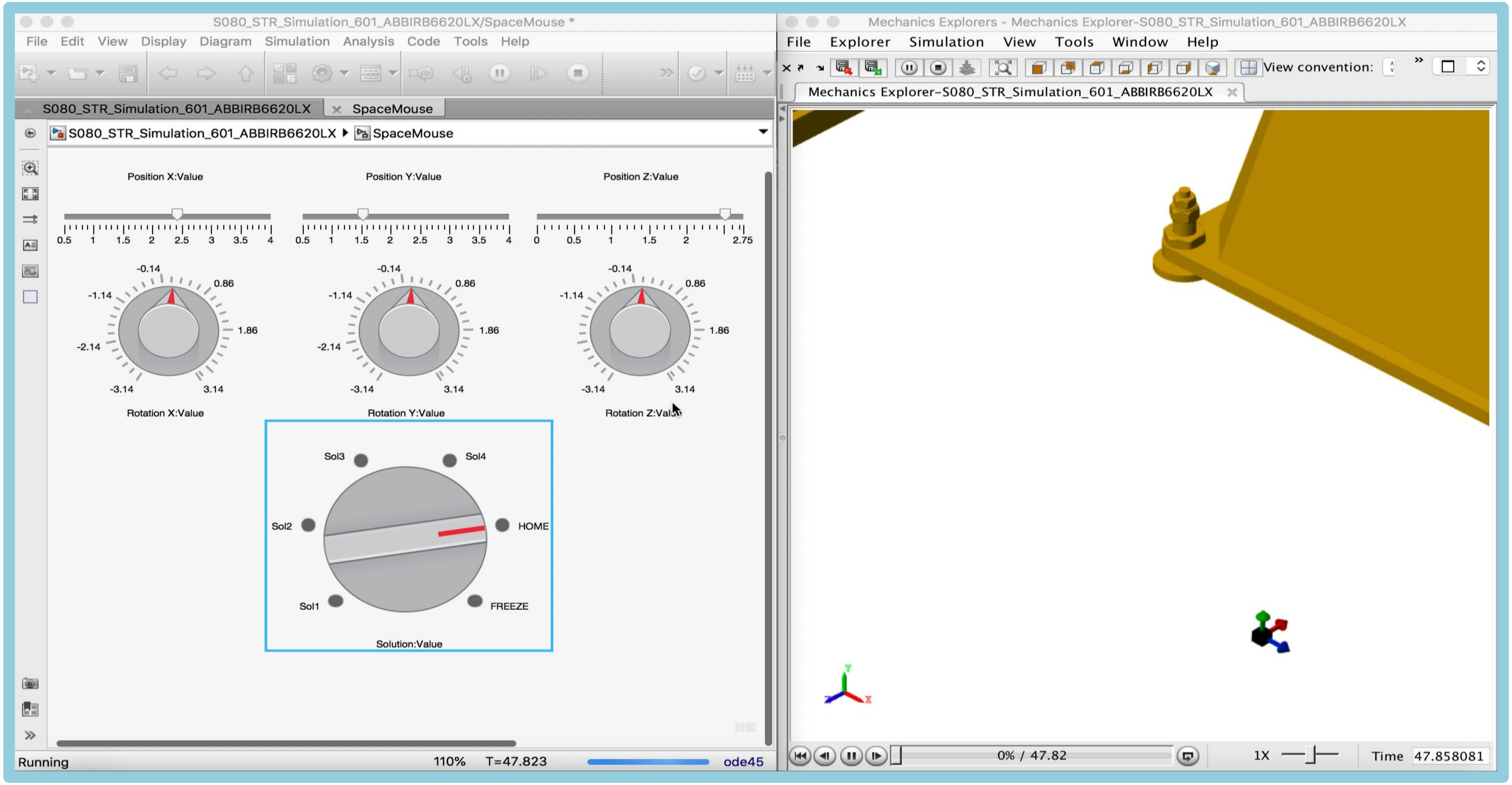
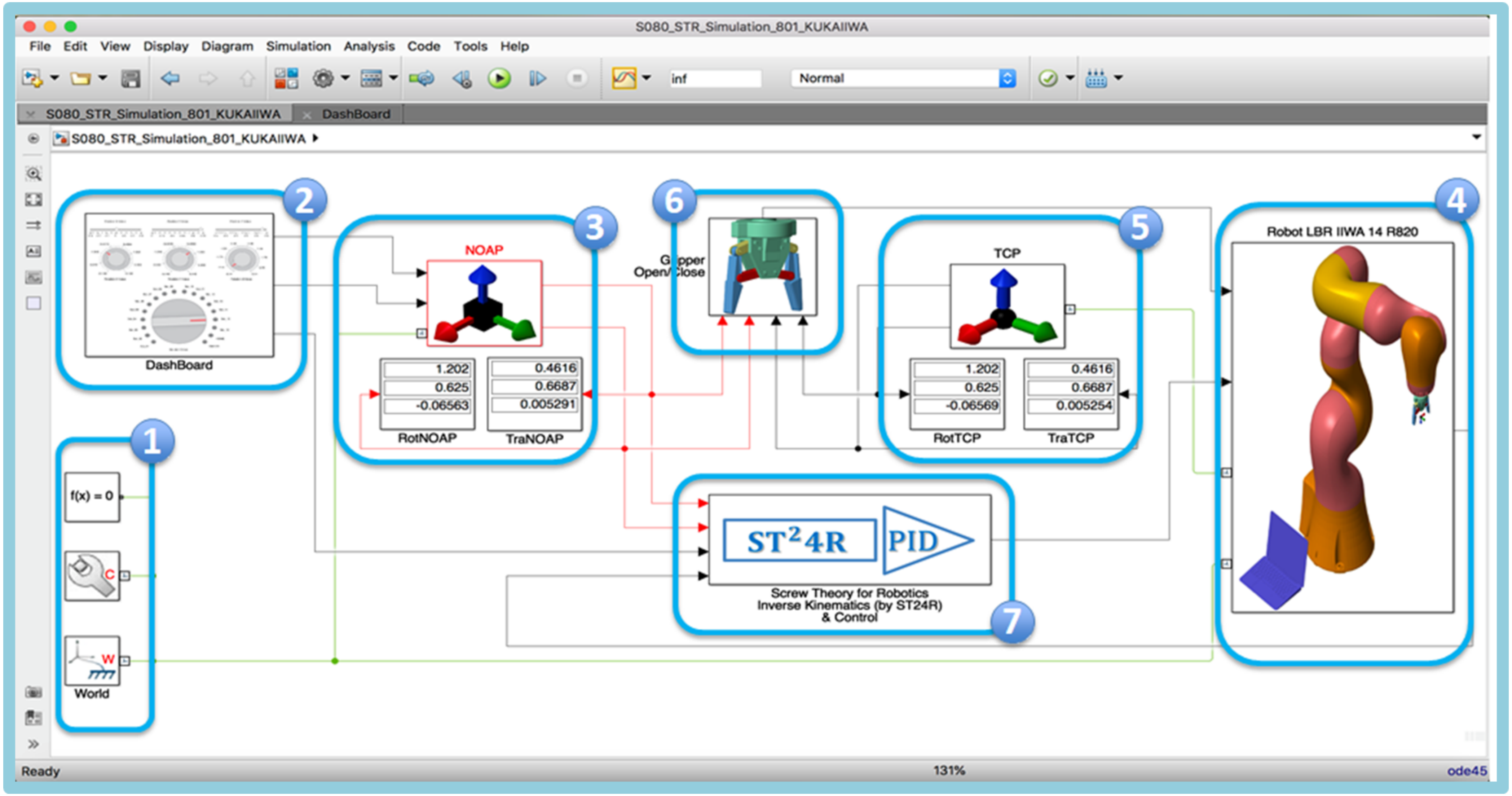


