

# Chapter 6: Basic BJT Amplifiers

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ELECTRICAL  
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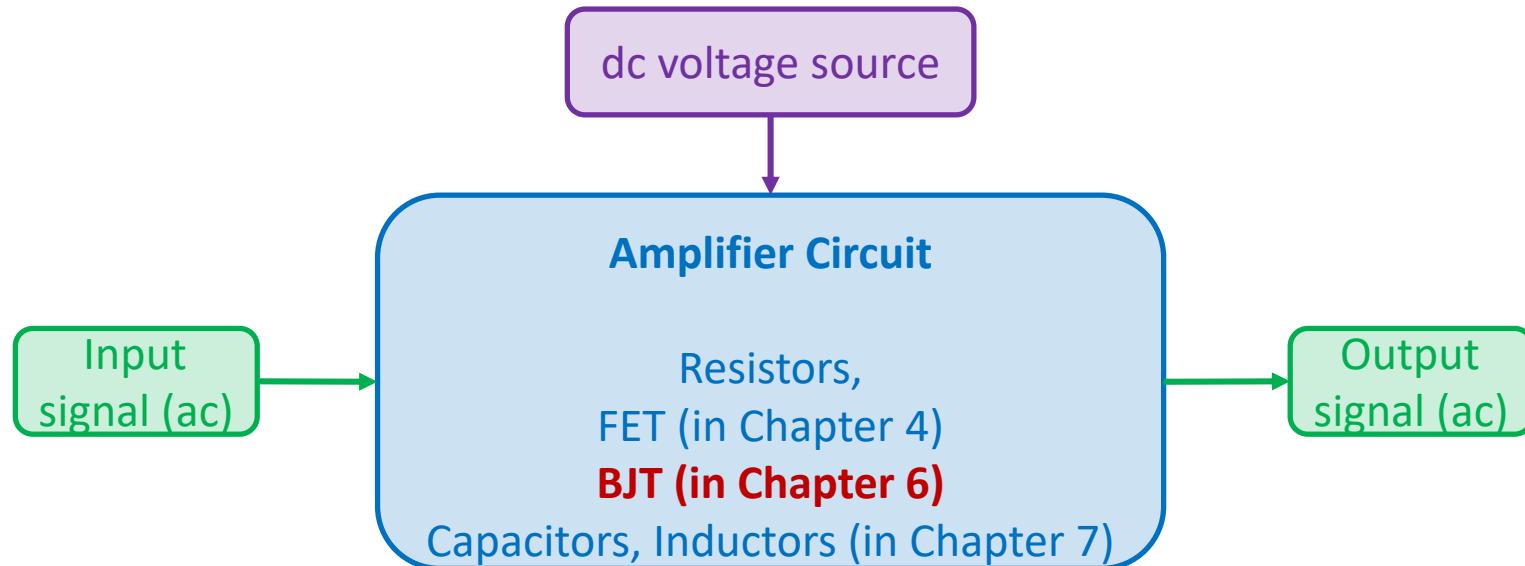
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# Preview

- Investigate how a **BJT transistor circuit** can amplify a small, time-varying input signal
- Discuss the **three** basic transistor amplifier configurations
  - Analyze the **common-emitter**, **common-collector**, and **common-base** amplifiers
  - Compare the general characteristics of the three basic amplifier configurations.
- Optional: analyze multitransistor amplifiers and understand the advantages of these circuits over single-transistor circuits

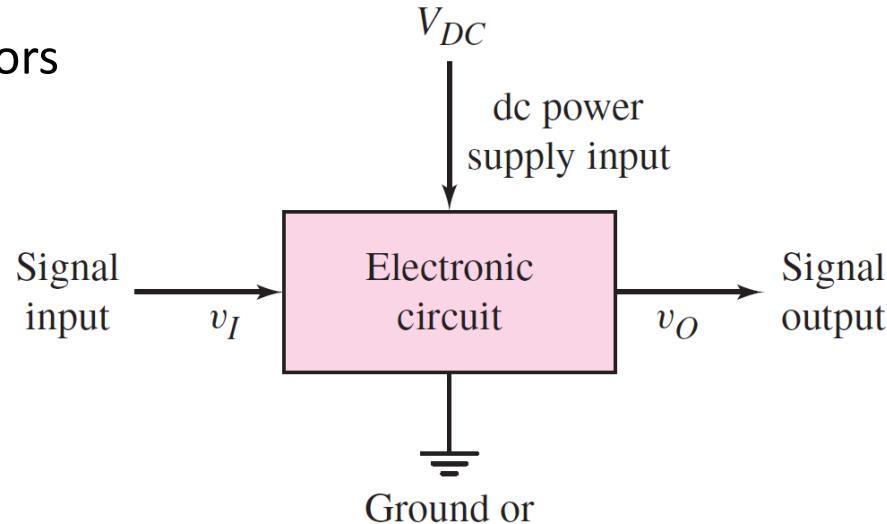
# Linear Amplifier

- Bipolar Junction Transistor (BJT) is used as the amplifying device
  - The magnitude of the output signal is **larger** and **directly proportional** to the input signal



# DC and AC Analysis

- **Superposition:** ac analysis and dc analyses must be applied
  - Total value = dc value + ac value
- **Step 1: dc analysis**
  - Large-signal analysis
  - Set **ac source** to zero
  - Establish the ***Q*-point** of the transistors
- **Step 2: ac analysis**
  - Small-signal analysis
  - Set **dc source** to zero
  - ac equivalent circuit
  - **Small-signal equivalent circuit**



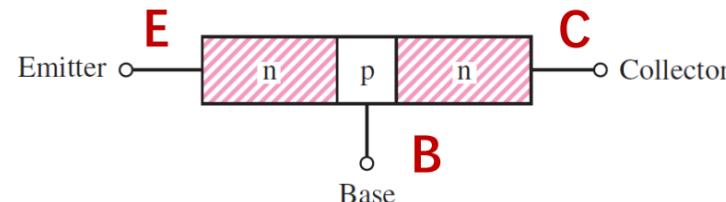
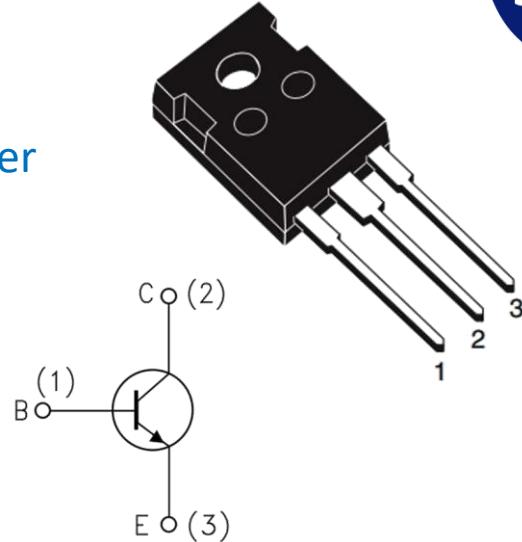
# The Bipolar Linear Amplifier

Investigate the process by which a single-transistor circuit can amplify a small, time-varying input signal and develop the small-signal models of the transistor that are used in the analysis of linear amplifiers.

# Operation Modes of a NPN BJT

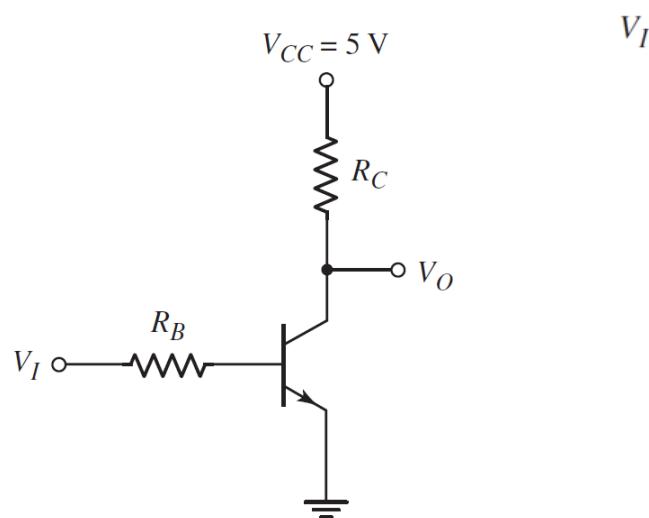
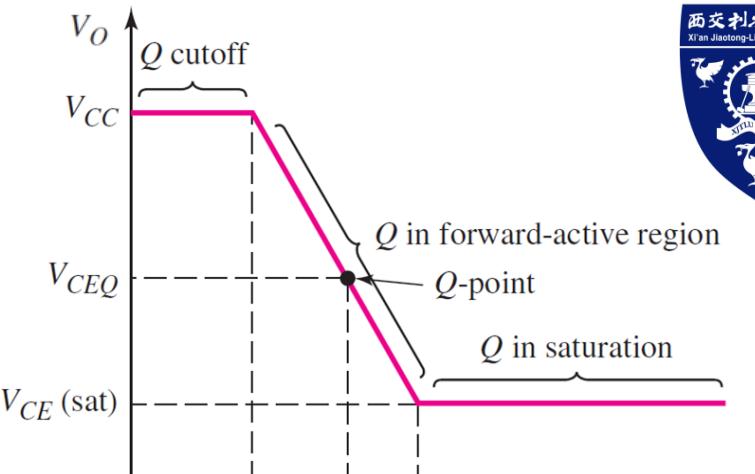
- Saturation region
  - B-E and B-C junctions are forward biased
- Cut-off region
  - B-E and B-C junction are reversed biased
- Forward-active region
  - B-E junction is forward biased
  - B-C junction is reverse biased
- Inverse-active region
  - B-E junction is reverse biased
  - B-C junction is forward biased

Inverter      Amplifier



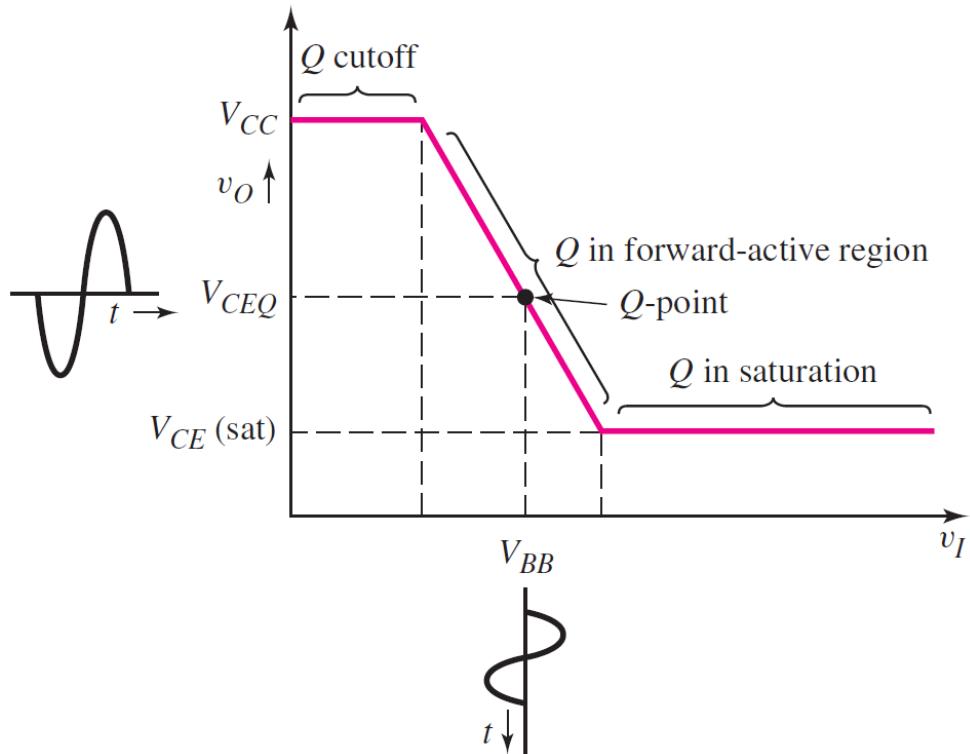
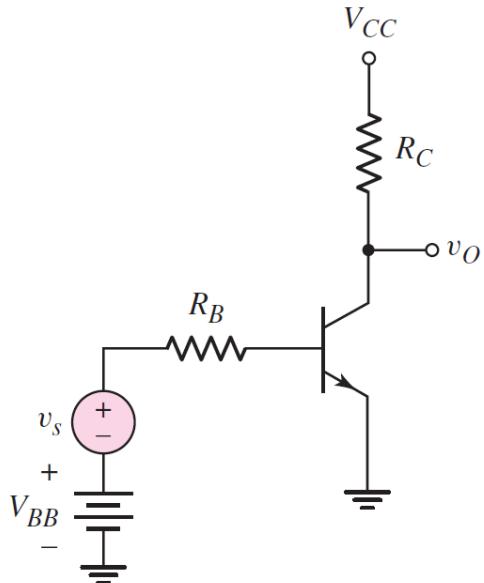
# BJT Inverter Circuit

- When the input  $V_I$  is **low** ( $V_I \leq 0.7V$ )
  - B-E and B-C are both reverse biased
  - The BJT is biased in the **cut-off region**
  - $I_B = I_C = 0$
  - The output is **high**  $\rightarrow V_O = V_{CC}$
- When the input  $V_I$  is **high** ( $V_I = V_{CC}$ )
  - B-E and B-C are both forward biased
  - The BJT is biased in the **saturation region**
  - The output  $V_O$  is **low**  $\rightarrow V_O = V_{CE}(\text{sat})$



# Basic BJT Amplifier Circuit

- When the BJT is biased in the **forward-active region**
  - A linear amplifier circuit
  - ac voltage  $v_s$  is to be amplified



# Basic BJT Amplifier Circuit

- When the BJT is biased in the forward-active region

- $i_C = \beta i_B \quad i_C = \alpha i_E \quad i_E = i_B + i_C$

- $\alpha$  is called the common-base current gain ( $\alpha < 1$ )
  - $\beta$  is called the common-emitter current gain ( $50 < \beta < 300$ )
  - $\beta = \frac{\alpha}{1-\alpha}$

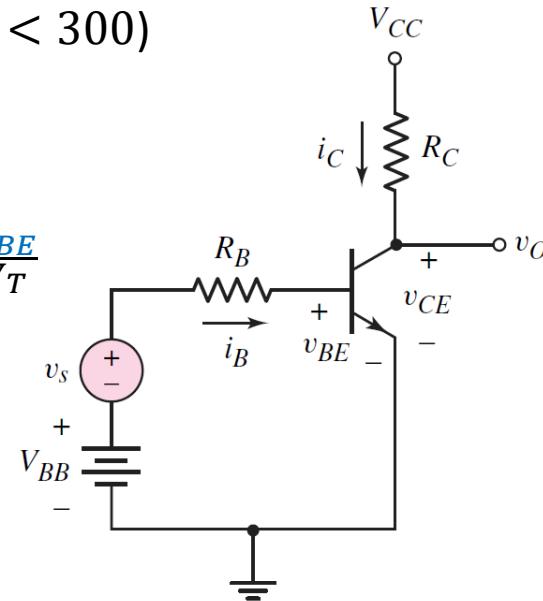
- B-E junction is **forward** biased

$$i_E = I_{EO} \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) \cong I_{EO} e^{\frac{v_{BE}}{V_T}} \quad i_B = I_{BO} \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) \cong I_{BO} e^{\frac{v_{BE}}{V_T}}$$

$$i_C = I_S \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) \cong I_S e^{\frac{v_{BE}}{V_T}}$$

- $I_{EO}$ ,  $I_{BO}$ , and  $I_S$  are parameters of the junction

- B-C junction is **reversed** biased

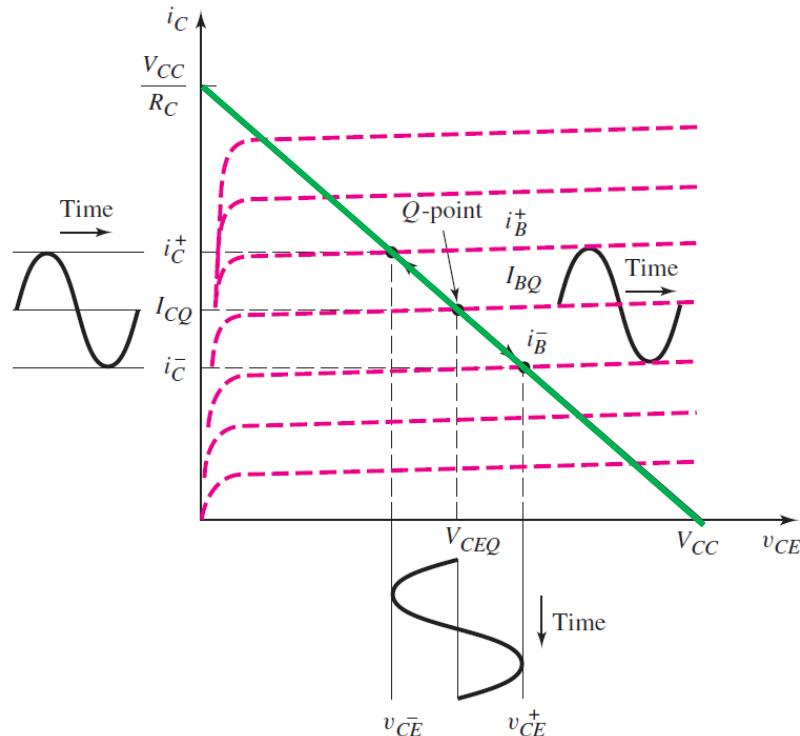
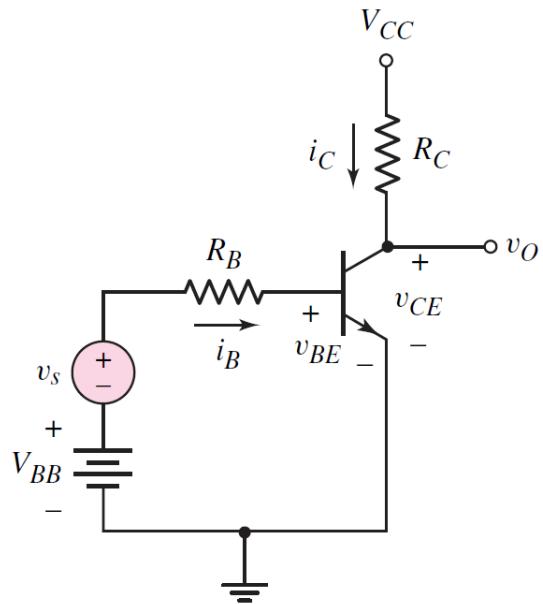


# Basic BJT Amplifier Circuit

- Based on the **KVL equation** from  $V_{CC}$  to C-E to ground

- $v_o = v_{CE} = V_{CC} - i_C R_C$

$$i_C \cong I_S e^{\frac{v_{BE}}{V_T}}$$



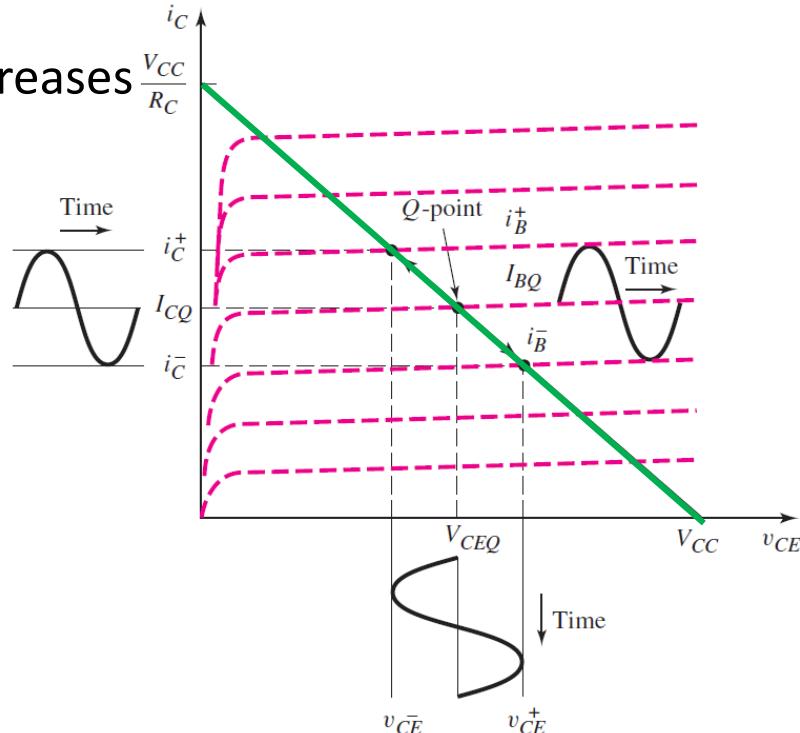
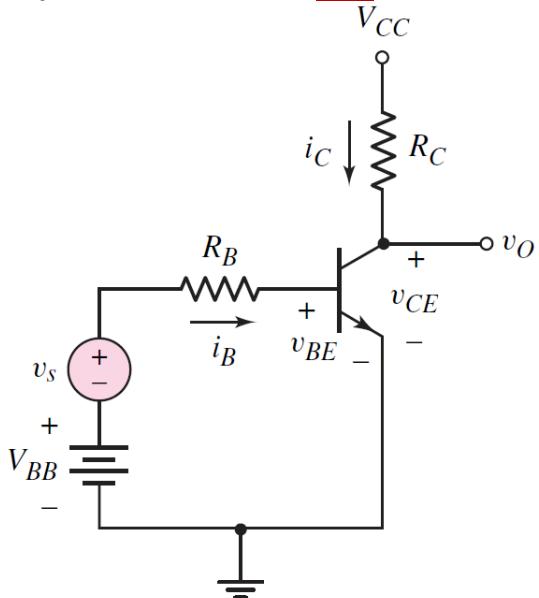
# Graphical Analysis

$$i_C \cong I_S e^{\frac{v_{BE}}{V_T}}$$

$$v_O = v_{CE} = V_{CC} - i_C R_C$$

- As  $v_s$  increases

- $v_{BE}$  increases  $\rightarrow i_C$  increases  $\rightarrow v_{CE}$  decreases
- Q-point moves up the load line



