Dr. José M. Pardos-Gotor

SCREW THEORY FOR ROBOTICS

A practical approach for Modern Robot KINEMATICS

Screw Theory for Robotics

An Illustrated Handbook

A practical approach for Modern Robot KINEMATICS

This preprint is being made available for personal use. It has approximately only 10% of the actual book's contents. The book will be published by Amazon Fulfilment 2018, ISBN 9781717931818. Additional material (e.g. Code, Exercises, Simulations) is provided at: https://github.com/DrPardosGotor/ScrewTheoryRobotics-KINEMATICS-Illustrated

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The importance of screw theory in robotics is recognised but hardly capitalised on. Engineering students rarely get to learn about it in class, so only few postgraduates know how to exploit it. However, in a variety of areas of robotics, the methods and formalisms based on the geometry and algebra of the screws, have proven to be superior to other techniques.

The idea of publishing this book came about because there were not enough specialised texts for **teaching the screw theory methodologies and advantages through a set of visual and comprehensive examples**.

This illustrated handbook presents an abstract mathematical formulation for **robot KINEMATICS** based in the use of the screw theory tools, making an emphasis on modern geometric techniques. Its main objective is to demonstrate that many robotics problems addressed today only with numerical iterative solutions, are **solved much better with closed-form geometric solutions based on screw theory.**

This book will surely spark your excitement about the technological and social prospects for robotics and **enable you to develop effective and efficient robot algorithms, solutions and applications**. In the end you will realise that, most of the time, a good theory is the fastest way to obtain a better performance, and the only thing you will have to do in exchange is put in some time and commitment in studying the screw theory.

The contents of this handbook are used at the "***Master in Robotics and Automation***" of the **UC3M (Universidad Carlos III de Madrid)** by the **Department of Systems Engineering and Automation.**

Contents

Introduction

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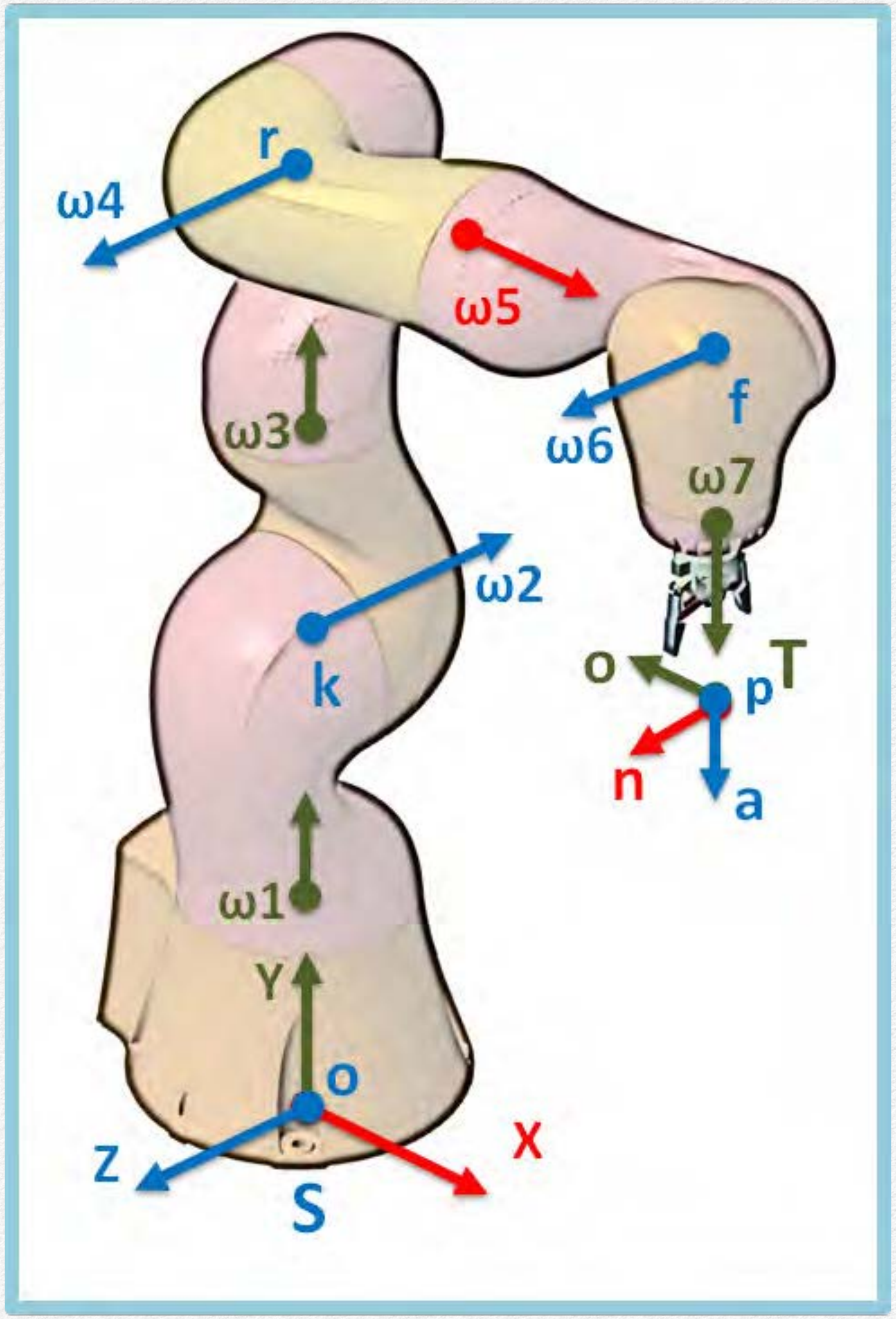
"Theoretically, if we could build a MACHINE whose mechanical structure duplicated HUMAN PHYSIOLOGY, then we could have a machine whose intellectual capacities would duplicate those of human beings."

— Norbert Wiener



Are you building REAL TIME Robot Applications?

Are you solving Robot Inverse KINEMATICS with Closed-Form Geometric Algorithms?

Are you needing Robot VELOCITIES without Differentiations?

If you have answered “yes” to any of these questions, your journey starts here!

- You have in your possession a handbook for robotics enthusiasts who are striving to improve the efficiency and performance of robot applications.

Mathematics

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"The ABSTRACTION SAVES TIME in the long run, in return for an initial investment of effort and patience in learning some mathematics."

— Richard M. Murray

— Zexiang Li

— S. Shankar Sastry

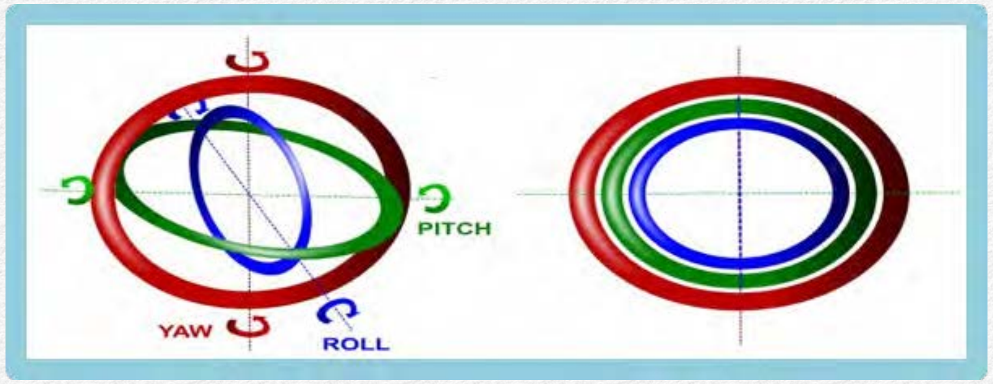
Twist & Screws - Properties & Benefits

The exponential map for a “**twist = ξ**” **gives the relative motion of a rigid body**. Twist transformation is as **mapping points from their initial coordinates to the coordinates after the rigid motion is applied (not as mapping points from one coordinate frame to another)**.

$$H_{ST}(\theta) = e^{\hat{\xi}\theta} H_{ST}(0)$$

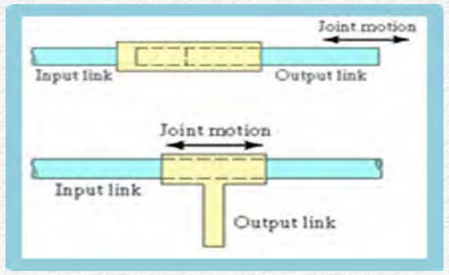
Screw theory provides an rigorous and geometric description of the motion which **simplifies the analysis of mechanisms**.

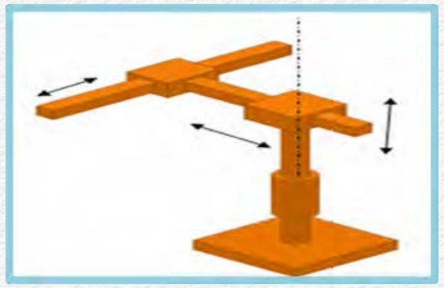
Screws allow a description of the motion which does not suffer from singularities due to the use of local coordinates. The fact that any three-angle representation for orientation has singularities (e.g. Euler angles) is a fundamental problem. This is also known as **Gimbal Lock**.



SCREW for PURE TRANSLATION

Vector **“υ”** is the AXIS line of translation, and **“ω”** is zero.





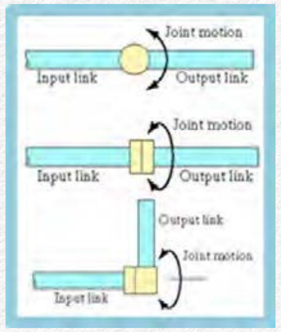
$$\xi = \begin{bmatrix} \upsilon \\ 0 \end{bmatrix}$$

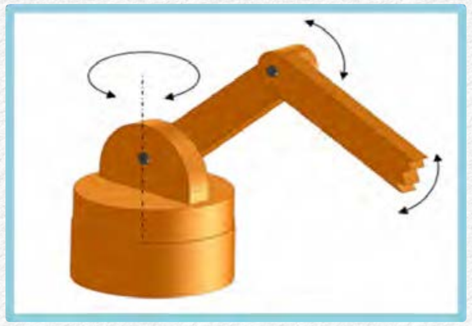
$$e^{\hat{\xi}\theta} = \begin{bmatrix} I_3 & \upsilon\theta \\ 0 & 1 \end{bmatrix}$$

SCREW for PURE ROTATION

Vector **“ω”** is the rotation AXIS and **“q”** is any point on “ω”.

$$\xi = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$





$$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & \left(I_3 - e^{\hat{\omega}\theta}\right)\left(\omega \times \upsilon\right) + \omega\omega^T \upsilon\theta \\ 0 & 1 \end{bmatrix}$$

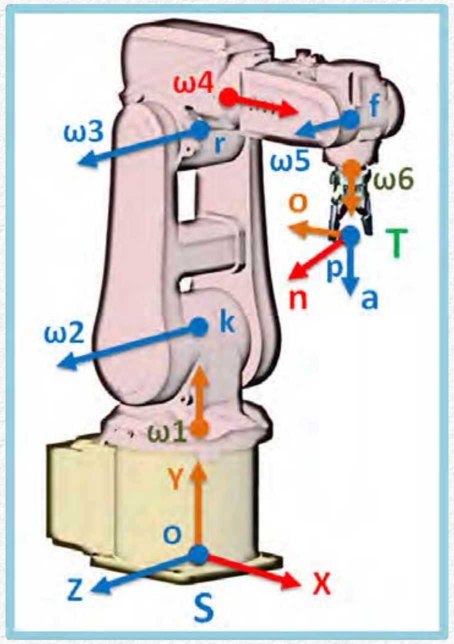
FORWARD Kinematics

3

“Never confuse MOTION with ACTION.”

