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FLUTTER OPTIMIZATION USING PARAMETRIC REDUCED ORDER AEROELASTIC MODEL

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In this paper, optimization is performed on an aeroelastic system for a maximum flutter speed within feasible limits of structural design variables (material density, Young's modulus and Poisson's ratio) and aerodynamic parameters (air density). The aeroelastic model under consideration is Goland wing described by finite element analysis and unsteady vortex lattice method. Gradient based optimization technique and global optimization method are used to seek for the maximum flutter speed. A parametric reduced order model (PROM) is generated using the parametric reduced order modeling technique developed previously by Kim and optimum results are compared against those of the full order model (FOM). It is shown that the optimization based on PROM results in 76.43% reduction in CPU time and yet yields the same optimum flutter result as the FOM.

Keyword: Parametric reduced order model (PROM), Optimization, Flutter speed

1. INTRODUCTION

Aeroelastic flutter is a rapid self-feeding motion, which involves interaction of aerodynamic forces, elastic structures and inertial forces. Flutter is a dynamic, potentially destructive phenomenon. It was first observed on Handley Page O/400 bomber in 1916, followed by many catastrophic failures of aircrafts and structures prone to fluid interaction over the years. Accurate models to describe aeroelastic behavior of real world complicated structures of aircraft are prone to non-linearities and uncertainties, and in most cases are computationally expensive. In recent years, attention has been directed to design optimization of structures for flutter stability, design optimization of wing/store configuration and flutter suppression. However, little attention has been given to optimization of structural parameters such as material density and elasticity coefficients for maximization of flutter speed. This paper presents a novel method of optimization of structural parameters and material properties for maximization of flutter speed within the feasible limits of modulation.

The aerodynamic model and structural design of wing must include wing flexibility and structural dynamics. Goland wing[1)] is used to model the aircraft wing and modal characteristics (mode shapes and mode shape slopes) are used to describe the dynamic behavior of the structure in the state of fluid-structure interaction. Gregory and Paidoussis[2)] used a Galerkin approximation in terms of cantilever beam eigenmodes, and showed that only the first few modes contribute to the motion. Vortex lattice method is used to generate elemental matrices to build unsteady aerodynamic equations. Generalized aerodynamic forces are calculated for the selected structural modes. Optimization process is done to calculate maximum flutter speed for the system with structural and material parameters (mass, elasticity, Poisson's ratio and air density according to the flight altitude).

Full-order model calculations are computationally expensive and time consuming. A significant progress has been made in the field of reduced order modeling in the past two decades. FOMs, typically in $O(10^5 \sim 10^7)$, can be successfully reduced to $O(10^1 \sim 10^2)$using reduced order models. Kim, 2011[3)] discusses the variety of model reduction methods available nowadays, especially in aeronautical applications including structural dynamics and aeroelasticity. Frangos et al.,[4)] give a comprehensive overview of all the recent developments in the so called Reduced-Order Surrogate Model (ROSM) techniques. In this paper, parametric reduced order modeling[5)] (PROM) is applied to the aeroelastic system to reduce the CPU time and

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optimization results and CPU time spent are compared with those of the FOM.

2. PROBLEM DESCRIPTION AND NUMERICAL APPROACHES

For the flutter optimization based on the parametric reduced order modeling, Goland wing model is considered. This is a benchmark model that many researchers in aeronautical community have studied and used for the purpose of comparisons and validations of various structural and aeroelastic methods. The heavy Goland wing is modeled by NASTRAN finite element (fig. 1). In this study, section 12, 14, 16, 18 and 20 in fig. 1 are the control region. Structure parameters are changed in that control region only.

