Credit Suisse

FinStats - 2017

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Structure of Presentation

1. Mathematical Modeling
   a. Modeling non-linear constraints as linear constraints using Integer Linear Programming
   b. Optimization Problem for Risk Averse and Risk Neutral
2. Optimal Portfolio Design and Results (Excel)
   a. Risk Neutral
   b. Risk Averse
   c. R pseudo - algorithm to determine weights in Risk Neutral Person
   d. Portfolio Performance over the Next 3 Months
3. Price Modeling and Accuracy Comparison
4. Pseudo Algorithm for Periodic Rebalancing in R

Mathematical Modeling

Mathematical Model

Variables

Form	Variable Explanation
X_i	indicator variable (binary) for investment in asset class I
Y_{ij}	indicator variable (binary) for investment in product j of asset class I
w_{ij}	weight for product j for the asset class i
r_{ij}	return for instrument of i class and j product
σ_{jk}	covariance of j instrument with k instrument
A	set of Asset classes = $\{ s, b, c, fo, f, a, c\}$
B	set of products = $\{A_s, B_s, C_s, D_s, P_b, Q_b, R_b, G_c, S_c, Co_c, UI_{fo}, EI_{fo}, CI_{fo}, J_f, F_f, L_f, A1_a, A2_a, C_c\}$ where U_V: U denotes the product and V denotes the asset class

Non Linear Constraints

Type	
Diversification in Asset classes	$\sum_i^A X_i \geq 3$
Diversification in products	$5 \leq \sum_i^A X_i \sum_j^B Y_{ij} \leq 7$
Investment in Asset class constraint	$X_i \sum_j^B Y_{ij} w_{ij} \leq 0.6 \; for \; i \; \in A$
Investment in particular asset	$0.05 \leq w_{ij} \leq 0.35 \; for \; all \; i, j$
Short Selling in product and asset classes	Ignored
Cash	$X_c Y_{cc} w_{cc} \leq 0.1$

Integer Linear Programing

- To transform the non-linear constraints between the indicator variables of X_j, Y_{ij} and w_{ij} to linear constraints

Type	Transformation
Linking X_i with Y_{ij}	<u>Logical Relationships</u> $X_i = \begin{cases} 1, & Y_{ij} = 1, \quad for\ any\ \ product\ j\ in\ class\ i \\ 0, & if\ \ Y_{ij} = 0, \quad for\ all\ j\ products\ in\ class\ i \end{cases}$ <u>Linear Transformation</u> $Y_{ij} \leq X_i \quad \forall\, i, j$ $X_i \leq \ \sum_{j=1}^{k} Y_{ij}$, for k products that belong to class I
Diversification in products	- $5 \leq \sum_{i,j} Y_{ij} \leq 7$, for all i,j
Investment in Asset class constraint	- $\sum_j^B w_{ij} \leq 0.6\ \ for\ i\ \ \in Asset\ Classes\ \&\ for\ \forall j\ that\ belong\ to\ class\ i$

Integer Linear Programing

Type	Transformation
Investment in particular asset product	- Linking RHS Constraints $$w_{ij} - 0.35Y_{ij} \leq 0$$ - Linking LHS Constraints $$0.05Y_{ij} - w_{ij} \leq 0$$
Investment in Cash	- Linking RHS $$w_{cc} - 0.1Y_{cc} \leq 0$$ - Linking LHS $$0.05Y_{cc} - w_{cc} \leq 0$$ - $Y_{cc} = X_c$
Weights Constraints	- $\sum_{i,j} w_{ij} = 1$, for all i,j

Portfolio Optimization

Investor Problem

Person	Statement
Mr X (Risk averse)	$Min\ \sigma^2(R) = \sum_{j}^{19} w_j^2 \sigma^2(r_j) + \sum_{j=1}^{19} (\sum_{k=1}^{19} 2\ w_j w_k cov(r_j, r_k)$ $E(R) = \sum_{i}^{7} X_i \sum_{j}^{19} Y_{ij}(r_{ij} w_{ij})$ $E(R) \geq 0.15$
Mr. Y (Risk neutral)	$Max\ E(R) = \sum_{i}^{A} X_i \sum_{j}^{B} Y_{ij}(r_{ij} w_{ij})$ $E(R) \geq 0.2$

