Homework N° 6

# The 6th Assignment of Robot Modeling and Control

## 6-1

Interpolation of a certain angular trajectory using modified cubic spline.

**Solution:**

Set

$$f_1(\theta) = a_{11}\theta^3 + a_{12}\theta^2 + a_{13}\theta + a_{14}$$
$$f_2(\theta) = a_{21}\theta^3 + a_{22}\theta^2 + a_{23}\theta + a_{24}$$

The function values must be equal at the interior knots (2n-2 conditions).

$$3^3a_{11} + 3^2a_{12} + 3a_{13} + a_{14} = 45$$

$$3^3a_{21} + 3^2a_{22} + 3a_{23} + a_{24} = 45$$

The first and last functions must pass through the end points(2 conditions).

$$a_{14} = 5$$

$$5^3a_{21} + 5^2a_{22} + 5a_{23} + a_{24} = 80$$

The first derivatives at the interior knots must be equal(n-1 conditions).

$$3 \times 3^2a_{11} + 2 \times 3a_{12} + a_{13} = 3 \times 3^2a_{21} + 2 \times 3a_{22} + a_{23}$$

The second derivatives at the interior knots must be equal(n-1 conditions).

$$6 \times 3a_{11} + 2 \times a_{12} = 6 \times 3a_{21} + 2 \times a_{22}$$

The first derivatives at the end knots are zero (2 conditions).

$$a_{13} = 0$$

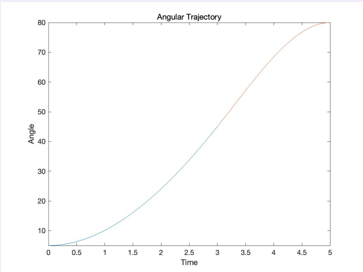
$$3 \times 5^2 a_{21} + 2 \times 5 a_{22} + a_{23} = 0$$

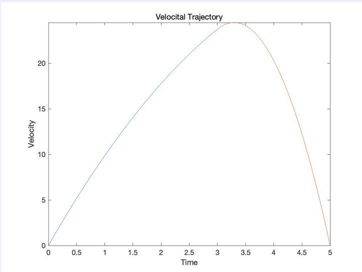
Write in the form of matrix,

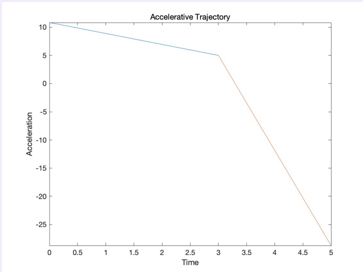
$$\begin{pmatrix} 27 & 9 & 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 27 & 9 & 3 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 125 & 25 & 5 & 1 \\ 27 & 6 & 1 & 0 & -27 & -6 & -1 & 0 \\ 18 & 2 & 0 & 0 & -18 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 75 & 10 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \end{pmatrix} = \begin{pmatrix} 45 \\ 45 \\ 5 \\ 80 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Would one obtain,

$$\begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \end{pmatrix} = \begin{pmatrix} -0.3241 \\ 5.4167 \\ 0 \\ 5 \\ -2.8125 \\ 27.8125 \\ -67.1875 \\ 72.1875 \end{pmatrix}$$




# 6-2

A greater example using modified quartic-cubic-quartic spline interpolation.

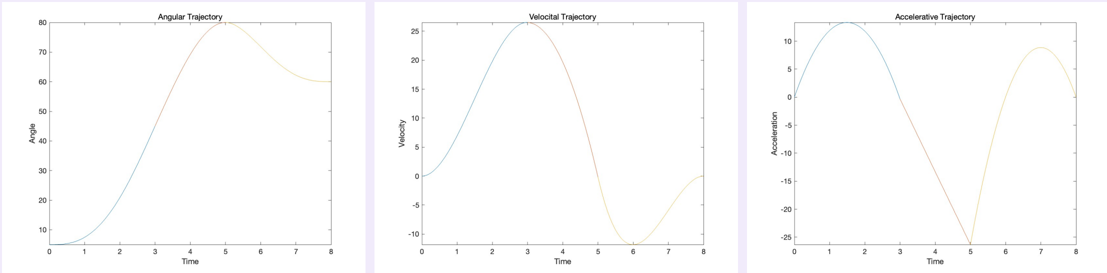
**Solution:**

For exactly the same reason, write in the form of a matrix.

