Short-term Forecasting Documentation

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Welcome to the short-term-forecasting wiki!

Short-term forecasting of electricity generation, demand and prices using machine learning.

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Background

The transition towards a future low-carbon economy is driven globally by the Paris Agreement [Pari15], which recognises the need for sustainable development worldwide to counter the threats of climate change. The European Union (EU) is committed to reduce greenhouse gas (GHG) emissions by 2050 to 80-90 % below 1990 levels [Ener12]. As the energy industry is responsible for the highest share of anthropogenic GHG emissions, importance is placed on how changes in energy systems can help achieve these GHG emission reduction targets [Ener12].

A number of opportunities exist for the decarbonisation of the energy industry. The International Renewable Energy Agency (IRENA), in their renewable energy

roadmap study, has identified renewable energy as having the highest potential in reducing energy-related carbon dioxide ($\rm CO_2$) emissions globally, which is closely followed by energy efficiency and electrification with renewable energy [Glob18]. In a 2018 political agreement, the EU member states agreed upon a target of at least 32 % of the demand being met with renewables by 2030, through national targets of the individual member states [ReneND]. The electricity demand in the transport sector is also expected to increase due to expected petrol and diesel engine bans and subsequently the electrification of road transport [Worl17].

The energy system is also transitioning towards a decentralised system with more consumer participation and new forms of flexibilities, including sector coupling, demand-side management (DSM), energy conversion and storage, cross-border interconnection and curtailment. This allows demand patterns to shift to better suit the generation patterns in systems with high penetration of variable renewable energy (VRE) resources, such as solar and wind [Lund17], [Towa18]. However, this requires cooperation involving many actors with various responsibilities and dependencies that interact within this energy system, and opens up the opportunity to perform interdisciplinary research work in the area of energy system analysis.

The ENSYSTRA - ENergy SYStems in TRAnsition Innovative Training Network has been established to address the challenges of the energy transition with interdisciplinary collaboration and regional cooperation involving academia, government and industry [AbouND]. ENSYSTRA is centred on the North Sea region and focusses on performing interdisciplinary modelling work involving technology, economics, social science and humanities, and combining various modelling approaches in different levels and resolutions. ENSYSTRA aims to keep an open science approach, which will allow the resulting models to be subject to full scientific scrutiny.

Energy systems models, which are tools used to project the future energy supply of a country or region [Herb12], is the centre of ENSYSTRA. The figure below explains the energy systems modelling process using a system analysis approach [Kroo15]. This process starts with creating a model of the actual energy system by simplifying and conceptualising the present system. This conceptualised system with all assumptions is then mathematically solved to produce numerical results. These results can then be interpreted and conclusions can be drawn regarding the future energy system. Such conclusions form the evidence-base for decision makers, resulting in policy implications or operational strategies that help achieve these climate targets.

The system analysis approach applied on the energy system modelling process, adapted from Krook-Riekkola 2015 [Kroo15].

There are 15 early-stage researchers (ESRs) across four work packages (WPs) in ENSYSTRA, as shown in the figure below. The research project entitled "Development of a real-time optimisation solution for dispatchable energy supply units" is conducted by ESR 9, who is enrolled as a PhD student at University of

Stavanger (UiS) in Norway. This project is within WP 2 (technology prospects and development pathways), which focusses on technological options for the energy transition, mainly in terms of techno-economic performance over time. For this research project, the technology focus is on the digitalisation of the electricity sector. As the electricity system transitions into smart systems, the system will have an increasing amount of sensors and controllers that continuously record measurements of the system [Lund17]. Advancements in these technologies mean that data that is fast, heterogeneous and high in volume from the electricity system will be generated. Data with these characteristics must be managed and analysed effectively to gain insights on the electricity system, which can then be converted to strategies that optimise the system [Mana12]. This project will specifically investigate how artificial intelligence (AI) can play a role in the transition to a low-carbon electricity system by utilising high resolution data of the system. The next section will investigate this, as well as explain what is meant by "real-time" and "dispatchable" in the context of electricity systems in this project.

Interactions between the four WPs of the ENSYSTRA project. Source: ENSYSTRA [AbouND].

Problem definition

Electricity system

The electricity system can be seen as having two components; the physical grid consisting of generators and transmission and distribution systems, and the electricity market consisting of a number of actors [Erba16].