Part A: Portfolio Optimization

Assumptions

- Short-selling is not allowed. Thus all the variables are assumed to be non-negative
- Ignored the transaction costs for optimization.
- For missing values in the data, simple average of the previous and next day returns has been used to impute the value

Risk Neutral Person (Mr.Y)

	Returns (in Annual Terms) based on $ln(\frac{P_t}{P_{t-1}})$	Weights
Price A2	0.3764964	0.05
Mutual Fund L	0.3958097	0.30
Mutual Fund J	0.3089988	0.05
Stock A	0.7417334	0.35
Stock C	0.5253977	0.25
Total Expected Return		**0.543973799**

Risk Averse Person (Mr. X)

	Returns (in Annual Terms) based on $ln(\frac{P_t}{P_{t-1}})$	**Weights**
Price A1	0.1449422	0.07
Price A2	0.3764964	0.09
AA Bond	0.111081	0.35
CC Bond	0.2531625	0.25
USD - INR	0.0183705	0.10
Mutual Fund J	0.3089988	0.05
Cash	0.0349981	0.10
	Total Expected Return	**0.165331905**
	Portfolio Standard Deviation	**0.058%**

NOTE: Here the convergence issues were faced, as standard deviation ranged from 0.058% to 0.3% with corresponding return ranging from 0.15 to 0.17. It may be because of GRG Non-linear excel optimization. The better solution could have been obtained by using R quad package to do quadratic optimization using linear constraints

Risk Neutral Algorithm in R

1. Get the input of named returns vectors which corresponds return of security with its name
2. To satisfy the Asset class diversification constraints, do as follows
    a. Get the best products from each of the asset class
    b. Select the best 3 products of now available best 7 products
    c. Give these 3 best products the weight of 0.35

    Now given that no. of asset product should range from 5 to 7 And since 3 has already been choosen. So Now select products in range of 2 to 4.
3. Run the for loop for j = 2 to 4, j = no. of remaining products to be choosen
    1. For j = 2, select best 2 out of 16(19-3), assign the weights using lp function in r, calculate, ER1

    The weights assigned should satisfy the weights constraints of products and classes

    1. For j = 3, select best 3 out of 16(19-3), assign the weights using lp function in r, calculate, ER2
    2. For j = 4, select best 4 out of 16(19-3), assign the weights using lp function in r, calculate, ER3
4. Now select the best ER of these three ERs, uses those weights at the outcome

Part D: Actual Return/Risk of Portfolio

		Weights	
Product Name	Return (91 Days)	Risk Neutrals	Risk Averse
Price A1	0.005330701	0.00	0.07
Price A2	0.037356782	0.05	0.09
AA bond	0.018105595	0.00	0.35
BB bond	0.026788918	0.00	0.00
CC Bond	0.03843964	0.00	0.25
Gold	0.039312212	0.00	0.00
Silver	0.029775574	0.00	0.00
Copper	0.110837119	0.00	0.00
Chy- INR	0.030042918	0.00	0.00
GBP - INR	0.044992212	0.00	0.00
USD - INR	0.010677809	0.00	0.10
Mutual Fund L	0.022058824	0.30	0.00
Mutual Fund K	0.062527815	0.00	0.00
Mutual Fund J	0.041758242	0.05	0.05
Stock A	0.039360873	0.35	0.00
Stock B	0.087347166	0.00	0.00
Stock C	0.154363518	0.25	0.00
Stock D	0.093012276	0.00	0.00
Cash	0.00861368	0.00	0.10
TOTAL RETURN of 91 Days		6.29%	2.36%
TOTAL RISK (of 63 Observations of out of 91 days)		0.01016%	0.0000446%

