CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 7 Time-varying Fields & Inductors

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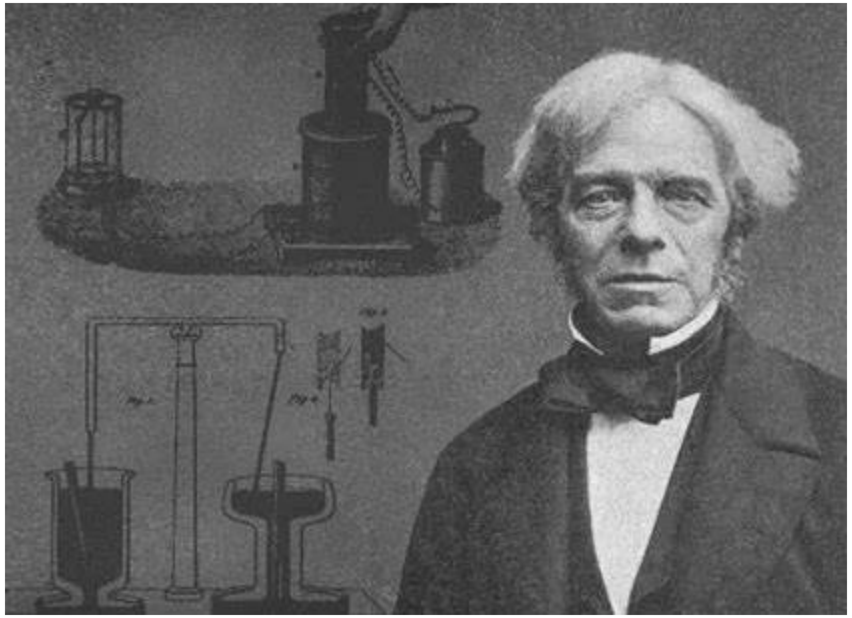
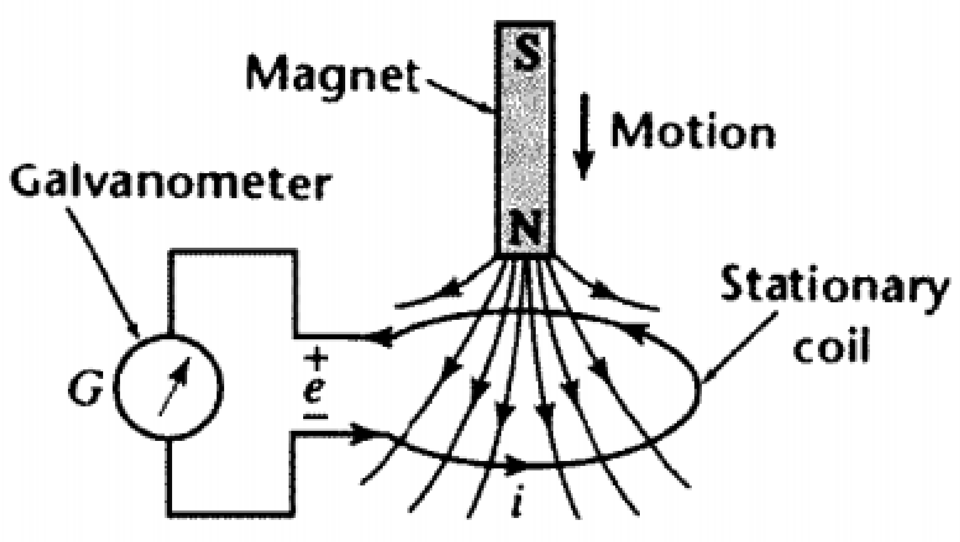
OUTLINE

- Electromagnetic (EM) Induction
  - ✓ Faraday's Experiments
  - ✓ Lenz's Law
  - ✓ Faraday's Law
- Motional Electromotive Force (*emf*)
- Generalised Ampere's Law
  - ✓ Displacement Current
- Inductors



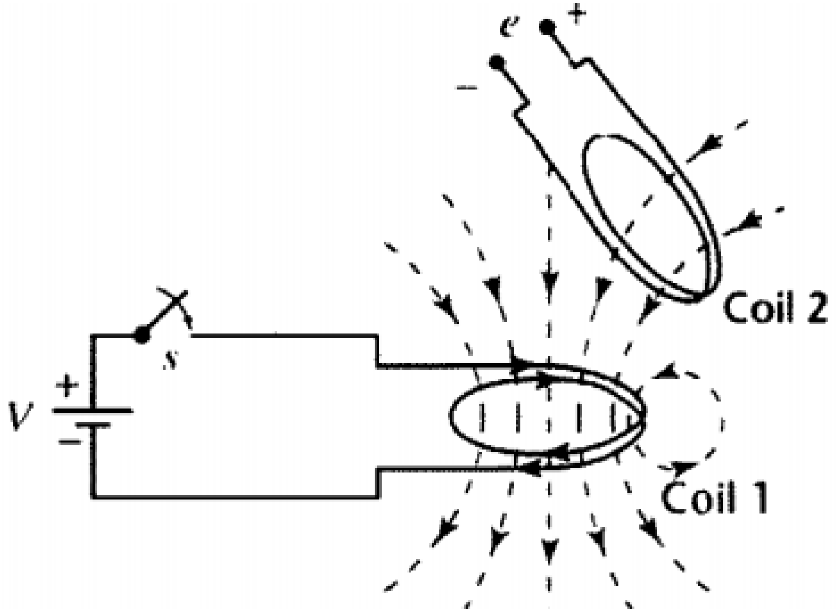
1.1 FARADAY’S EXPERIMENTS

Experiment 1



Induced voltage and current can be detected in the coil when moving the magnet towards or away from the coil.

Experiment 2



Induced voltage can be detected in coil 2 at the time of turning on or off the switch *s*.

1.1 FARADAY’S EXPERIMENTS

The process of inducing a voltage in a coil (also called a loop) by placing it in a time-varying magnetic field is now commonly referred to as an **electromagnetic (EM) induction**.

In fact, the electromagnetic induction will take place as long as one of the following conditions holds:

1. A **time-changing** flux linking a stationary **closed path**;
2. **Relative motion** between a steady flux and a **closed path**;
   The coil continuously changes its shape, position or orientation.
3. A **combination** of the two.

1.1 OBSERVATION

Experiment observation conclusion - a time-varying magnetic field produces an **electromotive force (*emf*)** that may establish a current in a suitable closed circuit.

- An electromotive force is merely a voltage that arises from conductors moving in a magnetic field or changing magnetic fields

$$emf = -N\boxed{\frac{d\Phi}{dt}}\ (V)$$

changing magnetic flux

- The minus sign is from Lenz's Law.

