So You Like Multivariable Calculus

A reference guide by a low level idiot verified by high level idiots

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Contents

1 Forward

Thanks for buying Multivaribale Calculus for Dumb... I mean the So You Like Multivariable Calculus Reference Guide. This is a reference guide made for university students by a university student. It was verified by multiple professors who are much more intelligent than that student; so, if it starts to get a little too pedantic, then you know who to blame. Because this is for broke poor college students, there is and will always be a free version in the right and wrong places (I don't care which one you use). However, if you would like to support me and more importantly the massive publishing conglomerate, then the physical version of this text is the best way to do it. Special thanks to professors XYZ who asked not to be named due to the horrible reputation they would acquire among their peers. This guide was also edited by XYZ, who was told not to touch the forward, so if it starts to feel more cohesive and have less grammatical errors and not have any run-on sentences, then you know who to blame. Alright, enough jibber jabbering let's get into the math that will dominate the next semester of your life. (Jokes in this forward were not S&P Approved.)

2 Introduction

This is an introduction to basic principles used throughout the guide.

2.1 Formulas

- **Distance Formula** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- **Midpoint Formula** $M = (\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}, \frac{z_2+z_1}{2})$
- **Standard Equation of a Sphere** $R^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$
- **General Equation of a Sphere** $V = Ax^2+Ay^2+Az^2+Bx+Cy+Dz+E$

2.2 Cylindrical Surfaces

If you're missing a coordinate then you get a cylindrical surface. First you graph the function in two dimensions, then extend the function forwards and backwards.

2.3 Introduction to Vectors

Displacement Vectors start at an initial point P and end at a terminal point Q. To make a vector given two points you subtract the first point from the second.

ex.

$$\begin{aligned}
P &= (x_1, y_1, z_1) \\
Q &= (x_2, y_2, z_2) \\
\overrightarrow{PQ} &= Q - P \\
\overrightarrow{PQ} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}
\end{aligned}$$

The Zero Vector is the only vector to have magnitude but arbitrary direction. It occurs when the initial point and terminal point are in the same location.

$$\begin{aligned}
\vec{p} + \vec{0} &= \vec{p} \\
(\vec{q})\vec{0} &= \vec{0}
\end{aligned}$$

Scalars can multiply vectors by a positive value which multiplies the magnitude and preserves direction; or, can multiply by a negative value which multiples the magnitude and changes to the opposite direction.

Vector addition adds both magnitude and direction of each vector.

Vector Norms look like $||\vec{v}||$ and are the magnitude of the vector (essentially the absolute value.)

Orthonormal Vectors are $\hat{i}, \hat{j}, \hat{k}$ and they represent a vector of magnitude 1 in the x, y, and z directions respectively.

2.3.1 Vector Operations

- **Norm of the Vector**

$$||\vec{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

- **Scalar Multiplication**

$$\begin{aligned}
C\vec{v} &= (Cv_1)\hat{i} + (Cv_2)\hat{j} + (Cv_3)\hat{k} \\
&= [Cv_1, Cv_2, Cv_3]
\end{aligned}$$

- **Scalar Addition**

$$\begin{aligned}
\vec{u} + \vec{v} &= u_1\hat{i} + u_2\hat{j} + v_1\hat{i} + v_2\hat{j} \\
&= (u_1 + v_1)\hat{i} + (u_2 + v_2)\hat{j}
\end{aligned}$$

2.3.2 Polar Form of Vectors in 2-D

$\vec{v} = ||\vec{v}|| \cos(\theta)\hat{i} + ||\vec{v}|| \sin(\theta)\hat{j}$

2.4 Dot Product

Dot Product - Multiplying components of two or more vectors together (results in a scalar.)

