



Dr. José M. Pardos-Gotor

SCREW THEORY

FOR

ROBOTICS

*A practical approach for
Modern Robot KINEMATICS*

Screw Theory for Robotics

An Illustrated Handbook

A practical approach for Modern Robot KINEMATICS

This preprint is being made available for personal use. It is only a sneak peak with a small sample of the actual book's contents. The book is published by Amazon Fulfilment 2018, ISBN 9781717931818.

Additional material (e.g. Code, Exercises, Simulations) is provided at:

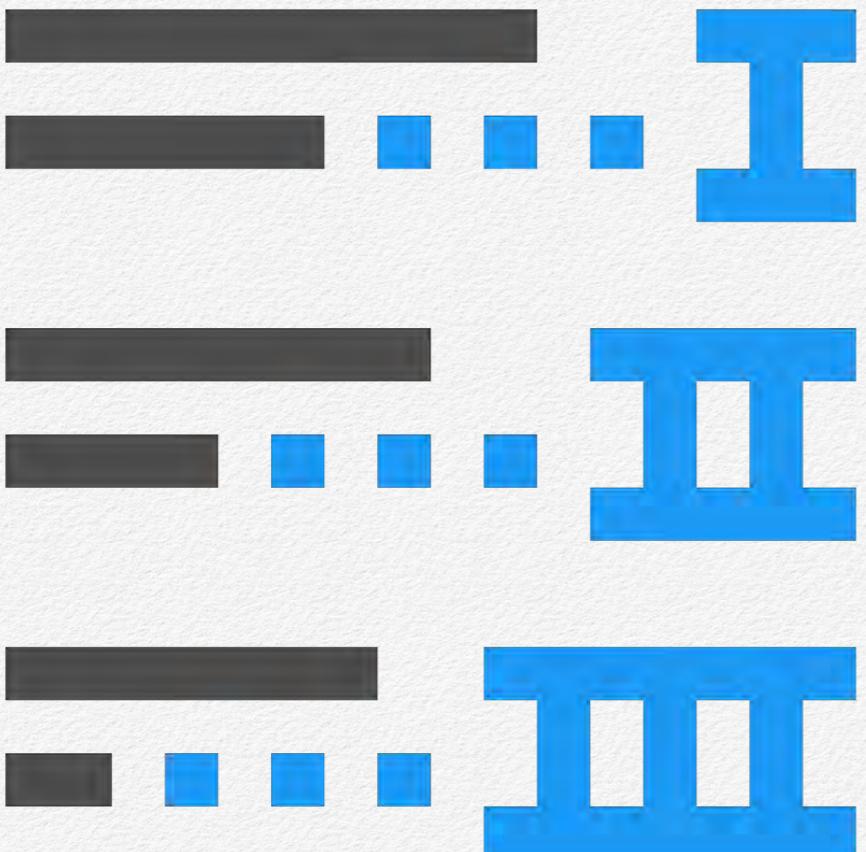
<https://github.com/DrPardosGotor/ScrewTheoryRobotics-KINEMATICS-Illustrated>

Written by Dr. José M. Pardos-Gotor

Associate Professor in the Department of Systems Engineering and Automation

UC3M - Universidad Carlos III de Madrid





Screw Theory for Robotics

• <i>Copyright</i>	v
• <i>Contents</i>	vi
• <i>Preface</i>	viii
• <i>Acknowledgements</i>	xi
• <i>Dedication</i>	xii

1. Introduction

1.1. Years of Robots	14
1.2. About this book	17
1.3. Preview	22
1.4. Audience	27
1.5. Further Reading	28

2. Mathematics

2.1. Rigid Body Motion	30
2.2. HOMOGENEOUS Representation	31
2.3. EXPONENTIAL Representation	38

3. FORWARD Kinematics

3.1. Robot Forward Kinematics	46
3.2. Denavit-Hartenberg (D-H)	47
3.3. Product of Exponentials (POE)	50
3.4. Puma Robot (e.g. ABB IRB120)	54
3.5. Gantry Robot (e.g. ABB IRB6620LX)	56
3.6. Scara Robot (e.g. ABB IRB910SC)	58

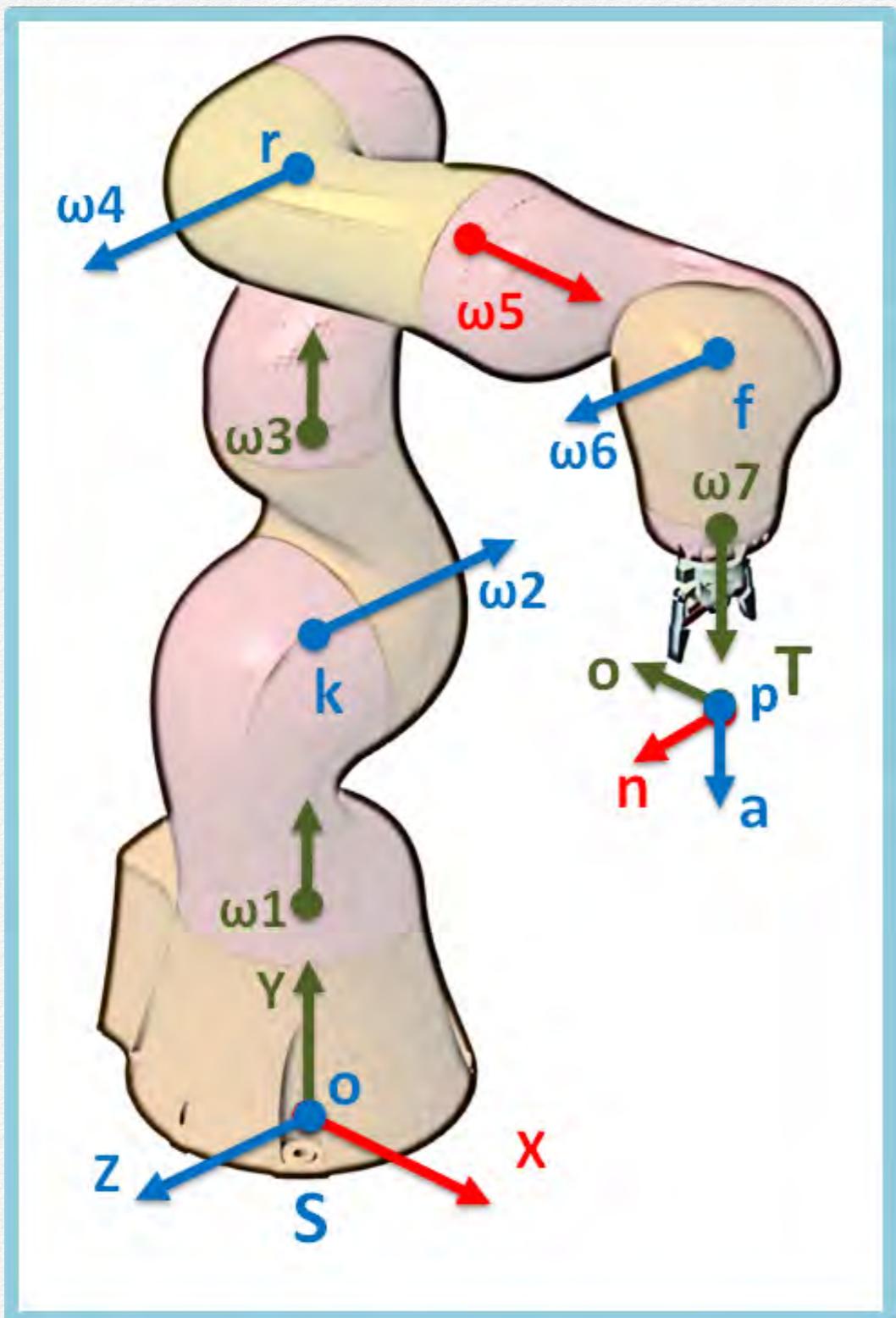
3.7. Redundant Robot (e.g. KUKA IIWA & RH0)	60	NEW	7.5. GEOMETRIC Jacobian (GJ)	146
4. INVERSE Kinematics		NEW	7.6. GJ Scara Robot (e.g. ABB IRB910SC)	149
4.1. Robot Inverse Kinematics	64	NEW	7.7. GJ Puma Robot (e.g. ABB IRB120)	156
4.2. NUMERIC Methods.	66			
4.3. GEOMETRIC Methods	68			
5. CANONICAL Inverse Kinematics			8. Simulation	
5.1. Canonical Kinematics Implementation	72	NEW	8.1. Robotics Simulation	164
5.2. Paden-Kahan subproblems	74	NEW	8.2. MATLAB & ST24R	166
NEW 5.3. Pardos-Gotor subproblems	80	NEW	8.3. Simulink & Simscape	170
		NEW	8.4. Puma Robot (e.g. ABB IRB120)	172
		NEW	8.5. Gantry Robot (e.g. ABB IRB6620LX)	184
		NEW	8.6. Scara Robot (e.g. ABB IRB910SC)	188
		NEW	8.7. Redundant Robot (e.g. KUKA IIWAR20)	192
6. EXAMPLES of Inverse Kinematics			9. Conclusions	
6.1. The problem of Inverse Kinematics	88		9.1. Lessons Learned.	198
6.2. Inverse Kinematics solution approach	89			
6.3. Puma Robot (e.g. ABB IRB120)	90			
NEW 6.4. Gantry Robot (e.g. ABB IRB6620LX)	98		Afterword	
NEW 6.5. Scara Robot (e.g. ABB IRB910SC)	108		• <i>Glossary</i>	204
NEW 6.6. Redundant Robot (e.g. KUKA IIWAR820)	116		• <i>References</i>	206
NEW 6.7. Geometric POE algorithms performance	126		• <i>Bio</i>	210
7. DIFFERENTIAL Kinematics				
7.1. Robot Velocity & Jacobian.	132			
7.2. ANALYTIC Jacobian (AJ)	136			
7.3. AJ Scara Robot (e.g. ABB IB910SC).	138			
7.4. AJ Puma Robot (e.g. ABB IRB120)	144			



Introduction

“Theoretically, if we could build a MACHINE whose mechanical structure duplicated HUMAN PHYSIOLOGY, then we could have a machine whose intellectual capacities would duplicate those of human beings.”

