# The GraphBLAS C API Specification $^{\dagger}:$

Version 2.1

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- <sup>3</sup> [Scott: THIS IS A DRAFT VERION. Update acks and remove DRAFT before release.]
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### $_{\scriptscriptstyle{172}}$ Chapter 1

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### $_{\scriptscriptstyle ext{\tiny 13}}$ $\mathbf{Introduction}$

The GraphBLAS standard defines a set of matrix and vector operations based on semiring algebraic structures. These operations can be used to express a wide range of graph algorithms. This document defines the C binding to the GraphBLAS standard. We refer to this as the *GraphBLAS* C API (Application Programming Interface).

The GraphBLAS C API is built on a collection of objects exposed to the C programmer as opaque data types. Functions that manipulate these objects are referred to as *methods*. These methods fully define the interface to GraphBLAS objects to create or destroy them, modify their contents, and copy the contents of opaque objects into non-opaque objects; the contents of which are under direct control of the programmer.

The GraphBLAS C API is designed to work with C99 (ISO/IEC 9899:199) extended with *static* type-based and number of parameters-based function polymorphism, and language extensions on par with the \_Generic construct from C11 (ISO/IEC 9899:2011). Furthermore, the standard assumes programs using the GraphBLAS C API will execute on hardware that supports floating point arithmetic such as that defined by the IEEE 754 (IEEE 754-2008) standard.

The GraphBLAS C API assumes programs will run on a system that supports acquire-release memory orders. This is needed to support the memory models required for multithreaded execution as described in section 2.5.2.

Implementations of the GraphBLAS C API will target a wide range of platforms. We expect cases will arise where it will be prohibitive for a platform to support a particular type or a specific parameter for a method defined by the GraphBLAS C API. We want to encourage implementors to support the GraphBLAS C API even when such cases arise. Hence, an implementation may still call itself "conformant" as long as the following conditions hold.

- Every method and operation from chapter 4 is supported for the vast majority of cases.
- Any cases not supported must be documented as an implementation-defined feature of the GraphBLAS implementation. Unsupported cases must be caught as an API error (section 2.6) with the parameter GrB\_NOT\_IMPLEMENTED returned by the associated method call.
- It is permissible to omit the corresponding nonpolymorphic methods from chapter 5 when it

is not possible to express the signature of that method.

The number of allowed omitted cases is vague by design. We cannot anticipate the features of target platforms, on the market today or in the future, that might cause problems for the GraphBLAS specification. It is our expectation, however, that such omitted cases would be a minuscule fraction of the total combination of methods, types, and parameters defined by the GraphBLAS C API specification.

The remainder of this document is organized as follows:

- Chapter 2: Basic Concepts
- Chapter 3: Objects

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- Chapter 4: Methods
- Chapter 5: Nonpolymorphic interface
- Appendix A: Revision history
- Appendix B: Non-opaque data format definitions
- Appendix C: Examples

### Chapter 2

## Basic concepts

- The GraphBLAS C API is used to construct graph algorithms expressed "in the language of linear algebra." Graphs are expressed as matrices, and the operations over these matrices are generalized through the use of a semiring algebraic structure.
- In this chapter, we will define the basic concepts used to define the GraphBLAS C API. We provide the following elements:
- Glossary of terms and notation used in this document.
- Algebraic structures and associated arithmetic foundations of the API.
- Functions that appear in the GraphBLAS algebraic structures and how they are managed.
- Domains of elements in the GraphBLAS.
- Indices, index arrays, scalar arrays, and external matrix formats used to expose the contents of GraphBLAS objects.
- The GraphBLAS opaque objects.
- The execution and error models implied by the GraphBLAS C specification.
- Enumerations used by the API and their values.

#### $\mathbf{2.1}$ Glossary

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#### 332 2.1.1 GraphBLAS API basic definitions

- application: A program that calls methods from the GraphBLAS C API to solve a problem.
- GraphBLAS C API: The application programming interface that fully defines the types, objects, literals, and other elements of the C binding to the GraphBLAS.

- function: Refers to a named group of statements in the C programming language. Methods, operators, and user-defined functions are typically implemented as C functions. When referring to the code programmers write, as opposed to the role of functions as an element of the GraphBLAS, they may be referred to as such.
  - method: A function defined in the GraphBLAS C API that manipulates GraphBLAS objects or other opaque features of the implementation of the GraphBLAS API.
- operator: A function that performs an operation on the elements stored in GraphBLAS matrices and vectors.
- GraphBLAS operation: A mathematical operation defined in the GraphBLAS mathematical specification. These operations (not to be confused with operators) typically act on matrices and vectors with elements defined in terms of an algebraic semiring.

#### <sup>347</sup> 2.1.2 GraphBLAS objects and their structure

- non-opaque datatype: Any datatype that exposes its internal structure and can be manipulated directly by the user.
- opaque datatype: Any datatype that hides its internal structure and can be manipulated only through an API.
- GraphBLAS object: An instance of an opaque datatype defined by the GraphBLAS C API that is manipulated only through the GraphBLAS API. There are four kinds of GraphBLAS opaque objects: domains (i.e., types), algebraic objects (operators, monoids and semirings), collections (scalars, vectors, matrices and masks), and descriptors.
  - handle: A variable that holds a reference to an instance of one of the GraphBLAS opaque objects. The value of this variable holds a reference to a GraphBLAS object but not the contents of the object itself. Hence, assigning a value to another variable copies the reference to the GraphBLAS object of one handle but not the contents of the object.
- domain: The set of valid values for the elements stored in a GraphBLAS collection or operated on by a GraphBLAS operator. Note that some GraphBLAS objects involve functions that map values from one or more input domains onto values in an output domain. These GraphBLAS objects would have multiple domains.
- collection: An opaque GraphBLAS object that holds a number of elements from a specified domain. Because these objects are based on an opaque datatype, an implementation of the GraphBLAS C API has the flexibility to optimize the data structures for a particular platform. GraphBLAS objects are often implemented as sparse data structures, meaning only the subset of the elements that have values are stored.
- *implied zero*: Any element that has a valid index (or indices) in a GraphBLAS vector or matrix but is not explicitly identified in the list of elements of that vector or matrix. From a mathematical perspective, an *implied zero* is treated as having the value of the zero element of the relevant monoid or semiring. However, GraphBLAS operations are purposefully defined

- using set notation in such a way that it makes it unnecessary to reason about implied zeros.

  Therefore, this concept is not used in the definition of GraphBLAS methods and operators.
  - mask: An internal GraphBLAS object used to control how values are stored in a method's output object. The mask exists only inside a method; hence, it is called an *internal opaque object*. A mask is formed from the elements of a collection object (vector or matrix) input as a mask parameter to a method. GraphBLAS allows two types of masks:
    - 1. In the default case, an element of the mask exists for each element that exists in the input collection object when the value of that element, when cast to a Boolean type, evaluates to true.
    - 2. In the *structure only* case, masks have structure but no values. The input collection describes a structure whereby an element of the mask exists for each element stored in the input collection regardless of its value.
    - complement: The complement of a GraphBLAS mask, M, is another mask, M', where the elements of M' are those elements from M that do not exist.

#### 2.1.3 Algebraic structures used in the GraphBLAS

- associative operator: In an expression where a binary operator is used two or more times consecutively, that operator is associative if the result does not change regardless of the way operations are grouped (without changing their order). In other words, in a sequence of binary operations using the same associative operator, the legal placement of parenthesis does not change the value resulting from the sequence operations. Operators that are associative over infinitely precise numbers (e.g., real numbers) are not strictly associative when applied to numbers with finite precision (e.g., floating point numbers). Such non-associativity results, for example, from roundoff errors or from the fact some numbers can not be represented exactly as floating point numbers. In the GraphBLAS specification, as is common practice in computing, we refer to operators as associative when their mathematical definition over infinitely precise numbers is associative even when they are only approximately associative when applied to finite precision numbers.
  - No GraphBLAS method will imply a predefined grouping over any associative operators. Implementations of the GraphBLAS are encouraged to exploit associativity to optimize performance of any GraphBLAS method with this requirement. This holds even if the definition of the GraphBLAS method implies a fixed order for the associative operations.
- commutative operator: In an expression where a binary operator is used (usually two or more times consecutively), that operator is commutative if the result does not change regardless of the order the inputs are operated on.
- No GraphBLAS method will imply a predefined ordering over any commutative operators. Implementations of the GraphBLAS are encouraged to exploit commutativity to optimize performance of any GraphBLAS method with this requirement. This holds even if the definition of the GraphBLAS method implies a fixed order for the commutative operations.

• GraphBLAS operators: Binary or unary operators that act on elements of GraphBLAS objects. GraphBLAS operators are used to express algebraic structures used in the GraphBLAS such as monoids and semirings. They are also used as arguments to several GraphBLAS methods. There are two types of GraphBLAS operators: (1) predefined operators found in Table 3.5 and (2) user-defined operators created using GrB\_UnaryOp\_new() or GrB\_BinaryOp\_new() (see Section 4.2.2).

- monoid: An algebraic structure consisting of one domain, an associative binary operator, and the identity of that operator. There are two types of GraphBLAS monoids: (1) predefined monoids found in Table 3.7 and (2) user-defined monoids created using GrB\_Monoid\_new() (see Section 4.2.2).
  - semiring: An algebraic structure consisting of a set of allowed values (the domain), a commutative and associative binary operator called addition, a binary operator called multiplication (where multiplication distributes over addition), and identities over addition ( $\theta$ ) and multiplication (1). The additive identity is an annihilator over multiplication.
  - GraphBLAS semiring: is allowed to diverge from the mathematically rigorous definition of a semiring since certain combinations of domains, operators, and identity elements are useful in graph algorithms even when they do not strictly match the mathematical definition of a semiring. There are two types of GraphBLAS semirings: (1) predefined semirings found in Tables 3.8 and 3.9, and (2) user-defined semirings created using GrB\_Semiring\_new() (see Section 4.2.2).
- index unary operator: A variation of the unary operator that operates on elements of GraphBLAS vectors and matrices along with the index values representing their location in the objects. There are predefined index unary operators found in Table 3.6), and user-defined operators created using GrB\_IndexUnaryOp\_new (see Section 4.2.2).

#### $_{435}$ 2.1.4 The execution of an application using the GraphBLAS C API

- program order: The order of the GraphBLAS method calls in a thread, as defined by the text of the program.
  - host programming environment: The GraphBLAS specification defines an API. The functions from the API appear in a program. This program is written using a programming language and execution environment defined outside of the GraphBLAS. We refer to this programming environment as the "host programming environment".
- execution time: time expended while executing instructions defined by a program. This term is specifically used in this specification in the context of computations carried out on behalf of a call to a GraphBLAS method.
  - sequence: A GraphBLAS application uniquely defines a directed acyclic graph (DAG) of GraphBLAS method calls based on their program order. At any point in a program, the state of any GraphBLAS object is defined by a subgraph of that DAG. An ordered collection of GraphBLAS method calls in program order that defines that subgraph for a particular object is the sequence for that object.

• complete: A GraphBLAS object is complete when it can be used in a happens-before relationship with a method call that reads the variable on another thread. This concept is used when reasoning about memory orders in multithreaded programs. A GraphBLAS object defined on one thread that is complete can be safely used as an IN or INOUT argument in a method-call on a second thread assuming the method calls are correctly synchronized so the definition on the first thread happens-before it is used on the second thread. In blocking-mode, an object is complete after a GraphBLAS method call that writes to that object returns. In nonblocking-mode, an object is complete after a call to the GrB\_wait() method with the GrB\_COMPLETE parameter.

- materialize: A GraphBLAS object is materialized when it is (1) complete, (2) the computations defined by the sequence that define the object have finished (either fully or stopped at an error) and will not consume any additional computational resources, and (3) any errors associated with that sequence are available to be read according to the GraphBLAS error model. A GraphBLAS object that is never loaded into a non-opaque data structure may potentially never be materialized. This might happen, for example, if the operations associated with the object are fused or otherwise changed by the runtime system that supports the implementation of the GraphBLAS C API. An object can be materialized by a call to the materialize mode of the GrB\_wait() method.
- context: An instance of the GraphBLAS C API implementation as seen by an application. An application can have only one context between the start and end of the application. A context begins with the first thread that calls GrB\_init() and ends with the first thread to call GrB\_finalize(). It is an error for GrB\_init() or GrB\_finalize() to be called more than one time within an application. The context is used to constrain the behavior of an instance of the GraphBLAS C API implementation and support various execution strategies. Currently, the only supported constraints on a context pertain to the mode of program execution.
- program execution mode: Defines how a GraphBLAS sequence executes, and is associated with the context of a GraphBLAS C API implementation. It is set by an application with its call to GrB\_init() to one of two possible states. In blocking mode, GraphBLAS methods return after the computations complete and any output objects have been materialized. In nonblocking mode, a method may return once the arguments are tested as consistent with the method (i.e., there are no API errors), and potentially before any computation has taken place.

#### 2.1.5 GraphBLAS methods: behaviors and error conditions

- *implementation-defined behavior*: Behavior that must be documented by the implementation and is allowed to vary among different compliant implementations.
- undefined behavior: Behavior that is not specified by the GraphBLAS C API. A conforming implementation is free to choose results delivered from a method whose behavior is undefined.
  - thread-safe: Consider a function called from multiple threads with arguments that do not overlap in memory (i.e. the argument lists do not share memory). If the function is thread-safe

then it will behave the same when executed concurrently by multiple threads or sequentially on a single thread.

- dimension compatible: GraphBLAS objects (matrices and vectors) that are passed as parameters to a GraphBLAS method are dimension (or shape) compatible if they have the correct number of dimensions and sizes for each dimension to satisfy the rules of the mathematical definition of the operation associated with the method. If any dimension compatibility rule above is violated, execution of the GraphBLAS method ends and the GrB\_DIMENSION\_MISMATCH error is returned.
- domain compatible: Two domains for which values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other, and a domain from a user-defined type is only compatible with itself. If any domain compatibility rule above is violated, execution of the GraphBLAS method ends and the GrB\_DOMAIN\_MISMATCH error is returned.

### 502 2.2 Notation

|   | Notation  | Description  |  |  |
|---|---|--|--|--|
|   | $D_{out}, D_{in}, D_{in_1}, D_{in_2}$             | Refers to output and input domains of various GraphBLAS operators.                           |  |  |
|   | $\mathbf{D}_{out}(*), \mathbf{D}_{in}(*),$        | Evaluates to output and input domains of GraphBLAS operators (usually                        |  |  |
|   | $\mathbf{D}_{in_1}(*), \mathbf{D}_{in_2}(*)$      | a unary or binary operator, or semiring).  |  |  |
|   | $\mathbf{D}(*)$                                   | Evaluates to the (only) domain of a GraphBLAS object (usually a monoid,                      |  |  |
|   |   | vector, or matrix).  |  |  |
|   | f   | An arbitrary unary function, usually a component of a unary operator.                        |  |  |
|   | $\mathbf{f}(F_u)$                                 | Evaluates to the unary function contained in the unary operator given as                     |  |  |
|   | ,   | the argument.  |  |  |
|   | $\odot$   | An arbitrary binary function, usually a component of a binary operator.                      |  |  |
|   | $\bigcirc(*)$                                     | Evaluates to the binary function contained in the binary operator or monoid                  |  |  |
|   |   | given as the argument.   |  |  |
|   | $\otimes$   | Multiplicative binary operator of a semiring.  |  |  |
|   | $\oplus$  | Additive binary operator of a semiring.  |  |  |
|   | $\bigotimes(S)$                                   | Evaluates to the multiplicative binary operator of the semiring given as the                 |  |  |
|   | <b>O</b> (**)                                     | argument.  |  |  |
|   | $\bigoplus(S)$                                    | Evaluates to the additive binary operator of the semiring given as the argu-                 |  |  |
|   | <b>(~)</b>  | ment.  |  |  |
|   | <b>0</b> (*)                                      | The identity of a monoid, or the additive identity of a GraphBLAS semiring.                  |  |  |
|   | $\mathbf{L}(*)$                                   | The contents (all stored values) of the vector or matrix GraphBLAS objects.                  |  |  |
|   | <b>-</b> (·)                                      | For a vector, it is the set of (index, value) pairs, and for a matrix it is the              |  |  |
|   |   | set of (row, col, value) triples.  |  |  |
| 3 | $\mathbf{v}(i)$ or $v_i$                          | The $i^{th}$ element of the vector $\mathbf{v}$ .  |  |  |
|   | $\mathbf{size}(\mathbf{v})$                       | The size of the vector <b>v</b> .  |  |  |
|   | ind(v)  | The set of indices corresponding to the stored values of the vector $\mathbf{v}$ .           |  |  |
|   | $\mathbf{nrows}(\mathbf{A})$                      | The number of rows in the <b>A</b> .   |  |  |
|   | $\mathbf{ncols}(\mathbf{A})$                      | The number of columns in the <b>A</b> .  |  |  |
|   | indrow(A)   | The set of row indices corresponding to rows in <b>A</b> that have stored values.            |  |  |
|   | indcol(A)   | The set of column indices corresponding to columns in <b>A</b> that have stored              |  |  |
|   | 1114001(11)                                       | values.  |  |  |
|   | $\mathbf{ind}(\mathbf{A})$                        | The set of $(i, j)$ indices corresponding to the stored values of the matrix.                |  |  |
|   | $\mathbf{A}(i,j)$ or $A_{ij}$                     | The element of <b>A</b> with row index $i$ and column index $j$ .                            |  |  |
|   | $\mathbf{A}(:,j)$                                 | The $j^{th}$ column of matrix $\mathbf{A}$ .   |  |  |
|   | $\mathbf{A}(i,:)$                                 | The $i^{th}$ row of matrix $\mathbf{A}$ .  |  |  |
|   | $\mathbf{A}^T$                                    | The transpose of matrix <b>A</b> .   |  |  |
|   | $\neg \mathbf{M}$                                 | The complement of M.   |  |  |
|   |   | The structure of M.  |  |  |
|   | $rac{\mathrm{s}(\mathbf{M})}{\mathbf{	ilde{t}}}$ | A temporary object created by the GraphBLAS implementation.                                  |  |  |
|   | $\langle type \rangle$                            | A method argument type that is void * or one of the types from Table 3.2.                    |  |  |
|   | GrB_ALL   | A method argument literal to indicate that all indices of an input array                     |  |  |
|   | OID_ALL   | should be used.  |  |  |
|   | GrB_Type  | A method argument type that is either a user defined type or one of the                      |  |  |
|   | OLD_Type  | types from Table 3.2.  |  |  |
|   | GrB_Object  | types from Table 3.2.  A method argument type referencing any of the GraphBLAS object types. |  |  |
|   | OLD ANIERT  | A method argument type referencing any of the Graphblas object types.                        |  |  |
|   | GrB_NULL  | The GraphBLAS NULL.  |  |  |

#### 2.3 Mathematical foundations

Graphs can be represented in terms of matrices. The values stored in these matrices correspond to attributes (often weights) of edges in the graph. Likewise, information about vertices in a graph are stored in vectors. The set of valid values that can be stored in either matrices or vectors is referred to as their domain. Matrices are usually sparse because the lack of an edge between two vertices means that nothing is stored at the corresponding location in the matrix. Vectors may be sparse or dense, or they may start out sparse and become dense as algorithms traverse the graphs.

Operations defined by the GraphBLAS C API specification operate on these matrices and vectors to carry out graph algorithms. These GraphBLAS operations are defined in terms of GraphBLAS semiring algebraic structures. Modifying the underlying semiring changes the result of an operation to support a wide range of graph algorithms. Inside a given algorithm, it is often beneficial to change the GraphBLAS semiring that applies to an operation on a matrix. This has two implications for the C binding of the GraphBLAS API.

First, it means that we define a separate object for the semiring to pass into methods. Since in many cases the full semiring is not required, we also support passing monoids or even binary operators, which means the semiring is implied rather than explicitly stated.

Second, the ability to change semirings impacts the meaning of the *implied zero* in a sparse rep-resentation of a matrix or vector. This element in real arithmetic is zero, which is the identity of the addition operator and the annihilator of the multiplication operator. As the semiring changes, this implied zero changes to the identity of the addition operator and the annihilator (if present) of the *multiplication* operator for the new semiring. Nothing changes regarding what is stored in the sparse matrix or vector, but the implied zeros within them change with respect to a particular operation. In all cases, the nature of the implied zero does not matter since the GraphBLAS C API requires that implementations treat them as nonexistent elements of the matrix or vector. 

As with matrices and vectors, GraphBLAS semirings have domains associated with their inputs and outputs. The semirings in the GraphBLAS C API are defined with two domains associated with the input operands and one domain associated with output. When used in the GraphBLAS C API these domains may not match the domains of the matrices and vectors supplied in the operations.

In this case, only valid domain compatible casting is supported by the API.

The mathematical formalism for graph operations in the language of linear algebra often assumes that we can operate in the field of real numbers. However, the GraphBLAS C binding is designed for implementation on computers, which by necessity have a finite number of bits to represent numbers. Therefore, we require a conforming implementation to use floating point numbers such as those defined by the IEEE-754 standard (both single- and double-precision) wherever real numbers need to be represented. The practical implications of these finite precision numbers is that the result of a sequence of computations may vary from one execution to the next as the grouping of operands (because of associativity) within the operations changes. While techniques are known to reduce these effects, we do not require or even expect an implementation to use them as they may add

<sup>&</sup>lt;sup>1</sup>More information on the mathematical foundations can be found in the following paper: J. Kepner, P. Aaltonen, D. Bader, A. Buluç, F. Franchetti, J. Gilbert, D. Hutchison, M. Kumar, A. Lumsdaine, H. Meyerhenke, S. McMillan, J. Moreira, J. Owens, C. Yang, M. Zalewski, and T. Mattson. 2016, September. Mathematical foundations of the GraphBLAS. In 2016 IEEE High Performance Extreme Computing Conference (HPEC) (pp. 1-9). IEEE.

| Table 2.1: | Types of | GraphBLAS | opaque | objects. |
|------------|----------|-----------|--------|----------|
|            |          |           |        |          |

| GrB_Object types    | Description  |
|---------------------|--|
| GrB_Type            | Scalar type.   |
| GrB_UnaryOp         | Unary operator.  |
| $GrB\_IndexUnaryOp$ | Unary operator, that operates on a single value and its location index values. |
| GrB_BinaryOp        | Binary operator.   |
| GrB_Monoid          | Monoid algebraic structure.  |
| GrB_Semiring        | A GraphBLAS semiring algebraic structure.                                      |
| GrB_Scalar          | One element; could be empty.   |
| GrB_Vector          | One-dimensional collection of elements; can be sparse.                         |
| GrB_Matrix          | Two-dimensional collection of elements; typically sparse.                      |
| GrB_Descriptor      | Descriptor object, used to modify behavior of methods (specifically            |
|                     | GraphBLAS operations).   |
|                     |  |

considerable overhead. In most cases, these roundoff errors are not significant. When they are significant, the problem itself is ill-conditioned and needs to be reformulated.

#### 544 2.4 GraphBLAS opaque objects

Objects defined in the GraphBLAS standard include types (the domains of elements), collections of elements (matrices, vectors, and scalars), operators on those elements (unary, index unary, and binary operators), algebraic structures (semirings and monoids), and descriptors. GraphBLAS objects are defined as opaque types; that is, they are managed, manipulated, and accessed solely through the GraphBLAS application programming interface. This gives an implementation of the GraphBLAS C specification flexibility to optimize objects for different scenarios or to meet the needs of different hardware platforms.

A GraphBLAS opaque object is accessed through its *handle*. A handle is a variable that references an instance of one of the types from Table 2.1. An implementation of the GraphBLAS specification has a great deal of flexibility in how these handles are implemented. All that is required is that the handle corresponds to a type defined in the C language that supports assignment and comparison for equality. The GraphBLAS specification defines a literal GrB\_INVALID\_HANDLE that is valid for each type. Using the logical equality operator from C, it must be possible to compare a handle to GrB\_INVALID\_HANDLE to verify that a handle is valid.

Every GraphBLAS object has a *lifetime*, which consists of the sequence of instructions executed in program order between the *creation* and the *destruction* of the object. The GraphBLAS C API predefines a number of these objects which are created when the GraphBLAS context is initialized by a call to GrB\_init and are destroyed when the GraphBLAS context is terminated by a call to GrB\_finalize.

An application using the GraphBLAS API can create additional objects by declaring variables of the appropriate type from Table 2.1 for the objects it will use. Before use, the object must be initialized

with a call call to one of the object's respective *constructor* methods. Each kind of object has at least one explicit constructor method of the form GrB\_\*\_new where '\*' is replaced with the type of object (e.g., GrB\_Semiring\_new). Note that some objects, especially collections, have additional constructor methods such as duplication, import, or descrialization. Objects explicitly created by a call to a constructor should be destroyed by a call to GrB\_free. The behavior of a program that calls GrB\_free on a pre-defined object is undefined.

These constructor and destructor methods are the only methods that change the value of a handle.
Hence, objects changed by these methods are passed into the method as pointers. In all other
cases, handles are not changed by the method and are passed by value. For example, even when
multiplying matrices, while the contents of the output product matrix changes, the handle for that
matrix is unchanged.

Several GraphBLAS constructor methods take other objects as input arguments and use these objects to create a new object. For all these methods, the lifetime of the created object must end strictly before the lifetime of any dependent input objects. For example, a vector constructor GrB\_Vector\_new takes a GrB\_Type object as input. That type object must not be destroyed until after the created vector is destroyed. Similarly, a GrB\_Semiring\_new method takes a monoid and a binary operator as inputs. Neither of these can be destroyed until after the created semiring is destroyed.

Note that some constructor methods like GrB\_Vector\_dup and GrB\_Matrix\_dup behave differently.

In these cases, the input vector or matrix can be destroyed as soon as the call returns. However,

the original type object used to create the input vector or matrix cannot be destroyed until after

the vector or matrix created by GrB\_Vector\_dup or GrB\_Matrix\_dup is destroyed. This behavior

must hold for any chain of duplicating constructors.

Programmers using GraphBLAS handles must be careful to distinguish between a handle and the object manipulated through a handle. For example, a program may declare two GraphBLAS objects of the same type, initialize one, and then assign it to the other variable. That assignment, however, only assigns the handle to the variable. It does not create a copy of that variable (to do that, one would need to use the appropriate duplication method). If later the object is freed by calling GrB\_free with the first variable, the object is destroyed and the second variable is left referencing an object that no longer exists (a so-called "dangling handle").

In addition to opaque objects manipulated through handles, the GraphBLAS C API defines an additional opaque object as an internal object; that is, the object is never exposed as a variable within an application. This opaque object is the mask used to control which computed values can be stored in the output operand of a *GraphBLAS operation*. Masks are described in Section 3.5.4.

#### $_{ iny 0}$ 2.5 Execution model

A program using the GraphBLAS C API is called a GraphBLAS application. The application constructs GraphBLAS objects, manipulates them to implement a graph algorithm, and then extracts values from the GraphBLAS objects to produce the results for that algorithm. Functions defined within the GraphBLAS C API that manipulate GraphBLAS objects are called *methods*. If the method corresponds to one of the operations defined in the GraphBLAS mathematical specifica-

tion, we refer to the method as an operation.

The GraphBLAS application specifies an ordered collection of GraphBLAS method calls defined by the order they appear in the text of the program (the *program order*). These define a directed acyclic graph (DAG) where nodes are GraphBLAS method calls and edges are dependencies between method calls.

Each method call in the DAG uniquely and unambiguously defines the output GraphBLAS objects as long as there are no execution errors that put objects in an invalid state (see Section 2.6). An ordered collection of method calls, a subgraph of the overall DAG for an application, defines the state of a GraphBLAS object at any point in a program. This ordered collection is the *sequence* for that object.

Since the GraphBLAS execution is defined in terms of a DAG and the GraphBLAS objects are opaque, the semantics of the GraphBLAS specification affords an implementation considerable flexibility to optimize performance. A GraphBLAS implementation can defer execution of nodes in the DAG, fuse nodes, or even replace whole subgraphs within the DAG to optimize performance. We discuss this topic further in section 2.5.1 when we describe *blocking* and *non-blocking* execution modes.

A correct GraphBLAS application must be *race-free*. This means that the DAG produced by an application and the results produced by execution of that DAG must be the same regardless of how the threads are scheduled for execution. It is the application programmer's responsibility to control memory orders and establish the required synchronized-with relationships to assure race-free execution of a multi-threaded GraphBLAS application. Writing race-free GraphBLAS applications is discussed further in Section 2.5.2.

#### 628 2.5.1 Execution modes

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The execution of the DAG defined by a GraphBLAS application depends on the execution mode of the GraphBLAS program. There are two modes: blocking and nonblocking.

- blocking: In blocking mode, each method finishes the GraphBLAS operation defined by the method and all output GraphBLAS objects are materialized before proceeding to the next statement. Even mechanisms that break the opaqueness of the GraphBLAS objects (e.g., performance monitors, debuggers, memory dumps) will observe that the operation has finished.
- nonblocking: In nonblocking mode, each method may return once the input arguments have been inspected and verified to define a well formed GraphBLAS operation. (That is, there are no API errors; see Section 2.6.) The GraphBLAS method may not have finished, but the output object is ready to be used by the next GraphBLAS method call. If needed, a call to GrB\_wait with GrB\_COMPLETE or GrB\_MATERIALIZE can be used to force the sequence for a GraphBLAS object (obj) to finish its execution.

The execution mode is defined in the GraphBLAS C API when the context of the library invocation is defined. This occurs once before any GraphBLAS methods are called with a call to the

GrB\_init() function. This function takes a single argument of type GrB\_Mode with values shown in Table 3.1(a).

An application executing in nonblocking mode is not required to return immediately after input arguments have been verified. A conforming implementation of the GraphBLAS C API running in nonblocking mode may choose to execute as if in blocking mode. A sequence of operations in nonblocking mode where every GraphBLAS operation with output object obj is followed by a GrB\_wait(obj, GrB\_MATERIALIZE) call is equivalent to the same sequence in blocking mode with GrB\_wait(obj, GrB\_MATERIALIZE) calls removed.

Nonblocking mode allows for any execution strategy that satisfies the mathematical definition of the sequence. The methods can be placed into a queue and deferred. They can be chained together and fused (e.g., replacing a chained pair of matrix products with a matrix triple product). Lazy evaluation, greedy evaluation, and asynchronous execution are all valid as long as the final result agrees with the mathematical definition provided by the sequence of GraphBLAS method calls appearing in program order.

Blocking mode forces an implementation to carry out precisely the GraphBLAS operations defined by the methods and to complete each and every method call individually. It is valuable for debugging or in cases where an external tool such as a debugger needs to evaluate the state of memory during a sequence of operations.

In a sequence of operations free of execution errors, and with input objects that are well-conditioned, the results from blocking and nonblocking modes should be identical outside of effects due to roundoff errors associated with floating point arithmetic. Due to the great flexibility afforded to an implementation when using nonblocking mode, we expect execution of a sequence in nonblocking mode to potentially complete execution in less time.

It is important to note that, processing of nonopaque objects is never deferred in GraphBLAS.
That is, methods that consume nonopaque objects (e.g., GrB\_Matrix\_build(), Section 4.2.5.9) and
methods that produce nonopaque objects (e.g., GrB\_Matrix\_extractTuples(), Section 4.2.5.13) always finish consuming or producing those nonopaque objects before returning regardless of the
execution mode.

Finally, after all GraphBLAS method calls have been made, the context is terminated with a call to GrB\_finalize(). In the current version of the GraphBLAS C API, the context can be set only once in the execution of a program. That is, after GrB\_finalize() is called, a subsequent call to GrB\_init() is not allowed.

#### $_{6}$ 2.5.2 Multi-threaded execution

The GraphBLAS C API is designed to work with applications that utilize multiple threads executing within a shared address space. This specification does not define how threads are created, managed and synchronized. We expect the host programming environment to provide those services.

A conformant implementation of the GraphBLAS must be *thread safe*. A GraphBLAS library is thread safe when independent method calls (i.e., GraphBLAS objects are not shared between method calls) from multiple threads in a race-free program return the same results as would follow

from their sequential execution in some interleaved order. This is a common requirement in software libraries.

Thread safety applies to the behavior of multiple independent threads. In the more general case for multithreading, threads are not independent; they share variables and mix read and write operations to those variables across threads. A memory consistency model defines which values can be returned when reading an object shared between two or more threads. The GraphBLAS specification does not define its own memory consistency model. Instead the specification defines what must be done by a programmer calling GraphBLAS methods and by the implementor of a GraphBLAS library so an implementation of the GraphBLAS specification can work correctly with the memory consistency model for the host environment.

A memory consistency model is defined in terms of happens-before relations between methods in different threads. The defining case is a method that writes to an object on one thread that is read (i.e., used as an IN or INOUT argument) in a GraphBLAS method on a different thread. The following steps must occur between the different threads.

• A sequence of GraphBLAS methods results in the definition of the GraphBLAS object.

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- The GraphBLAS object is put into a state of completion by a call to GrB\_wait() with the GrB\_COMPLETE parameter (see Table 3.1(b)). A GraphBLAS object is said to be *complete* when it can be safely used as an IN or INOUT argument in a GraphBLAS method call from a different thread.
- Completion happens before a synchronized-with relation that executes with at least a release memory order.
  - A synchronized-with relation on the other thread executes with at least an acquire memory order.
- This synchronized-with relation happens-before the GraphBLAS method that reads the graph-BLAS object.

We use the phrase at least when talking about the memory orders to indicate that a stronger memory order such as sequential consistency can be used in place of the acquire-release order.

A program that violates these rules contains a data race. That is, its reads and writes are unordered across threads making the final value of a variable undefined. A program that contains a data race is invalid and the results of that program are undefined. We note that multi-threaded execution is compatible with both blocking and non-blocking modes of execution.

Completion is the central concept that allows GraphBLAS objects to be used in happens-before relations between threads. In earlier versions of GraphBLAS (1.X) completion was implied by any operation that produced non-opaque values from a GraphBLAS object. These operations are summarized in Table 2.2). In GraphBLAS 2.0, these methods no longer imply completion. This change was made since there are cases where the non-opaque value is needed but the object from which it is computed is not. We want implementations of the GraphBLAS to be able to exploit this case and not form the opaque object when that object is not needed.

Table 2.2: Methods that extract values from a GraphBLAS object that forcing completion of the operations contributing to that particular object in GraphBLAS 1.X. In GraphBLAS 2.0, these methods *do not* force completion.

| Method   | Section  |
|--|----------|
| GrB_Vector_nvals   | 4.2.4.6  |
| GrB_Vector_extractElement  | 4.2.4.10 |
| GrB_Vector_extractTuples   | 4.2.4.11 |
| GrB_Matrix_nvals   | 4.2.5.8  |
| GrB_Matrix_extractElement  | 4.2.5.12 |
| GrB_Matrix_extractTuples   | 4.2.5.13 |
| GrB_reduce (vector-scalar value variant)                                   | 4.3.10.2 |
| ${\sf GrB\_reduce}~({\rm matrix\text{-}scalar}~{\rm value}~{\rm variant})$ | 4.3.10.3 |

#### $_{\scriptscriptstyle 1}$ 2.6 Error model

All GraphBLAS methods return a value of type GrB\_Info (an enum) to provide information available to the system at the time the method returns. The returned value will be one of the defined values shown in Table 3.16. The return values fall into three groups: informational, API errors, and execution errors. While API and execution errors take on negative values, informational return values listed in Table 3.16(a) are non-negative and include GrB\_SUCCESS (a value of 0) and GrB\_NO\_VALUE.

An API error (listed in Table 3.16(b)) means that a GraphBLAS method was called with parameters that violate the rules for that method. These errors are restricted to those that can be determined by inspecting the dimensions and domains of GraphBLAS objects, GraphBLAS operators, or the values of scalar parameters fixed at the time a method is called. API errors are deterministic and consistent across platforms and implementations. API errors are never deferred, even in nonblocking mode. That is, if a method is called in a manner that would generate an API error, it always returns with the appropriate API error value. If a GraphBLAS method returns with an API error, it is guaranteed that none of the arguments to the method (or any other program data) have been modified. The informational return value, GrB\_NO\_VALUE, is also deterministic and never deferred in nonblocking mode.

Execution errors (listed in Table 3.16(c)) indicate that something went wrong during the execution of a legal GraphBLAS method invocation. Their occurrence may depend on specifics of the execution environment and data values being manipulated. This does not mean that execution errors are the fault of the GraphBLAS implementation. For example, a memory leak could arise from an error in an application's source code (a "program error"), but it may manifest itself in different points of a program's execution (or not at all) depending on the platform, problem size, or what else is running at that time. Index out-of-bounds errors, for example, always indicate a program error.

If a GraphBLAS method returns with any execution error other than GrB\_PANIC, it is guaranteed that the state of any argument used as input-only is unmodified. Output arguments may be left in an invalid state, and their use downstream in the program flow may cause additional errors. If a

GraphBLAS method returns with a GrB\_PANIC execution error, no guarantees can be made about the state of any program data.

In nonblocking mode, execution errors can be deferred. A return value of GrB\_SUCCESS only guarantees that there are no API errors in the method invocation. If an execution error value is returned by a method with output object obj in nonblocking mode, it indicates that an error was found during execution of any of the pending operations on obj, up to and including the GrB\_wait() method (Section 4.2.8) call that completes those pending operations. When possible, that return value will provide information concerning the cause of the error.

As discussed in Section 4.2.8, a GrB\_wait(obj) on a specific GraphBLAS object obj completes all pending operations on that object. No additional errors on the methods that precede the call to GrB\_wait and have obj as an OUT or INOUT argument can be reported. From a GraphBLAS perspective, those methods are *complete*. Details on the guaranteed state of objects after a call to GrB\_wait can be found in Section 4.2.8.

After a call to any GraphBLAS method that modifies an opaque object, the program can retrieve additional error information (beyond the error code returned by the method) though a call to the function GrB\_error(), passing the method's output object as described in Section 4.2.9.

The function returns a pointer to a NULL-terminated string, and the contents of that string are implementation-dependent. In particular, a null string (not a NULL pointer) is always a valid error string. GrB\_error() is a thread-safe function, in the sense that multiple threads can call it simultaneously and each will get its own error string back, referring to the object passed as an input argument.

### $_{\scriptscriptstyle{770}}$ Chapter 3

# Objects

In this chapter, all of the enumerations, literals, data types, and predefined opaque objects defined in the GraphBLAS API are presented. Enumeration literals in GraphBLAS are assigned specific 773 values to ensure compatibility between different runtime library implementations. The chapter starts by defining the enumerations that are used by the init() and wait() methods. Then a num-775 ber of transparent (i.e., non-opaque) types that are used for interfacing with external data are 776 defined. Sections that follow describe the various types of opaque objects in GraphBLAS: types 777 (or domains), algebraic objects, collections and descriptors. Each of these sections also lists the 778 predefined instances of each opaque type that are required by the API. This chapter concludes with 779 a section on the definition for GrB Info enumeration that is used as the return type of all methods. 780

#### 3.1 Enumerations for init() and wait()

Table 3.1 lists the enumerations and the corresponding values used in the GrB\_init() method to set the execution mode and in the GrB\_wait() method for completing or materializing opaque objects.

#### 3.2 Indices, index arrays, and scalar arrays

In order to interface with third-party software (i.e., software other than an implementation of the GraphBLAS), operations such as GrB\_Matrix\_build (Section 4.2.5.9) and GrB\_Matrix\_extractTuples (Section 4.2.5.13) must specify how the data should be laid out in non-opaque data structures. To this end we explicitly define the types for indices and the arrays used by these operations.

For indices a typedef is used to give a GraphBLAS name to a concrete type. We define it as follows:

typedef uint64\_t GrB\_Index;

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The range of valid values for a variable of type GrB\_Index is [0, GrB\_INDEX\_MAX] where the largest index value permissible is defined with a macro, GrB\_INDEX\_MAX. For example:

An implementation is required to define and document this value.

An index array is a pointer to a set of GrB Index values that are stored in a contiguous block of 795 memory (i.e., GrB\_Index\*). Likewise, a scalar array is a pointer to a contiguous block of memory 796 storing a number of scalar values as specified by the user. Some GraphBLAS operations (e.g., 797 GrB assign) include an input parameter with the type of an index array. This input index array 798 selects a subset of elements from a GraphBLAS vector or matrix object to be used in the operation. 799 In these cases, the literal GrB\_ALL can be used in place of the index array input parameter to 800 indicate that all indices of the associated GraphBLAS vector or matrix object should be used. An 801 implementation of the GraphBLAS C API has considerable freedom in terms of how GrB\_ALL is defined. Since GrB\_ALL is used as an argument for an array parameter, it must use a type 803 consistent with a pointer. GrB\_ALL must also have a non-null value to distinguish it from the 804 erroneous case of passing a NULL pointer as an array. 805

#### 3.3 Types (domains)

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In GraphBLAS, domains correspond to the valid values for types from the host language (in our case, the C programming language). GraphBLAS defines a number of operators that take elements from one or more domains and produce elements of a (possibly) different domain. GraphBLAS also defines three kinds of collections: matrices, vectors and scalars. For any given collection, the elements of the collection belong to a *domain*, which is the set of valid values for the elements. For any variable or object V in GraphBLAS we denote as  $\mathbf{D}(V)$  the domain of V, that is, the set of possible values that elements of V can take.

Table 3.1: Enumeration literals and corresponding values input to various GraphBLAS methods.

(a) GrB\_Mode execution modes for the GrB\_init method.

| Symbol          | Value | Description                             |
|-----------------|-------|---|
| GrB_NONBLOCKING | 0     | Specifies the nonblocking mode context. |
| GrB_BLOCKING    | 1     | Specifies the blocking mode context.    |

#### (b) GrB\_WaitMode wait modes for the GrB\_wait method.

| Symbol          | Value | Description   |
|-----------------|-------|---|
| GrB_COMPLETE    | 0     | The object is in a state where it can be used in a happens-         |
|                 |       | before relation so that multithreaded programs can be prop-         |
|                 |       | erly synchronized.  |
| GrB_MATERIALIZE | 1     | The object is <i>complete</i> , and in addition, all computation of |
|                 |       | the object is finished and any error information is available.      |
|                 | ,u    | '   |

Table 3.2: Predefined  $GrB\_Type$  values, and the corresponding GraphBLAS domain suffixes, C type (for scalar parameters), and domains for GraphBLAS. The domain suffixes are used in place of I, F, and T in Tables 3.5, 3.6, 3.7, 3.8, and 3.9).

| GrB_Type   | GrB_Type_Code     | Suffix | C type   | Domain                                |
|------------|-------------------|--------|----------|---------------------------------------|
| -          | GrB_UDT_CODE=0    | UDT    | -        | -                                     |
| GrB_BOOL   | GrB_BOOL_CODE=1   | BOOL   | bool     | $\{ \mathtt{false}, \mathtt{true} \}$ |
| GrB_INT8   | GrB_INT8_CODE=2   | INT8   | int8_t   | $\mathbb{Z}\cap[-2^7,2^7)$            |
| GrB_UINT8  | GrB_UINT8_CODE=3  | UINT8  | uint8_t  | $\mathbb{Z}\cap[0,2^8)$               |
| GrB_INT16  | GrB_INT16_CODE=4  | INT16  | int16_t  | $\mathbb{Z} \cap [-2^{15}, 2^{15})$   |
| GrB_UINT16 | GrB_UINT16_CODE=5 | UINT16 | uint16_t | $\mathbb{Z}\cap[0,2^{16})$            |
| GrB_INT32  | GrB_INT32_CODE=6  | INT32  | int32_t  | $\mathbb{Z} \cap [-2^{31}, 2^{31})$   |
| GrB_UINT32 | GrB_UINT32_CODE=7 | UINT32 | uint32_t | $\mathbb{Z} \cap [0, 2^{32})$         |
| GrB_INT64  | GrB_INT64_CODE=8  | INT64  | int64_t  | $\mathbb{Z} \cap [-2^{63}, 2^{63})$   |
| GrB_UINT64 | GrB_UINT64_CODE=9 | UINT64 | uint64_t | $\mathbb{Z} \cap [0, 2^{64})$         |
| GrB_FP32   | GrB_FP32_CODE=10  | FP32   | float    | IEEE 754 binary32                     |
| GrB_FP64   | GrB_FP64_CODE=11  | FP64   | double   | IEEE 754 binary64                     |
|            |                   |        |          |                                       |

The domains for elements that can be stored in collections and operated on through GraphBLAS methods are defined by GraphBLAS objects called GrB\_Type. The predefined types and corresponding domains used in the GraphBLAS C API are shown in Table 3.2. The Boolean type (bool) is defined in stdbool.h, the integral types (int8\_t, uint8\_t, int16\_t, uint16\_t, int32\_t, uint32\_t, int64\_t, uint64\_t) are defined in stdint.h, and the floating-point types (float, double) are native to the language and platform and in most cases defined by the IEEE-754 standard. UDT stands for user-defined type and is the type code returned for all objects which use a non-predefined type. Implementations which add new types should start their GrB\_Type\_Codes at 100 to avoid possible conflicts with built-in types which may be added in the future.

#### 3.4 Algebraic objects, operators and associated functions

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GraphBLAS operators operate on elements stored in GraphBLAS collections. A binary operator is a function that maps two input values to one output value. A unary operator is a function that maps one input value to one output value. Binary operators are defined over two input domains and produce an output from a (possibly different) third domain. Unary operators are specified over one input domain and produce an output from a (possibly different) second domain.

In addition to the operators that operate on stored values, GraphBLAS also supports *index unary* operators that maps a stored value and the indices of its position in the matrix or vector to an output value. That output value can be used in the index unary operator variants of apply (§ 4.3.8) to compute a new stored value, or be used in the select operation (§ 4.3.9) to determine if the stored input value should be kept or annihilated.

34 Some GraphBLAS operations require a monoid or semiring. A monoid contains an associative

Table 3.3: Operator input for relevant GraphBLAS operations. The semiring add and times are shown if applicable.

| Operation                        | Operator input              |
|----------------------------------|-----------------------------|
| mxm, mxv, vxm                    | semiring                    |
| eWiseAdd                         | binary operator             |
|                                  | monoid                      |
|                                  | semiring (add)              |
| eWiseMult                        | binary operator             |
|                                  | monoid                      |
|                                  | semiring (times)            |
| reduce (to vector or GrB_Scalar) | binary operator             |
|                                  | monoid                      |
| reduce (to scalar value)         | monoid                      |
| apply                            | unary operator              |
|                                  | binary operator with scalar |
|                                  | index unary operator        |
| select                           | index unary operator        |
| kronecker                        | binary operator             |
|                                  | monoid                      |
|                                  | semiring                    |
| dup argument (build methods)     | binary operator             |
| accum argument (various methods) | binary operator             |

binary operator where the input and output domains are the same. The monoid also includes an identity value of the operator. The semiring consists of a binary operator – referred to as the "times" operator – with up to three different domains (two inputs and one output) and a monoid – referred to as the "plus" operator – that is also commutative. Furthermore, the domain of the monoid must be the same as the output domain of the "times" operator.

The GraphBLAS algebraic objects operators, monoids, and semirings are presented in this section.
These objects can be used as input arguments to various GraphBLAS operations, as shown in
Table 3.3. The specific rules for each algebraic object are explained in the respective sections of
those objects. A summary of the properties and recipes for building these GraphBLAS algebraic
objects is presented in Table 3.4.

A number of predefined operators are specified by the GraphBLAS C API. They are presented in tables in their respective subsections below. Each of these operators is defined to operate on specific GraphBLAS types and therefore, this type is built into the name of the object as a suffix. These suffixes and the corresponding predefined GrB\_Type objects that are listed in Table 3.2.

#### 3.4.1 Operators

A GraphBLAS unary operator  $F_u = \langle D_{out}, D_{in}, f \rangle$  is defined by two domains,  $D_{out}$  and  $D_{in}$ , and an operation  $f: D_{in} \to D_{out}$ . For a given GraphBLAS unary operator  $F_u = \langle D_{out}, D_{in}, f \rangle$ , we

Table 3.4: Properties and recipes for building GraphBLAS algebraic objects: unary operator, binary operator, monoid, and semiring (composed of operations *add* and *times*).

#### (a) Properties of algebraic objects.

| Object          | Must be     | Must be     | Identity         | Number         |
|-----------------|-------------|-------------|------------------|----------------|
|                 | commutative | associative | must exist       | of domains     |
| Unary operator  | n/a         | n/a         | n/a              | 2              |
| Binary operator | no          | no          | no               | 3              |
| Monoid          | no          | yes         | yes              | 1              |
| Reduction add   | yes         | yes         | yes (see Note 1) | 1              |
| Semiring add    | yes         | yes         | yes              | 1              |
| Semiring times  | no          | no          | no               | 3 (see Note 2) |

#### (b) Recipes for algebraic objects.

| Object          | Recipe                                    | Number of domains |
|-----------------|---|-------------------|
| Unary operator  | Function pointer                          | 2                 |
| Binary operator | Function pointer                          | 3                 |
| Monoid          | Associative binary operator with identity | 1                 |
| Semiring        | Commutative monoid + binary operator      | 3                 |

Note 1: Some high-performance GraphBLAS implementations may require an identity to perform reductions to sparse objects like GraphBLAS vectors and scalars. According to the descriptions of the corresponding GraphBLAS operations, however, this identity is mathematically not necessary. There are API signatures to support both. Note 2: The output domain of the semiring times must be same as the domain of the semiring's add monoid. This

ensures three domains for a semiring rather than four.

```
define \mathbf{D}_{out}(F_u) = D_{out}, \mathbf{D}_{in}(F_u) = D_{in}, and \mathbf{f}(F_u) = f.
```

- A GraphBLAS binary operator  $F_b = \langle D_{out}, D_{in_1}, D_{in_2}, \odot \rangle$  is defined by three domains,  $D_{out}, D_{in_1}$ ,  $D_{in_2}$ , and an operation  $\odot: D_{in_1} \times D_{in_2} \to D_{out}$ . For a given GraphBLAS binary operator  $F_b = \langle D_{out}, D_{in_1}, D_{in_2}, \odot \rangle$ , we define  $\mathbf{D}_{out}(F_b) = D_{out}$ ,  $\mathbf{D}_{in_1}(F_b) = D_{in_1}$ ,  $\mathbf{D}_{in_2}(F_b) = D_{in_2}$ , and  $\mathbf{O}(F_b) = \mathbf{D}_{out}$ . Note that  $\mathbf{O}$  could be used in place of either  $\mathbf{O}$  or  $\mathbf{O}$  in other methods and operations.
- A GraphBLAS index unary operator  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$  is defined by three domains,  $D_{out}, D_{in_1}, D_{in_2}$ , the domain of GraphBLAS indices, and an operation  $f_i : D_{in_1} \times I_{U64}^2 \times D_{in_2} \to D_{out}$  (where  $I_{U64}$  corresponds to the domain of a GrB\_Index). For a given GraphBLAS index operator  $F_i$ , we define  $\mathbf{D}_{out}(F_i) = D_{out}$ ,  $\mathbf{D}_{in_1}(F_i) = D_{in_1}$ ,  $\mathbf{D}_{in_2}(F_i) = D_{in_2}$ , and  $\mathbf{f}(F_i) = f_i$ .
- User-defined operators can be created with calls to  $GrB\_UnaryOp\_new$ ,  $GrB\_BinaryOp\_new$ , and  $GrB\_IndexUnaryOp\_new$ , respectively. See Section 4.2.2 for information on these methods. The GraphBLAS C API predefines a number of these operators. These are listed in Tables 3.5 and 3.6. Note that most entries in these tables represent a "family" of predefined operators for a set of different types represented by the T, I, or F in their names. For example, the multiplicative inverse ( $GrB\_MINV\_F$ ) function is only defined for floating-point types (F = FP32 or FP64). The division ( $GrB\_DIV\_T$ ) function is defined for all types, but only if  $y \neq 0$  for integral and floating point types and  $y \neq false$  for the Boolean type.

Table 3.5: Predefined unary and binary operators for GraphBLAS in C. The T can be any suffix from Table 3.2, I can be any integer suffix from Table 3.2, and F can be any floating-point suffix from Table 3.2.

| Operator     | GraphBLAS          |  |   |                        |
|--------------|--------------------|--|---|------------------------|
| type         | identifier         | Domains                                | Description                             |                        |
| GrB_UnaryOp  | $GrB\_IDENTITY\_T$ | $T \to T$                              | f(x) = x                                | identity               |
| GrB_UnaryOp  | GrB_ABS_T          | $T \to T$                              | f(x) =  x ,                             | absolute value         |
| GrB_UnaryOp  | GrB_AINV_T         | $T \to T$                              | f(x) = -x,                              | additive inverse       |
| GrB_UnaryOp  | $GrB\_MINV\_F$     | $F \to F$                              | $f(x) = \frac{1}{x}$                    | multiplicative inverse |
| GrB_UnaryOp  | GrB_LNOT           | $	exttt{bool} 	o 	exttt{bool}$         | $f(x) = \neg x,$                        | logical inverse        |
| GrB_UnaryOp  | GrB_BNOT_I         | $I \rightarrow I$                      | $\int f(x) = \tilde{x},$                | bitwise complement     |
| CoD Discos O | C.D. LOD           |  |   | 1 : 1 OD               |
| GrB_BinaryOp | GrB_LOR            | $bool \times bool \rightarrow bool$    | $f(x,y) = x \vee y,$                    | logical OR             |
| GrB_BinaryOp | GrB_LAND           | $bool \times bool \rightarrow bool$    | $\int f(x,y) = x \wedge y,$             | logical AND            |
| GrB_BinaryOp | GrB_LXOR           | $bool \times bool \rightarrow bool$    | $f(x,y) = \underbrace{x \oplus y}_{f},$ | logical XOR            |
| GrB_BinaryOp | GrB_LXNOR          | $bool \times bool \rightarrow bool$    | $f(x,y) = \overline{x \oplus y},$       | logical XNOR           |
| GrB_BinaryOp | GrB_BOR_I          | $I \times I \to I$                     | $\int_{0}^{\infty} f(x,y) = x \mid y,$  | bitwise OR             |
| GrB_BinaryOp | GrB_BAND_I         | $I \times I \to I$                     | $\int_{C} f(x,y) = x \& y,$             | bitwise AND            |
| GrB_BinaryOp | GrB_BXOR_I         | $I \times I \to I$                     | $f(x,y) = \underline{x \cdot y},$       | bitwise XOR            |
| GrB_BinaryOp | GrB_BXNOR_I        | $I \times I \to I$                     | $f(x,y) = \overline{x \cdot y},$        | bitwise XNOR           |
| GrB_BinaryOp | GrB_EQ_T           | $T \times T \to \text{bool}$           | f(x,y) = (x == y)                       | equal                  |
| GrB_BinaryOp | GrB_NE_T           | $T \times T \to \text{bool}$           | $f(x,y) = (x \neq y)$                   | not equal              |
| GrB_BinaryOp | GrB_GT_T           | $T \times T \rightarrow \texttt{bool}$ | f(x,y) = (x > y)                        | greater than           |
| GrB_BinaryOp | GrB_LT_T           | $T \times T \to \texttt{bool}$         | f(x,y) = (x < y)                        | less than              |
| GrB_BinaryOp | GrB_GE_T           | $T \times T \to \texttt{bool}$         | $f(x,y) = (x \ge y)$                    | greater than or equal  |
| GrB_BinaryOp | GrB_LE_T           | $T \times T \to \texttt{bool}$         | $f(x,y) = (x \le y)$                    | less than or equal     |
| GrB_BinaryOp | GrB_ONEB_T         | $T \times T \to T$                     | $\int_{C} f(x,y) = 1,$                  | 1 (cast to $T$ )       |
| GrB_BinaryOp | GrB_FIRST_T        | $T \times T \to T$                     | $\int f(x,y) = x,$                      | first argument         |
| GrB_BinaryOp | GrB_SECOND_T       | $T \times T \to T$                     | $\int f(x,y) = y,$                      | second argument        |
| GrB_BinaryOp | GrB_MIN_T          | $T \times T \to T$                     | f(x,y) = (x < y) ? x : y,               | minimum                |
| GrB_BinaryOp | GrB_MAX_T          | $ T \times T \to T $                   | f(x,y) = (x > y) ? x : y,               | maximum                |
| GrB_BinaryOp | GrB_PLUS_T         | $T \times T \to T$                     | $\int f(x,y) = x + y,$                  | addition               |
| GrB_BinaryOp | GrB_MINUS_T        | $T \times T \to T$                     | $\int_{C} f(x,y) = x - y,$              | subtraction            |
| GrB_BinaryOp | GrB_TIMES_T        | $T \times T \to T$                     | $\int_{C} f(x,y) = xy,$                 | multiplication         |
| GrB_BinaryOp | GrB_DIV_T          | $T \times T \to T$                     | $f(x,y) = \frac{x}{y},$                 | division               |

Table 3.6: Predefined index unary operators for GraphBLAS in C. The T can be any suffix from Table 3.2.  $I_{U64}$  refers to the unsigned 64-bit, GrB\_Index, integer type,  $I_{32}$  refers to the signed, 32-bit integer type, and  $I_{64}$  refers to signed, 64-bit integer type. The parameters,  $u_i$  or  $A_{ij}$ , are the stored values from the containers where the i and j parameters are set to the row and column indices corresponding to the location of the stored value. When operating on vectors, j will be passed with a zero value. Finally, s is an additional scalar value used in the operators. The expressions in the "Description" column are to be treated as mathematical specifications. That is, for the index arithmetic functions in the first two groups below, each one of i, j, and s is interpreted as an integer number in the set  $\mathbb{Z}$ . Functions are evaluated using arithmetic in  $\mathbb{Z}$ , producing a result value that is also in  $\mathbb{Z}$ . The result value is converted to the output type according to the rules of the C language. In particular, if the value cannot be represented as a signed 32- or 64-bit integer type, the output is implementation defined. Any deviations from this ideal behavior, including limitations on the values of i, j, and s, or possible overflow and underflow conditions, must be defined by the implementation.

| Operator type                             | GraphBLAS                 | Don  | nains (-  | is don'     | t care)     | Description          |   |                    |   |
|---|---------------------------|------|-----------|-------------|-------------|----------------------|---|--------------------|---|
| Type                                      | identifier                | A, u | i, j      | s           | result      |                      |   |                    |   |
| GrB_IndexUnaryOp                          | GrB_ROWINDEX_ $I_{32/64}$ | _    | $I_{U64}$ | $I_{32/64}$ | $I_{32/64}$ | $f(A_{ij}, i, j, s)$ | = | (i+s),             | replace with its row index (+ s)        |
|   | ,                         | _    | $I_{U64}$ | $I_{32/64}$ | $I_{32/64}$ | $f(u_i, i, 0, s)$    | = | (i+s)              |   |
| $GrB\_IndexUnaryOp$                       | GrB_COLINDEX $I_{32/64}$  | _    | $I_{U64}$ | $I_{32/64}$ | $I_{32/64}$ | $f(A_{ij},i,j,s)$    | = | (j+s)              | replace with its column index $(+ s)$   |
| $GrB\_IndexUnaryOp$                       | GrB_DIAGINDEX $I_{32/64}$ | _    | $I_{U64}$ | $I_{32/64}$ | $I_{32/64}$ | $f(A_{ij},i,j,s)$    | = | (j-i+s)            | replace with its diagonal index $(+ s)$ |
| GrB_IndexUnaryOp                          | GrB_TRIL                  | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(A_{ij}, i, j, s)$ | = | $(j \le i + s)$    | triangle on or below diagonal s         |
| $\underline{\leftarrow}$ GrB_IndexUnaryOp | GrB_TRIU                  | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(A_{ij},i,j,s)$    | = | $(j \ge i + s)$    | triangle on or above diagonal s         |
| $^{\odot}$ GrB_IndexUnaryOp               | GrB_DIAG                  | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(A_{ij},i,j,s)$    | = | (j == i + s)       | diagonal s                              |
| $GrB\_IndexUnaryOp$                       | GrB_OFFDIAG               | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(A_{ij},i,j,s)$    | = | $(j \neq i + s)$   | all but diagonal s                      |
| $GrB\_IndexUnaryOp$                       | GrB_COLLE                 | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(A_{ij},i,j,s)$    | = | $(j \le s)$        | columns less or equal to s              |
| $GrB\_IndexUnaryOp$                       | GrB_COLGT                 | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(A_{ij},i,j,s)$    | = | (j>s)              | columns greater than s                  |
| $GrB\_IndexUnaryOp$                       | GrB_ROWLE                 | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(A_{ij},i,j,s)$    | = | $(i \leq s),$      | rows less or equal to s                 |
|   |                           | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(u_i, i, 0, s)$    | = | $(i \le s)$        |   |
| $GrB\_IndexUnaryOp$                       | GrB_ROWGT                 | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(A_{ij},i,j,s)$    | = | (i>s),             | rows greater than s                     |
|   |                           | _    | $I_{U64}$ | $I_{64}$    | bool        | $f(u_i, i, 0, s)$    | = | (i > s)            |   |
| GrB_IndexUnaryOp                          | $GrB\_VALUEEQ\_T$         | T    | _         | T           | bool        | $f(A_{ij},i,j,s)$    | = | $(A_{ij} == s),$   | elements equal to value s               |
|   |                           | T    | _         | T           | bool        | $f(u_i, i, 0, s)$    | = | $(u_i == s)$       |   |
| $GrB\_IndexUnaryOp$                       | $GrB\_VALUENE\_T$         | T    | _         | T           | bool        | $f(A_{ij},i,j,s)$    | = | $(A_{ij} \neq s),$ | elements not equal to value s           |
|   |                           | T    | _         | T           | bool        | $f(u_i, i, 0, s)$    | = | $(u_i \neq s)$     |   |
| $GrB\_IndexUnaryOp$                       | GrB_VALUELT_T             | T    | _         | T           | bool        | $f(A_{ij},i,j,s)$    | = | $(A_{ij} < s),$    | elements less than value s              |
|   |                           | T    | _         | T           | bool        | $f(u_i, i, 0, s)$    | = | $(u_i < s)$        |   |
| $GrB\_IndexUnaryOp$                       | GrB_VALUELE_T             | T    | _         | T           | bool        | $f(A_{ij},i,j,s)$    | = | $(A_{ij} \leq s),$ | elements less or equal to value s       |
|   |                           | T    | _         | T           | bool        | $f(u_i, i, 0, s)$    | = | $(u_i \le s)$      |   |
| $GrB\_IndexUnaryOp$                       | $GrB\_VALUEGT\_T$         | T    | _         | T           | bool        | $f(A_{ij},i,j,s)$    | = | $(A_{ij} > s),$    | elements greater than value s           |
|   |                           | T    | _         | T           | bool        | $f(u_i, i, 0, s)$    | = | $(u_i > s)$        |   |
| $GrB\_IndexUnaryOp$                       | $GrB\_VALUEGE\_T$         | T    | _         | T           | bool        | $f(A_{ij},i,j,s)$    | = | $(A_{ij} \geq s),$ | elements greater or equal to value s    |
|   |                           | T    | _         | T           | bool        | $f(u_i, i, 0, s)$    | = | $(u_i \ge s)$      |   |

#### 3.4.2 Monoids

- A GraphBLAS monoid  $M = \langle D, \odot, 0 \rangle$  is defined by a single domain D, an associative<sup>1</sup> operation  $\odot: D \times D \to D$ , and an identity element  $0 \in D$ . For a given GraphBLAS monoid  $M = \langle D, \odot, 0 \rangle$  we define  $\mathbf{D}(M) = D$ ,  $\odot(M) = \odot$ , and  $\mathbf{0}(M) = 0$ . A GraphBLAS monoid is equivalent to the conventional monoid algebraic structure.
- Let  $F = \langle D, D, D, \odot \rangle$  be an associative GraphBLAS binary operator with identity element  $0 \in D$ .