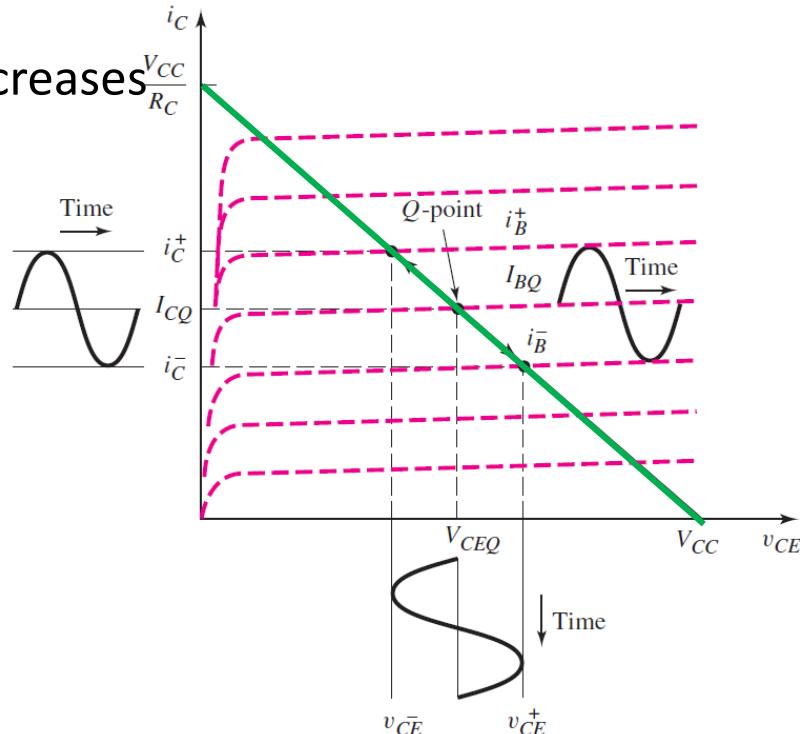
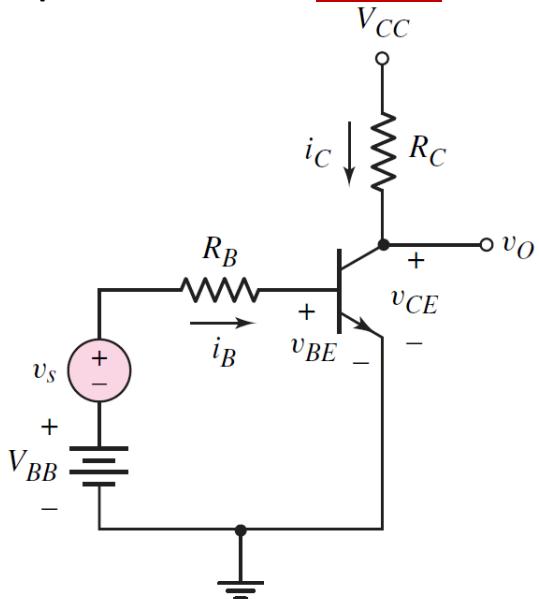
# Graphical Analysis

$$i_C \cong I_S e^{\frac{v_{BE}}{V_T}}$$

$$v_O = v_{CE} = V_{CC} - i_C R_C$$

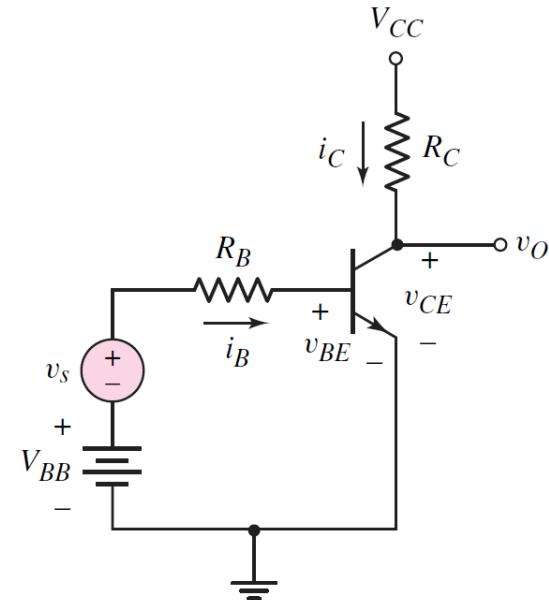
- As  $v_s$  decreases

- $v_{BE}$  decreases  $\rightarrow i_C$  decreases  $\rightarrow v_{CE}$  increases
- Q-point **moves down** the load line



# Basic BJT Amplifier Circuit

- As  $v_s$  increases
  - $v_{BE}$  increases  $\rightarrow i_C$  increases  $\rightarrow v_{CE}$  decreases
  - Q-point **moves up** the load line
- As  $v_s$  decreases
  - $v_{BE}$  decreases  $\rightarrow i_C$  decreases  $\rightarrow v_{CE}$  increases
  - Q-point **moves down** the load line
- A **smaller** change at the input ( $v_s$ ), produces a **larger** change at the output ( $v_{CE}$ , i.e.,  $v_o$ )



# Small Signal: Base Current

- The total instantaneous value of the base current is

$$i_B \cong I_{BO} e^{\frac{v_{BE}}{V_T}} = I_{BO} e^{\frac{V_{BEQ} + v_{be}}{V_T}} = I_{BO} e^{\frac{V_{BEQ}}{V_T}} e^{\frac{v_{be}}{V_T}} = I_{BQ} e^{\frac{v_{be}}{V_T}}$$

- $V_{BEQ}$  is the base-emitter turn-on voltage  $V_{BE}(\text{on}) \leftarrow$  a constant
- If  $v_{be} \ll V_T$  (small-signal), expand the exponential term in a Taylor series, keeping only the linear term

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots \cong 1 + x$$

$$i_B = I_{BQ} e^{\frac{v_{be}}{V_T}} \cong I_{BQ} \left( 1 + \frac{v_{be}}{V_T} \right) = I_{BQ} + \frac{I_{BQ}}{V_T} v_{be} = I_{BQ} + i_b$$

Total value = dc value + ac value

# Small Signal: Collector Current

- The ac component of the base current is

$$i_b = \left( \frac{I_{BQ}}{V_T} \right) v_{be} = \left( \frac{1}{r_\pi} \right) v_{be} \quad r_\pi = \frac{V_T}{I_{BQ}} = \frac{\beta V_T}{I_{CQ}}$$

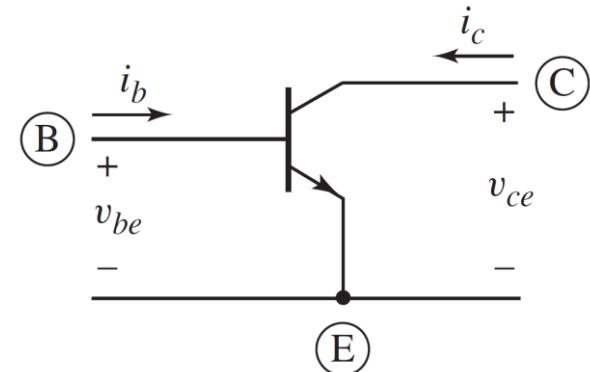
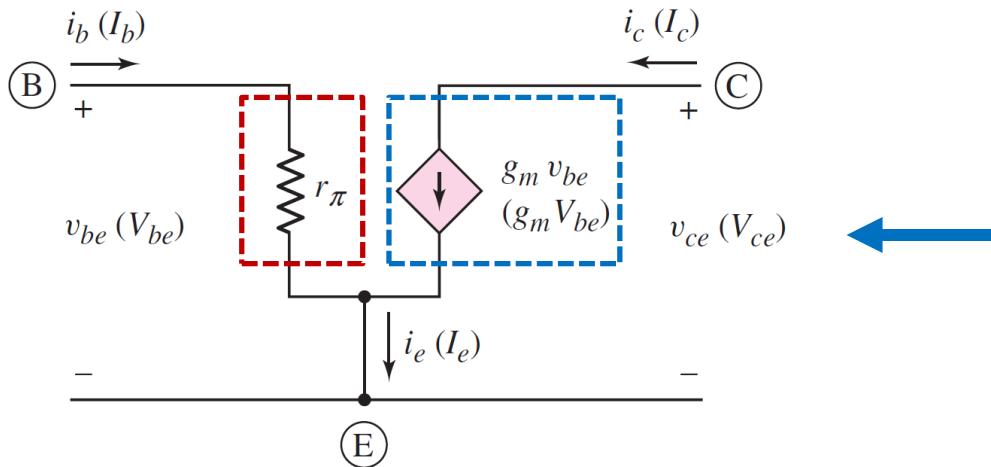
- $r_\pi$  is diffusion resistance or base-emitter input resistance
- $r_\pi$  is a function of the Q-point parameter
- The ac component of the collector current is

$$i_c = \beta i_b = \beta \left( \frac{I_{BQ}}{V_T} \right) v_{be} = \left( \frac{I_{CQ}}{V_T} \right) v_{be} = g_m v_{be}$$

- $g_m$  is transconductance

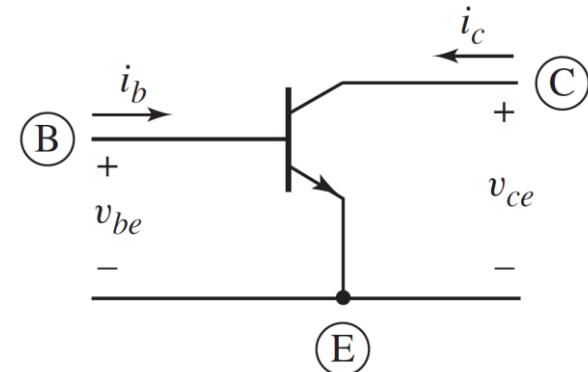
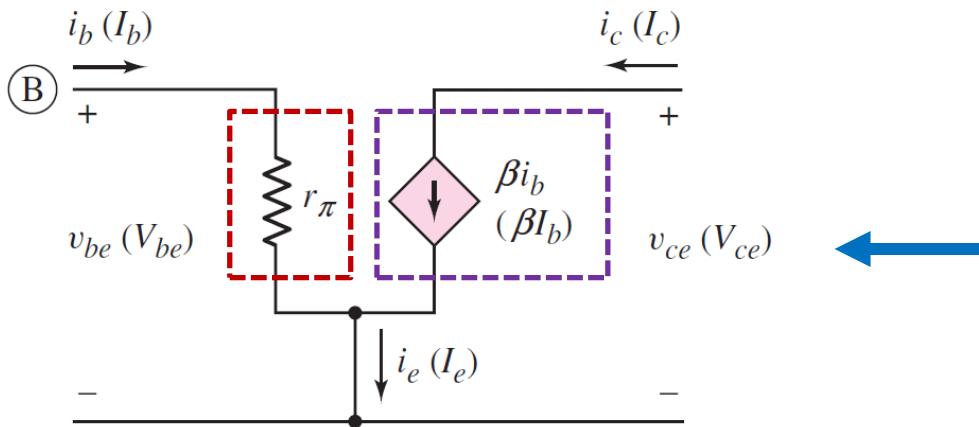
# Hybrid- $\pi$ Model for BJT

- A simplified **small-signal hybrid- $\pi$  model** for the npn transistor
  - The ac component of the **base current**
    - $v_{be} = r_\pi i_b$
  - The ac component of the **collector current**
    - $i_c = g_m v_{be}$



# Hybrid- $\pi$ Model for BJT

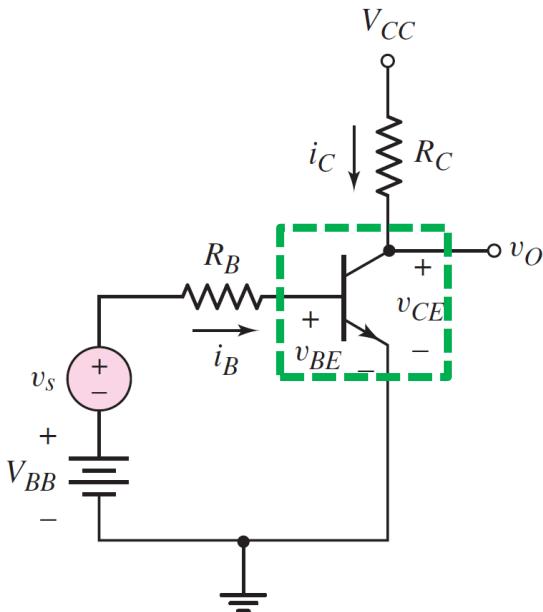
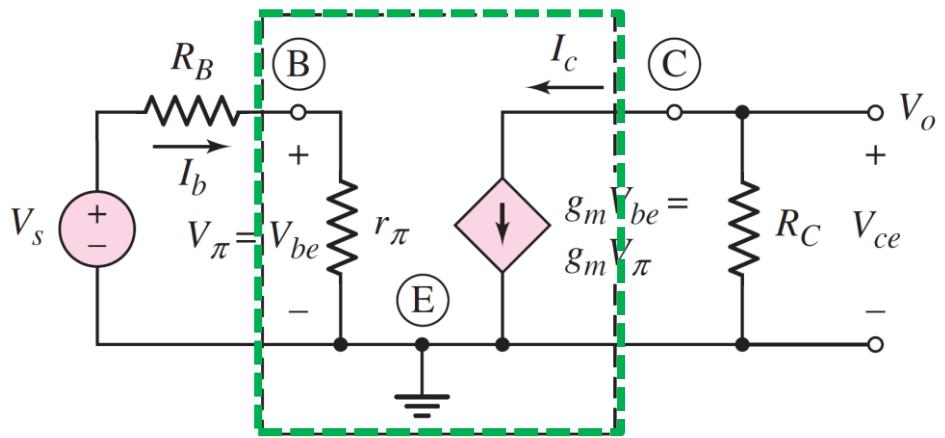
- A simplified **small-signal hybrid- $\pi$  model** for the npn transistor
  - The ac component of the **base current**
    - $v_{be} = r_\pi i_b$
  - The ac component of the **collector current**
    - $i_c = \beta i_b$



# Small-Signal Equivalent Circuit

- In ac analysis

- Set all dc sources to zero  $\rightarrow V_{CC} = 0$  and  $V_{BB} = 0$
- Hybrid- $\pi$  model of the BJT
- Small-signal equivalent circuit



# Small-Signal Voltage Gain

- The output voltage is

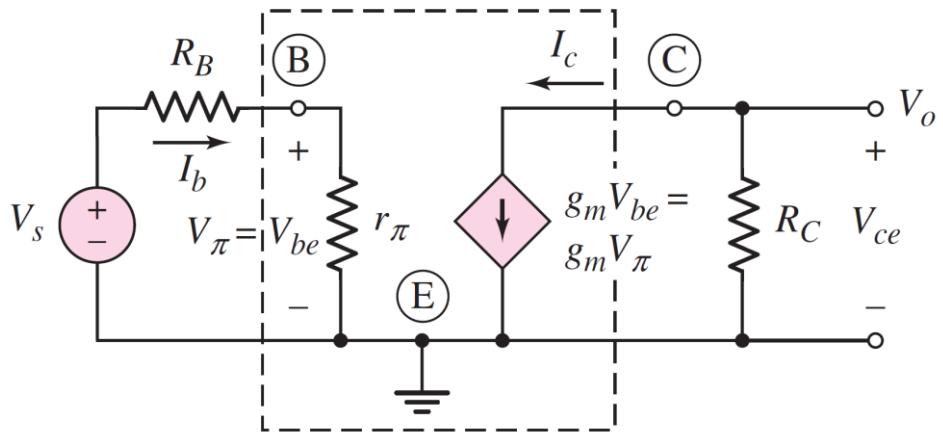
$$V_o = -(g_m V_\pi) R_C$$

- The source voltage is

$$V_\pi = \left( \frac{r_\pi}{r_\pi + R_B} \right) V_s$$

- The small-signal voltage gain is

$$A_v = \frac{V_o}{V_s} = -g_m R_C \left( \frac{r_\pi}{r_\pi + R_B} \right)$$



# Problem-Solving Technique

- The **dc** and **ac** analyses are performed **separately** (the same as FET amplifier circuit Chapter 5)
- The analysis of the **BJT amplifier** proceeds as follows:
  - **Step 1. dc analysis**
    - Analyze circuit with only the dc sources to find **quiescent solution**.
      - $V_{CEQ}$  and  $I_{CQ}$
    - Transistor must be biased in **forward-active region** for linear amplifier.
      - The B-E junction is forward biased, the B-C junction is reverse biased
  - **Step 2. ac analysis**
    - Replace BJT with **Hybrid- $\pi$  model**
    - Analyze **small-signal equivalent circuit**, setting **dc sources** to zero, to produce the circuit to the time-varying input signals only.

