CAN207 Continuous and Discrete Time Signals and Systems

Lecture-4

Introduction to Signals_Part 2

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Room EE322

Content

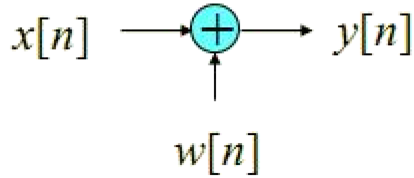
- 1. Introduction
  - signals, signal representation and examples.
- 2. Signal classification (properties)
  - continuity, periodicity, determinacy, symmetry, energy and power.
- 3. Signal operations (time-domain transformation)
  - time shifting, scaling and reversal.
- 4. Elementary signals and sequences
  - unit step, rectangular, signum, ramp, sinusoidal, sinc, exponential and unit impulse functions.

3. Operations

Classification	Elemetary Signals	Operations
• Continuous VS Discrete	• Unit step and rectangular func.	• Elementary operations
• Periodic VS Aperiodic	• Signum and ramp func.	• Time Shifting
• Deterministic VS Random	• Sinusoidal and sinc func.	• Time Scaling
• Symmetric VS Asymmetric	• Real and complex exponential func.	• Time Reversal (folding)
• Energy & Power	• Unit impulse func.	• Combined operations

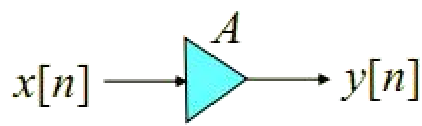
3.1 Operations - Elementary operations

- Addition
  - Adder



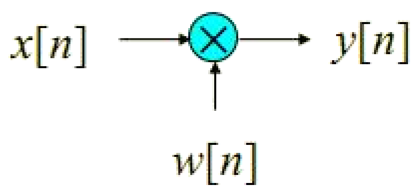
$$y[n] = x[n] + w[n]$$

- Multiplication
  - Multiplier



$$y[n] = A \cdot x[n]$$

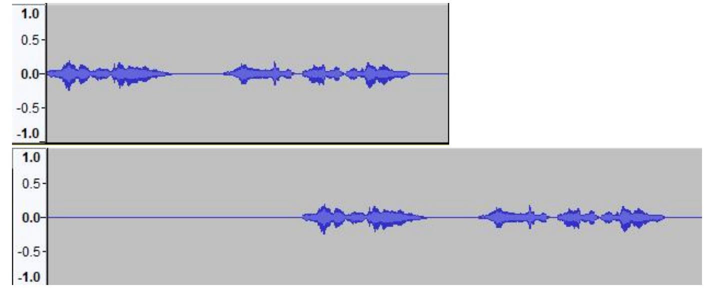
- Production
  - Productor



$$y[n] = x[n] \cdot w[n]$$

3.2 Time Shifting - CT

- Listen to this sound clip:

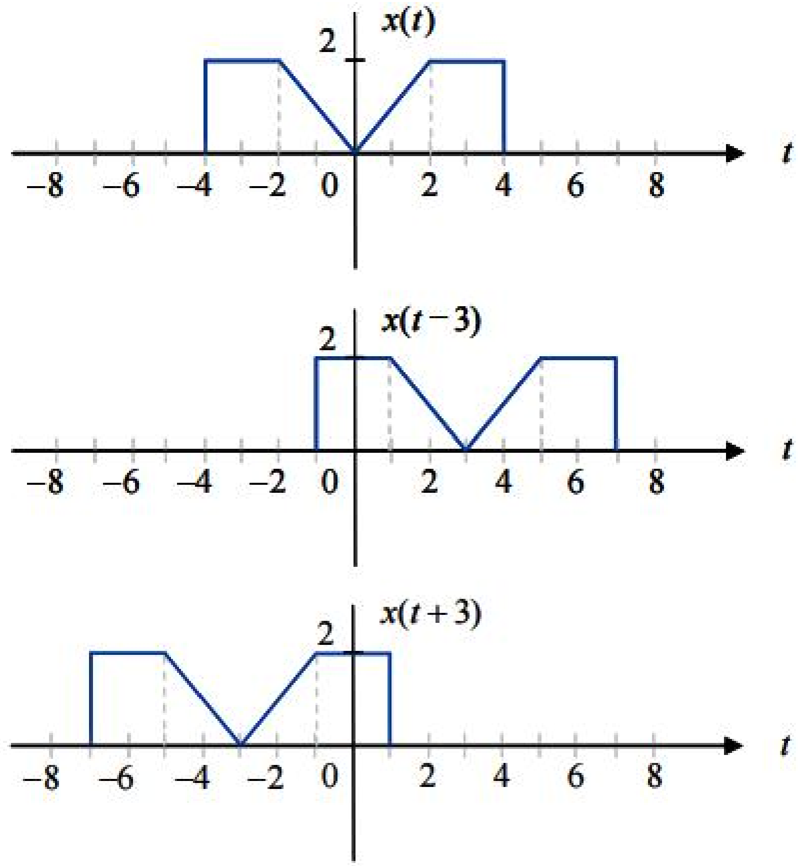


- The time-shifting operation delays or advances forward the input signal in time.

$$\varphi(t) = x(t - T)$$

  – if T > 0, shifted to the right (delayed)

  – if T < 0, shifted to the left (advanced)

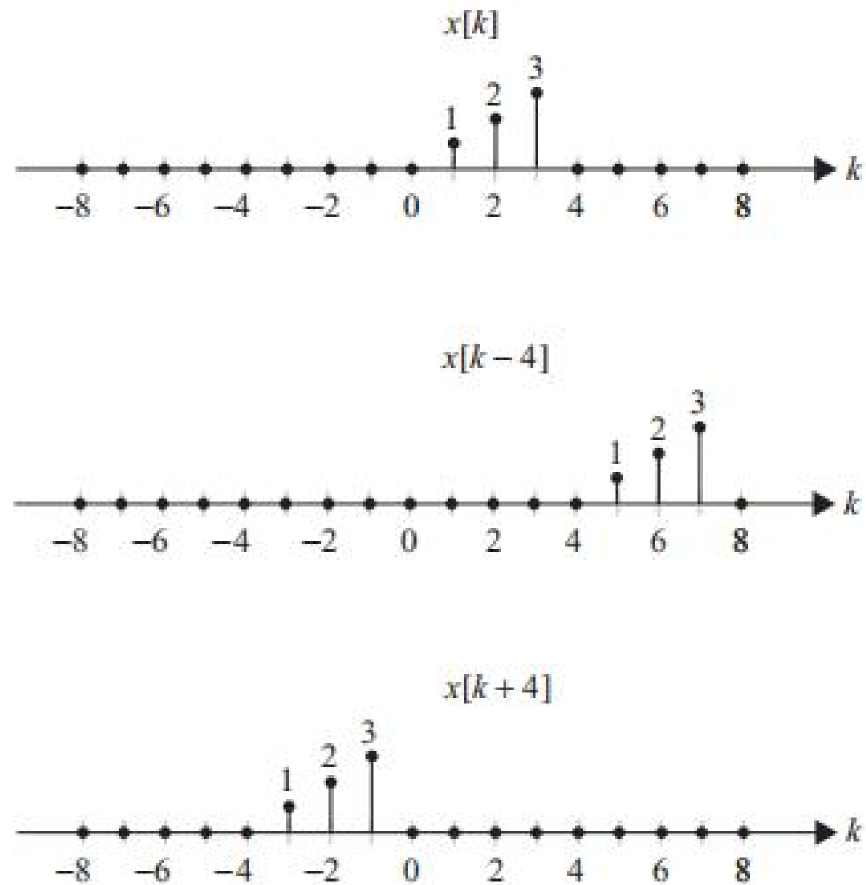


3.2 Time Shifting - DT

- When a DT signal x[k] is shifted by m time units, the delayed signal φ[k] is expressed as:

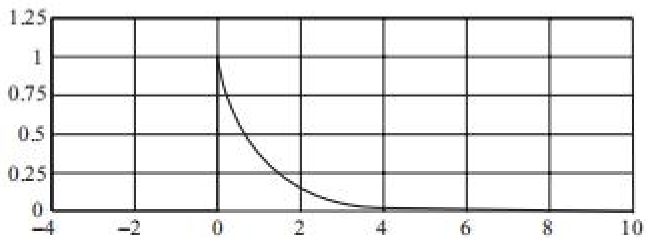
$$\varphi[k] = x[k - M]$$

  - if M > 0, shifted to the right (delayed)
  - if M < 0, shifted to the left (advanced)

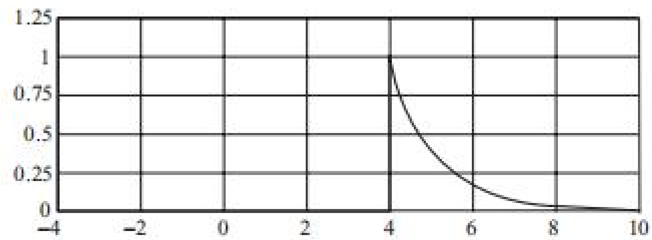


3.2 Time Shifting - Examples

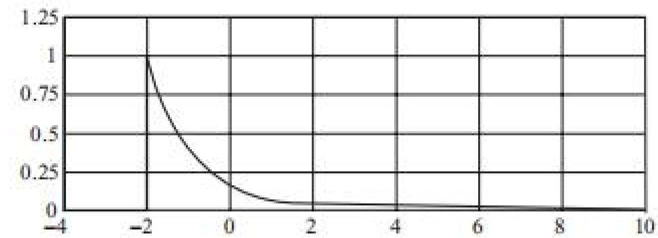
- 1. Consider the signal $x(t) = e^{-t}u(t)$. Determine and plot the time-shifted versions $x(t-4)$ and $x(t+2)$.



$x(t)$

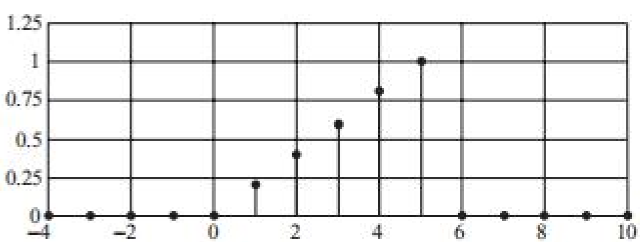


$x(t-4)$

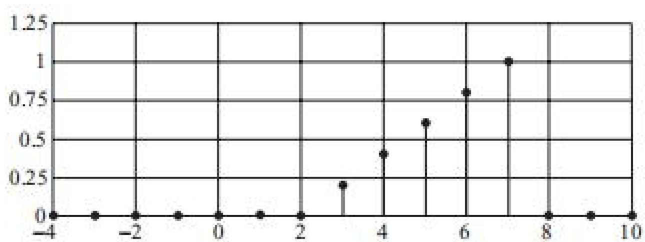


$x(t+2)$

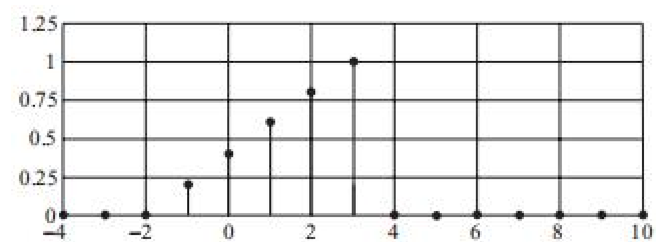
- 2. Consider the signal $x[n] = \begin{cases} 0.2n & 0 \leq n \leq 5 \\ 0 & \text{elsewhere} \end{cases}$. Determine and plot the time-shifted versions $x[n-2]$ and $x[n+2]$.



$x[n]$



$x[n-2]$



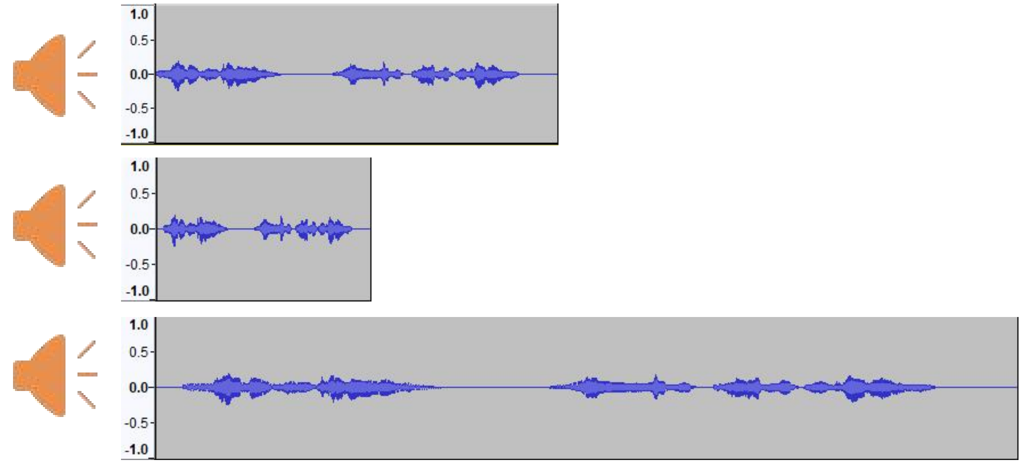
$x[n+2]$

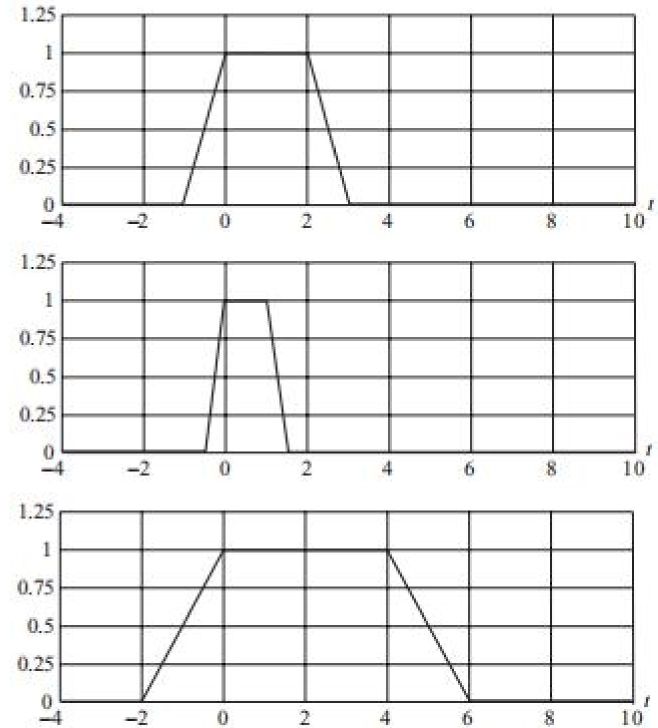
3.3 Time Scaling - CT

- The time-scaling operation compresses or expands the input signal in the time domain.
- A CT signal x(t) scaled by a factor c in the time domain is denoted by

$$\varphi(t) = x(ct)$$

  - if c > 1, the signal is compressed (shorter)
  - if 0 < c < 1, the signal is expanded (longer)