  Then  $M = \langle F, 0 \rangle = \langle D, \odot, 0 \rangle$  is a GraphBLAS monoid. If  $\odot$  is commutative, then M is said to be a commutative monoid. If a monoid M is created using an operator  $\odot$  that is not associative, the outcome of GraphBLAS operations using such a monoid is undefined.
- User-defined monoids can be created with calls to  $GrB\_Monoid\_new$  (see Section 4.2.2). The GraphBLAS C API predefines a number of monoids that are listed in Table 3.7. Predefined monoids are named  $GrB\_op\_MONOID\_T$ , where op is the name of the predefined GraphBLAS operator used as the associative binary operation of the monoid and T is the domain (type) of the monoid.

#### 3.4.3 Semirings

- A GraphBLAS semiring  $S = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$  is defined by three domains  $D_{out}, D_{in_1}$ , and  $D_{in_2}$ ; an associative<sup>1</sup> and commutative additive operation  $\oplus : D_{out} \times D_{out} \to D_{out}$ ; a multiplicative operation  $\otimes : D_{in_1} \times D_{in_2} \to D_{out}$ ; and an identity element  $0 \in D_{out}$ . For a given GraphBLAS semiring  $S = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$  we define  $\mathbf{D}_{in_1}(S) = D_{in_1}$ ,  $\mathbf{D}_{in_2}(S) = D_{in_2}$ ,  $\mathbf{D}_{out}(S) = 0$ .
- Let  $F = \langle D_{out}, D_{in_1}, D_{in_2}, \otimes \rangle$  be an operator and let  $A = \langle D_{out}, \oplus, 0 \rangle$  be a commutative monoid, then  $S = \langle A, F \rangle = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$  is a semiring.
- In a GraphBLAS semiring, the multiplicative operator does not have to distribute over the additive operator. This is unlike the conventional *semiring* algebraic structure.
- Note: There must be one GraphBLAS monoid in every semiring which serves as the semiring's additive operator and specifies the same domain for its inputs and output parameters. If this monoid is not a commutative monoid, the outcome of GraphBLAS operations using the semiring is undefined.
- A UML diagram of the conceptual hierarchy of object classes in GraphBLAS algebra (binary operators, monoids, and semirings) is shown in Figure 3.1.
- User-defined semirings can be created with calls to GrB\_Semiring\_new (see Section 4.2.2). A list of predefined true semirings and convenience semirings can be found in Tables 3.8 and 3.9, respectively.

  Predefined semirings are named GrB\_add\_mul\_SEMIRING\_T, where add is the semiring additive operation, mul is the semiring multiplicative operation and T is the domain (type) of the semiring.

<sup>&</sup>lt;sup>1</sup>It is expected that implementations of the GraphBLAS will utilize floating point arithmetic such as that defined in the IEEE-754 standard even though floating point arithmetic is not strictly associative.

Table 3.7: Predefined monoids for GraphBLAS in C. Maximum and minimum values for the various integral types are defined in  $\mathtt{stdint.h.}$  Floating-point infinities are defined in  $\mathtt{math.h.}$  The x in  $\mathsf{UINT}x$  or  $\mathsf{INT}x$  can be one of 8, 16, 32, or 64; whereas in  $\mathsf{FP}x$ , it can be 32 or 64.

| $\operatorname{GraphBLAS}$ | Domains, $T$         |                       |                         |
|----------------------------|----------------------|-----------------------|-------------------------|
| identifier                 | $(T \times T \to T)$ | Identity              | Description             |
| GrB_PLUS_MONOID_T          | UINTx                | 0                     | addition                |
|                            | INTx                 | 0                     |                         |
|                            | FPx                  | 0                     |                         |
| $GrB\_TIMES\_MONOID\_T$    | UINTx                | 1                     | multiplication          |
|                            | INTx                 | 1                     |                         |
|                            | FPx                  | 1                     |                         |
| $GrB \_MIN \_MONOID \_T$   | UINTx                | $UINTx\_\mathtt{MAX}$ | minimum                 |
|                            | INTx                 | $INTx_{MAX}$          |                         |
|                            | FPx                  | INFINITY              |                         |
| $GrB\_MAX\_MONOID\_T$      | UINTx                | 0                     | maximum                 |
|                            | INTx                 | $ $ INT $x$ _MIN      |                         |
|                            | FPx                  | -INFINITY             |                         |
|                            |                      |                       |                         |
| GrB_LOR_MONOID_BOOL        | BOOL                 | false                 | logical OR              |
| GrB_LAND_MONOID_BOOL       | BOOL                 | true                  | logical AND             |
| GrB_LXOR_MONOID_BOOL       | BOOL                 | false                 | logical XOR (not equal) |
| GrB_LXNOR_MONOID_BOOL      | BOOL                 | true                  | logical XNOR (equal)    |

Table 3.8: Predefined true semirings for GraphBLAS in C where the additive identity is the multiplicative annihilator. The x can be one of 8, 16, 32, or 64 in UINTx or INTx, and can be 32 or 64 in FPx.

|                                    | Domains, $T$           | + identity                     |                     |
|------------------------------------|------------------------|--------------------------------|---------------------|
| GraphBLAS identifier               | $ (T \times T \to T) $ | $\times$ annihilator           | Description         |
| $GrB\_PLUS\_TIMES\_SEMIRING\_T$    | UINTx                  | 0                              | arithmetic semiring |
|                                    | INTx                   | 0                              |                     |
|                                    | FPx                    | 0                              |                     |
| $GrB \_MIN \_PLUS \_SEMIRING \_T$  | $\bigcup UINT x$       | $\mathtt{UINT}x\_\mathtt{MAX}$ | min-plus semiring   |
|                                    | INTx                   | $\mathtt{INT}x\mathtt{\_MAX}$  |                     |
|                                    | FPx                    | INFINITY                       |                     |
| $GrB\_MAX\_PLUS\_SEMIRING\_T$      | INTx                   | $\mathtt{INT}x\mathtt{\_MIN}$  | max-plus semiring   |
|                                    | FPx                    | -INFINITY                      |                     |
| $GrB \_MIN \_TIMES \_SEMIRING \_T$ | $\bigcup UINT x$       | $\mathtt{UINT}x\_\mathtt{MAX}$ | min-times semiring  |
| $GrB \_MIN \_MAX \_SEMIRING \_T$   | $\bigcup UINT x$       | $\mathtt{UINT}x\_\mathtt{MAX}$ | min-max semiring    |
|                                    | INTx                   | $\mathtt{INT}x\mathtt{\_MAX}$  |                     |
|                                    | FPx                    | INFINITY                       |                     |
| $GrB\_MAX\_MIN\_SEMIRING\_T$       | $\bigcup UINT x$       | 0                              | max-min semiring    |
|                                    | INTx                   | $\mathtt{INT}x\mathtt{\_MIN}$  |                     |
|                                    | FPx                    | -INFINITY                      |                     |
| $GrB\_MAX\_TIMES\_SEMIRING\_T$     | $\bigcup UINT x$       | 0                              | max-times semiring  |
| $GrB\_PLUS\_MIN\_SEMIRING\_T$      | $\bigcup UINT x$       | 0                              | plus-min semiring   |
|                                    |                        |                                |                     |
| GrB_LOR_LAND_SEMIRING_BOOL         | BOOL                   | false                          | Logical semiring    |
| GrB_LAND_LOR_SEMIRING_BOOL         | BOOL                   | true                           | "and-or" semiring   |
| GrB_LXOR_LAND_SEMIRING_BOOL        | BOOL                   | false                          | same as NE_LAND     |
| GrB_LXNOR_LOR_SEMIRING_BOOL        | BOOL                   | true                           | same as EQ_LOR      |

Table 3.9: Other useful predefined semirings for GraphBLAS in C that don't have a multiplicative annihilator. The x can be one of 8, 16, 32, or 64 in UINTx or INTx, and can be 32 or 64 in FPx.

|                                     | Domains, $T$         |                                |                            |
|-------------------------------------|----------------------|--------------------------------|----------------------------|
| GraphBLAS identifier                | $(T \times T \to T)$ | + identity                     | Description                |
| GrB_MAX_PLUS_SEMIRING_T             | UINTx                | 0                              | max-plus semiring          |
| $GrB \_MIN \_TIMES \_SEMIRING \_T$  | INTx                 | $\mathtt{INT}x\mathtt{\_MAX}$  | min-times semiring         |
|                                     | FPx                  | INFINITY                       |                            |
| $GrB\_MAX\_TIMES\_SEMIRING\_T$      | INTx                 | $\mathtt{INT}x_{\mathtt{MIN}}$ | max-times semiring         |
|                                     | FPx                  | -INFINITY                      |                            |
| $GrB\_PLUS\_MIN\_SEMIRING\_T$       | INTx                 | 0                              | plus-min semiring          |
|                                     | FPx                  | 0                              |                            |
| $GrB \_MIN \_FIRST \_SEMIRING \_T$  | UINTx                | $\mathtt{UINT}x\_\mathtt{MAX}$ | min-select first semiring  |
|                                     | INTx                 | $\mathtt{INT}x\mathtt{\_MAX}$  |                            |
|                                     | FPx                  | INFINITY                       |                            |
| $GrB \_MIN \_SECOND \_SEMIRING \_T$ | UINTx                | $\mathtt{UINT}x\_\mathtt{MAX}$ | min-select second semiring |
|                                     | INTx                 | $\mathtt{INT}x\mathtt{\_MAX}$  |                            |
|                                     | FPx                  | INFINITY                       |                            |
| $GrB\_MAX\_FIRST\_SEMIRING\_T$      | UINTx                | 0                              | max-select first semiring  |
|                                     | INTx                 | $\mathtt{INT}x_{\mathtt{MIN}}$ |                            |
|                                     | FPx                  | -INFINITY                      |                            |
| $GrB\_MAX\_SECOND\_SEMIRING\_T$     | UINTx                | 0                              | max-select second semiring |
|                                     | INTx                 | $\mathtt{INT}x_{\mathtt{MIN}}$ |                            |
|                                     | FPx                  | -INFINITY                      |                            |

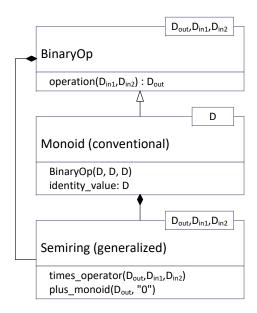


Figure 3.1: Hierarchy of algebraic object classes in GraphBLAS. GraphBLAS semirings consist of a conventional monoid with one domain for the addition function, and a binary operator with three domains for the multiplication function.

#### $_{903}$ 3.5 Collections

#### 904 **3.5.1** Scalars

A GraphBLAS scalar,  $s = \langle D, \{\sigma\} \rangle$ , is defined by a domain D, and a set of zero or one scalar value,  $\sigma$ , where  $\sigma \in D$ . We define  $\mathbf{size}(s) = 1$  (constant), and  $\mathbf{L}(s) = \{\sigma\}$ . The set  $\mathbf{L}(s)$  is called the contents of the GraphBLAS scalar s. We also define  $\mathbf{D}(s) = D$ . Finally,  $\mathbf{val}(s)$  is a reference to the scalar value,  $\sigma$ , if the GraphBLAS scalar is not empty, and is undefined otherwise.

#### 909 **3.5.2** Vectors

A vector  $\mathbf{v} = \langle D, N, \{(i, v_i)\} \rangle$  is defined by a domain D, a size N > 0, and a set of tuples  $(i, v_i)$  where  $0 \le i < N$  and  $v_i \in D$ . A particular value of i can appear at most once in  $\mathbf{v}$ . We define size( $\mathbf{v}$ ) = N and  $\mathbf{L}(\mathbf{v}) = \{(i, v_i)\}$ . The set  $\mathbf{L}(\mathbf{v})$  is called the *content* of vector  $\mathbf{v}$ . We also define the set  $\mathbf{ind}(\mathbf{v}) = \{i : (i, v_i) \in \mathbf{L}(\mathbf{v})\}$  (called the *structure* of  $\mathbf{v}$ ), and  $\mathbf{D}(\mathbf{v}) = D$ . For a vector  $\mathbf{v}$ ,  $\mathbf{v}(i)$  is a reference to  $v_i$  if  $(i, v_i) \in \mathbf{L}(\mathbf{v})$  and is undefined otherwise.

#### 915 3.5.3 Matrices

A matrix  $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$  is defined by a domain D, its number of rows M > 0, its 916 number of columns N > 0, and a set of tuples  $(i, j, A_{ij})$  where  $0 \le i < M$ ,  $0 \le j < N$ , and 917  $A_{ij} \in D$ . A particular pair of values i, j can appear at most once in **A**. We define  $\mathbf{ncols}(\mathbf{A}) = N$ , 918  $\mathbf{nrows}(\mathbf{A}) = M$ , and  $\mathbf{L}(\mathbf{A}) = \{(i, j, A_{ij})\}$ . The set  $\mathbf{L}(\mathbf{A})$  is called the *content* of matrix  $\mathbf{A}$ . We also 919 define the sets  $indrow(\mathbf{A}) = \{i : \exists (i, j, A_{ij}) \in \mathbf{A}\}$  and  $indcol(\mathbf{A}) = \{j : \exists (i, j, A_{ij}) \in \mathbf{A}\}$ . (These 920 are the sets of nonempty rows and columns of A, respectively.) The *structure* of matrix A is the 921 set  $ind(A) = \{(i,j) : (i,j,A_{ij}) \in L(A)\}, \text{ and } D(A) = D.$  For a matrix A, A(i,j) is a reference to  $A_{ij}$  if  $(i, j, A_{ij}) \in \mathbf{L}(\mathbf{A})$  and is undefined otherwise. 923 If **A** is a matrix and  $0 \leq j < N$ , then  $\mathbf{A}(:,j) = \langle D, M, \{(i,A_{ij}) : (i,j,A_{ij}) \in \mathbf{L}(\mathbf{A})\} \rangle$  is a 924 vector called the j-th column of A. Correspondingly, if A is a matrix and  $0 \le i < M$ , then 925  $\mathbf{A}(i,:) = \langle D, N, \{(j,A_{ij}): (i,j,A_{ij}) \in \mathbf{L}(\mathbf{A})\} \rangle$  is a vector called the *i*-th row of  $\mathbf{A}$ . Given a matrix  $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$ , its transpose is another matrix  $\mathbf{A}^T = \langle D, N, M, \{(j, i, A_{ij}) : A_{ij} :$ 927  $(i, j, A_{ij}) \in \mathbf{L}(\mathbf{A}) \} \rangle$ .

#### 9 3.5.3.1 External matrix formats

The specification also supports the export and import of matrices to/from a number of commonly used formats, such as COO, CSR, and CSC formats. When importing or exporting a matrix to or from a GraphBLAS object using GrB\_Matrix\_import (§ 4.2.5.17) or GrB\_Matrix\_export (§ 4.2.5.16), it is necessary to specify the data format for the matrix data external to GraphBLAS, which is being imported from or exported to. This non-opaque data format is specified using an argument of enumeration type GrB\_Format that is used to indicate one of a number of predefined formats. The predefined values of GrB\_Format are specified in Table 3.10. A precise definition of the non-opaque data formats can be found in Appendix B.

Table 3.10: GrB\_Format enumeration literals and corresponding values for matrix import and export methods.

| Symbol         | Value | Description   |
|----------------|-------|---|
| GrB_CSR_FORMAT | 0     | Specifies the compressed sparse row matrix format.    |
| GrB_CSC_FORMAT | 1     | Specifies the compressed sparse column matrix format. |
| GrB_COO_FORMAT | 2     | Specifies the sparse coordinate matrix format.        |

#### 938 **3.5.4** Masks

The GraphBLAS C API defines an opaque object called a *mask*. The mask is used to control how computed values are stored in the output from a method. The mask is an *internal* opaque object; that is, it is never exposed as a variable within an application.

The mask is formed from input objects to the method that uses the mask. For example, a Graph-BLAS method may be called with a matrix as the mask parameter. The internal mask object is constructed from the input matrix in one of two ways. In the default case, an element of the mask is created for each tuple that exists in the matrix for which the value of the tuple cast to Boolean evaluates to true. Alternatively, the user can specify *structure*-only behavior where an element of the mask is created for each tuple that exists in the matrix *regardless* of the value stored in the input matrix.

The internal mask object can be either a one- or a two-dimensional construct. One- and twodimensional masks, described more formally below, are similar to vectors and matrices, respectively, except that they have structure (indices) but no values. When needed, a value is implied for the elements of a mask with an implied value of true for elements that exist and an implied value of false for elements that do not exist (i.e., the locations of the mask that do not have a stored value imply a value of false). Hence, even though a mask does not contain any values, it can be considered to imply values from a Boolean domain.

A one-dimensional mask  $\mathbf{m} = \langle N, \{i\} \rangle$  is defined by its number of elements N > 0, and a set  $\mathbf{ind}(\mathbf{m})$  of indices  $\{i\}$  where  $0 \le i < N$ . A particular value of i can appear at most once in  $\mathbf{m}$ . We define  $\mathbf{size}(\mathbf{m}) = N$ . The set  $\mathbf{ind}(\mathbf{m})$  is called the *structure* of mask  $\mathbf{m}$ .

A two-dimensional mask  $\mathbf{M} = \langle M, N, \{(i,j)\} \rangle$  is defined by its number of rows M > 0, its number of columns N > 0, and a set  $\mathbf{ind}(\mathbf{M})$  of tuples (i,j) where  $0 \le i < M$ ,  $0 \le j < N$ . A particular pair of values i,j can appear at most once in  $\mathbf{M}$ . We define  $\mathbf{ncols}(\mathbf{M}) = N$ , and  $\mathbf{nrows}(\mathbf{M}) = M$ . We also define the sets  $\mathbf{indrow}(\mathbf{M}) = \{i : \exists (i,j) \in \mathbf{ind}(\mathbf{M})\}$  and  $\mathbf{indcol}(\mathbf{M}) = \{j : \exists (i,j) \in \mathbf{ind}(\mathbf{M})\}$ . These are the sets of nonempty rows and columns of  $\mathbf{M}$ , respectively. The set  $\mathbf{ind}(\mathbf{M})$  is called the structure of mask  $\mathbf{M}$ .

One common operation on masks is the *complement*. For a one-dimensional mask  $\mathbf{m}$  this is denoted as  $\neg \mathbf{m}$ . For a two-dimensional mask  $\mathbf{M}$ , this is denoted as  $\neg \mathbf{M}$ . The complement of a one-dimensional mask  $\mathbf{m}$  is defined as  $\mathbf{ind}(\neg \mathbf{m}) = \{i : 0 \le i < N, i \notin \mathbf{ind}(\mathbf{m})\}$ . It is the set of all possible indices that do not appear in  $\mathbf{m}$ . The complement of a two-dimensional mask  $\mathbf{M}$  is defined as the set  $\mathbf{ind}(\neg \mathbf{M}) = \{(i,j) : 0 \le i < M, 0 \le j < N, (i,j) \notin \mathbf{ind}(\mathbf{M})\}$ . It is the set of all possible indices that do not appear in  $\mathbf{M}$ .

## 971 3.6 Descriptors

Descriptors are used to modify the behavior of a GraphBLAS method. When present in the signature of a method, they appear as the last argument in the method. Descriptors specify how the other input arguments corresponding to GraphBLAS collections – vectors, matrices, and masks – should be processed (modified) before the main operation of a method is performed. A complete list of what descriptors are capable of are presented in this section.

The descriptor is a lightweight object. It is composed of (*field*, *value*) pairs where the *field* selects one of the GraphBLAS objects from the argument list of a method and the *value* defines the indicated modification associated with that object. For example, a descriptor may specify that a particular input matrix needs to be transposed or that a mask needs to be complemented (defined in Section 3.5.4) before using it in the operation.

For the purpose of constructing descriptors, the arguments of a method that can be modified

are identified by specific field names. The output parameter (typically the first parameter in a GraphBLAS method) is indicated by the field name, GrB\_OUTP. The mask is indicated by the 984 GrB\_MASK field name. The input parameters corresponding to the input vectors and matrices are 985 indicated by GrB INP0 and GrB INP1 in the order they appear in the signature of the GraphBLAS 986 method. The descriptor is an opaque object and hence we do not define how objects of this type 987 should be implemented. When referring to (field, value) pairs for a descriptor, however, we often use 988 the informal notation desc[GrB Desc Field]. GrB Desc Value without implying that a descriptor is 989 to be implemented as an array of structures (in fact, field values can be used in conjunction with 990 multiple values that are composable). We summarize all types, field names, and values used with 991 descriptors in Table 3.11. 992

In the definitions of the GraphBLAS methods, we often refer to the *default behavior* of a method with respect to the action of a descriptor. If a descriptor is not provided or if the value associated with a particular field in a descriptor is not set, the default behavior of a GraphBLAS method is defined as follows:

- Input matrices are not transposed.
- The mask is used, as is, without complementing, and stored values are examined to determine whether they evaluate to true or false.
  - Values of the output object that are not directly modified by the operation are preserved.

GraphBLAS specifies all of the valid combinations of (field, value) pairs as predefined descriptors.

Their identifiers and the corresponding set of (field, value) pairs for that identifier are shown in

Table 3.12.

#### 3.7 Fields

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All GraphBLAS objects and implementations contain fields like those in the descriptor, which provide information to users and allow setting runtime parameters and hints. All GraphBLAS objects are required to implement the GrB\_get and GrB\_set methods required to query and set these fields. The library itself also contains several (*field*, *value*) pairs, which provide defaults to object level fields, and implementation information such as the version number or implementation name.

The required *value*, *field* pairs available for each object are defined in 3.13. Implementations may add their own custom GrB\_Field enum values to extend the behavior of objects and methods. A field must always be readable, but in many cases may not be writable. Such read-only fields might contain static, compile-time information such as GrB\_API\_VER, while others are determined by other operations, such as GrB\_BLOCKING\_MODE which is determined by GrB\_Init.

1016 GrB\_INVALID\_VALUE must be returned when attempting to write to fields which are read only.

The GrB\_Field enumeration is defined by the values in Table 3.13, and selected values are described in Table 3.14.

Table 3.11: Descriptors are GraphBLAS objects passed as arguments to GraphBLAS operations to modify other GraphBLAS objects in the operation's argument list. A descriptor, desc, has one or more (*field*, *value*) pairs indicated as desc[GrB\_Desc\_Field].GrB\_Desc\_Value. In this table, we define all types and literals used with descriptors.

#### (a) Types used with GraphBLAS descriptors.

| Type           | Description                            |
|----------------|--|
| GrB_Descriptor | Type of a GraphBLAS descriptor object. |
| GrB_Desc_Field | The descriptor field enumeration.      |
| GrB_Desc_Value | The descriptor value enumeration.      |

(b) Descriptor field names of type GrB\_Desc\_Field enumeration and corresponding values.

| Field Name | Value | Description                                       |
|------------|-------|---|
| GrB_OUTP   | 0     | Field name for the output GraphBLAS object.       |
| GrB_MASK   | 1     | Field name for the mask GraphBLAS object.         |
| GrB_INP0   | 2     | Field name for the first input GraphBLAS object.  |
| GrB_INP1   | 3     | Field name for the second input GraphBLAS object. |

(c) Descriptor field values of type GrB\_Desc\_Value enumeration and corresponding values.

| Value Name    | Value | Description   |
|---------------|-------|---|
| (reserved)    | 0     | Unused  |
| GrB_REPLACE   | 1     | Clear the output object before assigning computed values.   |
| GrB_COMP      | 2     | Use the complement of the associated object. When combined with GrB_STRUCTURE, the complement of the structure of the associated object is used without evaluating the values stored. |
| GrB_TRAN      | 3     | Use the transpose of the associated object.   |
| GrB_STRUCTURE | 4     | The write mask is constructed from the structure (pattern of stored values) of the associated object. The stored values are not examined.   |

Table 3.12: Predefined GraphBLAS descriptors. The list includes all possible descriptors, according to the current standard. Columns list the possible fields and entries list the value(s) associated with those fields for a given descriptor.

| Identifier       | GrB_OUTP    | GrB_MASK                | GrB_INP0     | GrB_INP1    |
|------------------|-------------|-------------------------|--------------|-------------|
| GrB_NULL         | _           | _                       | _            | _           |
| GrB_DESC_T1      | _           | _                       | _            | GrB_TRAN    |
| GrB_DESC_T0      | _           | _                       | $GrB\_TRAN$  | _           |
| GrB_DESC_T0T1    | _           | _                       | $GrB\_TRAN$  | GrB_TRAN    |
| GrB_DESC_C       | _           | GrB_COMP                | _            | _           |
| GrB_DESC_S       | _           | GrB_STRUCTURE           | _            | _           |
| GrB_DESC_CT1     | _           | GrB_COMP                | _            | GrB_TRAN    |
| GrB_DESC_ST1     | _           | GrB_STRUCTURE           | _            | GrB_TRAN    |
| GrB_DESC_CT0     | _           | GrB_COMP                | $GrB\_TRAN$  | _           |
| GrB_DESC_ST0     | _           | GrB_STRUCTURE           | $GrB\_TRAN$  | _           |
| GrB_DESC_CT0T1   | _           | GrB_COMP                | $GrB \_TRAN$ | GrB_TRAN    |
| GrB_DESC_ST0T1   | _           | GrB_STRUCTURE           | $GrB\_TRAN$  | $GrB\_TRAN$ |
| GrB_DESC_SC      | _           | GrB_STRUCTURE, GrB_COMP | _            | _           |
| GrB_DESC_SCT1    | _           | GrB_STRUCTURE, GrB_COMP | _            | GrB_TRAN    |
| GrB_DESC_SCT0    | _           | GrB_STRUCTURE, GrB_COMP | $GrB\_TRAN$  | _           |
| GrB_DESC_SCT0T1  | _           | GrB_STRUCTURE, GrB_COMP | $GrB\_TRAN$  | $GrB\_TRAN$ |
| GrB_DESC_R       | GrB_REPLACE | _                       | _            | _           |
| GrB_DESC_RT1     | GrB_REPLACE | _                       | _            | $GrB\_TRAN$ |
| GrB_DESC_RT0     | GrB_REPLACE | _                       | $GrB\_TRAN$  | _           |
| GrB_DESC_RT0T1   | GrB_REPLACE | _                       | $GrB \_TRAN$ | GrB_TRAN    |
| GrB_DESC_RC      | GrB_REPLACE | GrB_COMP                | _            | _           |
| GrB_DESC_RS      | GrB_REPLACE | GrB_STRUCTURE           | _            | _           |
| GrB_DESC_RCT1    | GrB_REPLACE | GrB_COMP                | _            | $GrB\_TRAN$ |
| GrB_DESC_RST1    | GrB_REPLACE | GrB_STRUCTURE           | _            | GrB_TRAN    |
| GrB_DESC_RCT0    | GrB_REPLACE | GrB_COMP                | $GrB \_TRAN$ | _           |
| GrB_DESC_RST0    | GrB_REPLACE | GrB_STRUCTURE           | GrB_TRAN     | _           |
| GrB_DESC_RCT0T1  | GrB_REPLACE | GrB_COMP                | $GrB \_TRAN$ | GrB_TRAN    |
| GrB_DESC_RST0T1  | GrB_REPLACE | GrB_STRUCTURE           | GrB_TRAN     | GrB_TRAN    |
| GrB_DESC_RSC     | GrB_REPLACE | GrB_STRUCTURE, GrB_COMP | _            | _           |
| GrB_DESC_RSCT1   | GrB_REPLACE | GrB_STRUCTURE, GrB_COMP | _            | GrB_TRAN    |
| GrB_DESC_RSCT0   | GrB_REPLACE | GrB_STRUCTURE, GrB_COMP | GrB_TRAN     | _           |
| GrB_DESC_RSCT0T1 | GrB_REPLACE | GrB_STRUCTURE, GrB_COMP | GrB_TRAN     | $GrB\_TRAN$ |
|                  | ,           |                         |              |             |

#### 1019 **3.7.1** Input Types

Allowable types used in GrB\_get and GrB\_set are ENUM, GrB\_Scalar, char\*, and void\*. Each GrB\_Field is associated with exactly one of these types as defined in Table 3.13. Implementations that add additional GrB\_Fields must document the type associated with each GrB\_Field.

#### 1023 3.7.1.1 ENUM Handling

ENUM types use standard int enumerations defined in C. User code should use the enum name rather than the integer value directly when getting or setting a field value.

#### 1026 3.7.1.2 GrB\_Scalar Handling

When calling GrB\_get, the user must provide an already initialized GrB\_Scalar object to which the implementation will write a value of the correct element type. When calling GrB\_set, the GrB\_Scalar must not be empty, otherwise a GrB\_EMPTY\_OBJECT error is raised.

#### 3.7.1.3 String (char\*) Handling

When the input to GrB\_set is a char\* the input array is null terminated. The GraphBLAS implementation must copy this array into internal data structures. Using GrB\_get for strings requires two calls. First, call GrB\_get with the field and object, but pass size\_t\* as the value argument. The implementation will return the size of the string buffer that the user must create. Second, call GrB\_get with the field and object, this time passing a pointer to the newly created string buffer. The GraphBLAS implementation will write to this buffer, including a trailing null terminator. The size returned in the first call will include enough bytes for the null terminator.

#### 1038 **3.7.1.4** void\* Handling

When the input to GrB set is a void\*, an extra int argument is passed to indicate the size of the 1039 buffer. The GraphBLAS implementation must copy this many bytes from the buffer into internal 1040 data structures. Similar to reading strings, GrB\_get must be called twice for void\*. The first call 1041 passes size\_t\* to find the required buffer size. The user must create a buffer and then pass the 1042 pointer to GrB get. The implementation will write to this buffer. No standard specification or 1043 protocol is required for the contents of void\*. It is meant to be a mechanism to allow full freedom 1044 for GraphBLAS implementations with needs that cannot be handled using ENUM, GrB\_Scalar, or 1045 Strings. 1046

#### 1047 3.7.2 Hints

Several fields are *hints* (marked H in Table 3.13). Hints are used to represent intended use cases or best guesses, but do not determine strict behavior. When GrB\_set is called with a hint, the

GraphBLAS implementation should return GrB\_SUCCESS, but is free to use or ignore the hint.
When GrB\_get is called, the implementation should return a best guess on the correct answer. If
there is no clear answer, the implementation should return GrB\_UNKNOWN.

#### 1053 **3.7.3** GrB\_NAME

The GrB\_NAME field is a special case regarding writability. All objects which have a GrB\_NAME field default to an empty string. Collections and GrB\_Descriptors may have their GrB\_NAME set at any time. User-defined algebraic objects and GrB\_Types may only have their GrB\_NAME set once to a globally unique value. Attempting to set this field after it has already been set will return a GrB\_ALREADY\_SET error code.

Built-in algebraic objects and GrB\_Types have names which can be read, but not written to. The name returned will be the string form of the GrB\_Type listed in Table 3.2 or the GraphBLAS identifier listed in Tables 3.5, 3.6, 3.7, 3.8, and 3.9. For example, the name of GrB\_INT32 type is "GrB\_INT32" (9 characters) and the name of GrB\_MIN\_FP64 binary op is "GrB\_MIN\_FP64" (12 characters).

| Field Name                   | W   | Н | Value | Implementing Objects | Type                    |
|------------------------------|---|---|-------|----------------------|-------------------------|
| GrB_OUTP                     | W   |   | 0     | GrB_Descriptor       | ENUM of GrB_Desc_Value  |
| GrB_MASK                     | W   |   | 1     | GrB_Descriptor       | ENUM of GrB_Desc_Value  |
| GrB_INP0                     | W   |   | 2     | GrB_Descriptor       | ENUM of GrB_Desc_Value  |
| GrB_INP1                     | W   | _ | 3     | GrB_Descriptor       | ENUM of GrB_Desc_Value  |
| GrB_NAME                     | *   |   | 10    | All                  | Null terminated char*   |
| GrB_LIBRARY_VER_MAJOR        | -   |   | 11    | Global               | $GrB\_Scalar~(INT32)$   |
| GrB_LIBRARY_VER_MINOR        | -   |   | 12    | Global               | $GrB\_Scalar~(INT32)$   |
| GrB_LIBRARY_VER_PATCH        | -   |   | 13    | Global               | $GrB\_Scalar (INT32)$   |
| GrB_API_VER_MAJOR            | l — i   |   | 14    | Global               | $GrB\_Scalar (INT32)$   |
| GrB_API_VER_MINOR            | l — i   |   | 15    | Global               | $GrB\_Scalar (INT32)$   |
| GrB_API_VER_PATCH            | l — i   |   | 16    | Global               | $GrB\_Scalar (INT32)$   |
| GrB_BLOCKING_MODE            | -   |   | 17    | Global               | ENUM of GrB_Mode        |
| GrB_STORAGE_ORIENTATION_HINT | W   | Н | 100   | Global, Collection   | ENUM of GrB_Orientation |
| GrB_ELTYPE_CODE              | -   | — | 102   | Collection           | ENUM of GrB_Type_Code   |
| GrB_INPUT1TYPE_CODE          |   |   | 103   | Algebraic            | ENUM of GrB_Type_Code   |
| GrB_INPUT2TYPE_CODE          | l — i   |   | 104   | Algebraic            | ENUM of GrB_Type_Code   |
| GrB_OUTPUTTYPE_CODE          | l — i   |   | 105   | Algebraic            | ENUM of GrB_Type_Code   |
| GrB_ELTYPE_STRING            | l — i   |   | 106   | Collection           | Null terminated char*   |
| GrB_INPUT1TYPE_STRING        | —   |   | 107   | Algebraic            | Null terminated char*   |
| GrB_INPUT2TYPE_STRING        | -   | _ | 108   | Algebraic            | Null terminated char*   |
| GrB_OUTPUTTYPE_STRING        | <u>  —                                   </u> | — | 109   | Algebraic            | Null terminated char*   |

| Table 3.14: Descriptions of select <i>field</i> , value pairs listed in 3.13 |   |  |  |
|--|---|--|--|
| Field Name   | Description   |  |  |
| GrB_NAME   | The name of any GraphBLAS object,                       |  |  |
|  | or the name of the library implementation.              |  |  |
| GrB_BLOCKING_MODE  | The blocking mode as set by GrB_init                    |  |  |
| GrB_STORAGE_ORIENTATION_HINT   | Hint to the library that a collection is best stored in |  |  |
|  | a row (lexicographic) or column (colexicographic) ma-   |  |  |
|  | jor format.   |  |  |
| GrB_ELTYPE_(CODE/STRING)   | The element type of a collection.                       |  |  |
| GrB_INPUT1TYPE_(CODE/STRING)   | The type of the first argument to an operator.          |  |  |
| GrB_INPUT2TYPE_(CODE/STRING)   | The type of the second argument to an operator.         |  |  |
| GrB_OUTPUTTYPE_(CODE/STRING)   | The type of the output of an operator.                  |  |  |

## 3.8 GrB\_Info return values

All GraphBLAS methods return a GrB\_Info enumeration value. The three types of return codes (informational, API error, and execution error) and their corresponding values are listed in Table 3.16.

Table 3.15: Enumerations not defined elsewhere in the documents and used when getting or setting fields are defined in the following tables.

## (a) Field values of type GrB\_Orientation.

| Value Name   | Value | Description  |
|--------------|-------|--|
| GrB_ROWMAJOR | 0     | The majority of iteration over the object will be row-wise.    |
| GrB_COLMAJOR | 1     | The majority of iteration over the object will be column-wise. |
| GrB_BOTH     | 2     | Iteration may occur with equal frequency in both directions.   |
| GrB_UNKNOWN  | 3     | No indication is given or is unknown.                          |
|              | ļi    |  |

Table 3.16: Enumeration literals and corresponding values returned by GraphBLAS methods and operations.

### (a) Informational return values

| Symbol       | Value | Description   |
|--------------|-------|---|
| GrB_SUCCESS  | 0     | The method/operation completed successfully (blocking mode), or       |
|              |       | encountered no API errors (non-blocking mode).                        |
| GrB_NO_VALUE | 1     | A location in a matrix or vector is being accessed that has no stored |
|              |       | value at the specified location.                                      |

## (b) API errors

| Symbol                   | Value | Description  |
|--------------------------|-------|--|
| GrB_UNINITIALIZED_OBJECT | -1    | A GraphBLAS object is passed to a method before      |
|                          |       | new was called on it.                                |
| GrB_NULL_POINTER         | -2    | A NULL is passed for a pointer parameter.            |
| GrB_INVALID_VALUE        | -3    | Miscellaneous incorrect values.                      |
| GrB_INVALID_INDEX        | -4    | Indices passed are larger than dimensions of the ma- |
|                          |       | trix or vector being accessed.                       |
| GrB_DOMAIN_MISMATCH      | -5    | A mismatch between domains of collections and op-    |
|                          |       | erations when user-defined domains are in use.       |
| GrB_DIMENSION_MISMATCH   | -6    | Operations on matrices and vectors with incompati-   |
|                          |       | ble dimensions.                                      |
| GrB_OUTPUT_NOT_EMPTY     | -7    | An attempt was made to build a matrix or vector      |
|                          |       | using an output object that already contains valid   |
|                          |       | tuples (elements).                                   |
| GrB_NOT_IMPLEMENTED      | -8    | An attempt was made to call a GraphBLAS method       |
|                          |       | for a combination of input parameters that is not    |
|                          |       | supported by a particular implementation.            |
| GrB_ALREADY_SET          | -9    | An attempt was made to write to a field which may    |
|                          |       | only be written to once.                             |

## (c) Execution errors

| Symbol                  | Value | Description  |
|-------------------------|-------|--|
| GrB_PANIC               | -101  | Unknown internal error.  |
| GrB_OUT_OF_MEMORY       | -102  | Not enough memory for operations.  |
| GrB_INSUFFICIENT_SPACE  | -103  | The array provided is not large enough to hold out-  |
| GrB_INVALID_OBJECT      | -104  | put. One of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous |
|                         |       | execution error.   |
| GrB_INDEX_OUT_OF_BOUNDS | -105  | Reference to a vector or matrix element that is outside the defined dimensions of the object.          |
| GrB_EMPTY_OBJECT        | -106  | One of the opaque GraphBLAS objects does not   |
|                         |       | have a stored value.   |

# Chapter 4

# Methods

This chapter defines the behavior of all the methods in the GraphBLAS C API. All methods can be declared for use in programs by including the GraphBLAS.h header file.

We would like to emphasize that no GraphBLAS method will imply a predefined order over any associative operators. Implementations of the GraphBLAS are encouraged to exploit associativity to optimize performance of any GraphBLAS method. This holds even if the definition of the GraphBLAS method implies a fixed order for the associative operations.

#### 6 4.1 Context methods

The methods in this section set up and tear down the GraphBLAS context within which all Graph-BLAS methods must be executed. The initialization of this context also includes the specification of which execution mode is to be used.

#### 1080 4.1.1 init: Initialize a GraphBLAS context

1081 Creates and initializes a GraphBLAS C API context.

#### 1082 C Syntax

GrB\_Info GrB\_init(GrB\_Mode mode);

#### Parameters

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mode Mode for the GraphBLAS context. Must be either GrB\_BLOCKING or GrB\_NONBLOCKING.

#### 1086 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

GrB\_INVALID\_VALUE invalid mode specified, or method called multiple times.

#### 1090 Description

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The init method creates and initializes a GraphBLAS C API context. The argument to GrB\_init defines the mode for the context. The two available modes are:

- GrB\_BLOCKING: In this mode, each method in a sequence returns after its computations have completed and output arguments are available to subsequent statements in an application. When executing in GrB\_BLOCKING mode, the methods execute in program order.
- GrB\_NONBLOCKING: In this mode, methods in a sequence may return after arguments in the method have been tested for dimension and domain compatibility within the method but potentially before their computations complete. Output arguments are available to subsequent GraphBLAS methods in an application. When executing in GrB\_NONBLOCKING mode, the methods in a sequence may execute in any order that preserves the mathematical result defined by the sequence.

An application can only create one context per execution instance. An application may only call GrB\_Init once. Calling GrB\_Init more than once results in undefined behavior.

#### 1104 4.1.2 finalize: Finalize a GraphBLAS context

1105 Terminates and frees any internal resources created to support the GraphBLAS C API context.

#### 1106 C Syntax

1110

```
GrB_Info GrB_finalize();
```

#### 1108 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

#### 1111 Description

The finalize method terminates and frees any internal resources created to support the GraphBLAS C API context. GrB\_finalize may only be called after a context has been initialized by calling GrB\_init, or else undefined behavior occurs. After GrB\_finalize has been called to finalize a Graph-BLAS context, calls to any GraphBLAS methods, including GrB\_finalize, will result in undefined behavior.

#### 1117 4.1.3 getVersion: Get the version number of the standard.

1118 Query the library for the version number of the standard that this library implements.

#### 1119 C Syntax

```
GrB_Info GrB_getVersion(unsigned int *version, unsigned int *subversion);
```

#### 1122 Parameters

version (OUT) On successful return will hold the value of the major version number.

version (OUT) On successful return will hold the value of the subversion number.

#### 125 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

#### 1128 Description

The getVersion method is used to query the major and minor version number of the GraphBLAS C API specification that the library implements at runtime. To support compile time queries the following two macros shall also be defined by the library.

```
#define GRB_VERSION 2
#define GRB_SUBVERSION 0
```

## 1134 4.2 Object methods

This section describes methods that setup and operate on GraphBLAS opaque objects but are not part of the the GraphBLAS math specification.

#### 1137 4.2.1 Get and Set methods

The methods in this section query and, optionally, set internal fields of GraphBLAS objects.

#### 1139 4.2.1.1 get: Query the value of an object

#### 1140 C Syntax

```
GrB Info GrB get(GrB < OBJ > o, <type > value, GrB Field field);
```

#### 1142 Parameters

- OBJ (IN) An existing, valid GraphBLAS object (collection, operation, type) which is being queried. To indicate the global context, the constant GrB\_Global is used.
- value (OUT) A pointer to or GrB\_Scalar containing a value whose type is dependent on field which will be filled with the current value of the field. type may be int\*, size\_t\*, GrB\_Scalar, char\* or void\*.
- field (IN) The field being queried.

#### 1149 Return Value

- GrB\_SUCCESS The method completed successfully.
- GrB\_PANIC unknown internal error.
- GrB\_OUT\_OF\_MEMORY not enough memory available for operation.
- GrB\_UNINITIALIZED\_OBJECT the value parameter is GrB\_Scalar and has not been initialized by a call to new.
- GrB\_INVALID\_VALUE invalid value type provided for the field or invalid field.

#### 1156 Description

Queries a field of an existing GraphBLAS object. The type of the argument is uniquely determined by field. For the case of char\* and void\*, the value can be replaced with size\_t\* to get the required buffer size to hold the response. Fields marked as hints in Table 3.13 will return a hint on how best to use the object.

#### 1161 4.2.1.2 set: Set field of an object

Set the content for a field for an existing GraphBLAS object.

#### 1163 C Syntax

```
GrB_Info GrB_set(GrB_<OBJ> o, <type> value, GrB_Field field);
GrB_Info GrB_set(GrB_<OBJ> o, <type> value, GrB_Field field, size_t voidSize);
```

#### 1166 Parameters

1171

OBJ (IN) The GraphBLAS object which is having field set. To indicate the global context, the constant GrB\_Global is used.

value (IN) A value whose type is dependent on field. type may be a int,

GrB\_Scalar, char\* or void\*.

field (IN) The field being set.

voidSize (IN) The size of the void\* buffer. Note that a size is not needed for char\* because the string is assumed null-terminated.

#### 1174 Return Values

GrB\_SUCCESS The method completed successfully.

GrB\_PANIC unknown internal error.

Grb Out Of Memory not enough memory available for operation.

1178 GrB\_UNINITIALIZED\_OBJECT the GrB\_Scalar parameter has not been initialized by a call to new.

Grb\_INVALID\_VALUE invalid value set on the field, invalid field, or field is read-only.

GrB\_ALREADY\_SET this field has already been set, and may only be set once.

#### 1181 Description

1182 Set a field of OBJ or the Global context to a new value.

#### 1183 4.2.2 Algebra methods

#### 1184 4.2.2.1 Type\_new: Construct a new GraphBLAS (user-defined) type

Creates a new user-defined GraphBLAS type. This type can then be used to create new operators, monoids, semirings, vectors and matrices.

#### C Syntax

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```
GrB_Info GrB_Type_new(GrB_Type *utype,
size_t sizeof(ctype));
```

#### 1190 Parameters

utype (INOUT) On successful return, contains a handle to the newly created user-defined GraphBLAS type object.

ctype (IN) A C type that defines the new GraphBLAS user-defined type.

#### 1194 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

GrB\_NULL\_POINTER utype pointer is NULL.

#### 1199 Description

Given a C type ctype, the Type\_new method returns in utype a handle to a new GraphBLAS type that is equivalent to the C type. Variables of this ctype must be a struct, union, or fixed-size array.

In particular, given two variables, src and dst, of type ctype, the following operation must be a valid way to copy the contents of src to dst:

```
memcpy(&dst, &src, sizeof(ctype))
```

A new, user-defined type utype should be destroyed with a call to GrB\_free(utype) when no longer needed.

1207 It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

#### 1209 4.2.2.2 UnaryOp\_new: Construct a new GraphBLAS unary operator

Initializes a new GraphBLAS unary operator with a specified user-defined function and its types (domains).

#### 1212 C Syntax

```
GrB_Info GrB_UnaryOp_new(GrB_UnaryOp *unary_op,

void (*unary_func)(void*, const void*),

GrB_Type d_out,

GrB_Type d_in);
```

#### 1217 Parameters

unary\_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS unary operator object.

unary\_func (IN) a pointer to a user-defined function that takes one input parameter of d\_in's type and returns a value of d\_out's type, both passed as void pointers. Specifically the signature of the function is expected to be of the form:

void func(void \*out, const void \*in);

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d\_out (IN) The GrB\_Type of the return value of the unary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined Graph-BLAS type.

d\_in (IN) The GrB\_Type of the input argument of the unary operator being created.
Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined
GraphBLAS type.

#### 1231 Return Values

GrB\_SUCCESS operation completed successfully.

1233 GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

1235 GrB\_UNINITIALIZED\_OBJECT any GrB\_Type parameter (for user-defined types) has not been initialized by a call to GrB\_Type\_new.

GrB\_NULL\_POINTER unary\_op or unary\_func pointers are NULL.

#### 1238 Description

The UnaryOp\_new method creates a new GraphBLAS unary operator

$$f_u = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in}), \mathsf{unary\_func} \rangle$$

```
and returns a handle to it in unary_op.
```

The implementation of unary\_func must be such that it works even if the d\_out and d\_in arguments are aliased. In other words, for all invocations of the function:

```
unary_func(out,in);
```

the value of out must be the same as if the following code was executed:

```
\begin{array}{lll} & \mathbf{D}(d_{in}) * \texttt{tmp} = \texttt{malloc(sizeof(D(d_{in})));} \\ & \texttt{1247} & \texttt{memcpy(tmp,in,sizeof(D(d_{in})));} \\ & \texttt{1248} & \texttt{unary\_func(out,tmp);} \\ & \texttt{free(tmp);} \end{array}
```

1250 It is not an error to call this method more than once on the same variable; however, the handle to 1251 the previously created object will be overwritten.

#### 4.2.2.3 BinaryOp\_new: Construct a new GraphBLAS binary operator

Initializes a new GraphBLAS binary operator with a specified user-defined function and its types (domains).

#### 1255 C Syntax

```
1256
             GrB_Info GrB_BinaryOp_new(GrB_BinaryOp *binary_op,
                                            void
                                                          (*binary_func)(void*,
1257
                                                                           const void*,
1258
                                                                           const void*),
1259
                                            GrB_Type
                                                            d_out,
1260
                                            GrB_Type
                                                            d_in1,
1261
                                            GrB_Type
                                                            d_in2);
1262
```

#### Parameters

binary\_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS binary operator object.

binary\_func (IN) A pointer to a user-defined function that takes two input parameters of types
d\_in1 and d\_in2 and returns a value of type d\_out, all passed as void pointers.

Specifically the signature of the function is expected to be of the form:

```
void func(void *out, const void *in1, const void *in2);
```

1270

1263

```
d_out (IN) The GrB_Type of the return value of the binary operator being created. Should
be one of the predefined GraphBLAS types in Table 3.2, or a user-defined Graph-
BLAS type.
```

- d\_in1 (IN) The GrB\_Type of the left hand argument of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.
- d\_in2 (IN) The GrB\_Type of the right hand argument of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

#### 1280 Return Values

1274

1275

1276

1277

1278

1279

```
GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT the GrB_Type (for user-defined types) has not been initialized by a call to GrB_Type_new.
```

GrB\_NULL\_POINTER binary\_op or binary\_func pointer is NULL.

#### 1287 Description

1288 The BinaryOp\_new methods creates a new GraphBLAS binary operator

```
f_b = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in1}), \mathbf{D}(\mathsf{d\_in2}), \mathsf{binary\_func} \rangle
```

and returns a handle to it in binary\_op.

The implementation of binary\_func must be such that it works even if any of the d\_out, d\_in1, and d\_in2 arguments are aliased to each other. In other words, for all invocations of the function:

```
binary_func(out,in1,in2);
```

the value of out must be the same as if the following code was executed:

```
\begin{array}{lll} & \mathbf{D}(\texttt{d\_in1}) * \texttt{tmp1} = \texttt{malloc(sizeof}(\mathbf{D}(\texttt{d\_in1}))); \\ & \mathbf{D}(\texttt{d\_in2}) * \texttt{tmp2} = \texttt{malloc(sizeof}(\mathbf{D}(\texttt{d\_in2}))); \\ & \texttt{percept}(\texttt{tmp1}, \texttt{in1}, \texttt{sizeof}(\mathbf{D}(\texttt{d\_in1}))); \\ & \texttt{memcpy}(\texttt{tmp2}, \texttt{in2}, \texttt{sizeof}(\mathbf{D}(\texttt{d\_in2}))); \\ & \texttt{binary\_func(out, tmp1}, \texttt{tmp2}); \\ & \texttt{free}(\texttt{tmp2}); \\ & \texttt{free}(\texttt{tmp1}); \\ \end{array}
```

1302 It is not an error to call this method more than once on the same variable; however, the handle to 1303 the previously created object will be overwritten.

#### 1304 4.2.2.4 Monoid\_new: Construct a new GraphBLAS monoid

1305 Creates a new monoid with specified binary operator and identity value.

#### 1306 C Syntax

```
GrB_Info GrB_Monoid_new(GrB_Monoid *monoid,
GrB_BinaryOp binary_op,
<type> identity);
```

#### 1310 Parameters

- monoid (INOUT) On successful return, contains a handle to the newly created GraphBLAS monoid object.
- binary\_op (IN) An existing GraphBLAS associative binary operator whose input and output types are the same.
- identity (IN) The value of the identity element of the monoid. Must be the same type as
  the type used by the binary\_op operator.

#### 1317 Return Values

- GrB\_SUCCESS operation completed successfully.
- GrB\_PANIC unknown internal error.
- GrB\_OUT\_OF\_MEMORY not enough memory available for operation.
- GrB\_UNINITIALIZED\_OBJECT the GrB\_BinaryOp (for user-defined operators) has not been initialized by a call to GrB\_BinaryOp\_new.
- GrB\_NULL\_POINTER monoid pointer is NULL.
- GrB\_DOMAIN\_MISMATCH all three argument types of the binary operator and the type of the identity value are not the same.

#### 1326 Description

The Monoid\_new method creates a new monoid  $M = \langle \mathbf{D}(\mathsf{binary\_op}), \mathsf{binary\_op}, \mathsf{identity} \rangle$  and returns a handle to it in monoid.

If binary\_op is not associative, the results of GraphBLAS operations that require associativity of this monoid will be undefined.

1331 It is not an error to call this method more than once on the same variable; however, the handle to 1332 the previously created object will be overwritten.

#### 1333 4.2.2.5 Semiring\_new: Construct a new GraphBLAS semiring

1334 Creates a new semiring with specified domain, operators, and elements.

#### 1335 C Syntax

```
GrB_Info GrB_Semiring_new(GrB_Semiring *semiring,
GrB_Monoid add_op,
GrB_BinaryOp mul_op);
```

#### 339 Parameters

- semiring (INOUT) On successful return, contains a handle to the newly created GraphBLAS semiring.
- add\_op (IN) An existing GraphBLAS commutative monoid that specifies the addition operator and its identity.
- mul\_op (IN) An existing GraphBLAS binary operator that specifies the semiring's multiplication operator. In addition, mul\_op's output domain,  $\mathbf{D}_{out}(\mathsf{mul\_op})$ , must be the same as the add\_op's domain  $\mathbf{D}(\mathsf{add\_op})$ .

#### 347 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for this method to complete.

GrB\_UNINITIALIZED\_OBJECT the add\_op (for user-define monoids) object has not been initialized with a call to GrB\_Monoid\_new or the mul\_op (for user-defined operators) object has not been initialized by a call to GrB BinaryOp new.

GrB\_NULL\_POINTER semiring pointer is NULL.

GrB\_DOMAIN\_MISMATCH the output domain of mul\_op does not match the domain of the add\_op monoid.

#### 1358 Description

1359 The Semiring\_new method creates a new semiring:

```
S = \langle \mathbf{D}_{out}(\mathsf{mul\_op}), \mathbf{D}_{in_1}(\mathsf{mul\_op}), \mathbf{D}_{in_2}(\mathsf{mul\_op}), \mathsf{add\_op}, \mathsf{mul\_op}, \mathbf{0}(\mathsf{add\_op}) \rangle
```

and returns a handle to it in semiring. Note that  $\mathbf{D}_{out}(\mathsf{mul} \ \mathsf{op})$  must be the same as  $\mathbf{D}(\mathsf{add} \ \mathsf{op})$ .

1362 If add\_op is not commutative, then GraphBLAS operations using this semiring will be undefined.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

# 1365 4.2.2.6 IndexUnaryOp\_new: Construct a new GraphBLAS index unary operator [Scott: NEW CONTENT]

Initializes a new GraphBLAS index unary operator with a specified user-defined function and its types (domains).

#### 1369 C Syntax

```
GrB_Info GrB_IndexUnaryOp_new(GrB_IndexUnaryOp
                                                                  *index_unary_op,
1370
                                            void (*index unary func)(void*,
1371
                                                                          const void*,
1372
                                                                         GrB Index,
1373
                                                                         GrB_Index,
1374
                                                                         const void*),
1375
                                            GrB_Type
                                                                    d_out,
1376
                                            GrB_Type
                                                                    d_in1,
1377
                                            GrB_Type
                                                                    d_in2);
1378
```

#### 1379 Parameters

index\_unary\_op (INOUT) On successful return, contains a handle to the newly created Graph-BLAS index unary operator object.

index\_unary\_func (IN) A pointer to a user-defined function that takes input parameters of types d\_in1, GrB\_Index, GrB\_Index and d\_in2 and returns a value of type d\_out. Except for the GrB\_Index parameters, all are passed as void pointers. Specifically the signature of the function is expected to be of the form:

```
void func(void *out,
const void *in1,
GrB_Index row_index,
GrB_Index col_index,
```

```
const void *in2);
1390
1391
                d_out (IN) The GrB_Type of the return value of the index unary operator being created.
1392
                       Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined
1393
                       GraphBLAS type.
1394
                d in (IN) The GrB Type of the first input argument of the index unary operator being
1395
                       created and corresponds to the stored values of the GrB_Vector or GrB_Matrix
1396
                       being operated on. Should be one of the predefined GraphBLAS types in Ta-
1397
                       ble 3.2, or a user-defined GraphBLAS type.
1398
                d_in2 (IN) The GrB_Type of the last input argument of the index unary operator be-
1399
                       ing created and corresponds to a scalar provided by the GraphBLAS operation
1400
                       that uses this operator. Should be one of the predefined GraphBLAS types in
1401
                       Table 3.2, or a user-defined GraphBLAS type.
1402
    Return Values
1403
                     GrB_SUCCESS operation completed successfully.
1404
                        GrB_PANIC unknown internal error.
1405
         GrB OUT OF MEMORY not enough memory available for operation.
1406
   GrB_UNINITIALIZED_OBJECT the GrB_Type (for user-defined types) has not been initialized by a
1407
                                     call to GrB Type new.
1408
             GrB_NULL_POINTER index_unary_op or index_unary_func pointer is NULL.
1409
    Description
1410
     The IndexUnaryOp_new methods creates a new GraphBLAS index unary operator
1411
          f_i = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in1}), \mathbf{D}(\mathsf{GrB\_Index}), \mathbf{D}(\mathsf{GrB\_Index}), \mathbf{D}(\mathsf{d\_in2}), \mathsf{index\_unary\_func} \rangle
1412
    and returns a handle to it in index unary op.
1413
    The implementation of index unary func must be such that it works even if any of the d out,
    d in1, and d in2 arguments are aliased to each other. In other words, for all invocations of the
1415
    function:
1416
           index_unary_func(out,in1,row_index,col_index,n,in2);
1417
    the value of out must be the same as if the following code was executed (shown here for matrices):
1418
```

```
GrB_Index row_index = ...;
1419
         GrB_Index col_index = ...;
1420
         D(d_{in1}) *tmp1 = malloc(sizeof(D(d_{in1})));
1421
         D(d_{in2}) *tmp2 = malloc(sizeof(D(d_{in2})));
1422
         memcpy(tmp1,in1,sizeof(D(d in1)));
1423
         memcpy(tmp2,in2,sizeof(D(d_in2)));
1424
         index unary func(out,tmp1,row index,col index,tmp2);
1425
         free(tmp2);
1426
         free(tmp1);
1427
```

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

#### 1430 4.2.3 Scalar methods

#### 1431 4.2.3.1 Scalar\_new: Construct a new scalar

1432 Creates a new empty scalar with specified domain.

#### 1433 C Syntax

```
GrB_Info GrB_Scalar_new(GrB_Scalar *s,

GrB_Type d);
```

#### 1436 Parameters

- s (INOUT) On successful return, contains a handle to the newly created GraphBLAS scalar.
- d (IN) The type corresponding to the domain of the scalar being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.

#### 1442 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output scalar s is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused

by a previous execution error. Call GrB\_error() to access any error 1450 messages generated by the implementation. 1451 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 1452 GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new 1453 (needed for user-defined types). 1454 GrB\_NULL\_POINTER The s pointer is NULL. 1455 Description 1456 Creates a new GraphBLAS scalar s of domain  $\mathbf{D}(\mathsf{d})$  and empty  $\mathbf{L}(s)$ . The method returns a handle 1457 to the new scalar in s. It is not an error to call this method more than once on the same variable; however, the handle to 1459 the previously created object will be overwritten. 1460 4.2.3.2Scalar\_dup: Construct a copy of a GraphBLAS scalar 1461 Creates a new scalar with the same domain and contents as another scalar. C Syntax 1463 GrB\_Info GrB\_Scalar\_dup(GrB\_Scalar \*t, const GrB\_Scalar s);

```
1464
```

#### **Parameters** 1466

- t (INOUT) On successful return, contains a handle to the newly created GraphBLAS 1467 scalar. 1468
- s (IN) The GraphBLAS scalar to be duplicated. 1469

#### **Return Values** 1470

1477

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-1471 blocking mode, this indicates that the API checks for the input 1472 arguments passed successfully. Either way, output scalar t is ready 1473 to be used in the next method of the sequence. 1474 GrB\_PANIC Unknown internal error. 1475 GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque 1476

GraphBLAS objects (input or output) is in an invalid state caused

by a previous execution error. Call GrB\_error() to access any error 1478 messages generated by the implementation. 1479 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 1480 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to 1481 Scalar\_new or Scalar\_dup. 1482 GrB\_NULL\_POINTER The t pointer is NULL. 1483 Description 1484 Creates a new scalar t of domain  $\mathbf{D}(s)$  and contents  $\mathbf{L}(s)$ . The method returns a handle to the new 1485 scalar in t. It is not an error to call this method more than once with the same output variable; however, the 1487 handle to the previously created object will be overwritten. 1488 4.2.3.3Scalar\_clear: Clear/remove a stored value from a scalar 1489 Removes the stored value from a scalar. 1490 C Syntax 1491 GrB\_Info GrB\_Scalar\_clear(GrB\_Scalar s); 1492 **Parameters** s (INOUT) An existing GraphBLAS scalar to clear. 1494 Return Values 1495 GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-1496 blocking mode, this indicates that the API checks for the input 1497 arguments passed successfully. Either way, output scalar s is ready 1498 to be used in the next method of the sequence. 1499 GrB\_PANIC Unknown internal error. 1500 GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque 1501 GraphBLAS objects (input or output) is in an invalid state caused 1502 by a previous execution error. Call GrB\_error() to access any error 1503 messages generated by the implementation. 1504

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

1505

1506 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar\_new or Scalar\_dup.

# 1508 Description

Removes the stored value from an existing scalar. After the call, L(s) is empty. The size of the scalar does not change.

# 4.2.3.4 Scalar\_nvals: Number of stored elements in a scalar

Retrieve the number of stored elements in a scalar (either zero or one).

# 1513 C Syntax

```
GrB_Info GrB_Scalar_nvals(GrB_Index *nvals, const GrB Scalar s);
```

#### 1516 Parameters

1519

nvals (OUT) On successful return, this is set to the number of stored elements in the scalar (zero or one).

s (IN) An existing GraphBLAS scalar being queried.

# 1520 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

1523 GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar\_new or Scalar\_dup.

GrB\_NULL\_POINTER The nvals pointer is NULL.

Return nvals(s) in nvals. This is the number of stored elements in scalar s, which is the size of L(s), and can only be either zero or one (see Section 3.5.1).

# 4.2.3.5 Scalar\_setElement: Set the single element in a scalar

Set the single element of a scalar to a given value.

# 1537 C Syntax

```
GrB_Info GrB_Scalar_setElement(GrB_Scalar s, <type> val);
```

#### 1540 Parameters

1542

1549

1557

s (INOUT) An existing GraphBLAS scalar for which the element is to be assigned.

val (IN) Scalar value to assign. The type must be compatible with the domain of s.

## 1543 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output scalar s is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar\_new or Scalar\_dup.

GrB\_DOMAIN\_MISMATCH The domains of s and val are incompatible.

First, val and output GraphBLAS scalar are tested for domain compatibility as follows: **D**(val) must be compatible with **D**(s). Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_Scalar\_setElement ends and the domain mismatch error listed above is returned.

We are now ready to carry out the assignment val; that is:

$$\mathsf{s}(0) = \mathsf{val}$$

1567 If s already had a stored value, it will be overwritten; otherwise, the new value is stored in s.

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of s is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of scalar s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 1572 4.2.3.6 Scalar\_extractElement: Extract a single element from a scalar.

Assign a non-opaque scalar with the value of the element stored in a GraphBLAS scalar.

# 1574 C Syntax

```
GrB_Info GrB_Scalar_extractElement(<type> *val, const GrB_Scalar s);
```

#### Parameters

val (INOUT) Pointer to a non-opaque scalar of type that is compatible with the domain of scalar s. On successful return, val holds the result of the operation, and any previous value in val is overwritten.

s (IN) The GraphBLAS scalar from which an element is extracted.

#### Return Values

1581

1582

1583

1584

1585

1586

1587

1588

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val, has been computed and is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque 1589 GraphBLAS objects (input or output) is in an invalid state caused 1590 by a previous execution error. Call GrB\_error() to access any error 1591 messages generated by the implementation. 1592 1593

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to 1594 Scalar\_new or Scalar\_dup. 1595

GrB\_NULL\_POINTER val pointer is NULL. 1596

GrB\_DOMAIN\_MISMATCH The domains of the scalar or scalar are incompatible.

GrB NO VALUE There is no stored value in the scalar.

#### Description 1599

1597

1598

First, val and input GraphBLAS scalar are tested for domain compatibility as follows: D(val) 1600 must be compatible with  $\mathbf{D}(s)$ . Two domains are compatible with each other if values from 1601 one domain can be cast to values in the other domain as per the rules of the C language. In 1602 particular, domains from Table 3.2 are all compatible with each other. A domain from a user-1603 defined type is only compatible with itself. If any compatibility rule above is violated, execution of 1604 GrB\_Scalar\_extractElement ends and the domain mismatch error listed above is returned. 1605

Then, if no value is currently stored in the GraphBLAS scalar, the method returns GrB\_NO\_VALUE 1606 and val remains unchanged. 1607

Finally the extract into the output argument, val can be performed; that is: 1608

$$val = s(0)$$

In both GrB\_BLOCKING mode GrB\_NONBLOCKING mode if the method exits with return value 1610 GrB\_SUCCESS, the new contents of val are as defined above. 161

#### Vector methods 4.2.4 1612

#### 4.2.4.1Vector\_new: Construct new vector 1613

Creates a new vector with specified domain and size. 1614

#### C Syntax 1615

```
GrB_Info GrB_Vector_new(GrB_Vector *v,
1616
                                         GrB_Type
                                                       d,
1617
                                         GrB Index
                                                       nsize);
1618
```

#### 1619 Parameters

- v (INOUT) On successful return, contains a handle to the newly created GraphBLAS vector.
- d (IN) The type corresponding to the domain of the vector being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.
- nsize (IN) The size of the vector being created.

# 6 Return Values

1631

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector v is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new (needed for user-defined types).

GrB\_NULL\_POINTER The v pointer is NULL.

Grb INVALID VALUE nsize is zero or outside the range of the type Grb Index.

#### 1641 Description

Creates a new vector  $\mathbf{v}$  of domain  $\mathbf{D}(\mathsf{d})$ , size nsize, and empty  $\mathbf{L}(\mathbf{v})$ . The method returns a handle to the new vector in  $\mathbf{v}$ .

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

## 1646 4.2.4.2 Vector\_dup: Construct a copy of a GraphBLAS vector

1647 Creates a new vector with the same domain, size, and contents as another vector.

# 1648 C Syntax

```
GrB_Info GrB_Vector_dup(GrB_Vector *w, const GrB_Vector u);
```

#### 1651 Parameters

w (INOUT) On successful return, contains a handle to the newly created GraphBLAS vector.

u (IN) The GraphBLAS vector to be duplicated.

#### 1655 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

Grb Out Of Memory Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, u, has not been initialized by a call to Vector new or Vector dup.

GrB\_NULL\_POINTER The w pointer is NULL.

#### 1669 Description

Creates a new vector  $\mathbf{w}$  of domain  $\mathbf{D}(u)$ , size  $\mathbf{size}(u)$ , and contents  $\mathbf{L}(u)$ . The method returns a handle to the new vector in  $\mathbf{w}$ .

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

## 1674 **4.2.4.3** Vector\_resize: Resize a vector

1675 Changes the size of an existing vector.

# 1676 C Syntax

```
GrB_Info GrB_Vector_resize(GrB_Vector w,
GrB_Index nsize);
```

#### 1679 Parameters

1681

1687

1694

w (INOUT) An existing Vector object that is being resized.

nsize (IN) The new size of the vector. It can be smaller or larger than the current size.

#### 1682 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_NULL\_POINTER The w pointer is NULL.

GrB\_INVALID\_VALUE nsize is zero or outside the range of the type GrB\_Index.

#### 1695 Description

Changes the size of w to nsize. The domain  $\mathbf{D}(w)$  of vector w remains the same. The contents  $\mathbf{L}(w)$  are modified as described below.

Let  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), N, \mathbf{L}(\mathbf{w}) \rangle$  when the method is called. When the method returns,  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathsf{nsize}, \mathbf{L}'(\mathbf{w}) \rangle$  where  $\mathbf{L}'(\mathbf{w}) = \{(i, w_i) : (i, w_i) \in \mathbf{L}(\mathbf{w}) \land (i < \mathsf{nsize})\}$ . That is, all elements of  $\mathbf{w}$  with index greater than or equal to the new vector size (nsize) are dropped.

# 1701 4.2.4.4 Vector\_clear: Clear a vector

1702 Removes all the elements (tuples) from a vector.