— Norbert Wiener



*Are you building REAL
TIME Robot
Applications?*

*Are you solving Robot
Inverse KINEMATICS
with Closed-Form
Geometric Algorithms?*

*Are you needing Robot
VELOCITIES without
Differentiations?*

If you have answered “yes” to any of these questions, your journey starts here!

- You have in your possession a handbook for robotics enthusiasts who are striving to improve the efficiency and performance of robot applications.



Mathematics



*“The ABSTRACTION
SAVES TIME in the long
run, in return for an initial
investment of effort and
patience in learning some
mathematics.”*

— Richard M. Murray

— Zexiang Li

— S. Shankar Sastry

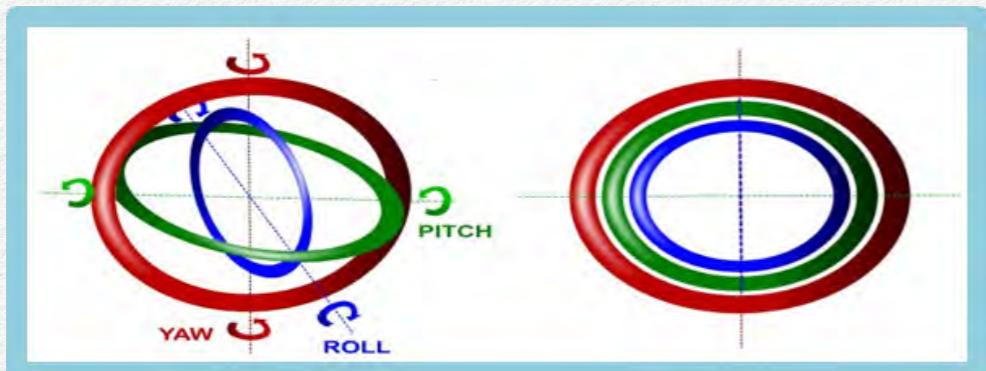
Twist & Screws - Properties & Benefits

The exponential map for a “twist” (ξ) gives the relative motion of a rigid body. Twist transformation is as mapping points from their initial coordinates to the coordinates after the rigid motion is applied (not as mapping points from one coordinate frame to another).

$$H_{ST}(\theta) = e^{\hat{\xi}\theta} H_{ST}(0)$$

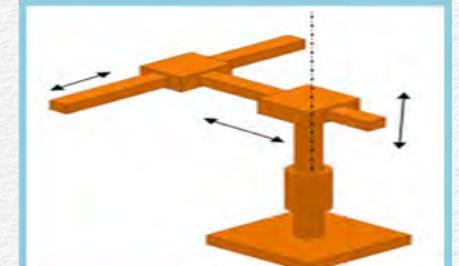
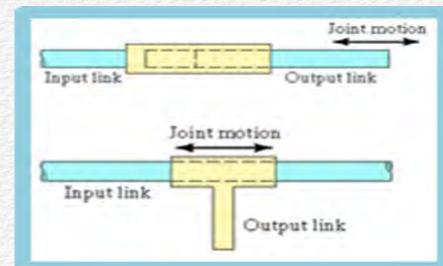
Screw theory provides an rigorous and geometric description of the motion which **simplifies the analysis of mechanisms**.

Screws allow a description of the motion which does not suffer from singularities due to the use of local coordinates. The fact that any three-angle representation for orientation has singularities (e.g. Euler angles) is a fundamental problem. This is also known as **Gimbal Lock**.



SCREW for PURE TRANSLATION

Vector “ v ” is the AXIS line of translation, and “ ω ” is zero.

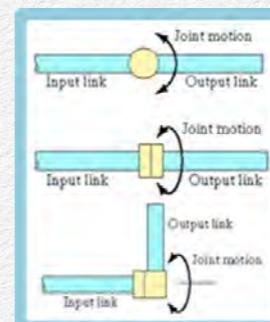
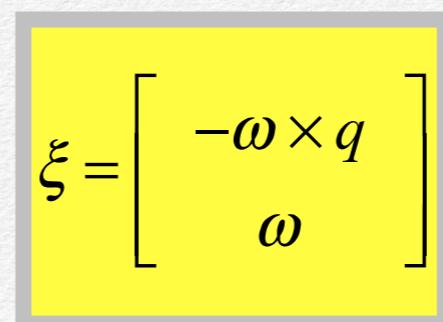


$$\xi = \begin{bmatrix} v \\ 0 \end{bmatrix}$$

$$e^{\hat{\xi}\theta} = \begin{bmatrix} I_3 & v\theta \\ 0 & 1 \end{bmatrix}$$

SCREW for PURE ROTATION

Vector “ ω ” is the rotation AXIS and “ q ” is any point on “ ω ”.



$$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I_3 - e^{\hat{\omega}\theta})(\omega \times v) + \omega \omega^T v \theta \\ 0 & 1 \end{bmatrix}$$

FORWARD Kinematics

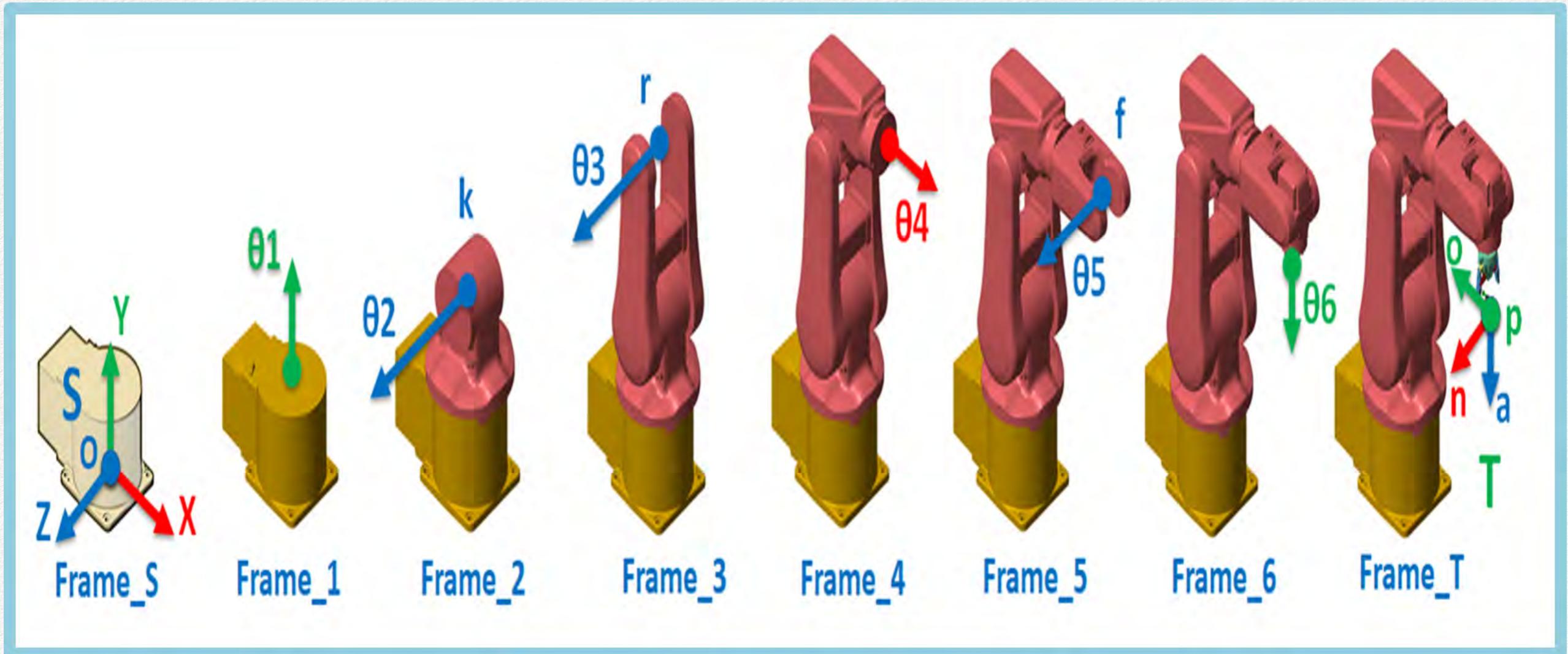


*“Never confuse MOTION
with ACTION.”*

— Benjamin Franklin

Screw Theory Forward Kinematics - Product Of Exponentials (POE)

For a typical 6 DOF manipulator, the expression for the forward kinematics has the form of a product of exponentials given by the **Twist (ξ)** and **Magnitude (θ)** associated to each rotation. **Each exponential represents the relative movement of a joint.**



$$H_{ST}(\theta) = \prod_{i=1}^n e^{\hat{\xi}_i \theta_i} H_{ST}(0) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} e^{\hat{\xi}_3 \theta_3} e^{\hat{\xi}_4 \theta_4} e^{\hat{\xi}_5 \theta_5} e^{\hat{\xi}_6 \theta_6} H_{ST}(0) = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

INVERSE Kinematics

*“In the beginning was
MECHANICS.”*

— Max von Laue



Example: Inverse Kinematics for a Puma Robot “ST24R vs. RST” (e.g. ABB IRB120)

