MA 454 HW4

Harrison Thomas
(hgthomas)

February 2025

1

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

a

$$\det\left(\begin{bmatrix} 1-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}\right) = (1-\lambda)(2-\lambda) = 0$$

$$\boldsymbol{\lambda_1 = 1}, \boldsymbol{\lambda_2 = 2}$$

b

$$\vec{\lambda_1} = \begin{bmatrix} 1-1 & 1 \\ 0 & 2-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \vec{\lambda_2} = \begin{bmatrix} 1-2 & 1 \\ 0 & 2-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{\lambda_1} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \vec{\lambda_2} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{\lambda_1} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad -x_1 + x_2 = 0$$

$$0x_1 + x_2 = 0$$
$$x_2 = 0,\ x_1 = 1$$

$$\vec{\boldsymbol{\lambda_1}} = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix}$$

$$x_2 = x_1$$
$$x_1 = 1,\ x_2 = 1$$

$$\vec{\boldsymbol{\lambda_2}} = \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \end{bmatrix}$$

c

$$\Phi(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^t, \; \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^2 t$$

$$\boldsymbol{\Phi(t) = \begin{bmatrix} e^t & e^{2t} \\ 0 & e^{2t} \end{bmatrix}}$$

$$\det(\Phi(t)) = e^t(e^{2t}) = e^{3t} \neq 0$$

Non-Singular

2

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

a

$$\det\left(\begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} \right) = (1-\lambda)^2 - 1 = 0$$

$$\Big((1-\lambda) - 1\Big)\Big((1-\lambda) + 1\Big) = 0$$
$$(-\lambda)(2-\lambda) = 0$$
$$\boldsymbol{\lambda_1 = 0}, \; \boldsymbol{\lambda_2 = 2}$$

b

$$\vec{\lambda_1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \vec{\lambda_2} = \begin{bmatrix} 1-2 & 1 \\ 1 & 1-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{\lambda_1} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \vec{\lambda_2} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_1 + x_2 = 0 \quad \vec{\lambda_2} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_1 = -x_2$$

$$\boldsymbol{\vec{\lambda_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$$\boldsymbol{\vec{\lambda_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

c

$$\Phi(t) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{0t},\ \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$$

$$\boldsymbol{\Phi(t) = \begin{bmatrix} 1 & e^{2t} \\ -1 & e^{2t} \end{bmatrix}}$$

$$\det(\Phi(t)) = e^{2t} + e^{2t} = 2e^{2t} \neq 0$$

Non-singular

3

a)

$$\Phi(0)C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

b)

$$\Phi(0)C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

4

$$\Phi(0)C = I$$
$$C = \Phi(0)^{-1}$$

$$\Phi(t)\Phi(0)^{-1} = e^{At}$$