$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3 + u_nv_n$$
$$\vec{u}\cdot\vec{v} = ||\vec{u}||\cdot||\vec{v}||\cos{(\theta)}$$

where θ is the acute angle between the vectors

ex.

$$\vec{u} = 3\hat{i} + 2\hat{j} - \hat{k}$$
$$\vec{v} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\begin{aligned}\vec{u}\cdot\vec{v} &= (3\cdot 2) + (2\cdot -1) + (-1\cdot 3)\\ &= 6 - 2 - 3 = 1\end{aligned}$$

****Note**

$$\text{if } (\vec{u}\cdot\vec{v}) > 0,\ 0^\circ \leq \theta < 90^\circ$$
$$\text{if } (\vec{u}\cdot\vec{v}) = 0,\ \theta = 90^\circ$$
$$\text{if } (\vec{u}\cdot\vec{v}) < 0,\ 90^\circ > \theta \geq 180^\circ$$

2.5 Projections

Parallel Projection

$$P_{||} = Proj_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{||\vec{v}||}(\vec{v})$$

Perpendicular Projection

$$P_{\perp} = \vec{u} - proj_{\vec{v}}\vec{u}$$

2.6 Cross Product

$$\vec{u}\text{ x }\vec{v} = det\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} = u_1(v_2) - u_2(v_1)$$

3x3 case

$$det\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \hat{\imath}\ det\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - \hat{\jmath}\ det\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + \hat{k}\ det\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

2.6.1 Geometric Properties of the Cross Product

- $\vec{u}$ x $\vec{v}$ is $\perp$ to both $\vec{u}$ and $\vec{v}$
- $\vec{u}$ x $\vec{v}$ is $=$ to the area of the parallelogram spanned by $\vec{u}$ and $\vec{v}$
- $\vec{u}$ x $\vec{v} = -(\vec{v}$ x $\vec{u})$
- if $\vec{u}$ x $\vec{v} = 0$ then $\vec{u}$ and $\vec{v}$ are parallel.

2.6.2 Cross Product with Basis Elements

- $\hat{\imath}$ x $\hat{\jmath} = \hat{k}$
- $\hat{\jmath}$ x $\hat{k} = \hat{\imath}$
- $\hat{k}$ x $\hat{\imath} = \hat{\jmath}$

ex

$$\begin{aligned} \hat{\jmath} \text{ x } (\hat{\imath} + \hat{\jmath} + \hat{k}) &= \hat{\jmath} \text{ x } \hat{\imath} + \hat{\jmath} \text{ x } \hat{\jmath} + \hat{\jmath} \text{ x } \hat{k} \\ &= \hat{\imath} - \hat{k} \end{aligned}$$

- Polar Formula for $\vec{u}$ x $\vec{v}$

$$||\vec{u} \text{ x } \vec{v}|| = ||\vec{u}|| \cdot ||\vec{v}|| \sin(\theta)$$

- Triple Scalar Product

$$\vec{a} \cdot (\vec{b} \text{ x } \vec{c}) = det\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \text{volume of the parallelpiped spanned by } \vec{a},\ \vec{b}, \text{ and } \vec{c}$$

2.7 Parametric Equations of a Line

The Parametric Form

The line that passes through (x_0, y_0, z_0) and is parallel to the vector $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ is given by:

$$\begin{aligned} x &= x_0 + v_1 t \\ y &= y_0 + v_2 t \\ z &= z_0 + v_3 t \end{aligned}$$

2.8 Parametric Representation of a Line

Parametric Form

$$\begin{aligned} x &= x_1 + \Delta x \cdot t \\ y &= y_1 + \Delta y \cdot t \\ z &= z_1 + \Delta z \cdot t \\ 0 &\leq t \leq 1 \end{aligned}$$

Vector Form

$$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$$

ex.

