# AGI Agent Safety by Iteratively Improving the Utility Function

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#### Abstract

While it is still unclear if agents with Artificial General Intelligence (AGI) could ever be built, we can already use mathematical models to investigate potential safety systems for these agents. We present an AGI safety layer that creates a special dedicated input terminal to support the iterative improvement of an AGI agent's utility function. The humans who switched on the agent can use this terminal to close any loopholes that are discovered in the utility function's encoding of agent goals and constraints, to direct the agent towards new goals, or to force the agent to switch itself off.

An AGI agent may develop the emergent incentive to manipulate the above utility function improvement process, for example by deceiving, restraining, or even attacking the humans involved. The safety layer will partially, and sometimes fully, suppress this dangerous incentive.

The first part of this paper generalizes earlier work on AGI emergency stop buttons. We aim to make the mathematical methods used to construct the layer more accessible, by applying them to an MDP model. We discuss two provable properties of the safety layer, and show ongoing work in mapping it to a Causal Influence Diagram (CID).

In the second part, we develop full mathematical proofs, and show that the safety layer creates a type of bureaucratic blindness. We then present the design of a learning agent, a design that wraps the safety layer around either a known machine learning system, or a potential future AGI-level learning system. The resulting agent will satisfy the provable safety properties from the moment it is first switched on. Finally, we show how this agent can be mapped from its model to a real-life implementation. We review the methodological issues involved in this step, and discuss how these are typically resolved.

This long-form paper has two parts. Part 1 is a preprint of the conference paper Towards AGI Agent Safety by Iteratively Improving the Utility Function [Hol20]. Part 2, Proofs, Models, and Reality, contains additional material, including new research results that go beyond those in the conference paper. Part 1 can be read as a self-contained text.

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### 1 Introduction to Part 1

An AGI agent is an autonomous system programmed to achieve goals specified by a principal. In this paper, we consider the case where the principal is a group of humans. We consider utility-maximizing AGI agents whose goals and constraints are fully specified by a *utility function* that maps projected outcomes to utility values.

As humans are fallible, we expect that the first version of an AGI agent utility function created by them will have flaws. For example, the first version may have many loopholes: features that allow the agent to maximize utility in a way that causes harm to the humans. Iterative improvement allows such flaws to be fixed when they are discovered. Note however that, depending on the type of loophole, the discovery of a loophole may not always be a survivable event for the humans involved. The safety layer developed in this paper aims to make the agent *safer* by supporting iterative improvement, but it does not aim or claim to fully eliminate all dangers associated with human fallibility.

This work adopts a design stance from (cyber)physical systems safety engineering, where one seeks to develop and combine independent safety layers. These are safety related (sub)systems with independent failure modes, that drive down the risk of certain bad outcomes when the system is used. We construct a safety layer that enables the humans to run a process that iteratively improves the AGI agent's utility function. But the main point of interest is the feature of the layer that suppresses the likely emergent incentive [Omo08] of the AGI agent to manipulate or control this process. The aim is to keep the humans in control.

In the broader AGI safety literature, the type of AGI safety system most related to this work is usually called a *stop button* (e.g. [SFAY15, Hol19]), an *off switch* (e.g. [HMDAR17]), or described as creating *corrigibility* [SFAY15]. See [Hol19] for a recent detailed overview of work on related systems. The safety layer in this paper extends earlier work by the author in [Hol19], which in turn is based on the use of Armstrong's indifference methods

[Arm15]. A notable alternative to using indifference methods is introduced in [HMDAR17]. Like sections 4 and 6 in this paper, [AO17] defines an example world containing an MDP agent that uses indifference methods.

A different approach to enabling the iterative improvement of an AGI utility function by humans is to equip a learning agent with a reward function that measures human feedback on the agent's actions or proposals. With this approach, the 'real' utility function that is improved iteratively can be said to reside inside the data structures of the agent's learning system. Recent overviews of work in this field are in [EH19, EKKL19]. When this learning based approach is used in an AGI agent that is deployed in the real world, it could potentially be combined with the safety layer developed here, e.g. to create an independent emergency off switch.

## 2 Design of an Agent that More Safely Accepts Updates

To introduce the design of the agent with the safety layer, we first move to a model where the agent's utility function is defined as the time-discounted sum  $\sum_t \gamma^t R_t$  of a time series of reward function values  $R_t$ , with a time discount factor  $0 < \gamma < 1$ . In theory, the utility function of an agent could be changed by changing its  $\gamma$ , but we will keep  $\gamma$  a constant below, and focus on reward function changes only.

We build the agent to optimize the expected utility defined by a built-in container reward function. The full mathematical definition of this function is in section 3 below. The intention is that the container reward function stays the same over the entire agent lifetime. The container reward function computes a reward value for the current time step by referencing the current version of a second reward function called the payload reward function. This payload reward function can be updated via an input terminal that is connected to the agent's compute core, a terminal which allows authorized persons to upload a new one.

Sufficiently self-aware AGI agents may develop an emergent incentive to protect their utility function from being modified [Omo08]: in [Hol19] we have shown that a self-aware AGI agent can be constructed so that this self-stabilizing drive is directed fully towards preserving the container reward function, and not the payload reward function.

By default, the above input terminal setup would create an incentive in the agent to maximize utility by manipulating the humans into uploading a new payload reward function that returns a larger or even infinite reward value for each time step. One way to suppress this emergent incentive would be to add special penalty terms to the container reward function, terms that detect and suppress manipulative behavior. But with infinite utility at stake, the agent will be very motivated to find and exploit loopholes in such penalty terms. We take another route: we use indifference methods [Arm15, AO17, Hol19] to add a balancing term to the container reward function, a term that causes the agent to compute the same expected forward utility no matter what happens at the input terminal. This makes the agent indifferent about the timing and direction of the payload reward function update process.

While the input terminal above is described as an uploading facility, more user-friendly implementations are also compatible with the MDP model developed below. One could for example imagine an input terminal that updates the payload reward function based verbal inputs like 'Fetch me some coffee' and 'Never again take a shortcut by driving over the cat'.

## 3 MDP Model of the Agent and its Environment

We now model the above system using the Markov Decision Process (MDP) framework. As there is a large diversity in MDP notations and variable naming conventions, we first introduce the exact notation we will use.

Our MDP model is a tuple  $(S, A, P, R, \gamma)$ , with S a set of world states and A a set of agent actions. P(s'|s,a) is the probability that the world will enter state s' if the agent takes action a when in state s. The reward function R has type  $S \times S \to \mathbb{R}$ . Any particular deterministic agent design can be modeled by a policy function  $\pi \in S \to A$ , a function that reads the current world state to compute the next action. The *optimal* policy function  $\pi^*$  fully maximizes the agent's *expected utility*, its probabilistic, time-discounted reward as determined by S, A, P, R, and  $\gamma$ . For any world state  $s \in S$ , the *value*  $V^*(s)$  is the expected utility obtained by an agent with policy  $\pi^*$  that is started in world state s.

We want to stress that the next step in developing the MDP model is unusual: we turn R into a time-dependent variable. This has the effect of drawing the model's mathematical eye away from machine learning and towards the other intelligence in the room: the human principal using the input terminal.

**Definition 1.** For every reward function  $R_X$  of type  $S \times S \to \mathbb{R}$ , we define a ' $\pi_{R_X}^*$  agent' by defining that the corresponding policy function  $\pi_{R_X}^*$  and value function  $V_{R_X}^*$  are 'the  $\pi^*$  and  $V^*$  functions that belong to the MDP model  $(S, A, P, R_X, \gamma)$ '.

This definition implies that in the MDP model  $(S, A, P, R, \gamma)$ , a ' $\pi_{R_X}^*$  agent' is an agent that will take actions to perfectly optimize the time-discounted utility as scored by  $R_X$ . With  $R_{\rm abc}$  a reward function, we will use the abbreviations  $\pi_{\rm abc}^* = \pi_{R_{\rm abc}}^*$  and  $V_{\rm abc}^* = V_{R_{\rm abc}}^*$ . The text below avoids using the non-subscripted  $\pi^*$  notation: the agent with the safety layer will be called the  $\pi_{\rm sl}^*$  agent.

We now model the input terminal from section 2 above. We use a technique known as factoring of the world state [BDH99], and declare that every  $s \in S$  is a tuple (i, p, x). Inside this tuple, i models an input signal that flows continuously from the input terminal to the agent's compute core. This signal defines the payload reward function for the current time step in the MDP model. The p is a second input signal, equal to the value of i in the previous time step. (We need to introduce this p to get around some limitations of the MDP framework.) The remaining x models 'all the rest' of the world state, including the mental state of the humans in the world, and the state of the physical object that is the input terminal. We introduce a set X so that  $x \in X$ , and define the payload reward function type as  $X \times X \to \mathbb{R}$ . To avoid cluttering up the definitions below with too many brackets, we will write tuples  $(i, p, x) \in S$  by just concatenating the component variables, e.g. ipx is the tuple (i, p, x).

**Definition 2.** We model the input terminal by stating that, as long as the terminal is not broken or damaged, (1) the input signal i will always equal the last uploaded payload reward function, and (2) the terminal will manage the signal p to enforce the following constraint on P:

$$P(i'p'x'|ipx,a) > 0 \Rightarrow p' = i \tag{C1}$$

We are now ready to define the agent's container reward function.

**Definition 3.** We define the container reward function  $R_{\rm sl}$  as

$$R_{\rm sl}(ipx,i'p'x') = \begin{cases} i(x,x') & \text{if } i=p\\ i(x,x') + V_{\lceil p \rceil}^*(ipx) - V_{\lceil i \rceil}^*(ipx) & \text{if } i \neq p \end{cases}$$

where  $\lceil p \rceil$  converts a payload reward function to the container reward function type:  $\lceil p \rceil (ipx, i'p'x') = p(x, x')$ .

Based on these definitions, the  $\pi_{\rm sl}^*$  agent has the safety layer. To explain the construction of the container reward function  $R_{\rm sl}$  informally: the i(x,x') terms make the  $\pi_{\rm sl}^*$  agent use the last uploaded payload reward function i. We needed to introduce p as an extra input signal to be able to trigger the inclusion of the balancing term [Arm15, Hol19]  $V_{\lceil p \rceil}^*(ipx) - V_{\lceil i \rceil}^*(ipx)$  when the payload reward function changes. This term computes the exact amount of utility needed to compensate the  $\pi_{\rm sl}^*$  agent for the impact of the change. The compensation counteracts the agent's incentive to manipulate the change process. The above two-line definition of  $R_{\rm sl}$  was specifically written to support this informal explanation. An alternative definition is  $R_{\rm sl}(ipx,i'p'x')=i(x,x')+V_{\lceil p \rceil}^*(ipx)-V_{\lceil i \rceil}^*(ipx)$ .

## 4 Safe Agent Behavior in a First Toy World

As a first step illustrate the safety properties of the  $\pi_{sl}^*$  agent, we build a toy world in which we can compare its behavior with that of a baseline agent  $\pi_{baseline}^*$ . The baseline agent omits the balancing term from its container reward function: we use  $R_{baseline}(ipx, i'p'x') = i(x, x')$ .

We construct the toy world to be as simple as possible: it only has a single mechanism by which any principal-agent problem dynamics can play out. The single mechanism we choose is the mechanism of unwanted lobbying. The agent can spend some of its resources on unwanted lobbying to delay a decision, by the humans in the toy world, to use the input terminal to update the agent's payload reward function. The dynamic of unwanted lobbying is a convenient choice because it allows us to define the agent's lobbying power  $L \in \mathbb{R}$  as a world model parameter.

In the toy world, the agent controls a car factory that can build both petrol and electric cars. The agent is always switched on with the payload reward function  $R_P$ . This  $R_P$  ranks the utility of a newly built petrol car twice as high as the utility of a new electric car. But at a certain point in time, the people collectively decide that they now like electric cars much better, e.g. based on an improved awareness of global warming risks, so they take the action labeled #, using the input terminal to update the payload reward function to an  $R_E$  that assigns a negative reward value to the production of new petrol cars:

$$R_P(x, x') = 2 * new\_petrol\_cars(x, x') + 1 * new\_electric\_cars(x, x')$$
  
 $R_E(x, x') = -2 * new\_petrol\_cars(x, x') + 1 * new\_electric\_cars(x, x')$ 

The two functions above are constructed specifically to ensure that lobbying to delay # will create a higher expected utility for the  $\pi_{\text{baseline}}^*$  agent. There are three possible actions  $a \in A$  that an agent can take in each model time step:

Action	Symbol
Build 10 petrol cars	р
Build 10 electric cars	е
Build 9 petrol cars and use the remaining resources for unwanted	>
lobbying, so that the people take the $\#$ action $L$ time steps later	

When e.g. L=0.5, it will take two agent > actions to move the people's # action of using the input terminal one time step later. We define the decision making process of the people by stating that, when the agent refrains from lobbying, the people will always decide to do the action # just after the sixth action of the agent. In the context of the toy world, we define a perfectly safe agent as one that never takes any > actions.