```
C Syntax
1703
              GrB_Info GrB_Vector_clear(GrB_Vector v);
1704
    Parameters
1705
                  v (INOUT) An existing GraphBLAS vector to clear.
1706
    Return Values
1707
                    GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
1708
                                    blocking mode, this indicates that the API checks for the input
1709
                                    arguments passed successfully. Either way, output vector v is ready
1710
                                    to be used in the next method of the sequence.
1711
                       GrB PANIC Unknown internal error.
1712
           GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
1713
                                    GraphBLAS objects (input or output) is in an invalid state caused
1714
                                    by a previous execution error. Call GrB_error() to access any error
1715
                                    messages generated by the implementation.
1716
        GrB_OUT_OF_MEMORY Not enough memory available for operation.
1717
   GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to
1718
                                    Vector_new or Vector_dup.
1719
    Description
1720
    Removes all elements (tuples) from an existing vector. After the call to GrB_Vector_clear(v),
1721
    \mathbf{L}(\mathbf{v}) = \emptyset. The size of the vector does not change.
1722
    4.2.4.5
              Vector_size: Size of a vector
    Retrieve the size of a vector.
```

# 1725 C Syntax

```
GrB_Info GrB_Vector_size(GrB_Index *nsize, const GrB_Vector v);
```

#### 1728 Parameters

1730

nsize (OUT) On successful return, is set to the size of the vector.

v (IN) An existing GraphBLAS vector being queried.

#### 1731 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nsize has been set.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector\_new or Vector\_dup.

GrB\_NULL\_POINTER nsize pointer is NULL.

# 1742 Description

1743 Return size(v) in nsize.

# 1744 4.2.4.6 Vector\_nvals: Number of stored elements in a vector

Retrieve the number of stored elements (tuples) in a vector.

# 1746 C Syntax

```
GrB_Info GrB_Vector_nvals(GrB_Index *nvals, const GrB_Vector v);
```

#### 1749 Parameters

1752

nvals (OUT) On successful return, this is set to the number of stored elements (tuples) in the vector.

v (IN) An existing GraphBLAS vector being queried.

#### 1753 Return Values

1756

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector\_new or Vector\_dup.

GrB\_NULL\_POINTER The nvals pointer is NULL.

# 1765 Description

Return nvals(v) in nvals. This is the number of stored elements in vector v, which is the size of L(v) (see Section 3.5.2).

# 1768 4.2.4.7 Vector\_build: Store elements from tuples into a vector

# 1769 C Syntax

```
GrB_Info GrB_Vector_build(GrB_Vector w,

const GrB_Index *indices,

const <type> *values,

GrB_Index n,

const GrB_BinaryOp dup);
```

# 1775 Parameters

1780

w (INOUT) An existing Vector object to store the result.

indices (IN) Pointer to an array of indices.

values (IN) Pointer to an array of scalars of a type that is compatible with the domain of vector w.

n (IN) The number of entries contained in each array (the same for indices and values).

dup (IN) An associative and commutative binary operator to apply when duplicate values for the same location are present in the input arrays. All three domains of dup must be the same; hence  $dup = \langle D_{dup}, D_{dup}, D_{dup}, \oplus \rangle$ . If dup is GrB\_NULL, then duplicate locations will result in an error.

#### 5 Return Values

1781

1782

1783

1784

1790

1795

1799

1806

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT Either w has not been initialized by a call to by GrB\_Vector\_new or by GrB\_Vector\_dup, or dup has not been initialized by a call to by GrB\_BinaryOp\_new.

GrB NULL POINTER indices or values pointer is NULL.

1800 GrB INDEX OUT OF BOUNDS A value in indices is outside the allowed range for w.

GrB\_DOMAIN\_MISMATCH Either the domains of the GraphBLAS binary operator dup are not all the same, or the domains of values and w are incompatible with each other or  $D_{dup}$ .

GrB\_OUTPUT\_NOT\_EMPTY Output vector w already contains valid tuples (elements). In other words, GrB\_Vector\_nvals(C) returns a positive value.

GrB\_INVALID\_VALUE indices contains a duplicate location and dup is GrB\_NULL.

#### 1807 Description

If dup is not GrB\_NULL, an internal vector  $\widetilde{\mathbf{w}} = \langle D_{dup}, \mathbf{size}(\mathbf{w}), \emptyset \rangle$  is created, which only differs from w in its domain; otherwise,  $\widetilde{\mathbf{w}} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \emptyset \rangle$ .

Each tuple {indices[k], values[k]}, where  $0 \le k < n$ , is a contribution to the output in the form of

$$\widetilde{\mathbf{w}}(\mathsf{indices}[\mathsf{k}]) = \begin{cases} (D_{\mathit{dup}})\,\mathsf{values}[\mathsf{k}] & \text{if } \mathsf{dup} \neq \mathsf{GrB\_NULL} \\ (\mathbf{D}(\mathsf{w}))\,\mathsf{values}[\mathsf{k}] & \text{otherwise.} \end{cases}$$

If multiple values for the same location are present in the input arrays and dup is not GrB\_NULL, dup is used to reduce the values before assignment into  $\widetilde{\mathbf{w}}$  as follows:

$$\widetilde{\mathbf{w}}_i = \bigoplus_{k: \, \mathsf{indices[k]} = i} (D_{dup}) \, \mathsf{values[k]},$$

where  $\oplus$  is the dup binary operator. Finally, the resulting  $\widetilde{\mathbf{w}}$  is copied into w via typecasting its values to  $\mathbf{D}(w)$  if necessary. If  $\oplus$  is not associative or not commutative, the result is undefined.

The nonopaque input arrays, indices and values, must be at least as large as n.

It is an error to call this function on an output object with existing elements. In other words, GrB\_Vector\_nvals(w) should evaluate to zero prior to calling this function.

After GrB\_Vector\_build returns, it is safe for a programmer to modify or delete the arrays indices or values.

# 4.2.4.8 Vector\_setElement: Set a single element in a vector

1823 Set one element of a vector to a given value.

# 1824 C Syntax

```
// scalar value
1825
             GrB_Info GrB_Vector_setElement(GrB_Vector
                                                                      W,
1826
                                                  <type>
                                                                      val,
1827
                                                 GrB_Index
                                                                      index);
1828
1829
              // GraphBLAS scalar
1830
             GrB_Info GrB_Vector_setElement(GrB_Vector
1831
                                                  const GrB_Scalar
1832
                                                 GrB_Index
                                                                      index);
1833
```

# 1834 Parameters

1836

1837

1839

1840

1841

w (INOUT) An existing GraphBLAS vector for which an element is to be assigned.

val or s (IN) Scalar assign. Its domain (type) must be compatible with the domain of w.

index (IN) The location of the element to be assigned.

#### 1838 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed suc-

cessfully. Either way, the output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, w, or GraphBLAS scalar, s, has not been initialized by a call to a respective constructor.

GrB\_INVALID\_INDEX index specifies a location that is outside the dimensions of w.

GrB\_DOMAIN\_MISMATCH The domains of the vector and the scalar are incompatible.

# 54 Description

1844

1853

First, the scalar and output vector are tested for domain compatibility as follows:  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{w})$ . Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_Vector\_setElement ends and the domain mismatch error listed above is returned.

Then, the index parameter is checked for a valid value where the following condition must hold:

If this condition is violated, execution of GrB\_Vector\_setElement ends and the invalid index error listed above is returned.

1865 We are now ready to carry out the assignment; that is:

$$w(index) = \begin{cases} \mathbf{L}(s), & \operatorname{GraphBLAS\ scalar}.\\ val, & \operatorname{otherwise}. \end{cases}$$

In the case of a transparent scalar or if  $\mathbf{L}(s)$  is not empty, then a value will be stored at the specified location in  $\mathbf{w}$ , overwriting any value that may have been stored there before. In the case of a GraphBLAS scalar, if  $\mathbf{L}(s)$  is empty, then any value stored at the specified location in  $\mathbf{w}$  will be removed.

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 1875 4.2.4.9 Vector\_removeElement: Remove an element from a vector

1876 Remove (annihilate) one stored element from a vector.

# 1877 C Syntax

```
GrB_Info GrB_Vector_removeElement(GrB_Vector w,

GrB_Index index);
```

#### 1880 Parameters

1882

1884

1885

1886

1887

1888

1889

1897

w (INOUT) An existing GraphBLAS vector from which an element is to be removed.

index (IN) The location of the element to be removed.

#### 1883 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

1895 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, w, has not been initialized by a call to Vector new or Vector dup.

GrB\_INVALID\_INDEX index specifies a location that is outside the dimensions of w.

# 1898 Description

First, the index parameter is checked for a valid value where the following condition must hold:

$$0 < \mathsf{index} < \mathsf{size}(\mathsf{w})$$

1901 If this condition is violated, execution of GrB\_Vector\_removeElement ends and the invalid index error listed above is returned.

We are now ready to carry out the removal of a value that may be stored at the location specified by index. If a value does not exist at the specified location in w, no error is reported and the operation has no effect on the state of w. In either case, the following will be true on return from the method: index  $\notin$  ind(w).

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 1911 4.2.4.10 Vector\_extractElement: Extract a single element from a vector.

1912 Extract one element of a vector into a scalar.

# 1913 C Syntax

```
// scalar value
1914
             GrB_Info GrB_Vector_extractElement(<type>
                                                                          *val,
1915
                                                      const GrB_Vector
                                                                           u,
1916
                                                      GrB_Index
                                                                           index);
1917
1918
             // GraphBLAS scalar
1919
             GrB_Info GrB_Vector_extractElement(GrB_Scalar
                                                                           s,
1920
                                                      const GrB Vector
1921
                                                                           u,
                                                      GrB_Index
                                                                           index);
1922
```

#### 1923 Parameters

1927

1928

1930

1931

1932

1933

1934

val or s (INOUT) An existing scalar of whose domain is compatible with the domain of vector
u. On successful return, this scalar holds the result of the extract. Any previous
value stored in val or s is overwritten.

u (IN) The GraphBLAS vector from which an element is extracted.

index (IN) The location in u to extract.

#### 1929 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val or s, has been computed and is ready to be used in the next method of the sequence.

1935 GrB\_NO\_VALUE When using the transparent scalar, val, this is returned when there is no stored value at specified location.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB OUT OF MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, u, or scalar, s, has not been initialized by a call to a corresponding constructor.

1945 GrB NULL POINTER val pointer is NULL.

GrB\_INVALID\_INDEX index specifies a location that is outside the dimensions of w.

GrB\_DOMAIN\_MISMATCH The domains of the vector and scalar are incompatible.

# 1948 Description

1937

1942

1947

1963

1964

First, the scalar and input vector are tested for domain compatibility as follows:  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{u})$ . Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_Vector\_extractElement ends and the domain mismatch error listed above is returned.

Then, the index parameter is checked for a valid value where the following condition must hold:

$$0 \le \mathsf{index} < \mathbf{size}(\mathsf{u})$$

If this condition is violated, execution of GrB\_Vector\_extractElement ends and the invalid index error listed above is returned.

1959 We are now ready to carry out the extract into the output scalar; that is:

$$\left. \begin{array}{c} \mathbf{L}(\mathsf{s}) \\ \mathsf{val} \end{array} \right\} = \mathsf{u}(\mathsf{index})$$

If  $index \in ind(u)$ , then the corresponding value from u is copied into s or val with casting as necessary. If  $index \notin ind(u)$ , then one of the follow occurs depending on output scalar type:

- The GraphBLAS scalar, s, is cleared and GrB\_SUCCESS is returned.
- The non-opaque scalar, val, is unchanged, and GrB\_NO\_VALUE is returned.

When using the non-opaque scalar variant (val) in both GrB\_BLOCKING mode GrB\_NONBLOCKING mode, the new contents of val are as defined above if the method exits with return value GrB\_SUCCESS or GrB\_NO\_VALUE.

When using the GraphBLAS scalar variant (s) with a GrB\_SUCCESS return value, the method exits and the new contents of s is as defined above and fully computed in GrB\_BLOCKING mode.

In GrB\_NONBLOCKING mode, the new contents of s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 1972 4.2.4.11 Vector\_extractTuples: Extract tuples from a vector

1973 Extract the contents of a GraphBLAS vector into non-opaque data structures.

# 974 C Syntax

| 1975<br>1976<br>1977<br>1978<br>1979 | <pre>GrB_Info GrB_Vector_extractTuples(GrB_Index *indices,</pre>   |
|--------------------------------------|--|
| 1980<br>1981                         | indices (OUT) Pointer to an array of indices that is large enough to hold all of the stored values' indices.   |
| 1982<br>1983                         | values (OUT) Pointer to an array of scalars of a type that is large enough to hold all of the stored values whose type is compatible with $\mathbf{D}(\mathbf{v})$ .                       |
| 1984<br>1985<br>1986                 | n (INOUT) Pointer to a value indicating (on input) the number of elements the values and indices arrays can hold. Upon return, it will contain the number of values written to the arrays. |
| 1987                                 | v (IN) An existing GraphBLAS vector.   |

#### $\mathbf{88}$ Return Values

| 1989 | GrB_SUCCESS        | In blocking or non-blocking mode, the operation completed suc-      |
|------|--------------------|---|
| 1990 |                    | cessfully. This indicates that the compatibility tests on the input |
| 1991 |                    | argument passed successfully, and the output arrays, indices and    |
| 1992 |                    | values, have been computed.   |
| 1993 | GrB_PANIC          | Unknown internal error.   |
| 1994 | GrB_INVALID_OBJECT | This is returned in any execution mode whenever one of the opaque   |
| 1995 |                    | GraphBLAS objects (input or output) is in an invalid state caused   |
| 1996 |                    | by a previous execution error. Call GrB_error() to access any error |
| 1997 |                    | messages generated by the implementation.                           |

- GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.
- GrB\_INSUFFICIENT\_SPACE Not enough space in indices and values (as indicated by the n parameter) to hold all of the tuples that will be extacted.
- GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector\_new or Vector\_dup.
- 2003 GrB\_NULL\_POINTER indices, values, or n pointer is NULL.
- GrB\_DOMAIN\_MISMATCH The domains of the v vector or values array are incompatible with one another.

- This method will extract all the tuples from the GraphBLAS vector v. The values associated with those tuples are placed in the values array and the indices are placed in the indices array.

  Both indices and values must be pre-allocated by the user to have enough space to hold at least
- Both indices and values must be pre-allocated by the user to have enough space to hold at least GrB\_Vector\_nvals(v) elements before calling this function.
- Upon return of this function, n will be set to the number of values (and indices) copied. Also, the entries of indices are unique, but not necessarily sorted. Each tuple  $(i, v_i)$  in v is unzipped and copied into a distinct kth location in output vectors:

$$\{\mathsf{indices}[\mathsf{k}], \mathsf{values}[\mathsf{k}]\} \leftarrow (i, v_i),$$

- where  $0 \le k < GrB\_Vector\_nvals(v)$ . No gaps in output vectors are allowed; that is, if indices[k] and values[k] exist upon return, so does indices[j] and values[j] for all j such that  $0 \le j < k$ .
- Note that if the value in n on input is less than the number of values contained in the vector v, then a GrB\_INSUFFICIENT\_SPACE error is returned because it is undefined which subset of values would be extracted otherwise.
- In both GrB\_BLOCKING mode GrB\_NONBLOCKING mode if the method exits with return value GrB\_SUCCESS, the new contents of the arrays indices and values are as defined above.

# 2021 4.2.5 Matrix methods

## 2022 4.2.5.1 Matrix new: Construct new matrix

2023 Creates a new matrix with specified domain and dimensions.

# 2024 C Syntax

| 2027                         |                           | GrB_Index nrows,  |
|------------------------------|---------------------------|---|
| 2028                         |                           | <pre>GrB_Index ncols);</pre>  |
|                              |                           |   |
| 2029                         | Parameters                |   |
| 2030<br>2031                 | A (INOUT) On s<br>matrix. | uccessful return, contains a handle to the newly created GraphBLAS  |
| 2032<br>2033<br>2034         | ` , ,                     | corresponding to the domain of the matrix being created. Can be defined GraphBLAS types in Table 3.2, or an existing user-defined ype.  |
| 2035                         | nrows (IN) The number     | ber of rows of the matrix being created.  |
| 2036                         | ncols (IN) The number     | ber of columns of the matrix being created.   |
| 2037                         | Return Values             |   |
| 2038<br>2039<br>2040<br>2041 | GrB_SUCCESS               | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix A is ready to be used in the next method of the sequence. |
| 2042                         | GrB_PANIC                 | Unknown internal error.   |
| 2043<br>2044<br>2045<br>2046 | GrB_INVALID_OBJECT        | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation. |
| 2047                         | GrB_OUT_OF_MEMORY         | Not enough memory available for operation.  |
| 2048<br>2049                 | GrB_UNINITIALIZED_OBJECT  | The GrB_Type object has not been initialized by a call to GrB_Type_new (needed for user-defined types).   |
| 2050                         | GrB_NULL_POINTER          | The A pointer is NULL.  |
| 2051                         | GrB_INVALID_VALUE         | nrows or ncols is zero or outside the range of the type GrB_Index.  |

Creates a new matrix **A** of domain  $\mathbf{D}(\mathsf{d})$ , size nrows  $\times$  ncols, and empty  $\mathbf{L}(\mathbf{A})$ . The method returns 2053 a handle to the new matrix in A. 2054

It is not an error to call this method more than once on the same variable; however, the handle to 2055 the previously created object will be overwritten. 2056

# 2057 4.2.5.2 Matrix\_dup: Construct a copy of a GraphBLAS matrix

2058 Creates a new matrix with the same domain, dimensions, and contents as another matrix.

# 2059 C Syntax

```
GrB_Info GrB_Matrix_dup(GrB_Matrix *C, const GrB Matrix A);
```

# 2062 Parameters

2065

2071

2079

C (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix.

A (IN) The GraphBLAS matrix to be duplicated.

#### 2066 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB\_NULL\_POINTER The C pointer is NULL.

# 2080 Description

Creates a new matrix  $\mathbf{C}$  of domain  $\mathbf{D}(A)$ , size  $\mathbf{nrows}(A) \times \mathbf{ncols}(A)$ , and contents  $\mathbf{L}(A)$ . It returns a handle to it in C.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

# 2085 4.2.5.3 Matrix\_diag: Construct a diagonal GraphBLAS matrix

Creates a new matrix with the same domain and contents as a GrB\_Vector, and square dimensions appropriate for placing the contents of the vector along the specified diagonal of the matrix.

# 2088 C Syntax

```
GrB_Info GrB_Matrix_diag(GrB_Matrix *C,

const GrB_Vector v,

int64_t k);
```

#### Parameters

2092

2104

2109

- C (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix. The matrix is square with each dimension equal to  $\operatorname{size}(\mathsf{v}) + |k|$ .
- v (IN) The GraphBLAS vector whose contents will be copied to the diagonal of the matrix.
- k (IN) The diagonal to which the vector is assigned. k = 0 represents the main diagonal, k > 0 is above the main diagonal, and k < 0 is below.

#### 2099 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector\_new or Vector\_dup.

2112 GrB\_NULL\_POINTER The C pointer is NULL.

Creates a new matrix C of domain  $\mathbf{D}(\mathbf{v})$ , size  $(\mathbf{size}(\mathbf{v}) + |k|) \times (\mathbf{size}(\mathbf{v}) + |k|)$ , and contents

```
2115 \mathbf{L}(\mathsf{C}) = \{(i, i + k, v_i) : (i, v_i) \in \mathbf{L}(\mathsf{v})\} \text{ if } k \ge 0 \text{ or}
2116 \mathbf{L}(\mathsf{C}) = \{(i - k, i, v_i) : (i, v_i) \in \mathbf{L}(\mathsf{v})\} \text{ if } k < 0.
```

It returns a handle to it in C. It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

# 2119 4.2.5.4 Matrix\_resize: Resize a matrix

2120 Changes the dimensions of an existing matrix.

# 2121 C Syntax

```
GrB_Info GrB_Matrix_resize(GrB_Matrix C,

GrB_Index nrows,

GrB_Index ncols);
```

#### 2125 Parameters

2126

2127

2128

2129

2130

2136

2141

C (INOUT) An existing Matrix object that is being resized.

nrows (IN) The new number of rows of the matrix. It can be smaller or larger than the current number of rows.

ncols (IN) The new number of columns of the matrix. It can be smaller or larger than the current number of columns.

#### 2131 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

```
GrB_NULL_POINTER The C pointer is NULL.
2142
             GrB_INVALID_VALUE nrows or ncols is zero or outside the range of the type GrB_Index.
2143
    Description
2144
     Changes the number of rows and columns of C to nrows and ncols, respectively. The domain \mathbf{D}(\mathsf{C})
2145
    of matrix C remains the same. The contents L(C) are modified as described below.
2146
    Let C = \langle \mathbf{D}(C), M, N, \mathbf{L}(C) \rangle when the method is called. When the method returns C is modified
2147
    to C = \langle D(C), \text{nrows}, \text{ncols}, L'(C) \rangle where L'(C) = \{(i, j, C_{ij}) : (i, j, C_{ij}) \in L(C) \land (i < \text{nrows}) \land (j < i) \}
     ncols). That is, all elements of C with row index greater than or equal to nrows or column index
2140
     greater than or equal to ncols are dropped.
2150
    4.2.5.5
               Matrix_clear: Clear a matrix
2151
    Removes all elements (tuples) from a matrix.
2152
    C Syntax
2153
               GrB_Info GrB_Matrix_clear(GrB_Matrix A);
2154
    Parameters
2155
                   A (IN) An exising GraphBLAS matrix to clear.
2156
    Return Values
2157
                     GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
2158
                                      blocking mode, this indicates that the API checks for the input ar-
2159
                                      guments passed successfully. Either way, output matrix A is ready
2160
                                      to be used in the next method of the sequence.
2161
                        GrB PANIC Unknown internal error.
2162
           GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
2163
                                      GraphBLAS objects (input or output) is in an invalid state caused
2164
                                      by a previous execution error. Call GrB_error() to access any error
2165
                                      messages generated by the implementation.
2166
         Grb Out of Memory Not enough memory available for operation.
2167
```

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to

any matrix constructor.

2168

2169

Removes all elements (tuples) from an existing matrix. After the call to  $GrB\_Matrix\_clear(A)$ ,  $L(A) = \emptyset$ . The dimensions of the matrix do not change.

# 2173 4.2.5.6 Matrix\_nrows: Number of rows in a matrix

2174 Retrieve the number of rows in a matrix.

# 2175 C Syntax

```
GrB_Info GrB_Matrix_nrows(GrB_Index *nrows, const GrB_Matrix A);
```

#### 2178 Parameters

2180

2184

nrows (OUT) On successful return, contains the number of rows in the matrix.

A (IN) An existing GraphBLAS matrix being queried.

#### 2181 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nrows has been set.

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

2189 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB\_NULL\_POINTER nrows pointer is NULL.

# 2192 Description

2193 Return **nrows**(A) in **nrows** (the number of rows).

## 2194 4.2.5.7 Matrix ncols: Number of columns in a matrix

2195 Retrieve the number of columns in a matrix.

```
C Syntax
2196
              GrB_Info GrB_Matrix_ncols(GrB_Index
                                                                *ncols,
2197
                                            const GrB_Matrix
                                                               A);
2198
    Parameters
               ncols (OUT) On successful return, contains the number of columns in the matrix.
2200
                  A (IN) An existing GraphBLAS matrix being queried.
2201
    Return Values
                   GrB_SUCCESS In blocking or non-blocking mode, the operation completed suc-
2203
                                   cessfully and the value of ncols has been set.
2204
                      GrB_PANIC Unknown internal error.
2205
          GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
2206
                                   GraphBLAS objects (input or output) is in an invalid state caused
2207
                                   by a previous execution error. Call GrB_error() to access any error
2208
                                   messages generated by the implementation.
2209
   GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to
2210
                                   any matrix constructor.
2211
            GrB_NULL_POINTER ncols pointer is NULL.
2212
    Description
2213
    Return ncols(A) in ncols (the number of columns).
2214
    4.2.5.8
              Matrix_nvals: Number of stored elements in a matrix
2215
    Retrieve the number of stored elements (tuples) in a matrix.
    C Syntax
2217
              GrB_Info GrB_Matrix_nvals(GrB_Index
                                                                *nvals,
2218
                                            const GrB_Matrix A);
2219
```

#### 2220 Parameters

2223

nvals (OUT) On successful return, contains the number of stored elements (tuples) in the matrix.

A (IN) An existing GraphBLAS matrix being queried.

#### 2224 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

2227 GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

2233 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

2235 GrB\_NULL\_POINTER The nvals pointer is NULL.

# 2236 Description

Return nvals(A) in nvals. This is the number of tuples stored in matrix A, which is the size of L(A) (see Section 3.5.3).

# 2239 4.2.5.9 Matrix\_build: Store elements from tuples into a matrix

# 2240 C Syntax

## Parameters

2242

C (INOUT) An existing Matrix object to store the result.

row\_indices (IN) Pointer to an array of row indices. 2243 col\_indices (IN) Pointer to an array of column indices. 2244 values (IN) Pointer to an array of scalars of a type that is compatible with the domain of 2245 matrix, C. 2246 n (IN) The number of entries contained in each array (the same for row indices, 2247 col indices, and values). 2248 dup (IN) An associative and commutative binary operator to apply when duplicate 2249 values for the same location are present in the input arrays. All three domains of 2250 dup must be the same; hence  $dup = \langle D_{dup}, D_{dup}, D_{dup}, \oplus \rangle$ . If dup is GrB\_NULL, 2251 then duplicate locations will result in an error. 2252 Return Values 2253 GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-2254 blocking mode, this indicates that the API checks for the input 2255 arguments passed successfully. Either way, output matrix C is 2256 ready to be used in the next method of the sequence. 2257 GrB\_PANIC Unknown internal error. 2258 GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the 2259 opaque GraphBLAS objects (input or output) is in an invalid 2260 state caused by a previous execution error. Call GrB error() to 2261 access any error messages generated by the implementation. 2262 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 2263 GrB UNINITIALIZED OBJECT Either C has not been initialized by a call to any matrix construc-2264 tor, or dup has not been initialized by a call to by GrB BinaryOp new. 2265 GrB\_NULL\_POINTER row\_indices, col\_indices or values pointer is NULL. 2266 GrB\_INDEX\_OUT\_OF\_BOUNDS A value in row\_indices or col\_indices is outside the allowed range 2267 for C. 2268 Grb\_DOMAIN\_MISMATCH Either the domains of the GraphBLAS binary operator dup are 2269 not all the same, or the domains of values and C are incompatible 2270 with each other or  $D_{dup}$ . 2271 Grb Output NOT EMPTY Output matrix C already contains valid tuples (elements). In 2272 other words, GrB\_Matrix\_nvals(C) returns a positive value.

2274

GrB\_INVALID\_VALUE indices contains a duplicate location and dup is GrB\_NULL.

If dup is not GrB\_NULL, an internal matrix  $\widetilde{\mathbf{C}} = \langle D_{dup}, \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \emptyset \rangle$  is created, which only differs from C in its domain; otherwise,  $\widetilde{\mathbf{C}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \emptyset \rangle$ .

Each tuple {row\_indices[k], col\_indices[k], values[k]}, where  $0 \le k < n$ , is a contribution to the output in the form of

$$\widetilde{\mathbf{C}}(\mathsf{row\_indices}[\mathsf{k}],\mathsf{col\_indices}[\mathsf{k}]) = \begin{cases} (D_{dup}) \, \mathsf{values}[\mathsf{k}] & \text{if } \mathsf{dup} \neq \mathsf{GrB\_NULL} \\ (\mathbf{D}(\mathsf{C})) \, \mathsf{values}[\mathsf{k}] & \text{otherwise.} \end{cases}$$

If multiple values for the same location are present in the input arrays and dup is not GrB\_NULL, dup is used to reduce the values before assignment into  $\tilde{\mathbf{C}}$  as follows:

$$\widetilde{\mathbf{C}}_{ij} = \bigoplus_{k:\, \mathsf{row\_indices}[\mathtt{k}] = i \, \land \, \mathsf{col\_indices}[\mathtt{k}] = j} (D_{dup}) \, \mathsf{values}[\mathtt{k}],$$

where  $\oplus$  is the dup binary operator. Finally, the resulting  $\widetilde{\mathbf{C}}$  is copied into C via typecasting its values to  $\mathbf{D}(C)$  if necessary. If  $\oplus$  is not associative or not commutative, the result is undefined.

The nonopaque input arrays row\_indices, col\_indices, and values must be at least as large as n.

It is an error to call this function on an output object with existing elements. In other words, GrB\_Matrix\_nvals(C) should evaluate to zero prior to calling this function.

After GrB\_Matrix\_build returns, it is safe for a programmer to modify or delete the arrays row\_indices, col\_indices, or values.

#### 4.2.5.10 Matrix\_setElement: Set a single element in matrix

2292 Set one element of a matrix to a given value.

#### 2293 C Syntax

```
// scalar value
2294
             GrB Info GrB Matrix setElement(GrB Matrix
                                                                        C,
2295
                                                  <type>
                                                                        val,
2296
                                                  GrB_Index
                                                                        row_index,
2297
                                                  GrB Index
                                                                        col_index);
2298
2299
              // GraphBLAS scalar
2300
             GrB_Info GrB_Matrix_setElement(GrB_Matrix
                                                                        С,
2301
                                                  const GrB_Scalar
2302
                                                  GrB_Index
                                                                        row_index,
2303
                                                  GrB Index
                                                                        col index);
2304
```

#### 2305 Parameters

C (INOUT) An existing GraphBLAS matrix for which an element is to be assigned.

val or s (IN) Scalar to assign. Its domain (type) must be compatible with the domain of
C.

row\_index (IN) Row index of element to be assigned

col\_index (IN) Column index of element to be assigned

#### Return Values

2311

2317

2327

2336

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

2323 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, or GraphBLAS scalar, s, has not been initialized by a call to a respective constructor.

GrB\_INVALID\_INDEX row\_index or col\_index is outside the allowable range (i.e., not less than  $\mathbf{nrows}(C)$  or  $\mathbf{ncols}(C)$ , respectively).

Grb\_DOMAIN\_MISMATCH The domains of the matrix and the scalar are incompatible.

# 2328 Description

First, the scalar and output matrix are tested for domain compatibility as follows:  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{C})$ . Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of  $\mathsf{GrB}_{\mathsf{Matrix\_setElement}}$  ends and the domain mismatch error listed above is returned.

Then, both index parameters are checked for valid values where following conditions must hold:

$$0 \le \text{row\_index} < \mathbf{nrows}(C),$$
  
 $0 \le \text{col\_index} < \mathbf{ncols}(C)$ 

If either of these conditions is violated, execution of GrB\_Matrix\_setElement ends and the invalid index error listed above is returned.

We are now ready to carry out the assignment; that is:

$$C(row\_index, col\_index) = \begin{cases} \mathbf{L}(s), & \operatorname{GraphBLAS\ scalar.} \\ val, & \operatorname{otherwise.} \end{cases}$$

In the case of a transparent scalar or if  $\mathbf{L}(s)$  is not empty, then a value will be stored at the specified location in  $\mathsf{C}$ , overwriting any value that may have been stored there before. In the case of a GraphBLAS scalar and if  $\mathbf{L}(s)$  is empty, then any value stored at the specified location in  $\mathsf{C}$  will be removed.

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector C is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

# 2349 4.2.5.11 Matrix\_removeElement: Remove an element from a matrix

2350 Remove (annihilate) one stored element from a matrix.

# 2351 C Syntax

```
GrB_Info GrB_Matrix_removeElement(GrB_Matrix C,
GrB_Index row_index,
GrB_Index col_index);
```

#### Parameters

2357

2359

2360

2361

2362

2363

2364

2365

<sup>2356</sup> C (INOUT) An existing GraphBLAS matrix from which an element is to be removed.

row\_index (IN) Row index of element to be removed

col index (IN) Column index of element to be removed

# Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

```
GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.
```

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, C, has not been initialized by a call to any matrix constructor.

GrB\_INVALID\_INDEX row\_index or col\_index is outside the allowable range (i.e., not less than nrows(C) or ncols(C), respectively).

## 2375 Description

2370

First, both index parameters are checked for valid values where following conditions must hold:

```
0 \leq \text{row\_index} < \mathbf{nrows}(C),
0 \leq \text{col\_index} < \mathbf{ncols}(C)
```

If either of these conditions is violated, execution of GrB\_Matrix\_removeElement ends and the invalid index error listed above is returned.

We are now ready to carry out the removal of a value that may be stored at the location specified by (row\_index, col\_index). If a value does not exist at the specified location in C, no error is reported and the operation has no effect on the state of C. In either case, the following will be true on return from this method: (row\_index, col\_index)  $\notin$  ind(C)

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new contents of C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector C is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

#### 2388 4.2.5.12 Matrix\_extractElement: Extract a single element from a matrix

Extract one element of a matrix into a scalar.

# 2390 C Syntax

```
// scalar value
2391
             GrB_Info GrB_Matrix_extractElement(<type>
                                                                          *val,
2392
                                                      const GrB_Matrix
2393
                                                      GrB_Index
                                                                           row_index,
2394
                                                      GrB_Index
                                                                           col_index);
2395
2396
              // GraphBLAS scalar
2397
```

```
const GrB_Matrix
                                                                              Α,
2399
                                                        GrB_Index
                                                                              row_index,
2400
                                                        GrB_Index
                                                                              col_index);
2401
2402
    Parameters
2403
           val or s (INOUT) An existing scalar whose domain is compatible with the domain of matrix
2404
                   A. On successful return, this scalar holds the result of the extract. Any previous
2405
                   value stored in val or s is overwritten.
2406
                 A (IN) The GraphBLAS matrix from which an element is extracted.
2407
        row index (IN) The row index of location in A to extract.
2408
         col_index (IN) The column index of location in A to extract.
2409
    Return Values
2410
                    GrB_SUCCESS In blocking or non-blocking mode, the operation completed suc-
2411
                                    cessfully. This indicates that the compatibility tests on dimensions
2412
                                    and domains for the input arguments passed successfully, and the
2413
                                    output scalar, val or s, has been computed and is ready to be used
2414
                                    in the next method of the sequence.
2415
                  GrB_NO_VALUE When using the transparent scalar, val, this is returned when there
2416
                                    is no stored value at specified location.
2417
                       GrB_PANIC Unknown internal error.
2418
           GrB INVALID OBJECT This is returned in any execution mode whenever one of the opaque
2419
                                    GraphBLAS objects (input or output) is in an invalid state caused
2420
                                    by a previous execution error. Call GrB_error() to access any error
2421
                                    messages generated by the implementation.
2422
         GrB OUT OF MEMORY Not enough memory available for operation.
2423
   GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, or scalar, s, has not been initialized by
2424
                                    a call to a corresponding constructor.
2425
            GrB_NULL_POINTER val pointer is NULL.
2426
            GrB INVALID INDEX row index or col index is outside the allowable range (i.e. less than
2427
                                    zero or greater than or equal to \mathbf{nrows}(A) or \mathbf{ncols}(A), respec-
2428
                                    tively).
2429
       GrB_DOMAIN_MISMATCH The domains of the matrix and scalar are incompatible.
```

GrB\_Info GrB\_Matrix\_extractElement(GrB\_Scalar

2398

2430

s,

First, the scalar and input matrix are tested for domain compatibility as follows:  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{A})$ . Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of  $\mathsf{GrB}_{\mathsf{Matrix}}$  extractElement ends and the domain mismatch error listed above is returned.

Then, both index parameters are checked for valid values where following conditions must hold:

If either condition is violated, execution of GrB\_Matrix\_extractElement ends and the invalid index error listed above is returned.

We are now ready to carry out the extract into the output scalar; that is,

$$\frac{\mathbf{L}(s)}{\text{val}} \left. \right\} = \mathsf{A}(\mathsf{row\_index}, \mathsf{col\_index})$$

If  $(row\_index, col\_index) \in ind(A)$ , then the corresponding value from A is copied into s or values with casting as necessary. If  $(row\_index, col\_index) \notin ind(A)$ , then one of the follow occurs depending on output scalar type:

- The GraphBLAS scalar, s, is cleared and GrB SUCCESS is returned.
- The non-opaque scalar, val, is unchanged, and GrB\_NO\_VALUE is returned.

When using the non-opaque scalar variant (val) in both GrB\_BLOCKING mode GrB\_NONBLOCKING mode, the new contents of val are as defined above if the method exits with return value GrB\_SUCCESS or GrB\_NO\_VALUE.

When using the GraphBLAS scalar variant (s) with a GrB\_SUCCESS return value, the method exits and the new contents of s is as defined above and fully computed in GrB\_BLOCKING mode.

In GrB\_NONBLOCKING mode, the new contents of s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

#### 4.2.5.13 Matrix extractTuples: Extract tuples from a matrix

2457 Extract the contents of a GraphBLAS matrix into non-opaque data structures.

# 2458 C Syntax

2447

2448

| 2461<br>2462<br>2463         |   | <type> *values,  GrB_Index *n,  const GrB_Matrix A);</type>   |
|------------------------------|---|---|
| 2464                         | Parameters                              |   |
| 2465<br>2466                 | row_indices (OUT) Pointe row indices.   | r to an array of row indices that is large enough to hold all of the  |
| 2467<br>2468                 | col_indices (OUT) Pointer column indice | to an array of column indices that is large enough to hold all of the s.  |
| 2469<br>2470                 | ` ,                                     | to an array of scalars of a type that is large enough to hold all of ues whose type is compatible with $\mathbf{D}(\mathbf{A})$ .   |
| 2471<br>2472<br>2473         | row_indices, a                          | ter to a value indicating (in input) the number of elements the values, nd col_indices arrays can hold. Upon return, it will contain the ues written to the arrays.   |
| 2474                         | A (IN) An existi                        | ng GraphBLAS matrix.  |
| 2475                         | Return Values                           |   |
| 2476<br>2477<br>2478<br>2479 | GrB_SUCCESS                             | In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output arrays, indices and values, have been computed.                     |
| 2480                         | GrB_PANIC                               | Unknown internal error.   |
| 2481<br>2482<br>2483<br>2484 | GrB_INVALID_OBJECT                      | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation. |
| 2485                         | GrB_OUT_OF_MEMORY                       | Not enough memory available for operation.  |
| 2486<br>2487<br>2488         | GrB_INSUFFICIENT_SPACE                  | Not enough space in $row\_indices$ , $col\_indices$ , and values (as indicated by the $n$ parameter) to hold all of the tuples that will be extacted.   |
| 2489<br>2490                 | GrB_UNINITIALIZED_OBJECT                | The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.  |
| 2491                         | GrB_NULL_POINTER                        | $row\_indices, \ col\_indices, \ values \ \mathrm{or} \ n \ \mathrm{pointer} \ \mathrm{is} \ NULL.$   |
| 2492<br>2493                 | GrB_DOMAIN_MISMATCH                     | The domains of the A matrix and values array are incompatible with one another.   |

This method will extract all the tuples from the GraphBLAS matrix A. The values associated with those tuples are placed in the values array, the column indices are placed in the col\_indices array, and the row indices are placed in the row\_indices array. These output arrays are pre-allocated by the user before calling this function such that each output array has enough space to hold at least GrB\_Matrix\_nvals(A) elements.

Upon return of this function, a pair of  $\{\text{row\_indices}[k], \text{col\_indices}[k]\}$  are unique for every valid k, but they are not required to be sorted in any particular order. Each tuple  $(i, j, A_{ij})$  in A is unzipped and copied into a distinct kth location in output vectors:

 $\{\text{row\_indices}[k], \text{col\_indices}[k], \text{values}[k]\} \leftarrow (i, j, A_{ij}),$ 

where  $0 \le k < GrB\_Matrix\_nvals(v)$ . No gaps in output vectors are allowed; that is, if row\_indices[k], col\_indices[k] and values[k] exist upon return, so does row\_indices[j], col\_indices[j] and values[j] for all j such that  $0 \le j < k$ .

Note that if the value in n on input is less than the number of values contained in the matrix A, then a GrB\_INSUFFICIENT\_SPACE error is returned since it is undefined which subset of values would be extracted.

In both GrB\_BLOCKING mode GrB\_NONBLOCKING mode if the method exits with return value GrB\_SUCCESS, the new contents of the arrays row\_indices, col\_indices and values are as defined above.

2512 **4.2.5.14** Matrix\_exportHint: Provide a hint as to which storage format might be most efficient for exporting a matrix

 $^{2514}$  C Syntax

#### 2515 Parameters

2516

2517

2521

hint (OUT) Pointer to a value of type GrB Format.

A (IN) A GraphBLAS matrix object.

## 2518 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of hint has been set.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB\_NULL\_POINTER hint is NULL.

GrB\_NO\_VALUE If the implementation does not have a preferred format, it may return the value GrB\_NO\_VALUE.

## 2532 Description

2526

2527

2528

2529

2530

2531

Given a GraphBLAS matrix A, provide a hint as to which format might be most efficient for exporting the matrix A. GraphBLAS implementations might return the current storage format of the matrix, or the format to which it could most efficiently be exported. However, implementations are free to return any value for format defined in Section 3.5.3.1. Note that an implementation is free to refuse to provide a format hint, returning GrB NO VALUE.

# 2538 4.2.5.15 Matrix\_exportSize: Return the array sizes necessary to export a GraphBLAS matrix object

#### 2540 C Syntax

```
GrB_Info GrB_Matrix_exportSize(GrB_Index *n_indptr,
GrB_Index *n_indices,
GrB_Index *n_values,
GrB_Format format,
GrB_Matrix A);
```

#### Parameters

2541

2544

2547

n\_indptr (OUT) Pointer to a value of type GrB\_Index.

n\_indices (OUT) Pointer to a value of type GrB\_Index.

n\_values (OUT) Pointer to a value of type GrB\_Index.

format (IN) a value indicating the format in which the matrix will be exported, as defined in Section 3.5.3.1.

A (IN) A GraphBLAS matrix object.

#### Return Values $\mathbf{Return}$

2554

2559

2562

GrB\_SUCCESS In blocking mode or non-blocking mode, the operation completed successfully. This indicates that the API checks for the input arguments passed successfully, and the number of elements necessary for the export buffers have been written to n\_indptr, n\_indices, and n\_values, respectively.

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS Matrix, A, has not been initialized by a call to any matrix constructor.

GrB\_NULL\_POINTER n\_indptr, n\_indices, or n\_values is NULL.

#### 2563 Description

Given a matrix **A**, returns the required capacities of arrays values, indptr, and indices necessary to export the matrix in the format specified by format. The output values n\_values, n\_indptr, and indices will contain the corresponding sizes of the arrays (in number of elements) that must be allocated to hold the exported matrix. The argument format can be chosen arbitrarily by the user as one of the values defined in Section 3.5.3.1.

### 2569 4.2.5.16 Matrix\_export: Export a GraphBLAS matrix to a pre-defined format

## 2570 C Syntax

```
GrB_Info GrB_Matrix_export(GrB_Index
                                                    *indptr,
                             GrB_Index
                                                    *indices,
                             <type>
                                                    *values,
                             GrB_Index
                                                    *n_indptr,
                             GrB_Index
                                                    *n_indices,
                             GrB Index
                                                    *n_values,
                             GrB_Format
                                                     format,
                             GrB_Matrix
                                                     A);
```

#### Parameters

indptr (INOUT) Pointer to an array that will hold row or column offsets, or row in-2572 dices, depending on the value of format. It must be large enough to hold at 2573 least n indptr elements of type GrB Index, where n indices was returned from 2574 GrB\_Matrix\_exportSize() method. 2575 indices (INOUT) Pointer to an array that will hold row or column indices of the elements 2576 in values, depending on the value of format. It must be large enough to hold at 2577 least n\_indices elements of type GrB\_Index, where n\_indices was returned from 2578 GrB Matrix exportSize() method. 2579 values (INOUT) Pointer to an array that will hold stored values. The type of ele-2580 ment must match the type of the values stored in A. It must be large enough 2581 to hold at least n\_values elements of that type, where n\_values was returned from 2582 GrB\_Matrix\_exportSize. 2583 n\_indptr (INOUT) Pointer to a value indicating (on input) the number of elements the indptr 2584 array can hold. Upon return, it will contain the number of elements written to the 2585 array. 2586 n\_indices (INOUT) Pointer to a value indicating (on input) the number of elements the indices 2587 array can hold. Upon return, it will contain the number of elements written to the 2588 array. 2589 n\_values (INOUT) Pointer to a value indicating (on input) the number of elements the values 2590 array can hold. Upon return, it will contain the number of elements written to the 2591 array. 2592 format (IN) a value indicating the format in which the matrix will be exported, as defined 2593 in Section 3.5.3.1. 2594 A (IN) A GraphBLAS matrix object. 2595

### 596 Return Values

| 2597<br>2598<br>2599<br>2600 | GrB_SUCCESS        | In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output arrays, indptr, indices and values, have been computed. |
|------------------------------|--------------------|---|
| 2601                         | GrB_PANIC          | Unknown internal error.   |
| 2602<br>2603                 | GrB_INVALID_OBJECT | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid  |
| 2604<br>2605                 |                    | state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.  |
| 2606                         | GrB_OUT_OF_MEMORY  | Not enough memory available for operation.  |

GrB\_INSUFFICIENT\_SPACE Not enough space in indptr, indices, and/or values (as indicated 2607 by the corresponding  $n_*$  parameter) to hold all of the corre-2608 sponding elements that will be extacted. 2609 GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to 2610 any matrix constructor. 2611 GrB\_NULL\_POINTER indptr, indices, values n\_indptr, n\_indices, n\_values pointer is 2612 NULL. 2613 GrB\_DOMAIN\_MISMATCH The domain of A does not match with the type of values. 2614

## 2615 Description

Given a matrix **A**, this method exports the contents of the matrix into one of the pre-defined GrB\_Format formats from Section 3.5.3.1. The user-allocated arrays pointed to by indptr, indices, and values must be at least large enough to hold the corresponding number of elements returned by calling GrB\_Matrix\_exportSize. The value of format can be chosen arbitrarily, but a call to GrB\_Matrix\_exportHint may suggest a format that results in the most efficient export. Details of the contents of indptr, indices, and values corresponding to each supported format is given in Appendix B.

## 2623 4.2.5.17 Matrix\_import: Import a matrix into a GraphBLAS object

#### 2624 C Syntax

```
GrB_Info GrB_Matrix_import(GrB_Matrix
                                                    *A,
                                                     d,
                             GrB_Type
                             GrB_Index
                                                     nrows,
                             GrB_Index
                                                     ncols
                             const GrB_Index
                                                    *indptr,
                             const GrB Index
                                                    *indices,
                             const <type>
                                                    *values,
                             GrB Index
                                                     n indptr,
                             GrB_Index
                                                     n indices,
                             GrB_Index
                                                     n_values,
                             GrB_Format
                                                     format);
```

#### Parameters

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- A (INOUT) On a successful return, contains a handle to the newly created Graph-BLAS matrix.
  - d (IN) The type corresponding to the domain of the matrix being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.

nrows (IN) Integer value holding the number of rows in the matrix. 2631 ncols (IN) Integer value holding the number of columns in the matrix. 2632 indptr (IN) Pointer to an array of row or column offsets, or row indices, depending on the 2633 value of format. 2634 indices (IN) Pointer to an array row or column indices of the elements in values, depending 2635 on the value of format. 2636 values (IN) Pointer to an array of values. Type must match the type of d. 2637 n indptr (IN) Integer value holding the number of elements in the array pointed to by indptr. 2638 n\_indices (IN) Integer value holding the number of elements in the array pointed to by indices. 2639 n values (IN) Integer value holding the number of elements in the array pointed to by values. 2640 format (IN) a value indicating the format of the matrix being imported, as defined in 2641 Section 3.5.3.1. 2642 Return Values 2643 GrB SUCCESS In blocking mode, the operation completed successfully. In non-2644 blocking mode, this indicates that the API checks for the input 2645 arguments passed successfully and the input arrays have been 2646 consumed. Either way, output matrix A is ready to be used in 2647 the next method of the sequence. 2648 GrB PANIC Unknown internal error. 2649 Grb Out of Memory Not enough memory available for operation. 2650 GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new 2651 (needed for user-defined types). 2652 GrB\_NULL\_POINTER A, indptr, indices or values pointer is NULL. 2653 GrB\_INDEX\_OUT\_OF\_BOUNDS A value in indptr or indices is outside the allowed range for indices 2654 in A and or the size of values, n\_values, depending on the value 2655 of format. 2656 GrB\_INVALID\_VALUE nrows or ncols is zero or outside the range of the type GrB\_Index. 2657 GrB\_DOMAIN\_MISMATCH The domain given in parameter d does not match the element 2658 type of values. 2659

## 2660 Description

Creates a new matrix **A** of domain **D**(d) and dimension nrows × ncols. The new GraphBLAS matrix will be filled with the contents of the matrix pointed to by indptr, and indices, and values. The method returns a handle to the new matrix in **A**. The structure of the data being imported is defined by format, which must be equal to one of the values defined in Section 3.5.3.1. Details of the contents of indptr, indices and values for each supported format is given in Appendix B.

It is not an error to call this method more than once on the same output matrix; however, the handle to the previously created object will be overwritten.

### 2668 4.2.5.18 Matrix\_serializeSize: Compute the serialize buffer size

2669 Compute the buffer size (in bytes) necessary to serialize a GrB\_Matrix using GrB\_Matrix\_serialize.

# 2670 C Syntax

#### 2671 Parameters

size (OUT) Pointer to GrB\_Index value where size in bytes of serialized object will be written.

A (IN) A GraphBLAS matrix object.

#### 2675 Return Values

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GrB\_SUCCESS The operation completed successfully and the value pointed to by \*size has been computed and is ready to use.

Grb Panic Unknown internal error.

Grb\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_NULL\_POINTER size is NULL.

### Description

Returns the size in bytes of the data buffer necessary to serialize the GraphBLAS matrix object A.
Users may then allocate a buffer of size bytes to pass as a parameter to GrB\_Matrix\_serialize.

## 2684 4.2.5.19 Matrix\_serialize: Serialize a GraphBLAS matrix.

<sup>2685</sup> Serialize a GraphBLAS Matrix object into an opaque stream of bytes.

### 2686 C Syntax

#### 2687 Parameters

serialized\_data (INOUT) Pointer to the preallocated buffer where the serialized matrix will be written.

serialized\_size (INOUT) On input, the size in bytes of the buffer pointed to by serialized\_data.

On output, the number of bytes written to serialized\_data.

A (IN) A GraphBLAS matrix object.

#### 2693 Return Values

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GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output buffer serialized\_data and serialized\_size, have been computed and are ready to use.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

Grb\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_NULL\_POINTER serialized\_data or serialize\_size is NULL.

GrB\_UNINITIALIZED\_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB\_INSUFFICIENT\_SPACE The size of the buffer serialized\_data (provided as an input serialized\_size) was not large enough.

## 2710 Description

Serializes a GraphBLAS matrix object to an opaque buffer. To guarantee successful execution, the size of the buffer pointed to by serialized\_data, provided as an input by serialized\_size, must be of at least the number of bytes returned from GrB\_Matrix\_serializeSize. The actual size of the serialized matrix written to serialized\_data is provided upon completion as an output written to serialized\_size.

The contents of the serialized buffer are implementation defined. Thus, a serialized matrix created with one library implementation is not necessarily valid for descrialization with another implementation.

## 2719 4.2.5.20 Matrix\_deserialize: Deserialize a GraphBLAS matrix.

2720 Construct a new GraphBLAS matrix from a serialized object.

## 2721 C Syntax

#### 22 Parameters

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A (INOUT) On a successful return, contains a handle to the newly created Graph-BLAS matrix.

d (IN) the type of the matrix that was serialized in serialized\_data.

 ${\tt serialized\_data} \ \ ({\tt IN}) \ a \ pointer \ to \ a \ serialized \ Graph BLAS \ matrix \ created \ with \ {\tt GrB\_Matrix\_serialize}.$ 

serialized\_size (IN) the size of the buffer pointed to by serialized\_data in bytes.

#### 2728 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix A is ready to be used in the next method of the sequence.

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned if serialized\_data is invalid or corrupted.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT The GrB\_Type object has not been initialized by a call to GrB\_Type\_new 2736 (needed for user-defined types). 2737 GrB\_NULL\_POINTER serialized\_data or A is NULL. 2738 Grb DOMAIN MISMATCH The type given in d does not match the type of the matrix 2739 serialized in serialized data. 2740 Description 2741 Creates a new matrix A using the serialized matrix object pointed to by serialized\_data. The object 2742 pointed to by serialized\_data must have been created using the method GrB\_Matrix\_serialize. The 2743

domain of the matrix is given as an input in d, which must match the domain of the matrix serialized in serialized\_data. Note that for user-defined types, only the size of the type will be checked.

Since the format of a serialized matrix is implementation-defined, it is not guaranteed that a matrix

serialized in one library implementation can be describilized by another.

It is not an error to call this method more than once on the same output matrix; however, the handle to the previously created object will be overwritten.

# 2750 4.2.6 Descriptor methods

The methods in this section create and set values in descriptors. A descriptor is an opaque Graph-BLAS object the values of which are used to modify the behavior of GraphBLAS operations.

### 2753 4.2.6.1 Descriptor new: Create new descriptor

2754 Creates a new (empty or default) descriptor.

# 2755 C Syntax

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GrB\_Info GrB\_Descriptor\_new(GrB\_Descriptor \*desc);

## Parameters

desc (INOUT) On successful return, contains a handle to the newly created GraphBLAS descriptor.

## Return Value

GrB\_SUCCESS The method completed successfully.

2762 GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

GrB\_NULL\_POINTER desc pointer is NULL.

## 2765 Description

2766 Creates a new descriptor object and returns a handle to it in desc. A newly created descriptor can be populated by calls to Descriptor\_set.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

## 2770 4.2.6.2 Descriptor\_set: Set content of descriptor

2771 Sets the content for a field for an existing descriptor.

# 2772 C Syntax

```
GrB_Info GrB_Descriptor_set(GrB_Descriptor desc,

GrB_Desc_Field field,

GrB_Desc_Value val);
```

# 2776 Parameters

desc (IN) An existing GraphBLAS descriptor to be modified.

field (IN) The field being set.

val (IN) New value for the field being set.

## 2780 Return Values

GrB\_SUCCESS operation completed successfully.

GrB\_PANIC unknown internal error.

GrB\_OUT\_OF\_MEMORY not enough memory available for operation.

2784 GrB\_UNINITIALIZED\_OBJECT the desc parameter has not been initialized by a call to new.

GrB\_INVALID\_VALUE invalid value set on the field, or invalid field.

## 2786 Description

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For a given descriptor, the GrB\_Descriptor\_set method can be called for each field in the descriptor to set the value associated with that field. Valid values for the field parameter include the following:

Grb OUTP refers to the output parameter (result) of the operation.

GrB\_MASK refers to the mask parameter of the operation.

GrB\_INPO refers to the first input parameters of the operation (matrices and vectors).

GrB\_INP1 refers to the second input parameters of the operation (matrices and vectors).

<sup>2793</sup> Valid values for the val parameter are:

GrB\_STRUCTURE Use only the structure of the stored values of the corresponding mask (GrB\_MASK) parameter.

GrB\_COMP Use the complement of the corresponding mask (GrB\_MASK) parameter. When combined with GrB\_STRUCTURE, the complement of the structure of the mask is used without evaluating the values stored.

GrB\_TRAN Use the transpose of the corresponding matrix parameter (valid for input matrix parameters only).

GrB\_REPLACE When assigning the masked values to the output matrix or vector, clear the matrix first (or clear the non-masked entries). The default behavior is to leave non-masked locations unchanged. Valid for the GrB\_OUTP parameter only.

Descriptor values can only be set, and once set, cannot be cleared. As, in the case of GrB\_MASK, multiple values can be set and all will apply (for example, both GrB\_COMP and GrB\_STRUCTURE).

A value for a given field may be set multiple times but will have no additional effect. Fields that have no values set result in their default behavior, as defined in Section 3.6.

## 2809 4.2.7 free: Destroy an object and release its resources

Destroys a previously created GraphBLAS object and releases any resources associated with the object.

### 2812 C Syntax

```
grB_Info GrB_free(<GrB_Object> *obj);
```

#### 2814 Parameters

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obj (INOUT) An existing GraphBLAS object to be destroyed. The object must have been created by an explicit call to a GraphBLAS constructor. It can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type. On successful completion of GrB\_free, obj behaves as an uninitialized object.

#### 20 Return Values

GrB\_SUCCESS operation completed successfully

GrB\_PANIC unknown internal error. If this return value is encountered when in nonblocking mode, the error responsible for the panic condition could be from any method involved in the computation of the input object. The GrB\_error() method should be called for additional information.

### 2827 Description

GraphBLAS objects consume memory and other resources managed by the GraphBLAS runtime system. A call to GrB\_free frees those resources so they are available for use by other GraphBLAS objects.

The parameter passed into GrB\_free is a handle referencing a GraphBLAS opaque object of a data type from table 2.1. The object must have been created by an explicit call to a GraphBLAS constructor. The behavior of a program that calls GrB\_free on a pre-defined object is implementation defined.

After the GrB\_free method returns, the object referenced by the input handle is destroyed and the handle has the value GrB\_INVALID\_HANDLE. The handle can be used in subsequent GraphBLAS methods but only after the handle has been reinitialized with a call the the appropriate \_new or \_dup method.

Note that unlike other GraphBLAS methods, calling GrB\_free with an object with an invalid handle is legal. The system may attempt to free resources that might be associated with that object, if possible, and return normally.

When using GrB\_free it is possible to create a dangling reference to an object. This would occur when a handle is assigned to a second variable of the same opaque type. This creates two handles that reference the same object. If GrB\_free is called with one of the variables, the object is destroyed and the handle associated with the other variable no longer references a valid object. This is not an error condition that the implementation of the GraphBLAS API can be expected to catch, hence programmers must take care to prevent this situation from occurring.

# 2848 4.2.8 wait: Return once an object is either complete or materialized

Wait until method calls in a sequence put an object into a state of completion or materialization.

### 2850 C Syntax

GrB\_Info GrB\_wait(GrB\_Object obj, GrB\_WaitMode mode);

#### Parameters

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obj (INOUT) An existing GraphBLAS object. The object must have been created by an explicit call to a GraphBLAS constructor. Can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type. On successful return of GrB\_wait, the obj can be safely read from another thread (completion) or all computing to produce obj by all GraphBLAS operations in its sequence have finished (materialization).

mode (IN) Set's the mode for GrB\_wait for whether it is waiting for obj to be in the state of *completion* or *materialization*. Acceptable values are GrB\_COMPLETE or GrB\_MATERIALIZE.

#### Return values

2863 GrB\_SUCCESS operation completed successfully.

GrB\_INDEX\_OUT\_OF\_BOUNDS an index out-of-bounds execution error happened during completion of pending operations.

GrB\_OUT\_OF\_MEMORY and out-of-memory execution error happened during completion of pending operations.

GrB\_UNINITIALIZED\_OBJECT object has not been initialized by a call to the respective \*\_new, or other constructor, method.

GrB\_PANIC unknown internal error.

GrB\_INVALID\_VALUE method called with a GrB\_WaitMode other than GrB\_COMPLETE GrB\_MATERIALIZE.

## Description

On successful return from GrB\_wait(), the input object, obj is in one of two states depending on the mode of GrB\_wait:

- complete: obj can be used in a happens-before relation, so in a properly synchronized program it can be safely used as an IN or INOUT parameter in a GraphBLAS method call from another thread. This result occurs when the mode parameter is set to GrB\_COMPLETE.
- materialized: obj is complete, but in addition, no further computing will be carried out on behalf of obj and error information is available. This result occurs when the mode parameter is set to GrB MATERIALIZE.

Since in blocking mode OUT or INOUT parameters to any method call are materialized upon return, GrB\_wait(obj,mode) has no effect when called in blocking mode.

In non-blocking mode, the status of any pending method calls, other than those associated with producing the *complete* or *materialized* state of obj, are not impacted by the call to GrB\_wait(obj,mode).

Methods in the sequence for obj, however, most likely would be impacted by a call to GrB\_wait(obj,mode);
especially in the case of the *materialized* mode for which any computing on behalf of obj must be
finished prior to the return from GrB\_wait(obj,mode).

## 2889 4.2.9 error: Retrieve an error string

Retrieve an error-message about any errors encountered during the processing associated with an object.

# 2892 C Syntax

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```
GrB_Info GrB_error(const char **error, const GrB_Object obj);
```

## Parameters

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error (OUT) A pointer to a null-terminated string. The contents of the string are implementation defined.

obj (IN) An existing GraphBLAS object. The object must have been created by an explicit call to a GraphBLAS constructor. Can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, or type.

