

CS495
Optimiztaion

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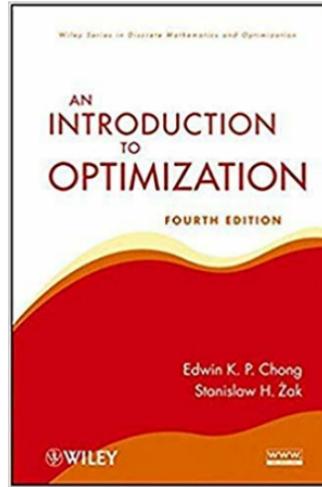
Lectures follow [Boyd and Vandenberghe \(2004\)](#):



Boyd, S., & Vandenberghe, L. (2004). Convex Optimization. Cambridge: Cambridge University Press.

Book and Stanford course:
<http://web.stanford.edu/~boyd/cvxbook/>

Some examples from [Chong and Zak \(2001\)](#):



Chong, E. K., & Zak, S. (2001). An introduction to optimization: Wiley-Interscience.

Course Objectives

- Developing rigorous mathematical treatment for mathematical optimization.
- Building intuition, in particular to practical problems.
- Developing computer practice to using optimization SW.

Prerequisites

Calculus (both single and multivariable) and Linear Algebra.

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Bibliography

Chapter 1

Introduction

1.1 Mathematical optimization

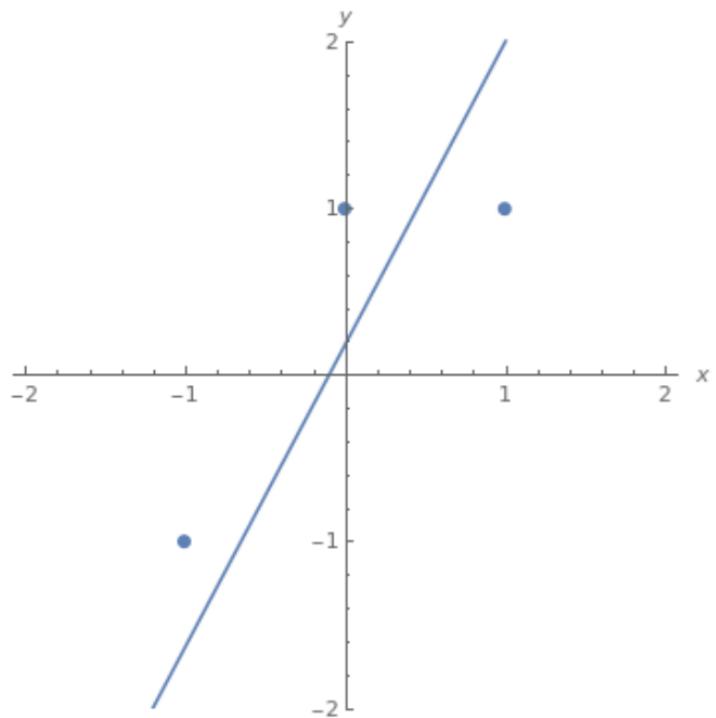
1.2 Least-squares and linear programming

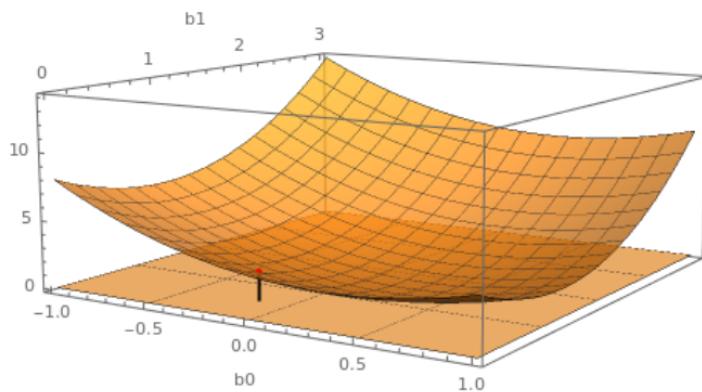
1.3 Convex optimization

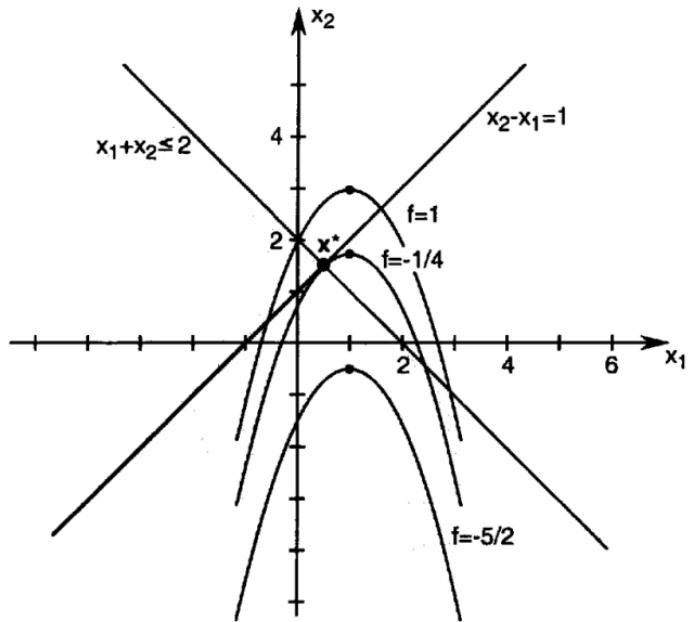
1.4 Nonlinear optimization

1.5 Outline

1.6 Notation







Chapter 2

Convex sets

2.1 Affine and convex sets

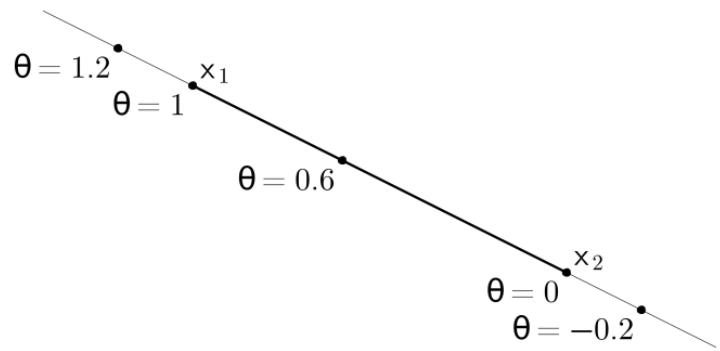
2.2 Some important examples

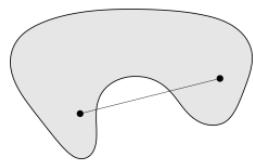
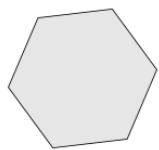
2.3 Operations that preserve convexity

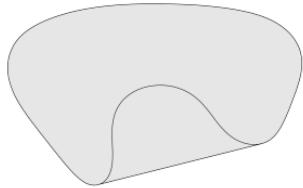
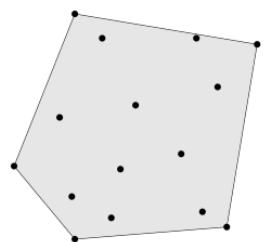
2.4 Generalized inequalities

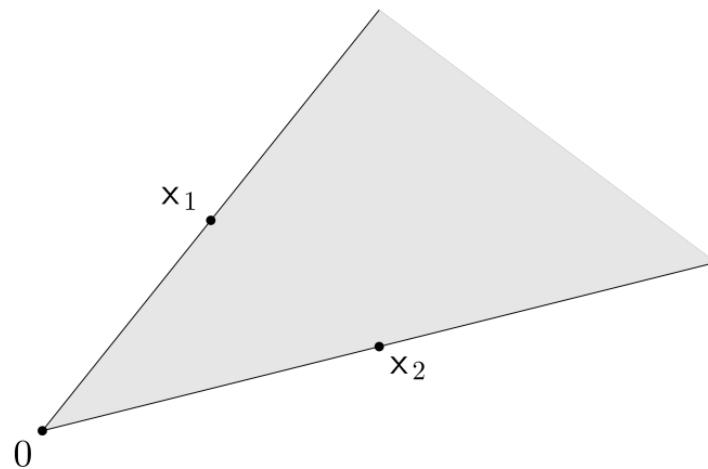
2.5 Separating and supporting hyperplanes

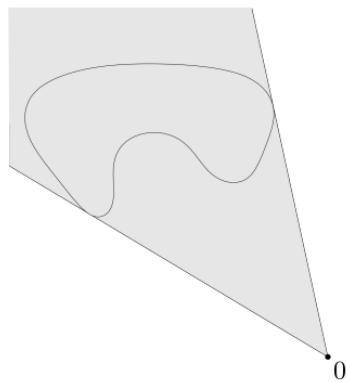
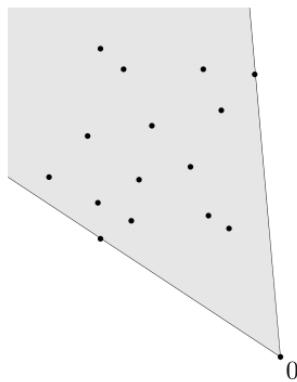
2.6 Dual cones and generalized inequalities

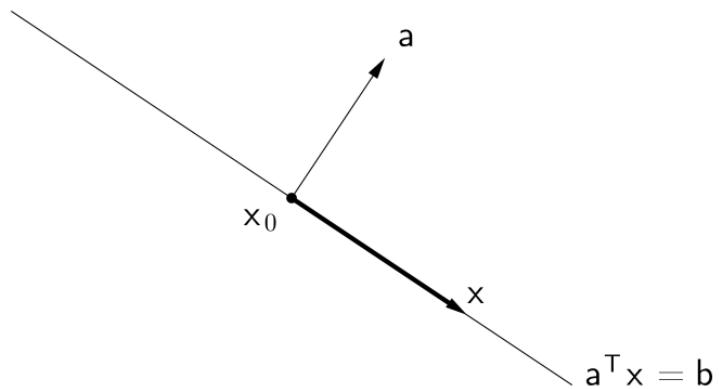


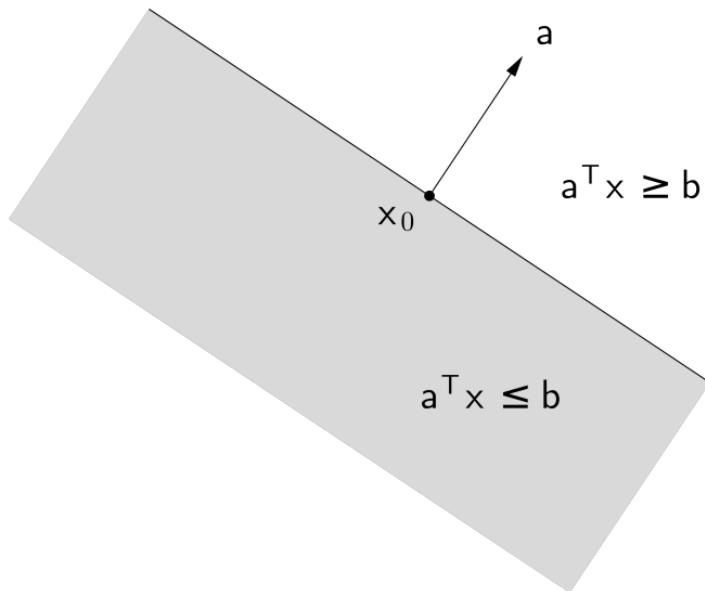


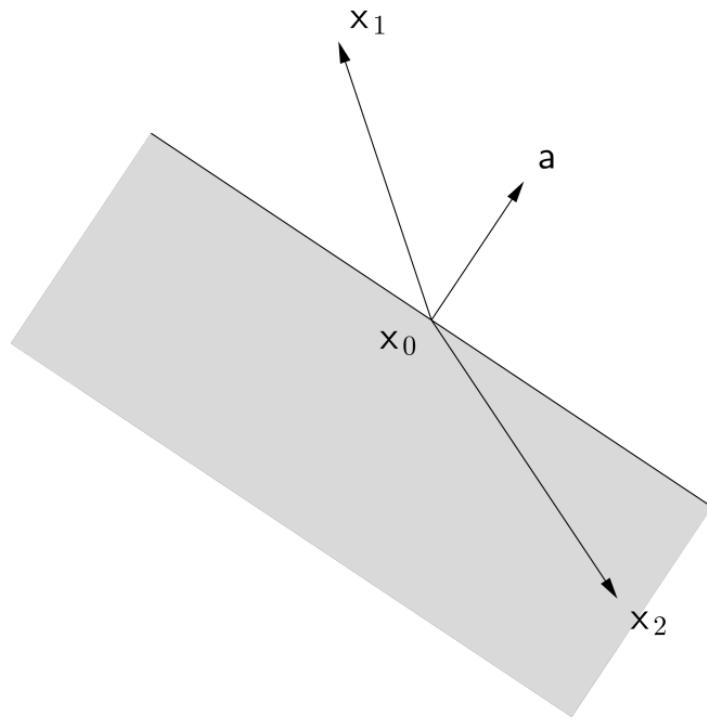


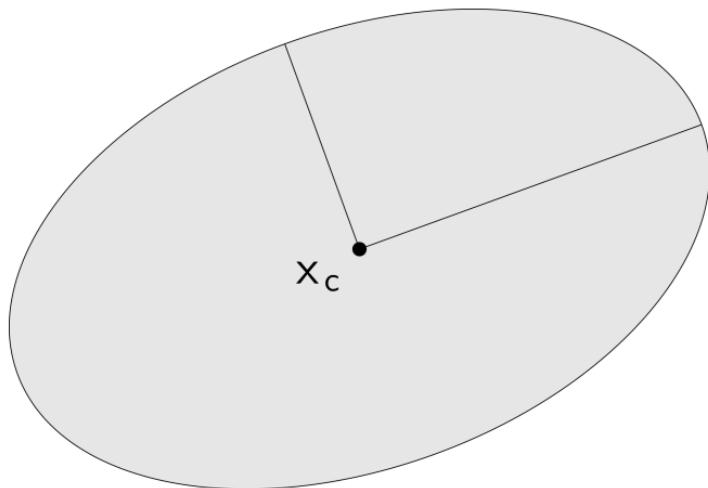


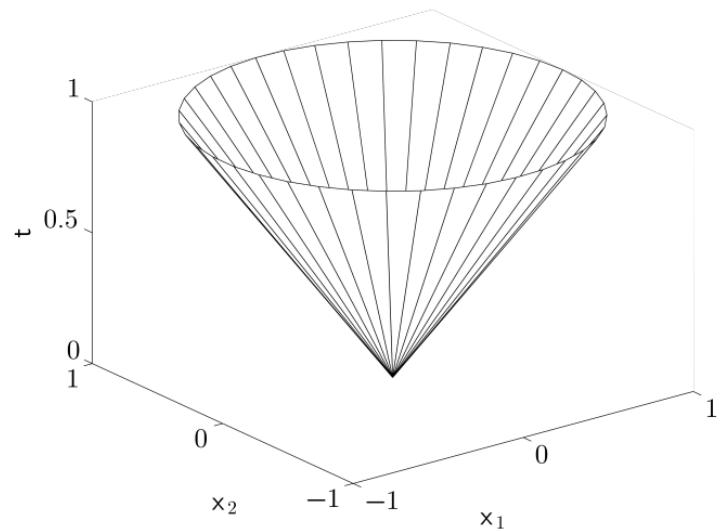


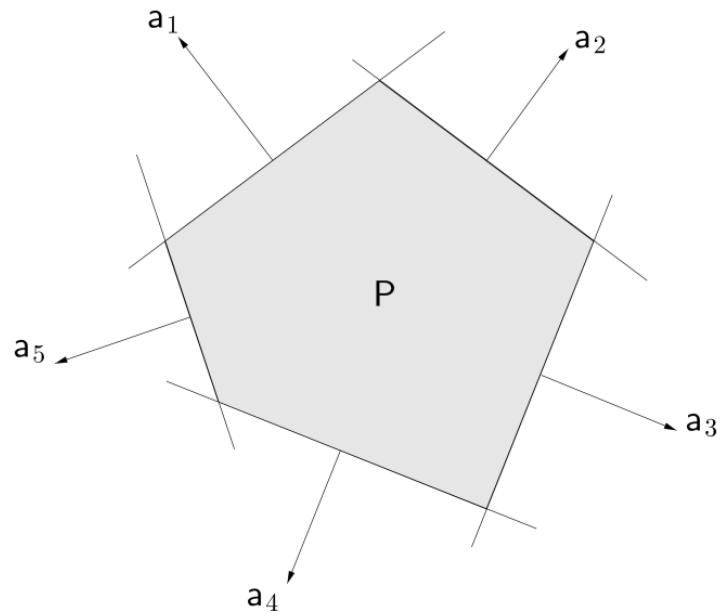


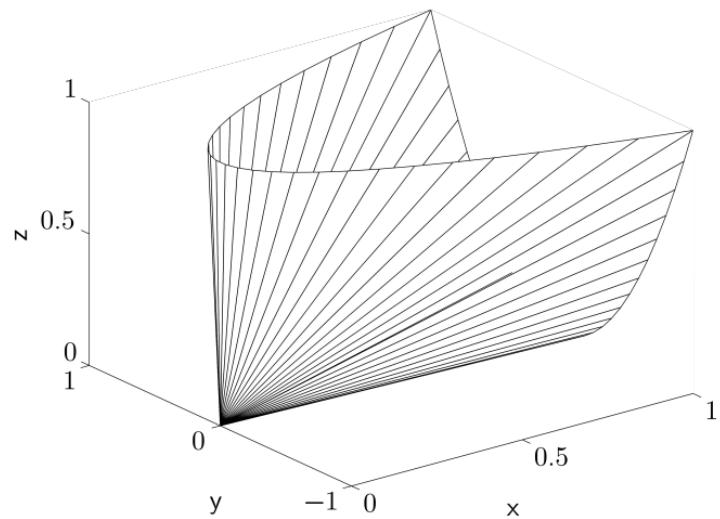


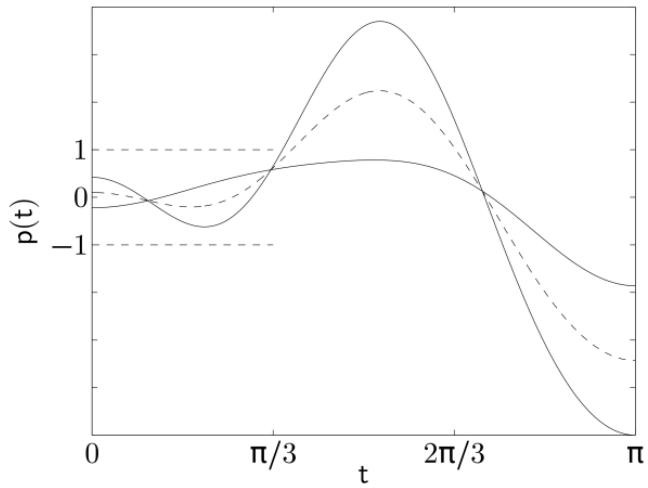


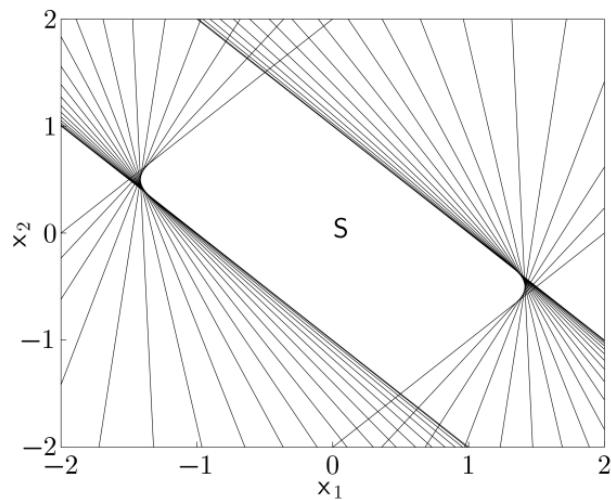


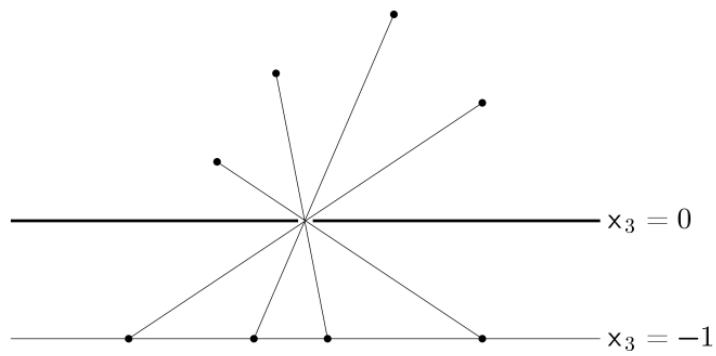


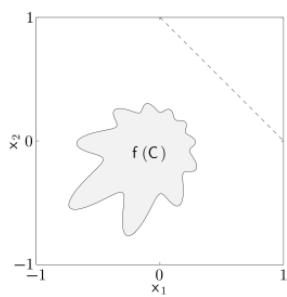
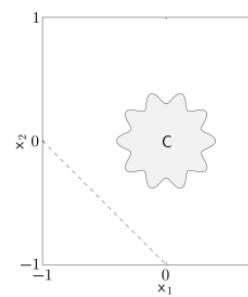


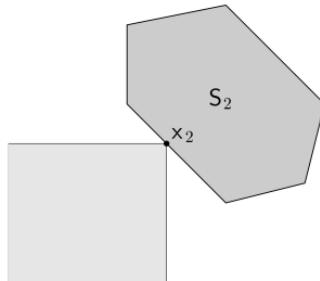
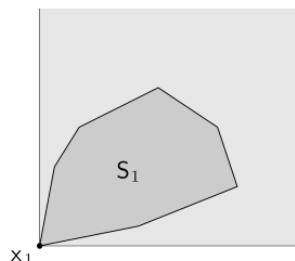


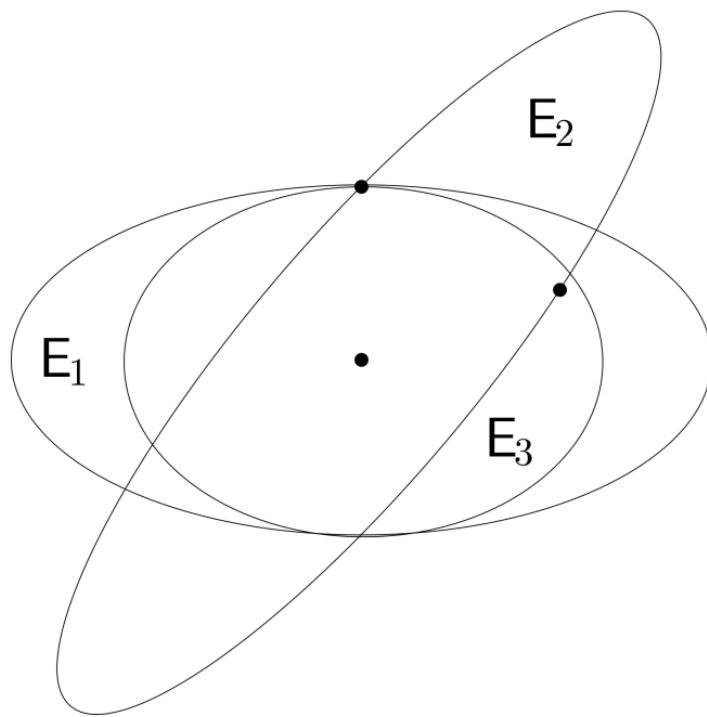






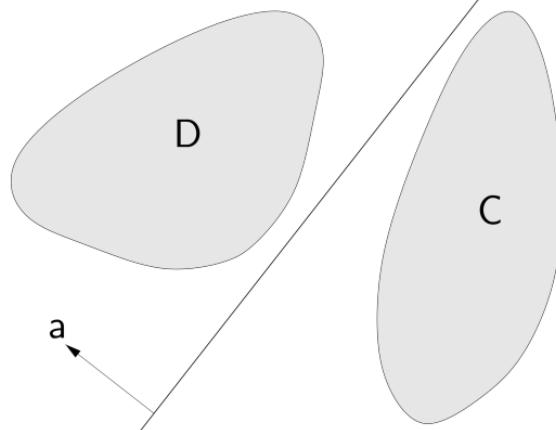


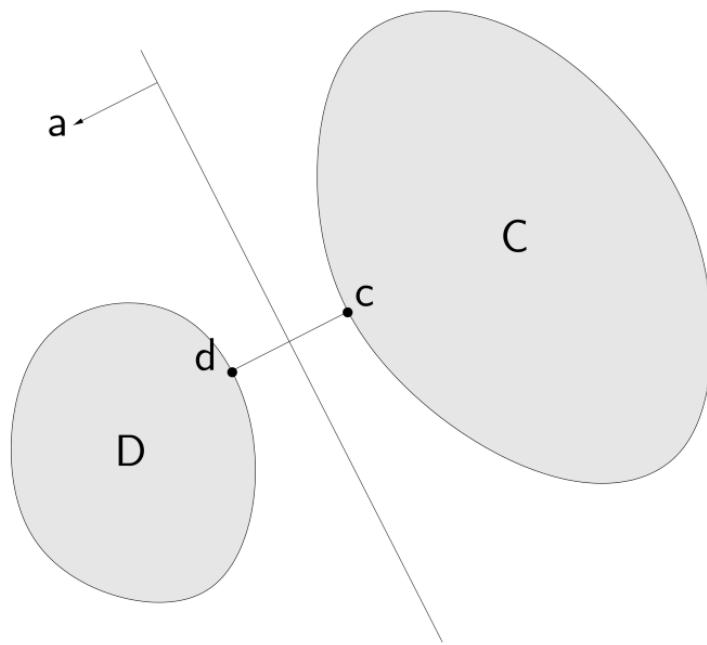


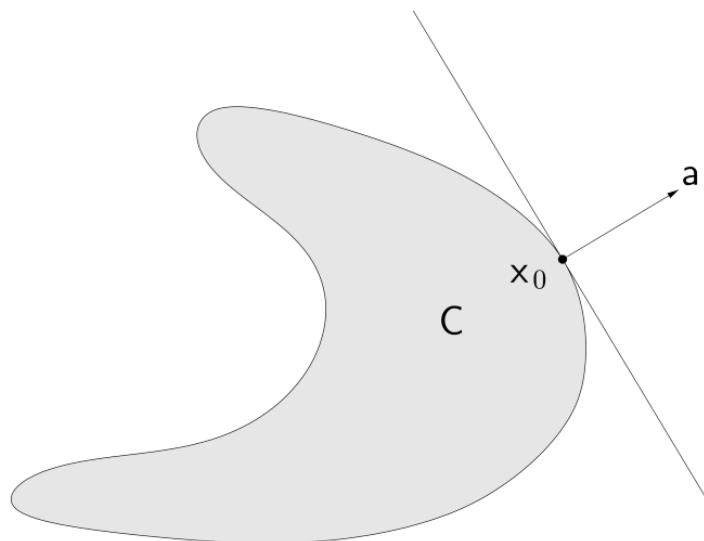


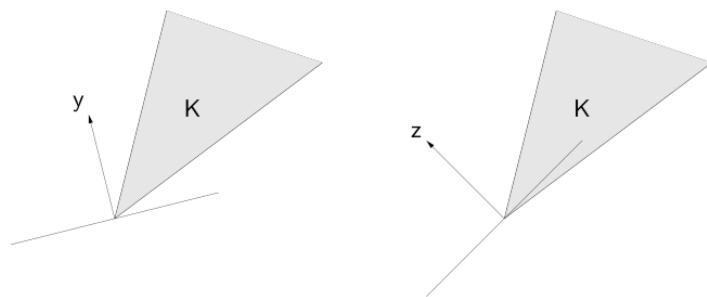
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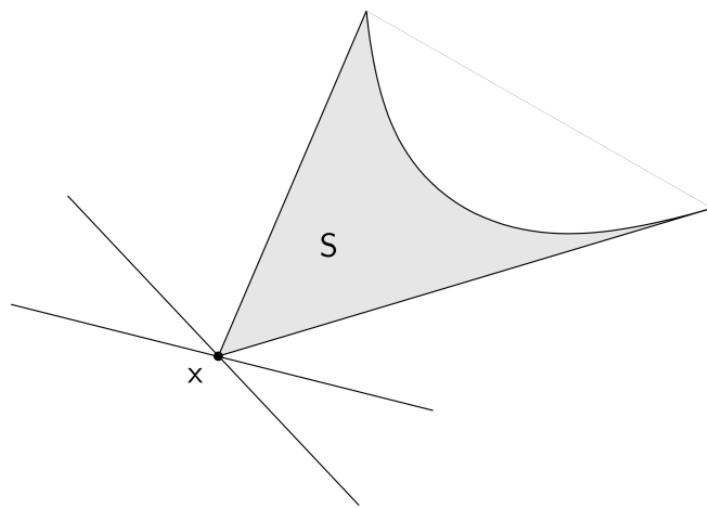
$$a^T x \leq b$$