P = (2, 3, 1), Q = (4, 5, 6)

$$\begin{aligned} x &= 2 + (4-2)t = 2 + 2t \\ y &= 3 + (5-3)t = 3 + 2t \\ z &= 1 + (6-1)t = 1 + 5t \end{aligned}$$

$$\vec{v} = (2+2t)\hat{i} + (3+2t)\hat{j} + (1+5t)\hat{k} \qquad \text{for } 0 \leq t \leq 1$$

2.9 Planes in 3-Space

A unique vector can be determined with a point (x_0, y_0, z_0) and a vector perpendicular to the plane denoted by $\vec{n}$.
Point-normal Formula: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Vector Form:

$$\overrightarrow{(v - v_0)} \cdot \vec{n} = 0, \qquad \vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$$

General Equation of a Plane: $ax + by + cz + d = 0$

ex.

Find the equation of the plane through point (1,2,3) and perpendicular to $\vec{n} = \langle 10, 20, 30 \rangle$

$$\begin{aligned} 10(x-1) + 20(y-2) + 30(z-3) &= 0 \\ 10x - 10 + 20y - 40 + 30z - 90 &= 0 \\ 10x + 20y + 30z - 140 &= 0 \\ \frac{1}{10}[10x + 20y + 30z - 140 &= 0] \\ \\ \mathbf{x + 2y + 3z} &\mathbf{= 14} \end{aligned}$$

ex.

Find the equation of the plane containing the points P = (1, 2, 3), Q = (4, 5, 6), and R = (9, 8, 7).

$\vec{n} = \overrightarrow{PQ}$ x $\overrightarrow{PR}$

$$\begin{aligned} \overrightarrow{PQ} &= \langle (4-1), (5-2), (6-3) \rangle \\ &= \langle 3, 3, 3 \rangle \end{aligned}$$

$$\begin{aligned} \overrightarrow{PR} &= \langle (9-1), (8-2), (7-3) \rangle \\ &= \langle 8, 6, 4 \rangle \end{aligned}$$

$$\overrightarrow{PQ} \text{ x } \overrightarrow{PR} = det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 3 \\ 8 & 6 & 4 \end{vmatrix} = -6\hat{i} + 12\hat{j} - 6\hat{k} = \langle -6, 12, -6 \rangle$$

$$
\begin{aligned}
-6(x-1)+12(y-2)-6(z-3)&=0\\
-6x+6+12y-24-6z+18&=0\\
-6x+12y-6z&=0\\
\frac{-1}{6}[-6x+12y-6z&=0]
\end{aligned}
$$

$$\mathbf{x-2y+z=0}$$

2.9.1 Distance between points and a plane

The distance between a point and a plane is given by the formula:

$$\mathbf{D=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}}$$

Given the parametric representation of a line and the equation of a plane:

- $$
\begin{aligned}
\langle(2-t),(1+t),3t\rangle \qquad 3x-2y+z&=10\\
3(2-t)-2(1+t)3t&=10\\
-3t+6-2t-2+3t&=10\\
-2t+4&=10
\end{aligned}
$$

  $$\mathbf{t=-3}$$

  *Indicates one point of intersection between the line and plane

- $$
\begin{aligned}
\langle(2t+1),(3t-2),(4t-1)\rangle \qquad x+2y-2z&=5\\
2t+1+2(3t-2)-2(4t+1)&=5\\
2t+1+6t-4-8t-2&=5
\end{aligned}
$$

  $$\mathbf{-1=5}$$

  *Indicates the line does not intersect with the plane

-

$$\langle (2t+1), (3t-2), (4t-1)\rangle \qquad x+2y-2z=-1$$
$$2t+1+2(3t-2)-2(4t+1)=5$$
$$2t+1+6t-4-8t-2=5$$

$$\mathbf{-1=-1}$$

*Indicates the line lies within the plane

2.10 Quadratic Surfaces

Ellipsoid

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Elliptic Cone

$f(x,y)=\pm\sqrt{(\frac{x^2}{a^2}+\frac{y^2}{b^2})}$

Elliptic Paralleloid

$f(x,y)=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Hyperbolic Paralleloid

$f(x,y)=\frac{x^2}{a^2}-\frac{y^2}{b^2}$

Hyperboloid of One Sheet

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Hyperboloid of Two Sheets

$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

2.11 Cylindrical and Spherical Coordinates

Polar Coordinates

Cylindrical Coordinates

Cylindrical Coordinates are the same as polar coordinates with the added z dimension.