# Hybrid- $\pi$ Model with Early Effect

- Early effect: the collector current does vary with collector-emitter voltage

- Nonezero slope in the  $i_d$  versus  $v_{ds}$  curve

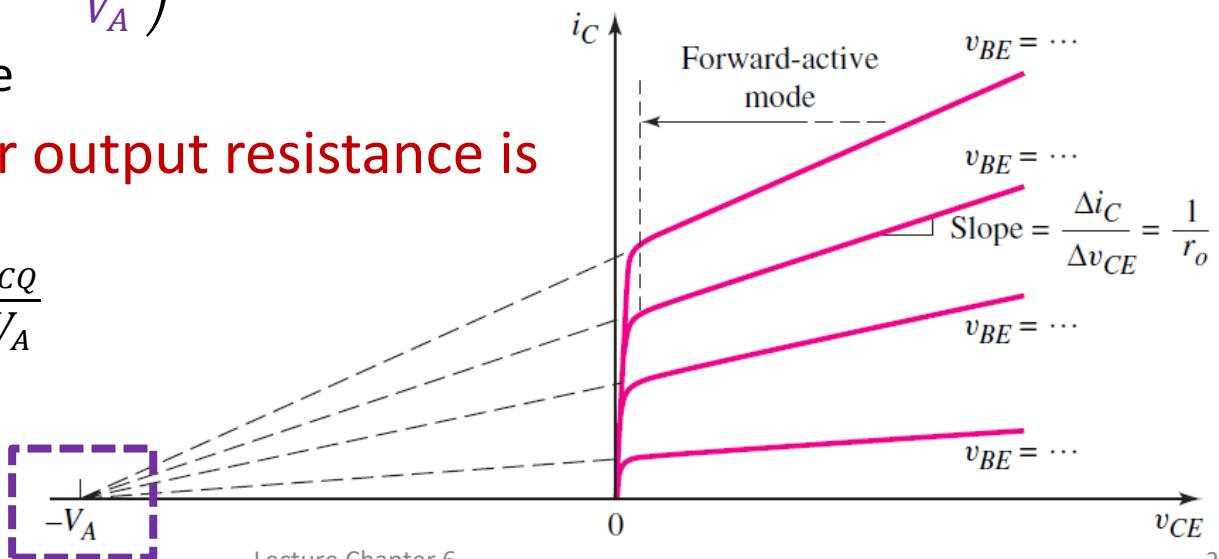
$$i_C = I_S e^{\frac{v_{BE}}{V_T}} \left( 1 + \frac{v_{CE}}{V_A} \right)$$

- $V_A$  is the early voltage

- Small-signal transistor output resistance is

$$\frac{1}{r_o} = \frac{\partial i_C}{\partial v_{CE}} = \frac{1}{V_A} I_S e^{\frac{v_{BE}}{V_T}} = \frac{I_{CQ}}{V_A}$$

$$r_o = \frac{V_A}{I_{CQ}}$$

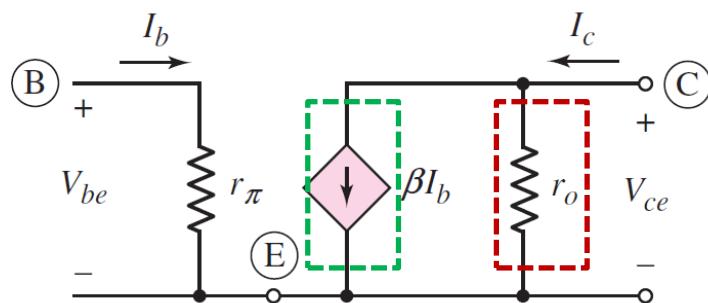
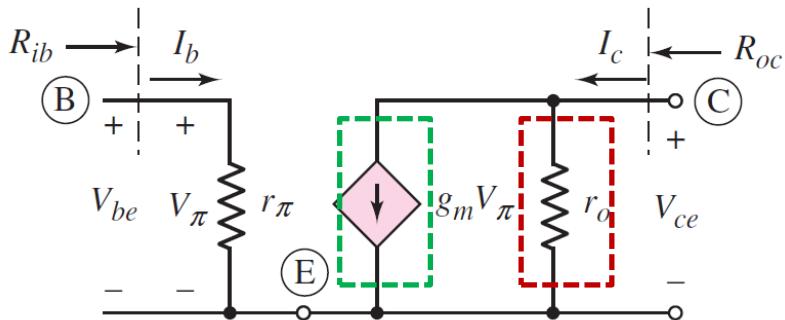


# Hybrid- $\pi$ Model with Early Effect

- The small-signal transistor output resistance is

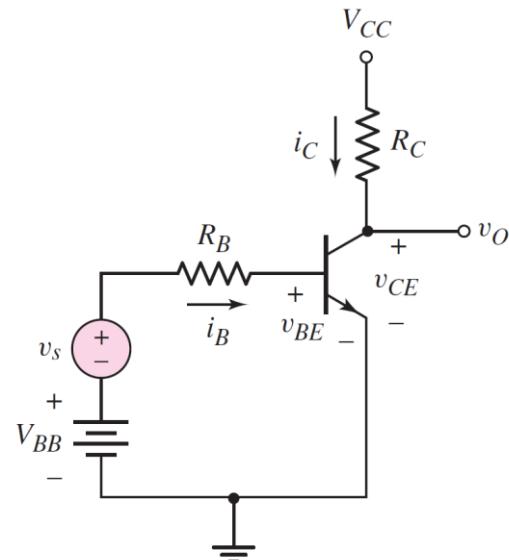
$$r_o = \frac{V_A}{I_{CQ}}$$

- $r_o$  is in parallel with the dependent current sources



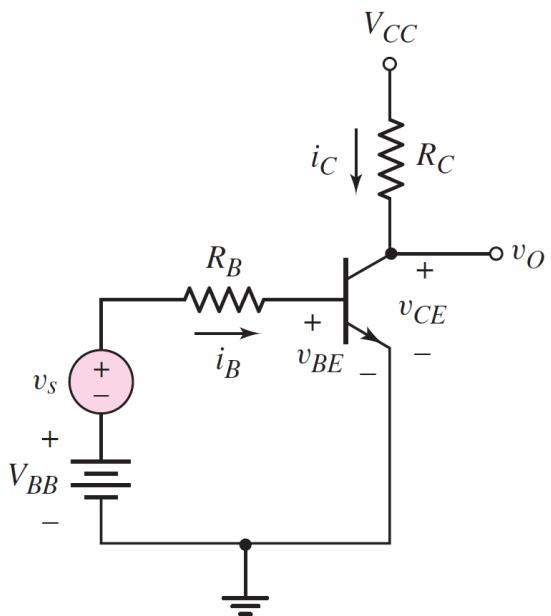
# Example 6.1

Calculate the small-signal voltage gain of the bipolar transistor circuit, including the Early effect  $r_o$ . Assume the transistor and circuit parameters are  $\beta = 100$ ,  $V_{CC} = 12 \text{ V}$ ,  $V_{BE} = 0.7 \text{ V}$ ,  $R_C = 6 \text{ k}\Omega$ ,  $R_B = 50 \text{ k}\Omega$ ,  $V_{BB} = 1.2 \text{ V}$ ,  $V_T = 0.026 \text{ V}$ , early voltage is  $V_A = 50 \text{ V}$ .

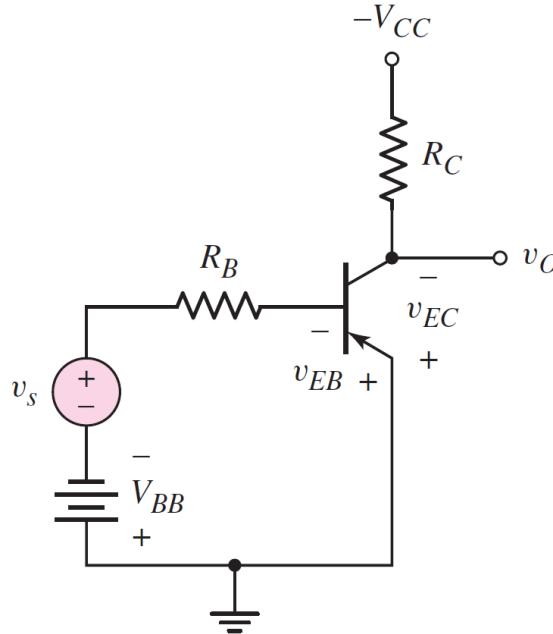


# NPN and PNP BJT Amplifier Circuits

- Current directions and voltage polarities are reversed



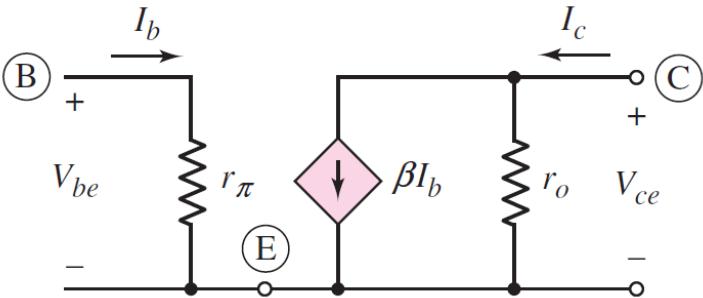
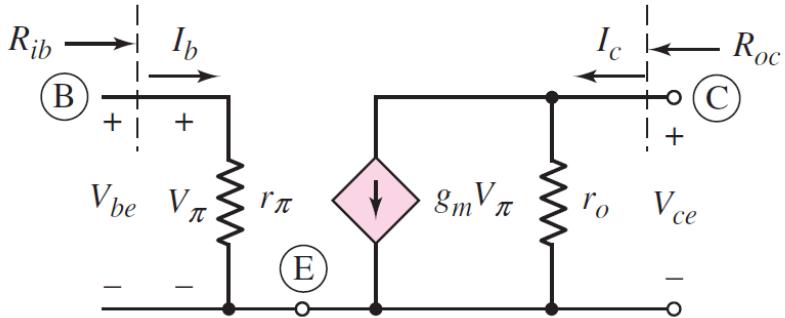
A NPN BJT amplifier circuit



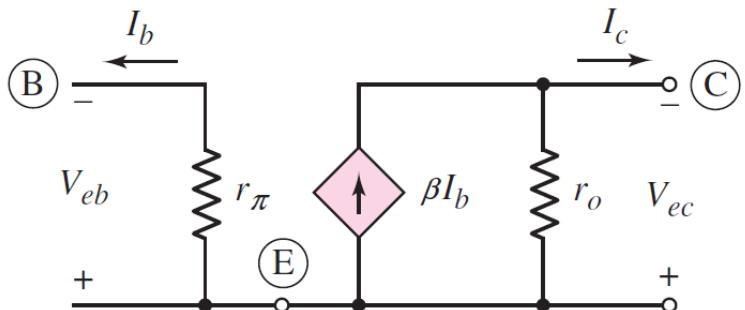
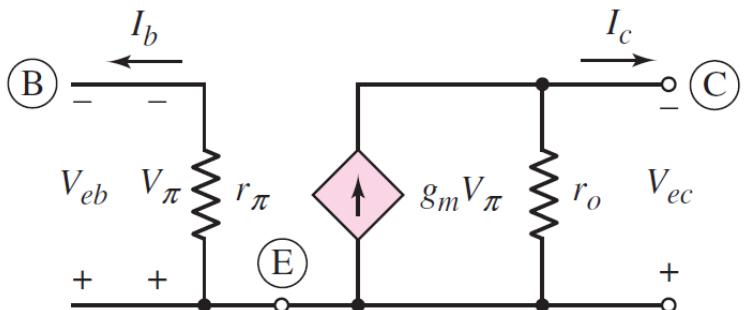
A PNP BJT amplifier circuit

# Hybrid- $\pi$ Model for a PNP Transistor

- NPN transistor



- PNP transistor

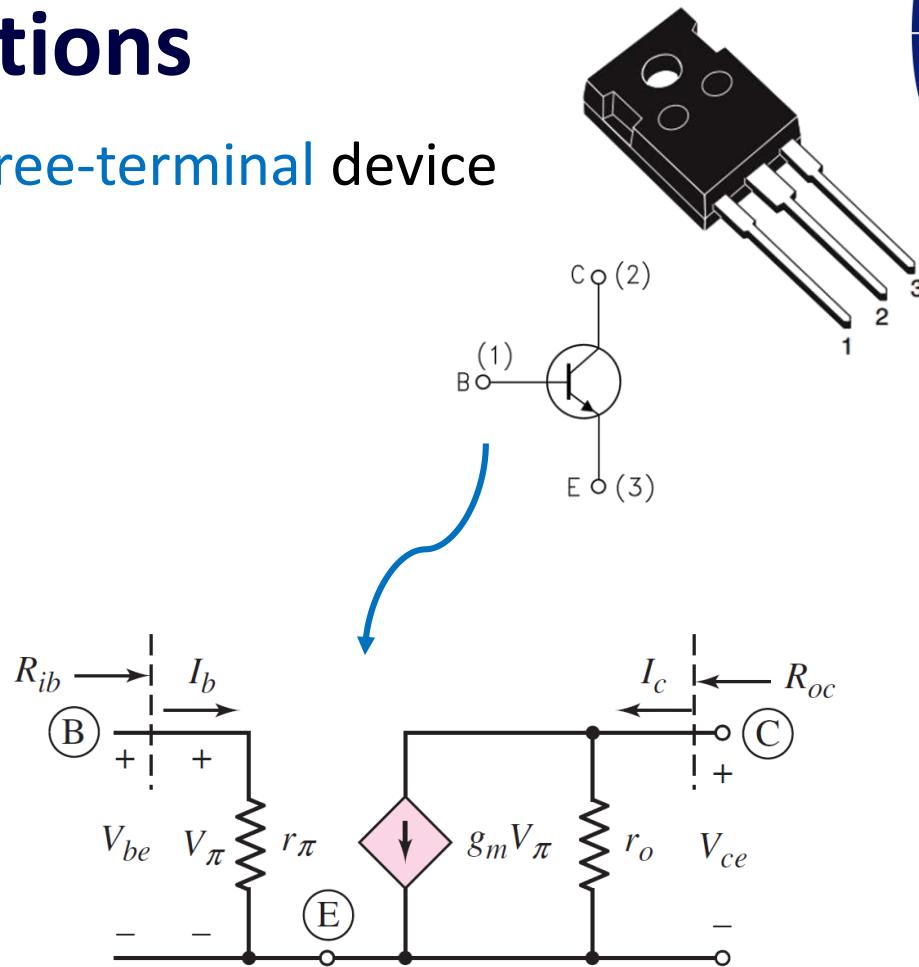


# Basic Transistor Amplifier Configuration

Discuss the three basic transistor amplifier configurations and discuss the four equivalent two-port networks.

# Amplifier Configurations

- The bipolar transistor is a **three-terminal** device
- Three basic configurations
  - Common-emitter
  - Common-collector
  - Common-base
- Characteristics of amplifiers
  - Small-signal voltage gain
  - Input resistance
  - Output resistance

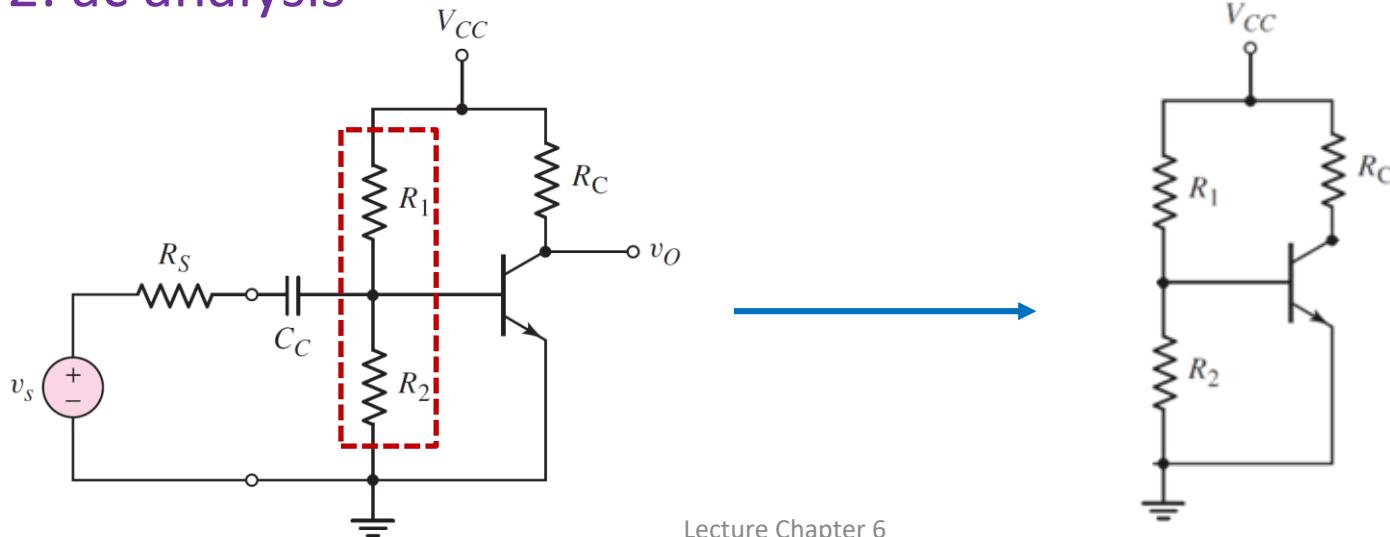


# Common-Emitter Amplifiers

Analyze the common-emitter amplifier and become familiar with the general characteristics of this circuit.

# Basic Common-Emitter Amplifier Circuit

- Basic common-emitter circuit with **voltage-divider biasing**
  - $R_1$  and  $R_2$  establish the **dc transistor biasing**: provides the base-emitter voltage to bias the transistor in the **forward-active region**
- Step 1: dc analysis
- Step 2: ac analysis

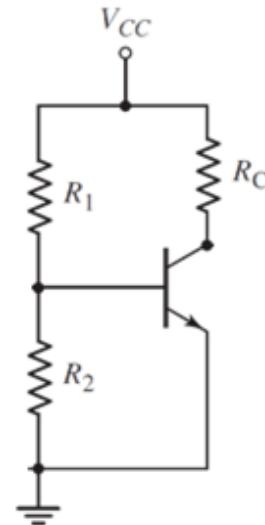


# Step 1: dc analysis

- Set all ac sources to zero, replace capacitors by open circuits
- Assume the BJT is biased in the forward-active region
  - B-E junction is **forward** biased  $\rightarrow V_{BE} = V_{BE}(\text{on})$
  - B-C junction is **reverse** biased  $\rightarrow V_C > V_B$
- To find  $V_{CEQ}$ , we write a KVL equation

$$-V_{CC} + I_{CQ}R_C + V_{CEQ} = 0$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C = V_{CC} - (\beta I_{BQ})R_C$$



# Step 1: Thevenin Equivalent Circuit

- To determine the base current at the quiescent point  $I_{BQ}$ , we need find the **Thevenin equivalent circuit**

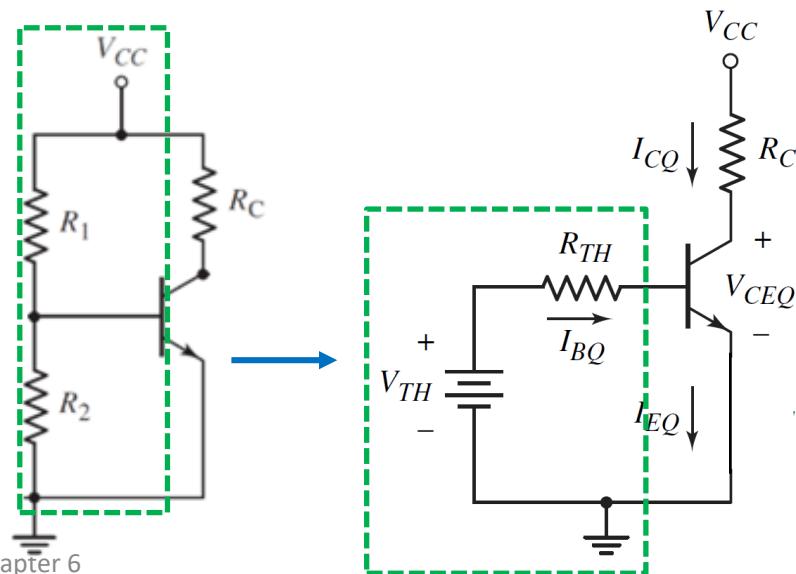
- $V_{TH}$  and  $R_{TH}$

- The Thevenin equivalent voltage

- $$V_{TH} = \frac{R_2}{R_1 + R_2} V_{CC}$$

- The Thevenin equivalent resistance

- Set independent source to zero
    - $V_{CC} = 0$
- $R_{TH} = R_1 \parallel R_2$



# Step 1: dc analysis

- A KVL equation in the loop

$$-V_{TH} + I_{BQ}R_{TH} + V_{BE}(\text{on}) = 0$$

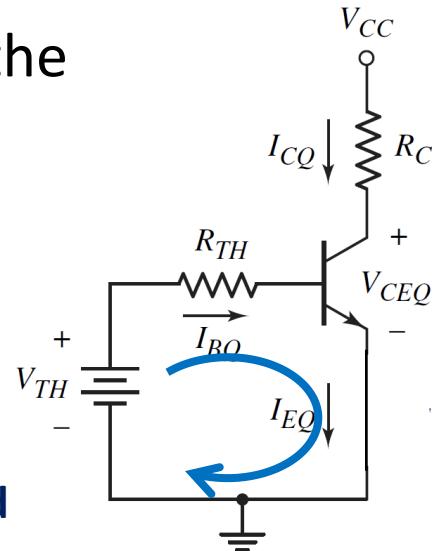
B-E junction is **forward biased**

- Verification: make sure the transistor is biased in the **forward-active region**

$$V_{CEQ} = V_{CC} - I_{CQ}R_C = V_{CC} - (\beta I_{BQ})R_C$$

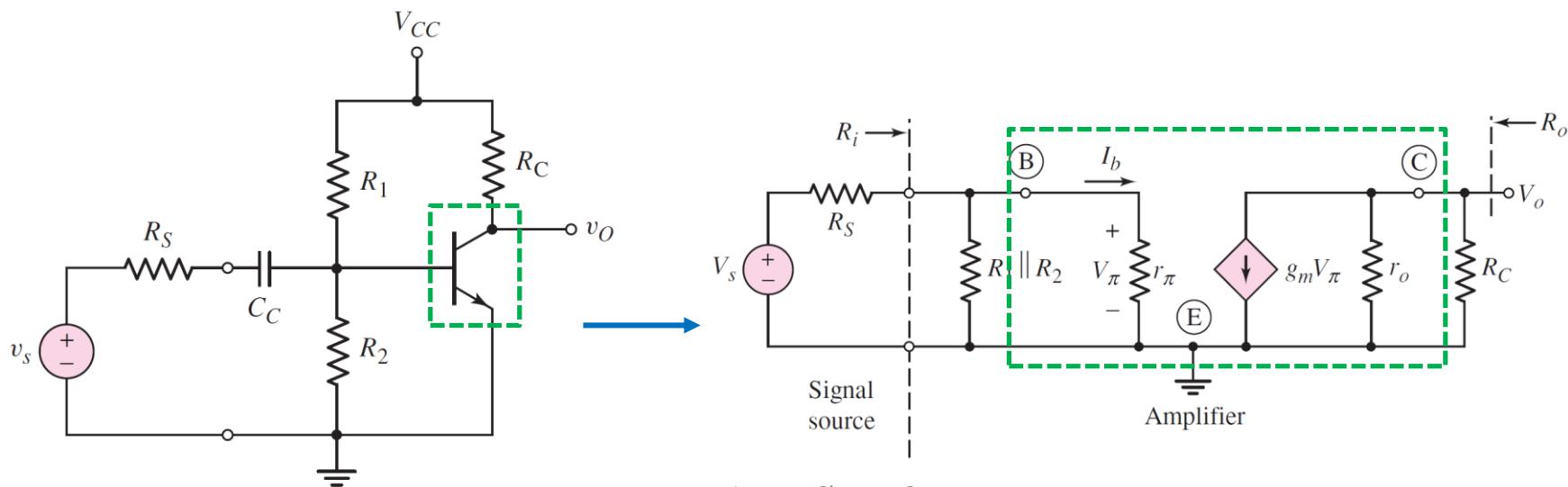
$V_{CEQ} > V_{BE}(\text{on})$

C-E junction is **reverse biased**



# Step 2: ac analysis

- Set all **dc sources** to zero, replace capacitors by **short circuits**
- **Hybrid- $\pi$  model** of the BJT
- Small signal equivalent circuit



