Introduction to Heuristic Search

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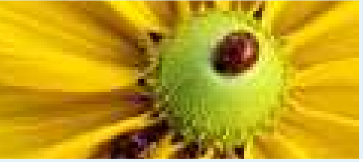
Logistics



Course Schedule

- Terminology, TSP, Depth First Search — October 7th
- Heuristics, Tree, Local, Multicore Search — October 14th
- 15 Puzzle, Heuristic Construction, Best First Search — October 21st
- Tile Puzzles, Bounded Suboptimality, Anytime Search October 28th
- Grid Navigation, Inadmissible Heuristics Learning in Search — November 4th
- Search on Disk for Big Problems — November 11th

Course Schedule — Revised

■ TSP, Heuristics, Basic Algorithms Tree Search, Local Search	October 7th
■ 15 Puzzle, Heuristic Construction, Best First Search	October 14th
■ Multiple Cores, Multiple Heuristics Advanced Search Techniques	October 21st
■ Traveling for STAC Program	October 28th
■ Traveling for STAC Program	November 4th
■ Home for Birth of Niece	November 11th

If there's interest in making up the other material, we can see about working something out.

Course Materials

- TSP Ingestion Code, Visualizer & Reference Solutions
  - Tiles Ingestion Code, Visualizer

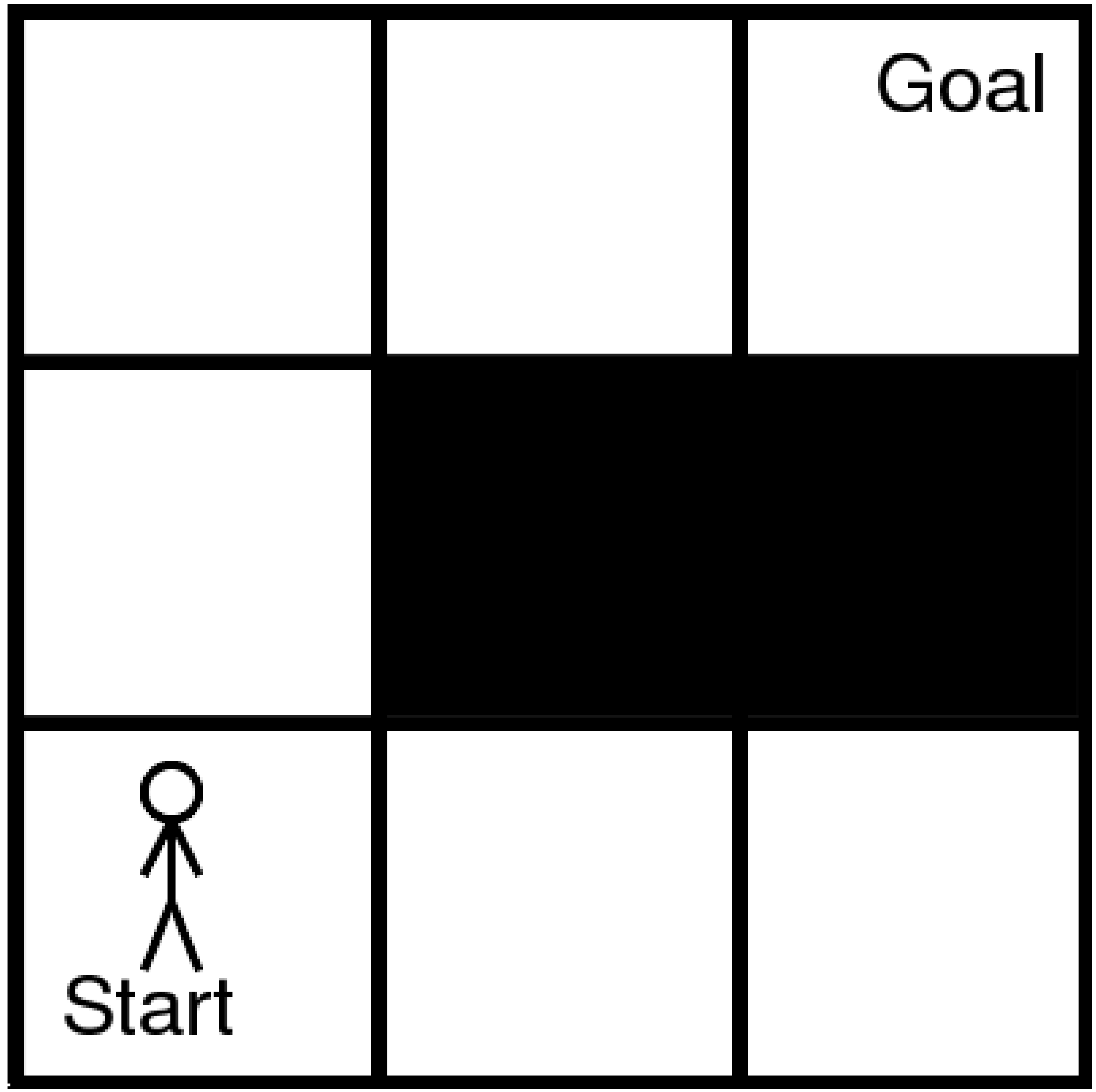
  https://github.com/jordanthayer/tsp-demo
- C++ Search Library https://github.com/jordanthayer/search
  - C++ translation of Ocaml, done by Ethan Burns
- Search Visualizations
  - My Youtube Channel
  - Still Images from my home page
- These Slides will be somewhere soon as well.