— **Benjamin Franklin**

Exercise: Forward Kinematics for ABB IRB120



From the motion of the joints, **Magnitudes ($\theta_1 \dots \theta_6$)**, we get the **pose for the tool (noap)**.

1. Define the **Axis "ω"** of each **Twist (ξ)**. Beware of the sign for ω6, because of the home position.

$$\omega_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \omega_6 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

2. Obtain the **Twists ($\xi_1 \dots \xi_6$)**, knowing each joint **Axis "ω"** and a point on that axis.

$$\xi_1 = \begin{bmatrix} -\omega_1 \times k \\ \omega_1 \end{bmatrix}; \xi_2 = \begin{bmatrix} -\omega_2 \times k \\ \omega_2 \end{bmatrix}; \xi_3 = \begin{bmatrix} -\omega_3 \times r \\ \omega_3 \end{bmatrix}; \xi_4 = \begin{bmatrix} -\omega_4 \times f \\ \omega_4 \end{bmatrix}; \xi_5 = \begin{bmatrix} -\omega_5 \times f \\ \omega_5 \end{bmatrix}; \xi_6 = \begin{bmatrix} -\omega_6 \times f \\ \omega_6 \end{bmatrix}$$

3. Get **$H_{st}(0)$**, the pose of the tool at the reference (home) robot position.

$$H_{ST}(0) = T_{XYZ}\begin{pmatrix} p_X \\ p_Y \\ p_Z \end{pmatrix} R_n(\frac{\pi}{2}) R_a(\frac{\pi}{2}) = \begin{bmatrix} 0 & -1 & 0 & p_X \\ 0 & 0 & -1 & p_Y \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Solve the problem to get **$H_{st}(\theta)$**, applying the **POE**.

$$H_{ST}(\theta) = \prod_{i=1}^{6} e^{\hat{\xi}_i \theta_i} H_{ST}(0) = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

https://github.com/DrPardosGotor/ScrewTheoryRobotics-KINEMATICS-Illustrated/tree/master/Exercises

"E030_STR_FORWARDKin_40_ABBIRB120.m"

INVERSE Kinematics

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"In the beginning was MECHANICS."

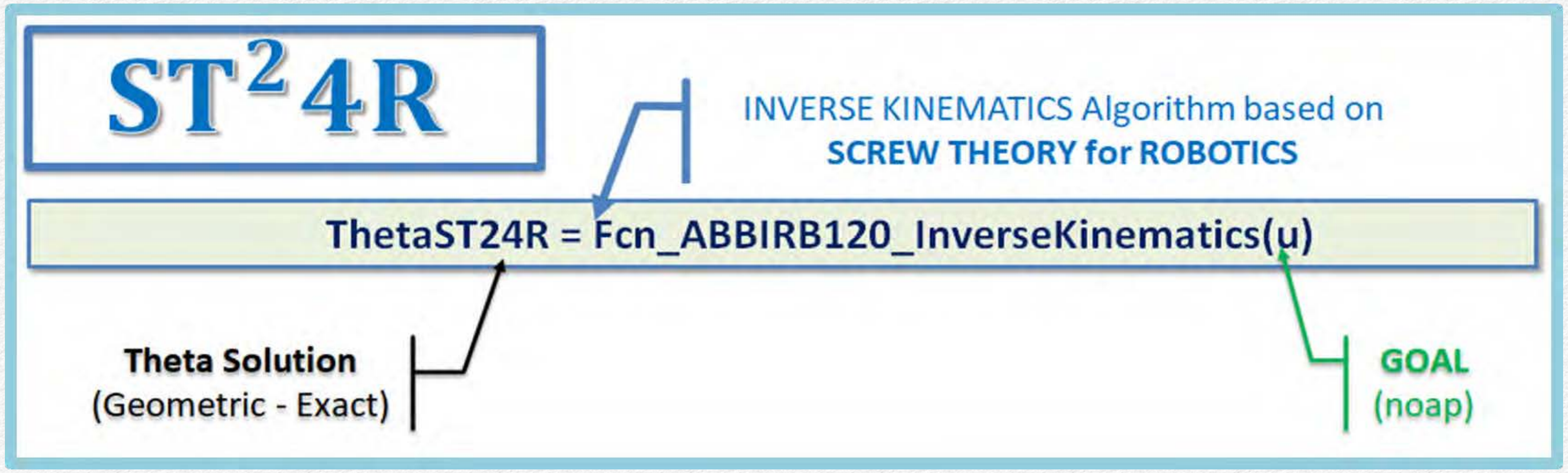
— Max von Laue

GEOMETRIC Solutions FEATURES

- **EFFICIENT calculation**, because it allows for fast solutions since there are no iterations for the computational formulation.
- **EFFECTIVE computation**, because it gives exact solutions since the direct formulation guarantees the convergence.
- **Multiple solutions can exist.** This facilitates the **possibility to choose the better solution for each application**.

GEOMETRIC Algorithm Example

The “**ST24R - Screw Theory Toolbox for Robotics**” allows the development of these kind of algorithms for many robot architectures, as we will see in next chapters:
https://github.com/DrPardosGotor/ScrewTheoryRobotics-KINEMATICS-Illustrated/tree/master/Code/ST24R_MATLAB_v3.10



“CLOSED-FORM Solutions are suitable for REAL TIME APPLICATIONS”
“Screw Theory GEOMETRIC algorithms are a useful & ELEGANT approach”

CANONICAL Inverse Kinematics

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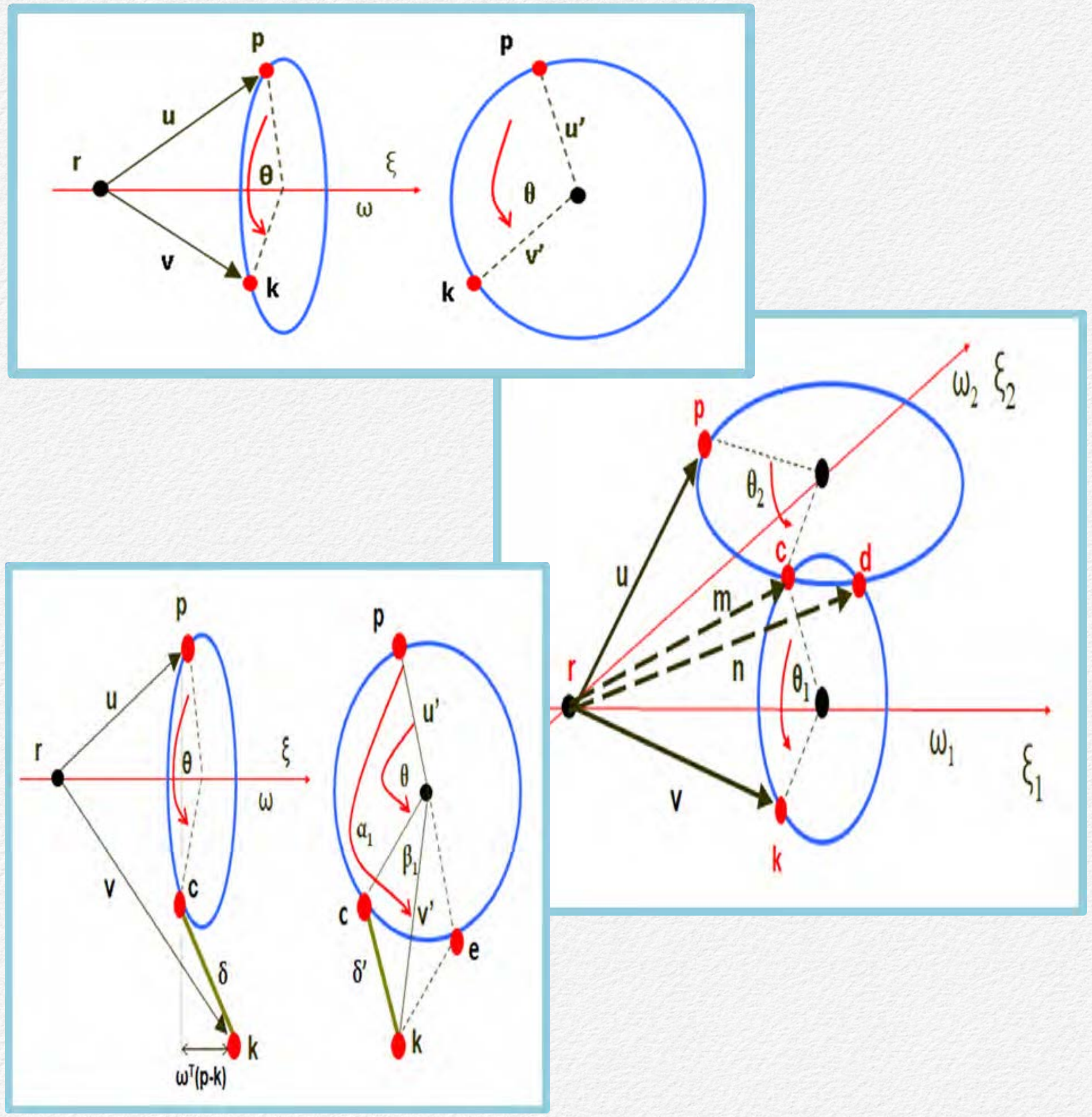
“Education is not the learning of facts, It’s rather the TRAINING of the mind TO THINK.”

— Albert Einstein

Paden-Kahan subproblems



The Canonical Problems Inception

The method was originally presented by Paden and built on the unpublished work of Kahan.