- $(x,y,z)_R$ - Denotes rectangular Coordinates
- $(r,\theta,z)_C$ - Denotes Cylindrical Coordinates

- Converting from Rectangular to Cylindrical Coordinates

$$r = \sqrt{x^2 + y^2} \qquad \theta = \frac{\pi}{2} \text{ if } x = 0 \text{ and } y > 0$$
$$\theta = \tan^{-1}(\frac{y}{x}) \qquad \theta = \frac{3\pi}{2} \text{ if } x = 0 \text{ and } y > 0$$

- Converting from Cylindrical to Rectangular Coordinates

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

Spherical Coordinates

- Converting from spherical to rectangular

$$x = \rho\sin(\phi)\cos(\theta)$$
$$y = \rho\sin(\phi)\sin(\theta)$$
$$z = \rho\cos(\phi)$$

- Converting from rectangular to spherical

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\phi = \cos^{-1}(\frac{z}{\rho})$$
$$\theta = \tan^{-1}(\frac{y}{x})$$

- Converting from spherical to cylindrical

$$r = \rho\sin(\phi)$$
$$z = \rho\cos(\phi)$$
$$\theta = \theta$$

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Contents

3 So, You like Vectors?

3.1 Vector Value Functions (of one independent variable)

Parametric Equations

$$\begin{aligned} x &= f(t) \\ y &= g(t) \\ z &= h(t) \\ a \leq\ &t \leq b \end{aligned}$$

Position Vector:

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$$

Ex:

$$C : \begin{cases} \vec{r}(t) = 2t^2\hat{\imath} + 3t\hat{\jmath} \\ 0 \leq t \leq 4 \end{cases}$$

$$C : \begin{cases} \langle 2t^2, 3t \rangle \\ 0 \leq t \leq 4 \end{cases}$$

3.2 Common Parametric Equations

1)

$$C : \begin{cases} x = t \\ y = f(t) \\ t_i \leq t \leq t_f \end{cases}$$

2) Line Segment

$$C : \begin{cases} x = x_0 + (x_1 - x_0)t \\ y = y_0 + (y_1 - y_0)t \\ z = z_0 + (z_1 - z_0)t \\ 0 \leq t \leq 1 \end{cases}$$

3) Counter-Clockwise Circle

$$C : \begin{cases} x = rcos(\theta)t \\ y = rsin(\theta)t \\ 0 \leq t \leq 2\pi \end{cases}$$

4) Clockwise Circle

$$C : \begin{cases} x = -rcos(\theta)t \\ y = -rsin(\theta)t \\ 0 \leq t \leq 2\pi \end{cases}$$

5) Circular Helix

$$C : \begin{cases} x = rcos(\theta)t \\ y = rsin(\theta)t \\ z = ct \\ -\infty < t < \infty \end{cases}$$

3.3 Definitions

Let $\vec{u}(t) = u_1(t)\hat{i} + u_2(t)\hat{j} + u_3(t)\hat{k}$

1)

$$\lim_{t \to t_0} \vec{u}(t) = \lim_{t \to t_0} \vec{u_1}(t)\hat{i} + \lim_{t \to t_0} \vec{u_2}(t)\hat{j} + \lim_{t \to t_0} \vec{u_3}(t)\hat{k}$$

2)

$$\frac{d}{dt}\vec{u}(t) = \frac{du_1}{dt}\hat{i} + \frac{du_2}{dt}\hat{j} + \frac{du_3}{dt}\hat{k}$$

