CAN102

Electromagnetism and Electromechanics

2023/24-S2

# Lecture 22 Review of Electromechanics

Jingchen Wang
SAT
jingchen.wang@xjtlu.edu.cn

# Exams (No MCQ)

## Week 8-12 Electromechanics (40%)

1 Magnetic Circuits

2 Transformers

3 Linear DC machine

**Question 1** → 20%

4 DC Machinery Fundamentals

5 DC motors

6 DC generators

**Question 2-a** → 10%

7 AC Machinery Fundamentals

8 Synchronous Generators

9 Synchronous /Induction Motors

**Question 2-b** → 10%

Equation list provided on LM

# Magnetic Circuits

Consider a coil with N turns, wound onto a core carries a current *I*, It will generate magnetomotive force (mmf/ $\mathcal{F}$)

1. Magnetomotive force

$$F = NI$$

SI Unit: Ampere-turns

2. Reluctance of the magnetic circuit

$$\Re = \frac{l}{\mu_r \mu_0 A}$$

*l* is the mean path length

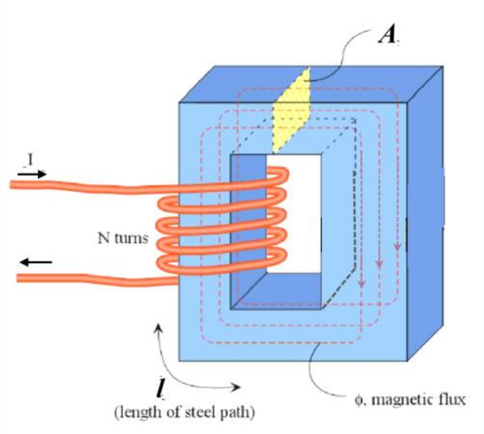
is the relative permeability of material

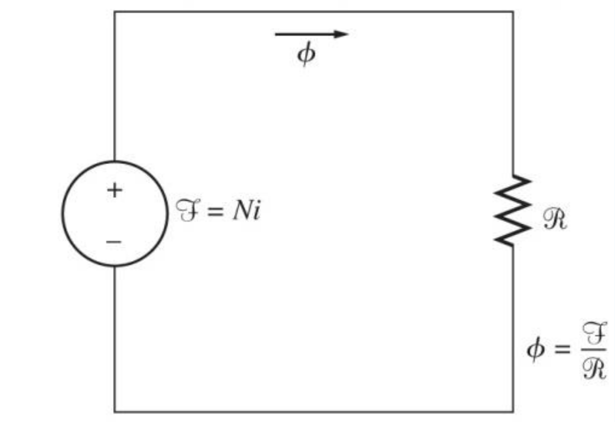
*A* is the perpendicular area to flux density

# Magnetic Circuits

## 3. Without air gap in the core

$$F = Hl = NI$$
$$= \Phi\Re$$



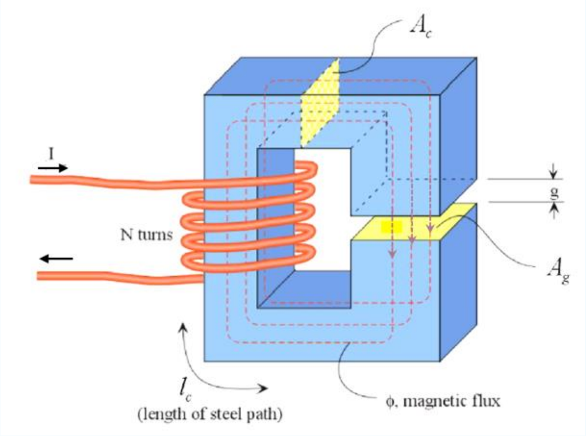



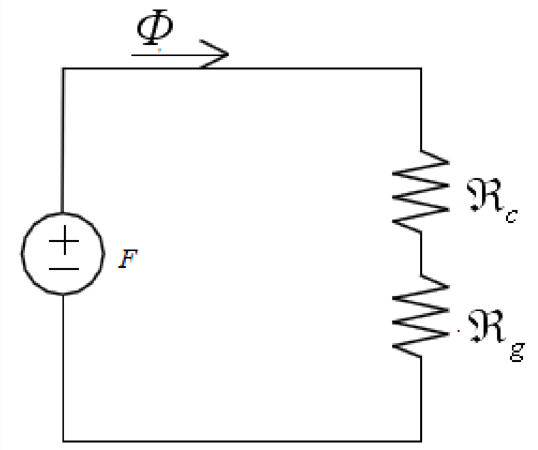

## 4. With air gap in the core

$$F = NI = H_c l_c + H_g l_g$$

where $H_c l_c = \dfrac{B_c}{\mu_c} l_c = \dfrac{\Phi_c}{\mu_c A_c} l_c = \Phi_c \Re_c$

and $\quad H_g l_g = \dfrac{B_g}{\mu_0} l_g = \dfrac{\Phi_g}{\mu_0 A_g} l_g = \Phi_g \Re_g$



# Magnetic Circuits

5. Neglecting the reluctance of the core $\Re_c << \Re_g$

high material permeability

small core reluctance

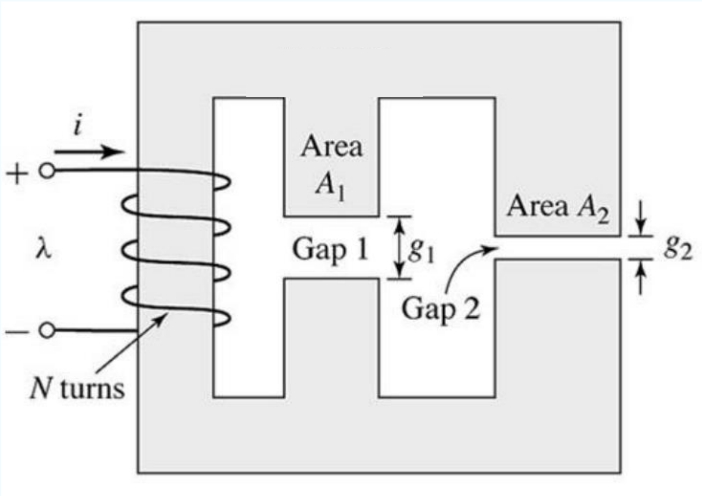
6. The reluctance of the core cannot be neglected

low material permeability
(2000~6000 times of air)

large core reluctance

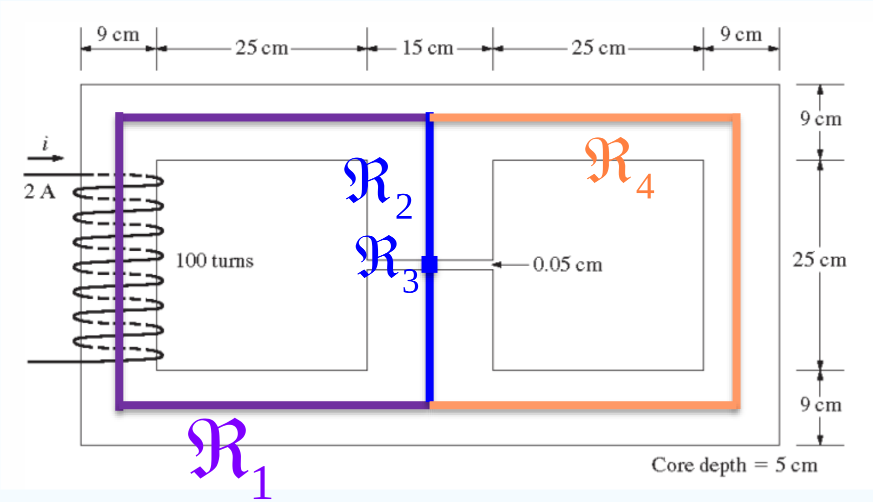
# Magnetic Circuits

## 7. Series and Parallel magnetic circuit




Series $\Re_{eq} = \Re_1 + \Re_2 + \ldots + \Re_n = \sum_{i=1}^{n} \Re_i$

Parallel $\frac{1}{\Re_{eq}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \ldots + \frac{1}{\Re_n} = \sum_{i=1}^{n} \frac{1}{\Re_i}$




