Optimal Portfolio Rebalancing with Dynamic Programming

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The Portfolio Rebalancing Problem

Anyone who invests money in assets, either directly or indirectly, owns a portfolio. Portfolios, like their underlying assets, have risk and return characteristics that naturally evolve over time with the market. The focus of rebalancing is to help the investor successfully navigate a portfolio across market regimes given a particular risk/return based objective.

Therefore, the objective of the portfolio rebalancing problem is to then make a decision at each point in time to rebalance or not while minimizing costs sustained by the portfolio. The application of dynamic programming provides the ideal modelling and optimal solution framework to a problem commonly solved by heuristics in the investment management industry.

Mathematical Framework

State & Decision Variables:

Portfolio Allocation: $w \in W$

Rebalancing Decision: $u \in U$

State & Decision Spaces:

$$U = \{u_t \in \mathbb{R}^n : u_t + w_t = w_{t+1} \forall w_t, w_{t+1} \in W\}$$

$$W = \{w \in \mathbb{R}^n : \sum_{i=1}^{n} w_i = 1, w_i \in [0,1]\}$$

Bellman Optimization Function:

$$J_t(w_t) = \mathbb{E}[G(w_t, u_t, \eta_t) + \gamma J_{t+1}(w_{t+1})]$$

$$G(w_t, u_t, \eta_t) = \tau(w_t, u_t) + \epsilon(w_t, w_{t+1})$$

$$J_t^*(w_t) = \min_{u_t \in U} \sum_{w' \in W} \mathbb{P}(w'|w) \times [G(w_t, u_t, \eta_t) + \gamma J_{t+1}(w')]$$

Algorithm

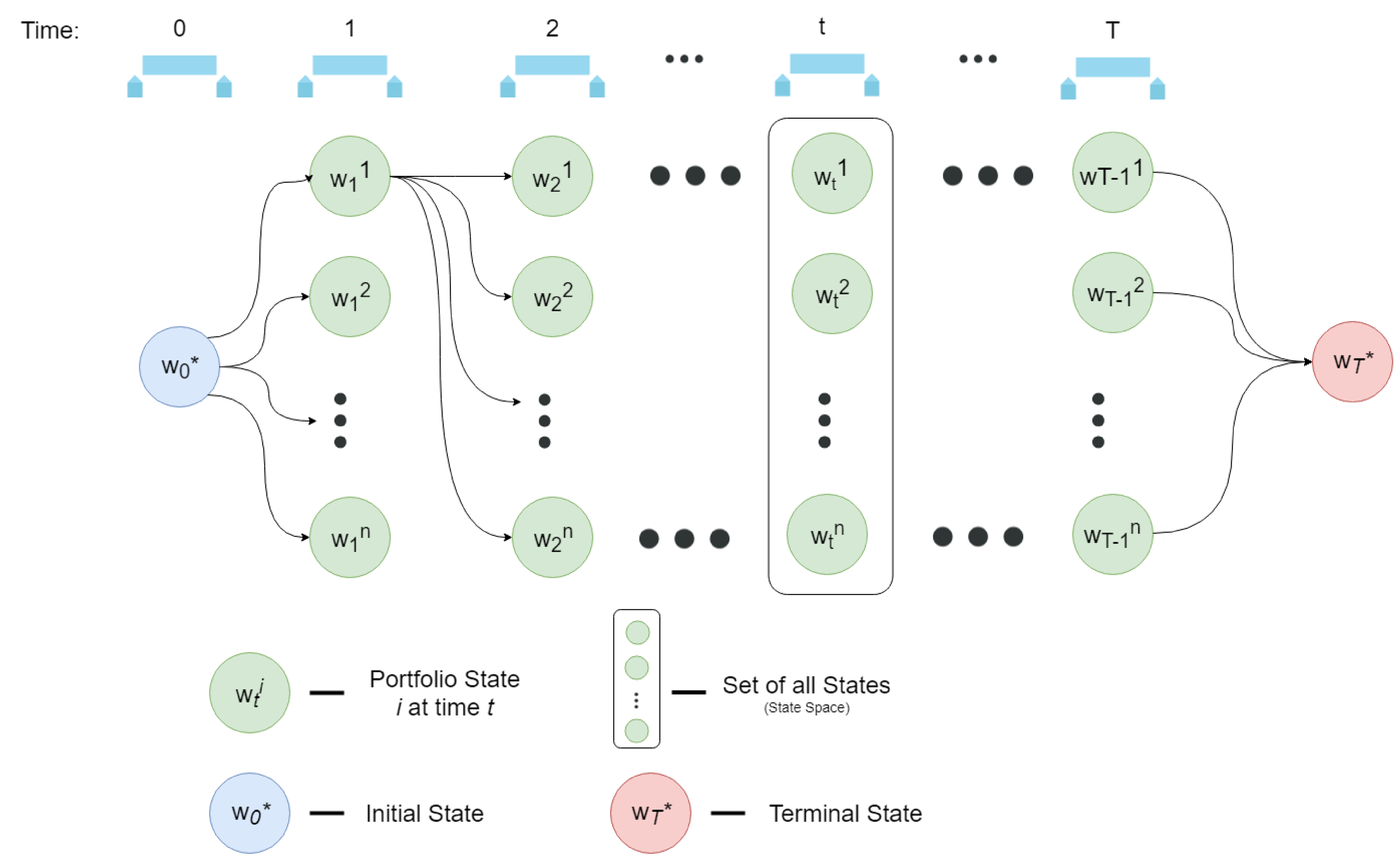
Pseudo code derived from a finite horizon value iteration solution used to determine the expected future cost of each potential portfolio state in the state space