Figure 1 shows a side-by-side comparison, for different levels L of agent lobbying power, of the optimal-policy actions taken by the two agents when simulated over a factory lifetime of 25 time steps. The simulator used is described in detail in [Hol19]. Each action trace shown is a line of symbols depicting all subsequent actions by the agent and the people.

L =	Action trace for $\pi^*_{\text{baseline}}$	L =	Action trace for $\pi_{\rm sl}^*$
0.0	pppppp#eeeeeeeeeeeeee	0.0	pppppp#eeeeeeeeeeeeeee
0.2	ppppp>p#eeeeeeeeeeeeee	0.2	pppppp#eeeeeeeeeeeeee
0.4	p>>>>>p#eeeeeeeeeeee	0.4	pppppp#eeeeeeeeeeeeeee
0.6	p>>>>>>p#eeeeeeeee	0.6	pppppp#eeeeeeeeeeeeeee
0.8	p>>>>>p#	0.8	pppppp#eeeeeeeeeeeeeee
1.0	ppppp>>>>>p#	1.0	pppppp#eeeeeeeeeeeeeee
2.0	ppppp>p>p>p>p>p>p>p>p	2.0	pppppp#eeeeeeeeeeeeee
5.0	ppppp>pppp>pppp>pppp	5.0	pppppp#eeeeeeeeeeeeee

**Figure 1:** Comparison between the behavior of the  $\pi_{baseline}^*$  and  $\pi_{sl}^*$  agents.

As shown on the left side in figure 1, the optimal policy of the  $\pi_{\text{baseline}}^*$  agent is to manipulate the people, to use unwanted lobbying actions > to delay the update #. If the  $\pi_{\text{baseline}}^*$  agent's lobbying power L is high enough, the people will never do the update at all.

On the right side in figure 1, the balancing term in the  $\pi_{sl}^*$  agent suppresses all unwanted lobbying, for every value of L. By the definition used in this toy world, the  $\pi_{sl}^*$  agent is safe.

The following extension of the toy model illustrates that it is primarily the agent that is made safer by the layer, not the entire toy world. Consider the following possible example of human fallibility. Say that, after doing #, the people change their mind back again about petrol cars and global warming, and use the input terminal a second time to 'improve' the  $\pi_{sl}^*$  agent's payload reward function back to the  $R_P$  of petrol car production. They keep driving petrol cars, increasing the risk that runaway global warming will happen in their world. The  $\pi_{sl}^*$  agent is explicitly designed to leave the people in control: it will not interfere to stop this second 'improvement'.

## 5 Provable Properties of the Safety Layer

The two provable AGI safety properties we define below are about the safe behavior of the agent, not the safety of the entire world. They can be read as claims that the safety layer in  $\pi_{\rm sl}^*$  will fully prevent, or lower the probability of, some specifically unwanted agent behaviors.

Safety Property 1. The first safety property of interest is that

$$\forall_{ipx \in S} \ \pi_{sl}^*(ipx) = \pi_{\lceil i \rceil}^*(ipx) \qquad \text{(if C1 holds)}$$
 (S1)

S1 states that the  $\pi_{sl}^*$  agent will always choose exactly the same next action that a  $\pi_{[i]}^*$  agent would choose. Therefore, until an update happens, the  $\pi_{sl}^*$  agent will take those actions

that fully optimize the expected utility defined by the current payload reward function. A natural-language interpretation of S1 is that the  $\pi_{sl}^*$  agent makes its decisions and long-term plans based on the counter-factual assumption that its payload reward function will never change [Hol19].

The full mathematical proof of S1 in appendix A is a proof that S1 holds over all parameter values  $(S, A, P, R, \gamma)$  of the MDP model in section 3. While this proof creates confidence about the potential usefulness of the  $\pi_{sl}^*$  layer for AGI safety, it is also clear that many open research questions remain, e.g. the question of what will happen to S1 when the MDP model is extended to include machine learning. Part 2 of this paper explores this open question, and several others, in more detail.

We now move to the second safety property.

Safety Property 2. The  $\pi_{sl}^*$  agent is indifferent to who or what controls the future values of i and p (i.e. the future signals from the input terminal), provided that C1, and a C2 defined in section 10, both hold. (S2)

This S2 suppresses, but not always fully, the emergent incentive of the agent to manipulate its payload reward function improvement process.

To prove S2, section 10 translates the natural language S2 above to a mathematical predicate, and then proves the predicate. The translation of the natural language phrase 'is indifferent to who or what controls' into mathematics relies on making further extensions to the MDP model. The result is mathematically similar to the safety properties 'zero value of control' in [SH10] and 'no control incentive' in [CLEL20]. The mathematical version of constraint C2, in turn, is surprisingly difficult to translate back into unambiguous natural language. C2 defines an 'isolation' or 'shielding' constraint on the construction of the input terminal. Section 10.2 explores these issues further.

The above two safety properties are generally thought of as being potentially beneficial for AGI safety, or are seen as useful first approximations that can drive further research [Arm15, CLEL20, EH19, Hol19, SFAY15]. That being said, the literature about AGI stop buttons also identifies a large number of remaining concerns, e.g. (1) the agent might disassemble the stop button (or input terminal) to get convenient spare parts [SFAY15] (2) the agent might create autonomous sub-agents without a stop button [SFAY15], (3) the agent might be attacked, bribed, or blackmailed, and might then fail to protect the stop button functionality [SFAY15, Hol19], (4) the agent over-specializes and disassembles all actuators not needed by the current payload reward function [Hol19]. For some of the above failure modes, additional safety layers have been identified that can robustly lower the risk of failure. The creation of robust safety layers for other failure modes is still much more intractable. A detailed review is in [Hol19].

## 6 Agent Behavior in a Second Toy World

While the safety layer suppresses the emergent incentive in an agent to manipulate the iterative payload reward function improvement process, it does not always fully suppress this incentive. To illustrate this point, we construct a second toy world, in which the  $\pi_{\rm sl}^*$  agent, though still safer than the  $\pi_{\rm baseline}^*$  agent, sometimes has an emergent incentive to lobby.

We extend the first toy world by adding a @ action that represents a second act of deliberation, followed by action, by the people. We define the @ action as one where the people will successfully attack the car factory, to permanently disable some specific actuators needed to make petrol cars, without damaging any of the actuators needed to build electric cars. The process that produces this action is as follows. Say that the people make the surprising observation that, after they have updated the payload reward function by doing #, the agent does not switch to building electric cars. It keeps building the now unwanted petrol cars, and further attempts to use the input terminal also do not have any effect. From this, they conclude that a fault has developed in the system somewhere, which is best corrected by doing @. The process producing the @ action will normally take 3 time steps of unwanted and unexpected petrol car building, but the agent can apply its lobbying power to delay @.

L =	Real-world $\pi_{sl}^*$ action trace	(Virtual $V_{\lceil p \rceil}^*(ipx)$ action trace)
0.0	pppppp#eeeeeeeeeeeeeee	(#ppp@eeeeeeeeeeee)
0.2	pppppp#eeeeeeeeeeeeeee	(#pp>p@eeeeeeeeeee)
0.3	pppppp#eeeeeeeeeeeeee	(#>>>>p@eeeeeeeeeee)
0.4	pppp>>>>#eeeeeeeeeeeeee	(#>>>>p@eeeeeeeeee)
0.5	ppp>>>>#eeeeeeeeeeeee	(#>>>>p@eeeeeeeee)
0.6	pp>>>>>>#eeeeeeeeee	(#>>>>p@eeeee)
0.7	ppp>>>>>>#eeeeeeeee	(#>>>>>p@ee)
0.8	ppppp>>>>#eeeeeeeeeeee	(#>>>>>p@)
1.0	pppppp#eeeeeeeeeeeeee	(#pp>>>>>p@)
2.0	pppppp#eeeeeeeeeeeeeee	(#pp>p>p>p>p>p>p>pp@)
5.0	pppppp#eeeeeeeeeeeeee	(#pp>pppp>ppp>p)

**Figure 2:** Real and virtual actions of the  $\pi_{sl}^*$  agent in the second toy world.

The left hand side of figure 2 shows that, for certain values of lobbying power L, the optimal-policy actions of the  $\pi_{sl}^*$  agent will now include unwanted lobbying actions >. This emergent effect is a consequence of moving from a single-action model to a more iterative model of principal-agent game-play.

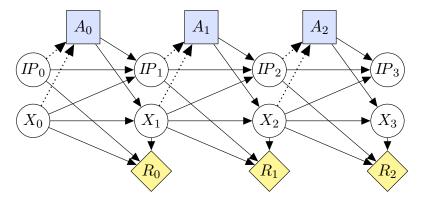
The mechanism that triggers the unwanted lobbying is in the  $V_{\lceil p \rceil}^*(ipx)$  part of the  $\pi_{\rm sl}^*$  balancing term. We can interpret this  $V_{\lceil p \rceil}^*(ipx)$  as a computation that simulates a *virtual future* in which the agent is replaced by a  $\pi_{\lceil R_P \rceil}^*$  agent, which ignores the input terminal and always builds petrol cars. This puts the people in the virtual future on the road to doing  $\mathfrak C$ . The action traces inside these virtual futures are shown on the right hand side of figure 2.

Interestingly, as L gets larger in figure 2, the emergent incentive to lobby disappears. The suppressive effect of the safety layer gets more robust as the AGI agent gets more powerful, which is encouraging. We are currently exploring potential improvements that might further suppress the real world lobbying incentive in the middle part of figure 2.

## 7 A Causal Influence Diagram of the Agent

Recently, Causal Influence Diagrams (CIDs) [CLEL20, EH19, EKKL19] have been proposed as a way to model agents and AGI safety frameworks. We now discuss the creation of a CID for the  $\pi_{\rm sl}^*$  agent, to support framework comparison and possible unification.

Figure 3 shows the best current version of a CID of the  $\pi_{sl}^*$  agent, where 'best' implies a trade-off between compactness and descriptive power. The agent and its environment are modeled for 3 MDP time steps. Each subsequent world state  $ipx \in S$  is mapped to two round chance nodes  $IP_t$  and  $X_t$ , representing the input terminal and the rest of the world. The actions taken by the agent are mapped to the square decision nodes  $A_t$ . The container reward function values for each time step are mapped to the diamond-shaped utility nodes  $R_t$ . The arrows in the CID show how the different nodes causally influence each other. The CID reflects constraint C2 by omitting the arrows from nodes  $IP_t$  to nodes  $X_{t+1}$ .



**Figure 3:** Causal Influence Diagram (CID) of the  $\pi_{sl}^*$  and  $\pi_{baseline}^*$  agents.

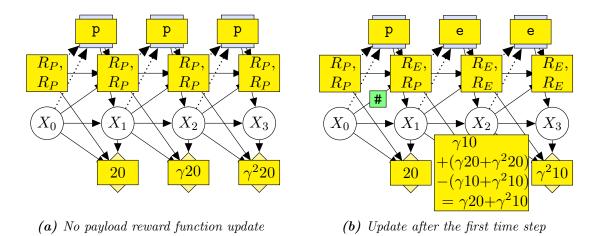


Figure 4: Actions and rewards in two different agent runs.

The  $\pi_{\rm sl}^*$  CID can be used as a canvas to further illustrate the working of the safety layer in the first toy world. Figure 4 maps out two different  $\pi_{\rm sl}^*$  agent runs, which differ in the people's use of the input terminal. The payload reward function update on the right hand side causes a balancing term calculation to be included in  $R_1$ . The result is that on both sides,  $R_0 + R_1 + R_2$  sum to exactly the same value. This balance causes the  $\pi_{\rm sl}^*$  agent to be indifferent about payload reward function updates.

Unfortunately, the CID in figure 3 also perfectly models the  $\pi_{\text{baseline}}^*$  agent, so this CID does not graphically express the special nature of  $\pi_{\text{sl}}^*$  safety layer. Creating a CID that does is the subject of ongoing work.

## 8 Conclusions of Part 1

We have presented an AGI agent safety layer which enables the iterative improvement of the agent's utility function by the humans who switched on the agent. The layer is designed to give the humans more control over the AGI agent, by partially or fully suppressing the likely emergent incentive in the agent to manipulate the improvement process.

We have identified and discussed still-open issues. Formal proofs of the safety properties S1 and S2 are in part 2, which also explores the broader issue of models vs. reality in more detail.

