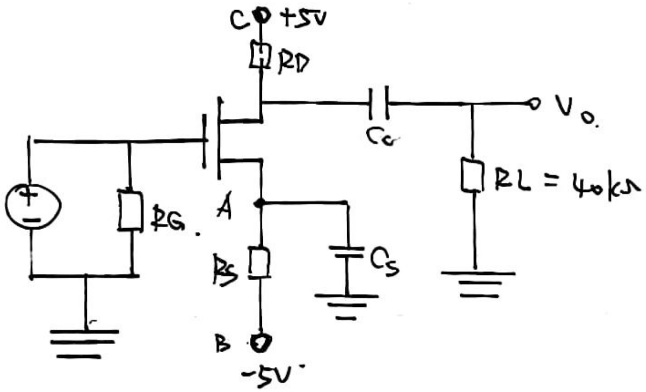
EEE 109 HW2

1.



$V_{TN} = 0.8V \quad k_n = 0.85 mA/V^2, \lambda = 0.02 V^{-1}$

(a) Assume that MosFET is biased in Saturation - Region

Such that $\begin{cases} V_{GSQ} > V_{TN} \\ V_{DSQ} \geq V_{GSQ} - V_{TN} \\ I_{DQ} = k_n (V_{GSQ} - V_{TN})^2 \end{cases}$

$\begin{cases} I_{DQ} = k_n (V_{GSQ} - V_{TN})^2 \\ I_{DQ} = 0.1 mA \\ k_n = 0.85 mA/V^2 \\ V_{TN} = 0.8 V \end{cases}$

$\Rightarrow 0.1 m = 0.85 m [V_{GSQ} - 0.8]^2$

$\Rightarrow V_{GSQ} = \begin{cases} 1.14 V \\ 0.46 V \end{cases}$

If $V_{GSQ} = 0.46V$

$\Rightarrow V_{GSQ} = 0.46V < V_{TN} = 0.8V$

$\Rightarrow$ this solution should be excluded

$\Rightarrow$ Therefore, $V_{GSQ} = 1.14V$

$\Rightarrow \begin{cases} V_{GQ} - V_{SQ} = 1.14V \\ V_{GQ} = 0V \end{cases}$

$\Rightarrow V_{SQ} = -1.14 V$

Apply ohm's Law to land AB

$\Rightarrow \begin{cases} I_{DQ} = \dfrac{V_{SQ}+5}{R_S} \\ V_{SQ} = -1.14V \\ \cancel{R_S =} \\ I_{DQ} = 0.1 mA \end{cases}$

$\Rightarrow R_S = \dfrac{5-1.14}{0.1 m}$

$\Rightarrow R_S = 38.6 k\Omega$

Apply KVL to loop CAB

$\Rightarrow \begin{cases} 5-(-5) = I_{DQ}(R_D+R_S) + V_{DSQ} \\ I_{DQ} = 0.1 mA \\ R_S = 38.6 k\Omega \\ V_{DSQ} = 5.5 V \end{cases}$

$\Rightarrow R_D = 6.4 k\Omega$

$\Rightarrow$ In Summary $\begin{cases} R_S = 38.6 k\Omega \\ R_D = 6.4 k\Omega \end{cases}$

(b) $g_m = \dfrac{i_d}{v_{gs}}$

$\begin{cases} = 2k_n [V_{GSQ} - V_{TN}] \\ k_n = 0.85 m A/V^2 \\ V_{GSQ} = 1.14V \\ V_{TN} = 0.8V \end{cases}$