The goal is to compare the performance between a geometric "**ST24R - Screw Theory Toolbox for Robotics**" and a numeric "**RST - Robotics System Toolbox**" approach. The focus here is not to enter in the details of the algorithm (as you will come to realise going through the next chapters), but rather to compare only the results. **Note that the geometric screw method is even three orders of magnitude FASTER**, and what is more important, it provides **EIGHT EXACT SOLUTIONS**. Meanwhile, the numeric method only gives (in the best case) one approximate solution.

```

noapST24R =
-0.8401  0.5318  0.1069 -0.4226
 0.5391  0.7968  0.2731  0.2161
 0.0600  0.2870 -0.9560  0.1709
    0       0       0     1.0000

ThetaST24R =
-2.5067 -1.4903  0.7965 -1.2848 -0.7433 -0.2789

time_IK_ST24R =
'Time to solve IK Screw Theory 371 µs'

TcpST24R =
-0.8401  0.5318  0.1069 -0.4226
 0.5391  0.7968  0.2731  0.2161
 0.0600  0.2870 -0.9560  0.1709
    0       0       0     1.0000

ThetaRSTn6 =
-1.9134 -0.1847 -0.3630 -0.1163 -1.0924  1.4402

time_IK_RST =
'Time to solve IK RS Toolbox 106 ms'

TcpRST =
-0.0502  0.9574  0.2843 -0.0700
 0.9969  0.0310  0.0719  0.4694
 0.0600  0.2870 -0.9560  0.1709
    0       0       0     1.0000

j5 >> |

```

The desired configuration POSE (rotation + translation) for the Tool end-effector (it is a reachable configuration)

IK Solutions ($\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$)
(in fact there are 8 solutions)

IK Efficiency $\sim \mu\text{s}$

IK Effectiveness
EXACT SOLUTION

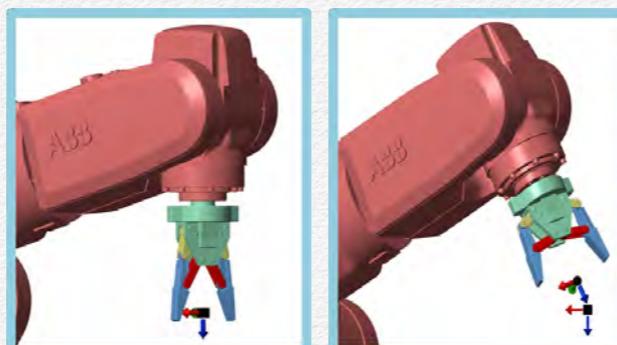
IK Solutions ($\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$)

IK Efficiency $\sim \text{ms}$

IK Effectiveness
APPROXIMATE SOLUTION

ST²4R





“E060_STR_EXAMPLESInvKin_47_ABBIRB120_ST24RandRST.m”

<https://github.com/DrPardosGotor/ScrewTheoryRobotics-KINEMATICS-Illustrated/tree/master/Exercises>

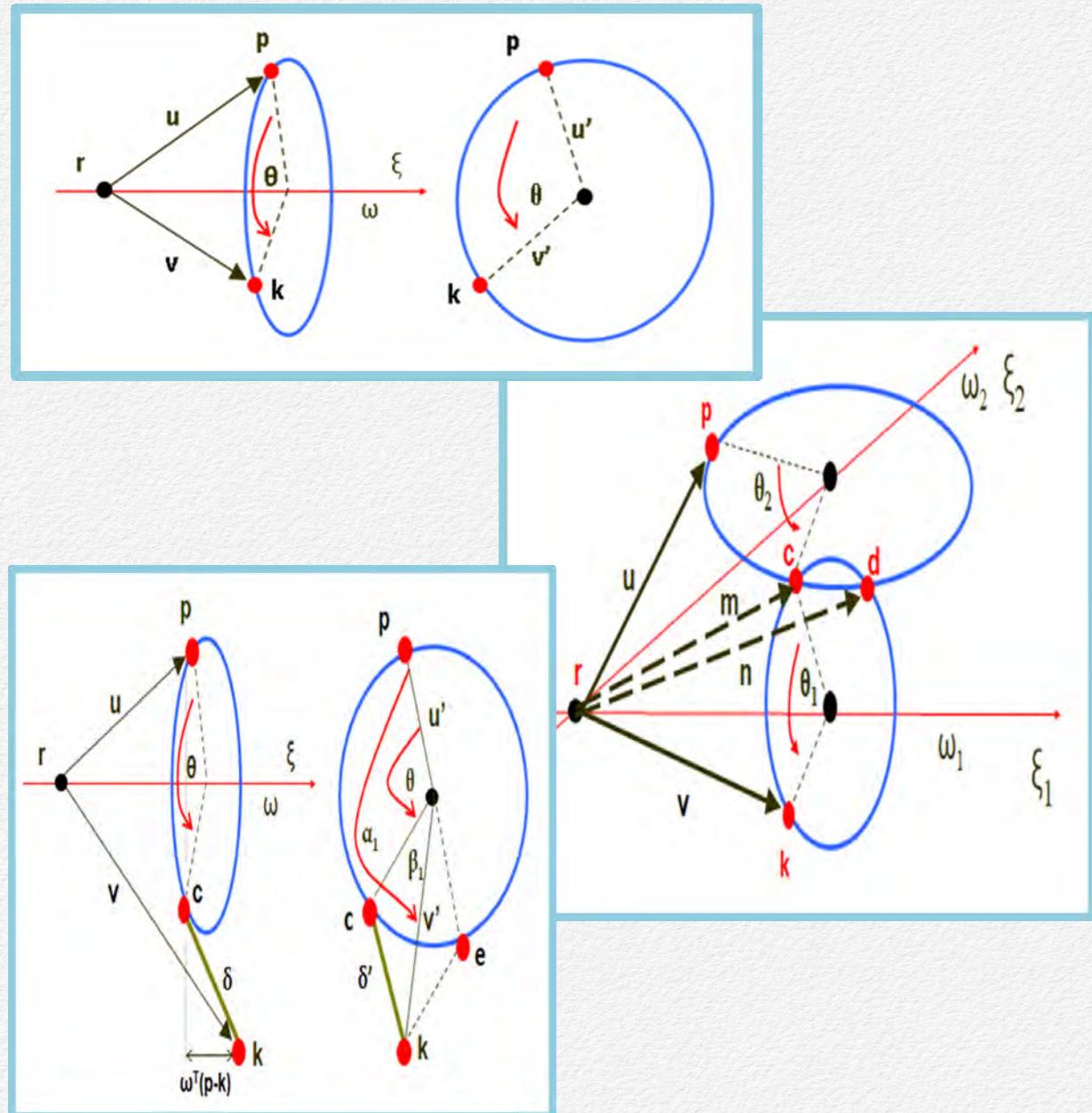
CANONICAL Inverse Kinematics



“Education is not the learning of facts, It’s rather the TRAINING of the mind TO THINK.”

— Albert Einstein

Paden-Kahan subproblems



The Canonical Problems Inception

The method was originally presented by Paden and built on the unpublished work of Kahan.

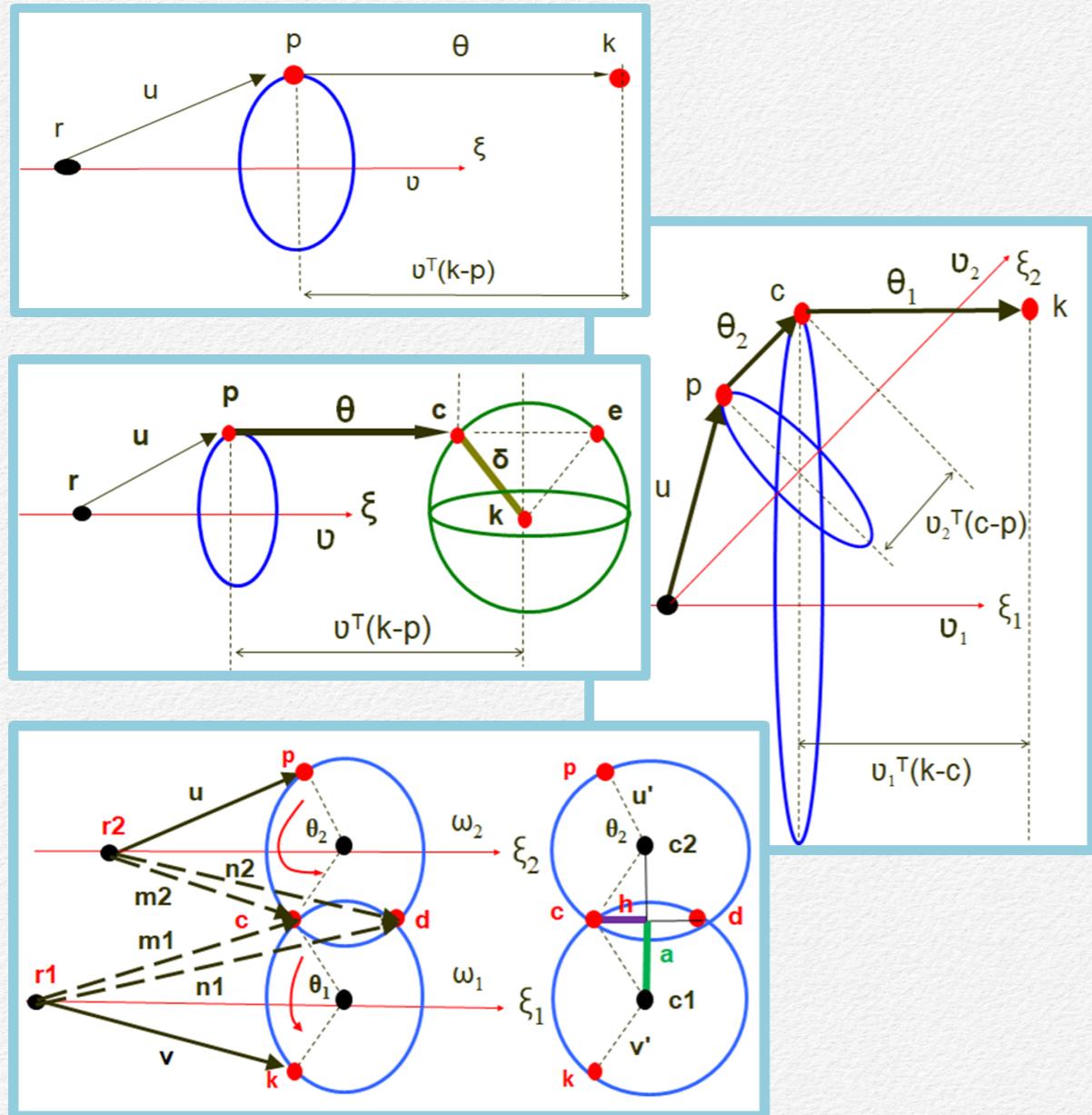
Paden-Kahan subproblems are a set of archetypical solved geometric problems which frequently occur in inverse kinematics of common robotic manipulators. Although the set of problems is not exhaustive, it may be used to simplify inverse kinematic analysis for many industrial robots.