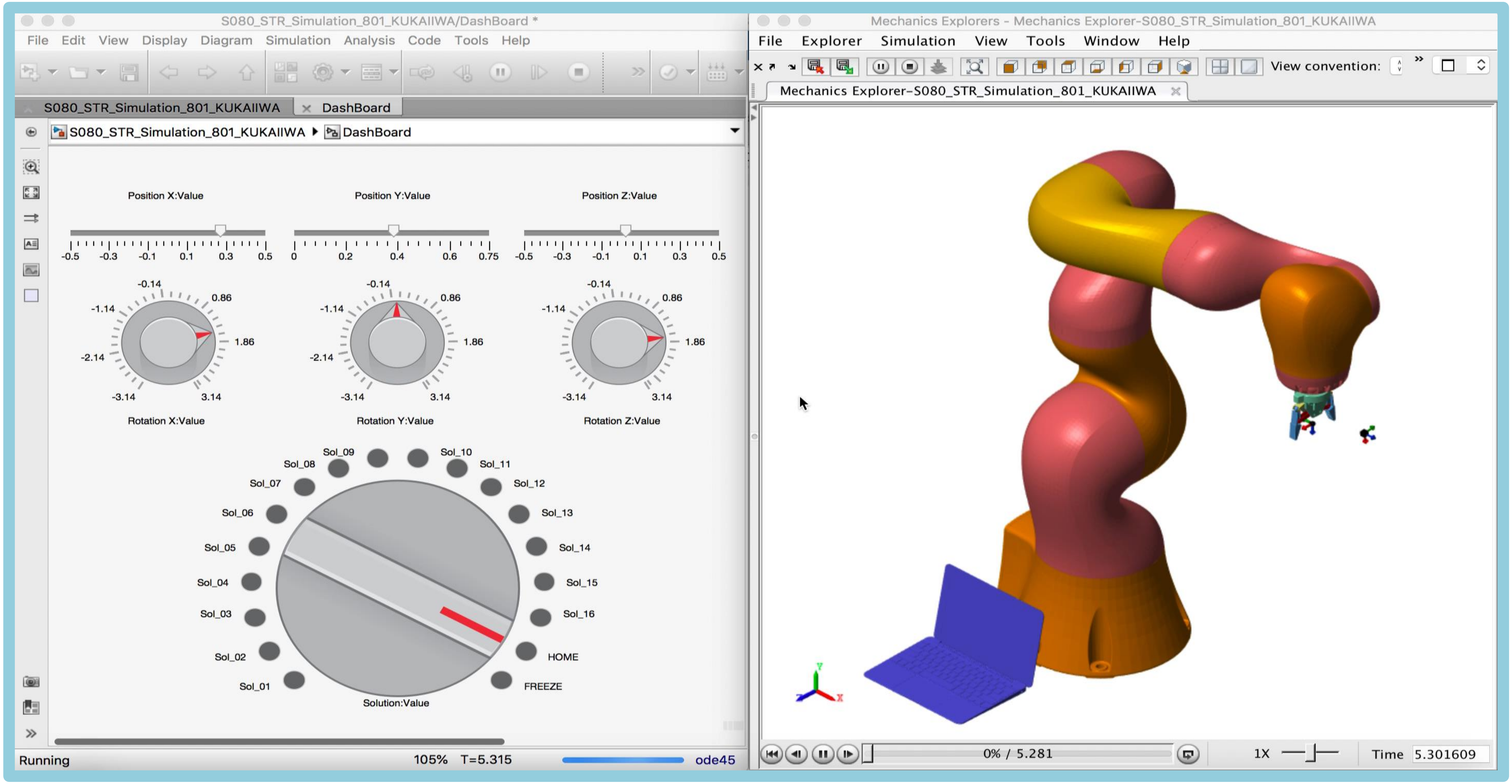
ABB 6620LX Inverse Kinematics Real Time Simulation with FOUR EXACT Geometric Solutions (θ1,θ2,θ3,θ4,θ5,θ6) -»

