Homework N° 9

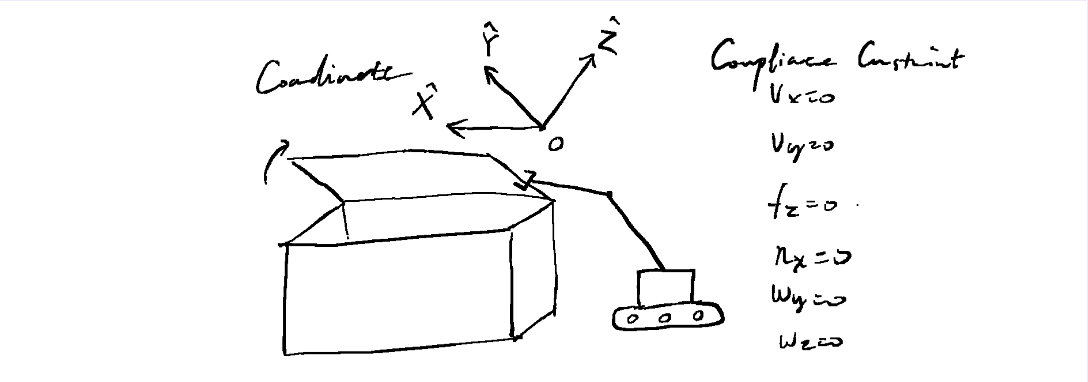
The 9th Assignment of Robot Modeling and Control

9-2

There is a box given with its cap hinged to its body. Formulate the compliant constrains and artificial constrains in its constraint coordinate.

Solution:

As the image goes:



With its compliance constraints being:

$$v_x = 0, v_y = 0, f_z = 0; n_x = 0, \omega_y = 0, \omega_z = 0$$

Therefore, for the duality condition the artificial constrain must comply:

$$\xi^{\mathrm{T}} F = 0$$

Hence the artificial constraints, its value determined by robot itself:

$$f_x = 0, f_y = 0, v_z = \omega_x r; \omega_x = a, n_y = 0, n_z = 0$$

9-3

Derivation of the joint-space control law for the impedance control of a robotic manipulator.

Solution:

The inner loop torque command is given as:

$$\tau = J_a^T(\Phi)\left(M_X(\Phi)a_d + V_X(\Phi,\dot{\Phi}) + G_X(\Phi) - F_a\right)$$

Using the definitions in (9-24):

$$\begin{aligned} M_X(\Phi) &= J_a^T(\Phi)M(\Phi)J_a^{-1}(\Phi), \\ V_X(\Phi,\dot{\Phi}) &= J_a^T(\Phi)V(\Phi,\dot{\Phi}) - M_X(\Phi)\dot{J}_a(\Phi)\dot{\Phi}, \\ G_X(\Phi) &= J_a^T(\Phi)G(\Phi) \end{aligned}$$

Substitute these into the inner loop control law:

$$\tau = J_a^T(\Phi)\left(\left(J_a^T M J_a^{-1}\right)a_d + \left(J_a^T V - M_X \dot{J}_a\dot{\Phi}\right) + J_a^T G - F_a\right)$$

Factor out $J_a^T(\Phi)$ and cancel terms:

$$\tau = J_a^T(\Phi)^2 M(\Phi)J_a^{-1}(\Phi)a_d + J_a^T(\Phi)^2 V(\Phi,\dot{\Phi}) - J_a^T(\Phi)M_X(\Phi)\dot{J}_a(\Phi)\dot{\Phi} + J_a^T(\Phi)^2 G(\Phi) - J_a^T(\Phi)F_a$$

Simplify $J_a^T J_a^T M J_a^{-1} = M J_a^{-1}$ (since $J_a^T J_a^{-1} = I$ for square Jacobians), leading to:

$$\tau = M(\Phi)J_a^{-1}(\Phi)a_d + V(\Phi,\dot{\Phi}) + G(\Phi) - J_a^T(\Phi)M_X(\Phi)\dot{J}_a(\Phi)\dot{\Phi} - J_a^T(\Phi)F_a$$

The outer loop defines a_d as:

$$a_d = \dot{X}_d + M_d^{-1}\left(-B_d\dot{X} - K_d\bar{X} + F_a\right).$$

Substitute this into the torque expression:

$$\tau = M(\Phi)J_a^{-1}(\Phi)\left(\dot{X}_d + M_d^{-1}\left(-B_d\dot{X} - K_d\bar{X} + F_a\right)\right) + V + G - M(\Phi)J_a^{-1}(\Phi)\dot{J}_a\dot{\Phi} - J_a^T(\Phi)F_a$$

