Multinomial Logit Models with Preference Space and Willingness-to-Pay Space Utility Specifications in R: The logitr Package

John Paul Helveston
George Washington University

Abstract

The abstract of the article.

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1. Introduction

In many applications of discrete choice models, modelers are interested in estimating consumer's marginal "willingness-to-pay" (WTP) for different attributes (cites).

2. The Random Utility Model

The random utility model is a well-established framework in many fields for estimating consumer preferences from observed consumer choices (Louviere, Hensher, and Swait 2000, Train (2009)). Random utility models assume that consumers choose the alternative j a set of alternatives that has the greatest utility u_j. Utility is a random variable that is modeled as $u_j = v_j + \varepsilon_j$, where v_j is the "observed utility" (a function of the observed attributes such that $v_j = f(\mathbf{x}_j)$) and ε_j is a random variable representing the portion of utility unobservable to the modeler.

Consider the following utility model:

$$u_j^* = \boldsymbol{\beta}^{*\prime}\mathbf{x}_j - \alpha^* p_j + \varepsilon_j^*, \qquad \varepsilon_j^* \sim \text{Gumbel}\left(0, \sigma^2 \frac{\pi^2}{6}\right) \tag{1}$$

where $\boldsymbol{\beta}^*$ is the vector of coefficients for non-price attributes $\mathbf{x}_j$, α^* is the coefficient for price p_j, and the error term, ε_j^*, is an IID random variable with a Gumbel extreme value distribution of mean zero and variance $\sigma^2(\pi^2/6)$. This model is not identified since there exists an infinite set of combinations of values for $\boldsymbol{\beta}^*$, α^*, and σ that produce the same choice probabilities. In order to specify an identifiable model, the modeler must normalize equation (1). One approach is to normalize the scale of the error term by dividing equation (1) by σ, producing the "preference space" utility specification (Train and Weeks 2005):

$$\left(\frac{u_j^*}{\sigma}\right) = \left(\frac{\boldsymbol{\beta}^*}{\sigma}\right)'\mathbf{x}_j - \left(\frac{\alpha^*}{\sigma}\right)p_j + \left(\frac{\varepsilon_j^*}{\sigma}\right), \qquad \left(\frac{\varepsilon_j^*}{\sigma}\right) \sim \text{Gumbel}\left(0, \frac{\pi^2}{6}\right) \tag{2}$$

The typical preference space parameterization of the multinomial logit (MNL) model can then be written by rewriting equation (2) with $u_j = (u^*_j/\sigma)$, $\boldsymbol{\beta} = (\boldsymbol{\beta}^*/\sigma)$, $\alpha = (\alpha^*/\sigma)$, and $\varepsilon_j = (\varepsilon^*_j/\sigma)$:

$$u_j = \boldsymbol{\beta}'\mathbf{x}_j - \alpha p_j + \varepsilon_j \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \frac{\pi^2}{6}\right) \tag{3}$$

The vector $\boldsymbol{\beta}$ represents the marginal utility for changes in each non-price attribute, and α represents the marginal utility obtained from price reductions. In addition, the coefficients $\boldsymbol{\beta}$ and α are measured in units of *utility*, which only has relative rather than absolute meaning.

The alternative normalization approach is to normalize equation (1) by α^* instead of σ, producing the "willingness-to-pay (WTP) space" utility specification (Train and Weeks 2005):

$$\left(\frac{u^*_j}{\alpha^*}\right) = \left(\frac{\boldsymbol{\beta}^*}{\alpha^*}\right)' \mathbf{x}_j - p_j + \left(\frac{\varepsilon^*_j}{\alpha^*}\right), \qquad \left(\frac{\varepsilon^*_j}{\alpha^*}\right) \sim \text{Gumbel}\left(0, \frac{\sigma^2}{(\alpha^*)^2}\frac{\pi^2}{6}\right) \tag{4}$$

Since the error term in equation is scaled by $\lambda^2 = \sigma^2/(\alpha^*)^2$, we can rewrite equation (4) by multiplying both sides by $\lambda = (\alpha^*/\sigma)$ and renaming $u_j = (\lambda u^*_j/\alpha^*)$, $\boldsymbol{\omega} = (\boldsymbol{\beta}^*/\alpha^*)$, and $\varepsilon_j = (\lambda\varepsilon^*_j/\alpha^*)$:

$$u_j = \lambda\left(\boldsymbol{\omega}'\mathbf{x}_j - p_j\right) + \varepsilon_j \qquad \varepsilon_j \sim \text{Gumbel}\left(0, \frac{\pi^2}{6}\right) \tag{5}$$

Here $\boldsymbol{\omega}$ represents the marginal WTP for changes in each non-price attribute, and λ represents the scale of the deterministic portion of utility relative to the standardized scale of the random error term.

References

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Affiliation:

John Paul Helveston
George Washington University
Science & Engineering Hall 800 22nd St NW Washington, DC 20052
E-mail: `john.helveston@gmail.com`
URL: `http://jhelvy.com`