			Weights	
Product Name	Mean Daily Return (of 63 observations)	Annual Return	Risk Neutrals	Risk Averse
Price A1	8.43896E-05	0.031280152	0.00	0.07
Price A2	0.000582157	0.236674176	0.05	0.09
AA bond	0.00028482	0.109538746	0.00	0.35
BB bond	0.000419625	0.165477657	0.00	0.00
CC Bond	0.000598718	0.244167603	0.00	0.25
Gold	0.00061205	0.250233044	0.00	0.00
Silver	0.000465728	0.185247121	0.00	0.00
Copper	0.001668474	0.83764655	0.00	0.00
Chy- INR	0.000469849	0.187030143	0.00	0.00
GBP - INR	0.000698562	0.290314866	0.00	0.00
USD - INR	0.000168591	0.06346278	0.00	0.10
Mutual Fu	0.000346334	0.134724688	0.30	0.00
Mutual Fu	0.000962711	0.420800609	0.00	0.00
Mutual Fu	0.000649364	0.267366048	0.05	0.05
Stock A	0.000612793	0.250572013	0.35	0.00
Stock B	0.001329221	0.623920981	0.00	0.00
Stock C	0.002278558	1.294987585	0.25	0.00
Stock D	0.001411705	0.673486387	0.00	0.00
Cash	0.00009425	0.034998139	0.00	0.10
	TOTAL ANNUAL RETURN		**47.71%**	**14.32%**

- For Mr. Y, the returns requirements are satisfied
- For Mr X, the expected return requirement of atleast equal to 15% is not satisfied
- The value of observed risk or standard deviation is too low. More careful analysis needs to be done

Price Modeling

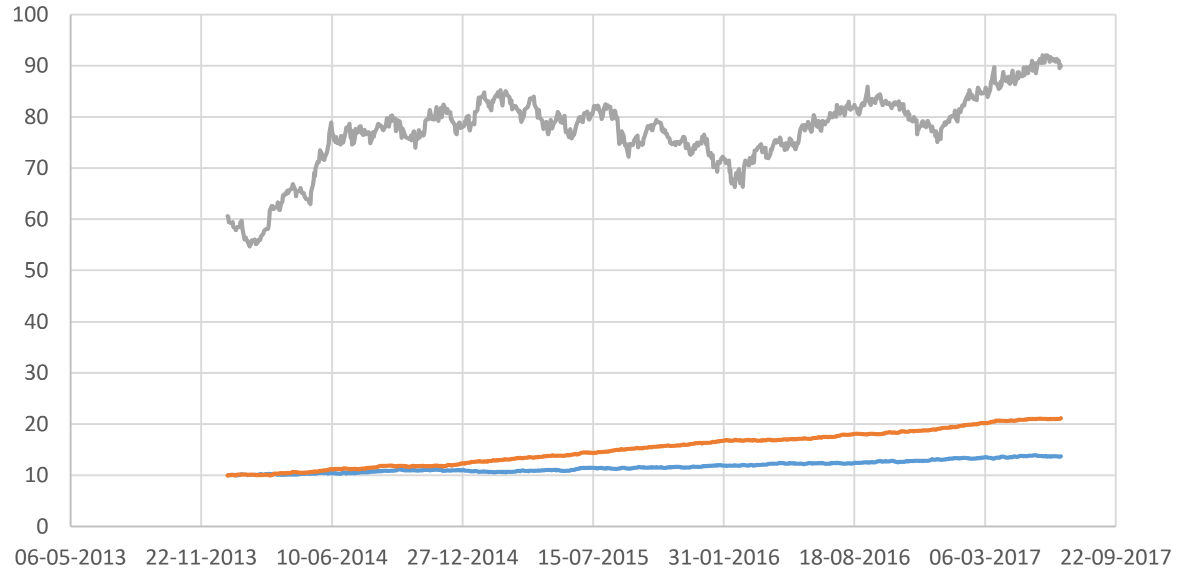
Comparison of Geometric Brownian Motion vs Regression Technique

- Geometric Brownian Motion is mostly preferred as compared to the regression techniques this is due the fact that regression provides an estimate of prices without taking in to account random variation of prices in the short run. Geometric Brownian Motion Method is able to correct this flaw by incorporating "μ dt" which takes in to account the expected return over period t and "σ δ sqrt(dt)" term which is representative of the random motion of prices around it's expected value. The expected volatility "σ" is multiplied by a random component "δ sqrt(dt)". The weight of the random component increases as "dt" increases.

Geometric Brownian Model

- $r_t = S(t) – S(t-1)/S(t-1)$
  $1 + r_t = S(t) / S(t-1)$

  Assumption
  $\log (1 + r_t) = r_t$
  $r_t = \log (S (t) / S(t-1))$
- We expect prices of assets to increase over time in the long run but in the short run the prices of the assets are expected to showcase random motion. The random motion is taken into account with the help of factor B(t).