$\Rightarrow g_m = 0.578 \ m \ (S)$

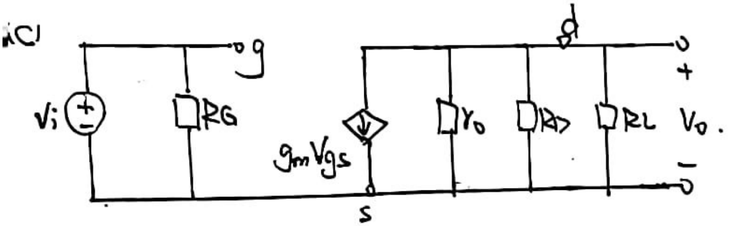
$r_o = \dfrac{1}{\lambda I_{DQ}}$

$r_o = \dfrac{1}{(0.02) \cdot 0.1 mA}$

$\Rightarrow r_o = 500 k\Omega$

In Summary $\begin{cases} g_m = 0.578 \ m \ (S) \\ r_o = 500 \ k \ (\Omega) \end{cases}$

(c)



(d) $V_{out} = -g_m V_{gs} \, (r_o \| R_D \| R_L)$

$r_o = 500\,k\Omega$; $R_D = 6.4\,k\Omega$; $R_L = 40\,k\Omega$

$g_m = 0.578\ m\ (S)$

$\Rightarrow V_{out} = -3.15\, V_{gs}$

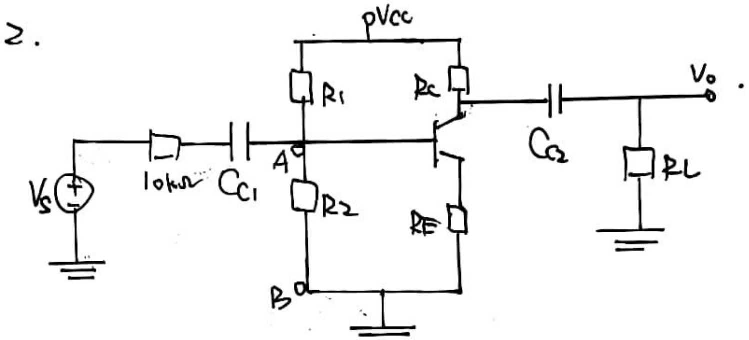
$V_{in} = V_{gs}$

$\frac{V_{in}}{V_s} = \frac{V_{in}}{V_i} = 1$

$A_{Vs} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_S}$

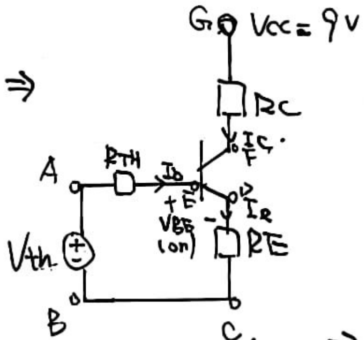
$\Rightarrow A_{Vs} = -3.15$

2.



(a) Take load AB as the load of Thevenin Equivalent Circuit And Set all the Ac Sources "Zero"

$\Rightarrow$



$V_{TH} = \frac{R_2}{R_1+R_2} \cdot V_{cc}$

$V_{cc} = 9V$

$R_1 = 27\,k\Omega$; $R_2 = 15\,k\Omega$

$\Rightarrow V_{TH} = \frac{15}{15+27} \cdot 9 = 3.21V$

$R_{TH} = R_1 \| R_2$

$= \frac{R_1 \cdot R_2}{R_1 + R_2}$

$R_1 = 27\,k\Omega$, $R_2 = 15\,k\Omega$

$\Rightarrow R_{TH} = \frac{27 \times 15}{27+15} k = 9.64\,k\Omega$

Apply kvl to loop "ABCDEA"

$\Rightarrow 3.21 = 9.64k \cdot I_{bQ} + 0.7 + 1.2k\, I_e$

$I_e = (1+\beta) I_b = 101\, I_b$

$\Rightarrow I_{bQ} = \frac{3.21 - 0.7}{9.64 + 101 \times 1.2}\ mA$

$\Rightarrow I_{bQ} = 0.01918\ mA$

$I_{bQ} = 19.18\ \mu A$

$I_{CQ} = \beta\, I_{bQ}$

$I_{EQ} = (1+\beta)\, I_{bQ}$

$\Rightarrow I_{CQ} = 1.918\ mA = 1.92\ mA$

$I_{EQ} = 1.937\ mA = 1.94\ mA$

Apply kvl to loop "GFDC"

