Measure Theory

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Contents

Chapter 1

Real Analysis

1.1

1.2 Differentiation and Integration

For conceptual simplicity, we study $\mathbb{R}$ instead of $\mathbb{R}^n$ in this section. Let us first recall what we learned in elementary calculus.

Theorem 1.2.1 (Newton-Leibniz)**.** *Let f be a Riemann integrable function on $[a, b]$.*

Definition 1.2.2 (Bounded Variation)**.**

Theorem 1.2.3 (Jordan Decomposition Theorem)**.**

$$f(x) = g(x) - h(x)$$

where $g(x)$ and $h(x)$

Bounded variable functions says that it has a length

Bounded variable functions are a.e. differentiable But it does not satisfies the desirable property: Length is equal to the integral of derivative, which is also known as the **fundamental theorem of calculus**.

Example 1.2.4.

Definition 1.2.5 (Absolute Continuous)**.**

Theorem 1.2.6. *If f is absolute continuous function on $[a, b]$, then*

$$f(x) - f(a) = \int_a^x f'(t)\mathrm{d}t, \quad x \in [a, b].$$

It is beneficial to compare it with the "classical" version (Theorem 1.2.1).