Electricity systems exist in different resolutions and levels of uncertainty. The figure below represents the different scales of electricity systems, mainly in terms of temporal resolution, but also uncertainty and spatial resolution [Glis18], [Pfen14]. Temporally, "real-time" is referred to as the time of dispatch. It can be observed that the operational planning scale has high spatial and temporal resolution, and relatively low uncertainty. Operational planning includes dispatch planning and plant scheduling (i.e., unit commitment), which ranges from a few minutes to a week before dispatch. Maintenance planning can take a few weeks to years, as it involves upgrade and maintenance work which may require shut-down of units or assets, in turn affecting the availability of generation units and grid infrastructure. Adequacy assessments, which takes years, involve assessing the existing generation and storage capacities and planning for new installations based on demand projections, to ensure this demand will be met in the future. Finally, grid investment decisions, including planning transmission and distribution grid networks, cross-border and regional interconnections and grid capacity expansions, take many years to decades and have very high uncertainty as a result.

The various scales of electricity systems in terms of their approximate temporal

resolution, as well as spatial resolution and uncertainty, adapted from Glismann 2018 and Pfenninger, et al. 2014 [Glis18], [Pfen14].

Generation technologies

The table below shows the characteristics of the main energy generation technologies, including their costs. These generation sources have different variabilities, fuel types, flexibilities, costs and carbon emissions. According to the EU reference scenario 2016 [EnerND], wind and solar energy resources, which are VRE resources, are expected to generate a total of 35 % of EU's electricity by 2050, which is a significant increase (23 %) from 2015 levels. Conversely, generation from nuclear and solids, which are not variable and provide base load generation, are expected to decrease significantly. Unlike conventional generators, VRE are intermittent as they are dependent on atmospheric conditions, such as wind speed and cloud cover, and they vary both spatially (i.e., location-dependent) and temporally [Josk11]. Therefore, VRE generation cannot be controlled to meet the demand patterns and needs of the energy system [Josk11], which is a challenge to electricity and energy system operators in general. The costs listed in this table are derived based on National Renewable Energy Laboratory (NREL)'s NREL-SEAC 2008 Data Set [Tidb10]. VRE generation technologies have high capital expenditure (CAPEX) compared to conventional fossil-powered and biomass generation. Conversely, the operational expenditure (OPEX), which includes fuel and fixed operational and maintenance (O&M) costs, is low for VRE generation technologies, as they have no fuel costs unlike conventional generators.

Table 1: Characteristics of the main energy generation technologies, adapted from Erbach 2016 [Erba16] and Tidball, et al. 2010 [Tidb10].

$\text{Type}^{[f1]}$	Variable	Fuel type	Flexibility	Low carbon	CAPEX	OPEX	$LCOE^{[f2]}$
Coal	no	fossil	medium	no	low	high	very low
Natural gas	no	fossil	high	no	very low	very high	low
Biomass	no	renewable	medium	$\mathrm{yes}^{[\mathrm{f3}]}$	low	very high	very high
Nuclear	no	nuclear	low	zero-emission	medium	medium	medium
Hydro	no	renewable	very high	zero-emission			
Solar	yes	renewable	very low	zero-emission	very high	very low	very high
Wind	yes	renewable	very low	zero-emission			
$On shore\ wind$					high	very low	very low
$Off shore\ wind$					very high	low	high
Geothermal	no	renewable	high	zero-emission	high	medium	high

[[]f1] Costs for natural gas, biomass, solar and geothermal are that of advanced combustion turbine, biomass gasification plant, utility-scale photovoltaic and

 $hydrothermal\ plant\ respectively$

- [f2] LCOE levelised cost of electricity
- [f3] regrowth of biomass compensates emissions

Electricity market

The actors in the electricity market include [Erba16], [PinsND], [TheC15]:

- Generator
- Retailer
- Large and small consumer
- Transmission system operator (TSO)
- Distribution network operator (DNO)
- Balance responsible party (BRP)
- Aggregator
- Regulator
- Market operator

There are two types of electricity markets; the retail market and the wholesale market [Erba16]. The retail market involves the retailers buying electricity from generators and selling it to consumers. The wholesale market involves generators, retailers and (large) consumers, who buy and sell electricity. Energy-only transactions in the wholesale market have different temporal resolutions [PinsND], [TheC15] and take place before dispatch, shown in green in the figure below. Balancing markets, shown in pink in the figure below, which involve both energy and services, operate both before and after dispatch [TheC15]. The energy-only markets are operated by the market operator or power exchanges, while the balancing market is operated by the system operator. The day-ahead and intra-day markets can be considered short-term electricity markets, as the former takes place 24 hours in advance of dispatch, while the latter takes place continuously after the day-ahead market, up to minutes before dispatch [Over16].