Expand the terms involving $M(\Phi)J_a^{-1}(\Phi)$:

$$\tau = MJ_a^{-1}\dot{X}_d + MJ_a^{-1}M_d^{-1}(-B_d\dot{X} - K_d\bar{X}) + MJ_a^{-1}M_d^{-1}F_a + V + G - MJ_a^{-1}\dot{J}_a\dot{\Phi} - J_a^T F_a$$

Combine the F_a-dependent terms:

$$MJ_a^{-1}M_d^{-1}F_a - J_a^T F_a = J_a^T\left(M_X M_d^{-1} - I\right)F_a \quad (\text{since } M_X = J_a^T M J_a^{-1})$$

Simplify the remaining terms:

$$MJ_a^{-1}\left(\dot{X}_d - \dot{J}_a\dot{\Phi} + M_d^{-1}(-B_d\dot{X} - K_d\bar{X})\right) + V + G + J_a^T\left(M_X M_d^{-1} - I\right)F_a$$

The final expression matches Equation (9-28):

$$\begin{aligned} \tau = {} & M(\Phi)J_a^{-1}(\Phi)\left(\dot{X}_d - \dot{J}_a(\Phi)\dot{\Phi} + M_d^{-1}\left(-B_d\dot{X} - K_d\bar{X}\right)\right) \\ & + V(\Phi,\dot{\Phi}) + G(\Phi) + J_a^T(\Phi)\left[M_X(\Phi)M_d^{-1} - I\right]F_a \end{aligned}$$

9-5

Select an impedance controller for the required robot with the transfer function between the position tracking error and the contact force having two identical overlapped poles $-\lambda$ in two decomposed directions x and y.

Solution:

For its prismatic essence, its generalized force must be equal to the actual environmental force, for there is no revolution section to begin with:

$$F_a = F$$

$$\mathbf{J_a} = \mathbf{J}$$

Therefore, its desired impedance relations must yields:

$$\mathbf{M_d}\ddot{e} + \mathbf{B_d}\dot{e} + \mathbf{K_d}e = F$$

Where:

$$\mathbf{M_d} = \begin{pmatrix} \mathbf{M_1} & 0 \\ 0 & \mathbf{M_2} \end{pmatrix}$$

Transferring the equation into s domain:

$$\mathbf{M_d}s^2 + \mathbf{B_d}s + \mathbf{K_d} = \frac{F(s)}{e(s)} = \begin{pmatrix} \mathbf{M_1}(s+\lambda)^2 & 0 \\ 0 & \mathbf{M_2}(s+\lambda)^2 \end{pmatrix}$$

Would we know:

$$\mathbf{B_d} = \begin{pmatrix} 2\mathbf{M_1}\lambda & 0 \\ 0 & 2\mathbf{M_2}\lambda \end{pmatrix}, \mathbf{K_d} = \begin{pmatrix} \mathbf{M_1}\lambda^2 & 0 \\ 0 & \mathbf{M_2}\lambda^2 \end{pmatrix}$$

Therefore, the next step is to clarify the dynamical model of the required robot. Its Lagrangian variables are:

$$L = T - V = \frac{1}{2}(m_1 + m_2)\dot{d_1}^2 + \frac{1}{2}m_2\dot{d_2}^2 - m_2 g d_2$$

With its Lagrangian equation(without environmental force):

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = \tau \rightarrow$$

$$\mathbf{M} = \begin{pmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{pmatrix}, \mathbf{G} = \begin{pmatrix} 0 & 0 \\ 0 & m_2 g \end{pmatrix}, \mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

With,

$$\mathbf{M_d} = \mathbf{M}$$

Hence, according to (9-30), the control rate shall be:

$$\begin{aligned} \tau &= MJ^{-1}\left(\ddot{x}_d - \dot{J}\dot{\Phi}\right) + V + G + J^{-\mathrm{T}}\left(-B_d\dot{e} - K_d e\right) \\ &= \begin{pmatrix} m_1 + m_2 & 0 \\ 0 & m_1 \end{pmatrix}\ddot{x}_d + \begin{pmatrix} 0 & 0 \\ 0 & m_2 g \end{pmatrix} - \begin{pmatrix} 2(m_1+m_2)\lambda & 0 \\ 0 & 2m_2\lambda \end{pmatrix}\dot{e} - \begin{pmatrix} (m_1+m_2)\lambda^2 & 0 \\ 0 & m_2\lambda^2 \end{pmatrix} e \end{aligned}$$