

EEE104 – Digital Electronics (I)

Lecture 9

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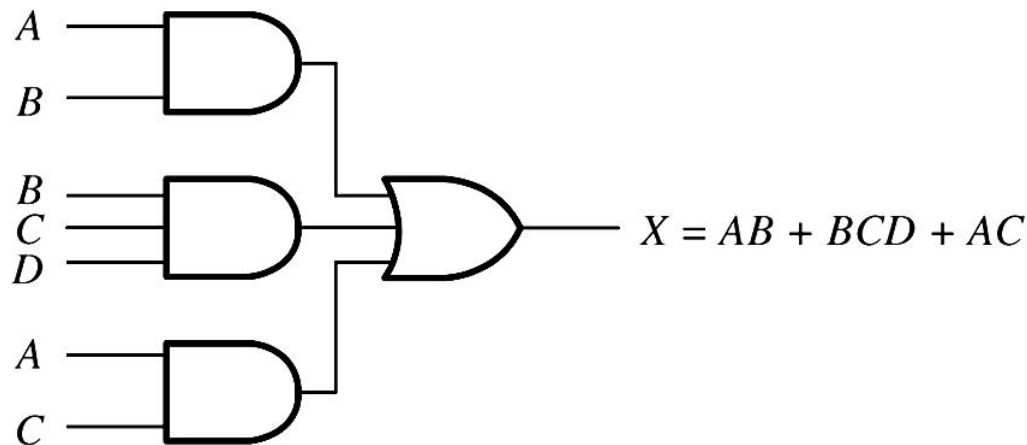
In This Session

- Boolean Algebra and Logic Simplification
 - Standard Form of Boolean Algebra

Standard Form of Boolean Expression

The Sum-of-Products (SOP) Form

- A **sum-of-products (SOP)** is the sum of product terms by Boolean addition, e.g. $AB + \bar{B}C\bar{D}$
- The **domain** of a Boolean expression is the set of variables in the expression, e.g. A, B, C, D.
- An SOP can be implemented by AND-OR logic.



Standard Form of Boolean Expression

The Sum-of-Products (SOP) Form

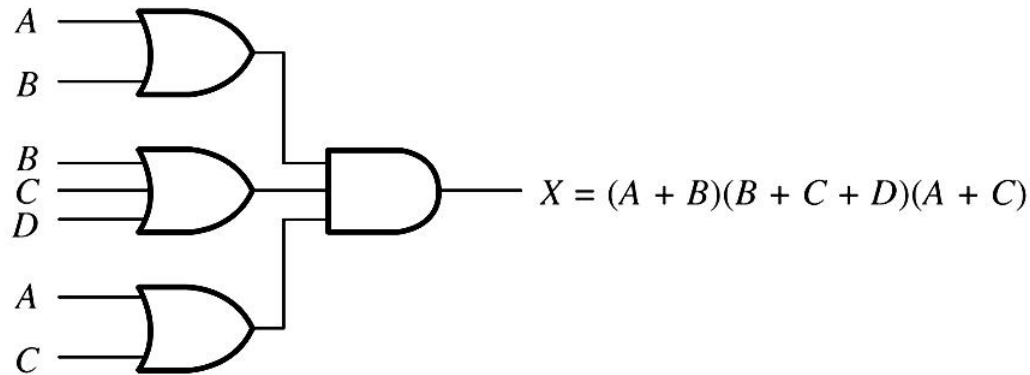
- A **standard SOP** expression is one in which **all** the variables in the domain appear in each product term, e.g. $\overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{D} + A\overline{B}C\overline{D}$
- A general SOP can be converted to standard SOP by repeatedly using Boolean algebra rule 6 ($A + \overline{A} = 1$)

$$\begin{aligned}\overline{A}\overline{B} + A\overline{B}C\overline{D} \\&= \overline{A}\overline{B}(C + \overline{C}) + A\overline{B}C\overline{D} \\&= \overline{A}\overline{B}C(D + \overline{D}) + \overline{A}\overline{B}C\overline{D}(D + \overline{D}) + A\overline{B}C\overline{D} \\&= \overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}C\overline{D} + A\overline{B}C\overline{D}\end{aligned}$$

Standard Form of Boolean Expression

The Product-of-Sums (POS) Form

- A **product-of-sums (POS)** is the product of sum terms by Boolean multiplication, e.g. $(A + B)(\bar{B} + C + \bar{D})$
- A POS expression can be implemented by OR-AND logic.