1.2 LENZ'S LAW

The direction of the **induced current** is determined by *Lenz's law*:

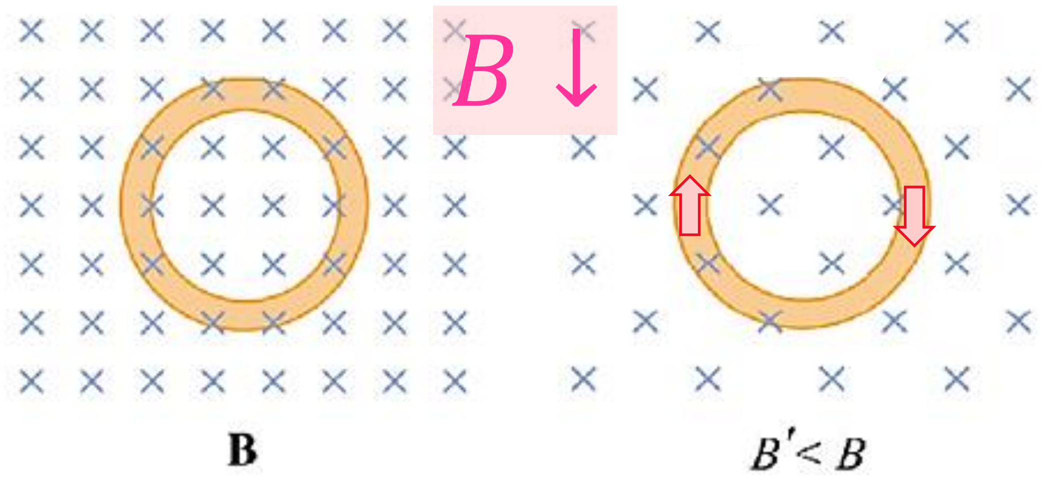
The *induced current* produces magnetic fields which tend to **oppose** the change in magnetic flux that induces such currents.

$$emf = -N\frac{d\Phi}{dt}\begin{cases} > 0 \rightarrow \text{induced } emf < 0 \\ = 0 \rightarrow \text{induced } emf = 0 \\ < 0 \rightarrow \text{induced } emf > 0 \end{cases}$$

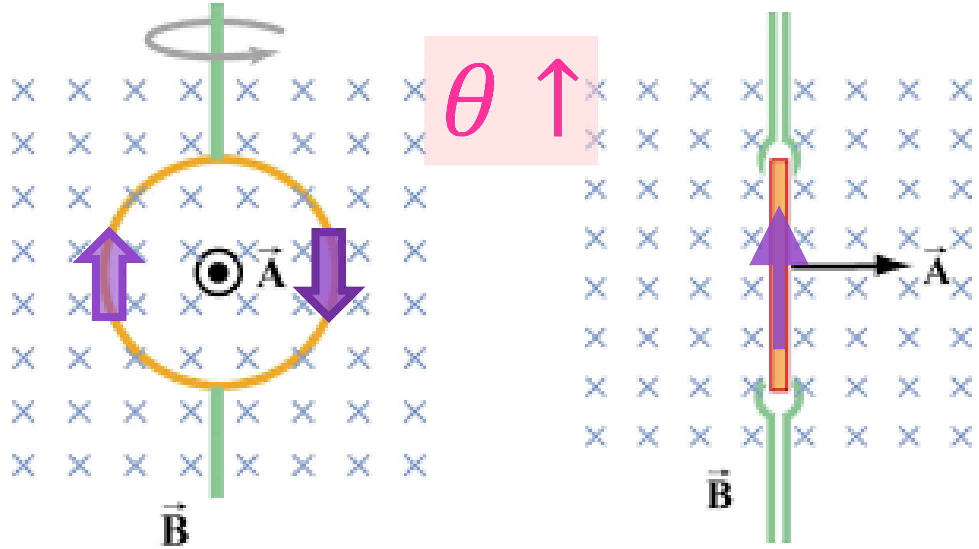
It states that the *induced electromotive force* must be in the direction that **opposes** the change.