3.3 Time Scaling - DT

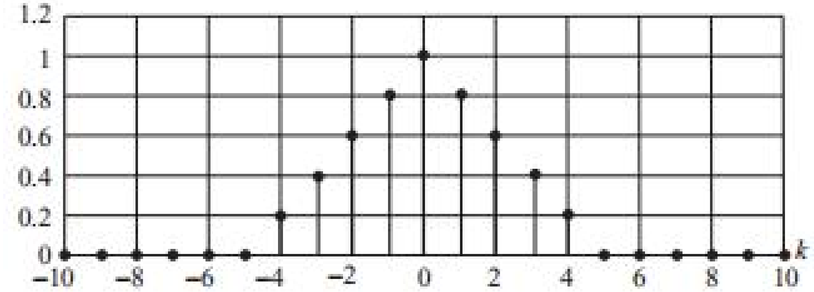
- For DT signal, the compression and expansion has different names:
  - If a sequence x[n] is compressed by a factor c, some data samples of x[n] are lost, this is called decimation.

    $$y[n] = x[Mn]$$

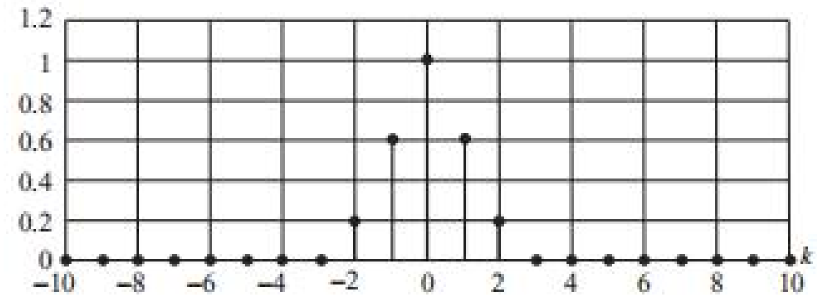
    - y[n] retains only the alternate samples given by x [0], x[M], x[2M], and so on.
  - The expansion (also referred to as interpolation) is defined as

    $$x^{(M)}[n] = \begin{cases} x\left[\frac{n}{M}\right] & \text{if } n \text{ is a multiple of } M \\ 0 & \text{otherwise} \end{cases}$$

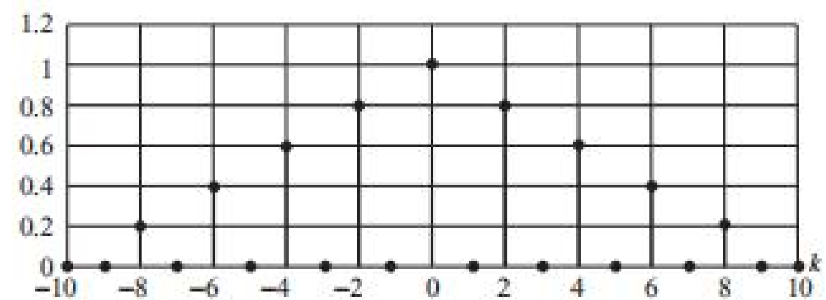
    - The interpolated inserts (M − 1) zeros between adjacent samples of x[n].



$x[n]$



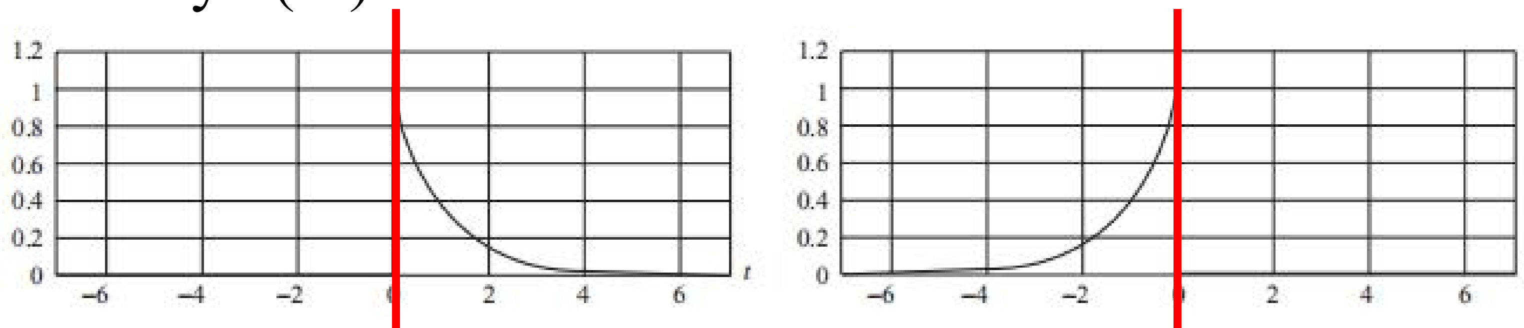
$x[2n]$



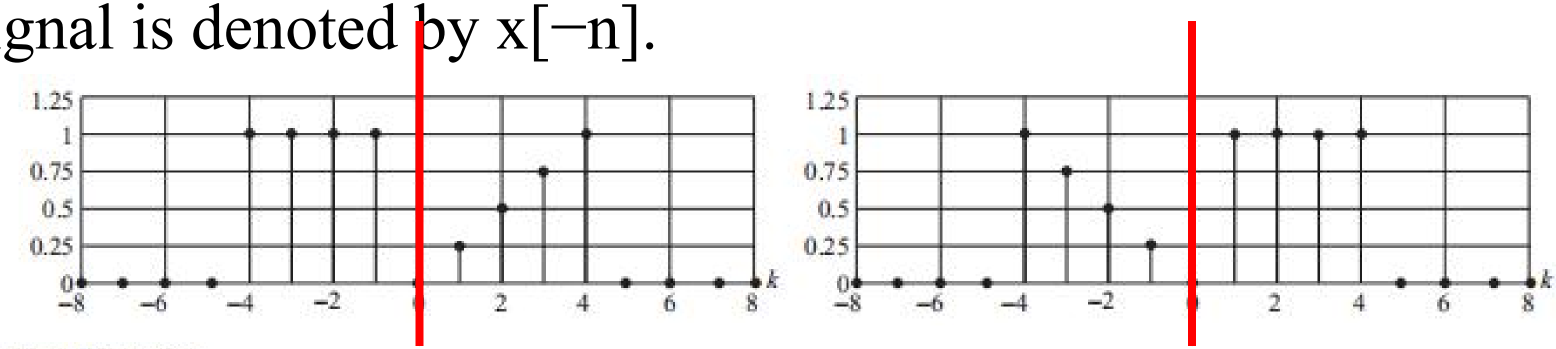
$x[n/2]$

3.4 Time Reversal / Inversion / Fold / Mirror / **Flip**

- The time inversion (also known as time reversal or folding) operation reflects the input signal about the vertical axis.
  - When a CT signal x(t) is time reversed, the inverted signal is denoted by $x(-t)$.



  - Likewise, when a DT signal x[n] is time-reversed, the inverted signal is denoted by $x[-n]$.