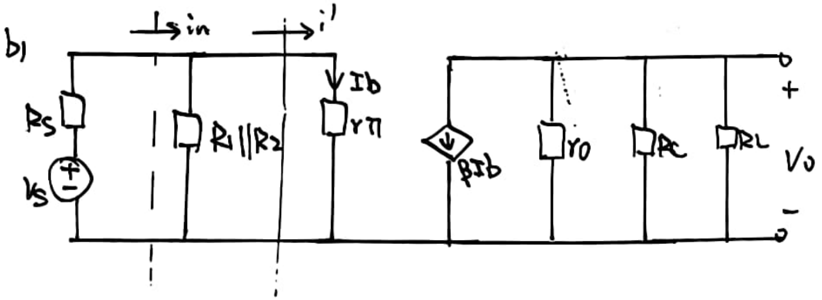
$\Rightarrow 9 - V_{CEQ} = I_{CQ} \cdot R_C + I_{EQ} \cdot R_E$

$R_C = 2.2\,k\Omega$; $R_E = 1.2\,k\Omega$.

$I_{CQ} = 1.92mA$, $I_{EQ} = \frac{101}{100} I_{CQ}$

$\Rightarrow V_{CEQ} = 2.45V \Rightarrow$ In summary $\begin{cases} I_{CQ} = 1.92mA \\ V_{CEQ} = 2.45V \end{cases}$

(b)



(c).

$$R_i' = \frac{V_i'}{I_b}$$

$$V_i' = I_b \cdot r_\pi$$

$$\Rightarrow R_i' = r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 26\,mV}{1.92\,mA} = 1.35\,k\Omega$$

$$R_{in} = (R_1 \parallel R_2) \parallel R_i'$$

$$\Rightarrow R_{in} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_\pi}}$$

$R_\pi = 1.35\,k\Omega$

$R_1 = 27\,k\Omega$

$R_2 = 15\,k\Omega$

$$\Rightarrow R_{in} = 1.18\,k\Omega$$

(d) $A_v = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{V_s}$

$$V_{out} = -\beta I_b (r_o \parallel R_C \parallel R_L)$$

$$V_{in} = I_b \cdot r_\pi$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = (-\beta) \cdot \frac{r_o \parallel R_C \parallel R_L}{r_\pi}$$

$$\frac{V_{in}}{V_S} = \frac{R_{in}}{R_{in} + R_S}$$

$$\Rightarrow A_v = (-\beta) \cdot \frac{R_{in}}{R_{in} + R_S} \cdot \frac{r_o \parallel R_C \parallel R_L}{r_\pi}$$

$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{1.92}\,k\Omega$.

$R_C = 2.2\,k\Omega$; $R_L = 2\,k\Omega$

$r_\pi = 1.35\,k\Omega$

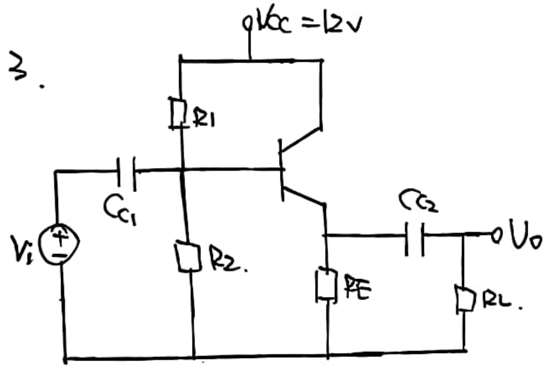
$R_{in} = 1.18\,k\Omega$.

$R_S = 10\,k\Omega$

$\beta = 100$

$$\Rightarrow A_v = -8.05$$

3.