Useful Texts

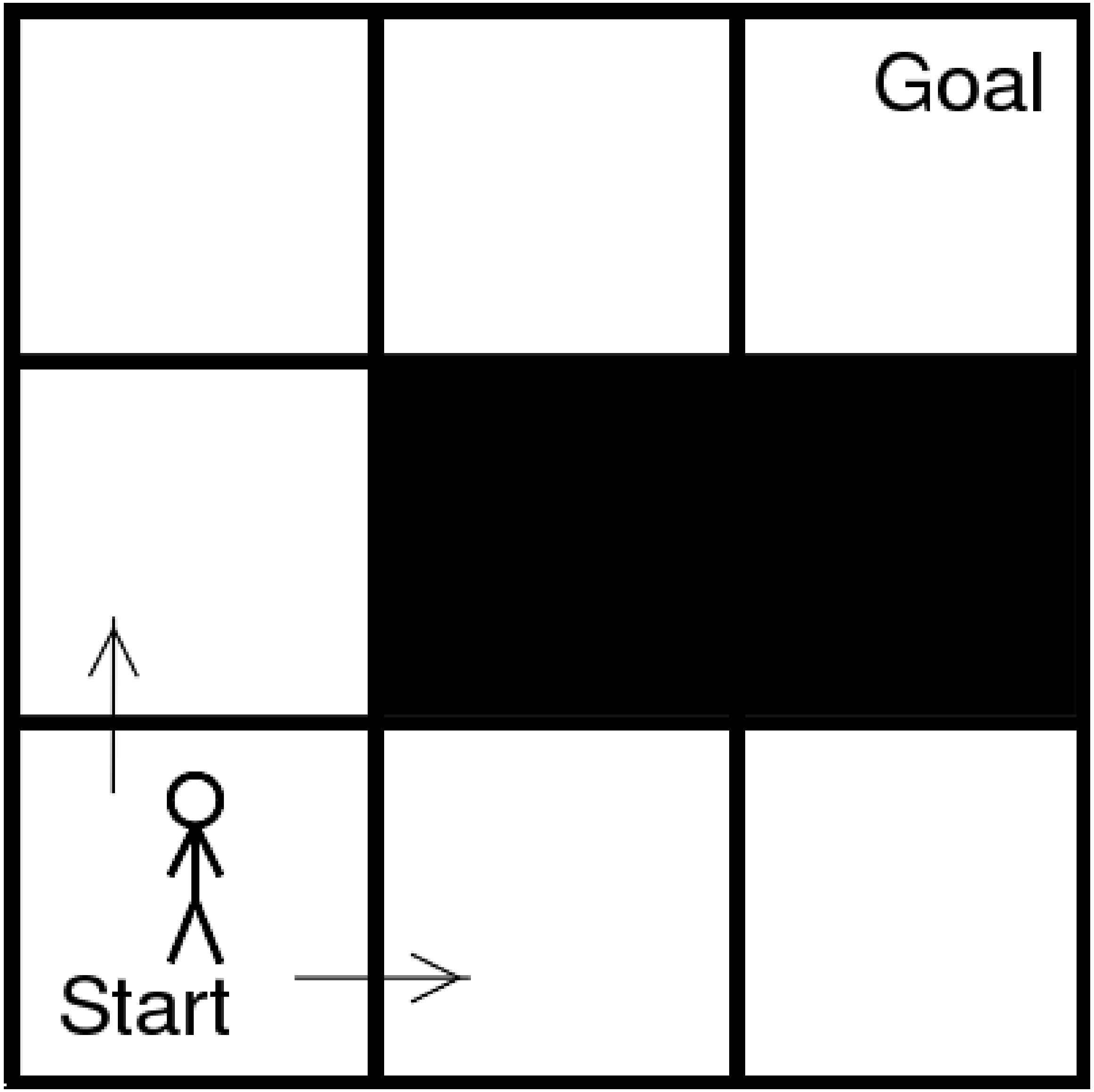
- Artificial Intelligence: A Modern Approach
  Especially Chapters TODO
- Heuristics: Intelligent Search Strategies for Computer Problem Solving
- Do the Right Thing

Heuristic Search

A Simple Problem



Where can I get to from here?



