

# Revision

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Dept. Electrical & Electronic Engineering

# Outline

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- ✓ Bipolar Junction Transistor (Lecture 02)
  - Introduction – Forward-Active Mode Operation
  - CE Current-Voltage Characteristics
  - DC Analysis of Transistor Circuits
  - Small-Signal Hybrid- $\pi$  Equivalent Circuit
  - Small-Signal Voltage Gain
  - Hybrid- $\pi$  Equivalent Circuit, including Early Effect

# Outline

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## ✓ BJT Amplifiers' (Lecture 03)

- Common-Emitter (CE) Amplifiers
- Common-Collector (CC) or Emitter-Follower Amplifier
- Common-Base (CB) Amplifier

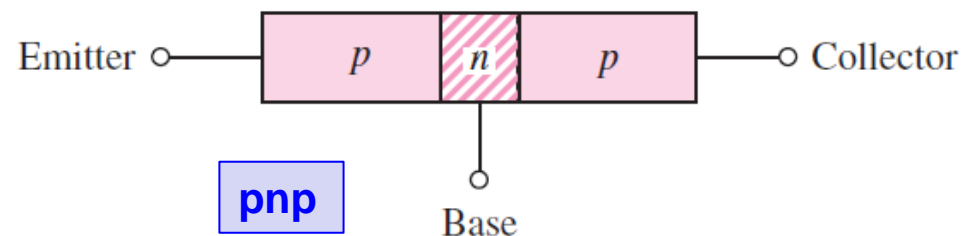
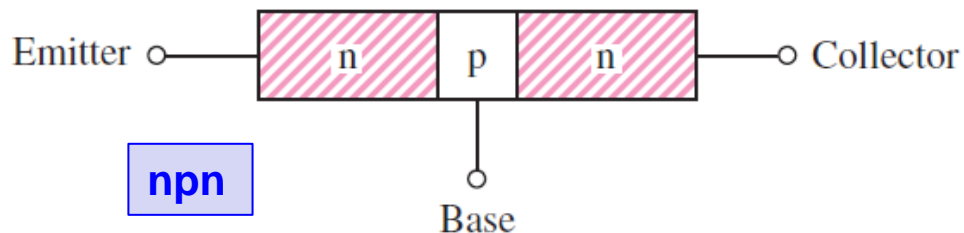
## ✓ Expanded Hybrid- $\pi$ Equivalent Circuit (Lecture 04)

- Short-Circuit Current Gain
- Cutoff Frequency
- Miller Effect and Miller Capacitance

# Basic Bipolar Junction Transistor (BJT)

Transistor principle is that *the voltage between two terminals controls the current through the third terminal.*

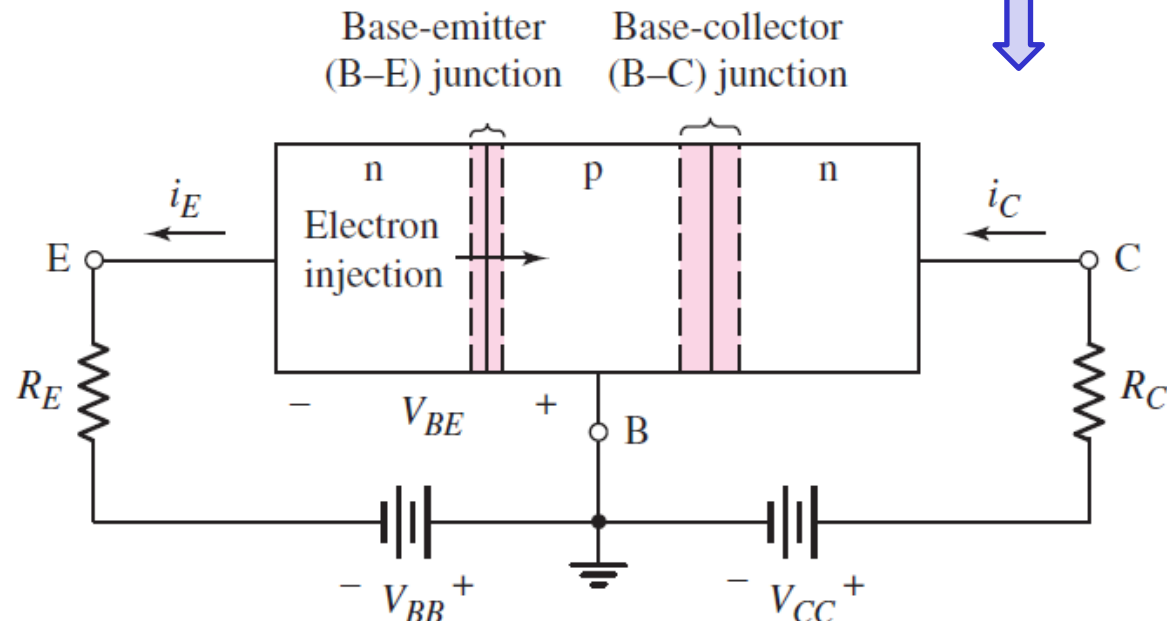
- ✓ BJT has 3 separately doped regions & contains 2 pn junctions. A single pn junction has two modes of operation – forward and reverse biases.
- ✓ Bipolar transistor with 2 pn junctions, therefore has 4 possible modes of operation, which is one reason for the versatility of the device.
- ✓ With 3 separately doped regions, the bipolar transistor is a 3 terminal device. Current in the transistor is due to the flow of both electrons and holes, hence the name *bipolar*.