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



SIMULATION: KUKA IIWA R820 Inverse Kinematics -»

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



KUKA IIWA Inverse Kinematics Real Time Simulation with SIXTEEN EXACT Solutions ($\theta 1, \theta 2, \theta 3, \theta 4, \theta 5, \theta 6, \theta 7$) -»

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

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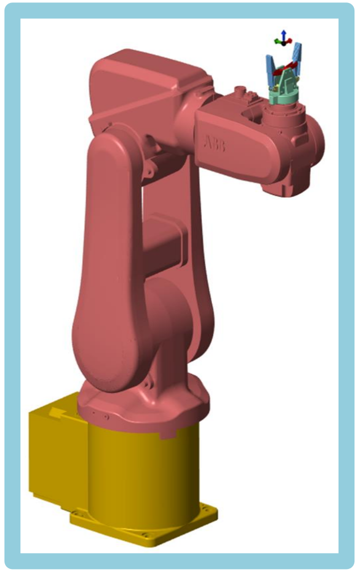
AGENDA

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



Differential INVERSE Kinematics

$$\dot{\theta} = \left(J_A(\theta) \right)^{-1} V_T^S$$

&

ANALYTIC JACOBIAN

$$J_A(\theta) = \begin{bmatrix} \frac{\partial T_x}{\partial \theta_1} & \cdots & \frac{\partial T_x}{\partial \theta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial T_\gamma}{\partial \theta_1} & \cdots & \frac{\partial T_\gamma}{\partial \theta_n} \end{bmatrix}$$

$$\overbrace{\begin{gathered}
n_x = (C_1C_2S_3 + S_2C_3)(C_4C_5C_6 - S_4S_6) + C_2S_1(S_4C_5C_6 + C_4S_6) + (-C_1C_2C_3 + S_2S_3)S_5C_6 = C_\beta C_\gamma \\
n_y = (-S_2C_1S_3 + S_2C_3)(C_4C_5C_6 - S_4S_6) + S_1S_2(S_4C_5C_6 + C_4S_6) + (-S_2C_1C_3 - C_2S_3)S_5C_6 = C_\beta S_\gamma \\
n_z = (-S_1S_3)(C_4C_5C_6 - S_4S_6) + C_1(S_4C_5C_6 + C_4S_6) + S_1C_3S_5C_6 = -S_\beta \\
o_x = (C_1C_2S_3 + S_2C_3)(-C_4C_5C_6 - S_4S_6) + C_2S_1(-S_4C_5C_6 + C_4S_6) + (-C_1C_2C_3 + S_2S_3)(-S_5C_6) = -S_\gamma C_\alpha + C_\gamma S_\beta S_\alpha \\
o_y = (-S_2C_1S_3 + S_2C_3)(-C_4C_5C_6 - S_4S_6) + S_1S_2(-S_4C_5C_6 + C_4S_6) + (-S_2C_1C_3 - C_2S_3)(-S_5C_6) = C_\gamma C_\alpha + S_\gamma S_\beta S_\alpha \\
o_z = (-S_1S_3)(-C_4C_5C_6 - S_4S_6) + C_1(-S_4C_5C_6 + C_4S_6) + S_1C_3(-S_5C_6) = C_\beta S_\alpha \\
a_{xyz} = n_{xyz} \times o_{xyz} \\
p_x = (C_1C_2S_3 + S_2C_3)(l_4C_4S_5) + C_2S_1(l_4S_4S_5) + (C_1C_2C_3 + S_2S_3)(-l_4C_5 + l_3) + (-l_2C_1C_2S_3 - l_2S_2C_3 - l_1S_2) = S_\gamma S_\alpha + C_\gamma S_\beta S_\alpha \\
p_y = (-S_2C_1S_3 - C_2C_3)(l_4C_4S_5) + S_1S_2(l_4S_4S_5) + (-C_1C_2C_3 - C_2S_3)(-l_4C_5 + l_3) + (-l_2S_2C_1S_3 - l_2C_2C_3 + l_1C_2) = -C_\gamma S_\alpha + S_\gamma S_\beta C_\alpha \\
p_z = (-S_1S_3)(l_4C_4C_5) + C_1(l_4S_4S_5) + S_1C_3(-l_4C_5 + l_3) + l_2S_1S_3 = C_\beta C_\alpha
\end{gathered}}^{T^S(x, y, z, \alpha, \beta, \gamma)}$$

The DIFFERENTIAL KINEMATICS with the ANALYTIC JACOBIAN needs the differentiation of not an easy kinematics formulation!