For any mechanism analysed and defined by a product of exponentials (POE) equation, Paden-Kahan subproblems may be used to simplify the problem. Generally, subproblems are applied to solve particular points in the inverse kinematics problem (e.g. the intersection of joint axes), in order to solve the joint movement magnitudes (i.e. angles).

We will see the three famous **Paden-Kahan subproblems**, with the general solution, and some examples which can help you practice the screw theory concepts. The three PK subproblems are:

- **ONE: Rotation about a Single Axis (PK1).**
- **TWO: Rotation about Two Subsequent Crossing Axes (PK2).**
- **THREE: Rotation to a given Distance (PK3).**

Pardos-Gotor subproblems



Following in Paden-Kahan's Footsteps

Screw theory allows us to develop new canonical subproblems to solve complex inverse kinematics problems. For instance, I needed closed-form geometric solutions for some well-known robot architectures (e.g. Gantry, Scara). Then, in addition to Paden-Kahan subproblems, it was necessary to design new subproblems for translation joints. **Following Paden-Kahan's example, I proposed three translation subproblems with a similar concept.** Besides, in this chapter I also present a new subproblem for rotation joints with parallel axes.

Please allow me to give these subproblems my surname, not because I believe they are great algorithms, quite on the contrary, but because I am responsible for them. Furthermore, this way you will be able to distinguish my algorithms from other equivalent or even more efficient solutions, which you can find in literature or create by yourself.

Here you will find the four **Pardos-Gotor subproblems (PG)**: translation along a single axis (PG1), two translations along two subsequent crossing axes (PG2), translation to a given distance (PG3), and rotation about two subsequent parallel axes (PG4).

Don't forget that you too can design and develop your own subproblems for new robot applications!

EXAMPLES of Inverse Kinematics



“Take to KINEMATICS. It will repay you. It is more fecund than geometry; it adds a fourth dimension to space.”

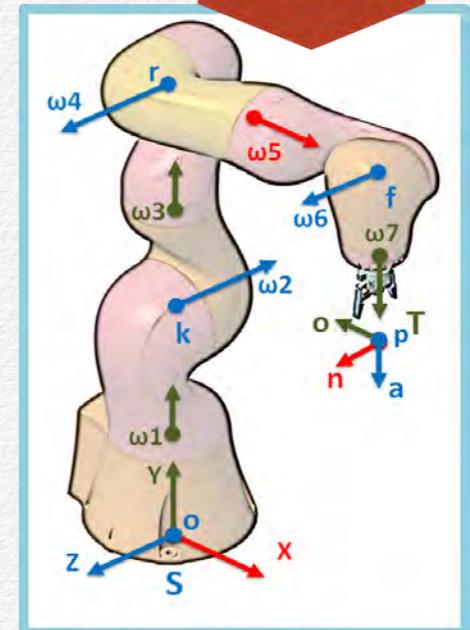
— Pafnuti Chebyshov

NEW

IK - REDUNDANT KUKA IIWA R820: θ_7 Solution

We pass $H_{st}(0)$ and the exponentials of the first six screws, which we already know from the previous steps of the algorithm (i.e. we already got " θ_1 ", " θ_2 ", " θ_3 ", " θ_4 ", " θ_5 " and " θ_6 "), to the right hand side of the problem definition equation. Then, we apply both sides of the equation to point "o" (origin of the spatial reference frame "S"). The right hand side of the equation is now a known value " k_4 ". The resulting equation is exactly the definition for the canonic **Paden-Kahan subproblem one PK1**. And we know the geometric inverse kinematics solutions for PK1. Finally, for each set of " $\theta_1-\theta_2-\theta_3-\theta_4-\theta_5-\theta_6$ " values, we solve the motion for the seventh joint of the robot, obtaining the rotation magnitude for " θ_7 " single solution.

$$e^{\hat{\xi}_7 \theta_7} o = e^{-\hat{\xi}_6 \theta_6} e^{-\hat{\xi}_5 \theta_5} e^{-\hat{\xi}_4 \theta_4} e^{-\hat{\xi}_3 \theta_3} e^{-\hat{\xi}_2 \theta_2} e^{-\hat{\xi}_1 \theta_1} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} o$$



SOLUTION "θ₇"

The θ_6 , θ_5 , θ_4 , θ_3 , θ_2 and θ_1 values
are already known from previous
calculations

**CANONIC Problem
PADEN-KAHAN-ONE
(PK1)**

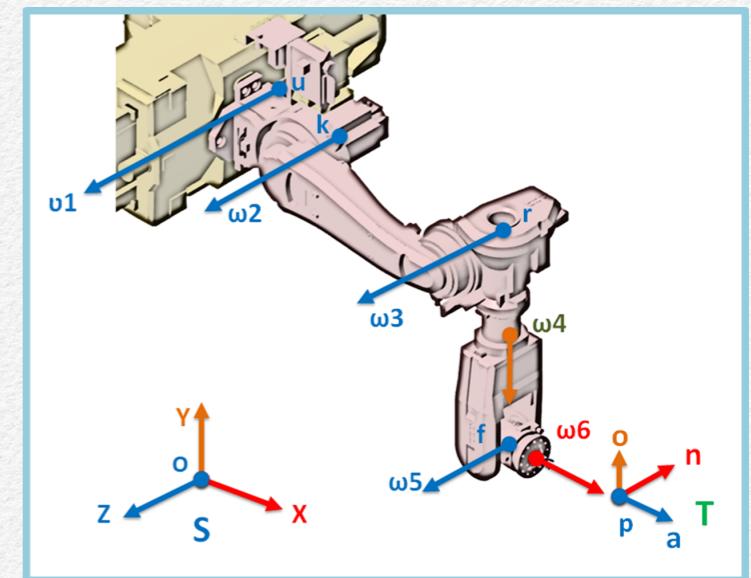
$$e^{\hat{\xi}_7 \theta_7} o = k_4$$

$$\begin{aligned} \theta_1^{01}, \theta_2^{01}, \theta_3^{in}, \theta_4^{01}, \theta_5^{01}, \theta_6^{01} &\Rightarrow \theta_7^{01} \\ \theta_1^{01}, \theta_2^{01}, \theta_3^{in}, \theta_4^{01}, \theta_5^{02}, \theta_6^{02} &\Rightarrow \theta_7^{02} \\ \theta_1^{02}, \theta_2^{02}, \theta_3^{in}, \theta_4^{01}, \theta_5^{03}, \theta_6^{03} &\Rightarrow \theta_7^{03} \\ \theta_1^{02}, \theta_2^{02}, \theta_3^{in}, \theta_4^{01}, \theta_5^{04}, \theta_6^{04} &\Rightarrow \theta_7^{04} \\ \theta_1^{03}, \theta_2^{03}, \theta_3^{in}, \theta_4^{02}, \theta_5^{05}, \theta_6^{05} &\Rightarrow \theta_7^{05} \\ \theta_1^{03}, \theta_2^{03}, \theta_3^{in}, \theta_4^{02}, \theta_5^{06}, \theta_6^{06} &\Rightarrow \theta_7^{06} \\ \theta_1^{04}, \theta_2^{04}, \theta_3^{in}, \theta_4^{02}, \theta_5^{07}, \theta_6^{07} &\Rightarrow \theta_7^{07} \\ \theta_1^{04}, \theta_2^{04}, \theta_3^{in}, \theta_4^{02}, \theta_5^{08}, \theta_6^{08} &\Rightarrow \theta_7^{08} \end{aligned}$$

$$\begin{aligned} \theta_1^{in}, \theta_2^{05}, \theta_3^{01}, \theta_4^{01}, \theta_5^{09}, \theta_6^{09} &\Rightarrow \theta_7^{09} \\ \theta_1^{in}, \theta_2^{05}, \theta_3^{01}, \theta_4^{01}, \theta_5^{10}, \theta_6^{10} &\Rightarrow \theta_7^{10} \\ \theta_1^{in}, \theta_2^{06}, \theta_3^{02}, \theta_4^{01}, \theta_5^{11}, \theta_6^{11} &\Rightarrow \theta_7^{11} \\ \theta_1^{in}, \theta_2^{06}, \theta_3^{02}, \theta_4^{01}, \theta_5^{12}, \theta_6^{12} &\Rightarrow \theta_7^{12} \\ \theta_1^{in}, \theta_2^{07}, \theta_3^{03}, \theta_4^{02}, \theta_5^{13}, \theta_6^{13} &\Rightarrow \theta_7^{13} \\ \theta_1^{in}, \theta_2^{07}, \theta_3^{03}, \theta_4^{02}, \theta_5^{14}, \theta_6^{14} &\Rightarrow \theta_7^{14} \\ \theta_1^{in}, \theta_2^{08}, \theta_3^{04}, \theta_4^{02}, \theta_5^{15}, \theta_6^{15} &\Rightarrow \theta_7^{15} \\ \theta_1^{in}, \theta_2^{08}, \theta_3^{04}, \theta_4^{02}, \theta_5^{16}, \theta_6^{16} &\Rightarrow \theta_7^{16} \end{aligned}$$