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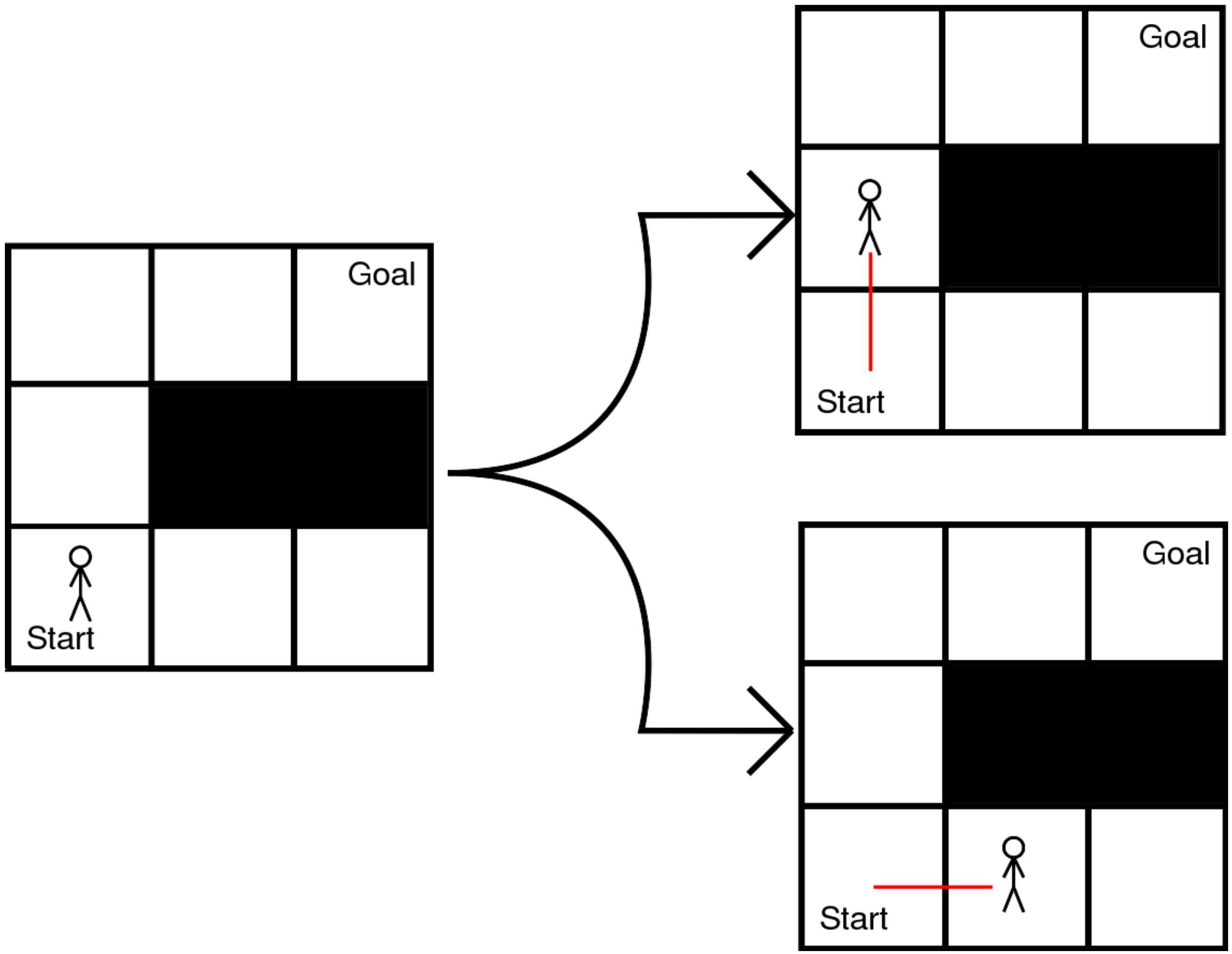
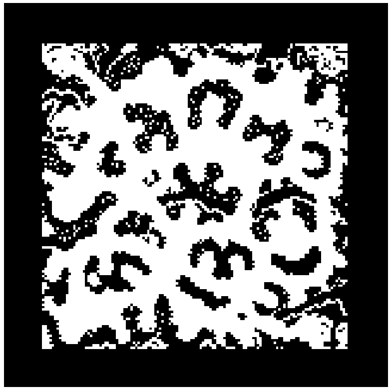
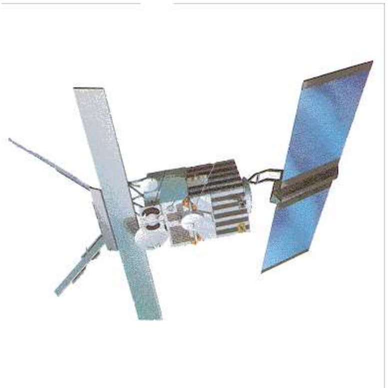


Other Heuristic Search Problems





- A starting configuration
- Primitive operations to move between configurations
- A goal test