3)

$$\int_a^b \vec{u}(t)dt = \left[\int_a^b \vec{u_1}(t)dt\right]\hat{i} + \left[\int_a^b \vec{u_2}(t)dt\right]\hat{j} + \left[\int_a^b \vec{u_3}(t)dt\right]\hat{k}$$

4) Velocity

$$\frac{d}{dt}\vec{r}(t) = \vec{v}(t)$$

5) Acceleration

$$\frac{d}{dt}\vec{v}(t) = \vec{a}(t)$$

6) Jerk

$$\frac{d}{dt}\vec{a}(t) = \vec{g}(t)$$

7) Snap, Crackle, Pop...

I'm not kidding and it keeps going

3.4 Product Rule

1)
$$\frac{d}{dt}\Big[f(t)\vec{u}(t)\Big] = \frac{df(t)}{dt}\vec{u}(t) + \frac{d\vec{u}(t)}{dt}f(t)$$

2)
$$\frac{d}{dt}\Big[\vec{v}(t)\cdot\vec{u}(t)\Big] = \frac{d}{dt}\vec{v}(t)\cdot\vec{u}(t) + \frac{d}{dt}\vec{u}(t)\cdot\vec{v}(t)$$

3)
$$\frac{d}{dt}\Big[\vec{v}(t)\times\vec{u}(t)\Big] = \frac{d}{dt}\vec{v}(t)\times\vec{u}(t) + \frac{d}{dt}\vec{u}(t)\times\vec{v}(t)$$

3.5 Arc Length Parametrization

Essentially, this is a tool to find the arc length of a curve when the given function $f(t)$ has a variable rate of change.

3.5.1 Cartesian

$$L = \int_a^b ||\frac{d\vec{r}(t)}{dt}||ds,\ a \le t \le b$$

3.5.2 Polar/Cylindrical

$$L = \int_a^b \sqrt{(\frac{dr}{dt})^2 + r^2(\frac{d\theta}{dt})^2 + (\frac{dz}{dt})^2}ds,\ a \le t \le b$$

3.5.3 Spherical

$$L = \int_a^b \sqrt{(\frac{d\rho}{dt})^2 + \rho^2 sin^2(\phi)(\frac{d\theta}{dt})^2 + \rho^2(\frac{d\phi}{dt})^2}ds,\ a \le t \le b$$

Ex:
$$\begin{cases} \vec{r}(t) = 3cos(t)\hat{i} + 3sin(t)\hat{j} \\ 0 \le t \le \pi \end{cases}$$

1^{st}) Differentiate the Vector Function

$$\frac{d\vec{r}(t)}{dt} = -3sin(t) + 3cos(t)$$

2^{nd}) Find the Magnitude of the Derivative

$$||\frac{d\vec{r}(t)}{dt}|| = \sqrt{9sin^2(t) + 9cos^2(t)}$$
$$= \sqrt{9(sin^2(t) + cos^2(t))} = \sqrt{9} = 3$$

3^{rd}) Integrate the Magnitude

$$\int_0^{\pi} 3ds = 3\pi - 3(0) = 3\pi$$

To check this result, we can use the circumference formula for a circle and divide it by two because the range of t is only from 0 to π. $L = \frac{C}{2} = \frac{6\pi}{2} = 3\pi$

Ex:

$$\begin{cases} \vec{r}(t) = (3t+5)\hat{\imath} + (4t+2)\hat{\jmath} \\ 0 \leq t \leq t \end{cases}$$

$$\frac{d\vec{r}(t)}{dt} = 3\hat{\imath} + 4\hat{\jmath}$$
$$||3\hat{\imath} + 4\hat{\jmath}|| = \sqrt{9+16} = \sqrt{25} = 5$$
$$s = \int_0^t 5du = 5u\Big]_0^t = 5t - 5(0) = 5t$$
$$s = 5t$$
$$t = \frac{s}{5}$$

$$\vec{r}(t(s)) = (\frac{3}{5}s + 5)\hat{\imath} + (\frac{4}{5}s + 2)\hat{\jmath}$$