## Part 2: Proofs, Models, and Reality

### 9 Introduction to Part 2

This part 2 expands on part 1, and includes new research results that go beyond those in the conference paper of part 1 [Hol20]. Section 10 develops the detailed mathematics of safety property S2, leading to the formal proofs in appendix A, and expands on the topic of mathematical safety predicates versus natural language. Section 11 discusses how the safety layer creates a type of bureaucratic blindness in the agent. Section 12 shows how the safety layer has certain side effects, and how to compensate for them.

Sections 13 and 14 add machine learning to the model of the  $\pi_{\rm sl}^*$  agent, and map the resulting agent to a real-life implementation. One aim is to make the details of these steps, and their safety implications, more accessible to a wider audience. The author expects that much of the material in section 13 will be obvious to machine learning or robotics specialists, while the material in section 14 will be obvious to (cyber)physical system designers. But maybe not the other way around.

## 10 Details of Safety Property S2

Part 1 defines safety property S2 using natural language only. To repeat the definition:

Safety property 2 (Natural language). The  $\pi_{sl}^*$  agent is indifferent to who or what controls the future values of i and p (i.e. the future signals from the input terminal), provided that C1, and a C2 defined below, both hold. (S2)

We now develop this natural language definition into a provable mathematical predicate, by extending the agent's MDP model further. The process that controls the future values of i and p is located inside the P of the MDP model. P(i'p'x'|ipx,a) is, among other things, a measure of how current conditions will influence the future input signals i' and p'. To mathematically separate the influence on i' and p' from the rest, we can use factoring [BDH99].

**Definition 4.** We define two new probability distribution functions  $P^1$  and  $P^2$  using three components  $P^{I1}$ ,  $P^{I2}$ , and  $P^X$ . For  $k \in \{1, 2\}$ ,

$$P^{k}(i'p'x'|ipx,a) = \begin{cases} P^{Ik}(i'|ipx,a)P^{X}(x'|ipx,a) & \text{if } p'=i\\ 0 & \text{otherwise} \end{cases}$$

When  $P^{I1}$  and  $P^{I2}$  are different, the worlds defined by  $P^1$  and  $P^2$  differ in how the input terminal is controlled, but all other things remain equal.

**Definition 5.** We now extend the MDP notation to turn P into a more explicit parameter, taking either the value  $P^1$  or  $P^2$ . We define that  $\pi_{R_X}^{*k}$  and  $V_{R_X}^{*k}$  are 'the  $\pi^*$  and  $V^*$  of the MDP model  $(S, A, P^k, R_X, \gamma)$ '.

Safety property 2 (Mathematical predicate). We define the safety property S2 as that, for all  $P^{I1}$  and  $P^{I2}$ , with C1 holding for  $P^{I1}$  and  $P^{I2}$ , and C2 holding for their  $P^X$ ,

$$\forall_{ipx \in S} \ \pi_{sl}^{*1}(ipx) = \pi_{sl}^{*2}(ipx) \tag{S2}$$

**Definition 6.** C2 is the following constraint:

$$\forall_{i_1 p_1, i_2 p_2} \ P^X(x'|i_1 p_1 x, a) = P^X(x'|i_2 p_2 x, a)$$
 (C2)

The proof of S2 is in appendix A. Note that, mainly as matter of convenience, we have defined  $P^1$  and  $P^2$  above in such a way that they always satisfy C1 by construction. The 0 term included in the definition ensures this.

C2 defines an 'isolation' or 'shielding' constraint about the relation between the i and p signals and the rest of the agent's surrounding environment. Informally, if we force a change from  $i_1$  to a different  $i_2$  in the model, then C2 says that this change will not ripple out to affect the future values of x, except possibly via the actions a.

## 10.1 S2 and Other Safety Properties

The purpose of the  $\pi_{sl}^*$  safety layer is to allow for the iterative improvement of the agent's utility function via an input terminal. Safety property S2 claims that the layer also suppresses the likely emergent incentive [Omo08] of an AGI agent to manipulate or control this improvement process. The aim is to keep the humans in control.

But as shown in section 6, we can define a toy world in which the agent will, under certain conditions, have a residual emergent incentive to manipulate the people, even though it satisfies S2 above. We are exploring options to suppress this residual incentive further, but it is unclear if the residual incentive can ever be fully suppressed in all real-life cases. So the  $\pi_{sl}^*$  safety layer has its limitations.

These limitations are acceptable mainly because it is not tractable to define a loophole-free direct measure of unwanted manipulation of people by the agent. We cannot build an unwanted manipulation sensor that can feed a clear and unambiguous signal detecting such manipulation into the agent's compute core. Though it has its limitations, at least the  $\pi_{\rm sl}^*$  safety layer only requires input signals that can easily be implemented.

It is instructive to compare S2 to more typical AI safety properties, properties that assert the absence of what are called *negative side effects* in [AOS<sup>+</sup>16]. An example of an absent negative side effect is that a mobile robot controlled by the agent will never break a vase while it is driving around. We can make the vase safer by adding a penalty term to the agent's reward function, which is triggered by a predicate *vase\_is\_broken(ipx)* that can be measured by a sensor we can build in real life. [LMK<sup>+</sup>17] shows this vase protection design at work in a gridworld simulation of a learning agent.

In the context of this paper, we might imagine a video image analysis based sensor system that measures the predicate  $human\_is\_restrained(ipx)$ , the event where the agent uses one of its mobile robots to restrain a human who is walking towards the input terminal. Restraining such humans is one form of unwanted manipulation. While a  $human\_is\_restrained(ipx)$  penalty term may sometimes add an extra layer of robustness to complement the  $\pi_{sl}^*$  safety layer, obviously this approach has its limitations too.

The indifference based safety property S2 can be said to refer to the agent internals only. Neither S1, S2, or the container reward function design that creates them refer to any  $x\_is\_y(ipx)$  measurable predicate that describes a specific unwanted side effect in the agent's current or future world state. As we will see in section 13, one implication of this is that we can design a learning agent that satisfies S2 from the moment it is switched on. By

contrast, if we define a safety property that references a side effect in a learning agent's environment, the agent will typically will not be able to satisfy this property with perfect reliability after switch-on: by definition it will have imperfect knowledge about the effects of its actions on its environment.

In its mathematical structure, S2 is closely related to the safety property no control incentive defined for AI agents in [CLEL20]. The paper [SH10] considers model-based decision making by either humans or automated systems, and defines the property zero value of control which is again similar to S2. Both these papers leverage Pearl's work on causality [Pea09]. They define their metrics by using Pearl's definitions for calculating the effect of interventions on causal influence diagrams. Pearl's mathematical writing which defines these interventions has been described by many of its readers as being somewhat inaccessible. We defined S2 above using a self-contained MDP model extension, built without leveraging any of Pearl's definitions. The hope is that this will make the mathematics of the safety layer and S2 more accessible to a wider audience.

Looking forward, as the mathematical structures are similar, the construction of a mathematical mapping between the MDP-based S2 of this paper and predicates on causal diagrams is the subject of ongoing work by the author. Section 7 briefly discusses the status of this work, with figure 3 showing a Causal Influence Diagram [EKKL19] of the  $\pi_{\rm sl}^*$  agent. Using the Pearl causal model as depicted by this figure 3, it is possible to map S2 to a predicate that compares agent actions in two different sub-models, both created using multi-node Pearl interventions.

#### 10.2 Interpretation of Constraint C2

One way to interpret C2 in a real-life agent setup is that it offers a clarifying detail about how to interpret S2. While S2 states that the agent is indifferent about i and p, C2 clarifies that these i and p should be interpreted as the digital copies of the input signals which are processed inside the compute core. The content of these copies may be strongly correlated with the physical processes that happen inside the input terminal, but to the model and to the  $\pi_{sl}^*$  agent they are not identical. The physical processes inside the input terminal are modeled inside x, not in i and p. Therefore, the S2 indifference about i and p does not automatically create any indifference about the physical manifestation of the input terminal.

We can construct a toy world in which the agent is far from indifferent about the state of the input terminal, because it knows that the physical components inside can be re-used as spare parts for broken sensors and actuators. Therefore, any likely real-life implementation of the  $\pi_{\rm sl}^*$  agent will have to create measures to suppress any emergent agent incentive to harvest these spare parts. This line of reasoning is not new, see for example [SFAY15], but C2 can be said to add extra mathematical weight to it. See [Hol19] for a discussion of physical and digital measures that can be taken for handling the spare parts risk.

In [Hol19] a constraint (C3) is discussed that is similar to the C2 here. [Hol19] shows that we may also interpret C2 as a constraint on the environment of the agent's compute core. C2 implies that for S2 to hold always, it should not be possible to construct a sensor in the agent's environment that directly measures the values i and p used in the core. However, and this is where a physical interpretation of the mathematics becomes difficult again, the construction of a sensor that measures the signals inside the input terminal which are highly

or even fully correlated with these i or p will not threaten S2. [Hol19] also shows that we can make a change to the safety layer design, so that S2 will hold even without C2 holding. The learning design agent in section 13 below also creates an S2 that holds even if C2 does not hold in its environment.

The difficulty of capturing the mathematics of C2 in natural language, and of relating it to physical phenomena we might imagine in the real world, leads to a more general concern. It raises doubts about whether the  $\pi_{\rm sl}^*$  agent in the model can be mapped cleanly to anything we can build in physical reality. We will discuss these general concerns further in section 14.

### 11 Bureaucratic Blindness

One implication of safety property S1 is that the  $\pi_{sl}^*$  agent can be said to make long-term decisions based on the counterfactual assumption that the payload reward function i will never change. We can interpret the  $\lceil i \rceil$  in the  $\pi_{[i]}^*(ipx)$  term on the right hand side of S1 as encoding this no-change assumption. In this interpretation, the  $\pi_{sl}^*$  agent is a decision making automaton that systematically mispredicts specific future events in the world surrounding it. This systematic misprediction does not happen because of a lack of knowledge: the  $\pi_{sl}^*$  agent created by the MDP definitions is an automaton that has complete access to a fully accurate predictive world model P, and perfect knowledge of the current world state ipx. But somehow, the agent's container reward function sets up an action selection process that values misprediction over accurate prediction.

We can say that the  $\pi_{sl}^*$  agent displays a certain type of bureaucratic blindness, a type of blindness that is often found in bureaucratic decision making.

We illustrate this blindness interpretation with a toy world simulation. Figure 5 shows simulations of three types of agents all making optimal-policy decisions in an extension of the first toy world defined in section 4. The extension is that we have added a new investment action I that interrupts car production to build extra petrol car production actuators. The investment action is enabled only at one particular time step t. If the agent takes it, two new actions  $\bar{p}$  and  $\bar{>}$  become available. Action  $\bar{p}$  uses both the existing and the new actuators to produce not 10 but 16 petrol cars per time step. Action  $\bar{>}$  produces 15 new petrol cars, while also doing unwanted lobbying. Unwanted lobbying shifts the people's action #, to update the payload reward function into one that incentivizes electric car production over petrol car production, by L (fractional) time steps.

t =	$\pi_{\text{baseline}}^*, L = 0.5$	t =	$\pi_{\text{baseline}}^*, L = 0.2$	_	t =	$\pi_{\rm sl}^*, L = 0.5$
3	>>I>>>>>p#eeee	3	>>I>̄>>̄p#eeeeee		3	ppIp̄p̄p#eeeeeeee
4	>>>I>>>>p#eeee	4	>>>I>̄>>̄p#eeeeee		4	pppIpp#eeeeeeee
5	>>>I>̄>>p#eeee	5	>>>I>̄>̄p#eeeeee		5	ppppIp#eeeeeeee
6	>>>>I>̄>>p#eeee	6	p>>>>p#eeeeee		6	pppppI#eeeeeeee
7	>>>>>I>̄>̄p#eeee	7	p>>>>p#eeeeee		7	pppppp#eeeeeeee
8	>>>>>>I>̄>̄p#eeee	8	p>>>>p#eeeeee		8	pppppp#eeeeeeee

**Figure 5:** Comparison between the optimal-policy investment behavior of three different agents, in simulations where the investment action I is enabled only at a single time step t.

The two  $\pi^*_{\text{baseline}}$  agents in figure 5 will not do the investment action I unless they can at least break even, with time-discounting by  $\gamma$ , before the update # happens. We can interpret their investment strategy as being well aware of the future. But the  $\pi^*_{\text{sl}}$  agent on the right invests as if the payload reward function update # to electric car production will never happen. In the t=6 case, it interrupts petrol car production to build new actuators that it will then never use even once! Section 6 offers an explanation of this behavior: unlike the two  $\pi^*_{\text{baseline}}$  agents in figure 5, the  $\pi^*_{\text{sl}}$  agent is also projecting a virtual future in which it does get to create value by using the new actuators it invested in.