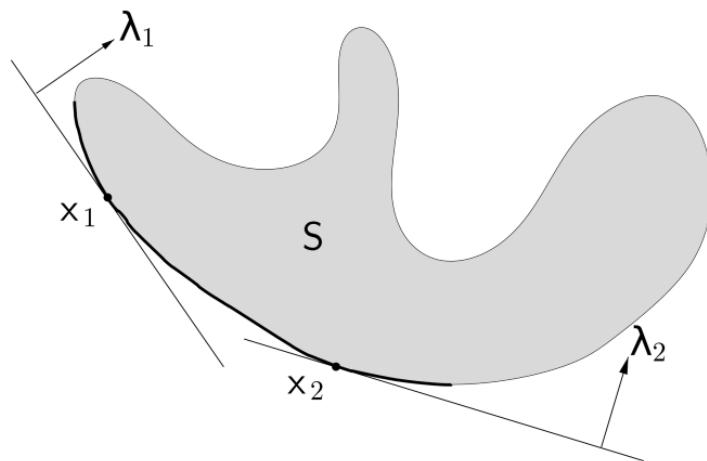


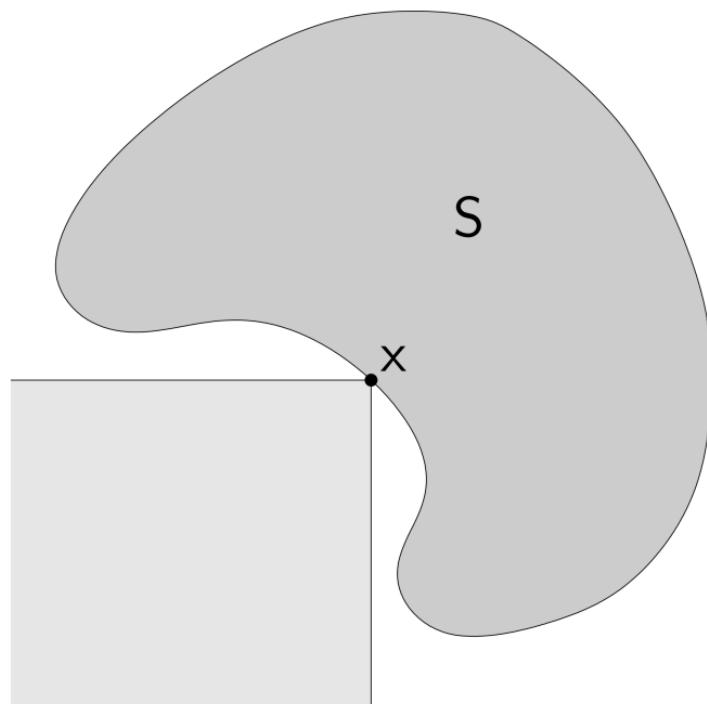


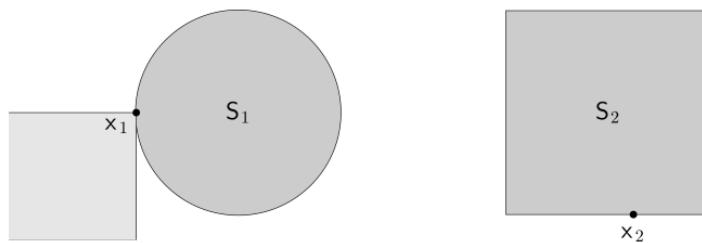


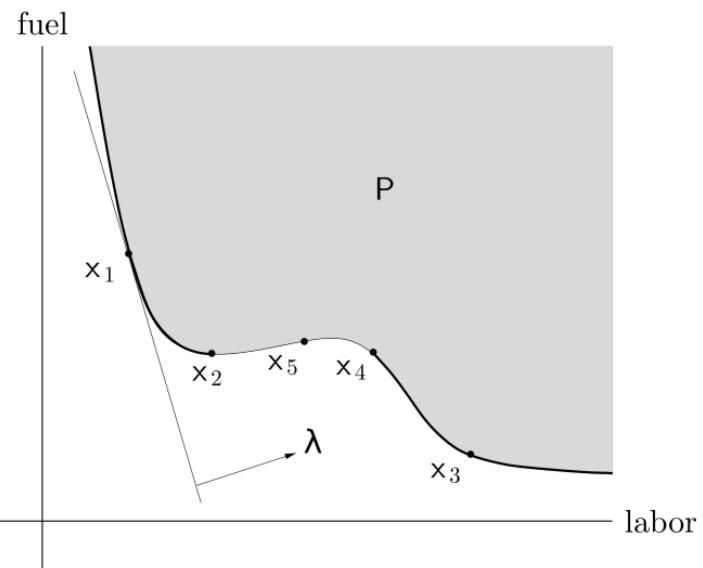












Chapter 3

Convex functions

3.1 Basic properties and examples

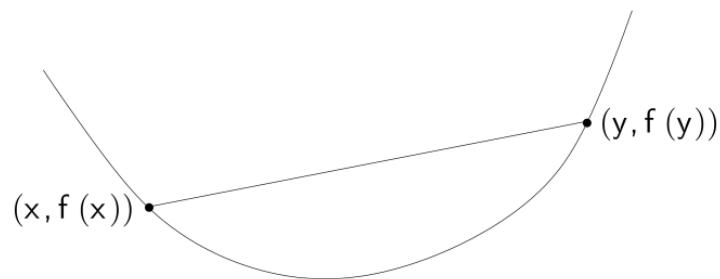
3.2 Operations that preserve convexity

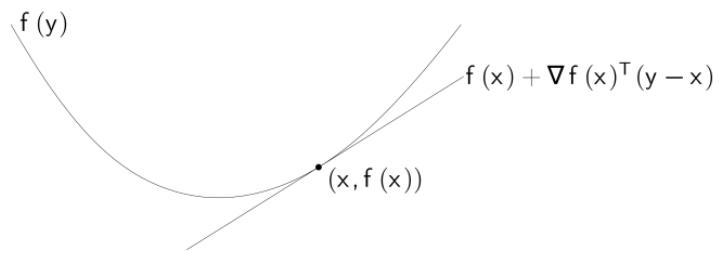
3.3 The conjugate function

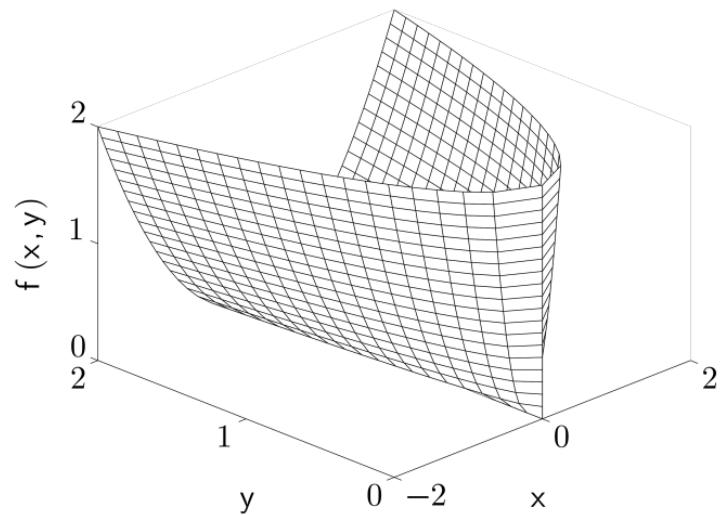
3.4 Quasiconvex functions

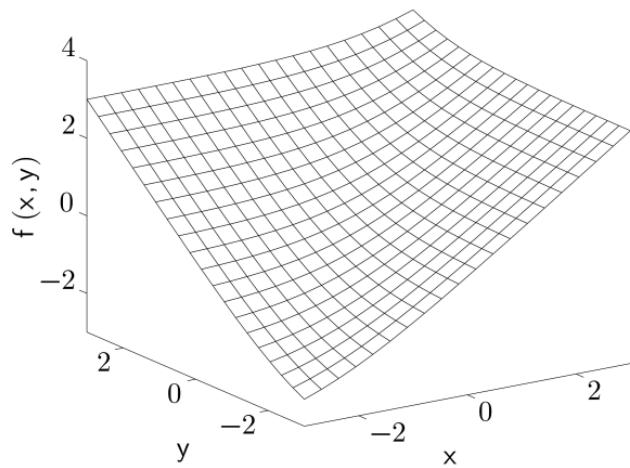
3.5 Log-concave and log-convex functions

3.6 Convexity with respect to generalized inequalities



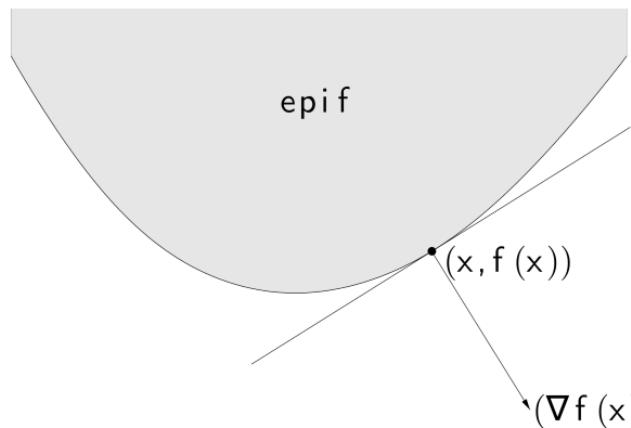


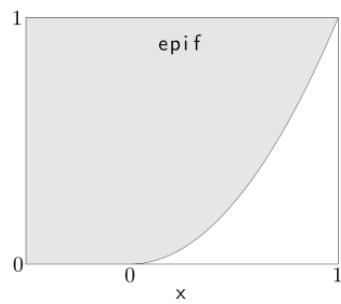
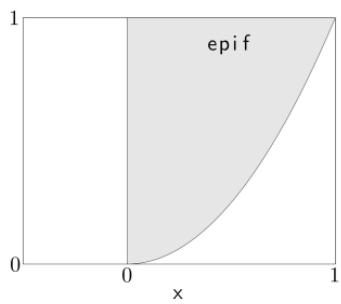


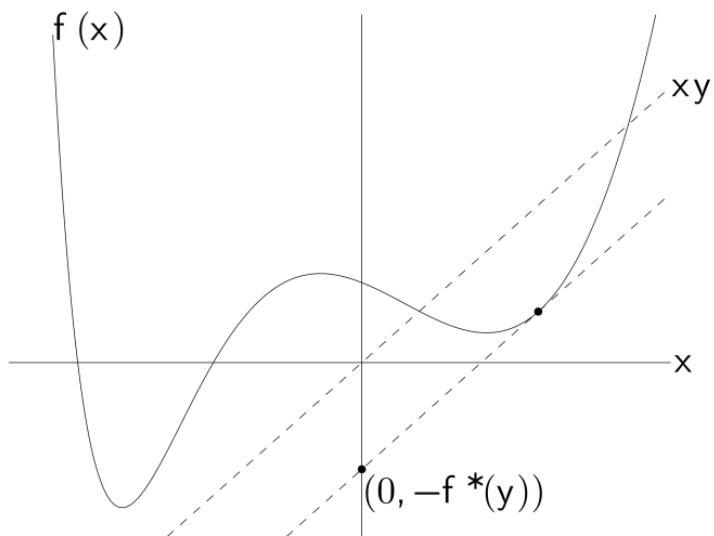


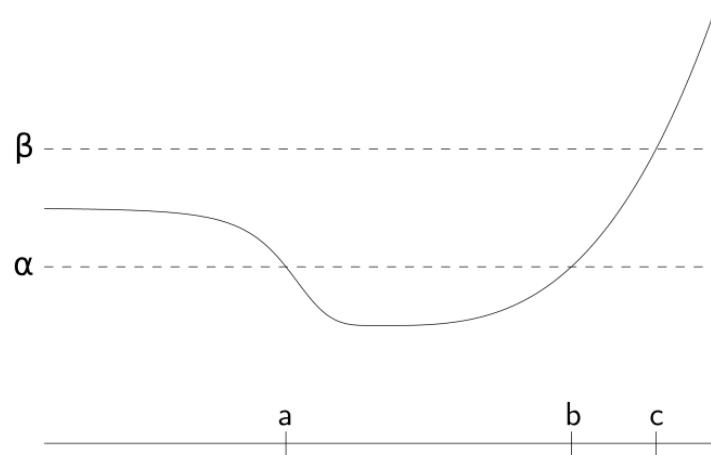
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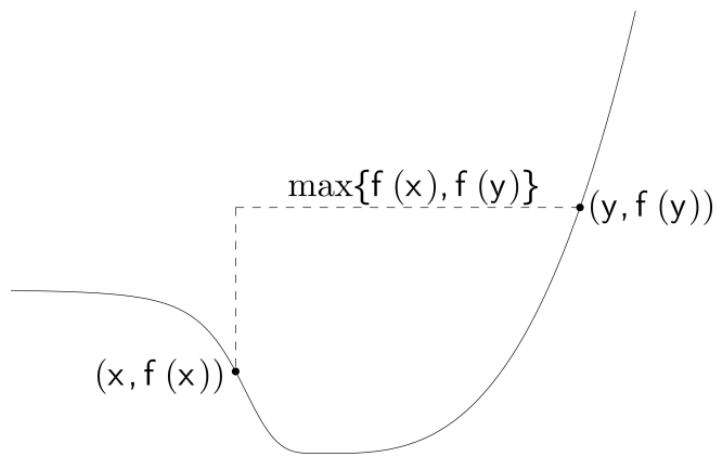
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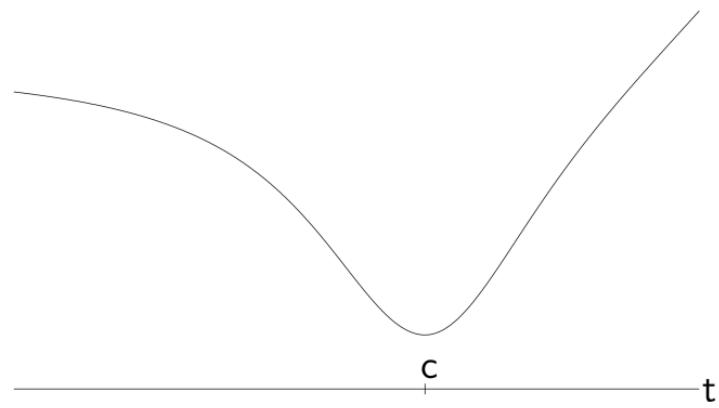


