Homework N° #

The 3rd Assignment of Robot Modeling and Control

3-1

Calculate the link length a_1 by two end axes.

Solution:

According to description,

$$o_0 = \mathbf{O}, z_0 = (1, -2, 3)^{\mathbf{T}}, o_1 = (-4, 3, 0)^{\mathbf{T}}, z_1 = (-1, -2, -1)^{\mathbf{T}}$$

Thus the normal vector of z_0, z_1 could be deducted,

$$n = z_0 \times z_1 = (8, -2, -4)^{\mathbf{T}}$$

Alongside with a_1,

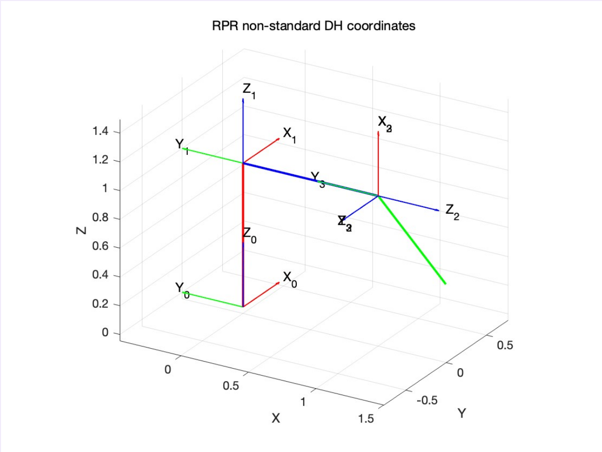
$$a_1 = \frac{|\boldsymbol{o_0 o_1} \cdot n|}{\|n\|} = \frac{13}{\sqrt{21}}$$

3-3

Formulate a non-standard DH coordinate of a RPR robot arm and calculate the transition matrices accordingly.

Solution:

1. Binding of the coordinates are shown on following graph.



with its parameter table formed.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1
2	$\frac{\pi}{2}$	0	d_2	$\frac{\pi}{2}$
3	$-\frac{\pi}{2}$	0	0	θ_3

2. Thus the kinematical matrix could be formulated,

$$\mathbf{T}_1^0 = \begin{pmatrix} c_{\theta_1} & -s_{\theta_1} & 0 & 0 \\ s_{\theta_1} & c_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \mathbf{T}_2^1 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \mathbf{T}_3^2 = \begin{pmatrix} c_{\theta_3} & -s_{\theta_3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{\theta_3} & c_{\theta_3} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Hence

$$\mathbf{T}_W^B = \mathbf{T}_1^0\mathbf{T}_2^1\mathbf{T}_3^2 = \begin{pmatrix} s_{\theta_1}s_{\theta_3} & -c_{\theta_3}s_{\theta_1} & -c_{\theta_1} & -d_2 s_{\theta_1} \\ c_{\theta_1}s_{\theta_3} & c_{\theta_1}c_{\theta_3} & -s_{\theta_1} & -d_2 c_{\theta_1} \\ c_{\theta_3} & -s_{\theta_2} & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3-5

Calculate the two dimensional coordinate of the end effector.

Solution:

The structure of the robot arm is relatively easy, thus calculable through simple geometric measurement.

$$x = 0.3 \times \cos(30^\circ) + 0.3 \times \cos(60^\circ) \approx 0.41$$

$$y = 0.3 \times \sin(30^\circ) + 0.3 \times \sin(60^\circ) \approx 0.41$$

3-6

A bunch of stuff related with a 4R robot arm.

Solution:

1.
$$\mathbf{T}_1^0 = \begin{pmatrix} c_1 & -s_1 & l_1c_1 \\ s_1 & c_1 & l_1s_1 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{T}_2^1 = \begin{pmatrix} c_2 & -s_2 & l_2c_2 \\ s_2 & c_2 & l_2s_2 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\mathbf{T}_3^2 = \begin{pmatrix} c_3 & -s_3 & l_3c_3 \\ s_3 & c_3 & l_3s_3 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{T}_4^3 = \begin{pmatrix} c_4 & -s_4 & l_4c_4 \\ s_4 & c_4 & l_4s_4 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Due to complexity of matrix method, Q2 and Q3 are solved by traditional geometric method.
$$x_W^B = l_1c_1 + l_2c_{12} + l_3c_{123-} + l_4c_{123-4}$$

$$y_W^B = l_1s_1 + l_2s_{12} + l_3s_{123-} + l_4s_{123-4}$$

3.
$$\varphi = \theta_1 + \theta_2 - \theta_3 + \theta_4$$