3.5 Combined Operations

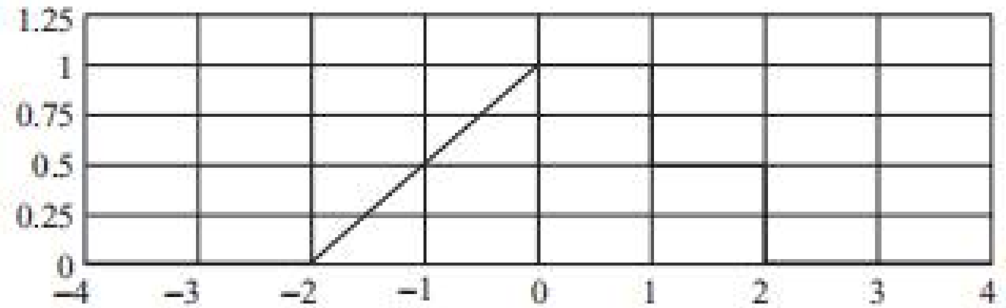
Plot $x(\alpha t + \beta)$ from $x(t)$:

- Express $x(\alpha t + \beta)$ as $x\left(\alpha\left(t + \frac{\beta}{\alpha}\right)\right)$;
- Scale $x(t)$ by $|\alpha|$. The resulting waveform represents $x(|\alpha|t)$;
- If α is negative, invert the scaled signal $x(|\alpha|t)$ with respect to the $n = 0$ axis, which produces the waveform for $x(\alpha t)$;
- Shift the waveform for $x(\alpha t)$ by $\left|\frac{\beta}{\alpha}\right|$ time units (left-hand side if positive, right-hand side otherwise), which will result in the required representation.

3.5 Combined Operations - CT Example

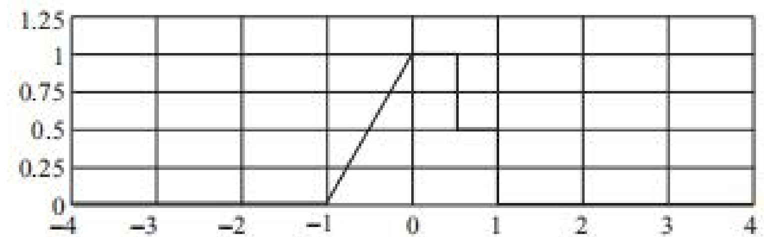
- Determine x(4 − 2t), where the waveform for x(t) is plotted on the right.

$$x(4-2t) = x(-2(t-2))$$



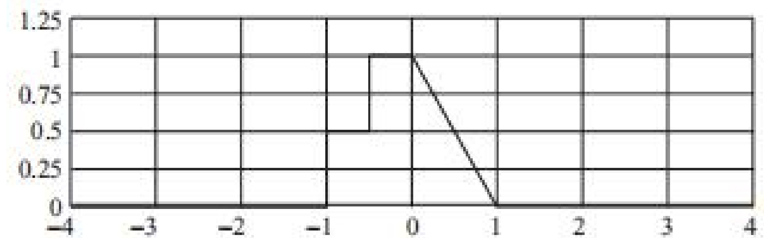
  - Step 1: scale by 2

$$=> x(2t)$$



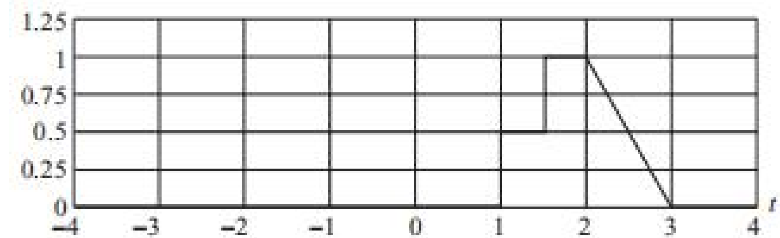
  - Step 2: flip x(2t)

$$=> x(-2t)$$



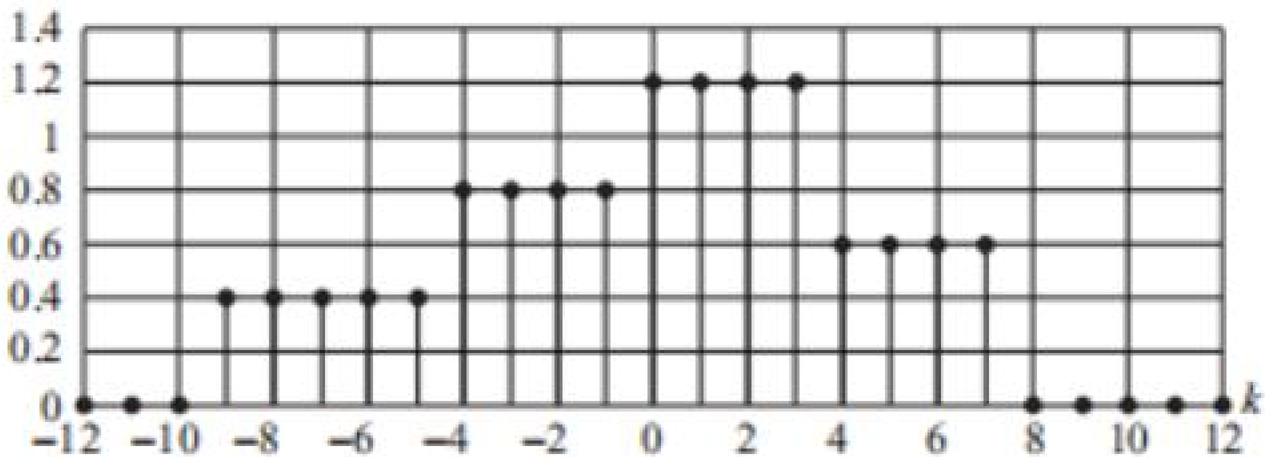
  - Step 3: shift by 2 (to right)

$$=> x(-2(t-2))$$



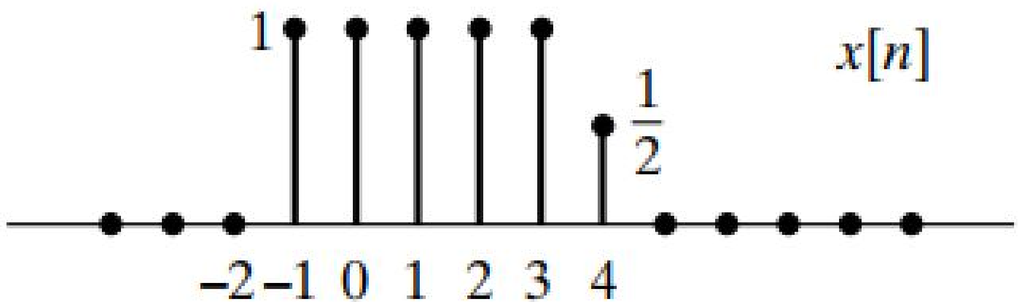
3.5 Combined Operations - DT Example

- Sketch the waveform for x[−15 – 3k] for the DT sequence x[k] plotted below:



Quiz 5

- A DT signal x[n] is shown on the right:
- Sketch and label carefully each of the following signals:
  - a) $x[n-2]$;
  - b) $x[4-n]$;
  - c) $x[2n]$;
  - d) $x[n]u[2-n]$;
  - e) $x[n-1]\delta[n-3]$.