# Step 2: ac analysis

- The find the small-signal voltage gain

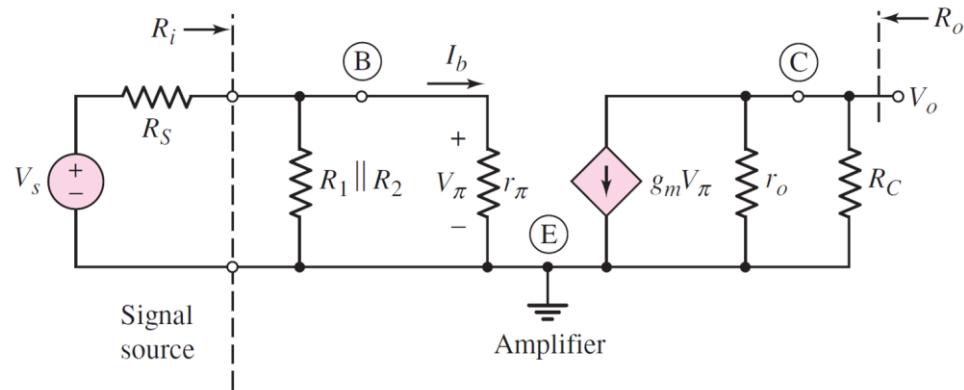
- $V_o \leftrightarrow V_\pi \leftrightarrow V_s$

- The output voltage is

$$V_o = -g_m V_\pi (r_o \parallel R_C)$$

- A voltage divider equation

$$V_\pi = \frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} V_s$$



# Step 2: ac analysis

- The small-signal voltage gain is

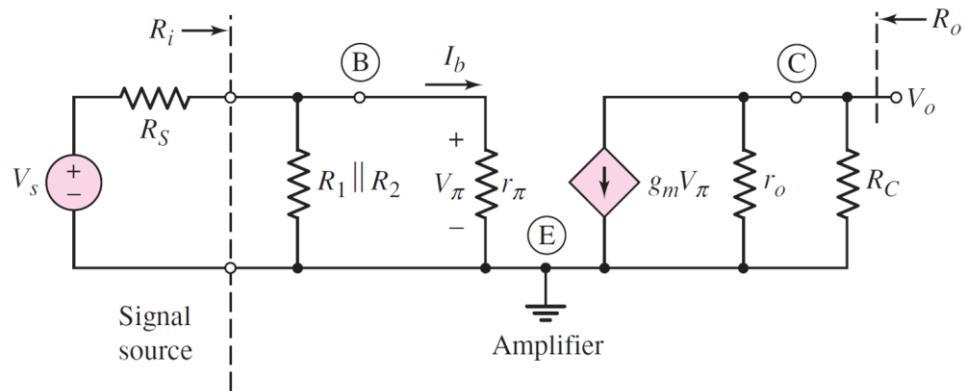
$$A_v = \frac{V_o}{V_s} = -g_m \left( \frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} \right) (r_o \parallel R_C)$$

- The voltage gain depends on transistor characteristics

$$g_m = \frac{I_{CQ}}{V_T}$$

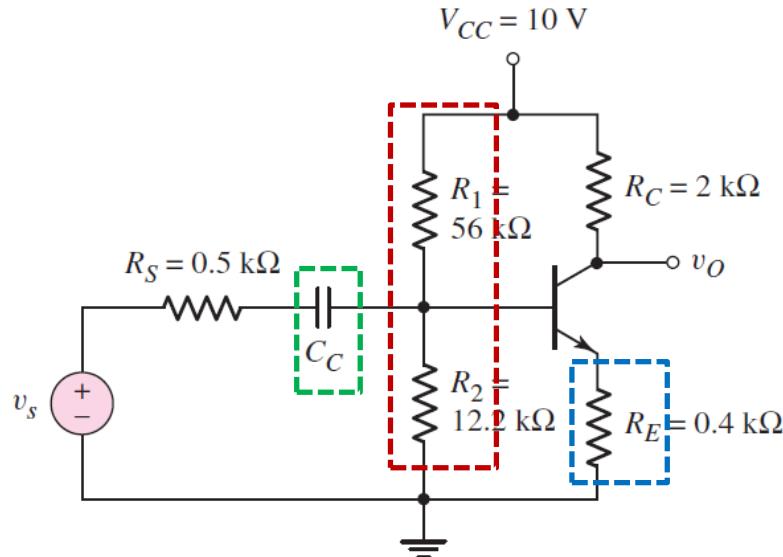
$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$

$$r_o = \frac{V_A}{I_{CQ}}$$



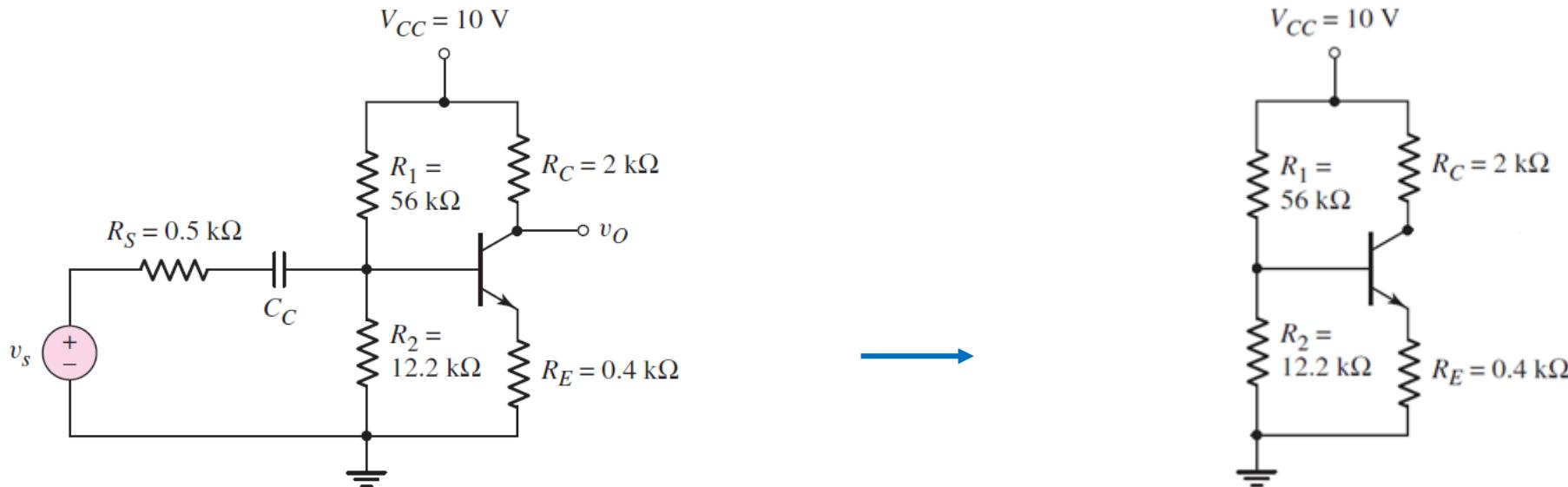
# Circuit with Emitter Resistor

- An npn common-emitter circuit with an **emitter resistor**, a **voltage-divider biasing** circuit, and a **coupling capacitor**
- Even though the emitter of this circuit is not at ground potential, this circuit is still referred to as a common-emitter circuit
- With a **emitter resistor**, the voltage gain of a circuit is less dependent on the transistor characteristics



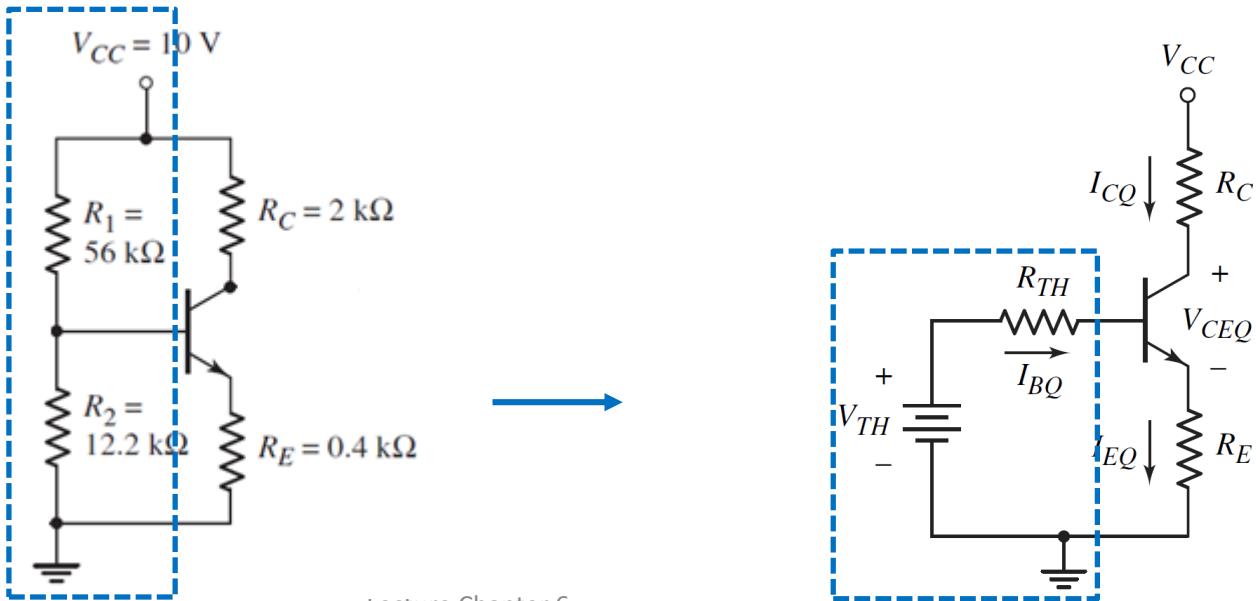
# Step 1: dc analysis

- Set all ac sources to zero, replace capacitors by open circuits



# Step 1: dc analysis

- Assume the transistor is biased in the forward-active region
- Find the Thevenin equivalent circuit
  - $V_{TH} = \frac{R_2}{R_1 + R_2} V_{CC}$  and  $R_{TH} = R_1 \parallel R_2$



# Step 1: dc analysis

- A KVL equation in the loop

$$-V_{TH} + I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E = 0$$

$$-V_{TH} + I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E = 0$$

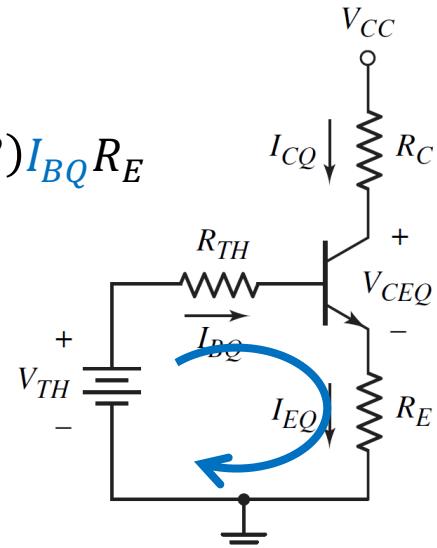
- To find the  $V_{CEQ}$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E = V_{CC} - (\beta I_{BQ})R_C - (1 + \beta)I_{BQ}R_E$$

- Verification:**

- The transistor is biased in the forward-active region

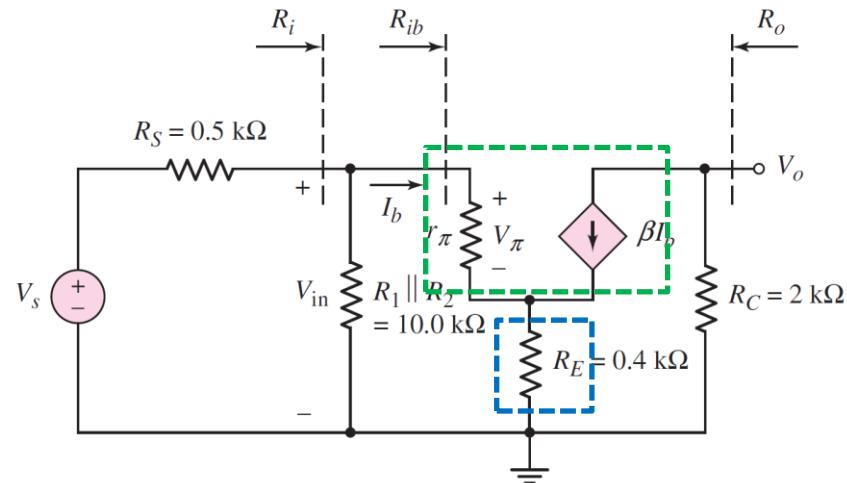
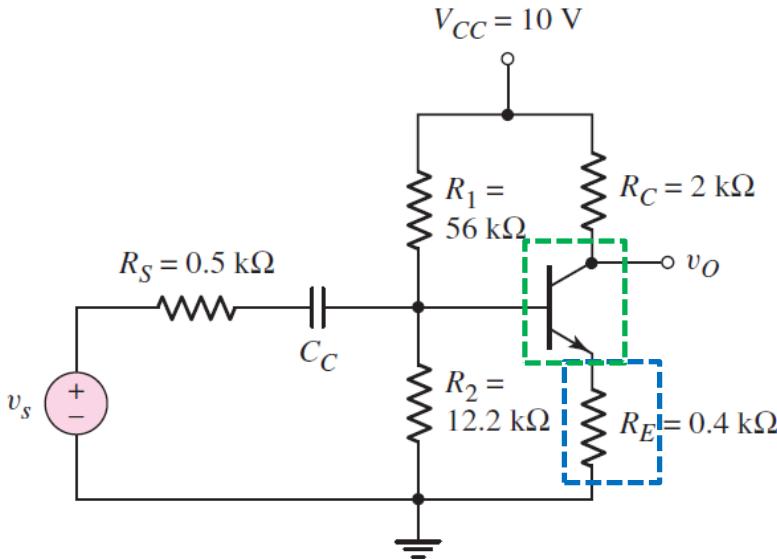
$$V_{CEQ} > V_{BE}(\text{on})$$



# Step 2: ac analysis

- Hybrid- $\pi$  model of the BJT
- Small signal equivalent circuit

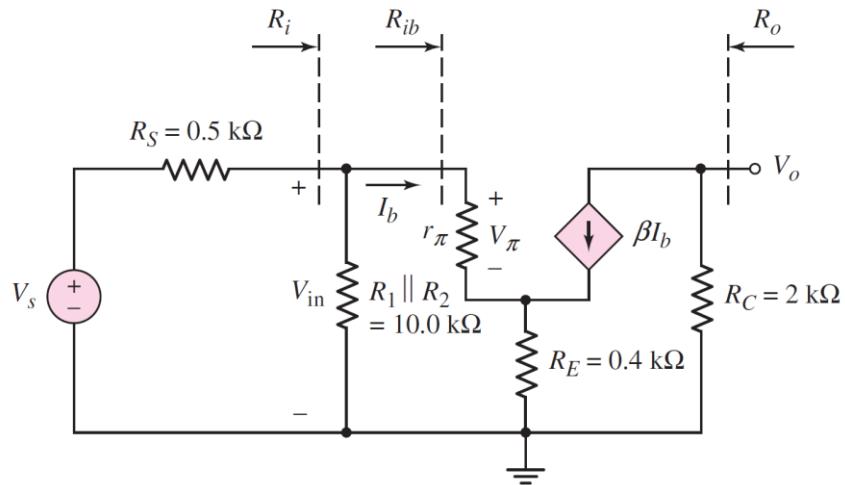
- The find the small-signal voltage gain
  - $V_o \leftrightarrow I_b (V_\pi) \leftrightarrow V_{in} \leftrightarrow V_s$



# Step 2: Small-Signal Voltage Gain

- To find the small-signal voltage gain
  - $V_o \leftrightarrow I_b \leftrightarrow V_{in} \leftrightarrow V_s$
- The output voltage is

$$V_o = -(\beta I_b) R_C$$



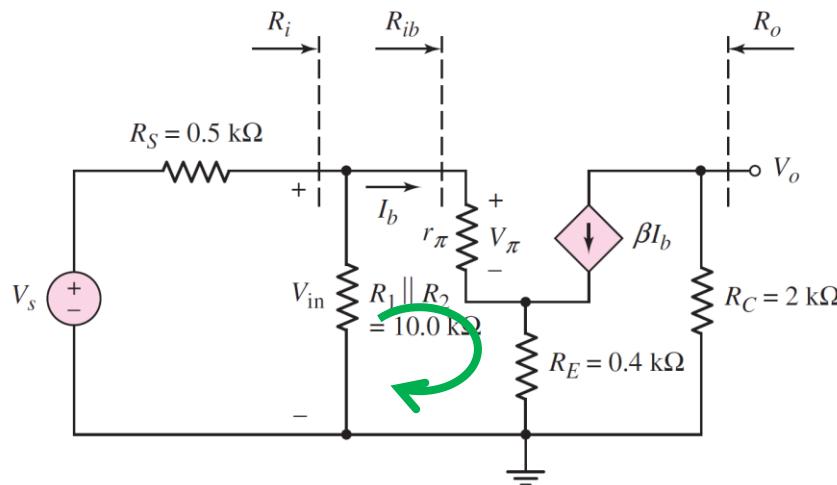
# Step 2: Small-Signal Voltage Gain

- A KVL equation in the loop

$$-V_{\text{in}} + I_b r_\pi + (1 + \beta) I_b R_E = 0$$

$$I_b = \frac{V_{\text{in}}}{r_\pi + (1 + \beta) R_E}$$

- $V_o \leftrightarrow I_b \leftrightarrow V_{\text{in}} \leftrightarrow V_s$

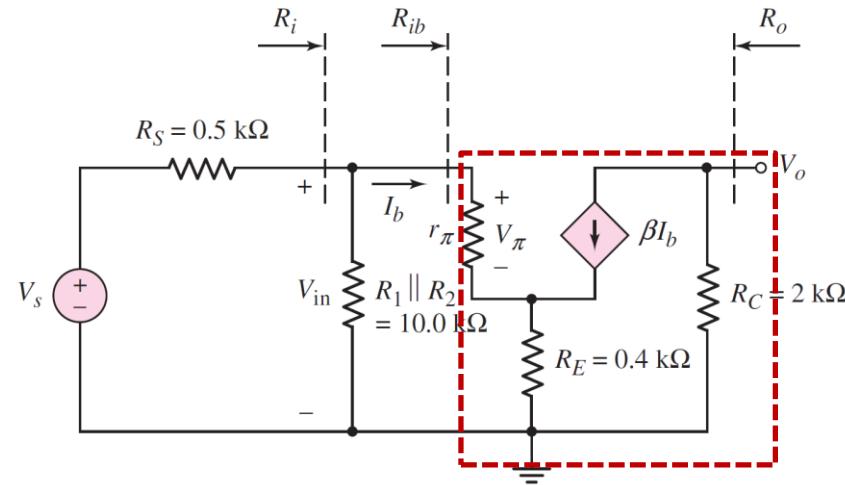
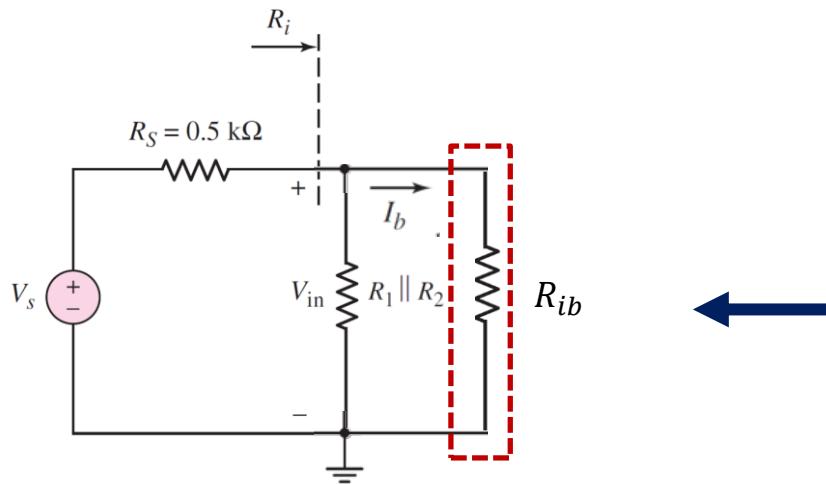


