Taylor Series Assignment

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Date 2017-06-12T20:20:26

Project 9189c752-e334-4311-afa9-605b6159620a

Location <u>15 - Taylor Series Assignment/Taylor Series Assignment.sagews</u>

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Taylor Series Assignment

Question 0

Watch the lecture video here.

Did you watch the video? [Type yes or no.]

Question 1

Use Taylor polynomials to approximate π using the following steps:

•
$$A=\int_0^1rac{1}{1+x^2}~dx=rctan(1)-rctan(0)=rac{\pi}{4}$$

$$ullet$$
 $T(x)=$ Taylor polynomial of degree 100 of $\dfrac{1}{1+x^2}$ centered at $x=0$

•
$$B=\int_0^1 T(x)\,dx$$

• Since A and B are approximately equal, $\pi \approx 4B$. So calculate 4B and convert to a decimal.

Question 2

Estimate the value of $\int_0^1 e^{-x^2} dx$ as follows:

- Define T20(x)= the Taylor polynomial of degree 20 of e^{-x^2} centered at x=0 .
- Calculate $\int_0^1 T20(x) dx$.

- Define T50(x)= the Taylor polynomial of degree 50 of e^{-x^2} centered at x=0.
- $\bullet \ \ {\rm Calculate} \ \int_0^1 T50(x) \ dx.$
- Compare your results with the output from Sage's numerical_integral command: 0.746824132812427. [Use the N() command to convert to decimals.]

Question 3

Let $f(x)=e^{\sin(x)}$, T5(x)= the 5th-degree Taylor polynomial of f centered at $x=\pi$, and T10(x)= the 10th-degree Taylor polynomial of f centered at $x=\pi$.

Graph all three on the window $0 \le x \le 2\pi$, $0 \le y \le 3$. Use black for f, blue for T5, and red for T10.

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