Optimal Portfolio Rebalancing using Reinforcement Learning

John-Craig Borman, Professor Somayeh Moazeni

STEVENS INSTITUTE of TECHNOLOGY
THE INNOVATION UNIVERSITY

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The Portfolio Rebalancing Problem

- Portfolios, like their underlying assets, have risk and return characteristics that naturally evolve over time with the market
- Rebalancing helps investors successfully navigate a portfolio across market regimes given a particular risk/return based objective
- The objective of the portfolio rebalancing problem is to make a decision at each point in time to rebalance or not while minimizing costs sustained by the portfolio
- Reinforcement learning provides the ideal modelling and optimal solution framework to a problem commonly solved by heuristics in the investment management industry

The Model

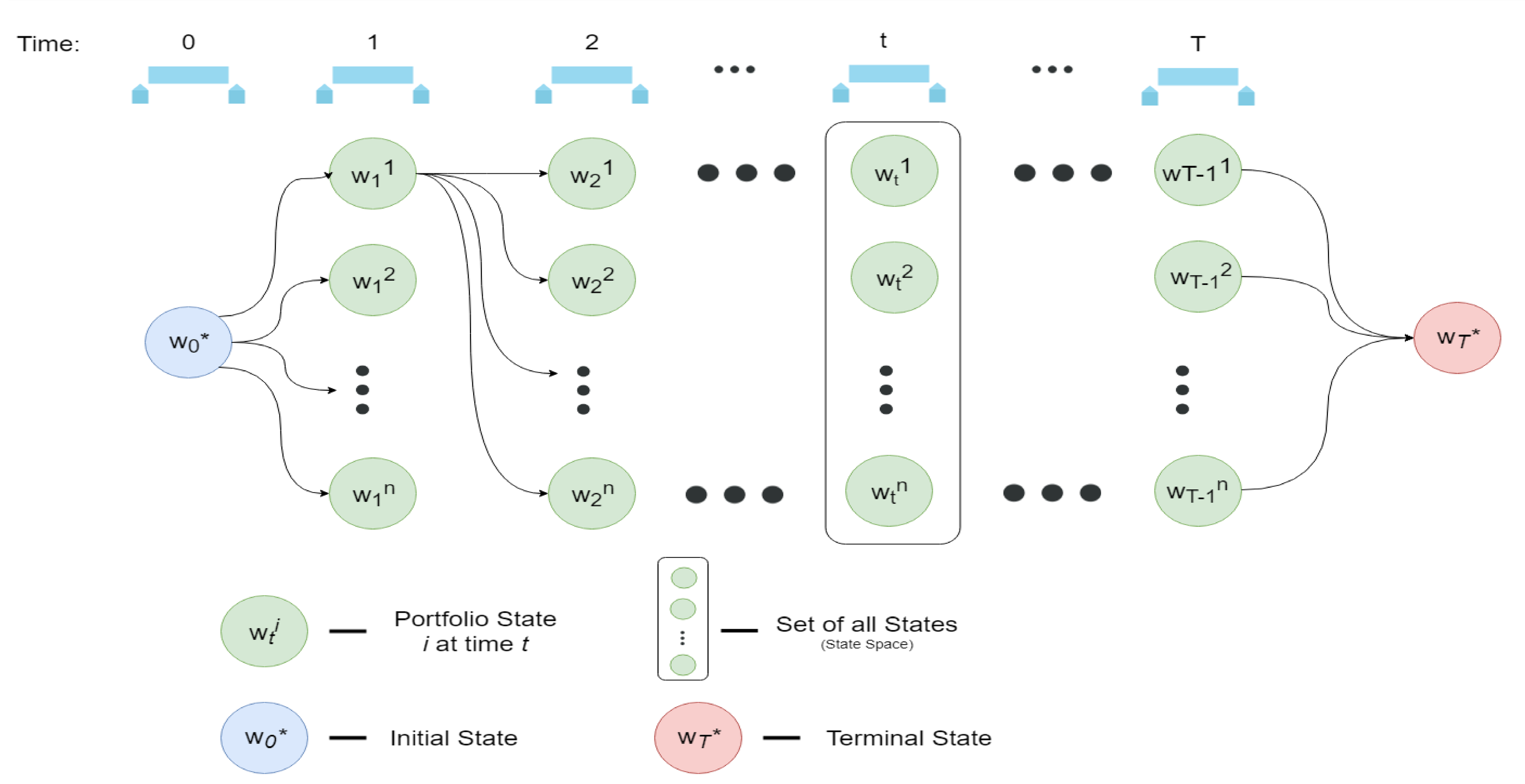
- Given a portfolio of N assets with portfolio weights $w^* = [w_1, \dots, w_N]$, our goal is to maintain a portfolio that tracks the target portfolio as closely as possible while minimizing transaction costs
- The portfolio can be rebalanced every month
- Normal returns are assumed:

  $w_{t+1} = (1+\eta_t)(w_t + u_t)$, where $\eta_t \sim N(\mu, \sigma)$
- The objective to be minimized is the sum of (i) tracking error, (ii) transaction costs, (iii) expected future costs

Methodology: Reinforcement Learning

Algorithm 1 Calculate $J_t(w_t) \forall w_t \in W, t \in \{0, 1, \dots, T\}$

Let $w_{init} \in W$ be the initial allocation
$T = 240, \gamma = 0.9$
$J_T(w_T) = 0, \forall w_T \in W$
for $t = T-1$ **to** 0 **do**
  $J_t(w_t) = \infty, \forall w_t \in W$
  for $i = 1$ **to** $|W|$ **do**
    $J_t(w_i) = \sum_{w' \in W} \mathbb{P}(w'|w_i) \times [G(w_i, u_t, \eta_t) + \gamma J_{t+1}(w')]$
  end for
end for
$J_0^*(w^*) = \min_{w \in W} J_0(w)$
$u_0^* = w^* - w_{init}$
return u_0^*



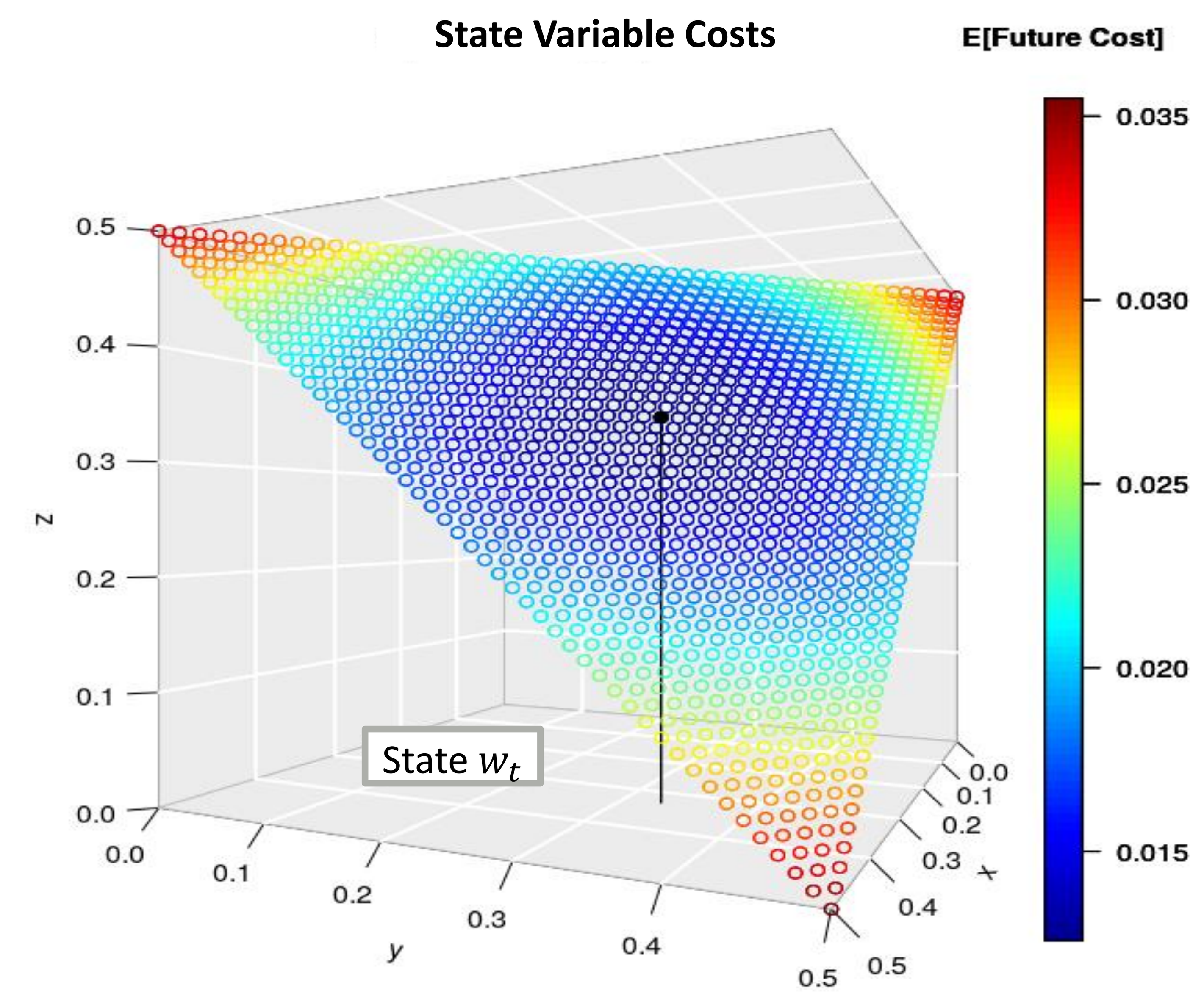
Computational Results

- Q-Learning can deal with the cures of dimensionality as the number of assets N grows
- Actions: Rebalancing Decisions (u)
- State Variable: Portfolio Allocation (w)
- Stage Cost:

$$G(w_t, u_t, \eta_t) = \tau(w_t, u_t) + \epsilon(w_t, w_{t+1})$$

$\tau(w_t, u_t)$ - Transaction Costs

$\epsilon(w_t, w_{t+1})$ – Tracking Error



Conclusion

- Reinforcement learning provides an objective *optimal* decision as a solution while heuristic methods provide ad-hoc sub-optimal decisions
- The solution model can be flexibly adapted to meet an investor's unique constraints
- Future research should look to implement the reinforcement learning model with alternative assumptions to the normal multiplicative dynamic model as well as higher dimensional portfolios

References

Sutton, Richard S., and Andrew G. Barto. Reinforcement Learning: an Introduction. The MIT Press, 2012.

Jaconetti, Colleen M, et al. Best Practices for Portfolio Rebalancing. Vangaurd, July 2010.

Sun, Walter, et al. "Optimal Rebalancing for Institutional Portfolios." Journal of Portfolio Management, vol. 32, no. 2, 2006, pp. 33–43.

Davis, Joseph H, et al. The Asset Allocation Debate: Provocative Questions, Enduring Realities. 2007, The Asset Allocation Debate: Provocative Questions, Enduring Realities.

Brinson, Gary P., L. Randolph Hood, and Gilbert L. Beebower, 1986. Determinants of Portfolio Performance. Financial Analysts Journal 42(4): 39–48.

Pula, Justina & Berisha, Gentrit & Ahmeti, Skender. (2012). The Impact of Portfolio Diversification in the Performance and the Risk of Investments of Kosovo Pension Savings Trust. Journal of Business and Economics.