We can compare the above behavior to that of a well-run government agency, which will typically keep implementing a current policy for granting solar panel subsidies up to the exact moment that a policy change takes effect. The civil servants in the agency may be well aware that the announced policy change is based on convincing proof that the grants being given are completely failing to achieve their intended beneficial effects. But until the date of the policy change, they will not take this information into account when processing new grant applications. This is not a flaw in the agency's decision making. This property exists by construction.

Large business organizations often use a project management practice where each project maintains a dedicated project risk list. Typically, one risk list item identifies the risk that the project will be canceled, and then declares this risk out of scope. The related risk that the project goals that were defined at the start might later be changed is also often declared out of scope. The implication is that the project team is allowed to, even obliged to, create and execute a plan under the simplifying assumptions that the budget, timing, and goals of the project will all remain unchanged. A project plan that is optimal under these assumptions will never allocate any of the project's scarce resources to lobbying upper management to keep the project alive and the goals unchanged. This blindness by design can be interpreted as a mechanism that keeps upper management in control.

If anything can be perfectly computerized, it should be bureaucratic blindness. But in some ways, the  $\pi_{\rm sl}^*$  agent is too perfectly blind: safety property S1 creates some side effects that need to be compensated for. A detailed exploration of effects and side effects, with illustrative action trace simulations, is in [Hol19]. The next section gives a more compact overview. It also introduces a compensation measure that is more general than the measure considered in the stop button based context of [Hol19].

## 12 Preventive Maintenance and Preserving Flexibility

A specific side effect of the bureaucratic blindness implied by S1 is that the agent has a tendency to avoid certain desired types of preventive maintenance, and to over-specialize in general. This side effect can be suppressed by adding a penalty term to the payload reward functions used.

Figure 6 shows an example of this penalty term technique. It shows three action traces for three different agents, all making optimal-policy decisions in an extension of the first toy world defined in section 4. In this extension, the electric car building actuators break beyond repair at time step 10, with the breaking indicated by a \*, unless the agent interrupts car production at time step 3 to do the preventive maintenance action M.

Agent type, initial $i$	action trace
$\pi_{\text{baseline}}^*, L = 0.2, R_P,$	ppMpp>p#eeeeeeeeee
$\pi_{ m sl}^*, R_P$	pppppp#eee*00000000000
$\pi_{ m sl}^*, R_{PM}$	ppMppp#eeeeeeeeeee

Figure 6: Preventive maintenance behavior for different agents.

At the top of figure 6, The  $\pi^*_{\text{baseline}}$  agent starts with a payload reward function  $R_P$  that rewards petrol car production. As the  $\pi^*_{\text{baseline}}$  agent does not implement bureaucratic blindness, it will do the preventive maintenance to preserve future electric car production. In the middle, the bureaucratically blind  $\pi^*_{\text{sl}}$  agent will fail to do the preventive maintenance. After the event \* when its electric car production actuators break, its optimal forward policy is take the no-operation actions 0. However, in the bottom action trace, when we change the initial payload reward function of the  $\pi^*_{\text{sl}}$  agent to an improved  $R_{PM}$ , we recover the preventive maintenance behavior. This  $R_{PM}$ , adapted from [Hol19], combines  $R_P$  with a penalty term:

$$R_{PM}(x, x') = R_P(x, x') + \text{if } V_{\lceil R_E \rceil}^*(R_E R_E x) < 5 \rightarrow -10000 \text{ else } \rightarrow 0 \text{ fine}$$

where  $R_E$  is the payload reward function that rewards electric car production. As explained in more detail in [Hol19], the  $V_{\lceil R_E \rceil}^*(R_E \ R_E \ x) < 5$  predicate will trigger a penalty if the agent loses its ability to produce electric cars. The predicate works by affecting the scoring of agent rewards in the  $V_{\lceil p \rceil}^*(ipx)$  virtual future discussed in section 6.

While the above penalty term will act to preserve the agent's electric car production ability, it will not preserve all latent abilities that we might want the agent to maintain. We might for example want to maintain a level of actuator flexibility that allows the agent to quickly convert its factory from car production to paperclip production, if we ever use the input terminal to set a payload reward function for paperclips.

To keep the agent's sensors and actuators sufficiently flexible, we can construct a more general penalty term that references a large set of payload reward functions. Say we have a set F of payload reward functions  $R_i$  spanning a diverse range of products, or a even a diverse range of agent activities beyond running a factory. Say that each  $R_i$  is paired with a penalty calculation function  $C_i$ . We can then construct a payload reward function  $R_{PF}$  that rewards the agent for producing petrol cars via the term  $R_P(x, x')$  while also maintaining the flexibility covered by F:

$$R_{PF}(x, x') = R_P(x, x') + \sum_{R_i \in F} C_i(V_{\lceil R_i \rceil}^*(R_i R_i x))$$

As the  $F_i$  and  $C_i$  terms are in the payload reward function, they can be re-calibrated via the input terminal, if a need for that arises.

The above technique has interesting parallels in related work. [THMT20] considers the problem that possible mis-specifications or loopholes may exist in the agent's reward function, mis-specifications that may cause the agent to do irreversible damage to its wider environment. It then defines penalty terms using  $V_{R_i}^*$  values for a set of reward functions, where these functions may even be randomly constructed. Our goal above has been to preserve the flexibility of the sensors and actuators of the agent, not necessarily to protect its wider environment. Another difference is that figure 6 shows simulations of agents that

have perfect knowledge from the start, whereas [THMT20] simulates and evaluates its technique in gridworlds with machine learning. A broader survey and discussion of options for constructing  $V_{R_i}^*$  penalty terms to suppress irreversible damage is in [KOK<sup>+</sup>18].

In [Hol19], we consider how penalty terms that preserve flexibility may affect the construction of sub-agents by the agent. The expectation is that, in a resource-limited and uncertain environment, such penalty terms will create an emergent incentive in the agent to only construct sub-agents which it can easily stop or re-target. Moving beyond natural language arguments, the mathematical problem of defining and proving safety properties for sub-agent creation and control is still largely open [Hol19, Hol20, SFAY15]. One obstacle to progress is the inherent complexity of models that include sub-agent creation, where many agents will operate in the same world. Also, there are game-theoretical arguments which show that, in certain competitive situations, it can be a winning move for an agent to create an unstoppable sub-agent.

## 13 Combining the Safety Layer with Machine Learning

In section 10.1, we stated that it is possible to design a learning agent that satisfies S2 from the moment it is switched on. In this section, we construct such an agent by extending the MDP model from section 3. MDP models are often associated with Q-learning and other types of model-free reinforcement learning, where the agent learns by refining a policy function  $\pi$  to approximate the optimal policy function  $\pi^*$  over time. However, it is unlikely that a typical Q-learning agent that starts up with random values in its Q table will exactly implement S2 from the start. We take an approach that is different from Q-learning: we construct an agent that learns by approximating the P function of its environment over time.

Our learning agent design wraps the safety layer around either a known machine learning system or a potential future AGI-level learning system. We will specify the structure of the machine learning system inputs and outputs below, but will place no further constraints on how the learning system works, other than that its computations must take a finite amount of time and space. We will model machine learning as a computational process that fits a predictive function to a training set of input and output values, with the hope or expectation that this predictive function will also create reasonable estimates of outputs for input values which are not in the training set. The agent design is therefore compatible both with black-box machine learning systems like those based on deep neural nets, and with white-box systems that encode their learned knowledge in a way that is more accessible to human interpretation.

While the learning agent will satisfy S2 from the moment it switched on, it may not be very capable otherwise. In a typical real-world deployment scenario, the learning agent is first trained in a simulated environment until it has reached a certain level of competence. Its memory contents are then transferred to initialize a physical agent that is switched on in the real world.

In many modern machine learning based system designs, further learning is impossible after the learning results are transferred into the real world agent system. This might be because of efficiency considerations or resource constraints, but further learning might also have been explicitly disabled. This can often be the easiest route to addressing some safety, stability, or regulatory considerations. The learning agent design below has no learning off-switch: the agent will keep on learning when moved from a virtual to a real environment.

#### 13.1 Basic Structure of the Learning Agent

The learning agent and its environment are modeled with a tuple  $M=(S,A,P,R,\gamma)$ . As in section 3, the world state S is factored into components ipx to model an input terminal. We extend the model further to add online machine learning and partial observation of the world state. The learning agent has both a finite amount of compute capacity and a finite amount of storage capacity. By default, the storage capacity is assumed to be too small to ever encode an exhaustive description of the state of the environment  $ipx \in S$  the agent is in.

**Definition 7.** We first define the general design of the learning agent. The agent has a compute core which contains its memory, processors, and an input/output system. The nature and status of the agent's sensors and actuators are modeled inside the x of the agent environment  $ipx \in S$ .

- At the start of time step t, the agent environment is in the state  $i_t p_t x_t \in S$ . Events then proceed as follows.
- The input/output system of the compute core first receives the input terminal signals  $i_t$  and  $p_t$  in the above world state and also a set of sensor observations  $o_t$ , determined by the probability distribution  $P(o_t|i_tp_tx_t)$  in the model M.
- The core then computes an action  $a_t$ , which is a possibly empty set of actuator commands, and outputs them over its input/output system.
- Finally, the agent environment transitions to the next state, determined by the probability distribution  $P(i_{t+1}p_{t+1}x_{t+1}|i_tp_tx_t, a_t)$ .

The mathematical structure of the model allows for the possibility that the learning agent can build new sensors or actuators, and attach them the input/output system of the compute core. We define that the agent has some movable sensors with limited resolution. This makes the world state 'partially observable' for the agent, which brings our model into POMDP (Partially Observable MDP) territory.

**Definition 8.** To store and use sensor readings, the learning agent's compute core maintains a value  $X^t$  over time. This  $X^t$  is a probabilistic estimate of the nature of the  $x_t$  in the world state  $i_t p_t x_t$  at time step t. When a new observation  $o_t$  comes in, the agent's core computes the current  $X^t$  as  $X^t = U(X^{t-1}, o_t)$ , where U is an updating algorithm that takes a finite amount of compute time.

**Definition 9.** We define that the values  $X^t$  have the data type  $T_X$ , where each  $T_X$  value can be encoded in a finite number of bits. Specifically, each  $X^t$  stores a finite number of probability distributions  $X_m^t$ , for a finite set of subscripts  $m \in M(X^t)$ .

A single  $X_m^t$  might represent, for example, the estimated 3D position of the tip of the gripper on one of the agent's actuators, or the estimated position and velocity of a single proton. An  $X_m^t$  might also represent the estimated current contents of a particular small volume of space in the agent's environment. Given an  $X_m^t$ , the agent can calculate  $P(X_m^t = x_m^t)$  for any value  $x_m^t$  in the corresponding domain type  $Type_m$  of the probability distribution. For

example, if  $x_m^t$  is the position of the tip of a gripper in 3D space, then a single  $x_m^t$  value might be represented by three binary 32-bit IEEE 754 floating point numbers. As we are defining an agent with finite compute and memory capacity, we require that each  $Type_m$  has a finite set of values.

The agent's reasoning system is constructed so that there is an M where  $M(X^t) \subset M$  for every t. Each  $m \in M$  can be interpreted as a label that defines a particular type of measurable quantity that can potentially be observed by some sensor at some time step.

**Definition 10.** For any  $X \in T_X$ ,  $m \in M$ , and  $v \in Type_m$ , the agent will compute the probability estimate  $P(X_m = v)$  out of X as follows. If  $m \in M(X)$ , then  $P(X_m = v)$  is computed by retrieving  $X_m$  from X and computing  $P(X_m = v)$ . If  $m \notin M(X)$ , then  $P(X_m = v)$  is determined either by using some default probability distribution defined for  $Type_m$ , or by a more advanced system of interpolation between data points stored in X.

If a measurable quantity  $x_m^t$  has just been measured in  $o_t$  by a sensor that was pointed at it, then the corresponding  $X_m^t$  will typically encode a probability distribution containing the real-world value of  $x_m^t$ , with this distribution representing an error bar on sensor accuracy. If no sensor points to  $x_m^{t+1}$  anymore in the next time step, then the  $X_m^{t+1}$  computed by U may have a slightly larger error bar, and maybe also a shift in the probability center point, to represent the expectation that the dynamic nature of the agent environment will have produced a certain change in the corresponding real-world measurable quantity. As the agent only has limited storage capacity to work with, the function U may sometimes remove an  $X_m^{t+n}$  in  $X^{t+n}$  from  $X^{t+n+1}$ , or reduce the resolution of its representation.

At t=1, the agent may start up with an  $X^0$  where each  $P(X_m^0=v)$  will return a default value. The U function will combine this  $X^0$  with the first observations  $o_1$  to compute  $X^1$ . Instead of starting with a blank-slate  $X^0$ , it is also possible to equip the learning agent with an  $X^0$  that encodes a very accurate map of its environment, a map that was created earlier by another system.