IK - GANTRY ABB IRB6620LX: θ_6 Solution

We pass $H_{st}(0)$ and the exponentials of the first five screws, which we already know from the previous steps of the algorithm (i.e. we already have " θ_1 ", " θ_2 ", " θ_3 ", " θ_4 " and " θ_5 "), to the right hand side of the problem definition equation. Then, we apply both sides of the equation to point "o" (origin of the spatial reference frame). The right hand side of the equation is now a known value " k_4 ". The resulting equation is exactly the definition for the canonical **Paden-Kahan subproblem one PK1**. And we know the geometric inverse kinematics solutions for PK1. Finally, for each set of " $\theta_1-\theta_2-\theta_3-\theta_4-\theta_5$ " values, we solve the motion for the sixth joint of the robot, obtaining the rotation magnitude for " θ_6 " solution.



$$e^{\hat{\xi}_6 \theta_6} o = \boxed{e^{-\hat{\xi}_5 \theta_5} e^{-\hat{\xi}_4 \theta_4} e^{-\hat{\xi}_3 \theta_3} e^{-\hat{\xi}_2 \theta_2} e^{-\hat{\xi}_1 \theta_1}} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} H_{ST}(0)^{-1} o$$

The θ_5 , θ_4 , θ_3 , θ_2 and θ_1 values are already known from previous calculations

$$e^{\hat{\xi}_6 \theta_6} o = k_4$$

CANONIC Problem
PADEN-KAHAN-ONE
(PK1)

SOLUTION " θ_6 "

$$\begin{aligned} \theta_1^{01}, \theta_2^{01}, \theta_3^{01}, \theta_4^{01}, \theta_5^{01} &\Rightarrow \theta_6^{01} \\ \theta_1^{01}, \theta_2^{01}, \theta_3^{01}, \theta_4^{02}, \theta_5^{02} &\Rightarrow \theta_6^{02} \\ \theta_1^{01}, \theta_2^{02}, \theta_3^{02}, \theta_4^{03}, \theta_5^{03} &\Rightarrow \theta_6^{03} \\ \theta_1^{01}, \theta_2^{02}, \theta_3^{02}, \theta_4^{04}, \theta_5^{04} &\Rightarrow \theta_6^{04} \end{aligned}$$

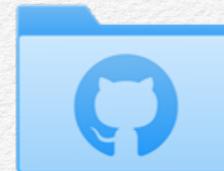
NEW

Inverse Kinematics REDUNDANT Collaborative Robot (KUKA IIWA R820) Problem: solution implementation

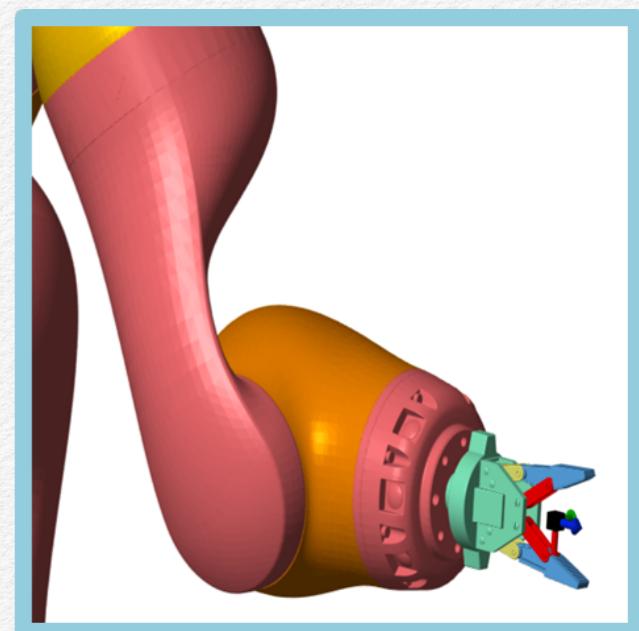
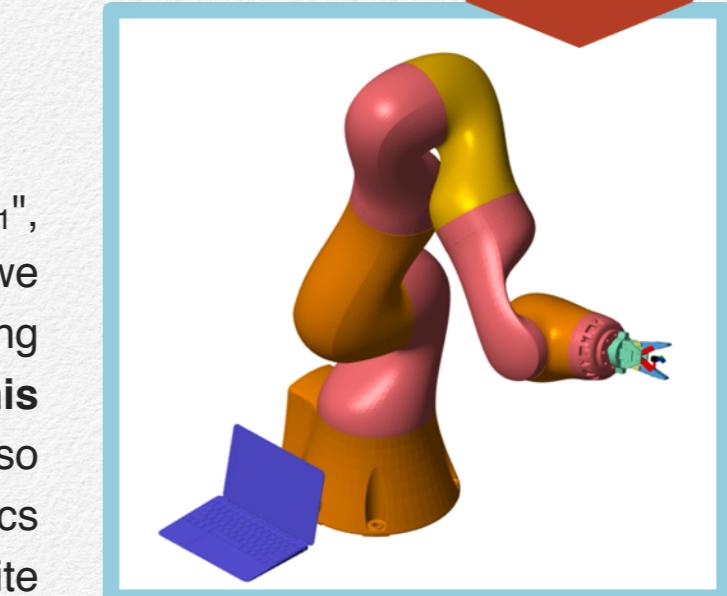
The geometric approach for solving this problem with POE implies that we first solve the " θ_1 ", " θ_2 ", " θ_3 " and " θ_4 " magnitudes in order to position the "f" wrist point in space, and afterwards we solve " θ_5 ", " θ_6 " and " θ_7 ". This algorithm uses Paden-Kahan subproblems solutions for positioning "f", assuming that the robot has a spherical wrist. However, be **careful not to confuse this algorithm with the “kinematics uncoupling”**. It is evident from the figure that " θ_5 " and " θ_6 " also imply translation for TCP in the 3D space, and not only rotation, as it happens with the kinematics uncoupling assumption. Here there is no kinematics simplification and the algorithm is quite elegant and useful, providing **16 exact geometric solutions**. With this broad set of closed-form results for the inverse kinematics of this robot, it is easier to implement a variety of applications. We can exploit the workspace given by this redundant mechanism much better. Even if there is no exact solution, it is quite convenient to find an approximate solution out of this set.

Due to the fact that the ST24R "Screw Theory Toolbox for Robotics" is programmed for giving always the most approximate solutions for the inverse kinematics canonical problems, the results of the algorithm could be inexact (but approximate) when the TCP target is out of the robot's dexterous workspace. This, in any case proves very useful for most applications. If you need another behaviour for your robot, you only need to reprogram the inverse kinematics canonical problems with your own approach.

The INVERSE KINEMATICS of this REDUNDANT
COLLABORATIVE robot KUKA IIWA is solved
with SIXTEEN EXACT CLOSED-FORM (geometric)
SOLUTIONS for the set $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7)$



“E060_STR_EXAMPLESInvKin_80_KUKAIWAR820.m”



<https://github.com/DrPardosGotor/ScrewTheoryRobotics-KINEMATICS-Illustrated/tree/master/Exercises>

DIFFERENTIAL Kinematics



“In turning from the smaller instruments in frequent use to the larger and more important machines, the economy arising from the increase of VELOCITY becomes more striking.”

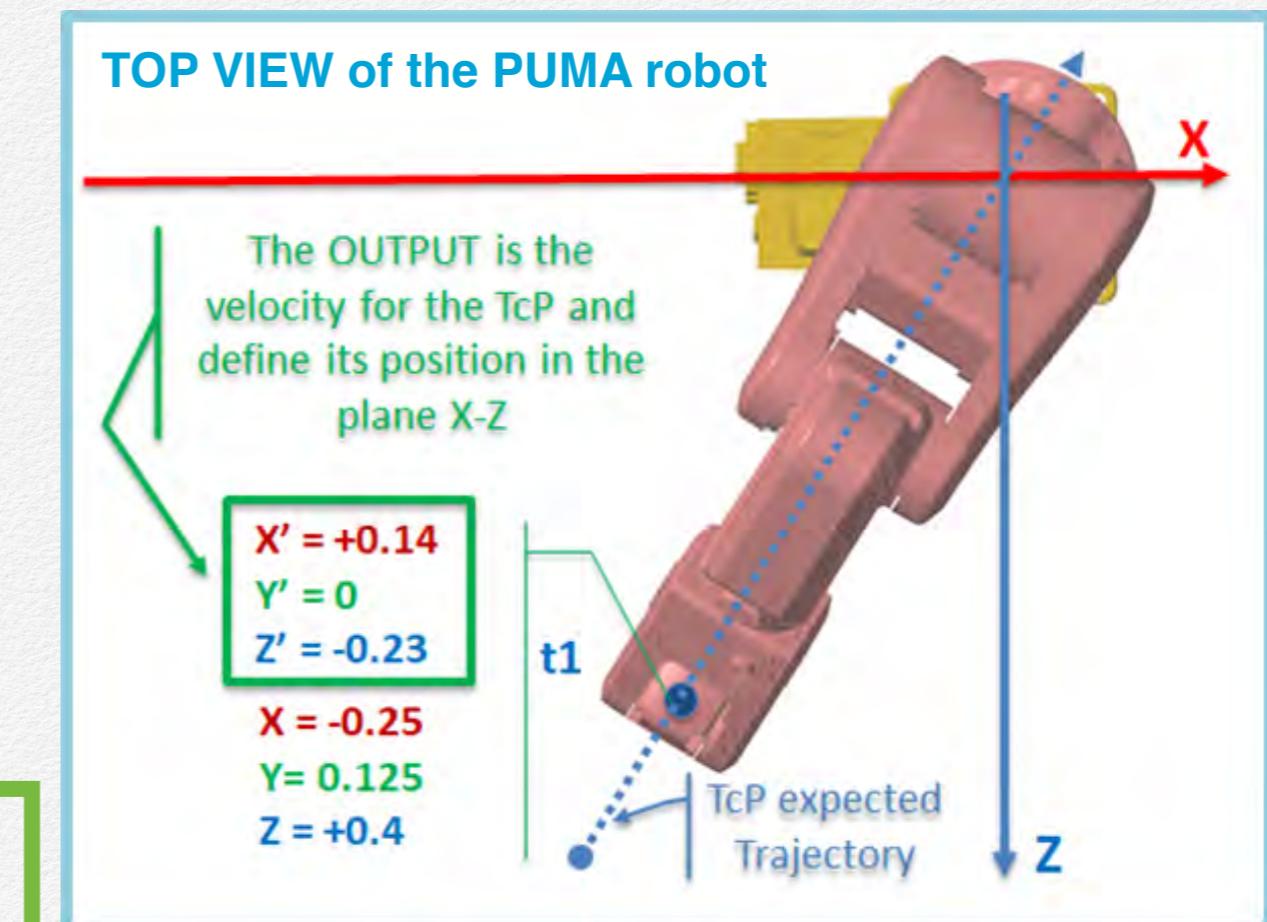
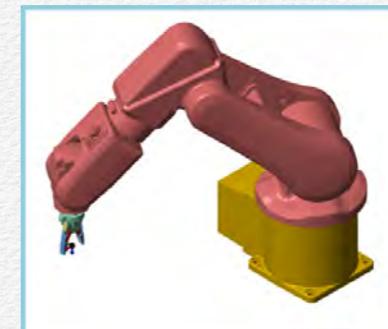
— Charles Babbage

Differential FORWARD Kinematics - (ABB IRB120): VELOCITY of the TOOL with GEOMETRIC Jacobian: (cont.)

