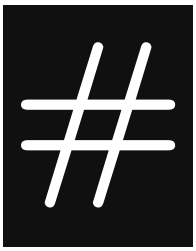
A11 1
A14 1
A16 2
A17 2
A18 2
A19 3
A20 3
A22 3
A23 4
A25 4
B3 4
B4 5
B5 5
B6 5

HOMEWORK N°



Assignment 2

A11

Calculate the probability of two student sharing the same birthday among a total of six.

Solution:

$$P(AtLeastTwo) = P(S) - P(NoneTheSame) = 1 - 1 \times \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} \times \frac{7}{12} \approx 0.78$$

✓

A14

A drawing-ball-without-replacement problem.

Solution:

1. $P = \frac{\mathbf{C}_2^1 \mathbf{C}_3^1 \mathbf{C}_2^1}{\mathbf{C}_7^3} = \frac{12}{35}$
2. $P = \frac{\mathbf{C}_3^3}{\mathbf{C}_7^3} = \frac{1}{35}$
3. $P = \frac{\mathbf{C}_2^1 \mathbf{C}_3^1 \mathbf{C}_2^1}{\mathbf{A}_7^3} = \frac{2}{35}$

✓

A16

A typical ball-placing problem.

Solution:

$$P = \frac{2^3 \mathbf{C}_5^2}{3^5} = \frac{80}{243}$$

✓

A17

A conditional probability problem.

Solution:

$P(AB) = P(A) + P(B) - P(A + B) = 0.3,$
$P(A'B') = P(A') + P(B') - P(A' + B') = 0.5 + 0.6 - P((AB)') = 0.4.$

1. $P(A|B) = \frac{P(AB)}{P(B)} = \frac{3}{4}$
2. $P(B|A) = \frac{P(AB)}{P(A)} = \frac{3}{5}$
3. $P(A'|B') = \frac{P(A'B')}{P(B')} = \frac{2}{3}$
4. $P(B'|A') = \frac{P(A'B')}{P(A')} = \frac{4}{5}$

✓

A18

A probability deduction problem.

Solution:

1. $P(AB) = P(A)P(B|A) = 0.24$ ✓
2. $P(B) = \frac{P(AB)}{P(A|B)} = 0.48$ ✓
3. $P(A + B) = P(A) + P(B) - P(AB) = 0.84$ ✓

A19

More probability deduction problems.

Solution:

$P(AB) = P(A) - P(AB') = 0.2, P(A+B) = P(A) + P(B) - P(AB) = 0.9$
$P(A'B) = P(B) - P(AB) = 0.2, P(A'+B) = P(A') + P(B) - P(A'B) = 0.5$

1. $P(A|A+B) = \frac{P(A)}{P(A+B)} = \frac{7}{9}$
2. $P(A|A'+B) = \frac{P(A(A'+B))}{P(A'+B)} = \frac{P(AB)}{P(A'+B)} = \frac{2}{5}$
3. $P(AB|A+B) = \frac{P(AB)}{P(A+B)} = \frac{2}{9}$

✓

A20

Several ball-drawing problems.

Solution:

1. $P = (\frac{\mathbf{C}_3^1\mathbf{C}_2^1}{\mathbf{C}_5^2} + \frac{\mathbf{C}_2^1\mathbf{C}_4^1}{\mathbf{C}_6^2}) \times \frac{1}{2} = \frac{17}{30}$
2. $P = \frac{3}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{2}{7} = \frac{13}{35}$
3. $P = \frac{3}{5} \times \frac{\mathbf{C}_3^1\mathbf{C}_4^1}{\mathbf{C}_7^2} + \frac{2}{5} \times \frac{\mathbf{C}_2^1\mathbf{C}_5^1}{\mathbf{C}_7^2} = \frac{8}{15}$ 官方答案认真的？

✓

A22

Factory yield certification.

Solution:

$P_{good} = \frac{6}{15} \times 85\% + \frac{5}{15} \times 90\% + \frac{4}{15} \times 80\% = \frac{64}{75}$

- $P_{ciaq|good} = \frac{P_{ciaq \cdot good}}{P_{good}} = \frac{\frac{17}{50}}{\frac{64}{75}} = \frac{51}{128}$
- $P_{iq|good} = \frac{45}{128}$
- $P_{bin|good} = \frac{32}{128}$

Thus, it is most likely that the factory **ciaq** produced this product.

✓

A23

Hit chance of unadjusted and adjusted guns.

Solution:

1. $P_{hit} = \frac{3}{4} \times 60\% + \frac{1}{4} \times 5\% = 46.25\%$
2. $P_{unadjusted|hit} = \frac{\frac{1}{4} \times 5\%}{46.25\%} = 2.70\%$

✓

A25

The correlation between smoking and birth defection observed through probability.

Solution:

1. $P_{dys} = 0.4 \times 0.8\% + 0.18 \times 1.4\% + 0.42 \times 2.1\% = 1.454\%$
2. $P_{\geq 10|dys} = \frac{0.42 \times 2.1\%}{1.454\%} = 60.66\%$

✓

B3

Astonishing complex problems on newspaper ordering rate.

Solution:

Below are some critical condition given by the description.

$$P(B|A) = 0.2$$

$$P(A+B|C) = 0.6$$

The target probability yields,

$$P(A+B+C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$$

It could be deducted from previous conditions that,

$$P(AB) = P(A)P(B|A) = 0.1$$

$$P((A+B)C) = P(AC+BC) = P(AC) + P(BC) - P(ABC) = 0.24$$

Subsituting the equations back to target would one obtain the final result,

$$P(A+B+C) = 0.5 + 0.4 + 0.3 - 0.1 - 0.24 = 0.86$$

✓

B4

Staff member administration post rate.

Solution:

1. $P_{admin|female} = \frac{P_{admin \cdot female}}{P_{female}} = \frac{5\%}{45\%} = \frac{1}{9}$
2. $P_{male|admin} = \frac{P_{admin \cdot male}}{P_{admin}} = \frac{5\%}{10\%} = \frac{1}{2}$

✓

B5

Precision of current sampling algorithm.

Solution:

$$P_P = 0.8 \times 1 + 0.15 \times \frac{\mathbf{C}_{11}^2}{\mathbf{C}_{12}^2} + 0.05 \times \frac{\mathbf{C}_{10}^2}{\mathbf{C}_{12}^2} = 0.9591$$

$$P_{TP} = 0.8$$

$$P_{TP|P} = \frac{P_{TP}}{P_P} = \frac{0.8}{0.9591} = 83.41\%$$

✓

B6

Non-replacing sampling problem.

Solution:

1. $P_{1P} = \frac{1}{2} \times (\frac{10}{30} + \frac{15}{20}) = \frac{13}{24}$
2. $P_{1P \cdot 2N} = \frac{1}{2} \times (\frac{\frac{1}{2}\mathbf{C_{10}^1 C_{20}^1}}{\mathbf{C}_{30}^2} + \frac{\frac{1}{2}\mathbf{C_{15}^1 C_5^1}}{\mathbf{C}_{20}^2}) = \frac{2825}{13224}$

   $P_{2N|1P} = \frac{P_{1P \cdot 2N}}{P_{1P}} = \frac{2825}{7163}$

✓