# Step 2: Input Resistance

- The input resistance looking into the base  $R_{ib}$

$$R_{ib} = \frac{\text{Voltage at the base}}{\text{Current into the base}} = \frac{V_{in}}{I_b} = \frac{I_b r_\pi + I_b(1 + \beta)R_E}{I_b}$$

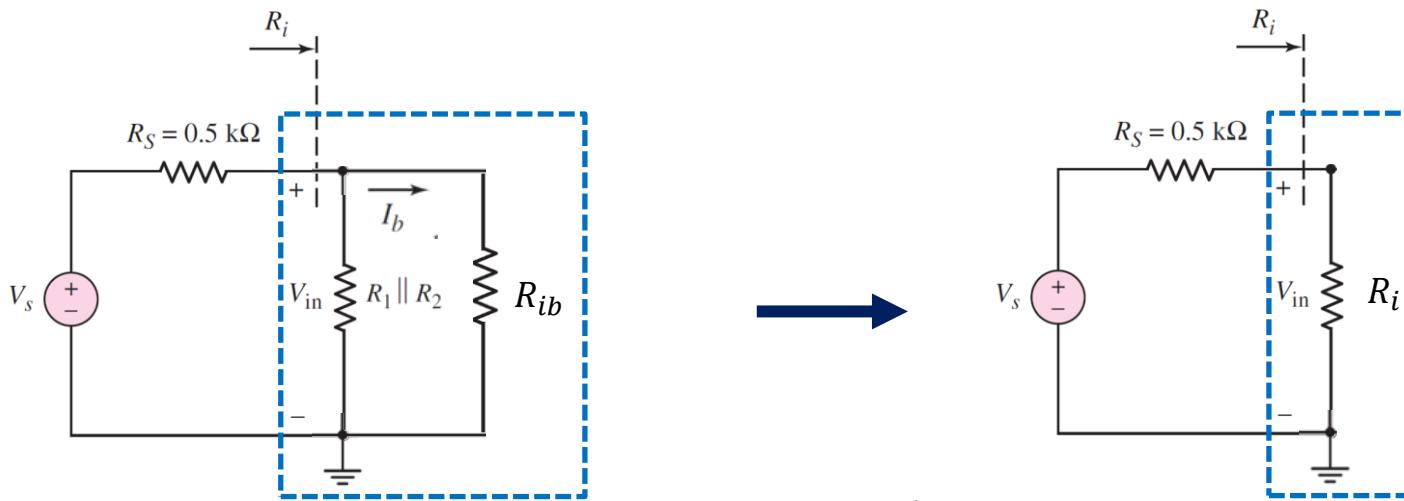
$$R_{ib} = r_\pi + (1 + \beta)R_E$$



# Step 2: Input Resistance

- The input resistance is

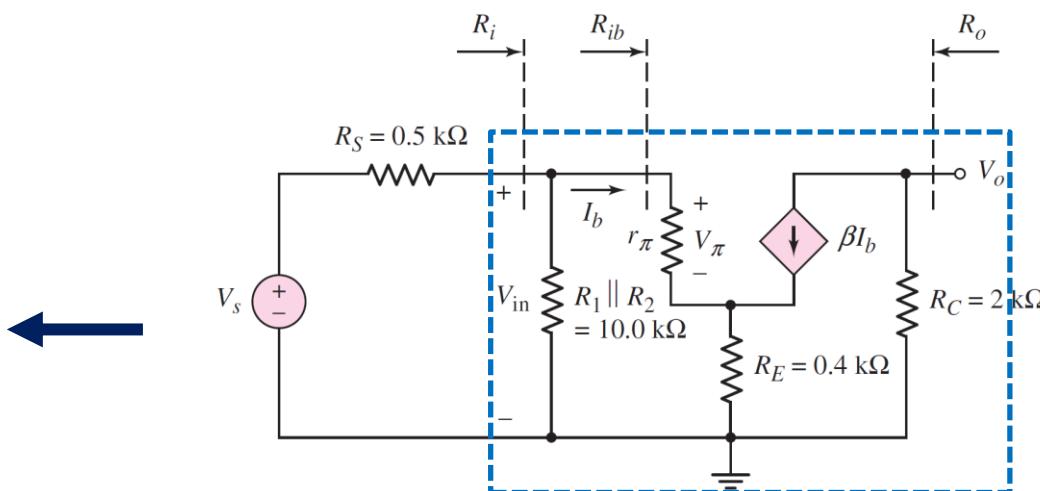
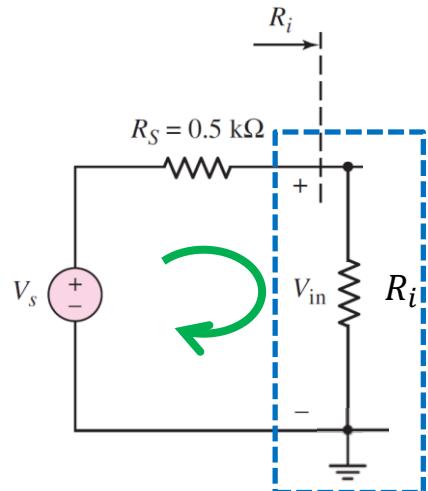
$$R_i = R_1 \parallel R_2 \parallel R_{ib}$$



# Step 2: Small-Signal Voltage Gain

- Based on the voltage-divider equation in the loop
  - $V_o \leftrightarrow I_b \leftrightarrow V_{in} \leftrightarrow V_s$

$$V_{in} = \left( \frac{R_i}{R_i + R_S} \right) V_s$$



# Step 2: Small-Signal Voltage Gain

- The small-signal voltage gain is

- $V_o \leftrightarrow I_b \leftrightarrow V_{in} \leftrightarrow V_s$

$$A_v = \frac{V_o}{V_s} = \frac{-(\beta I_b)R_C}{V_s} = -\beta R_C \left( \frac{V_{in}}{R_{ib}} \right) \left( \frac{1}{V_s} \right) = -\beta R_C \left( \frac{1}{R_{ib}} \right) \left( \frac{V_{in}}{V_s} \right)$$

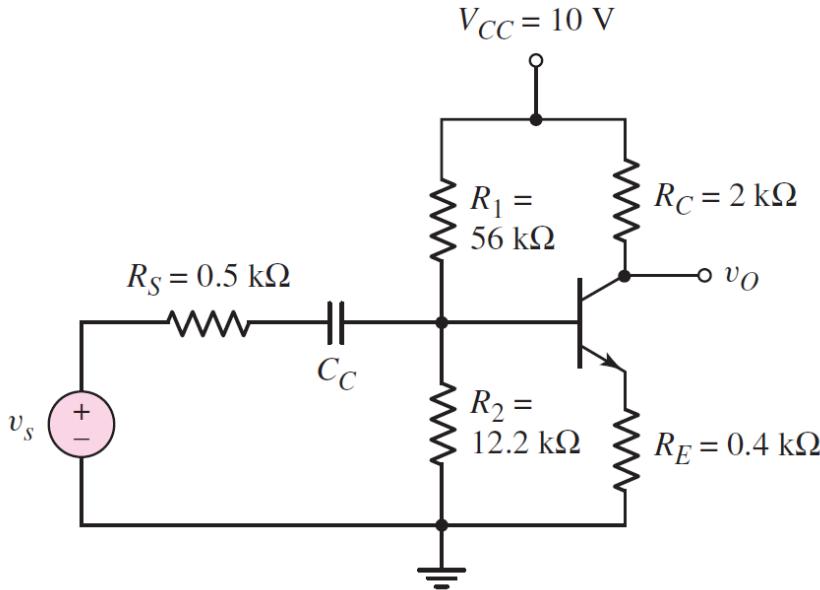
- With a emitter resistor, the small-signal voltage gain is less dependent on the transistor parameters

$$A_v = -\beta R_C \left( \frac{1}{r_\pi + (1 + \beta)R_E} \right) \left( \frac{R_i}{R_i + R_s} \right) \cong \frac{-\beta R_C}{(1 + \beta)R_E} \cong \frac{-R_C}{R_E}$$



# Example 6.2

Determine the small-signal voltage gain and input resistance of a common-emitter circuit with an emitter resistor. The transistor parameters are:  $\beta = 100$ ,  $V_{BE}(\text{on}) = 0.7 \text{ V}$ , and  $V_A = \infty$ .

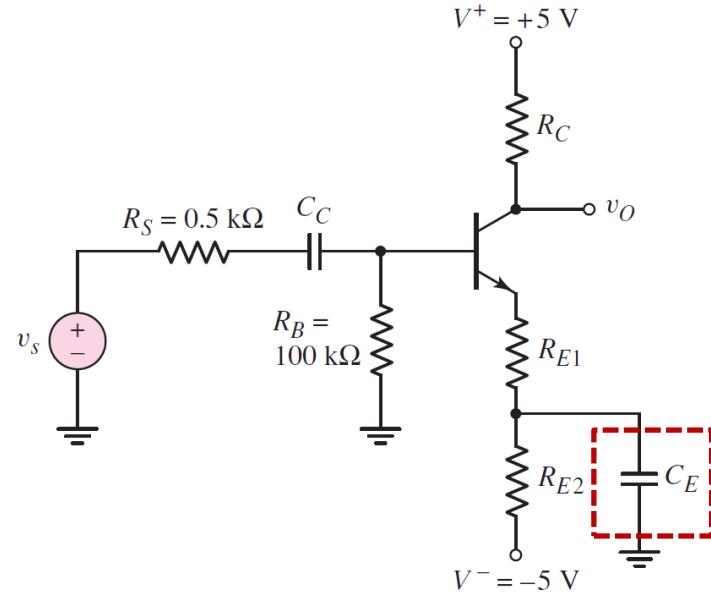


# Circuit with Emitter Bypass Capacitor

- Sometime  $R_E$  must be large for the purpose of dc design
  - the small-signal voltage gain is degraded

$$A_v \cong \frac{-R_C}{R_E}$$

- By introducing a **bypass capacitor  $C_E$** 
  - In **dc analysis** ( $C_E$  is an **open circuit**)
    - $R_E \rightarrow R_{E1} + R_{E2}$
  - In **ac analysis** ( $C_E$  is an **short circuit**)
    - $R_E \rightarrow R_{E1}$



- The small-signal voltage gain can be maintained in a higher value

# AC Load Line Analysis (Optional)

Understand the concept of the ac load line

# AC Load Line

- Write KVL equation around the C-E loop

- dc analysis

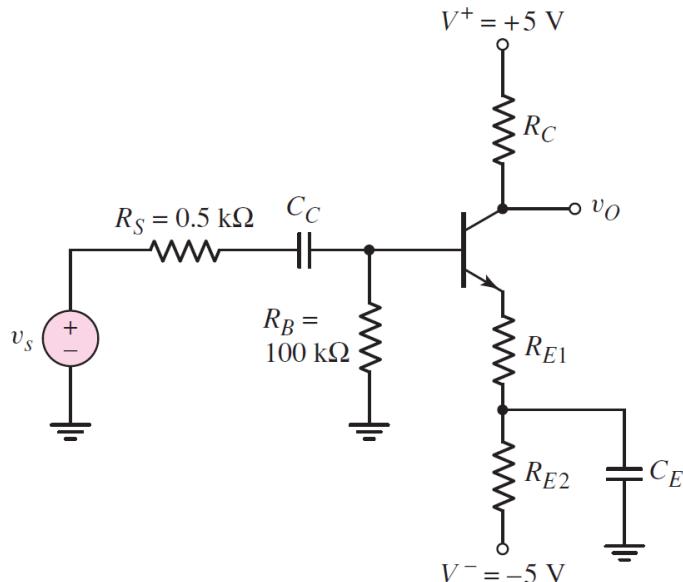
$$-V^+ + I_C R_C + V_{CE} + I_E (R_{E1} + R_{E2}) + V^- = 0$$

$$V_{CE} = V^+ - V^- - I_C \left[ R_C + \left( \frac{1 + \beta}{\beta} \right) (R_{E1} + R_{E2}) \right]$$

- ac analysis

$$i_c R_C + v_{ce} + i_e R_{E1} = 0$$

$$v_{ce} \cong -i_c (R_C + R_{E1})$$



# AC Load Line

- dc load line

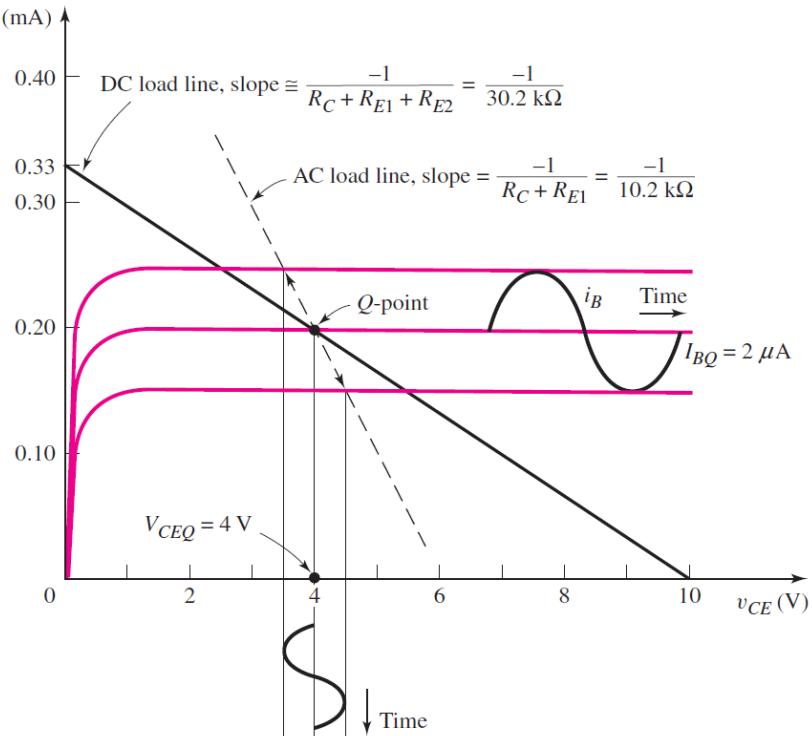
$$V_{CE} = V^+ - V^- - I_C \left[ R_C + \left( \frac{1 + \beta}{\beta} \right) (R_{E1} + R_{E2}) \right]$$

- Slope  $\cong \frac{-1}{R_C + R_{E1} + R_{E2}}$

- ac load line

$$v_{ce} \cong -i_c(R_C + R_{E1})$$

- Slope  $= \frac{-1}{R_C + R_{E1}}$

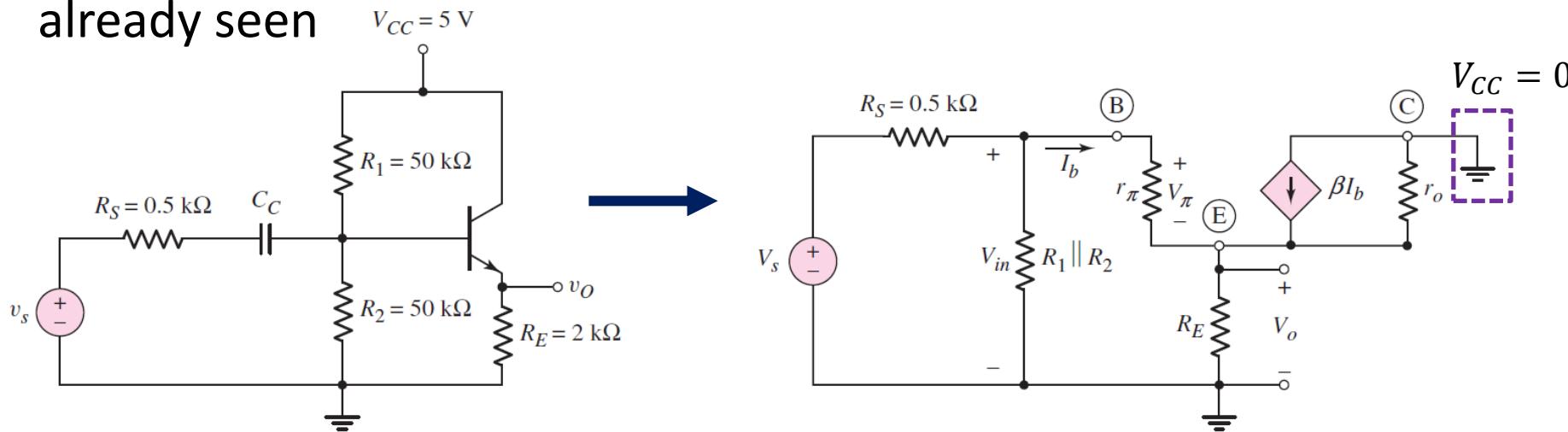


# Common-Collector (Emitter-Follower) Amplifier

Analyze the emitter-follower amplifier and become familiar with the general characteristics of this circuit.

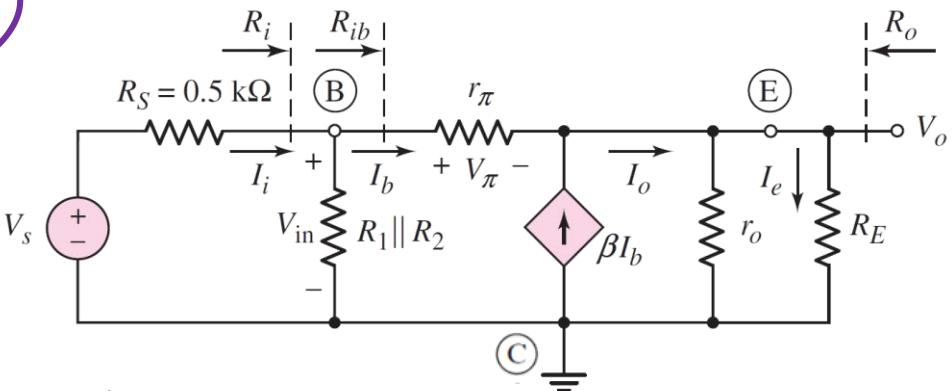
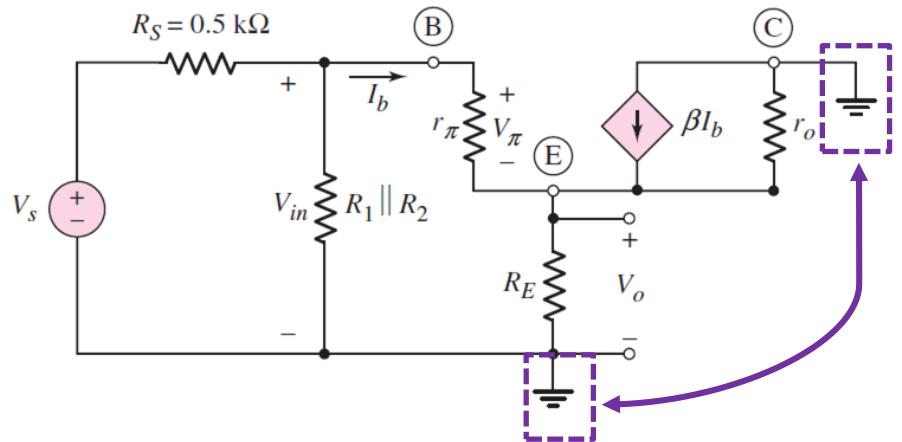
# Common-Collector Amplifier

- The output signal is taken off of the emitter with respect to ground
- The collector is connected to  $V_{CC}$  (ac signal ground)
- The dc analysis of the circuit is exactly the same as we have already seen