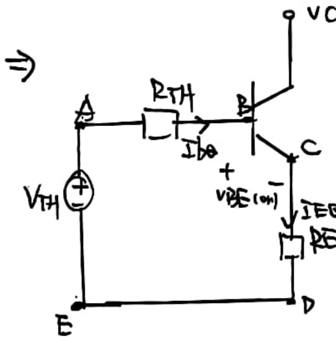


(a) DC Analysis

Turn off AC sources and

Apply Thevenin Equivalent Method

⇒



$$\begin{cases} V_{TH} = V_{CC}\cdot\dfrac{R_2}{R_1+R_2} \\ R_{TH} = 0.1(1+\beta)R_E \\ \quad = R_1 \parallel R_2 \end{cases}$$

Assume BJT is biased in Saturation

Apply KVL to loop ABCDEA

$$\Rightarrow V_{TH} = I_{BQ}\cdot R_{TH} + V_{BE(on)} + I_{EQ}\cdot R_E$$

$$\Rightarrow \left.\begin{aligned} V_{TH} &= 0.7 + 1.1\, I_{EQ}\cdot R_E \\ I_{EQ} &= 1.5\,mA ;\ R_E = 4k\Omega \\ V_{TH} &= V_{CC}\cdot\frac{R_2}{R_1+R_2} \end{aligned}\right\}$$

$$\Rightarrow \frac{R_2}{R_1+R_2} = \frac{73}{120} \quad ①$$

$$\left.\begin{aligned} R_{TH} &= 0.1(1+\beta)R_E \\ \beta &= 120,\ R_E = 4k\Omega \\ R_{TH} &= R_1 \parallel R_2 \end{aligned}\right\}$$

$$\Rightarrow \frac{R_1\cdot R_2}{R_1+R_2} = 48.4\ k. \quad ②$$

Combine ① and ②

$$\Rightarrow \begin{cases} R_1 = 79.56\ k\Omega. \\ R_2 = 123.57\ k\Omega. \end{cases}$$

$$V_{CEQ} = V_{CC} - I_{EQ}\cdot R_E$$

$$= 6V > V_{BE(on)} = 0.7V$$

⇒ BJT is biased in Saturation.

⇒ In Summary

$$\begin{cases} R_1 = 79.56\ k\Omega \\ R_2 = 123.57\ k\Omega \end{cases}$$

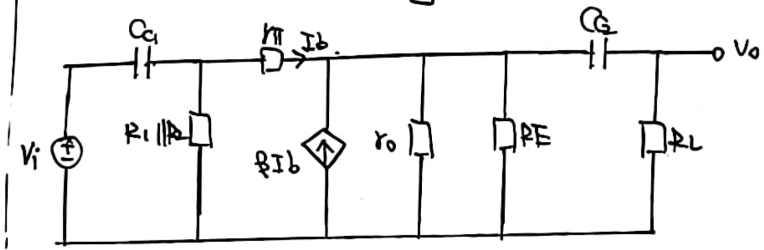
(b) From (a) $\left.\begin{aligned} I_{EQ} &= 1.5mA \\ \beta &= 120 \end{aligned}\right\}$

$$\Rightarrow I_{CQ} = \frac{\beta}{1+\beta} I_{EQ} = \frac{120}{121}\cdot 1.5mA = 1.49\ mA$$

$$\Rightarrow \begin{cases} r_\pi = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{100\cdot 26m}{1.49m} = 1.74\ k\Omega \\ r_o = \dfrac{V_A}{I_{CQ}} = \dfrac{50}{1.49m} = 33.56\ k\Omega \end{cases}$$