$\Re_1 + ((\Re_2 + \Re_3) // \Re_4)$

# Magnetic Circuit

magnetomotive force 磁动势 $F = NI$

$$F = Hl = \frac{B}{\mu_r \mu_0} l = \frac{\Phi}{A \mu_r \mu_0} l$$

Flux density **B**
磁通密度
Flux **Φ**
磁通量
Magnetic field intensity **H**
磁场强度
Magnetic reluctance $\Re$
磁阻

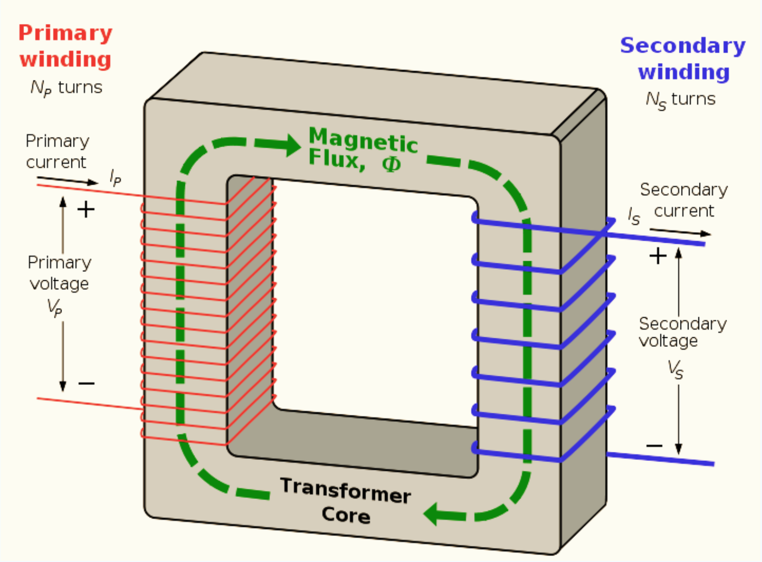
relative permeability $\mu_r$
相对磁导率
Mean path length of the core/air gap $l$
平均路径长度
Cross-sectional area $A$
横截面积

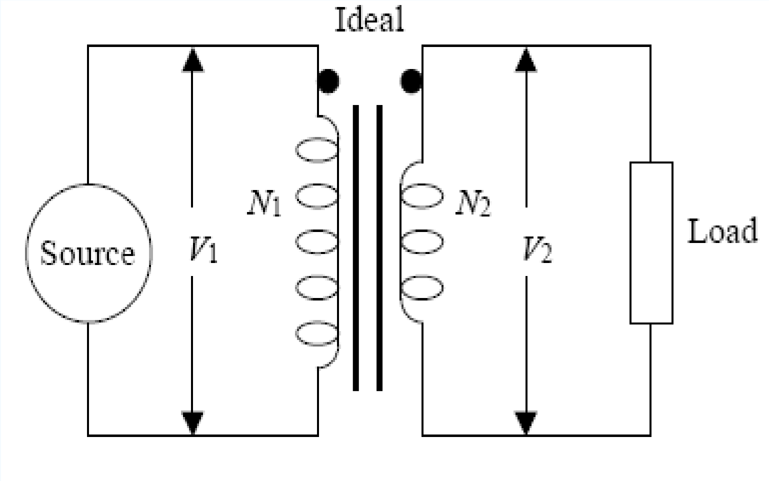
$$F = \Phi \Re = BA\Re = \mu_r \mu_0 HA\Re$$

