Worksheet 4

Introduction

This sheet continues the work on straight lines, especially the mathematics of linear inequalities found in Problem 8. The application of such inequalities to simple linear programming problems is investigated.

Problem 9

In Problem 8 we encountered the three inequalities:

$$3y + 2x \leq 6 \qquad 4x + 4y \leq 9 \qquad y + 2x \leq 4$$

Plus the fact that neither x nor y are negative. Additionally we found the values of x and y that satisfied the inequalities and made $P = 2y + 3x$ a maximum. If you haven't done this problem yet then do it now!

The solution to Problem 8 solves the following *linear programming problem*:

A company produces two products X and Y processing each in three departments A, B, and C. The following table indicates how long it takes to process each item in each department and the weekly capacity of each department.

Department	Hours to produce 1 item of X	Hours to produce 1 item of Y	Daily capacity in hrs of each department
A	0.02	0.03	6
B	0.04	0.04	9
C	0.02	0.01	4

If we denote by x and y the production levels of the two products X and Y respectively, measured in hundreds of items (i.e., if $x = 2$ then we produce 200 items etc.) then show that the 3 rows of the table lead to the 3 inequalities above.

If additionally we know that the profit from production is given by $P = 2y + 3x$, measured in $£1,000$s, use your result from Problem 8 to write down the production levels, to the nearest item, that give the maximum profit and calculate this profit to the nearest pound. (If you haven't done Problem 8 then do so now.)

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Problem 10

A factory produces two types of screws: wood screws for use in wood and metal screws for use in metal. The wood screws sell for $£20$ per box and the metal screws for $£25$ per box. In addition to a $£5,000$ weekly overhead carried by the factory, the cost of producing a box of wood screws is $£10$ and the cost of producing a box of metal screws is $£8$. If x denotes the number of boxes of wood screws produced each week and y the number of boxes of metal screws produced each week:

- Show that the profit is given by $P = 10x + 17y - 5000$. (Assuming that everything that is produced is sold.)

All the screws have to pass through a slotting machine and a threading machine. A box of wood screws requires 3 minutes in the slotting machine and 2 minutes in the threading machine whereas a box of metal screws requires 2 minutes in the slotting machine and 8 minutes in the threading machine. In a given week each machine is available for 60 hours.

- Construct a table similar to the one in Problem 9 replacing Department A with the slotting process and Department B with the threading process. Omit Department C.

- Show that $3x + 2y \leq 3600$ and $2x + 8y \leq 3600$.
- Use DERIVE to plot a feasible polygon (region).
- Plot a sample profit line when $P = 2000$ (i.e., plot $7000 = 10x + 17y$).
- Proceed as in Problem 8 to obtain the maximum profit and the levels of production of the two types of screw.

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Problem 11

This problem differs from the above in that it minimizes a cost subject to a set of inequalities rather than maximizing a profit.

At a particular locality in the USA there is a plan to reclaim all, or part, of a 20,000 acre area of land that has previously been used for industrial purposes. The cost of reclaiming the land and using it for agricultural purposes is \$300 per acre and for residential purposes \$400 per acre.

- Obtain an expression for the cost C of reclaiming x acres for residential purposes and y acres for agricultural purposes.

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- Write down an inequality, involving both x and y, that expresses the fact that there is only 20,000 acres of land in total that can be reclaimed.

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Reclaiming land is always a political issue and the needs of several groups have to be taken into account when deciding how to reclaim the land. The following requirements of the three main groups are to be satisfied:

(i) A housing association requires that at least 4,000 acres of land be reclaimed for residential use.

(ii) A farming lobby insist that at least 5,000 acres of land be reclaimed for agriculture.

(iii) A third group is interested in reclaiming as much land as possible but insists that at least 10,000 acres of land in total are reclaimed.

- Write down the 3 inequalities defined by these three requirements.

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- Use DERIVE to determine the feasible region.
- Plot 2 sample cost lines with $C = 4,000,000$ and $C = 3,000,000$.
- Proceed as in Problem 8 to obtain the minimum cost and the areas of the two types of land reclaimed.

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Problem 12

This problem not only asks you to maximize a profit but to answer other questions relating to the feasible region.

A coach capable of seating 44 passengers is offered for hire for a children's outing for $£40$. The organizer charges $£1.75$ for an adult and $£1$ for a child. Each adult is responsible for at least one child and at most three. No child is unaccompanied. The trip will be cancelled if the hire charge cannot be covered.

Let x stand for the number of children and y the number of adults. The above information can be translated into the following 4 inequalities involving x and y.

- Write down the reason for each inequality:

  (i) $x + y \leq 44$ because . . . .

  (ii) $40 \leq x + 1.75y$ because . . . .

  (iii) $y \leq x \leq 3y$ because . . . .

  Note that (iii) can be written as 2 inequalities: $y \leq x$ and $x \leq 3y$.

- Show that the profit is given by
$$P = \frac{7}{4}y + x - 40.$$

- Use DERIVE to plot the feasible region and hence answer the following questions:
  Given that the trip is viable find

  (i) The greatest and least number of adults: . . . .

  (ii) The greatest and least number of children: . . . .

  (iii) The greatest profit: . . . .