# Forward-Active Mode Operation

- ✓ Since the transistor has 2 pn junctions, 4 possible bias combinations may be applied to the device, depending on whether a forward or reverse bias is applied to each junction.
- ✓ For example, if the transistor is used as an amplifying device, the **base-emitter (B-E) junction** is forward biased and the **base-collector (B-C) junction** is reverse biased, in a configuration called *forward-active operating mode*, or simply called the *active region*.

Since B-E junction is forward-biased, electrons from the emitter are injected across B-E junction into the base, creating excess minority carrier concentration in base. Since B-C junction is reverse-biased, the electron concentration at the edge of junction is 0.



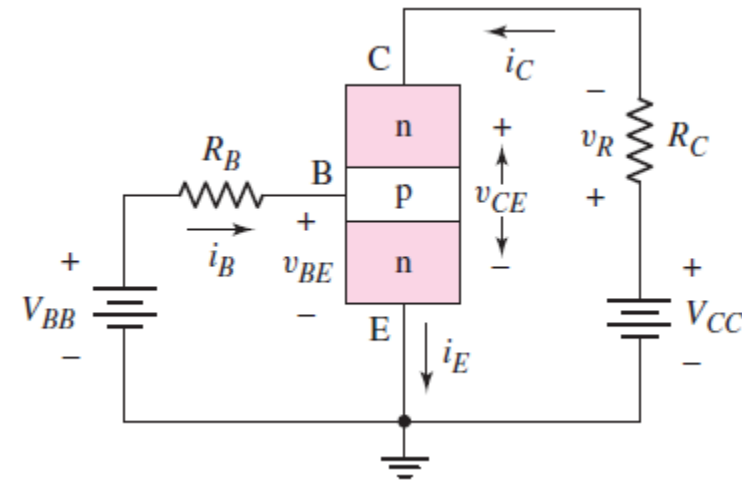