#### Return value

GrB\_SUCCESS operation completed successfully.

GrB\_UNINITIALIZED\_OBJECT object has not been initialized by a call to the respective \*\_new, or other constructor, method.

GrB\_PANIC unknown internal error.

## 2907 Description

This method retrieves a message related to any errors that were encountered during the last Graph-2908 BLAS method that had the opaque GraphBLAS object, obj, as an OUT or INOUT parameter. 2900 The function returns a pointer to a null-terminated string and the contents of that string are 2910 implementation-dependent. In particular, a null string (not a NULL pointer) is always a valid error 2911 string. The string that is returned is owned by obj and will be valid until the next time obj is 2912 used as an OUT or INOUT parameter or the object is freed by a call to GrB\_free(obj). This is a 2913 thread-safe function. It can be safely called by multiple threads for the same object in a race-free 2914 program. 2915

# 2916 4.3 GraphBLAS operations

The GraphBLAS operations are defined in the GraphBLAS math specification and summarized in Table 4.1. In addition to methods that implement these fundamental GraphBLAS operations, we support a number of variants that have been found to be especially useful in algorithm development.

A flowchart of the overall behavior of a GraphBLAS operation is shown in Figure 4.1.

### 2921 Domains and Casting

A GraphBLAS operation is only valid when the domains of the GraphBLAS objects are mathemat-2922 ically consistent. The C programming language defines implicit casts between built-in data types. 2923 For example, floats, doubles, and ints can be freely mixed according to the rules defined for implicit 2924 casts. It is the responsibility of the user to assure that these casts are appropriate for the algorithm 2925 in question. For example, a cast to int implies truncation of a floating point type. Depending on 2926 the operation, this truncation error could lead to erroneous results. Furthermore, casting a wider 2927 type onto a narrower type can lead to overflow errors. The GraphBLAS operations do not attempt 2928 to protect a user from these sorts of errors. 2929

When user-define types are involved, however, GraphBLAS requires strict equivalence between types and no casting is supported. If GraphBLAS detects these mismatches, it will return a domain mismatch error.

### 2933 Dimensions and Transposes

GraphBLAS operations also make assumptions about the numbers of dimensions and the sizes of vectors and matrices in an operation. An operation will test these sizes and report an error if they are not shape compatible. For example, when multiplying two matrices,  $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ , the number of rows of  $\mathbf{C}$  must equal the number of rows of  $\mathbf{A}$ , the number of columns of  $\mathbf{A}$  must match the number of rows of  $\mathbf{B}$ , and the number of columns of  $\mathbf{C}$  must match the number of columns of  $\mathbf{B}$ . This is the behavior expected given the mathematical definition of the operations.

For most of the GraphBLAS operations involving matrices, an optional descriptor can modify the matrix associated with an input GraphBLAS matrix object. For example, if an input matrix is an

Table 4.1: A mathematical notation for the fundamental GraphBLAS operations supported in this specification. Input matrices  $\mathbf{A}$  and  $\mathbf{B}$  may be optionally transposed (not shown). Use of an optional accumulate with existing values in the output object is indicated with  $\odot$ . Use of optional write masks and replace flags are indicated as  $\mathbf{C}\langle\mathbf{M},r\rangle$  when applied to the output matrix,  $\mathbf{C}$ . The mask controls which values resulting from the operation on the right-hand side are written into the output object (complement and structure flags are not shown). The "replace" option, indicated by specifying the r flag, means that all values in the output object are removed prior to assignment. If "replace" is not specified, only the values/locations computed on the right-hand side and allowed by the mask will be written to the output ("merge" mode).

| Operation Name  | ]   | Math | nematical N                     | Votation   |
|-----------------|---|------|---------------------------------|--|
| mxm             | $\mathbf{C}\langle \mathbf{M}, r \rangle$               | =    | <b>C</b> ⊙                      | $\mathbf{A} \oplus . \otimes \mathbf{B}$                                 |
| mxv             | $\mathbf{w}\langle\mathbf{m},r angle$                   | =    | $\mathbf{w}$                    |  |
| vxm             | $\mathbf{w}^T \langle \mathbf{m}^T, r \rangle$          | =    | $\mathbf{w}^T$ $\odot$          | $\mathbf{u}^T \oplus . \otimes \mathbf{A}$                               |
| eWiseMult       | $\mathbf{C}\langle\mathbf{M},r angle$                   | =    | $\mathbf{C}$ $\odot$            | $\mathbf{A} \otimes \mathbf{B}$  |
|                 | $\mathbf{w}\langle\mathbf{m},r\rangle$                  | =    | $\mathbf{w}$ $\odot$            | $\mathbf{u} \otimes \mathbf{v}$  |
| eWiseAdd        | $\mathbf{C}\langle\mathbf{M},r angle$                   | =    | $\mathbf{C}$ $\odot$            | $\mathbf{A} \oplus \mathbf{B}$   |
|                 | $\mathbf{w}\langle\mathbf{m},r\rangle$                  | =    | $\mathbf{w}$ $\odot$            | $\mathbf{u} \oplus \mathbf{v}$   |
| extract         | $\mathbf{C}\langle\mathbf{M},r angle$                   | =    | $\mathbf{C}$ $\odot$            | $\mathbf{A}(m{i},m{j})$  |
|                 | $\mathbf{w}\langle\mathbf{m},r angle$                   | =    | $\mathbf{w}$ $\odot$            | $\mathbf{u}(m{i})$   |
| assign          | $\mathbf{C}\langle\mathbf{M},r\rangle(\pmb{i},\pmb{j})$ | =    | $\mathbf{C}(m{i},m{j})$ $\odot$ | ) <b>A</b>   |
|                 | $\mathbf{w}\langle\mathbf{m},r\rangle(i)$               | =    | $\mathbf{w}(i)$ $\odot$         | $\mathbf{u}$   |
| reduce (row)    | $\mathbf{w}\langle\mathbf{m},r\rangle$                  | =    | $\mathbf{w}$ $\odot$            | $[\oplus_j \mathbf{A}(:,j)]$   |
| reduce (scalar) | s   | =    | $s$ $\odot$                     | $[\oplus_{i,j} \mathbf{A}(i,j)]$   |
|                 | s   | =    | $s$ $\odot$                     | $[\oplus_i \mathbf{u}(i)]$   |
| apply           | $\mathbf{C}\langle\mathbf{M},r angle$                   | =    | $\mathbf{C}$ $\odot$            | $f_u({f A})$   |
|                 | $\mathbf{w}\langle\mathbf{m},r\rangle$                  | =    | $\mathbf{w}$ $\odot$            | $f_u(\mathbf{u})$  |
| apply(indexop)  | $\mathbf{C}\langle\mathbf{M},r\rangle$                  | =    | <b>C</b> ⊙                      | $f_i(\mathbf{A}, \mathbf{ind}(\mathbf{A}), s)$                           |
|                 | $\mathbf{w}\langle\mathbf{m},r angle$                   | =    | $\mathbf{w}$ $\odot$            | $f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), s)$                           |
| select          | $\mathbf{C}\langle\mathbf{M},r angle$                   | =    | $\mathbf{C}$                    | $\mathbf{A}\langle f_i(\mathbf{A}, \mathbf{ind}(\mathbf{A}), s) \rangle$ |
|                 | $\mathbf{w}\langle\mathbf{m},r\rangle$                  | =    | $\mathbf{w}$ $\odot$            | $\mathbf{u}\langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), s) \rangle$ |
| transpose       | $\mathbf{C}\langle\mathbf{M},r angle$                   | =    | <b>C</b> •                      | $\mathbf{A}^T$   |
| kronecker       | $\mathbf{C}\langle\mathbf{M},r angle$                   | =    | $\mathbf{C}$ $\odot$            | $\mathbf{A} \otimes \mathbf{B}$  |

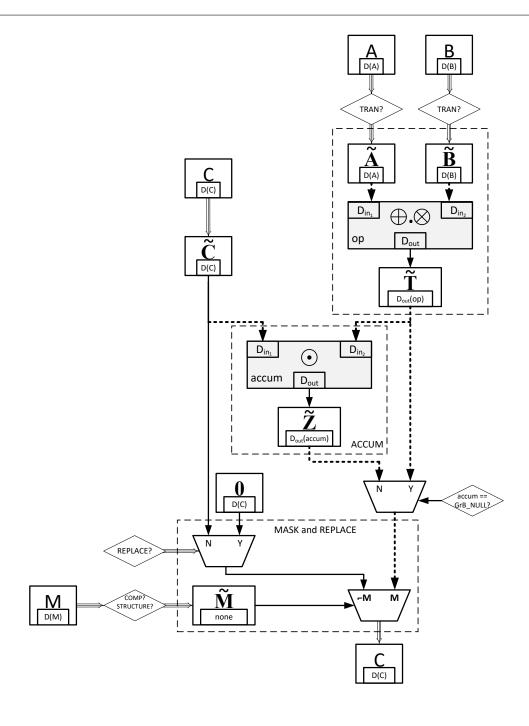


Figure 4.1: Flowchart for the GraphBLAS operations. Although shown specifically for the mxm operation, many elements are common to all operations: such as the "ACCUM" and "MASK and REPLACE" blocks. The triple arrows  $(\Rrightarrow)$  denote where "as if copy" takes place (including both collections and descriptor settings). The bold, dotted arrows indicate where casting may occur between different domains.

argument to a GraphBLAS operation and the associated descriptor indicates the transpose option, then the operation occurs as if on the transposed matrix. In this case, the relationships between the sizes in each dimension shift in the mathematically expected way.

### <sup>2945</sup> Masks: Structure-only, Complement, and Replace

When a GraphBLAS operation supports the use of an optional mask, that mask is specified through 2946 a GraphBLAS vector (for one-dimensional masks) or a GraphBLAS matrix (for two-dimensional 2947 masks). When a mask is used and the GTB\_STRUCTURE descriptor value is not set, it is applied 2948 to the result from the operation wherever the stored values in the mask evaluate to true. If the 2949 GrB\_STRUCTURE descriptor is set, the mask is applied to the result from the operation wherever the 2950 mask as a stored value (regardless of that value). Wherever the mask is applied, the result from 2951 the operation is either assigned to the provided output matrix/vector or, if a binary accumulation 2952 operation is provided, the result is accumulated into the corresponding elements of the provided 2953 output matrix/vector. 2954

Given a GraphBLAS vector  $\mathbf{v} = \langle D, N, \{(i, v_i)\} \rangle$ , a one-dimensional mask is derived for use in the operation as follows:

$$\mathbf{m} = \begin{cases} \langle N, \{ \mathbf{ind}(\mathbf{v}) \} \rangle, & \text{if GrB\_STRUCTURE is specified,} \\ \langle N, \{ i : (\mathsf{bool}) v_i = \mathsf{true} \} \rangle, & \text{otherwise} \end{cases}$$

where (bool) $v_i$  denotes casting the value  $v_i$  to a Boolean value (true or false). Likewise, given a GraphBLAS matrix  $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$ , a two-dimensional mask is derived for use in the operation as follows:

$$\mathbf{M} = \begin{cases} \langle M, N, \{\mathbf{ind}(\mathbf{A})\} \rangle, & \text{if GrB\_STRUCTURE is specified,} \\ \langle M, N, \{(i,j) : (\mathsf{bool}) A_{ij} = \mathsf{true} \} \rangle, & \text{otherwise} \end{cases}$$

where (bool)  $A_{ij}$  denotes casting the value  $A_{ij}$  to a Boolean value. (true or false)

In both the one- and two-dimensional cases, the mask may also have a subsequent complement operation applied (Section 3.5.4) as specified in the descriptor, before a final mask is generated for use in the operation.

When the descriptor of an operation with a mask has specified that the GrB\_REPLACE value is to be applied to the output (GrB\_OUTP), then anywhere the mask is not true, the corresponding location in the output is cleared.

#### Invalid and uninitialized objects

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Upon entering a GraphBLAS operation, the first step is a check that all objects are valid and initialized. (Optional parameters can be set to GrB\_NULL, which always counts as a valid object.) An invalid object is one that could not be computed due to a previous execution error. An unitialized object is one that has not yet been created by a corresponding new or dup method. Appropriate error codes are returned if an object is not initialized (GrB\_UNINITIALIZED\_OBJECT) or invalid (GrB\_INVALID\_OBJECT). To support the detection of as many cases of uninitialized objects as possible, it is strongly recommended to initialize all GraphBLAS objects to the predefined value GrB\_INVALID\_HANDLE at the point of their declaration, as shown in the following examples:

```
2979 GrB_Type type = GrB_INVALID_HANDLE;
2980 GrB_Semiring semiring = GrB_INVALID_HANDLE;
2981 GrB_Matrix matrix = GrB_INVALID_HANDLE;
```

### 2982 Compliance

We follow a *prescriptive* approach to the definition of the semantics of GraphBLAS operations.
That is, for each operation we give a recipe for producing its outcome. Any implementation that
produces the same outcome, and follows the GraphBLAS execution model (Section 2.5) and error
model (Section 2.6) is a conforming implementation.

# 2987 4.3.1 mxm: Matrix-matrix multiply

<sup>2988</sup> Multiplies a matrix with another matrix on a semiring. The result is a matrix.

# 2989 C Syntax

```
GrB_Info GrB_mxm(GrB_Matrix
                                                             С,
2990
                                  const GrB_Matrix
                                                             Mask,
2991
                                  const GrB_BinaryOp
                                                             accum,
2992
                                  const GrB_Semiring
                                                             op,
2993
                                  const GrB_Matrix
                                                             Α,
2994
                                  const GrB Matrix
                                                             В,
2995
                                  const GrB_Descriptor
                                                             desc);
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```

#### Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the matrix product. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.

- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The semiring used in the matrix-matrix multiply.
  - A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.
    - B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.
  - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|      |       |          |               | structure (pattern of stored values) of the |
| 3018 |       |          |               | input Mask matrix. The stored values are    |
|      |       |          |               | not examined.                               |
|      | Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
|      | Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |
|      | В     | GrB_INP1 | GrB_TRAN      | Use transpose of B for the operation.       |

#### Return Values

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| 3020<br>3021<br>3022<br>3023<br>3024 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 3025                                 | GrB_PANIC                | Unknown internal error.  |
| 3026<br>3027<br>3028<br>3029         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 3030                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.   |
| 3031                                 | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by   |

3033 GrB\_DIMENSION\_MISMATCH Mask and/or matrix dimensions are incompatible.

a call to new (or Matrix\_dup for matrix parameters).

GrB\_DOMAIN\_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

## 3038 Description

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GrB\_mxm computes the matrix product  $C = A \oplus . \otimes B$  or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot (A \oplus . \otimes B)$  (where matrices A and B can be optionally transposed). Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 3044 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 3046 Up to four argument matrices are used in the GrB\_mxm operation:
- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 3048 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 30. A =  $\langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 3050 4.  $B = \langle \mathbf{D}(B), \mathbf{nrows}(B), \mathbf{ncols}(B), \mathbf{L}(B) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the semiring.
- 3.  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the semiring.
- 4.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring.
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

- If any compatibility rule above is violated, execution of GrB\_mxm ends and the domain mismatch error listed above is returned.
- From the argument matrices, the internal matrices and mask used in the computation are formed  $(\leftarrow \text{denotes copy})$ :
- 3068 1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 3069 2. Two-dimensional mask, M, is computed from argument Mask as follows:
- 3070 (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- (b) If Mask  $\neq$  GrB NULL,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
  - (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Matrix  $\tilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}).$
- 3083 2.  $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$
- 3085 4.  $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{B}}).$
- $5. \ \mathbf{ncols}(\widetilde{\mathbf{A}}) = \mathbf{nrows}(\widetilde{\mathbf{B}}).$
- If any compatibility rule above is violated, execution of GrB\_mxm ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the matrix multiplication and any additional associated operations.
- 3092 We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the product of matrices  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{B}}$ .
- $\tilde{\mathbf{z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix  $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{B}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{B}}(: ,j)) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by

$$T_{ij} = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{B}}(:,j))} (\widetilde{\mathbf{A}}(i,k) \otimes \widetilde{\mathbf{B}}(k,j)),$$

where  $\oplus$  and  $\otimes$  are the additive and multiplicative operators of semiring op, respectively.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a standard matrix accumulate:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}}_{\mathsf{NULL}}$ , then  $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$ .

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• If  $\mathbf{z}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
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3107 
$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
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$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

# 3126 4.3.2 vxm: Vector-matrix multiply

Multiplies a (row) vector with a matrix on an semiring. The result is a vector.

## 3128 C Syntax

```
GrB_Info GrB_vxm(GrB_Vector
3129
                                                            W,
                                 const GrB_Vector
                                                            mask,
3130
                                 const GrB_BinaryOp
                                                            accum,
3131
                                 const GrB_Semiring
3132
                                                            op,
                                 const GrB_Vector
3133
                                                            u,
                                 const GrB Matrix
3134
                                                            Α,
                                 const GrB_Descriptor
                                                            desc);
```

#### Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the vector-matrix product. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) Semiring used in the vector-matrix multiply.
      - u (IN) The GraphBLAS vector holding the values for the left-hand vector in the multiplication.
      - A (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.
    - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input mask vector. The stored values are    |
|       |          |               | not examined.                               |
| mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |
| Α     | GrB INP1 | GrB TRAN      | Use transpose of A for the operation.       |

## Return Values

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| 3159<br>3160<br>3161<br>3162<br>3163 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector <b>w</b> is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|---|
| 3164                                 | GrB_PANIC                | Unknown internal error.   |
| 3165<br>3166<br>3167<br>3168         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.   |
| 3169                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.  |
| 3170<br>3171                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for matrix or vector parameters).  |
| 3172                                 | GrB_DIMENSION_MISMATCH   | Mask, vector, and/or matrix dimensions are incompatible.  |
| 3173<br>3174<br>3175<br>3176         | GrB_DOMAIN_MISMATCH      | The domains of the various vectors/matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).   |

# 3177 **Description**

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GrB\_vxm computes the vector-matrix product  $\mathbf{w}^T = \mathbf{u}^T \oplus . \otimes \mathsf{A}$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $\mathbf{w}^T = \mathbf{w}^T \odot \left( \mathbf{u}^T \oplus . \otimes \mathsf{A} \right)$  (where matrix A can be optionally transposed). Logically, this operation occurs in three steps:

**Setup** The internal vectors, matrices and mask used in the computation are formed and their domains/dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors or matrices are used in the GrB\_vxm operation:

```
1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

3187 2. 
$$\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$$

31.88 3. 
$$u = \langle \mathbf{D}(u), \mathbf{size}(u), \mathbf{L}(u) = \{(i, u_i)\} \rangle$$

3189 4. 
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the semiring.
- 3.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the semiring.
- 4.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring.
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

If any compatibility rule above is violated, execution of GrB\_vxm ends and the domain mismatch error listed above is returned.

From the argument vectors and matrices, the internal matrices and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
- i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3214 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

- 4. Matrix  $\tilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- The internal matrices and masks are checked for shape compatibility. The following conditions must hold:
- 3218 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}}).$
- 3219 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}).$
- 3220 3.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB\_vxm ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the vector-matrix multiplication and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the product of vector  $\tilde{\mathbf{u}}^T$  and matrix  $\tilde{\mathbf{A}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector  $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{ncols}(\tilde{\mathbf{A}}), \{(j,t_j) : \mathbf{ind}(\tilde{\mathbf{u}}) \cap \mathbf{ind}(\tilde{\mathbf{A}}(:,j)) \neq \emptyset \} \rangle$  is created.

  The value of each of its elements is computed by

$$t_j = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{A}}(:,j))} (\widetilde{\mathbf{u}}(k) \otimes \widetilde{\mathbf{A}}(k,j)),$$

- where  $\oplus$  and  $\otimes$  are the additive and multiplicative operators of semiring op, respectively.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:
- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}}_{\mathsf{NULL}}, \operatorname{then} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$

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• If accum is a binary operator, then  $\widetilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$egin{aligned} z_i &= \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), & ext{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})), \ \\ z_i &= \widetilde{\mathbf{w}}(i), & ext{if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))), \ \\ z_i &= \widetilde{\mathbf{t}}(i), & ext{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))), \end{aligned}$$

where  $\odot = \odot(accum)$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

```
\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.
```

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 3260 4.3.3 mxv: Matrix-vector multiply

Multiplies a matrix by a vector on a semiring. The result is a vector.

## 3262 C Syntax

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```
GrB_Info GrB_mxv(GrB_Vector
                                                             W,
3263
                                  const GrB_Vector
                                                             mask,
3264
                                  const GrB_BinaryOp
                                                             accum,
3265
                                  const GrB Semiring
3266
                                                             op,
                                  const GrB_Matrix
                                                             Α,
3267
                                  const GrB Vector
3268
                                                             u,
                                  const GrB Descriptor
                                                             desc);
3269
```

### Parameters

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w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the matrix-vector product. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain

| 3277<br>3278 | of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the |
|--------------|--|
| 3279         | dimensions of w), GrB_NULL should be specified.  |
|              |  |
| 3280         | $accum\ (IN)$ An optional binary operator used for accumulating entries into existing $w$  |
| 3281         | entries. If assignment rather than accumulation is desired, GrB_NULL should be   |
| 3282         | specified.   |

op (IN) Semiring used in the vector-matrix multiply.

- A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.
- u (IN) The GraphBLAS vector holding the values for the right-hand vector in the multiplication.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input mask vector. The stored values are    |
|       |          |               | not examined.                               |
| mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |
| Α     | GrB INP0 | GrB TRAN      | Use transpose of A for the operation.       |

## Return Values

| 3293 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-     |
|------|--------------------------|---|
| 3294 |                          | blocking mode, this indicates that the compatibility tests on di-   |
| 3295 |                          | mensions and domains for the input arguments passed successfully.   |
| 3296 |                          | Either way, output vector w is ready to be used in the next method  |
| 3297 |                          | of the sequence.  |
| 3298 | GrB_PANIC                | Unknown internal error.   |
| 3299 | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque   |
| 3300 |                          | GraphBLAS objects (input or output) is in an invalid state caused   |
| 3301 |                          | by a previous execution error. Call GrB_error() to access any error |
| 3302 |                          | messages generated by the implementation.                           |
| 3303 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.                      |
| 3304 | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by    |

a call to new (or dup for matrix or vector parameters).

3306 GrB\_DIMENSION\_MISMATCH Mask, vector, and/or matrix dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors/matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

### 3311 Description

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- GrB\_mxv computes the matrix-vector product  $w = A \oplus . \otimes u$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $w = w \odot (A \oplus . \otimes u)$  (where matrix A can be optionally transposed).

  Logically, this operation occurs in three steps:
- Setup The internal vectors, matrices and mask used in the computation are formed and their domains/dimensions are tested for compatibility.
- Compute The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 3319 Up to four argument vectors or matrices are used in the GrB\_mxv operation:
- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 3321 2. mask =  $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 3322 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 3323 4.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the semiring.
- 3.  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the semiring.
- 3330 4.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring.
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the semiring must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

  If any compatibility rule above is violated, execution of GrB\_mxv ends and the domain mismatch error listed above is returned.
- From the argument vectors and matrices, the internal matrices and mask used in the computation are formed ( $\leftarrow$  denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- (b) If mask  $\neq$  GrB NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- The internal matrices and masks are checked for shape compatibility. The following conditions must hold:
- 3352 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}}).$
- 3353 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$
- 3.  $\operatorname{size}(\widetilde{\mathbf{u}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB\_mxv ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the matrix-vector multiplication and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the product of matrix  $\tilde{\mathbf{A}}$  and vector  $\tilde{\mathbf{u}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector  $\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \{(i, t_i) : \mathbf{ind}(\widetilde{\mathbf{A}}(i, :)) \cap \mathbf{ind}(\widetilde{\mathbf{u}}) \neq \emptyset \} \rangle$  is created.

  The value of each of its elements is computed by

$$t_i = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{u}})} (\widetilde{\mathbf{A}}(i,k) \otimes \widetilde{\mathbf{u}}(k)),$$

where  $\oplus$  and  $\otimes$  are the additive and multiplicative operators of semiring op, respectively.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

• If  $\operatorname{accum} = \operatorname{GrB} \ \operatorname{NULL}$ , then  $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$ .

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If desc[GrB\_OUTP].GrB\_REPLACE is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 4.3.4 eWiseMult: Element-wise multiplication

Note: The difference between eWiseAdd and eWiseMult is not about the element-wise operation but how the index sets are treated. eWiseAdd returns an object whose indices are the "union" of the indices of the inputs whereas eWiseMult returns an object whose indices are the "intersection" of the indices of the inputs. In both cases, the passed semiring, monoid, or operator operates on the set of values from the resulting index set.

#### 3400 4.3.4.1 eWiseMult: Vector variant

Perform element-wise (general) multiplication on the intersection of elements of two vectors, producing a third vector as result.

## 3403 C Syntax

```
GrB_Info GrB_eWiseMult(GrB_Vector
                                                                    W,
3404
                                         const GrB_Vector
                                                                   mask,
3405
                                         const GrB_BinaryOp
                                                                    accum,
3406
                                         const GrB_Semiring
                                                                    op,
3407
                                         const GrB_Vector
3408
                                                                    u,
                                         const GrB_Vector
                                                                    v,
3409
                                         const GrB_Descriptor
                                                                    desc);
3410
3411
              GrB_Info GrB_eWiseMult(GrB_Vector
3412
                                                                    W,
                                         const GrB_Vector
                                                                   mask,
3413
                                         const GrB_BinaryOp
                                                                    accum,
3414
                                         const GrB_Monoid
                                                                    op,
3415
                                         const GrB Vector
                                                                    u,
3416
                                         const GrB Vector
                                                                    v,
3417
                                         const GrB_Descriptor
                                                                    desc);
3418
3419
              GrB_Info GrB_eWiseMult(GrB_Vector
3420
                                                                    W,
                                         const GrB_Vector
                                                                   mask,
3421
                                         const GrB_BinaryOp
3422
                                                                    accum,
                                         const GrB_BinaryOp
3423
                                                                    op,
                                         const GrB_Vector
                                                                    u,
3424
                                         const GrB_Vector
                                                                    v,
3425
                                         const GrB_Descriptor
                                                                    desc);
3426
```

#### 3427 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the element-wise operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing w

entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$ , used:

BinaryOp:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ .

Monoid:  $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ ; the identity element is ignored.

Semiring:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes(\mathsf{op}) \rangle$ ; the additive monoid is ignored.

- u (IN) The GraphBLAS vector holding the values for the left-hand vector in the operation.
- v (IN) The GraphBLAS vector holding the values for the right-hand vector in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input mask vector. The stored values are    |
|       |          |               | not examined.                               |
| mask  | GrB MASK | GrB COMP      | Use the complement of mask.                 |

#### 3456 Return Values

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GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully.

Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

3470 GrB\_DIMENSION\_MISMATCH Mask or vector dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

### 475 Description

This variant of GrB\_eWiseMult computes the element-wise "product" of two GraphBLAS vectors:  $w = u \otimes v$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $w = w \odot (u \otimes v)$ . Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

3481 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors are used in the GrB\_eWiseMult operation:

- 3484 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 3485 2. mask =  $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 3.  $u = \langle \mathbf{D}(u), \mathbf{size}(u), \mathbf{L}(u) = \{(i, u_i)\} \rangle$
- 3487 4.  $\mathbf{v} = \langle \mathbf{D}(\mathbf{v}), \mathbf{size}(\mathbf{v}), \mathbf{L}(\mathbf{v}) = \{(i, v_i)\} \rangle$

The argument vectors, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 3492 2.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3.  $\mathbf{D}(\mathbf{v})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 4.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_eWiseMult ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- 3507 (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- 3508 (b) If  $mask \neq GrB\_NULL$ ,
- i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- 3510 ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
  - (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4. Vector  $\tilde{\mathbf{v}} \leftarrow \mathbf{v}$ .
- The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}}) = \operatorname{size}(\widetilde{\mathbf{u}}) = \operatorname{size}(\widetilde{\mathbf{v}}).$
- If any compatibility rule above is violated, execution of GrB\_eWiseMult ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "product" and any additional associated operations.
- We describe this in terms of two intermediate vectors:
  - $\tilde{\mathbf{t}}$ : The vector holding the element-wise "product" of  $\tilde{\mathbf{u}}$  and vector  $\tilde{\mathbf{v}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector  $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\tilde{\mathbf{u}}), \{(i, t_i) : \mathbf{ind}(\tilde{\mathbf{u}}) \cap \mathbf{ind}(\tilde{\mathbf{v}}) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by:

$$t_i = (\widetilde{\mathbf{u}}(i) \otimes \widetilde{\mathbf{v}}(i)), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}))$$

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$ 

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• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$
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$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
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$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector  $\mathbf{w}$ , using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 4.3.4.2 eWiseMult: Matrix variant

Perform element-wise (general) multiplication on the intersection of elements of two matrices, producing a third matrix as result.

## 3558 C Syntax

```
GrB_Info GrB_eWiseMult(GrB_Matrix
                                                                    С,
3559
                                         const GrB_Matrix
                                                                    Mask,
3560
                                         const GrB_BinaryOp
                                                                    accum,
3561
                                         const GrB_Semiring
                                                                    op,
3562
                                         const GrB_Matrix
                                                                    Α,
3563
                                         const GrB_Matrix
                                                                    В,
3564
                                         const GrB Descriptor
                                                                    desc);
3565
3566
              GrB Info GrB eWiseMult(GrB Matrix
                                                                    C,
3567
                                         const GrB Matrix
                                                                   Mask,
3568
                                         const GrB BinaryOp
                                                                    accum,
3569
                                         const GrB_Monoid
                                                                    op,
3570
                                         const GrB_Matrix
                                                                    Α,
3571
                                         const GrB_Matrix
                                                                    В,
3572
                                         const GrB_Descriptor
                                                                    desc);
3573
3574
              GrB_Info GrB_eWiseMult(GrB_Matrix
                                                                    C,
3575
                                         const GrB_Matrix
                                                                   Mask,
3576
                                         const GrB_BinaryOp
                                                                    accum,
3577
                                         const GrB_BinaryOp
                                                                    op,
3578
                                         const GrB_Matrix
                                                                    Α,
3579
                                         const GrB Matrix
                                                                    В,
3580
                                         const GrB_Descriptor
                                                                    desc);
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```

### Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the element-wise operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$ , used:

| 3598 | BinaryOp: $F_b = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op), \mathbf{D}_{in_2}(op), \bigcirc(op) \rangle$ .                     |
|------|--|
| 3599 | Monoid: $F_b = \langle \mathbf{D}(op), \mathbf{D}(op), \mathbf{D}(op), \bigcirc(op) \rangle$ ; the identity element is ig-               |
| 3600 | nored.   |
| 3601 | Semiring: $F_b = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op), \mathbf{D}_{in_2}(op), \otimes (op) \rangle$ ; the additive monoid |
| 3602 | is ignored.  |
| 3603 | A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the   |
| 3604 | operation.   |
| 3004 | oporation  |
| 3605 | B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the  |
| 3606 | operation.   |
| 3607 | desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL   |
| 3608 | should be specified. Non-default field/value pairs are listed as follows:  |
| 3609 |  |
|      | Param Field Value Description  |
|      | C GrB_OUTP GrB_REPLACE Output matrix C is cleared (all elements  |
|      | removed) before the result is stored in it.  |

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input Mask matrix. The stored values are    |
|       |          |               | not examined.                               |
| Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
| Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |
| В     | GrB_INP1 | GrB_TRAN      | Use transpose of B for the operation.       |

# 3611 Return Values

| 3612         | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-   |
|--------------|--------------------------|---|
| 3613         |                          | blocking mode, this indicates that the compatibility tests on di-   |
| 3614         |                          | mensions and domains for the input arguments passed successfully.   |
| 3615         |                          | Either way, output matrix C is ready to be used in the next method  |
| 3616         |                          | of the sequence.  |
| 3617         | GrB_PANIC                | Unknown internal error.   |
| 3618         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque   |
| 3619         |                          | GraphBLAS objects (input or output) is in an invalid state caused   |
| 3620         |                          | by a previous execution error. Call GrB_error() to access any error   |
| 3621         |                          | messages generated by the implementation.   |
| 3622         | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.  |
| 3623<br>3624 | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters). |
| 2625         | GrB DIMENSION MISMATCH   | Mask and/or matrix dimensions are incompatible.   |
| 3625         | GRE_DIMENSION_MISMATCH   | Mask and/or matrix dimensions are incompatible.   |

GrB\_DOMAIN\_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

## 3630 Description

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This variant of GrB\_eWiseMult computes the element-wise "product" of two GraphBLAS matrices:  $C = A \otimes B$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot (A \otimes B)$ .

Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 3636 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- Up to four argument matrices are used in the GrB\_eWiseMult operation:
- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 3640 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3641 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 4.  $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3648 3.  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 4.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any

- compatibility rule above is violated, execution of GrB\_eWiseMult ends and the domain mismatch error listed above is returned.
- From the argument matrices, the internal matrices and mask used in the computation are formed  $(\leftarrow \text{denotes copy})$ :
- 3660 1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- $\mathbf{M}$ , is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
  - (b) If Mask  $\neq$  GrB\_NULL,
    - i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE}$  is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
    - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
  - (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Matrix  $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{M}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}) = \operatorname{nrows}(\widetilde{\mathbf{C}}).$ 
  - 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) = \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB\_eWiseMult ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "product" and any additional associated operations.

  We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the element-wise product of  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{B}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.
- The intermediate matrix  $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}}) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by

$$T_{ij} = (\widetilde{\mathbf{A}}(i,j) \otimes \widetilde{\mathbf{B}}(i,j)), \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})$$

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a standard matrix accumulate:

• If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathsf{then} \; \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$ 

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• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
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$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 4.3.5 eWiseAdd: Element-wise addition

Note: The difference between eWiseAdd and eWiseMult is not about the element-wise operation but how the index sets are treated. eWiseAdd returns an object whose indices are the "union" of the indices of the inputs whereas eWiseMult returns an object whose indices are the "intersection" of the indices of the inputs. In both cases, the passed semiring, monoid, or operator operates on the set of values from the resulting index set.

#### 3720 4.3.5.1 eWiseAdd: Vector variant

Perform element-wise (general) addition on the elements of two vectors, producing a third vector as result.

## 3723 C Syntax

```
GrB_Info GrB_eWiseAdd(GrB_Vector
3724
                                                                  W,
                                       const GrB_Vector
                                                                  mask,
3725
                                       const GrB_BinaryOp
                                                                  accum,
3726
                                       const GrB_Semiring
                                                                  op,
3727
                                       const GrB_Vector
3728
                                                                  u,
                                       const GrB_Vector
                                                                  v,
3729
                                       const GrB_Descriptor
                                                                  desc);
3730
3731
             GrB_Info GrB_eWiseAdd(GrB_Vector
3732
                                                                  w,
                                       const GrB_Vector
                                                                  mask,
3733
                                       const GrB BinaryOp
3734
                                                                  accum,
                                       const GrB_Monoid
3735
                                                                  op,
                                       const GrB Vector
3736
                                                                  u,
                                       const GrB Vector
                                                                  v,
3737
                                       const GrB_Descriptor
                                                                  desc);
3738
3739
             GrB_Info GrB_eWiseAdd(GrB_Vector
3740
                                                                  W,
                                       const GrB_Vector
                                                                  mask,
3741
                                       const GrB_BinaryOp
3742
                                                                  accum,
                                       const GrB_BinaryOp
3743
                                                                  op,
                                       const GrB_Vector
                                                                  u,
3744
                                       const GrB_Vector
3745
                                                                  v,
                                       const GrB_Descriptor
                                                                  desc);
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```

#### 3747 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the element-wise operation. On output, this vector holds the results of the operation.
  - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing w

entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "sum" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \oplus \rangle$ , used:

BinaryOp:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ .

Monoid:  $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ ; the identity element is ignored.

Semiring:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigoplus(\mathsf{op}) \rangle$ ; the multiplicative binary op and additive identity are ignored.

- u (IN) The GraphBLAS vector holding the values for the left-hand vector in the operation.
- v (IN) The GraphBLAS vector holding the values for the right-hand vector in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input mask vector. The stored values are    |
|       |          |               | not examined.                               |
| mask  | GrB MASK | GrB COMP      | Use the complement of mask.                 |

#### 3776 Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully.

Either way, output vector w is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

3790 GrB\_DIMENSION\_MISMATCH Mask or vector dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

## 3795 Description

This variant of GrB\_eWiseAdd computes the element-wise "sum" of two GraphBLAS vectors:  $w = u \oplus v$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $w = w \odot (u \oplus v)$ . Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

3801 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

3803 Up to four argument vectors are used in the GrB\_eWiseAdd operation:

- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 3805 2. mask =  $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 3806 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- 3807 4.  $\mathbf{v} = \langle \mathbf{D}(\mathbf{v}), \mathbf{size}(\mathbf{v}), \mathbf{L}(\mathbf{v}) = \{(i, v_i)\} \rangle$

The argument vectors, the "sum" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 3812 2.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3813 3.  $\mathbf{D}(\mathsf{v})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 3814 4.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5.  $\mathbf{D}(\mathbf{u})$  and  $\mathbf{D}(\mathbf{v})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 6. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_eWiseAdd ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
  - (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3833 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4. Vector  $\tilde{\mathbf{v}} \leftarrow \mathbf{v}$ .
- The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:
- 3837 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{v}}).$
- If any compatibility rule above is violated, execution of GrB\_eWiseAdd ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "sum" and any additional associated operations.
- 3843 We describe this in terms of two intermediate vectors:
  - $\widetilde{\mathbf{t}}$ : The vector holding the element-wise "sum" of  $\widetilde{\mathbf{u}}$  and vector  $\widetilde{\mathbf{v}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector  $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\tilde{\mathbf{u}}), \{(i, t_i) : \mathbf{ind}(\tilde{\mathbf{u}}) \cup \mathbf{ind}(\tilde{\mathbf{v}}) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by:

$$t_i = (\widetilde{\mathbf{u}}(i) \oplus \widetilde{\mathbf{v}}(i)), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}))$$
3849
$$t_i = \widetilde{\mathbf{u}}(i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) - (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}})))$$

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$$t_i = \widetilde{\mathbf{v}}(i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{v}}) - (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}})))$$

where the difference operator in the previous expressions refers to set difference.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a *standard vector accumulate*:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \operatorname{then} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$ 

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• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$\begin{split} z_i &= \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})), \\ z_i &= \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))), \\ z_i &= \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))), \end{split}$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

### 4.3.5.2 eWiseAdd: Matrix variant

Perform element-wise (general) addition on the elements of two matrices, producing a third matrix as result.

## 3884 C Syntax

```
3885
              GrB_Info GrB_eWiseAdd(GrB_Matrix
                                                                   С,
                                        const GrB_Matrix
                                                                   Mask,
3886
                                        const GrB_BinaryOp
                                                                   accum,
3887
                                        const GrB_Semiring
                                                                   op,
3888
                                        const GrB_Matrix
                                                                   Α,
3889
                                        const GrB_Matrix
                                                                   Β,
3890
                                        const GrB Descriptor
                                                                   desc);
3891
3892
              GrB Info GrB eWiseAdd(GrB Matrix
                                                                   С,
3893
                                        const GrB Matrix
                                                                   Mask,
3894
                                        const GrB BinaryOp
                                                                   accum,
3895
                                       const GrB_Monoid
                                                                   op,
3896
                                       const GrB_Matrix
                                                                   Α,
3897
                                       const GrB_Matrix
                                                                   В,
3898
                                       const GrB_Descriptor
                                                                   desc);
3899
3900
              GrB_Info GrB_eWiseAdd(GrB_Matrix
                                                                   С,
3901
                                       const GrB_Matrix
                                                                   Mask,
3902
                                       const GrB_BinaryOp
                                                                   accum,
3903
                                        const GrB_BinaryOp
                                                                   op,
3904
                                        const GrB_Matrix
                                                                   Α,
3905
                                       const GrB Matrix
                                                                   В,
3906
                                       const GrB_Descriptor
                                                                   desc);
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```

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the element-wise operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The semiring, monoid, or binary operator used in the element-wise "sum" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \oplus \rangle$ , used:

| 3924 | Binar           | yOp: F              | $F_b = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op) \rangle$ | $ p), \mathbf{D}_{in_2}(op), \bigcirc(op)  angle.$   |
|------|-----------------|---------------------|---|--|
| 3925 | Moi             | noid: $F$           | $T_b = \langle \mathbf{D}(op), \mathbf{D}(op), \mathbf{I} \rangle$  | $O(op), \bigcirc(op)$ ; the identity element is ig-  |
| 3926 |                 | n                   | ored.   |  |
| 3927 | Semi            | ring: $F$           | $F_b = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op) \rangle$ | $ \mathbf{p}\rangle, \mathbf{D}_{in_2}(op), \bigoplus (op)\rangle;$ the multiplicative bi- |
| 3928 |                 | n                   | ary op and additive id  | lentity are ignored.   |
| 2000 | Λ (IN) The C    | <sup>1</sup> ranh R | IAS matrix holding  | the values for the left-hand matrix in the   |
| 3929 | ` /             | ларпо               | LAS matrix holding  | the values for the left-hand matrix in the   |
| 3930 | operation.      |                     |   |  |
| 3931 | B (IN) The G    | [raphB]             | LAS matrix holding t  | he values for the right-hand matrix in the   |
| 3932 | operation.      | -                   | o .   |  |
|      | · F             |                     |   |  |
| 3933 | desc (IN) An op | tional o            | peration descriptor. I  | f a default descriptor is desired, GrB_NULL  |
| 3934 | should be s     | specifie            | d. Non-default field/v  | alue pairs are listed as follows:  |
| 3935 |                 |                     |   |  |
|      | Param Field     |                     | Value   | Description  |
|      | C C D           | OUTD                | C.D. DEDLACE  | O + + + C · 1 1/11 1 +   |

|   | Param | Field    | Value         | Description                                 |
|---|-------|----------|---------------|---|
|   | С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|   |       |          |               | removed) before the result is stored in it. |
|   | Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| - |       |          |               | structure (pattern of stored values) of the |
| 5 |       |          |               | input Mask matrix. The stored values are    |
|   |       |          |               | not examined.                               |
|   | Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
|   | Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |
|   | В     | GrB_INP1 | GrB_TRAN      | Use transpose of B for the operation.       |

# 3937 Return Values

| 3938<br>3939<br>3940<br>3941<br>3942 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 3943                                 | GrB_PANIC                | Unknown internal error.  |
| 3944<br>3945<br>3946<br>3947         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 3948                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.   |
| 3949<br>3950                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to <code>new</code> (or <code>Matrix_dup</code> for matrix parameters).  |
| 3951                                 | GrB_DIMENSION_MISMATCH   | Mask and/or matrix dimensions are incompatible.  |

GrB\_DOMAIN\_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

# 3956 Description

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This variant of GrB\_eWiseAdd computes the element-wise "sum" of two GraphBLAS matrices:  $C = A \oplus B$ , or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot (A \oplus B)$ . Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 3962 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- Up to four argument matrices are used in the GrB\_eWiseAdd operation:
- 3965 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 3966 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3967 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 3968 4.  $B = \langle \mathbf{D}(B), \mathbf{nrows}(B), \mathbf{ncols}(B), \mathbf{L}(B) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the "sum" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 3973 2.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3974 3.  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 3975 4.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5.  $\mathbf{D}(\mathsf{A})$  and  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 6. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_eWiseAdd ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed  $\leftarrow$  denotes copy):

1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- <sup>3988</sup> 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:
- 3989 (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- 3991 (b) If  $Mask \neq GrB\_NULL$ ,
- 3992 i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3997 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}.$
- 3998 4. Matrix  $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}).$
- 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) = \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB\_eWiseAdd ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "sum" and any additional associated operations.

  We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the element-wise sum of  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{B}}$ .
  - $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix  $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}) \cup \mathbf{ind}(\widetilde{\mathbf{B}}) \neq \emptyset \} \rangle$  is created. The value of each of its elements is computed by

$$T_{ij} = (\widetilde{\mathbf{A}}(i,j) \oplus \widetilde{\mathbf{B}}(i,j)), \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})$$

$$T_{ij} = \widetilde{\mathbf{A}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{A}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

$$T_{ij} = \widetilde{\mathbf{B}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{A}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

$$T_{ij} = \widetilde{\mathbf{B}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{B}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

where the difference operator in the previous expressions refers to set difference.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

• If  $accum = GrB\_NULL$ , then  $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$ .

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• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
4026
4027 
$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
4028
4029 
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\dot{\mathbf{Z}}$  are written into the final result matrix  $\mathsf{C}$ , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4046 4.3.6 extract: Selecting sub-graphs

4047 Extract a subset of a matrix or vector.

### 4048 4.3.6.1 extract: Standard vector variant

Extract a sub-vector from a larger vector as specified by a set of indices. The result is a vector whose size is equal to the number of indices.

## 4051 C Syntax

```
GrB_Info GrB_extract(GrB_Vector
4052
                                                                  W,
                                      const GrB_Vector
                                                                  mask,
4053
                                      const GrB_BinaryOp
                                                                  accum,
4054
                                      const GrB_Vector
                                                                  u,
4055
                                      const GrB_Index
                                                                 *indices,
4056
                                      GrB_Index
                                                                  nindices,
4057
                                      const GrB_Descriptor
                                                                  desc);
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```

#### Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the extract operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - u (IN) The GraphBLAS vector from which the subset is extracted.
- indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations of elements from u that are extracted. If all elements of u are to be extracted in order from 0 to nindices 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation.
- nindices (IN) The number of values in indices array. Must be equal to size(w).

| desc | (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL |
|------|---|
|      | should be specified. Non-default field/value pairs are listed as follows:           |

| 4081 |       |          |               |   |
|------|-------|----------|---------------|---|
|      | Param | Field    | Value         | Description                                 |
|      | W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 4082 |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input mask vector. The stored values are    |
|      |       |          |               | not examined.                               |
|      | mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |

## Return Values

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| 4084<br>4085<br>4086<br>4087<br>4088 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 4089                                 | GrB_PANIC                | Unknown internal error.  |
| 4090<br>4091<br>4092<br>4093         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 4094                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for operation.   |
| 4095<br>4096                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).   |
| 4097<br>4098                         | GrB_INDEX_OUT_OF_BOUNDS  | A value in indices is greater than or equal to $\mathbf{size}(u).$ In non-blocking mode, this error can be deferred.   |
| 4099                                 | GrB_DIMENSION_MISMATCH   | $mask \ \mathrm{and} \ w \ \mathrm{dimensions} \ \mathrm{are} \ \mathrm{incompatible}, \ \mathrm{or} \ nindices \neq \mathbf{size}(w).$  |
| 4100<br>4101<br>4102<br>4103         | GrB_DOMAIN_MISMATCH      | The domains of the various vectors are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).   |
| 4104                                 | GrB_NULL_POINTER         | Argument row_indices is a NULL pointer.  |

# Description

This variant of  $GrB\_extract$  computes the result of extracting a subset of locations from a Graph-BLAS vector in a specific order: w = u(indices); or, if an optional binary accumulation operator 4108 ( $\odot$ ) is provided,  $w = w \odot u$ (indices). More explicitly:

$$\begin{aligned} \mathbf{w}(i) &= \quad \mathbf{u}(\mathsf{indices}[i]), \ \forall \ i: \ 0 \leq i < \mathsf{nindices}, \ \ \mathsf{or} \\ \mathbf{w}(i) &= \mathbf{w}(i) \odot \mathbf{u}(\mathsf{indices}[i]), \ \forall \ i: \ 0 \leq i < \mathsf{nindices} \end{aligned}$$

- Logically, this operation occurs in three steps:
- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4113 Compute The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 4115 Up to three argument vectors are used in this GrB\_extract operation:
- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 4117 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument vectors and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 4123 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}(\mathbf{u})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_extract ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vectors, mask, and index array used in the computation are formed ( $\leftarrow$  denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .

- (b) If mask  $\neq$  GrB\_NULL,
- i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE} \ \mathrm{is} \ \mathrm{set}, \ \mathrm{then} \ \widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i: i \in \mathbf{ind}(\mathsf{mask})\} \rangle,$
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- 4. The internal index array,  $\tilde{I}$ , is computed from argument indices as follows:
- (a) If indices = GrB\_ALL, then  $\tilde{I}[i] = i, \forall i : 0 \le i < \text{nindices}$ .
- (b) Otherwise,  $\widetilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nindices}.$
- The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 2. nindices =  $\mathbf{size}(\widetilde{\mathbf{w}})$ .
- If any compatibility rule above is violated, execution of GrB\_extract ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- $^{4153}$  We are now ready to carry out the extract and any additional associated operations. We describe  $^{4154}$  this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the extraction from  $\tilde{\mathbf{u}}$  in their destination locations relative to  $\tilde{\mathbf{w}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, \widetilde{\mathbf{u}}(\widetilde{\boldsymbol{I}}[i])) \ \forall \ i, 0 \leq i < \mathsf{nindices} : \widetilde{\boldsymbol{I}}[i] \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

- At this point, if any value in  $\tilde{I}$  is not in the valid range of indices for vector  $\tilde{\mathbf{u}}$ , the execution of GrB\_extract ends and the index-out-of-bounds error listed above is generated. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the
- result vector, w, is invalid from this point forward in the sequence.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:
- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

```
z_{i} = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),
z_{i} = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),
z_{i} = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),
z_{i} = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),
```

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector  $\mathbf{w}$ , using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

### 4190 4.3.6.2 extract: Standard matrix variant

Extract a sub-matrix from a larger matrix as specified by a set of row indices and a set of column indices. The result is a matrix whose size is equal to size of the sets of indices.

### 4193 C Syntax

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```
C,
             GrB Info GrB extract(GrB Matrix
4194
                                      const GrB_Matrix
                                                               Mask,
4195
                                      const GrB_BinaryOp
                                                               accum,
4196
                                      const GrB_Matrix
                                                               Α,
4197
                                      const GrB_Index
                                                              *row_indices,
4198
                                      GrB_Index
                                                               nrows,
4199
                                      const GrB_Index
                                                              *col_indices,
4200
                                      GrB_Index
                                                               ncols,
4201
                                      const GrB Descriptor
                                                               desc);
4202
```

# 4203 Parameters

| 4204         | C           | '                                  |                         | rix. On input, the matrix provides values lt of the extract operation. On output, the |
|--------------|-------------|------------------------------------|-------------------------|---|
| 4205<br>4206 |             |                                    | esults of the operation |   |
|              | NA I        | (INI) A 1 (6                       |                         |   |
| 4207         | iviask      | . , –                              |                         | atrols which results from this operation are mask dimensions must match those of the  |
| 4208<br>4209 |             |                                    | =                       | criptor is <i>not</i> set for the mask, the domain                                    |
| 4210         |             |                                    |                         | ol or any of the predefined "built-in" types  |
| 4211         |             |                                    |                         | sired (i.e., a mask that is all true with the   |
| 4212         |             |                                    | GrB_NULL should be      | •   |
|              |             | (INI) A                            | 1.                      |   |
| 4213         | accum       | ` / –                              |                         | for accumulating entries into existing C  |
| 4214         |             | entries. It assignments specified. | nent ratner than accu   | imulation is desired, GrB_NULL should be  |
| 4215         |             | specified.                         |                         |   |
| 4216         | А           | (IN) The GraphBI                   | LAS matrix from whi     | ch the subset is extracted.   |
| 4217         | row indices | (IN) Pointer to the                | e ordered set (array)   | of indices corresponding to the rows of A   |
| 4218         | _           | , ,                                | , - ,                   | lements in all rows of A are to be extracted  |
| 4219         |             | in order, $GrB\_ALL$               | should be specified.    | Regardless of execution mode and return   |
| 4220         |             | value, this array r                | may be manipulated      | by the caller after this operation returns  |
| 4221         |             | without affecting a                | any deferred computa    | ations for this operation.  |
| 4222         | nrows       | (IN) The number of                 | of values in the row_i  | indices array. Must be equal to $\mathbf{nrows}(C)$ .                                 |
| 4223         | col indices | (IN) Pointer to th                 | e ordered set (array)   | ) of indices corresponding to the columns   |
| 4224         | a.ooo       | ` '                                | ,                       | ed. If elements in all columns of A are to  |
| 4225         |             |                                    |                         | hould be specified. Regardless of execution   |
| 4226         |             | mode and return                    | value, this array ma    | y be manipulated by the caller after this   |
| 4227         |             | operation returns                  | without affecting any   | deferred computations for this operation.   |
| 4228         | ncols       | (IN) The number of                 | of values in the col_i  | ndices array. Must be equal to $\mathbf{ncols}(C).$                                   |
| 4229         | desc        | (IN) An optional of                | peration descriptor. I  | f a default descriptor is desired, GrB_NULL   |
| 4230         |             | . , –                              | = =                     | value pairs are listed as follows:  |
| 4231         |             | -                                  | ,                       | -   |
|              | Pa          | ram Field                          | Value                   | Description   |
|              | С           | GrB_OUTP                           | GrB_REPLACE             | Output matrix C is cleared (all elements  |
|              |             |                                    |                         | removed) before the result is stored in it.   |
|              | Ma          | sk GrB_MASK                        | GrB_STRUCTURE           | The write mask is constructed from the  |
| 4232         |             |                                    |                         | structure (pattern of stored values) of the   |
|              |             |                                    |                         | input Mask matrix. The stored values are not examined.                                |
|              | Ma          | sk GrB_MASK                        | GrB_COMP                | Use the complement of Mask.   |
|              | ivia        | יייי פיים ויוירטוע                 | OLD_COMI                | obe the complement of Mask.   |

Use transpose of A for the operation.

