Lattice and Partition Function

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November 30, 2006

Abstract

This paper describes how to get from the lattice to the partition function [1, 2]

Contents

1 The lattice explained

Lets take an simple example of an $n \times m$ lattice, This consists of $n \times m$ points (or **spins**) that can hold q different values. Where $n, m, q \in \mathbb{N}$. I have taken $q = 2$, so the spins can only hold a one of two values. ie a or b, -1 or 1, positive or negative, north or south.

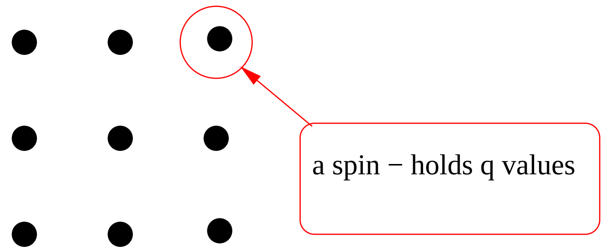

Figure 1: A 3×3 lattice

A spin will be green if it holds the value a, else red if b. Below is an example of a 2×2 lattice in some of the different states it can be.

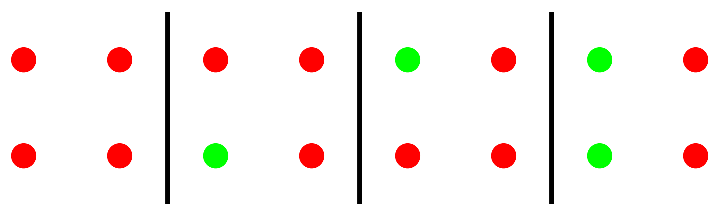

Figure 2: Different spins of a 2×2 lattice

As we can see there are 16 states for a 2×2 lattice. In general the number of states for an $n \times m$ lattice is $q^{n \times m}$.

We proceed by connecting each spin to its nearest neighbours with a line (or **edge**).See figure 3

1.1 The delta value

If two connected spins hold the same value then the edge connecting them has an energy of one unit. Otherwise the edge has an energy of 0. Thus we refer to **delta** as the sum of all the energies of the lattice in a fixed state.

As we can see figure 3 has a $delta = 7$ units. This leads us on to the **Kronecker delta function**

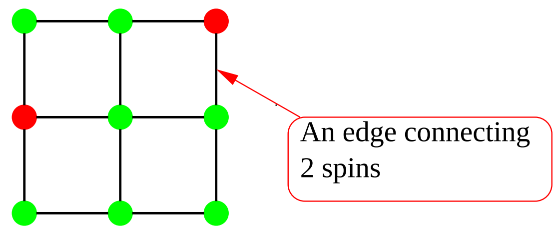

Figure 3: A lattice with each spin connected to it neighbour by edges

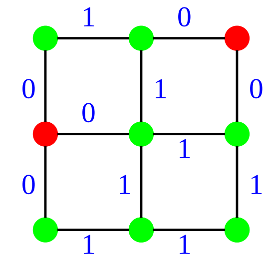

Figure 4: Shows the energy values for figure 3

1.2 Kronecker delta function

The Kronecker delta function is defined as follows.

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Where i and j are usually integers.[1]

The mapping of a lattice s_i to a real number is known as the **Hamitonian**.

1.3 The Hamiltonian

Let S be the set of all states. $s_i \in S$. Then the hamitonian H(s) is given by

$$H(s) = \sum_{(i,j)} \delta_{s_i, s_j}$$

For $s \in S$ and $\delta_{x,y}$ is the kronecker delta function.

1.4 The Partition Function

To do!!

[1]Discuss with ppm what i and j are in terms of the lattice.

1.5 Whats next

Ok up to this point, I should have explained how to get from a lattice to the partition function, Z(x). My aim is to calculate the partition function for a $5 \times 5 \times 5$ lattice. That means the lattice can take on $2^{125} = 4.25 \times 10^{37}$ different states! It would probably takes years for present day computers to calculate this, so now I will describe some technics used to calculate this.

2 Groups and lattices

2.1 Functions

A formal definitions of functions will go here.