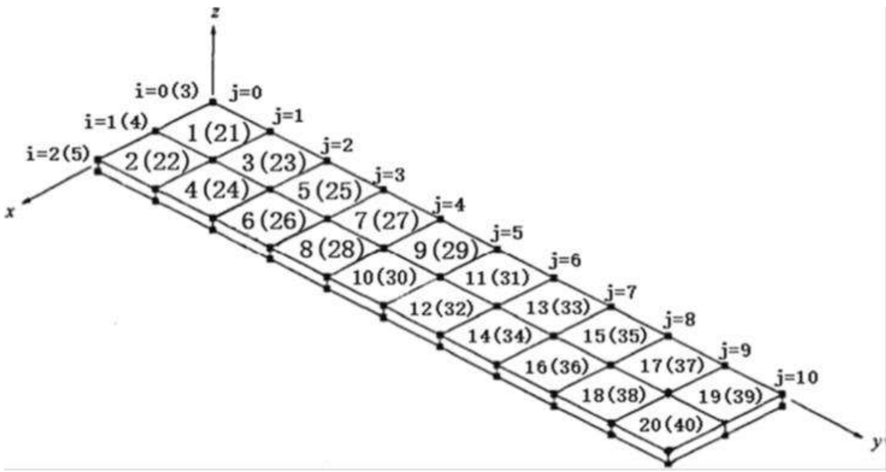

Figure 1 : Finite element model of Goland wing

Nominal specs of the Goland wing are given as follows:

l ≡ span = 20 ft	E ≡ Young's Modulus = 1.4976 × 10^2 slugs/ft^2
c ≡ chord = 4 ft	ν ≡ Poisson Ratio = .33
t ≡ thickness = .33 ft	g ≡ structural damping coefficient = .03
ρ ≡ density = .0001 slugs/ft^3	

The structural dynamic system is described by a discrete-time, state-space equation,

$$\boldsymbol{z}^{n+1} = \boldsymbol{A}_s \boldsymbol{z}^n + \boldsymbol{B}_s \boldsymbol{y}^n + \boldsymbol{B}_i \boldsymbol{u}^n \tag{1}$$

and similarly an unsteady aerodynamic model is given as,

$$\boldsymbol{x}^{n+1} = \boldsymbol{A}_a \boldsymbol{x}^n + \boldsymbol{B}_a \boldsymbol{z}^n \tag{2}$$

$$\boldsymbol{y}^n = q_D\,(\boldsymbol{C}_a \boldsymbol{x}^n + \boldsymbol{D}_a \boldsymbol{z}^n) \tag{3}$$

Where

$\boldsymbol{x}$ ≡ *aerodynamic states vector*

$\boldsymbol{y}$ ≡ *aerodynamic forces vector*

$\boldsymbol{z}$ ≡ *structural states vector*

$$\boldsymbol{u} \equiv \textit{inputs vector}$$

$$q_D \equiv \textit{dynamic pressure} \; (\equiv \tfrac{1}{2}\rho V^2)$$

For a validation of finite element modeling, natural frequencies of the first four natural modes are computed and compared with the results reported previously[6] (fig.2).