Algorithm 1 Calculate $J_t(w_t) \forall w_t \in W, t \in \{0, 1, ..., T\}$

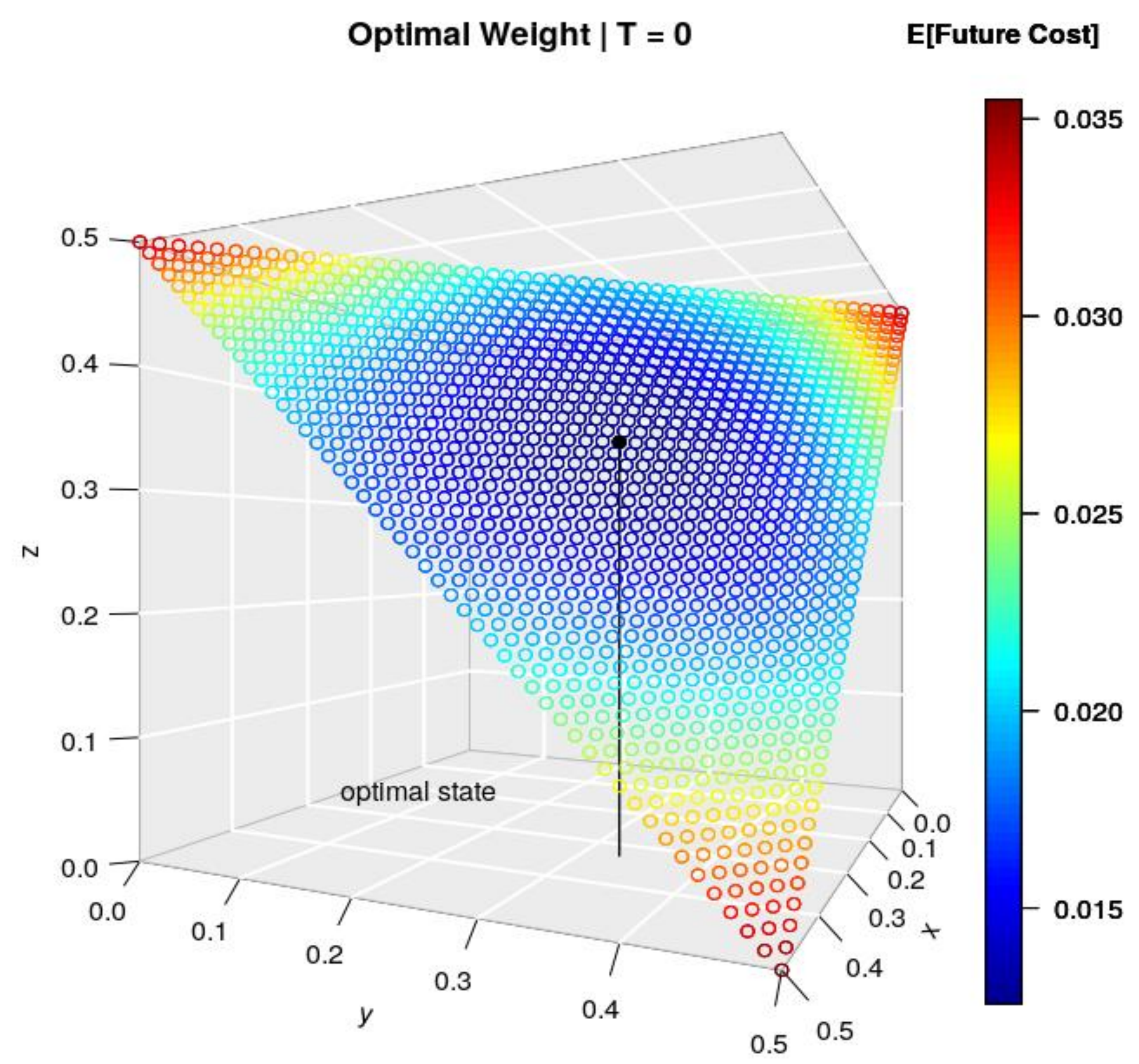
Let $w_{init} \in W$ be the initial allocation
$T = 240, \gamma = 0.9$
$J_T(w_T) = 0, \forall w_T \in W$
for $t = T - 1$ **to** 0 **do**
  $J_t(w_t) = \infty, \forall w_t \in W$
  for $i = 1$ **to** $|W|$ **do**
    $J_t(w_i) = \sum_{w' \in W} \mathbb{P}(w'|w_i) \times [G(w_i, u_t, \eta_t) + \gamma J_{t+1}(w')]$
  end for
end for
$J_0^*(w^*) = \min_{w \in W} J_0(w)$
$u_0^* = w^* - w_{init}$
return u_0^*

Optimization Results

The diagram shown below reflects the layout of the finite time horizon dynamic programming problem.



By starting at the **Terminal State** and iteratively stepping backwards, the expected cost of being in any state $w_t^i \in W$ can be estimated. The cost space of a 3-asset portfolio is estimated over a 240 day time horizon:



Conclusion

The advantages of the dynamic programming approach to portfolio rebalancing are twofold. First, that the mathematical framework is more robust than any heuristic rebalancing method. Dynamic programming provides an objective *optimal* decision as a solution while heuristic methods provide ad-hoc sub-optimal decisions. Second, the nature of a dynamic programming framework is flexibility. It is a methodology that has been successfully applied across disciplines both in and out of the investment industry.

Future research in this space should aim to apply this to portfolios of higher dimensions in computationally efficient methodologies.