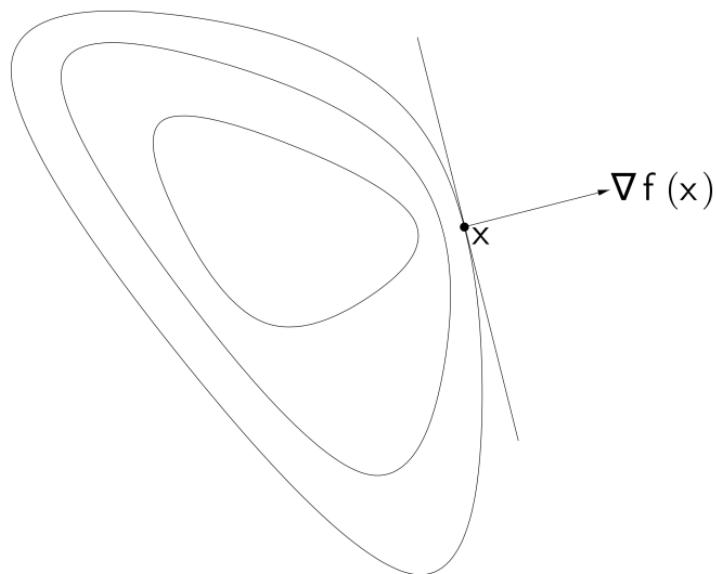


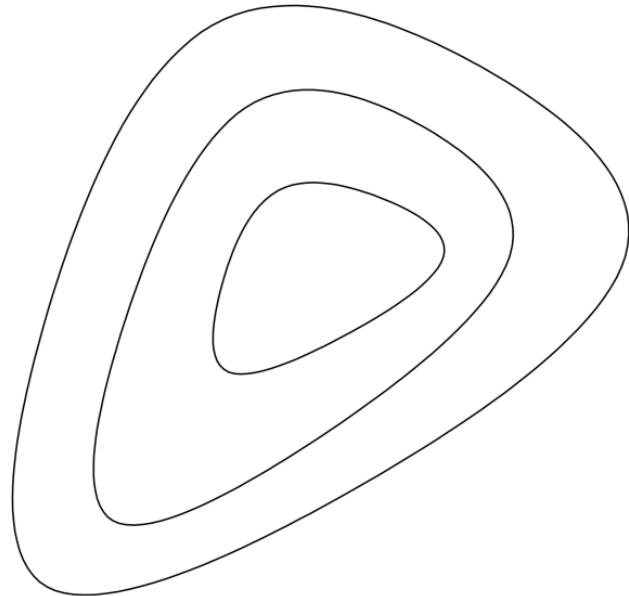














Chapter 4

Convex optimization problems

4.1 Optimization problems

4.2 Convex optimization

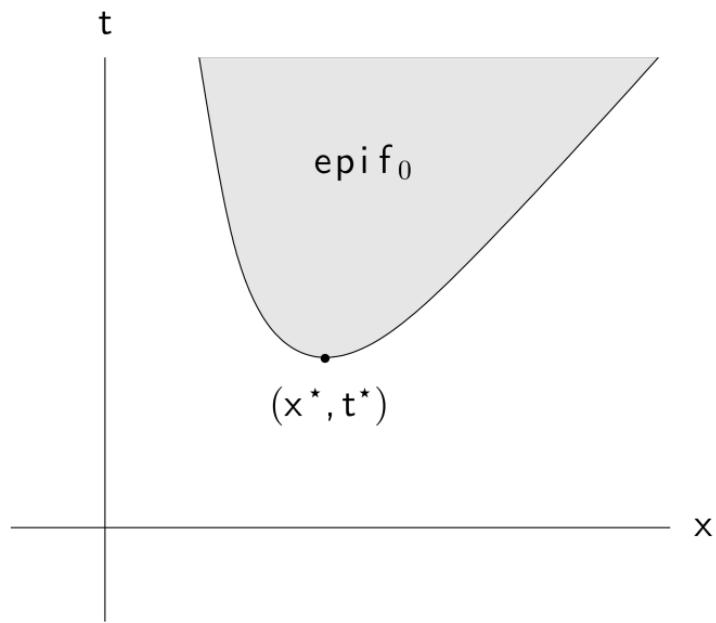
4.3 Linear optimization problems

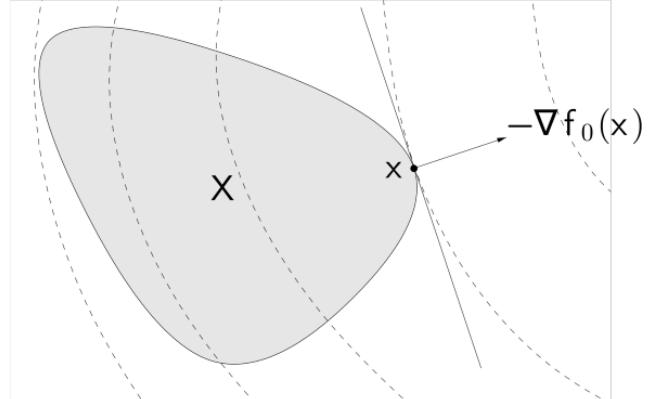
4.4 Quadratic optimization problems

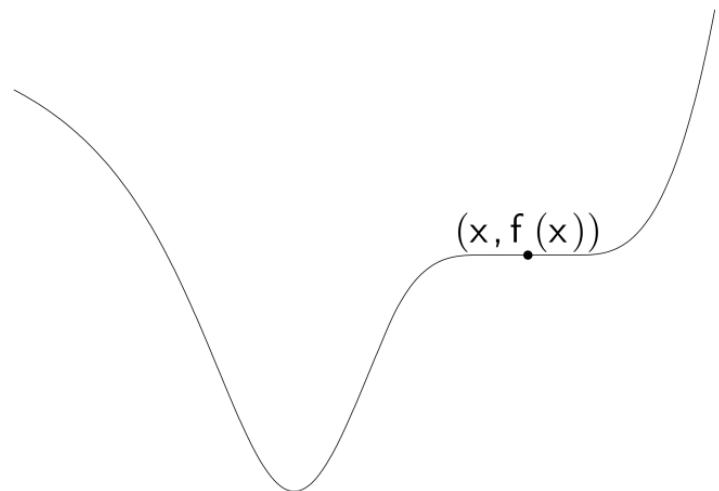
4.5 Geometric programming

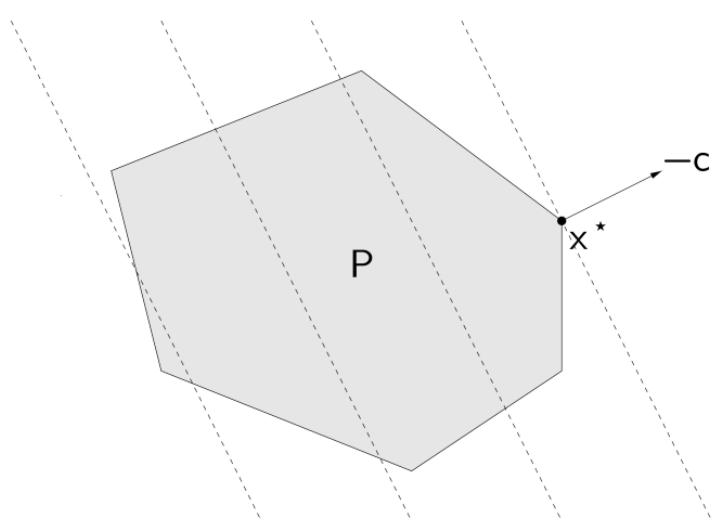
4.6 Generalized inequality constraints

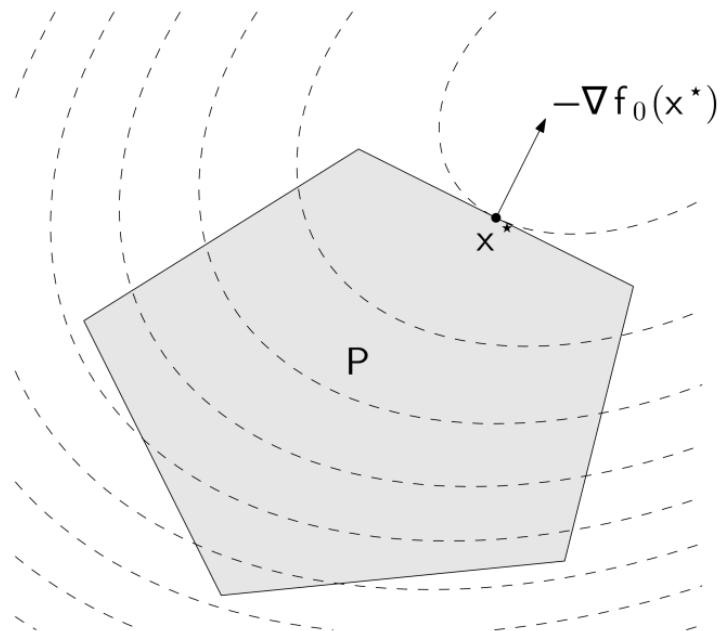
4.7 Vector optimization

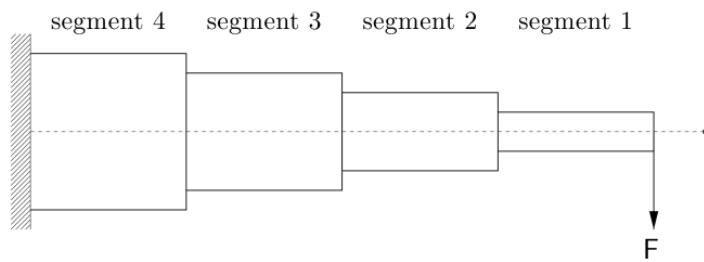


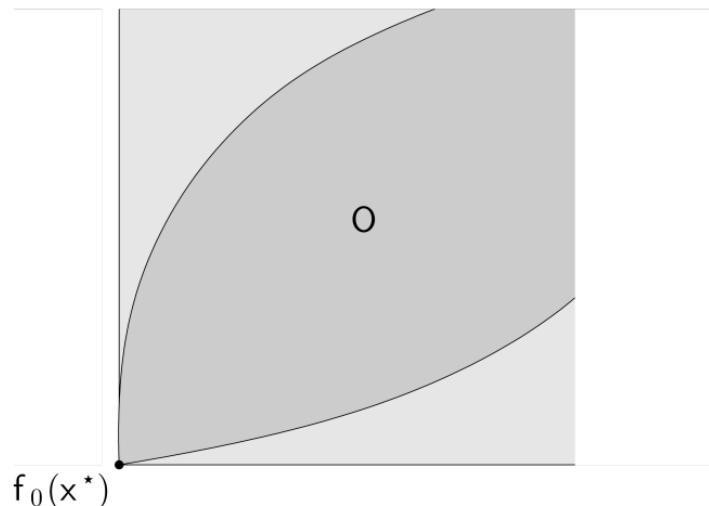


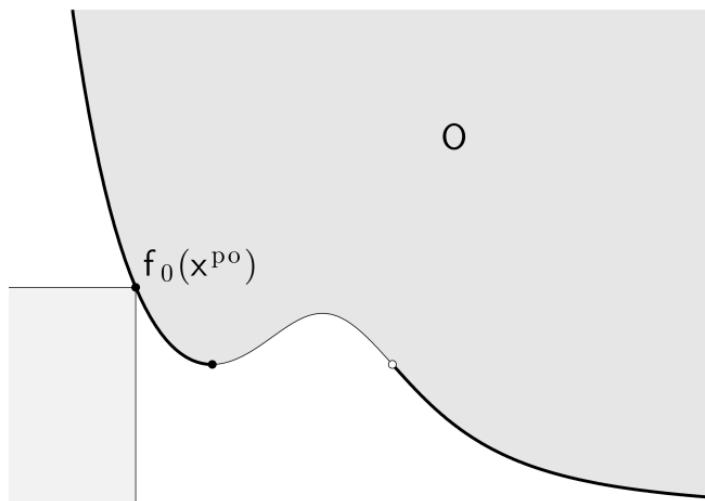


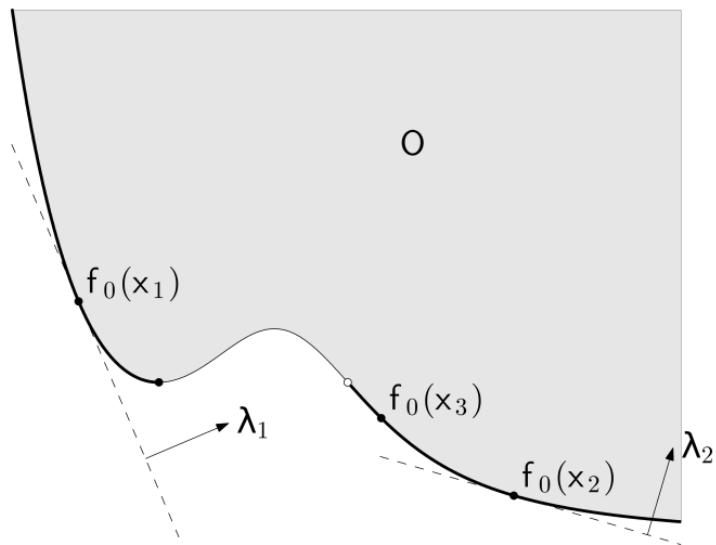


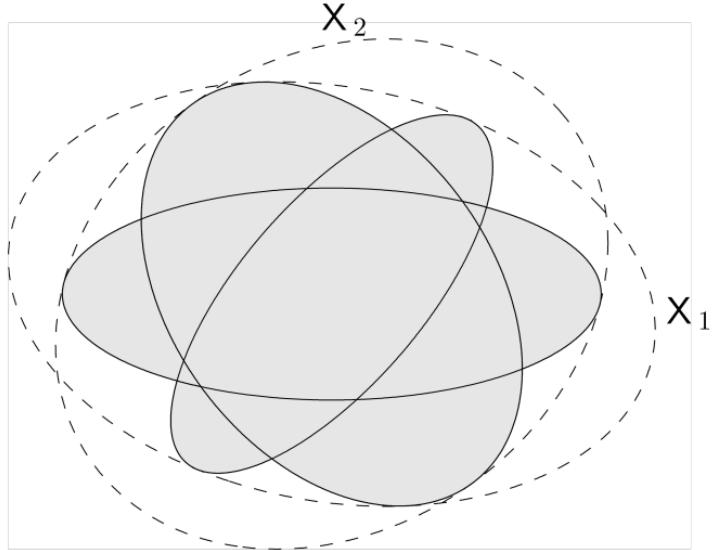


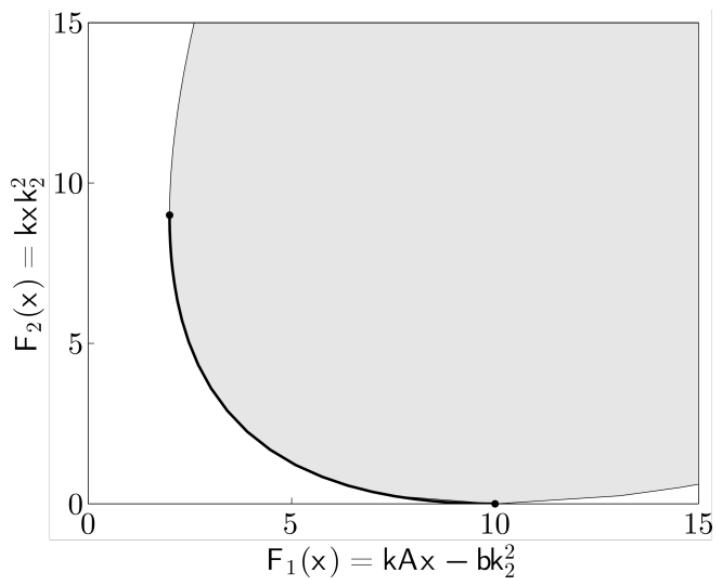


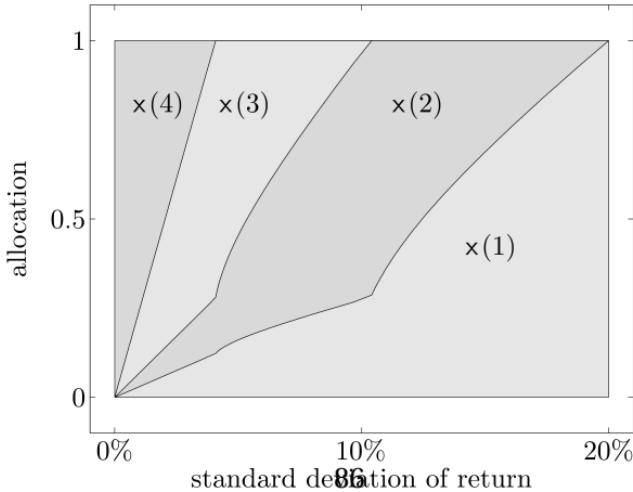
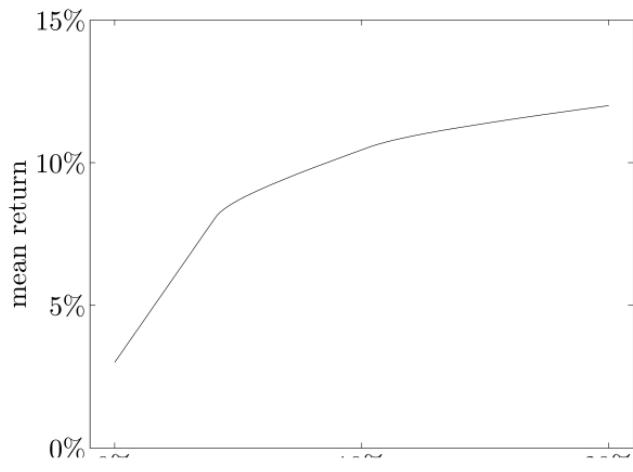


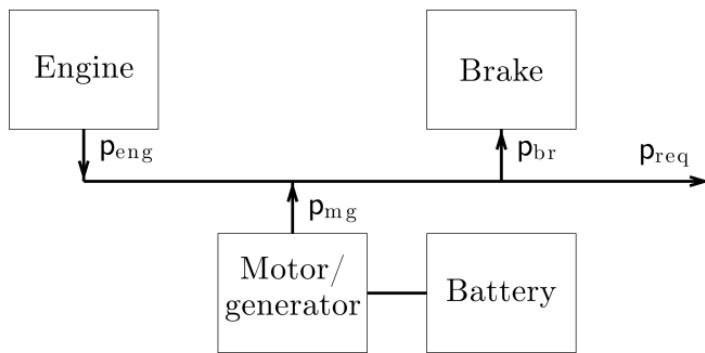












Chapter 5

Duality

5.1 The Lagrange dual function

5.2 The Lagrange dual problem

5.3 Geometric interpretation

5.4 Saddle-point interpretation

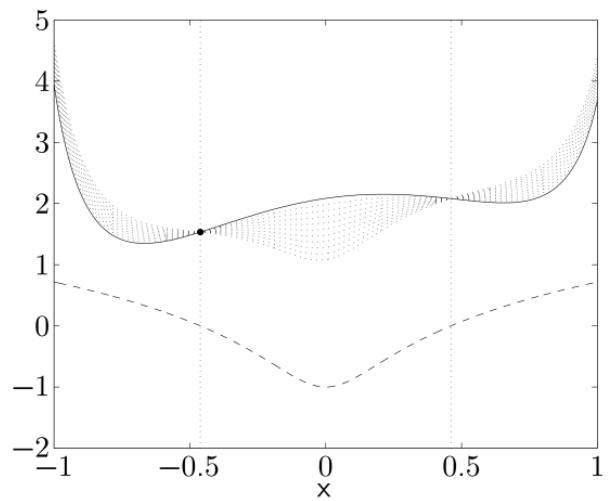
5.5 Optimality conditions

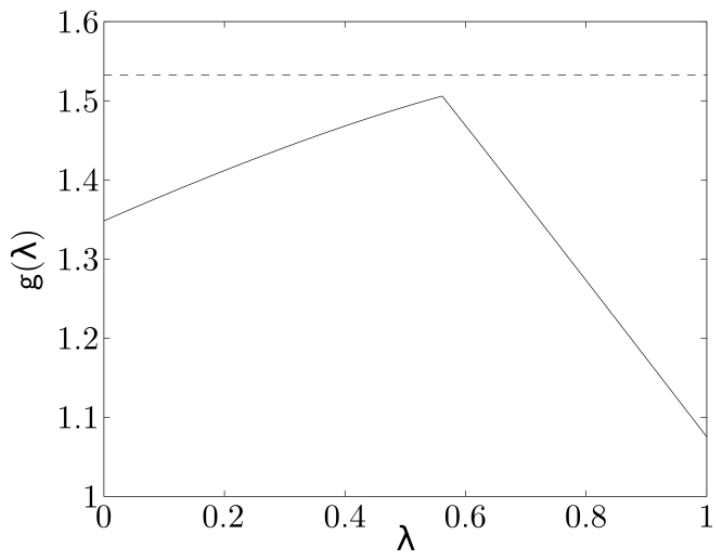
5.6 Perturbation and sensitivity analysis

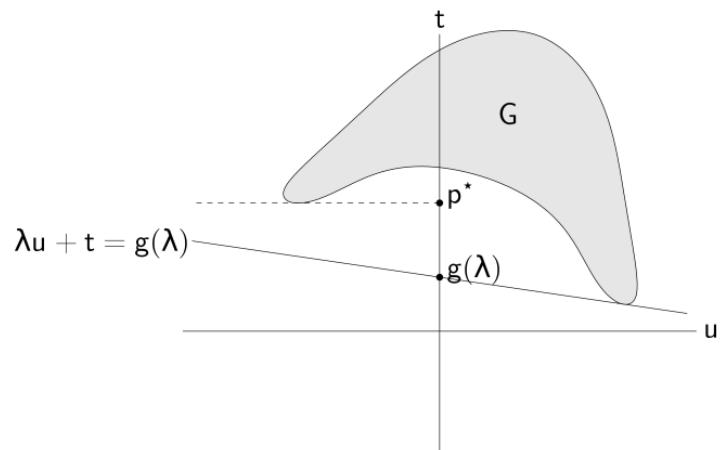
5.7 Examples

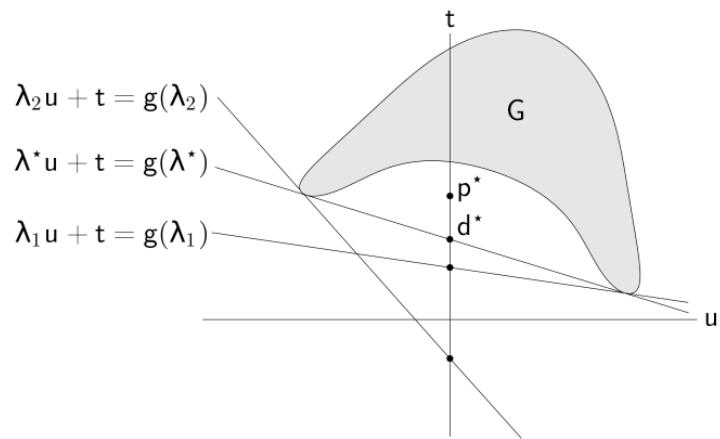
5.8 Theorems of alternatives

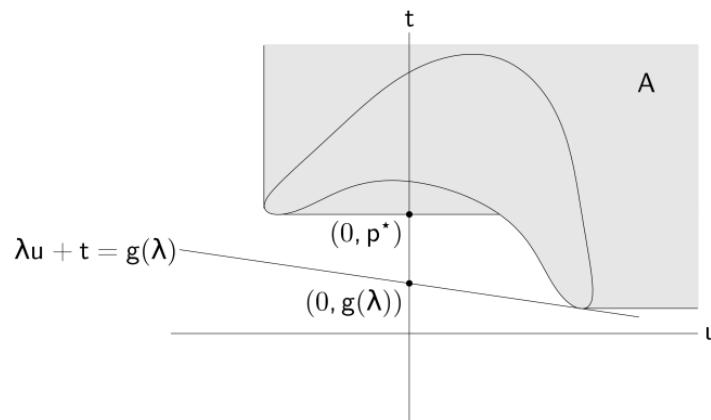
5.9 Generalized inequalities

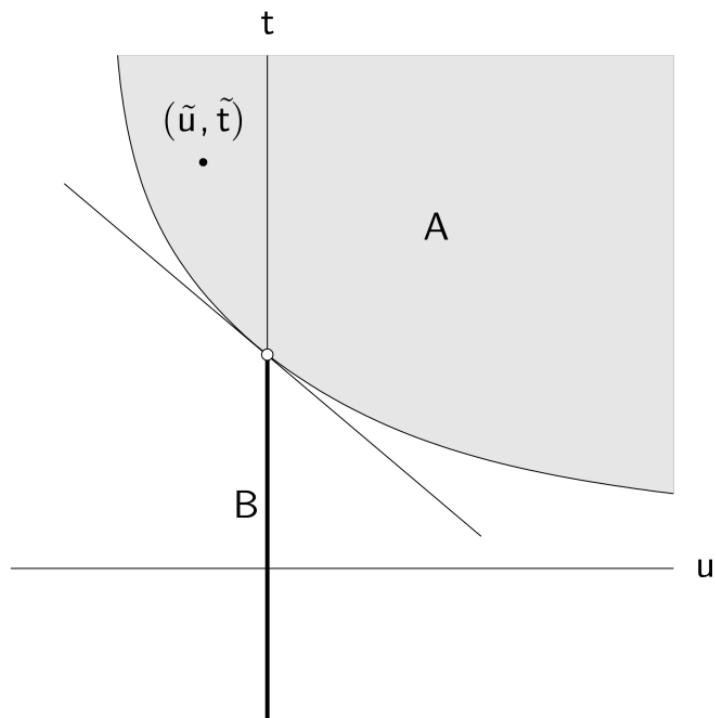


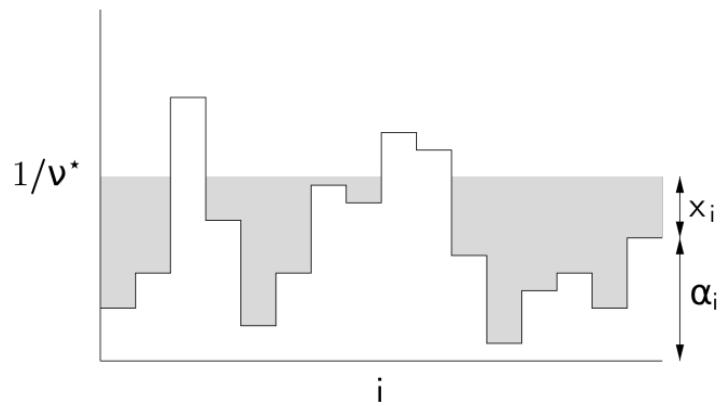


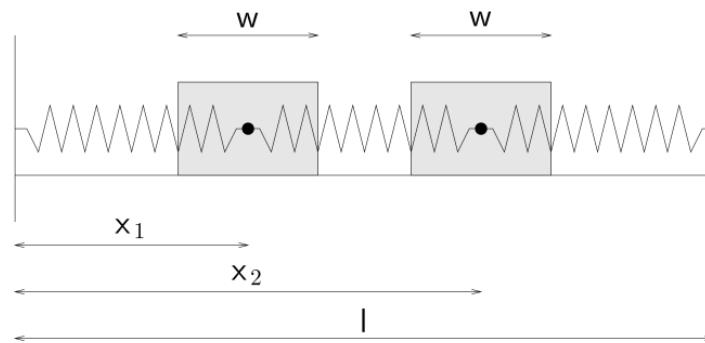


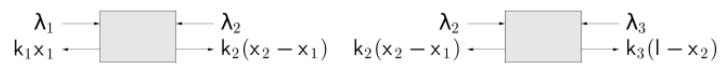


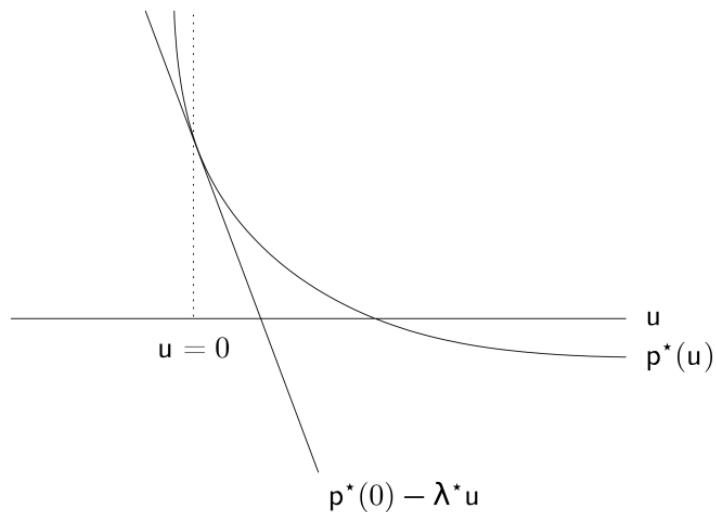












Chapter 6

Approximation and fitting

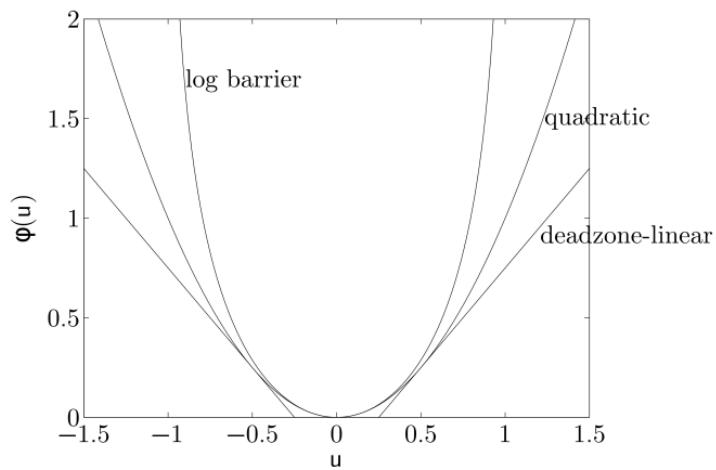
6.1 Norm approximation

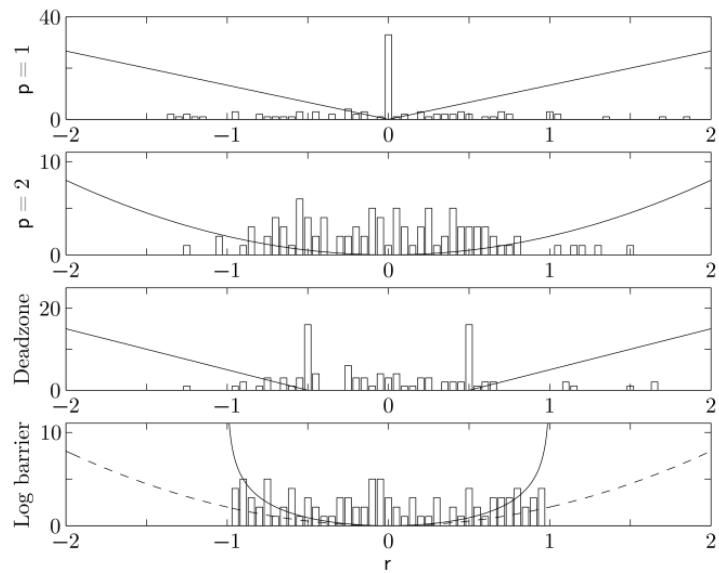
6.2 Least-norm problems

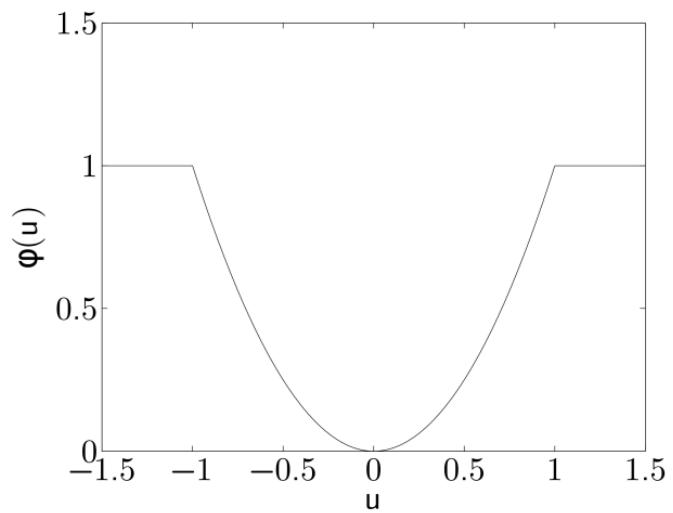
6.3 Regularized approximation

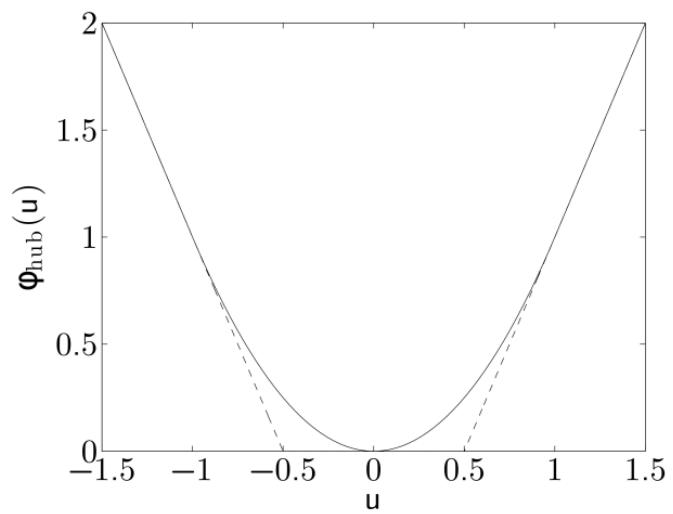
6.4 Robust approximation

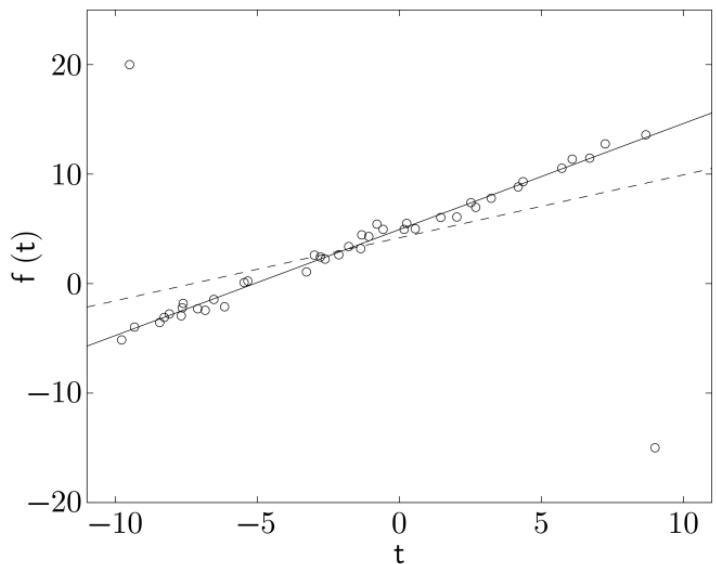
6.5 Function fitting and interpolation

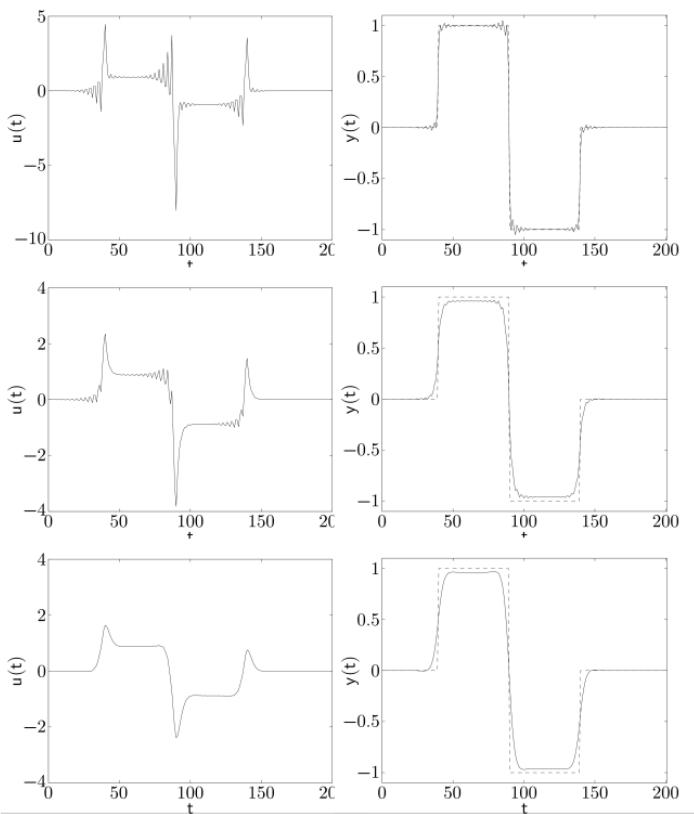


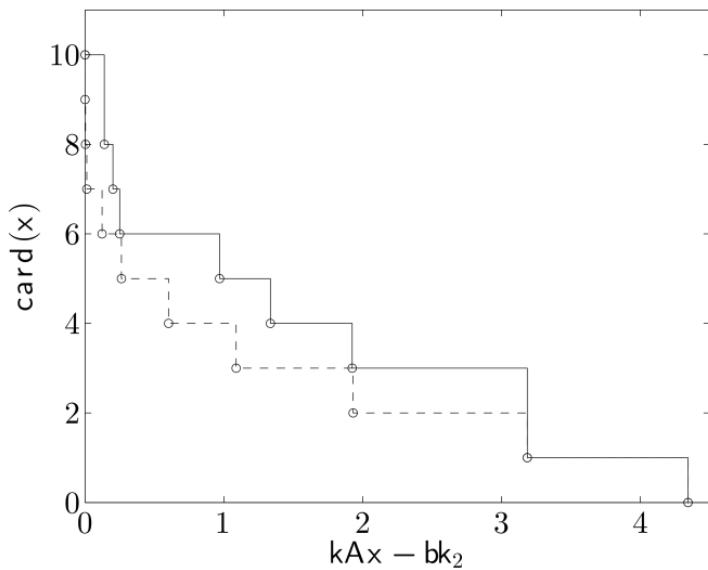


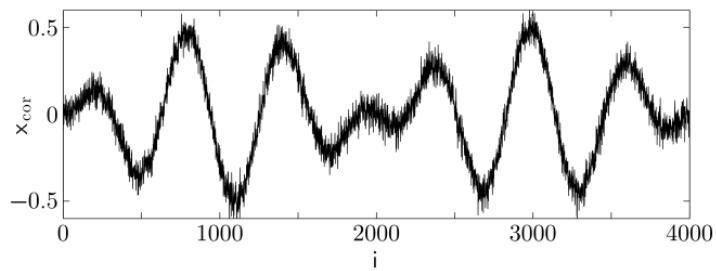
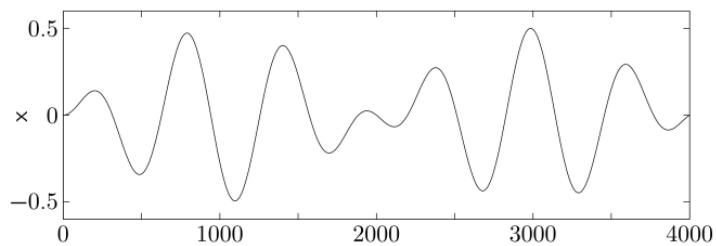