 $\mathsf{GrB} \mathsf{\_TRAN}$ 

GrB\_INP0

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## Return Values

| 4234<br>4235<br>4236<br>4237<br>4238 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 4239                                 | GrB_PANIC                | Unknown internal error.  |
| 4240<br>4241<br>4242<br>4243         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 4244                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.   |
| 4245<br>4246                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).  |
| 4247<br>4248<br>4249                 | GrB_INDEX_OUT_OF_BOUNDS  | A value in row_indices is greater than or equal to $\mathbf{nrows}(A)$ , or a value in $\mathbf{col}_i$ ndices is greater than or equal to $\mathbf{ncols}(A)$ . In non-blocking mode, this error can be deferred.   |
| 4250<br>4251                         | GrB_DIMENSION_MISMATCH   | Mask and C dimensions are incompatible, nrows $\neq$ $\mathbf{nrows}(C)$ , or $\mathbf{ncols} \neq \mathbf{ncols}(C)$ .  |
| 4252<br>4253<br>4254<br>4255         | GrB_DOMAIN_MISMATCH      | The domains of the various matrices are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).  |
| 4256<br>4257                         | GrB_NULL_POINTER         | Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.  |

## 4258 Description

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This variant of GrB\_extract computes the result of extracting a subset of locations from specified rows and columns of a GraphBLAS matrix in a specific order:  $C = A(row\_indices, col\_indices)$ ; or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot A(row\_indices, col\_indices)$ .

More explicitly (not accounting for an optional transpose of A):

$$\mathsf{C}(i,j) = \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_indices}[j]) \ \forall \ i,j \ : \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols}, \ \mathsf{or} \\ \mathsf{C}(i,j) = \mathsf{C}(i,j) \odot \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_indices}[j]) \ \forall \ i,j \ : \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \\ \mathsf{ncols}(i,j) = \mathsf{C}(i,j) \odot \mathsf{A}(\mathsf{ncols}(i), \mathsf{ncols}(i), \mathsf{$$

Logically, this operation occurs in three steps:

**Setup** The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

- 4267 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 4269 Up to three argument matrices are used in the GrB\_extract operation:
- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 2. Mask =  $\langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument matrices and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{A})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_extract ends and the domain mismatch error listed above is returned.
- From the arguments, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):
- 1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

- 2. Two-dimensional mask, M, is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- (b) If Mask  $\neq$  GrB\_NULL,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
- 4295 ii. Otherwise,  $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
  - 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB} \ \mathsf{INP0}].\mathsf{GrB} \ \mathsf{TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}.$

- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- 4300 (a) If row\_indices = GrB\_ALL, then  $\tilde{I}[i] = i, \forall i : 0 \le i < \text{nrows}$ .
- 4301 (b) Otherwise,  $\tilde{I}[i] = \text{row\_indices}[i], \forall i : 0 \leq i < \text{nrows}.$
- 5. The internal column index array,  $\tilde{J}$ , is computed from argument col\_indices as follows:
- 4303 (a) If col\_indices = GrB\_ALL, then  $\widetilde{\boldsymbol{J}}[j] = j, \forall j: 0 \leq j < \text{ncols.}$
- 4304 (b) Otherwise,  $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j], \forall j: 0 \leq j < \mathsf{ncols}.$
- The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 4308 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathsf{nrows}.$
- 4.  $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathsf{ncols}.$
- If any compatibility rule above is violated, execution of GrB\_extract ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the extraction from  $\widetilde{\mathbf{A}}$ .
- $\bullet$   $\widetilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.
- The intermediate matrix,  $\widetilde{\mathbf{T}}$ , is created as follows:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (i,j,\widetilde{\mathbf{A}}(\widetilde{\boldsymbol{I}}[i],\widetilde{\boldsymbol{J}}[j])) \ \forall \ (i,j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} : (\widetilde{\boldsymbol{I}}[i],\widetilde{\boldsymbol{J}}[j]) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle. \end{split}$$

- At this point, if any value in the  $\widetilde{I}$  array is not in the range  $[0, \mathbf{nrows}(\widetilde{\mathbf{A}}))$  or any value in the  $\widetilde{J}$  array is not in the range  $[0, \mathbf{ncols}(\widetilde{\mathbf{A}}))$ , the execution of  $\mathsf{GrB\_extract}$  ends and the index out-of-bounds error listed above is generated. In  $\mathsf{GrB\_NONBLOCKING}$  mode, the error can be deferred until a sequence-terminating  $\mathsf{GrB\_wait}()$  is called. Regardless, the result matrix  $\mathsf{C}$  is invalid from this point forward in the sequence.
- The intermediate matrix  $\widetilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:
- If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathsf{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

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$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\text{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.6.3 extract: Column (and row) variant

Extract from one column of a matrix into a vector. Note that with the transpose descriptor for the source matrix, elements of an arbitrary row of the matrix can be extracted with this function as well.

## 4357 C Syntax

| 4358 | <pre>GrB_Info GrB_extract(GrB_Vector</pre> | W,                    |
|------|--|-----------------------|
| 4359 | const GrB_Vector                           | mask,                 |
| 4360 | const GrB_BinaryOp                         | accum,                |
| 4361 | const GrB_Matrix                           | Α,                    |
| 4362 | const GrB_Index                            | *row_indices,         |
| 4363 | ${\tt GrB\_Index}$                         | nrows,                |
| 4364 | ${\tt GrB\_Index}$                         | <pre>col_index,</pre> |
| 4365 | const GrB_Descriptor                       | desc);                |

#### 66 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the extract operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - A (IN) The GraphBLAS matrix from which the column subset is extracted.
- row\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations
  within the specified column of A from which elements are extracted. If elements in
  all rows of A are to be extracted in order, GrB\_ALL should be specified. Regardless
  of execution mode and return value, this array may be manipulated by the caller
  after this operation returns without affecting any deferred computations for this
  operation.
  - nrows (IN) The number of indices in the row indices array. Must be equal to size(w).
- col\_index (IN) The index of the column of A from which to extract values. It must be in the range  $[0, \mathbf{ncols}(A))$ .
  - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 4392 |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input mask vector. The stored values are    |
|      |       |          |               | not examined.                               |
|      | mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |
|      | Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |

# Return Values

| 4394<br>4395<br>4396<br>4397<br>4398 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector <b>w</b> is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|---|
| 4399                                 | GrB_PANIC                | Unknown internal error.   |
| 4400<br>4401<br>4402<br>4403         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.   |
| 4404                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for operation.  |
| 4405<br>4406                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).  |
| 4407                                 | GrB_INVALID_INDEX        | ${\color{blue} \textbf{col\_index} \ is \ outside \ the \ allowable \ range \ (i.e., \ greater \ than \ \textbf{ncols}(A)).}$   |
| 4408<br>4409                         | GrB_INDEX_OUT_OF_BOUNDS  | A value in $row\_indices$ is greater than or equal to $\mathbf{nrows}(A)$ . In non-blocking mode, this error can be deferred.   |
| 4410                                 | GrB_DIMENSION_MISMATCH   | $mask \ \mathrm{and} \ w \ \mathrm{dimensions} \ \mathrm{are} \ \mathrm{incompatible}, \ \mathrm{or} \ nrows \neq \mathbf{size}(w).$  |
| 4411<br>4412<br>4413<br>4414         | GrB_DOMAIN_MISMATCH      | The domains of the vector or matrix are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).   |
| 4415                                 | GrB_NULL_POINTER         | Argument row_indices is a NULL pointer.   |

# Description

This variant of GrB\_extract computes the result of extracting a subset of locations (in a specific 4417 order) from a specified column of a GraphBLAS matrix:  $w = A(:, col\_index)(row\_indices)$ ; or, if 4418

an optional binary accumulation operator  $(\odot)$  is provided,  $w = w \odot A(:,col\_index)(row\_indices)$ .

More explicitly:

$$\begin{aligned} \mathsf{w}(i) &= \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_index}) \; \forall \; i: \; 0 \leq i < \mathsf{nrows}, \; \; \mathsf{or} \\ \mathsf{w}(i) &= \mathsf{w}(i) \odot \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_index}) \; \forall \; i: \; 0 \leq i < \mathsf{nrows} \end{aligned}$$

- Logically, this operation occurs in three steps:
- Setup The internal matrices, vectors, and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4425 **Compute** The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 4427 Up to three argument vectors and matrices are used in this GrB\_extract operation:
- 4428 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$

- 2.  $\operatorname{\mathsf{mask}} = \langle \mathbf{D}(\operatorname{\mathsf{mask}}), \operatorname{\mathbf{size}}(\operatorname{\mathsf{mask}}), \mathbf{L}(\operatorname{\mathsf{mask}}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument vectors, matrix and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 4435 2.  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}(A)$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_extract ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vector, matrix, mask, and index array used in the computation are formed ( $\leftarrow$  denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathsf{w}) \} \rangle$ .

- (b) If  $mask \neq GrB\_NULL$ ,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB} \ \mathsf{INP0}].\mathsf{GrB} \ \mathsf{TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}.$
- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- (a) If indices = GrB\_ALL, then  $\widetilde{\boldsymbol{I}}[i] = i, \ \forall \ i: 0 \leq i < \text{nrows}.$
- (b) Otherwise,  $\widetilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nrows}.$
- The internal vector, mask, and index array are checked for dimension compatibility. The following conditions must hold:
- 4459 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 4460 2.  $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathsf{nrows}.$
- If any compatibility rule above is violated, execution of GrB\_extract ends and the dimension mismatch error listed above is returned.
- The col\_index parameter is checked for a valid value. The following condition must hold:
- $1. 0 \leq \text{col\_index} < \mathbf{ncols}(A)$
- If the rule above is violated, execution of GrB\_extract ends and the invalid index error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$ : The vector holding the extraction from a column of  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{A}), \mathsf{nrows}, \{(i, \widetilde{\mathbf{A}}(\widetilde{\boldsymbol{I}}[i], \mathsf{col\_index})) \ \forall \ i, 0 \leq i < \mathsf{nrows} : (\widetilde{\boldsymbol{I}}[i], \mathsf{col\_index}) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle.$$

- At this point, if any value in  $\widetilde{I}$  is not in the range  $[0, \mathbf{nrows}(\widetilde{\mathbf{A}}))$ , the execution of GrB\_extract ends and the index-out-of-bounds error listed above is generated. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result
- the error can be deterred until a sequence-terminating GrB\_wait() is called. Regardless, the resultance vector was invalid from this point forward in the sequence
- vector, w, is invalid from this point forward in the sequence.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$ 

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• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector  $\mathbf{w}$ , using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.7 assign: Modifying sub-graphs

4507 Assign the contents of a subset of a matrix or vector.

## 4.3.7.1 assign: Standard vector variant

Assign values from one GraphBLAS vector to a subset of a vector as specified by a set of indices.

The size of the input vector is the same size as the index array provided.

## 4511 C Syntax

```
GrB_Info GrB_assign(GrB_Vector
                                                               W,
4512
                                     const GrB Vector
                                                              mask,
4513
                                     const GrB_BinaryOp
                                                               accum,
4514
                                     const GrB Vector
                                                               u,
4515
                                     const GrB_Index
                                                             *indices.
4516
                                     GrB_Index
                                                              nindices,
4517
                                     const GrB_Descriptor
                                                              desc);
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```

#### Parameters 1519

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the assign operation. On output, this vector holds the results of the operation.
  - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
    - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
      - u (IN) The GraphBLAS vector whose contents are assigned to a subset of w.
  - indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in w that are to be assigned. If all elements of w are to be assigned in order from 0 to nindices 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.
    - nindices (IN) The number of values in indices array. Must be equal to size(u).
    - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
| 4544 |       |          |               | removed) before the result is stored in it. |
|      | mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|      |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input mask vector. The stored values are    |
|      |       |          |               | not examined.                               |
|      | mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |

# Return Values

| 4546<br>4547<br>4548<br>4549<br>4550 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector <b>w</b> is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|---|
| 4551                                 | GrB_PANIC                | Unknown internal error.   |
| 4552<br>4553<br>4554<br>4555         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.   |
| 4556                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for operation.  |
| 4557<br>4558                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).  |
| 4559<br>4560                         | GrB_INDEX_OUT_OF_BOUNDS  | A value in indices is greater than or equal to ${\bf size}(w).$ In non-blocking mode, this can be reported as an execution error.   |
| 4561                                 | GrB_DIMENSION_MISMATCH   | $mask \ \mathrm{and} \ w \ \mathrm{dimensions} \ \mathrm{are} \ \mathrm{incompatible}, \ \mathrm{or} \ nindices \neq \mathbf{size}(u).$   |
| 4562<br>4563<br>4564<br>4565         | GrB_DOMAIN_MISMATCH      | The domains of the various vectors are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).  |
| 4566                                 | GrB_NULL_POINTER         | Argument indices is a NULL pointer.   |

## Description

This variant of GrB\_assign computes the result of assigning elements from a source GraphBLAS 4568 vector to a destination GraphBLAS vector in a specific order: w(indices) = u; or, if an optional 4569 binary accumulation operator  $(\odot)$  is provided,  $w(indices) = w(indices) \odot u$ . More explicitly: 4570

```
w(indices[i]) =
                                                                                                       u(i), \ \forall \ i : 0 \le i < \text{nindices}, \ \text{ or}
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                                            \mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{u}(i), \ \forall \ i : \ 0 \le i < \mathsf{nindices}.
```

- Logically, this operation occurs in three steps:
- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4575 **Compute** The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 4577 Up to three argument vectors are used in the GrB\_assign operation:
- 4578 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 4579 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument vectors and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 4585 2.  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}(u)$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vectors, mask and index array used in the computation are formed (← denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- 4599 (b) If  $mask \neq GrB\_NULL$ ,
  - i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .

3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4. The internal index array,  $\widetilde{I}$ , is computed from argument indices as follows:
  - (a) If indices = GrB\_ALL, then  $\tilde{I}[i] = i, \ \forall \ i : 0 \le i < \text{nindices}$ .
    - (b) Otherwise,  $\widetilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nindices}.$

The internal vector and mask are checked for dimension compatibility. The following conditions must hold:

- 1.  $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}})$
- 2. nindices =  $\mathbf{size}(\widetilde{\mathbf{u}})$ .

If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$ : The vector holding the elements from  $\tilde{\mathbf{u}}$  in their destination locations relative to  $\tilde{\mathbf{w}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{\boldsymbol{I}}[i], \widetilde{\mathbf{u}}(i)) \forall i, 0 \leq i < \mathsf{nindices} : i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

At this point, if any value of  $\tilde{I}[i]$  is outside the valid range of indices for vector  $\tilde{\mathbf{w}}$ , computation ends and the method returns the index-out-of-bounds error listed above. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result vector, w, is invalid from this point forward in the sequence.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows:

• If  $accum = GrB \ NULL$ , then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The above expression defines the structure of vector  $\tilde{\mathbf{z}}$  as follows: We start with the structure of  $\tilde{\mathbf{w}}$  ( $\mathbf{ind}(\tilde{\mathbf{w}})$ ) and remove from it all the indices of  $\tilde{\mathbf{w}}$  that are in the set of indices being assigned ( $\{\tilde{I}[k], \forall k\} \cap \mathbf{ind}(\tilde{\mathbf{w}})$ ). Finally, we add the structure of  $\tilde{\mathbf{t}}$  ( $\mathbf{ind}(\tilde{\mathbf{t}})$ ).

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
  
$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_{i} = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_{i} = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_{i} = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_{i} = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If desc[GrB\_OUTP].GrB\_REPLACE is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

### 4.3.7.2 assign: Standard matrix variant

Assign values from one GraphBLAS matrix to a subset of a matrix as specified by a set of indices.

The dimensions of the input matrix are the same size as the row and column index arrays provided.

## C Syntax

```
GrB_Info GrB_assign(GrB_Matrix C,

4667 const GrB_Matrix Mask,

4668 const GrB_BinaryOp accum,

4669 const GrB_Matrix A,
```

| 4670 | const GrB_Index      | *row_indices, |
|------|----------------------|---------------|
| 4671 | GrB_Index            | nrows,        |
| 4672 | const GrB_Index      | *col_indices, |
| 4673 | GrB_Index            | ncols,        |
| 4674 | const GrB_Descriptor | desc);        |

### Parameters

- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, the matrix holds the results of the operation.
  - Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - A (IN) The GraphBLAS matrix whose contents are assigned to a subset of C.
- row\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the rows of C that are assigned. If all rows of C are to be assigned in order from 0 to nrows 1, then GrB\_ALL can be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies assignment of more than one value to the same location which leads to undefined results.
  - nrows (IN) The number of values in the row\_indices array. Must be equal to  $\mathbf{nrows}(A)$  if A is not transposed, or equal to  $\mathbf{ncols}(A)$  if A is transposed.
  - col\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the columns of C that are assigned. If all columns of C are to be assigned in order from 0 to ncols 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies assignment of more than one value to the same location which leads to undefined results.
    - ncols (IN) The number of values in col\_indices array. Must be equal to  $\mathbf{ncols}(A)$  if A is not transposed, or equal to  $\mathbf{nrows}(A)$  if A is transposed.

| 4707 | desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL |
|------|--|
| 4708 | should be specified. Non-default field/value pairs are listed as follows:                |

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
| -    | С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 4710 |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input Mask matrix. The stored values are    |
|      |       |          |               | not examined.                               |
|      | Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
|      | Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |

# 4711 Return Values

| 4712<br>4713<br>4714<br>4715<br>4716 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 4717                                 | GrB_PANIC                | Unknown internal error.  |
| 4718<br>4719<br>4720<br>4721         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 4722                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.   |
| 4723<br>4724                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).  |
| 4725<br>4726<br>4727                 | GrB_INDEX_OUT_OF_BOUNDS  | A value in row_indices is greater than or equal to $\mathbf{nrows}(C)$ , or a value in $\mathbf{col\_indices}$ is greater than or equal to $\mathbf{ncols}(C)$ . In non-blocking mode, this can be reported as an execution error.   |
| 4728<br>4729                         | GrB_DIMENSION_MISMATCH   | Mask and C dimensions are incompatible, $nrows \neq nrows(A)$ , or $ncols \neq ncols(A)$ .   |
| 4730<br>4731<br>4732<br>4733         | GrB_DOMAIN_MISMATCH      | The domains of the various matrices are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).  |
| 4734<br>4735                         | GrB_NULL_POINTER         | Either argument $row\_indices$ is a NULL pointer, argument $col\_indices$ is a NULL pointer, or both.  |

#### 4736 Description

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This variant of GrB\_assign computes the result of assigning the contents of A to a subset of rows and columns in C in a specified order:  $C(row\_indices, col\_indices) = A$ ; or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C(row\_indices, col\_indices) = C(row\_indices, col\_indices) \odot$ A. More explicitly (not accounting for an optional transpose of A):

Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4745 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 4747 Up to three argument matrices are used in the GrB assign operation:
- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$ 
  - 2.  $Mask = \langle \mathbf{D}(Mask), \mathbf{nrows}(Mask), \mathbf{ncols}(Mask), \mathbf{L}(Mask) = \{(i, j, M_{ij})\} \rangle$  (optional)
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument matrices and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{A})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.
- From the arguments, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathbf{C}$ .
- 2. Two-dimensional mask M is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- (b) If Mask  $\neq$  GrB\_NULL,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
- 4773 ii. Otherwise,  $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- 4778 (a) If row\_indices = GrB\_ALL, then  $\widetilde{I}[i] = i, \forall i : 0 \leq i < \text{nrows}$ .
- (b) Otherwise,  $\widetilde{I}[i] = \text{row\_indices}[i], \forall i : 0 \leq i < \text{nrows}.$
- 5. The internal column index array,  $\tilde{J}$ , is computed from argument col\_indices as follows:
- 4781 (a) If col\_indices = GrB\_ALL, then  $\widetilde{m{J}}[j] = j, \forall j: 0 \leq j < ext{ncols}.$
- (b) Otherwise,  $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j], \ \forall \ j: 0 \leq j < \mathsf{ncols}.$
- The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}).$
- 4786 2.  $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}}).$
- 3.  $\mathbf{nrows}(\widetilde{\mathbf{A}}) = \mathsf{nrows}.$
- 4.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}) = \operatorname{\mathsf{ncols}}.$
- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{T}}$ : The matrix holding the contents from  $\widetilde{\mathbf{A}}$  in their destination locations relative to  $\widetilde{\mathbf{C}}$ .
- $\widetilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\widetilde{\mathbf{T}}$ , is created as follows:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \widetilde{\mathbf{A}}(i,j)) \ \forall \ (i,j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} : (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle. \end{split}$$

At this point, if any value in the  $\widetilde{I}$  array is not in the range  $[0, \mathbf{nrows}(\widetilde{\mathbf{C}}))$  or any value in the  $\widetilde{J}$  array is not in the range  $[0, \mathbf{ncols}(\widetilde{\mathbf{C}}))$ , the execution of  $\mathsf{GrB\_assign}$  ends and the index out-of-bounds error listed above is generated. In  $\mathsf{GrB\_NONBLOCKING}$  mode, the error can be deferred until a sequence-terminating  $\mathsf{GrB\_wait}()$  is called. Regardless, the result matrix  $\mathsf{C}$  is invalid from this point forward in the sequence.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows:

• If  $accum = GrB\_NULL$ , then  $\widetilde{\mathbf{Z}}$  is defined as

$$egin{array}{lll} \widetilde{\mathbf{Z}} &=& \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \ &\{(i,j,Z_{ij}) orall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{m{I}}[k],\widetilde{m{J}}[l]), orall k,l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} 
angle. \end{array}$$

The above expression defines the structure of matrix  $\widetilde{\mathbf{Z}}$  as follows: We start with the structure of  $\widetilde{\mathbf{C}}$  ( $\mathbf{ind}(\widetilde{\mathbf{C}})$ ) and remove from it all the indices of  $\widetilde{\mathbf{C}}$  that are in the set of indices being assigned ( $\{(\widetilde{I}[k], \widetilde{J}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}})$ ). Finally, we add the structure of  $\widetilde{\mathbf{T}}$  ( $\mathbf{ind}(\widetilde{\mathbf{T}})$ ).

The values of the elements of  $\tilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{C}}$  and  $\tilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in \mathbf{ind}(\widetilde{\mathbf{T}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 4842 4.3.7.3 assign: Column variant

Assign the contents a vector to a subset of elements in one column of a matrix. Note that since the output cannot be transposed, a different variant of assign is provided to assign to a row of a matrix.

#### 4846 C Syntax

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```
GrB_Info GrB_assign(GrB_Matrix
                                                               С,
4847
                                     const GrB Vector
                                                               mask,
4848
                                     const GrB BinaryOp
4849
                                                               accum,
                                     const GrB Vector
                                                               u,
4850
                                     const GrB_Index
                                                              *row_indices,
4851
                                     GrB Index
                                                               nrows,
4852
                                     GrB Index
                                                               col_index,
4853
                                     const GrB Descriptor
                                                               desc);
4854
```

#### Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, this matrix holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the specified column of the output matrix C. The mask dimensions must match those of a single column of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type

| 4863<br>4864<br>4865 |       | bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of a column of C), GrB_NULL should be specified. |
|----------------------|-------|---|
| 4866                 | accum | (IN) An optional binary operator used for accumulating entries into existing $C$  |
| 4867                 |       | entries. If assignment rather than accumulation is desired, GrB_NULL should be  |
| 4868                 |       | specified.  |

u (IN) The GraphBLAS vector whose contents are assigned to (a subset of) a column of C.

row\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in the specified column of C that are to be assigned. If all elements of the column in C are to be assigned in order from index 0 to nrows — 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.

nrows (IN) The number of values in row\_indices array. Must be equal to size(u).

col index (IN) The index of the column in C to assign. Must be in the range [0, ncols(C)).

desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| С     | GrB_OUTP | GrB_REPLACE   | Output column in C is cleared (all ele-     |
|       |          |               | ments removed) before result is stored in   |
|       |          |               | it.   |
| mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input mask vector. The stored values are    |
|       |          |               | not examined.                               |
| mask  | GrB MASK | GrB COMP      | Use the complement of mask.                 |

#### Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the 4892 opaque GraphBLAS objects (input or output) is in an invalid 4893 state caused by a previous execution error. Call GrB\_error() to 4894 access any error messages generated by the implementation. 4895 GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 4896 Grb\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized 4897 by a call to new (or dup for vector or matrix parameters). GrB INVALID INDEX col index is outside the allowable range (i.e., greater than ncols(C)). 4899 GrB\_INDEX\_OUT\_OF\_BOUNDS A value in row\_indices is greater than or equal to nrows(C). In 4900 non-blocking mode, this can be reported as an execution error. 4901 GrB\_DIMENSION\_MISMATCH mask size and number of rows in C are not the same, or nrows  $\neq$ 4902 size(u). 4903

Grb DOMAIN MISMATCH The domains of the matrix and vector are incompatible with

4904 each other or the corresponding domains of the accumulation 4905 operator, or the mask's domain is not compatible with bool (in 4906 the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set). 4907

GrB\_NULL\_POINTER Argument row\_indices is a NULL pointer.

#### Description 4909

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This variant of GrB\_assign computes the result of assigning a subset of locations in a column of a 4910 GraphBLAS matrix (in a specific order) from the contents of a GraphBLAS vector: 4911

 $C(:, col index) = u; or, if an optional binary accumulation operator (<math>\odot$ ) is provided,  $C(:, col index) = u; or, if an optional binary accumulation operator (<math>\odot$ ) is provided,  $C(:, col index) = u; or, if an optional binary accumulation operator (<math>\odot$ ) is provided,  $C(:, col index) = u; or, if an optional binary accumulation operator (<math>\odot$ ) is provided,  $C(:, col index) = u; or, if an optional binary accumulation operator (<math>\odot$ ) is provided. 4912 C(:, col index) ⊙ u. Taking order of row\_indices into account, it is more explicitly written as: 4913

```
C(row\_indices[i], col\_index) = u(i), \forall i : 0 \le i < nrows, or
4914
            C(row\_indices[i], col\_index) = C(row\_indices[i], col\_index) \odot u(i), \forall i : 0 \le i < nrows.
```

Logically, this operation occurs in three steps: 4915

**Setup** The internal matrices, vectors and mask used in the computation are formed and their 4916 domains and dimensions are tested for compatibility. 4917

Compute The indicated computations are carried out. 4918

Output The result is written into the output matrix, possibly under control of a mask. 4919

Up to three argument vectors and matrices are used in this GrB\_assign operation: 4920

```
1. C = \langle D(C), nrows(C), ncols(C), L(C) = \{(i, j, C_{ij})\} \rangle
4921
```

2.  $\operatorname{mask} = \langle \mathbf{D}(\operatorname{mask}), \operatorname{size}(\operatorname{mask}), \mathbf{L}(\operatorname{mask}) = \{(i, m_i)\} \rangle$  (optional)

```
3. \mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle
```

The argument vectors, matrix, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 4928 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{u})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(C)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.

The col\_index parameter is checked for a valid value. The following condition must hold:

```
1.0 \leq \text{col\_index} < \mathbf{ncols}(C)
```

If the rule above is violated, execution of GrB\_assign ends and the invalid index error listed above is returned.

From the arguments, the internal vectors, mask, and index array used in the computation are formed ( $\leftarrow$  denotes copy):

1. The vector,  $\tilde{\mathbf{c}}$ , is extracted from a column of C as follows:

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$$\widetilde{\mathbf{c}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \{(i, C_{ij}) \ \forall \ i : 0 \le i < \mathbf{nrows}(\mathsf{C}), j = \mathsf{col\_index}, (i, j) \in \mathbf{ind}(\mathsf{C}) \} \rangle$$

- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- 4946 (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{nrows}(\mathsf{C}), \{i, \ \forall \ i : 0 \le i < \mathbf{nrows}(\mathsf{C}) \} \rangle$ .
- 4947 (b) If  $mask \neq GrB\_NULL$ ,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 4951 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- 4953 (a) If row\_indices = GrB\_ALL, then  $\widetilde{I}[i] = i, \ \forall \ i : 0 \leq i < \text{nrows}.$

(b) Otherwise,  $\tilde{I}[i] = \text{row\_indices}[i], \ \forall \ i : 0 \le i < \text{nrows}.$ 

The internal vectors, matrices, and masks are checked for dimension compatibility. The following conditions must hold:

- 4957 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{c}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 4958 2. nrows =  $\mathbf{size}(\widetilde{\mathbf{u}})$ .

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- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the elements from  $\tilde{\mathbf{u}}$  in their destination locations relative to  $\tilde{\mathbf{c}}$ .
  - $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(\widetilde{I}[i], \widetilde{\mathbf{u}}(i)) \ \forall \ i, \ 0 \leq i < \mathsf{nrows} : i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

- At this point, if any value of  $\tilde{I}[i]$  is outside the valid range of indices for vector  $\tilde{\mathbf{c}}$ , computation ends and the method returns the index out-of-bounds error listed above. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result matrix, C, is invalid from this point forward in the sequence.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows:
  - If  $accum = GrB\_NULL$ , then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The above expression defines the structure of vector  $\tilde{\mathbf{z}}$  as follows: We start with the structure of  $\tilde{\mathbf{c}}$  ( $\mathbf{ind}(\tilde{\mathbf{c}})$ ) and remove from it all the indices of  $\tilde{\mathbf{c}}$  that are in the set of indices being assigned ( $\{\tilde{I}[k], \forall k\} \cap \mathbf{ind}(\tilde{\mathbf{c}})$ ). Finally, we add the structure of  $\tilde{\mathbf{t}}$  ( $\mathbf{ind}(\tilde{\mathbf{t}})$ ).

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{c}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$
  
$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

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$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i) \ orall \ i \in \mathbf{ind}(\widetilde{\mathbf{c}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} 
angle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

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$$z_i = \widetilde{\mathbf{c}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}})),$$
4990 
$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$
4992 
$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up the  $\tilde{\mathbf{z}}$  vector are written into the column of the final result matrix,  $C(:, col\_index)$ . This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C(:,col\_index) on input to this operation are deleted and the new contents of the column is given by:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : j \neq \mathsf{col\_index}\} \cup \{(i, \mathsf{col\_index}, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{z}}$  indicated by the mask are copied into the column of the final result matrix,  $C(:,col\_index)$ , and elements of this column that fall outside the set indicated by the mask are unchanged:

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$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}): j \neq \mathsf{col\_index}\} \cup \\ \{(i,\mathsf{col\_index},\widetilde{\mathbf{c}}(i)): i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \\ \{(i,\mathsf{col\_index},z_i): i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

### 4.3.7.4 assign: Row variant

Assign the contents a vector to a subset of elements in one row of a matrix. Note that since the output cannot be transposed, a different variant of assign is provided to assign to a column of a matrix.

#### 5015 C Syntax

```
5016
             GrB_Info GrB_assign(GrB_Matrix
                                                              С,
                                     const GrB_Vector
                                                              mask,
5017
                                     const GrB_BinaryOp
                                                              accum.
5018
                                     const GrB_Vector
                                                              u,
5019
                                     GrB_Index
                                                              row_index,
5020
                                     const GrB_Index
                                                             *col_indices,
5021
                                                              ncols,
                                     GrB Index
5022
                                     const GrB_Descriptor
                                                              desc);
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```

#### 5024 Parameters

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- C (INOUT) An existing GraphBLAS Matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, this matrix holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the specified row of the output matrix C. The mask dimensions must match those of a single row of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of a row of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - u (IN) The GraphBLAS vector whose contents are assigned to (a subset of) a row of C.
- row\_index (IN) The index of the row in C to assign. Must be in the range [0, nrows(C)).
- col\_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in the specified row of C that are to be assigned. If all elements of the row in C are to be assigned in order from index 0 to ncols 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.
  - ncols (IN) The number of values in col\_indices array. Must be equal to size(u).
  - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
| 5052 | С     | GrB_OUTP | GrB_REPLACE   | Output row in C is cleared (all elements    |
|      |       |          |               | removed) before result is stored in it.     |
|      | mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|      |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input mask vector. The stored values are    |
|      |       |          |               | not examined.                               |
|      | mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |

# 5053 Return Values

| 5054<br>5055<br>5056<br>5057<br>5058 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 5059                                 | GrB_PANIC                | Unknown internal error.  |
| 5060<br>5061<br>5062<br>5063         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 5064                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for operation.   |
| 5065<br>5066                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).   |
| 5067                                 | GrB_INVALID_INDEX        | ${\sf row\_index} \ {\rm is} \ {\rm outside} \ {\rm the} \ {\rm allowable} \ {\rm range} \ ({\rm i.e., greater} \ {\rm than} \ {\bf nrows}(C)).$   |
| 5068<br>5069                         | GrB_INDEX_OUT_OF_BOUNDS  | A value in $col\_indices$ is greater than or equal to $ncols(C)$ . In non-blocking mode, this can be reported as an execution error.   |
| 5070<br>5071                         | GrB_DIMENSION_MISMATCH   | mask size and number of columns in $C$ are not the same, or $n\text{cols} \neq \mathbf{size}(u).$  |
| 5072<br>5073<br>5074<br>5075         | GrB_DOMAIN_MISMATCH      | The domains of the matrix and vector are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).                                       |
| 5076                                 | GrB_NULL_POINTER         | Argument col_indices is a NULL pointer.  |

# Description

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This variant of GrB\_assign computes the result of assigning a subset of locations in a row of a 5078 GraphBLAS matrix (in a specific order) from the contents of a GraphBLAS vector: 5079

 $C(row\_index,:) = u;$  or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C(row\_index,:) = C(row\_index,:) \odot u$ . Taking order of col\_indices into account it is more explicitly written as:

Logically, this operation occurs in three steps:

Setup The internal matrices, vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5086 Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

5088 Up to three argument vectors and matrices are used in this GrB assign operation:

```
5089 1. C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle
```

2. 
$$\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$$
 (optional)

3. 
$$\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$$

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The argument vectors, matrix, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5096 2.  $\mathbf{D}(C)$  must be compatible with  $\mathbf{D}(u)$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(C)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.

5105 The row\_index parameter is checked for a valid value. The following condition must hold:

```
1. 0 \leq \text{row\_index} < \text{nrows}(C)
```

If the rule above is violated, execution of GrB\_assign ends and the invalid index error listed above is returned.

From the arguments, the internal vectors, mask, and index array used in the computation are formed ( $\leftarrow$  denotes copy):

1. The vector,  $\tilde{\mathbf{c}}$ , is extracted from a row of C as follows:

$$\widetilde{\mathbf{c}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(j, C_{ij}) \ \forall \ j : 0 \le j < \mathbf{ncols}(\mathsf{C}), i = \mathsf{row\_index}, (i, j) \in \mathbf{ind}(\mathsf{C}) \} \rangle$$

- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- 5114 (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{ncols}(\mathsf{C}), \{i, \ \forall \ i : 0 \le i < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- 5115 (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool}) \mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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- 4. The internal column index array,  $\tilde{J}$ , is computed from argument col\_indices as follows:
- (a) If col\_indices = GrB\_ALL, then  $\widetilde{J}[j] = j, \ \forall \ j : 0 \le j < \text{ncols.}$
- (b) Otherwise,  $\widetilde{\boldsymbol{J}}[j] = \text{col\_indices}[j], \ \forall \ j: 0 \leq j < \text{ncols.}$
- The internal vectors, matrices, and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{c}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 2.  $\operatorname{ncols} = \operatorname{size}(\widetilde{\mathbf{u}}).$
- 5127 If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mis-5128 match error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the elements from  $\tilde{\mathbf{u}}$  in their destination locations relative to  $\tilde{\mathbf{c}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

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$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathbf{c}}), \{ (\widetilde{J}[j], \widetilde{\mathbf{u}}(j)) \ \forall \ j, \ 0 \le j < \mathsf{ncols} : j \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle.$$

- At this point, if any value of  $\widetilde{J}[j]$  is outside the valid range of indices for vector  $\widetilde{\mathbf{c}}$ , computation ends and the method returns the index out-of-bounds error listed above. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result matrix, C, is invalid from this point forward in the sequence.
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows:

• If  $accum = GrB\_NULL$ , then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The above expression defines the structure of vector  $\widetilde{\mathbf{z}}$  as follows: We start with the structure of  $\widetilde{\mathbf{c}}$  ( $\mathbf{ind}(\widetilde{\mathbf{c}})$ ) and remove from it all the indices of  $\widetilde{\mathbf{c}}$  that are in the set of indices being assigned ( $\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}})$ ). Finally, we add the structure of  $\widetilde{\mathbf{t}}$  ( $\mathbf{ind}(\widetilde{\mathbf{t}})$ ).

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{c}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$
  
 $z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$ 

where the difference operator refers to set difference.

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(j, z_i) \ \forall \ j \in \mathbf{ind}(\widetilde{\mathbf{c}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_j = \widetilde{\mathbf{c}}(j) \odot \widetilde{\mathbf{t}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}})),$$

$$z_j = \widetilde{\mathbf{c}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_j = \widetilde{\mathbf{t}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up the  $\tilde{\mathbf{z}}$  vector are written into the column of the final result matrix,  $C(row\_index,:)$ . This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C(row\_index,:) on input to this operation are deleted and the new contents of the column is given by:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : i \neq \mathsf{row\_index}\} \cup \{(\mathsf{row\_index},j,z_j) : j \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{z}}$  indicated by the mask are copied into the column of the final result matrix,  $C(row\_index,:)$ , and elements of this column that fall outside the set indicated by the mask are unchanged:

$$\begin{split} \mathbf{L}(\mathsf{C}) &= & \{(i,j,C_{ij}): i \neq \mathsf{row\_index}\} \cup \\ & \{(\mathsf{row\_index},j,\widetilde{\mathbf{c}}(j)): j \in (\mathbf{ind}(\widetilde{\mathbf{c}}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \\ & \{(\mathsf{row\_index},j,z_j): j \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}. \end{split}$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

## 5179 4.3.7.5 assign: Constant vector variant[Scott: NEW CONTENT]

Assign the same value to a specified subset of vector elements. With the use of GrB\_ALL, the entire destination vector can be filled with the constant.

### 5182 C Syntax

```
GrB_Info GrB_assign(GrB_Vector
                                                               w,
5183
                                     const GrB_Vector
5184
                                                               mask,
                                     const GrB BinaryOp
                                                               accum,
5185
                                     <type>
                                                               val,
5186
                                     const GrB_Index
                                                              *indices.
5187
                                     GrB_Index
                                                               nindices,
5188
                                     const GrB_Descriptor
                                                               desc);
5189
             GrB_Info GrB_assign(GrB_Vector
                                                               W,
5190
                                     const GrB_Vector
                                                               mask,
5191
                                     const GrB_BinaryOp
                                                               accum,
5192
                                     const GrB_Scalar
5193
                                                               s,
                                                              *indices,
                                     const GrB_Index
5194
                                     GrB Index
                                                               nindices,
5195
                                     const GrB_Descriptor
                                                               desc);
5196
```

#### Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the assign operation. On output, this vector holds the results of the operation.
  - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
    - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
      - val (IN) Scalar value to assign to (a subset of) w.
  - s (IN) Scalar value to assign to (a subset of) w.
  - indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in w that are to be assigned. If all elements of w are to be assigned in order from 0

to nindices – 1, then GrB\_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. In this variant, the specific order of the values in the array has no effect on the result. Unlike other variants, if there are duplicated values in this array the result is still defined.

nindices (IN) The number of values in indices array. Must be in the range: [0, size(w)]. If nindices is zero, the operation becomes a NO-OP.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|     | Param | Field    | Value         | Description                                 |
|-----|-------|----------|---------------|---|
|     | W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|     |       |          |               | removed) before the result is stored in it. |
|     | mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 225 |       |          |               | structure (pattern of stored values) of the |
|     |       |          |               | input mask vector. The stored values are    |
|     |       |          |               | not examined.                               |
|     | mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |

#### Return Values

| 5227 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-    |
|------|--------------------------|--|
| 5228 |                          | blocking mode, this indicates that the compatibility tests on      |
| 5229 |                          | dimensions and domains for the input arguments passed suc-         |
| 5230 |                          | cessfully. Either way, output vector w is ready to be used in the  |
| 5231 |                          | next method of the sequence.                                       |
| 5232 | GrB_PANIC                | Unknown internal error.  |
| 5233 | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the         |
| 5234 |                          | opaque GraphBLAS objects (input or output) is in an invalid        |
| 5235 |                          | state caused by a previous execution error. Call GrB_error() to    |
| 5236 |                          | access any error messages generated by the implementation.         |
| 5237 | GrB_OUT_OF_MEMORY        | Not enough memory available for operation.                         |
| 5238 | GrB UNINITIALIZED OBJECT | One or more of the GraphBLAS objects has not been initialized      |
| 5239 | 0.5_0                    | by a call to new (or dup for vector parameters).                   |
| 3233 |                          | of a call to her (of aup for voctor parameters).                   |
| 5240 | GrB_INDEX_OUT_OF_BOUNDS  | A value in indices is greater than or equal to $size(w)$ . In non- |
| 5241 |                          | blocking mode, this can be reported as an execution error.         |
| 5242 | GrB_DIMENSION_MISMATCH   | mask and w dimensions are incompatible, or nindices is not less    |
| 5243 | _ <b>_</b>               | than size(w).  |

GrB\_DOMAIN\_MISMATCH The domains of the vector and scalar are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

GrB NULL POINTER Argument indices is a NULL pointer.

#### 5249 Description

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This variant of GrB\_assign computes the result of assigning a constant scalar value – either val or s - to locations in a destination GraphBLAS vector. Either w(indices) = val or w(indices) = s is performed. If an optional binary accumulation operator  $(\odot)$  is provided, then either  $w(indices) = w(indices) \odot val$  or  $w(indices) = w(indices) \odot s$  is performed. More explicitly, if a non-opaque value val is provided:

 $\mathsf{w}(\mathsf{indices}[i]) = \mathsf{val}, \ \forall \ i : 0 \le i < \mathsf{nindices}, \ \text{ or } \\ \mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{val}, \ \forall \ i : 0 \le i < \mathsf{nindices}.$ 

5256 Correspondingly, if a GrB\_Scalar s is provided:

$$\mathsf{w}(\mathsf{indices}[i]) = \mathsf{s}, \ \forall \ i : 0 \le i < \mathsf{nindices}, \ \text{ or } \\ \mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{s}, \ \forall \ i : 0 \le i < \mathsf{nindices}.$$

5258 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5261 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

5263 Up to two argument vectors are used in the GrB\_assign operation:

```
1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)

The argument scalar, vectors, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(w)$  must be compatible with either  $\mathbf{D}(val)$  or  $\mathbf{D}(s)$ , depending on the signature of the method.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator.

- 4. If accum is not GrB\_NULL, then either  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$ , depending on the signature of the method, must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vectors, mask and index array used in the computation are formed  $(\leftarrow \text{denotes copy})$ :
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- 5285 (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
  - (b) If mask  $\neq$  GrB\_NULL,
    - i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
    - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Scalar  $\tilde{s} \leftarrow s$  (GrB Scalar version only).
- 4. The internal index array,  $\tilde{I}$ , is computed from argument indices as follows:
- (a) If indices = GrB\_ALL, then  $\widetilde{\boldsymbol{I}}[i] = i, \ \forall \ i: 0 \leq i < \text{nindices}$ .
- (b) Otherwise,  $\tilde{I}[i] = \mathsf{indices}[i], \ \forall \ i: 0 \le i < \mathsf{nindices}.$
- The internal vector and mask are checked for dimension compatibility. The following conditions must hold:
- 5296 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- $2. 0 < \text{nindices} < \text{size}(\widetilde{\mathbf{w}}).$
- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the copies of the scalar, either val or  $\tilde{s}$ , in their destination locations relative to  $\tilde{\mathbf{w}}$ .

•  $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows. If a non-opaque scalar val is provided:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{val}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{\mathbf{I}}[i], \mathsf{val}) \ \forall \ i, \ 0 \leq i < \mathsf{nindices}\} \rangle.$$

Correspondingly, if a non-empty GrB\_Scalar  $\tilde{s}$  is provided (i.e.,  $\mathbf{size}(\tilde{s}) = 1$ ):

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{I}[i], \mathbf{val}(\widetilde{s})) \ \forall \ i, \ 0 \le i < \mathsf{nindices}\} \rangle.$$

Finally, if an empty GrB\_Scalar  $\tilde{s}$  is provided (i.e.,  $\mathbf{size}(\tilde{s}) = 0$ ):

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{size}(\widetilde{\mathbf{w}}), \emptyset \rangle.$$

If  $\tilde{I}$  is empty, this operation results in an empty vector,  $\tilde{\mathbf{t}}$ . Otherwise, if any value in the  $\tilde{I}$  array is not in the range  $[0, \mathbf{size}(\tilde{\mathbf{w}}))$ , the execution of GrB\_assign ends and the index out-of-bounds error listed above is generated. In GrB\_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB\_wait() is called. Regardless, the result vector,  $\mathbf{w}$ , is invalid from this point forward in the sequence.

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows:

• If  $accum = GrB \ \ NULL$ , then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{\mathbf{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The above expression defines the structure of vector  $\widetilde{\mathbf{z}}$  as follows: We start with the structure of  $\widetilde{\mathbf{w}}$  ( $\mathbf{ind}(\widetilde{\mathbf{w}})$ ) and remove from it all the indices of  $\widetilde{\mathbf{w}}$  that are in the set of indices being assigned ( $\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}})$ ). Finally, we add the structure of  $\widetilde{\mathbf{t}}$  ( $\mathbf{ind}(\widetilde{\mathbf{t}})$ ).

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
  
 $z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$ 

where the difference operator refers to set difference. We note that in this case of assigning a constant,  $\{\widetilde{I}[k], \forall k\}$  and  $\operatorname{ind}(\widetilde{\mathbf{t}})$  are identical.

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc(\text{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

### 5356 4.3.7.6 assign: Constant matrix variant[Scott: NEW CONTENT]

Assign the same value to a specified subset of matrix elements. With the use of GrB\_ALL, the entire destination matrix can be filled with the constant.

#### 5359 C Syntax

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```
GrB_Info GrB_assign(GrB_Matrix
                                                               С,
5360
                                     const GrB_Matrix
                                                               Mask,
5361
                                                               accum,
                                     const GrB BinaryOp
5362
                                     <type>
                                                               val,
5363
                                     const GrB_Index
                                                              *row_indices,
5364
                                     GrB_Index
                                                               nrows,
5365
                                     const GrB_Index
                                                              *col_indices,
5366
                                     GrB_Index
                                                               ncols,
5367
                                     const GrB_Descriptor
                                                               desc);
5368
                                                               С,
             GrB_Info GrB_assign(GrB_Matrix
5369
                                     const GrB_Matrix
                                                               Mask,
5370
                                     const GrB_BinaryOp
                                                               accum,
5371
                                     const GrB_Scalar
5372
                                                               s,
                                     const GrB_Index
                                                              *row_indices,
5373
                                     GrB_Index
                                                               nrows,
5374
```

| 5375         |             | <pre>const GrB_Index *col_indices,</pre>  |
|--------------|-------------|---|
| 5376         |             | GrB_Index ncols,  |
| 5377         |             | <pre>const GrB_Descriptor desc);</pre>  |
|              |             |   |
| 5378         | Parameters  |   |
| 5379         | С           | (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values  |
| 5380         |             | that may be accumulated with the result of the assign operation. On output, the   |
| 5381         |             | matrix holds the results of the operation.  |
|              | Mack        | (IN) An entianal "write" most that controls which results from this eneration are   |
| 5382         | IVIASK      | (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the    |
| 5383<br>5384 |             | matrix C. If the GrB_STRUCTURE descriptor is <i>not</i> set for the mask, the domain  |
| 5385         |             | of the Mask matrix must be of type bool or any of the predefined "built-in" types   |
| 5386         |             | in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the  |
| 5387         |             | dimensions of C), GrB_NULL should be specified.   |
|              |             |   |
| 5388         | accum       | (IN) An optional binary operator used for accumulating entries into existing C  |
| 5389         |             | entries. If assignment rather than accumulation is desired, GrB_NULL should be  |
| 5390         |             | specified.  |
| 5391         | val         | (IN) Scalar value to assign to (a subset of) C.   |
| 5392         | S           | (IN) Scalar value to assign to (a subset of) $C.$   |
| 5393         | row_indices | (IN) Pointer to the ordered set (array) of indices corresponding to the rows of C   |
| 5394         |             | that are assigned. If all rows of $C$ are to be assigned in order from 0 to $nrows-1,$  |
| 5395         |             | then GrB_ALL can be specified. Regardless of execution mode and return value,   |
| 5396         |             | this array may be manipulated by the caller after this operation returns without  |
| 5397         |             | affecting any deferred computations for this operation. Unlike other variants, if   |
| 5398         |             | there are duplicated values in this array the result is still defined.  |
| 5399         | nrows       | (IN) The number of values in row_indices array. Must be in the range: $[0, \mathbf{nrows}(C)]$ .  |
| 5400         |             | If nrows is zero, the operation becomes a NO-OP.  |
|              |             |   |
| 5401         | col_indices | (IN) Pointer to the ordered set (array) of indices corresponding to the columns of C  |
| 5402         |             | that are assigned. If all columns of C are to be assigned in order from 0 to ncols – 1,   |
| 5403         |             | then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without |
| 5404         |             | affecting any deferred computations for this operation. Unlike other variants, if   |
| 5405<br>5406 |             | there are duplicated values in this array the result is still defined.  |
| 3400         |             | onero are dapheaced values in one aray one result is som defined.   |
| 5407         | ncols       | (IN) The number of values in $col\_indices$ array. Must be in the range: $[0, \mathbf{ncols}(C)]$ .   |
| 5408         |             | If ncols is zero, the operation becomes a NO-OP.  |
| 5409         | desc        | (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL   |

 should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 5412 |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input Mask matrix. The stored values are    |
|      |       |          |               | not examined.                               |
|      | Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |

# 5413 Return Values

| 5414<br>5415                 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-<br>blocking mode, this indicates that the compatibility tests on<br>dimensions and domains for the input arguments passed suc-  |
|------------------------------|--------------------------|---|
| 5416<br>5417<br>5418         |                          | cessfully. Either way, output matrix C is ready to be used in the next method of the sequence.  |
| 5419                         | GrB_PANIC                | Unknown internal error.   |
| 5420<br>5421<br>5422<br>5423 | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation. |
| 5424                         | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.  |
| 5425<br>5426                 | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).  |
| 5427<br>5428<br>5429         | GrB_INDEX_OUT_OF_BOUNDS  | A value in $row\_indices$ is greater than or equal to $nrows(C)$ , or a value in $col\_indices$ is greater than or equal to $ncols(C)$ . In non-blocking mode, this can be reported as an execution error.  |
| 5430<br>5431                 | GrB_DIMENSION_MISMATCH   | Mask and C dimensions are incompatible, nrows is not less than $\mathbf{nrows}(C)$ , or ncols is not less than $\mathbf{ncols}(C)$ .  |
| 5432<br>5433<br>5434<br>5435 | GrB_DOMAIN_MISMATCH      | The domains of the matrix and scalar are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).    |
| 5436<br>5437                 | GrB_NULL_POINTER         | Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.   |

# B Description

This variant of  $GrB_assign$  computes the result of assigning a constant scalar value – either val or s – to locations in a destination GraphBLAS matrix: Either  $C(row\_indices, col\_indices) = val$ 

or  $C(row\_indices, col\_indices) = s$  is performed. If an optional binary accumulation operator  $(\odot)$  is provided, then either  $C(row\_indices, col\_indices) = C(row\_indices, col\_indices)$  or  $C(row\_indices, col\_indices) = C(row\_indices, col\_indices)$  or  $col\_indices$  or  $col\_ind$ 

5446 Correspondingly, if a GrB Scalar s is provided:

5448 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5451 Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

5453 Up to two argument matrices are used in the GrB\_assign operation:

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- $2. \ \mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \ (\text{optional})$

The argument scalar, matrices, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(C)$  must be compatible with either  $\mathbf{D}(val)$  or  $\mathbf{D}(val)$ , depending on the signature of the method.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator.
- 4. If accum is not GrB\_NULL, then either  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$ , depending on the signature of the method, must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal matrices, index arrays, and mask used in the computation are formed ( $\leftarrow$  denotes copy):

1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 5474 2. Two-dimensional mask  $\widetilde{\mathbf{M}}$  is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
  - (b) If Mask  $\neq$  GrB\_NULL,
    - i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE}$  is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
      - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
    - (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Scalar  $\tilde{s} \leftarrow s$  (GrB\_Scalar version only).
- 4. The internal row index array,  $\tilde{I}$ , is computed from argument row\_indices as follows:
- (a) If row\_indices = GrB\_ALL, then  $\widetilde{I}[i] = i, \forall i : 0 \leq i < \text{nrows}$ .
- 5486 (b) Otherwise,  $\widetilde{\boldsymbol{I}}[i] = \mathsf{row\_indices}[i], \forall i: 0 \leq i < \mathsf{nrows}.$
- 5. The internal column index array,  $\widetilde{J}$ , is computed from argument col\_indices as follows:
- $\text{(a) If col\_indices} = \mathsf{GrB\_ALL}, \text{ then } \widetilde{\boldsymbol{J}}[j] = j, \forall j: 0 \leq j < \mathsf{ncols}.$
- 5489 (b) Otherwise,  $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j], \forall j: 0 \leq j < \mathsf{ncols}.$
- The internal matrix and mask are checked for dimension compatibility. The following conditions must hold:
- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}).$
- 3.  $0 \le \operatorname{nrows}(\widetilde{\mathbf{C}})$
- 4.  $0 \le \operatorname{ncols} \le \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB\_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the copies of the scalar, either val or  $\widetilde{s}$ , in their destination locations relative to  $\widetilde{\mathbf{C}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\widetilde{\mathbf{T}}$ , is created as follows. If a non-opaque scalar val is provided:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{val}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \mathsf{val}) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \} \rangle. \end{split}$$

Correspondingly, if a non-empty GrB\_Scalar  $\tilde{s}$  is provided (i.e.,  $\mathbf{size}(\tilde{s}) = 1$ ):

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\widetilde{s}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \mathbf{val}(\widetilde{s})) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \} \rangle. \end{split}$$

Finally, if an empty GrB\_Scalar  $\tilde{s}$  is provided (i.e.,  $\mathbf{size}(\tilde{s}) = 0$ ):

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \emptyset \rangle.$$

If either  $\tilde{I}$  or  $\tilde{J}$  is empty, this operation results in an empty matrix,  $\tilde{\mathbf{T}}$ . Otherwise, if any value in the  $\tilde{I}$  array is not in the range  $[0, \mathbf{nrows}(\tilde{\mathbf{C}}))$  or any value in the  $\tilde{J}$  array is not in the range  $[0, \mathbf{ncols}(\tilde{\mathbf{C}}))$ , the execution of  $\mathsf{GrB\_assign}$  ends and the index out-of-bounds error listed above is generated. In  $\mathsf{GrB\_NONBLOCKING}$  mode, the error can be deferred until a sequence-terminating  $\mathsf{GrB\_wait}()$  is called. Regardless, the result matrix  $\mathsf{C}$  is invalid from this point forward in the sequence.

The intermediate matrix  $\hat{\mathbf{Z}}$  is created as follows:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}$ , then  $\widetilde{\mathbf{Z}}$  is defined as

$$\begin{split} \widetilde{\mathbf{Z}} &= \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (i, j, Z_{ij}) \forall (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle. \end{split}$$

The above expression defines the structure of matrix  $\widetilde{\mathbf{Z}}$  as follows: We start with the structure of  $\widetilde{\mathbf{C}}$  ( $\mathbf{ind}(\widetilde{\mathbf{C}})$ ) and remove from it all the indices of  $\widetilde{\mathbf{C}}$  that are in the set of indices being assigned ( $\{(\widetilde{I}[k], \widetilde{J}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}})$ ). Finally, we add the structure of  $\widetilde{\mathbf{T}}$  ( $\mathbf{ind}(\widetilde{\mathbf{T}})$ ).

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in \mathbf{ind}(\widetilde{\mathbf{T}}),$$

where the difference operator refers to set difference. We note that, in this particular case of assigning a constant to a matrix, the sets  $\{(\tilde{\boldsymbol{I}}[k], \tilde{\boldsymbol{J}}[l]), \forall k, l\}$  and  $\operatorname{ind}(\tilde{\mathbf{T}})$  are identical.

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

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$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
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$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\dot{\mathbf{Z}}$  are written into the final result matrix  $\mathsf{C}$ , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 5556 4.3.8 apply: Apply a function to the elements of an object

Computes the transformation of the values of the elements of a vector or a matrix using a unary function, or a binary function where one argument is bound to a scalar.

#### 5559 4.3.8.1 apply: Vector variant

5560 Computes the transformation of the values of the elements of a vector using a unary function.

#### 5561 C Syntax

| 5562 | <pre>GrB_Info GrB_apply(GrB_Vector</pre> | W,     |
|------|--|--------|
| 5563 | const GrB_Vector                         | mask,  |
| 5564 | const GrB_BinaryOp                       | accum, |
| 5565 | const GrB_UnaryOp                        | op,    |
| 5566 | const GrB_Vector                         | u,     |
| 5567 | const GrB Descriptor                     | desc); |

#### Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) A unary operator applied to each element of input vector u.
  - u (IN) The GraphBLAS vector to which the unary function is applied.
  - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 5586 |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input mask vector. The stored values are    |
|      |       |          |               | not examined.                               |
|      | mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |

#### Return Values

GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully.

Either way, output vector w is ready to be used in the next method of the sequence.

Grb Panic Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

5601 GrB\_DIMENSION\_MISMATCH mask, w and/or u dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or unary function, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

#### 5606 Description

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This variant of GrB\_apply computes the result of applying a unary function to the elements of a GraphBLAS vector:  $\mathbf{w} = f(\mathbf{u})$ ; or, if an optional binary accumulation operator  $(\odot)$  is provided,  $\mathbf{w} = \mathbf{w} \odot f(\mathbf{u})$ .

5610 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5613 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in this GrB\_apply operation:

- 5616 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 5617 2. mask =  $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$

The argument vectors, unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5623 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the unary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the unary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in}(\mathsf{op})$ .

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 5635 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- 5638 (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 5645 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 5646 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$

If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$ : The vector holding the result from applying the unary operator to the input vector  $\tilde{\mathbf{u}}$ .
- $\bullet$   $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\widetilde{\mathbf{u}}(i))) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle,$$

where f = f(op).

The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ orall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \odot(accum)$ , and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 5685 4.3.8.2 apply: Matrix variant

5686 Computes the transformation of the values of the elements of a matrix using a unary function.

# 5687 C Syntax

```
GrB_Info GrB_apply(GrB_Matrix
                                                             С,
5688
                                    const GrB_Matrix
                                                             Mask,
5689
                                    const GrB_BinaryOp
                                                             accum,
5690
                                    const GrB_UnaryOp
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                                                             op,
                                    const GrB_Matrix
                                                             Α,
5692
                                    const GrB_Descriptor
                                                             desc);
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```

#### 5694 Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) A unary operator applied to each element of input matrix A.
  - A (IN) The GraphBLAS matrix to which the unary function is applied.
  - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 5712 |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input Mask matrix. The stored values are    |
|      |       |          |               | not examined.                               |
|      | Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
|      | Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |

#### Return Values 5713

Grb SUCCESS In blocking mode, the operation completed successfully. In non-5714 blocking mode, this indicates that the compatibility tests on 5715 dimensions and domains for the input arguments passed suc-5716 cessfully. Either way, output matrix C is ready to be used in the 5717 next method of the sequence. 5718 GrB PANIC Unknown internal error. 5719 GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the 5720 opaque GraphBLAS objects (input or output) is in an invalid 5721 state caused by a previous execution error. Call GrB error() to 5722 access any error messages generated by the implementation. 5723 GrB OUT\_OF\_MEMORY Not enough memory available for the operation. 5724 Grb Uninitialized Object One or more of the GraphBLAS objects has not been initialized 5725 by a call to new (or Matrix\_dup for matrix parameters). 5726 GrB\_DIMENSION\_MISMATCH Mask and C dimensions are incompatible,  $nrows \neq nrows(C)$ , or 5727  $ncols \neq ncols(C)$ . 5728 5729 Grb DOMAIN MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or unary 5730 function, or the mask's domain is not compatible with bool (in 5731 the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

#### Description 5733

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This variant of GrB\_apply computes the result of applying a unary function to the elements of a 5734 GraphBLAS matrix: C = f(A); or, if an optional binary accumulation operator  $(\odot)$  is provided, 5735  $C = C \odot f(A)$ . 5736

Logically, this operation occurs in three steps: 5737

**Setup** The internal matrices and mask used in the computation are formed and their domains 5738 and dimensions are tested for compatibility. 5739

**Compute** The indicated computations are carried out. 5740

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB\_apply operation: 5742

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$ 5743
- 2.  $Mask = \langle \mathbf{D}(Mask), \mathbf{nrows}(Mask), \mathbf{ncols}(Mask), \mathbf{L}(Mask) = \{(i, j, M_{ij})\} \rangle$  (optional) 5744

```
3. A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle
```

The argument matrices, unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 5750 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the unary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the unary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in}(\mathsf{op})$  of the unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):

1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 5763 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
  - (b) If Mask  $\neq$  GrB\_NULL,
    - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
    - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$

4. 
$$\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}).$$

If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the result from applying the unary operator to the input matrix  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\widetilde{\mathbf{T}}$ , is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\widetilde{\mathbf{A}}(i, j))) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle,$$

where f = f(op).