4 (cont.).- To finally obtain the tool velocity in the spatial frame “ V_T ”, we use the spatial velocity just calculated and the **tool position in that pose “t1”, this is “TCP (X = -0.25, Y = 0.125, Z = 0.4)”.**

It can seem a bit surprising that the velocity result for the differential forward kinematics of this Puma robot is exactly the same as the one obtained for the Scara robot. This is due to the fact that we have intended for the same trajectory velocity of the tool for both robots. Apparently, there is no difference so far, but once they develop the trajectory, you will realise how different the behaviour of those mechanisms are. We will see this in the next exercise for differential inverse kinematics.

$$V_T^S = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \left(v_{ST}^S \right) + \left(\omega_{ST}^S \right)^{\wedge} \begin{bmatrix} TcP_X \\ TcP_Y \\ TcP_Z \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0 \\ -0.23 \\ 0 \\ 0 \\ -0.92 \end{bmatrix}$$



<https://github.com/DrPardosGotor/ScrewTheoryRobotics-KINEMATICS-Illustrated/tree/master/Exercises>

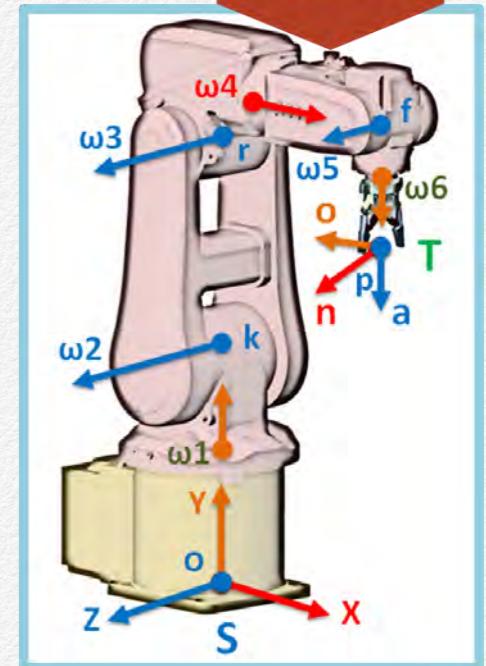
“E070_STR_DIFFERENTIALKin_40_ABBIRB120_GeoForJac.m”

NEW

Differential INVERSE Kinematics - (ABB IRB120): VELOCITY of the JOINTS with GEOMETRIC Jacobian

5.- We want to get the velocity of the joints as a function of the inverse spatial Jacobian and the desired tool velocities "V_T". Given the POSITION velocity for the TCP (i.e. v_{TCP}=(x',y',z')) and the ROTATION velocity for the tool system (i.e. ω_{ST}=(α',β',γ')), we obtain the robot joint velocities (i.e. θ₁', θ₂', θ₃', θ₄', θ₅', θ₆'), in the spatial frame.

$$\dot{\theta} = (J_{ST}^S(\theta))^{-1} V_{ST}^S \Rightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} = [\xi'_1 \ \xi'_2 \ \xi'_3 \ \xi'_4 \ \xi'_5 \ \xi'_6]^{-1} \begin{bmatrix} v_{ST}^S \\ \omega_{ST}^S \end{bmatrix}$$



6.- Then we get this general joint velocity (i.e. θ₁', θ₂', θ₃', θ₄', θ₅', θ₆') expression, based on the screw theory geometric Jacobian, but considering the information that usually we have, which is the tool velocity at the pose (i.e. x',y',z',α',β',γ').

POSITION of the TCP from the desired trajectory or forward kinematics of the robot

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} = [\xi'_1 \ \xi'_2 \ \xi'_3 \ \xi'_4 \ \xi'_5 \ \xi'_6]^{-1} \begin{bmatrix} (v_{TCP}^S) - (\omega_{ST}^S)^{\wedge} TcP(\theta) \\ \omega_{ST}^S \end{bmatrix}$$

$$= [\xi'_1 \ \xi'_2 \ \xi'_3 \ \xi'_4 \ \xi'_5 \ \xi'_6]^{-1} \begin{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} - \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}^{\wedge} \begin{bmatrix} TcP_x \\ TcP_y \\ TcP_z \end{bmatrix} \\ \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -\dot{\gamma} & \dot{\beta} \\ \dot{\gamma} & 0 & -\dot{\alpha} \\ -\dot{\beta} & \dot{\alpha} & 0 \end{bmatrix}$$