2.2 Applying functions to lattices

Look at the lattice in figure 5. We can see that they both have the same delta value. So state a is equivalent b, and c to d.

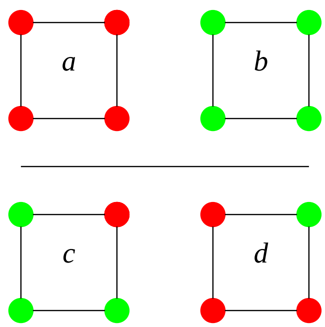

Figure 5: Shows two states of a 2×2 lattice

Lets define a function F, as:

$$F(x) = \begin{cases} G & \text{if } x = \text{R} \\ R & \text{if } x = \text{G} \end{cases} \quad where\ G,\ R\ stand\ for\ green\ and\ red\ respectively. \tag{1}$$

Applying (1) to all spins in a, will give change it to state equal to b.

What other reflections could we have? Look at figure 6. The dashed lines could be mirrors, reflecting the lattice in state a. As we can see a reflected in mirror y changes its state to b. Also a to c using mirror x. And finally reflecting a in x then in y produces state d.

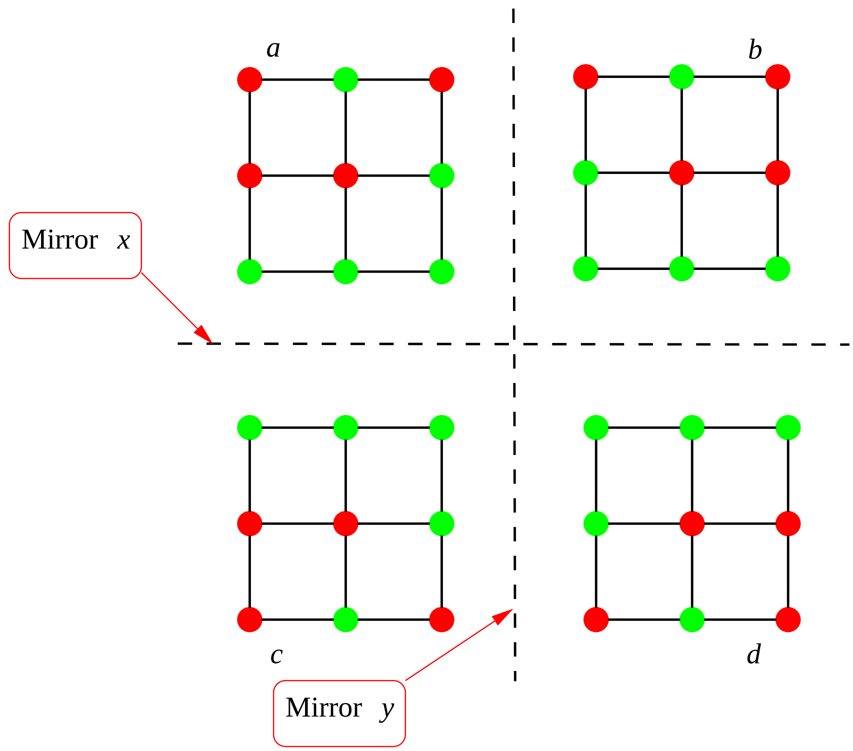

Figure 6: A 3×3 lattice reflected in two directions

And these two mirrors could be applied to many more lattice states. So how can we represent these two mirrors as functions. For this we need to be able uniquely identify each spin in the lattice. For simplicity lets just label the spins $1...(n \times m) - 1$ as shown in figure 7.

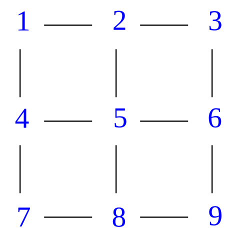

Figure 7: Spins labeled in a 3×3 lattice

References

[1] George D. Greenwade. The Comprehensive Tex Archive Network (CTAN). *TUGBoat*, 14(3):342–351, 1993.

[2] Prof Paul P. Martin. Statistical mechanics overview.