3.6 Frenet Frame for a Curve

Unit Tangent Vector:

$$\hat{T}(t) = \frac{\vec{v}(t)}{||\vec{v}(t)||}$$

Unit Normal Vector:

$$\hat{N}(t) = \hat{B} \times \hat{T}$$

Unit Binomial Vector:

$$\hat{B}(t) = \frac{\vec{v}(t) \times \vec{a}(t)}{||\vec{v}(t) \times \vec{a}(t)||}$$

Notes:

$\hat{N}$ Always points in the direction direction of concavity.
$\hat{N} = \vec{0}$ at points of inflection. This can be confirmed by the fact that points of inflection don't have any concavity because this is the point where the concavity switches from one direction to the other.

$\hat{B} = \vec{0}$ at points of inflection. This can be deduced from the formula for the Unit Normal Vector since $\vec{T}$ does not equal $\vec{0}$ unless the function is a point. The direction of $\hat{B}$ can be found with the Right Hand Rule with $\hat{N}$ being the index finger and $\hat{T}$ being the thumb.

3.6.1 Radius of Curvature

There exists a unique circle that best fits a curve at a point which can be represented by the radius ρ of the circle.
The Curvature of a path $k(t) = 1/\rho$.

$$k(t) = \frac{||\vec{v}(t) \times \vec{a}(t)||}{||\vec{v}(t)||}$$

$$k(t) = \frac{||\frac{d}{dt}\vec{r}(t) \times \frac{d^2}{dt^2}\vec{r}(t)||}{||\frac{d}{dt}\vec{r}(t)||}$$

3.7 Torsion

Torsion $\tau(t)$ is the measure of how much a curve is twisting out of the plane in the direction of $\hat{B}$.

$$\tau(t) = \frac{\left[\vec{v}(t) \times \vec{a}(t)\right] \cdot \frac{d^3}{dt^3}\vec{r}(t)}{||\vec{v}(t) \times \vec{a}(t)||^2}$$

Ex:

$$\vec{r}(t) = 2cos(t)\hat{i} + 2sin(t)\hat{j} + (\sqrt{5}t)\hat{k}$$

Find $\vec{v}(t)$, $\vec{a}(t)$, $\frac{d^3}{dt^3}\vec{r}(t)$, $\hat{T}$, $\hat{N}$, $\hat{B}$, $k(t)$, and $\tau(t)$.

$$\vec{v}(t) = -2sin(t)\hat{i} + 2cos(t)\hat{j} + \sqrt{5}\hat{k}$$

$$\vec{a}(t) = -2cos(t)\hat{i} - 2sin(t)\hat{j} + 0\hat{k}$$

$$\frac{d^3}{dt^3}\vec{r}(t) = 2sin(t)\hat{i} - 2cos(t)\hat{j}$$

$$
\begin{aligned}
\hat{T} &= \frac{\vec{v}(t)}{||\vec{v}(t)||} \\
&= \frac{-2sin(t)\hat{i} + 2cos\hat{j} + \sqrt{5}\hat{k}}{\sqrt{4sin^2(t) + 4cos^2(t) + \sqrt{5}^2}} \\
&= \frac{-2sin(t)\hat{i} + 2cos\hat{j} + \sqrt{5}\hat{k}}{\sqrt{4(sin^2(t) + cos^2(t)) + 5}} \\
&= \frac{-2sin(t)\hat{i} + 2cos\hat{j} + \sqrt{5}\hat{k}}{\sqrt{4+5}} \\
&= \frac{-2sin(t)\hat{i} + 2cos\hat{j} + \sqrt{5}\hat{k}}{\sqrt{9}} \\
&= \frac{-2sin(t)\hat{i} + 2cos\hat{j} + \sqrt{5}\hat{k}}{3}
\end{aligned}
$$