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & & & & & & \\
81 & 27 & 9 & 3 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 27 & 9 & 3 & 1 & 0 \\
0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 125 & 25 & 5 & 1 & 0 \\
0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 625 \\
125 & 25 & 5 & 1 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4096 \\
512 & 64 & 8 & 1 & & & & & & \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & & & & & & \\
108 & 27 & 6 & 1 & 0 & -27 & -6 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 75 & 10 & 1 & 0 & -500 \\
-75 & -10 & -1 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2048 \\
192 & 16 & 1 & 0 & & & & & & \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & & & & & & \\
108 & 18 & 2 & 0 & 0 & -18 & -2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 30 & 2 & 0 & 0 & -300 \\
-30 & -2 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 768 \\
48 & 2 & 0 & 0 & & & & & &
\end{pmatrix}
\begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{15} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{31} \\ a_{32} \\ a_{33} \\ a_{34} \\ a_{35} \end{pmatrix}
=
\begin{pmatrix} 5 \\ 45 \\ 45 \\ 80 \\ 80 \\ 60 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
$$

From which the $\mathbf{a}$ matrix is solved,

$$
\begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{15} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{31} \\ a_{32} \\ a_{33} \\ a_{34} \\ a_{35} \end{pmatrix}
=
\begin{pmatrix} -0.5 \\ 2.9815 \\ 0 \\ 0 \\ 5 \\ -2.1667 \\ 19.3333 \\ -31 \\ 22.5 \\ -0.7346 \\ 20.5617 \\ -211.4074 \\ 939.0617 \\ -1441.2346 \end{pmatrix}
$$




# 6-7

Linear interpolation with parabolic bends.

**Solution:**

The exact positions of the pseudo-points are,

$$\theta_l = 35 - 5 \times \frac{1}{2} = 32.5, t_{d0l} = 2.5$$

$$\theta_r = 37.5, t_{dlr} = 1$$

Thus,

$$\ddot{\theta}_0 t_0 = \frac{\theta_l - \theta_0}{t_{d0l} - \frac{1}{2}t_0} \rightarrow t_0 = 0.168$$

$$\dot{\theta}_{0l} = 13.45$$

Rinse and repeat.

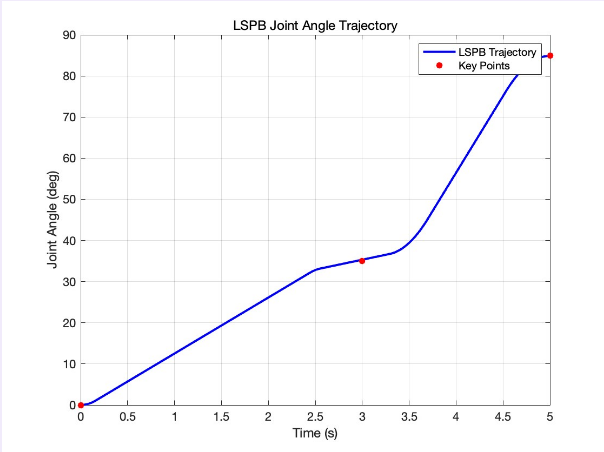
$$t_l = \frac{5 - 13.45}{-80} = 0.106$$

$$\ddot{\theta}_2 t_2 = \frac{\theta_2 - \theta_l}{t_{dr2} - \frac{1}{2}t_2} \rightarrow t_2 = 0.469$$

$$\dot{\theta}_{r2} = 37.5$$

$$t_r = \frac{37.5 - 5}{80} = 0.406$$

Plotting.




# 6-11

Quaternion spherical linear interpolation with non-constant series angular velocity.

**Solution:**

Since the **SLERF** rate is effectively controlled by $t$, is it plausible that such parameter could be modified into any non-linear but still monotonic function regarding $t$,with its derivative at both tips being 0. For instance, this would do:

$$\mathbf{r} = \frac{\sin{(1-3t^2+2t^3)\theta}}{\sin\theta}\mathbf{r_1} + \frac{\sin{(3t^2-2t^3)\theta}}{\sin\theta}\mathbf{r_2}$$