Homework N° 8

The 8th Assignment of Robot Modeling and Control

8-1

Closed loop differential equation.

Solution:

Boring.

$$m(\ddot{\theta}_d + k_r\dot{\theta} + k_p e) + \cos\theta + \sin e = m\ddot{\theta} + b\dot{\theta}^2 + c\dot{\theta} + q\theta^2 - k\theta$$

Which is equivalent to,

$$m\ddot{e} + b\dot{\theta}^2 + (c - mk_r)\dot{\theta} + q\theta^2 - k\theta - \cos\theta - mk_p e - \sin e = 0$$

8-3

Trajectory tracking control.

Solution:

The closed-loop system obtains such a two-order differential equation.

$$\ddot{e} + k_p\dot{e} + k_v e = 0$$

With its closed-loop poles being $s = -10$, its characteristic equation must satisfy the following.

$$s^2 + 20s + 100 = 0 \to k_p = 20, k_v = 100$$

Therefore,

$$\ddot{\theta} = \ddot{\theta}_d + 20\dot{e} + 100e$$

Substituting it back into the original differential equation would one obtain the desired control law.

$$\tau = 2[\ddot{\theta}_d - 20(\dot{\theta} - \dot{\theta}_d) - 100(\theta - \theta_d) + \frac{1}{2}\dot{\theta} + \frac{7}{2}\cos\theta - \frac{3}{2}\sin\theta]$$

8-8

Linearization of a dynamical equation of a two-joint robot.

Solution:

It is relatively easy to deduce that there are a total of four variables conducting Ψ.

$$\Psi = \begin{pmatrix} m_1 l_1^2 + I_{zz1} + I_{yy2} \\ m_2 \\ m_1 l_1 g \\ m_2 g \end{pmatrix}$$

Therefore,

$$\tau = \begin{pmatrix} \ddot{\theta}_1 & d_2^2\ddot{\theta}_1 + 2d_2\dot{\theta}_1\dot{d}_2 & \cos\theta_1 & d_2\cos\theta_1 \\ 0 & \ddot{d}_2 - d_2\dot{\theta}_1^2 & 0 & \sin\theta_1 \end{pmatrix} \Psi$$

8-9

Prove that among the deduction of self-adapting control, $\dot{\varphi} = \overline{A}\varphi + \overline{D}\widetilde{\Psi}$.

Solution:

Since,

$$\mathbf{M}\ddot{\Phi} + \mathbf{C}\dot{\Phi} + \mathbf{L}\Phi = \mathbf{Y}\Psi = \tau_d$$

$$\hat{\mathbf{M}}a_\Phi + \hat{\mathbf{C}}\dot{\Phi} + \hat{\mathbf{L}}\Phi = \tau_d$$

It is clear from substituting one into another that,

$$\hat{\mathbf{M}}a_\Phi + \hat{\mathbf{C}}\dot{\Phi} + \hat{\mathbf{L}}\Phi = \mathbf{Y}\Psi$$

From (8-93) would one derive the following equation.

$$a_\Phi = \ddot{\Phi}_d + K_d\dot{\widetilde{\Phi}} + K_p\widetilde{\Phi}$$

Therefore

$$\hat{\mathbf{M}}(\ddot{\Phi}_d + K_d\dot{\widetilde{\Phi}} + K_p\widetilde{\Phi}) + \hat{\mathbf{C}}\dot{\Phi} + \hat{\mathbf{L}}\Phi = \mathbf{Y}\Psi$$

And, it is also safe to assume that

$$\hat{\mathbf{M}}\ddot{\Phi} + \hat{\mathbf{C}}\dot{\Phi} + \hat{\mathbf{L}}\Phi = \mathbf{Y}\hat{\Psi}$$

Which means subtracting the two equations would result in such a equation

$$\hat{\mathbf{M}}(\ddot{\widetilde{\Phi}} + K_d\dot{\widetilde{\Phi}} + K_p\widetilde{\Phi}) = \mathbf{Y}\widetilde{\Psi}$$

(It is clarification needed that according to the original definition in equation (8-82), $\widetilde{\Phi} = \Phi_d - \Phi$)

Therefore, arrange the equation to match the desired matrix form where

$$\overline{\mathbf{A}} = \begin{pmatrix} 0 & \mathbf{I} \\ -K_p & -K_d \end{pmatrix}, \overline{\mathbf{B}} = \begin{pmatrix} 0 \\ \mathbf{I} \end{pmatrix}, \varphi = \begin{pmatrix} \widetilde{\Phi} \\ \dot{\widetilde{\Phi}} \end{pmatrix}$$

Would one get the final result.

$$\dot{\varphi} = \overline{\mathbf{A}}\varphi + \overline{\mathbf{B}}\mathbf{Y}\widetilde{\Psi} = \overline{\mathbf{A}}\varphi + \overline{\mathbf{D}}\widetilde{\Psi}$$

Q.E.D.

8-10

Validate $\dot{V}_L(\varphi, \widetilde{\Psi}) = -\varphi^{\mathrm{T}}Q_L\varphi$.

Solution:

One has already obtained a fine Lyapunov function

$$V = \varphi^{\mathrm{T}}P\varphi + \widetilde{\Psi}^{\mathrm{T}}\Gamma\widetilde{\Psi}$$

Differentiate the function against time would one get

$$\dot{V} = \dot{\varphi}^{\mathrm{T}}P\varphi + \varphi^{\mathrm{T}}P\dot{\varphi} + 2\widetilde{\Psi}^{\mathrm{T}}\Gamma\dot{\widetilde{\Psi}}$$

And because of

$$\dot{\varphi} = \overline{\mathbf{A}}\varphi + \overline{\mathbf{D}}\widetilde{\Psi}, -Q = \overline{\mathbf{A}}^{\mathrm{T}}P + P\overline{\mathbf{A}}$$

One would get

$$\dot{V} = \varphi^{\mathrm{T}}(\overline{\mathbf{A}}^{\mathrm{T}}P + P\overline{\mathbf{A}})\varphi + 2\widetilde{\Psi}^{\mathrm{T}}\overline{\mathbf{D}}^{\mathrm{T}}P\varphi + 2\widetilde{\Psi}^{\mathrm{T}}\Gamma\dot{\widetilde{\Psi}} = -\varphi^{\mathrm{T}}Q\varphi + 2\widetilde{\Psi}^{\mathrm{T}}(\overline{\mathbf{D}}^{\mathrm{T}}P\varphi + \Gamma\dot{\widetilde{\Psi}})$$

For $\dot{\Psi} = 0$, one would get $\dot{\widetilde{\Psi}}$ according to (8-124),

$$\dot{\widetilde{\Psi}} = -\dot{\hat{\Psi}} = -\Gamma^{-1}\overline{\mathbf{D}}^{\mathrm{T}}P\varphi$$

Substituting back would one get the desired result.

$$\dot{V} = - - \varphi^{\mathrm{T}}Q\varphi$$