A9 1
A10 1
A11 2
A13 2
A14 3
B16 3
B17 4
B19 4
B28 4
B29 5
B30 6
B31 6

HOMEWORK N° 4

Chapter 2

A9

Formulate a probability distribution rate.

Solution:

Trivial.

X	-1	1	3
P	0.3	0.4	0.3

✓

A10

Several problems regarding the probability distribution rate.

Solution:

1. For,

$$\int_0^2 c(2-x)dx = 2c = 1$$

The constant must comply,

$$c = 0.5$$

✓

2. Thus its cumulative distribution function is,

$$F(X) = \begin{cases} 0, & x < 0 \\ x - \frac{1}{4}x^2, & 0 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

3. $P(0.5 < X < 1) = F(1) - F(0.5) = \frac{5}{16}$

✓

A11

Idem.

Solution:

1.
$$\lim_{X \to 2^-} F(X) = \lim_{X \to 2^+} F(X) \to 1 - \frac{c}{2} = 0 \to c = 2$$

2. When $X \ge 2$,
$$f(x) = \frac{d}{dx}(1 - \frac{2}{x}) = \frac{2}{x^2}$$
Hence,
$$f(x) = \begin{cases} 0, & x < 2 \\ \frac{2}{x^2}, & x \ge 2 \end{cases}$$

3.
$$P(X \le 4) = F(4) - F(-\infty) = 0.5$$

✓

A13

A uniform distribution example.

Solution:

It is trivial that X satisfies a uniform distribution on (-1,3),

$$f_X(x) = \begin{cases} \frac{1}{4}, & -1 < x < 3 \\ 0, & x = \text{others} \end{cases}$$

Therefore, the probability distribution when randomly picking numbers should yield,

$$P(X \ge 0) = \frac{3}{4}, P(X < 0) = \frac{1}{4}$$

It coulde be deducted that $Y \sim B(n, 0.75)$, thus,

✓

$$P(Y=k)=\mathbf{C}_n^k(\frac{3}{4})^k(\frac{1}{4})^{n-k}, k=0,1,...,n$$

✓

A14

The distribution rate of an exponential distribution that satisfies the product.

Solution:

$X \sim Exp(\lambda=0.2)$, Thus,

1.
$$F(x)=\begin{cases}1-e^{-0.2x}, & x\geq 0\\ 0, & x<0\end{cases}$$

2.
$$P(X>5)=P(+\infty)-P(5)=e^{-1}$$

3. Due to the memory-less property, the expression could be re-arranged as,
$$P(X\leq 10|X>5)=1-P(X>5)=1-e^{-1}$$

✓

B16

Calculate distribution function.

Solution:

The stated expression indicates that,

$$P(0\leq X\leq 1)=\frac{1}{2}, P(2\leq X\leq 3)=\frac{1}{2}$$

Thus, X would only fall in the region of $(0,1)\cup(2,3)$.

1.
$$F(x)=\begin{cases}0, & x<0\\ \frac{x}{2}, & 0\leq x<1\\ \frac{1}{2}, & 1\leq x<2\\ \frac{x-1}{2}, & 2\leq x<3\\ 1, & x\geq 3\end{cases}$$

2.
$$P(X\leq 2.5)=F(2.5)-F(-\infty)=\frac{3}{4}$$

✓

B17

Some problems regarding a certain distribution.

Solution:

1. $$\int_0^2 c(4-x^2)dx = \frac{16}{3}c = 1 \to c = \frac{3}{16}$$

2. $$F(x) = \begin{cases} 0, & x < 0 \\ \frac{3}{16}(4x - \frac{1}{3}x^3), & 0 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

3. $$P(-1 < X < 1) = F(1) - F(-1) = \frac{11}{16}$$

4. Let the occurrence number of the event $-1 < X < 1$ to be Y. Thus,
$$P(Y=2) = \mathbf{C}_5^2(\frac{11}{16})^2(\frac{5}{16})^3 = 0.144$$

✓

B19

A problem regarding a student catching the school bus.