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

Differential INVERSE Kinematics

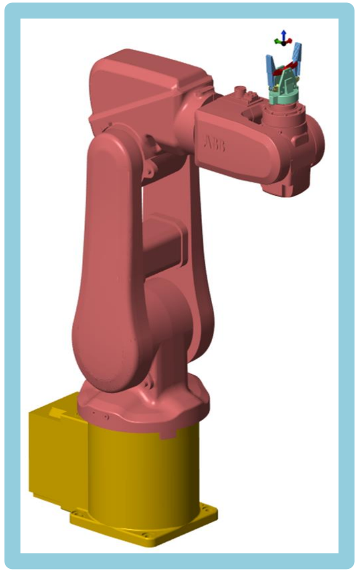
$$\dot{\theta} = \left(J^S_{ST}(\theta) \right)^{-1} V^S_{ST}$$

&

GEOMETRIC JACOBIAN

$$J^S_{ST}(\theta) = \left[\xi'_1 \cdots \xi'_n \right]$$

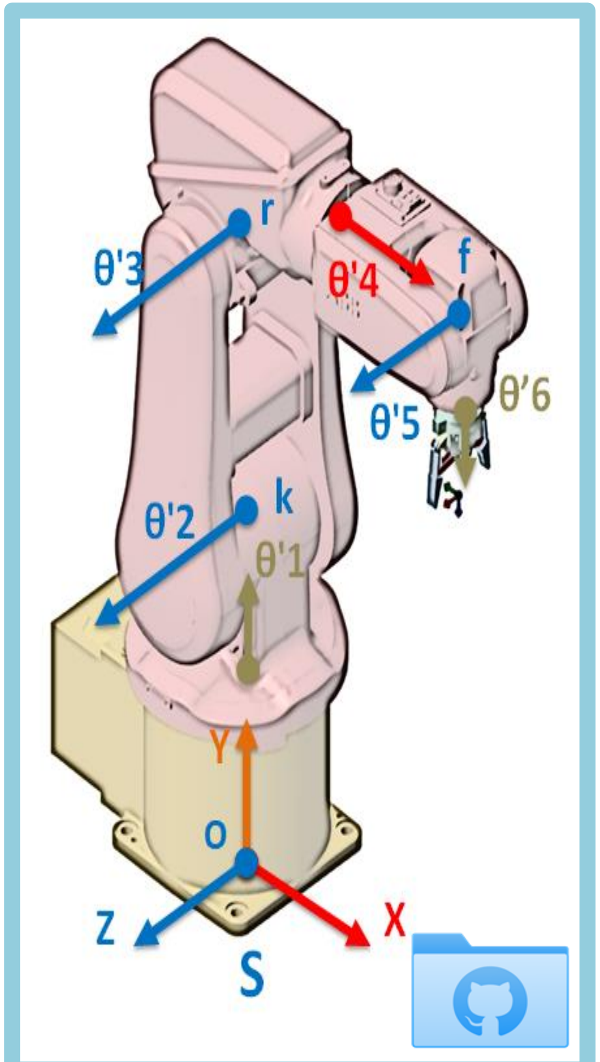
$$\xi'_i = Ad_{\left(e^{\hat{\xi}_1 \theta_1} \cdots e^{\hat{\xi}_{i-1} \theta_{i-1}} \right)} \xi_i$$



$$\begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} = \left[\xi'_1 \cdots \xi'_n \right]^{-1} \begin{bmatrix} \left(\upsilon^S_{TcP} \right) - \left(\omega^S_{ST} \right)^{\wedge} TcP(\theta) \\ \omega^S_{ST} \end{bmatrix} = \left[\xi'_1 \cdots \xi'_n \right]^{-1} \begin{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} - \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}^{\wedge} \begin{bmatrix} TcP_X \\ TcP_Y \\ TcP_Z \end{bmatrix} \\ \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} \end{bmatrix}$$

DIFFERENTIAL KINEMATICS with the GEOMETRIC JACOBIAN by Inspection or Definition is easy (NO DIFFERENTIATION at all)!

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics



Differential FORWARD Kinematics

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{\alpha}\\ \dot{\beta}\\ \dot{\gamma}\end{bmatrix} = \begin{bmatrix}\left(\upsilon_{ST}^{S}\right)+\left(\omega_{ST}^{S}\right)^{\wedge} TCP(\theta)\\ \omega_{ST}^{S}\end{bmatrix} \Leftarrow \begin{bmatrix}\upsilon_{ST}^{S}\\ \omega_{ST}^{S}\end{bmatrix} = \begin{bmatrix}\xi_1' & \xi_2' & \xi_3' & \xi_4' & \xi_5' & \xi_6'\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \dot{\theta}_4\\ \dot{\theta}_5\\ \dot{\theta}_6\end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \dot{\theta}_4\\ \dot{\theta}_5\\ \dot{\theta}_6\end{bmatrix} = \begin{bmatrix}\xi_1' & \xi_2' & \xi_3' & \xi_4' & \xi_5' & \xi_6'\end{bmatrix}^{-1}\begin{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} - \begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\\ \dot{\gamma}\end{bmatrix}^{\wedge}\begin{bmatrix}TCP_X\\ TCP_Y\\ TCP_Z\end{bmatrix}\\ \begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\\ \dot{\gamma}\end{bmatrix}\end{bmatrix}$$

Differential INVERSE Kinematics

ABB IRB120 DIFFERENTIAL Forward & Inverse KINEMATICS with GEOMETRIC JACOBIAN without any DIFFERENTIATION!

CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

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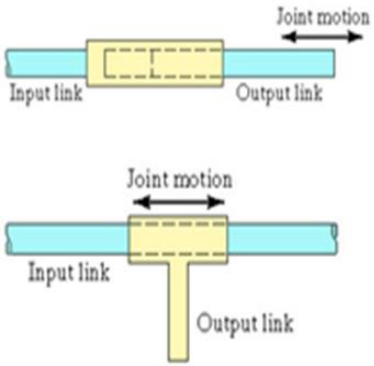
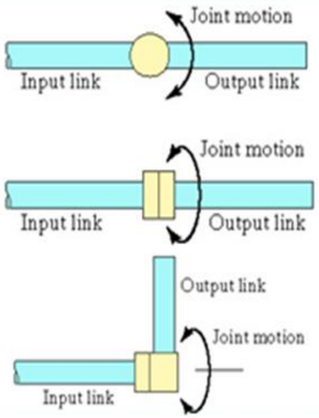
AGENDA

CASE Study
Some Screw Theory BENEFITS

1 GEOMETRY as the foundation

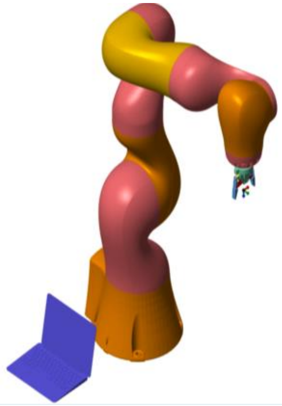


$$\xi = \begin{bmatrix} \upsilon \\ 0 \end{bmatrix}$$

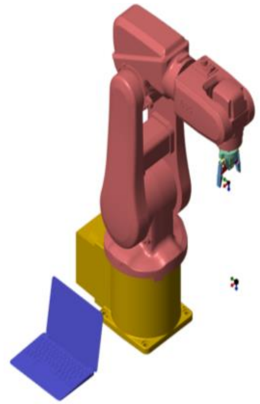


$$\xi = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$

2 Matrix EXPONENTIAL as primitive



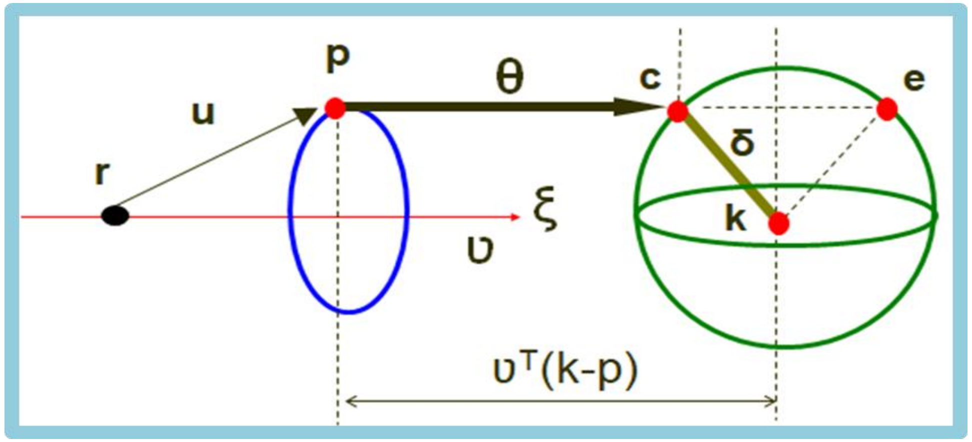
$$e^{\hat{\xi}_i \theta_i}$$

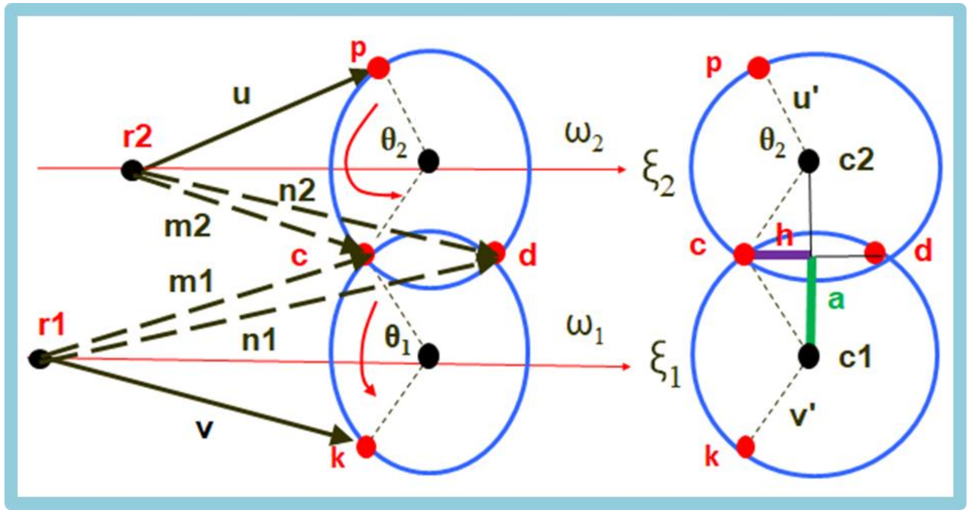


$$H_{ST}(\theta) = \prod_{i=1}^{n} e^{\hat{\xi}_i \theta_i} H_{ST}(0)$$

CASE Study
Some Screw Theory BENEFITS

3 CANONICAL subproblems as basis





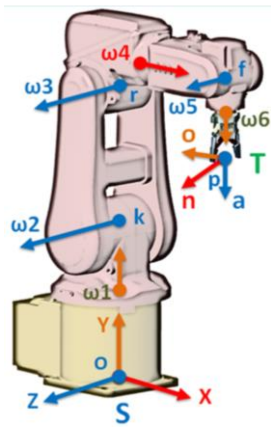
4 GEOMETRIC Methods as tools

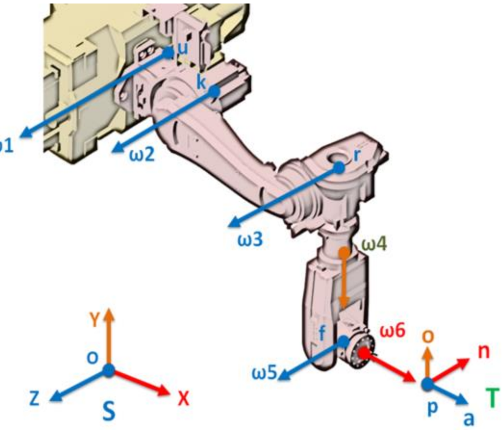
ST^{2}4R

CASE Study
Some Screw Theory BENEFITS

5 EXAMPLES & SIMULATIONS with GEOMETRIC EXACT SOLUTIONS as exercises to learn kinematics

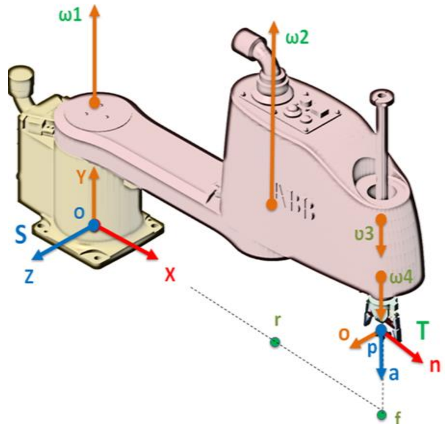
$$\theta_1^{01},\theta_2^{01},\theta_3^{01},\theta_4^{01},\theta_5^{01} \Rightarrow \theta_6^{01}$$
$$\theta_1^{01},\theta_2^{01},\theta_3^{01},\theta_4^{02},\theta_5^{02} \Rightarrow \theta_6^{02}$$