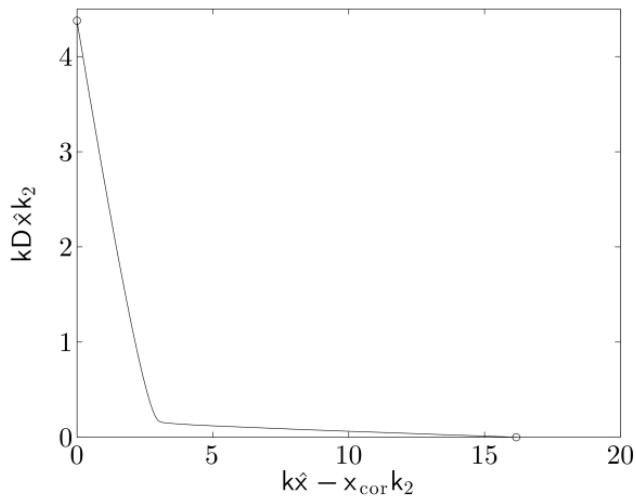


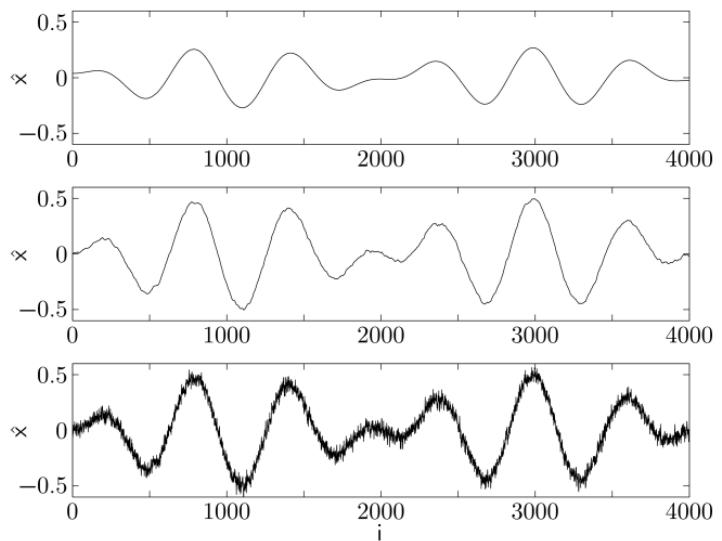


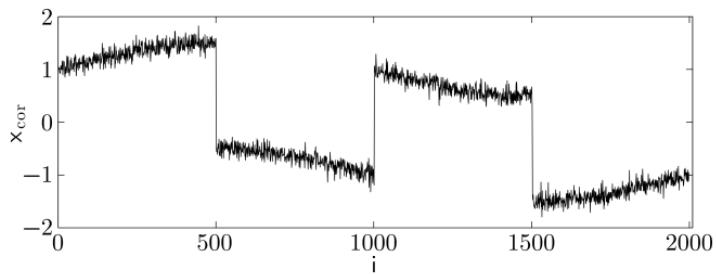
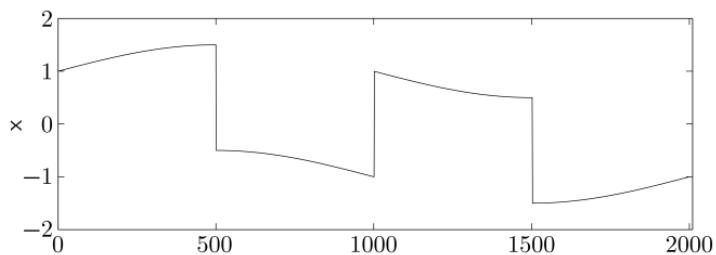


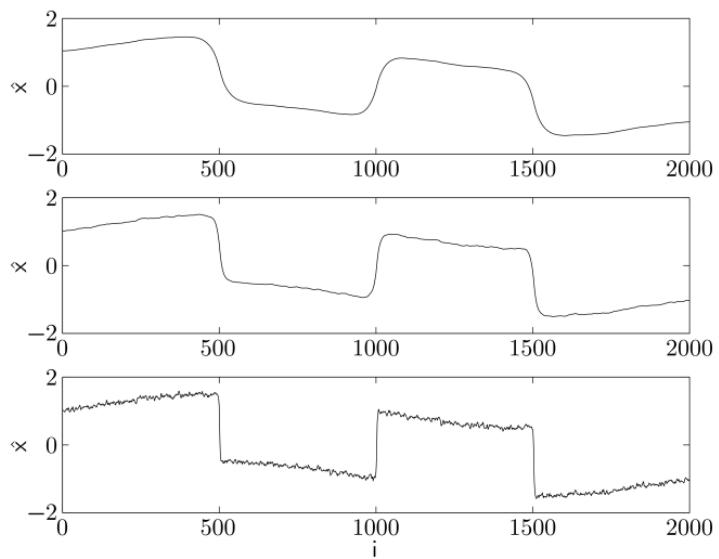


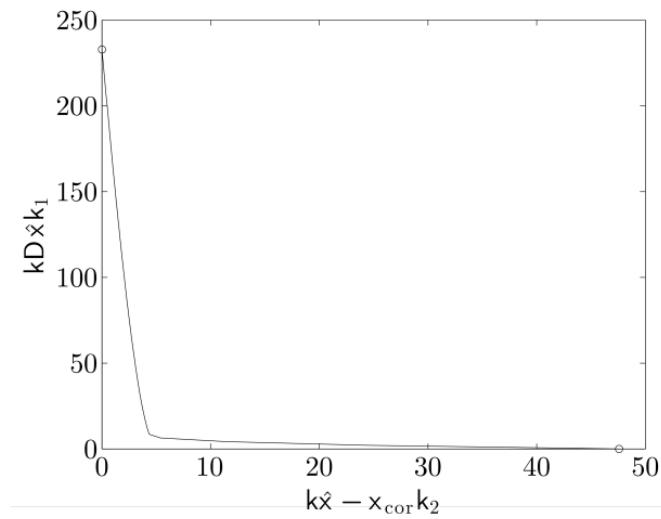


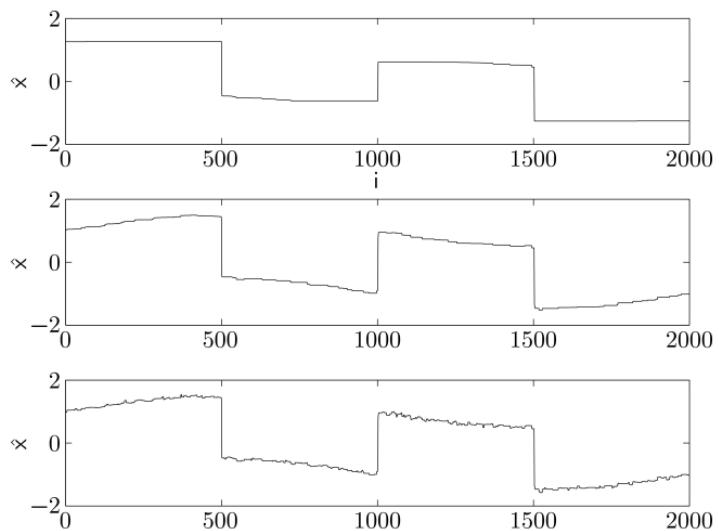


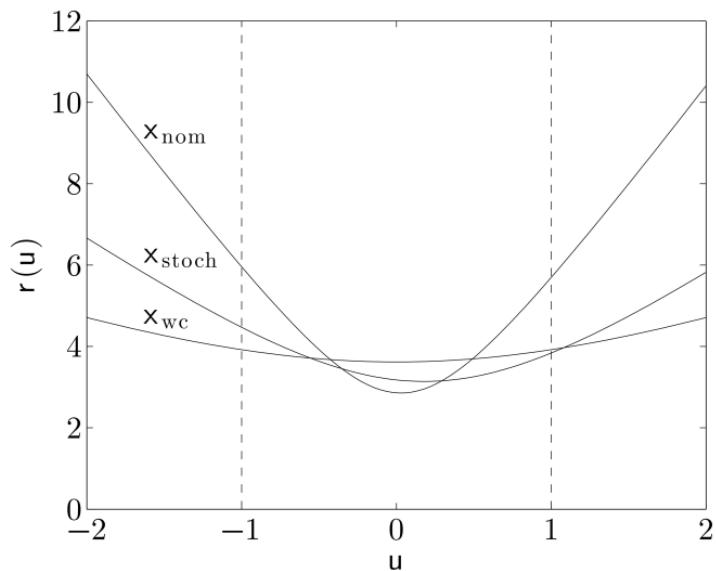


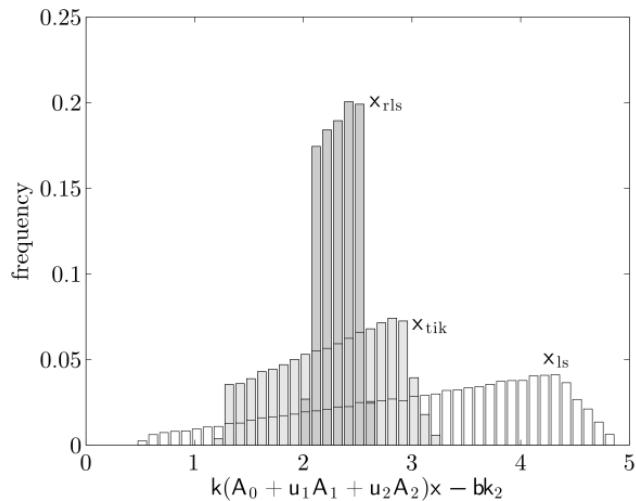


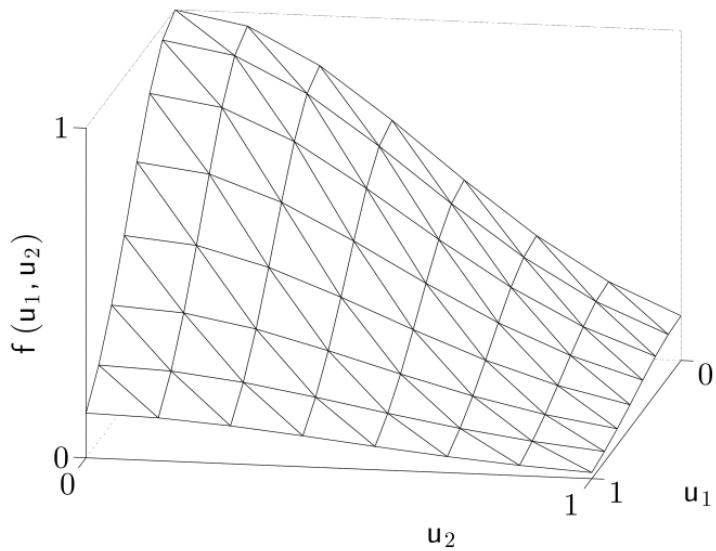


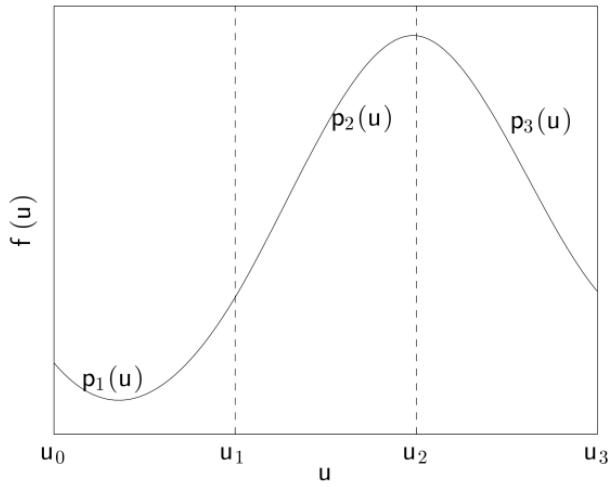


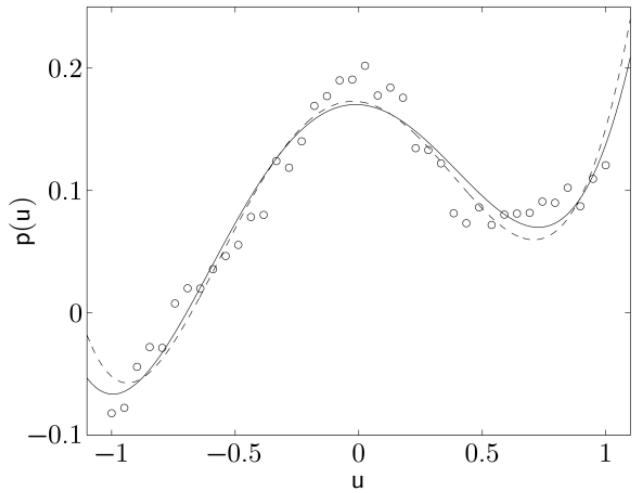


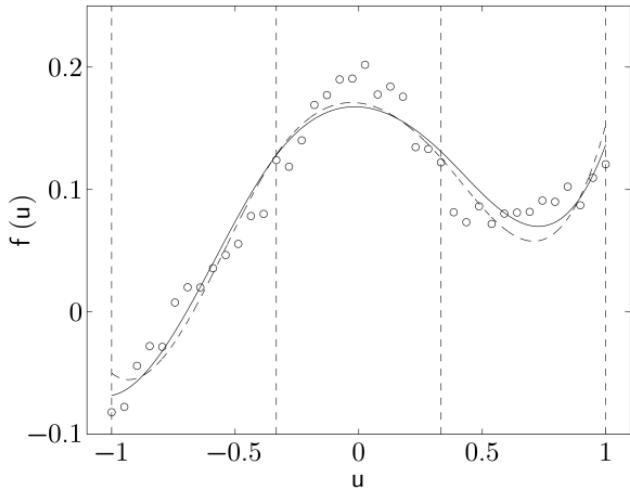


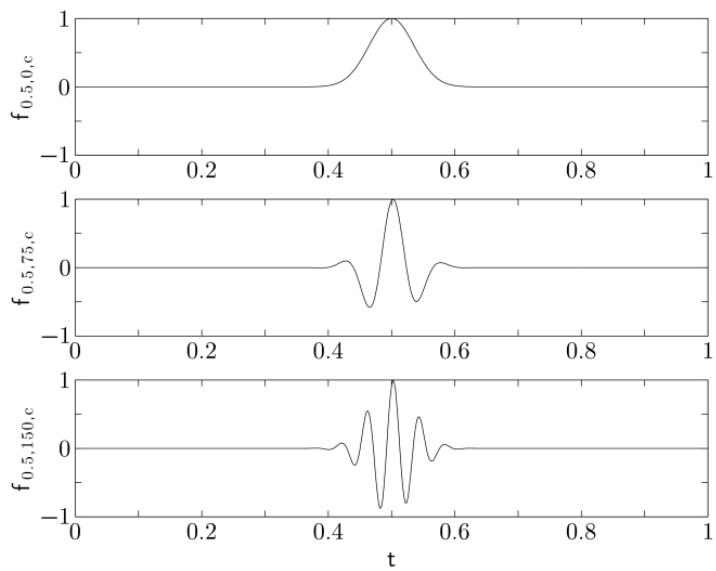


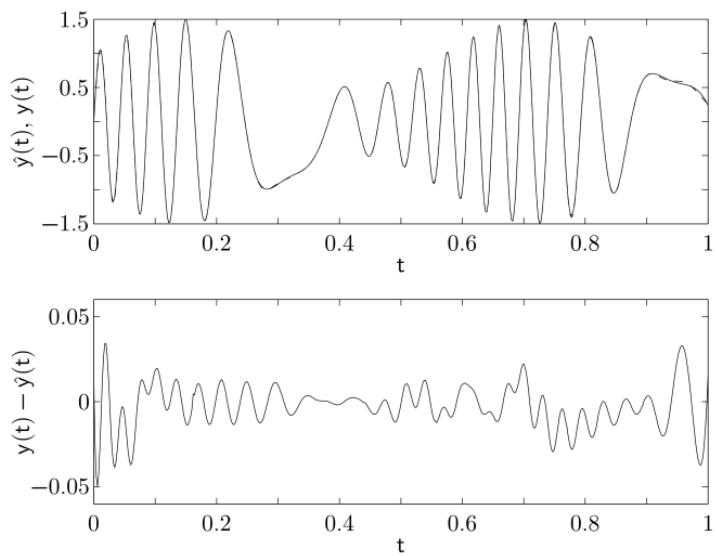


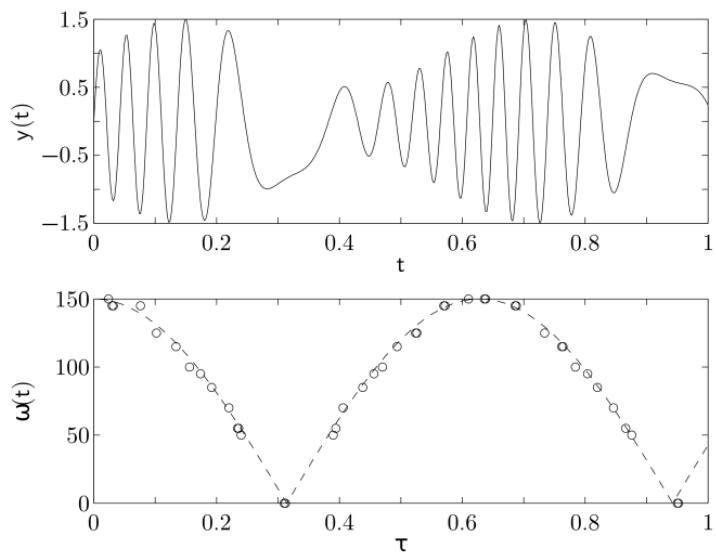


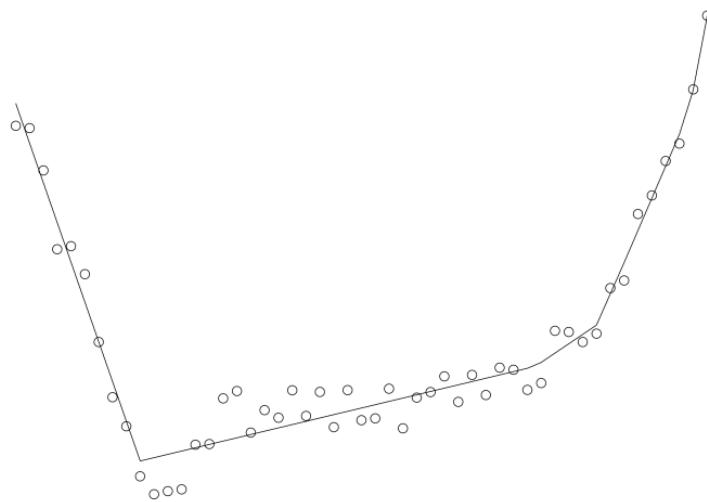


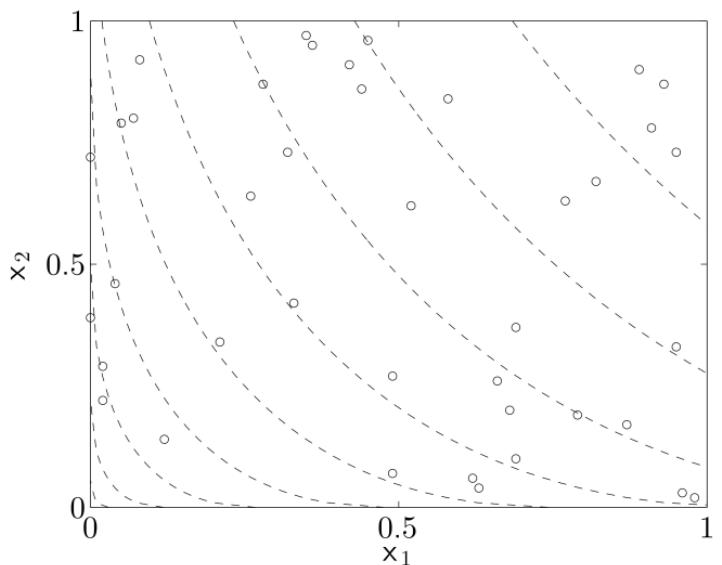


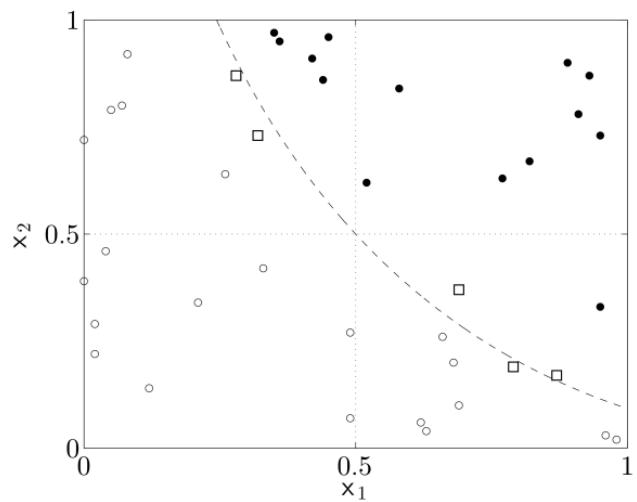












Chapter 7

Statistical estimation

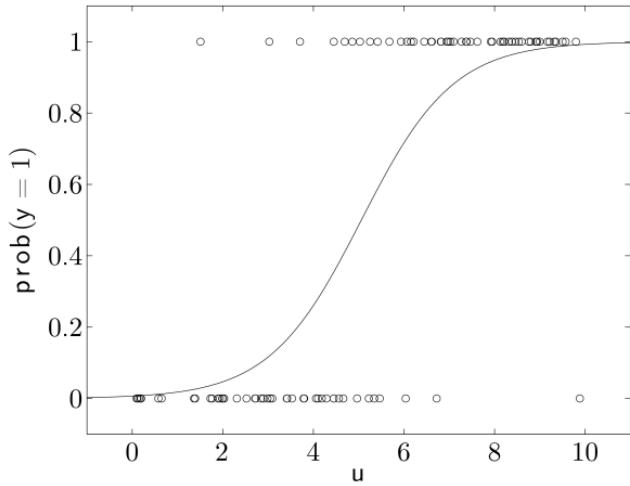
7.1 Parametric distribution estimation

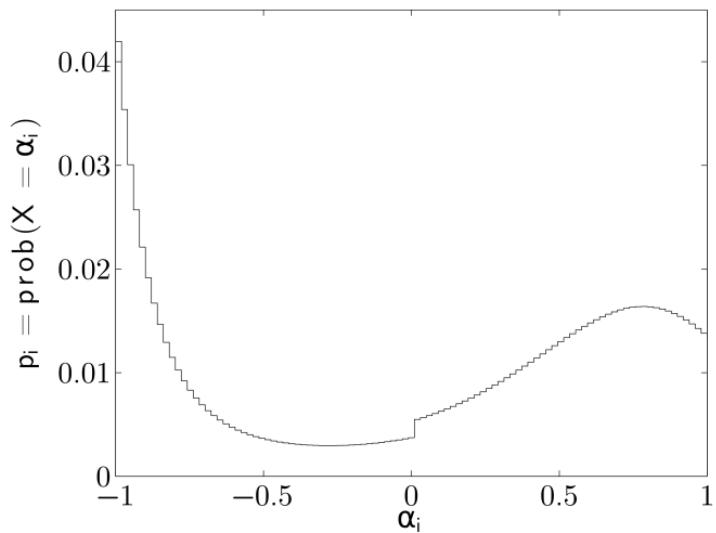
7.2 Nonparametric distribution estimation