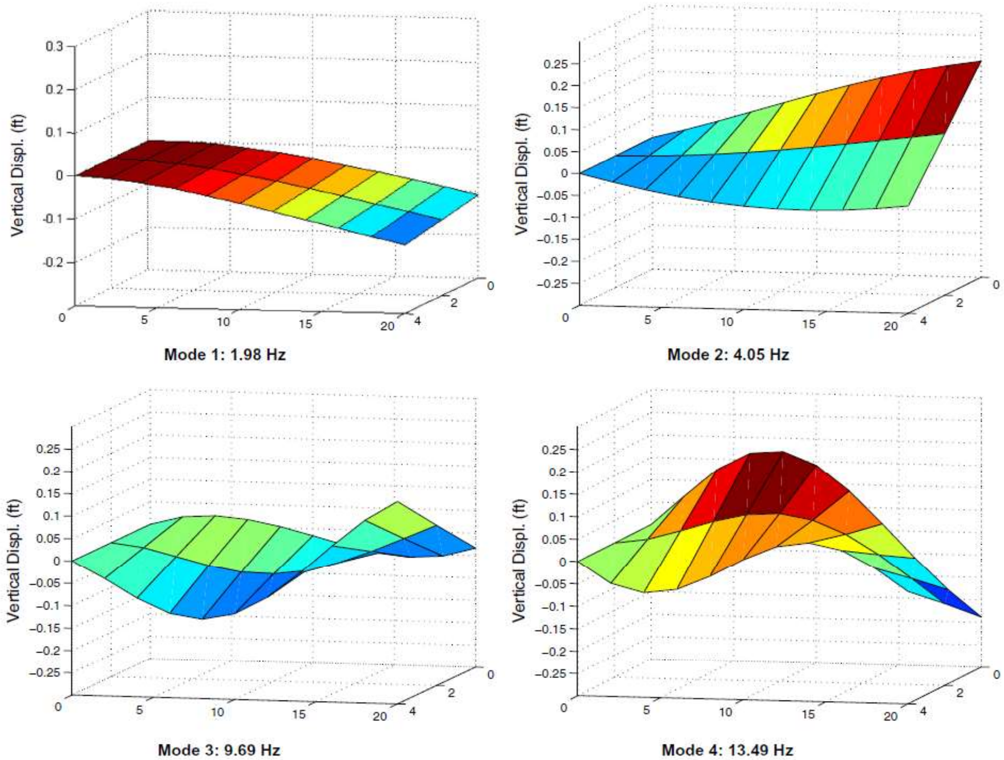

Figure 2 : Vibrational modes of Goland wing

Certain mode shapes can be found with very small displacement and hence do not contribute to overall behavior of the wing. Such ‘spurious modes’ must be eliminated and replaced with the next available mode with higher frequency. An example of ‘spurious mode’ is shown in fig.3.

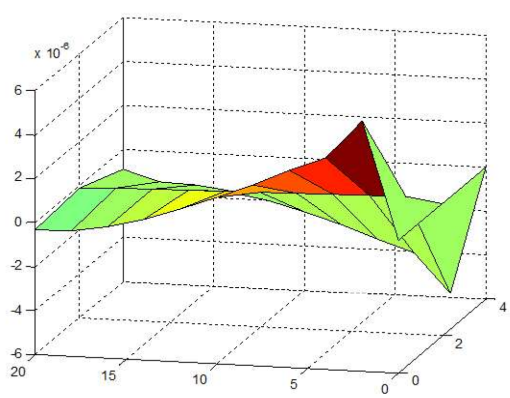

Figure 3 : Shape of garbage mode

The unsteady flow field around the wing is modeled by the linear, inviscid vortex lattice model (fig 4) in the discrete-time, linear state space format[7] as described by Eqs. (2) and (3). Generalized Aerodynamic Forces (GAF) are calculated on the six structural modes and through the discrete-time equations (2) and (3) are coupled with the structural dynamic equation (1) which is also cast in the state space format. This results in a linear states-space aeroelastic system with 1,012 degrees of freedom, 1,000 for the aerodynamics, 2×6=12 for the structure.

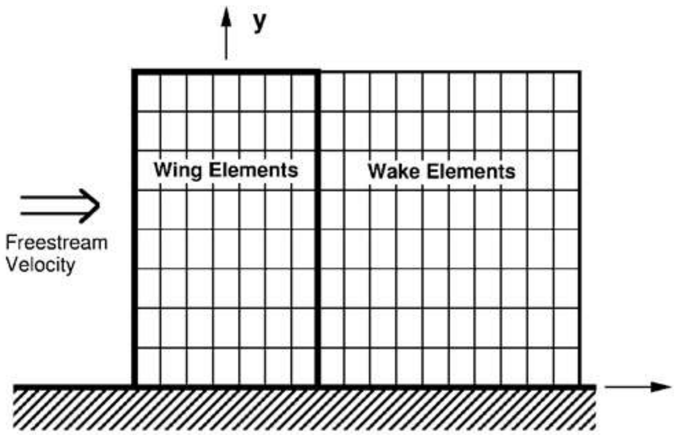

Figure 4 : Vortex lattice grids for rectangular semi-span

The full-order model (1,012 × 1,012) of the aeroelastic system is used for minimization of the objective function i.e. negative of flutter speed using nonlinear multivariable programming solver in MATLAB. However, full-order model (FOM) is computationally expensive. In order to save the computational time, a reduced order model (ROM) is constructed using the so called modally equivalent perturbed system (MEPS)[8], which reduces the linear system with parameter changes (4)