# Small-Signal Equivalent Circuit

- Rearrange the circuit to make all signal grounds are at the same point



# Small-Signal Voltage Gain

- To find the small-signal voltage gain

- $V_o \leftrightarrow I_b \leftrightarrow V_{in} \leftrightarrow V_s$

- The output voltage is

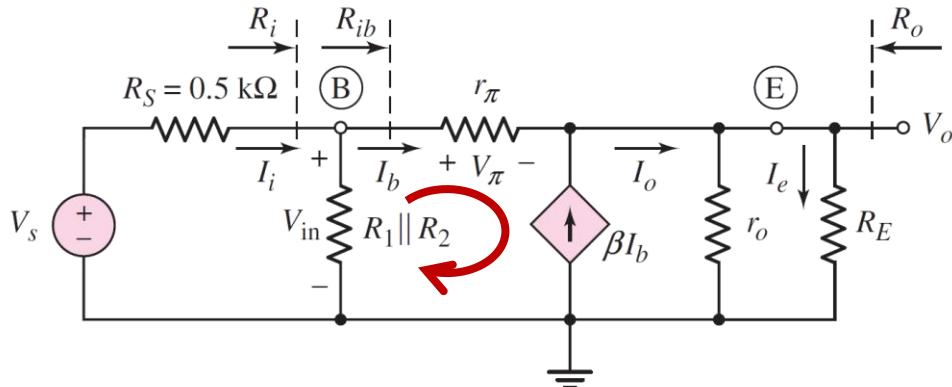
$$V_o = I_o(r_o \parallel R_E) = (1 + \beta)I_b(r_o \parallel R_E)$$

- Write KVL equation around the B-E loop

$$-V_{in} + I_b r_\pi + V_o = 0$$

$$-V_{in} + I_b r_\pi + (1 + \beta)I_b(r_o \parallel R_E) = 0$$

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1 + \beta)(r_o \parallel R_E)$$



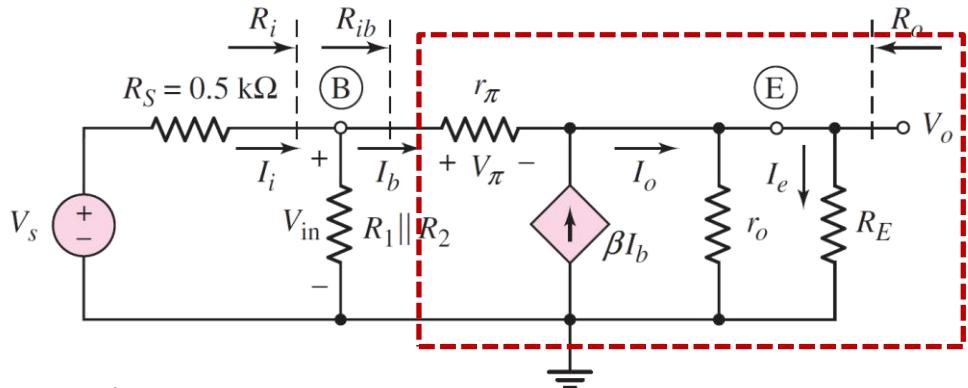
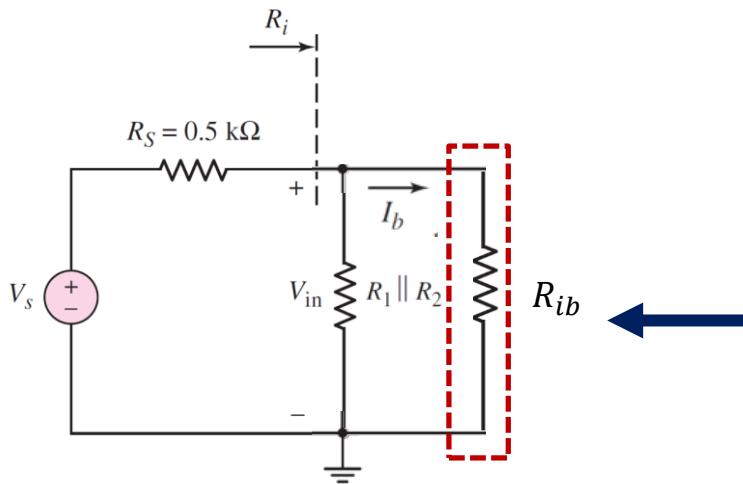
# Input Resistance

- The input resistance looking into the base is

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1 + \beta)(r_o \parallel R_E)$$

- The input resistance is

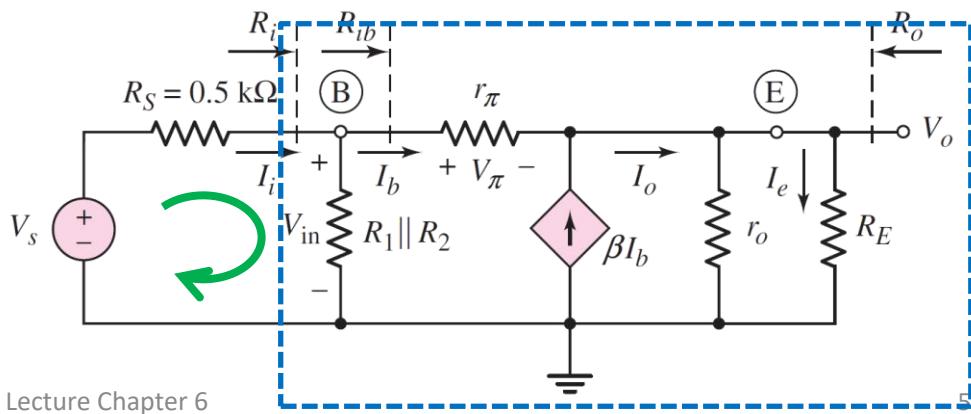
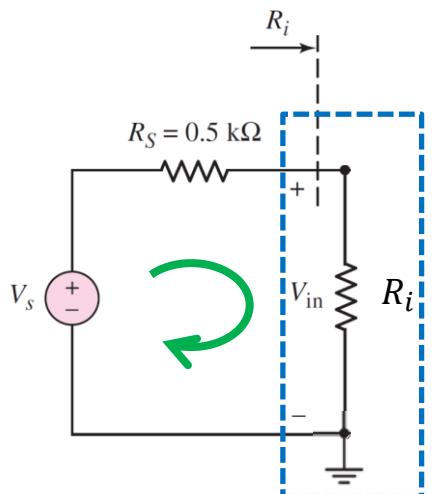
$$R_i = R_1 \parallel R_2 \parallel R_{ib}$$



# Small Signal Voltage Gain

- Write a voltage divider equation is the loop

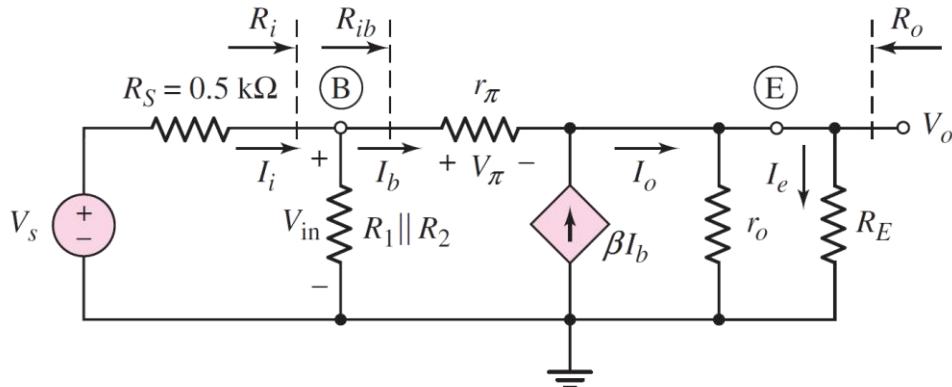
$$V_{in} = \left( \frac{R_i}{R_i + R_S} \right) V_s$$



# Small Signal Voltage Gain

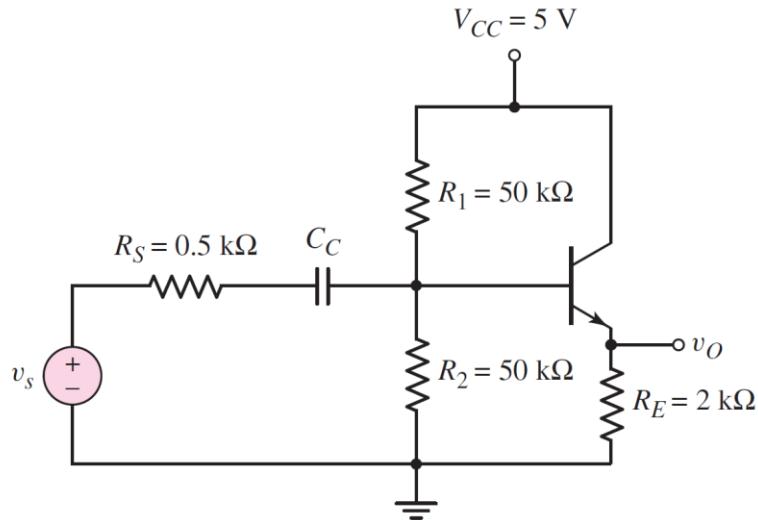
- We have found the relationship between  $V_o$  with  $V_s$ 
  - $V_o \leftrightarrow I_b \leftrightarrow V_{in} \leftrightarrow V_s$
- The small-signal voltage gain is

$$A_v = \frac{V_o}{V_s} = \frac{(1 + \beta)I_b(r_o \parallel R_E)}{\frac{V_{in}}{\left(\frac{R_i}{R_i + R_S}\right)}} = \frac{(1 + \beta)(r_o \parallel R_E)}{r_\pi + (1 + \beta)(r_o \parallel R_E)} \left( \frac{R_i}{R_i + R_S} \right) \cong 1$$



# Example 6.3

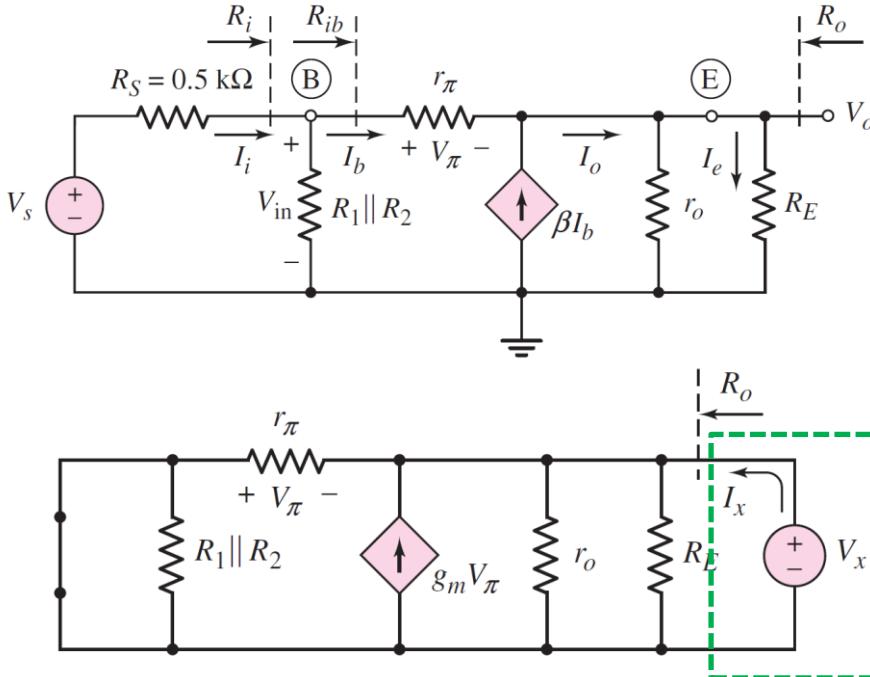
Calculate the small-signal voltage gain of an emitter-follower circuit. Assume the transistor parameters are:  $\beta = 100$ ,  $V_{BE}(\text{on}) = 0.7 \text{ V}$ , and  $V_A = 80 \text{ V}$ .



# Output Resistance

- Assume the signal source is ideal  $\rightarrow R_S = 0$
- Set the independent voltage source  $V_S$  to zero
- A test voltage source is applied to the output terminal
  - The voltage is  $V_x$
  - The current is  $I_x$
- The output resistance is

$$R_o = \frac{V_x}{I_x}$$



# Output Resistance

- Write a KCL equation at the output node

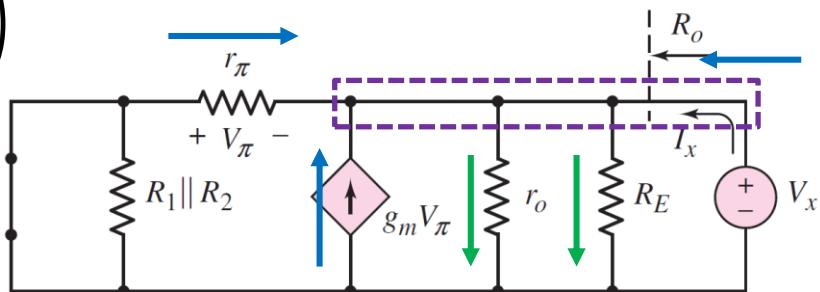
$$-\frac{V_x}{R_E} - \frac{V_x}{r_o} + \frac{V_\pi}{r_\pi} + I_x + g_m V_\pi = 0$$

$$-\frac{V_x}{R_E} - \frac{V_x}{r_o} + \frac{(-V_x)}{r_\pi} + I_x + g_m(-V_x) = 0$$

- The output resistance is

$$\frac{1}{R_o} = \frac{I_x}{V_x} = \frac{1}{R_E} + \frac{1}{r_o} + \frac{1}{r_\pi} + g_m = \frac{1}{R_E} + \frac{1}{r_o} + \left( \frac{1 + \beta}{r_\pi} \right)$$

$$R_o = \frac{r_\pi}{1 + \beta} \parallel R_E \parallel r_o$$



# Small-Signal Current Gain

- The small-signal current gain is

$$A_i = \frac{\text{Output current}}{\text{Input current}} = \frac{I_e}{I_i}$$

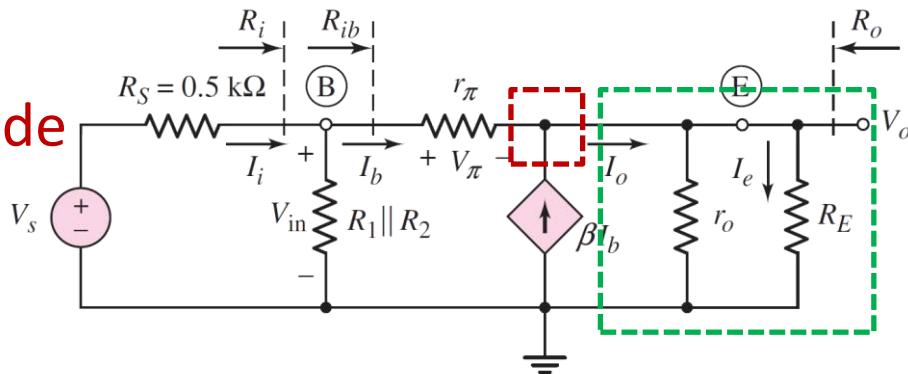
- $I_e \leftrightarrow I_o \leftrightarrow I_b \leftrightarrow I_i$

- Based on the current divider equation

$$I_e = \left( \frac{r_o}{r_o + R_E} \right) I_o$$

- A KCL equation at the emitter node

$$I_o = I_b + \beta I_b = (1 + \beta) I_b$$



# Small-Signal Current Gain

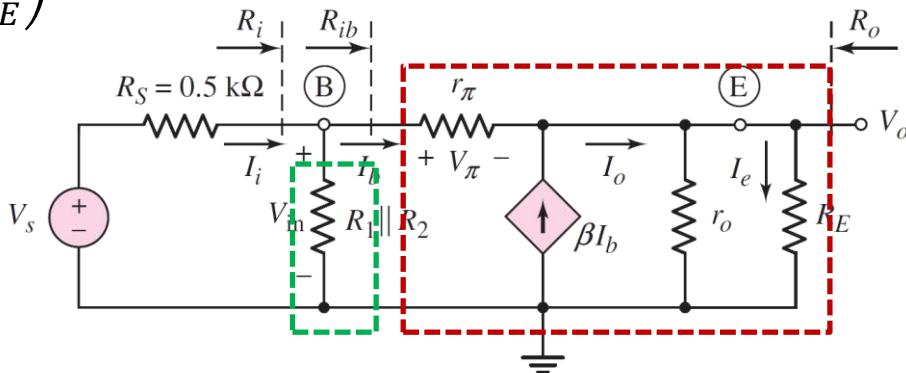
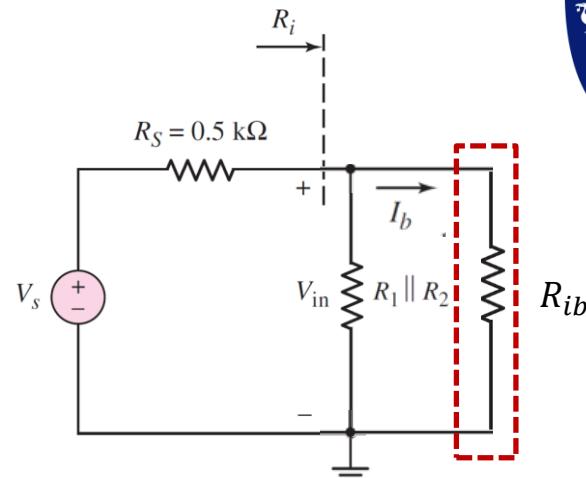
- A KCL equation at the base node

$$I_b = \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) I_i$$

- The small-signal current gain is

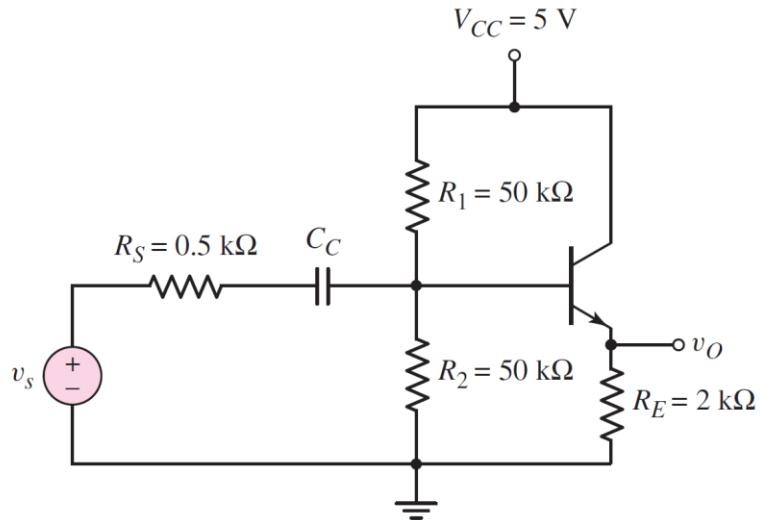
$$A_i = \frac{I_e}{I_i} = (1 + \beta) \left( \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{ib}} \right) \left( \frac{r_o}{r_o + R_E} \right) \cong 1 + \beta$$

- $R_1 \parallel R_2 \gg R_{ib}, r_o \gg R_E$



# Example 6.4

Calculate the input and output resistance of the emitter-follower circuit. Assume  $R_s = 0$ .

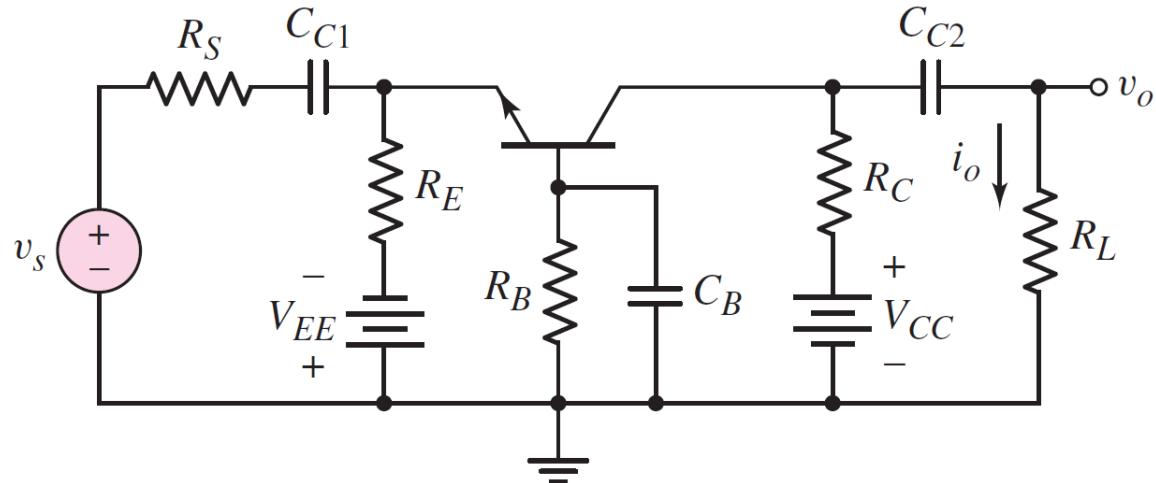


# Common-Base Amplifier

Analyze the common-base amplifier and become familiar with the general characteristics of this circuit.