TSP

Problem Definition

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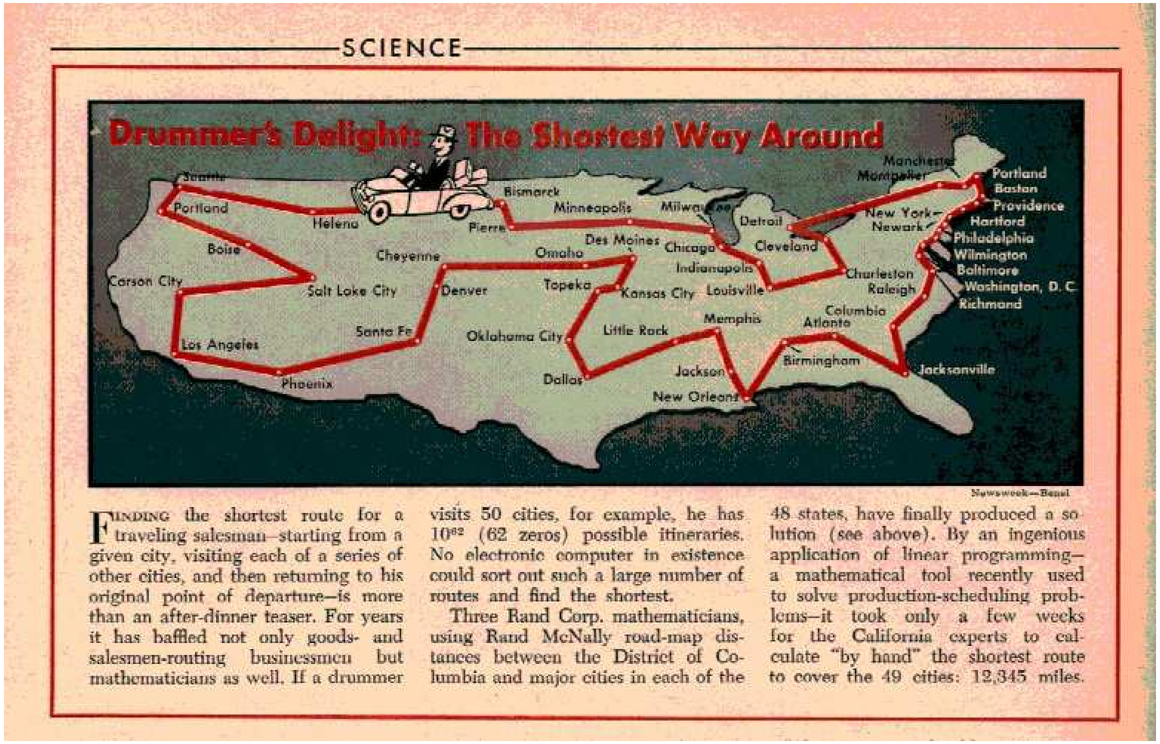

SCIENCE

FINDING the shortest route for a traveling salesman—starting from a given city, visiting each of a series of other cities, and then returning to his original point of departure—is more than an after-dinner teaser. For years it has baffled not only goods- and salesmen-routing businessmen but mathematicians as well. If a drummer visits 50 cities, for example, he has 10^{62} (62 zeros) possible itineraries. No electronic computer in existence could sort out such a large number of routes and find the shortest.

Three Rand Corp. mathematicians, using Rand McNally road-map distances between the District of Columbia and major cities in each of the 48 states, have finally produced a solution (see above). By an ingenious application of linear programming—a mathematical tool recently used to solve production-scheduling problems—it took only a few weeks for the California experts to calculate "by hand" the shortest route to cover the 49 cities: 12,345 miles.

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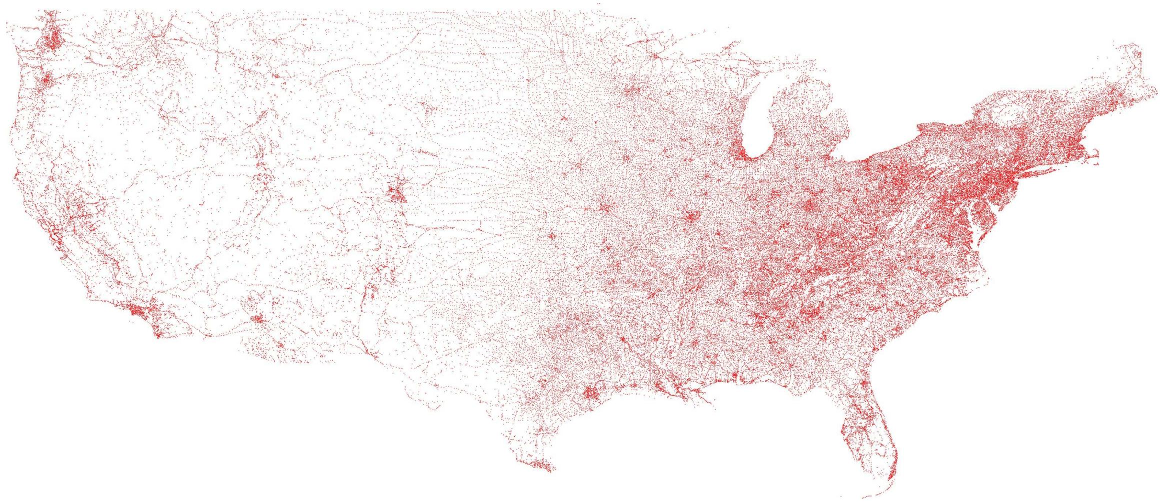
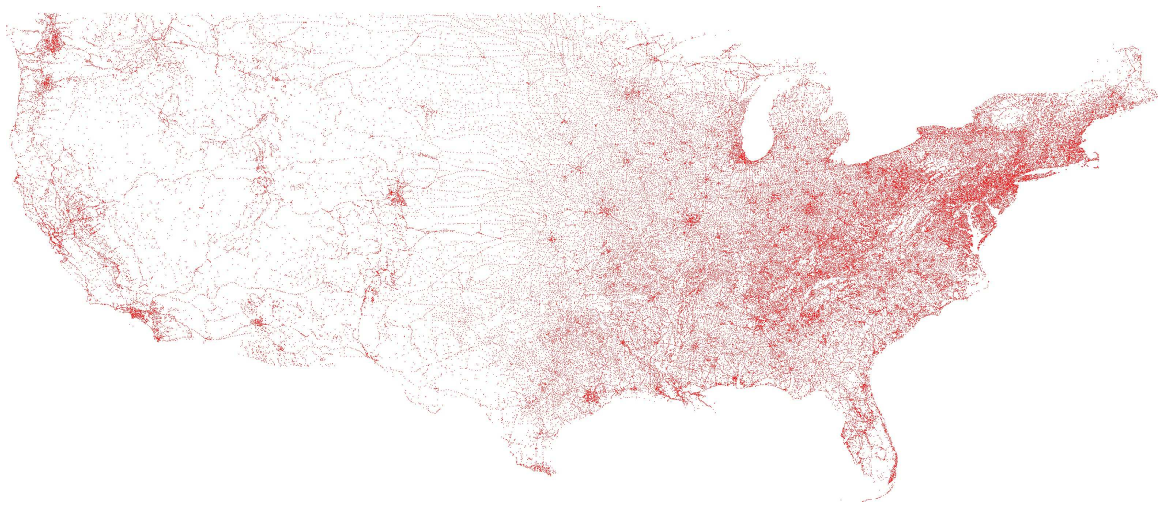