$$\Re = \frac{l}{\mu_r \mu_0 A}$$

# Transformers

## 1. Ideal Transformers



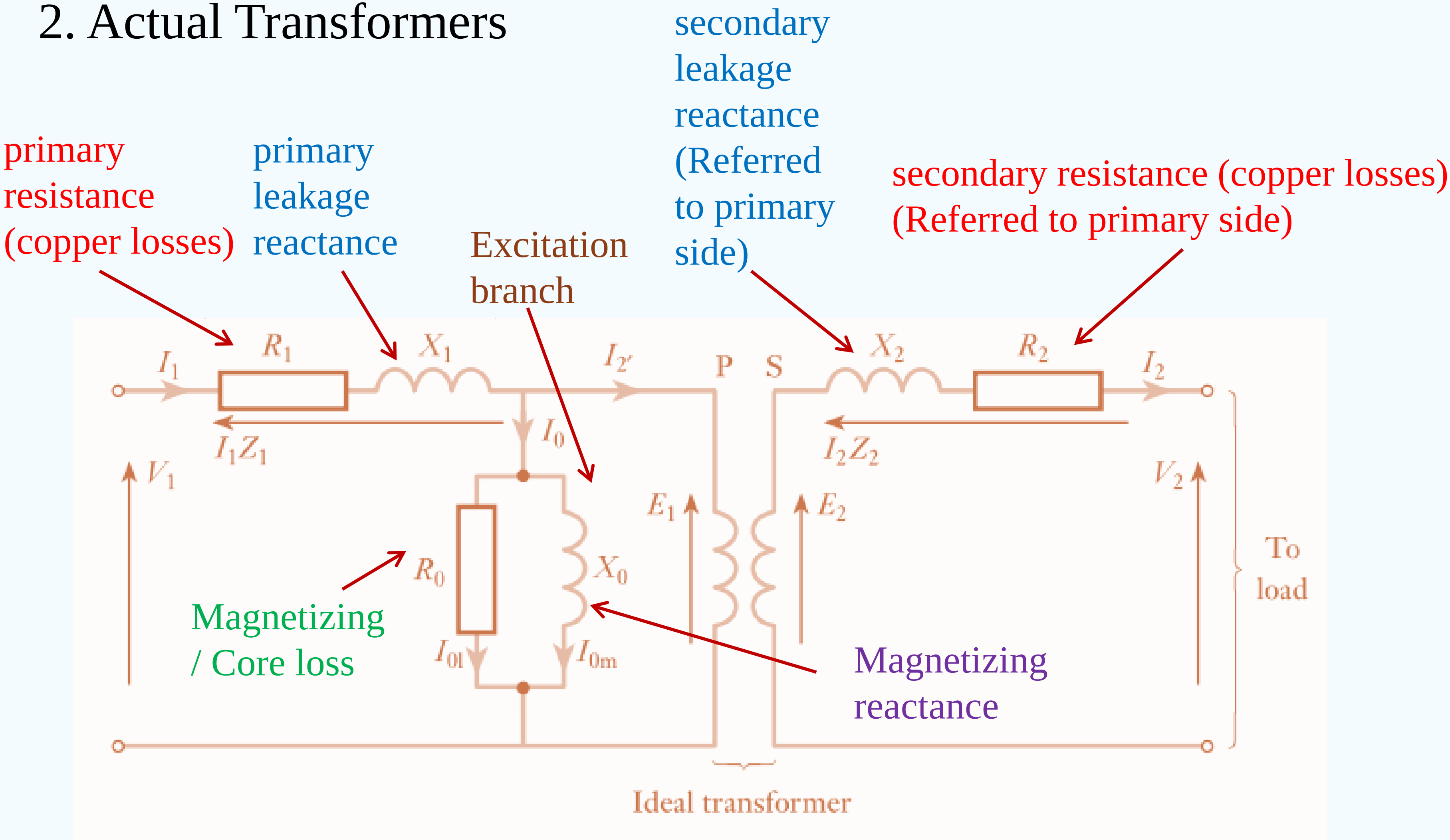




$$\frac{V_p}{V_s} = a \qquad \frac{I_p}{I_s} = \frac{1}{a} \qquad \frac{Z_p}{Z_s} = a^2$$

$$P_{in} = P_{out}$$

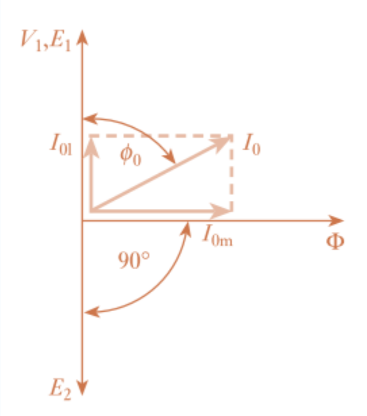
# Transformers

## 2. Actual Transformers



# Transformers

a) Phasor Diagram of Transformers at no load



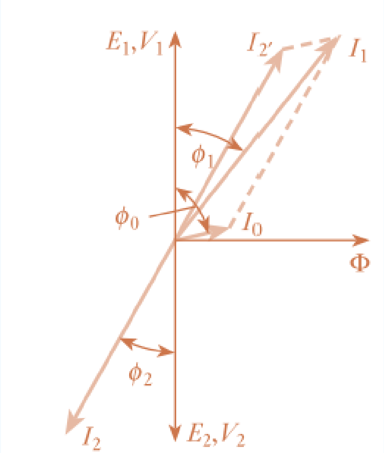

$$I_0 = \sqrt{{I_{01}}^2 + {I_{0m}}^2}$$

$$\cos\phi_0 = I_{01}V_1 / I_0V_1 = I_{01} / I_0$$

**Power factor**

b) Phasor Diagram of Transformers at full load

**Lagging or Leading**




$$I_1V_1\cos\phi_1 \simeq I_2V_2\cos\phi_2$$

$$I_1\cos\phi_1 = I_0\cos\phi_0 + I_{2'}\cos\phi_2$$
$$I_1\sin\phi_1 = I_0\sin\phi_0 + I_{2'}\sin\phi_2$$

# Transformers

$$\frac{V_p}{V_s} = a \qquad \frac{I_p}{I_s} = \frac{1}{a} \qquad \frac{Z_p}{Z_s} = a^2$$

c) Two important performance characteristics:

Voltage regulation:

$$VR = \frac{V_{2,nl} - V_{2,fl}}{V_{2,fl}} \times 100\%$$

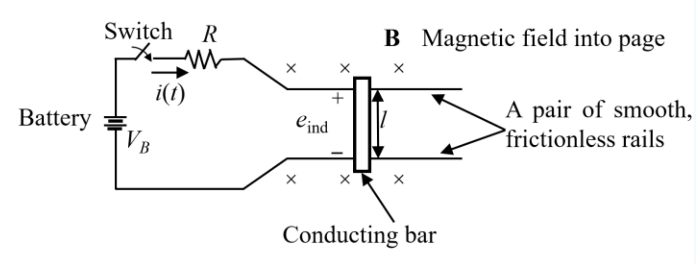
Efficiency:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

- Copper ($I^2R$) Losses
- Core Losses:
  - Eddy Current Losses
  - Hysteresis Losses
- Leakage Flux

# Linear DC Machines

◆ Four basic equations




## 1. Force on a wire

The force on a wire in the presence of a magnetic field:

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$$

## 2. Induced voltage in a moving wire

The voltage induced in a wire moving in a magnetic field

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$

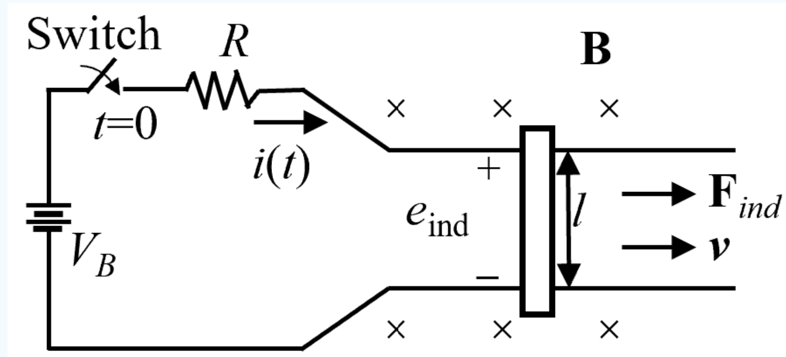
## 3. Kirchhoff's voltage law: $V_B = e_{ind} + iR$

## 4. Newton's law: For the bar: $F_{net} = ma$

# Linear DC Machines

Starting a linear DC machine

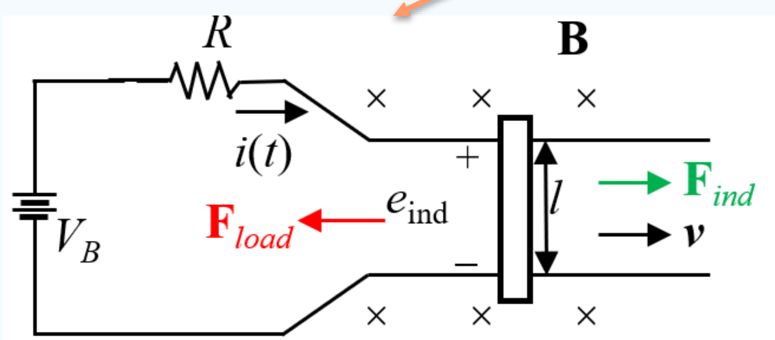
*Starting current* $i = V_B/R$



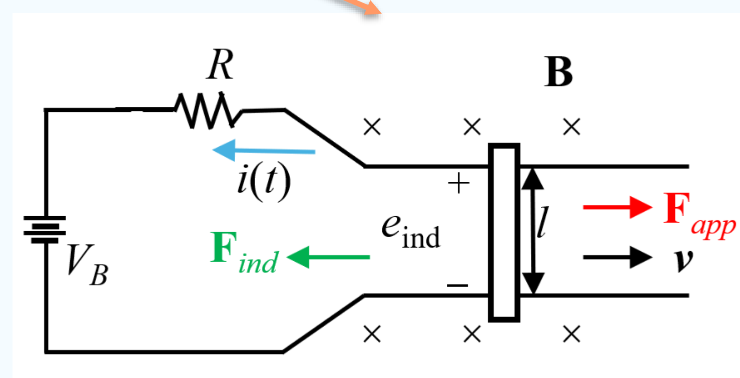

No-load condition

$F_{ind} = 0$, $e_{ind} = V_B$, $i = 0$, and $v_{nl} = V_B/Bl$.