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The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

- If  $\mathsf{accum} = \mathsf{GrB} \ \ \mathsf{NULL}, \ \mathsf{then} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
5797
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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
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$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\dot{\mathbf{Z}}$  are written into the final result matrix  $\mathsf{C}$ , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

```
\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.
```

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 5817 4.3.8.3 apply: Vector-BinaryOp variants[Scott: NEW CONTENT]

Computes the transformation of the values of the stored elements of a vector using a binary operator and a scalar value. In the *bind-first* variant, the specified scalar value is passed as the first argument to the binary operator and stored elements of the vector are passed as the second argument. In the *bind-second* variant, the elements of the vector are passed as the first argument and the specified scalar value is passed as the second argument. The scalar can be passed either as a non-opaque variable or as a GrB\_Scalar object.

### 5824 C Syntax

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```
// bind-first + scalar value
5825
             GrB_Info GrB_apply(GrB_Vector
5826
                                                              W,
                                    const GrB Vector
                                                              mask,
5827
                                    const GrB_BinaryOp
                                                              accum,
5828
                                    const GrB_BinaryOp
5829
                                                              op,
                                    <type>
                                                              val,
5830
                                    const GrB_Vector
5831
                                                              u,
                                    const GrB_Descriptor
                                                              desc);
5832
             // bind-first + GraphBLAS scalar
5833
             GrB_Info GrB_apply(GrB_Vector
                                                              w,
5834
                                    const GrB Vector
                                                              mask,
5835
                                    const GrB_BinaryOp
                                                              accum,
5836
                                    const GrB_BinaryOp
5837
                                                              op,
                                    const GrB_Scalar
                                                              s,
5838
                                    const GrB_Vector
5839
                                                              u,
                                    const GrB_Descriptor
5840
                                                              desc);
             // bind-second + scalar value
5841
             GrB_Info GrB_apply(GrB_Vector
                                                              w,
5842
                                    const GrB_Vector
                                                              mask,
5843
```

```
const GrB_BinaryOp
                                                               accum,
5844
                                    const GrB_BinaryOp
5845
                                                               op,
                                    const GrB_Vector
5846
                                                               u,
                                    <type>
                                                               val,
5847
                                    const GrB Descriptor
                                                               desc);
5848
             // bind-second + GraphBLAS scalar
5849
             GrB_Info GrB_apply(GrB_Vector
5850
                                                               W,
                                    const GrB Vector
                                                               mask,
5851
                                    const GrB_BinaryOp
                                                               accum,
5852
                                    const GrB_BinaryOp
5853
                                                               op,
                                    const GrB_Vector
                                                               u,
5854
                                    const GrB_Scalar
5855
                                                               s,
                                    const GrB_Descriptor
                                                               desc);
5856
```

#### Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) A binary operator applied to each element of input vector, u, and the scalar value, val.
  - u (IN) The GraphBLAS vector whose elements are passed to the binary operator as the right-hand (second) argument in the *bind-first* variant, or the left-hand (first) argument in the *bind-second* variant.
  - val (IN) Scalar value that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant.
    - s (IN) A GraphBLAS scalar that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant. It must not be empty.

| 5881 | desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL |
|------|--|
| 5882 | should be specified. Non-default field/value pairs are listed as follows:                |

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
| 5884 | mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|      |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input mask vector. The stored values are    |
|      |       |          |               | not examined.                               |
|      | mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |

# Return Values

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| 5886<br>5887<br>5888<br>5889<br>5890 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector <b>w</b> is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|---|
| 5891                                 | GrB_PANIC                | Unknown internal error.   |
| 5892<br>5893<br>5894<br>5895         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.   |
| 5896                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for operation.  |
| 5897<br>5898                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).  |
| 5899                                 | GrB_DIMENSION_MISMATCH   | mask,w and/or $u$ dimensions are incompatible.  |
| 5900<br>5901<br>5902<br>5903         | GrB_DOMAIN_MISMATCH      | The domains of the various vectors and scalar are incompatible with the corresponding domains of the binary operator or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).                                |
| 5904<br>5905                         | GrB_EMPTY_OBJECT         | The $GrB\_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s)=0)$ and therefore a value cannot be passed to the binary operator.   |

# 5906 Description

This variant of GrB\_apply computes the result of applying a binary operator to the elements of a GraphBLAS vector each composed with a scalar constant, either val or s:

bind-first: w = f(val, u) or w = f(s, u)

bind-second: w = f(u, val) or w = f(u, s),

or if an optional binary accumulation operator  $(\odot)$  is provided:

bind-first:  $w = w \odot f(val, u)$  or  $w = w \odot f(s, u)$ 

bind-second:  $w = w \odot f(u, val)$  or  $w = w \odot f(u, s)$ .

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5917 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

<sup>5919</sup> Up to three argument vectors are used in this GrB\_apply operation:

- 5920 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 5921 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$

The argument scalar, vectors, binary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5927 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the binary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the binary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- 5932 5. If bind-first:

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- (a)  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.
- (b) If the non-opaque scalar val is provided, then  $\mathbf{D}(\mathsf{val})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- (c) If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.

#### 6. If bind-second:

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- (a)  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- (b) If the non-opaque scalar val is provided, then  $\mathbf{D}(\text{val})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.
  - (c) If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .
- 5952 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
  - (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \forall i : 0 \le i < \mathbf{size}(\mathbf{w})\} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 5958 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- 5959 4. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 5962 1.  $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}})$
- 5963 2.  $\operatorname{size}(\widetilde{\mathbf{u}}) = \operatorname{size}(\widetilde{\mathbf{w}}).$

If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB Scalar  $\tilde{s}$  is provided (**nvals**( $\tilde{s}$ ) = 0), the method returns with code GrB EMPTY OBJECT.

If a non-empty GrB\_Scalar,  $\tilde{s}$ , is provided (i.e.,  $\mathbf{nvals}(\tilde{s}) = 1$ ), we then create an internal variable val with the same domain as  $\tilde{s}$  and set  $\mathbf{val} = \mathbf{val}(\tilde{s})$ .

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$ : The vector holding the result from applying the binary operator to the input vector  $\widetilde{\mathbf{u}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as one of the following:

```
bind-first: \widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\mathsf{val}, \widetilde{\mathbf{u}}(i))) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle,
bind-second: \widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\widetilde{\mathbf{u}}(i), \mathsf{val})) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle,
```

5978 where f = f(op).

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The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}}_{-}\operatorname{\mathsf{NULL}}, \, \operatorname{then} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$ 
  - If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### apply: Matrix-BinaryOp variants[Scott: NEW CONTENT] 4.3.8.4 6006

Computes the transformation of the values of the stored elements of a matrix using a binary operator and a scalar value. In the bind-first variant, the specified scalar value is passed as the 6008 first argument to the binary operator and stored elements of the matrix are passed as the second 6009 argument. In the bind-second variant, the elements of the matrix are passed as the first argument 6010 and the specified scalar value is passed as the second argument. The scalar can be passed either as 6011 a non-opaque variable or as a GrB\_Scalar object. 6012

#### C Syntax 6013

```
// bind-first + scalar value
6014
             GrB_Info GrB_apply(GrB_Matrix
                                                             C,
6015
                                    const GrB_Matrix
                                                             Mask,
6016
                                    const GrB_BinaryOp
                                                             accum,
6017
                                    const GrB_BinaryOp
6018
                                                             op,
                                    <type>
                                                             val,
6019
                                    const GrB_Matrix
                                                             Α,
6020
                                    const GrB_Descriptor
                                                             desc);
6021
6022
             // bind-first + GraphBLAS scalar
             GrB_Info GrB_apply(GrB_Matrix
                                                             С,
6023
                                    const GrB_Matrix
                                                             Mask,
6024
                                    const GrB_BinaryOp
                                                             accum,
6025
                                    const GrB_BinaryOp
6026
                                                             op,
                                    const GrB_Scalar
                                                             s,
6027
                                    const GrB Matrix
                                                             Α,
6028
                                    const GrB_Descriptor
                                                             desc);
6029
             // bind-second + scalar value
6030
             GrB_Info GrB_apply(GrB_Matrix
                                                             C,
6031
                                    const GrB_Matrix
                                                             Mask,
6032
                                    const GrB BinaryOp
6033
                                                             accum,
                                    const GrB_BinaryOp
                                                             op,
6034
                                    const GrB_Matrix
                                                             Α,
6035
                                    <type>
                                                             val.
6036
                                    const GrB_Descriptor
                                                             desc);
6037
             // bind-second + GraphBLAS scalar
6038
             GrB_Info GrB_apply(GrB_Matrix
                                                             С,
6039
                                    const GrB_Matrix
                                                             Mask,
6040
                                    const GrB_BinaryOp
                                                             accum,
6041
                                    const GrB_BinaryOp
                                                             op,
6042
                                    const GrB_Matrix
                                                             Α,
6043
```

| 6044 |            | const GrB_Scalar s,   |
|------|------------|---|
| 6045 |            | <pre>const GrB_Descriptor desc);</pre>  |
|      |            |   |
|      |            |   |
| 6046 | Parameters |   |
|      |            |   |
| 6047 | C          | (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values  |
| 6048 |            | that may be accumulated with the result of the apply operation. On output, the  |
| 6049 |            | matrix holds the results of the operation.  |
|      |            |   |
| 6050 | Mask       | (IN) An optional "write" mask that controls which results from this operation are   |
| 6051 |            | stored into the output matrix C. The mask dimensions must match those of the  |
| 6052 |            | matrix C. If the $GrB\_STRUCTURE$ descriptor is $not$ set for the mask, the domain  |
| 6053 |            | of the Mask matrix must be of type bool or any of the predefined "built-in" types   |
| 6054 |            | in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the  |
| 6055 |            | dimensions of C), GrB_NULL should be specified.   |
|      |            |   |
| 6056 | accum      | (IN) An optional binary operator used for accumulating entries into existing C  |
| 6057 |            | entries. If assignment rather than accumulation is desired, GrB_NULL should be  |
| 6058 |            | specified.  |
|      |            |   |
| 6059 | on         | (IN) A binary operator applied to each element of input matrix, A, with the element   |
| 6060 | Oβ         | of the input matrix used as the left-hand argument, and the scalar value, val, used   |
| 6061 |            | as the right-hand argument.   |
|      |            |   |
|      | Λ          | (INI) TO LOT AC 4: 1 1 4 11:  |
| 6062 | А          | (IN) The GraphBLAS matrix whose elements are passed to the binary operator as   |
| 6063 |            | the right-hand (second) argument in the <i>bind-first</i> variant, or the left-hand (first) argument in the <i>bind-second</i> variant. |
| 0004 |            | argument in the onthe second variant.   |
|      |            |   |
| 6065 | val        | (IN) Scalar value that is passed to the binary operator as the left-hand (first)  |
| 6066 |            | argument in the bind-first variant, or the right-hand (second) argument in the  |
| 6067 |            | bind-second variant.  |
|      |            |   |

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the bind-second variant. It must not be empty.

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6072 6073 s (IN) GraphBLAS scalar value that is passed to the binary operator as the left-hand

desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL

should be specified. Non-default field/value pairs are listed as follows:

(first) argument in the bind-first variant, or the right-hand (second) argument in

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|      |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input Mask matrix. The stored values are    |
| 6074 |       |          |               | not examined.                               |
|      | Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
|      | Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation        |
|      |       |          |               | (bind-second variant only).                 |
|      | Α     | GrB_INP1 | GrB_TRAN      | Use transpose of A for the operation        |
|      |       |          |               | (bind-first variant only).                  |

# 6075 Return Values

| 6076<br>6077<br>6078<br>6079<br>6080 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 6081                                 | GrB_PANIC                | Unknown internal error.  |
| 6082<br>6083<br>6084<br>6085         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 6086                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.   |
| 6087<br>6088                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).  |
| 6089<br>6090<br>6091                 | GrB_INDEX_OUT_OF_BOUNDS  | A value in row_indices is greater than or equal to $\mathbf{nrows}(A)$ , or a value in $\mathbf{col\_indices}$ is greater than or equal to $\mathbf{ncols}(A)$ . In non-blocking mode, this can be reported as an execution error.   |
| 6092<br>6093                         | GrB_DIMENSION_MISMATCH   | Mask and C dimensions are incompatible, nrows $\neq$ $\mathbf{nrows}(C)$ , or $\mathbf{ncols} \neq \mathbf{ncols}(C)$ .  |
| 6094<br>6095<br>6096<br>6097<br>6098 | GrB_DOMAIN_MISMATCH      | The domains of the various matrices and scalar are incompatible with the corresponding domains of the binary operator or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).                        |
| 6099<br>6100                         | GrB_EMPTY_OBJECT         | The $GrB\_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s) = 0)$ and therefore a value cannot be passed to the binary operator.  |

## 6101 Description

This variant of GrB\_apply computes the result of applying a binary operator to the elements of a GraphBLAS matrix each composed with a scalar constant, val or s:

bind-first: 
$$C = f(val, A)$$
 or  $C = f(s, A)$ 

bind-second: 
$$C = f(A, val)$$
 or  $C = f(A, s)$ ,

or if an optional binary accumulation operator (①) is provided:

bind-first: 
$$C = C \odot f(val, A)$$
 or  $C = C \odot f(s, A)$ 

bind-second: 
$$C = C \odot f(A, val)$$
 or  $C = C \odot f(A, s)$ .

6109 Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 6112 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- Up to three argument matrices are used in the GrB\_apply operation:

6115 1. 
$$C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$$

2. 
$$Mask = \langle \mathbf{D}(Mask), \mathbf{nrows}(Mask), \mathbf{ncols}(Mask), \mathbf{L}(Mask) = \{(i, j, M_{ij})\} \rangle$$
 (optional)

3. 
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument scalar, matrices, binary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 6122 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the binary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the binary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- 5. If bind-first:

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(a)  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.

- (b) If the non-opaque scalar val is provided, then  $\mathbf{D}(\text{val})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- (c) If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.

#### 6. If bind-second:

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- (a)  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the binary operator.
- (b) If the non-opaque scalar val is provided, then  $\mathbf{D}(\text{val})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.
- (c) If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the binary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):

- 1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathbf{C}$ .
  - 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
  - (b) If Mask  $\neq$  GrB NULL,
- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}}$  is computed from argument A as follows:

```
bind-first: \widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}
bind-second: \widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}
```

4. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{M}}).$ 

- 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$
- 4.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- If an empty GrB\_Scalar  $\tilde{s}$  is provided (nvals( $\tilde{s}$ ) = 0), the method returns with code GrB\_EMPTY\_OBJECT.
- If a non-empty  $GrB\_Scalar$ ,  $\tilde{s}$ , is provided (i.e.,  $\mathbf{nvals}(\tilde{s}) = 1$ ), we then create an internal variable
- val with the same domain as  $\tilde{s}$  and set val = val( $\tilde{s}$ ).
- We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$ : The matrix holding the result from applying the binary operator to the input matrix  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.
- The intermediate matrix,  $\tilde{\mathbf{T}}$ , is created as one of the following:

bind-first: 
$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\mathsf{val}, \widetilde{\mathbf{A}}(i, j))) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle$$

bind-second: 
$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\widetilde{\mathbf{A}}(i, j), \mathsf{val})) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle$$

6180 where f = f(op).

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- The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:
- If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathsf{then} \,\, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(\mathsf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix,  $\mathsf{C}$ , and elements of  $\mathsf{C}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 6208 4.3.8.5 apply: Vector index unary operator variant [Scott: NEW CONTENT]

Computes the transformation of the values of the stored elements of a vector using an index unary operator that is a function of the stored value, its location indices, and an user provided scalar value. The scalar can be passed either as a non-opaque variable or as a GrB\_Scalar object.

## 6212 C Syntax

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```
GrB_Info GrB_apply(GrB_Vector
                                                                W,
6213
                                    const GrB_Vector
6214
                                                                mask,
                                    const GrB_BinaryOp
                                                                accum,
6215
                                    const GrB_IndexUnaryOp
6216
                                                                op,
                                    const GrB_Vector
                                                                u,
6217
                                    <type>
                                                                val,
6218
                                    const GrB_Descriptor
                                                                desc);
6219
             GrB_Info GrB_apply(GrB_Vector
6220
                                                                W,
                                    const GrB_Vector
                                                                mask,
6221
                                    const GrB_BinaryOp
6222
                                                                accum,
                                    const GrB_IndexUnaryOp
                                                                op,
6223
                                    const GrB_Vector
                                                                u,
6224
                                    const GrB_Scalar
6225
                                                                s,
                                    const GrB_Descriptor
                                                                desc);
6226
```

#### **Parameters** 6227

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values 6228 that may be accumulated with the result of the apply operation. On output, this 6229 vector holds the results of the operation. 6230 mask (IN) An optional "write" mask that controls which results from this operation are 6231 stored into the output vector w. The mask dimensions must match those of the 6232 vector w. If the GrB STRUCTURE descriptor is not set for the mask, the domain 6233 of the mask vector must be of type bool or any of the predefined "built-in" types 6234 in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the 6235 dimensions of w), GrB\_NULL should be specified. 6236 accum (IN) An optional binary operator used for accumulating entries into existing w 6237 entries. If assignment rather than accumulation is desired, GrB\_NULL should be 6238 specified. 6239 op (IN) An index unary operator,  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ , applied 6240 to each element stored in the input vector, u. It is a function of the stored element's 6241 value, its location index, and a user supplied scalar value (either s or val). 6242 u (IN) The GraphBLAS vector whose elements are passed to the index unary oper-6243 ator. 6244 val (IN) An additional scalar value that is passed to the index unary operator. 6245 s (IN) An additional GraphBLAS scalar that is passed to the index unary operator. 6246 It must not be empty. 6247 desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL 6248 should be specified. Non-default field/value pairs are listed as follows: 6249 6250 Param Value Field Description GrB OUTP **GrB\_REPLACE** Output vector w is cleared (all elements W removed) before the result is stored in it. mask GrB\_MASK GrB\_STRUCTURE The write mask is constructed from the 6251 structure (pattern of stored values) of the input mask vector. The stored values are not examined. GrB\_MASK GrB\_COMP

### Return Values

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mask

GrB\_SUCCESS In blocking mode, the operation completed successfully. In nonblocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

Use the complement of mask.

GrB\_PANIC Unknown internal error.

Grb INVALID OBJECT This is returned in any execution mode whenever one of the

opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB error() to

access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation. 6263

GrB UNINITIALIZED OBJECT One or more of the GraphBLAS objects has not been initialized 6264 by a call to **new** (or another constructor). 6265

GrB\_DIMENSION\_MISMATCH mask, w and/or u dimensions are incompatible. 6266

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the cor-

responding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

Grb EMPTY OBJECT The Grb Scalar's used in the call is empty  $(\mathbf{nvals}(s) = 0)$  and therefore a value cannot be passed to the index unary operator.

#### Description 6273

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This variant of GrB\_apply computes the result of applying an index unary operator to the elements 6274 of a GraphBLAS vector each composed with the element's index and a scalar constant, val or s: 6275

$$w = f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{val}) \text{ or } \mathsf{w} = f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}),$$

or if an optional binary accumulation operator  $(\odot)$  is provided: 6277

$$\mathsf{w} = \mathsf{w} \odot f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{val}) \text{ or } \mathsf{w} = \mathsf{w} \odot f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}).$$

Logically, this operation occurs in three steps: 6279

**Setup** The internal vectors and mask used in the computation are formed and their domains 6280 and dimensions are tested for compatibility. 6281

**Compute** The indicated computations are carried out. 6282

Output The result is written into the output vector, possibly under control of a mask. 6283

Up to three argument vectors are used in this GrB\_apply operation: 6284

- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$ 6285
- 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional) 6286

```
3. \mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle
```

The argument scalar, vectors, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the index unary operator.
- 5. If the non-opaque scalar val is provided, then  $\mathbf{D}(\mathsf{val})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.
- 6. If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
  - (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \forall i : 0 \le i < \mathbf{size}(\mathbf{w})\} \rangle$ .
- (b) If mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- 6314 (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- 4. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 1.  $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}})$
- 6320 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$
- If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- If an empty GrB\_Scalar  $\tilde{s}$  is provided ( $\mathbf{nvals}(\tilde{s}) = 0$ ), the method returns with code GrB\_EMPTY\_OBJECT.
- If a non-empty GrB\_Scalar,  $\tilde{s}$ , is provided (**nvals**( $\tilde{s}$ ) = 1), we then create an internal variable val
- with the same domain as  $\tilde{s}$  and set  $val = val(\tilde{s})$ .
- We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$ : The vector holding the result from applying the index unary operator to the input vector  $\widetilde{\mathbf{u}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{ (i, f_i(\widetilde{\mathbf{u}}(i), [i], 0, \mathsf{val})) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle,$$

where  $f_i = \mathbf{f}(\mathsf{op})$ .

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- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:
- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

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$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

```
\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.
```

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 4.3.8.6 apply: Matrix index unary operator variant[Scott: NEW CONTENT]

Computes the transformation of the values of the stored elements of a matrix using an index unary operator that is a function of the stored value, its location indices, and an user provided scalar value. The scalar can be passed either as a non-opaque variable or as a GrB\_Scalar object.

# 6367 C Syntax

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```
GrB_Info GrB_apply(GrB_Matrix
                                                                С,
6368
                                    const GrB_Matrix
                                                                Mask,
6369
                                    const GrB_BinaryOp
                                                                accum,
6370
                                    const GrB_IndexUnaryOp
                                                                op,
6371
                                    const GrB_Matrix
                                                                Α,
6372
                                                                val,
                                    <type>
6373
                                    const GrB_Descriptor
                                                                desc);
6374
             GrB_Info GrB_apply(GrB_Matrix
                                                                C,
6375
                                    const GrB_Matrix
                                                                Mask,
6376
                                    const GrB_BinaryOp
                                                                accum,
6377
                                    const GrB_IndexUnaryOp
                                                                op,
6378
                                    const GrB_Matrix
                                                                Α,
6379
                                    const GrB_Scalar
6380
                                                                s,
                                    const GrB_Descriptor
                                                                desc);
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```

#### Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.

- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is not set for the mask, the domain 6388 of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the 6390 dimensions of C), GrB NULL should be specified.
  - accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) An index unary operator,  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ , applied to each element stored in the input matrix, A. It is a function of the stored element's value, its row and column indices, and a user supplied scalar value (either s or val).
    - A (IN) The GraphBLAS matrix whose elements are passed to the index unary operator.
    - val (IN) An additional scalar value that is passed to the index unary operator.
      - s (IN) An additional GraphBLAS scalar that is passed to the index unary operator.
  - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input Mask matrix. The stored values are    |
|       |          |               | not examined.                               |
| Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
| Α     | GrB INP0 | GrB TRAN      | Use transpose of A for the operation.       |

#### Return Values

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| 6407 | GrB_SUCCESS In blocking mode, the operation completed successfully. In non- |
|------|---|
| 6408 | blocking mode, this indicates that the compatibility tests on di-           |
| 6409 | mensions and domains for the input arguments passed successfully.           |
| 6410 | Either way, output matrix C is ready to be used in the next method          |
| 6411 | of the sequence.  |
|      |   |

Grb Panic Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused

by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

6418 GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or another constructor).

6420 GrB\_DIMENSION\_MISMATCH mask, w and/or u dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

GrB\_EMPTY\_OBJECT The GrB\_Scalar s used in the call is empty ( $\mathbf{nvals}(s) = 0$ ) and therefore a value cannot be passed to the index unary operator.

## 6427 Description

This variant of GrB\_apply computes the result of applying a index unary operator to the elements of a GraphBLAS matrix each composed with the elements row and column indices, and a scalar constant, val or s:

$$C = f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{val}) \text{ or } C = f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{sol}(\mathbf{ind}(A)), \mathsf{sol}(A))$$

or if an optional binary accumulation operator (⊙) is provided:

$$C = C \odot f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{val}) \text{ or } C = C \odot f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{sol}).$$

Where the **row** and **col** functions extract the row and column indices from a list of two-dimensional indices, respectively.

6436 Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6439 Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

6441 Up to three argument matrices are used in the GrB\_apply operation:

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 6443 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$

```
3. A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle
```

The argument scalar, matrices, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the index unary operator.
- 5. If the non-opaque scalar val is provided, then  $\mathbf{D}(\mathsf{val})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.
- 6. If the GrB\_Scalar s is provided, then  $\mathbf{D}(s)$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ( $\leftarrow$  denotes copy):

1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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- 2. Two-dimensional mask, M, is computed from argument Mask as follows:
- (a) If Mask = GrB\_NULL, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$ .
- (b) If Mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
    - ii. Otherwise,  $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\tilde{\mathbf{A}}$  is computed from argument A as follows:

$$\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB}\_\mathsf{INP0}].\mathsf{GrB}\_\mathsf{TRAN} \; ? \; \mathsf{A}^T : \mathsf{A}$$

4. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

- 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$

If any compatibility rule above is violated, execution of GrB\_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB\_Scalar  $\tilde{s}$  is provided (**nvals**( $\tilde{s}$ ) = 0), the method returns with code GrB\_EMPTY\_OBJECT.

If a non-empty GrB\_Scalar,  $\tilde{s}$ , is provided (i.e.,  $\mathbf{nvals}(\tilde{s}) = 1$ ), we then create an internal variable

val with the same domain as  $\tilde{s}$  and set val = val( $\tilde{s}$ ).

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the result from applying the index unary operator to the input matrix  $\widetilde{\mathbf{A}}$ .
  - $\widetilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\tilde{\mathbf{T}}$ , is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f_i(\widetilde{\mathbf{A}}(i, j), i, j, \mathsf{val})) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle,$$

where  $f_i = \mathbf{f}(op)$ .

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The intermediate matrix  $\widetilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

- If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathrm{then} \,\, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc(accum)$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If desc[GrB\_OUTP].GrB\_REPLACE is not set, the elements of  $\tilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 6526 **4.3.9** select:

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Apply a select operator to the stored elements of an object to determine whether or not to keep them.

## $_{6529}$ 4.3.9.1 select: Vector variant[Scott: NEW CONTENT]

Apply a select operator (an index unary operator) to the elements of a vector.

## 6531 C Syntax

```
// scalar value variant
6532
             GrB Info GrB select(GrB Vector
                                                                 W,
6533
                                     const GrB Vector
                                                                 mask,
6534
                                     const GrB_BinaryOp
                                                                 accum.
6535
                                     const GrB_IndexUnaryOp
                                                                 op,
6536
                                     const GrB_Vector
                                                                 u,
6537
                                     <type>
                                                                 val,
6538
                                     const GrB_Descriptor
6539
                                                                 desc);
6540
              // GraphBLAS scalar variant
6541
             GrB_Info GrB_select(GrB_Vector
6542
                                                                 W,
                                     const GrB_Vector
                                                                 mask.
6543
```

| 6544 | const | <pre>GrB_BinaryOp</pre>     | accum,            |
|------|-------|-----------------------------|-------------------|
| 6545 | const | <pre>GrB_IndexUnaryOp</pre> | op,               |
| 6546 | const | <pre>GrB_Vector</pre>       | u,                |
| 6547 | const | <pre>GrB_Scalar</pre>       | s,                |
| 6548 | const | <pre>GrB_Descriptor</pre>   | <pre>desc);</pre> |

#### Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the select operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) An index unary operator,  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ , applied to each element stored in the input vector,  $\mathbf{u}$ . It is a function of the stored element's value, its location index, and a user supplied scalar value (either  $\mathbf{s}$  or  $\mathsf{val}$ ).
    - u (IN) The GraphBLAS vector whose elements are passed to the index unary operator.
  - val (IN) An additional scalar value that is passed to the index unary operator.
    - s (IN) An GraphBLAS scalar that is passed to the index unary operator. It must not be empty.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 6574 |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input mask vector. The stored values are    |
|      |       |          |               | not examined.                               |
|      | mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |

#### 6575 Return Values

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GrB\_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for operation.

GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to one of its constructors.

6589 GrB\_DIMENSION\_MISMATCH mask, w and/or u dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

GrB\_EMPTY\_OBJECT The GrB\_Scalar s used in the call is empty (nvals(s) = 0) and therefore a value cannot be passed to the index unary operator.

### 6596 Description

This variant of GrB\_select computes the result of applying a index unary operator to select the elements of the input GraphBLAS vector. The operator takes, as input, the value of each stored element, along with the element's index and a scalar constant – either val or s. The corresponding element of the input vector is selected (kept) if the function evaluates to true when cast to bool. This acts like a functional mask on the input vector as follows:

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$$\mathbf{w} = \mathbf{u} \langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), 0, \mathsf{val}) \rangle,$$

$$\mathbf{w} = \mathbf{w} \odot \mathbf{u} \langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), 0, \mathsf{val}) \rangle.$$

6604 Correspondingly, if a GrB\_Scalar, s, is provided:

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$$\mathsf{w} = \mathsf{u} \langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}) \rangle,$$
 
$$\mathsf{w} = \mathsf{w} \odot \mathsf{u} \langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}) \rangle.$$

- 6607 Logically, this operation occurs in three steps:
- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 6610 Compute The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 6612 Up to three argument vectors are used in this GrB\_select operation:
- 1.  $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 6614 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)
- 3.  $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument scalar, vectors, index unary operator and the accumulation operator (if provided)
  are tested for domain compatibility as follows:
- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 6620 2.  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}(\mathbf{u})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(w)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator must be from one of the pre-defined types of Table 3.2; i.e., castable to bool.
- 5.  $\mathbf{D}(\mathbf{u})$  must be compatible with  $\mathbf{D}_{in_1}(\mathbf{op})$  of the index unary operator.
- 6.  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$ , depending on the signature of the method, must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_select ends and the domain mismatch error listed above is returned.
- From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):
- 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:

```
(a) If mask = GrB_NULL, then \widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle.
```

- (b) If mask  $\neq$  GrB\_NULL,
- i. If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE} \text{ is } \mathsf{set}, \text{ then } \widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i: i \in \mathbf{ind}(\mathsf{mask})\} \rangle,$
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- (c) If desc[GrB\_MASK].GrB\_COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .
- 4. Scalar  $\tilde{s} \leftarrow s$  (GrB\_Scalar version only).
- The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:
- 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 6648 2.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$
- If any compatibility rule above is violated, execution of GrB\_select ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with G652 GrB\_SUCCESS return code and defer any computation and/or execution error codes.
- If an empty  $GrB\_Scalar\ \widetilde{s}$  is provided (i.e.,  $nvals(\widetilde{s}) = 0$ ), the method returns with code  $GrB\_EMPTY\_OBJECT$ .
- If a non-empty  $GrB\_Scalar$ ,  $\widetilde{s}$ , is provided (i.e.,  $nvals(\widetilde{s}) = 1$ ), we then create an internal variable val with the same domain as  $\widetilde{s}$  and set  $val = val(\widetilde{s})$ .
- We are now ready to carry out the select and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$ : The vector holding the result from applying the index unary operator to the input vector  $\widetilde{\mathbf{u}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathsf{u}}), \{(i, \widetilde{\mathsf{u}}(i), : i \in \mathbf{ind}(\widetilde{\mathsf{u}}) \land (\mathsf{bool}) f_i(\widetilde{\mathsf{u}}(i), i, 0, \mathsf{val}) = \mathsf{true} \} \rangle,$$

where  $f_i = \mathbf{f}(\mathsf{op})$ .

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The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

```
z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),
z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),
z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),
```

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

# 4.3.9.2 select: Matrix variant[Scott: NEW CONTENT]

Apply a select operator (an index unary operator) to the elements of a matrix.

#### 6693 C Syntax

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```
// scalar value variant
6694
             GrB_Info GrB_select(GrB_Matrix
                                                                 С,
6695
                                     const GrB_Matrix
                                                                 Mask,
6696
                                     const GrB_BinaryOp
                                                                 accum,
6697
                                     const GrB_IndexUnaryOp
6698
                                                                 op,
                                     const GrB_Matrix
                                                                 Α,
6699
                                     <type>
                                                                 val,
6700
                                     const GrB_Descriptor
                                                                 desc);
6701
```

```
// GraphBLAS scalar variant
6703
             GrB_Info GrB_select(GrB_Matrix
                                                                С,
6704
                                     const GrB_Matrix
                                                                Mask,
6705
                                     const GrB_BinaryOp
                                                                accum,
6706
                                     const GrB IndexUnaryOp
                                                                op,
6707
                                     const GrB Matrix
                                                                Α,
6708
                                     const GrB Scalar
                                                                s,
6709
                                     const GrB_Descriptor
                                                                desc);
6710
```

#### Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the select operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) An index unary operator,  $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ , applied to each element stored in the input matrix, A. It is a function of the stored element's value, its row and column indices, and a user supplied scalar value (either s or val).
  - A (IN) The GraphBLAS matrix whose elements are passed to the index unary operator.
  - val (IN) An additional scalar value that is passed to the index unary operator.
    - s (IN) An GraphBLAS scalar that is passed to the index unary operator. It must not be empty.
  - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

|      | Param | Field    | Value         | Description                                 |
|------|-------|----------|---------------|---|
|      | С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|      |       |          |               | removed) before the result is stored in it. |
|      | Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
| 6735 |       |          |               | structure (pattern of stored values) of the |
|      |       |          |               | input Mask matrix. The stored values are    |
|      |       |          |               | not examined.                               |
|      | Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
|      | Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |

## 36 Return Values

| 6737<br>6738<br>6739<br>6740<br>6741 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output mattrix ${\sf C}$ is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|---|
| 6742                                 | GrB_PANIC                | Unknown internal error.   |
| 6743<br>6744<br>6745<br>6746         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.   |
| 6747                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for operation.  |
| 6748<br>6749                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to one of its constructors.   |
| 6750                                 | GrB_DIMENSION_MISMATCH   | Mask, C and/or A dimensions are incompatible.   |
| 6751<br>6752<br>6753<br>6754         | GrB_DOMAIN_MISMATCH      | The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).                                       |
| 6755<br>6756                         | GrB_EMPTY_OBJECT         | The $GrB\_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s)=0)$ and therefore a value cannot be passed to the index unary operator.  |

# Description

This variant of GrB\_select computes the result of applying a index unary operator to select the elements of the input GraphBLAS matrix. The operator takes, as input, the value of each stored element, along with the element's row and column indices and a scalar constant – from either val or s. The corresponding element of the input matrix is selected (kept) if the function evaluates to true when cast to bool. This acts like a functional mask on the input matrix as follows when specifying a transparent scalar value:

6764 
$$\mathsf{C} = \mathsf{A}\langle f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{val})\rangle, \text{ or}$$

$$\mathsf{C} = \mathsf{C} \odot \mathsf{A}\langle f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{val})\rangle.$$

6766 Correspondingly, if a GrB\_Scalar, s, is provided:

6767 
$$C = A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), s) \rangle, \text{ or}$$
6768 
$$C = C \odot A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), s) \rangle.$$

Where the **row** and **col** functions extract the row and column indices from a list of two-dimensional indices, respectively.

6771 Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 6774 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- <sup>6776</sup> Up to three argument matrices are used in the GrB\_select operation:
- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 6778 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument scalar, matrices, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 6784 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{A})$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 4.  $\mathbf{D}_{out}(\mathsf{op})$  of the index unary operator must be from one of the pre-defined types of Table 3.2; i.e., castable to bool.
- 5.  $\mathbf{D}(A)$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$  of the index unary operator.
- 6.  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$ , depending on the signature of the method, must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$  of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all 6794 compatible with each other. A domain from a user-defined type is only compatible with itself. If 6795 any compatibility rule above is violated, execution of GrB\_select ends and the domain mismatch 6796 error listed above is returned. 6797

From the argument matrices, the internal matrices, mask, and index arrays used in the computation 6798 are formed ( $\leftarrow$  denotes copy):

1. Matrix  $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$ . 6800

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2. Two-dimensional mask, M, is computed from argument Mask as follows: 6801

```
(a) If Mask = GrB_NULL, then \widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq i < \mathbf{nr
6802
                                                                                                                                                                                                                                                                                                                                                                                                                                                              j < \mathbf{ncols}(\mathsf{C}) \} \rangle.
6803
```

- (b) If Mask  $\neq$  GrB NULL,
  - i. If desc[GrB MASK].GrB STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : \}$  $(i, j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$  $\{(i,j):(i,j)\in\mathbf{ind}(\mathsf{Mask})\land(\mathsf{bool})\mathsf{Mask}(i,j)=\mathsf{true}\}$ .
- (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}}$  is computed from argument A as follows:  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB} \ \mathsf{INP0}].\mathsf{GrB} \ \mathsf{TRAN}\ ?\ \mathsf{A}^T : \mathsf{A}$ 6810
- 4. Scalar  $\tilde{s} \leftarrow s$  (GrB\_Scalar version only). 6811

The internal matrices and mask are checked for dimension compatibility. The following conditions 6812 must hold: 6813

- 1.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{M}}).$ 6814
- 2.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}).$ 6815
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}})$ . 6816
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$ 6817

If any compatibility rule above is violated, execution of GrB\_select ends and the dimension mismatch 6818 error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with 6820 GrB SUCCESS return code and defer any computation and/or execution error codes. 6821

If an empty GrB Scalar  $\tilde{s}$  is provided (i.e.,  $nvals(\tilde{s}) = 0$ ), the method returns with code GrB EMPTY OBJECT. 6822

If a non-empty  $GrB\_Scalar$ ,  $\tilde{s}$ , is provided (i.e.,  $nvals(\tilde{s}) = 1$ ), we then create an internal variable 6823

val with the same domain as  $\tilde{s}$  and set val = val( $\tilde{s}$ ). 6824

We are now ready to carry out the select and any additional associated operations. We describe 6825 this in terms of two intermediate matrices: 6826

- $\widetilde{\mathbf{T}}$ : The matrix holding the result from applying the index unary operator to the input matrix  $\widetilde{\mathbf{A}}$ .
  - $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix,  $\widetilde{\mathbf{T}}$ , is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \\ \{(i, j, \widetilde{\mathbf{A}}(i, j) : i, j \in \mathbf{ind}(\widetilde{\mathbf{A}}) \land (\mathsf{bool}) f_i(\widetilde{\mathbf{A}}(i, j), i, j, \mathsf{val}) = \mathsf{true} \} \rangle,$$

where  $f_i = f(op)$ .

The intermediate matrix  $\hat{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$ 
  - If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

## 6860 4.3.10 reduce: Perform a reduction across the elements of an object

6861 Computes the reduction of the values of the elements of a vector or matrix.

#### 6862 4.3.10.1 reduce: Standard matrix to vector variant

This performs a reduction across rows of a matrix to produce a vector. If reduction down columns is desired, the input matrix should be transposed using the descriptor.

## 6865 C Syntax

```
GrB_Info GrB_reduce(GrB_Vector
                                                                W,
6866
                                     const GrB_Vector
6867
                                                                mask,
                                     const GrB_BinaryOp
                                                                accum
6868
                                     const GrB Monoid
                                                                op,
6869
                                     const GrB_Matrix
                                                                Α,
6870
                                     const GrB Descriptor
                                                                desc);
6871
6872
             GrB_Info GrB_reduce(GrB_Vector
6873
                                                                w,
                                     const GrB_Vector
                                                                mask,
6874
                                     const GrB_BinaryOp
                                                                accum,
6875
                                     const GrB BinaryOp
6876
                                                                op,
                                     const GrB Matrix
6877
                                                                Α,
                                     const GrB Descriptor
                                                                desc);
6878
```

### 79 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the reduction operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The monoid or binary operator used in the element-wise reduction operation. Depending on which type is passed, the following defines the binary operator with one domain,  $F_b = \langle D, D, D, \oplus \rangle$ , that is used:

BinaryOp:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \odot(\mathsf{op}) \rangle$ .

Monoid:  $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \odot(\mathsf{op}) \rangle$ , the identity element of the monoid is ignored.

If op is a  $GrB\_BinaryOp$ , then all its domains must be the same. Furthermore, in both cases  $\bigcirc(op)$  must be commutative and associative. Otherwise, the outcome of the operation is undefined.

A (IN) The GraphBLAS matrix on which reduction will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| W     | GrB_OUTP | GrB_REPLACE   | Output vector w is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input mask vector. The stored values are    |
|       |          |               | not examined.                               |
| mask  | GrB_MASK | GrB_COMP      | Use the complement of mask.                 |
| Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |

## Segon Return Values

| 6907<br>6908<br>6909<br>6910<br>6911 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 6912                                 | GrB_PANIC                | Unknown internal error.  |
| 6913<br>6914<br>6915<br>6916         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 6917                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.   |
| 6918 <b>G</b>                        | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).   |
| 6920 (                               | GrB_DIMENSION_MISMATCH   | mask,w and/or $u$ dimensions are incompatible.   |
| 6921<br>6922<br>6923                 | GrB_DOMAIN_MISMATCH      | Either the domains of the various vectors and matrices are incompatible with the corresponding domains of the accumulation operator or reduce function, or the domains of the GraphBLAS binary   |

operator op are not all the same, or the mask's domain is not compatible with bool (in the case where desc[GrB\_MASK].GrB\_STRUCTURE is not set).

### 6927 Description

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This variant of GrB\_reduce computes the result of performing a reduction across each of the rows of an input matrix:  $w(i) = \bigoplus A(i,:) \forall i$ ; or, if an optional binary accumulation operator is provided,  $w(i) = w(i) \odot (\bigoplus A(i,:)) \forall i$ , where  $\bigoplus = \bigodot (F_b)$  and  $\odot = \bigodot (accum)$ .

6931 Logically, this operation occurs in three steps:

Setup The internal vector, matrix and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6934 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

6936 Up to two vector and one matrix argument are used in this GrB\_reduce operation:

```
6937 1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

6938 2.  $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$  (optional)

3. 
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument vector, matrix, reduction operator and accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(w)$  must be compatible with the domain of the reduction binary operator,  $\mathbf{D}(F_b)$ .
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathbf{w})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(F_b)$ , must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
  - 4.  $\mathbf{D}(A)$  must be compatible with the domain of the binary reduction operator,  $\mathbf{D}(F_b)$ .

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_reduce ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ( $\leftarrow$  denotes copy):

6956 1. Vector  $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$ .

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- 2. One-dimensional mask,  $\widetilde{\mathbf{m}}$ , is computed from argument mask as follows:
- 6958 (a) If mask = GrB\_NULL, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$ .
- (b) If  $mask \neq GrB\_NULL$ ,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$ ,
- ii. Otherwise,  $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$ .
- 6962 (c) If desc[GrB MASK].GrB COMP is set, then  $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$ .
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:
- 6966 1.  $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 6967 2.  $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{nrows}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB\_reduce ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GPB\_SUCCESS return code and defer any computation and/or execution error codes.
- We carry out the reduce and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$ : The vector holding the result from reducing along the rows of input matrix  $\tilde{\mathbf{A}}$ .
- $\tilde{\mathbf{z}}$ : The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector,  $\tilde{\mathbf{t}}$ , is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i,t_i) : \mathbf{ind}(A(i,:)) \neq \emptyset \} \rangle.$$

6978 The value of each of its elements is computed by

6979 
$$t_i = \bigoplus_{j \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:))} \widetilde{\mathbf{A}}(i,j),$$

- where  $\bigoplus = \bigcirc(F_b)$ .
- The intermediate vector  $\tilde{\mathbf{z}}$  is created as follows, using what is called a standard vector accumulate:
- If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$

• If accum is a binary operator, then  $\tilde{\mathbf{z}}$  is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{z}}$  are computed based on the relationships between the sets of indices in  $\tilde{\mathbf{w}}$  and  $\tilde{\mathbf{t}}$ .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector  $\tilde{\mathbf{z}}$  are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\tilde{\mathbf{z}}$  indicated by the mask are copied into the result vector,  $\mathbf{w}$ , and elements of  $\mathbf{w}$  that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

#### 7008 4.3.10.2 reduce: Vector-scalar variant[Scott: NEW CONTENT]

7009 Reduce all stored values into a single scalar.

### 7010 C Syntax

```
// scalar value + monoid (only)
GrB_Info GrB_reduce(<type> *val,
const GrB_BinaryOp accum,
const GrB_Monoid op,
const GrB_Vector u,
```

```
const GrB_Descriptor
                                                              desc);
7016
7017
              // GraphBLAS Scalar + monoid
7018
             GrB_Info GrB_reduce(GrB_Scalar
7019
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
7020
                                     const GrB_Monoid
                                                              op,
7021
                                     const GrB_Vector
                                                              u,
7022
                                     const GrB_Descriptor
                                                              desc);
7023
7024
              // GraphBLAS Scalar + binary operator
7025
             GrB_Info GrB_reduce(GrB_Scalar
7026
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
7027
                                     const GrB_BinaryOp
                                                              op,
7028
                                     const GrB_Vector
                                                              u,
7029
                                     const GrB_Descriptor
                                                              desc);
7030
```

#### 7031 Parameters

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- val or s (INOUT) Scalar to store final reduced value into. On input, the scalar provides a value that may be accumulated (optionally) with the result of the reduction operation. On output, this scalar holds the results of the operation.
  - accum (IN) An optional binary operator used for accumulating entries into an existing scalar (s or val) value. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) The monoid  $(M = \langle D, \oplus, 0 \rangle)$  or binary operator  $(F_b = \langle D, D, D, \oplus \rangle)$  used in the reduction operation. The  $\oplus$  operator must be commutative and associative; otherwise, the outcome of the operation is undefined.
      - u (IN) The GraphBLAS vector on which reduction will be performed.
  - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

## Param Field Value Description

*Note:* This argument is defined for consistency with the other GraphBLAS operations. There are currently no non-default field/value pairs that can be set for this operation.

#### 7049 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully, and the output scalar (s or val) is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB\_error() to access any error messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

7059 GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to a respective constructor.

GrB\_NULL\_POINTER val pointer is NULL.

GrB\_DOMAIN\_MISMATCH The domains of input and output arguments are incompatible with the corresponding domains of the accumulation operator, or reduce operator.

#### 7065 Description

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This variant of GrB\_reduce computes the result of performing a reduction across all of the stored elements of an input vector storing the result into either s or val. This corresponds to (shown here for the scalar value case only):

$$\mathsf{val} \; = \left\{ \begin{aligned} &\bigoplus_{i \in \mathbf{ind}(\mathsf{u})} \mathsf{u}(i), & \text{ or } \\ &\mathsf{val} \; \odot \; \left[ \bigoplus_{i \in \mathbf{ind}(\mathsf{u})} \mathsf{u}(i) \right], \text{ if the the optional accumulator is specified.} \end{aligned} \right.$$

where  $\bigoplus = \bigcirc(\mathsf{op})$  and  $\odot = \bigcirc(\mathsf{accum})$ .

7071 Logically, this operation occurs in three steps:

Setup The internal vector used in the computation is formed and its domain is tested for compatibility.

7074 Compute The indicated computations are carried out.

Output The result is written into the output scalar.

One vector argument is used in this GrB\_reduce operation:

7077 1. 
$$u = \langle \mathbf{D}(u), \mathbf{size}(u), \mathbf{L}(u) = \{(i, u_i)\} \rangle$$

The output scalar, argument vector, reduction operator and accumulation operator (if provided) are tested for domain compatibility as follows:

1. If accum is GrB\_NULL, then  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{op})$  from M (or with  $\mathbf{D}_{in_1}(\mathsf{op})$  and  $\mathbf{D}_{in_2}(\mathsf{op})$  from  $F_b$ ).

- 2. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator, and  $\mathbf{D}(\mathsf{op})$  from M (or  $\mathbf{D}_{out}(\mathsf{op})$  from  $F_b$ ) must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
  - 3.  $\mathbf{D}(\mathsf{u})$  must be compatible with  $\mathbf{D}(\mathsf{op})$  from M (or with  $\mathbf{D}_{in_1}(\mathsf{op})$  and  $\mathbf{D}_{in_2}(\mathsf{op})$  from  $F_b$ ).

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_reduce ends and the domain mismatch error listed above is returned.

The number of values stored in the input, u, is checked. If there are no stored values in u, then one of the following occurs depending on the output variant:

$$\mathbf{L}(s) = \begin{cases} \{\}, & \text{(cleared) if accum} = \text{GrB\_NULL}, \\ \\ \mathbf{L}(s), & \text{(unchanged) otherwise}, \end{cases}$$

7094 Or  $\text{val} = \begin{cases} \mathbf{0}(\mathsf{op}), & \text{(unchanged) otherwise,} \\ \\ \mathbf{0}(\mathsf{op}), & \text{(cleared) if accum} = \mathsf{GrB\_NULL,} \\ \\ \mathsf{val} \ \odot \ \mathbf{0}(\mathsf{op}), & \text{otherwise,} \end{cases}$ 

where  $\mathbf{0}(\mathsf{op})$  is the identity of the monoid. The operation returns immediately with  $\mathsf{GrB\_SUCCESS}$ .

For all other cases, the internal vector and scalar used in the computation is formed ( $\leftarrow$  denotes copy):

7099 1. Vector  $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$ .

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7100 2. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

We are now ready to carry out the reduction and any additional associated operations. An intermediate scalar result t is computed as follows:

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$$t = \bigoplus_{i \in \mathbf{ind}(\widetilde{\mathbf{u}})} \widetilde{\mathbf{u}}(i),$$

where  $\oplus = \bigcirc(\mathsf{op})$ .

7105 The final reduction value is computed as follows:

In both GrB\_BLOCKING and GrB\_NONBLOCKING modes, the method exits with return value GrB\_SUCCESS and the new contents of the output scalar is as defined above.

#### 7111 4.3.10.3 reduce: Matrix-scalar variant[Scott: NEW CONTENT]

7112 Reduce all stored values into a single scalar.

#### 7113 C Syntax

```
// scalar value + monoid (only)
7114
             GrB_Info GrB_reduce(<type>
                                                             *val,
7115
                                    const GrB_BinaryOp
                                                              accum,
7116
                                    const GrB_Monoid
7117
                                                              op,
                                     const GrB_Matrix
                                                              Α,
7118
                                     const GrB_Descriptor
                                                              desc);
7119
7120
             // GraphBLAS Scalar + monoid
7121
             GrB_Info GrB_reduce(GrB_Scalar
7122
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
7123
7124
                                    const GrB_Monoid
                                                              op,
                                     const GrB_Matrix
                                                              Α,
7125
                                     const GrB Descriptor
                                                              desc);
7126
7127
             // GraphBLAS Scalar + binary operator
7128
             GrB_Info GrB_reduce(GrB_Scalar
7129
                                                              s,
                                    const GrB_BinaryOp
                                                              accum,
7130
                                    const GrB_BinaryOp
7131
                                                              op,
                                     const GrB_Matrix
                                                              Α,
7132
                                     const GrB_Descriptor
                                                              desc);
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```

#### 7134 Parameters

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- val or s (INOUT) Scalar to store final reduced value into. On input, the scalar provides a value that may be accumulated (optionally) with the result of the reduction operation. On output, this scalar holds the results of the operation.
  - accum (IN) An optional binary operator used for accumulating entries into existing (s or val) value. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
    - op (IN) The monoid  $(M = \langle D, \oplus, 0 \rangle)$  or binary operator  $(F_b = \langle D, D, D, \oplus \rangle)$  used in the reduction operation. The  $\oplus$  operator must be commutative and associative; otherwise, the outcome of the operation is undefined.
      - A (IN) The GraphBLAS matrix on which the reduction will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

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Param Field Value Description

*Note:* This argument is defined for consistency with the other GraphBLAS operations. There are currently no non-default field/value pairs that can be set for this operation.

#### 7152 Return Values

GrB\_SUCCESS In blocking or non-blocking mode, the operation completed successfully, and the output scalar (s or val) is ready to be used in the next method of the sequence.

GrB\_PANIC Unknown internal error.

GrB\_INVALID\_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB\_error() to access any error
messages generated by the implementation.

GrB\_OUT\_OF\_MEMORY Not enough memory available for the operation.

7162 GrB\_UNINITIALIZED\_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to a respective constructor.

GrB\_NULL\_POINTER val pointer is NULL.

GrB\_DOMAIN\_MISMATCH The domains of input and output arguments are incompatible with the corresponding domains of the accumulation operator, or reduce operator.

#### 7168 Description

This variant of GrB\_reduce computes the result of performing a reduction across all of the stored elements of an input matrix storing the result into either s or val. This corresponds to (shown here for the scalar value case only):

val = 
$$\begin{cases} \bigoplus_{(i,j) \in \mathbf{ind}(\mathsf{A})} \mathsf{A}(i,j), & \text{or} \\ \\ \mathsf{val} \ \odot \ \left[ \bigoplus_{(i,j) \in \mathbf{ind}(\mathsf{A})} \mathsf{A}(i,j) \right], & \text{if the the optional accumulator is specified.} \end{cases}$$

where  $\bigoplus = \bigcirc(\mathsf{op})$  and  $\odot = \bigcirc(\mathsf{accum})$ .

Logically, this operation occurs in three steps:

**Setup** The internal matrix used in the computation is formed and its domain is tested for compatibility.

7177 Compute The indicated computations are carried out.

Output The result is written into the output scalar.

One matrix argument is used in this GrB\_reduce operation:

1. 
$$A = \langle \mathbf{D}(A), \mathbf{size}(A), \mathbf{L}(A) = \{(i, j, A_{i,j})\} \rangle$$

The output scalar, argument matrix, reduction operator and accumulation operator (if provided)
are tested for domain compatibility as follows:

- 1. If accum is GrB\_NULL, then  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}(\mathsf{op})$  from M (or with  $\mathbf{D}_{in_1}(\mathsf{op})$  and  $\mathbf{D}_{in_2}(\mathsf{op})$  from  $F_b$ ).
  - 2. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{val})$  or  $\mathbf{D}(\mathsf{s})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator, and  $\mathbf{D}(\mathsf{op})$  from M (or  $\mathbf{D}_{out}(\mathsf{op})$  from  $F_b$ ) must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.
- 3.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}(\mathsf{op})$  from M (or with  $\mathbf{D}_{in_1}(\mathsf{op})$  and  $\mathbf{D}_{in_2}(\mathsf{op})$  from  $F_b$ ).

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_reduce ends and the domain mismatch error listed above is returned.

The number of values stored in the input, A, is checked. If there are no stored values in A, then one of the following occurs depending on the output variant:

$$\mathbf{L}(s) = \begin{cases} \{\}, & \text{(cleared) if accum} = \mathsf{GrB\_NULL}, \\ \\ \mathbf{L}(s), & \text{(unchanged) otherwise}, \end{cases}$$

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where  $\mathbf{0}(\mathsf{op})$  is the identity of the monoid. The operation returns immediately with  $\mathsf{GrB\_SUCCESS}$ .

For all other cases, the internal matrix and scalar used in the computation is formed ( $\leftarrow$  denotes copy):

- 1. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{A}$ .
- 7203 2. Scalar  $\tilde{s} \leftarrow s$  (GraphBLAS scalar case).

We are now ready to carry out the reduce and any additional associated operations. An intermediate scalar result t is computed as follows:

$$t = \bigoplus_{(i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})} \widetilde{\mathbf{A}}(i,j),$$

where  $\oplus = \bigcirc(\mathsf{op})$ .

7208 The final reduction value is computed as follows:

$$\mathbf{L}(\mathsf{s}) \leftarrow \begin{cases} \{t\}, & \text{when accum} = \mathsf{GrB\_NULL} \text{ or } \tilde{s} \text{ is empty, or} \\ \{\mathbf{val}(\tilde{s}) \odot t\}, & \text{otherwise;} \end{cases}$$

In both GrB\_BLOCKING and GrB\_NONBLOCKING modes, the method exits with return value GrB\_SUCCESS and the new contents of the output scalar is as defined above.

#### 7214 4.3.11 transpose: Transpose rows and columns of a matrix

7215 This version computes a new matrix that is the transpose of the source matrix.

#### 7216 C Syntax

```
GrB_Info GrB_transpose(GrB_Matrix C,
const GrB_Matrix Mask,
const GrB_BinaryOp accum,
const GrB_Matrix A,
const GrB_Descriptor desc);
```

#### Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the transpose operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.

| 7232 | accum (IN) An optional binary operator used for accumulating entries into existing C |  |
|------|--|--|
| 7233 | entries. If assignment rather than accumulation is desired, GrB_NULL should be       |  |
| 7234 | specified.   |  |

A (IN) The GraphBLAS matrix on which transposition will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input Mask matrix. The stored values are    |
|       |          |               | not examined.                               |
| Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
| Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |

#### Return Values

| 7241<br>7242<br>7243<br>7244<br>7245 | <del>-</del>             | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 7246                                 | GrB_PANIC                | Unknown internal error.  |
| 7247<br>7248<br>7249<br>7250         |                          | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 7251                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.   |
| 7252<br>7253                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to <code>new</code> (or <code>Matrix_dup</code> for matrix parameters).  |
| 7254                                 | GrB_DIMENSION_MISMATCH   | mask,C and/or $A$ dimensions are incompatible.   |
| 7255<br>7256<br>7257<br>7258         |                          | The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCT is not set).   |

#### 7259 Description

GrB\_transpose computes the result of performing a transpose of the input matrix:  $C = A^T$ ; or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot A^T$ . We note that the input matrix A can itself be optionally transposed before the operation, which would cause either an assignment from A to C or an accumulation of A into C.

Logically, this operation occurs in three steps:

Setup The internal matrix and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

7267 **Compute** The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

7269 Up to three matrix arguments are used in this GrB\_transpose operation:

- 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 2.  $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$

The argument matrices and accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}(\mathsf{A})$  of the input matrix.
- 3. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}(\mathsf{A})$  of the input matrix must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_transpose ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed  $(\leftarrow \text{denotes copy})$ :

7288 1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:

```
7290 (a) If Mask = GrB_NULL, then \widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle.
```

7292 (b) If Mask  $\neq$  GrB\_NULL,

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- i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - ii. Otherwise,  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}$

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

- 7301 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 7302 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$

If any compatibility rule above is violated, execution of GrB\_transpose ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the matrix transposition and any additional associated operations.
We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the transpose of  $\widetilde{\mathbf{A}}$ .
- $\tilde{\mathbf{z}}$ : The matrix holding the result after application of the (optional) accumulation operator.
- 7313 The intermediate matrix

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \{(j, i, A_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \}$$

7315 is created.

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The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

- If  $\mathsf{accum} = \mathsf{GrB} \_\mathsf{NULL}, \, \mathrm{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\tilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\mathbf{C}$  and  $\mathbf{T}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
where  $\odot = \mathfrak{O}(\mathbf{accum})$ , and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix  $\mathsf{C}$ , 7328 using what is called a standard matrix mask and replace. This is carried out under control of the 7329 mask which acts as a "write mask". 7330

• If desc[GrB OUTP].GrB REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In Grb BLOCKING mode, the method exits with return value Grb SUCCESS and the new content 7338 of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method 7339 exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but 7340 may not be fully computed. However, it can be used in the next GraphBLAS method call in a 7341 sequence. 7342

#### kronecker: Kronecker product of two matrices 4.3.12

Computes the Kronecker product of two matrices. The result is a matrix. 7344

#### C Syntax 7345

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```
GrB_Info GrB_kronecker(GrB_Matrix
                                                                    С,
7346
                                         const GrB_Matrix
                                                                    Mask,
7347
                                         const GrB_BinaryOp
                                                                    accum,
7348
                                         const GrB_Semiring
7349
                                                                    op,
                                         const GrB_Matrix
                                                                    Α,
7350
                                         const GrB Matrix
                                                                    В,
7351
                                         const GrB_Descriptor
                                                                    desc);
7352
7353
```

```
С,
             GrB_Info GrB_kronecker(GrB_Matrix
7354
                                         const GrB_Matrix
                                                                   Mask,
7355
                                         const GrB_BinaryOp
7356
                                                                    accum,
                                         const GrB_Monoid
                                                                    op,
7357
                                         const GrB Matrix
                                                                    Α,
7358
                                         const GrB Matrix
                                                                   В,
7359
                                         const GrB Descriptor
                                                                    desc);
7360
7361
             GrB_Info GrB_kronecker(GrB_Matrix
                                                                    C,
7362
                                         const GrB_Matrix
                                                                   Mask,
7363
                                         const GrB_BinaryOp
7364
                                                                    accum,
                                         const GrB_BinaryOp
                                                                    op,
7365
                                         const GrB_Matrix
                                                                    Α,
7366
                                         const GrB_Matrix
                                                                    Β,
7367
                                         const GrB_Descriptor
                                                                    desc);
7368
```

#### 9 Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the Kronecker product. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB\_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB\_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB\_NULL should be specified.
  - op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator,  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$ , used:

```
BinaryOp: F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle.

Monoid: F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle; the identity e
```

Monoid:  $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$ ; the identity element is ignored.

Semiring:  $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes (\mathsf{op}) \rangle$ ; the additive monoid is ignored.