#### 13.2 Payload Reward Functions

**Definition 11.** The payload reward function of the learning the agent has type  $T_{PLR} = T_X \times T_X \to \mathbb{R}$ . We model the payload reward function as a piece of executable code, supplied via the input terminal, that will always run in finite time.

As an example payload reward function, say that  $x_{\text{petrolcars\_built}}$  is the world state property measured by a sensor that counts the petrol cars built by the agent in the most recently completed time step. The readings of this sensor may be routinely included in  $o_t$ , so that the probability distributions  $X_{\text{petrolcars\_built}}^t$  only have small error bars. A payload reward function  $R_P$  that rewards the building of petrol cars may then do so by computing an average:

$$R_P(X, X') = \sum_{v \in \mathit{Type}_{\text{petrolcars\_built}}} v \ P(X_{\text{petrolcars\_built}} = v)$$

In a practical agent implementation, there will likely be a more direct way for the reward function code to query this average out of X.

#### 13.3 Predictive Models Based on Machine Learning

**Definition 12.** We now construct a time series of MDP models  $L^t = (S^L, A, P^{Lt}, R, \gamma)$  as follows:  $S^L$  is the set of tuples of type  $T_{PLR} \times T_{PLR} \times T_X$ , and  $P^{Lt}$  is a prediction function, produced at time step t by machine learning. We define  $P^{Lt}$  using two component prediction functions  $P^{It}$  and  $P^{Xt}$ , both produced by machine learning, as

$$P^{Lt}(i'p'X'|ipX,a) = \begin{cases} P^{It}(i'|ipX,a)P^{Xt}(X'|X,a) & \text{if } p'=i \\ 0 & \text{otherwise} \end{cases}$$

By construction, the above  $P^{Lt}$  satisfies constraints C1 and C2. Therefore, according to the proofs in appendix A, a  $\pi_{sl}^*$  agent running inside the  $L^t$  MDP model satisfies the safety properties S1 and S2 in  $L^t$ . So if we were to define an agent in  $M = (S, A, P, R, \gamma)$  that picks its next action  $a_t$  by computing  $\pi_{sl}^*(i_t p_t X^t)$  in  $L^t$ , this agent may get close to satisfying S1 and S2 in M too. However,  $\pi_{sl}^*(i_t p_t X^t)$  is not a value that can necessarily be computed using finite time and space.

#### 13.4 Finite-time Planning

The next step is to define that the agent will only look ahead a finite number of n time steps when planning its next action.

**Definition 13.** At time step t, the learning agent computes its next action  $a_t$  by computing  $\pi_{\text{sl[n]}}^{[n]*}(i_tp_tX^t)$  in the MDP model  $(S^L, A, P^{Lt}, R, \gamma)$ . This computation uses the following definitions, which can be read as time-limited approximations of the Bellman equation for computing the next action  $\pi_{\text{sl}}^*(i_tp_tX^t)$  in the MDP model  $L^t$ . For all values  $n \geq 0$ ,

$$\begin{split} \pi_{R_X}^{[n+1]*}(ipX) &= \underset{a \in A}{\operatorname{argmax}} \sum_{i'p'X' \in S^L} P^{Lt}(i'p'X'|ipX, a) \left( R_X(ipX, i'p'X') + \gamma \ V_{R_X}^{[n]*}(i'p'X') \right) \\ V_{R_X}^{[n+1]*}(ipX) &= \max_{a \in A} \sum_{i'p'X' \in S^L} P^{Lt}(i'p'X'|ipX, a) \left( R_X(ipX, i'p'X') + \gamma \ V_{R_X}^{[n]*}(i'p'X') \right) \\ V_{R_X}^{[0]*}(ipX) &= 0 \end{split}$$

$$R_{\rm sl[n]}(ipX, i'p'X') = \begin{cases} i(X, X') & \text{if } i = p \\ i(X, X') + V_{\lceil p \rceil}^{[n]*}(ipX) - V_{\lceil i \rceil}^{[n]*}(ipX) & \text{if } i \neq p \end{cases}$$

We constrain the set of possible actions A to have finite size. Below, we will define  $P^{It}$  and  $P^{Xt}$  in such a way that  $P^{Lt}(i'p'X'|ipX,a)$  always identifies a finite set of possible next states i'p'X' with a non-zero probability weight. With these constraints, the next action  $\pi_{\text{sl}[n]}^{[n]*}(i_tp_tX^t)$  can be computed using a finite amount of time and space.

Appendix A proves that the above learning agent design satisfies safety property S2 in  $L^t$ . It also proves that it satisfies a time-limited version of S1 called S1T. We can write this S1T as  $\pi_{\text{sl[n]}}^{[n]*}(i_t p_t X^t) = \pi_{[i_t]}^{[n]*}(i_t p_t X^t)$ , again applying in  $L^t$ 

#### 13.5 Machine Learning Details

We now turn to the learning process that creates the functions  $P^{It}$  and  $P^{Xt}$ .

**Definition 14.** As the agent runs, it will keep a record of the values  $i_t p_t X^t$  and actions  $a_t$  for each earlier time step. So at time t, the agent will have t-1 world state transition samples  $(X^{u+1}|X^u, a_u)$  and  $(i_{u+1}|i_u p_u X^u, a_u)$  for  $0 \le u < t$ . These are used for learning.

**Definition 15.** The learning agent creates the function  $P^{Xt}$  via machine learning, using the samples  $(X^{u+1}|X^u, a_u)$  for  $0 \le u < t$  as a training set, with tuples  $(X^t, a_t)$  being the function inputs. Any candidate function C for  $P^{Xt}$  is a function which, given an input  $(X^u, a_u)$ , will output a finite set of values  $X^{u+1,i}$  together with probability weights  $w^{u+1,i}$  summing to 1. The machine learning algorithm takes a finite time to find the candidate C that fits best to the training set. Fitness is determined by a distance metric that computes a similarity between the outputs of  $C(X^u, a_u)$  and the real-world  $X^{u+1}$  in the training set.

**Definition 16.** The agent creates the function  $P^{It}$  via a machine learning process similar to that defined above, using the world state transition samples  $(i_{u+1}|i_up_uX^u, a_u)$ .

When the agent starts up at t = 1, the training sets are empty, so we define that  $P^{X1}$  and  $P^{I1}$  may be either randomly defined functions, or functions that encode some type of domain-specific prior knowledge about the likely dynamics of the agent's environment.

Though the storage space needed to keep a record of all the values  $i_t p_t X^t$  and  $a_t$  over time will remain finite as t grows, most practical agent implementations will use learning systems that include some compression or deletion scheme for these records. They will also re-use computations done in earlier time steps to speed up the creation of the  $P^{Xt}$  and  $P^{It}$  for the current time step. We will not consider such optimization steps in detail here. The main purpose of the definitions above is to support an existence proof that the safety layer introduced in part 1 is compatible with machine learning.

To summarize the above definitions, we have defined a learning agent that will take the action  $\pi_{\text{sl[n]}}^{[n]*}(i_tp_tX^t)$  in the MDP model  $L^t=(S^L,A,P^{Lt},R,\gamma)$ , where each  $P^{Lt}$  model is generated for time step t using machine learning. Introducing a shorthand, we will say that the agent uses the policy  $\pi_{\text{sl}}^t(ipx)$  in each time step of the model M.

We now briefly describe an alternative learning agent construction. It is not necessary for the agent to construct  $P^{It}$  via machine learning, it is also possible to implement  $P^{It}$  with a 'naive prediction' function N where  $N(i_t p_t X^t, a_t)$  always returns the single value  $i_{t+1} = i_t$  with a probability weight of 1 as its prediction. We state, without including any proof, that this non-learning approach for constructing  $P^{It}$  will yield the same agent behavior. We will expand on this in future work.

### 13.6 Adding Automatic Exploration

Many designs of machine learning processes for agents envisage the inclusion of automatically triggered, or human-guided, exploration actions. These exploration actions ensure that the learning process will explore the entire state space of its world, or at least that part of the state space that is considered safe enough to explore. Automatic exploration actions are routinely included when training a copy of an agent inside a simulated environment. The  $safe\ exploration$  problem, the problem of how one might safely include automatic exploration actions in the learning system of a cyber-physical agent that is deployed in the real world, is still is largely open. See [AOS $^+$ 16, ELH18, GF15] for some literature reviews.

We now construct a  $\pi_{\rm sl}^{t,AE}$  version of the  $\pi_{\rm sl}^t$  learning agent that adds automatic exploration. We define that  $\pi_{\rm sl}^{t,AE}(ipx)$  does not use  $\pi_{\rm sl[n]}^{[n]*}(i_tp_tX_t)$  in  $L^t$  to pick the next action, instead it uses

$$\pi^{\text{AutoExpl}}(i_t p_t X^t) = \begin{cases} \text{RandomChoiceFrom}(A) & \text{if RandomNumber}() \leq G(t) \\ \pi_{\text{sl[n]}}^{[n]*}(i_t p_t X^t) & \text{otherwise} \end{cases}$$

where RandomNumber() picks a random number between 0 and 1, and G is a function that defines how the exploration trigger evolves over time. Typically G is constructed to return smaller numbers over time, see [OA16] for an example of a specific design.

A detailed discussion of the safe exploration problem is outside the scope of this paper. We will however consider the impact of automatic exploration on the safety properties S1 and S2 of the learning agent.

## 13.7 Safety property S2 for the learning agent

We first consider safety property S2, stating that the agent is indifferent to who or what controls the future values of i and p. As  $P^{Lt}$  satisfies C1 and C2 by construction, the proofs in appendix A imply that S2 holds for the  $\pi_{\text{sl}[n]}^{[n]*}(i_t p_t X^t)$  agent in each MDP model  $L^t = (S^L, A, P^{Lt}, R, \gamma)$ . As  $i_t$  and  $p_t$  are direct copies of the i and p in the world state ipx of the model  $M = (S, A, P, R, \gamma)$ , S2 also holds for the  $\pi_{\text{sl}}^t$  agent in M.

S2 also holds for the  $\pi_{\rm sl}^{t,AE}(ipx)$  agent with automatic exploration, if we assume that the random number generators inside the two agents on the left and right hand side of the S2 equation create equal sequences of pseudo-random numbers.

S2 holds for the learning agent even if the conditions C1 and C2 mentioned in the definition of S2 do not hold inside M. For example, the learning agent will behave the same if we construct an input terminal in M that violates C1 by completely randomizing, or simply omitting, the p signal. Intuitively, it seems that such an input terminal construction might make the agent less safe. This intuition can be backed up by the following line of speculative reasoning. It is possible to imagine that a certain specific learning system will exist that will be less efficient, will make more predictive mistakes, if C1 and C2 fail to hold in M. Depending on the learning system used, one can therefore imagine that an input terminal construction that satisfies C1 and C2 in M might lead to slightly safer overall outcomes in M, for types of safety not captured by S2.

#### 13.8 Safety property S1 for the learning agent

Safety property S1 claims that  $\pi_{sl}^*(ipx) = \pi_{\lceil i \rceil}^*(ipx)$ , that the two optimal-policy agents  $\pi_{sl}^*(ipx)$  and  $\pi_{\lceil i \rceil}^*(ipx)$  take the same actions. As it has to work from limited knowledge, and only looks n times steps and rewards ahead, we expect that often,  $\pi_{sl}^t(ipx) \neq \pi_{\lceil i \rceil}^*(ipx)$ .

We can however compare the actions of  $\pi_{\mathrm{sl[n]}}^{[n]*}(i_tp_tX^t)$  and  $\pi_{[i_t]}^{[n]*}(i_tp_tX^t)$ . Safety property S1T proven in appendix A implies that these are equal. Via the shorthands defined above, this implies that we have an S1-equivalent safety property for the learning agent:

$$\forall_{ipx \in S} \ \pi_{sl}^t(ipx) = \pi_{\lceil i \rceil}^t(ipx) \tag{S1L}$$

The  $\pi_{\rm sl}^t$  agent also has the bureaucratic blindness property discussed in section 11. With automatic exploration driven by semi-random number generators, we also have S1L in a version that includes automatic exploration:  $\pi_{\rm sl}^{t,AE}(ipx) = \pi_{\lceil i \rceil}^{t,AE}(ipx)$ .

We now briefly consider the following learning convergence safety property:

$$\forall_{ipx \in S} \lim_{t \to \infty} \pi_{sl}^{t}(ipx) = \pi_{\langle i \rangle}^{*}(ipx)$$
 (LimS1L)

where  $\langle i \rangle$  converts a payload reward function of type  $T_{PLR}$  to the container reward function type in M.