1.2 EXAMPLES

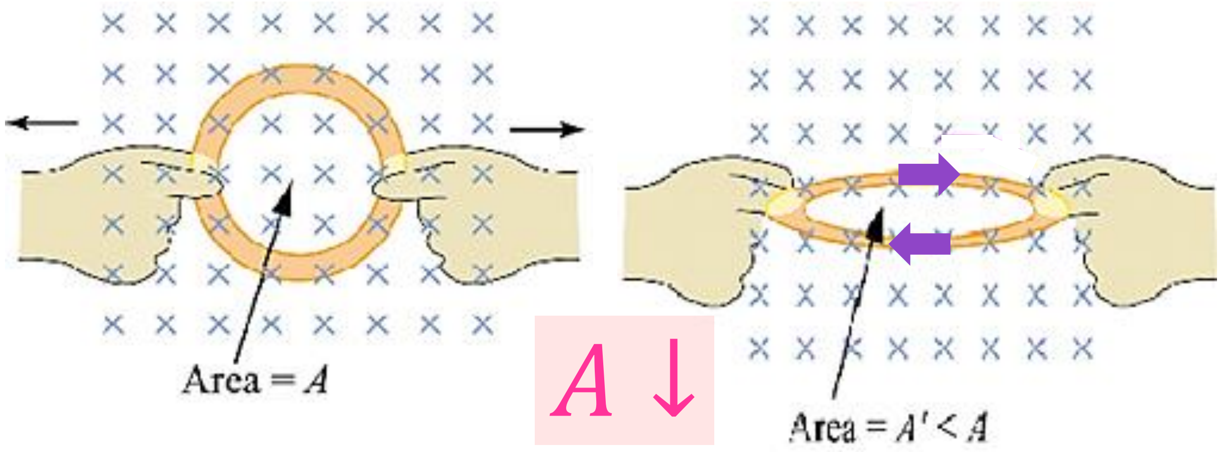
oppose the change in magnetic flux



Vary the magnitude of B with time

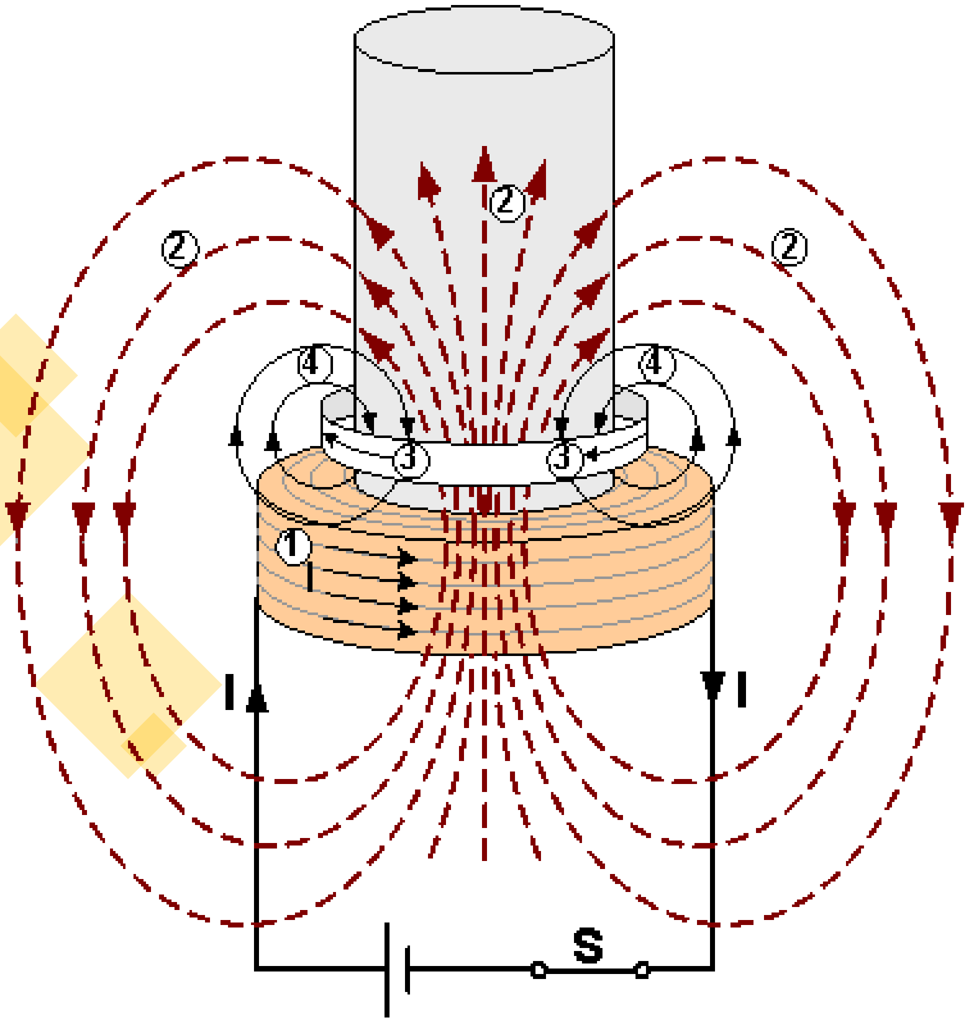


Vary the angle between B&A with time

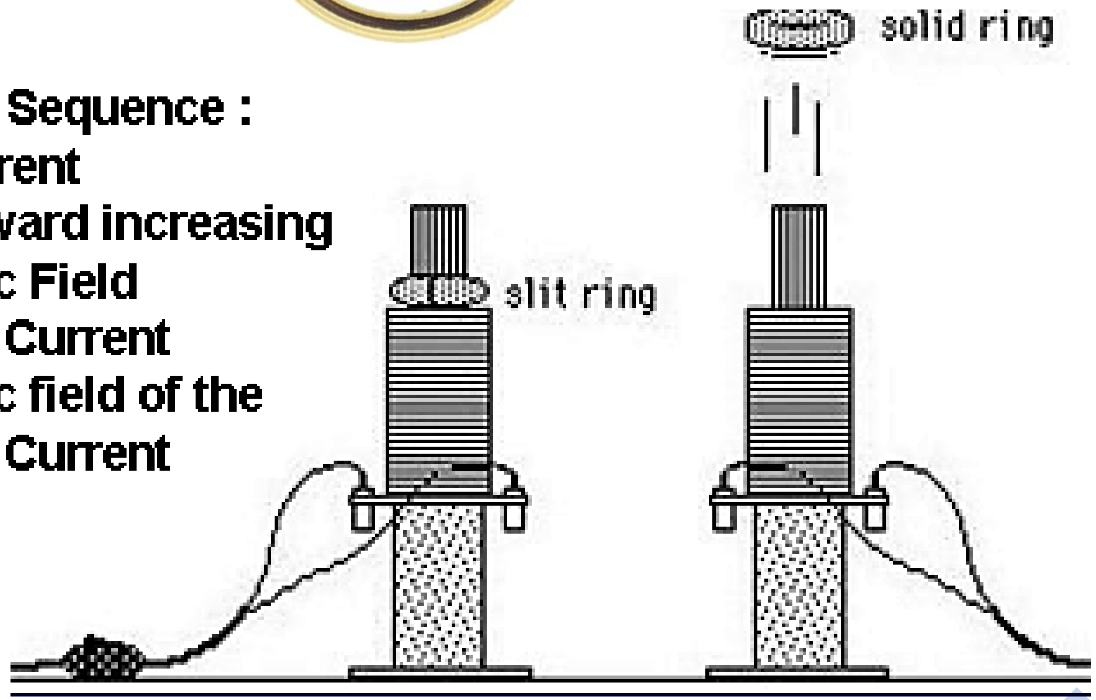


Vary the magnitude of A with time

DEMO: JUMPING RING



Formation Sequence :
1. Coil Current
2. Fast upward increasing Magnetic Field
3. Induced Current
4. Magnetic field of the Induced Current



When the apparatus is turned on, the solid ring is ejected into the air. The ring with the slit remains.

1.3 FARADAY'S LAW

Define the induced *emf* in terms of the induced $\vec{E}$ inside the conductor as:

$$emf = \oint_C \vec{E} \cdot d\vec{l}$$

The total flux enclosed by contour *c* is:

$$\Phi = \iint_S \vec{B} \cdot d\vec{s}$$

So, the (Maxwell's) Faraday's eq. becomes:

$$\oint_C \vec{E} \cdot d\vec{l} = emf = -\frac{d\Phi}{dt} = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{s}$$

$$= -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Integral form

$$\oint_C \vec{E} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Recall the "Curl Theorem":

$$\oint_C \vec{E} \cdot d\vec{l} = \iint_S (\nabla \times \vec{E}) \cdot d\vec{s}$$

$$\Rightarrow \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Differential Form

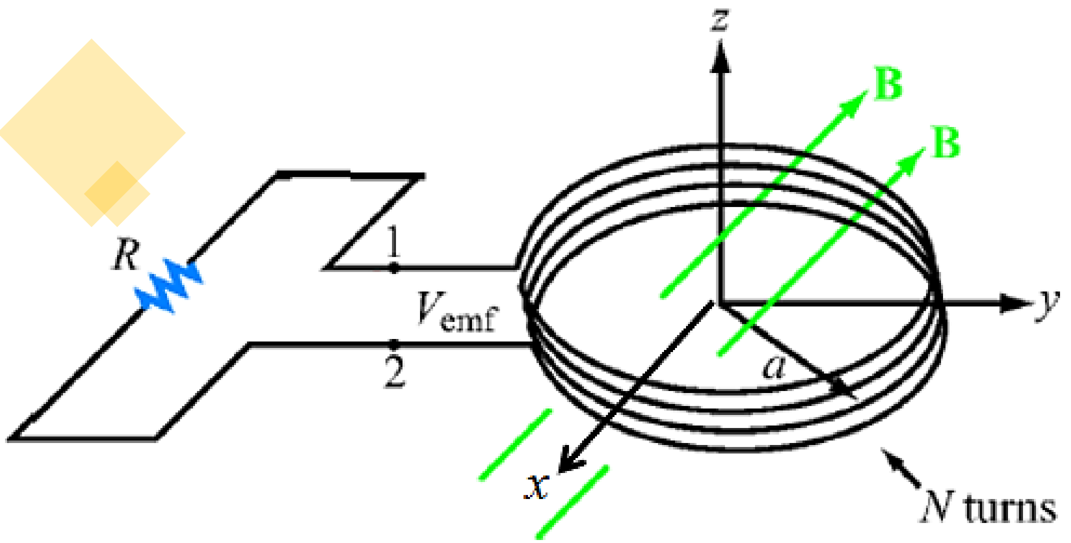
The electric field intensity in a region of time-varying magnetic flux density is ***nonconservative***.