# Forward-Active Mode Operation

## Common-Emitter (CE) Current Gain:

- ✓ If bipolar transistor is considered as a single node, by using KCL, we get,

$$i_E = i_C + i_B$$

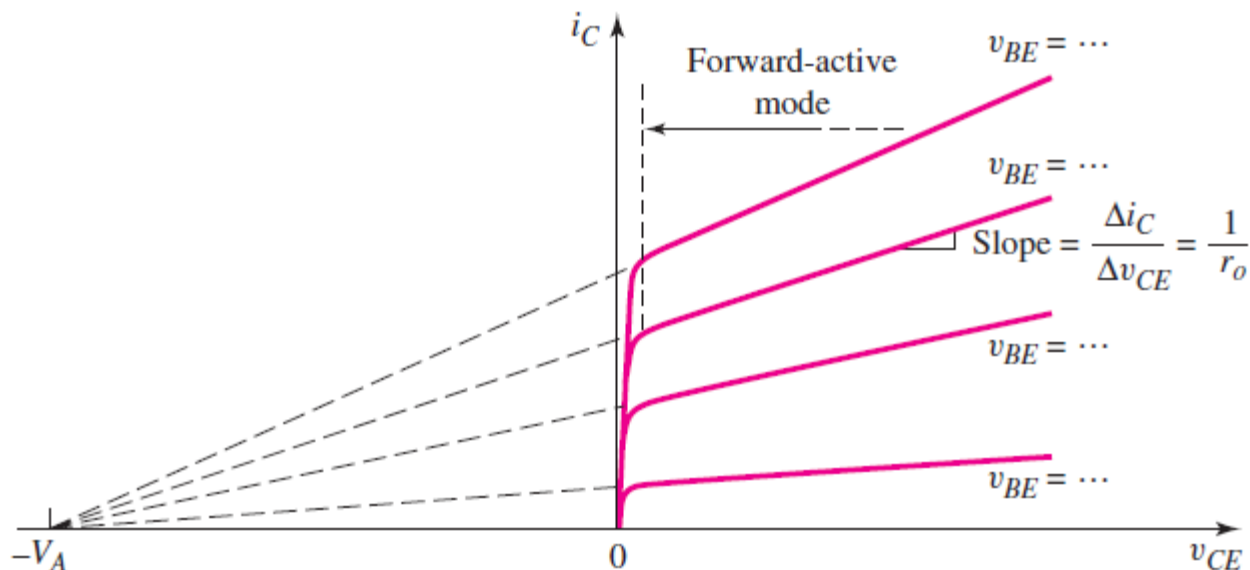
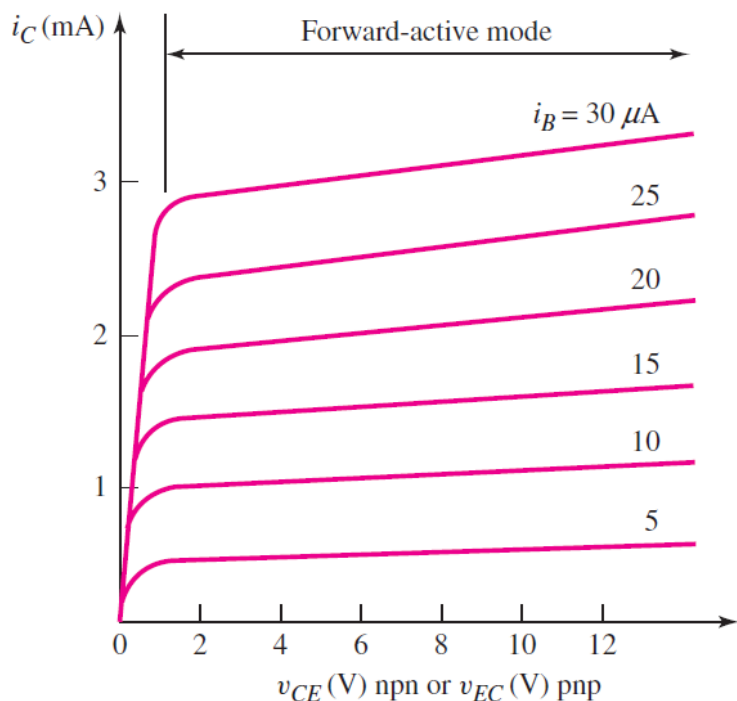
- ✓ If the transistor is biased in the forward-active mode, then  $i_C = \beta i_B$ .
- ✓ From the above equations, we can get the relationship,  $i_E = (1 + \beta)i_B$ .
- ✓ Moreover,  $i_C$  &  $i_E$  are related as,  $i_C = \left(\frac{\beta}{1+\beta}\right) i_E$
- ✓ Recall  $i_C = \alpha i_E \rightarrow \alpha = \frac{\beta}{1+\beta}$



Common-emitter configuration

# CE Circuit Current-Voltage Characteristics

The collector current is plotted against the collector–emitter voltage, for various constant values of the base current.