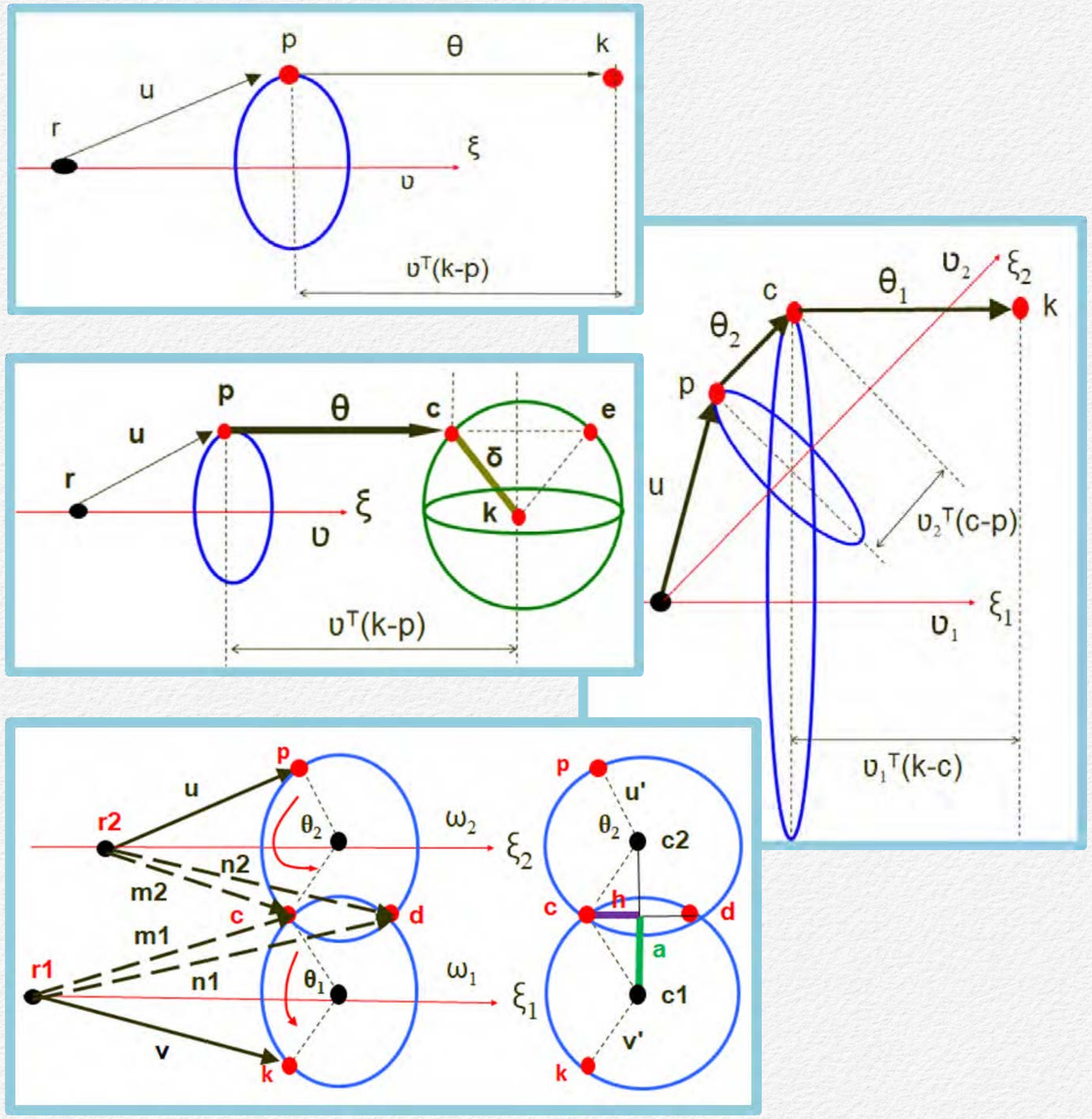
Paden-Kahan subproblems are a set of solved geometric problems which frequently occur in inverse kinematics of common robotic manipulators. Although the set of problems is not exhaustive, it may be used to simplify inverse kinematic analysis for many industrial robots.

For any mechanism defined by the product of exponentials equation, Paden-Kahan subproblems may be used to simplify and solve the problem. Generally, subproblems are applied to solve particular points in the inverse kinematics problem (e.g., the intersection of joint axes) in order to solve the joint movement magnitudes (i.e. angles).

We will see the three famous **Paden-Kahan subproblems**, with the general solution and some examples which can help you practice the screw theory concepts. The three PK subproblems are:

- **ONE: Rotation about a Single Axis (PK1).**
- **TWO: Rotation about Two Subsequent Crossing Axes (PK2).**
- **THREE: Rotation to a given Distance (PK3).**

Pardos-Gotor subproblems



Following in Paden-Kahan's Footsteps

Screw theory allows us to develop new canonical subproblems to solve complex inverse kinematics. For instance, I needed geometric solutions for some well-known robot architectures (e.g. Gantry, Scara). In addition to Paden-Kahan subproblems, it was necessary to design new subproblems for translation joints. **Following Paden-Kahan's example, I proposed three translation subproblems with a similar concept**. In this chapter I also present a new subproblem for rotation joints with parallel axes.

Please allow me to give these subproblems my surname, not because I believe they are great algorithms, quite on the contrary, but simply because I am responsible for them. Besides, this way you will be able to distinguish my algorithms from other equivalent or more efficient solutions, which you can find in literature or create by yourself.

Here are the the four **Pardos-Gotor subproblems (PG)**: translation along a single axis (PG1), translation along two subsequent crossing axes (PG2), translation to a given distance (PG3) and totation about two subsequent parallel axes (PG4).

<u>Don't forget that you too can develop your own subproblems for your new robot applications!</u>

EXAMPLES of Inverse Kinematics

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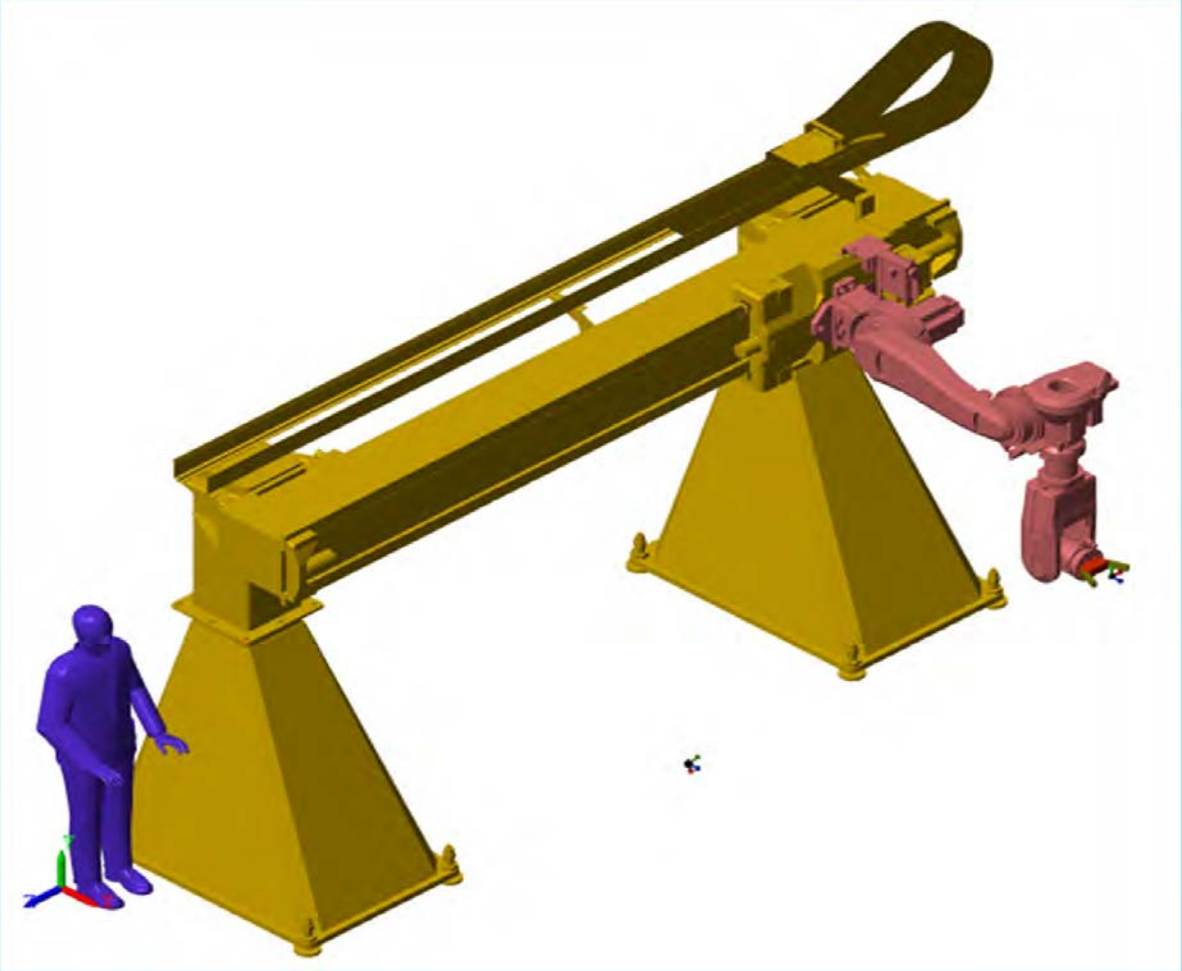
"Take to KINEMATICS. It will repay you. It is more fecund than geometry; it adds a fourth dimension to space."

— **Pafnuti Chebyshov**

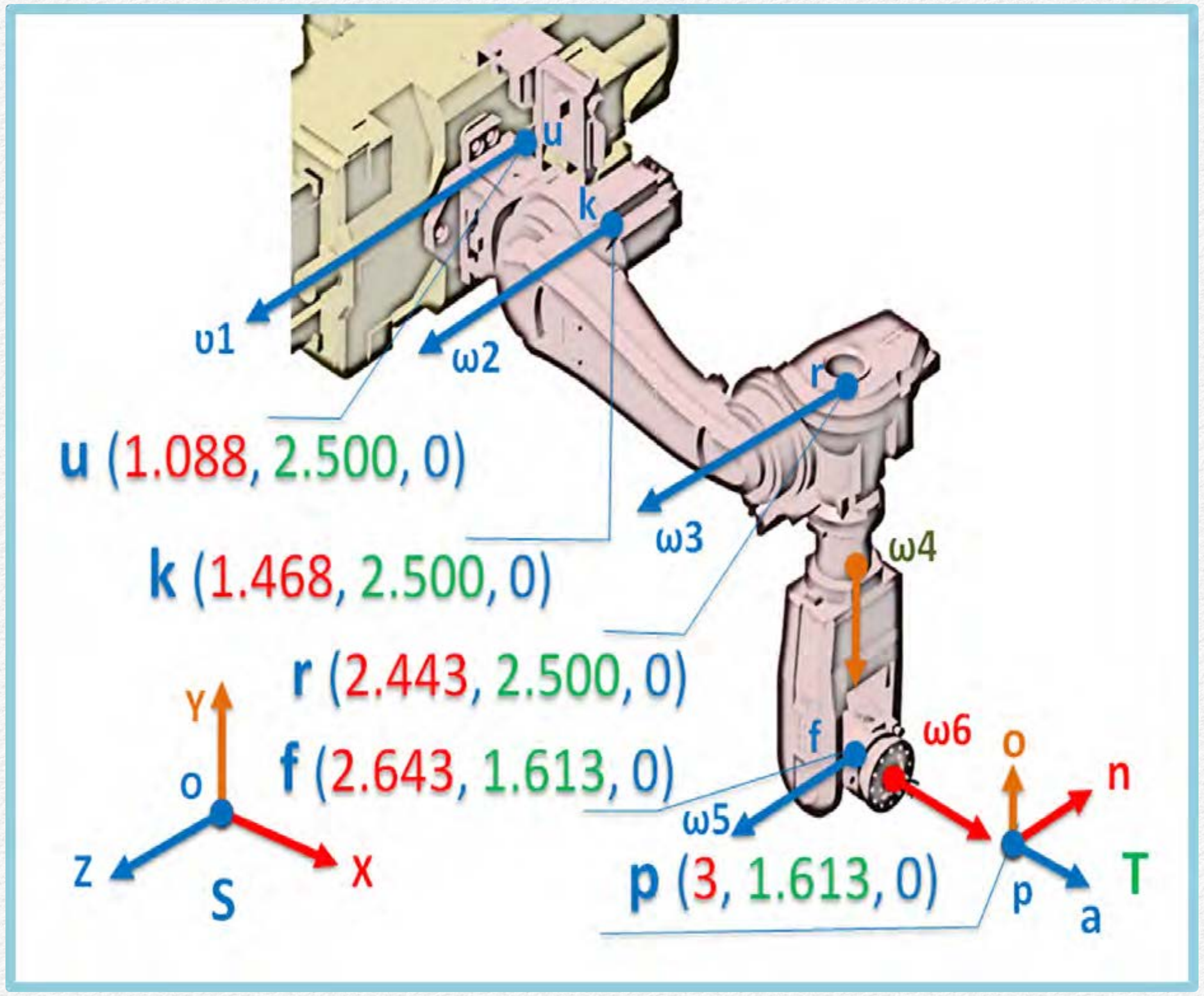
Gantry Robot (e.g. ABB IRB6620LX)



Inverse Kinematics (IK) GANTRY Robot

The exercise is based on the ABB IRB6620LX, which is a very illustrative typical GANTRY robot geometry.