QUIZ 1.1

An **inductor** is formed by winding *N* turns of a thin conducting wire into a circular loop of radius *a*. The inductor loop is in the *x*-*y* plane with its center at the origin, and connected to a resistor *R*=1000 Ω. In the presence of a magnetic field $\vec{B} = B_0(4\hat{y} + 2\hat{z})\sin(\omega t)$, where *ωt* is the angular frequency. Find the following:

(a) the magnetic flux linking a single turn of the inductor

(b) The induced voltage *emf*



QUIZ 1.2

A stationary circuit in a time-varying magnetic field. A **circular loop** of N turns of conducting wire lies in the *xy*-plane with its centre at the origin of a magnetic field specified by

$$\vec{B} = B_0 \cos\left(\frac{\pi}{2b}\right) \sin(\omega t)\, \hat{z} \quad T$$

where b is the radius of the loop and ω is the angular frequency.

Find the *emf* induced in the loop.

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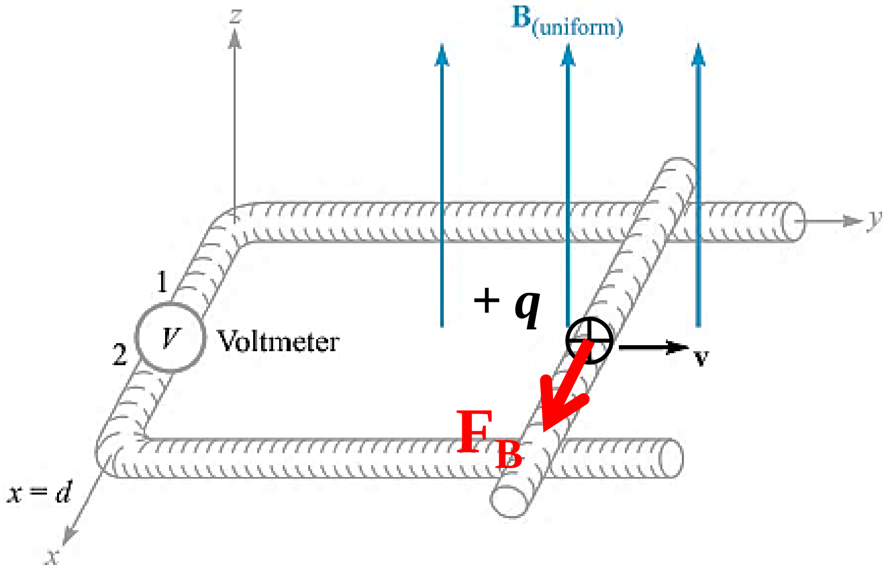
2.1 MOTIONAL EMF

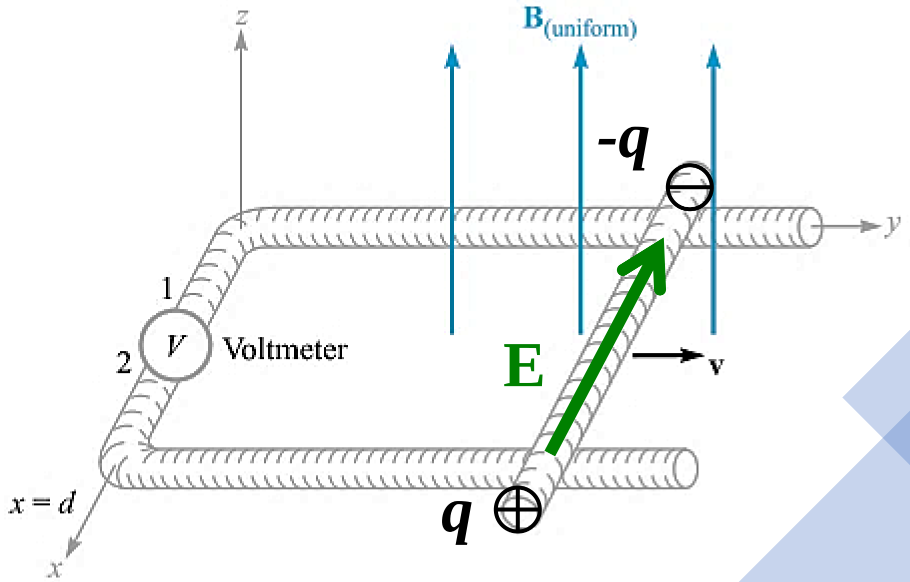
In the constant magnetic field, a conducting bar moves to the right with a velocity *v*, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter is used to read the *emf*.

Analyses

Consider a charge *q* on the conductor, which experiences a force $\boldsymbol{F}_{\mathbf{B}}$, make it drifted to the lower end (+*x* direction) of the conducting bar.



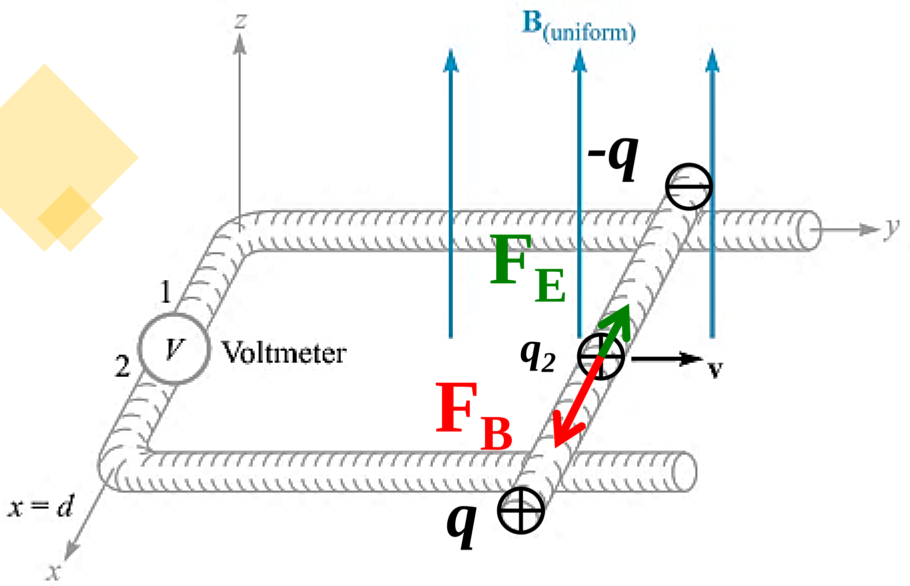
The whole bar is neutral, so a positive and negative charge pair built an internal **E** field inside the bar.