Exaggerated view plotted for constant values of B-E voltage

The slope in these characteristics is due to an effect called base-width modulation that was first analyzed by J. M. Early – called *Early effect*.

# CE Circuit Current-Voltage Characteristics

- ✓ When the curves are extrapolated to zero current, they meet at a point on the negative voltage axis, at  $v_{CE} = -V_A$ . The voltage  $V_A$  is positive, called *Early voltage*. Typical values are in the range  $50 < V_A < 300 \text{ V}$ .
- ✓ The linear dependence of  $i_C$  vs  $v_{CE}$  in the forward-active mode can be as,  
$$i_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{v_{CE}}{V_A} \right)$$
- ✓ Non-zero slope of the curves indicates that the *output resistance*  $r_o$  looking into the collector is finite. This  $r_o$  is determined from,  $\frac{1}{r_o} = \left. \frac{\partial i_C}{\partial v_{CE}} \right|_{v_{BE}=\text{Const}}$
- ✓ Therefore, we can show that,  $r_o \cong \frac{V_A}{I_C}$

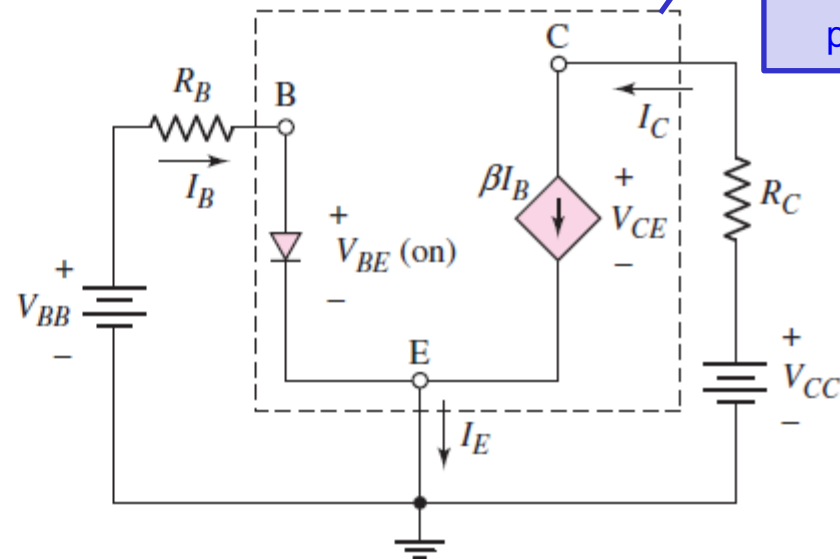
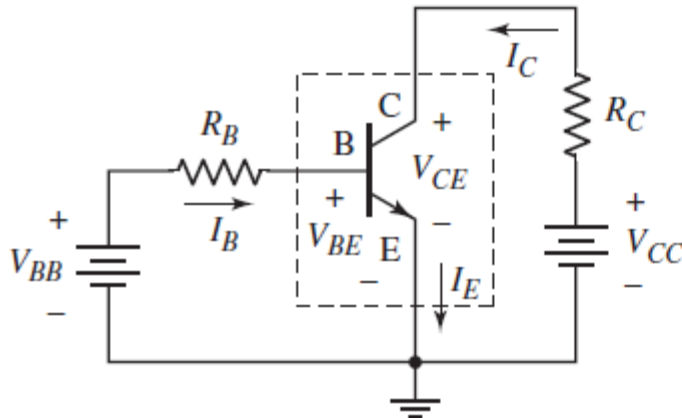
$I_C$  is the quiescent collector current when  $v_{BE}$  is constant and  $v_{CE} < v_A$ .



# DC Analysis of Transistor Circuits

- ✓ Now analyze and design the dc biasing of transistor circuits – an important part of designing bipolar amplifiers which is the focus of later syllabus.
- ✓ The *piecewise linear model of a pn junction* can be used for the dc analysis of bipolar transistor circuit.
- ✓ One of the basic transistor circuit configuration is *common-emitter (CE) circuit* – the *emitter terminal* is obviously at *ground potential*.