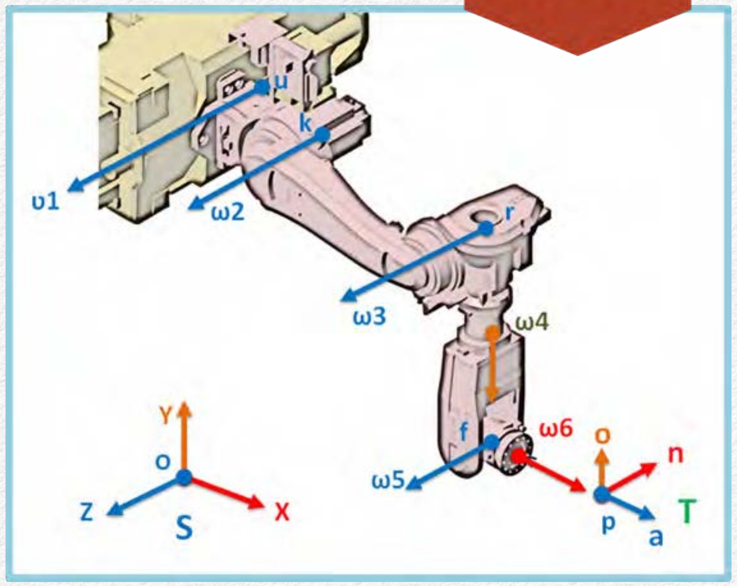
You will appreciate how flexible and elegant the POE approach is to apply different Pardos-Gotor and Paden-Kahan subproblems and get solutions. Pay attention to the two possible ways to solve this problem: **PG1+PG4+PK2+PK1** or the alternative **PG1+PK3+PK1+PK2+PK1**.



NEW

IK - GANTRY ABB IRB6620LX: θ_1 Solution

We pass **H_{st}(0)** to the right hand side of the problem definition equation. Then, we apply both sides of the equation to point "**f**" (crossing of the last three axes: "θ_4", "θ_5", "θ_6"). In the same way we can apply the **PK1 simplification case** to the left hand side of the equation. Bear in mind, that **the screw rotations θ_2, θ_3, DO NOT CHAGE the plane where "f" moves, which is perpendicular to the axis of υ_1, and therefore do not affect θ_1**. The right hand side of the equation is a known value "**k_1**". The resulting equation is exactly the definition for the canonic **Pardos-Gotor subproblem one PG1**. And we know the geometric inverse kinematics solutions for PG1, solving the motion for the first joint and obtaining the translation magnitude for "**θ_1**".



$$e^{\hat{\xi}_1\theta_1}\boxed{e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}}\boxed{e^{\hat{\xi}_4\theta_4}e^{\hat{\xi}_5\theta_5}e^{\hat{\xi}_6\theta_6}}f=\begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} f$$

The screw rotations θ_4, θ_5, θ_6, have NO EFFECT when applied to any point "f" which is on its axis

The screw rotations θ_2 and θ_3 DO NOT CHAGE the plane where "f" moves, which is perpendicular to the axis of υ_1, and therefore do not affect θ_1

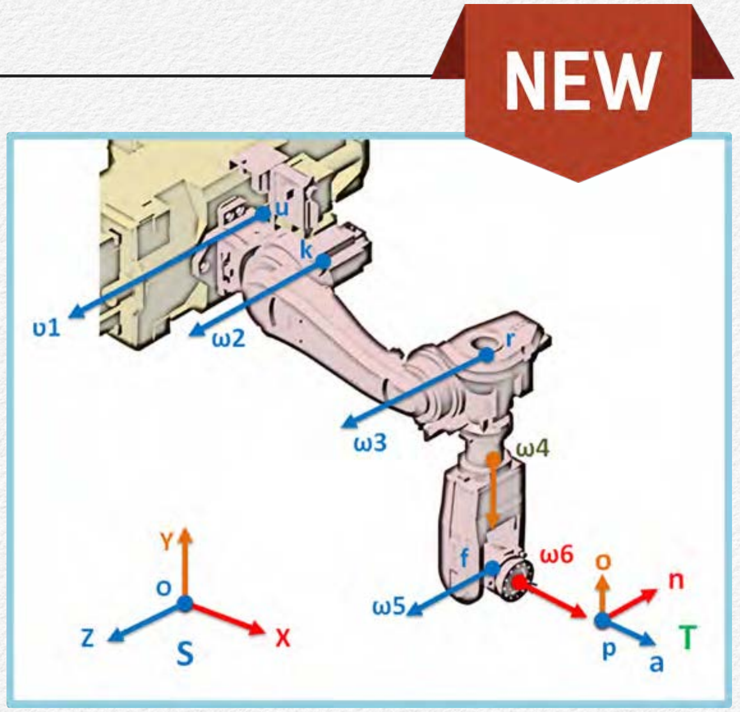
$$\boxed{e^{\hat{\xi}_1\theta_1} f = k_1}$$

CANONIC Problem PARDOS-GOTOR-ONE (PG1)

SOLUTION "θ_1"

$$\boxed{\theta_1^{01}}$$

IK - GANTRY ABB IRB6620LX: θ_3 & θ_2 Solutions



We pass $\mathbf{H_{st}(0)}$ and the exponential of the first screw, which we already know from the previous step of the algorithm (we got "θ_1"), to the right hand side of the problem definition equation. Then, we apply both sides of the equation to point "**f**" (crossing of the last three axes: "θ_4", "θ_5", "θ_6"). The right hand side of the equation is now a known value "$\mathbf{k_2}$". We can cancel the unknowns "$\boldsymbol{\theta_6}$", "$\boldsymbol{\theta_5}$" and "$\boldsymbol{\theta_4}$" **by PK1 simplification case**. The resulting equation is exactly the definition for the canonic **Pardos-Gotor subproblem four PG4**. And we know the geometric inverse kinematics solutions for PG4, which can be none, one or two double solutions. Knowing "θ_1" value, we now solve the motion for the second and third joints of the robot, obtaining the rotation magnitudes (if they exist) for "$\boldsymbol{\theta_3}$-$\boldsymbol{\theta_2}$" double solutions.

$$e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} \boxed{e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6}} f = e^{-\hat{\xi}_1\theta_1} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} f$$

The screw rotations θ_4, θ_5, θ_6, have NO EFFECT when applied to any point "f" which is on its axis

$$\boxed{e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} f = k_2}$$

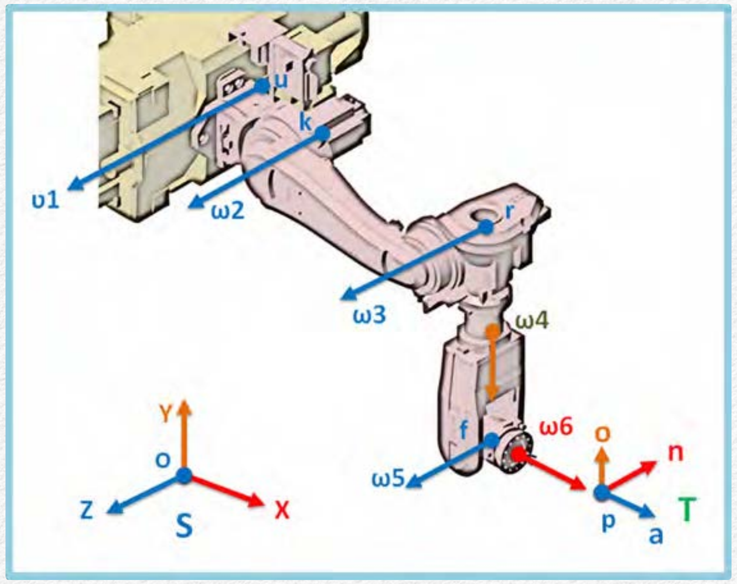
CANONIC Problem PARDOS-GOTOR-FOUR (PG4)

SOLUTIONS "θ_3-θ_2" double

$$\boxed{\theta_1^{01} \Rightarrow \theta_2^{01}, \theta_3^{01} \;\&\; \theta_2^{02}, \theta_3^{02}}$$

IK - GANTRY ABB IRB6620LX: θ_6 Solution

We pass **H_{st}(0)** and the exponentials of the first five screws, which we already know from the previous steps of the algorithm (we already have "θ_1", "θ_2", "θ_3", "θ_4" and "θ_5") to the right hand side of the problem definition equation. Then, we apply both sides of the equation to point "**o**" (origin of the spatial reference frame). The right hand side of the equation is now a known value "**k_4**". The resulting equation is exactly the definition for the canonic **Paden-Kahan subproblem one PK1**. And we know the geometric inverse kinematics solutions for PK1. Finally, for each set o "θ_1-θ_2-θ_3-θ_4-θ_5"" values, we solve the motion for the sixth joint of the robot, obtaining the rotation magnitude for "**θ_6**" solution.



$$e^{\hat{\xi}_6\theta_6} o = \boxed{e^{-\hat{\xi}_5\theta_5} e^{-\hat{\xi}_4\theta_4} e^{-\hat{\xi}_3\theta_3} e^{-\hat{\xi}_2\theta_2} e^{-\hat{\xi}_1\theta_1}} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} o$$