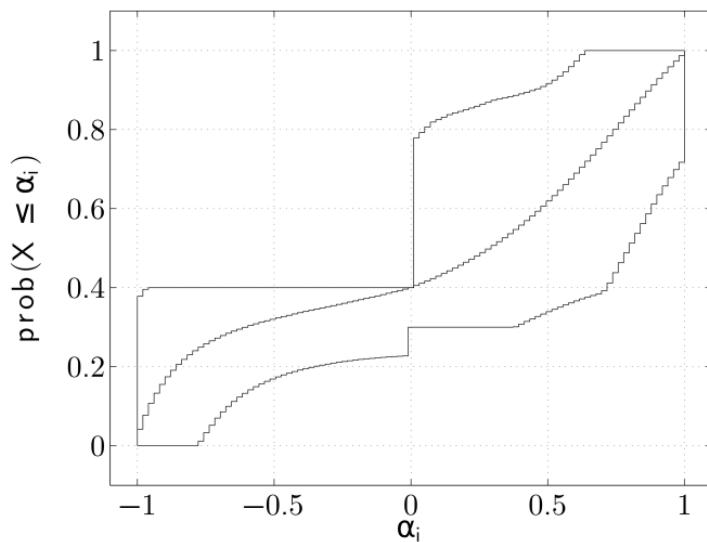
7.3 Optimal detector design and hypothesis testing