External force




The linear DC machine as a motor




The linear DC machine as a generator

# Linear DC Machines

- How to determine if the machine is a generator or a motor
  - The only difference between the two is whether the externally applied forces are in the direction of motion (generator) or opposite to the direction of motion (motor)
  - Electrically, when $e_{ind} > V_B$, the machine acts as a generator, and when $e_{ind} < V_B$, the machine acts as a motor
  - The machine is a generator when it moved rapidly ($v_{app}>v_{nl}$) and a motor when it moved more slowly ($v_{load}<v_{nl}$) , but whether it was a motor or a generator, **it always moved in the same direction**

- Power conversion

$$P_{conv} = P_e = P_m$$

$$P_{conv} = e_{ind} i = F_{ind} v_{load}$$

$$\boxed{P_{conv} = \tau_{ind}\omega}$$

# Linear DC Machines

- Methods of speed control

  - *reducing* the flux density ***B*** of the machine *increases* the steady-state speed, and

  - *reducing* the battery voltage $V_B$ *decreases* the stead-state speed of the machine.

# Electrical Machines

- AC Machines
  - 同步 Synchronous
    - Generators (popular)
    - Motors
  - 感应/异步 Induction
    - Motors (Three-phase)
- DC Machines
  - Generators
  - Motors

armature windings on rotor

field windings on stator

armature windings on stator (rotating magnetic field)

field windings on rotor (DC source)

stator winding (rotating magnetic field)

rotor winding (induced)

# DC Machinery Fundamentals

The total induced voltage

$$e_{ind} = \begin{cases} 2rlB\omega & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$e_{ind} = \begin{cases} \dfrac{2}{\pi}\Phi\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

where $\Phi = \pi rlB = A_p B$

The induced voltage in any real machine will depend on the same 3 factors:

1. The flux in the machine
2. The speed of rotation
3. A constant representing the construction of the machine.

$$e_{ind} = K\Phi\omega$$

The total induced torque

$$\tau_{ind} = \begin{cases} 2rlBi & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

$$\tau_{ind} = \begin{cases} \dfrac{2}{\pi}\Phi i & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

where $\Phi = \pi rlB = A_p B$

The torque in any real machine will depend on the same 3 factors:

1. The flux in the machine
2. The current in the machine
3. A constant representing the construction of the machine.

$$\tau_{ind} = K\Phi i$$

# DC Motors

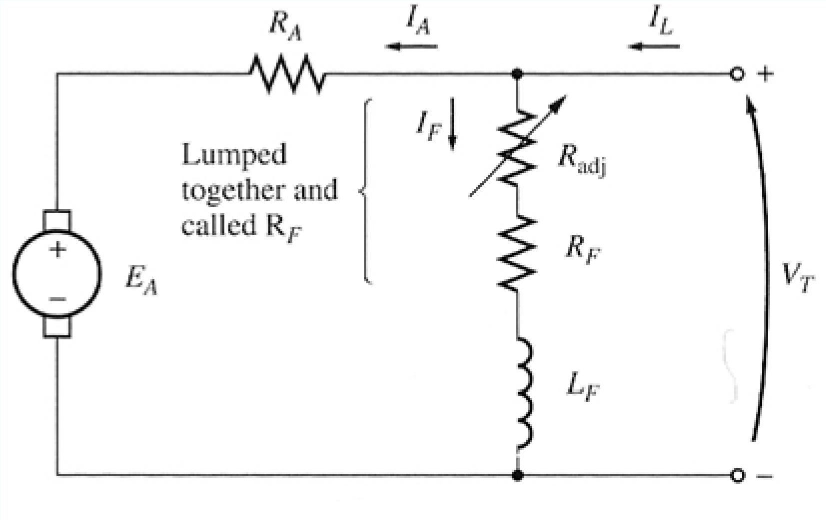
There are five types of DC motors:

separately excited （自励）, shunt （并励）, permanent-magnet（永磁）, series（串励）, and compounded（复励）.

- Equivalent Circuits
- Terminal Characteristic: torque-speed characteristic
- Non-linear Analysis
- Methods of Speed Control
- Power Flow

# DC Motors-Shunt

1. Equivalent circuit



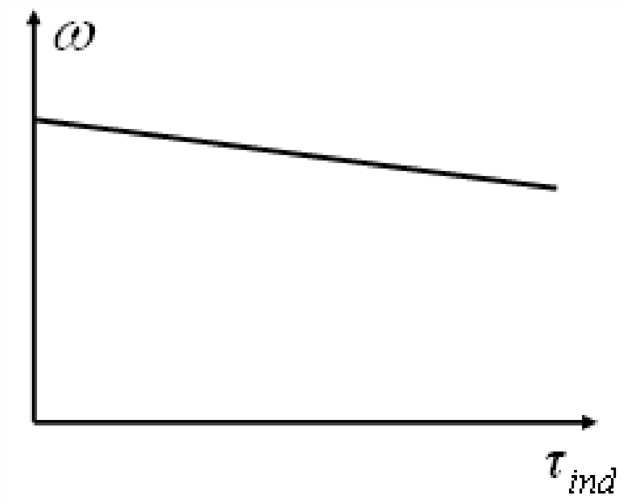

$I_F = V_T/R_F$

$V_T = E_A + I_AR_A$

$I_L = I_A + I_F$

2. Terminal characteristic

$$\omega = \frac{V_T}{K\Phi} - \frac{R_A}{(K\Phi)^2}\tau_{ind}$$



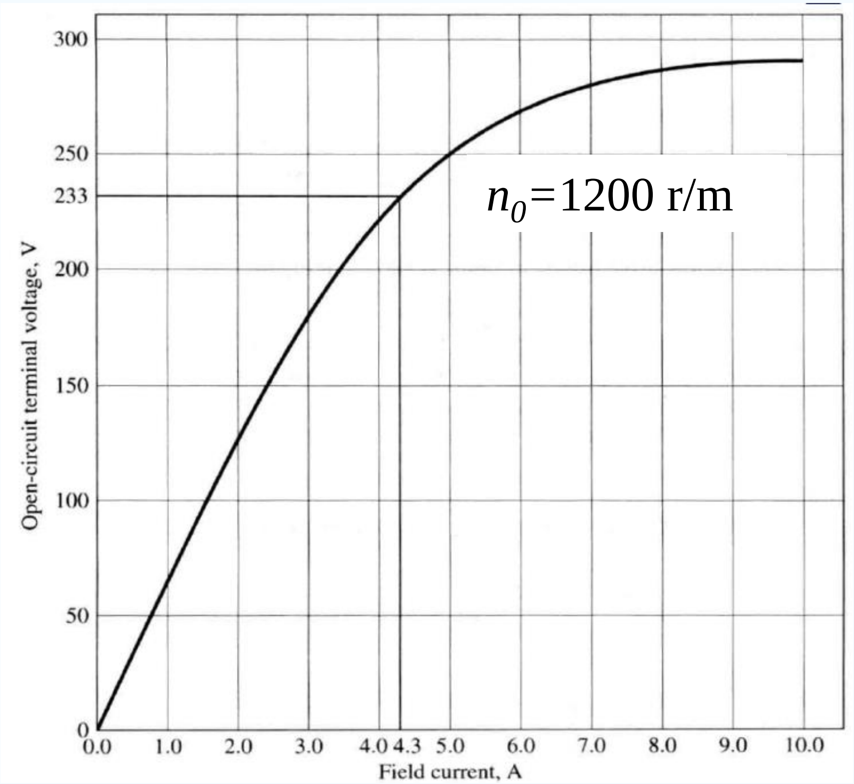
# DC Motors-Shunt

## 3. Nonlinear Analysis

- For a given effective filed current, the flux in a machine is fixed, so the internal generated voltage is related to speed by

$$\boxed{\frac{E_A}{E_{A0}} = \frac{\omega_m}{\omega_0} = \frac{n_m}{n_0}}$$

Where $E_{A0}$ and $n_0$ represent the reference values of voltage and speed, respectively




the magnetization curve is a direct plot of $E_A$ versus $I_F$ for a given speed.