$$\boldsymbol{x}\,(\boldsymbol{\mu}, t) = \boldsymbol{A}(\boldsymbol{\mu})\,\boldsymbol{x}\,(\boldsymbol{\mu}, t) + \mathbf{B}(\boldsymbol{\mu})\,\boldsymbol{u}(t)$$
$$\boldsymbol{y}\,(\boldsymbol{\mu}, t) = \boldsymbol{C}\,(\boldsymbol{\mu})\,\boldsymbol{x}\,(\boldsymbol{\mu}, t) \tag{4}$$

to (5) PROM via Galerkin's projection

$$\boldsymbol{q}\,(\boldsymbol{\mu}, t) = \boldsymbol{A}R\,(\boldsymbol{\mu})\,\boldsymbol{q}\,(\boldsymbol{\mu}, t) + \boldsymbol{B}R\,(\boldsymbol{\mu})\,\boldsymbol{u}(t)$$
$$\boldsymbol{y}\,(\boldsymbol{\mu}, t) = \boldsymbol{C}R\,(\boldsymbol{\mu})\,\boldsymbol{q}\,(\boldsymbol{\mu}, t) \tag{5}$$

where

$$\boldsymbol{A}R\,(\boldsymbol{\mu}) \equiv \quad T\,\boldsymbol{A}(\boldsymbol{\mu}) \quad (R \times R)$$
$$\boldsymbol{B}R\,(\boldsymbol{\mu}) \equiv \quad T\,\boldsymbol{B}\,(\boldsymbol{\mu})\;(R \times I\,)$$
$$\boldsymbol{C}R\,(\boldsymbol{\mu}) \equiv \boldsymbol{C}\,(\boldsymbol{\mu}) \quad (L \times R) \tag{6}$$

where R is dimension of the reduced order model, which in this case is (171 x 171). The graphical conceptualization of the MEPS is shown in fig. 5.

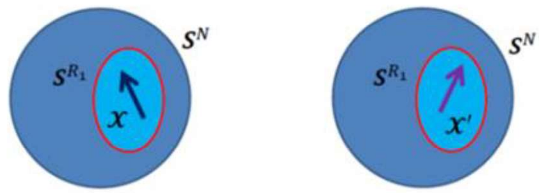

Figure 5 : Solution space ($\boldsymbol{S^{R_1}}$) shared by original (left) and modally equivalent (right) perturbed solutions

In this study, an optimization problem is defined with four parameters and single object function. It is defined as below,

$$\text{Maximize} \quad f(\boldsymbol{x}) = v_f \text{ (flutter speed)}$$

$$\boldsymbol{x} = [E, \nu, M, \rho_{air}]$$

$$\text{subject to} \quad v_f - 300 > 0$$

where E is Young's modulus, ν is Poission`s ratio, M the mass of half-quarter region and ρ_{air} is the air density which is determined by flight altitude. The optimization process is performed by using thefunction 'fmincon' on MATLAB1 for FOM optimization and PROM validation. In that function, an interior-point algorithm is used for finding the optima. However, in general the gradient based optimization tends to converge to a local optimum. To remedy this and ensure a global optima, the global optimization method (direct method) such as Genetic Algorithm(GA) is preferred. A direct method requires a significant amount of computation and for this reason, applying GA using the FOM is inefficient and is not recommended. Thus, in this study the GA is adopted to PROM optimization case only. Parameters and design space for optimization are defined as tab.1. Note that these parameters are allowed to vary only in the areas 12, 14, 16, 18, 20, i.e., 25% of the wing. Mass in Goland model is added as a lumped mass at the structure nodes. The lumped masses on controlling area are controlled with parameter M.

