HOMEWORK N° #

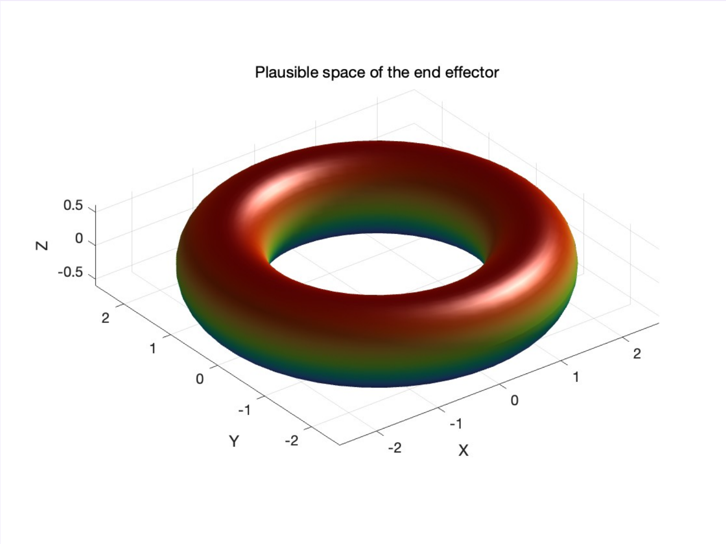
# The 4th Assignment of Robot Modeling and Control

## 4-2

The end workspace and inverse kinematics of a RRR robot arm.

**Solution:**

The end workspace of the RRR robot arm is trivial. A donut, that is to assume $L_2 + L_3 \leq L_1$, that acquired a radius of $L_1$ and a dough width of $2(L_2 + L_3)$, as shown below.




For the inherent essence of this geometric structure is relatively easy, one could simply imply the geometrical solution to obtain its inverse kinematical result, as deprecated in the following steps.

$$\mathbf{X}_{ee}^{2} = l_2 c_2 + l_3 c_3, \mathbf{Y}_{ee}^{2} = l_2 s_2 + l_3 s_3 \tag{1.1}$$

Thus clarifying,

$$\mathbf{X}_{W}^{B} = (l_1 + \mathbf{X}_{ee}^{2}) c_1, \mathbf{Y}_{W}^{B} = (l_1 + \mathbf{X}_{ee}^{2}) s_1, \mathbf{Z}_{W}^{B} = \mathbf{Y}_{ee}^{2} \tag{1.2}$$

Hence, according to law of cosines,

$$\theta_2 = \begin{cases} \beta + \psi \\ \beta - \psi \end{cases}, \beta = \arctan 2(\mathbf{Y}_{ee}^{2}, \mathbf{X}_{ee}^{2}), \psi = \arccos \frac{{\mathbf{Y}_{ee}^{2}}^{2} + {\mathbf{X}_{ee}^{2}}^{2} + l_2^2 - l_3^2}{2 l_2 \sqrt{{\mathbf{Y}_{ee}^{2}}^{2} + {\mathbf{X}_{ee}^{2}}^{2}}}$$

With $\theta_3$ and $\mathbf{X}_{ee}^{2}, \mathbf{Y}_{ee}^{2}$ solvable by (1.1) and (1.2) respectively. Furthermore, $\theta_1$ could be easily deduced as reflected.

$$\theta_1 = \arctan 2(\mathbf{Y}_{W}^{B}, \mathbf{X}_{W}^{B})$$