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## 第八章作业

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6. 满射: (4)

单射: (1) (3)

双射: (6)

一致的函数: (2) (3) (7) (8)

$$8: f \circ g(x) = g(f(x)) = 2x + 6$$

$$g \circ f(x) = f(g(x)) = 2x + 11$$

$$h \circ f(x) = f(h(x)) = x + 1$$

$$f \circ (h \circ g)(x) = (h \circ g)(f(x)) = x + \frac{11}{2}$$

$$g \circ (h \circ f)(x) = (h \circ f)(g(x)) = x + 6$$

12.  $f$  是满射, 对于  $\forall a \in A, \exists a_1 \in A,$ 使得  $f(a_1) = a \therefore f$  是函数

$$\therefore f(f(a_1)) = f(a), \text{ 即 } f \circ f(a_1) = f(a)$$

$$\text{因 } f \circ f = f$$

$$\therefore f(a_1) = f(a)$$

$$\therefore f(a_1) = a$$

$$\therefore f(a) = a$$

 $\therefore \forall a$  的任意性

$$f = \text{id}_A$$

13. (1)  $\forall b \in B, \exists c \in C$  使  $g(b) = c$  $\therefore f \circ g$  是满射对于  $C, \exists a \in A$  使  $f \circ g(a) = c$ 

$$\text{即 } g(f(a)) = c \therefore g(b) = c$$

$$\therefore g(f(a)) = g(b) = c \therefore g \text{ 是单射 } \therefore f(a) = b$$

即  $\forall b \in B, \exists a \in A$  使  $f(a) = b$  $\therefore f$  是满射

$$(2) \forall b_1, b_2 \in B$$

$$\because f \text{ 是满射 } \exists a_1, a_2 \in A$$

$$\text{使 } f(a_1) = b_1, f(a_2) = b_2 \quad \because b_1 \neq b_2 \quad \therefore a_1 \neq a_2$$

$$\therefore f \circ g \text{ 是单射 } \therefore f \circ g(a_1) \neq f \circ g(a_2)$$

$$\therefore g(b_1) \neq g(b_2)$$

$$\therefore g \text{ 是单射}$$

$$15. \forall b_1, b_2 \in B \quad \because f \text{ 是满射 } \therefore \exists a_1, a_2 \in A$$

$$\text{使 } f(a_1) = b_1, f(a_2) = b_2$$

$$\text{且 } f(a_1) \neq f(a_2) \quad \text{即 } a_1 \neq a_2$$

$$\therefore g(b_1) = \{x \mid (x \in A) \wedge (f(x) = b_1)\}$$

$$g(b_2) = \{x \mid (x \in A) \wedge (f(x) = b_2)\}$$

$$\therefore a_1 \in g(b_1), a_2 \in g(b_2)$$

$$\therefore a_1 \notin g(b_2), a_2 \notin g(b_1)$$

$$\therefore g(b_1) \neq g(b_2)$$

$$\therefore g \text{ 为单射}$$