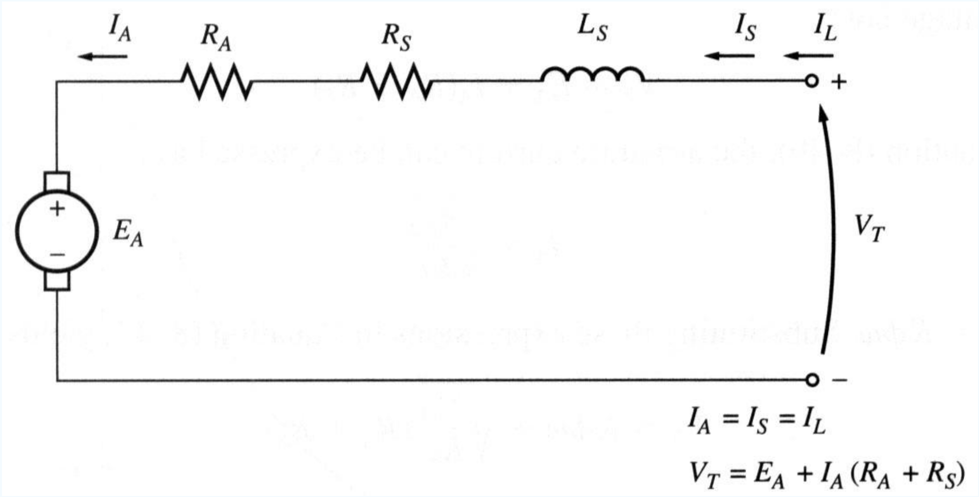
# DC Motors-Shunt

- Adjusting the field resistance $R_F$ (and thus the field flux) $R_F \uparrow \quad \omega \uparrow$
- Adjusting the terminal voltage applied to the armature. $V_A \uparrow \quad \omega \uparrow$
- Inserting a resistor in series with the armature circuit.

***How and Why ?***

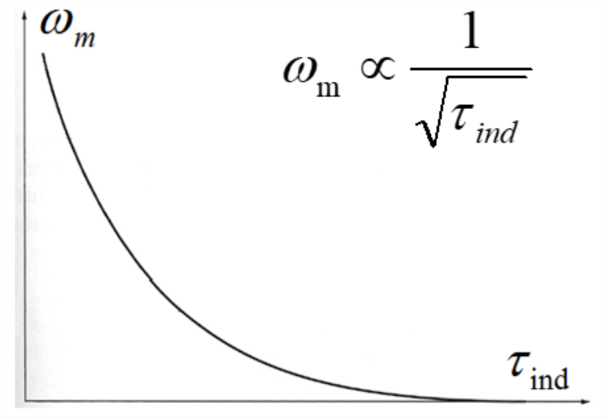
# DC Motors-Series

1. Equivalent circuit




2. Terminal characteristic

$$\omega_m = \frac{V_T}{\sqrt{Kc}}\frac{1}{\sqrt{\tau_{ind}}} - \frac{R_A + R_s}{Kc}$$

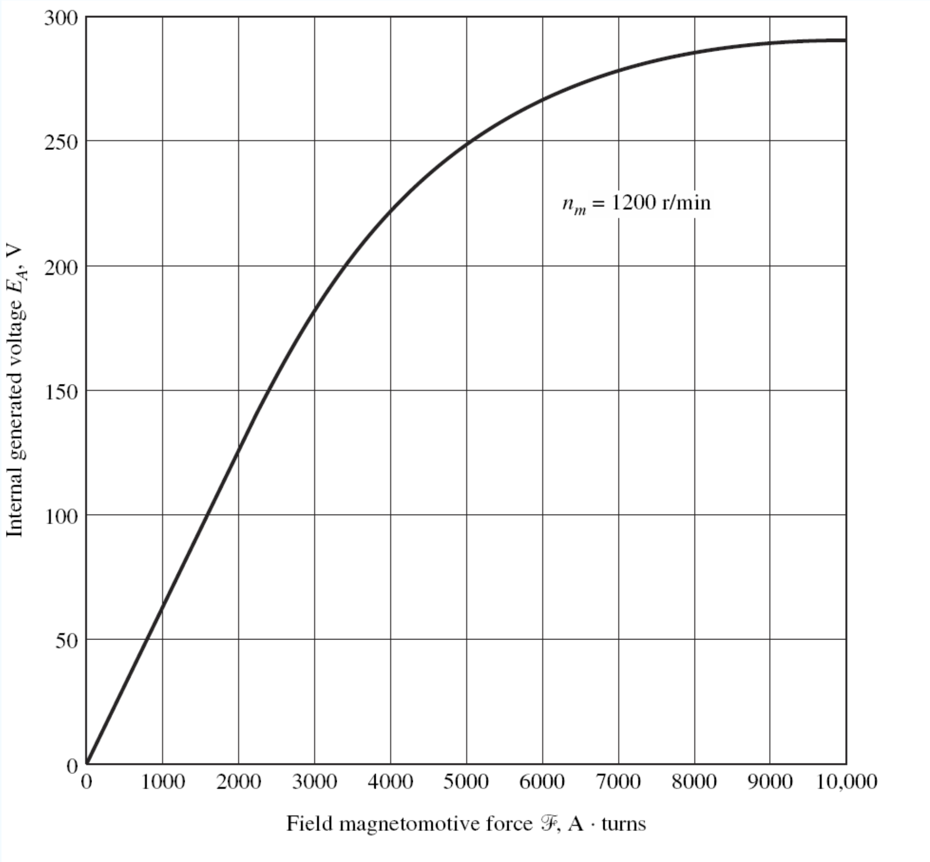


# DC Motors-Series

## 3. Nonlinear Analysis

- The ratio of the internal generated voltage at one speed to the internal generated voltage at another speed is given

$$\frac{E_{A2}}{E_{A1}} = \frac{\omega_{m2}}{\omega_{m1}} = \frac{n_{m2}}{n_{m2}}$$