CE circuit with npn transistor



Piecewise linear transistor parameters

DC equivalent circuit

# DC Analysis of Transistor Circuits

- ✓ Assume that the B-E junction is forward-biased so that the voltage drop across that junction is cut-in or turn-on voltage  $V_{BE(on)}$ .
- ✓ When the transistor is biased in forward-active mode, the *collector current* is represented as a *dependent current source, function of base current*.
- ✓ Neglect the reverse-biased junction leakage current and Early effect.

The base current is,  $I_B = \frac{V_{BB} - V_{BE(on)}}{R_B}$

It is implicit that  $V_{BB} > V_{BE(on)} \rightarrow I_B > 0$ . If  $V_{BB} < V_{BE(on)}$ , transistor is cut off &  $I_B = 0$ .

In the collector-emitter portion, we can write

$$I_C = \beta I_B, \text{ and } V_{CC} = I_C R_C + V_{CE} \quad (\text{or}) \quad V_{CE} = V_{CC} - I_C R_C$$

It is also implicit that  $V_{CE} > V_{BE(on)}$ , which means B-C junction is reverse biased and the transistor is biased in the forward-active mode.



# Hybrid- $\pi$ Equivalent Circuit

We can now *develop an equivalent circuit model* of BJT – voltage controlled current source and explicitly includes input resistance looking into base ( $r_\pi$ ).

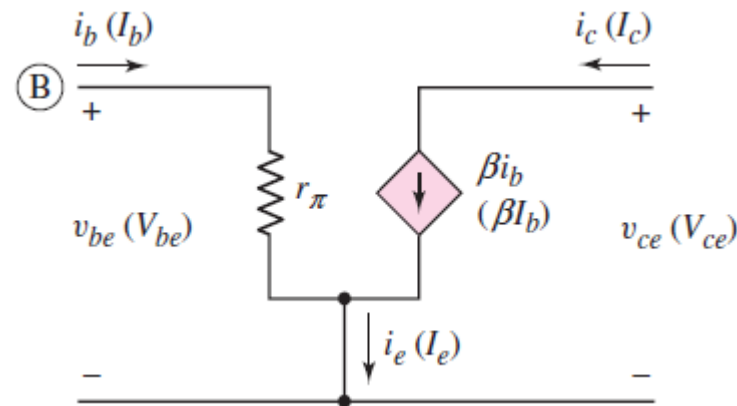
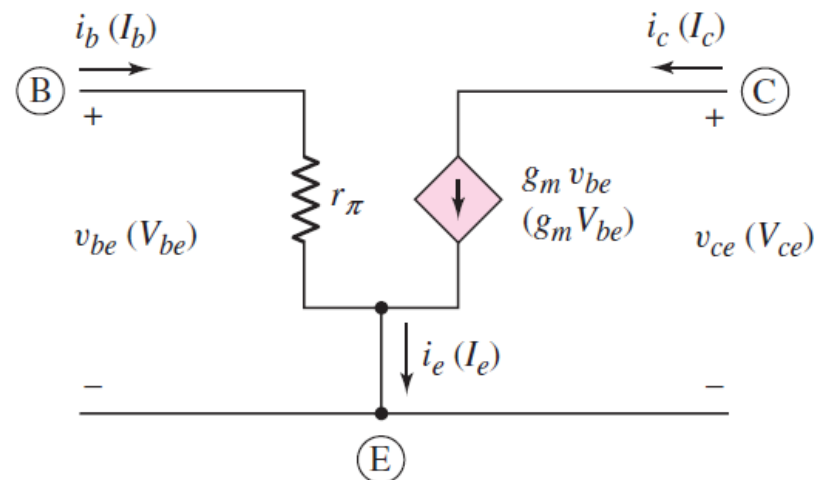
At emitter node, we have

$$i_e = \frac{v_{be}}{r_\pi} + g_m v_{be} = \frac{v_{be}}{r_\pi} (1 + g_m r_\pi) = \frac{v_{be}}{r_\pi} (1 + \beta)$$

We can relate small-signal collector current to the small-signal base current as,