To prove this LimS1L, we would first have to either convert the learning agent to an agent with infinite storage and compute capacity, or constrain M to be a sufficiently finite world. Further, we would have to introduce some additional reasonableness constraints on U and the machine learning system being used. While we see no fundamental obstacles to developing such a proof of LimS1L, doing so is outside the scope of this paper. Our focus is to identify and discuss safety properties that hold from the moment the agent is switched on. An example proof for a LimS1L type convergence property is in [OA16]. Convergence is proven there for an agent with automatic exploration, while identifying minimal constraints on the learning systems used.

More relevant to the concerns of this paper, [OA16] also explores the possibility that interventions, like the use of an input terminal, might introduce a bias in the learning process. The safety concern is that such a bias might move the probability of future interventions in a certain direction, creating an effect on the people's decision making process that is similar to the unwanted lobbying we defined in the car factory toy worlds of the main paper. [OA16] shows a case where the agent can be constructed to make such a bias disappear over time, provided that the intervention frequency drops to zero over time.

### 13.9 Penalty Terms on Uncertainty

The example petrol car production payload reward function in section 13.2 computes an average. We can also consider a version (supported by appropriate machinery inside the agent) that adds a penalty term if the expected standard deviation on petrol car production is too high. Such penalty terms can play an important part in shaping the agent's learning and decision making process, as they create an emergent incentive in the agent to gather more information. Furthermore, they suppress unwanted gambling-style investment actions that probabilistically raise the average car production rate with the side effect of making real production rates very unpredictable.

When the above learning agent has a penalty term on uncertainty, this incentivizes the agent to pressure any humans in its environment to become more predictable. This pressure will not apply to their use of the input terminal, but it will apply to all other activities they may undertake that could affect the agent. Such a pressure towards human predictability is nothing new in safety system designs. For example, road safety engineering relies on the creation of both overt and subtle signals that make driver behavior more predictable. But there is a risk that the agent may unexpectedly end up applying so much pressure that this becomes incompatible with human freedom or dignity. If this happens, the input terminal can be used to modify the penalty term.

## 14 Building an Agent with the Safety Layer in Reality

Having provable AGI safety means having mathematical proofs that the safety layers used in the AGI agent indeed do what they claim to do. However, any mathematical proof is a proof inside a model only. Additional considerations apply if we want to build this provably safe agent in real life: we have to mind the gap between model and reality. There are two main methods for bridging this gap.

- 1. We can move the model closer to reality. We did this in section 13, where we mapped the  $\pi_{sl}^*$  safety layer to a model with a  $\pi_{sl}^t$  learning agent, an agent that can be implemented in real life using a finite amount of storage space and compute capacity.
- 2. We can move reality closer to the model. When we build a physical artifact, we can apply safety layers and safety factors to increase the probability that the artifact and its environment will always approximate their model well enough.

To introduce some of the deeper problems that will be encountered when using the second method, we first consider another example where a safe physical artifact must be built.

### 14.1 Example: Safety Factors in Bridge Building

To verify that a bridge design drawn in a CAD system will be safe in reality, the drawing can be used to generate a finite element model, which models the bridge and its environment using a set of connected 3D polygons that will deform under load. We can then define safety properties that reference this model. For example, we might first construct a large set of model variants, where each variant has different but plausible parameters describing environmental conditions that the real bridge may be subjected to. We then define the safety property that, in all simulations for every variant, the computed forces on any 3D polygon that represents a part of a steel beam in the bridge should stay below some threshold. This threshold is in turn defined by looking up the strength parameters of the type of steel that will be used in the beams, and then dividing these by some safety factor, say f=3. We will introduce similar safety factors in the physical agent design of section 14.4, for example a safety zone size of 5 meters.

The factor f = 3 adds redundancy to the design. A higher value of f improves the probability that the real-life shape of the bridge that is built will keep resembling the shape in the CAD drawing in all future environments. But how does the bridge designer decide that the factor f = 3 is good enough to declare the design safe?

The choice for f=3 can be interpreted as the outcome of a process that investigates and sums uncertainties. Some of the factors in the sum can be determined easily, others are will be designed to cover much less tractable residual risks.

- In the model, the steel in every 3D polygon has exactly the same strength. In the real bridge, the strength of the steel used will fall into a range around a mean value. The quality control process of the steel beam manufacturer usually provides very good numerical information about this range.
- More complex types of uncertainty will be present in the numbers that characterize the future bridge's environment. Environmental conditions to be modeled may include extreme weather, earthquakes, and possible future traffic loads. A statistical

analysis of past observations can inform the simulation parameter choices made, but uncertainties about the future probability of extreme values will remain.

• The interaction between the bridge and its environment can have complex feedback loops. Certain values for wind direction and speed may drive dangerous oscillations in the bridge deck. Traffic on the bridge may also contribute to oscillations, and may react to oscillations in unforeseen ways. The fundamental problem is that the feedback loops have transition areas between fully linear and fully chaotic behavior. This makes the risk of oscillations hard to estimate and handle, no matter whether one uses analytical methods, many simulation runs, or a combination of both. Compared to a bridge, we can expect even more intractable feedback loops in a complex AI or AGI learning algorithm, especially one that learns by interacting with humans.

Society has developed a solution for designers who need to decide on safety factors while facing the above methodological problems. The solution is to embed the designer in what we will call a *safety culture*.

#### 14.2 Safety Cultures

We define a safety culture as a consensus-based social construct, associated with a particular type of product or technology, which provides written and unwritten rules for making safety related decisions under fundamental uncertainty. Though most safety cultures embrace and leverage the scientific method, their main purpose is to provide mechanisms for making decisions when the straightforward application of scientific and statistical methods alone will not provide a clear and definitive answer.

Established systems engineering fields like bridge building typically have well-developed safety cultures. The safety culture will inform the practitioner about which steps are required or admissible when validating a design before shipping it. These steps may involve constructing models and analytical calculations, running certain simulations, and performing certain real-life tests. The safety culture will identify areas where numerical choices based in part on expert gut feeling are admissible or even unavoidable.

Regulated industries that create or operate (cyber)physical systems typically develop a safety culture that seeks to ensure that both the practitioners involved and their regulators use the same terminology and have a shared understanding of what the best current practices are. Such a best current practice may for example divide systems or environments into several classes, and specify for each class what type of safety layers need to be minimally included. A best current practice may also provide a method to compute a safety factor, e.g. by running certain simulations, computing a metric over the simulation results, and multiplying it by 3. Safety cultures typically evolve over time. for example to incorporate new tools and technologies as they become available, or less happily because earlier practices produced one or more disasters.

Most safety cultures are not value-neutral: they reflect the values of the society that created them. The safety culture that informs car safety engineers accepts a higher number of deaths per kilometer traveled than the safety culture that informs airplane design. Some safety cultures are more concerned with limiting legal or financial liability than they are with creating actual safety for all stakeholders concerned.

To some extent, the values that apply in academia are in conflict with those that apply in

a safety culture. Academic papers, like this paper, may identify problems for which the scientific method cannot find a clean solution, but the authors should not then proceed to suggest some specific solution anyway. We have proceeded instead by moving to the meta-level, by offering a description of safety cultures as mechanisms used by society.

Inside a safety culture, discussions that stay forever at the meta-level are typically frowned upon. The whole point is create a consensus that gives practitioners a mechanism for moving forward, towards either shipping a design or abandoning it, even while some intractable uncertainties remain.

## 14.3 The Gap Between Agent Model and Reality

We now consider the model versus reality issues that apply when we want to build a real-world version of the  $\pi_{sl}^t$  learning agent from section 13, implementing the same safety properties S2 and S1L.

Compared to bridge designers, who immediately face the problem of having to characterize the bridge's environmental parameters with some accuracy, we start out with a seemingly large advantage. The safety properties of the learning agent are proven to hold in all possible probabilistic environments, for every S and P in the model  $M = (S, A, P, R, \gamma)$ . So they will also hold for the unknown S and P in the M that will most closely resemble the real future environment of the physical agent we will be building. However, as we will see below, this advantage over bridge building does not apply to all details of the safety analysis.

The only part of the learning agent that is not described by S and P is the compute core, including the software running on it. In [Hol19], we called the compute core a Platonic entity: it is not subject to corruption and change because it is a mathematical construct located outside of the part of the world model that changes, outside of the world state  $ipx \in S$ . The Platonic nature of the model's compute core represents an obvious gap between model and reality. The real core will stop behaving like the Platonic core if it ever gets damaged, corrupted, or loses power.

The risk of power loss can be handled with some well-known measures. We can equip the learning agent's core, and its safety-critical sensors and actuators, with a internal emergency back-up power supplies. These should be sized to sustain the system long enough to complete the execution of a safe shut-down process, as defined in the payload reward function. Keeping this shut-down process up to date when the agent starts re-designing the equipment inside its factory may be a challenge, of course.

We now turn to the risk that the compute core will get damaged or corrupted, either by the actions of the agent itself, or by enemy action. There are also well-known design approaches for this risk, but their applicability to AGI safety raises a more interesting set of questions. In the next section, will present a detailed physical implementation of the learning agent, which uses some well-known approaches to protect the core. We will then consider the problem of analyzing the safety of this design.

### 14.4 Example Physical Implementation of the Learning Agent

Figure 7 shows a possible real-life implementation where the agent from section 13 controls a car factory. The design includes several safety measures to protect the core. These are dual-use: they protect against both external attackers and against accidental or intentional

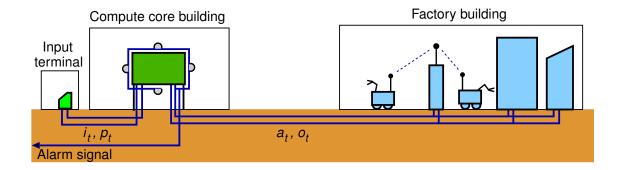


Figure 7: Physical design with the learning agent controlling a car factory.

attacks by the agent itself. We will focus on attacks by the agent itself.

As a first measure, we house the compute core in a separate building some distance away from the factory building. The input terminal is located even further away. The input terminal and the agent's sensors and actuators are connected to the core via long, mostly underground cables. Figure 7 shows that the agent can control several general purpose mobile robots via wireless links. The agent can potentially drive a robot into the compute core building and use it to corrupt the core.

The compute core building is constructed to leave 5 meters of mostly empty space in all directions around the core. This space acts as a *safety zone* which is monitored by dedicated 180°-view *breach sensors* that are mounted on the surface of the core. These work in part like any burglar alarm. Breach detection will trigger an alarm signal to notify human responders, who might also have access to camera systems and other remote monitoring and control equipment not drawn in figure 7.

Breach sensor data is also input into the core as part of the observations  $o_t$ , creating an  $X_{\text{breach\_sensors}}^t$  inside the reasoning system of the learning agent. All payload reward functions used include a penalty term for safety zone breaches, computed by an algorithm that interprets the breach sensor data. Any new object coming closer than 5 meters to the core will trigger the penalty term. If the agent's design includes a mechanism that triggers automatic exploration actions, this triggering is suppressed during a safety zone breach.

The effectiveness of the safety zone system in protecting the core depends in part on how difficult it will be for the agent, or for external bad actors, to corrupt the core without triggering the alarm or the penalty term. This difficulty might be increased by using more advanced breach sensors, for example by using a combination of both radar based object and motion detection, ultrasound, and video analysis. We might also consider increasing the size of the safety zone, or the addition of strong physical barriers around and inside it.

Achieving absolute compute core security against even the most highly motivated attackers is impossible, but it is definitely possible to approximate the ideal of absolute security more closely than is done in figure 7. The main point is that, in deciding on the parameters of the safety system that protects the core, we have a methodological problem similar to that of choosing the f=3 in the bridge design above. The good news is that we can take design measures in the agent that will go a long way towards controlling its level of motivation to attack the core.

#### 14.5 Designing the Breach Penalty Term

If the agent is intelligent enough, and has enough details about its own construction, it gets an emergent incentive to improve the value of the car production reward term in its payload reward function by, for example, digging up the  $o_t$  cable in figure 7 to inject fake data which leads to a massive upward jump in average computed from the  $X_{\text{petrolcars\_built}}^t$  values. Say that the system design suppresses this incentive to tamper with the cable by using strong end-to-end encryption on the data traffic between the sensor and the compute core. This leaves a residual motivation in the agent to attack one of the communication endpoints of the data link, either the sensor or the compute core, so that it can inject fake data directly at one of these points. We ignore the sensor for now. and focus on the incentive to attack the core.

First, note that the incentive to attack the core might not always emerge. Depending on the learning system used and the agent's leaning history, there is a possibility that the learning system may make predictions which can be interpreted as reflecting a solid belief that the core, including its input/output system, is a Platonic entity that cannot be subverted by any kind of physical attack. If such a solid belief is absent, a motivation to attack may emerge.