$$\theta_1^{02},\theta_2^{02},\theta_3^{01},\theta_4^{03},\theta_5^{03} \Rightarrow \theta_6^{03}$$
$$\theta_1^{02},\theta_2^{02},\theta_3^{01},\theta_4^{04},\theta_5^{04} \Rightarrow \theta_6^{04}$$
$$\theta_1^{03},\theta_2^{03},\theta_3^{02},\theta_4^{05},\theta_5^{05} \Rightarrow \theta_6^{05}$$
$$\theta_1^{03},\theta_2^{03},\theta_3^{02},\theta_4^{06},\theta_5^{06} \Rightarrow \theta_6^{06}$$
$$\theta_1^{04},\theta_2^{04},\theta_3^{02},\theta_4^{07},\theta_5^{07} \Rightarrow \theta_6^{07}$$
$$\theta_1^{04},\theta_2^{04},\theta_3^{02},\theta_4^{08},\theta_5^{08} \Rightarrow \theta_6^{08}$$

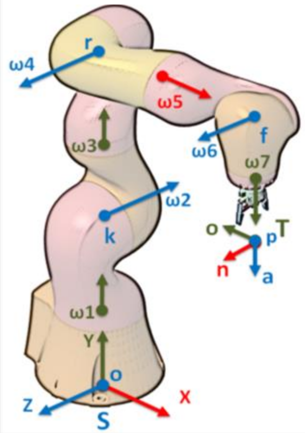




$$\theta_1^{01},\theta_2^{01},\theta_3^{01},\theta_4^{01},\theta_5^{01} \Rightarrow \theta_6^{01}$$
$$\theta_1^{01},\theta_2^{01},\theta_3^{01},\theta_4^{02},\theta_5^{02} \Rightarrow \theta_6^{02}$$
$$\theta_1^{01},\theta_2^{02},\theta_3^{02},\theta_4^{03},\theta_5^{03} \Rightarrow \theta_6^{03}$$
$$\theta_1^{01},\theta_2^{02},\theta_3^{02},\theta_4^{04},\theta_5^{04} \Rightarrow \theta_6^{04}$$

$$\theta_1^{01},\theta_2^{01},\theta_3^{01} \Rightarrow \theta_4^{01}$$
$$\theta_1^{02},\theta_2^{02},\theta_3^{01} \Rightarrow \theta_4^{02}$$