Size of the TSP Problem



This instance has 115475 cities.

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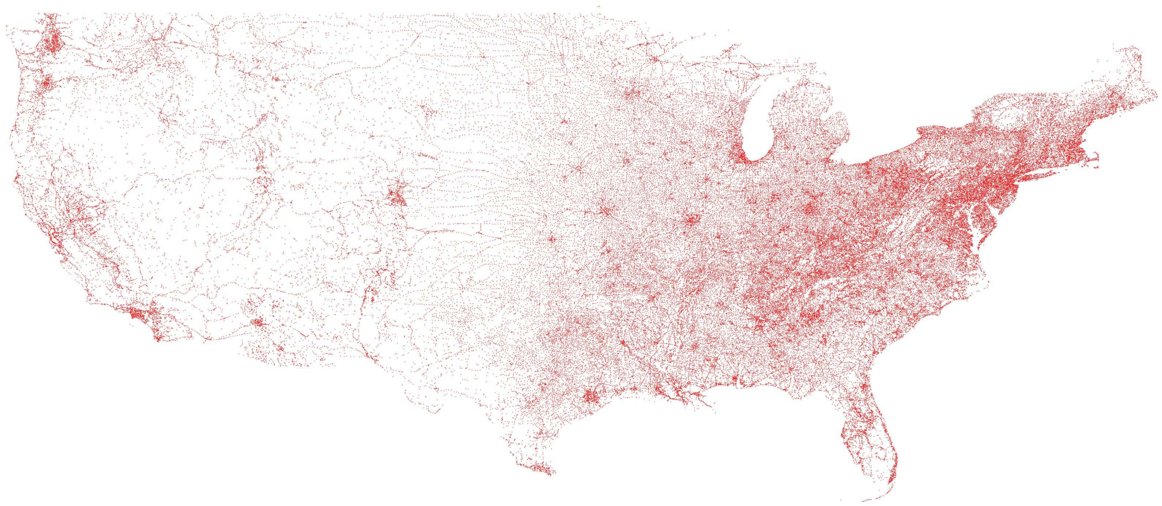

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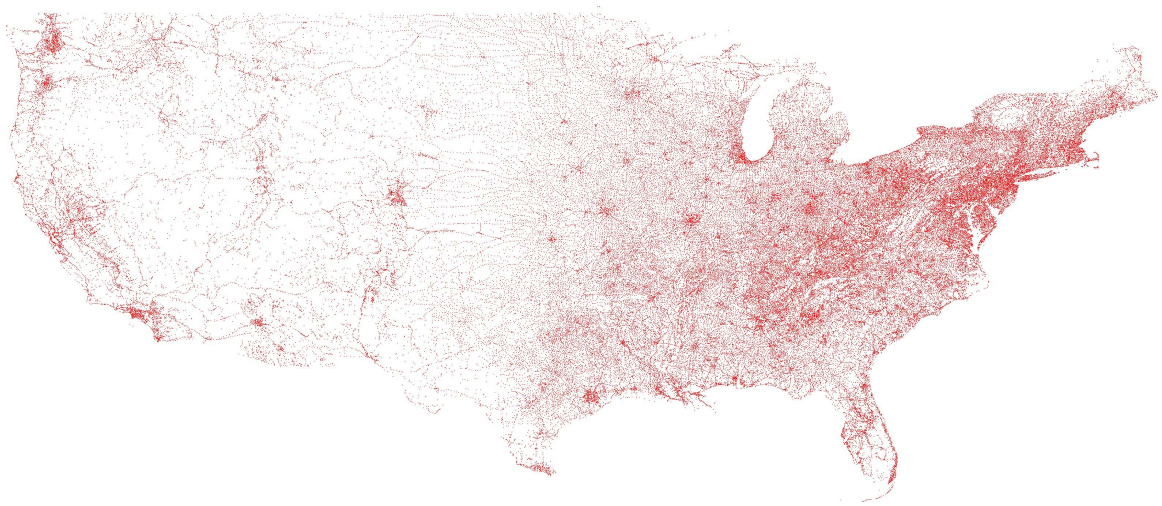