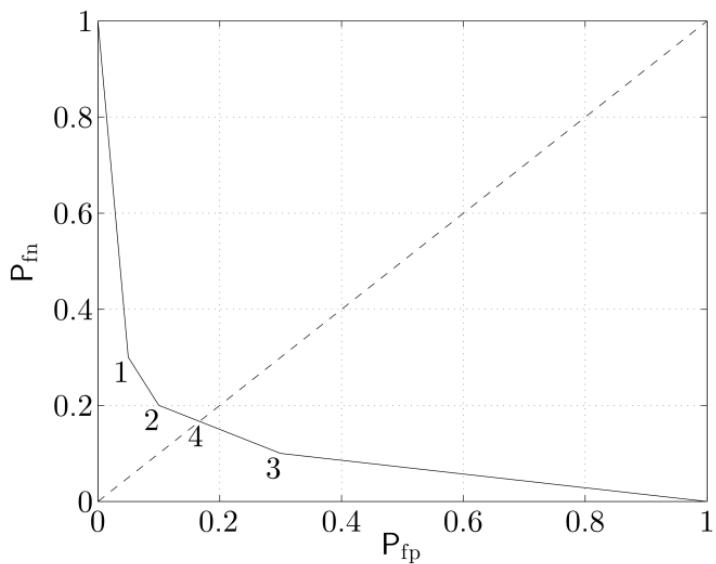
7.4 Chebyshev and Chernoff bounds

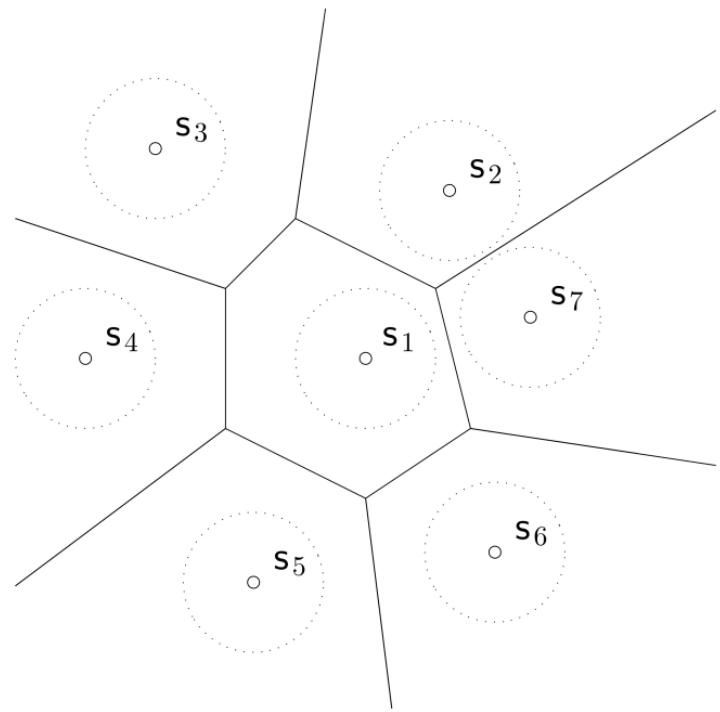
7.5 Experiment design

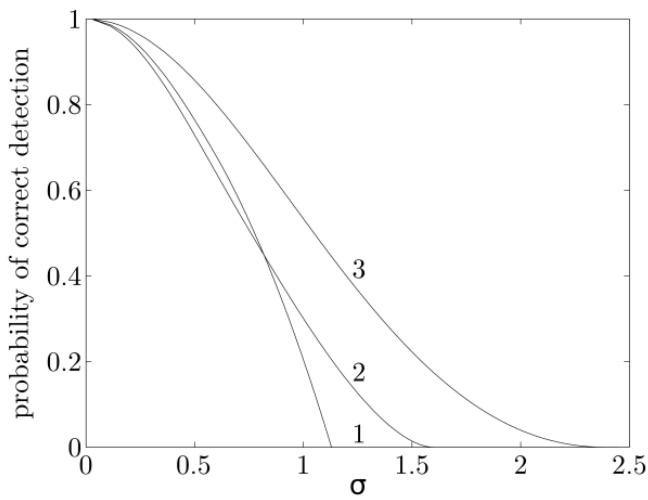


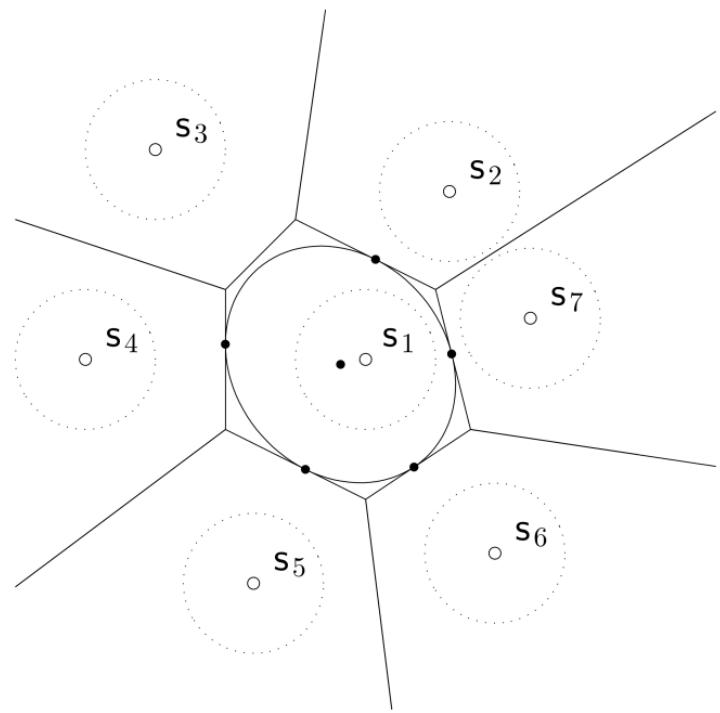


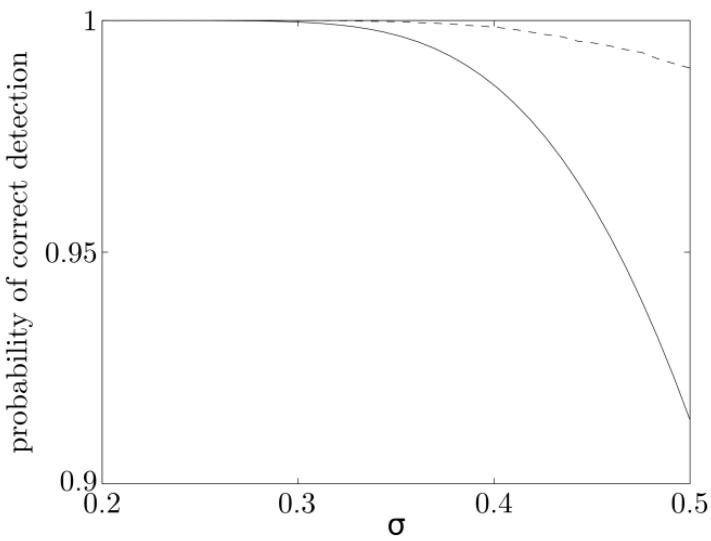


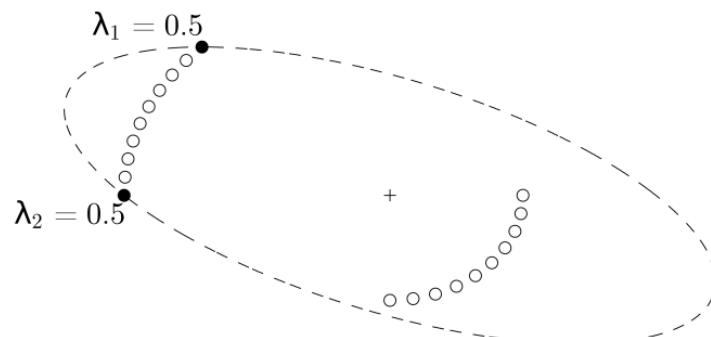


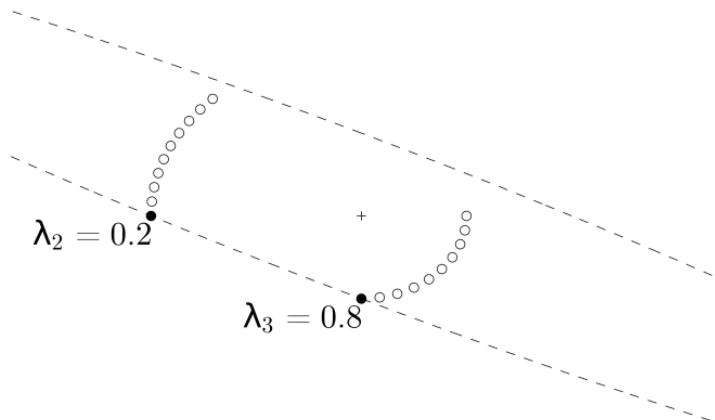


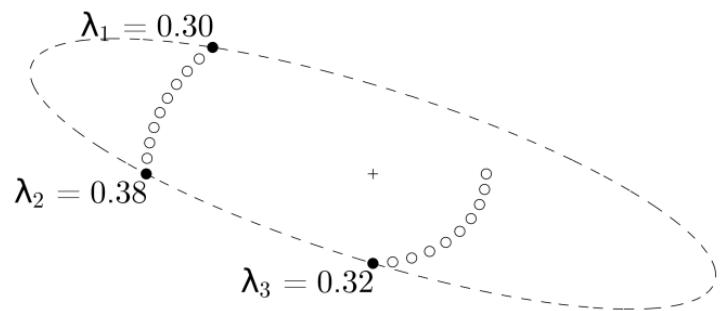


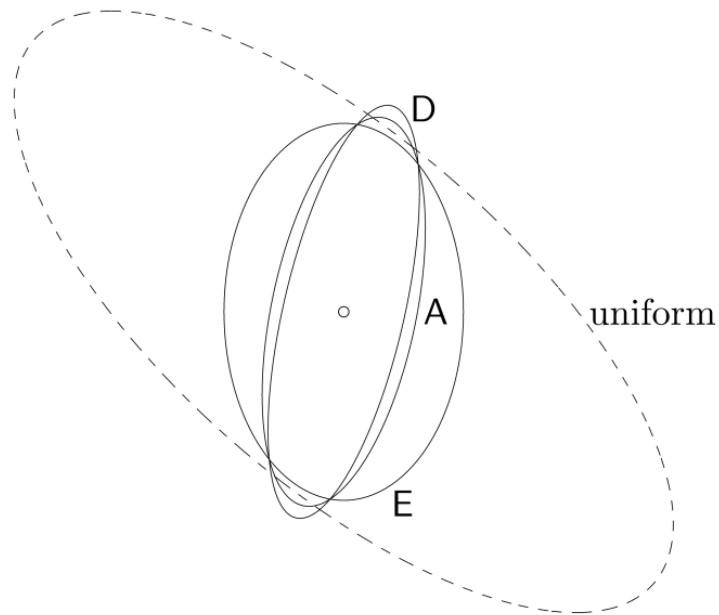












Chapter 8

Geometric problems

8.1 Projection on a set

8.2 Distance between sets

8.3 Euclidean distance and angle problems

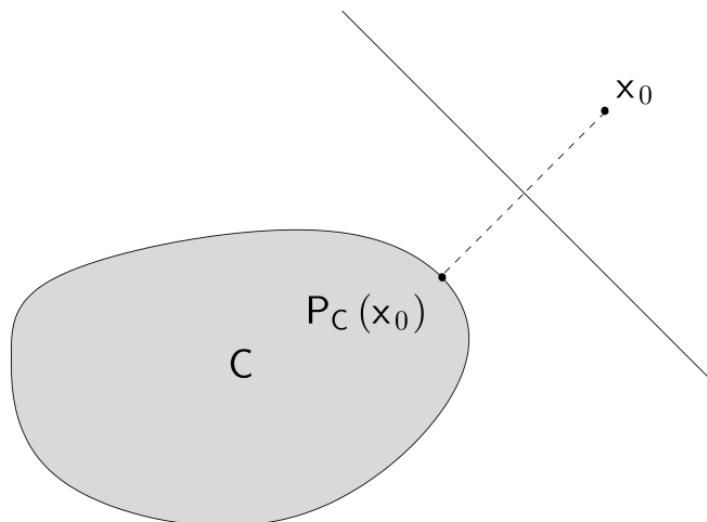
8.4 Extremal volume ellipsoids

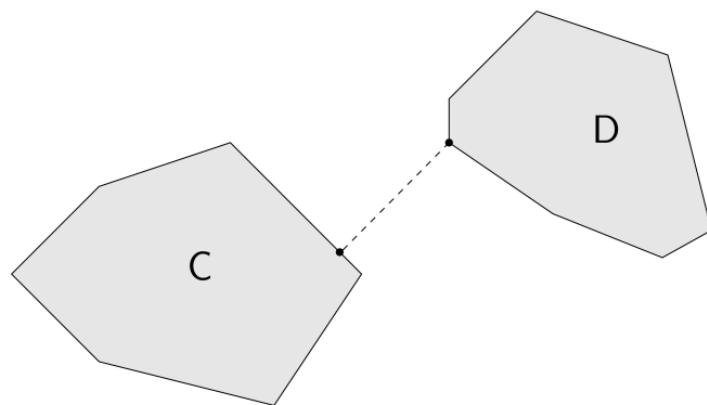
8.5 Centering

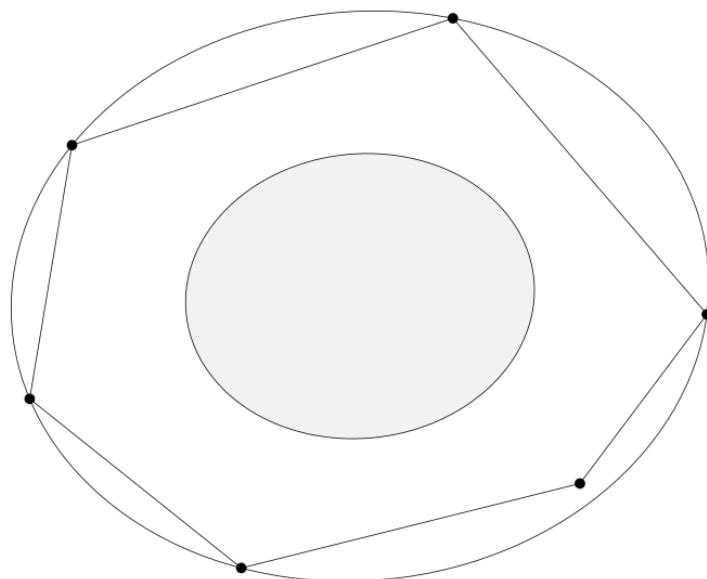
8.6 Classification

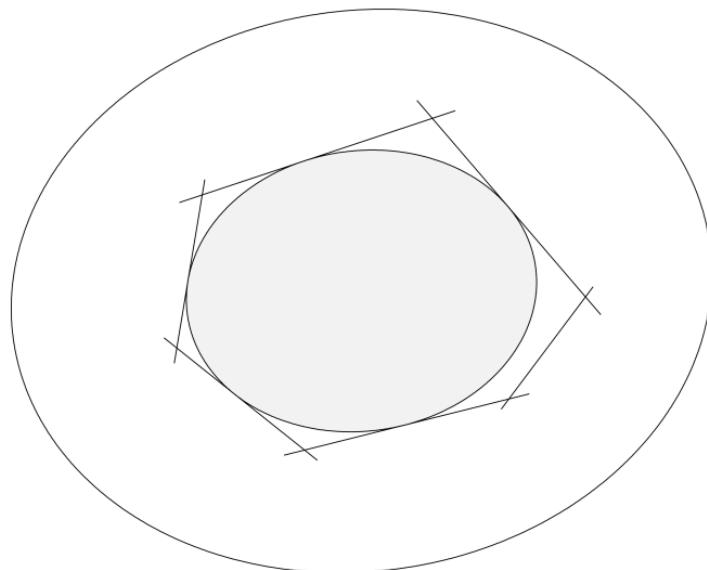
8.7 Placement and location

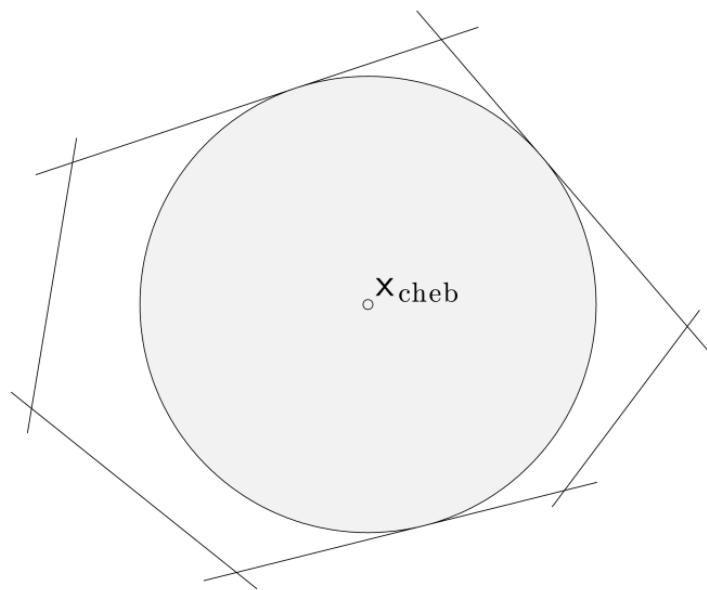
8.8 Floor planning

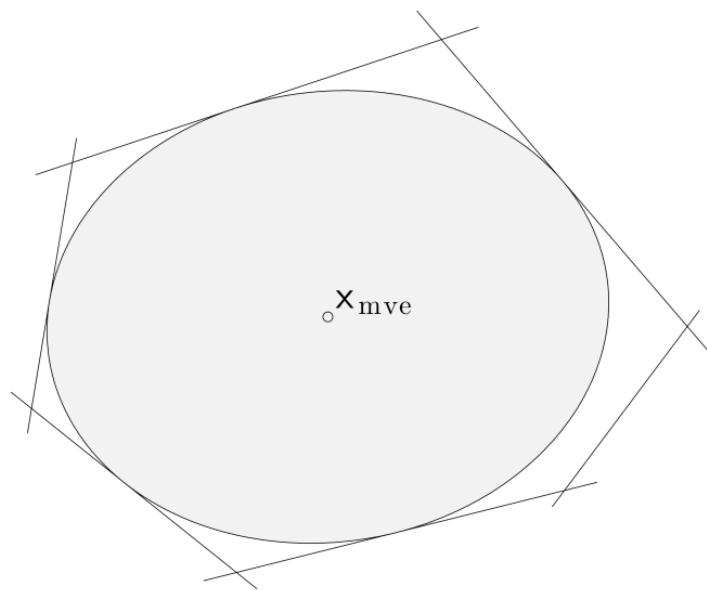


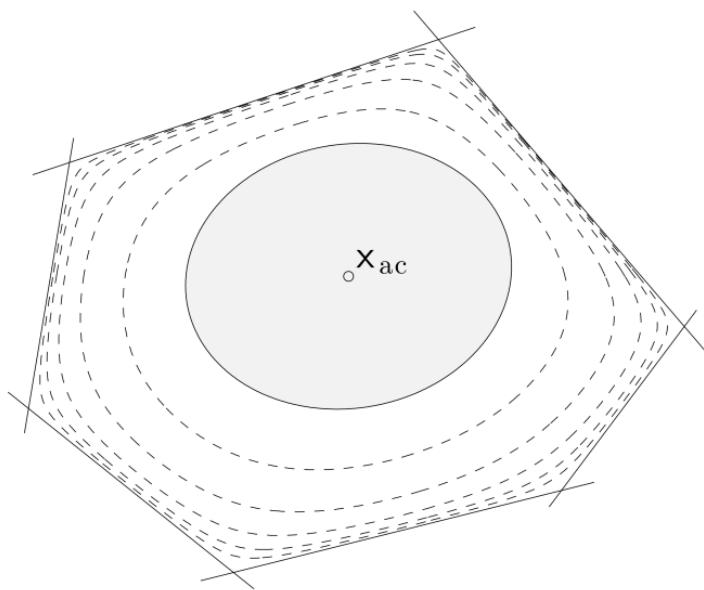


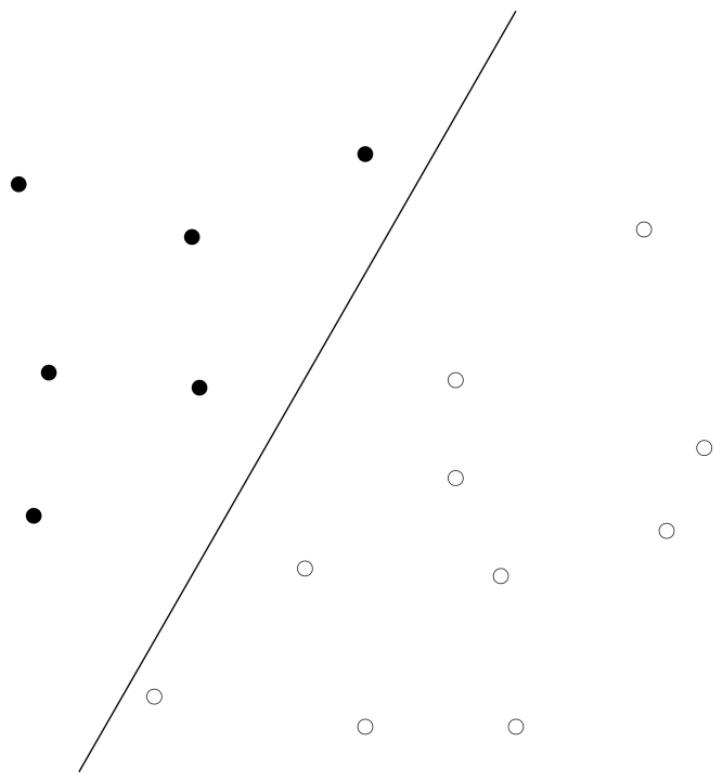


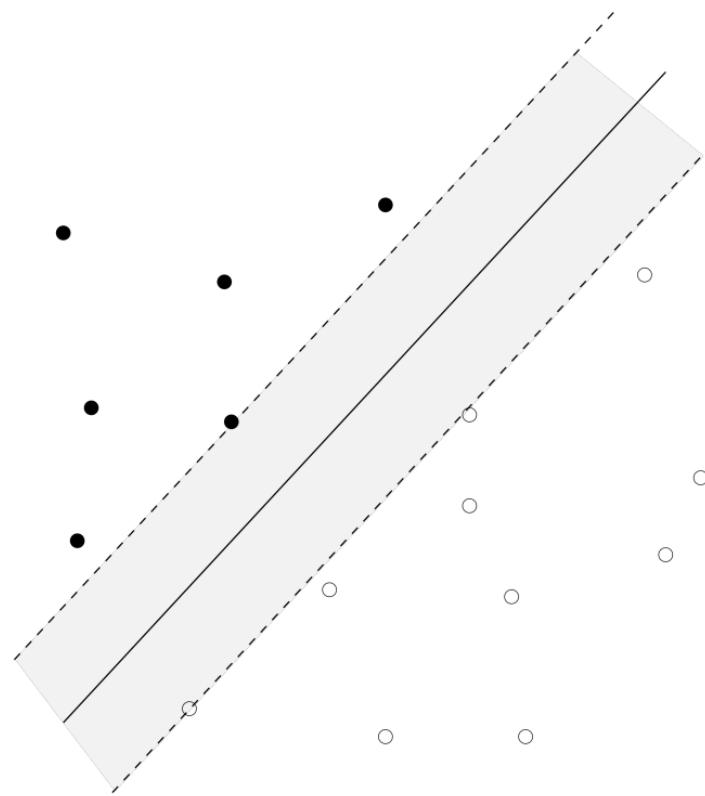


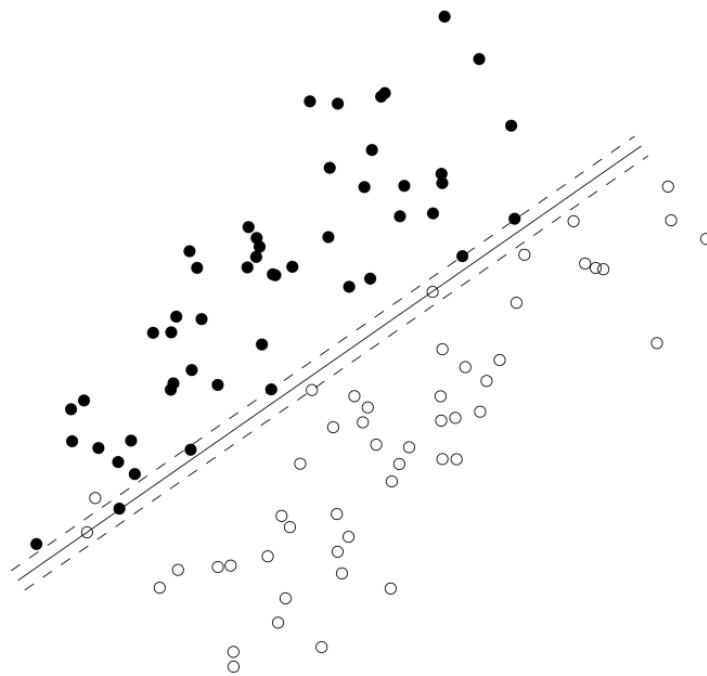


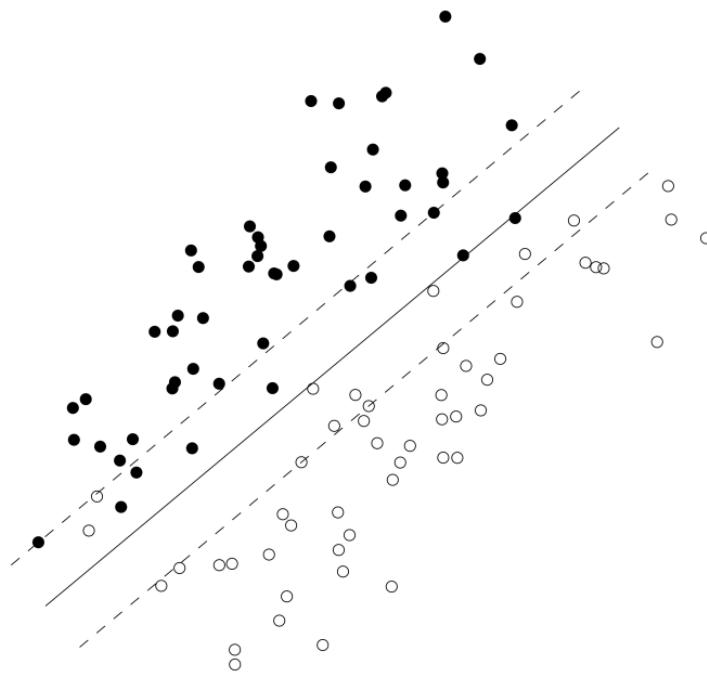


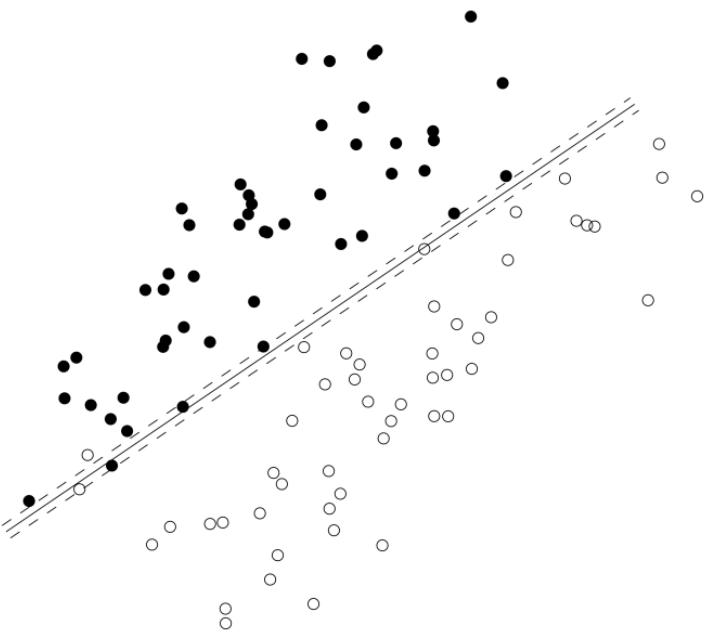


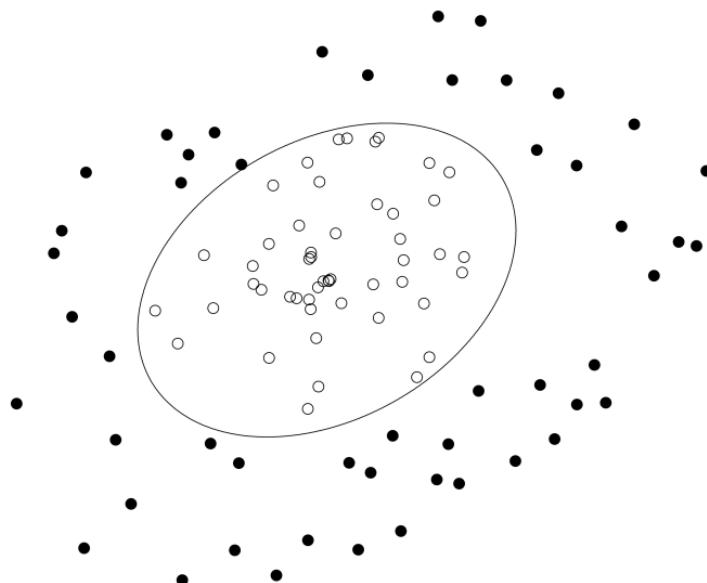


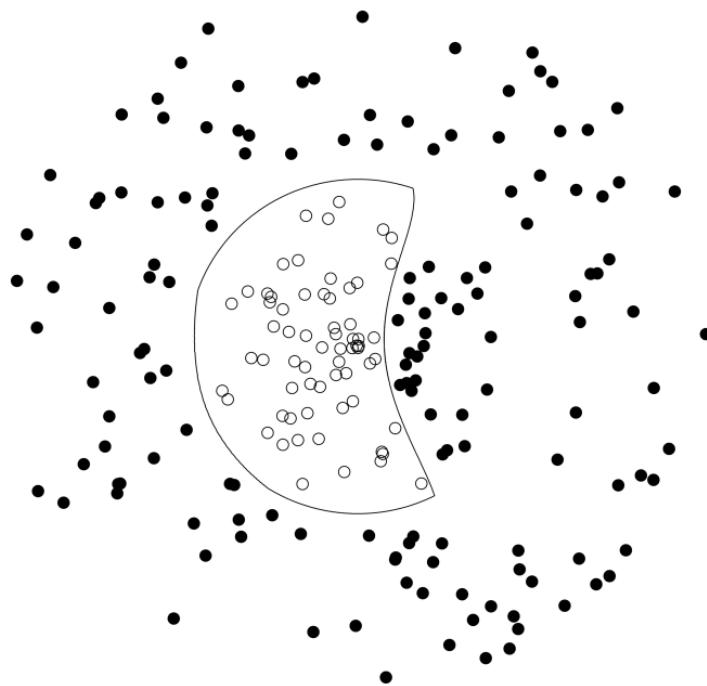


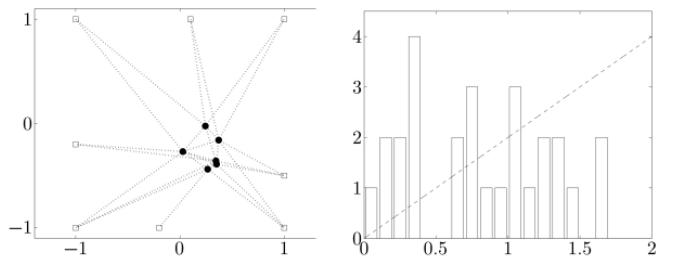


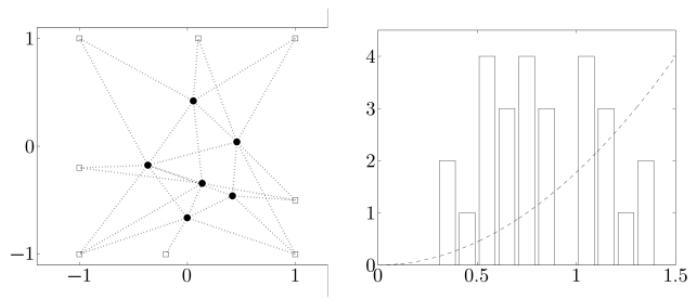


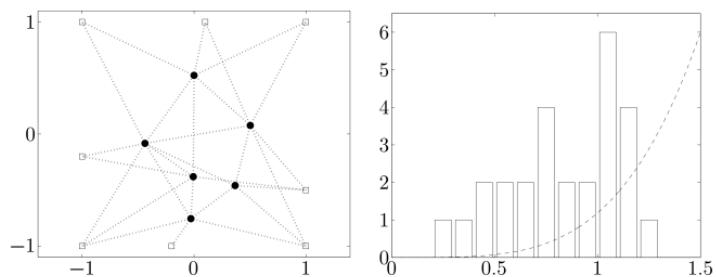


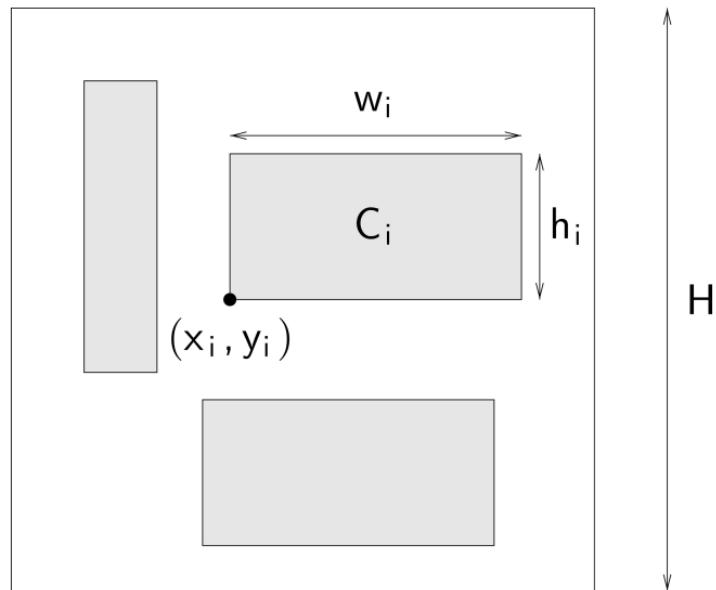




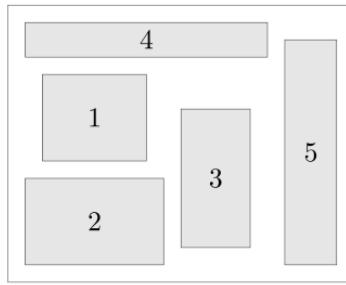
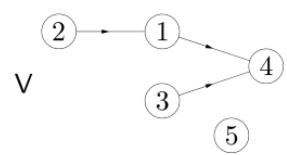
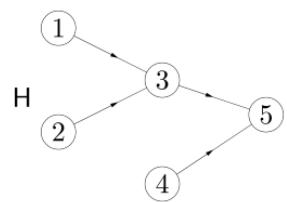


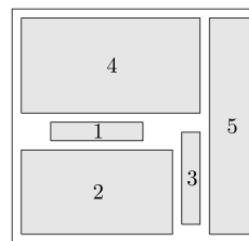
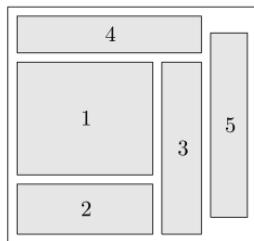
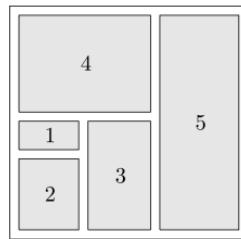
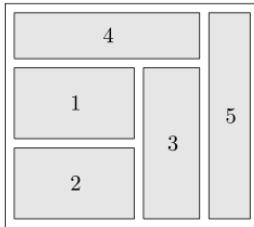