$\theta_1^{01},\theta_2^{01},\theta_3^{in},\theta_4^{01},\theta_5^{01},\theta_6^{01} \Rightarrow \theta_7^{01}$	$\theta_1^{in},\theta_2^{05},\theta_3^{01},\theta_4^{01},\theta_5^{09},\theta_6^{09} \Rightarrow \theta_7^{09}$
$\theta_1^{01},\theta_2^{01},\theta_3^{in},\theta_4^{01},\theta_5^{02},\theta_6^{02} \Rightarrow \theta_7^{02}$	$\theta_1^{in},\theta_2^{05},\theta_3^{01},\theta_4^{01},\theta_5^{10},\theta_6^{10} \Rightarrow \theta_7^{10}$
$\theta_1^{02},\theta_2^{02},\theta_3^{in},\theta_4^{01},\theta_5^{03},\theta_6^{03} \Rightarrow \theta_7^{03}$	$\theta_1^{in},\theta_2^{06},\theta_3^{02},\theta_4^{01},\theta_5^{11},\theta_6^{11} \Rightarrow \theta_7^{11}$
$\theta_1^{02},\theta_2^{02},\theta_3^{in},\theta_4^{01},\theta_5^{04},\theta_6^{04} \Rightarrow \theta_7^{04}$	$\theta_1^{in},\theta_2^{06},\theta_3^{02},\theta_4^{01},\theta_5^{12},\theta_6^{12} \Rightarrow \theta_7^{12}$
$\theta_1^{03},\theta_2^{03},\theta_3^{in},\theta_4^{02},\theta_5^{05},\theta_6^{05} \Rightarrow \theta_7^{05}$	$\theta_1^{in},\theta_2^{07},\theta_3^{03},\theta_4^{02},\theta_5^{13},\theta_6^{13} \Rightarrow \theta_7^{13}$
$\theta_1^{03},\theta_2^{03},\theta_3^{in},\theta_4^{02},\theta_5^{06},\theta_6^{06} \Rightarrow \theta_7^{06}$	$\theta_1^{in},\theta_2^{07},\theta_3^{03},\theta_4^{02},\theta_5^{14},\theta_6^{14} \Rightarrow \theta_7^{14}$
$\theta_1^{04},\theta_2^{04},\theta_3^{in},\theta_4^{02},\theta_5^{07},\theta_6^{07} \Rightarrow \theta_7^{07}$	$\theta_1^{in},\theta_2^{08},\theta_3^{04},\theta_4^{02},\theta_5^{15},\theta_6^{15} \Rightarrow \theta_7^{15}$
$\theta_1^{04},\theta_2^{04},\theta_3^{in},\theta_4^{02},\theta_5^{08},\theta_6^{08} \Rightarrow \theta_7^{08}$	$\theta_1^{in},\theta_2^{08},\theta_3^{04},\theta_4^{02},\theta_5^{16},\theta_6^{16} \Rightarrow \theta_7^{16}$

CASE Study
Some Screw Theory BENEFITS

6 SPATIAL VELOCITY

as a generality

$$V_{ST}^{S} = J_{ST}^{S}(\theta)\dot{\theta}$$

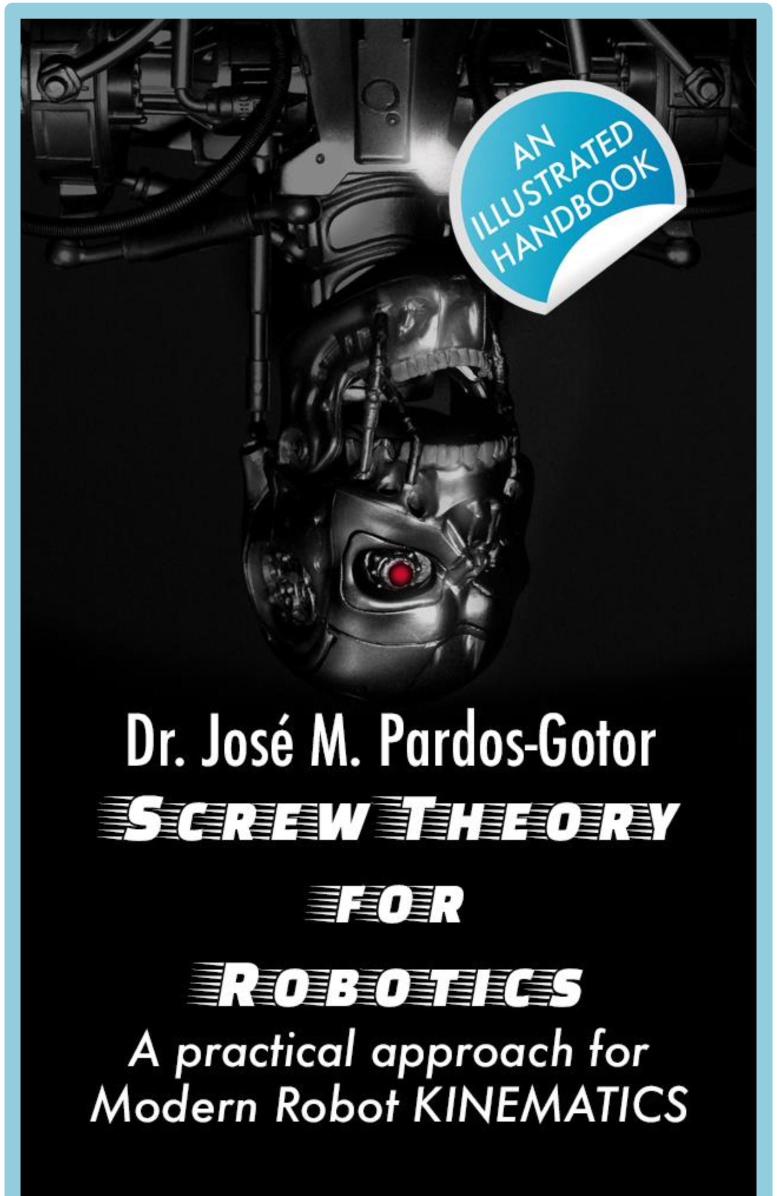
$$J_{ST}^{S}(\theta) = \left[\xi_1' \cdots \xi_n'\right]$$

