Homework N° #

Problem A's

A2

Elaborate the sample space of the following A and B on dice.

Solution:

1. $A = \{1, 2\}, B = \{4, 5, 6\}$
2. $A = \{(1,3),(3,1),(2,4),(4,2),(3,5),(5,3),(4,6),(6,4)\}, B = \{(3,1),(6,2)\}$
3. $A = \{(x, y) : 0 < x < \frac{1}{2}, x \leq y < 1 - x\}, B = \{(x, y) : x < \frac{1}{2}, \frac{1}{2} < y < 1 - x\}$

✓

A4

Let A, B, C be three random events. Express particular events using event operation relations.

Solution: hint:$A + B \xrightarrow{eq} A \cup B$, $AB \xrightarrow{eq} A \cap B$, simplified by Karnaugh Graph

Applying boolean operation rules would one obtain,

1. $AB + AC + BC + ABC = AB + AC + BC$
2. $A'B'C + A'BC' + AB'C' + A'B'C' = A'B' + A'C' + B'C'$
3. $A'BC + AB'C + ABC'$ (could not be simplified)
4. $A'BC + AB'C + ABC' + A'B'C + A'BC' + AB'C' + A'B'C' = A' + C' + AB'$

✓

此处需要说明：按照卡诺图的最小相加项化简是正确的，书本上的答案也没有错

A6

Elementary probability calculation when $P(A) = 0.5, P(B) = 0.4$ under different circumstances.

Solution:

1. $P(A + B) = P(A) + P(B) = 0.9, P(AB') = P(A) = 0.5$
2. $P(A + B) = P(A) = 0.5, P(AB') = P(A) - P(B) = 0.1$

✓

A7

Elementary probability calculation when $P(A) = 0.3, P(B) = 0.5$ and there is no overlap.

Solution:

1. $P(A'B + AB' + AB) = P(S) - P(A'B') = 1 - 0.2 = 0.8$
2. $P(A'B') = P(S) - P(A) - P(B) = 1 - 0.8 = 0.2$
3. $P(A'B) = P(B) = 0.5$

✓

A8

Same with $P(A) = 0.5, P(B) = 0.4$ and $P(A + B) = 0.6$.

Solution:

$P(AB) = P(A) + P(B) - P(A + B) = 0.5 + 0.4 - 0.6 = 0.3$

1. $P(AB') = P(A) - P(AB) = 0.5 - 0.3 = 0.2$
2. $P(A'B') = P(S) - P(A + B) = 1 - 0.6 = 0.4$
3. $P(A' + B') = P(S) - P(AB) = 1 - 0.3 = 0.7$(De Morgan's Law)

✓

A9

Calculation of basic classical probability model problems.

Solution:

1. $P(oneS + twoS) = \frac{C(4,1)C(2,1)+C(2,2)}{C(6,2)} = \frac{3}{5}$
2. $P(oneSatmost) = P(S) - \frac{C(2,2)}{C(6,2)} = \frac{14}{15}$

✓

A12

Calculation of basic classical probability model problems on sampling difference.

Solution: Notice the third problem.

- sampling with replacement
  1. $P(RR) = P(R)^2 = 0.8^2 = 0.64$
  2. $P(RX + XR) = 2P(R)P(X) = 0.32$
  3. $P(RR + XR) = P(R) = 0.8$
- sampling without replacement
  1. $P(RR) = P(R)P(R_2) = \frac{4}{5} \times \frac{7}{9} = \frac{28}{45}$
  2. $P(RX + XR) = \frac{4}{5} \times \frac{2}{9} + \frac{1}{5} \times \frac{8}{9} = \frac{16}{45}$
  3. $P(RR + XR) = \frac{28}{45} + \frac{1}{5} \times \frac{8}{9} = \frac{4}{5}$

✓

B1

Calculation of basic classical probability model problems with event A,B and C.

Solution: Heavy implementation on De Morgan's law.

1. P(A'C)=P(C)-P(A)=0.4-0.3=0.1
2. P(A'B+AB'+AB)=P(S)-P(A'B')=P(A+B)=P(A)+P(B)=0.6
3. P(A'B'C')=P(S)-P(A+B+C)=P(S)-P(B)-P(C)=0.3

✓