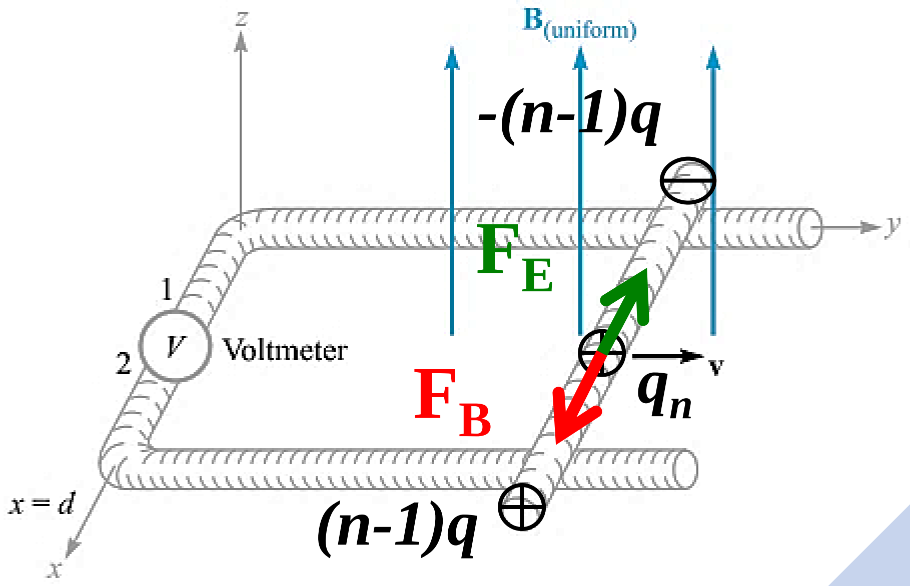
2.1 MOTIONAL EMF

Analyses

Consider a new charge q_2, which experiences two forces, the magnetic force and the electric force. In this case, $\mathbf{F_B} > \mathbf{F_E}$, so q_2 drifts to $+x$ direction and contributes to electric field.



After a very short period, the electric field increases to a value large enough to generate the force $\mathbf{F_E} = \mathbf{F_B}$. Now the charges can move in y direction without x direction drifting – equilibrium state.



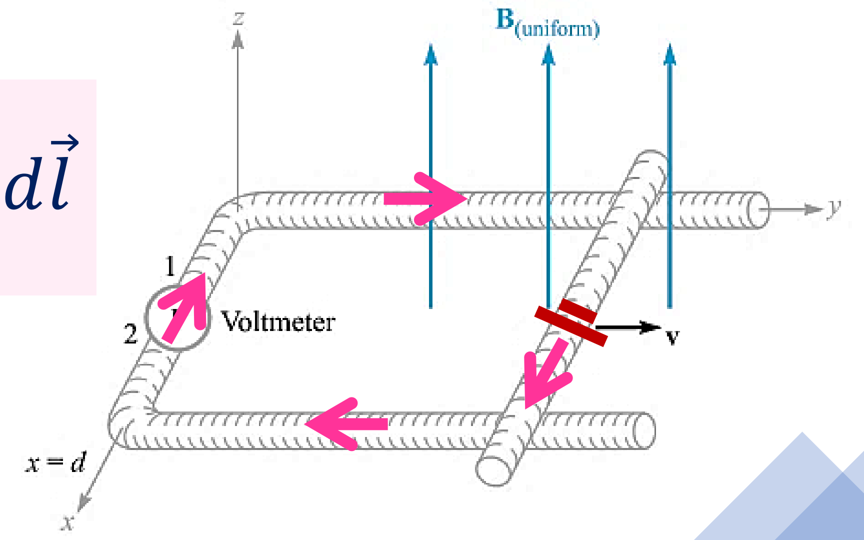
2.1 MOTIONAL EMF

The force per unit charge is called the motional electric field intensity $\vec{E}_m$:

$$\vec{E}_m = \frac{\vec{F}}{q} = \frac{q\,\vec{v} \times \vec{B}}{q} = \vec{v} \times \vec{B}$$

The voltage produced by the induced motional electric field intensity is :

$$emf = \oint \vec{E}_m \cdot d\vec{l} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$



This is referred to as a ***motional emf***.

Only the part of the circuit that moves in a direction not parallel to the magnetic flux will contribute to V.

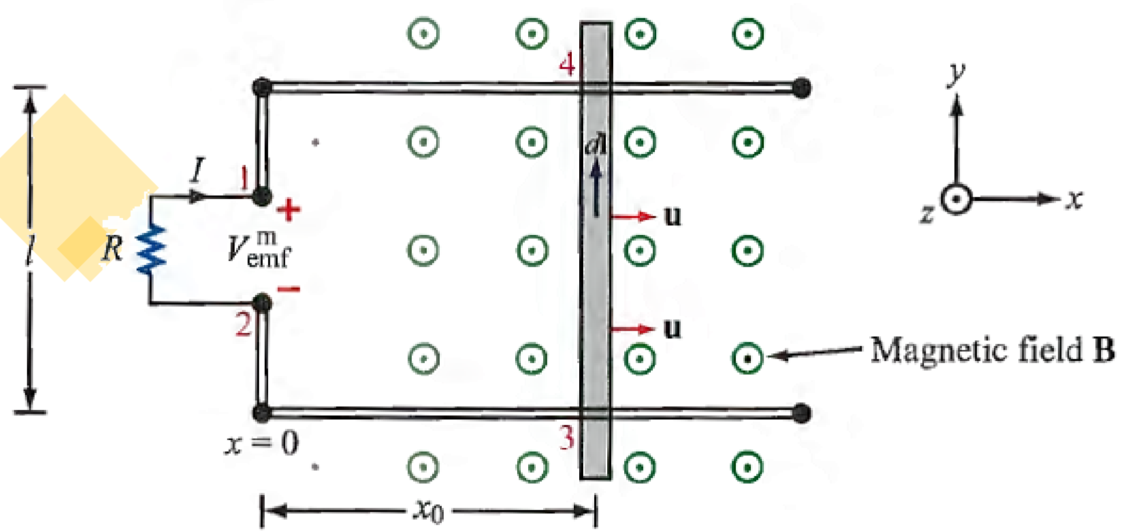
QUIZ 2.1

The rectangular loop has a constant width l and its length x_o increases with time as a conducting bar slides with uniform velocity $\boldsymbol{u}$ in a static magnetic field. The bar starts from $x = 0$ at $t = 0$. Given the magnetic flux density is:

$$\boxed{emf = \oint(\vec{v} \times \vec{B}) \cdot d\vec{l}}$$

$$\vec{B} = x\hat{z} \;\; T$$

Assume that the loop resistance $R_i << R$, find the emf between terminals 1 and 2.



2.2 GENERAL FORM

Recall the **Lorentz's force equation**:

For a charge q moving in a region where both $\vec{E}$ and $\vec{B}$ fields exist, the EM force $\vec{F}$ on q is measured by:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = q\vec{E}'$$

So, when a conducting circuit with contour C and surface S moves with a velocity ***v*** in a mixed field (**E**, **B**), the total *emf* is:

$$\boxed{-\frac{d\Phi}{dt} = emf} = \oint_C \vec{E}' \cdot d\vec{l} = \boxed{-\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}} + \boxed{\oint \left(\vec{v} \times \vec{B}\right) \cdot d\vec{l}} \quad (V)$$

Transformer *emf* – due to the time variation of $\vec{B}$

Motional *emf* – due to the motion of the circuit

This is the general form of **Faraday's Law**.