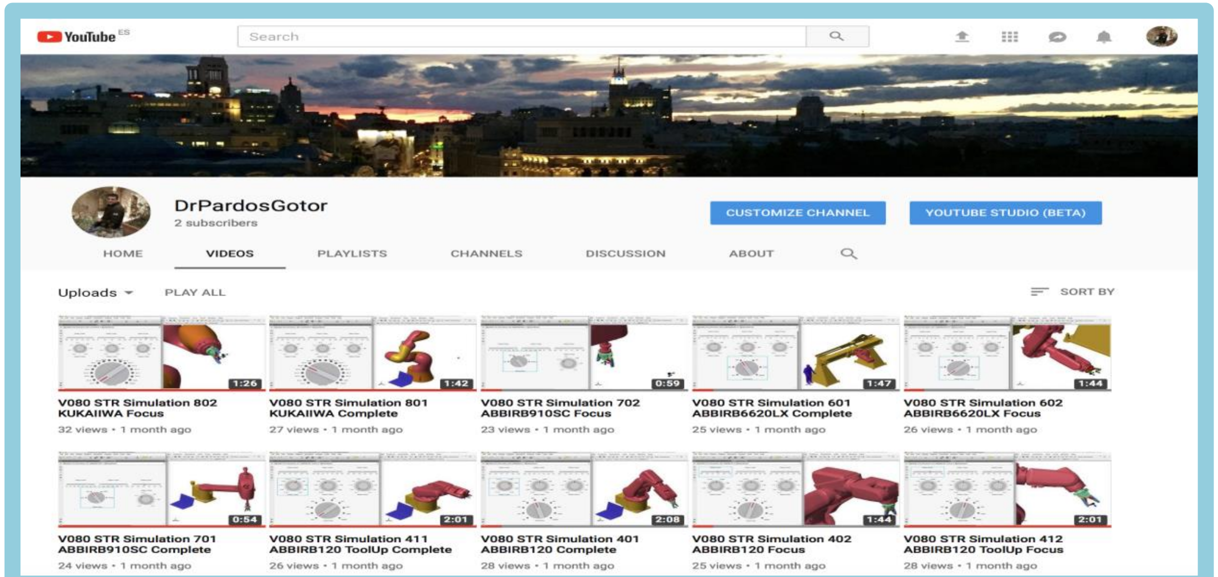
$$\xi_i' = Ad_{\left(e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_{i-1}\theta_{i-1}}\right)}\xi_i$$

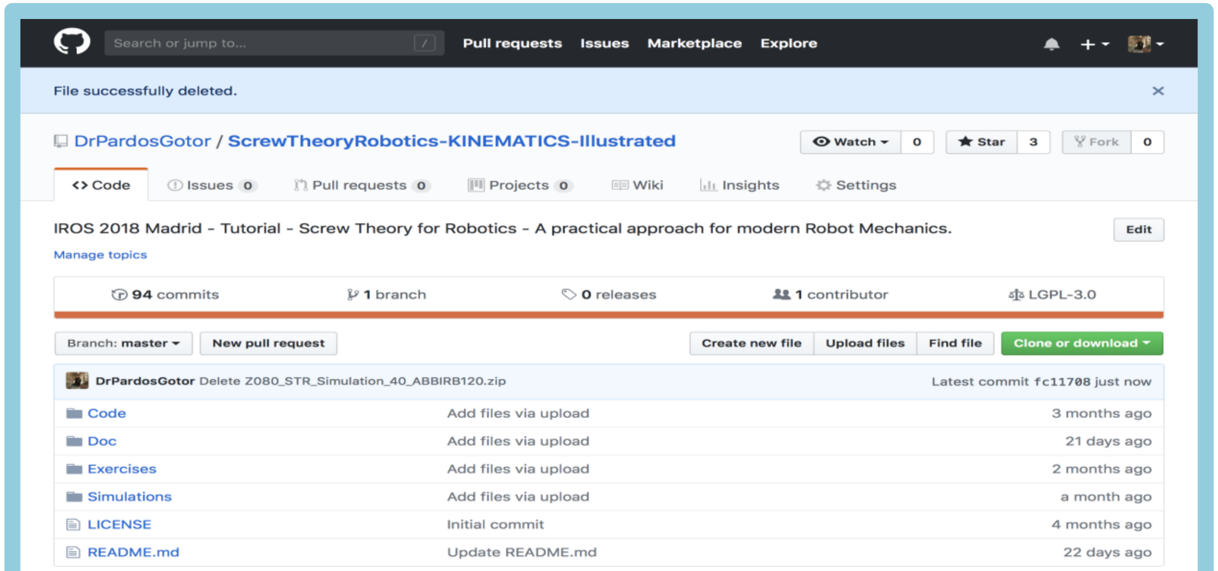
7 SCREW THEORY

as best practice

CASE Study Reference Book & Additional Material







CASE Study: A Compelling Computational Approach of Screw Theory for Robotics

Tutorial

Screw Theory for Robotics

A practical approach for modern Robot Mechanics

(5th October - Room 1.L3 BERLIN)

GAME
OVER