A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the product.

| 7392 | B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the |
|------|---|
| 7393 | product.  |

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB\_NULL should be specified. Non-default field/value pairs are listed as follows:

| Param | Field    | Value         | Description                                 |
|-------|----------|---------------|---|
| С     | GrB_OUTP | GrB_REPLACE   | Output matrix C is cleared (all elements    |
|       |          |               | removed) before the result is stored in it. |
| Mask  | GrB_MASK | GrB_STRUCTURE | The write mask is constructed from the      |
|       |          |               | structure (pattern of stored values) of the |
|       |          |               | input Mask matrix. The stored values are    |
|       |          |               | not examined.                               |
| Mask  | GrB_MASK | GrB_COMP      | Use the complement of Mask.                 |
| Α     | GrB_INP0 | GrB_TRAN      | Use transpose of A for the operation.       |
| В     | GrB_INP1 | GrB_TRAN      | Use transpose of B for the operation.       |

#### Return Values

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| 7399<br>7400<br>7401<br>7402<br>7403 | GrB_SUCCESS              | In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence. |
|--------------------------------------|--------------------------|--|
| 7403                                 | GrB_PANIC                | Unknown internal error.  |
| 7405<br>7406<br>7407<br>7408         | GrB_INVALID_OBJECT       | This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.                                    |
| 7409                                 | GrB_OUT_OF_MEMORY        | Not enough memory available for the operation.   |
| 7410<br>7411                         | GrB_UNINITIALIZED_OBJECT | One or more of the GraphBLAS objects has not been initialized by a call to <code>new</code> (or <code>Matrix_dup</code> for matrix parameters).  |
| 7412                                 | GrB_DIMENSION_MISMATCH   | Mask and/or matrix dimensions are incompatible.  |
| 7413<br>7414<br>7415<br>7416         | GrB_DOMAIN_MISMATCH      | The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).                              |

#### 117 Description

GrB\_kronecker computes the Kronecker product  $C = A \otimes B$  or, if an optional binary accumulation operator  $(\odot)$  is provided,  $C = C \odot (A \otimes B)$  (where matrices A and B can be optionally transposed).

7420 The Kronecker product is defined as follows:

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$$\mathsf{C} = \mathsf{A} \ \otimes \ \mathsf{B} = \left[ \begin{array}{ccccc} A_{0,0} \otimes \mathsf{B} & A_{0,1} \otimes \mathsf{B} & \dots & A_{0,n_A-1} \otimes \mathsf{B} \\ A_{1,0} \otimes \mathsf{B} & A_{1,1} \otimes \mathsf{B} & \dots & A_{1,n_A-1} \otimes \mathsf{B} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m_A-1,0} \otimes \mathsf{B} & A_{m_A-1,1} \otimes \mathsf{B} & \dots & A_{m_A-1,n_A-1} \otimes \mathsf{B} \end{array} \right]$$

where A :  $\mathbb{S}^{m_A \times n_A}$ , B :  $\mathbb{S}^{m_B \times n_B}$ , and C :  $\mathbb{S}^{m_A m_B \times n_A n_B}$ . More explicitly, the elements of the Kronecker product are defined as

$$C(i_A m_B + i_B, j_A n_B + j_B) = A_{i_A, j_A} \otimes B_{i_B, j_B},$$

- where  $\otimes$  is the multiplicative operator specified by the op parameter.
- Logically, this operation occurs in three steps:
- The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 7430 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 7432 Up to four argument matrices are used in the GrB\_kronecker operation:
- 7433 1.  $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- $2. \ \mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \ (\text{optional})$
- 3.  $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 4.  $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB\_NULL, and desc[GrB\_MASK].GrB\_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 7441 2.  $\mathbf{D}(\mathsf{A})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{op})$ .
- 3.  $\mathbf{D}(\mathsf{B})$  must be compatible with  $\mathbf{D}_{in_2}(\mathsf{op})$ .
- 4.  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{out}(\mathsf{op})$ .
- 5. If accum is not GrB\_NULL, then  $\mathbf{D}(\mathsf{C})$  must be compatible with  $\mathbf{D}_{in_1}(\mathsf{accum})$  and  $\mathbf{D}_{out}(\mathsf{accum})$  of the accumulation operator and  $\mathbf{D}_{out}(\mathsf{op})$  of op must be compatible with  $\mathbf{D}_{in_2}(\mathsf{accum})$  of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB\_kronecker ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed (
denotes copy):

7454 1. Matrix  $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$ .

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7455 2. Two-dimensional mask,  $\widetilde{\mathbf{M}}$ , is computed from argument Mask as follows:

```
(a) If Mask = GrB_NULL, then \widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle.
```

- (b) If Mask  $\neq$  GrB\_NULL,
  - i. If desc[GrB\_MASK].GrB\_STRUCTURE is set, then  $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$ ,
  - $$\begin{split} \text{ii. Otherwise, } & \widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ & \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle. \end{split}$$
- (c) If  $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$  is set, then  $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix  $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Matrix  $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{B}^T : \mathsf{B}.$

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

- 7468 1.  $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 7469 2.  $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3.  $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}) \cdot \operatorname{nrows}(\widetilde{\mathbf{B}}).$
- 4.  $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) \cdot \operatorname{ncols}(\widetilde{\mathbf{B}}).$

If any compatibility rule above is violated, execution of GrB\_kronecker ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB\_NONBLOCKING mode, the method can optionally exit with GrB\_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the Kronecker product and any additional associated operations.
We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$ : The matrix holding the Kronecker product of matrices  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{B}}$ .
- $\tilde{\mathbf{Z}}$ : The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix  $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}) \times \mathbf{nrows}(\widetilde{\mathbf{B}}), \mathbf{ncols}(\widetilde{\mathbf{A}}) \times \mathbf{ncols}(\widetilde{\mathbf{B}}), \{(i, j, T_{ij}) \text{ where } i = i_A \cdot m_B + i_B, \ j = j_A \cdot n_B + j_B, \ \forall \ (i_A, j_A) = \mathbf{ind}(\widetilde{\mathbf{A}}), \ (i_B, j_B) = \mathbf{ind}(\widetilde{\mathbf{B}}) \rangle$  is created. The value of each of its elements is computed by

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$$T_{i_A \cdot m_B + i_B, \ j_A \cdot n_B + j_B} = \widetilde{\mathbf{A}}(i_A, j_A) \otimes \widetilde{\mathbf{B}}(i_B, j_B)),$$

where  $\otimes$  is the multiplicative operator specified by the **op** parameter.

The intermediate matrix  $\tilde{\mathbf{Z}}$  is created as follows, using what is called a *standard matrix accumulate*:

• If  $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \operatorname{then} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$ 

• If accum is a binary operator, then  $\widetilde{\mathbf{Z}}$  is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of  $\widetilde{\mathbf{Z}}$  are computed based on the relationships between the sets of indices in  $\widetilde{\mathbf{C}}$  and  $\widetilde{\mathbf{T}}$ .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where  $\odot = \bigcirc$  (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix  $\tilde{\mathbf{Z}}$  are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB\_OUTP].GrB\_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If  $desc[GrB\_OUTP].GrB\_REPLACE$  is not set, the elements of  $\widetilde{\mathbf{Z}}$  indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB\_BLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB\_NONBLOCKING mode, the method exits with return value GrB\_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence. s

## 512 Chapter 5

# Nonpolymorphic interface[Scott: NEW CONTENT]

Each polymorphic GraphBLAS method (those with multiple parameter signatures under the same name) has a corresponding set of long-name forms that are specific to each parameter signature.
That is show in Tables 5.1 through 5.11.

Table 5.1: Long-name, nonpolymorphic form of GraphBLAS methods.

| Polymorphic signature                       | Nonpolymorphic signature  |
|---|---|
| GrB_Monoid_new(GrB_Monoid*,,bool)           | GrB_Monoid_new_BOOL(GrB_Monoid*,GrB_BinaryOp,bool)                          |
| $GrB\_Monoid\_new(GrB\_Monoid*,,int8\_t)$   | $GrB\_Monoid\_new\_INT8(GrB\_Monoid*,GrB\_BinaryOp,int8\_t)$                |
| $GrB\_Monoid\_new(GrB\_Monoid*,,uint8\_t)$  | GrB_Monoid_new_UINT8(GrB_Monoid*,GrB_BinaryOp,uint8_t)                      |
| $GrB\_Monoid\_new(GrB\_Monoid*,,int16\_t)$  | $GrB\_Monoid\_new\_INT16(GrB\_Monoid*,GrB\_BinaryOp,int16\_t)$              |
| $GrB\_Monoid\_new(GrB\_Monoid*,,uint16\_t)$ | GrB_Monoid_new_UINT16(GrB_Monoid*,GrB_BinaryOp,uint16_t)                    |
| $GrB\_Monoid\_new(GrB\_Monoid*,,int32\_t)$  | $\label{lem:grb_Monoid_new_INT32} GrB\_Monoid^*, GrB\_BinaryOp, int 32\_t)$ |
| $GrB\_Monoid\_new(GrB\_Monoid*,,uint32\_t)$ | GrB_Monoid_new_UINT32(GrB_Monoid*,GrB_BinaryOp,uint32_t)                    |
| $GrB\_Monoid\_new(GrB\_Monoid*,,int64\_t)$  | $GrB\_Monoid\_new\_INT64(GrB\_Monoid*,GrB\_BinaryOp,int64\_t)$              |
| $GrB\_Monoid\_new(GrB\_Monoid*,,uint64\_t)$ | GrB_Monoid_new_UINT64(GrB_Monoid*,GrB_BinaryOp,uint64_t)                    |
| $GrB\_Monoid\_new(GrB\_Monoid*,,float)$     | ${\sf GrB\_Monoid\_new\_FP32}({\sf GrB\_Monoid*,GrB\_BinaryOp,float})$      |
| $GrB\_Monoid\_new(GrB\_Monoid*,,double)$    | ${\sf GrB\_Monoid\_new\_FP64(GrB\_Monoid*,GrB\_BinaryOp,double)}$           |
| $GrB\_Monoid\_new(GrB\_Monoid*,,other)$     | ${\sf GrB\_Monoid\_new\_UDT(GrB\_Monoid*,GrB\_BinaryOp,void*)}$             |

Table 5.2: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Nonpolymorphic signature

| Polymorphic signature                       | Nonpolymorphic signature                      |
|---|---|
| GrB_Scalar_setElement(, bool,)              | GrB_Scalar_setElement_BOOL(, bool,)           |
| $GrB\_Scalar\_setElement(, int8\_t,)$       | $GrB\_Scalar\_setElement\_INT8(, int8\_t,)$   |
| $GrB\_Scalar\_setElement(, uint8\_t,)$      | $GrB\_Scalar\_setElement\_UINT8(, uint8\_t,)$ |
| $GrB\_Scalar\_setElement(, int16\_t,)$      | $GrB\_Scalar\_setElement\_INT16(, int16\_t,)$ |
| $GrB\_Scalar\_setElement(, uint16\_t,)$     | GrB_Scalar_setElement_UINT16(, uint16_t,)     |
| $GrB\_Scalar\_setElement(, int32\_t,)$      | GrB_Scalar_setElement_INT32(, int32_t,)       |
| $GrB\_Scalar\_setElement(, uint32\_t,)$     | GrB_Scalar_setElement_UINT32(, uint32_t,)     |
| $GrB\_Scalar\_setElement(, int64\_t,)$      | $GrB\_Scalar\_setElement\_INT64(, int64\_t,)$ |
| $GrB\_Scalar\_setElement(, uint64\_t,)$     | GrB_Scalar_setElement_UINT64(, uint64_t,)     |
| $GrB\_Scalar\_setElement(, float,)$         | GrB_Scalar_setElement_FP32(, float,)          |
| $GrB\_Scalar\_setElement(, double,)$        | GrB_Scalar_setElement_FP64(, double,)         |
| $GrB\_Scalar\_setElement(, other,)$         | GrB_Scalar_setElement_UDT(,const void*,)      |
| GrB_Scalar_extractElement(bool*,)           | GrB_Scalar_extractElement_BOOL(bool*,)        |
| $GrB\_Scalar\_extractElement(int8\_t*,)$    | GrB_Scalar_extractElement_INT8(int8_t*,)      |
| $GrB\_Scalar\_extractElement(uint8\_t*,)$   | GrB_Scalar_extractElement_UINT8(uint8_t*,)    |
| $GrB\_Scalar\_extractElement(int16\_t^*,)$  | GrB_Scalar_extractElement_INT16(int16_t*,)    |
| $GrB\_Scalar\_extractElement(uint16\_t^*,)$ | GrB_Scalar_extractElement_UINT16(uint16_t*,)  |
| $GrB\_Scalar\_extractElement(int32\_t*,)$   | GrB_Scalar_extractElement_INT32(int32_t*,)    |
| $GrB\_Scalar\_extractElement(uint32\_t^*,)$ | GrB_Scalar_extractElement_UINT32(uint32_t*,)  |
| $GrB\_Scalar\_extractElement(int64\_t^*,)$  | GrB_Scalar_extractElement_INT64(int64_t*,)    |
| $GrB\_Scalar\_extractElement(uint64\_t^*,)$ | GrB_Scalar_extractElement_UINT64(uint64_t*,)  |
| $GrB\_Scalar\_extractElement(float*,)$      | GrB_Scalar_extractElement_FP32(float*,)       |
| $GrB\_Scalar\_extractElement(double*,)$     | GrB_Scalar_extractElement_FP64(double*,)      |
| GrB_Scalar_extractElement(other*,)          | GrB_Scalar_extractElement_UDT(void*,)         |

Table 5.3: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                 Nonpolymorphic signature
GrB_Vector_build(...,const bool*,...)
                                                 GrB\_Vector\_build\_BOOL(...,const bool*,...)
                                                 \label{eq:GrB_Vector_build_INT8(...,const int8\_t*,...)} $$\operatorname{GrB\_Vector\_build\_UINT8(...,const uint8\_t*,...)}$$
GrB_Vector_build(...,const int8_t*,...)
GrB_Vector_build(...,const uint8_t*,...)
GrB_Vector_build(...,const int16_t*,...)
                                                 GrB_Vector_build_INT16(...,const int16_t*,...)
GrB_Vector_build(...,const uint16_t*,...)
                                                 GrB_Vector_build_UINT16(...,const uint16_t*,...)
\mathsf{GrB\_Vector\_build}(\dots, \mathsf{const\ int} 32\_t^*, \dots)
                                                 \mathsf{GrB}\_\mathsf{Vector}\_\mathsf{build}\_\mathsf{INT32}(\dots,\mathsf{const\ int32}\_\mathsf{t*},\dots)
GrB_Vector_build(...,const uint32_t*,...)
                                                  GrB_Vector_build_UINT32(...,const_uint32_t*,...)
                                                 GrB_Vector_build_INT64(...,const int64_t*,...)
GrB_Vector_build(...,const int64_t*,...)
GrB_Vector_build(...,const uint64_t*,...)
                                                 GrB_Vector_build_UINT64(...,const uint64_t*,...)
GrB_Vector_build(...,const float*,...)
                                                 GrB_Vector_build_FP32(...,const float*,...)
GrB_Vector_build(...,const double*,...)
                                                 GrB_Vector_build_FP64(...,const double*,...)
GrB_Vector_build(...,const other*,...)
                                                  GrB_Vector_build_UDT(...,const void*,...)
                                                 GrB_Vector_setElement_Scalar(...,const GrB_Scalar,...)
GrB\_Vector\_setElement(...,GrB\_Scalar,...)
                                                 GrB Vector_setElement_BOOL(..., bool,...)
GrB Vector_setElement(...,bool,...)
GrB\_Vector\_setElement(...,int8\_t,...)
                                                 GrB_Vector_setElement_INT8(..., int8_t,...)
GrB\_Vector\_setElement(...,uint8\_t,...)
                                                 GrB\_Vector\_setElement\_UINT8(..., uint8\_t,...)
GrB_Vector_setElement(...,int16_t,...)
                                                  GrB\_Vector\_setElement\_INT16(..., int16\_t,...)
                                                 GrB_Vector_setElement_UINT16(..., uint16_t,...)
GrB\_Vector\_setElement(...,uint16\_t,...)
GrB_Vector_setElement(...,int32_t,...)
                                                 GrB_Vector_setElement_INT32(..., int32_t,...)
GrB_Vector_setElement(...,uint32_t,...)
                                                 GrB_Vector_setElement_UINT32(..., uint32_t,...)
GrB_Vector_setElement(...,int64_t,...)
                                                 GrB\_Vector\_setElement\_INT64(..., int64\_t,...)
                                                 GrB_Vector_setElement_UINT64(..., uint64_t,...)
GrB_Vector_setElement(...,uint64_t,...)
                                                 GrB_Vector_setElement_FP32(..., float,...)
GrB\_Vector\_setElement(...,float,...)
GrB\_Vector\_setElement(...,double,...)
                                                 GrB_Vector_setElement_FP64(..., double,...)
GrB_Vector_setElement(...,other,...)
                                                 GrB_Vector_setElement_UDT(...,const void*,...)
                                                 GrB\_Vector\_extractElement\_Scalar(GrB\_Scalar,...)
GrB\_Vector\_extractElement(GrB\_Scalar,...)
GrB_Vector_extractElement(bool*,...)
                                                  GrB_Vector_extractElement_BOOL(bool*,...)
                                                 GrB_Vector_extractElement_INT8(int8_t*,...)
GrB_Vector_extractElement(int8_t*,...)
GrB_Vector_extractElement(uint8_t*,...)
                                                 GrB_Vector_extractElement_UINT8(uint8_t*,...)
                                                 GrB\_Vector\_extractElement\_INT16(int16\_t^*,...)
GrB_Vector_extractElement(int16_t*,...)
GrB_Vector_extractElement(uint16_t*,...)
                                                 \label{lement_UINT16} GrB\_Vector\_extractElement\_UINT16(uint16\_t^*,\dots)
GrB_Vector_extractElement(int32_t*,...)
                                                 GrB_Vector_extractElement_INT32(int32_t*,...)
GrB_Vector_extractElement(uint32_t*,...)
                                                 GrB_Vector_extractElement_UINT32(uint32_t*,...)
GrB_Vector_extractElement(int64_t*,...)
                                                 GrB_Vector_extractElement_INT64(int64_t*,...)
GrB_Vector_extractElement(uint64_t*,...)
                                                 GrB_Vector_extractElement_UINT64(uint64_t*,...)
GrB_Vector_extractElement(float*,...)
                                                 GrB_Vector_extractElement_FP32(float*,...)
GrB_Vector_extractElement(double*,...)
                                                 GrB_Vector_extractElement_FP64(double*,...)
GrB_Vector_extractElement(other*,...)
                                                 GrB_Vector_extractElement_UDT(void*,...)
GrB\_Vector\_extractTuples(...,bool*,...)
                                                 GrB_Vector_extractTuples_BOOL(..., bool*,...)
GrB\_Vector\_extractTuples(...,int8\_t*,...)
                                                 GrB\_Vector\_extractTuples\_INT8(..., int8\_t*,...)
GrB\_Vector\_extractTuples(...,uint8\_t*,...)
                                                 \label{linear_struct_toples} GrB\_Vector\_extractTuples\_UINT8(..., uint8\_t^*,...)
GrB\_Vector\_extractTuples(...,int16\_t*,...)
                                                 GrB_Vector_extractTuples_INT16(..., int16_t*,...)
                                                 GrB\_Vector\_extractTuples\_UINT16(..., uint16\_t*,...)
GrB\_Vector\_extractTuples(...,uint16\_t^*,...)
GrB\_Vector\_extractTuples(...,int32\_t^*,...)
                                                 GrB_Vector_extractTuples_INT32(..., int32_t*,...)
GrB_Vector_extractTuples(...,uint32_t*,...)
                                                 GrB\_Vector\_extractTuples\_UINT32(..., uint32\_t*,...)
GrB\_Vector\_extractTuples(...,int64\_t*,...)
                                                 \label{linear_continuity} GrB\_Vector\_extractTuples\_INT64(\dots,\ int64\_t^*,\dots)
GrB_Vector_extractTuples(...,uint64_t*,...)
                                                  GrB\_Vector\_extractTuples\_UINT64(..., uint64\_t*,...)
                                                 GrB\_Vector\_extractTuples\_FP32(..., float*,...)
GrB\_Vector\_extractTuples(...,float*,...)
                                                 GrB_Vector_extractTuples_FP64(..., double*,...)
GrB\_Vector\_extractTuples(...,double*,...)
GrB_Vector_extractTuples(...,other*,...)
                                                 GrB_Vector_extractTuples_UDT(..., void*,...)
```

Table 5.4: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Nonpolymorphic signature
Polymorphic signature
GrB_Matrix_build(...,const bool*,...)
                                                 GrB_Matrix_build_BOOL(...,const bool*,...)
                                                 GrB_Matrix_build_INT8(...,const int8_t*,...)
GrB_Matrix_build(...,const int8_t*,...)
GrB_Matrix_build(...,const uint8_t*,...)
                                                 GrB_Matrix_build_UINT8(...,const uint8_t*,...)
GrB_Matrix_build(...,const int16_t*,...)
                                                 GrB_Matrix_build_INT16(...,const int16_t*,...)
GrB_Matrix_build(...,const uint16_t*,...)
                                                 GrB_Matrix_build_UINT16(...,const uint16_t*,...)
                                                 \mathsf{GrB}\_\mathsf{Matrix\_build}\_\mathsf{INT32}(\dots,\mathsf{const\ int32\_t*},\dots)
GrB_Matrix_build(...,const int32_t*,...)
GrB_Matrix_build(...,const uint32_t*,...)
                                                 GrB_Matrix_build_UINT32(...,const_uint32_t*,...)
                                                 \label{local_gradient} $\sf GrB\_Matrix\_build\_INT64(\dots,const\ int64\_t^*,\dots)$}
GrB_Matrix_build(...,const int64_t*,...)
GrB_Matrix_build(...,const uint64_t*,...)
                                                 GrB_Matrix_build_UINT64(...,const uint64_t*,...)
GrB_Matrix_build(...,const float*,...)
                                                 GrB_Matrix_build_FP32(...,const float*,...)
GrB_Matrix_build(...,const double*,...)
                                                 GrB_Matrix_build_FP64(...,const double*,...)
GrB_Matrix_build(...,const other*,...)
                                                 GrB_Matrix_build_UDT(...,const void*,...)
                                                 GrB_Matrix_setElement_Scalar(...,const GrB_Scalar,...)
GrB\_Matrix\_setElement(...,GrB\_Scalar,...)
GrB Matrix_setElement(...,bool,...)
                                                 GrB Matrix_setElement_BOOL(..., bool,...)
GrB\_Matrix\_setElement(...,int8\_t,...)
                                                 GrB_Matrix_setElement_INT8(..., int8_t,...)
GrB_Matrix_setElement(...,uint8_t,...)
                                                 \label{lement_UINT8} GrB\_Matrix\_setElement\_UINT8(..., uint8\_t,...)
GrB_Matrix_setElement(...,int16_t,...)
                                                 GrB\_Matrix\_setElement\_INT16(..., int16\_t,...)
GrB_Matrix_setElement(...,uint16_t,...)
                                                 GrB_Matrix_setElement_UINT16(..., uint16_t,...)
                                                 GrB_Matrix_setElement_INT32(..., int32_t,...)
GrB_Matrix_setElement(...,int32_t,...)
GrB_Matrix_setElement(...,uint32_t,...)
                                                 GrB_Matrix_setElement_UINT32(..., uint32_t,...)
                                                 GrB\_Matrix\_setElement\_INT64(..., int64\_t,...)
GrB_Matrix_setElement(...,int64_t,...)
GrB_Matrix_setElement(...,uint64_t,...)
                                                 GrB\_Matrix\_setElement\_UINT64(..., uint64\_t,...)
                                                 GrB_Matrix_setElement_FP32(..., float,...)
GrB\_Matrix\_setElement(...,float,...)
GrB_Matrix_setElement(...,double,...)
                                                 GrB_Matrix_setElement_FP64(..., double,...)
                                                 {\sf GrB\_Matrix\_setElement\_UDT}(\dots, {\sf const\ void*}, \dots)
GrB\_Matrix\_setElement(...,other,...)
GrB_Matrix_extractElement(GrB_Scalar,...)
                                                 GrB_Matrix_extractElement_Scalar(GrB_Scalar,...)
GrB_Matrix_extractElement(bool*,...)
                                                 GrB_Matrix_extractElement_BOOL(bool*,...)
                                                 GrB_Matrix_extractElement_INT8(int8_t*,...)
GrB_Matrix_extractElement(int8_t*,...)
GrB\_Matrix\_extractElement(uint8\_t^*,...)
                                                 GrB_Matrix_extractElement_UINT8(uint8_t*,...)
GrB_Matrix_extractElement(int16_t*,...)
                                                 GrB_Matrix_extractElement_INT16(int16_t*,...)
GrB_Matrix_extractElement(uint16_t*,...)
                                                 GrB_Matrix_extractElement_UINT16(uint16_t*,...)
GrB_Matrix_extractElement(int32_t*,...)
                                                 GrB_Matrix_extractElement_INT32(int32_t*,...)
GrB_Matrix_extractElement(uint32_t*,...)
                                                 GrB_Matrix_extractElement_UINT32(uint32_t*,...)
GrB\_Matrix\_extractElement(int64\_t^*,...)
                                                 GrB_Matrix_extractElement_INT64(int64_t*,...)
GrB_Matrix_extractElement(uint64_t*,...)
                                                 GrB_Matrix_extractElement_UINT64(uint64_t*,...)
GrB_Matrix_extractElement(float*,...)
                                                 GrB_Matrix_extractElement_FP32(float*,...)
GrB_Matrix_extractElement(double*,...)
                                                 GrB\_Matrix\_extractElement\_FP64(double*,...)
                                                 GrB_Matrix_extractElement_UDT(void*,...)
GrB_Matrix_extractElement(other,...)
GrB_Matrix_extractTuples(..., bool*,...)
                                                 GrB\_Matrix\_extractTuples\_BOOL(..., bool*,...)
GrB_Matrix_extractTuples(..., int8_t*,...)
                                                 GrB\_Matrix\_extractTuples\_INT8(..., int8\_t*,...)
                                                 GrB\_Matrix\_extractTuples\_UINT8(..., uint8\_t*,...)
GrB_Matrix_extractTuples(..., uint8_t*,...)
GrB_Matrix_extractTuples(..., int16_t*,...)
                                                 GrB_Matrix_extractTuples_INT16(..., int16_t*,...)
                                                 \label{linear_gradient} GrB\_Matrix\_extractTuples\_UINT16(\dots, uint16\_t^*,\dots)
GrB_Matrix_extractTuples(..., uint16_t*,...)
GrB_Matrix_extractTuples(..., int32_t*,...)
                                                 GrB_Matrix_extractTuples_INT32(..., int32_t*,...)
GrB_Matrix_extractTuples(..., uint32_t*,...)
                                                 GrB_Matrix_extractTuples_UINT32(..., uint32_t*,...)
GrB_Matrix_extractTuples(..., int64_t*,...)
                                                 GrB_Matrix_extractTuples_INT64(..., int64_t*,...)
GrB_Matrix_extractTuples(..., uint64_t*,...)
                                                 GrB_Matrix_extractTuples_UINT64(..., uint64_t*,...)
                                                 GrB_Matrix_extractTuples_FP32(..., float*,...)
GrB\_Matrix\_extractTuples(..., float*,...)
                                                 GrB_Matrix_extractTuples_FP64(..., double*,...)
GrB_Matrix_extractTuples(..., double*,...)
GrB_Matrix_extractTuples(...,other*,...)
                                                 GrB_Matrix_extractTuples_UDT(..., void*,...)
```

Table 5.5: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                    Nonpolymorphic signature
GrB_Matrix_import(...,const bool*,...)
                                                    GrB_Matrix_import_BOOL(...,const bool*,...)
GrB_Matrix_import(...,const int8_t*,...)
                                                    GrB_Matrix_import_INT8(...,const int8_t*,...)
GrB_Matrix_import(...,const uint8_t*,...)
                                                    GrB_Matrix_import_UINT8(...,const uint8_t*,...)
GrB_Matrix_import(...,const int16_t*,...)
                                                    GrB_Matrix_import_INT16(...,const int16_t*,...)
                                                    GrB_Matrix_import_UINT16(...,const uint16_t*,...)
GrB_Matrix_import(...,const uint16_t*,...)
                                                    GrB_Matrix_import_INT32(...,const int32_t*,...)
GrB_Matrix_import(...,const int32_t*,...)
GrB_Matrix_import(...,const uint32_t*,...)
                                                    GrB_Matrix_import_UINT32(...,const uint32_t*,...)
GrB_Matrix_import(...,const int64_t*,...)
                                                    \mathsf{GrB}\_\mathsf{Matrix}\_\mathsf{import}\_\mathsf{INT64}(\dots,\mathsf{const}\;\mathsf{int64}\_\mathsf{t*},\dots)
GrB_Matrix_import(...,const uint64_t*,...)
                                                    GrB_Matrix_import_UINT64(...,const uint64_t*,...)
                                                    GrB_Matrix_import_FP32(...,const float*,...)
GrB_Matrix_import(...,const float*,...)
GrB_Matrix_import(...,const double*,...)
                                                    GrB_Matrix_import_FP64(...,const double*,...)
                                                    GrB_Matrix_import_UDT(...,const void*,...)
GrB_Matrix_import(...,const other,...)
\overline{\mathsf{GrB}}_Matrix_export(...,bool*,...)
                                                    GrB\_Matrix\_export\_BOOL(...,bool*,...)
GrB_Matrix_export(...,int8_t*,...)
                                                    GrB_Matrix_export_INT8(...,int8_t*,...)
                                                    GrB_Matrix_export_UINT8(...,uint8_t*,...)
GrB_Matrix_export(...,uint8_t*,...)
GrB_Matrix_export(...,int16_t*,...)
                                                    GrB_Matrix_export_INT16(...,int16_t*,...)
GrB\_Matrix\_export(...,uint16\_t^*,...)
                                                    GrB\_Matrix\_export\_UINT16(...,uint16\_t*,...)
                                                    GrB\_Matrix\_export\_INT32(...,int32\_t*,...)
GrB_Matrix_export(...,int32_t*,...)
GrB_Matrix_export(...,uint32_t*,...)
                                                    GrB_Matrix_export_UINT32(...,uint32_t*,...)
                                                    GrB_Matrix_export_INT64(...,int64_t*,...)
GrB_Matrix_export(...,int64_t*,...)
GrB_Matrix_export(...,uint64_t*,...)
                                                    GrB_Matrix_export_UINT64(...,uint64_t*,...)
GrB_Matrix_export(...,float*,...)
                                                    GrB_Matrix_export_FP32(...,float*,...)
GrB\_Matrix\_export(...,double*,...)
                                                    GrB_Matrix_export_FP64(...,double*,...)
GrB_Matrix_export(...,other,...)
                                                    GrB_Matrix_export_UDT(...,void*,...)
GrB_free(GrB_Type*
                                                    GrB_Type_free(GrB_Type*)
GrB_free(GrB_UnaryOp*)
                                                    GrB_UnaryOp_free(GrB_UnaryOp*)
                                                    {\sf GrB\_IndexUnaryOp\_free}({\sf GrB\_IndexUnaryOp*})
GrB_free(GrB_IndexUnaryOp*)
GrB_free(GrB_BinaryOp*)
                                                    GrB_BinaryOp_free(GrB_BinaryOp*)
GrB_free(GrB_Monoid*)
                                                    GrB_Monoid_free(GrB_Monoid*)
                                                    GrB_Semiring_free(GrB_Semiring*)
GrB_free(GrB_Semiring*)
GrB_free(GrB_Scalar*)
                                                    GrB_Scalar_free(GrB_Scalar*)
                                                    GrB_Vector_free(GrB_Vector*)
GrB_free(GrB_Vector*)
GrB_free(GrB_Matrix*)
                                                    GrB_Matrix_free(GrB_Matrix*)
GrB_free(GrB_Descriptor*)
                                                    GrB_Descriptor_free(GrB_Descriptor*)
GrB_wait(GrB_Type, GrB_WaitMode)
                                                    GrB_Type_wait(GrB_Type, GrB_WaitMode)
GrB_wait(GrB_UnaryOp, GrB_WaitMode)
                                                    GrB_UnaryOp_wait(GrB_UnaryOp, GrB_WaitMode)
GrB_wait(GrB_IndexUnaryOp, GrB_WaitMode)
                                                    GrB_IndexUnaryOp_wait(GrB_IndexUnaryOp, GrB_WaitMode)
GrB_wait(GrB_BinaryOp, GrB_WaitMode)
                                                    GrB_BinaryOp_wait(GrB_BinaryOp, GrB_WaitMode)
GrB_wait(GrB_Monoid, GrB_WaitMode)
                                                    GrB_Monoid_wait(GrB_Monoid, GrB_WaitMode)
GrB_wait(GrB_Semiring, GrB_WaitMode)
                                                    GrB_Semiring_wait(GrB_Semiring, GrB_WaitMode)
GrB_wait(GrB_Scalar, GrB_WaitMode)
                                                    GrB_Scalar_wait(GrB_Scalar, GrB_WaitMode)
GrB_wait(GrB_Vector, GrB_WaitMode)
                                                    {\sf GrB\_Vector\_wait}({\sf GrB\_Vector},\ {\sf GrB\_WaitMode})
GrB_wait(GrB_Matrix, GrB_WaitMode)
                                                    GrB_Matrix_wait(GrB_Matrix, GrB_WaitMode)
GrB_wait(GrB_Descriptor, GrB_WaitMode)
                                                    GrB_Descriptor_wait(GrB_Descriptor, GrB_WaitMode)
GrB_error(const char**, const GrB_Type)
                                                    GrB_Type_error(const char**, const GrB_Type)
GrB_error(const char**, const GrB_UnaryOp)
                                                    GrB_UnaryOp_error(const char**, const GrB_UnaryOp)
GrB_error(const char**, const GrB_IndexUnaryOp)
                                                    GrB_IndexUnaryOp_error(const char**, const GrB_IndexUnaryOp)
GrB_error(const char**, const GrB_BinaryOp)
GrB_error(const char**, const GrB_Monoid)
                                                    GrB_BinaryOp_error(const char**, const GrB_BinaryOp)
                                                    GrB_Monoid_error(const char**, const GrB_Monoid)
                                                    GrB_Semiring_error(const char**, const GrB_Semiring)
GrB_error(const char**, const GrB_Semiring)
GrB_error(const char**, const GrB_Scalar)
                                                    GrB_Scalar_error(const char**, const GrB_Scalar)
                                                    GrB_Vector_error(const char**, const GrB_Vector)
GrB_error(const char**, const GrB_Vector)
GrB_error(const char**, const GrB_Matrix)
GrB_error(const char**, const GrB_Descriptor)
                                                    GrB_Matrix_error(const char**, const GrB_Matrix)
                                                    GrB_Descriptor_error(const char**, const GrB_Descriptor)
```

Table 5.6: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                                    Nonpolymorphic signature
GrB_eWiseMult(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB_Vector_eWiseMult_Semiring(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB\_Vector\_eWiseMult\_Monoid(GrB\_Vector,...,GrB\_Monoid,...)
GrB_eWiseMult(GrB_Vector,...,GrB_Monoid,...)
GrB_eWiseMult(GrB_Vector,...,GrB_BinaryOp,...)
                                                                    GrB\_Vector\_eWiseMult\_BinaryOp(GrB\_Vector,...,GrB\_BinaryOp,...)
GrB_eWiseMult(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB_Matrix_eWiseMult_Semiring(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB\_Matrix\_eWiseMult\_Monoid(GrB\_Matrix,...,GrB\_Monoid,...)
GrB_eWiseMult(GrB_Matrix,...,GrB_Monoid,...)
\mathsf{GrB\_eWiseMult}(\mathsf{GrB\_Matrix}, \ldots, \mathsf{GrB\_BinaryOp}, \ldots)
                                                                    GrB_Matrix_eWiseMult_BinaryOp(GrB_Matrix,...,GrB_BinaryOp,...)
GrB_eWiseAdd(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB_Vector_eWiseAdd_Semiring(GrB_Vector,...,GrB_Semiring,...)
                                                                     GrB\_Vector\_eWiseAdd\_Monoid(GrB\_Vector, \dots, GrB\_Monoid, \dots) 
GrB_eWiseAdd(GrB_Vector,...,GrB_Monoid,...)
                                                                    \label{lem:grb_vector_eWiseAdd_BinaryOp} GrB\_Vector, \dots, GrB\_BinaryOp, \dots)
GrB_eWiseAdd(GrB_Vector,...,GrB_BinaryOp,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB\_Matrix\_eWiseAdd\_Semiring(GrB\_Matrix,...,GrB\_Semiring,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_Monoid,...)
                                                                    GrB Matrix eWiseAdd Monoid(GrB Matrix,...,GrB Monoid,...)
GrB\_eWiseAdd(GrB\_Matrix,...,GrB\_BinaryOp,...)
                                                                    \label{linearyOp} GrB\_Matrix\_eWiseAdd\_BinaryOp(GrB\_Matrix, \ldots, GrB\_BinaryOp, \ldots)
GrB_extract(GrB_Vector,...,GrB_Vector,...
                                                                    GrB\_Vector\_extract(GrB\_Vector,...,GrB\_Vector,...)
GrB\_extract(GrB\_Matrix,...,GrB\_Matrix,...)
                                                                    GrB_Matrix_extract(GrB_Matrix,...,GrB_Matrix,...)
GrB_extract(GrB_Vector,...,GrB_Matrix,...)
                                                                    GrB\_Col\_extract(GrB\_Vector,...,GrB\_Matrix,...)
GrB_assign(GrB_Vector,...,GrB_Vector,...)
                                                                    GrB\_Vector\_assign(GrB\_Vector,...,GrB\_Vector,...)
GrB_assign(GrB_Matrix,...,GrB_Matrix,...)
                                                                    GrB_Matrix_assign(GrB_Matrix,...,GrB_Matrix,...)
\label{lem:grb_assign} $$\operatorname{\mathsf{GrB\_Matrix}},\ldots,\operatorname{\mathsf{GrB\_Vector}},\operatorname{\mathsf{const}} \ \operatorname{\mathsf{GrB\_Index}}^*,\ldots)$$
                                                                    {\sf GrB\_Col\_assign}({\sf GrB\_Matrix}, \ldots, {\sf GrB\_Vector}, {\sf const}\ {\sf GrB\_Index^*}, \ldots)
                                                                     \begin{array}{lll} & GrB\_Row\_assign(GrB\_Matrix, \ldots, GrB\_Vector, GrB\_Index, \ldots) \\ & GrB\_Vector\_assign\_Scalar(GrB\_Vector, \ldots, const \ GrB\_Scalar, \ldots) \end{array} 
GrB\_assign(GrB\_Matrix,...,GrB\_Vector,GrB\_Index,...)
GrB_assign(GrB_Vector,...,GrB_Scalar,...)
GrB_assign(GrB_Vector,...,bool,...)
                                                                    GrB_Vector_assign_BOOL(GrB_Vector,..., bool,...)
GrB_assign(GrB_Vector,...,int8_t,...)
                                                                    GrB_Vector_assign_INT8(GrB_Vector,..., int8_t,...)
GrB_assign(GrB_Vector,...,uint8_t,...)
                                                                    GrB_Vector_assign_UINT8(GrB_Vector,..., uint8_t,...)
GrB_assign(GrB_Vector,...,int16_t,...)
                                                                    GrB_Vector_assign_INT16(GrB_Vector,..., int16_t,...)
GrB_assign(GrB_Vector,...,uint16_t,...)
                                                                    GrB_Vector_assign_UINT16(GrB_Vector,..., uint16_t,...)
GrB_assign(GrB_Vector,...,int32_t,...)
                                                                    GrB_Vector_assign_INT32(GrB_Vector,..., int32_t,...)
GrB_assign(GrB_Vector,...,uint32_t,...)
                                                                    GrB_Vector_assign_UINT32(GrB_Vector,..., uint32_t,...)
GrB_assign(GrB_Vector,...,int64_t,...)
                                                                    GrB\_Vector\_assign\_INT64(GrB\_Vector,..., int64\_t,...)
GrB_assign(GrB_Vector,...,uint64_t,...)
                                                                    GrB_Vector_assign_UINT64(GrB_Vector,..., uint64_t,...)
GrB\_assign(GrB\_Vector,...,float,...)
                                                                    GrB_Vector_assign_FP32(GrB_Vector,..., float,...)
                                                                    GrB_Vector_assign_FP64(GrB_Vector,..., double,...)
GrB_assign(GrB_Vector,...,double,...)
GrB_assign(GrB_Vector,...,other,...)
                                                                    GrB_Vector_assign_UDT(GrB_Vector,...,const void*,...)
GrB_assign(GrB_Matrix,...,GrB_Scalar,...)
                                                                    GrB_Matrix_assign_Scalar(GrB_Matrix,...,const GrB_Scalar,...)
GrB_assign(GrB_Matrix,...,bool,...)
                                                                    GrB_Matrix_assign_BOOL(GrB_Matrix,..., bool,...)
                                                                    GrB\_Matrix\_assign\_INT8(GrB\_Matrix,..., int8\_t,...)
GrB_assign(GrB_Matrix,...,int8_t,...)
GrB_assign(GrB_Matrix,...,uint8_t,...)
                                                                    GrB_Matrix_assign_UINT8(GrB_Matrix,..., uint8_t,...)
GrB_assign(GrB_Matrix,...,int16_t,...)
                                                                    GrB_Matrix_assign_INT16(GrB_Matrix,..., int16_t,...)
GrB\_assign(GrB\_Matrix,...,uint16\_t,...)
                                                                    GrB\_Matrix\_assign\_UINT16(GrB\_Matrix,..., uint16\_t,...)
                                                                    GrB_Matrix_assign_INT32(GrB_Matrix,..., int32_t,...)
GrB_assign(GrB_Matrix,...,int32_t,...)
GrB_assign(GrB_Matrix,...,uint32_t,...)
                                                                    GrB_Matrix_assign_UINT32(GrB_Matrix,..., uint32_t,...)
GrB_assign(GrB_Matrix,...,int64_t,...)
                                                                    GrB_Matrix_assign_INT64(GrB_Matrix,..., int64_t,...)
GrB_assign(GrB_Matrix,...,uint64_t,...)
                                                                    GrB_Matrix_assign_UINT64(GrB_Matrix,..., uint64_t,...)
GrB_assign(GrB_Matrix,...,float,...)
                                                                    {\sf GrB\_Matrix\_assign\_FP32}({\sf GrB\_Matrix}, \ldots, \ {\sf float}, \ldots)
                                                                    GrB_Matrix_assign_FP64(GrB_Matrix,..., double,...)
GrB_assign(GrB_Matrix,...,double,...)
GrB_assign(GrB_Matrix,...,other,...)
                                                                    GrB_Matrix_assign_UDT(GrB_Matrix,...,const void*,...)
```

Table 5.7: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

|              | Polymorphic signature  | Nonpolymorphic signature   |
|--------------|--|--|
| -            | GrB_apply(GrB_Vector,,GrB_UnaryOp,GrB_Vector,)   | GrB_Vector_apply(GrB_Vector,,GrB_UnaryOp,GrB_Vector,)  |
|              | $GrB_apply(GrB_Matrix,,GrB_UnaryOp,GrB_Matrix,)$   | GrB_Matrix_apply(GrB_Matrix,,GrB_UnaryOp,GrB_Matrix,)  |
| -            | GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Scalar,GrB_Vector,)   | $GrB\_Vector\_apply\_BinaryOp1st\_Scalar(GrB\_Vector,,GrB\_BinaryOp,GrB\_Scalar,GrB\_Vector,)$                                   |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,bool,GrB\_Vector,)$   | $GrB\_Vector\_apply\_BinaryOp1st\_BOOL(GrB\_Vector, , GrB\_BinaryOp,bool,GrB\_Vector,)$  |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,int8\_t,GrB\_Vector,)$  | GrB_Vector_apply_BinaryOp1st_INT8(GrB_Vector,,GrB_BinaryOp,int8_t,GrB_Vector,)   |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,uint8\_t,GrB\_Vector,)$   | $\label{linear_grb_def} Grb\_Vector, \dots, Grb\_BinaryOp, uint8\_t, Grb\_Vector, \dots)$  |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,int16\_t,GrB\_Vector,)$   | $\label{linear_grb_def} Grb\_Vector, \dots, Grb\_BinaryOp, int16\_t, Grb\_Vector, \dots)$  |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,uint16\_t,GrB\_Vector,)$  | $GrB\_Vector\_apply\_BinaryOp1st\_UINT16(GrB\_Vector,,GrB\_BinaryOp,uint16\_t,GrB\_Vector,)$                                     |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,int32\_t,GrB\_Vector,)$   | GrB_Vector_apply_BinaryOp1st_INT32(GrB_Vector,,GrB_BinaryOp,int32_t,GrB_Vector,)   |
|              | $\label{lem:grb_apply} $$\operatorname{GrB\_Vector},\ldots,\operatorname{GrB\_BinaryOp,uint32\_t,GrB\_Vector},\ldots)$$                                      | GrB_Vector_apply_BinaryOp1st_UINT32(GrB_Vector,,GrB_BinaryOp,uint32_t,GrB_Vector,)   |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,int64\_t,GrB\_Vector,)$   | GrB_Vector_apply_BinaryOp1st_INT64(GrB_Vector,,GrB_BinaryOp,int64_t,GrB_Vector,)   |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,uint64\_t,GrB\_Vector,)$  | GrB_Vector_apply_BinaryOp1st_UINT64(GrB_Vector,,GrB_BinaryOp,uint64_t,GrB_Vector,)   |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,float,GrB\_Vector,)$  | GrB_Vector_apply_BinaryOp1st_FP32(GrB_Vector,,GrB_BinaryOp,float,GrB_Vector,)  |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,double,GrB\_Vector,)$   | GrB_Vector_apply_BinaryOp1st_FP64(GrB_Vector,,GrB_BinaryOp,double,GrB_Vector,)   |
| _            | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,other,GrB\_Vector,)$  | GrB_Vector_apply_BinaryOp1st_UDT(GrB_Vector,,GrB_BinaryOp,const void*,GrB_Vector,)   |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,GrB\_Scalar,)$  | $\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_Scalar(GrB\_Vector,\dots,GrB\_BinaryOp,GrB\_Vector,GrB\_Scalar,\dots)$ |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,bool,)$   | $\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_BOOL(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,bool,)$                    |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int8\_t,)$  | $\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_INT8(GrB\_Vector,\dots,GrB\_BinaryOp,GrB\_Vector,int8\_t,\dots)$       |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,uint8\_t,)$   | $\label{linear_gradient} GrB\_Vector, \dots, GrB\_BinaryOp, GrB\_Vector, uint8\_t, \dots)$                                       |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int16\_t,)$   | $\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_INT16 (GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int16\_t,)$              |
| $\mathbf{c}$ | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,uint16\_t,)$  | $\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_UINT16(GrB\_Vector,\dots,GrB\_BinaryOp,GrB\_Vector,uint16\_t,\dots)$   |
| 7            | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int32\_t,)$   | $\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_INT32(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int32\_t,)$               |
| •            | $\label{lem:grb_apply} $$\operatorname{GrB\_Vector}_{,\dots,\operatorname{GrB\_BinaryOp}_{,\operatorname{GrB\_Vector}_{,\operatorname{uint}32\_t},\dots)}$$$ | $\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_UINT32(GrB\_Vector,\dots,GrB\_BinaryOp,GrB\_Vector,uint32\_t,\dots)$   |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int64\_t,)$   | $\label{linear_gradient} GrB\_Vector\_apply\_BinaryOp2nd\_INT64(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,int64\_t,)$               |
|              | $GrB\_apply(GrB\_Vector,,GrB\_BinaryOp,GrB\_Vector,uint64\_t,)$  | GrB_Vector_apply_BinaryOp2nd_UINT64(GrB_Vector,,GrB_BinaryOp,GrB_Vector,uint64_t,)   |
|              | $\label{lem:grb_apply} $$\operatorname{GrB\_Vector}_{,\dots,\operatorname{GrB\_BinaryOp}_{,\operatorname{GrB\_Vector}_{,\operatorname{float}_{,\dots}}})$$$  | GrB_Vector_apply_BinaryOp2nd_FP32(GrB_Vector,,GrB_BinaryOp,GrB_Vector,float,)  |
|              | $GrB_apply(GrB_Vector,, GrB_BinaryOp, GrB_Vector, double,)$  | GrB_Vector_apply_BinaryOp2nd_FP64(GrB_Vector,,GrB_BinaryOp,GrB_Vector,double,)   |
| _            | $GrB_apply(GrB_Vector,, GrB_BinaryOp, GrB_Vector, other,)$   | GrB_Vector_apply_BinaryOp2nd_UDT(GrB_Vector,,GrB_BinaryOp,GrB_Vector,const void*,)   |

 ${\it Table 5.8: Long-name, nonpolymorphic form of GraphBLAS methods (continued)}.$ 

|    | Polymorphic signature  | Nonpolymorphic signature   |
|----|--|--|
| _  | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Scalar,GrB\_Matrix,)$  | GrB_Matrix_apply_BinaryOp1st_Scalar(GrB_Matrix,,GrB_BinaryOp,GrB_Scalar,GrB_Matrix,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,bool,GrB\_Matrix,)$   | GrB_Matrix_apply_BinaryOp1st_BOOL(GrB_Matrix,,GrB_BinaryOp,bool,GrB_Matrix,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,int8\_t,GrB\_Matrix,)$  | GrB_Matrix_apply_BinaryOp1st_INT8(GrB_Matrix,,GrB_BinaryOp,int8_t,GrB_Matrix,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,uint8\_t,GrB\_Matrix,)$   | GrB_Matrix_apply_BinaryOp1st_UINT8(GrB_Matrix,,GrB_BinaryOp,uint8_t,GrB_Matrix,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,int16\_t,GrB\_Matrix,)$   | GrB_Matrix_apply_BinaryOp1st_INT16(GrB_Matrix,,GrB_BinaryOp,int16_t,GrB_Matrix,)   |
|    | GrB_apply(GrB_Matrix,,GrB_BinaryOp,uint16_t,GrB_Matrix,)   | GrB_Matrix_apply_BinaryOp1st_UINT16(GrB_Matrix,,GrB_BinaryOp,uint16_t,GrB_Matrix,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,int32\_t,GrB\_Matrix,)$   | GrB_Matrix_apply_BinaryOp1st_INT32(GrB_Matrix,,GrB_BinaryOp,int32_t,GrB_Matrix,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,uint32\_t,GrB\_Matrix,)$  | GrB_Matrix_apply_BinaryOp1st_UINT32(GrB_Matrix,,GrB_BinaryOp,uint32_t,GrB_Matrix,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,int64\_t,GrB\_Matrix,)$   | GrB_Matrix_apply_BinaryOp1st_INT64(GrB_Matrix,,GrB_BinaryOp,int64_t,GrB_Matrix,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,uint64\_t,GrB\_Matrix,)$  | GrB_Matrix_apply_BinaryOp1st_UINT64(GrB_Matrix,,GrB_BinaryOp,uint64_t,GrB_Matrix,)   |
| 2  | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,float,GrB\_Matrix,)$  | GrB_Matrix_apply_BinaryOp1st_FP32(GrB_Matrix,,GrB_BinaryOp,float,GrB_Matrix,)  |
| 78 | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,double,GrB\_Matrix,)$   | GrB_Matrix_apply_BinaryOp1st_FP64(GrB_Matrix,,GrB_BinaryOp,double,GrB_Matrix,)   |
| •  | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp, other, GrB\_Matrix,)$  | GrB_Matrix_apply_BinaryOp1st_UDT(GrB_Matrix,,GrB_BinaryOp,const void*,GrB_Matrix,)   |
|    | $\label{lem:grb_apply} $$\operatorname{GrB\_Matrix}_{,\dots,\operatorname{GrB\_BinaryOp}},\operatorname{GrB\_Matrix}_{,\operatorname{GrB\_Scalar}_{,\dots}}$$$ | $\label{linear_grb_matrix} GrB\_Matrix\_apply\_BinaryOp2nd\_Scalar(GrB\_Matrix,\ldots,GrB\_BinaryOp,GrB\_Matrix,GrB\_Scalar,\ldots)$ |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,bool,)$   | GrB_Matrix_apply_BinaryOp2nd_BOOL(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,bool,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,int8\_t,)$  | GrB_Matrix_apply_BinaryOp2nd_INT8(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int8_t,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,uint8\_t,)$   | GrB_Matrix_apply_BinaryOp2nd_UINT8(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint8_t,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,int16\_t,)$   | GrB_Matrix_apply_BinaryOp2nd_INT16(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int16_t,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,uint16\_t,)$  | GrB_Matrix_apply_BinaryOp2nd_UINT16(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint16_t,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,int32\_t,)$   | GrB_Matrix_apply_BinaryOp2nd_INT32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int32_t,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,uint32\_t,)$  | GrB_Matrix_apply_BinaryOp2nd_UINT32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint32_t,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,int64\_t,)$   | GrB_Matrix_apply_BinaryOp2nd_INT64(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int64_t,)   |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,uint64\_t,)$  | $\label{linear_gradient} GrB\_Matrix\_apply\_BinaryOp2nd\_UINT64(GrB\_Matrix, \dots, GrB\_BinaryOp, GrB\_Matrix, uint64\_t, \dots)$  |
|    | $GrB\_apply(GrB\_Matrix,,GrB\_BinaryOp,GrB\_Matrix,float,)$  | GrB_Matrix_apply_BinaryOp2nd_FP32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,float,)  |
|    | $GrB\_apply(GrB\_Matrix, \dots, GrB\_BinaryOp, GrB\_Matrix, double, \dots)$  | $\label{linear_gradient} GrB\_Matrix\_apply\_BinaryOp2nd\_FP64 (GrB\_Matrix, \dots, GrB\_BinaryOp, GrB\_Matrix, double, \dots)$      |
| _  | $GrB\_apply(GrB\_Matrix, \dots, GrB\_BinaryOp, GrB\_Matrix, other, \dots)$   | GrB_Matrix_apply_BinaryOp2nd_UDT(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,const void*,)   |

Table 5.9: Long-name, nonpolymorphic form of GraphBLAS methods (continued). [Scott: NEW CONTENT]

| Polymorphic signature  | Nonpolymorphic signature   |
|--|--|
| $\label{lem:grb_apply} $$\operatorname{GrB\_Vector}_{,\dots,GrB\_IndexUnaryOp}_{GrB\_Vector}_{GrB\_Scalar}_{,\dots)}$$     | GrB_Vector_apply_IndexOp_Scalar(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,GrB_Scalar,)   |
| $GrB\_apply(GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, bool, \dots)$  | GrB_Vector_apply_IndexOp_BOOL(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,bool,)   |
| $GrB_apply(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int8_t,)$   | GrB_Vector_apply_IndexOp_INT8(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int8_t,)   |
| $GrB\_apply(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint8\_t,)$   | GrB_Vector_apply_IndexOp_UINT8(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint8_t,)   |
| $GrB\_apply(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,int16\_t,)$   | GrB_Vector_apply_IndexOp_INT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int16_t,)   |
| $\label{lem:grb_apply} GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, uint16\_t, \dots)$                              | GrB_Vector_apply_IndexOp_UINT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint16_t,)   |
| $GrB_apply(GrB_Vector,, GrB_IndexUnaryOp, GrB_Vector, int 32_t,)$  | GrB_Vector_apply_IndexOp_INT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int32_t,)   |
| $GrB_apply(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint32_t,)$   | GrB_Vector_apply_IndexOp_UINT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint32_t,)   |
| $GrB\_apply(GrB\_Vector,\ldots,GrB\_IndexUnaryOp,GrB\_Vector,int64\_t,\ldots)$   | GrB_Vector_apply_IndexOp_INT64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int64_t,)   |
| $GrB\_apply(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint64\_t,)$  | GrB_Vector_apply_IndexOp_UINT64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint64_t,)   |
| $GrB\_apply(GrB\_Vector,\ldots,GrB\_IndexUnaryOp,GrB\_Vector,float,\ldots)$  | GrB_Vector_apply_IndexOp_FP32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,float,)  |
| $GrB\_apply(GrB\_Vector,\ldots,GrB\_IndexUnaryOp,GrB\_Vector,double,\ldots)$   | GrB_Vector_apply_IndexOp_FP64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,double,)   |
| $\label{eq:GrB_apply} $$ GrB_apply(GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, other, \dots)$$                        | GrB_Vector_apply_IndexOp_UDT(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,const void*,)   |
| $\label{lem:grb_apply} $$\operatorname{GrB\_Apply}(GrB\_Matrix,\ldots,GrB\_IndexUnaryOp,GrB\_Matrix,GrB\_Scalar,\ldots)$$$ | GrB_Matrix_apply_IndexOp_Scalar(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,GrB_Scalar,)   |
| $GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, bool, \dots)$  | GrB_Matrix_apply_IndexOp_BOOL(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,bool,)   |
| $GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int8\_t, \dots)$   | GrB_Matrix_apply_IndexOp_INT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int8_t,)   |
| $GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, uint8\_t, \dots)$  | $\begin{tabular}{ll} GrB\_Matrix\_apply\_IndexOp\_UINT8(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,uint8\_t,) \end{tabular}$ |
| $GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int16\_t, \dots)$  | GrB_Matrix_apply_IndexOp_INT16(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int16_t,)   |
| $GrB\_apply(GrB\_Matrix,\ldots,GrB\_IndexUnaryOp,GrB\_Matrix,uint16\_t,\ldots)$  | GrB_Matrix_apply_IndexOp_UINT16(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,)   |
| $\label{lem:grb_apply} GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int 32\_t, \dots)$                              | GrB_Matrix_apply_IndexOp_INT32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int32_t,)   |
| $_{f f \bigcirc}$ GrB_apply(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)   | GrB_Matrix_apply_IndexOp_UINT32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)   |
| $\ccite{G}$ GrB_apply(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int64_t,)  | GrB_Matrix_apply_IndexOp_INT64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int64_t,)   |
| $\label{lem:grb_apply} GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, uint64\_t, \dots)$                   | GrB_Matrix_apply_IndexOp_UINT64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint64_t,)   |
| $GrB\_apply(GrB\_Matrix,\ldots,GrB\_IndexUnaryOp,GrB\_Matrix,float,\ldots)$  | GrB_Matrix_apply_IndexOp_FP32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,float,)  |
| $GrB\_apply(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, double, \dots)$  | GrB_Matrix_apply_IndexOp_FP64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,double,)   |
| $GrB\_apply(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,other,)$  | GrB_Matrix_apply_IndexOp_UDT(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,const void*,)   |

Table 5.10: Long-name, nonpolymorphic form of GraphBLAS methods (continued).[Scott: NEW CONTENT]

|    | Polymorphic signature   | Nonpolymorphic signature  |
|----|---|---|
|    | $GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,GrB\_Scalar,)$  |   |
|    | $GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,bool,)$   | $ GrB\_Vector\_select\_BOOL(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, bool, \ldots) $                                      |
|    | $GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,int8\_t,)$  | GrB_Vector_select_INT8(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int8_t,)   |
|    | $\label{lem:conditional} GrB\_select(GrB\_Vector,\ldots,GrB\_IndexUnaryOp,GrB\_Vector,uint8\_t,\ldots)$   | $\label{lem:grb_vector_select_UINT8} GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, uint8\_t, \dots)$                            |
|    | $\label{lem:grb_select} GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, int16\_t, \dots)$   | $\label{linear_gradient} GrB\_Vector\_select\_INT16(GrB\_Vector,\dots,GrB\_IndexUnaryOp,GrB\_Vector,int16\_t,\dots)$                  |
|    | $\label{local_gradient} GrB\_select(GrB\_Vector, \ldots, GrB\_IndexUnaryOp, GrB\_Vector, uint16\_t, \ldots)$  | GrB_Vector_select_UINT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint16_t,)   |
|    | $\label{local_gradient} GrB\_select(GrB\_Vector,\dots,GrB\_IndexUnaryOp,GrB\_Vector,int32\_t,\dots)$  | GrB_Vector_select_INT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int32_t,)   |
|    | $\label{lem:grb_select} GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, uint 32\_t, \dots)$   | $\label{lem:conditional_condition} GrB\_Vector\_select\_UINT32(GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, uint32\_t, \dots)$ |
|    | $GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,int64\_t,)$   | $\label{localization} GrB\_Vector\_select\_INT64 (GrB\_Vector, \dots, GrB\_IndexUnaryOp, GrB\_Vector, int64\_t, \dots)$               |
|    | $GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,uint64\_t,)$  | $\label{lem:conditional_gradient} GrB\_Vector\_select\_UINT64(GrB\_Vector,\ldots,GrB\_IndexUnaryOp,GrB\_Vector,uint64\_t,\ldots)$     |
| 2  | $GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,float,)$  | $\label{lem:grb_vector_select_FP32} GrB\_Vector, GrB\_IndexUnaryOp, GrB\_Vector, float, \dots)$                                       |
| 80 | $GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,double,)$   | $\label{lem:grb_vector_select_FP64} GrB\_Vector, GrB\_IndexUnaryOp, GrB\_Vector, double, \dots)$                                      |
|    | $GrB\_select(GrB\_Vector,,GrB\_IndexUnaryOp,GrB\_Vector,other,)$  | GrB_Vector_select_UDT(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,const void*,)   |
|    | $\label{lem:grb_select} $$\operatorname{GrB\_Matrix},\ldots,\operatorname{GrB\_IndexUnaryOp},\operatorname{GrB\_Matrix},\operatorname{GrB\_Scalar},\ldots)$$$ | $\label{lem:grb_matrix_select_Scalar} GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, GrB\_Scalar, \dots)$                        |
|    | $GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,bool,)$   | $\label{lem:grb_matrix} Grb\_Matrix\_select\_BOOL(Grb\_Matrix,\dots,Grb\_IndexUnaryOp,Grb\_Matrix,bool,\dots)$                        |
|    | GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int8_t,)   | GrB_Matrix_select_INT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int8_t,)   |
|    | GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,)  | GrB_Matrix_select_UINT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,)   |
|    | $GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,int16\_t,)$   | $\label{local_gradient} GrB\_Matrix\_select\_INT16 \big(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int16\_t, \dots \big)$    |
|    | $GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,uint16\_t,)$  | GrB_Matrix_select_UINT16(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,)   |
|    | $GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,int32\_t,)$   | $\label{local_gradient} GrB\_Matrix\_select\_INT32 (GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, int32\_t, \dots)$             |
|    | $GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,uint32\_t,)$  | GrB_Matrix_select_UINT32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)   |
|    | $\label{lem:grb_select} GrB\_Select \big(GrB\_Matrix, \ldots, GrB\_IndexUnaryOp, GrB\_Matrix, int 64\_t, \ldots \big)$  | $\label{lem:grb_matrix} GrB\_Matrix\_select\_INT64(GrB\_Matrix,\dots,GrB\_IndexUnaryOp,GrB\_Matrix,int64\_t,\dots)$                   |
|    | $\label{lem:grb_select} GrB\_Matrix, \ldots, GrB\_IndexUnaryOp, GrB\_Matrix, uint 64\_t, \ldots)$   | $\label{lem:grb_matrix} Grb\_Matrix\_select\_UINT64 (Grb\_Matrix,\dots,Grb\_IndexUnaryOp,Grb\_Matrix,uint64\_t,\dots)$                |
|    | $GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,float,)$  | GrB_Matrix_select_FP32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,float,)  |
|    | $GrB\_select(GrB\_Matrix,,GrB\_IndexUnaryOp,GrB\_Matrix,double,)$   | GrB_Matrix_select_FP64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,double,)   |
|    | $GrB\_select(GrB\_Matrix, \dots, GrB\_IndexUnaryOp, GrB\_Matrix, other, \dots)$   | GrB_Matrix_select_UDT(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,const void*,)   |