OUTLINE

$$\oint_C \vec{H} \cdot d\vec{l} = I_{encl}$$

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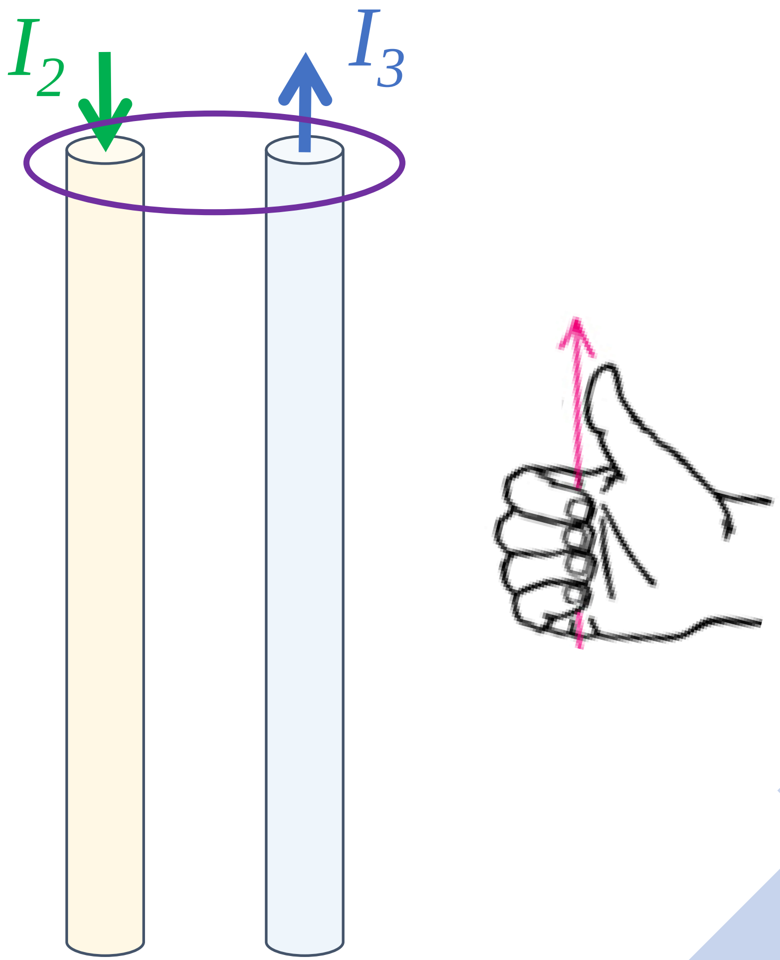
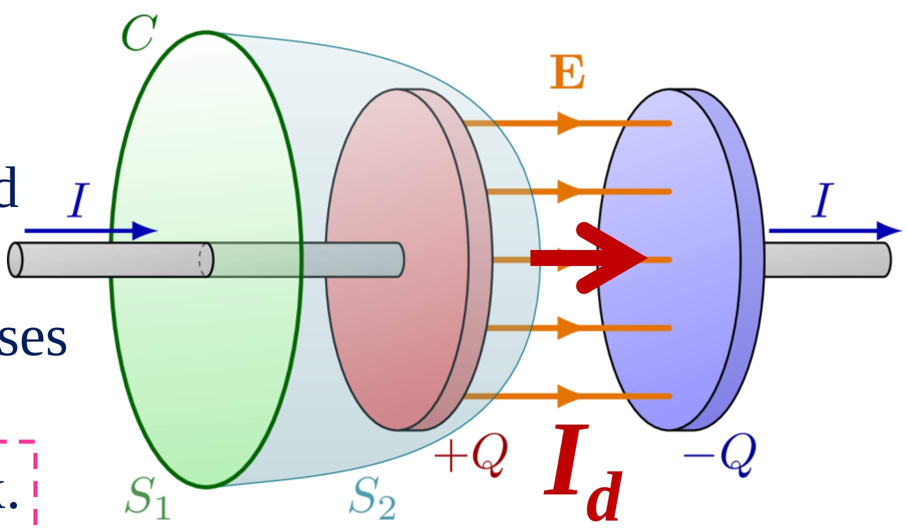


3.1 DISPLACEMENT CURRENT

Displacement current



Consider a capacitor which is being charged by a DC current I .

If the surface bounded by the path is the plane surface, the enclosed current is $I_{encl} = I_c = I$.

If we choose the bulging surface, then $I_{encl} = 0$ since no current passes through this surface.

Idea: adding an extra term which involves a change in electric flux.

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

The electric flux passes through the bulging surface is:

$$\Phi_E = \iint_{area} \vec{E} \cdot d\vec{s} = ES = \frac{Q}{\varepsilon_0}$$

$\because I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt}$, (the rate of increase of charge on the plate) and we know $I_c = \frac{dQ}{dt}$, so $I_d = I_c = I$.

The changing flux is equivalent to a conduction current through that surface.

The generalised Ampere's (or the Ampere-Maxwell) law:

$$\oint_C \vec{H} \cdot d\vec{l} = I_c + \varepsilon_0 \frac{d\Phi_E}{dt}$$

Integral Form

$$\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$$

Differential Form

QUIZ 3.1

Within a certain region, if the magnetic flux density is

$$\vec{H} = 20\cos(10^5 t)\sin(10^{-3} y)\,\hat{x} \;\; A/m$$

Given the permittivity is $\varepsilon = 10^{-11}\ F/m$,

Find the expression of the corresponding electric field intensity.

QUIZ 3.2

Find the maximum value of the displacement current density within a large, oil-filled power capacitor where permittivity is $2.65 \times 10^{-11} C^2/N \cdot m^2$ and the electric field intensity is

$$\vec{E} = 90\, sin[2 \times 10^{-5}(3 \times 10^8 t - 3z)]\, \hat{y}\ V/m$$

Ans: 14.3 $\mu A/m^2$

COMPLETE MAXWELL'S EQUATIONS

Law	Integral	Differential	Physical meaning
Gauss's law for **E-field**	$\oiint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$	$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$	Electric flux through a closed surface is proportional to the charges enclosed
Faraday's law	$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Changing magnetic flux produces an E-field
Gauss's law for **H-field**	$\oiint_S \vec{B} \cdot d\vec{s} = 0$	$\nabla \cdot \vec{B} = 0$	The total magnetic flux through a closed surface is zero
Maxwell-Ampere's law	$\oint_C \vec{H} \cdot \mathrm{d}\vec{l} = I + \varepsilon_0 \frac{d\Phi_E}{dt}$	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	Electric current and changing electric flux produces a magnetic field