Simulation

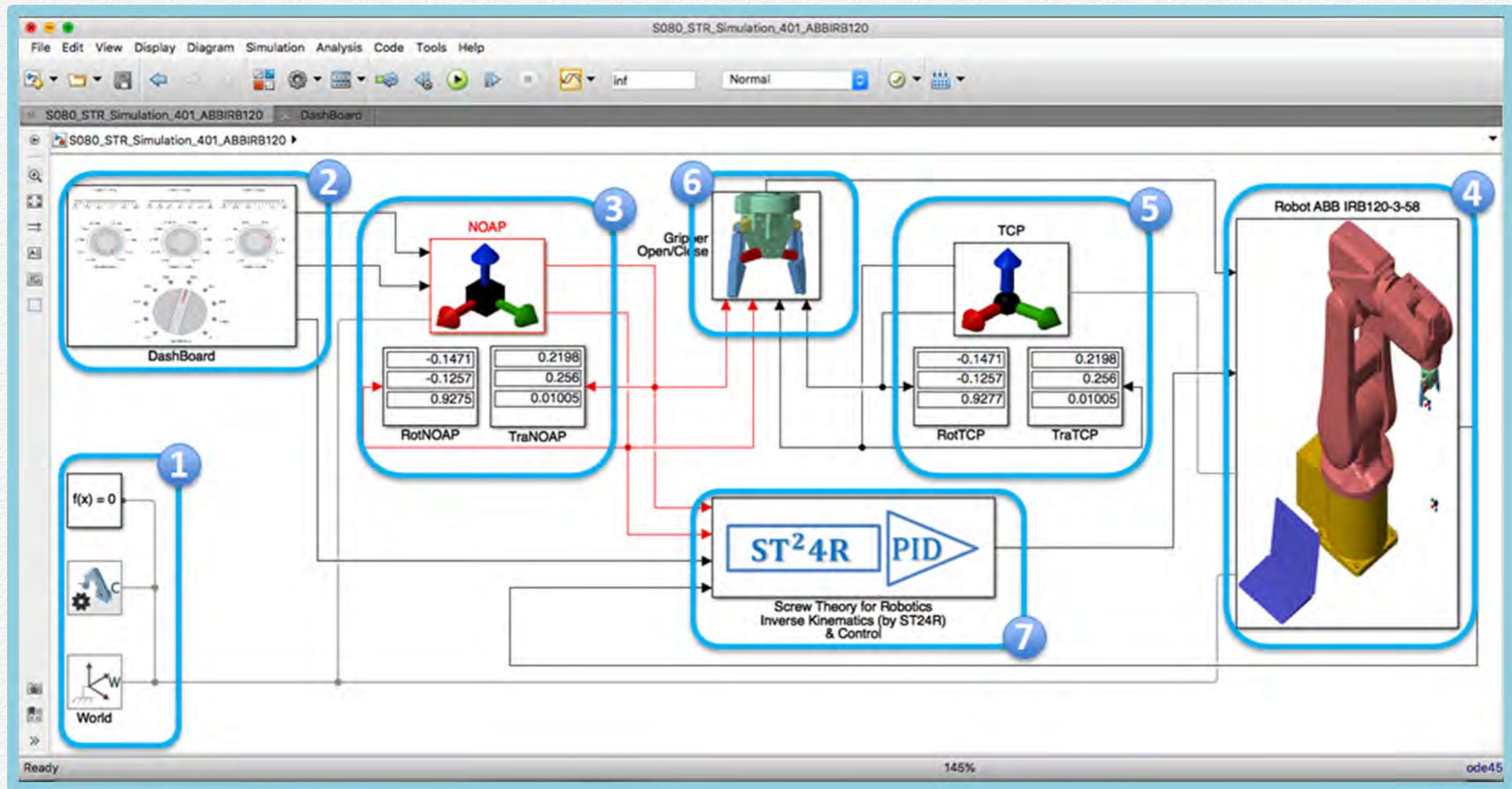


*“Without LABORATORIES
men of science are soldiers
without arms.”*

— Louis Pasteur

40.0.- Simulation for ABB IRB120 - Complete model

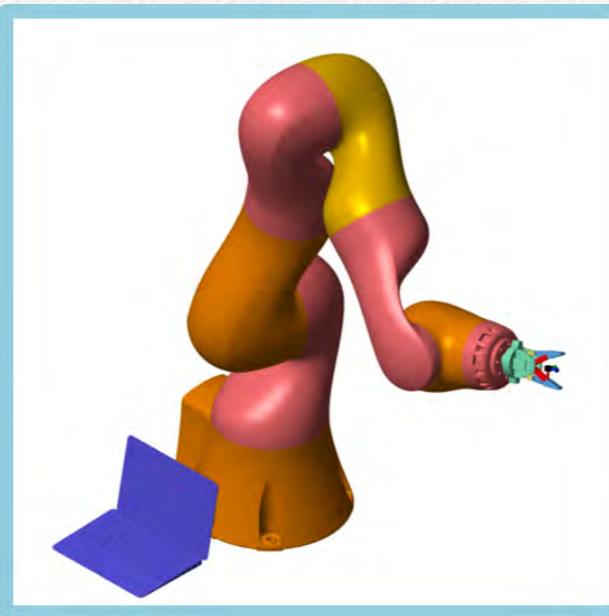
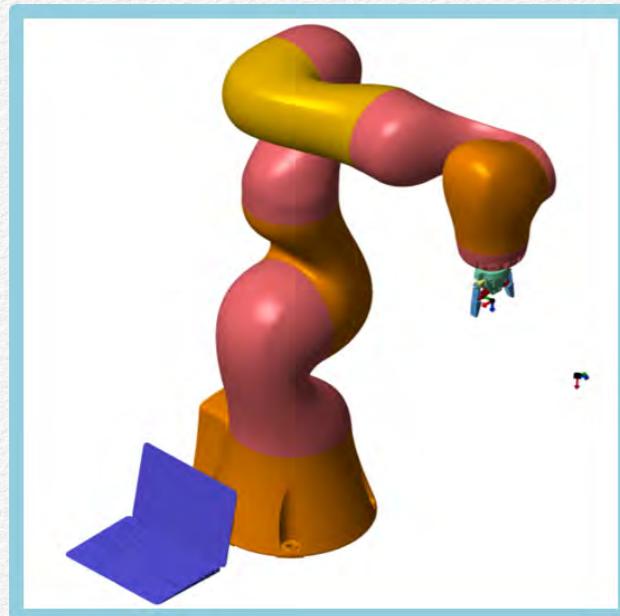
You will find all files you need for the construction of the complete robotics simulation in the corresponding folder (.zip file in the previous page). You will have a Simulink® file with the complete model “S080_STR_Simulation_401_ABBIRB120.slx”. Now we will go through the seven main blocks of this simulator model.



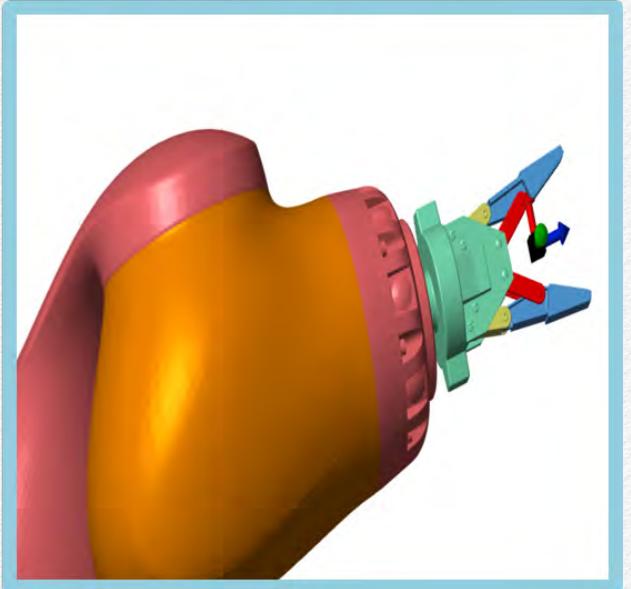
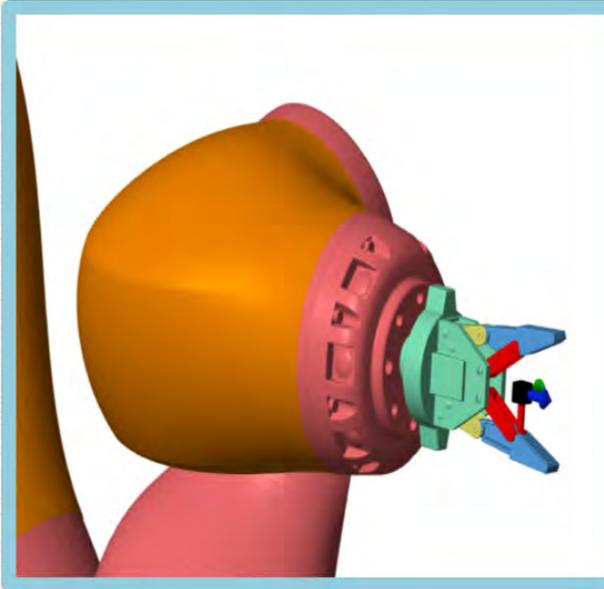
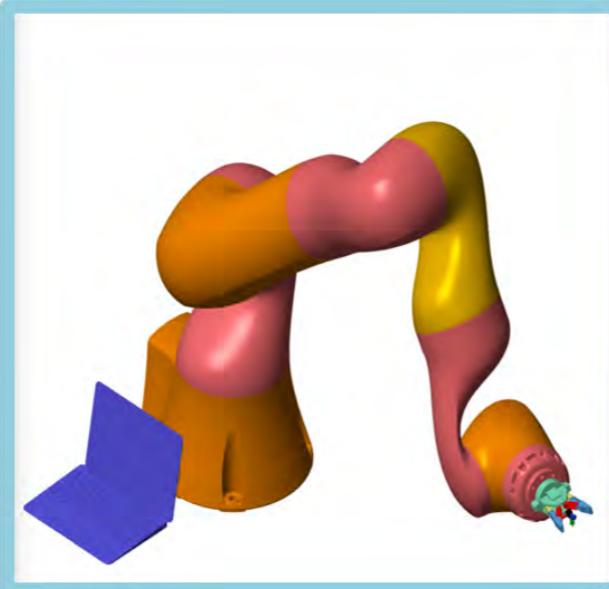
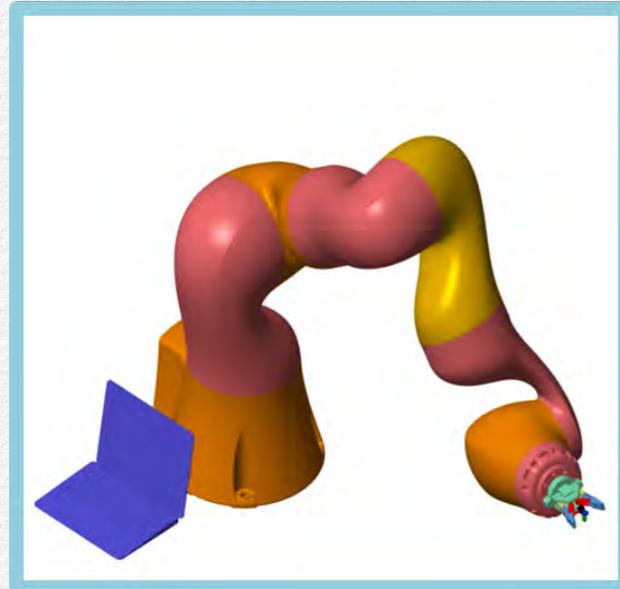
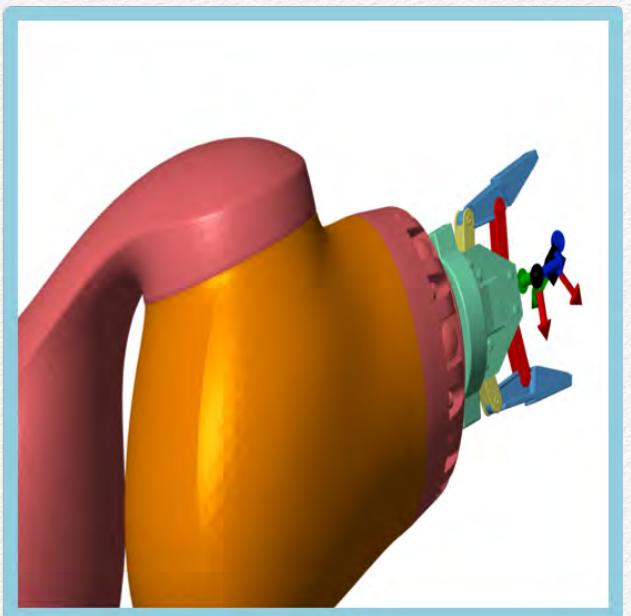
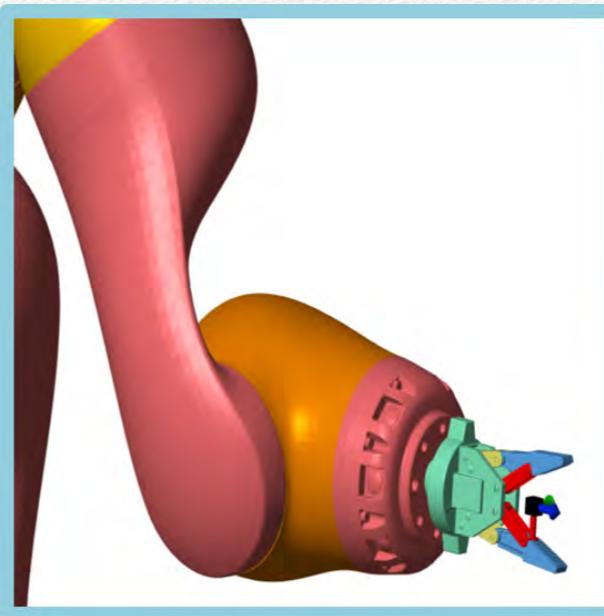
80.9.- Simulation for KUKA IIWA - Video Creator and Recording

To test the algorithms it is really useful to have videos which allow you to test very quickly the performance of the developments. In addition, the visual material comes in handy for communication purposes. For this exercise, two videos are available in YouTube. The first one shows the 16 geometric solutions for the inverse kinematics with a general view. The second is focused on the tool details.