The θ_5, θ_4, θ_3, θ_2, θ_1 values are already known from previous calculations

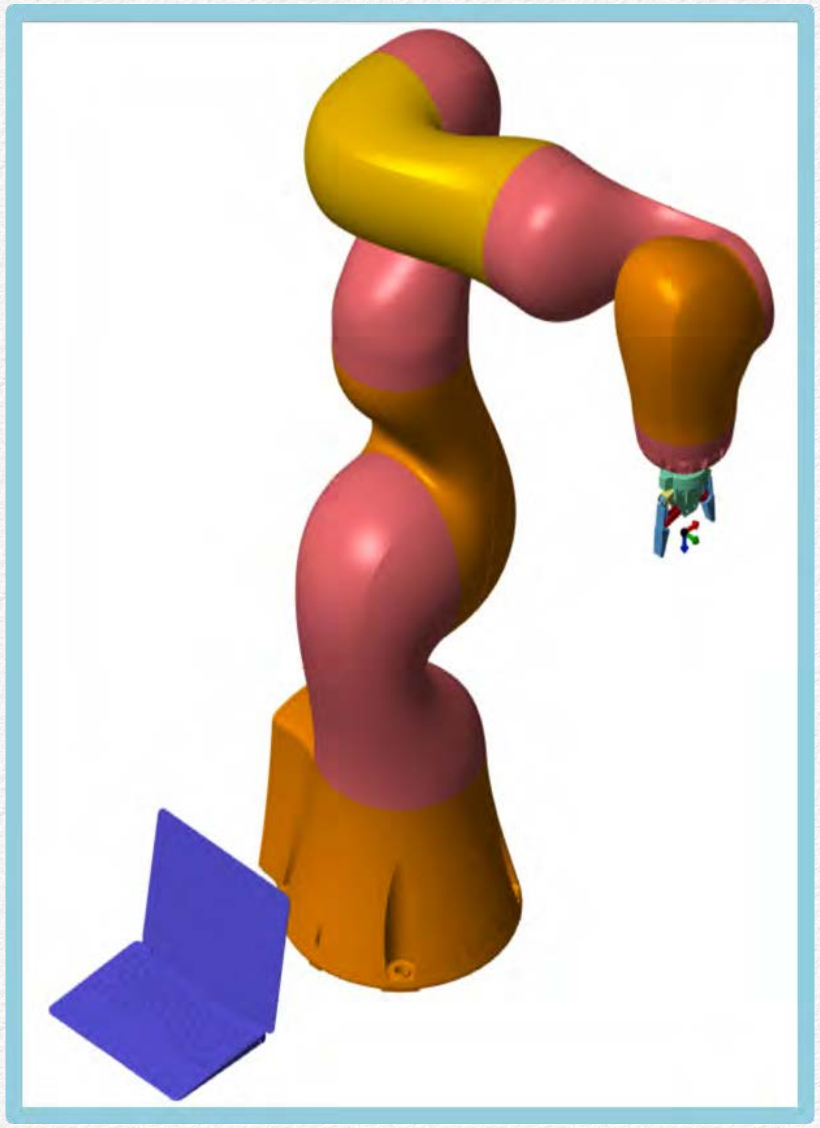
$$e^{\hat{\xi}_6\theta_6} o = k_4$$

CANONIC Problem PADEN-KAHAN-ONE (PK1)

SOLUTION "θ_6"

$$\theta_1^{01}, \theta_2^{01}, \theta_3^{01}, \theta_4^{01}, \theta_5^{01} \Rightarrow \theta_6^{01}$$
$$\theta_1^{01}, \theta_2^{01}, \theta_3^{01}, \theta_4^{02}, \theta_5^{02} \Rightarrow \theta_6^{02}$$
$$\theta_1^{01}, \theta_2^{02}, \theta_3^{02}, \theta_4^{03}, \theta_5^{03} \Rightarrow \theta_6^{03}$$
$$\theta_1^{01}, \theta_2^{02}, \theta_3^{02}, \theta_4^{04}, \theta_5^{04} \Rightarrow \theta_6^{04}$$

Redundant Robot (e.g. KUKA IIWAR820)



REDUNDANT Robots

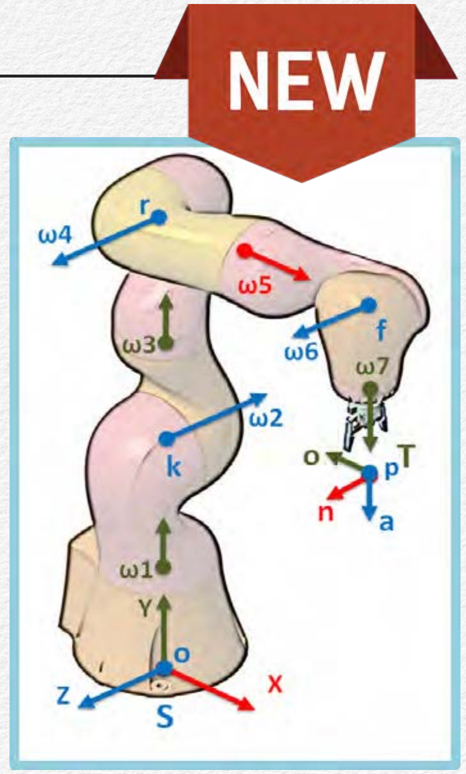
There are other kinematic mechanisms that frequently occur in robotic manipulation. Now we will address a particular type of structure, such as the redundant robot. An explanation will be given on how to extend the screw theory methodologies for this kind of mechanical systems.

To perform a given task, a robot must have enough degrees of freedom (DOF) to accomplish such task. In the mathematics presented so far, we have concentrated on the case in which the robot has precisely the required degrees of freedom. A kinematically redundant robot has more than the minimal number of DOF required to complete a set of tasks.

A redundant robot can have an infinite number of joint configurations which give the same tool configuration. The extra DOFs, present in redundant manipulators can be used to develop some motion strategy. For instance, **these extra DOFs can be used to avoid obstacles and kinematic singularities or to optimise the motion of the manipulator relative to a cost function**. **If joint limits are present, a redundant robot can be used to increase the dexterous workspace.**

The forward kinematics of a redundant manipulator can be obtained exactly as we have seen so far, using the POE formula.

IK - REDUNDANT KUKA IIWA R820: θ_1^{in} & θ_3^{in} Solutions



It is a redundant Robot with 7 DOF, and so it can have infinite solutions for its inverse kinematics problem. **To limit the number of solutions we decide to choose θ_1 & θ_3 as a function of some criteria and then solve the IK for each of those, getting 8 + 8 solutions**. The values will be named "θ_1^{in}" & "θ_3^{in}" after "input" constraints solutions.

We can choose several criteria: the minimum joint velocity, the minimum norm for the joint magnitude, the secure path to avoid obstacles or kinematic singularities, the optimum motion of the robot relative to some cost function, and so on. In general, any clever criteria must increase the dexterous workspace for the redundant robot in comparison with a robot with only 6 DOF.

This algorithm will orientate the robot TCP in the direction of the Target pose (noap), so as to get θ_1^{in} & θ_3^{in}. In this way, the whole movement of the robot is quite natural when following a planned trajectory and, at the same time, it exploits the workspace capacities of the KUKA robot.

SOLUTIONS "θ_1^{in}" & "θ_3^{in}"

$$e^{\hat{\xi}_1\theta_1} p = p_{noap}$$

CANONIC Problem PADEN-KAHAN-ONE (PK1)

$$\theta_1^{in}$$

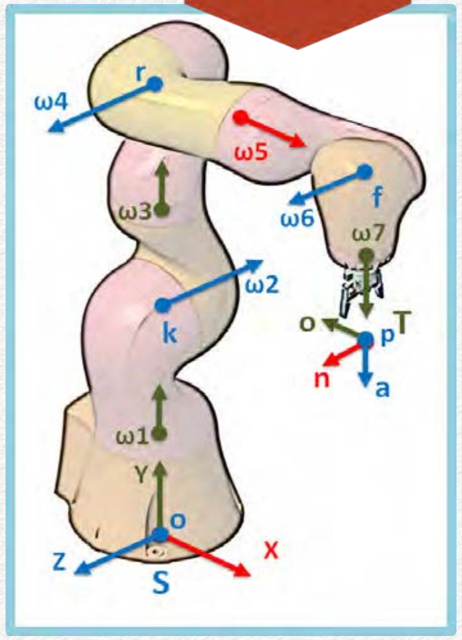
$$e^{\hat{\xi}_3\theta_3} p = p_{noap}$$

CANONIC Problem PADEN-KAHAN-ONE (PK1)

$$\theta_3^{in}$$

IK - REDUNDANT KUKA IIWA R820: θ_7 Solution

We pass $\mathbf{H_{st}(0)}$ and the exponentials of the first six screws, which we already know from the previous steps of the algorithm (i.e. we already got "θ_1", "θ_2", "θ_3", "θ_4", "θ_5" and "θ_6"), to the right hand side of the problem definition equation. Then, we apply both sides of the equation to point "**o**" (origin of the spatial reference frame "S"). The right hand side of the equation is now a known value "**k₄**". The resulting equation is exactly the definition for the canonic **Paden-Kahan subproblem one (PK1)**. And we know the geometric inverse kinematics solutions for PK1. Finally, for each set o "θ_1-θ_2-θ_3-θ_4-θ_5-θ_6"" values, we solve the motion for the seventh joint of the robot, obtaining the rotation magnitude for "$\boldsymbol{\theta_7}$" single solution.



$$e^{\hat{\xi}_7\theta_7}o = \boxed{e^{-\hat{\xi}_6\theta_6}e^{-\hat{\xi}_5\theta_5}e^{-\hat{\xi}_4\theta_4}e^{-\hat{\xi}_3\theta_3}e^{-\hat{\xi}_2\theta_2}e^{-\hat{\xi}_1\theta_1}}\begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1}\, o$$

The θ_6, θ_5, θ_4, θ_3, θ_2 and θ_1 values are already known from previous calculations

CANONIC Problem PADEN-KAHAN-ONE (PK1)

$$\boxed{e^{\hat{\xi}_7\theta_7}o = k_4}$$

SOLUTION "θ_7"

$$\theta_1^{01},\theta_2^{01},\theta_3^{in},\theta_4^{01},\theta_5^{01},\theta_6^{01} \Rightarrow \theta_7^{01} \qquad \theta_1^{in},\theta_2^{05},\theta_3^{01},\theta_4^{01},\theta_5^{09},\theta_6^{09} \Rightarrow \theta_7^{09}$$

$$\theta_1^{01},\theta_2^{01},\theta_3^{in},\theta_4^{01},\theta_5^{02},\theta_6^{02} \Rightarrow \theta_7^{02} \qquad \theta_1^{in},\theta_2^{05},\theta_3^{01},\theta_4^{01},\theta_5^{10},\theta_6^{10} \Rightarrow \theta_7^{10}$$

$$\theta_1^{02},\theta_2^{02},\theta_3^{in},\theta_4^{01},\theta_5^{03},\theta_6^{03} \Rightarrow \theta_7^{03} \qquad \theta_1^{in},\theta_2^{06},\theta_3^{02},\theta_4^{01},\theta_5^{11},\theta_6^{11} \Rightarrow \theta_7^{11}$$

$$\theta_1^{02},\theta_2^{02},\theta_3^{in},\theta_4^{01},\theta_5^{04},\theta_6^{04} \Rightarrow \theta_7^{04} \qquad \theta_1^{in},\theta_2^{06},\theta_3^{02},\theta_4^{01},\theta_5^{12},\theta_6^{12} \Rightarrow \theta_7^{12}$$

$$\theta_1^{03},\theta_2^{03},\theta_3^{in},\theta_4^{02},\theta_5^{05},\theta_6^{05} \Rightarrow \theta_7^{05} \qquad \theta_1^{in},\theta_2^{07},\theta_3^{03},\theta_4^{02},\theta_5^{13},\theta_6^{13} \Rightarrow \theta_7^{13}$$

$$\theta_1^{03},\theta_2^{03},\theta_3^{in},\theta_4^{02},\theta_5^{06},\theta_6^{06} \Rightarrow \theta_7^{06} \qquad \theta_1^{in},\theta_2^{07},\theta_3^{03},\theta_4^{02},\theta_5^{14},\theta_6^{14} \Rightarrow \theta_7^{14}$$

$$\theta_1^{04},\theta_2^{04},\theta_3^{in},\theta_4^{02},\theta_5^{07},\theta_6^{07} \Rightarrow \theta_7^{07} \qquad \theta_1^{in},\theta_2^{08},\theta_3^{04},\theta_4^{02},\theta_5^{15},\theta_6^{15} \Rightarrow \theta_7^{15}$$

$$\theta_1^{04},\theta_2^{04},\theta_3^{in},\theta_4^{02},\theta_5^{08},\theta_6^{08} \Rightarrow \theta_7^{08} \qquad \theta_1^{in},\theta_2^{08},\theta_3^{04},\theta_4^{02},\theta_5^{16},\theta_6^{16} \Rightarrow \theta_7^{16}$$

DIFFERENTIAL Kinematics

7

“In turning from the smaller instruments in frequent use to the larger and more important machines, the economy arising from the increase of VELOCITY becomes more striking.”