# DC Motors-Series

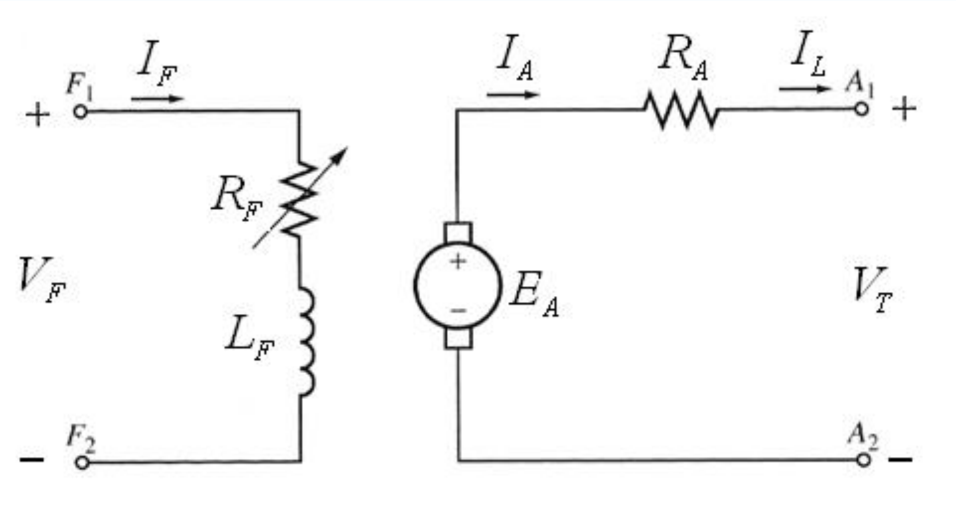
4. Methods of speed control

there is only **one efficient way** to change the speed of a series DC motor:

to **change the terminal voltage of the motor**, since the motor speed is directly proportional to its terminal voltage for any given torque.

# DC Generators-Separately Excited

1. Equivalent circuit

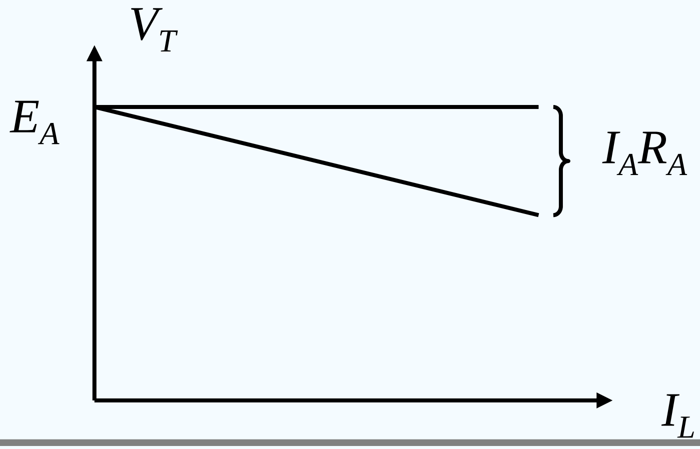



$$I_F = V_F/R_F$$

$$V_T = E_A - I_A R_A$$

$$I_L = I_A$$

2. Terminal characteristic



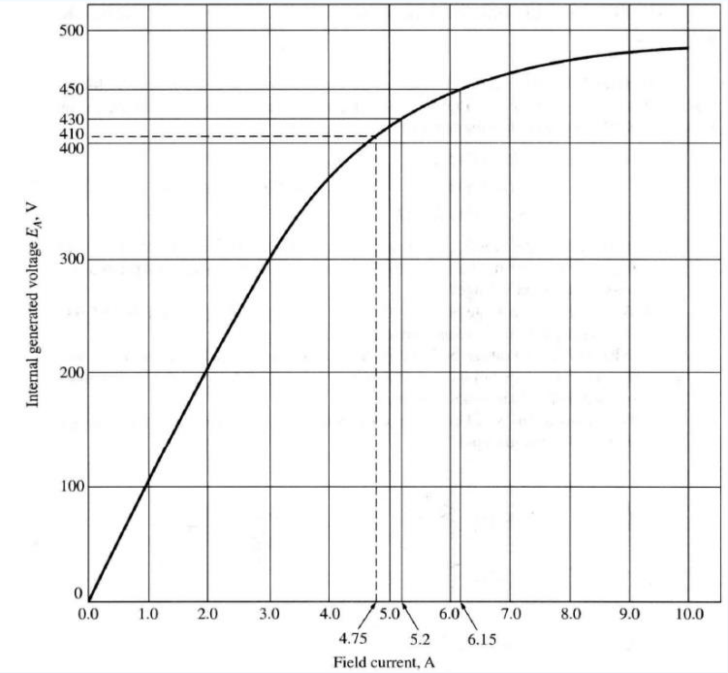
# DC Generators-Separately Excited

## 3. Nonlinear Analysis

The difference between the speed of the magnetization curve and the real speed of the generator must be taken into account

$$\frac{E_A}{E_{A0}} = \frac{\omega_m}{\omega_0} = \frac{n_m}{n_0}$$

Where $E_{A0}$ and $n_0$ represent the reference values of voltage and speed, respectively




If a DC machine has compensating windings, armature reaction can be neglected.