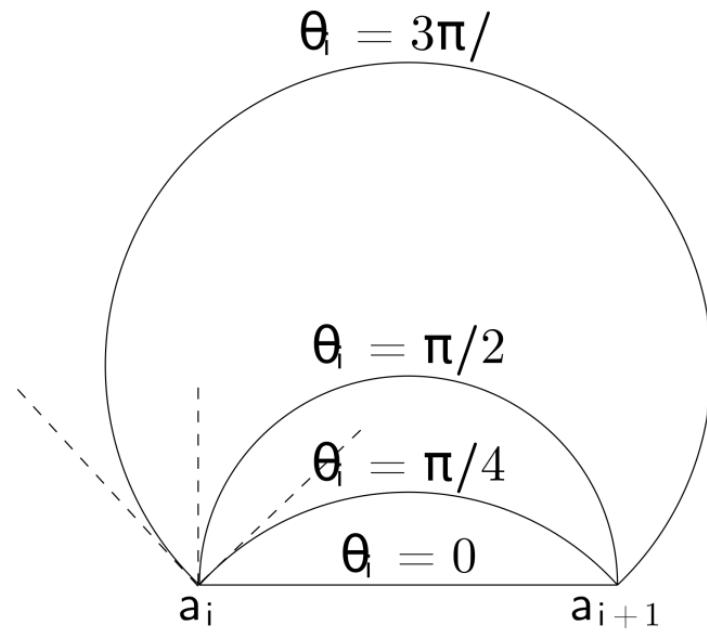


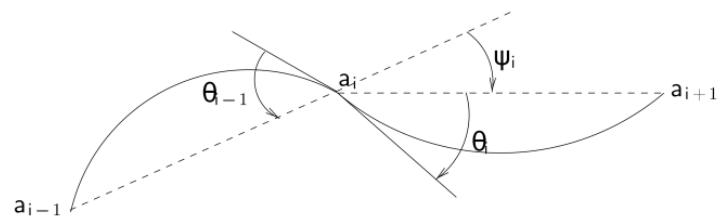


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Chapter 9

Unconstrained minimization

9.1 Unconstrained minimization

9.2 Descent methods

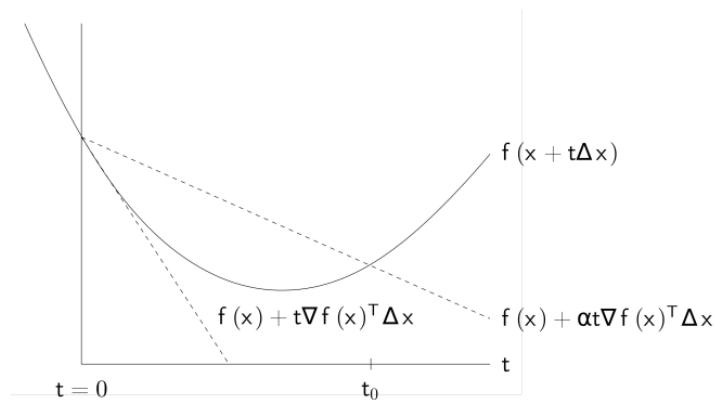
9.3 Gradient descent method

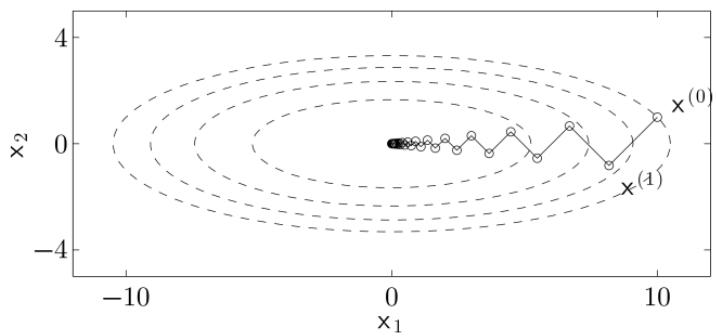
9.4 Steepest descent method

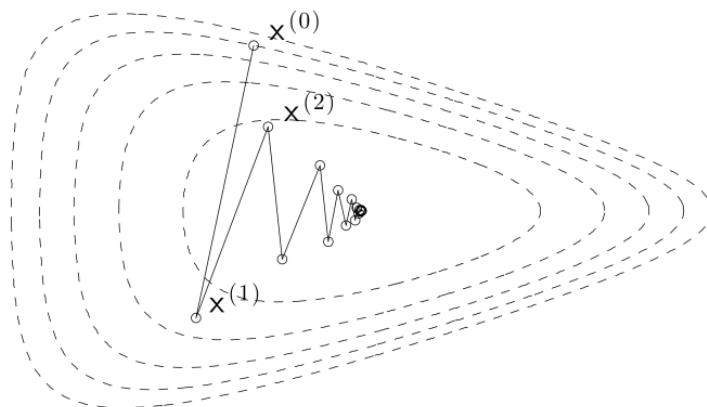
9.5 Newton's method

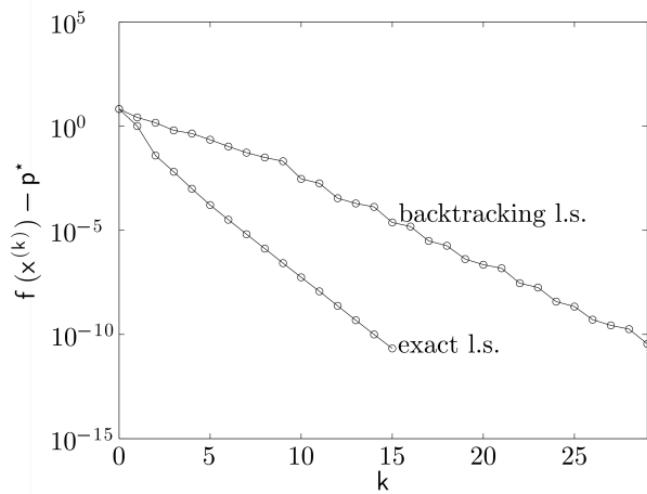
9.6 Self-concordance

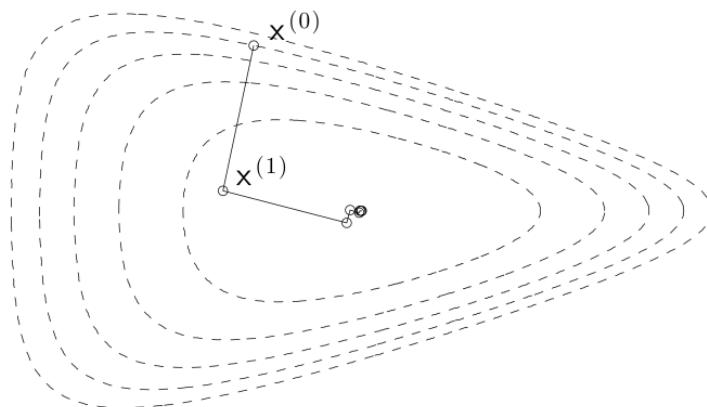
9.7 Implementation

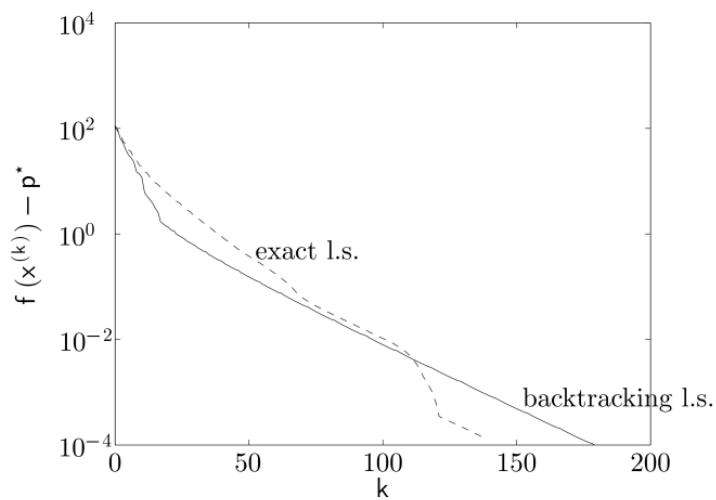


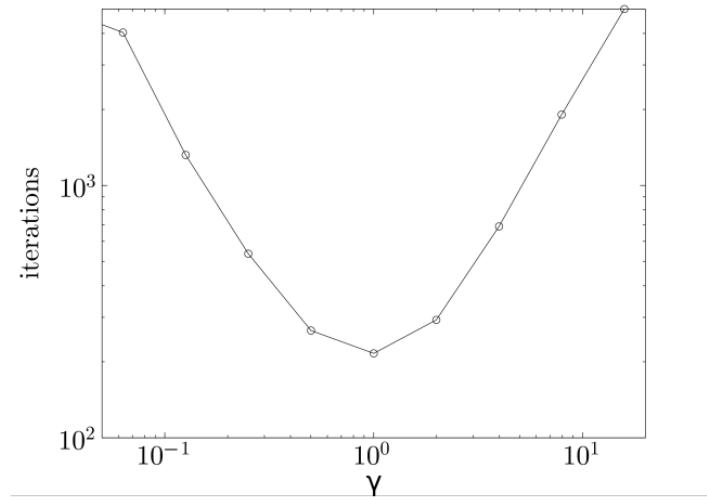


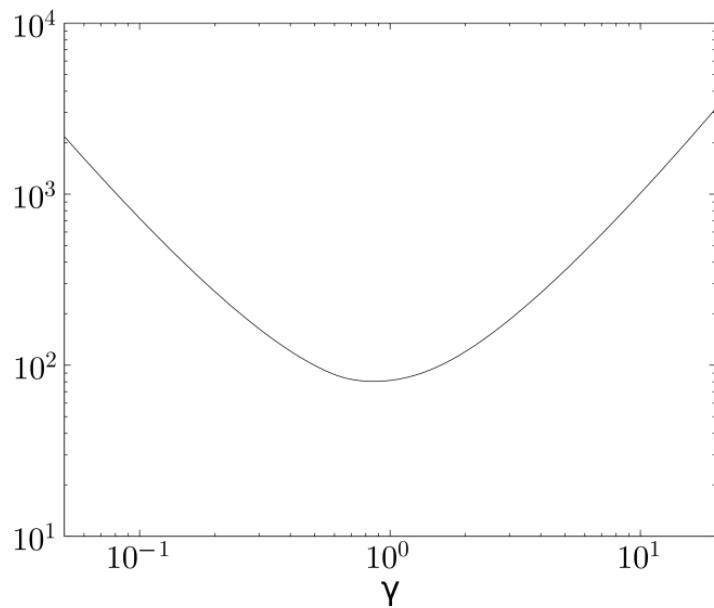


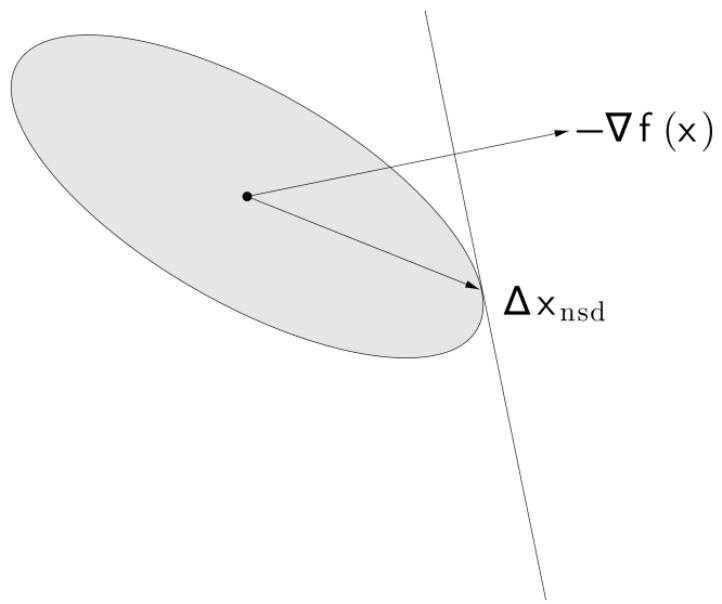


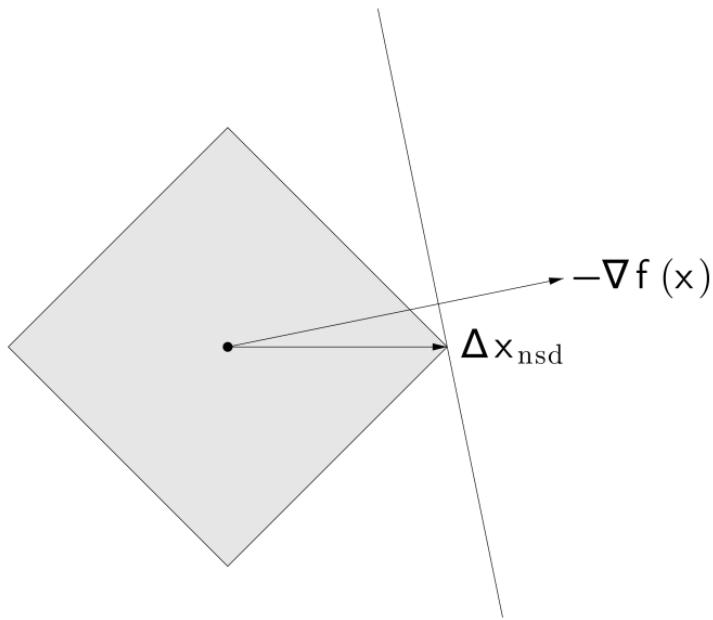


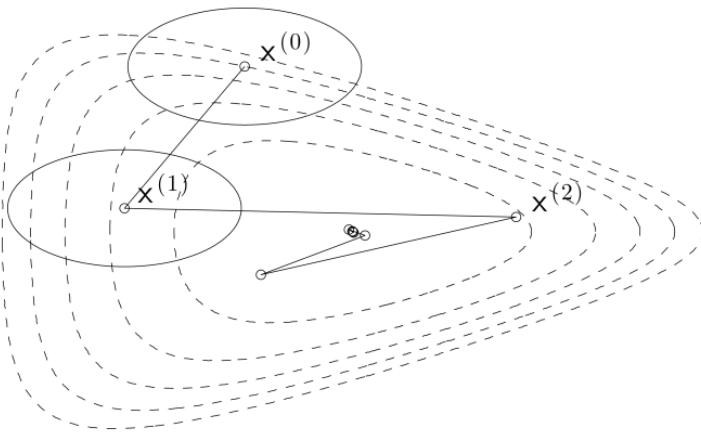


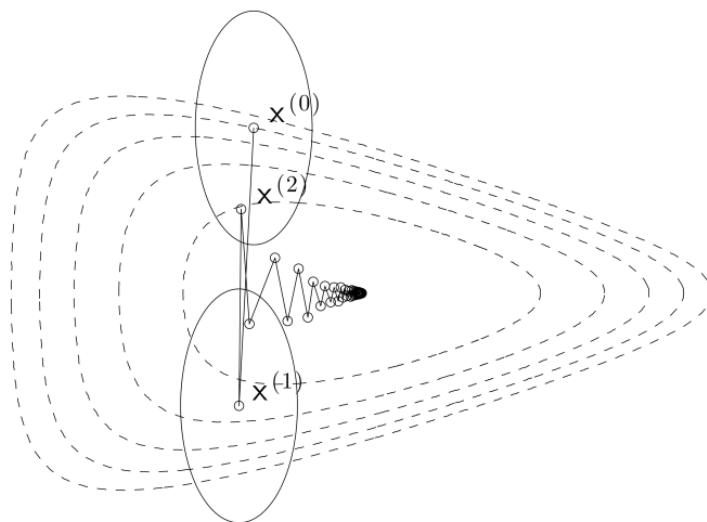


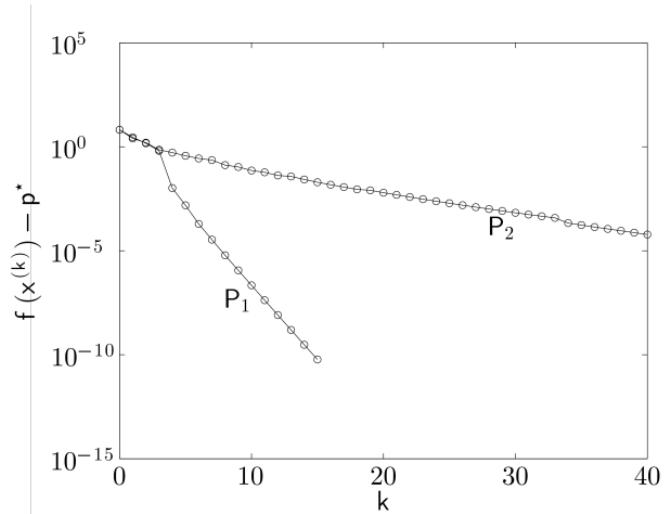


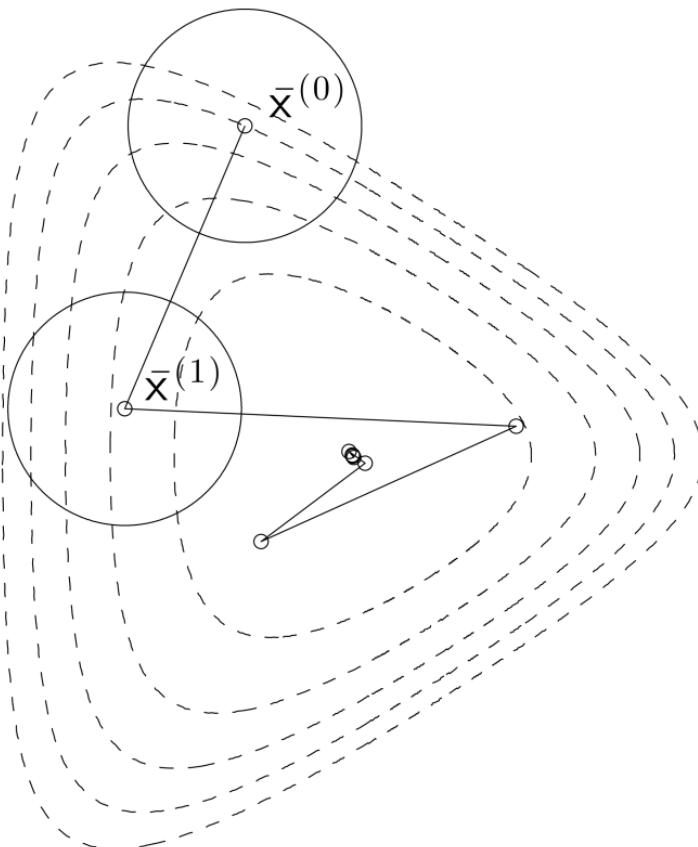


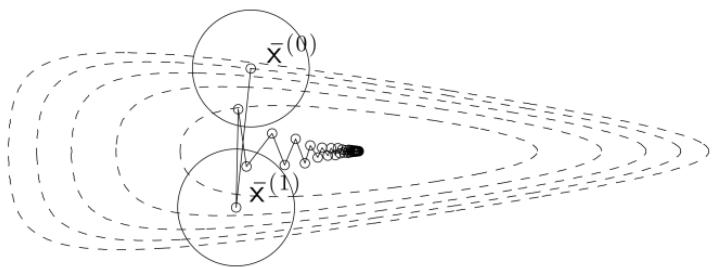


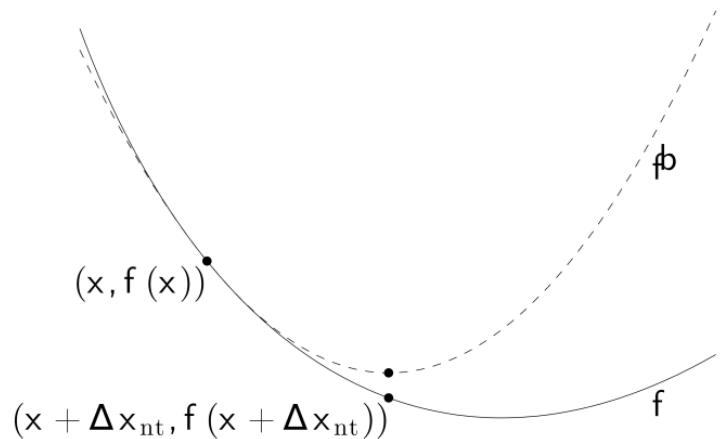


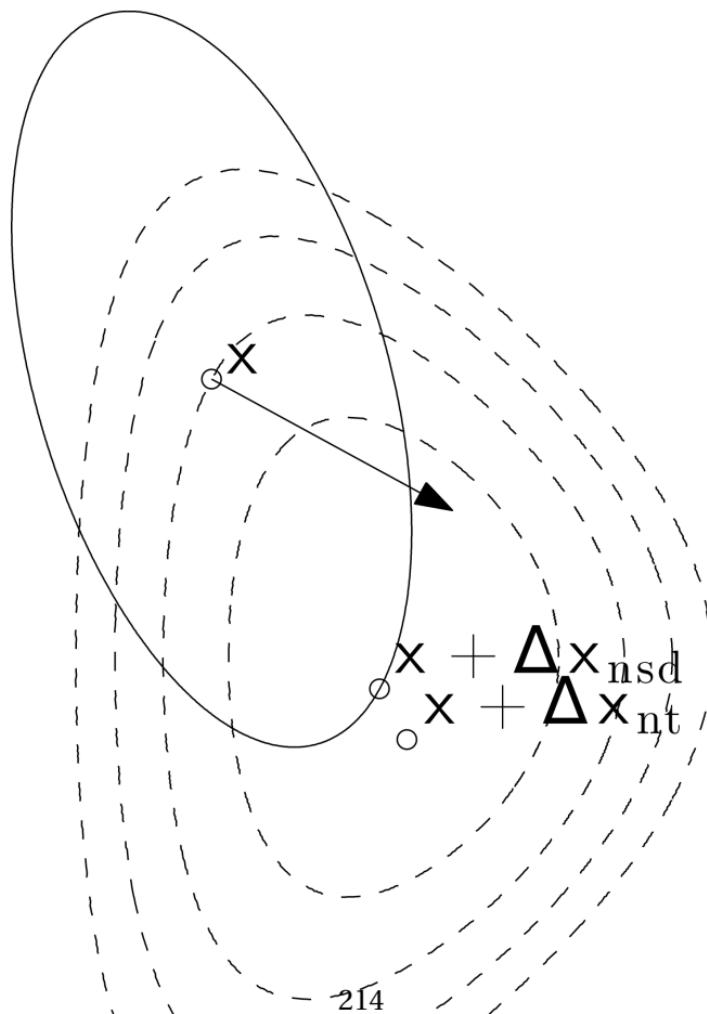


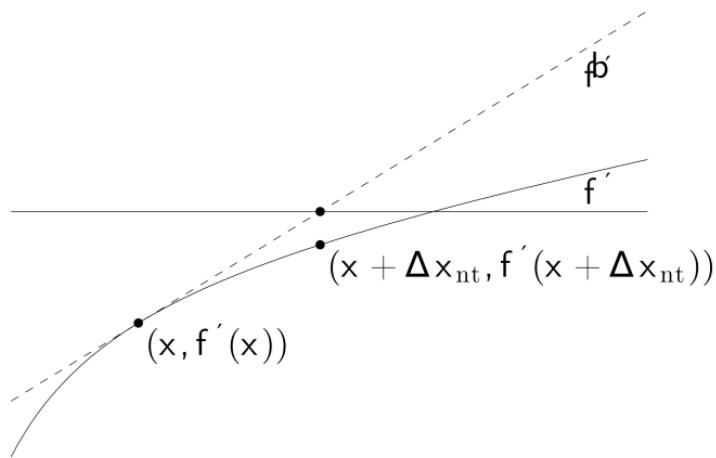


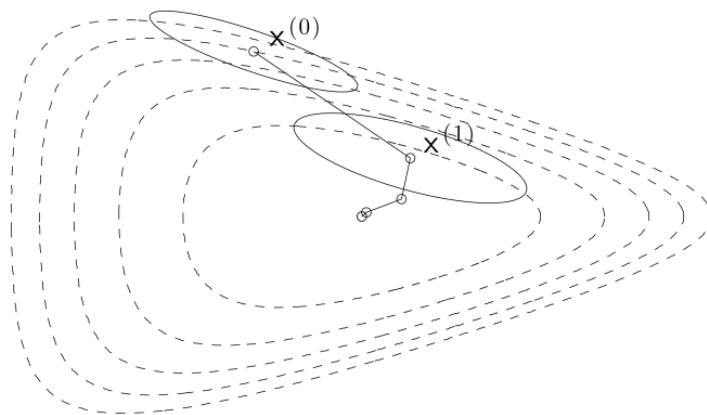


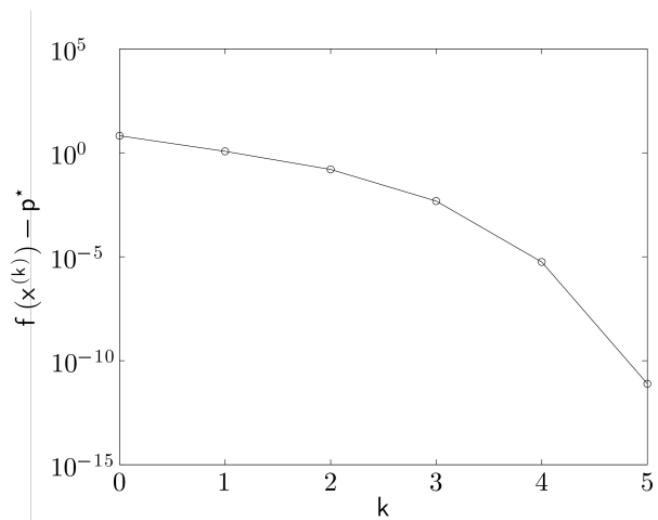


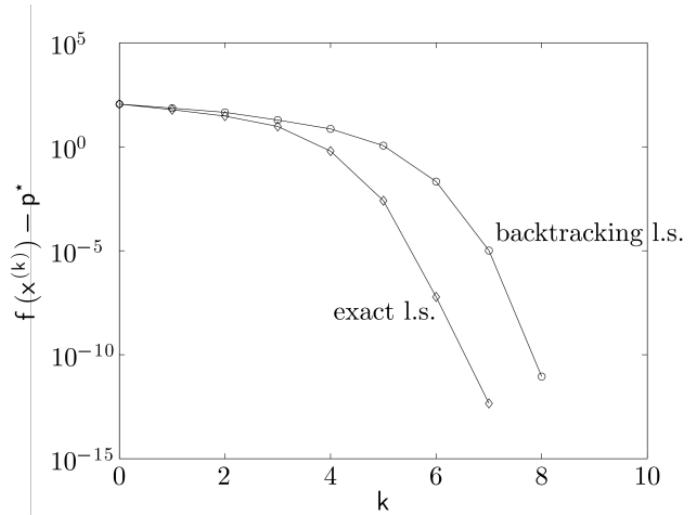


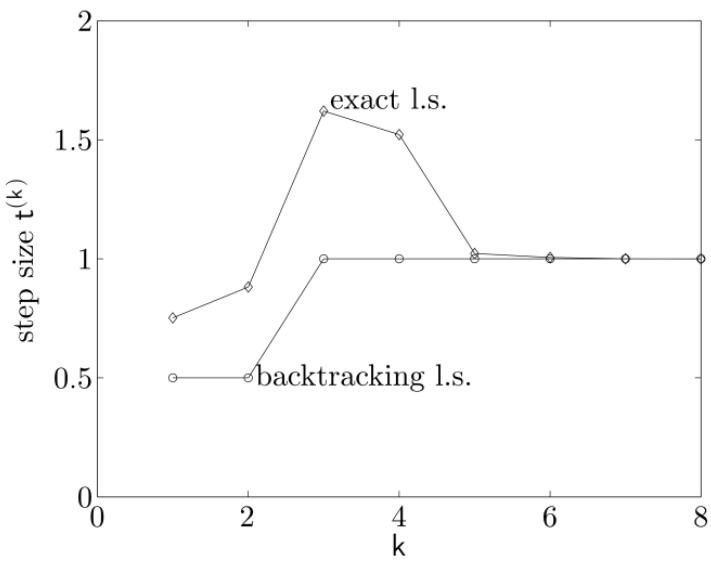


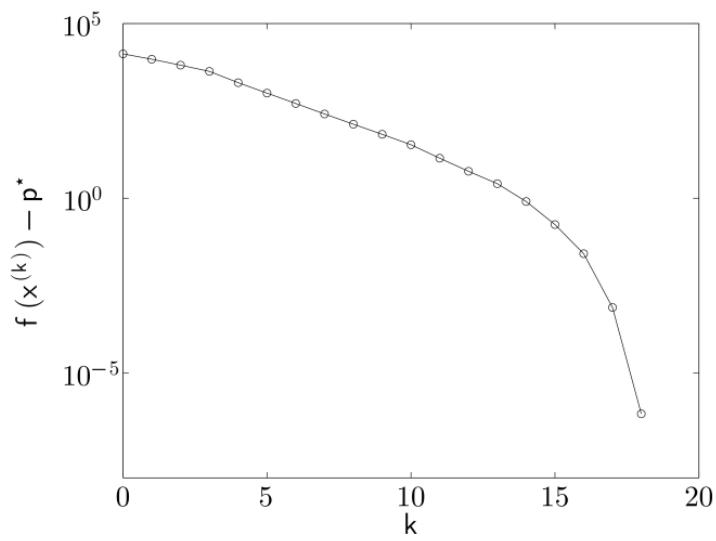


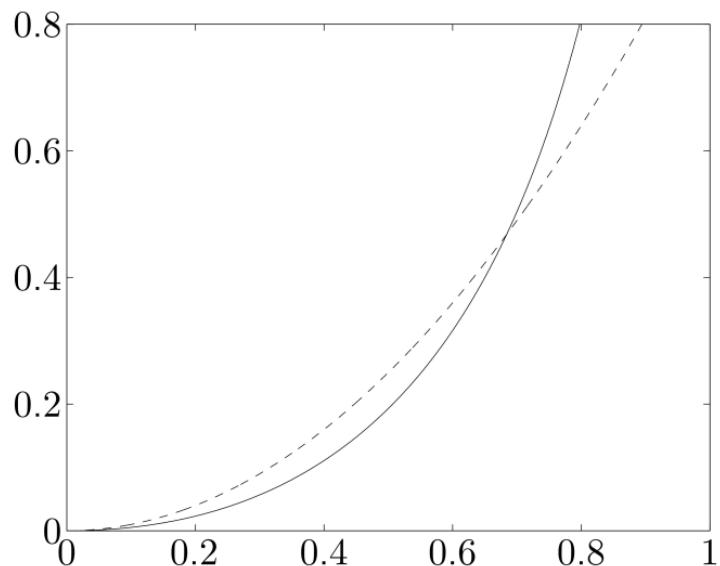


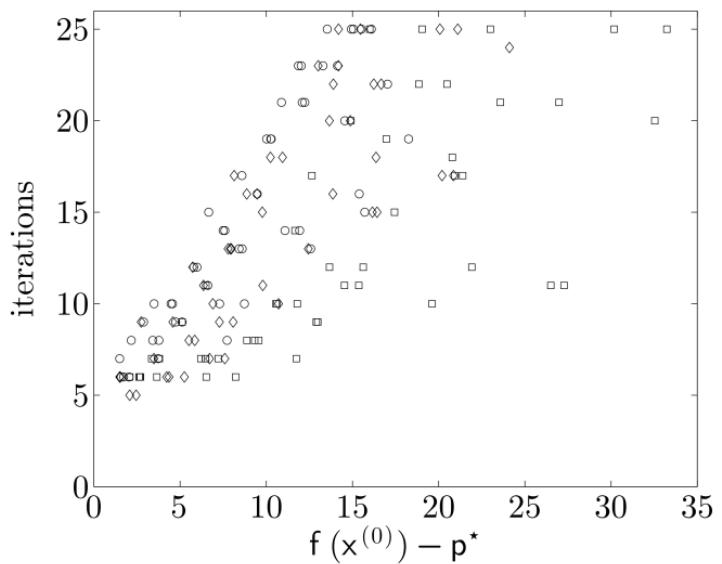












Chapter 10

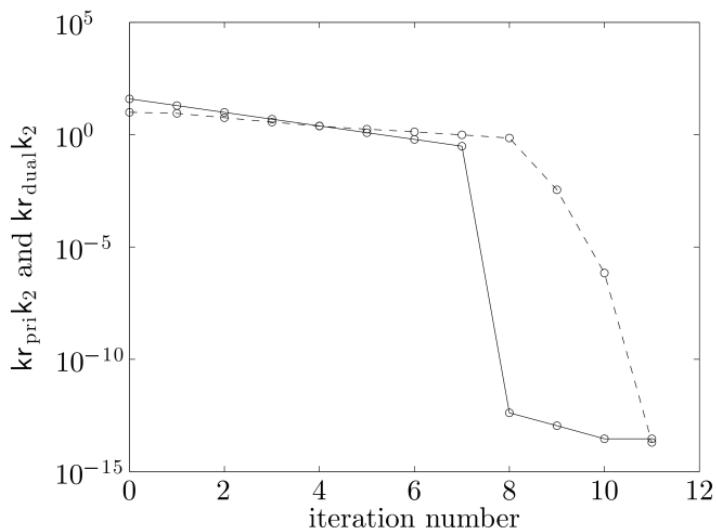
Equality constrained minimization

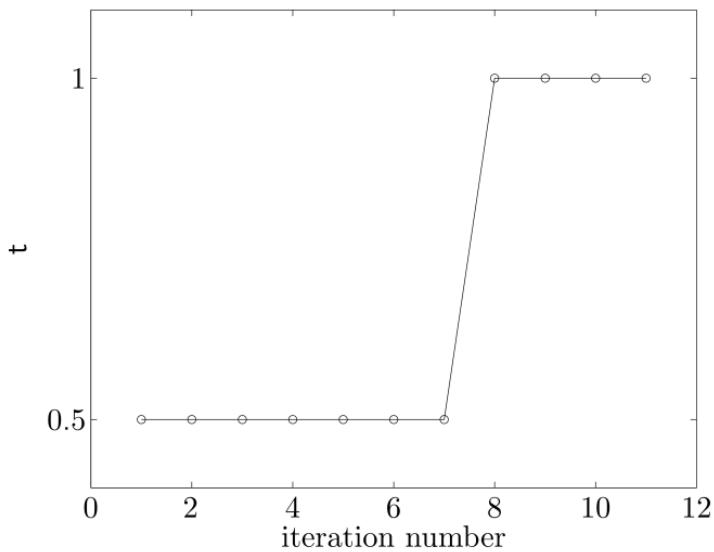
10.1 Equality constrained minimization problems

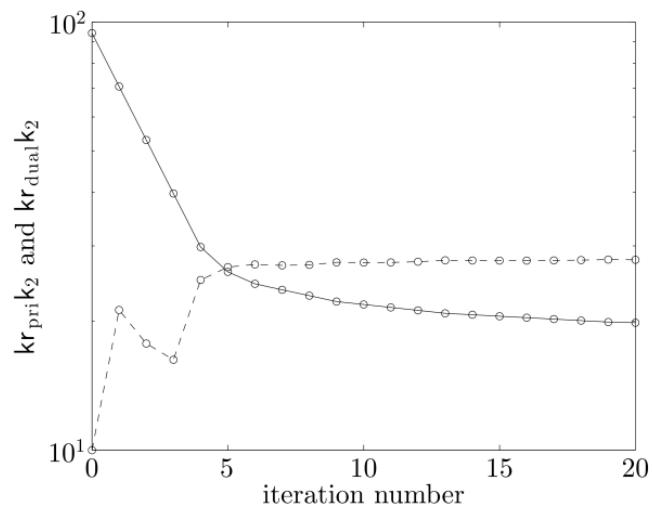
10.2 Newton's method with equality constraints

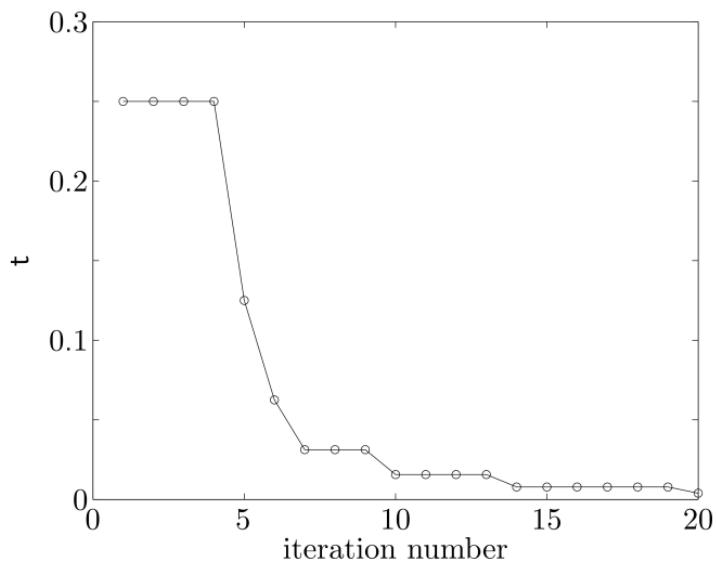
10.3 Infeasible start Newton method

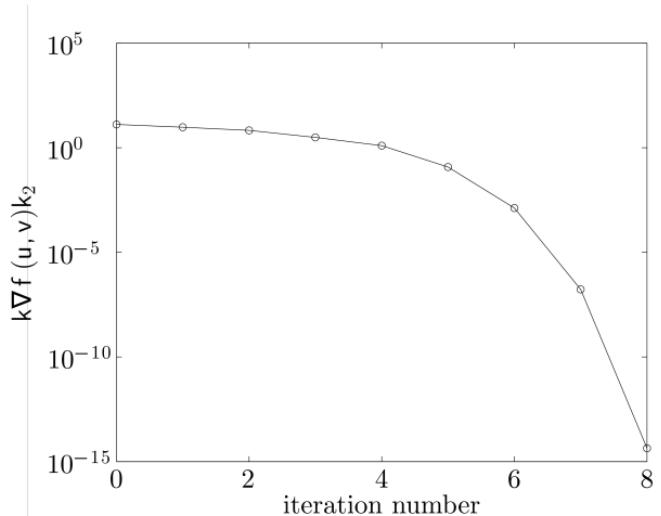
10.4 Implementation

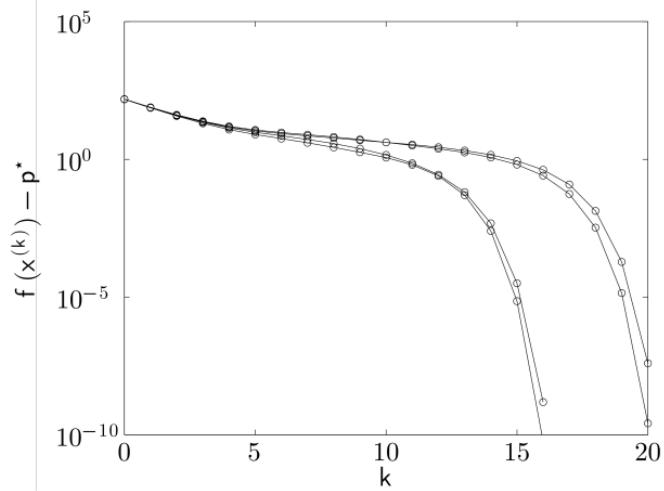


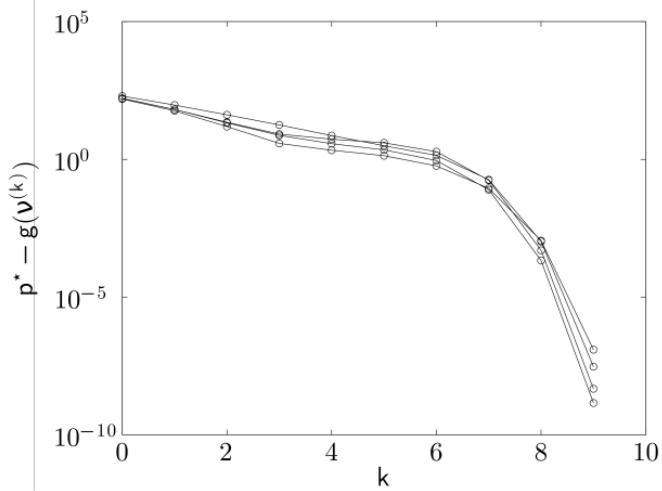


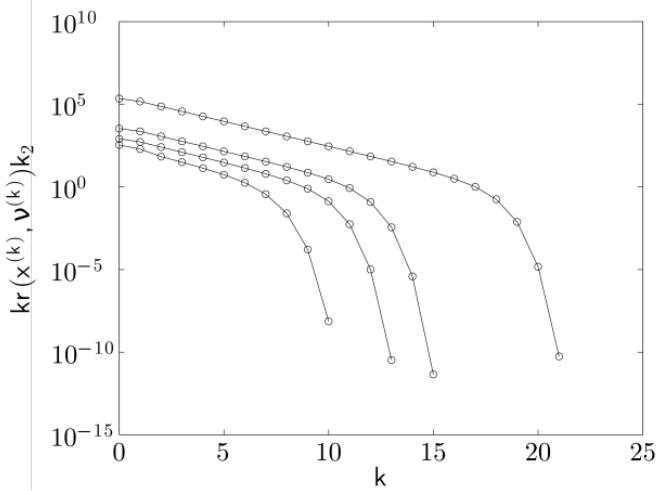












Chapter 11

Interior-point methods

11.1 Inequality constrained minimization problems

11.2 Logarithmic barrier function and central path

11.3 The barrier method

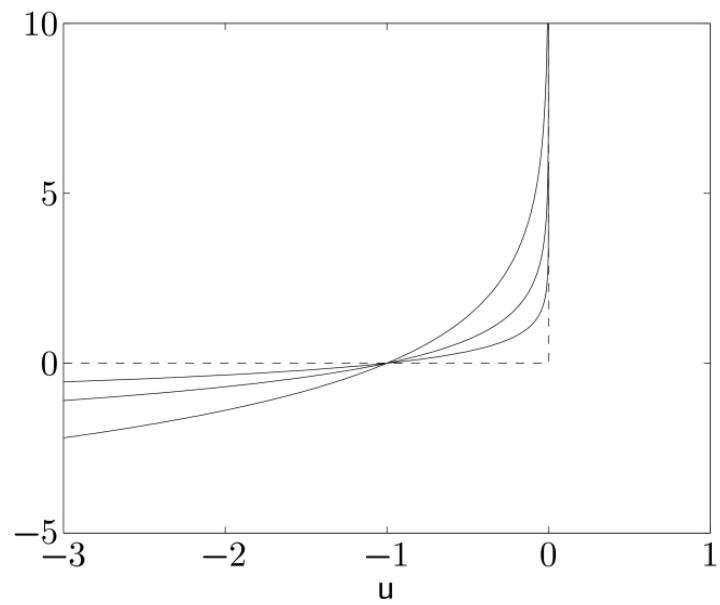
11.4 Feasibility and phase I methods

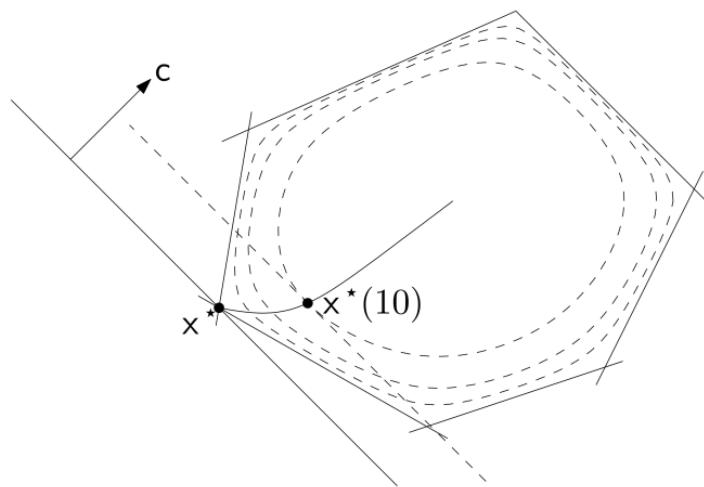
11.5 Complexity analysis via self-concordance