The various electricity markets in terms of operator and temporal resolution, before and after dispatch, adapted from KU Leuven Energy Institute 2015 and Pinson 2018 [PinsND], [TheC15].

Objectives

The main research objective of this project is:

To develop an open-source, machine learning-based electricity market model for the North Sea region which will help electricity generators, retailers, large consumers, BRPs and system operators in short-term electricity markets (i.e., day-ahead and intra-day markets) to develop operational and bidding strategies that maximise their profits under uncertainty of VRE generation. The model will consist of a

forecaster based on machine learning, which will use high resolution time series weather forecasts for the upcoming period, and recent historical measurements of electricity generation, demand and market prices, to forecast the latter three quantities for the upcoming period. These forecasts will serve as inputs to an optimiser, which maximises social welfare in the electricity market.

Based on the main research objective, the following research questions have been derived:

- What methods and resources are needed to process and store the large volume of high resolution data required for this model?
- What type of machine learning algorithms are suited for the time series forecasting of electricity prices, demand and generation?
- What optimisation methods are suitable for maximising the social welfare problem in the electricity market, and what are the constraints to this optimisation problem?
- What methods can be used to analyse the inputs and outputs of the model and translate them into operational strategies relevant to the market participant?
- How can this model be standardised and published so that it is available for use openly by any participant in the electricity market, as well as other interested parties, such as policymakers?
- How can this high resolution electricity market operational model be integrated with the overall North Sea energy systems model to provide insights on long-term planning and investments in the energy sector?

Regions

North Sea countries

As per the definition provided by the European MSP Platform [NortND] and the CPMR North Sea Commission [Memb15], the North Sea region consists of eight countries: Belgium, Denmark, France, Germany, Netherlands, Norway, Sweden and United Kingdom.

NUTS (Nomenclature of territorial units for statistics)

See the Jupyter notebook.

Bidding zones

Definition

According to [Bidd14]:

• The largest geographical area within which market participants are able to exchange energy without capacity allocation.

- The majority of bidding zones in Europe are defined by national borders (e.g., France or the Netherlands).
- Some are larger than national borders (e.g., Austria, Germany and Luxembourg or the Single Electricity Market for the island of Ireland)
- Some are smaller zones within individual countries (e.g., Italy, Norway or Sweden).

Bidding zones in the North Sea region

The bidding zones in the European electricity market are illustrated in the map below [Tren17].

Bidding zones in the European electricity market. Source: Polskie Sieci Elektroenergetyczne [Tren17].

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Table 2:	Bidding	ZONES	ın	the	North	500	region
Table 2.	Didding	ZOIICS	111	ULIC	1101011	\mathcal{DCa}	region.

Country	Market(s)	Zone(s)
Belgium (BE)	EPEX SPOT	BE
Germany (DE / GE)	EPEX SPOT	$\mathrm{DE} ext{-}\mathrm{AT} ext{-}\mathrm{LU}^{[\mathrm{f4}]}$
Denmark (DK)	Nord Pool	DK1, DK2
France (FR)	EPEX SPOT	FR
Netherlands (NL)	EPEX SPOT	NL
Norway (NO)	Nord Pool	NO1, NO2, NO3, NO4, NO5
Sweden (SE / SW)	Nord Pool	SE1, SE2, SE3, SE4
United Kingdom (UK)	EPEX SPOT, Nord Pool	$GB, I-SEM^{[f4]}$

[f4] Austria (AT / AU); Luxembourg (LU); Great Britain (GB); Irish single electricity market (I-SEM), which includes Republic of Ireland (IE) and UK's Northern Ireland (NI).

Transmission system operators and interconnections

The power exchanges that operate in the North Sea region are EPEX SPOT (Belgium, France, Germany, Netherlands, United Kingdom) and Nord Pool (Denmark, Norway, Sweden, United Kingdom) [Over16], [SeeMND], [EPEXND]. The day-ahead market takes place generally as an hourly auction 24 hours prior to dispatch [Over16]. The intra-day market has continuous trading and will operate until two hours and up to five minutes before dispatch [Over16]. The North Sea region consists of multiple TSOs, cross-border interconnections and bidding zones, as listed in the table below.