Standard Form of Boolean Expression

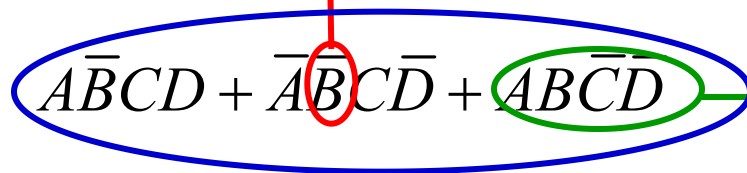
The Product-of-Sums (POS) Form

- A **standard POS** expression is one in which **all** the variables in the domain appear in each sum term, e.g.
 $(A + \bar{B} + C + D)(\bar{A} + \bar{B} + C + \bar{D})(A + B + \bar{C} + \bar{D})$
- A general POS can be converted to standard POS by repeatedly using Boolean algebra rule 8 ($A \cdot \bar{A} = 0$) and rule 12 ($(A + B)(A + C) = A + BC$)

$$\begin{aligned} &(\bar{A} + \bar{B})(A + B + \bar{C}) \\ &= (\bar{A} + \bar{B} + C\bar{C})(A + B + \bar{C}) \\ &= (\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})(A + B + \bar{C}) \end{aligned}$$

Boolean Expressions: A Review

- A **literal** is a variable or its complement.



The diagram shows the Boolean expression $\bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D}$ enclosed in a blue oval. A red circle highlights the \bar{B} literal in the second term, with a red arrow pointing up to the definition of a literal. A green circle highlights the entire third term $A\bar{B}\bar{C}\bar{D}$, with a green arrow pointing right to the definition of a product term. A blue arrow points down from the blue oval to the definition of a sum-of-products (SOP).

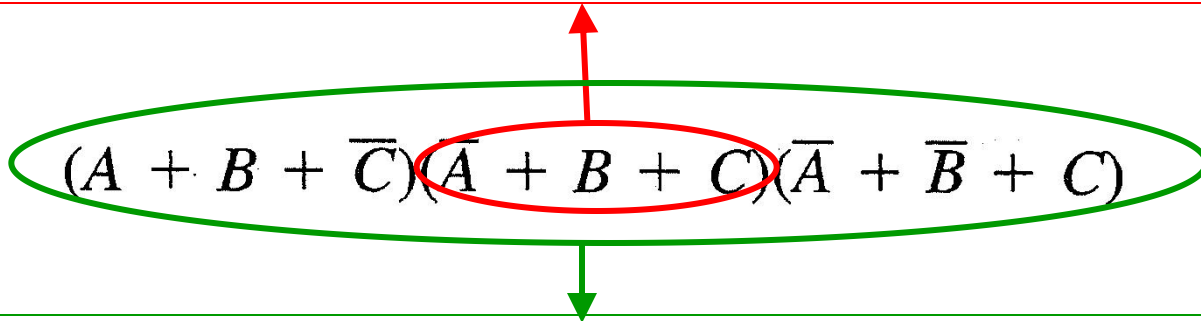
$$\bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D}$$

- A **product term** is the product of literals.

- A **sum-of-products (SOP)** is the sum of product terms.
- A **standard SOP** is one in which all variables appear in each product term.

Boolean Expressions: A Review

- A **sum term** is the sum of literals.



- A **product-of-sums (POS)** is the product of sum terms.
- A **standard POS** is one in which all variables appear in each sum term.

Standard Form of Boolean Expression

Binary Representation

- The **binary value of a standard product term** is one which makes the product term equal to 1, e.g. it is 1010 for $A\bar{B}C\bar{D}$
- The **binary value of a sum term** is one which makes the sum term equal to 0, e.g. it is 0101 for $A + \bar{B} + C + \bar{D}$

Boolean Expressions and Truth Tables

Converting SOP to Truth Table

1. Convert the SOP to standard form.

$$\overline{A}\overline{B}C + A\overline{B}\overline{C} + ABC$$

2. Find the binary values for each product term.

3. Set its output to 1.

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Boolean Expressions and Truth Tables

Converting POS to Truth Table

$$(A + B + C)(A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

1. Convert the POS to standard form.
2. Find the binary value for each sum term.
3. Set its output to 0.

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Boolean Expressions and Truth Tables

Determine SOP expressions from Truth Tables

1. List the binary values of the input variables for which the output is 1.
2. Replace each 1 with the variable and each 0 with the variable complement.

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$$011 \longrightarrow \bar{A}BC$$

$$100 \longrightarrow A\bar{B}\bar{C}$$

$$110 \longrightarrow AB\bar{C}$$

$$111 \longrightarrow ABC$$

$$X = \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

Boolean Expressions and Truth Tables

Determine POS expressions from Truth Tables

1. List the binary values of the input variables for which the output is 0.
2. Replace each 1 with the variable complement and each 0 with the variable.

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$$000 \longrightarrow A + B + C$$

$$001 \longrightarrow A + B + \bar{C}$$

$$010 \longrightarrow A + \bar{B} + C$$

$$101 \longrightarrow \bar{A} + B + \bar{C}$$

$$X = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + \bar{C})$$

Standard Form of Boolean Expression

Converting Standard SOP to Standard POS

$$\overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

- Determine the binary values which represent the product terms, e.g. 000 + 010 + 011 + 101 + 111
- Determine all of the binary values not included, e.g. 001, 100, and 110.
- Write the equivalent sum terms in POS form, e.g.
 $(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$