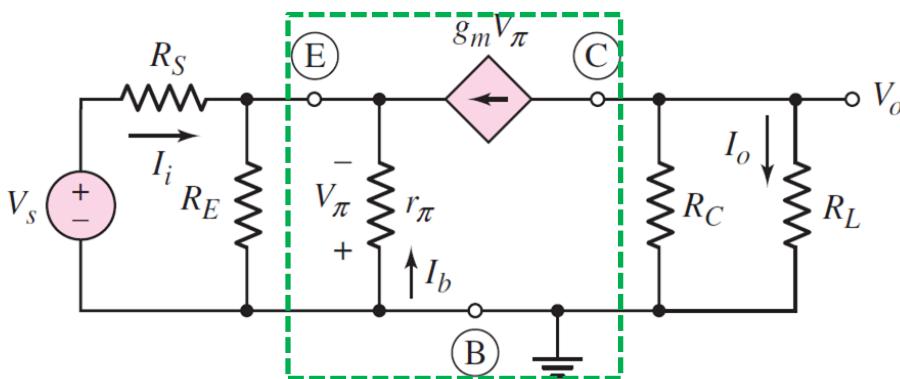
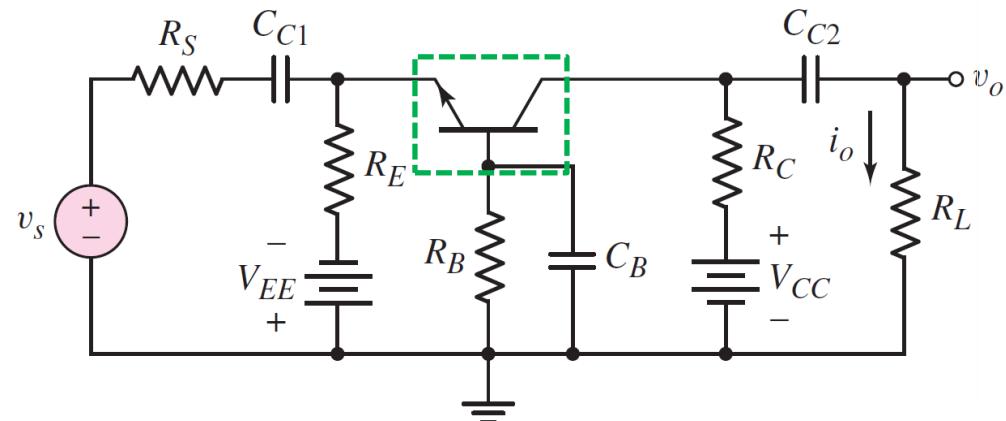
# Common-Base Amplifier

- The **base** is at signal ground
- The **input signal** is applied to the **emitter**
- The **output signal** is measured at the collector terminal
- The **load** is connected to the output through a coupling capacitor



# Small-Signal Equivalent Circuit

- The dc analysis of the common-base circuit is essentially the same as for the common-emitter circuit.
- Small-signal equivalent circuit**
  - Hybrid- $\pi$  model of the BJT
  - with the transistor resistance to be infinite  $\rightarrow r_o = \infty$



# Small-Signal Voltage Gain

- To find the small-signal voltage gain

- $V_s \leftrightarrow V_\pi \leftrightarrow V_o$

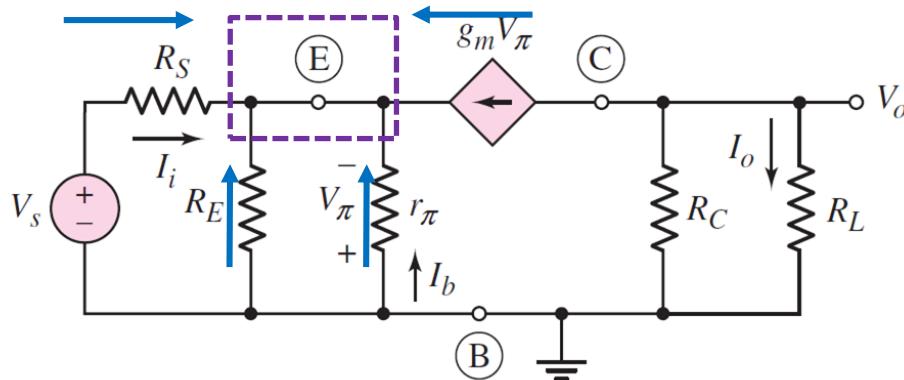
- The output voltage is

$$V_o = -(g_m V_\pi) (R_C \parallel R_L)$$

- Apply KCL at the emitter node

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + I_i = 0$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_s - (-V_\pi)}{R_S} = 0$$



# Small-Signal Voltage Gain

- Rearrange the KCL equation

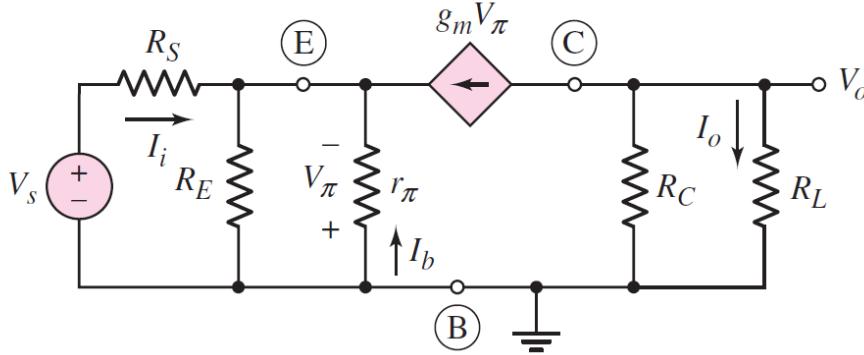
$$\left( g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right) V_\pi + \frac{V_S}{R_S} = 0$$

- $g_m$  and  $r_\pi$  are small-signal parameters
- $g_m = \frac{I_{CQ}}{V_T}$  and  $r_\pi = \frac{\beta V_T}{I_{CQ}} \rightarrow g_m \cdot r_\pi = \beta$

- Then

$$\left( \frac{\beta + 1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right) V_\pi + \frac{V_S}{R_S} = 0$$

$$V_\pi = -\frac{1}{R_S} \left[ \left( \frac{r_\pi}{1 + \beta} \right) \parallel R_E \parallel R_S \right] V_S$$



# Small-Signal Voltage Gain

- The small-signal voltage gain is

$$A_v = \frac{V_o}{V_s} = \frac{-(g_m V_\pi)(R_C \parallel R_L)}{V_\pi + \frac{1}{R_S} \left[ \left( \frac{r_\pi}{1 + \beta} \right) \parallel R_E \parallel R_S \right]} = g_m \frac{1}{R_S} \left[ \left( \frac{r_\pi}{1 + \beta} \right) \parallel R_E \parallel R_S \right] (R_C \parallel R_L)$$

- As the source resistor  $R_S$  approaches 0, the small-signal voltage gain becomes

$$A_v = g_m \frac{\left( \frac{r_\pi}{1 + \beta} \right) \parallel R_E \parallel R_S}{R_S} (R_C \parallel R_L) \cong g_m (R_C \parallel R_L)$$

# Small-Signal Current Gain

- Apply KCL at the emitter node

$$I_i + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + g_m V_\pi = 0$$

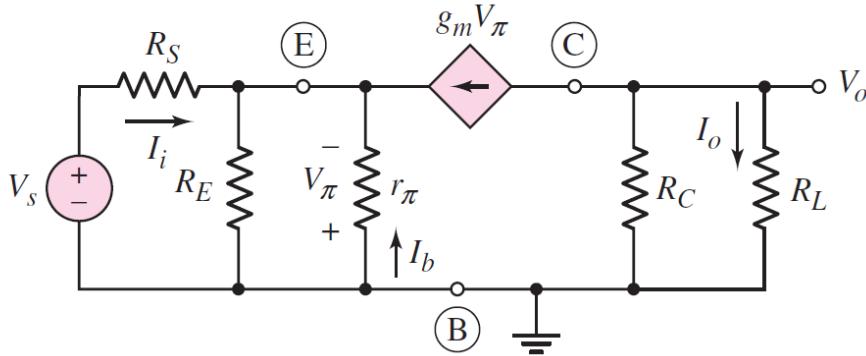
$$V_\pi \left( g_m + \frac{1}{r_\pi} + \frac{1}{R_E} \right) = -I_i$$

$$V_\pi \left( \frac{1 + \beta}{r_\pi} + \frac{1}{R_E} \right) = -I_i$$

$$V_\pi = -I_i \left[ \left( \frac{r_\pi}{1 + \beta} \right) \parallel R_E \right]$$

- The load current is

$$I_o = -(g_m V_\pi) \left( \frac{R_C}{R_C + R_L} \right)$$



# Small-Signal Current Gain

- The small signal current gain is

$$A_i = \frac{I_o}{I_i} = \frac{-(g_m V_\pi) \left( \frac{R_C}{R_C + R_L} \right)}{-\left[ \left( \frac{r_\pi}{1 + \beta} \right) \parallel R_E \right]} = g_m \left[ \left( \frac{r_\pi}{1 + \beta} \right) \parallel R_E \right] \left( \frac{R_C}{R_C + R_L} \right)$$

- We take the limits  $R_E$  approaches infinity and  $R_L$  approaches zero

$$A_i = g_m \frac{r_\pi}{1 + \beta} = \frac{\beta}{1 + \beta} = \alpha \quad \text{← common-base current gain}$$

- The small-signal voltage gain is usually greater than 1
- The small-signal current gain is slightly less than 1

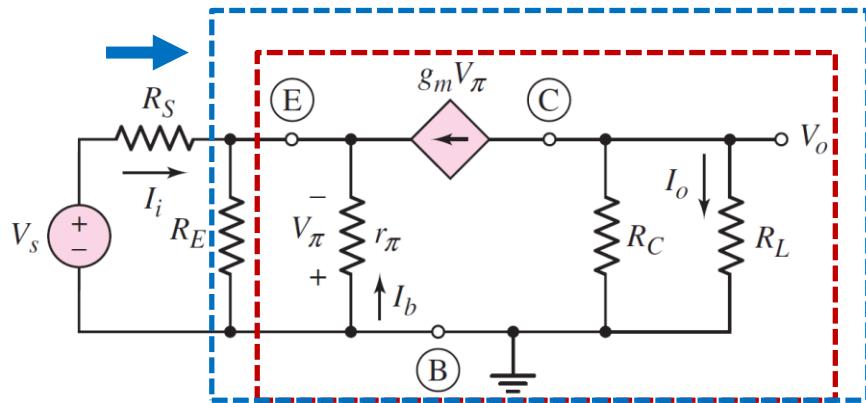
# Input Resistance (Optional)

- The **input resistance looking into the emitter** is defined as

$$R_{ie} = \frac{\text{voltage at emitter}}{\text{current at emitter}} = \frac{V_\pi}{I_b + g_m V_\pi} = \frac{V_\pi}{\frac{V_\pi}{r_\pi} + g_m V_\pi} = \frac{r_\pi}{1 + g_m r_\pi} = \frac{r_\pi}{1 + \beta}$$

- The **input resistance** is

$$R_i = R_E \parallel R_{ie} \cong \frac{r_\pi}{1 + \beta}$$

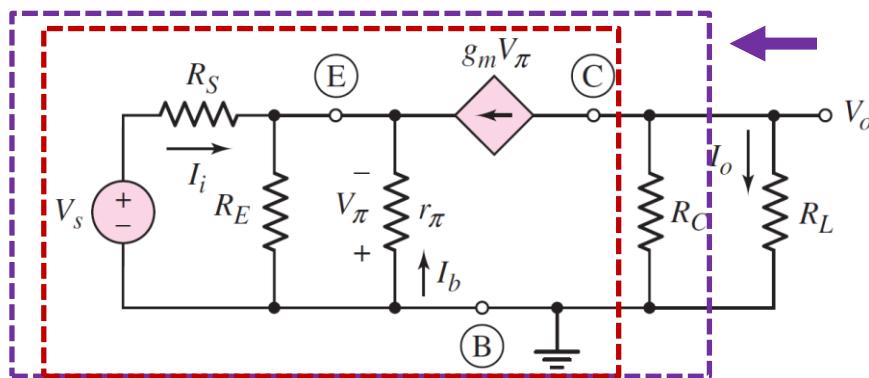
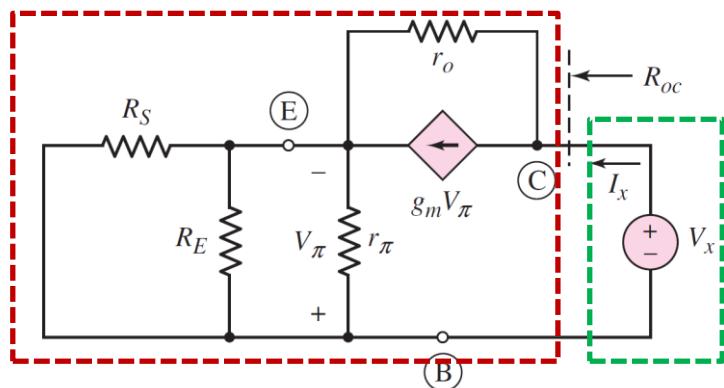


# Output Resistance (Optional)

- The output resistance is

$$R_o = R_C \parallel R_{oc}$$

- $R_{oc}$  is the output resistance looking into the collector
- To find  $R_{oc}$  ( $r_o$  is included), we need to set  $V_s$  to zero, and apply a test source  $V_x$  at the collector



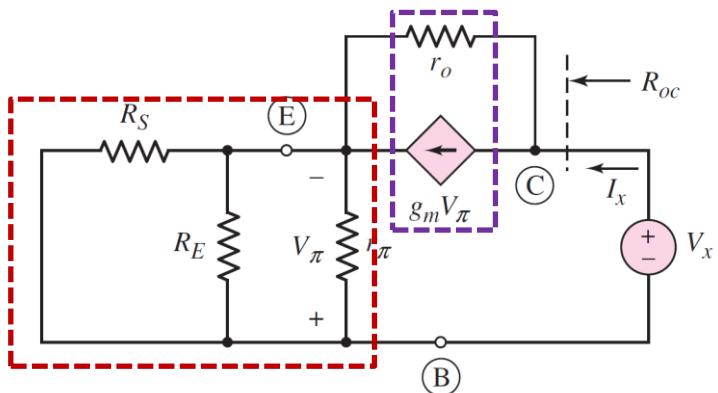
# Output Resistance (Optional)

- The output resistance looking into the collector

$$R_{oc} = \frac{V_x}{I_x}$$

- Write a KCL equation at the emitter

$$\frac{-V_\pi}{r_\pi} + \frac{-V_\pi}{R_E} + \frac{-V_\pi}{R_S} = \frac{V_x - (-V_\pi)}{r_o} + g_m V_\pi$$



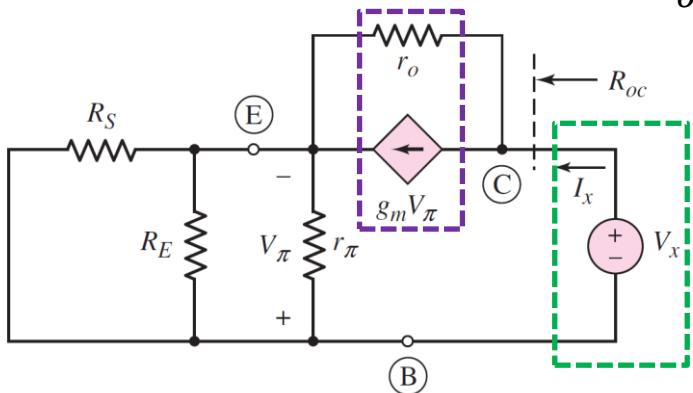
# Output Resistance (Optional)

- Write a KCL equation at the collector terminal

$$\frac{V_x - (-V_\pi)}{r_o} + g_m V_\pi = I_x$$

- Combine two KCL equations, we find the output resistance looking into the collector is

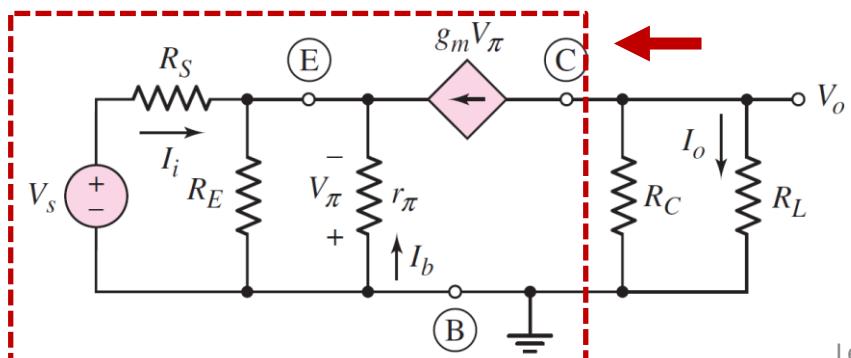
$$R_{oc} = \frac{V_x}{I_x} = r_o [1 + g_m(R_S || R_E || r_\pi)] + (R_S || R_E || r_\pi)$$



# Output Resistance (Optional)

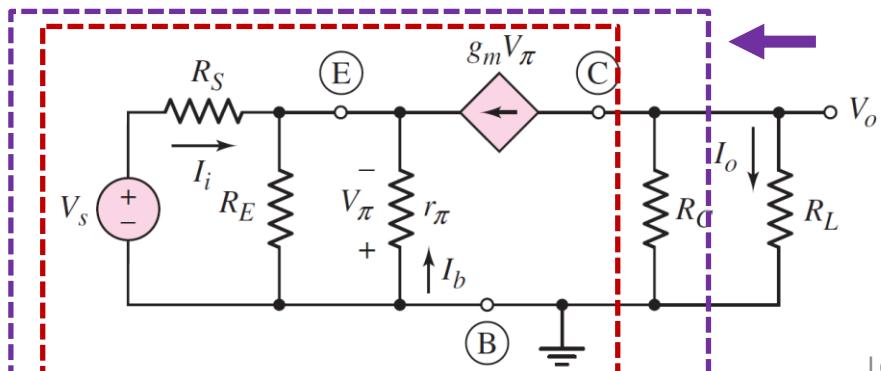
$$R_{oc} = \frac{V_x}{I_x} = r_o[1 + g_m(R_S||R_E||r_\pi)] + (R_S||R_E||r_\pi)$$

- For ideal source,  $R_S = 0$ , then  $R_{eq} = 0$
- The output resistance looking back into the collector is
  - $R_{oc} = r_o \rightarrow$  a very large value



# Output Resistance (Optional)

- Including a collector resistor, the **output resistance** looking back into the output terminal is
  - $R_o = R_{oc} \parallel R_C = r_o \parallel R_C$
- The common-base circuit is very useful when the input signal is a current



# The Three Basic Amplifier: Summary and Comparison

Compare the general characteristics of the three basic amplifier configurations.

