HOMEWORK N° 14

Chapter 8

B5

An octahedral die has faces labeled with the numbers 1, 2, 3, 4, 5, 6, 7, and 8. To test whether the die is fair (i.e., whether each face has an equal probability of landing face up), the die is rolled 600 times. The observed frequencies for each face are shown in a chart:

At a significance level of 0.05, test the hypothesis H_0: The octahedral die is fair.

Solution:

To test whether the octahedral die is fair, we will perform a chi-square goodness-of-fit test.

1. Null hypothesis H_0: The die is fair, meaning each face has an equal probability of $\frac{1}{8}$.
2. Alternative hypothesis H_1: The die is not fair (at least one face has a different probability).

Under H_0, the expected frequency for each face is $600 \times \frac{1}{8} = 75$.
The formula for the chi-square statistic is:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the observed frequency and E_i is the expected frequency for each face.

$$\chi^2 = 0.12 + 0.853 + 0.12 + 3 + 0.333 + 0.213 + 1.613 + 0.12 = 6.372$$

Since $\chi^2 = 6.372 < 14.067$, we fail to reject H_0.
There is not enough evidence at the 0.05 significance level to conclude that the octahedral die is unfair. The data is consistent with the die being fair.

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B8

Let X be the time interval (in minutes) between two consecutive customers at an ATM. Observations were made 120 times, yielding a certain table of data.

At a significance level of $\alpha = 0.05$, test the hypothesis H_0: The time interval follows an exponential distribution with a mean of 10.

Solution:

To test whether the time intervals follow an exponential distribution with mean $\mu = 10$, we will perform a chi-square goodness-of-fit test.

1. Null Hypothesis H_0: The data follows an exponential distribution with mean $\mu = 10$.
2. Alternative Hypothesis H_1: The data does not follow this distribution.

The exponential distribution has the cumulative distribution function (CDF):

$$F(x) = 1 - e^{-\lambda x}, \quad \text{where } \lambda = \frac{1}{\mu} = \frac{1}{10}.$$

Compute the probabilities for each interval:

$$P(0 \leq x \leq 5) = F(5) - F(0) = (1 - e^{-5/10}) - 0 = 1 - e^{-0.5} \approx 0.3935$$

. ...

$$P(x > 30) = 1 - F(30) = e^{-3} \approx 0.0498$$

.

Multiply each probability by the total number of observations (120):
Expected for $0 \leq x \leq 5$: $120 \times 0.3935 \approx 47.22$.
...
Expected for $x > 30$: $120 \times 0.0498 \approx 5.98$.

$$\chi^2 = \frac{4.93}{47.22} + \frac{2.69}{28.64} + \frac{8.41}{27.90} + \frac{45.56}{16.25} \approx 0.104 + 0.094 + 0.301 + 2.804 = 3.303$$

Degrees of freedom $df = k - 1 - m$, where $k = 4$ (intervals) and $m = 1$ (estimated parameter, but here $\lambda = 1/10$ is given, so $m = 0$).Thus, $df = 4 - 1 = 3$.From the chi-square table, the critical value for $\alpha = 0.05$ and $df = 3$ is 7.815.

Since $\chi^2 = 3.303 < 7.815$, we fail to reject H_0.

There is insufficient evidence at the 0.05 level to conclude that the time intervals do not follow an exponential distribution with mean 10.

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B9

The height X (in cm) of adult males in a certain region was observed. A random sample of 200 men was taken, yielding a sample mean of $\overline{x} = 169.9$ and a sample standard deviation of $s = 9.6$. The frequency distribution of heights is described in a chart.

At a significance level of $\alpha = 0.05$, test the hypothesis H_0: The height of adult males in this region follows a normal distribution.

Solution:

1. Null Hypothesis H_0: The data follows a normal distribution $N(169.9, 9.6^2)$.
2. Alternative Hypothesis H_1: The data does not follow this normal distribution.

Convert each interval boundary to a z-score using $z = \frac{x-\mu}{\sigma}$, then find the probabilities using the standard normal distribution table. Multiply each probability by the total number of observations (200).

$$\chi^2 \approx 0.807 + 0.768 + 1.814 + 0.060 + 0.101 + 0.042 + 4.040 = 7.632$$

Degrees of freedom $df = k - 1 - m$, where $k = 7$ (intervals) and $m = 2$ (estimated parameters: mean and standard deviation). Thus, $df = 7 - 1 - 2 = 4$.

From the chi-square table, the critical value for $\alpha = 0.05$ and $df = 4$ is 9.488.

Since $\chi^2 = 7.632 < 9.488$, we fail to reject H_0. There is insufficient evidence at the 0.05 level to conclude that the heights do not follow a normal distribution with mean 169.9 cm and standard deviation 9.6 cm.