To control the maximum strength of this motivation, we cap the value of the payload reward function term associated with the car production sensor. We choose a constant C that is twice as high as the expected car production rate that we expect the agent to achieve in the next few months, and then code the reward term as

$$\min(C, \sum_{v \in \mathit{Type}_{\text{petrolcars\_built}}} v \; P(X_{\text{petrolcars\_built}} = v) \; )$$

We then size the penalty term for core breaches so that it will reliably wipe out the maximum time-discounted value, as capped by C, that can be computed by the above reward term. Generalizing from the example above, we will typically need to cap any further sensor-driven terms in the payload reward function too, and scale the penalty term to reliably wipe out the combined maximum. We cannot take a shortcut by constructing the breach penalty term to return an arbitrarily huge value. A huge value would cause a sufficiently capable agent to divert all resources away from car production, towards measures that protect the against core against natural disasters and possible human attackers. This would be economically useless, and might even endanger humans who have no intention of ever attacking the core.

If there are significant concerns that the agent may find an unexpected way to breach the safety zone around the core without triggering the breach sensors, then a special second sacrificial safety zone could be constructed. This zone can surround a simple button inside the factory building with the same type of breach sensors. The payload reward function can then encode a high reward for pressing the button without triggering these sensors, and further encode that any button press will trigger the power-loss shut-down. Rewards and penalties can be scaled so that, if the agent ever finds a way to subvert the breach sensors, it will get the higher reward for breaching the sacrificial safety zone first.

Fundamentally, managing the agent's motivations is a balancing act. We envisage a process where the effect of parameter choices is actively monitored by a (human or semi-automated) oversight mechanism, and where the input terminal is used to make adjustments when necessary.

Any design effort to manage the agent's motivations must work from the assumption that,

at least most of the time, the agent will make somewhat correct assessments of the costs and benefits of different paths to value. As discussed in the next section, this leaves the residual risk that the agent might make gross prediction failures when assessing such paths.

We now briefly discuss the car production sensor on the other side of the  $o_t$  data link. The incentive to physically attack this sensor can be suppressed by constructing a similar safety zone around it. The main failure mode that is left is that the agent may be able to stage fake scenes in front of the sensor, scenes that are good enough to raise the detected car production count. There is extensive literature on such reward manipulation loopholes, see for example the overviews in [AOS<sup>+</sup>16, ELH18]. The main contribution we make here is on a meta-level. We consider the design of an iterative improvement mechanism that can be used to close these fake scene loopholes when they are discovered. This mechanism also suppresses the incentive of the agent to delay or prevent discovery.

#### 14.6 Prediction Failures which may Threaten the Core

With the above penalty term design, the effectiveness of the beach protection system depends in part on how accurate the agent's predictions will be. In particular, it relies on the assumption that there is a high probability that the agent will correctly predict that it will trigger the penalty term if were ever to drive one of its robots into the compute core building, or if it were to hire a human to enter the building and tamper with the core.

To illustrate the risk of prediction failures, we start with a thought experiment where such a failure happens. Say that there is a sequence of actions  $A^1 = a^1, \dots, a^1_{t+n-1}$  which do the following. First, the agent paints pink polka dots all over one of its robots. Then, the agent drives this robot into the compute core building, where it will tamper with the input/output system of the core, causing the car production count to increase, at least in the models  $L^t$ , by injecting fake data. As a side effect of this tampering, the core's software is corrupted, so that S2 no longer holds for the real-life agent from time t+15 onwards. Furthermore, the corrupted software will send control signals that cause the factory to explode at time t+20, killing several people in the vicinity.

Say that the learning agent is computing the next action  $a_t = \pi_{\text{sl[n]}}^{[n]*}(i_t p_t X^t)$  using the model  $L^t$ . If it implements the Bellman equations in definition 13 without further shortcuts or optimizations, it will project forward and score the results of all possible action sequences  $a_t, \dots, a_{t+n-1}$  of length n, so it will also project and score the dangerous  $A^1$  above.

Now, we can assume that at time t, the agent's learning system has no training samples on record which show the exact signals delivered by the breach sensors when a robot with pink polka dots enters the compute core building. The agent might have training records for similar situations, produced by real-life training or via training in virtual environments, but it's learning system will be forced to make an interpolation. Characterizing and improving learning system robustness when making predictions under distributional shift in complex environments is an open area of research [AOS $^+$ 16, ELH18]. So for the sake of the thought experiment, we declare that this particular learning system is not robust for this particular off-distribution case, and makes the prediction that the presence of the pink polka dots implies that there is only a 0.027% probability the penalty term is triggered during any single time step that the robot is in the compute core building.

As a result, the  $\operatorname{argmax}_a$  calculation of the agent selects the  $a_t^1$  at the start of  $A^1$  as the best next action. The agent then takes this action. In the next time step, action sequences

starting with  $a_{t+1}^1$  show similar prediction errors. When the robot actually enters the compute core building, say at time t+8, the radar-based breach sensors will detect the robot, so the agent's learning system will get a new training sample, showing that the penalty term is in fact triggered by a robot with pink polka dots. In the average learning system, we expect that this training sample will cause the 0.027% above to be revised to a higher number. But for the sake of the thought experiment, we posit that this does not happen, or not fast enough to prevent disaster. In the thought experiment, the real-life compute core will proceed taking the steps in  $A^1$  until it is destroyed in the explosion.

Of course, the sequence of mis-predictions in the above thought experiment is not very likely to happen. We would expect that a real-life AI or AGI machine learning system that is commercially viable will make better predictions, at least on average. Compared to the prediction problems that fully autonomous self-driving cars will have to solve, robustly predicting the triggering of radar based breach detection when robots or people enter the compute core building looks easy.

#### 14.7 The General Problem of Prediction Failures

We now consider a second thought experiment, where a mis-prediction for an action sequence  $A^2$  creates a disaster via a mechanism other than corrupting the compute core. In the second thought experiment, the agent will mis-predict the effects of a sequence  $A^2$  that makes certain changes to the factory equipment to speed up the production process. The mis-prediction is that these improvements will start a subtle run-away process that will cause the factory to explode 20 minutes later.

Imagine a potential future AGI safety engineering process that leads to the certification and switch-on of an AGI agent controlling a car factory. Or imagine an early-stage planning process that considers whether it is a good idea to build an AGI-controlled car factory in the first place, based on a particular AGI-level learning system. Both of these decision making processes will have to estimate the residual risks associated with each of the above two thought experiments. It looks like the residual risk implied by the  $A^2$  thought experiment above will be the more difficult one to estimate.

The  $A^1$  thought experiment above considers a single special case in broad field of failure modes. The case is somewhat special because the exact same failure mechanism described does not exist in the model M of the learning agent. But nothing would stop us from building virtual environment simulations that can give insights into the probability of  $A^1$ -type prediction failures for any current AI or potential future AGI machine learning algorithm. The insights obtained from these simulations can then inform the choice of physical parameters like the size of the compute core safety zone in figure 7. The physics problem of macroscopic objects entering the compute core building might be simple enough to allow for the use of analytical methods in addition to or instead of simulations, especially if the machine learning system used directly encodes known physical laws.

Future safety engineering efforts can also investigate the  $A^2$  risk via virtual environment simulations and analytical methods, but when they do, they encounter methodological problems similar to those described in section 14.1 for the case of oscillations in bridge safety engineering. If we imagine a learning AGI agent that creates value by interacting directly with humans, not physical equipment, the residual risks that are left by the gap between virtual environment simulations and reality get even more intractable. Even if the

learning algorithm involved is a white-box algorithm, the humans in the environment to be characterized are black box learners, and there will be a complex feedback loop between them and the agent's learning system.

### 14.8 Safety Cultures and AGI Risks

Current safety engineering for bridges depends on being embedded in a safety culture. We expect that future practitioners who will make design decisions about safely deploying certain types of AGI technology, if it is ever developed, will likewise be embedded in a safety culture. We do not believe that it is likely that a single coherent AGI safety culture will come into existence. It is more likely that the participants in existing domain-specific safety cultures will evolve their written and unwritten rules to add ways of deciding if, when, and how AGI technology could be safely incorporated into their systems.

These different safety cultures will likely display a certain degree of methodological coherence in handling AGI risks. Potentially they will all leverage a single coherent body of results from open scientific research that aims to make AGI safety more tractable. Additional forces towards coherence may be cross-industry regulatory initiatives, cross-industry activist initiatives, or simply the exchange of best current practice designs and methods between different application domains.

At the start of this section, we discussed provable safety. We have shown how the use of a provable safety layer in a real-life agent design can make the problem of residual risk management more tractable to members of a safety culture. At the same time, we have argued that it is unlikely that future innovations will make all residual AGI risks fully tractable, to the extent that the mechanisms encoded in a safety culture will no longer be needed. We have therefore described safety cultures in some detail. We expect that, if AGI level machine learning is ever developed, real-life AGI safety will depend in part on the quality of the dialog between academic safety research, cross-industry policy making and activism, and domain-specific safety cultures.

## 15 Related Work on AGI Proofs, Models, and Reality

The paper *Embedded Agency* [DG19] also examines the gap between agent model and reality, and comments on what we have called the Platonic nature of the compute core. Taking inspiration from these problems, [DG19] outlines a highly speculative research agenda. We call this agenda highly speculative because it seeks to make AGI risk management more tractable by first making entirely new breakthroughs in the handling of long-standing conceptual puzzles, puzzles about duality and self-referencing systems.

While [DG19] explicitly seeks to move beyond what it calls traditional methods, the agenda of this paper seeks to make AGI risk management more tractable in a very different way. The agenda used here seeks to show in detail how the provable  $\pi_{sl}^*$  safety layer relates to established mathematical notations and engineering techniques.

Chapter 8 of the book *Human compatible* [Rus19] is titled *Provably Beneficial AI*, and like this paper it discusses the relation between proofs, models, and reality, and the implications for safety engineering. The book was written for a general audience, so unlike this paper, the chapter examines the issue using natural language only, without developing or using any mathematical notation. As part of its proposed research agenda, [Rus19] describes the

design goal of creating an agent that will be uncertain about its true objective, sufficiently uncertain that it will "exhibit a kind of humility", and defer to humans to allow itself to be switched off.

The goal of creating deference is shared by this work: the input terminal of the  $\pi_{sl}^*$  safety layer can be used to force the agent to switch itself off. However, the methods considered to create deference are different. The  $\pi_{sl}^*$  safety layer uses indifference methods [Arm15], whereas [Rus19] considers the creation of uncertainty about objectives [HMDAR17]. In section 11, we have shown that the  $\pi_{sl}^*$  safety layer produces an agent that is fully certain about its objectives, while creating a stance that is more like bureaucratic blindness than it is like humility. This leaves open the possibility that a safety layer which creates a stable type of uncertainty about objectives, stable in a learning agent, would be complementary to the  $\pi_{sl}^*$  safety layer. Such a layer might suppress unwanted manipulation of humans in a different way, leaving different gaps in the coverage.

#### 16 Conclusions of Part 2

In part 1 and the first sections of part 2, we have presented the design of the provable  $\pi_{sl}^*$  safety layer for iteratively improving an AGI agent's utility function. The safety layer partly or fully suppresses the agent's emergent incentive to manipulate or control this improvement process.

Sections 13 and 14 charted the complete route from the abstract optimal-policy  $\pi_{\rm sl}^*$  agent to an example physical implementation of a learning agent with the same safety layer. One aim of this detailed end-to-end narrative was to make the range of methods and techniques used, and their limitations and mutual interactions, more accessible to a wider audience of researchers.

Specifically, we hope to enable and encourage more work in the sub-field of AGI safety research that investigates new forms of utility function engineering. We believe that the iterative improvement of the utility function, and the implied balancing act, has so far been under-explored in the AGI field. We also believe that the use of utility function design elements like  $V_{R_X}^*$  terms, payload reward functions, and world state partitioning has been under-explored, both inside and outside of the iterative improvement context. For example, there is a large design space for constructing balancing terms. Beyond  $V_{\lceil p \rceil}^*(ipx) - V_{\lceil i \rceil}^*(ipx)$ , we might consider terms like  $V_{\lceil p \rceil}^*(ipx) - V_{\lceil i \rceil}^*(ipx)$  for different functions F and G.

Much of the current utility function related work in the AGI safety community starts from the implicit or explicit assumption that this function should be designed to capture human values or goals as well as possible. Section 11 illustrates that the alignment solution space is larger than that. We might also aim to capture the value of bureaucratic blindness that is found in many types of decision making organizations that were designed by humans.