# DC Generators-Separately Excited

## 4. Methods of terminal voltage control

- Change the speed of rotation: when $\omega\uparrow$

$$V_T\uparrow = E_A\uparrow - I_A R_A$$

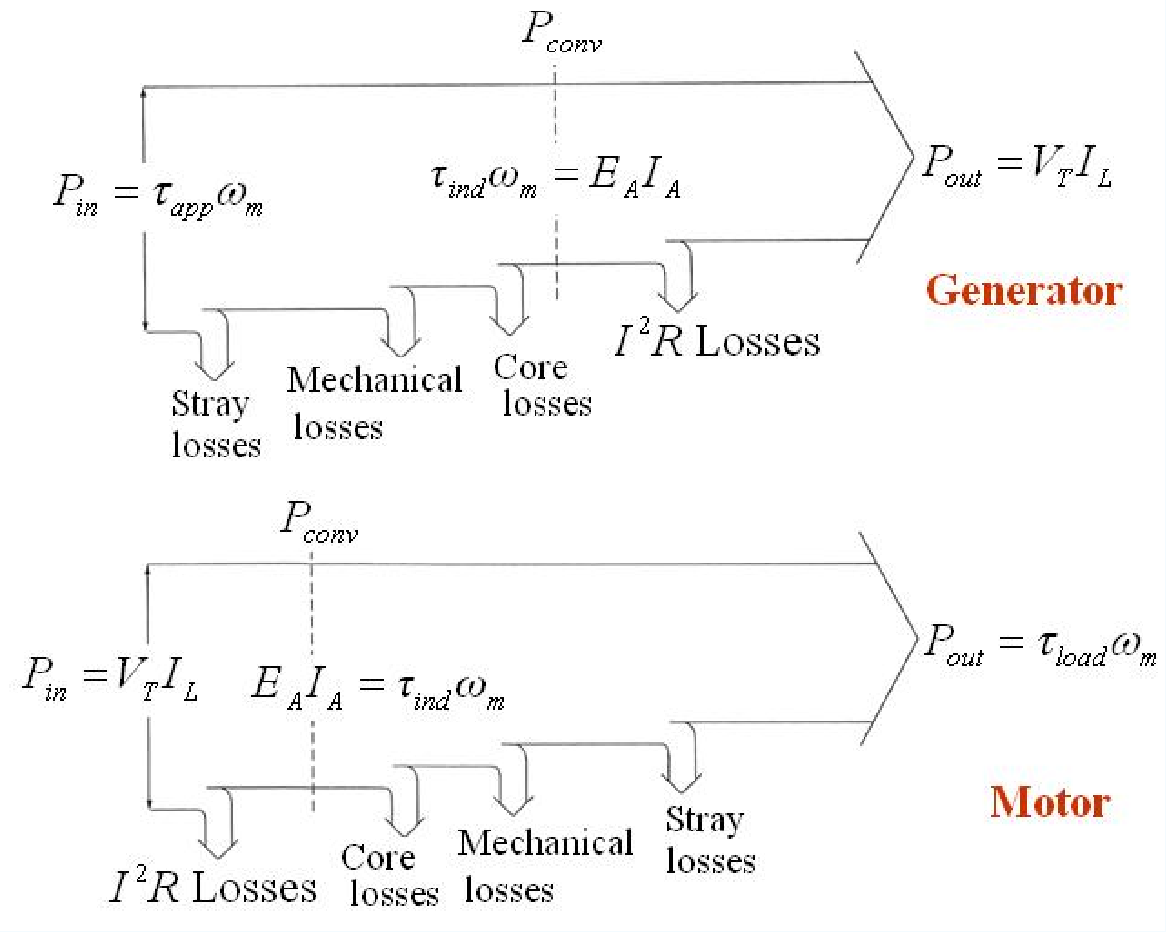
$$E_A\uparrow = K\Phi\omega\uparrow$$

- Change the field current: when $R_F\downarrow$

$$I_F\uparrow = V_F/R_F\downarrow$$

$$\Phi\uparrow \dashrightarrow E_A\uparrow \dashrightarrow V_T\uparrow$$

# Power Flow in DC Machines




DC generators convert mechanical energy to DC electric energy.

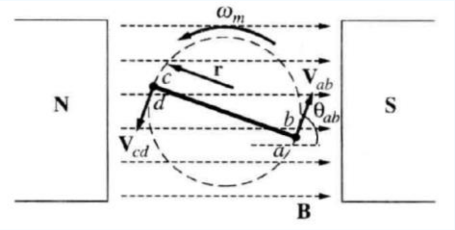
DC motors convert DC electric energy to mechanical energy.

There is no real difference between a generator and a motor except for the direction of power flow.

# AC Machinery Fundamentals

✓ A simple loop in a uniform magnetic field

- Induced voltage $e_{ind} = 2\,vBl\,\sin\theta$
- Induced torque $\tau_{ind} = 2rilB\sin\theta$




✓ The rotating magnetic field $f_e = \dfrac{n_m P}{120}$

$$f_m = \frac{\omega_m}{2\pi} \qquad \theta_e = \frac{P}{2}\theta_m$$

$$n_m = 60\,f_m \qquad f_e = \frac{P}{2}\,f_m$$

$$\omega_m = \frac{2\pi n_m}{60} \qquad \omega_e = \frac{P}{2}\omega_m$$

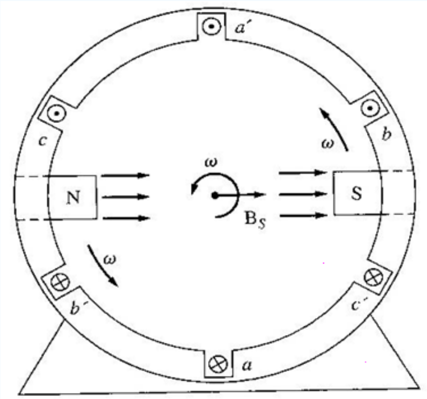
✓ Induced voltage in AC machines

$$\boxed{\begin{aligned} e_{aa'} &= N\Phi\omega_e \sin\omega_e t \\ e_{bb'} &= N\Phi\omega_e \sin(\omega_e t - 120^0) \\ e_{cc'} &= N\Phi\omega_e \sin(\omega_e t - 240^0) \end{aligned}}$$

$$E_{max} = KN\,\Phi\omega_e = N_C\Phi\omega_e$$

✓ Induced torque in AC machines

$$\boxed{\boldsymbol{\tau}_{ind} = k\mathbf{B}_{loop} \times \mathbf{B}_S}$$

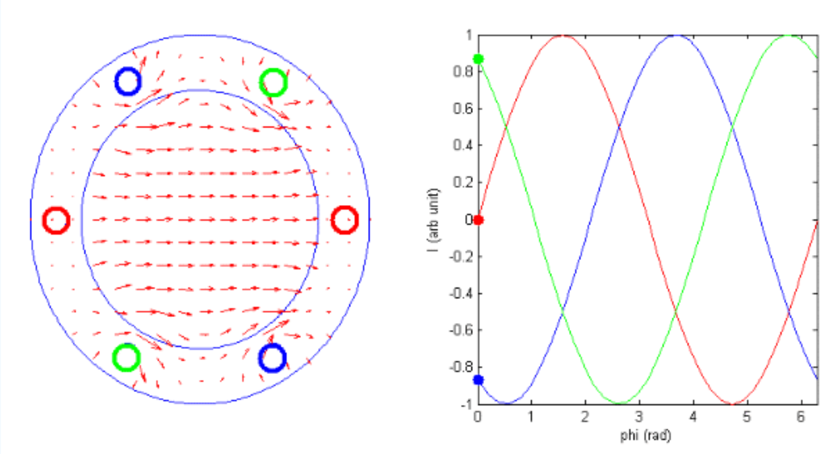



✓ Voltage regulation and speed regulation

# AC Machinery Fundamentals

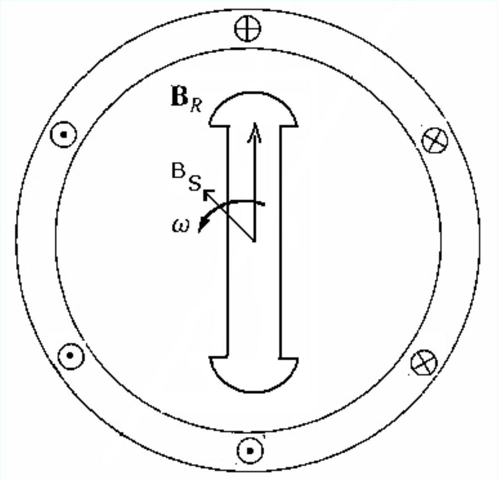
A three-phase set of currents can generate a uniform rotating magnetic field in a machine stator ⟺ A uniform rotating magnetic field can generate a three-phase set of voltage in such a stator

Rotating magnetic field ⟺ Three-phase voltage



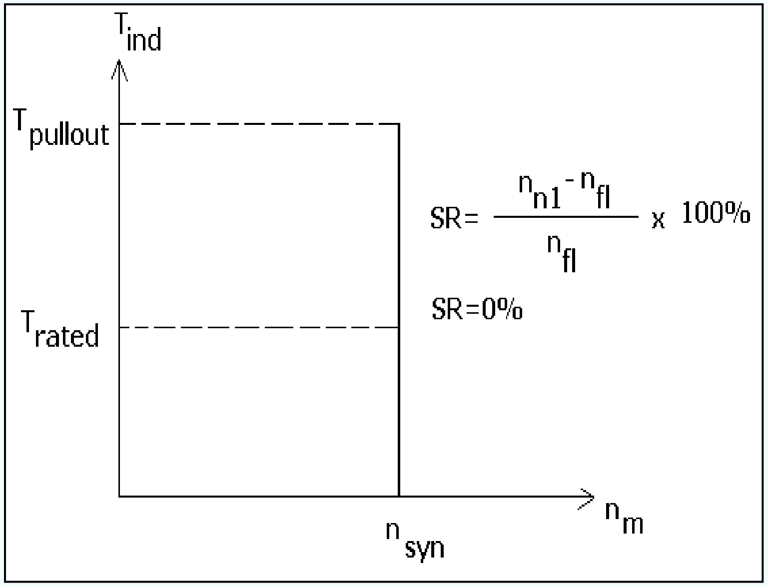
# Synchronous Machines

Magnetic field current is supplied by a separate DC power source




$$\boldsymbol{\tau}_{ind} = k\mathbf{B}_R \times \mathbf{B}_S \text{ counterclockwise}$$