Table 3: TSOs and cross-border interconnections in the North Sea region. Data: European Network of Transmission System Operators for Electricity [ENTSND], [RegiND].

Ctry.[f5	TSOs	Cross-border interconnection ^{[f5],[}	$_{\mathrm{f6}_{\mathrm{zones}^{[f5],[f6]}}}^{\mathrm{Bidding}}$
$\overline{\mathrm{BE}}$	Elia System Operator	FR, LU, NL, UK	BE
DK	Energinet	DE, NO, SE	DK1, DK2
DE	TransnetBW, TenneT TSO,	AT, CH, CZ, DK,	CZ+DE+SK,
	Amprion, 50Hertz	FR, LU, NL, PL,	DE-AT-LU,
	Transmission	SE	DE-LU
FR	Réseau de Transport	BE, CH, DE, ES,	FR
	d'Electricité	IT, UK	
NL	TenneT TSO	BE, DE, NO, UK	NL
NO	Statnett	DK, FI, NL, SE	NO1, NO2, NO3,
SE	Svenska Kraftnät	DK, FI, DE, LT, NO, PL	NO4, NO5 SE1, SE2, SE3, SE4
UK	National Grid Electricity	BE, FR, IE, NL	GB, IE (SEM)
	Transmission, System		
	Operator for Northern		
	Ireland, Scottish Hydro		
	Electric Transmission,		
	ScottishPower Transmission		

[[]f5] Ctry. - Country; AT - Austria; BE - Belgium; CH - Switzerland; CZ - Czech Republic; DE - Germany; DK - Denmark; ES - Spain; FI - Finland; FR - France; GB - Great Britain; IE - Ireland; IT - Italy; LT - Lithuania; LU - Luxembourg; NL - Netherlands; NO - Norway; PL - Poland; SE - Sweden; SK - Slovakia; UK - United Kingdom; SEM - Single electricity market.

Data

Data folder navigation

- ENTSO-E
 - generation and load data for each bidding zone in the North Sea region, grouped by country
- Meteo meteorological data, grouped by country
- Market market data for the North Sea region
- NUTS territorial units
- output output or modified data from this project

[[]f6] These countries are not part of the North Sea region: AT, CH, CZ, ES, FI, IE, IT, LT, LU, PL.

Met data

Deutscher Wetterdienst

- CDC (Climate Data Center) portal
- CDC OpenData
- Terms of use for data on the CDC ftp server
- Data set descriptions
 - Hourly station observations of air temperature at 2 m above ground in °C for Germany
 - Hourly station observations of relative humidity in % for Germany
 - Hourly station observations of precipitation amount in mm for Germany
 - Hourly station observations of form of precipitation (WR code) for Germany
 - Hourly station observations of index whether precipitation has fallen for Germany
 - Hourly mean of station observations of wind speed ca. 10 m above ground in m/s for Germany
 - Hourly mean of station observations of wind direction at ca. 10 m above ground in degree for Germany
 - Hourly station observations of air pressure at station level in hpa for Germany
 - Hourly station observations of air pressure at mean sea level in hpa for Germany
 - Hourly station observations of cloud coverage in eighths for Germany
- Hourly wind data

Royal Netherlands Meteorological Institute

Met Office

Norwegian Meteorological Institute

Swedish Meteorological and Hydrological Institute

Danish Meteorological Institute

Météo-France

The Royal Meteorological Institute of Belgium

Generation and demand data

ENTSO-E Transparency Platform

- GENERAL TERMS AND CONDITIONS FOR THE USE OF THE ENTSO-E TRANSPARENCY PLATFORM
- LIST OF DATA AVAILABLE FOR FREE RE-USE
- Downloaded data:
 - Actual Generation per Production Type
 - Total Load Day Ahead / Actual

Market data

Nord Pool

- Membership list Nord Pool
- Terms and conditions for use

EPEX Spot

• EPEX SPOT Exchange Members

Other data

NUTS (Nomenclature of territorial units for statistics)

Methodology

Modelling framework

Modelling framework. License: CC BY-SA 4.0.

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