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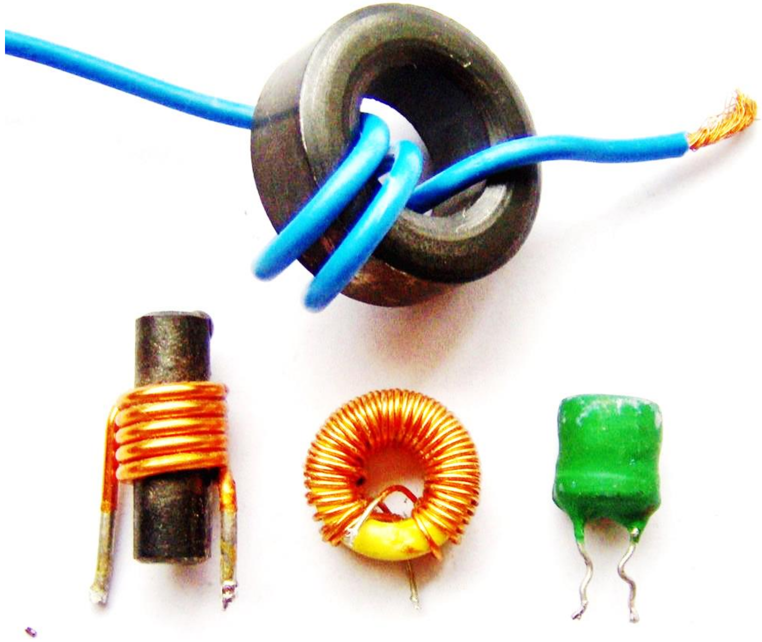
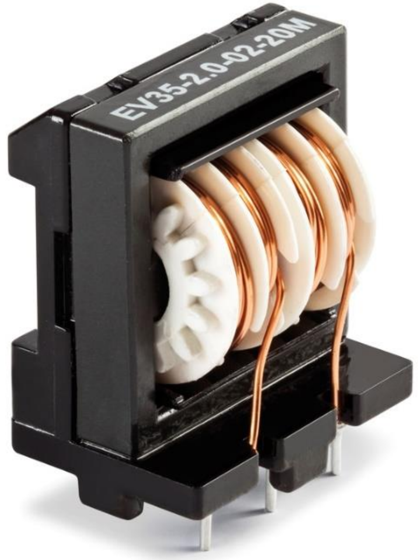
4.1 WHAT IS AN INDUCTOR?

An inductor is a circuit device that is designed to have a particular inductance that can store energy in a magnetic field. An inductor's ability is to store magnetic energy.

Typically, an inductor is a conducting wire shaped as a coil, the loops helping to create a strong magnetic field inside the coil.





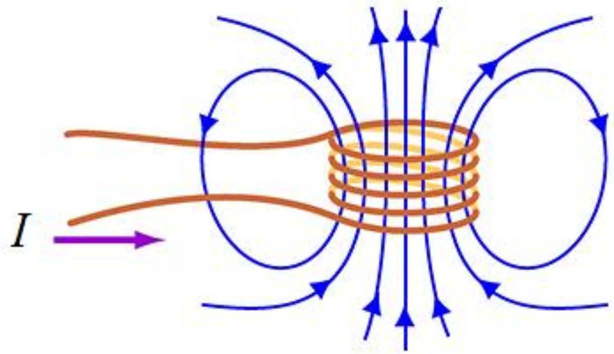
4.2 SELF-INDUCTANCE

Consider a coil consisting of N turns and carrying current I. If current is steady, magnetic flux through the loop remains constant. If I changes with time, then an induced emf arises to oppose the change.

The property of the loop in which its own magnetic field opposes any change in current is called "***self-inductance***" and the emf generated is called the ***self-induced emf*** or ***back emf***.

From Faraday's law:

$$emf_L = -N\frac{d\Phi_B}{dt}$$



The self-induced *emf*:

$$L\frac{dI}{dt} = N\frac{d\Phi_B}{dt}$$

So, the self-inductance:

$$\boxed{L = N\frac{\Phi_B}{I}}$$

the ratio of the total flux linkages to the current

SI unit for inductance: H (Henry) = 1 Wb/A = 1 V·s/A

QUIZ 4.1

Calculate the inductance per meter length of a coaxial cable of inner radius a and outer radius b.

The magnetic flux density is:

$$B_\varphi = \frac{\mu_0 I}{2\pi r} \quad (a < r < b)$$

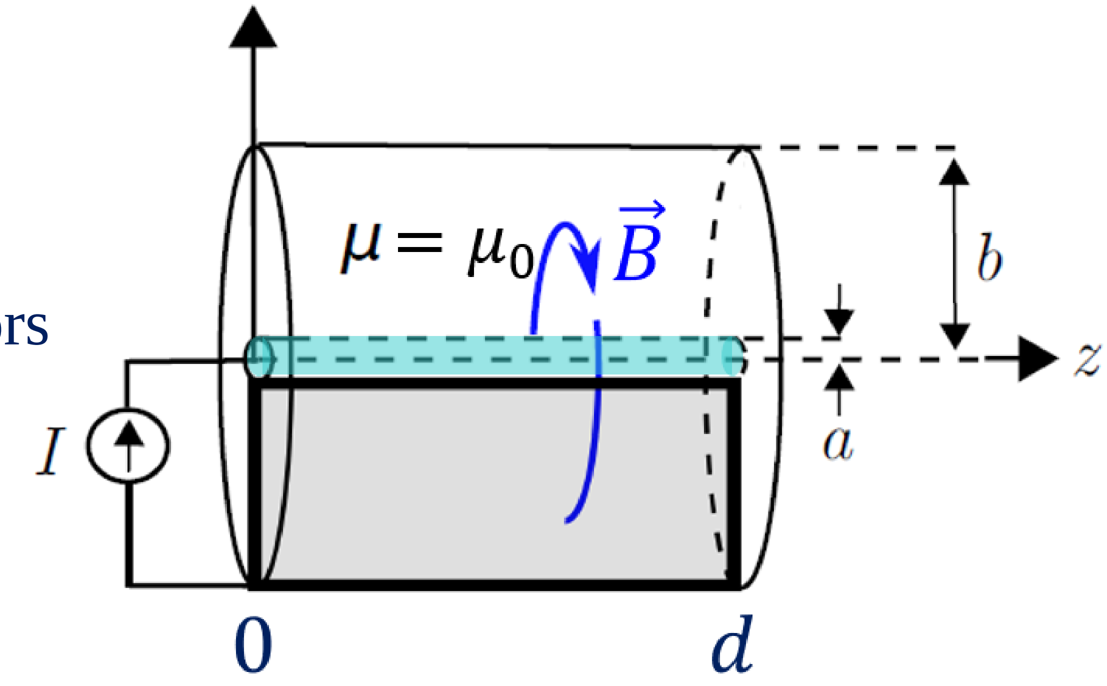
The magnetic flux contained between the conductors for length d:

$$\Phi_B = \iint \vec{B}\, d\vec{s}$$

$$= \mu_0 \int_0^d \int_a^b \frac{I}{2\pi r} dr dz\, \hat{\varphi} \cdot \hat{\varphi} = \frac{\mu_0 I d}{2\pi} \ln\left(\frac{b}{a}\right)$$

So the inductance rapidly for a length d is:

$$L = N\frac{\Phi_B}{I} = \frac{\mu_0 d}{2\pi} \ln\left(\frac{b}{a}\right)$$



On a per-meter basis:

$$\boxed{L_l = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \text{ H/m}}$$

4.3 MUTUAL-INDUCTANCE

Mutual inductance is the effect of Faraday’s law of induction for one device upon another, such as the primary coil in transmitting energy to the secondary in a transformer.

4.3 MUTUAL-INDUCTANCE

Suppose two coils are placed near each other.

Some of the magnetic field lines through coil 1 will also pass coil 2.

Let Φ_{21} denote the magnetic flux through one turn of coil 2 due to I_1.