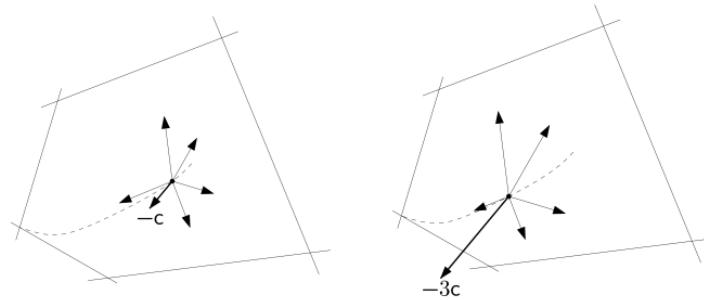
11.6 Problems with generalized inequalities

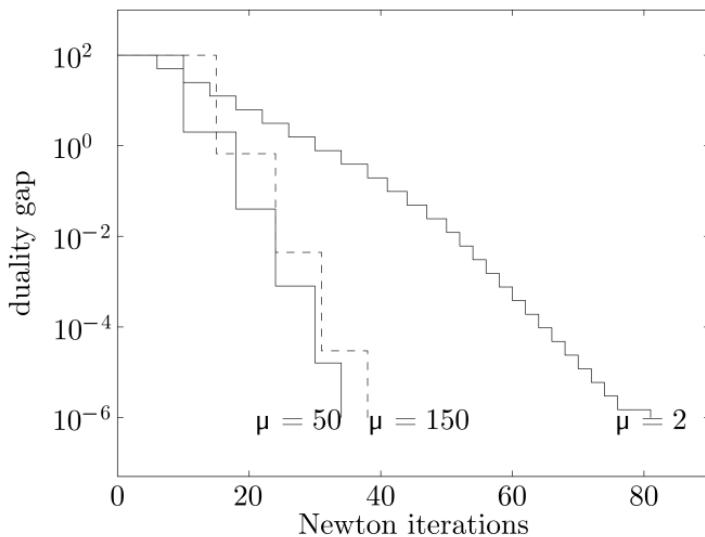
11.7 Primal-dual interior-point methods

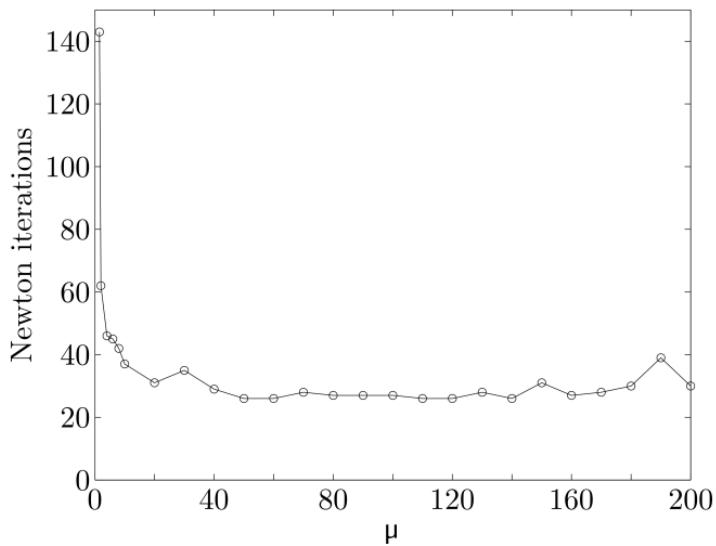
11.8 Implementation

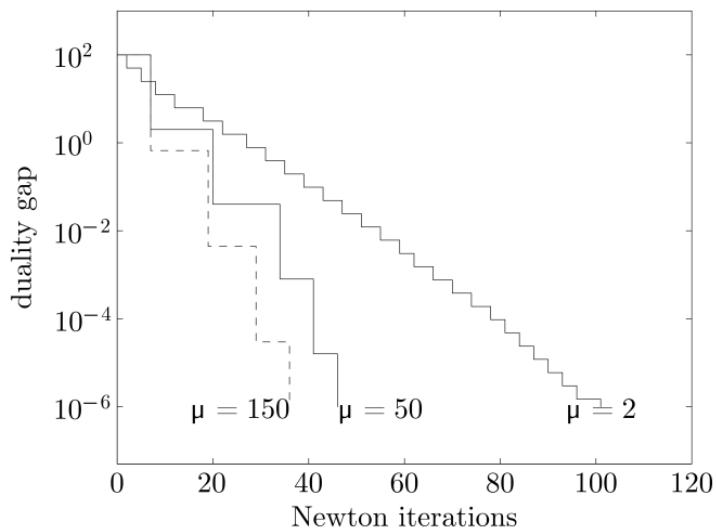


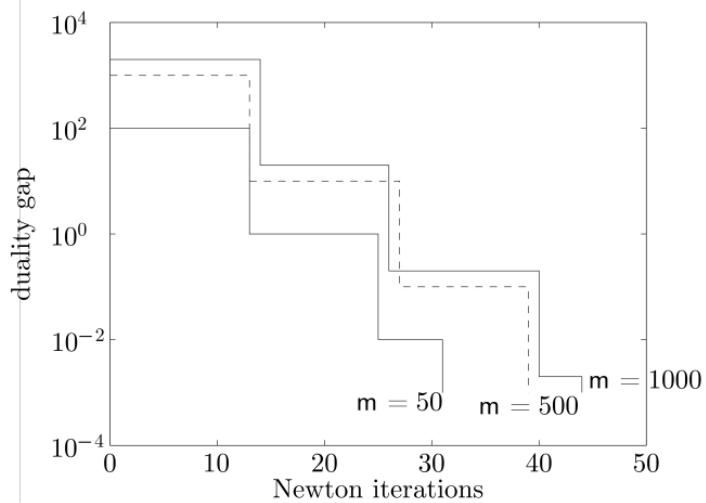


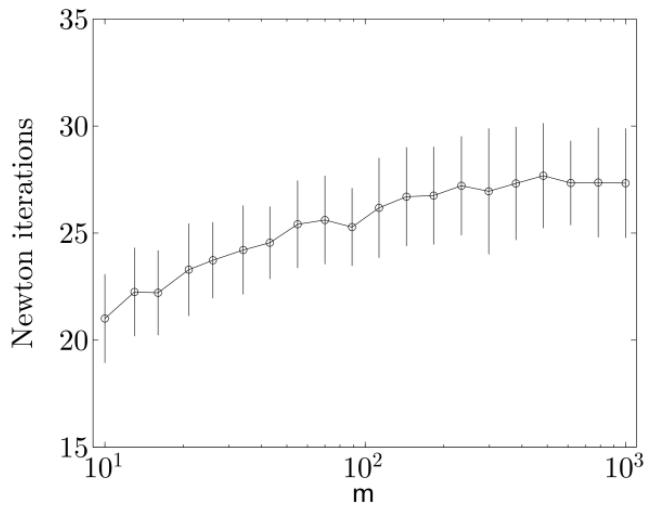


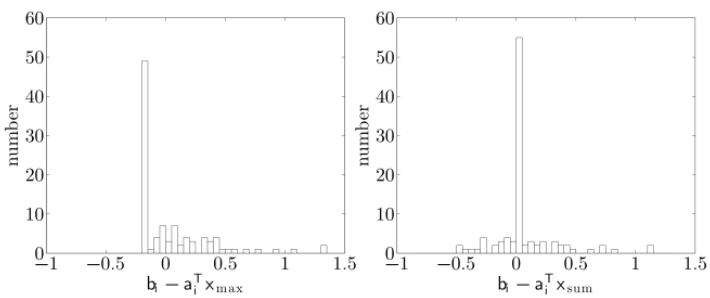


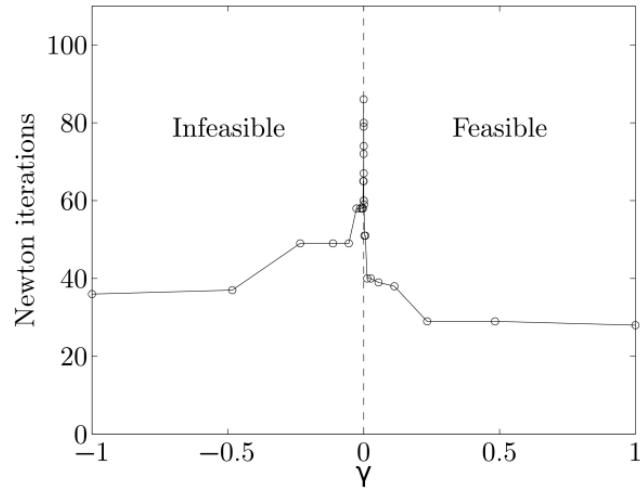


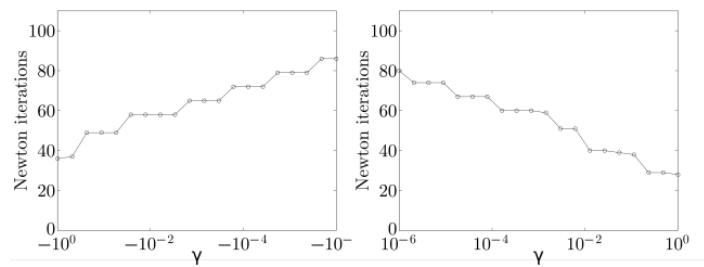


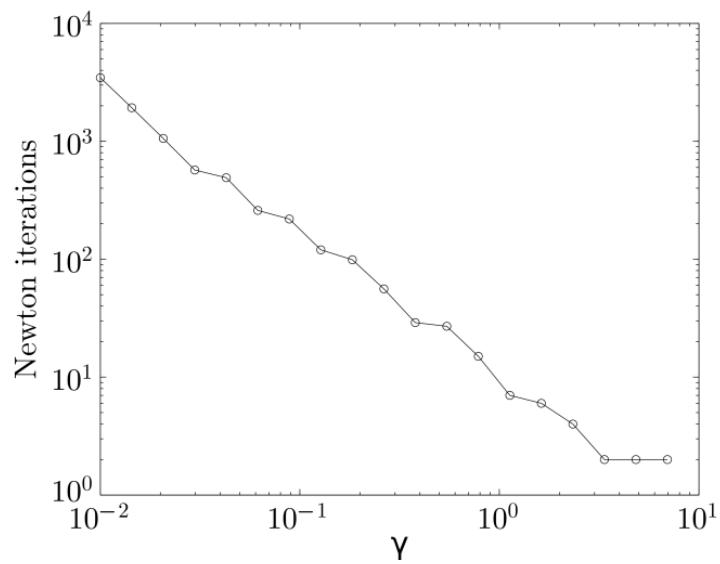


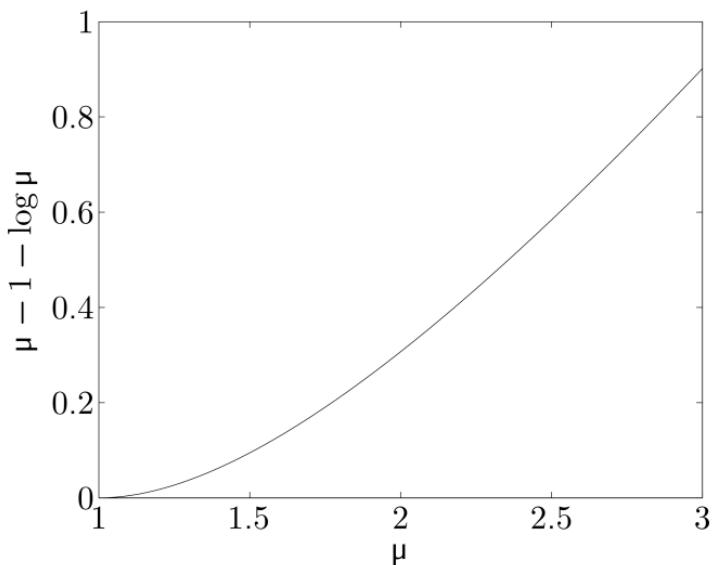


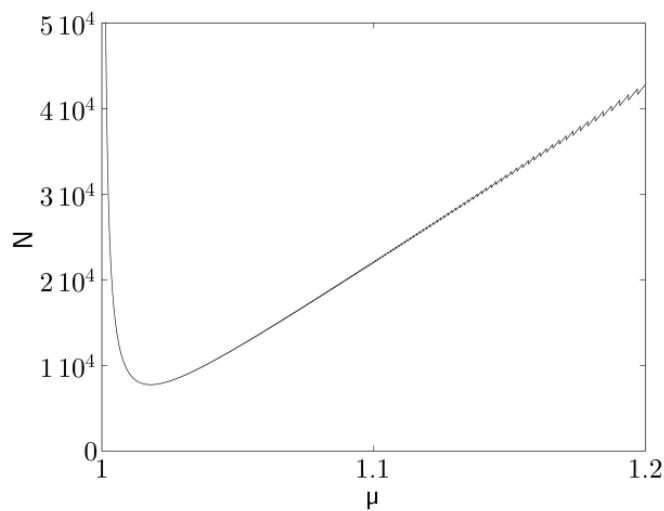


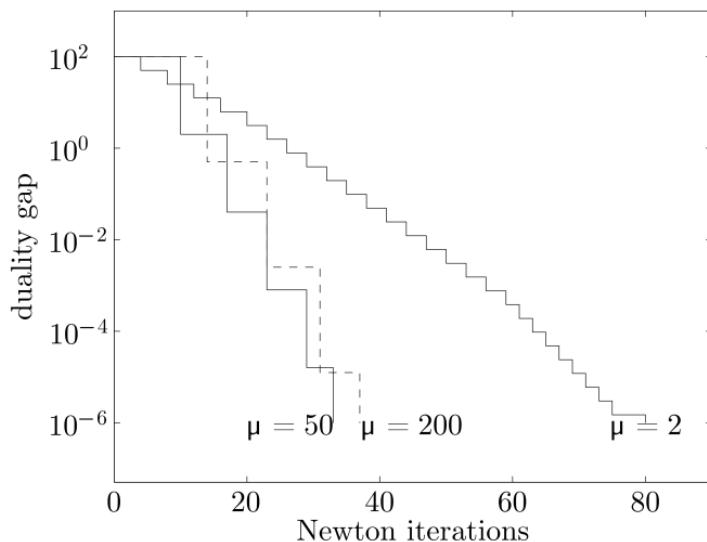


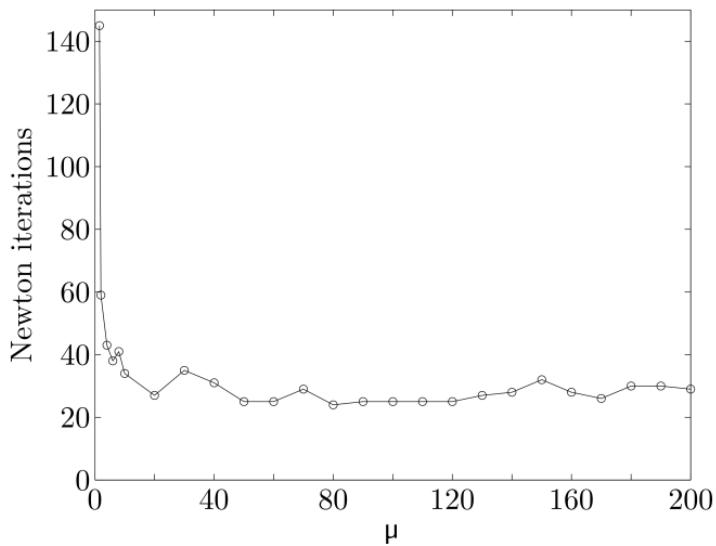


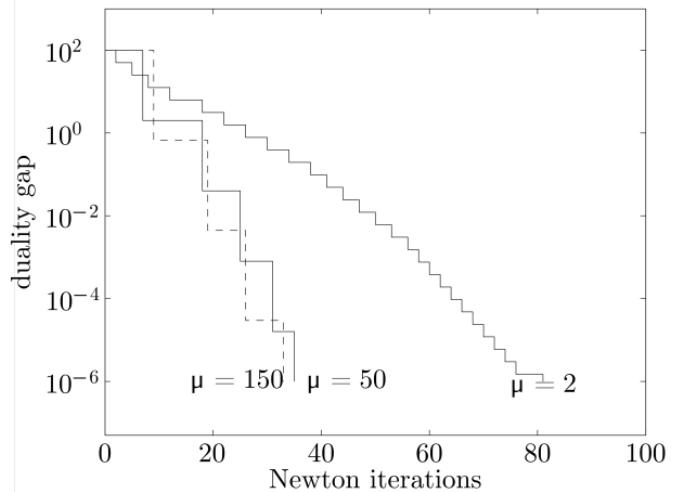


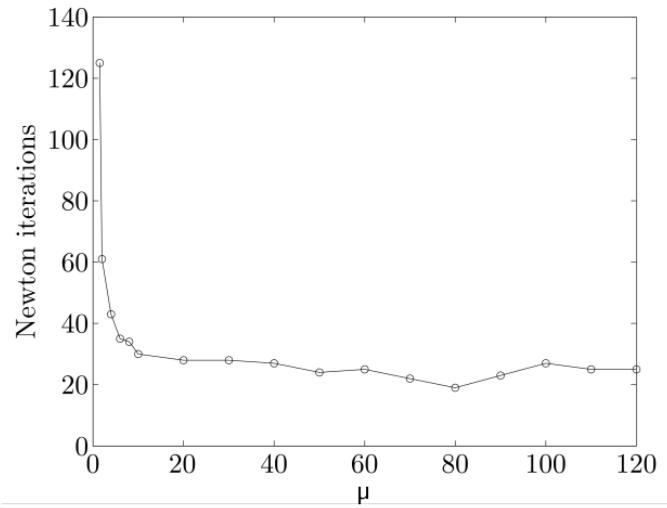


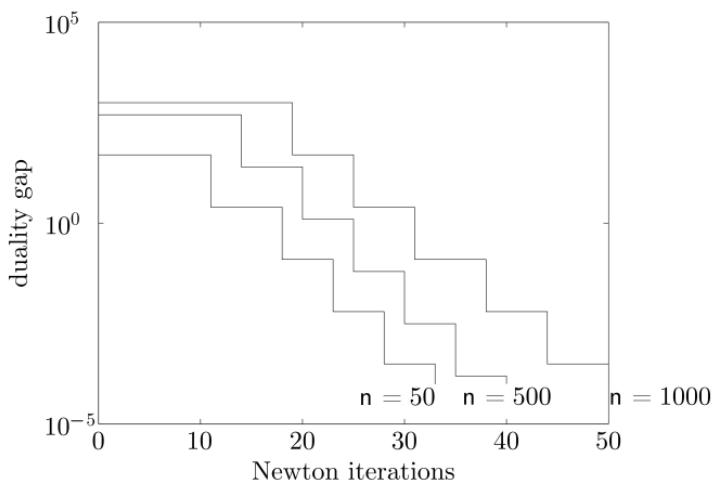


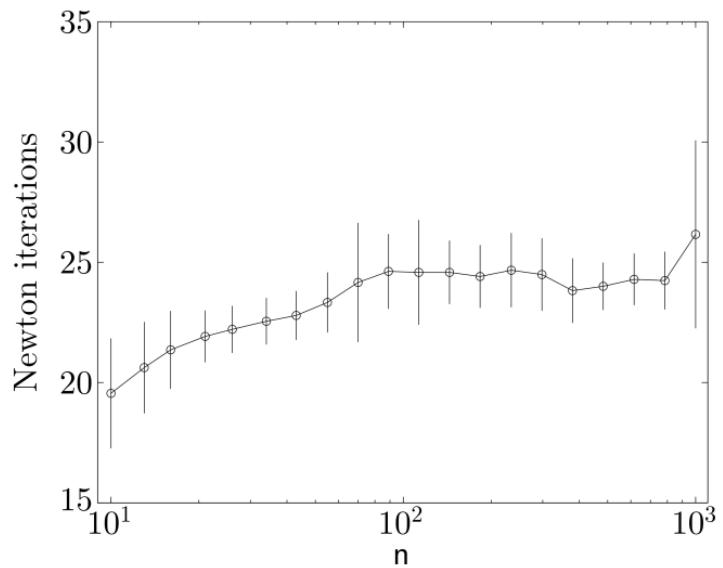


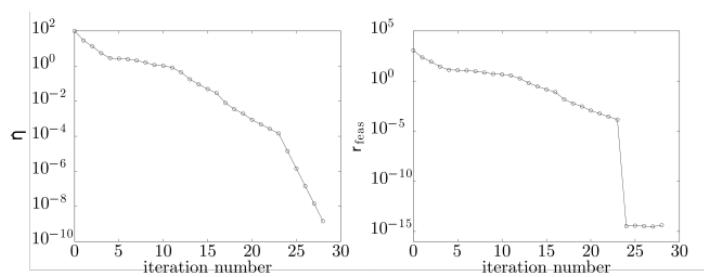


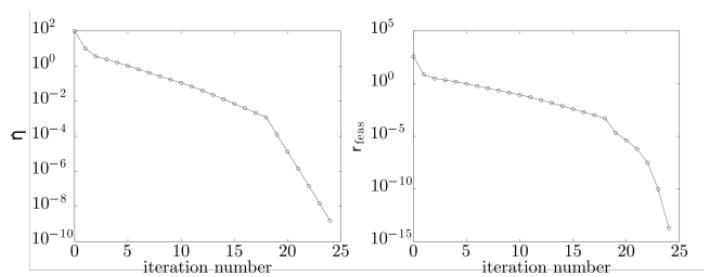


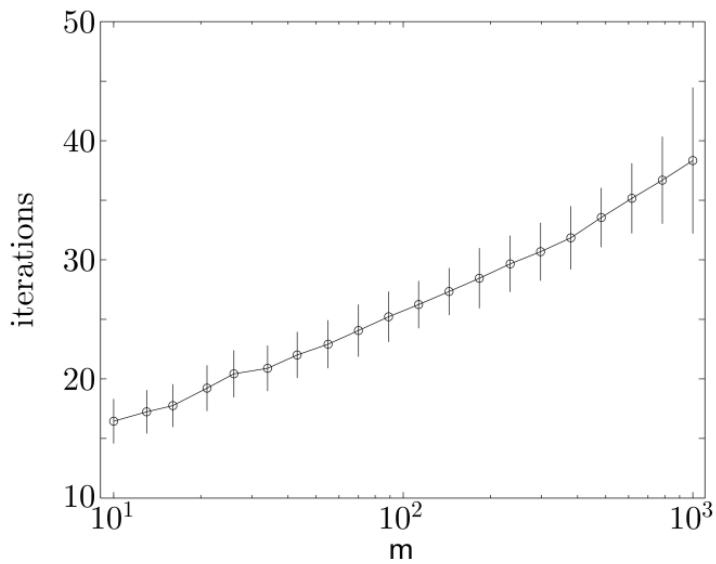












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