4. Elementary signals

Classification	Elemetary Signals	Operations
• Continuous VS Discrete	• Unit step and rectangular func.	• Elementary operations
• Periodic VS Aperiodic	• Signum and ramp func.	• Time Shifting
• Deterministic VS Random	• Sinusoidal and sinc func.	• Time Scaling
• Symmetric VS Asymmetric	• Real and complex exponential func.	• Time Reversal (folding)
• Energy & Power	• Unit impulse func.	• Combined operations

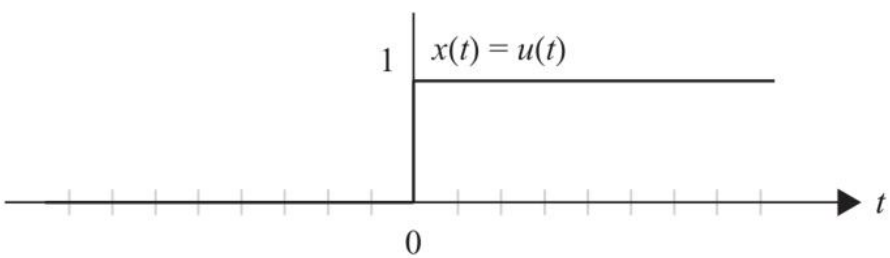
4.1 Unit step function

- The CT unit step function u(t) is defined as follows:

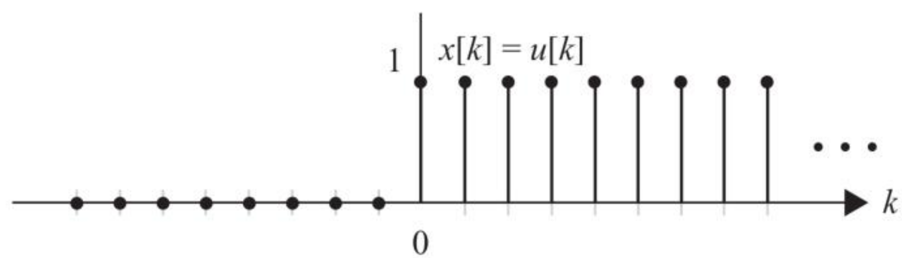
$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

- The DT unit step function u[n] is defined as follows:

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



$u(t)$



$u[n]$

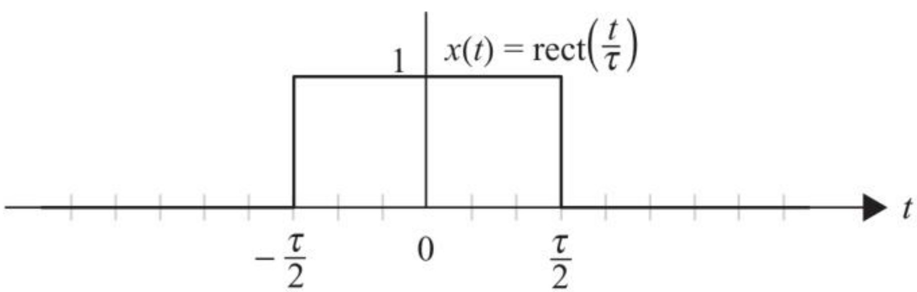
4.2 Rectangular pulse function

- The CT rectangular pulse rect(t/τ) is defined as follows:

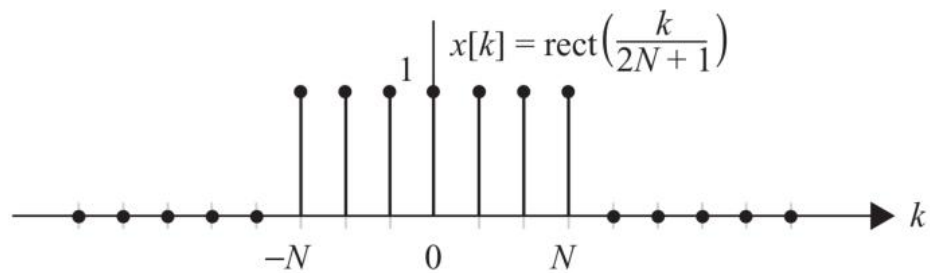
$$\operatorname{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & |t| < \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$$

- The DT rectangular pulse rect(n/(2N + 1)) is defined as follows:

$$\operatorname{rect}\left(\frac{n}{2N+1}\right) = \begin{cases} 1 & |n| \le N \\ 0 & |n| > N \end{cases}$$



$$\operatorname{rect}\left(\frac{t}{\tau}\right)$$



$$\operatorname{rect}\left(\frac{n}{2N+1}\right)$$

4.3 Signum (sign) function

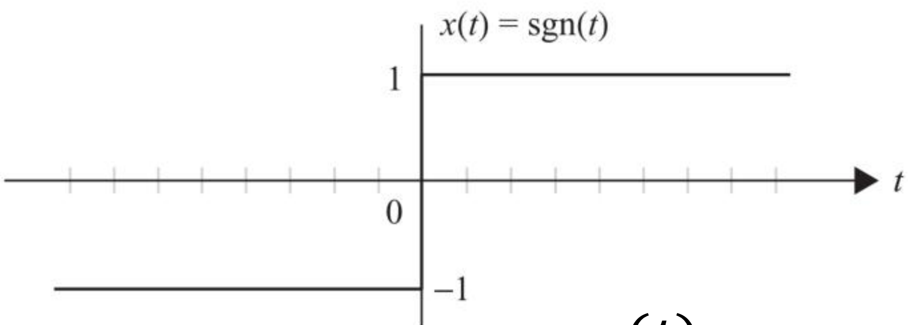
- The signum (or sign) function, denoted by sgn(t), is defined as follows:

$$\operatorname{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

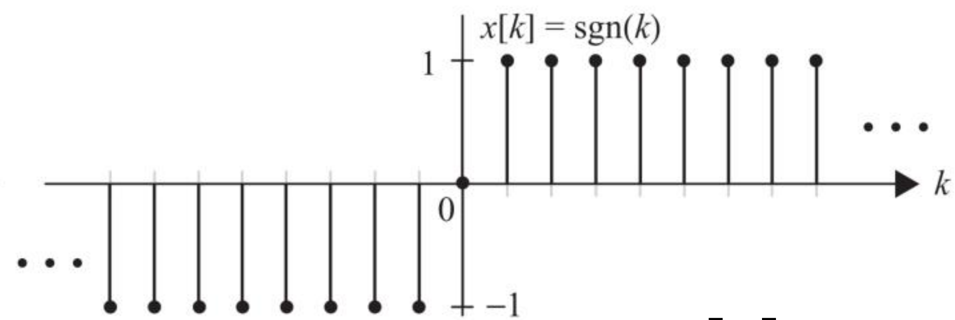
  - Note that the operation sgn(·) can be used to output the sign of the input argument.