Solution:

Let the time-waiting variant be X, the exact time be T.

1. $$P(X < 10) = \frac{10+5}{25} = \frac{3}{5}$$

2. $$P(5 < X < 15) = \frac{5+10}{25} = \frac{3}{5}$$

3. $$P(T < 7:30 | X > 5) = \frac{5}{20} = \frac{1}{4}$$

✓

B28

The time distribution of a customer waiting at the counter for an another people to finish its affair.

Solution:

Set the service time variant to be T. $T \sim Exp(\frac{1}{8})$. Due to its memory-less property,

$$P(T > t_0 + t | T > t_0) = P(T > t)$$

And the brilliant part unfolds. By definition,

$$P(T > t_0 + t | T > t_0) = P(X > t)$$

Thus, two variant X, T must have identical distribution law.

$$P(X > t) = P(T > t) \to X \sim Exp(\frac{1}{8})$$

1.
$$f_X(x) = \begin{cases} \frac{1}{8}e^{-\frac{x}{8}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

2.
$$P(X > 10) = F_X(+\infty) - F_X(10) = e^{-\frac{5}{4}}$$

3.
$$P(8 < X < 16) = F_X(16) - F_X(8) = e^{-1} - e^{-2}$$

✓

B29

The service life distribution of two kinds of product mixed in certain proportion.

Solution:

Let the service life variable be X, with the separate components in factory A and B's X yielding $X|A \sim Exp(\frac{1}{3}), X|B \sim Exp(\frac{1}{6})$.

1.
$$P(X > 6) = P(X > 6|A)P(A) + P(X > 6|B)P(B) = 0.4e^{-2} + 0.6e^{-1}$$

2.
$$P(X > 12 | X > 4, A) = P(X > 8|A) = e^{-\frac{1}{3}\times\frac{2}{3}} = e^{-\frac{2}{9}}$$ 写法非常重要。

3.
$$P(X > 12|X > 4) = \frac{P(X > 12)}{P(X > 4)} = \frac{P(X > 12|A)P(A) + P(X > 12|B)P(B)}{P(X > 4|A)P(A) + P(X > 4|B)P(B)}$$
$$= \frac{0.4e^{-\frac{1}{3}} + 0.6e^{-\frac{1}{6}}}{0.4e^{-\frac{1}{9}} + 0.6e^{-\frac{1}{18}}}$$

✓

B30

The time for waiting a customer.

Solution:

$X \sim Exp(0.2)$

1.

$$f(x) = \begin{cases} 0.2e^{-0.2x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

2.

$$P(5 < X < 10) = F(10) - F(5) = e^{-1} - e^{-2}$$

3.

$$P(X \leq 5) = F(5) - F(-\infty) = 1 - e^{-1}$$

Let the number of date when $X \leq 5$ be Y. $Y \sim B(7, 1 - e^{-1})$. One would obtain,

$$P(Y \geq 6) = \mathbf{C}_7^6 (1 - e^{-1})^6 (e^{-1}) + (1 - e^{-1})^7 = (1 - e^{-1})(6e^{-1} + 1)$$

✓

B31

The probability of someone re-buying the same components.

Solution:

The life expectancy of one component satisfies $X \sim Exp(0.01)$. Let the number of components that successfully endures 150 hours be Y. $Y \sim B(n, p)$.

1.

$$P(X > 150) = e^{-1.5}$$

Thus, when buying three components, the probability of two working as expected would be,

$$P(Y = 2) = \mathbf{C}_3^2 (e^{-1.5})^2 (1 - e^{-1.5}) = 3e^{-3}(1 - e^{-1.5})$$

2.

$$P(Y \geq 2) = 3e^{-3}(1 - e^{-1.5}) + e^{-4.5} = 3e^{-3} - 2e^{-4.5}$$

✓