

# Symmetric Three-body Problem Notes

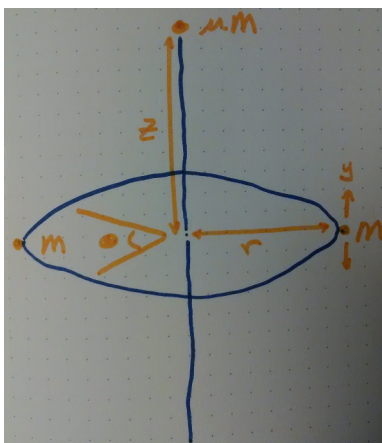
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## 1 Equations of Motion

From Ekeland page 59 (paperback edition):

Imagine two stars of equal mass, rotating around their common center of gravity. Newton's law asserts that this is possible, and that the orbit of the two stars will be a circle, along which they travel with equal speed, being exactly opposite to each other at all times. [...] A third body, with very small mass—a comet, for instance—moves along [the axis].



Potentials,

$$U_{mm} = \frac{-Gmm}{2r} \quad (1)$$

$$U_{\mu m} = \frac{-2G\mu mm}{\sqrt{r^2 + (z - y)^2}} \quad (2)$$

$$(3)$$

Kinetic Energy,

$$T = \frac{1}{2}\mu m\dot{z}^2 + m\dot{y}^2 + m\dot{r}^2 + m(r\dot{\theta})^2 \quad (4)$$

Lagrangian,

$$\mathcal{L} = T - V \quad (5)$$

$$= \frac{1}{2}\mu m\dot{z}^2 + m\dot{y}^2 + m\dot{r}^2 + m(r\dot{\theta})^2 + \frac{-Gmm}{2r} + \frac{-2G\mu mm}{\sqrt{r^2 + (z-y)^2}} \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{z}} = \mu m \dot{z} \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = 2m\dot{r} \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{y}} = 2m\dot{y} \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = 2mr^2\dot{\theta} \quad (10)$$

$$\frac{\partial \mathcal{L}}{\partial z} = \frac{2G\mu m^2(z-y)}{(r^2 + (z-y)^2)^{3/2}} \quad (11)$$

$$\frac{\partial \mathcal{L}}{\partial r} = 2mr\dot{\theta}^2 - \frac{Gm^2}{r^2} - \frac{2G\mu m^2 r}{(r^2 + (z-y)^2)^{3/2}} \quad (12)$$

$$\frac{\partial \mathcal{L}}{\partial y} = -\frac{2G\mu m^2(z-y)}{(r^2 + (z-y)^2)^{3/2}} \quad (13)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0 \quad (14)$$

Equations of motion,

$$\ddot{z} = \frac{2Gm(z-y)}{(r^2 + (z-y)^2)^{3/2}} \quad (15)$$

$$\ddot{y} = \frac{G\mu m(z-y)}{(r^2 + (z-y)^2)^{3/2}} \quad (16)$$

$$\ddot{r} = r\dot{\theta}^2 - Gm\left(\frac{1}{2r^2} + \frac{\mu r}{(r^2 + (z-y)^2)^{3/2}}\right) \quad (17)$$

$$\ddot{\theta} = \frac{-2\dot{r}\dot{\theta}}{r} \quad (18)$$

## 2 Sources and Licensing

For code and example output, please visit and fork or clone <https://github.com/DrSkippy/Gravitational-Three-Body-Symmetric>. If you find errors or have comments, please email [scott@drskippy.net](mailto:scott@drskippy.net). This work is licensed under a Creative Commons CC0 1.0 Universal (CC0 1.0) <http://creativecommons.org/publicdomain/zero/1.0/>.

## References

- [Ekeland1990] Ivar Ekeland. Mathematics and the Unexpected. <http://www.amazon.com/Mathematics-Unexpected-Ivar-Ekeland/dp/0226199908> 1990.
- [Drexel] The Leapfrog Integrator, [http://einstein.drexel.edu/courses/Comp\\_Phys/Integrators/leapfrog/](http://einstein.drexel.edu/courses/Comp_Phys/Integrators/leapfrog/).