- The DT signum function, denoted by sgn(n), is defined as follows:

$$\operatorname{sgn}[\mathrm{n}] = \begin{cases} 1 & n > 0 \\ 0 & n = 0 \\ -1 & n < 0 \end{cases}$$



$\operatorname{sgn}(t)$



$\operatorname{sgn}[n]$

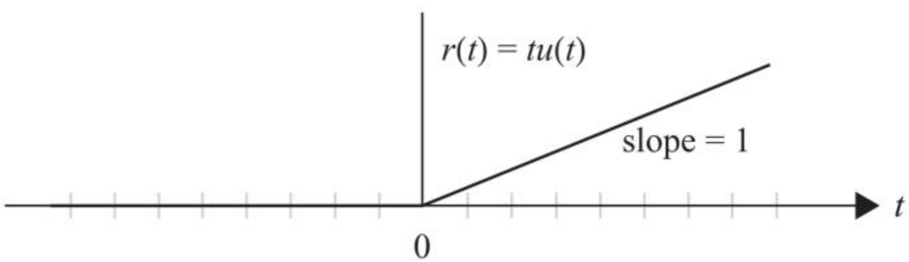
4.4 Ramp function

- The CT ramp function r(t) is defined as follows:

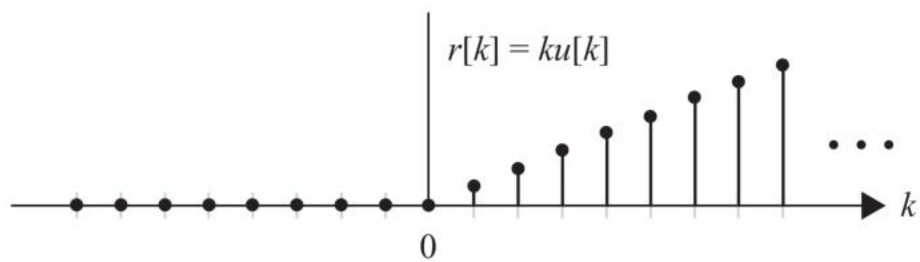
$$r(t) = tu(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- The DT ramp function r[n] is defined as follows:

$$r[n] = nu(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



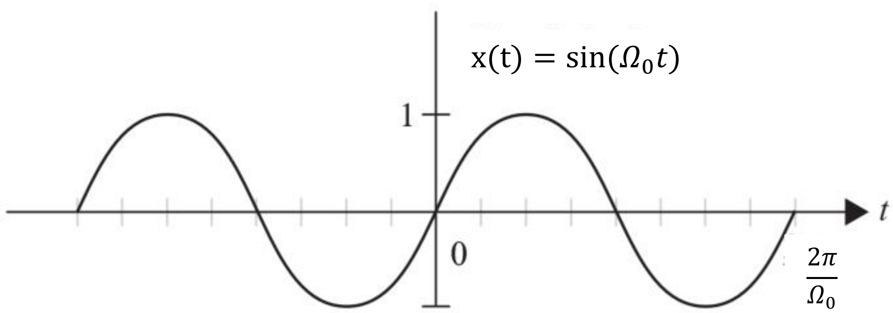
r(t)



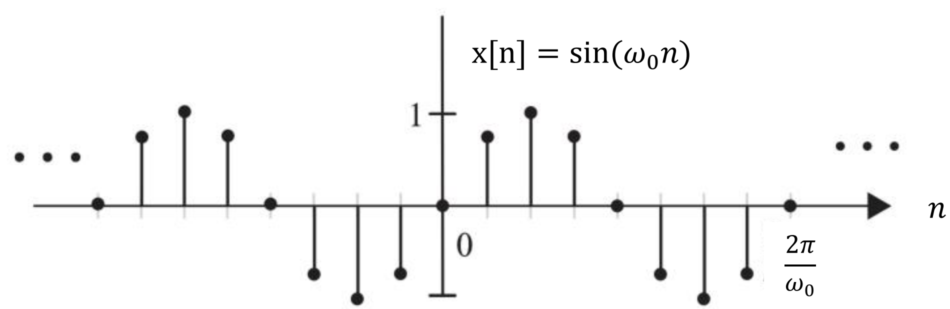
r$[n]$

4.5 Sinusoidal functions

- The CT sinusoid of frequency F_0 (or, equivalently, an angular frequency $\Omega_0 = 2\pi F_0$) is defined as follows:
$$x(t) = A\sin(\Omega_0 t + \theta) = A\sin(2\pi F_0 t + \theta)$$
  – It's always periodic.
- The DT sinusoid is defined as follows:
$$x[n] = A\sin(\omega_0 n + \theta) = A\sin(2\pi f_0 n + \theta)$$
  – It is periodic only if the fraction $\omega_0/2\pi$ is a rational number



$x(t)$



$x[n]$

4.6 Sinc functions

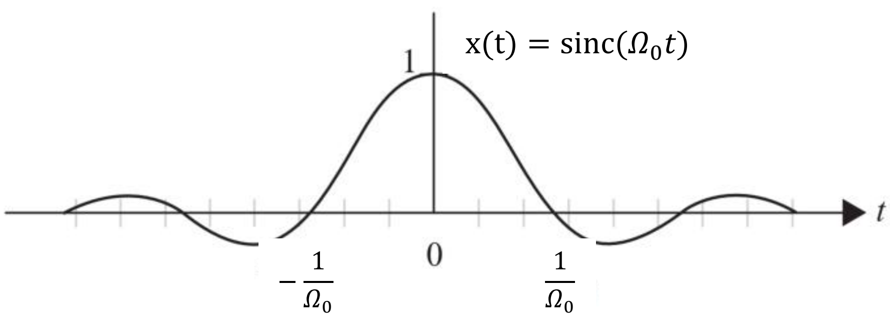
- The CT sinc function is defined as follows:

$$\mathrm{sinc}(\Omega_0 t) = \frac{\sin(\pi\Omega_0 t)}{\pi\Omega_0 t} \quad or \quad \frac{\sin(\Omega_0 t)}{\Omega_0 t}$$

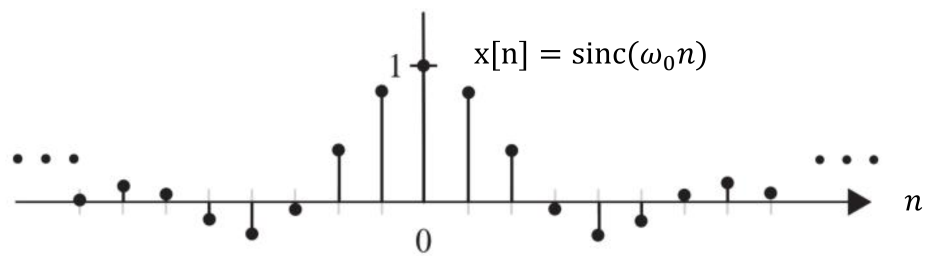
  – In this module, we use the first form of sinc function.

- The DT sinc function is defined as follows:

$$\mathrm{sinc}(\omega_0 n) = \frac{\sin(\pi\omega_0 n)}{\pi\omega_0 n}$$



$\mathrm{sinc}(t)$

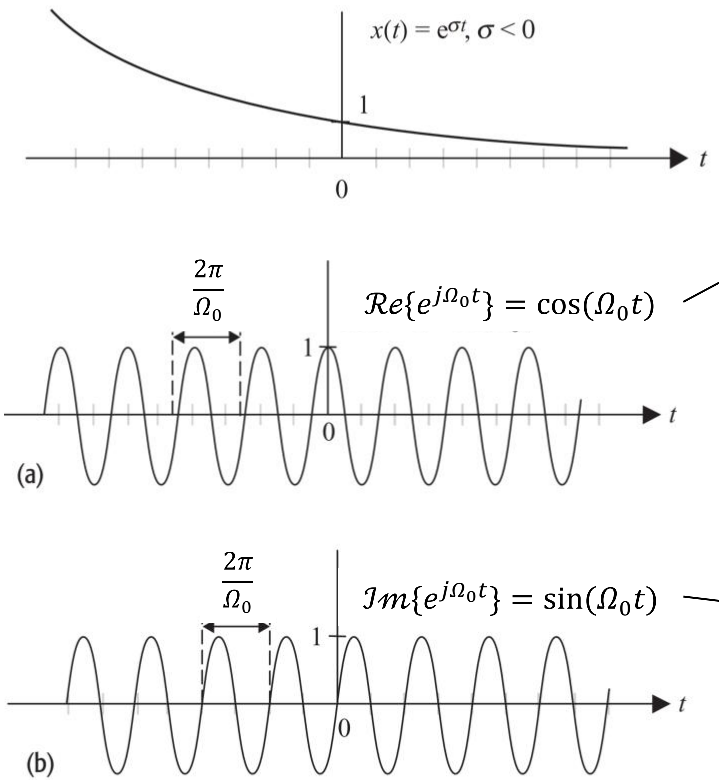


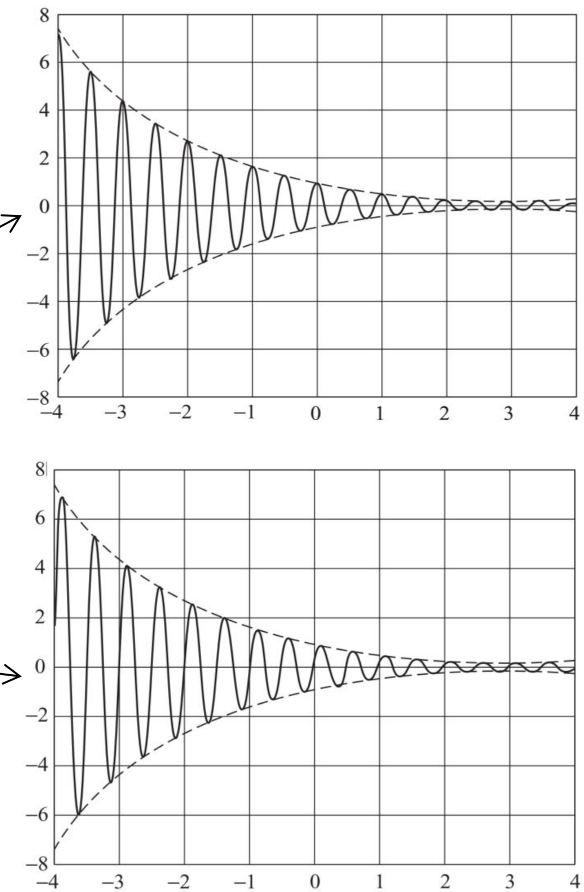
$\mathrm{sinc}[n]$

4.7 Exponential functions - CT

- A CT exponential function, with complex frequency $s = \sigma + j\Omega_0$, is represented by

$$x(t) = e^{st} = e^{(\sigma + j\Omega_0)t} = e^{\sigma t}(\cos \Omega_0 t + j \sin \Omega_0 t)$$





4.7 Exponential functions - DT

- The DT complex exponential function with radian frequency ω_0 is defined as follows:

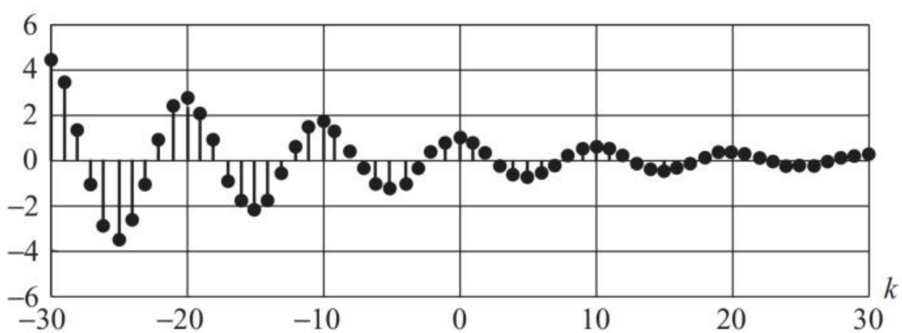
$$x[n] = e^{(\sigma + j\omega_0)n} = e^{\sigma n}(\cos \omega_0 n \,+\, j \sin \omega_0 n)$$

  – It is periodic iff. $\omega_0/2\pi$ is a rational number.

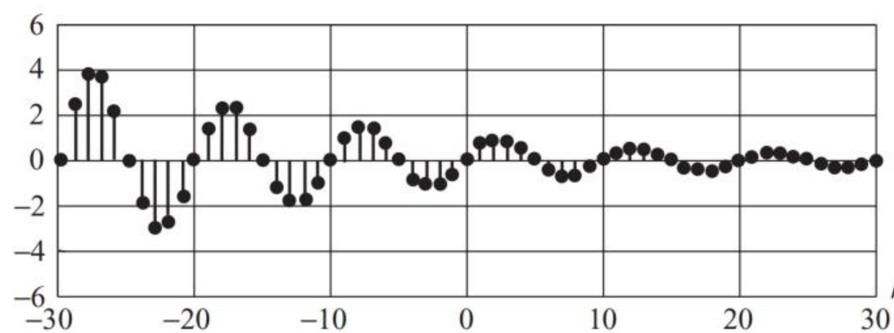
- An alternative representation of the DT complex exponential function is obtained by expanding:

$$x[n] = \left(e^{(\sigma + j\omega_0)}\right)^n = z^n$$

  – where $z = \sigma + j\omega_0$ is a complex number.



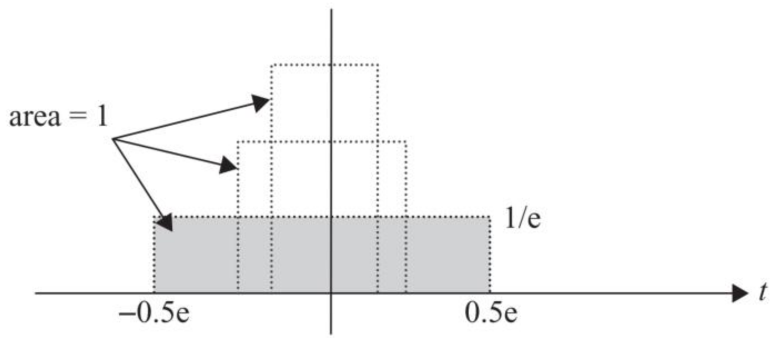
$\mathcal{Re}\{x[n]\}$

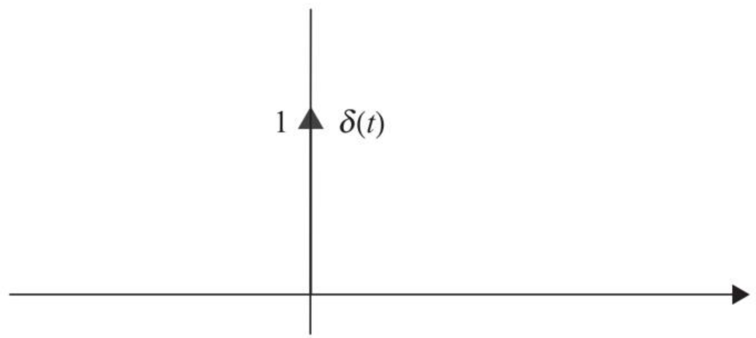


$\mathcal{Im}\{x[n]\}$

4.8 Unit Impulse function - CT

- The unit impulse function δ(t), also known as the Dirac delta function or simply the delta function, is defined in terms of two properties:
  - 1) $\delta(t) = 0, \;\; t \neq 0$
  - 2) $\int_{-\infty}^{\infty} \delta(t)dt = 1$
- Visualization of unit impulse is difficult:
  - 1. A tall narrow rectangle with width *e* and height 1/*e*, such that the area enclosed by the rectangular function equals 1;
  - 2. As the width *e* → 0, the rectangular function converges to δ(t) with an infinite amplitude at t = 0, but its area is still finite and equals 1.