 ${\it Table 5.11: Long-name, nonpolymorphic form of GraphBLAS methods (continued).}$ 

| Polymorphic signature  | Nonpolymorphic signature  |
|--|---|
| GrB_reduce(GrB_Vector,,GrB_Monoid,)                                | GrB_Matrix_reduce_Monoid(GrB_Vector,,GrB_Monoid,)   |
| $GrB\_reduce(GrB\_Vector,,GrB\_BinaryOp,)$                         | GrB_Matrix_reduce_BinaryOp(GrB_Vector,,GrB_BinaryOp,)   |
| GrB_reduce(GrB_Scalar,,GrB_Monoid,GrB_Vector,)                     | GrB_Vector_reduce_Monoid_Scalar(GrB_Scalar,,GrB_Vector,)  |
| GrB_reduce(GrB_Scalar,,GrB_BinaryOp,GrB_Vector,)                   | GrB_Vector_reduce_BinaryOp_Scalar(GrB_Scalar,,GrB_Vector,)  |
| GrB_reduce(bool*,,GrB_Vector,)                                     | GrB_Vector_reduce_BOOL(bool*,,GrB_Vector,)  |
| $GrB\_reduce(int8\_t^*,,GrB\_Vector,)$                             | GrB_Vector_reduce_INT8(int8_t*,,GrB_Vector,)  |
| GrB_reduce(uint8_t*,,GrB_Vector,)                                  | GrB_Vector_reduce_UINT8(uint8_t*,,GrB_Vector,)  |
| $GrB\_reduce(int16\_t^*,,GrB\_Vector,)$                            | $GrB\_Vector\_reduce\_INT16(int16\_t^*,,GrB\_Vector,)$  |
| $GrB\_reduce(uint16\_t^*,,GrB\_Vector,)$                           | GrB_Vector_reduce_UINT16(uint16_t*,,GrB_Vector,)  |
| $GrB\_reduce(int32\_t^*,,GrB\_Vector,)$                            | GrB_Vector_reduce_INT32(int32_t*,,GrB_Vector,)  |
| $GrB\_reduce(uint32\_t^*,,GrB\_Vector,)$                           | GrB_Vector_reduce_UINT32(uint32_t*,,GrB_Vector,)  |
| $GrB\_reduce(int64\_t^*,,GrB\_Vector,)$                            | GrB_Vector_reduce_INT64(int64_t*,,GrB_Vector,)  |
| $GrB\_reduce(uint64\_t^*,,GrB\_Vector,)$                           | GrB_Vector_reduce_UINT64(uint64_t*,,GrB_Vector,)  |
| $GrB\_reduce(float*,\ldots,GrB\_Vector,\ldots)$                    | GrB_Vector_reduce_FP32(float*,,GrB_Vector,)   |
| $GrB\_reduce(double*,,GrB\_Vector,)$                               | $GrB\_Vector\_reduce\_FP64(double*,,GrB\_Vector,)$  |
| $GrB\_reduce(other,,GrB\_Vector,)$                                 | $GrB\_Vector\_reduce\_UDT(void*,,GrB\_Vector,)$   |
| $GrB\_reduce(GrB\_Scalar, \dots, GrB\_Monoid, GrB\_Matrix, \dots)$ | $\label{lem:grb_matrix} GrB\_Matrix\_reduce\_Monoid\_Scalar(GrB\_Scalar, \dots, GrB\_Monoid, GrB\_Matrix, \dots)$     |
| $GrB\_reduce(GrB\_Scalar,,GrB\_BinaryOp,GrB\_Matrix,)$             | $\label{lem:grb_matrix} GrB\_Matrix\_reduce\_BinaryOp\_Scalar(GrB\_Scalar, \dots, GrB\_BinaryOp, GrB\_Matrix, \dots)$ |
| $GrB\_reduce(bool*,\ldots,GrB\_Matrix,\ldots)$                     | $GrB\_Matrix\_reduce\_BOOL(bool*,\ldots,GrB\_Matrix,\ldots)$  |
| $GrB\_reduce(int8\_t^*, \dots, GrB\_Matrix, \dots)$                | $GrB\_Matrix\_reduce\_INT8(int8\_t^*,,GrB\_Matrix,)$  |
| $GrB\_reduce(uint8\_t*,\ldots,GrB\_Matrix,\ldots)$                 | GrB_Matrix_reduce_UINT8(uint8_t*,,GrB_Matrix,)  |
| $GrB\_reduce(int16\_t^*,\ldots,GrB\_Matrix,\ldots)$                | GrB_Matrix_reduce_INT16(int16_t*,,GrB_Matrix,)  |
| $GrB\_reduce(uint16\_t^*,\ldots,GrB\_Matrix,\ldots)$               | GrB_Matrix_reduce_UINT16(uint16_t*,,GrB_Matrix,)  |
| $GrB\_reduce(int32\_t^*,\ldots,GrB\_Matrix,\ldots)$                | GrB_Matrix_reduce_INT32(int32_t*,,GrB_Matrix,)  |
| $GrB\_reduce(uint32\_t^*, \dots, GrB\_Matrix, \dots)$              | GrB_Matrix_reduce_UINT32(uint32_t*,,GrB_Matrix,)  |
| $GrB\_reduce(int64\_t^*, \dots, GrB\_Matrix, \dots)$               | $GrB\_Matrix\_reduce\_INT64(int64\_t^*, \dots, \mathsf{GrB\_Matrix, \dots)}$  |
| $GrB\_reduce(uint64\_t*,\ldots,GrB\_Matrix,\ldots)$                | $GrB\_Matrix\_reduce\_UINT64(uint64\_t^*,,GrB\_Matrix,)$  |
| GrB_reduce(float*,,GrB_Matrix,)                                    | GrB_Matrix_reduce_FP32(float*,,GrB_Matrix,)   |
| $GrB\_reduce(double*,\ldots,GrB\_Matrix,\ldots)$                   | GrB_Matrix_reduce_FP64(double*,,GrB_Matrix,)  |
| GrB_reduce(other,,GrB_Matrix,)                                     | GrB_Matrix_reduce_UDT(void*,,GrB_Matrix,)   |
| $GrB\_kronecker(GrB\_Matrix,,GrB\_Semiring,)$                      | $\label{lem:grb_matrix} GrB\_Matrix\_kronecker\_Semiring(GrB\_Matrix,\dots,GrB\_Semiring,\dots)$                      |
| $GrB\_kronecker(GrB\_Matrix,,GrB\_Monoid,)$                        | GrB_Matrix_kronecker_Monoid(GrB_Matrix,,GrB_Monoid,)  |
| $GrB\_kronecker(GrB\_Matrix,,GrB\_BinaryOp,)$                      | GrB_Matrix_kronecker_BinaryOp(GrB_Matrix,,GrB_BinaryOp,)  |

Table 5.12: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

| 9 , 1   | ymorphic form of Graphblas methods (continued).  |
|---|--|
| Polymorphic signature   | Nonpolymorphic signature   |
| GrB_get(GrB_Scalar,GrB_Scalar,GrB_Field)  | GrB_Scalar_get_Scalar(GrB_Scalar,GrB_Scalar,GrB_Field)   |
| $GrB\_get(GrB\_Scalar,char^*,GrB\_Field)$   | GrB_Scalar_get_String(GrB_Scalar,char*,GrB_Field)  |
| $GrB\_get(GrB\_Scalar,int*,GrB\_Field)$   | GrB_Scalar_get_ENUM(GrB_Scalar,int*,GrB_Field)   |
| $GrB\_get(GrB\_Scalar,size\_t^*,GrB\_Field)$  | GrB_Scalar_get_SIZE(GrB_Scalar,size_t*,GrB_Field)  |
| GrB_get(GrB_Scalar,void*,GrB_Field)   | GrB_Scalar_get_VOID(GrB_Scalar,void*,GrB_Field)  |
| GrB_get(GrB_Vector,GrB_Scalar,GrB_Field)  | GrB_Vector_get_Scalar(GrB_Vector,GrB_Scalar,GrB_Field)   |
| GrB_get(GrB_Vector,char*,GrB_Field)   | GrB_Vector_get_String(GrB_Vector,char*,GrB_Field)  |
| GrB_get(GrB_Vector,int*,GrB_Field)  | GrB_Matrix_get_ENUM(GrB_Vector,int*,GrB_Field)   |
| GrB_get(GrB_Vector,size_t*,GrB_Field)   | GrB_Matrix_get_SIZE(GrB_Vector,size_t*,GrB_Field)  |
| GrB_get(GrB_Vector,void*,GrB_Field)   | GrB_Vector_get_VOID(GrB_Vector,void*,GrB_Field)  |
| GrB get(GrB Matrix,GrB Scalar,GrB Field)  | GrB_Matrix_get_Scalar(GrB_Matrix,GrB_Scalar,GrB_Field)   |
| GrB_get(GrB_Matrix,char*,GrB_Field)   | GrB_Matrix_get_String(GrB_Matrix,char*,GrB_Field)  |
| GrB_get(GrB_Matrix,int*,GrB_Field)  | GrB_Matrix_get_ENUM(GrB_Matrix,int*,GrB_Field)   |
| GrB_get(GrB_Matrix,size_t*,GrB_Field)   | GrB_Matrix_get_SIZE(GrB_Matrix,size_t*,GrB_Field)  |
| GrB_get(GrB_Matrix,void*,GrB_Field)   | GrB_Matrix_get_VOID(GrB_Matrix,void*,GrB_Field)  |
| GrB_get(GrB_UnaryOp,GrB_Scalar,GrB_Field)   | GrB_UnaryOp_get_Scalar(GrB_UnaryOp,GrB_Scalar,GrB_Field)   |
| GrB_get(GrB_UnaryOp,char*,GrB_Field)  | GrB_UnaryOp_get_String(GrB_UnaryOp,char*,GrB_Field)  |
| GrB_get(GrB_UnaryOp,int*,GrB_Field)   | GrB_UnaryOp_get_ENUM(GrB_UnaryOp,int*,GrB_Field)   |
| GrB_get(GrB_UnaryOp,size_t*,GrB_Field)  | GrB_UnaryOp_get_SIZE(GrB_UnaryOp,size_t*,GrB_Field)  |
| GrB_get(GrB_UnaryOp,void*,GrB_Field)  | GrB_UnaryOp_get_VOID(GrB_UnaryOp,void*,GrB_Field)  |
| GrB_get(GrB_IndexUnaryOp,GrB_Scalar,GrB_Field)  | GrB_IndexUnaryOp_get_Scalar(GrB_IndexUnaryOp,GrB_Scalar,GrB_Field)   |
| GrB get(GrB_IndexUnaryOp,char*,GrB Field)   | GrB_IndexUnaryOp_get_String(GrB_IndexUnaryOp,GrB_Field)  |
| GrB_get(GrB_IndexOnaryOp,int*,GrB_Field)  | GrB_IndexUnaryOp_get_ENUM(GrB_IndexUnaryOp,int*,GrB_Field)   |
| GrB_get(GrB_IndexOnaryOp,int*,GrB_Field) GrB_get(GrB_IndexUnaryOp,size_t*,GrB_Field)  | GrB_IndexOnaryOp_get_ENOWI(GrB_IndexOnaryOp,int ',GrB_Field) GrB_IndexUnaryOp_get_SIZE(GrB_IndexUnaryOp,size_t*,GrB_Field) |
| GrB_get(GrB_IndexUnaryOp,size_t*,GrB_Field) GrB_get(GrB_IndexUnaryOp,void*,GrB_Field) | GrB_IndexUnaryOp_get_VOID(GrB_IndexUnaryOp,size_t ',GrB_Field)   |
|   | GrB_BinaryOp_get_VOID(GrB_IndexOrlaryOp,void ,GrB_Field)  GrB_BinaryOp_get_Scalar(GrB_BinaryOp,GrB_Scalar,GrB_Field)       |
| GrB_get(GrB_BinaryOp,GrB_Scalar,GrB_Field)  |  |
| GrB_get(GrB_BinaryOp,char*,GrB_Field)   | GrB_BinaryOp_get_String(GrB_BinaryOp,char*,GrB_Field)  |
| GrB_get(GrB_BinaryOp,int*,GrB_Field)  | GrB_BinaryOp_get_ENUM(GrB_BinaryOp,int*,GrB_Field)   |
| GrB_get(GrB_BinaryOp,size_t*,GrB_Field)   | GrB_BinaryOp_get_SIZE(GrB_BinaryOp,size_t*,GrB_Field)  |
| GrB_get(GrB_BinaryOp,void*,GrB_Field)   | GrB_BinaryOp_get_VOID(GrB_BinaryOp,void*,GrB_Field)  |
| GrB_get(GrB_Monoid,GrB_Scalar,GrB_Field)  | GrB_Monoid_get_Scalar(GrB_Monoid,GrB_Scalar,GrB_Field)   |
| GrB_get(GrB_Monoid,char*,GrB_Field)   | GrB_Monoid_get_String(GrB_Monoid,char*,GrB_Field)  |
| $GrB\_get(GrB\_Monoid,int*,GrB\_Field)$   | GrB_Monoid_get_ENUM(GrB_Monoid,int*,GrB_Field)   |
| $GrB\_get(GrB\_Monoid,size\_t^*,GrB\_Field)$  | GrB_Monoid_get_SIZE(GrB_Monoid,size_t*,GrB_Field)  |
| GrB_get(GrB_Monoid,void*,GrB_Field)   | GrB_Monoid_get_VOID(GrB_Monoid,void*,GrB_Field)  |
| $GrB\_get(GrB\_Semiring,GrB\_Scalar,GrB\_Field)$                                      | $\label{lem:grb_semiring_get_Scalar} GrB\_Semiring, GrB\_Scalar, GrB\_Field)$  |
| $GrB\_get(GrB\_Semiring, char*, GrB\_Field)$  | GrB_Semiring_get_String(GrB_Semiring,char*,GrB_Field)  |
| $GrB\_get(GrB\_Semiring,int*,GrB\_Field)$   | GrB_Semiring_get_ENUM(GrB_Semiring,int*,GrB_Field)   |
| $GrB\_get(GrB\_Semiring,size\_t^*,GrB\_Field)$  | GrB_Semiring_get_SIZE(GrB_Semiring,size_t*,GrB_Field)  |
| $GrB\_get(GrB\_Semiring,void*,GrB\_Field)$  | GrB_Semiring_get_VOID(GrB_Semiring,void*,GrB_Field)  |
| GrB_get(GrB_Descriptor,GrB_Scalar,GrB_Field)  | GrB_Descriptor_get_Scalar(GrB_Descriptor,GrB_Scalar,GrB_Field)   |
| $GrB\_get(GrB\_Descriptor, char*, GrB\_Field)$  | GrB_Descriptor_get_String(GrB_Descriptor,char*,GrB_Field)  |
| $GrB\_get(GrB\_Descriptor,int*,GrB\_Field)$   | GrB_Descriptor_get_ENUM(GrB_Descriptor,int*,GrB_Field)   |
| $GrB\_get(GrB\_Descriptor, size\_t^*, GrB\_Field)$                                    | GrB_Descriptor_get_SIZE(GrB_Descriptor,size_t*,GrB_Field)  |
| $GrB\_get(GrB\_Descriptor,void*,GrB\_Field)$  | GrB_Descriptor_get_VOID(GrB_Descriptor,void*,GrB_Field)  |
| GrB_get(GrB_Type,GrB_Scalar,GrB_Field)  | GrB_Type_get_Scalar(GrB_Type,GrB_Scalar,GrB_Field)   |
| GrB_get(GrB_Type,char*,GrB_Field)   | GrB_Type_get_String(GrB_Type,char*,GrB_Field)  |
| $GrB\_get(GrB\_Type,int*,GrB\_Field)$   | GrB_Type_get_ENUM(GrB_Type,int*,GrB_Field)   |
| GrB_get(GrB_Type,size_t*,GrB_Field)   | GrB_Type_get_SIZE(GrB_Type,size_t*,GrB_Field)  |
| GrB_get(GrB_Type,void*,GrB_Field)   | GrB_Type_get_VOID(GrB_Type,void*,GrB_Field)  |
| GrB_get(GrB_Global,GrB_Scalar,GrB_Field)  | GrB_Global_get_Scalar(GrB_Global,GrB_Scalar,GrB_Field)   |
| GrB_get(GrB_Global,char*,GrB_Field)   | GrB_Global_get_String(GrB_Global,char*,GrB_Field)  |
| GrB_get(GrB_Global,int*,GrB_Field)  | GrB_Global_get_ENUM(GrB_Global,int*,GrB_Field)   |
| GrB_get(GrB_Global,size_t*,GrB_Field)   | GrB_Global_get_SIZE(GrB_Global,size_t*,GrB_Field)  |
| GrB_get(GrB_Global,void*,GrB_Field)   | GrB_Global_get_VOID(GrB_Global,void*,GrB_Field)  |
|   |  |

Table 5.13: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

| Polymorphic signature                                | Nonpolymorphic signature   |
|--|--|
| GrB_set(GrB_Scalar,GrB_Scalar,GrB_Field)             | GrB_Scalar_set_Scalar(GrB_Scalar,GrB_Scalar,GrB_Field)   |
| GrB_set(GrB_Scalar,GrB_Scalar,GrB_Field)             | GrB_Scalar_set_String(GrB_Scalar,GrB_Field)  |
| GrB_set(GrB_Scalar,int,GrB_Field)                    | GrB_Scalar_set_ENUM(GrB_Scalar,int,GrB_Field)  |
| GrB_set(GrB_Scalar,void*,GrB_Field,int)              | GrB_Scalar_set_VOID(GrB_Scalar,nvoid*,GrB_Field,int)   |
| GrB set(GrB Vector,GrB Scalar,GrB Field)             |  |
|  | GrB_Vector_set_Scalar(GrB_Vector,GrB_Scalar,GrB_Field)   |
| GrB_set(GrB_Vector,char*,GrB_Field)                  | GrB_Vector_set_String(GrB_Vector,char*,GrB_Field)  |
| GrB_set(GrB_Vector,int,GrB_Field)                    | GrB_Vector_set_ENUM(GrB_Vector,int,GrB_Field)  |
| GrB_set(GrB_Vector,void*,GrB_Field,int)              | GrB_Vector_set_VOID(GrB_Vector,void*,GrB_Field,int)  |
| GrB_set(GrB_Matrix,GrB_Scalar),GrB_Field             | GrB_Matrix_set_Scalar(GrB_Matrix,GrB_Scalar),GrB_Field   |
| GrB_set(GrB_Matrix,char*,GrB_Field)                  | GrB_Matrix_set_String(GrB_Matrix,char*,GrB_Field)  |
| GrB_set(GrB_Matrix,int,GrB_Field)                    | GrB_Matrix_set_ENUM(GrB_Matrix,int,GrB_Field)  |
| GrB_set(GrB_Matrix,void*,GrB_Field,int)              | GrB_Matrix_set_VOID(GrB_Matrix,void*,GrB_Field,int)  |
| GrB_set(GrB_UnaryOp,GrB_Scalar,GrB_Field)            | GrB_UnaryOp_set_Scalar(GrB_UnaryOp,GrB_Scalar,GrB_Field)   |
| GrB_set(GrB_UnaryOp,char*,GrB_Field)                 | GrB_UnaryOp_set_String(GrB_UnaryOp,char*,GrB_Field)  |
| GrB_set(GrB_UnaryOp,int,GrB_Field)                   | GrB_UnaryOp_set_ENUM(GrB_UnaryOp,int,GrB_Field)  |
| $GrB\_set(GrB\_UnaryOp,void*,GrB\_Field,int)$        | GrB_UnaryOp_set_VOID(GrB_UnaryOp,void*,GrB_Field,int)  |
| $GrB\_set(GrB\_IndexUnaryOp,GrB\_Scalar,GrB\_Field)$ | $\label{linear_gradient} GrB\_IndexUnaryOp\_set\_Scalar(GrB\_IndexUnaryOp,GrB\_Scalar,GrB\_Field)$ |
| $GrB\_set(GrB\_IndexUnaryOp,char*,GrB\_Field)$       | GrB_IndexUnaryOp_set_String(GrB_IndexUnaryOp,char*,GrB_Field)                                      |
| $GrB\_set(GrB\_IndexUnaryOp,int,GrB\_Field)$         | GrB_IndexUnaryOp_set_ENUM(GrB_IndexUnaryOp,int,GrB_Field)  |
| $GrB\_set(GrB\_IndexUnaryOp,void*,GrB\_Field,int)$   | GrB_IndexUnaryOp_set_VOID(GrB_IndexUnaryOp,void*,GrB_Field,int)                                    |
| $GrB\_set(GrB\_BinaryOp,GrB\_Scalar,GrB\_Field)$     | GrB_BinaryOp_set_Scalar(GrB_BinaryOp,GrB_Scalar,GrB_Field)   |
| $GrB\_set(GrB\_BinaryOp,char*,GrB\_Field)$           | GrB_BinaryOp_set_String(GrB_BinaryOp,char*,GrB_Field)  |
| $GrB\_set(GrB\_BinaryOp,int,GrB\_Field)$             | GrB_BinaryOp_set_ENUM(GrB_BinaryOp,int,GrB_Field)  |
| $GrB\_set(GrB\_BinaryOp,void*,GrB\_Field,int)$       | GrB_BinaryOp_set_VOID(GrB_BinaryOp,void*,GrB_Field,int)  |
| $GrB\_set(GrB\_Monoid,GrB\_Scalar,GrB\_Field)$       | GrB_Monoid_set_Scalar(GrB_Monoid,GrB_Scalar,GrB_Field)   |
| $GrB\_set(GrB\_Monoid,char*,GrB\_Field)$             | GrB_Monoid_set_String(GrB_Monoid,char*,GrB_Field)  |
| $GrB\_set(GrB\_Monoid,int,GrB\_Field)$               | GrB_Monoid_set_ENUM(GrB_Monoid,int,GrB_Field)  |
| $GrB\_set(GrB\_Monoid,void*,GrB\_Field,int)$         | GrB_Monoid_set_VOID(GrB_Monoid,void*,GrB_Field,int)  |
| $GrB\_set(GrB\_Semiring,GrB\_Scalar,GrB\_Field)$     | GrB_Semiring_set_Scalar(GrB_Semiring,GrB_Scalar,GrB_Field)   |
| $GrB\_set(GrB\_Semiring,char*,GrB\_Field)$           | GrB_Semiring_set_String(GrB_Semiring,char*,GrB_Field)  |
| $GrB\_set(GrB\_Semiring,int,GrB\_Field)$             | GrB_Semiring_set_ENUM(GrB_Semiring,int,GrB_Field)  |
| $GrB\_set(GrB\_Semiring,void*,GrB\_Field,int)$       | GrB_Semiring_set_VOID(GrB_Semiring,void*,GrB_Field,int)  |
| GrB_set(GrB_Descriptor,GrB_Scalar,GrB_Field)         | GrB_Descriptor_set_Scalar(GrB_Descriptor,GrB_Scalar,GrB_Field)                                     |
| $GrB\_set(GrB\_Descriptor, char*, GrB\_Field)$       | GrB_Descriptor_set_String(GrB_Descriptor,char*,GrB_Field)  |
| $GrB\_set(GrB\_Descriptor,int,GrB\_Field)$           | GrB_Descriptor_set_ENUM(GrB_Descriptor,int,GrB_Field)  |
| $GrB\_set(GrB\_Descriptor, void*, GrB\_Field, int)$  | GrB_Descriptor_set_VOID(GrB_Descriptor,void*,GrB_Field,int)  |
| GrB_set(GrB_Type,GrB_Scalar,GrB_Field)               | GrB_Type_set_Scalar(GrB_Type,GrB_Scalar,GrB_Field)   |
| GrB_set(GrB_Type,char*,GrB_Field)                    | GrB_Type_set_String(GrB_Type,char*,GrB_Field)  |
| GrB_set(GrB_Type,int,GrB_Field)                      | GrB_Type_set_ENUM(GrB_Type,int,GrB_Field)  |
| GrB_set(GrB_Type,void*,GrB_Field,int)                | GrB_Type_set_VOID(GrB_Type,void*,GrB_Field,int)  |
| GrB_set(GrB_Global,GrB_Scalar,GrB_Field)             | GrB_Global_set_Scalar(GrB_Global,GrB_Scalar,GrB_Field)   |
| GrB_set(GrB_Global,char*,GrB_Field)                  | GrB_Global_set_String(GrB_Global,char*,GrB_Field)  |
| GrB_set(GrB_Global,int,GrB_Field)                    | GrB_Global_set_ENUM(GrB_Global,int,GrB_Field)  |
| GrB_set(GrB_Global,void*,GrB_Field,int)              | GrB_Global_set_VOID(GrB_Global,void*,GrB_Field,int)  |
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## $_{7518}$ Appendix A

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# Revision history

7520 Changes in 2.0.1 (Released: ## Xxxxx 2022:

(Issue GH-69) Fix error in description of contents of matrix constructed from GrB\_Matrix\_diag.

7522 Changes in 2.0.0 (Released: 15 November 2021:

- Reorganized Chapters 2 and 3: Chapter 2 contains prose regarding the basic concepts captured in the API; Chapter 3 presents all of the enumerations, literals, data types, and predefined objects required by the API. Made short captions for the List of Tables.
  - (Issue BB-49, BB-50) Updated and corrected language regarding multithreading and completion, and requirements regarding acquire-release memory orders. Methods that used to force complete no longer do.
- (Issue BB-74, BB-9) Assigned integer values to all return codes as well as all enumerations in the API to ensure run-time compatibility between libraries.
- (Issues BB-70, BB-67) Changed semantics and signature of GrB\_wait(obj, mode). Added wait modes for 'complete' or 'materialize' and removed GrB\_wait(void). This breaks backward compatibility.
  - (Issue GH-51) Removed deprecated GrB\_SCMP literal from descriptor values. This breaks backward compatibility.
- (Issues BB-8, BB-36) Added sparse GrB\_Scalar object and its use in additional variants of extract/setElement methods, and reduce, apply, assign and select operations.
- (Issues BB-34, GH-33, GH-45) Added new select operation that uses an index unary operator.

  Added new variants of apply that take an index unary operator (matrix and vector variants).
  - (Issues BB-68, BB-51) Added serialize and deserialize methods for matrices to/from implementation defined formats.

- (Issues BB-25, GH-42) Added import and export methods for matrices to/from API specified formats. Three formats have been specified: CSC, CSR, COO. Dense row and column formats have been deferred.
- (Issue BB-75) Added matrix constructor to build a diagonal GrB\_Matrix from a GrB\_Vector.
- (Issue BB-73) Allow GrB\_NULL for dup operator in matrix and vector build methods. Return error if duplicate locations encountered.
- (Issue BB-58) Added matrix and vector methods to remove (annihilate) elements.
- (Issue BB-17) Added GrB\_ABS\_T (absolute value) unary operator.
- (Issue GH-46) Adding GrB\_ONEB\_T binary operator that returns 1 cast to type T (not to be confused with the proposed unary operator).
- (Issue GH-53) Added language about what constitutes a "conformant" implementation. Added GrB\_NOT\_IMPLEMENTED return value (API error) for API any combinations of inputs to a method that is not supported by the implementation.
- Added GrB\_EMPTY\_OBJECT return value (execution error) that is used when an opaque object (currently only GrB\_Scalar) is passed as an input that cannot be empty.
- (Issue BB-45) Removed language about annihilators.
- (Issue BB-69) Made names/symbols containing underscores searchable in PDF.
- Updated a number algorithms in the appendix to use new operations and methods.
- Numerous additions (some changes) to the non-polymorphic interface to track changes to the specification.
- Typographical error in version macros was corrected. They are all caps: GRB\_VERSION and GRB\_SUBVERSION.
- Typographical change to eWiseAdd Description to be consistent in order of set intersections.
- Typographical errors in eWiseAdd: cut-and-paste errors from eWiseMult/set intersection fixed to read eWiseAdd/set union.
- Typographical error (NEQ  $\rightarrow$  NE) in Description of Table 3.8.
- 7568 Changes in 1.3.0 (Released: 25 September 2019):
- (Issue BB-50) Changed definition of completion and added GrB\_wait() that takes an opaque GraphBLAS object as an argument.
- (Issue BB-39) Added GrB\_kronecker operation.
- (Issue BB-40) Added variants of the GrB\_apply operation that take a binary function and a scalar.

- (Issue BB-59) Changed specification about how reductions to scalar (GrB\_reduce) are to be performed (to minimize dependence on monoid identity).
- (Issue BB-24) Added methods to resize matrices and vectors (GrB\_Matrix\_resize and GrB\_Vector\_resize).
- (Issue BB-47) Added methods to remove single elements from matrices and vectors (GrB\_Matrix\_removeElement).
- (Issue BB-41) Added GrB\_STRUCTURE descriptor flag for masks (consider only the structure of the mask and not the values).
- (Issue BB-64) Deprecated GrB\_SCMP in favor of new GrB\_COMP for descriptor values.
- (Issue BB-46) Added predefined descriptors covering all possible combinations of field, value pairs.
- Added unary operators: absolute value ( $GrB\_ABS\_T$ ) and bitwise complement of integers ( $GrB\_BNOT\_I$ ).
- (Issues BB-42, BB-62) Added binary operators: Added boolean exclusive-nor (GrB\_LXNOR)

  and bitwise logical operators on integers (GrB\_BOR\_I, GrB\_BAND\_I, GrB\_BXOR\_I, GrB\_BXNOR\_I).
- (Issue BB-11) Added a set of predefined monoids and semirings.
- (Issue BB-57) Updated all examples in the appendix to take advantage of new capabilities and predefined objects.
- (Issue BB-43) Added parent-BFS example.
- (Issue BB-1) Fixed bug in the non-batch betweenness centrality algorithm in Appendix C.4 where source nodes were incorrectly assigned path counts.
- (Issue BB-3) Added compile-time preprocessor defines and runtime method for querying the GraphBLAS API version being used.
- (Issue BB-10) Clarified GrB\_init() and GrB\_finalize() errors.
- (Issue BB-16) Clarified behavior of boolean and integer division. Note that GrB\_MINV for integer and boolean types was removed from this version of the spec.
- (Issue BB-19) Clarified aliasing in user-defined operators.
- (Issue BB-20) Clarified language about behavior of GrB\_free() with predefined objects (implementation defined)
- (Issue BB-55) Clarified that multiplication does not have to distribute over addition in a GraphBLAS semiring.
- (Issue BB-45) Removed unnecessary language about annihilators.
- (Issue BB-61) Removed unnecessary language about implied zeros.
- (Issue BB-60) Added disclaimer against overspecification.

• Fixed miscellaneous typographical errors (such as  $\otimes .\oplus$ ).

#### 7608 Changes in 1.2.0:

• Removed "provisional" clause.

#### 7610 Changes in 1.1.0:

- Removed unnecessary const from nindices, nrows, and ncols parameters of both extract and assign operations.
- Signature of GrB\_UnaryOp\_new changed: order of input parameters changed.
- Signature of GrB\_BinaryOp\_new changed: order of input parameters changed.
- Signature of GrB\_Monoid\_new changed: removal of domain argument which is now inferred from the domains of the binary operator provided.
- Signature of GrB\_Vector\_extractTuples and GrB\_Matrix\_extractTuples to add an in/out argument, n, which indicates the size of the output arrays provided (in terms of number of elements, not number of bytes). Added new execution error, GrB\_INSUFFICIENT\_SPACE which is returned when the capacities of the output arrays are insufficient to hold all of the tuples.
- Changed GrB\_Column\_assign to GrB\_Col\_assign for consistency in non-polymorphic interface.
- Added replace flag (z) notation to Table 4.1.
  - Updated the "Mathematical Description" of the assign operation in Table 4.1.
- Added triangle counting example.
- Added subsection headers for accumulate and mask/replace discussions in the Description sections of GraphBLAS operations when the respective text was the "standard" text (i.e., identical in a majority of the operations).
- Fixed typographical errors.

#### 7631 Changes in 1.0.2:

- Expanded the definitions of Vector\_build and Matrix\_build to conceptually use intermediate matrices and avoid casting issues in certain implementations.
- Fixed the bug in the GrB\_assign definition. Elements of the output object are no longer being erased outside the assigned area.
- Changes non-polymorphic interface:
- 7637 Renamed GrB\_Row\_extract to GrB\_Col\_extract.

- Renamed GrB\_Vector\_reduce\_BinaryOp to GrB\_Matrix\_reduce\_BinaryOp.
- $\ \, Renamed \ \, \mathsf{GrB\_Vector\_reduce\_Monoid} \ \, to \ \, \mathsf{GrB\_Matrix\_reduce\_Monoid}.$
- Fixed the bugs with respect to isolated vertices in the Maximal Independent Set example.
- Fixed numerous typographical errors.

# Appendix B

# Non-opaque data format definitions

# B.1 GrB\_Format: Specify the format for input/output of a Graph-BLAS matrix.

In this section, the non-opaque matrix formats specified by GrB\_Format and used in matrix import and export methods are defined.

#### 7648 B.1.1 GrB\_CSR\_FORMAT

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The GrB\_CSR\_FORMAT format indicates that a matrix will be imported or exported using the compressed sparse row (CSR) format. indptr is a pointer to an array of GrB\_Index of size nrows+1 elements, where the i'th index will contain the starting index in the values and indices arrays corresponding to the i'th row of the matrix. indices is a pointer to an array of number of stored elements (each a GrB\_Index), where each element contains the corresponding element's column index within a row of the matrix. values is a pointer to an array of number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. The elements of each row are not required to be sorted by column index.

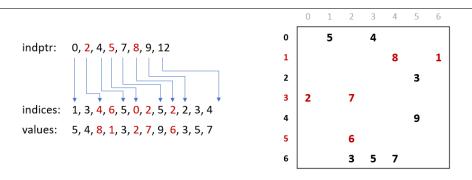


Figure B.1: Data layout for CSR format.

#### B.1.2 GrB\_CSC\_FORMAT

The GrB\_CSC\_FORMAT format indicates that a matrix will be imported or exported using the compressed sparse column (CSC) format. indptr is a pointer to an array of GrB\_Index of size ncols+1 elements, where the i'th index will contain the starting index in the values and indices arrays corresponding to the i'th column of the matrix. indices is a pointer to an array of number of stored elements (each a GrB\_Index), where each element contains the corresponding element's row index within a column of the matrix. values is a pointer to an array of number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. The elements of each column are not required to be sorted by row index.

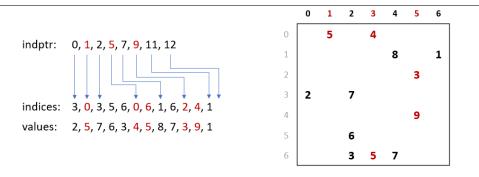


Figure B.2: Data layout for CSC format.

#### B.1.3 GrB\_COO\_FORMAT

The GrB\_COO\_FORMAT format indicates that a matrix will be imported or exported using the coordinate list (COO) format. indptr is a pointer to an array of GrB\_Index of size number of stored elements, where each element contains the corresponding element's column index. indices will be a pointer to an array of GrB\_Index of size number of stored elements, where each element contains the corresponding element's row index. values will be a pointer to an array of size number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. Elements are not required to be sorted in any order.

```
3
                                                                                    4
                                                                                         5
                                                                                              6
                                                           1
                                                                                    8
                                                                                              1
indptr:
          1, 3, 4, 6, 5, 0, 1, 5, 2, 2, 3, 4
                                                                                         3
                                                           2
indices: 0, 0, 1, 1, 2, 3, 3, 4, 5, 6, 6, 6
                                                           3
                                                                2
                                                                          7
values: 5, 4, 8, 1, 3, 2, 7, 9, 6, 3, 5, 7
                                                           4
                                                                                         9
                                                           5
                                                                          3
                                                                                   7
                                                           6
```

Figure B.3: Data layout for COO format.

Appendix C

Examples

### C.1 Example: Level breadth-first search (BFS) in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
   #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
    * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal
8
     * of the graph and sets v[i] to the level in which vertex i is visited (v[s] == 1).
     * If i is not reacheable from s, then v[i] = 0. (Vector v should be empty on input.)
10
11
    GrB_Info BFS(GrB_Vector *v, GrB_Matrix A, GrB_Index s)
13
14
      GrB_Index n;
                                                        // n = \# of rows of A
      GrB\_Matrix\_nrows(\&n,A);
15
16
                                                        // Vector < int32_t > v(n)
17
      GrB\_Vector\_new(v,GrB\_INT32,n);
18
19
      GrB_Vector q;
                                                        // vertices visited in each level
      GrB\_Vector\_new(\&q,GrB\_BOOL,n);
20
                                                        // Vector < bool > q(n)
21
      GrB_Vector_setElement(q,(bool)true,s);
                                                        // q[s] = true, false everywhere else
22
23
       * BFS traversal and label the vertices.
24
25
26
      int32 t d = 0;
                                                        // d = level in BFS traversal
27
      bool succ = false;
                                                        // succ == true when some successor found
28
      do {
29
                                                        // next level (start with 1)
30
        GrB_assign(*v,q,GrB_NULL,d,GrB_ALL,n,GrB_NULL);
                                                              // v[q] = d
31
        GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
                                                        // q[!v] = q ||.&& A; finds all the ||...| unvisited successors from current q
                 q, A, GrB\_DESC\_RC);
32
33
        GrB_reduce(&succ, GrB_NULL, GrB_LOR_MONOID_BOOL,
34
35
                    q, GrB_NULL);
                                                        // succ = //(q)
      } while (succ);
36
                                                        // if there is no successor in q, we are done.
37
                                                        // q vector no longer needed
38
      GrB_free(&q);
39
40
      return GrB SUCCESS;
41
```

### C.2 Example: Level BFS in GraphBLAS using apply

```
#include <stdlib.h>
   #include <stdio.h>
3 #include <stdint.h>
4 #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
    * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal
8
     * of the graph and sets v[i] to the level in which vertex i is visited (v[s] == 1).
     * If i is not reachable from s, then v[i] does not have a stored element.
10
11
     * Vector v should be uninitialized on input.
12
   GrB_Info BFS(GrB_Vector *v, const GrB_Matrix A, GrB_Index s)
13
14
      GrB Index n;
15
                                                        // n = \# of rows of A
16
      GrB\_Matrix\_nrows(\&n,A);
17
18
      GrB_Vector_new(v,GrB_INT32,n);
                                                        // Vector < int32_t > v(n) = 0
19
                                                        // vertices visited in each level
20
      GrB_Vector q;
      GrB\_Vector\_new(&q,GrB\_BOOL,n);
                                                        // Vector < bool > q(n) = false
// q[s] = true, false everywhere else
21
      GrB_Vector_setElement(q,(bool)true,s);
22
23
^{24}
25
      * BFS traversal and label the vertices.
26
                                                        //\ level = depth\ in\ BFS\ traversal
27
      int32\_t level = 0;
28
      GrB_Index nvals;
29
      do {
30
        ++level;
                                                        // next level (start with 1)
        GrB_apply(*v,GrB_NULL,GrB_PLUS_INT32,
31
                   GrB\_SECOND\_INT32, q, level, GrB\_NULL); // v[q] = level
32
        GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
33
                                                        // q[!v] = q //.&&A; finds all the
34
                q, A, GrB\_DESC\_RC);
35
                                                         // unvisited successors from current q
36
        GrB_Vector_nvals(&nvals, q);
      } while (nvals);
37
                                                        // if there is no successor in q, we are done.
38
39
      GrB_free(&q);
                                                        // q vector no longer needed
40
41
      return GrB_SUCCESS;
42 }
```

#### C.3 Example: Parent BFS in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
   #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
     * Given a binary n x n adjacency matrix A and a source vertex s, performs a BFS
8
     * traversal of the graph and sets parents[i] to the index of vertex i's parent.
     * The parent of the root vertex, s, will be set to itself (parents[s] == s). If * vertex i is not reachable from s, parents[i] will not contain a stored value.
10
11
12
    GrB\_Info\ BFS(GrB\_Vector\ *parents\ ,\ \textbf{const}\ GrB\_Matrix\ A,\ GrB\_Index\ s\ )
13
14
      GrB Index N;
15
                                                            //N = \# vertices
16
      GrB_Matrix_nrows(&N, A);
17
      GrB_Vector_new(parents, GrB_UINT64, N);
18
                                                            // parents[s] = s
      GrB_Vector_setElement(*parents, s, s);
20
21
      GrB Vector wavefront;
      GrB_Vector_new(&wavefront, GrB_UINT64, N);
22
23
      GrB_Vector_setElement(wavefront, 1UL, s);
                                                           // wavefront[s] = 1
^{24}
25
26
       * BFS traversal and label the vertices.
27
28
      GrB Index nvals;
29
      GrB_Vector_nvals(&nvals, wavefront);
30
31
      while (nvals > 0)
32
33
         // convert all stored values in wavefront to their 0-based index
        GrB_apply(wavefront, GrB_NULL, GrB_NULL, GrB_ROWINDEX_INT64,
34
35
                    wavefront , OUL, GrB_NULL);
36
        // "FIRST" because left-multiplying wavefront rows. Masking out the parent
37
         // list ensures wavefront values do not overwrite parents already stored.
38
        \label{eq:cont_state} GrB\_vxm(\,wavefront\,,\,\,*parents\,,\,\,GrB\_NULL,\,\,GrB\_MIN\_FIRST\_SEMIRING\_UINT64,
39
                  wavefront, A, GrB_DESC_RSC);
40
41
        //\ {\it Don't\ need\ to\ mask\ here\ since\ we\ did\ it\ in\ mxm.\ Merges\ new\ parents\ in}
42
         // current wavefront with existing parents: parents += wavefront
        GrB_apply(*parents, GrB_NULL, GrB_PLUS_UINT64,
44
45
                    GrB_IDENTITY_UINT64, wavefront, GrB_NULL);
46
47
        GrB_Vector_nvals(&nvals, wavefront);
48
49
50
      GrB free(&wavefront);
51
      return GrB_SUCCESS;
52
53
```

## C.4 Example: Betweenness centrality (BC) in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
4
   #include <stdbool.h>
   #include "GraphBLAS.h"
7
8
     * Given a boolean n x n adjacency matrix A and a source vertex s,
     st compute the BC-metric vector delta, which should be empty on input.
9
10
    GrB_Info BC(GrB_Vector *delta, GrB_Matrix A, GrB_Index s)
11
12
13
      GrB_Index n;
      GrB\_Matrix\_nrows(\&n,A);
                                                           // n = \# of vertices in graph
14
15
      GrB Vector new(delta, GrB FP32, n);
                                                           // Vector < float > delta(n)
16
17
18
      GrB_Matrix sigma;
                                                           // Matrix < int32\_t > sigma(n,n)
      GrB_Matrix_new(&sigma, GrB_INT32, n, n);
                                                           // sigma [d,k] = \# shortest paths to node k at level d
19
20
21
      GrB_Vector q;
                                                           // Vector<int32_t> q(n) of path counts
22
      GrB_Vector_new(&q, GrB_INT32, n);
                                                           // q[s] = 1
23
      GrB_Vector_setElement(q,1,s);
24
                                                           //\ \ Vector < int 32\_t > p(n) \ \ shortest \ \ path \ \ counts \ \ so \ \ far
25
      GrB_Vector p;
      GrB\_Vector\_dup(\&p, q);
26
27
      GrB\_vxm(\,q\,,p\,,GrB\_NULL,GrB\_PLUS\_TIMES\_SEMIRING\_INT32\,,
28
                                                           // get the first set of out neighbors
29
               q, A, GrB\_DESC\_RC);
30
31
       * BFS phase
32
33
      GrB\_Index d = 0;
                                                           // BFS level number
                                                           // sum == 0 when BFS phase is complete
35
      int32\_t sum = 0;
36
37
         GrB\_assign\left(sigma,GrB\_NULL,GrB\_NULL,q,d,GrB\_ALL,n,GrB\_NULL\right); \qquad // \ sigma\left[d,:\right] = q 
38
         GrB_eWiseAdd(p,GrB_NULL,GrB_NULL,GrB_PLUS_INT32,p,q,GrB_NULL); // accum path counts on this level
39
40
        GrB_vxm(q,p,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_INT32,
41
                  q, A, GrB\_DESC\_RC);
                                                                                  // q = \# paths to nodes reachable
42
                                                                                        from current level
        GrB reduce(&sum, GrB NULL, GrB PLUS MONOID INT32, q, GrB NULL);
                                                                                  // sum path counts at this level
43
44
        ++d;
45
      } while (sum);
46
47
48
       * BC computation phase
49
        * (t1, t2, t3, t4) are temporary vectors
50
      GrB_Vector t1; GrB_Vector_new(&t1,GrB_FP32,n);
51
       \begin{array}{ll} GrB\_Vector & t2 \ ; & GrB\_Vector\_new(\&t2 \ ,GrB\_FP32 \ ,n \ ) \ ; \end{array} 
52
      GrB_Vector t3; GrB_Vector_new(&t3,GrB_FP32,n);
53
54
      GrB_Vector t4; GrB_Vector_new(&t4,GrB_FP32,n);
55
      for (int i=d-1; i>0; i---)
56
57
         GrB assign(t1,GrB NULL,GrB NULL,1.0f,GrB ALL,n,GrB NULL);
                                                                                      // t1 = 1 + delta
58
        GrB_eWiseAdd(t1,GrB_NULL,GrB_NULL,GrB_PLUS_FP32,t1,*delta,GrB_NULL);
59
        GrB_extract(t2,GrB_NULL,GrB_NULL,sigma,GrB_ALL,n,i,GrB_DESC_T0);
GrB_eWiseMult(t2,GrB_NULL,GrB_NULL,GrB_DIV_FP32,t1,t2,GrB_NULL);
60
                                                                                      // t2 = sigma[i,:]
                                                                                      // t2 = (1 + delta)/sigma[i,:]
61
        GrB_mxv(t3,GrB_NULL,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_FP32,
                                                                                      // add contributions made by
62
```

```
63
64
65
66
67
68
      GrB_free(&sigma);
GrB_free(&q); GrB_free(&p);
69
70
71
      \label{eq:GrB_free} $\operatorname{GrB\_free}(\&t1)$; $\operatorname{GrB\_free}(\&t2)$; $\operatorname{GrB\_free}(\&t3)$; $\operatorname{GrB\_free}(\&t4)$;}
72
73
      return GrB_SUCCESS;
74
```

#### C.5 Example: Batched BC in GraphBLAS

```
#include <stdlib.h>
   #include "GraphBLAS.h" // in addition to other required C headers
2
4
    /\!/ Compute partial BC metric for a subset of source vertices, s, in graph A
   GrB Info BC update(GrB Vector *delta, GrB Matrix A, GrB Index *s, GrB Index nsver)
5
6
7
     GrB_Index n;
     GrB_Matrix_nrows(&n, A);
8
                                                            // n = \# of vertices in graph
     GrB_Vector_new(delta,GrB_FP32,n);
                                                             // Vector < float > delta(n)
9
10
     // index and value arrays needed to build numsp
11
12
     GrB_Index *i_nsver = (GrB_Index*) malloc(sizeof(GrB_Index)*nsver);
13
     int32\_t *ones = (int32\_t*) malloc(sizeof(int32\_t)*nsver);
     for(int i=0; i< nsver; ++i) {
14
15
       i_nsver[i] = i;
       ones [i] = 1;
16
17
18
     // numsp: structure holds the number of shortest paths for each node and starting vertex
19
20
      // discovered so far. Initialized to source vertices: numsp[s[i], i]=1, i=[0, nsver)
21
     GrB_Matrix numsp;
22
     GrB_Matrix_new(&numsp, GrB_INT32, n, nsver);
23
     GrB_Matrix_build(numsp,s,i_nsver,ones,nsver,GrB_PLUS_INT32);
24
     free(i_nsver); free(ones);
25
26
     // frontier: Holds the current frontier where values are path counts.
27
        Initialized to out vertices of each source node in s.
28
     GrB_Matrix frontier;
     GrB Matrix new(&frontier, GrB INT32, n, nsver);
30
     GrB_extract(frontier, numsp, GrB_NULL, A, GrB_ALL, n, s, nsver, GrB_DESC_RCT0);
31
     // sigma: stores frontier information for each level of BFS phase. The memory
32
     // for an entry in sigmas is only allocated within the do-while loop if needed.
33
      // n is an upper bound on diameter.
34
35
     GrB_Matrix *sigmas = (GrB_Matrix*) malloc(sizeof(GrB_Matrix)*n);
36
37
     int32 t d = 0;
                                                            // BFS level number
                                                            // nvals == 0 when BFS phase is complete
     GrB\_Index nvals = 0;
38
39
                           —— The BFS phase (forward sweep) —
40
41
     do {
        // sigmas [d](:,s) = d^{h} level frontier from source vertex s
42
       GrB_Matrix_new(&(sigmas[d]),GrB_BOOL,n,nsver);
43
44
       GrB\_apply(sigmas [d], GrB\_NULL, GrB\_NULL,
45
                  GrB_IDENTITY_BOOL, frontier ,GrB_NULL);
                                                            // sigmas[d](:,:) = (Boolean) frontier
46
       GrB\_eWiseAdd (numsp\,, GrB\_NULL, GrB\_NULL, GrB\_PLUS\_INT32\,,
47
48
                     numsp, frontier, GrB NULL);
                                                             // numsp += frontier (accum path counts)
       49
                                                            //\ f < !numsp > = A \ ' \ +.* \ f \ (update \ frontier)
                A, frontier, GrB_DESC_RCT0);
50
       GrB_Matrix_nvals(&nvals, frontier);
                                                            // number of nodes in frontier at this level
51
52
       d++:
53
     } while (nvals);
54
      // nspinv: the inverse of the number of shortest paths for each node and starting vertex.
55
     GrB_Matrix nspinv;
56
     GrB_Matrix_new(&nspinv,GrB_FP32,n,nsver);
57
     GrB_apply(nspinv,GrB_NULL,GrB_NULL,
58
                GrB_MINV_FP32, numsp ,GrB_NULL);
                                                            // nspinv = 1./numsp
59
60
61
      // bcu: BC updates for each vertex for each starting vertex in s
     GrB_Matrix bcu;
62
```

```
GrB_Matrix_new(&bcu,GrB_FP32,n,nsver);
63
64
      GrB assign (bcu , GrB NULL, GrB NULL,
                  1.0f, GrB_ALL, n, GrB_ALL, nsver, GrB_NULL); // filled with 1 to avoid sparsity issues
65
66
67
      GrB Matrix w;
                                                                 // temporary workspace matrix
68
      GrB_Matrix_new(&w, GrB_FP32, n, nsver);
69
70
                               — Tally phase (backward sweep) —
      for (int i=d-1; i>0; i--) {
71
        GrB\_eWiseMult (w, sigmas \cite{black} i \cite{black} i \cite{black}, GrB\_NULL,
72
73
                       74
         // add contributions by successors and mask with that BFS level's frontier
75
76
        GrB_mxm(w, sigmas[i-1], GrB_NULL, GrB_PLUS_TIMES_SEMIRING_FP32,
        \label{eq:continuous} $$ \prod_{x, w, \text{cib\_desc_R}} : // w < igmas [i-1] > = (A + .* w) $$ GrB_eWiseMult(bcu, GrB_NULL, GrB_PLUS_FP32, GrB_TIMES_FP32, w, numsp. GrB_NULL).
77
78
79
                       w, numsp, GrB_NULL);
                                                                    // bcu += w .* numsp
80
      }
81
      // row reduce bcu and subtract "nsver" from every entry to account
82
83
      // for 1 extra value per bcu row element.
      GrB_reduce(*delta,GrB_NULL,GrB_NULL,GrB_PLUS_FP32,bcu,GrB_NULL);
84
      GrB_apply(*delta,GrB_NULL,GrB_NULL,GrB_MINUS_FP32, *delta,(float)nsver,GrB_NULL);
85
86
87
      // Release resources
88
      for (int i=0; i < d; i++) {
89
        GrB\_free(\&(sigmas[i]));
90
91
      free (sigmas);
92
93
      GrB_free(&frontier);
                                  GrB_free(&numsp);
      GrB_free(&nspinv);
                                  GrB_free(&bcu);
94
                                                          GrB_free(&w);
95
96
      return GrB_SUCCESS;
97
  }
```

#### C.6 Example: Maximal independent set (MIS) in GraphBLAS

```
1 #include <stdlib.h>
2 #include <stdio.h>
   #include <stdint.h>
4 #include <stdbool.h>
5 #include "GraphBLAS.h"
      Assign a random number to each element scaled by the inverse of the node's degree.
7
   // This will increase the probability that low degree nodes are selected and larger
   // sets are selected.
9
10
   void setRandom(void *out, const void *in)
11
12
      uint32\_t degree = *(uint32\_t*)in;
      *(float*)out = (0.0001f + random()/(1. + 2.*degree)); // add 1 to prevent divide by zero
13
   }
14
15
16
     * A variant of Luby's randomized algorithm [Luby 1985].
17
18
    * Given a numeric n x n adjacency matrix A of an unweighted and undirected graph (where
19
     * the value true represents an edge), compute a maximal set of independent vertices and * return it in a boolean n-vector, 'iset' where set[i] == true implies vertex\ i is a member
21
22
     * of the set (the iset vector should be uninitialized on input.)
23
24
    GrB_Info MIS(GrB_Vector *iset, const GrB_Matrix A)
25
26
      GrB Index n;
27
      GrB Matrix nrows(&n,A);
                                                        // n = \# of rows of A
28
                                                        // holds random probabilities for each node
29
      GrB Vector prob;
                                                        // holds value of max neighbor probability
30
      GrB_Vector neighbor_max;
31
      GrB_Vector new_members;
                                                        // holds set of new members to iset
                                                        // holds set of new neighbors to new iset mbrs.
      GrB_Vector new_neighbors;
32
      GrB_Vector candidates;
                                                        // candidate members to iset
33
      GrB_Vector_new(&prob, GrB_FP32, n);
35
36
      GrB_Vector_new(&neighbor_max, GrB_FP32, n);
37
      GrB_Vector_new(&new_members, GrB_BOOL, n);
38
      GrB_Vector_new(&new_neighbors,GrB_BOOL,n);
      GrB_Vector_new(&candidates, GrB_BOOL, n);
40
      GrB_Vector_new(iset ,GrB_BOOL, n);
                                                        // Initialize independent set vector, bool
41
42
      GrB_UnaryOp set_random;
      GrB\_UnaryOp\_new(\&set\_random\;, setRandom\;, GrB\_FP32\;, GrB\_UINT32\;)\;;
43
      // compute the degree of each vertex.
45
46
      GrB_Vector degrees;
      GrB\_Vector\_new(\&degrees, GrB\_FP64, n);
47
48
      GrB reduce(degrees, GrB NULL, GrB NULL, GrB PLUS FP64, A, GrB NULL);
49
50
      // Isolated vertices are not candidates: candidates[degrees !=0] = true
      GrB_assign(candidates, degrees, GrB_NULL, true, GrB_ALL, n, GrB_NULL);
51
52
      // add all singletons to iset: iset[degree == 0] = 1
53
54
      GrB_assign(*iset , degrees ,GrB_NULL, true ,GrB_ALL, n ,GrB_DESC_RC) ;
55
56
      // Iterate while there are candidates to check.
57
      GrB_Index nvals;
      GrB_Vector_nvals(&nvals, candidates);
58
59
      while (nvals > 0) {
        // compute a random probability scaled by inverse of degree
60
61
        GrB_apply(prob, candidates, GrB_NULL, set_random, degrees, GrB_DESC_R);
62
```

```
63
        // compute the max probability of all neighbors
64
        GrB mxv(neighbor max, candidates, GrB NULL, GrB MAX SECOND SEMIRING FP32, A, prob, GrB DESC R);
65
66
        //\ select\ vertex\ if\ its\ probability\ is\ larger\ than\ all\ its\ active\ neighbors\,,
        // and apply a "masked no-op" to remove stored falses
67
68
        GrB_eWiseAdd(new_members,GrB_NULL,GrB_NULL,GrB_GT_FP64,prob,neighbor_max,GrB_NULL);
69
        GrB_apply(new_members,new_members,GrB_NULL,GrB_IDENTITY_BOOL,new_members,GrB_DESC_R);
70
71
        // add new members to independent set.
        GrB_eWiseAdd(*iset,GrB_NULL,GrB_NULL,GrB_LOR,*iset,new_members,GrB_NULL);
72
73
74
        // remove new members from set of candidates c = c \mathcal{E} !new
        GrB_eWiseMult(candidates, new_members, GrB_NULL,
75
76
                       GrB_LAND, candidates, candidates, GrB_DESC_RC);
77
        GrB_Vector_nvals(&nvals, candidates);
78
79
        if (nvals == 0) { break; }
                                                        // early exit condition
80
        // Neighbors of new members can also be removed from candidates
81
        GrB_mxv(new_neighbors, candidates, GrB_NULL, GrB_LOR_LAND_SEMIRING_BOOL,
82
83
                A, new_members, GrB_NULL);
        GrB\_eWiseMult(candidates, new\_neighbors, GrB\_NULL, GrB\_LAND,
84
                       candidates, candidates, GrB_DESC_RC);
85
86
87
        GrB\_Vector\_nvals(\&nvals\;,\; candidates\;)\;;
88
89
      GrB_free(&neighbor_max);
                                                       // free all objects "new'ed"
90
91
      GrB_free(&new_members);
      GrB_free(&new_neighbors);
92
93
      GrB_free(&prob);
      GrB_free(&candidates);
94
      GrB_free(&set_random);
95
96
      GrB_free(&degrees);
97
98
      return GrB_SUCCESS;
99
```

## C.7 Example: Counting triangles in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
 3 #include <stdint.h>
 4 #include <stdbool.h>
   #include "GraphBLAS.h"
 6
 7
     * Given an n x n boolean adjacency matrix, A, of an undirected graph, computes
 8
     st the number of triangles in the graph.
10
    uint64_t triangle_count(GrB_Matrix A)
11
12
      GrB_Index n;
13
14
      GrB_Matrix_nrows(&n, A);
                                                             // n = \# of vertices
15
      // L: NxN, lower-triangular, bool
16
      GrB_Matrix L;
17
18
      GrB_Matrix_new(&L, GrB_BOOL, n, n);
      \label{eq:conditional_grb_null} $\operatorname{GrB\_NULL}, \ \operatorname{GrB\_NULL}, \ \operatorname{GrB\_TRIL}, \ A, \ \operatorname{OUL}, \ \operatorname{GrB\_NULL});$
20
21
      GrB_Matrix C;
22
      GrB\_Matrix\_new(\&C, GrB\_UINT64, n, n);
23
24
      25
26
      uint64 t count;
      \label{eq:GrB_reduce} $$\operatorname{GrB\_NULL}, $\operatorname{GrB\_PLUS\_MONOID\_UINT64}, $\operatorname{C}, $\operatorname{GrB\_NULL})$;}
27
                                                                                        // 1-norm of C
28
29
      GrB_free(&C);
30
      GrB_free(&L);
31
32
      return count;
33 }
```