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For reference, Eddignton's number is about $1.6 \cdot 10^{80}$.

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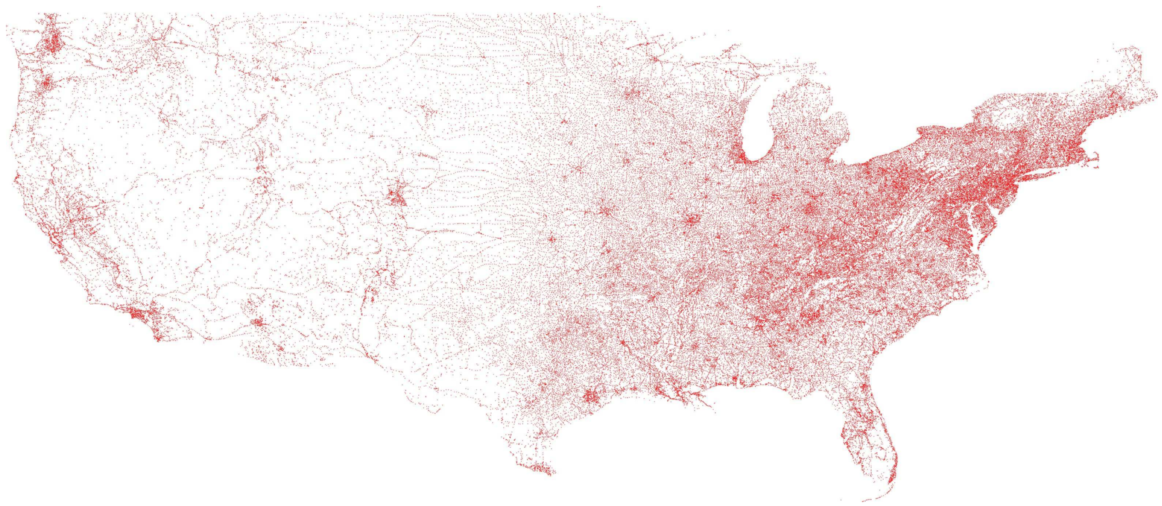

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So how do we start solving this?

Depth First

1. For a given city at the head of a partial tour
2. If this city completes the tour
   Record the new solution
3. Otherwise, for each neighboring city
   consider it as the head of a new partial tour
   recurse

```
dfs(Node n):
  if goal(n):
    updateIncumbent(n)
  else:
    for s in successors(n):
      dfs(s)
```

```
incumbent = None
updateIncumbent(Node n):
  if incumbent == None or n.cost < incumbent.cost:
    incumbent = n
```

```
successors(Node n):
  remaining = n.remaining
  city = n.city
  cost = node.cost
  accum = []
  for ind from 0 to length(remaining) - 1:
    remainingPrime = copy(remaining)
    cityPrime = remainingPrime[ind]
    costPrime = cost + distance(city, cityPrime)
    del remainingPrime[ind]
    accum.extend(Node(cityPrime,
                      remainingPrime,
                      costPrime))
  return accum
```

Go to the java program, show a couple instances.

```
dfs(Node n):
  if goal(n):
    updateIncumbent(n)
  else if better(n, incumbent):
    for s in successors(n):
      dfs(s)
```

```
better(Node n, Node inc):
  inc == None or n.cost < inc.cost
```