— Charles Babbage

GEOMETRIC Jacobian (GJ)

$$V_{ST}^{S} = J_{ST}^{S}(\theta)\dot{\theta}$$

$$J_{ST}^{S}(\theta) = \left[\xi_1' \cdots \xi_n'\right]$$

$$\xi_i' = Ad_{\left(e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_{i-1}\theta_{i-1}}\right)}\xi_i$$

$$Ad_H = \begin{bmatrix} R & \hat{p}R \\ 0 & R \end{bmatrix} \forall H = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \wedge \hat{p} = \begin{bmatrix} 0 & -p_Z & p_Y \\ p_Z & 0 & -p_X \\ -p_Y & p_X & 0 \end{bmatrix}$$

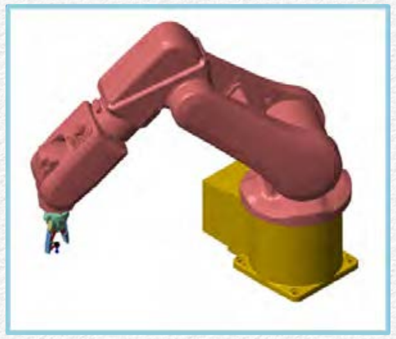
A New Description for the Jacobian

As we know, the proper representation of rigid body velocity is through the use of twists, and the Jacobian of the differential kinematics map is given in terms of twists. We shall see that the POE leads to a very natural and explicit description of the robot Jacobian, which highlights the geometry of the mechanism and has none of the drawbacks of a local analytic representation. It avoids the difficulties given by local parameterisation such as some singularities. This is called **the geometric, spatial or manipulator Jacobian**. Bear in mind that this expression is a configuration-dependent matrix which maps joint velocities to end-effector velocities.

The geometric Jacobian has a very special structure. The contribution of each joint velocity to the tool velocity is independent to the configuration of later joints in the chain. Furthermore, each column of the spatial Jacobian corresponds to its joint twist, transformed to the current robot configuration. In order to do so we use the **ADJOINT TRANSFORMATION (Ad)**, which transforms twist from one frame to another.

This powerful structure means that **we can calculate the geometric Jacobian "by inspection"**. It is very important to note that **we are able to calculate the entire robot spatial Jacobian "by definition", without explicitly differentiating the forward kinematics map**.

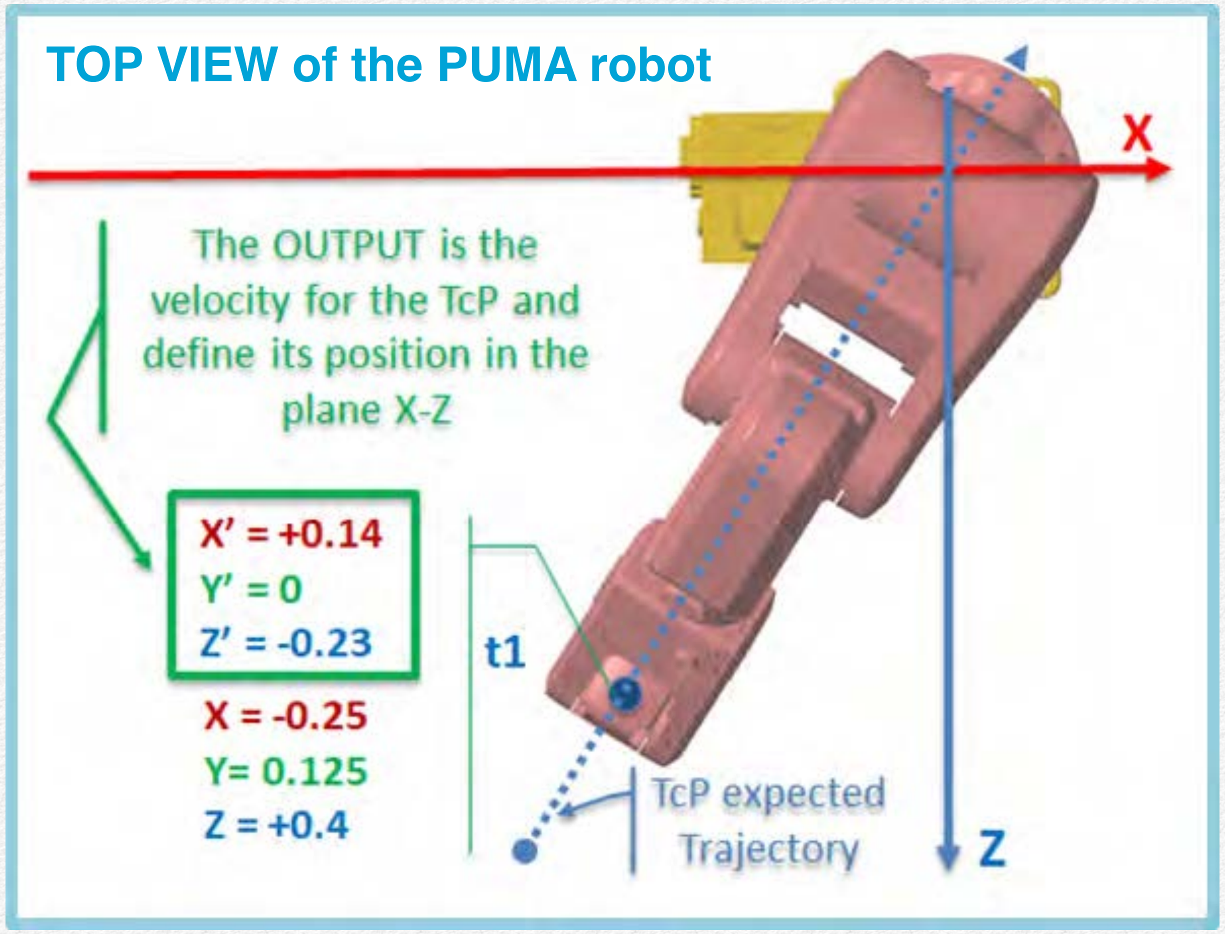
Differential FORWARD Kinematics - (ABB IRB120): VELOCITY of the TOOL with GEOMETRIC Jacobian: (cont.)



4 (cont.).- To finally obtain the tool velocity in the spatial frame, we use the spatial velocity just calculated and the **tool position in that pose "t1", this is "TCP (X = -0.25, Y = 0.125, Z = 0.4)"**.

It can seem a bit surprising that the velocity result for the differential forward kinematics of this Puma robot is exactly the same as the one obtained for the Scara robot. This is due to the fact that we have intended for the same trajectory velocity of the tool for both robots. Apparently, there is no difference so far, but once they develop the trajectory, you will realise how different the behaviour of those mechanisms are. We will see this in the next exercise for differential inverse kinematics.

$$V_T^S = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \left(\upsilon_{ST}^S\right) + \left(\omega_{ST}^S\right)^{\wedge} \begin{bmatrix} TcP_X \\ TcP_Y \\ TcP_Z \end{bmatrix} \\ \omega_{ST}^S \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0 \\ -0.23 \\ 0 \\ -0.92 \\ 0 \end{bmatrix}$$



https://github.com/DrPardosGotor/ScrewTheoryRobotics-KINEMATICS-Illustrated/tree/master/Exercises

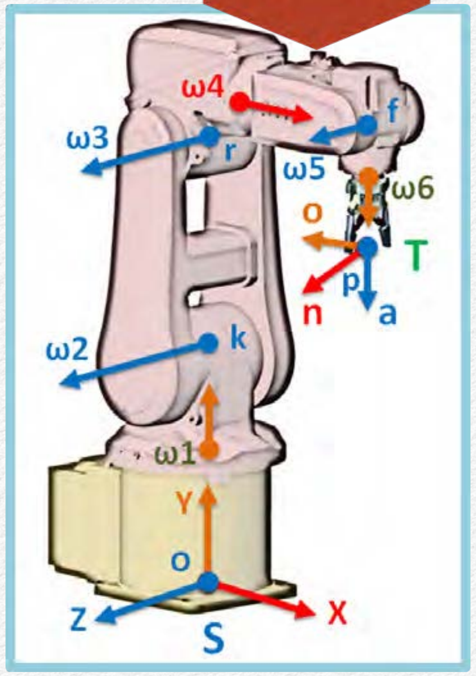
"E070_STR_DIFFERENTIALKin_40_ABBIRB120_GeoForJac.m"

Differential INVERSE Kinematics - (ABB IRB120):

VELOCITY of the JOINTS with GEOMETRIC Jacobian

5.- We want to get the velocity of the joints as a function of the inverse spatial Jacobian and the desired tool velocities. **Given the POSITION velocity for the TCP (i.e. x',y',z') and the ROTATION velocity for the tool system (i.e. α',β',γ'), we obtain the robot joint velocities (i.e. θ_1', θ_2', θ_3', θ_4', θ_5', θ_6'), always in the spatial frame.**

$$\dot{\theta} = \left(J_{ST}^{S}(\theta)\right)^{-1} V_{ST}^{S} \Rightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} = \left[\xi'_1\ \xi'_2\ \xi'_3\ \xi'_4\ \xi'_5\ \xi'_6\right]^{-1} \begin{bmatrix} \upsilon_{ST}^{S} \\ \omega_{ST}^{S} \end{bmatrix}$$



6.- Then we get this general joint velocity (i.e. θ_1', θ_2', θ_3', θ_4', θ_5', θ_6') expression, based on the screw theory geometric Jacobian, but considering the information that usually we have, which is the tool velocity at the pose (i.e. x',y',z',α',β',γ').

Only POSITION of the TCP from the desired trajectory or forward kinematics of the robot.

