HOMEWORK N° 3

Chapter 2

A28

A bunch of probabilities within three independent random events.

Solution:

1. $$P(ABC) = P(A)P(B)P(C) = 0.08$$

2. $$P(A + B + C) = P(A) + P(B) + P(C) - P(A)P(B) - P(B)P(C) + P(ABC) = 0.82$$

3. $$P(AB|AC) = \frac{P(ABC)}{P(AC)} = 0.4$$

✓

A30

Stacking independent event's probabilities.

Solution:

1. $P(H_1H_2) = P(H)^2 = 0.64$
2. $P(h \geq 1) = 1 - 0.2^3 = 0.992$

✓

A32

Coin flipping problems.

Solution:

1. $P(A_i) = (1-p)^{i-1}p, P(B_4) = p^2(1-p)$
2. $P(B_4|A_1) = \frac{P(A_1B_4)}{P(A_1)} = \frac{p^3(1-p)}{p} = p^2(1-p)$ ✓
3. $P(A_1|B_4) = \frac{p^3(1-p)}{p^2(1-p)} = p$

A34

A dart problem.

Solution:

The probability of hitting at least once is correlated with the number of trials in a series.

$$P(k \geq 1) = 1 - 0.95^n$$

Hence the equation.

$$1 - 0.95^n \geq 0.5 \rightarrow n \geq 14$$

Thus, the competitor should dart at least 14 times. ✓

B7

Effective rate of a system composed of several components.

Solution:

1. $\alpha = p_1p_2p_3(1-p_4) + p_1p_2p_4(1-p_3) + p_1p_3p_4(1-p_2) + p_2p_3p_4(1-p_1) + p_1p_2p_3p_4$
2. $\beta = \frac{p_1p_2p_3p_4}{\alpha}$
3. $\gamma = \mathbf{C}_3^2\alpha^2(1-\alpha) = 3\alpha^2(1-\alpha)$ ✓

B8

The mask rate of citizens on hazy days and non-hazy days.

Solution:

1. $P(M) = P(M|H)P(H) + P(M|N)P(N) = 0.2 \times 0.4 + 0.01 \times 0.6 = 0.086$

2. $$P(k \geq 1|H) = 1 - P(k = 0|H) = 1 - 0.8^3 = 0.488$$

   Vise versa,
   $$P(k \geq 1|N) = 1 - 0.99^3 \approx 0.0297$$

   Thus according to the law of total probability, ✓
   $$P(k \geq 1) = 0.4 \times 0.488 + 0.6 \times 0.0297 \approx 0.213$$

Homework N° 3

Chapter 3

A2

Calculation of the normalization constant of a probability distribution.

Solution:

$$P(S) = \sum P(i) = c \times (4 + 3 + 2 + 1) = 1$$

Hence,

$$c = 0.1$$

✓

$$P(1.5 < X \leq 3) = P(X = 2) + P(X = 3) = 0.1 \times (3 + 2) = 0.5$$

A3

The hit rate of slam dunk.

Solution:

$X \sim B(n, p)$.

1. $P(X \geq 2) = \mathbf{C}_3^2 \times 0.4^2 \times 0.6 + 0.4^3 = 0.352$
2. $P(X \leq 2) = 1 - P(X = 3) = 1 - 0.064 = 0.936$

✓

A4

Deduction of slam dunk hit rate by observing the probability of hitting at least once.

Solution:

$X \sim B(n, p)$.

$$P(X \geq 1) \ = 1 - P(X = 0) = 1 - (1 - p)^4 = 0.9375 \to p = 0.5$$

✓

A6

Poisson distribution calculation.

Solution:

$X \sim Pois(2)$.

1. $P(X \leq 2) = e^{-2}(1 + 2 + 2) = 5e^{-2}$
2. $P(X \geq 2) = 1 - e^{-2}(1 + 2) = 1 - 3e^{-2}$
3. $P(X \leq 1 | X \leq 2) = \frac{P(X \leq 1)}{P(X \leq 2)} = \frac{3}{5}$

✓

A7

Shop revenue that satisfies the Poisson distribution.

Solution:

$X \sim Pois(3)$.

1. $P(X \geq 4) = 1 - P(X < 4) = 1 - e^{-3}(1 + 3 + \frac{9}{2} + \frac{9}{2}) = 1 - 13e^{-3}$
2. $P(2 \leq X \leq 4) = e^{-3}(\frac{9}{2} + \frac{9}{2} + \frac{27}{8}) = \frac{99}{8}e^{-3}$

✓

B1

Distribution of the middle number in a certain extraction method.

Solution:

It is equivalent to fixing one number in 2,3,4,5,6 in the middle, and calculating the possibility of each scenario. Thus,

$$P(X = i) = \frac{\mathbf{C}_{i-1}^{1}\mathbf{C}_{7-i}^{1}}{\mathbf{C}_{7}^{3}}, i = 2, 3, 4, 5, 6$$

✓

B2

Computer game credit bonus probability distribution.

Solution:

X	0	1	2	4
p	0.8	0.16	0.032	0.008

- $P(X > 2) = P(X = 4) = 0.008$
- $P(X \geq 2) = P(X = 2) + P(X = 4) = 0.04, P(X = 4|X \geq 2) = \frac{1}{5} = 0.2$

✓

B3

Distribution rate of non-replacing ball choosing.

Solution:

X	0	1	2
p	$\frac{\mathbf{C}_3^2}{\mathbf{C}_6^2} = 0.2$	$\frac{\mathbf{C}_3^1\mathbf{C}_3^1}{\mathbf{C}_6^2} = 0.6$	$\frac{\mathbf{C}_3^2}{\mathbf{C}_6^2} = 0.2$

✓

B6

Effective rate of working components.

Solution:

$X \sim B(5, 0.8)$.

1. $P(X = 3) = \mathbf{C}_5^3 \times 0.8^3 \times 0.2^2 = 0.2048$
2. $P(X \geq 4) = \mathbf{C}_5^4 \times 0.8^4 \times 0.2^1 + \mathbf{C}_5^5 \times 0.8^5 = 0.73728$
3. $P(X \leq 2) = 0.2^5 + \mathbf{C}_5^1 \times 0.2^4 \times 0.8 + \mathbf{C}_5^2 \times 0.2^3 \times 0.8^2 = 0.05792$

✓

B9

ATM service rate distribution.

Solution:

$X \sim Pois(1)$.

1. $P(X \geq 2) = 1 - P(X < 2) = 1 - e^{-1}(1 + 1) = 1 - 2e^{-1}$
2. $P(X \leq 3 | X \geq 2) = \frac{\frac{2}{3}e^{-1}}{1-2e^{-1}} = \frac{2}{3e-6}$

✓

B11

Distribution of the number of people waiting in the bus station.

Solution:

1. One could immediately recognize that $\lambda = 4.5$. Thus, $X \sim Pois(4.5)$ and,

$$P(X \geq 2) = 1 - (1 + 4.5)e^{-4.5} = 1 - 5.5e^{-4.5}$$

2. $P(X=1|X\geq 1)=\frac{3.2e^{-3.2}}{1-e^{-3.2}}$

✓