Geometric Brownian Model

- $S(t) = S_0$ (exp(r(t))
  r(t) log normally distributed with mean μ and variance σ
  $r(t) = \sigma B(dt) + \mu dt$ {return over period t}
  $B(t) = \delta \text{ sqrt}(t)$ {Random volatility}
  σ – expected volatility
  δ – random draw from Standard Normal Distribution
- $\{S(t) - S(t-1)\} / (S(t-1) = r_t$
  $r(t) = \sigma B(dt) + \mu dt$
  $S(t) = S(t-1) \{1+ \sigma \delta \text{ sqrt}(t) + \mu dt\}$
  dt – weight of time period difference given to each price observation {lies between 0 and 1}
  dt = 1/no. of days of simulation
  dt = 1/20

Accuracy Check

- Mean Absolute Deviation Percentage Error $= \frac{100}{n} * \sum \frac{Actual\ value - Forecast\ value}{Actual\ value}$

Price A1	Price A2	AA bond	BB bond	CC Bond	Gold	Silver	Copper	Chy- INR	GBP - INR
1.0960947	0.5644629	0.24569329	0.3978416	0.5372793	2.298514	5.513423	2.341474	0.65248377	1.01996644

USD - INR	Index	Mutual Fund L	Mutual Fund K	Mutual Fund J	Stock A	Stock B	Stock C	Stock D
0.77878692	2.13078	1.145578111	2.444517683	1.316122966	4.796227	3.154198	1.85684	3.258942

Periodic Rebalancing Algorithm

Part E: Periodic Rebalancing Algorithm

Instruction

- Choosing monthly periodic rebalancing.
- Deciding the optimal weights monthly
  - But the rebalancing should only be done if following constraint is satisfied

$$\begin{aligned}&Money\ received\ from\ selling\ assets\\&\geq\ Money\ needed\ for\ purchasing\ the\ additonal\ asset + transaction\ cost\end{aligned}$$

  - Then repeat this exercise for 2 more times for t = 2 and 3
- Assuming the initial purchase of portfolio is cost less for the investor.
- Ignored the appendix constraints except the transaction cost

Algorithm

- $At\ t = 1,$ the weights were $w_{1i}\ for\ the\ ith\ product.$ The weights are obtained by running the same optimization exercise that was done for the Part A.
- Let the w_t the weights vector at time $= t$ where corresponding asset product weight given by w_{ti}.
- Let P_t denotes the prices vector at time $= t$ where corresponding asset product price given by p_{ti}.
- Let t denotes the transaction cost where corresponding asset transaction cost is given by t_i.
- Let N_t denotes the no. of asset product vector that a person has on time t such that N_{ti} denotes the absolute amount of asset product that a person has.
- Let M_t denotes the budget of person at time t

$$N_{0i} = M_0 * \frac{w_{0i}}{P_{0i}}$$

$$Portfolio\ Value(V_0)\ at\ (t = 0) = \sum P_{0i}N_0 = V_0 = M_0$$

Algorithm

- Let the valuation of portfolio at time $t = 1$ be V_1. Then $N_{1i} = V_1 * \frac{w_{1i}}{P_{1i}}$
- $Surplus\ Revenue = \begin{cases} \sum_i (N_{0i} - N_{1i}) P_{1i}(1 - t_i),\ N_{1i} - N_{0i} < 0 \\ \sum_i (N_{1i} - N_{0i}) P_{1i}(1 + t_i),\ N_{1i} - N_{0i} \geq 0 \end{cases}$
- For Trade to take place

$$Surplus\ Revenue = Money\ obtained\ from\ selling\ - Money\ needed\ for\ buying$$
$$\sum_{i \in Selling\ Asset} (N_{0i} - N_{1i}) P_{1i}(1 - t_i) \geq \sum_{i \in Buying\ Asset} (N_{1i} - N_{0i}) P_{1i}(1 + t_i)$$
$$Surplus\ Revenue \geq 0$$

- Now if SR is satisfied. That weights is chosen.
- Now we do the similar exercise for t =2 and t = 3.
- Then based upon these weights, returns and variance-covariance matrix, we could obtain the expected rate of return and risk for both Mr. X and Mr. Y

Questions and Answers