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} = \left[\xi'_1\ \xi'_2\ \xi'_3\ \xi'_4\ \xi'_5\ \xi'_6\right]^{-1} \begin{bmatrix} \left(\upsilon_{TcP}^{S}\right) - \left(\omega_{ST}^{S}\right)^{\wedge} TcP(\theta) \\ \omega_{ST}^{S} \end{bmatrix} = \left[\xi'_1\ \xi'_2\ \xi'_3\ \xi'_4\ \xi'_5\ \xi'_6\right]^{-1} \begin{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} - \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}^{\wedge} \begin{bmatrix} TcP_X \\ TcP_Y \\ TcP_Z \end{bmatrix} \\ \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -\dot{\gamma} & \dot{\beta} \\ \dot{\gamma} & 0 & -\dot{\alpha} \\ -\dot{\beta} & \dot{\alpha} & 0 \end{bmatrix}$$

Simulation

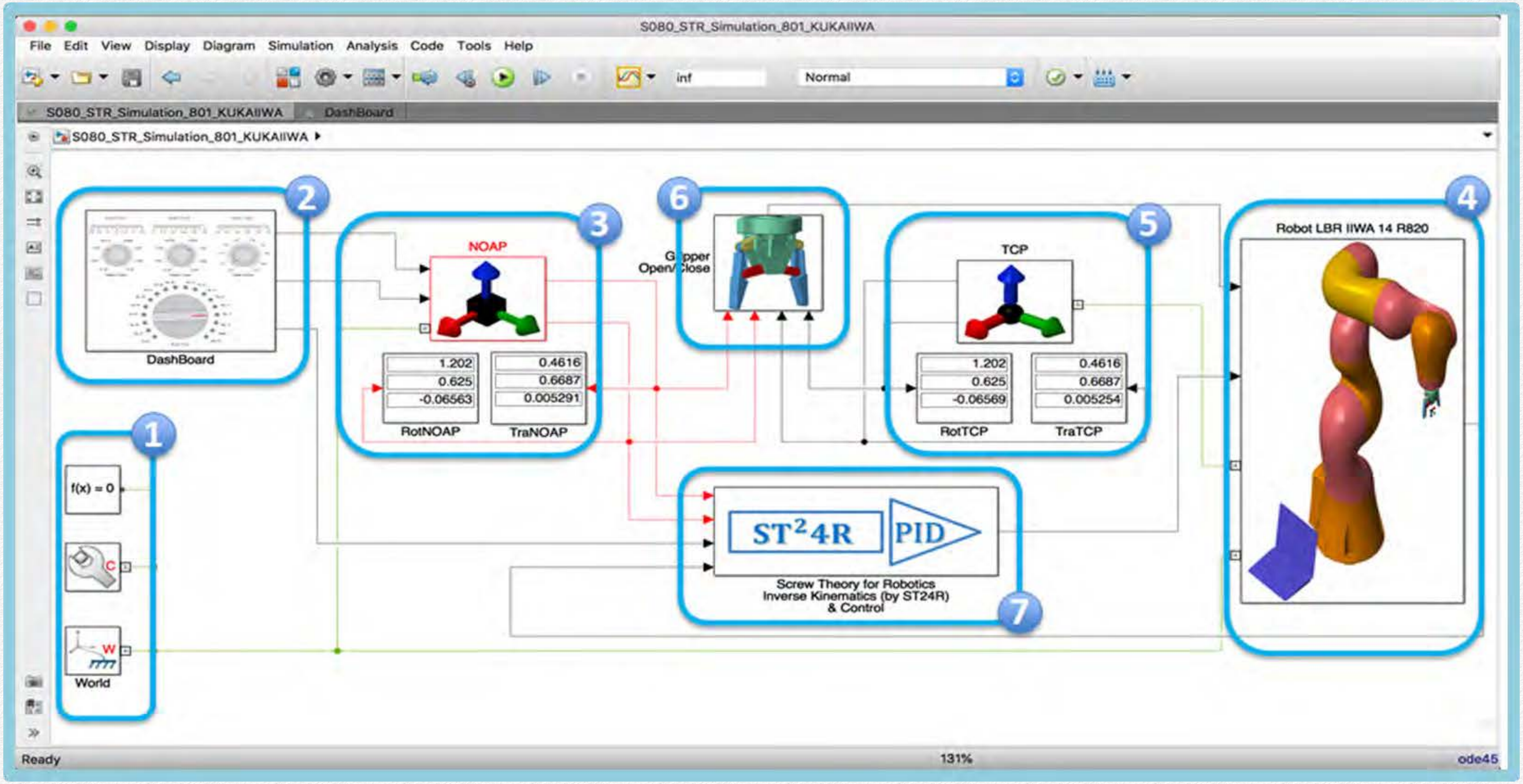
8

"Without LABORATORIES men of science are soldiers without arms."

— Louis Pasteur

80.0.- Simulation for KUKA IIWA - Complete model

In the folder (see .zip in the previous page) you will find a directory with all the material needed to build this robotic simulation. The file with the complete model in Simulink® is: "**S080_STR_Simulation_801_KUKAIIWA.slx**". I recommend you get started by studying all blocks and details of this simulation. Many of them have already been reviewed in previous examples.

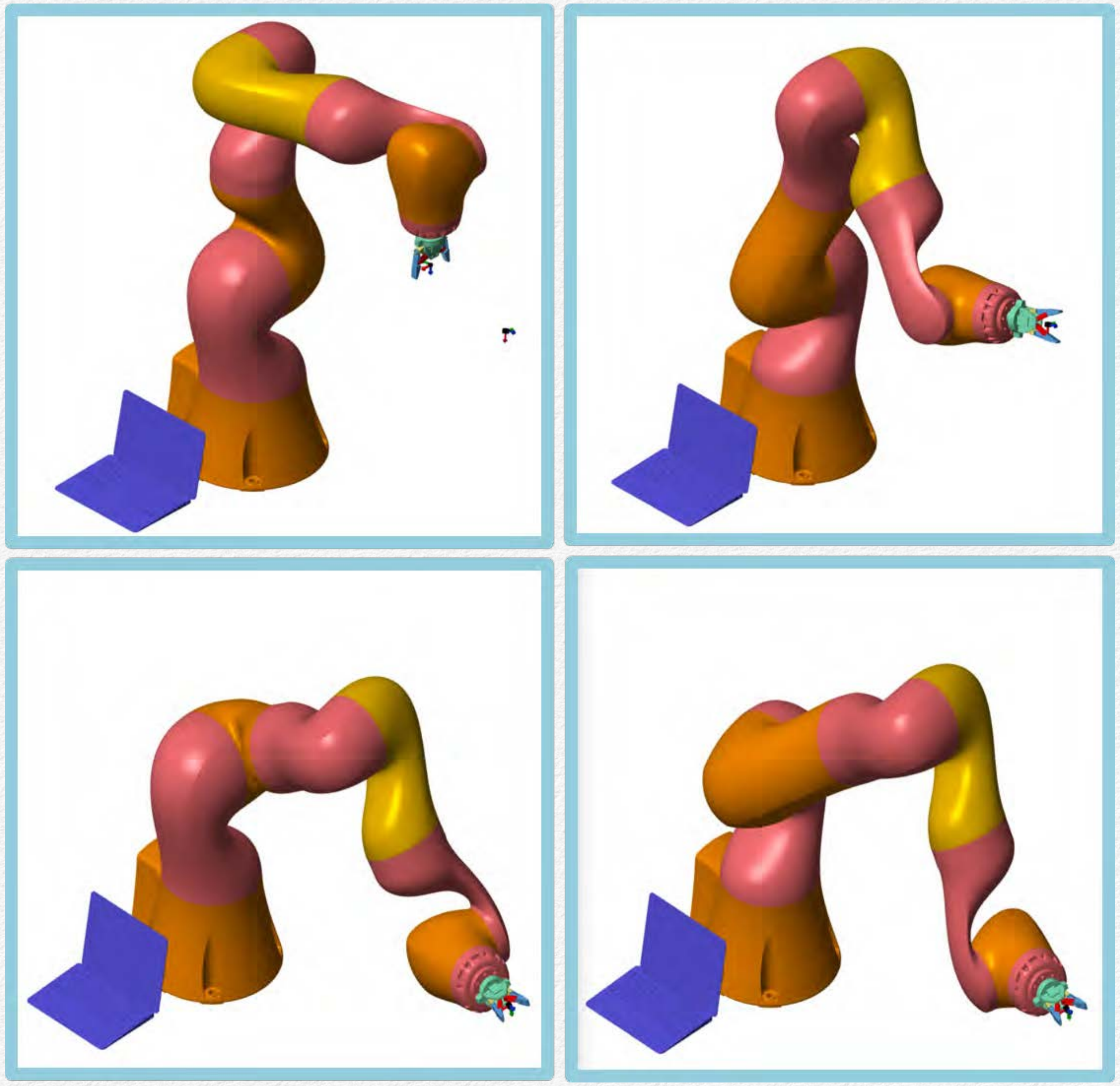


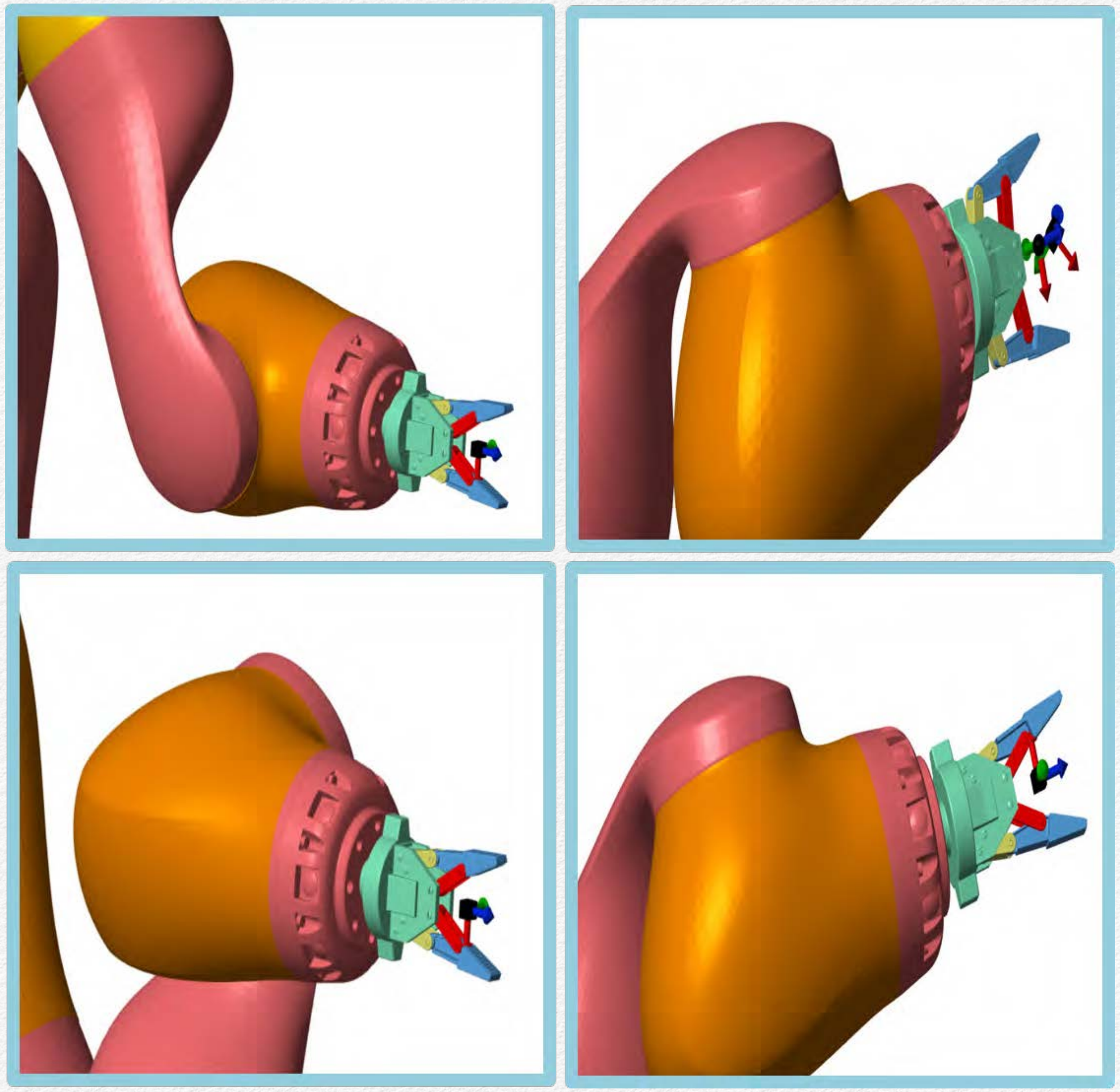
80.9.- Simulation for KUKA IIWA - Video Creator and Recording

To test the algorithms it is really useful to have videos which allow you to view very quickly the performance of the developments. In addition, the visual material comes in handy for communication purposes. For this exercise, two videos are available in YouTube. The first one shows the 16 geometric solutions for the inverse kinematics with a general view. The second is focused on the tool details.