4.8 Unit Impulse function - CT

- Properties of CT unit impulse function:
  - 1. The impulse function is an even function, i.e. $\delta(t) = \delta(-t)$;
  - 2. The product of an arbitrary function $\varphi(t)$ and an impulse function:

$$\varphi(t)\delta(t) = \varphi(0)\delta(t)$$

    - if the impulse function is shifted:

$$\varphi(t)\delta(t-t_0) = \varphi(t_0)\delta(t-t_0)$$

    - therefore:

$$\int_{-\infty}^{\infty} \varphi(t)\delta(t-t_0)dt = \varphi(t_0)\int_{-\infty}^{\infty} \delta(t-t_0)dt = \varphi(t_0)$$

  - 3. Relationship to the unit step function:
    - unit impulse function is the derivative of the unit step function:

$$\delta(t) = \frac{du(t)}{dt}$$

    - unit step function is obtained by integrating the unit impulse function

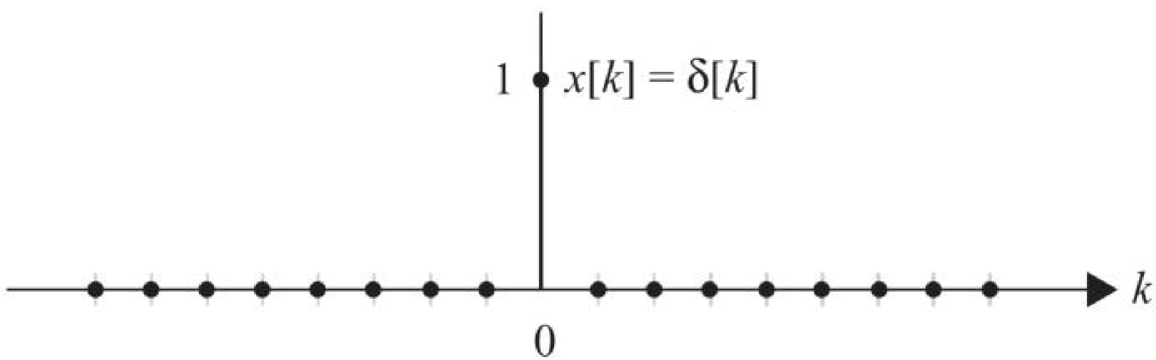
$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

4.8 Unit Impulse function - DT

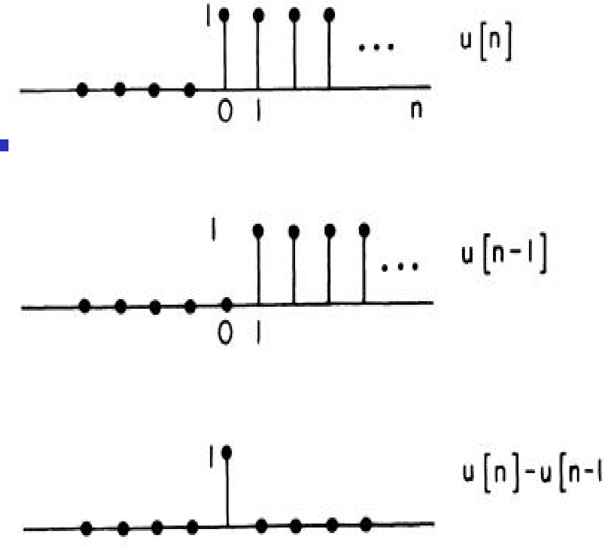
- The DT impulse function, also referred to as the Kronecker delta function, is defined as:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

  - Unlike the CT unit impulse function, the DT impulse function is well defined for all values of n.



4.8 Unit Impulse function - DT



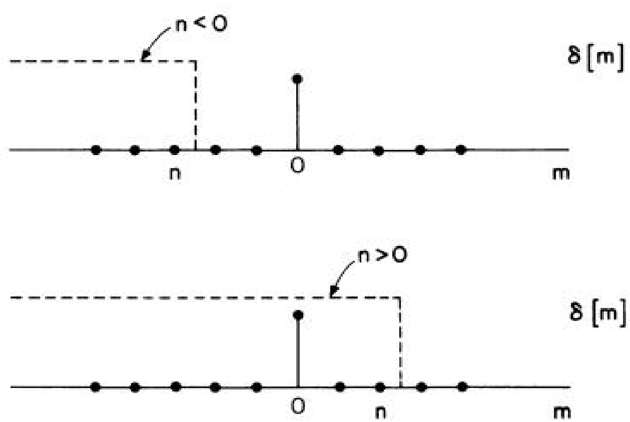
- Relationship to unit step function $u[n]$:
  - 1. unit impulse is the ***first difference*** of the unit step:

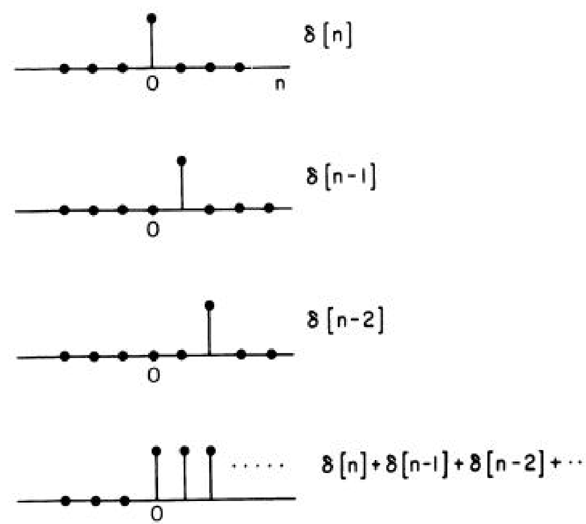
$$\delta[n] = u[n] - u[n-1]$$

  - 2. the unit step is the ***running sum*** of the unit impulse:

$$\boxed{u[n] = \sum_{m=-\infty}^{n} \delta[m].}$$

$$\boxed{u[n] = \sum_{k=0}^{\infty} \delta[n-k].}$$





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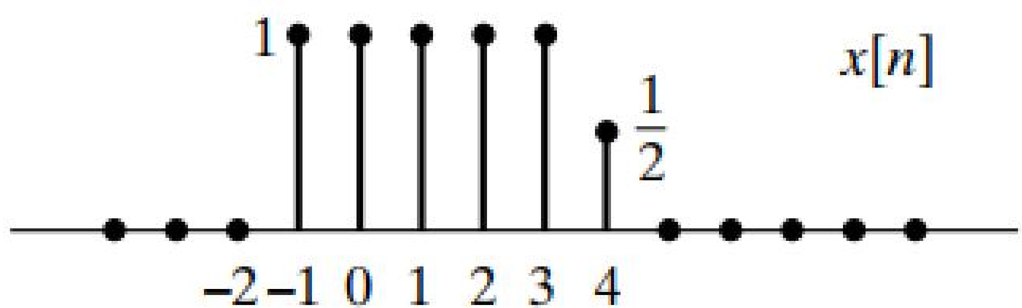
Quiz 6

- 1. Simplify the following expressions:
  - a) $\frac{5-jt}{7+t^2}\delta(t)$;
  - b) $\int_{-\infty}^{\infty}(t+5)\delta(t-2)dt$;

- 2. A DT signal x[n] is shown on the right. Sketch and label carefully each of the following signals:
  - d) x[n]$u[2-n]$;
  - e) x[n − 1]$\delta[n-3]$.



Next ...

- Introduction to Systems
  - What is a system?
  - Classification of systems
  - Interconnection of systems