***the rotor field will tend to line up with the stator field***




The speed of rotation of the applied mechanical fields is locked to the applied electrical frequency , so *the speed of the synchronous motor will be constant regardless of the load.*

# Synchronous Machines

$$f_e = \frac{n_m P}{120} \quad \text{or} \quad n_m = \frac{120\, f_e}{P}$$

where is $f_e$ electric frequency, in Hz

$n_m$ : mechanical speed of magnetic field (= rotor speed), in r/min

$P$ : number of poles

| | Rotor speed $n_m$ (r/min) | |
|---|---|---|
| No. of poles ($P$) | 60 Hz | 50 Hz |
| 2 | 3600 | 3000 |
| 4 | 1800 | 1500 |
| 6 | 1200 | 1000 |
| 8 | 900 | 750 |
| 10 | 720 | 600 |
| 12 | 600 | 500 |
| 16 | 450 | 375 |
| 18 | 400 | 333 |
| 20 | 360 | 300 |
| 24 | 300 | 250 |
| 32 | 225 | 188 |
| 40 | 180 | 150 |

# Induction motors

the rotor voltage that produces the rotor current and the rotor magnetic field is **induced in the rotor windings** rather than being physically connected by wires. **No DC field current is required to run the machine.**

synchronous speed ( the speed of rotating magnetic field produced by three-phase current on stator)

$$n_{sync} = \frac{120 f_{se}}{P}$$

Mechanical speed of rotor

**Slip speed**: $n_{slip} = n_{sync} - n_m$

$$n_m = (1-s)\, n_{sync}$$

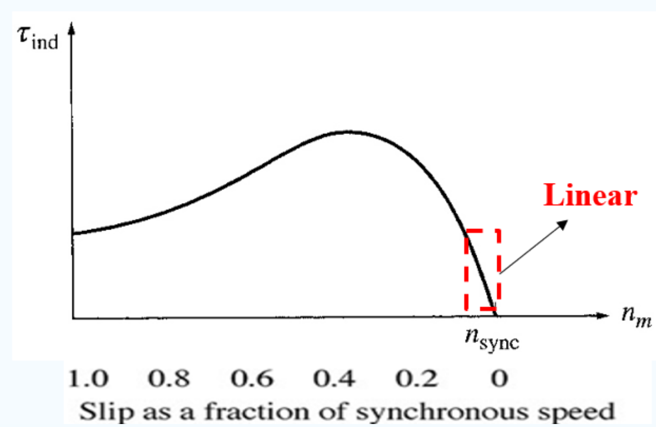
$$\omega_m = (1-s)\, \omega_{sync}$$

**Slip** $s = \dfrac{n_{slip}}{n_{sync}} \times 100\% = \dfrac{n_{sync} - n_m}{n_{sync}} \times 100\%$

rotor frequency $f_{re} = s f_{se}$ System electric frequency

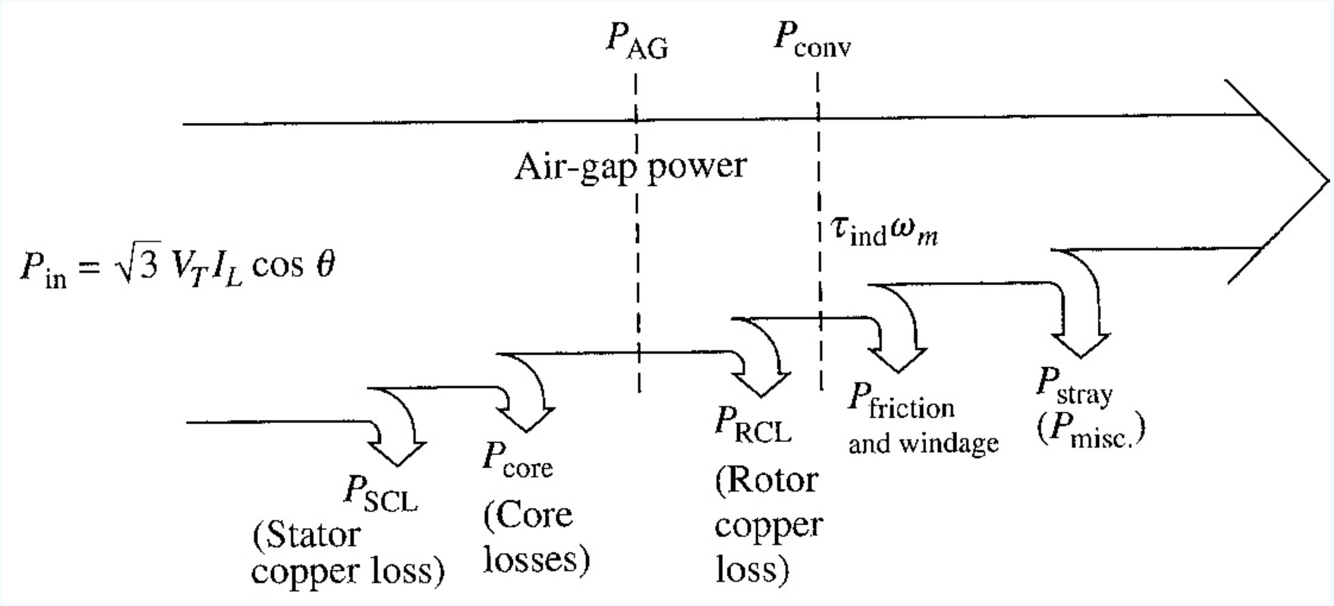
$$f_{re} = \frac{(n_{sync} - n_m)P}{120}$$

$$f_{se} = \frac{n_{sync} P}{120}$$



# Induction Motors-Power flow

The relationship between the input electric power and the output mechanical power of this motor is shown below:



# Suggestions

For revision:

- Independently complete examples/questions from problem sheet and lecture slides.

- Analyze issues using both forward and reverse reasoning.

Office hours in week 13: Thursday 9:00-11:00 and 13:00-15:00
Friday 9:00-11:00 and 13:00-15:00

# Suggestions

For exams:

- Bring your own calculator if needed, only the university approved calculator-Casio FS82ES/83ES is allowed. (**No extra calculator** is provided during the exam)
- Total time allowed: **10 minutes reading time (No writing or annotating is allowed)** + 180 minutes writing time

Quickly scan exam papers and preliminarily judge which questions are easier or more challenging.

- Answer the easier questions first and then focus on tackling the more difficult ones
- Select and write down all relevant equations from the equation list.
- Don’t give up

**Good Luck**

# Next

Review of Electromagnetism

Thanks for your attention