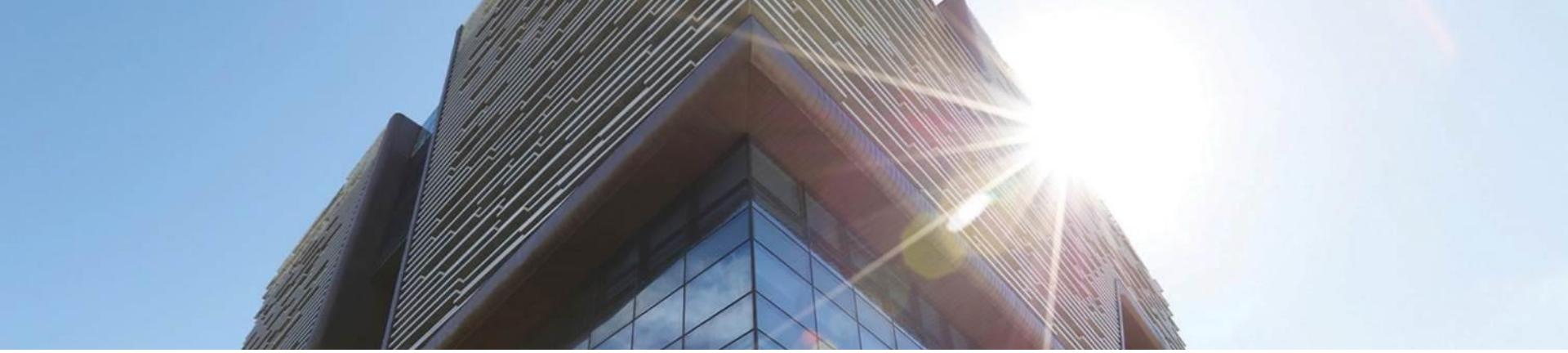
# Summary of BJT Amplifier Configurations

Characteristics of the three BJT amplifier configurations

Configuration	Voltage gain	Current gain	Input resistance	Output resistance
Common-emitter	$A_v > 1$	$A_i > 1$	Moderate $R_1 \parallel R_2 \parallel R_{ib}$	$R_C \parallel r_o$
Common-collector (emitter-follower)	$A_v \approx 1$	$A_i \approx 1 + \beta$	Moderate $R_1 \parallel R_2 \parallel R_{ib}$	Low $\frac{r_\pi}{1 + \beta} \parallel R_E \parallel r_o$
Common-base	$A_v > 1$	$A_i \approx 1$	Low $\frac{r_\pi}{1 + \beta}$	Moderate to high $r_o \parallel R_C$

# Summary

- ✓ Explain the **amplification process** in a simple BJT amplifier circuit
  - ✓ The BJT must be biased in the forward-active region → dc analysis
  - ✓ Describe **small-signal equivalent circuit** of the BJT → ac analysis
    - ✓ **hybrid- $\pi$  model** and **small-signal parameters**
- ✓ Analyze three BJT amplifier circuits
  - ✓ Common-collector amplifier
  - ✓ Common-emitter (emitter-follower)amplifier
  - ✓ Common-base amplifier
- ✓ Characterize the **small-signal voltage/current gains**, and the **input/output resistances** of three amplifiers
- Optional: apply the bipolar small-signal equivalent circuit in the analysis of multistage amplifier circuits



# Thank You

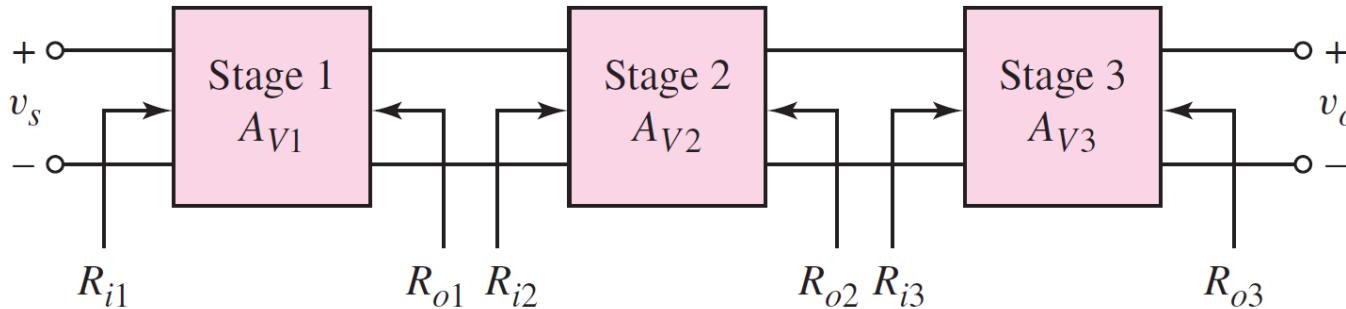


# Optional: Multistage Amplifiers

Analyze multitransistor or multistage amplifiers and understand the advantages of these circuits over single-transistor amplifiers.

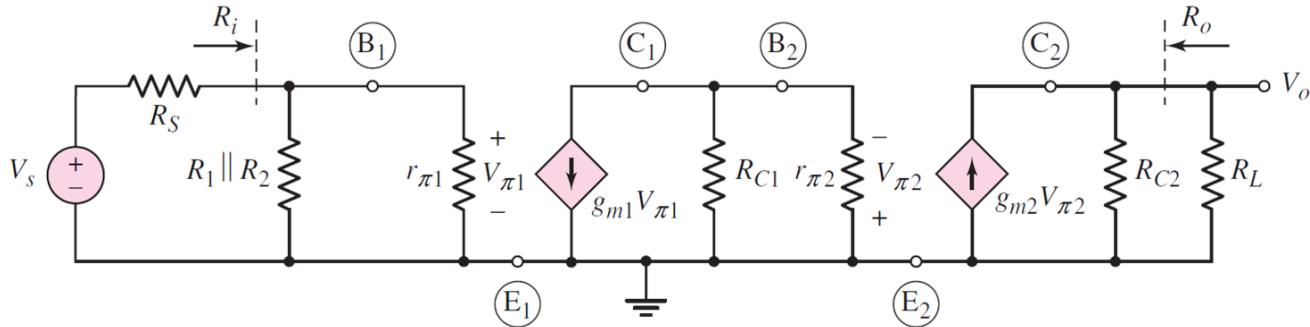
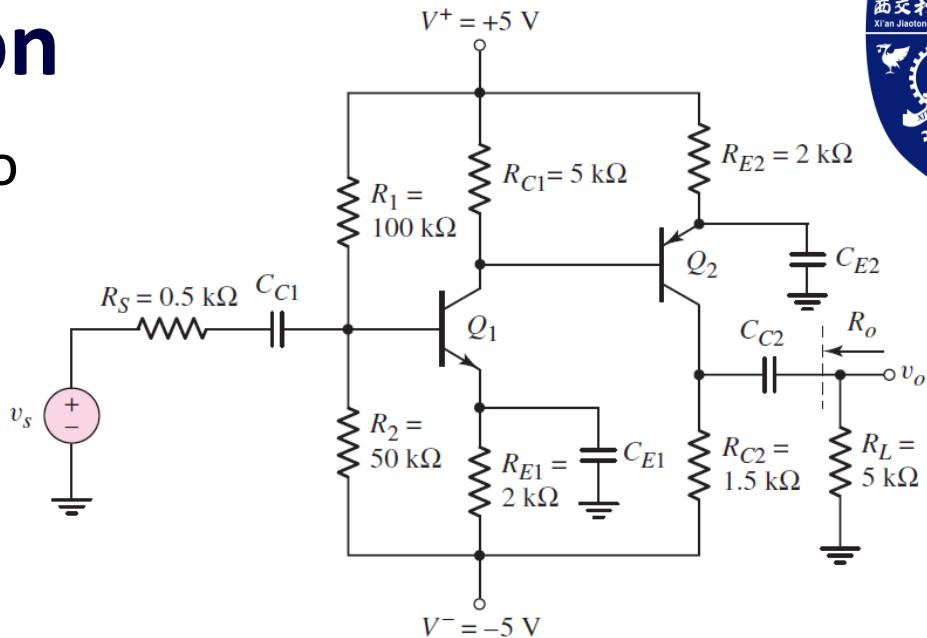
# Multistage Amplifiers

- A single transistor amplifier will not be able to meet the combined specifications
  - Amplification factor
  - Input/output resistance
- Transistor amplifier circuits can be connected in series, or **cascaded**



# Cascade Configuration

- A cascade configuration of two common-emitter circuits
  - $Q_1$  is a npn transistor
  - $Q_2$  is a pnp transistor
- Both are biased in the **forward-active mode**



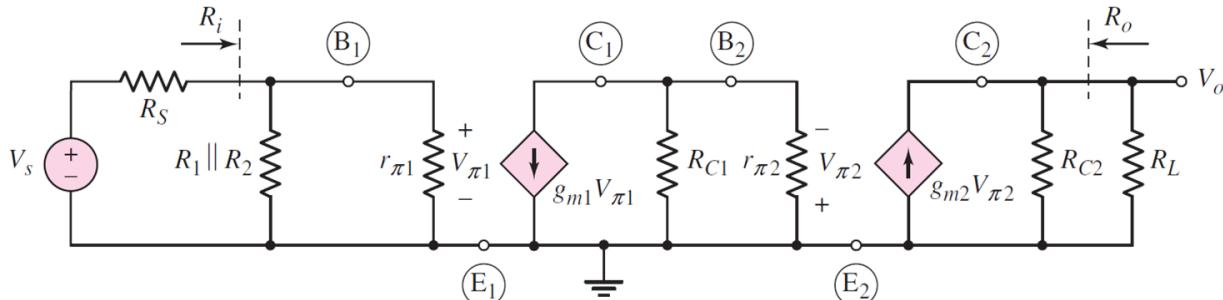
# Small-Signal Voltage Gain

$$V_o = g_{m2} V_{\pi 2} (R_{C2} \parallel R_L) = g_{m2} [g_{m1} V_{\pi 1} (R_{C1} \parallel r_{\pi 2})] (R_{C2} \parallel R_L)$$

$$V_{\pi 1} = \left( \frac{R_i}{R_i + R_S} \right) V_s$$

- The small-signal voltage gain is

$$A_v = \frac{V_o}{V_s} = g_{m1} g_{m2} (R_{C1} \parallel r_{\pi 2}) (R_{C2} \parallel R_L) \left( \frac{R_i}{R_i + R_S} \right)$$



# Input and Output Resistance

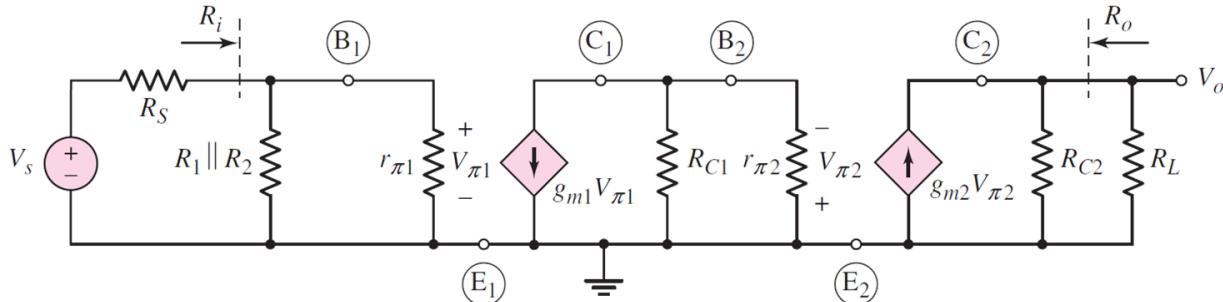
- The input resistance is

$$R_i = R_1 \parallel R_2 \parallel r_{\pi 1}$$

- The output resistance is

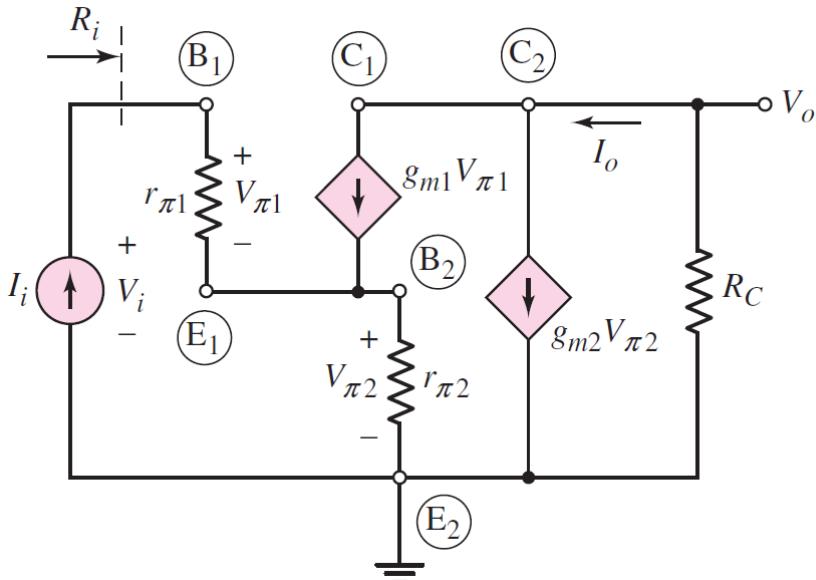
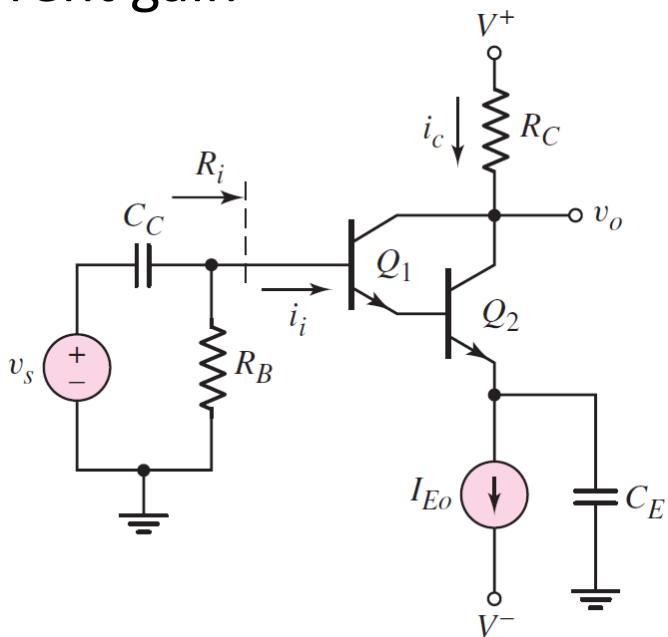
$$R_o = R_{C2}$$

- Set  $V_s$  to zero



# Darlington Pair Configuration

- Darlington pair or a Darlington configuration provides large current gain



# Small-Signal Current Gain

$$g_{m1}V_{\pi1} = g_{m1}I_i r_{\pi1} = \beta_1 I_i$$

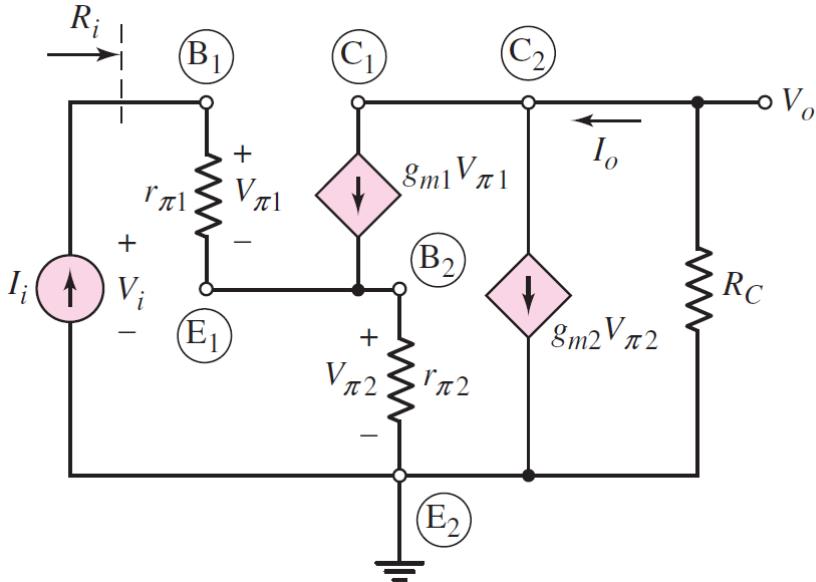
$$V_{\pi2} = (I_i + \beta_1 I_i) r_{\pi2}$$

- The output current is

$$I_o = g_{m1}V_{\pi1} + g_{m2}V_{\pi2} = \beta_1 I_i + \beta_2(1 + \beta_1)I_i$$

- The overall current gain is

$$A_i = \frac{I_o}{I_i} = \beta_1 + \beta_2(1 + \beta_1) \cong \beta_1 \beta_2$$



# Input Resistance

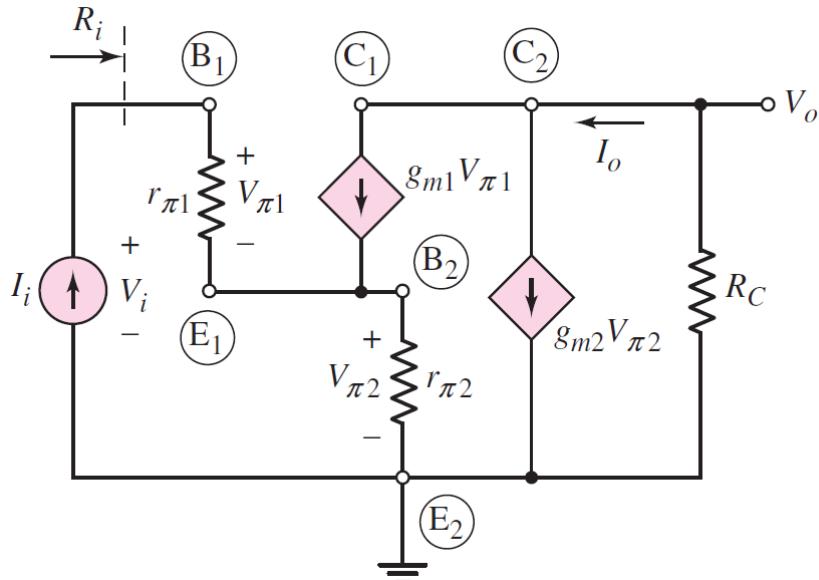
$$V_i = V_{\pi 1} + V_{\pi 2} = I_i r_{\pi 1} + I_i (1 + \beta_1) r_{\pi 1}$$

- The input resistance is

$$R_i = \frac{V_i}{I_i} = r_{\pi 1} + (1 + \beta_1) r_{\pi 1}$$

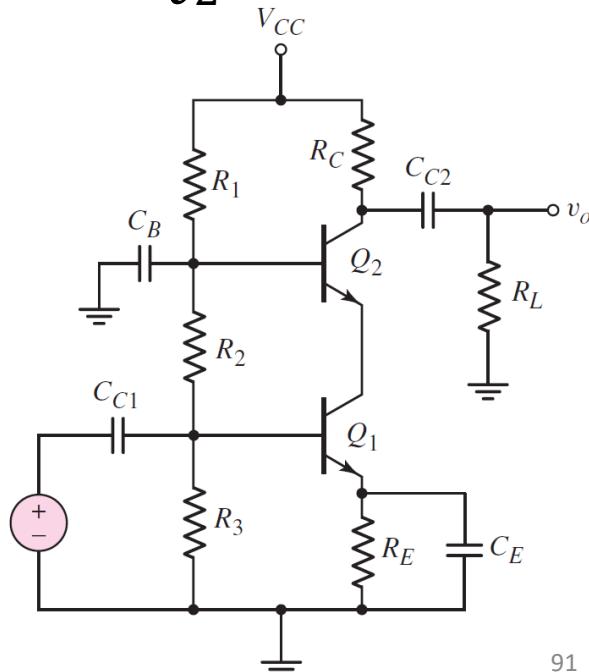
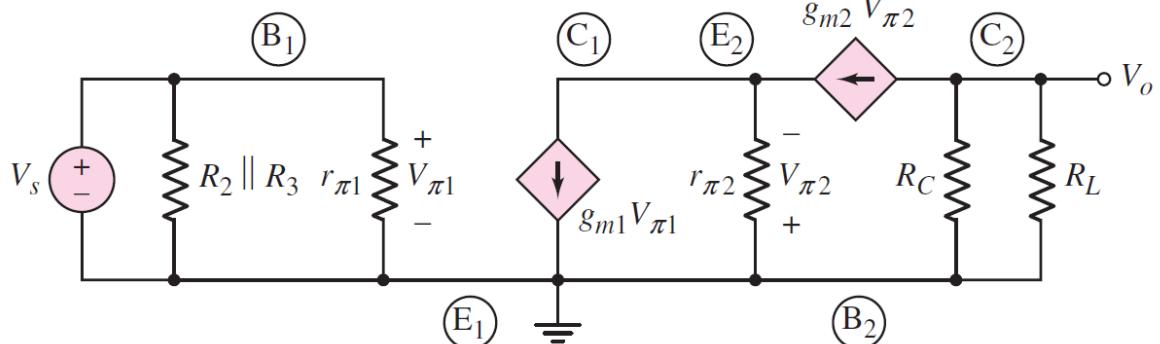
$$r_{\pi 1} = \frac{\beta_1 V_T}{I_{CQ1}} \cong \frac{\beta_1 V_T}{\left( \frac{I_{CQ2}}{\beta_2} \right)} = \frac{\beta_1 \beta_2 V_T}{I_{CQ2}} = \beta_1 r_{\pi 2}$$

$$R_i \cong \beta_1 r_{\pi 2} + (1 + \beta_1) r_{\pi 2} \cong 2\beta_1 r_{\pi 2}$$



# Cascode Configuration

- The input is into a common-emitter amplifier ( $Q_1$ ), which drives a common-base amplifier ( $Q_2$ )
- The output signal current of  $Q_1$  is the input signal of  $Q_2$



# Small-Signal Voltage Gain

- Write KCL equation at  $E_2$

$$g_{m2}V_{\pi2} + \frac{V_{\pi2}}{r_{\pi2}} - g_{m1}V_{\pi1} = 0 \quad V_{\pi2} = \left( \frac{1}{g_{m2} + \frac{1}{r_{\pi2}}} \right) g_{m1}V_{\pi1} = \left( \frac{r_{\pi2}}{1 + \beta_2} \right) g_{m1}V_s$$

$$V_o = -g_{m2}V_{\pi2}(R_C \parallel R_L) = -g_{m2} \left( \frac{r_{\pi2}}{1 + \beta_2} \right) g_{m1}V_s(R_C \parallel R_L) \cong g_{m1}V_s(R_C \parallel R_L)$$

- The small signal voltage gain is

$$A_v = \frac{V_o}{V_s} \cong -g_{m1}(R_C \parallel R_L)$$