AC Analysis

⇒ Draw small signal model



Due to $\begin{cases} C_{C1} \to \infty . \\ C_{C2} = 2\mu F \end{cases}$

$$\Rightarrow \begin{cases} V_i = V_{in} = I_b\cdot r_\pi + I_b\cdot(1+\beta)\cdot\left[r_o \parallel R_E \parallel \left(\dfrac{1}{sC_{C2}} + R_L\right)\right] \\ V_o = I_b\cdot(1+\beta)\left[r_o \parallel R_E \parallel \left(\dfrac{1}{sC_{C2}} + R_L\right)\right]\cdot\dfrac{R_L}{R_L+\frac{1}{sC_{C2}}} \end{cases}$$

$$\Rightarrow A_v(s) = \frac{V_o}{V_i} = \frac{r_\pi + (1+\beta)\left[r_o \parallel R_E \parallel \left(\frac{1}{sC_{C2}}+R_L\right)\right]}{(1+\beta)\left[r_o \parallel R_E \parallel \left(\frac{1}{sC_{C2}}+R_L\right)\right]}\cdot\frac{1+sC_{C2}R_L}{sC_{C2}R_L}$$

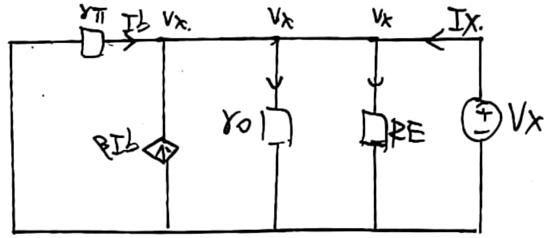
$r_\pi = 1.74k\Omega$, $\beta = 120$, $r_o = 33.56k\Omega$, $R_E = 4k\Omega$, $R_L = 4k\Omega$

$C_{C2} = 2\mu F$

$$\Rightarrow A_{vi}(s) = \frac{1+s\cdot g_m}{s\cdot g_m} + \frac{1+s\cdot g_m}{s\cdot g_m}\cdot\frac{4090.87s + 270059.50}{536.96s + 67120}$$

(c) Apply a test voltage source

$\Rightarrow$



Apply KCL

$$\left.\begin{aligned} I_b + \beta I_b + I_x &= \frac{V_x}{r_o} + \frac{V_x}{R_E} \\ I_b &= \frac{-V_x}{r_\pi} \end{aligned}\right\}$$

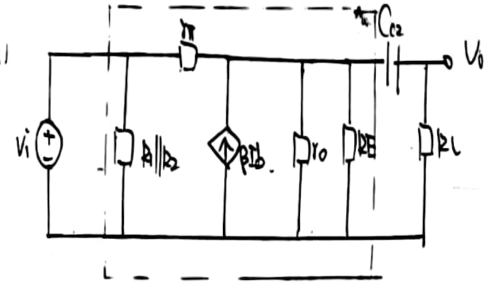
$$\Rightarrow \frac{V_x}{I_x} = \frac{1}{\frac{1}{r_o} + \frac{1}{R_E} + \frac{1+\beta}{r_\pi}}$$

$r_o = 33.56\ k\Omega$, $\beta = 120$

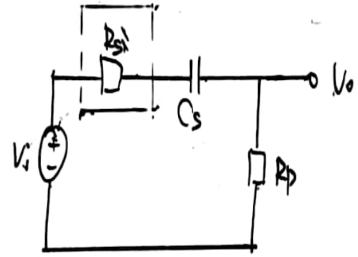
$R_E = 4\ k\Omega$, $r_\pi = 1.74 k\Omega$

$$\Rightarrow \frac{V_x}{I_x} = R_o = 14.32\Omega$$

(d)



Notice that Capacitor C_{c1} is treated as Short Circuit.



Compared with High-Pass Model

$$\begin{cases} C_s = C_{c2} = 2\mu F \\ R_p = R_L = 4k\Omega \\ V_i = v_i \\ R_s = R_o = 14.32\Omega \end{cases}$$

$$\Rightarrow \tau_s = C_s(R_s + R_p)$$

$$= 2\mu F(4k\Omega + 14.32\Omega) = 8.03\ ms$$

$$f_L = \frac{1}{2\pi\tau_s} = \frac{1}{2\pi \times 8.03 ms} = 19.82\ Hz$$