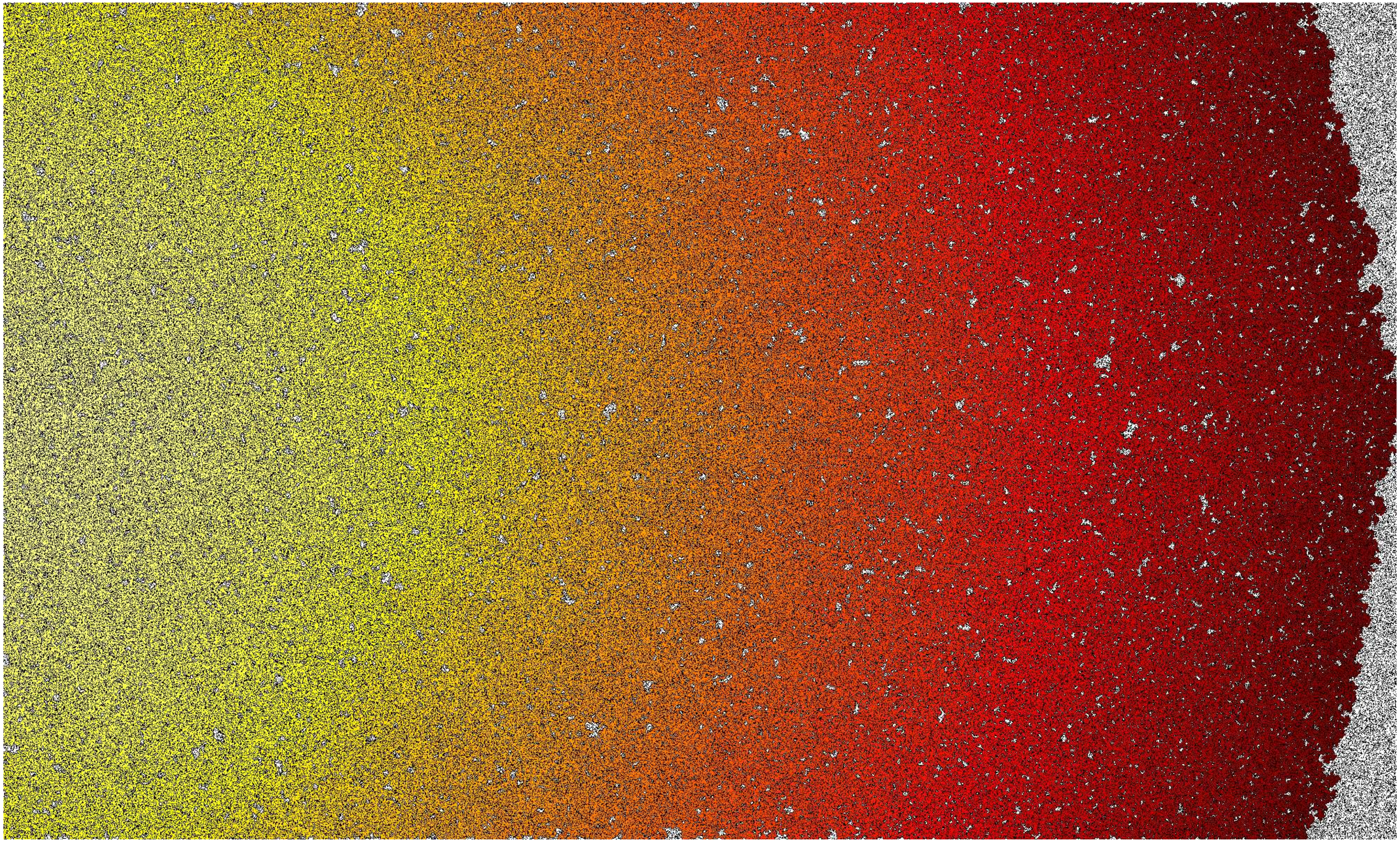
This is still complete and converges on optimal!

```
dfs(Node n):
  if goal(n):
    updateIncumbent(n)
  else:
    for delta in successorDeltas(n):
      applyDelta(delta,n)
      dfs(n)
      undoDelta(n)
```