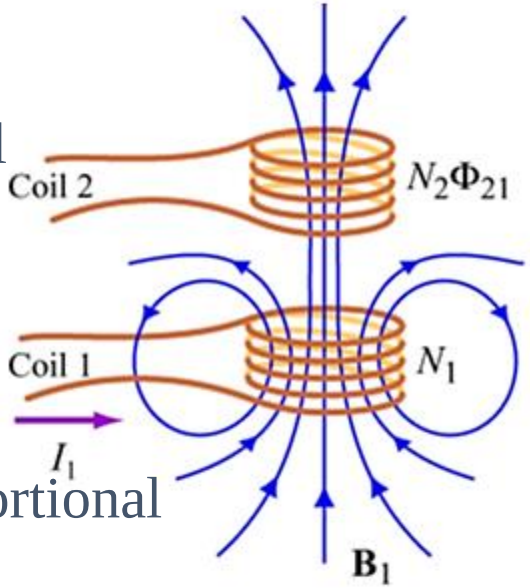
By varying I_1 with t, there will be an induced *emf* associated with the changing magnetic flux in coil 2:

$$v_2 = N_2 \frac{d\Phi_{21}}{dt}$$

The rate of change of Φ_{21} in coil 2 is proportional to the time rate of the change of the current in coil 1:

$$N_2 \frac{d\Phi_{21}}{dt} = N_2 \frac{d\Phi_{21}}{dI_1} \cdot \frac{dI_1}{dt} = M_{21} \cdot \frac{dI_1}{dt}$$

Similarly, the induced emf in coil 1 due to current change in coil 2:



$$v_1 = N_1 \frac{d\Phi_{12}}{dt}$$

This changing flux is also proportional to the changing current in coil 2:

$$N_1 \frac{d\Phi_{12}}{dt} = N_1 \frac{d\Phi_{12}}{dI_2} \cdot \frac{dI_2}{dt} = M_{12} \cdot \frac{dI_2}{dt}$$

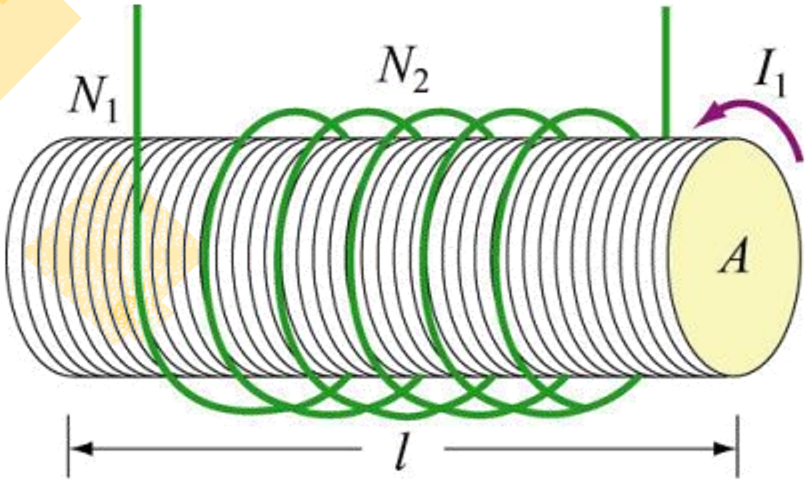
The proportionality constant M_{12} and M_{21} are equal:

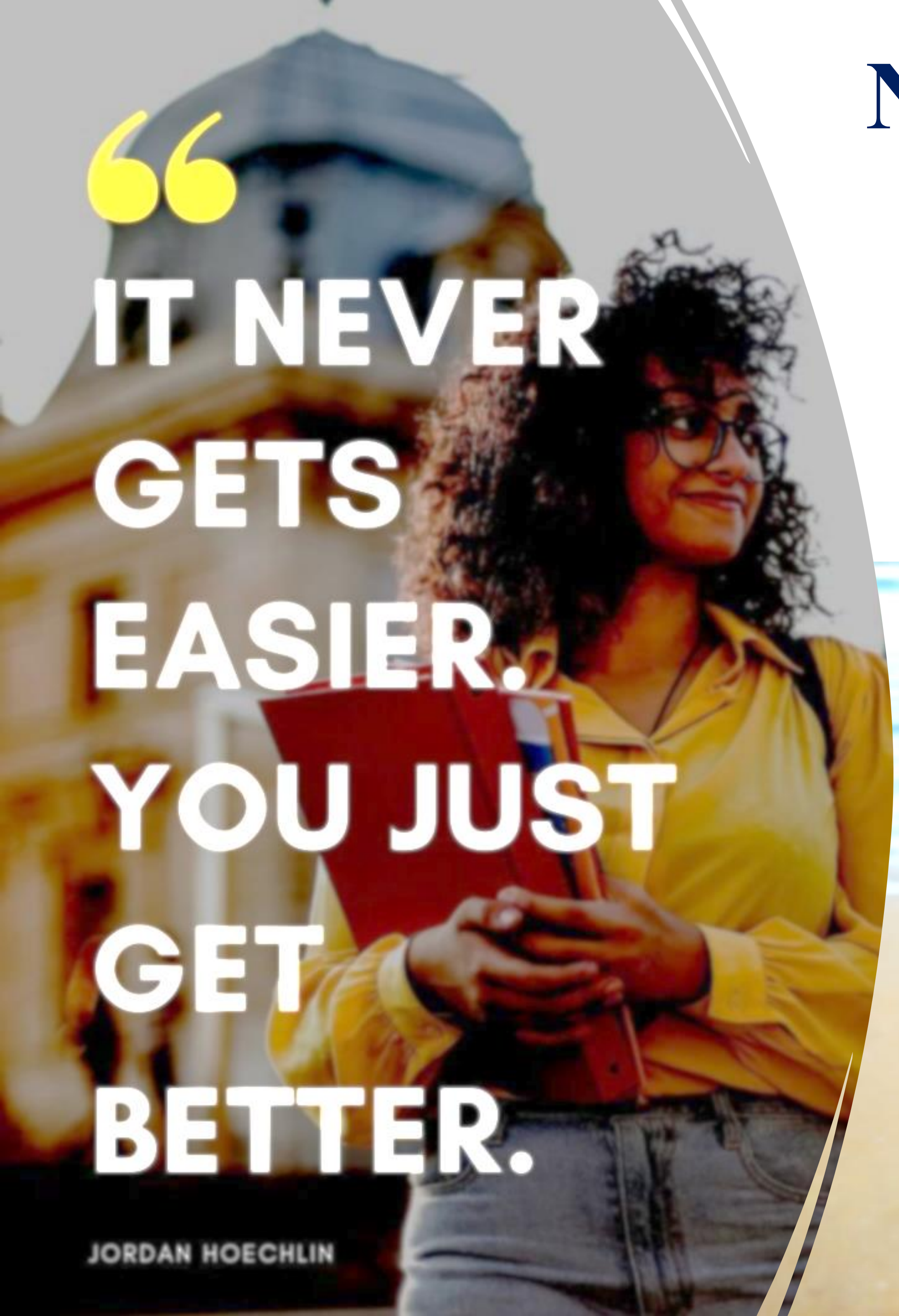
$$M_{12} = M_{21} \equiv M$$

mutual inductance

QUIZ 4.2

A long solenoid with length l and a cross-sectional area A consists of N_1 turns of wire. An insulated coil of N_2 turns is wrapped around it. Given that the relative permeability is μ_r=1 and all the flux from the solenoid passes through the outer coil, calculate the mutual inductance between the coils.





NEXT…

Sinusoidal Fields

&

EM Wave Propagation