 ["V080 STR Simulation 801 KUKAIIWA Complete"](#)



 ["V080 STR Simulation 802 KUKAIIWA Focus"](#)



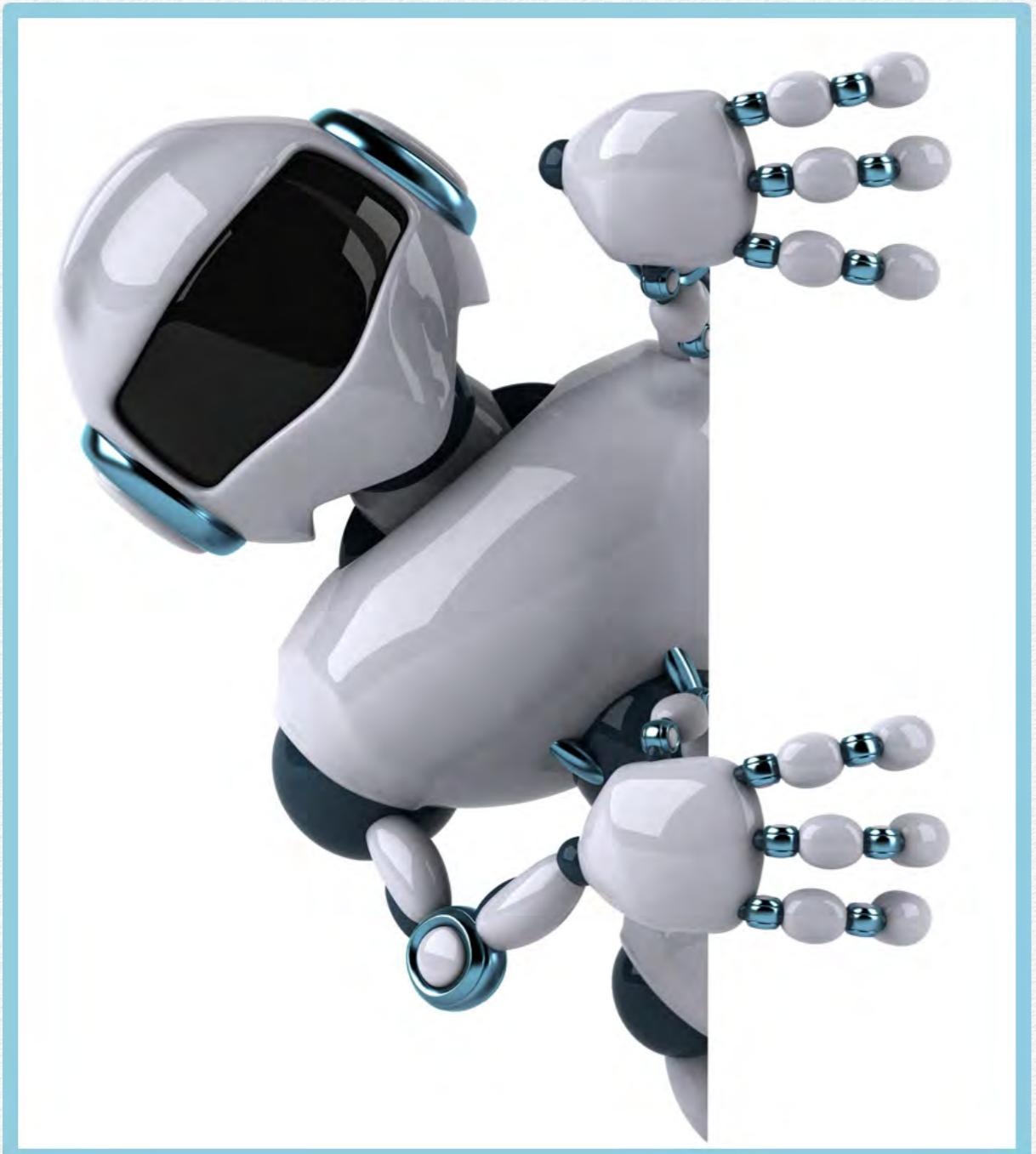
Conclusions



*“There’s nothing more
PRACTICAL than a good
THEORY.”*

— Gilbert K. Chesterton

Lessons Learned



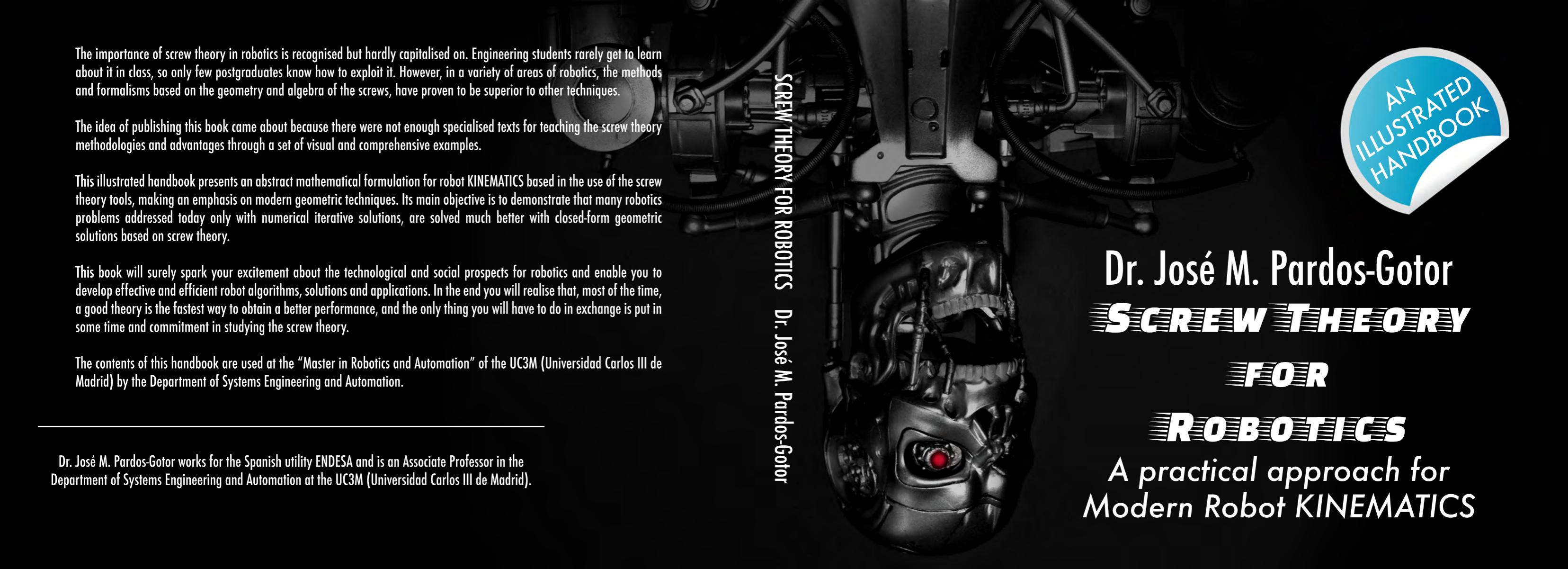
Screw Theory for Robotics Benefits

The importance of screw theory is recognised in applications for robotics, because it allows a better capture of the physical features of a robot by a geometric description. Nevertheless, these tools have remained quite inaccessible to many because they require a new language (e.g. screws, twists). This book has displayed and exhibited a clear route to overcome these difficulties, with an accessible and visual approach to these methodologies and derived tools. The practical examples and graphical structures, facilitate the access to KINEMATICS.

Most robot algorithms rely on standard algebraic alternatives and end up buried in the details of calculation. Conversely, at the heart of the screw theory there is a simple, but high level and meaningful geometric interpretation of mechanics. **Many robotics problems addressed today only with numerical iterative solutions, are solved much better with closed-form geometric solutions based on screw theory**, which are far more convenient for real time applications.

The main lesson learned is that **the elegance of the underlying screw theory makes for more effective and efficient robotics algorithms and applications**.

To conclude, have a look at these seven key takeaways regarding screw theory learned along this handbook:



The importance of screw theory in robotics is recognised but hardly capitalised on. Engineering students rarely get to learn about it in class, so only few postgraduates know how to exploit it. However, in a variety of areas of robotics, the methods and formalisms based on the geometry and algebra of the screws, have proven to be superior to other techniques.

The idea of publishing this book came about because there were not enough specialised texts for teaching the screw theory methodologies and advantages through a set of visual and comprehensive examples.

This illustrated handbook presents an abstract mathematical formulation for robot KINEMATICS based in the use of the screw theory tools, making an emphasis on modern geometric techniques. Its main objective is to demonstrate that many robotics problems addressed today only with numerical iterative solutions, are solved much better with closed-form geometric solutions based on screw theory.

This book will surely spark your excitement about the technological and social prospects for robotics and enable you to develop effective and efficient robot algorithms, solutions and applications. In the end you will realise that, most of the time, a good theory is the fastest way to obtain a better performance, and the only thing you will have to do in exchange is put in some time and commitment in studying the screw theory.

The contents of this handbook are used at the "Master in Robotics and Automation" of the UC3M (Universidad Carlos III de Madrid) by the Department of Systems Engineering and Automation.

Dr. José M. Pardos-Gotor works for the Spanish utility ENDESA and is an Associate Professor in the Department of Systems Engineering and Automation at the UC3M (Universidad Carlos III de Madrid).

SCREW THEORY FOR ROBOTICS Dr. José M. Pardos-Gotor

Dr. José M. Pardos-Gotor
SCREW THEORY
FOR
ROBOTICS
A practical approach for
Modern Robot KINEMATICS