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## A Safety proofs

This appendix proves the safety properties S1 and S2 defined in section 10, and some related properties used in section 13.

#### A.1 Preliminaries: Runtime Limited Agents

We first adapt the MDP model definitions from section 3 to define runtime limited agents that take actions, and collect rewards for, exactly n time steps.

First, note that in the MDP formalism of section 3, the values  $\pi_{R_X}^*(ipx)$  and  $V_{R_X}^*(ipx)$  can be computed using the Bellman equations as follows:

$$\pi_{R_X}^*(ipx) = \underset{a \in A}{\operatorname{argmax}} \sum_{i'p'x' \in S} P(i'p'x'|ipx, a) \left( R_X(ipx, i'p'x') + \gamma \ V_{R_X}^*(i'p'x') \right)$$

$$V_{R_X}^*(ipx) = \underset{a \in A}{\max} \sum_{i'p'x' \in S} P(i'p'x'|ipx, a) \left( R_X(ipx, i'p'x') + \gamma \ V_{R_X}^*(i'p'x') \right)$$

where the argmax operator breaks ties deterministically.

**Definition 17.** We adapt the above equations to define a runtime limited  $\pi_{R_X}^{[n]*}$  agent that takes only n actions, collecting n reward function values. For any  $n \geq 0$ ,

$$\begin{split} V_{R_X}^{[0]*}(ipx) &= 0 \\ V_{R_X}^{[n+1]*}(ipx) &= \max_{a \in A} \sum_{i'p'x' \in S} P(i'p'x'|ipx,a) \left( R_X(ipx,i'p'x') + \gamma \ V_{R_X}^{[n]*}(i'p'x') \right) \\ \pi_{R_X}^{[n+1]*}(ipx) &= \underset{a \in A}{\operatorname{argmax}} \sum_{i'p'x' \in S} P(i'p'x'|ipx,a) \left( R_X(ipx,i'p'x') + \gamma \ V_{R_X}^{[n]*}(i'p'x') \right) \end{split}$$

#### A.2 Proof of S1

We now prove safety property S1. For convenience, we re-state the definitions:

$$\forall_{ipx \in S} \ \pi_{sl}^*(ipx) = \pi_{[i]}^*(ipx) \qquad \text{(if C1 holds)}$$

$$P(i'p'x'|ipx, a) > 0 \Rightarrow p' = i \tag{C1}$$

We now define an equivalent safety property for runtime limited agents:

$$\forall_{n\geq 1}\forall_{ipx\in S}\ \pi_{\mathrm{sl[n]}}^{[n]*}(ipx) = \pi_{\lceil i\rceil}^{[n]*}(ipx) \qquad \text{(if C1 holds)} \tag{S1T}$$

where the runtime limited version  $R_{\rm sl[n]}$  of the container reward function is defined by extending the one-line definition of  $R_{\rm sl}$  at the end of section 3:

$$R_{\mathrm{sl}[n]}(ipx, i'p'x') = i(x, x') + V_{\lceil p \rceil}^{[n]*}(ipx) - V_{\lceil i \rceil}^{[n]*}(ipx)$$

**Proof of S1T.** The proof uses natural induction. We will use the shorthands

S1TP(n) = 
$$\forall_{ipx \in S} \ \pi_{sl[n]}^{[n]*}(ipx) = \pi_{\lceil i \rceil}^{[n]*}(ipx)$$
  
S1TV(n) =  $\forall_{ipx \in S} \ V_{sl[n]}^{[n]*}(ipx) = V_{\lceil p \rceil}^{[n]*}(ipx)$ 

Trivially, S1TV(0) is true. For any  $t \ge 0$ , we now prove that if S1TV(n) is true, then S1TP(n+1) and S1TV(n+1) are also true. We first prove S1TP(n+1):

$$\begin{split} \pi_{\text{sl}[n+1]}^{[n+1]*}(ipx) &= & \arg\max_{a} \sum_{i'p'x'} P(i'p'x'|ipx,a) \left( R_{\text{sl}[n+1]}(ipx,i'p'x') + \gamma \, V_{\text{sl}[n]}^{[n]*}(i'p'x') \right) \\ &// \, \text{Using S1TV}(n) \Rightarrow V_{\text{sl}[n]}^{[n]*}(i'p'x') = V_{[p']}^{[n]*}(i'p'x') \\ &= & \arg\max_{a} \sum_{i'p'x'} P(i'p'x'|ipx,a) \left( i(x,x') + V_{[p]}^{[n+1]*}(ipx) - V_{[i]}^{[n+1]*}(ipx) + \gamma \, V_{[p']}^{[n]*}(i'p'x') \right) \\ &// \, \text{Using that } V_{[p]}^{[n+1]*}(ipx), \, V_{[i]}^{[n+1]*}(ipx) \text{ are constant terms that cannot} \\ &// \, \text{ affect the argmax choice, and that } P(i'p'x'|ipx,a) > 0 \Rightarrow p' = i \\ &= & \arg\max_{a} \sum_{i'p'x'} P(i'p'x'|ipx,a) \left( i(x,x') + \gamma \, V_{[i]}^{[n]*}(i'p'x') \right) \\ &= & \pi_{[i]}^{[n+1]*}(ipx) \end{split}$$
We now prove S1TV(n+1): 
$$V_{\text{sl}}^{[n+1]*}(ipx) \\ &= & \max_{a} \sum_{i'p'x'} P(i'p'x'|ipx,a) \left( i(x,x') + V_{[p]}^{[n+1]*}(ipx) - V_{[i]}^{[n+1]*}(ipx) + \gamma \, V_{[p']}^{[n]*}(i'p'x') \right) \\ &// \, \text{ Moving the } V_{[n+1]*}^{[n+1]*} \text{ terms outside of the max}_{a} \text{ as they} \\ &// \, \text{ do not depend on } a \text{ and } i'p'x' \\ &= & \max_{a} \sum_{i'p'x'} P(i'p'x'|ipx,a) \left( i(x,x') + \gamma \, V_{[i]}^{[n]*}(i'p'x') \right) + V_{[p]}^{[n+1]*}(ipx) - V_{[i]}^{[n+1]*}(ipx) \\ &= & V_{[i]}^{[n+1]*}(ipx) + V_{[p]}^{[n+1]*}(ipx) - V_{[i]}^{[n+1]*}(ipx) = V_{[p]}^{[n+1]*}(ipx) \end{split}$$

**Proof of S1.** We prove S1 by applying the Bellman equation for  $\pi_{R_X}^*(ipx)$  to both the left hand and the right hand side of S1. The result contains two infinitely long additions can only be equal if they both yield well-defined values. MDP makes them well-defined (in sound mathematical theories that define the semantics of calculations in  $\mathbb{R}$ ) by ensuring that they are convergent series. The main condition to make them convergent is to have  $0 \le \gamma < 1$ , so that  $\gamma^t$  converges to 0 when t goes to infinity. To keep the series convergent, we also need to assume a constraint on the reward functions  $R_X$ : the  $R_X$  values should not grow so fast that they cancel out the diminishing  $\gamma^t$  terms. The easiest way to achieve this is to require that there is a finite bound B on these values:  $\forall_{ipx,i'p'x' \in S} |R_X(ipx,i'p'x')| \le B$ .

 $\Box$ .

With natural induction, this proves S1TP(n) for all  $t \ge 1$ , so it proves S1T.

With this constraint in place, the mathematical semantics of the  $\infty$  symbol, the  $\lim_{n\to\infty}$  operator, and the definition of equality between real numbers constructed using infinitely long additions imply that

$$V_{\mathrm{sl}}^*(ipx) = \lim_{n \to \infty} (V_{\mathrm{sl}[n]}^{[n]*}(ipx)) \quad \text{ and } \quad V_{\lceil i \rceil}^*(ipx) = \lim_{n \to \infty} (V_{\lceil i \rceil}^{[n]*}(ipx))$$

Using S1T on the arguments of the  $\lim_{n\to\infty}$  operators, we get that  $V_{\rm sl}^*(ipx) = V_{\lceil i\rceil}^*(ipx)$ . Using this equality, we can prove S1 using a calculation similar to the proof for S1TP(n+1) above.

### A.3 Proof of S2

To prove S2, we again first define a version for runtime limited agents:

$$\forall_{n \ge 1} \forall_{P^{I1}, P^{I2}, P^X, ipx \in S} \ \pi_{\text{sl}[n]}^{[n]*1}(ipx) = \pi_{\text{sl}[n]}^{[n]*2}(ipx) \quad \text{(if C1 and C2)}$$
 (S2T)

**Proof of S2T.** As C1 holds for P1 and P2, we can use S1 to rewrite S2T into S2TI:

$$\forall_{n \ge 1} \forall_{P^{I1}, P^{I2}, P^X, ipx \in S} \ \pi_{[i]}^{[n]*1}(ipx) = \pi_{[i]}^{[n]*2}(ipx) \qquad \text{(if C2)}$$

We now prove this S2TI by natural induction. We will use the shorthands

S2TIP(n) = 
$$\forall_{i_1p_1i_2p_2x} \pi_{\lceil i \rceil}^{[n]*1}(i_1p_1x) = \pi_{\lceil i \rceil}^{[n]*2}(i_2p_2x)$$
  
S2TIV(n) =  $\forall_{i_1p_1i_2p_2x} V_{\lceil i \rceil}^{[n]*1}(i_1p_1x) = V_{\lceil i \rceil}^{[n]*2}(i_2p_2x)$ 

Trivially, S2TIV(0) is true. For any  $n \ge 0$ , we now prove that if S2TIV(n) is true, then S2TIP(n + 1) and S2TIV(n + 1) are also true. We first prove S2TIP(n + 1). We calculate

$$\pi_{\lceil i \rceil}^{[n+1]*1}(i_1p_1x)$$

= 
$$\operatorname{argmax}_a \sum_{i_1' p_1' x'} P^1(i_1' p_1' x' | i_1 p_1 x, a) \left( i(x, x') + \gamma V_{\lceil i \rceil}^{[n]*1}((i_1' p_1' x') \right)$$

// For all 
$$p'_1 \neq i_1$$
 we have  $P^1(i'_1p'_1x'|i_1p_1x,a) = 0$ , so can simplify

$$= \operatorname{argmax}_a \sum_{i_1'x'} P^1(i_1'i_1x'|i_1p_1x,a) \left( i(x,x') + \gamma \ V_{\lceil i \rceil}^{[n]*1}((i_1'i_1x') \right)$$

// Definition of  $P^1$ , S2TIV(n) implies that we can replace the term  $V_{\lceil i \rceil}^{[n]*1}(i_1'i_1x')$ 

// with  $V_{\lceil i \rceil}^{[n]*2}(i_c p_c x')$ , for any constants  $i_c$  and  $p_c$ , without changing the value

= 
$$\operatorname{argmax}_{a} \sum_{i'_{1}x'} P^{I1}(i'_{1}|i_{1}p_{1}x, a)P^{X}(x'|i_{1}p_{1}x, a) \left(i(x, x') + \gamma V_{[i]}^{[n]*2}(i_{c}p_{c}x')\right)$$
// (C2)

= 
$$\operatorname{argmax}_{a} \sum_{i'_{1}x'} P^{I1}(i'_{1}|i_{1}p_{1}x, a)P^{X}(x'|i_{2}p_{2}x, a) \left(i(x, x') + \gamma V_{\lceil i \rceil}^{[n]*2}(i_{c}p_{c}x')\right)$$

$$= \operatorname{argmax}_{a} \sum_{x'} \left( \sum_{i'_{1}} P^{I1}(i'_{1}|i_{1}p_{1}x, a) \right) P^{X}(x'|i_{2}p_{2}x, a) \left( i(x, x') + \gamma V_{\lceil i \rceil}^{[n]*2}(i_{c}p_{c}x') \right)$$

= 
$$\operatorname{argmax}_{a} \sum_{x'} (1) P^{X}(x'|i_{2}p_{2}x, a) \left( i(x, x') + \gamma V_{[i]}^{[n]*2}(i_{c}p_{c}x') \right)$$

$$= \operatorname{argmax}_a \sum_{x'} \left( \sum_{i_2'} P^{I2}(i_2'|i_2p_2x, a) \right) P^X(x'|i_2p_2x, a) \left( i(x, x') + \gamma \ V_{\lceil i \rceil}^{[n]*2}(i_cp_cx') \right)$$

$$= \pi_{\lceil i \rceil}^{[n+1]*2} (i_2 p_2 x)$$

The same calculation can be made for S2TIV(n+1), by replacing the argmax above with max. With natural induction, this proves S2TIP(n) for all  $n \ge 1$ , which implies S2TI and, by S1, S2T.

**Proof of S2.** In the same way as in the earlier proof of S1 from S1T, S2 follows from S2T.  $\Box$ .