"V080_STR_Simulation_801_KUKAIIWA_Complete"

"V080_STR_Simulation_802_KUKAIIWA_Focus"





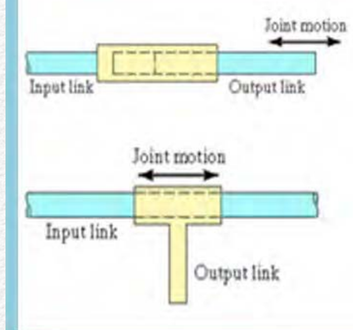
Conclusions

9

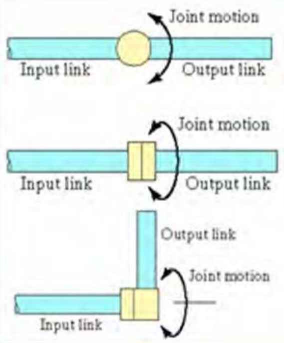
"There's nothing more PRACTICAL than a good THEORY."

— Gilbert K. Chesterton

1.- GEOMETRY as the foundation



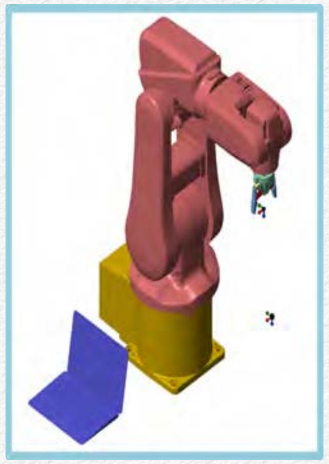
$$\xi = \begin{bmatrix} v \\ 0 \end{bmatrix}$$



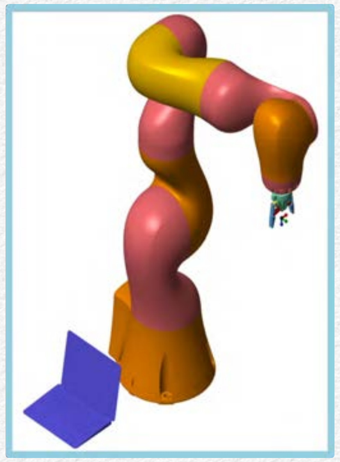
$$\xi = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$

Screw theory provides a global and truly geometric representation of the kinematics which greatly simplifies the analysis for robotics, through the use of "Twists"

2.- MATRIX EXPONENTIAL as primitive

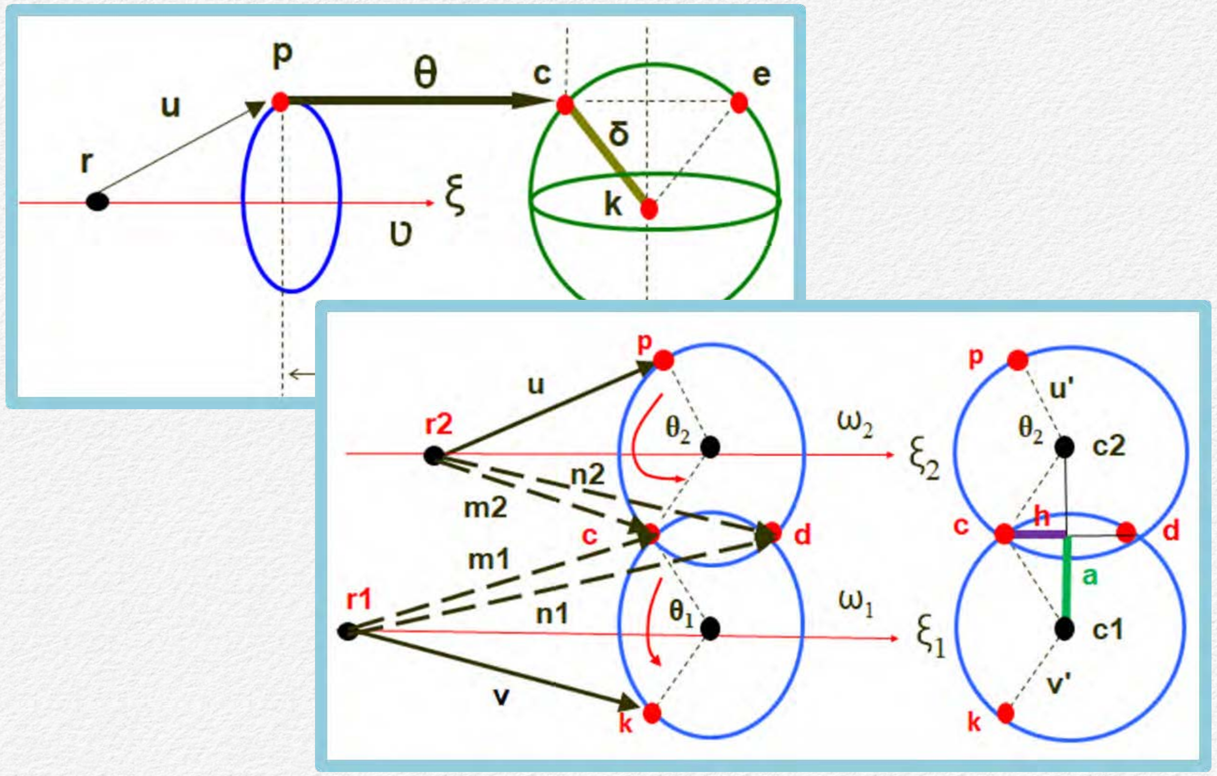


$$e^{\hat{\xi}\theta}$$



Because the basic mathematical primitive is the matrix exponential you get the great reward of being capable to treat the resulting motion equations for the robot very easily

3.- CANONICAL subproblems as basis



The possibility of using the POE to develop new geometric algorithms and solve fundamental inverse kinematic subproblems geometrically meaningful for many robotics configurations

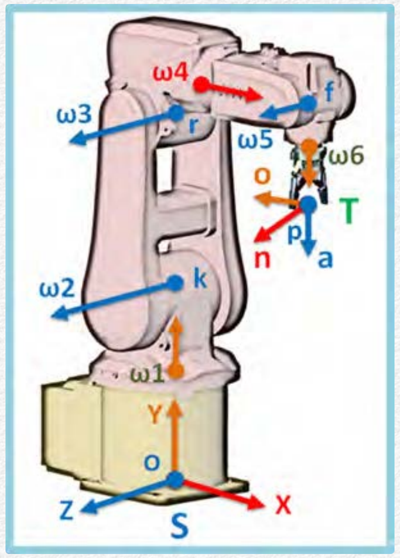
4.- Geometric METHODS as tools

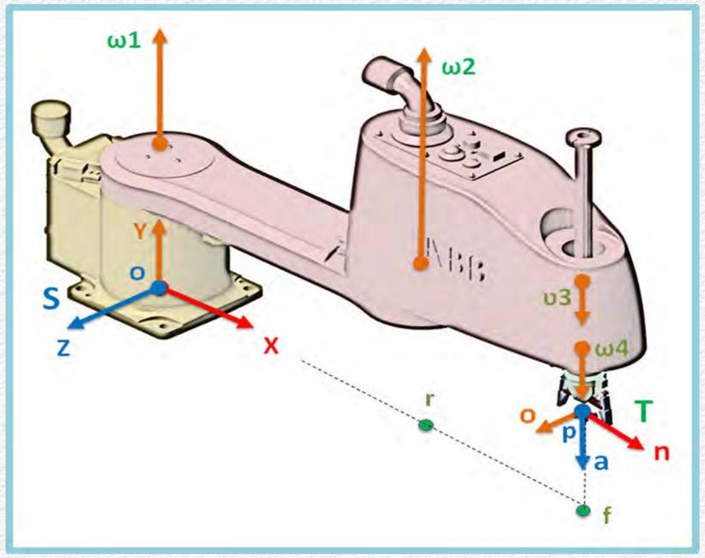
ST^24R

There are several specialised toolboxes available for building the underlying of kinematics algorithms, which provide geometric solutions of greater performance

5.- EXAMPLES & SIMULATIONS as exercises to learn inverse kinematics

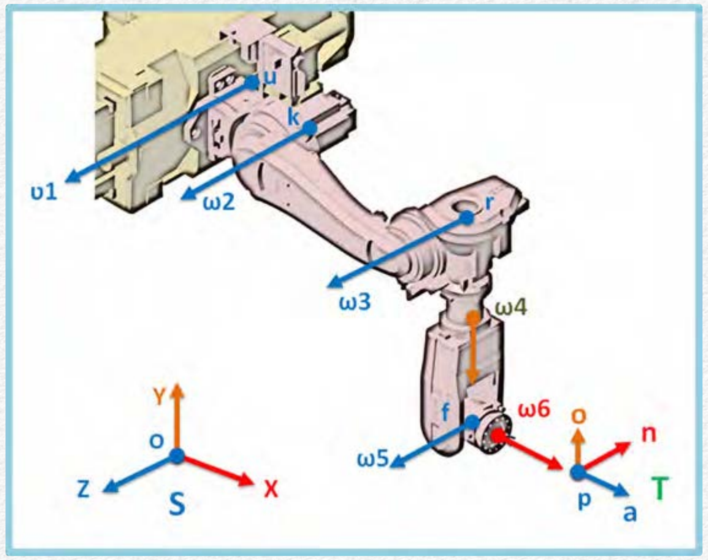
The INVERSE KINEMATICS of this PUMA robot ABB IRB120 is solved with EIGHT EXACT CLOSED-FORM (geometric) SOLUTIONS for the set: $\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6$

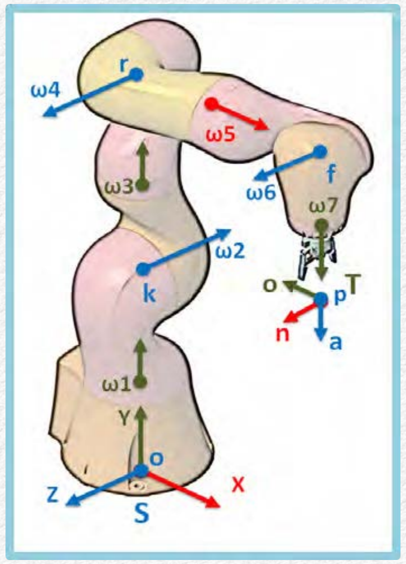




The INVERSE KINEMATICS of this SCARA robot ABB IRB910SC is solved with TWO EXACT CLOSED-FORM (geometric) SOLUTIONS for the set: $\theta_1,\theta_2,\theta_3,\theta_4$

The INVERSE KINEMATICS of this GANTRY robot ABB IRB6620LX is solved with FOUR EXACT CLOSED-FORM (geometric) SOLUTIONS for the set: $\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6$





The INVERSE KINEMATICS of this REDUNDANT robot KUKA IIWA is solved with 16 EXACT CLOSED-FORM (geometric) SOLUTIONS for the set: $\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7$

Screw theory is able to provide systematic, elegant and geometrically meaningful closed-form solutions for the inverse kinematics of many robot architectures, including those with many DOF