Solution

$n!$ can be factored into its prime components. Notice that the largest prime factor of $n!$ is less than or equal to n. So you can find the set of all prime numbers less than or equal to n and the multiplicity of each prime factor in $n!$.

You can use the Sieve of Eratosthenes to find all primes numbers less than or equal to n. You can then use Legendre's formula to find the multiplicity of a prime p in $n!$:

$$\upsilon_p(n!) = \sum_{i=1}^{\infty} \lfloor \frac{n}{p^i} \rfloor = \sum_{i=1}^{\lfloor \log_p(n) \rfloor + 1} \lfloor \frac{n}{p^i} \rfloor$$

To find the number of trailing zeros in the base-b representation you will need to factorize b into its prime factors. You will then need to determine the largest number m you can raise b by which still divides $n!$.

$$b^m = (\prod_p p^{b_p})^m = \prod_p p^{b_p \cdot m} | n! = \prod_p p^{\upsilon_p(n!)}$$

$$\forall p : b_p \cdot m \leq \upsilon_p(n!)$$

$$\forall p : m \leq \frac{\upsilon_p(n!)}{b_p}$$

$$\exists p : \upsilon_p(n!) < b_p \cdot (m+1)$$

$$\exists p : \frac{\upsilon_p(n!)}{b_p} < m + 1$$

$$m = \min_p (\lfloor \frac{\upsilon_p(n!)}{b_p} \rfloor)$$