Table 1. Design Space for Flutter Speed Optimization

	Lower	0	Upper
E	0.5990	1.4976	2.6957
ν	0.20	0.33	0.45
M	-1	0	1
ρ_{air}	0.001267 (for 20,000 ft)	0.001756 (for 10,000 ft)	0.002377 (ground)

To get the flutter speed, flow speed is changed iteratively until the real part of any eigenvalue becomes zero or close to zero. The convergence to a flutter speed is shown in fig.6 for the nominal case of Kim's paper[5)]. Parameters for nominal case are $\mathrm{M} = \text{initial lumped mass}$, $\mathrm{E} = 1.4976 \times 10^2\ \mathrm{slugs/ft^2}$, $\nu = 0.33$, $\rho = 2.3769\ \mathrm{slugs/ft^3}$. Occasionally, flutter speeds of PROM are compared against those of FOM for validation and it is shown that they are well matched for all the cases checked.

[1] MATLAB is registered trademark of The Math Works, Inc.

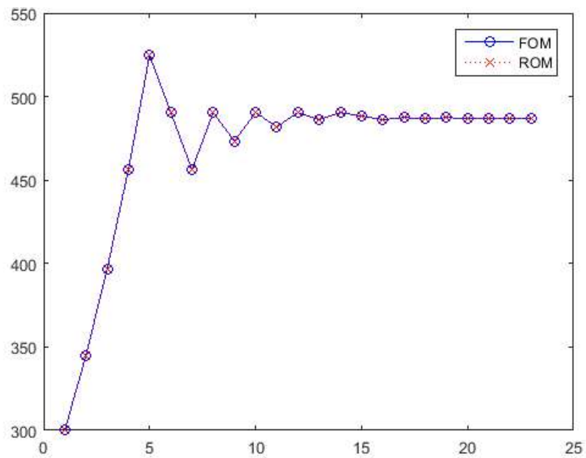

Figure 7 : Flutter speed estimation by iteration method

3. RESULTS

First, for the FOM case optimization was done with gradient based method, fmincon, and this gradient based optimization result was validated with the PROM (but fmincon was not repeated with the reduced order model). The optimum results are shown in tab.2. Parameters are normalized in -1 to 1 design spaces. x1 is Young's modulus, x2 is Poisson's ratio, x3 is parameter for lumped mass and x4 is air density.

Table 2. Gradient based optimization results of FOM and PROM

V_f	x1	x2	x3	x4	Time for FOM	Time for PROM
574.0735	0.6344	0.0852	0.3684	0.0495	77664.312 (21:34:24)	26578.044 (est) (7:22:58)

FOM and PROM with same optimum parameter set are converged to same optimal solution with only difference in computation time. FOM case took 21 hours 34 minutes and 24 seconds. On the contrary, the PROM took 34.2% of the time to get the flutter speed for the one iteration. Note that the total time, 7 hours 22 minutes and 58 seconds, for GBM with PROM is an estimated time. The eigenvalues at flutter speed (V_f= 574.0735 ft/sec) is shown in fig.7.