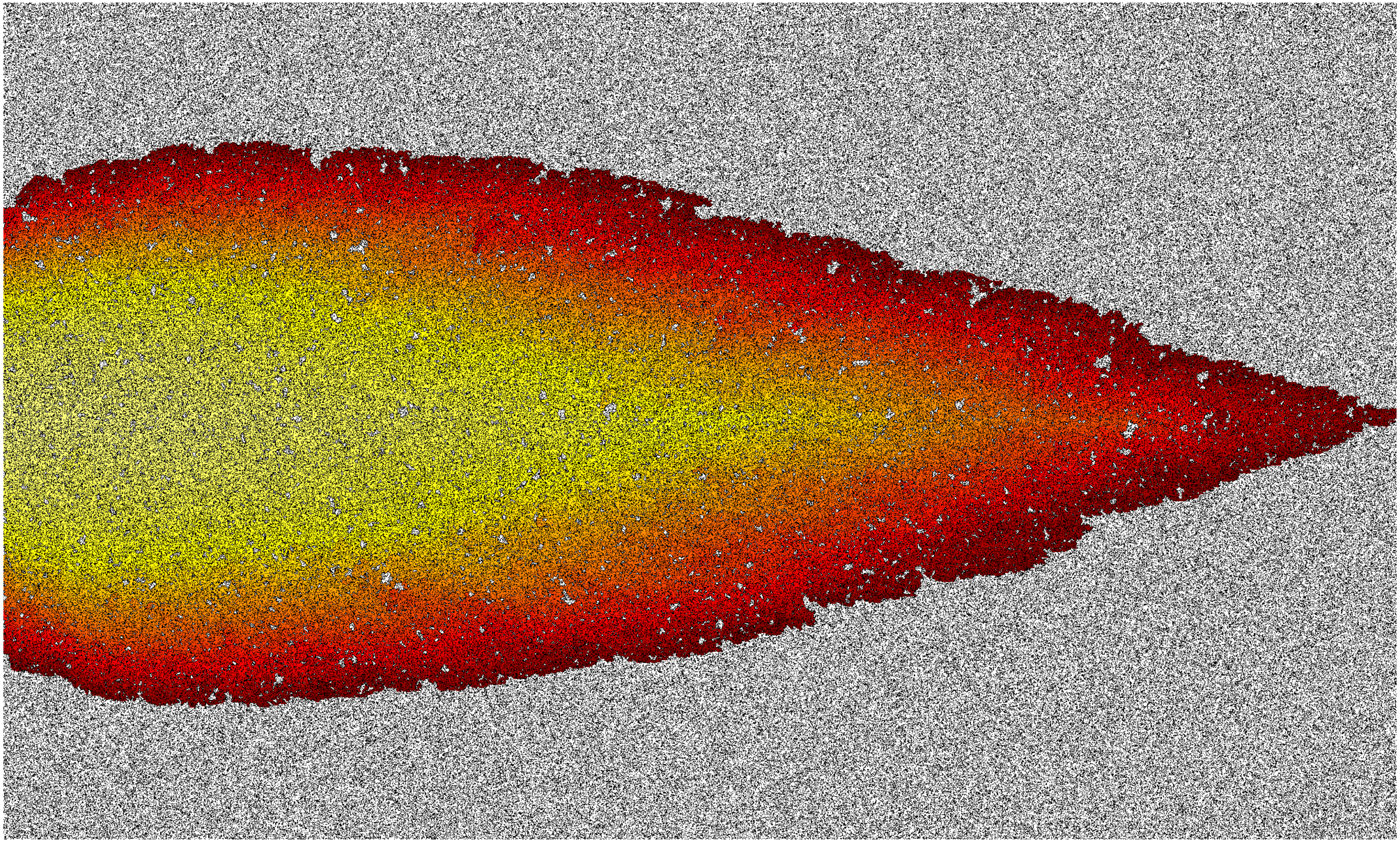
```
successors(Node n):
  remaining = n.remaining
  city = n.city
  cost = node.cost
  accum = []
  for ind from 0 to length(remaining) - 1:
    remainingPrime = copy(remaining) ## Expensive!
    cityPrime = remainingPrime[ind]
    costPrime = cost + distance(city, cityPrime)
    del remainingPrime[ind]
    accum.extend(Node(cityPrime,
                      remainingPrime,
                      costPrime))
  return accum
```

By working with a single logical state, and making and undoing changes, we can touch less memory and generally go faster.

Heuristics



Why Heuristics?



Where Do Heuristics Come From?

- Solutions to a Relaxed Problem
- Solutions to an Abstract Problem
- General Rules of Thumb from Domain Experts
- Machine Learning

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What is $g(n)$ for the TSP?
What are some potential $h(n)$s?

- The distance to the nearest city.
- Of all remaining cities, the furthest distance in latitude and the furthest distance in longitude from either the tour head or the starting city.
- The minimum spanning tree of the remaining cities

Minimum Spanning Tree as an admissible estimate of cost to go.

Preference Heuristics

A preference heuristic simply tells us, among two solutions or partial solutions, or in general, two search alternatives, which should the algorithm consider first.

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- Given a current partial tour, how should I extend it?

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- Given a current partial tour, how should I extend it?

The nearest neighbor is a very natural intuition, and it works pretty well in practice.

Using Heuristics in DFS


```
dfs(Node n):
  if goal(n):
    updateIncumbent(n)
  else if feasible(n, incumbent):
    next = successors(n)
    sort(next, nearest(n.city)) # Visit Children in
    for s in next:              # heuristic order.
      dfs(s)


feasible(Node n, Node inc):
 if inc == None:
   return True
 else:
   return inc.cost > n.cost + mst(n) # g(inc) > f(n)
```

Another Demo!

Show the child ordering DFS implementation against the non-child ordering implementation.

Local Search

Local Search in General

```
localSearch(currentSolution):
  updateIncumbent(currentSolution)
  if outOfTime:
    return currentSolution
  else:
    possible = neigbors(currentSolution)
    next = select(possible, currentSolution)
    localSearch(next)
```

Random Walk

```
randomWalk(currentSolution):
  updateIncumbent(currentSolution)
  if outOfTime:
    return currentSolution
  else:
    possible = neigbors(currentSolution)
    next = randomElt(possible)
    randomWalk(next)
```

Hill Climbing


```
hillClimb(currentSolution):
  updateIncumbent(currentSolution)
  if outOfTime:
    return currentSolution
  else:
    possible = neigbors(currentSolution)
    next = None
    for p in possible:
      if p.cost < currentSolution.cost:
        next = p
        current = p
    if next:
      hillClimb(next)
    else:
      return hillClimb(randomSolution())
```

Completeness Proofs

Wrap-Up

Relationship Between Techniques

Properties of Problem Beneficial to These Techniques

- Solutions Exist at a Fixed Depth
- Feasible Solutions Are Easy To Find
- Moving Between Solutions is Easy
- Good Selection of Heuristics

Similar Relevant Things I Won't Talk About

Next Week

- Graphs Vs. Trees
- Best First Search
- Heuristic Construction
- Dealing With Non-Assignment Problems