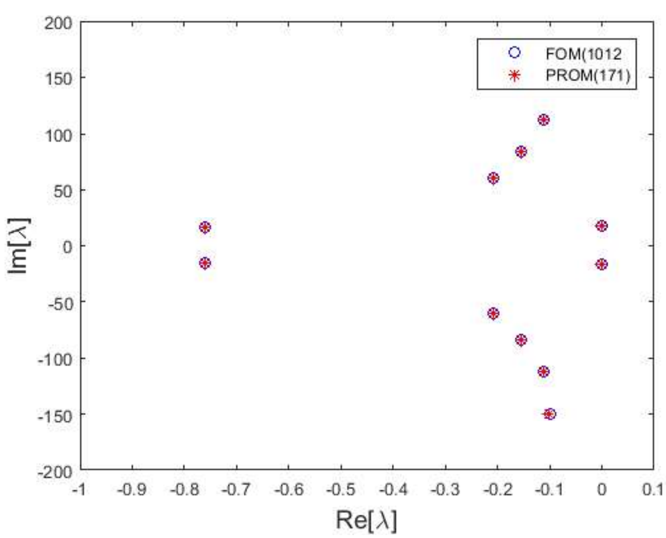

Figure 8 : Eigenvalues of FOM and PROM at U=574.07 ft/sec

To obtain a global optimum solution, GA method is used for PROM case. Initial population of GA is 30 and maximum generation of GA is 50 for four parameters. The results of GA are shown in tab.3 and the eigenvalues are shown in fig.8.

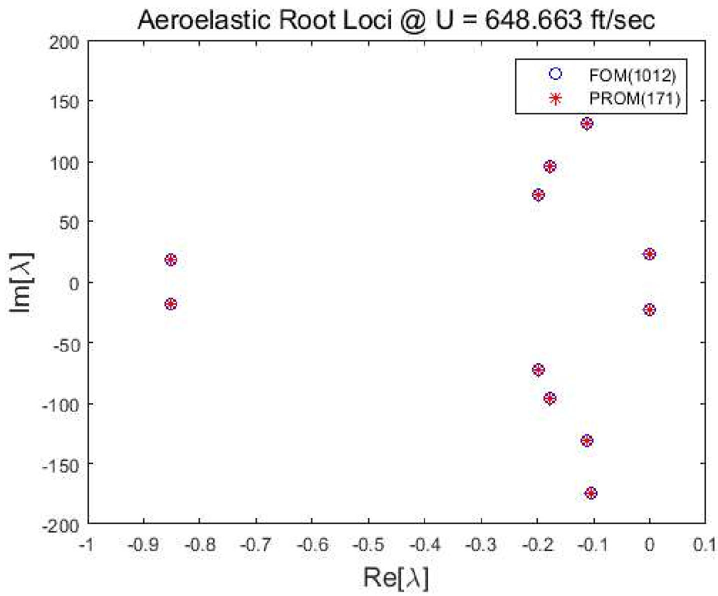

Figure 9 : Eigenvalues of FOM and PROM at GA1 (left) and GA2(right)

Table 3. Results of GA for PROM case

	Vf	x1	x2	x3	x4	Time
GA-PROM	648.6627	1.0	1.0	-1.0	-1.0	12317.858 (3:25:18)
GA-FOM	648.6627	1.0	1.0	-1.0	-1.0	47339.983 (13:09:00)

Comparing with gradient based results, GA optimal results are converged to different, higher flutter speed. It shows that gradient based method actually converged to the local optima. During the global optimization process, algorithm in flutter speed estimation was improved to become more efficient. It resulted in a saving about 80% of the time at each iteration. Without the improvement, GA with FOM can take few days to get the result. GA with PROM and GA with FOM converged to the same flutter speed with same parameters set. Not surprisingly, the parameters are converged to lower and upper boundaries in this case. The flutter speed is increased about 160 ft/sec from the nominal case in previous research[5)] and it is 33.319% improvement. The GA with PROM took about 3 hours 25 minutes while the GA with FOM took about 13 hours 9 minutes to converge to the same optimum point. Thus, PROM saves 76.43% of the computation time.

4. CONCLUSIONS

In this study, the PROM was applied successfully to the Goland wing aeroelastic system. Gradient based and global optimization procedures were established and applied to this system in both FOM and PROM. The optimum flutter speed using the gradient based optimization is 575.0735 ft/sec and takes 21 hours and 30 minutes for FOM and 7 hours and 20 minutes for PROM. PROM result and FOM result were perfectly matched. Global optimization results were obtained with GA method. The flutter speed of global optima is 648.6627 ft/sec and it converged in 3 hours and 25 minutes with PROM and 13 hours and 9 minutes with

FOM. Hence, it was shown that PROM cane save CPU time up to about 75%. Therefore, with the proposed PROM, global optima can be obtained with acceptable amount of calculation time.

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