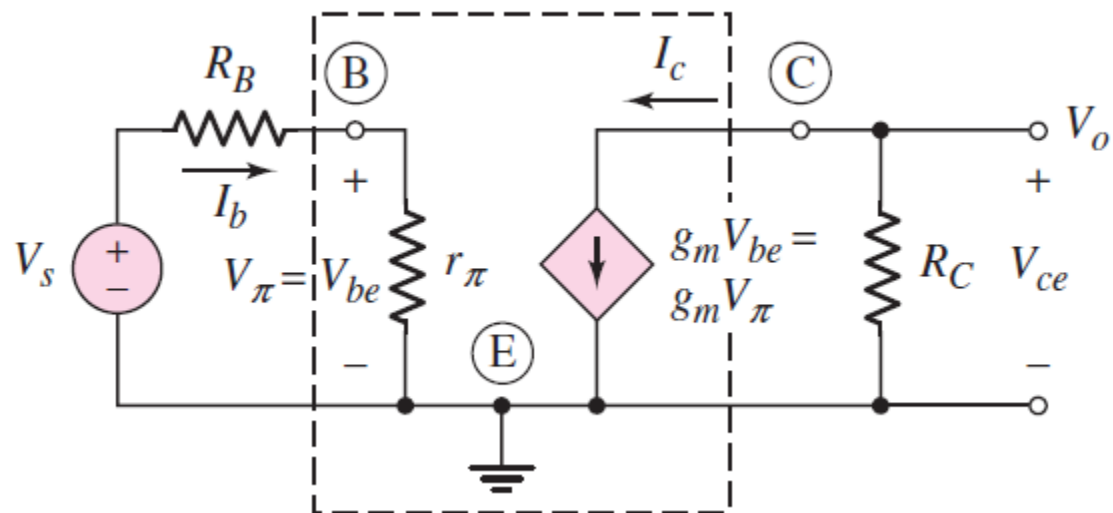
$$i_c = g_m v_{be} = g_m (i_b r_\pi) = (g_m r_\pi) i_b = \beta i_b$$

Consider,  $g_m r_\pi = \frac{I_{CQ}}{V_T} \times \frac{\beta V_T}{I_{CQ}} = \beta$



# Small-Signal Voltage Gain

We can now *incorporate the small-signal hybrid- $\pi$*  model into the ac equivalent circuit – replace the equivalent model of the transistor.



Small-signal voltage gain

$$A_v = \frac{V_o}{V_s} = \frac{V_{ce}}{V_s} = \frac{-(g_m V_\pi) R_C}{V_\pi / \left( \frac{r_\pi}{r_\pi + R_B} \right)} = -(g_m R_C) \left( \frac{r_\pi}{r_\pi + R_B} \right)$$

Note that,  $V_\pi = \left( \frac{r_\pi}{r_\pi + R_B} \right) V_s$  . . .

Voltage  
divider rule

# Problem-Solving Technique

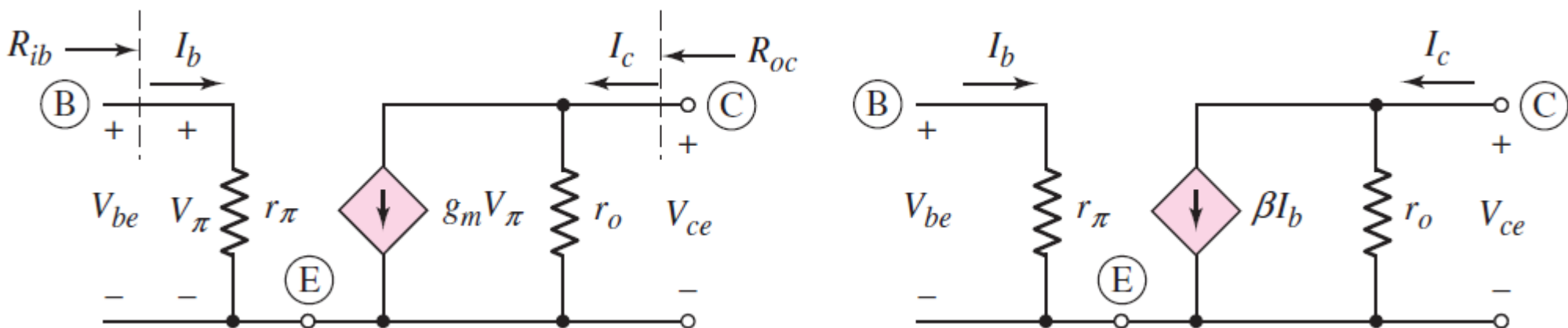
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Since we are dealing with linear amplifier circuits, superposition applies, which means that we can perform the *dc* and *ac* analyses separately. The procedure is as follows:

1. Analyze the circuit with *only the dc sources present*. This solution is the dc or quiescent solution, which uses the dc signal models. The transistor must be biased in the forward-active region in order to produce linear amplifier.
2. Replace each element in the circuit with its *small-signal model*. The small-signal model *hybrid- $\pi$*  applies to the transistor.
3. Analyze the small-signal equivalent circuit, *setting the dc source components equal to zero*, to produce the response of the circuit to the time-varying input signals only.

# Hybrid- $\pi$ Equivalent Circuit with Early Effect

So far we have assumed that the collector current is independent of the collector-emitter voltage in the small-signal equivalent circuit. Recall Early Effect, where the collector current varies with collector-emitter voltage.



$i_c = I_S e^{v_{BE}/V_T} \left(1 + \frac{v_{CE}}{V_A}\right)$  &  $r_o \cong \frac{V_A}{I_C}$  is *small-signal transistor output resistance*.

This resistance is an equivalent *Norton resistance*, which means that  $r_o$  is in parallel with the dependent current sources.

Input resistance,  $r_\pi$  (ohms); current gain,  $\beta$  (dimensionless)  
 output resistance,  $r_o$  (ohms); transconductance,  $g_m$  (mhos);

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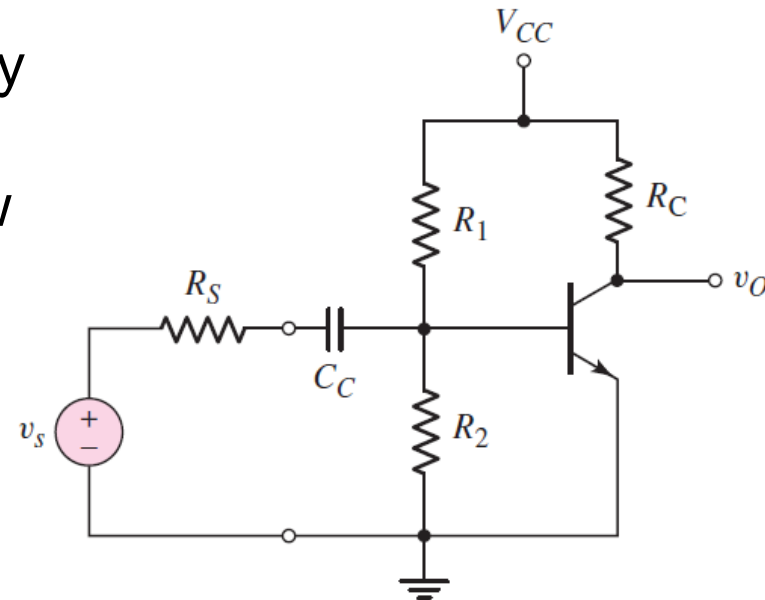
# Basic BJT Amplifiers (Lecture 03)

# Common-Emitter (CE) Amplifier

- ✓ Note that the *emitter is at ground potential* – hence called common-emitter.
- ✓ Signal from the signal source is coupled into the base of the transistor through the coupling capacitor  $C_C$ , which provides *dc isolation* between the amplifier and the signal source.
- ✓ The *DC transistor biasing* is established by  $R_1$  and  $R_2$ , and is not disturbed when the signal source is capacitively coupled to the amplifier.

Assume that the signal frequency is 1) sufficiently high that any *coupling capacitance* acts as a perfect short circuit, and 2) is also sufficiently low that the *transistor capacitances* are neglected.

Neglect any capacitance effects within the transistor.



Basic CE circuit with coupling capacitor

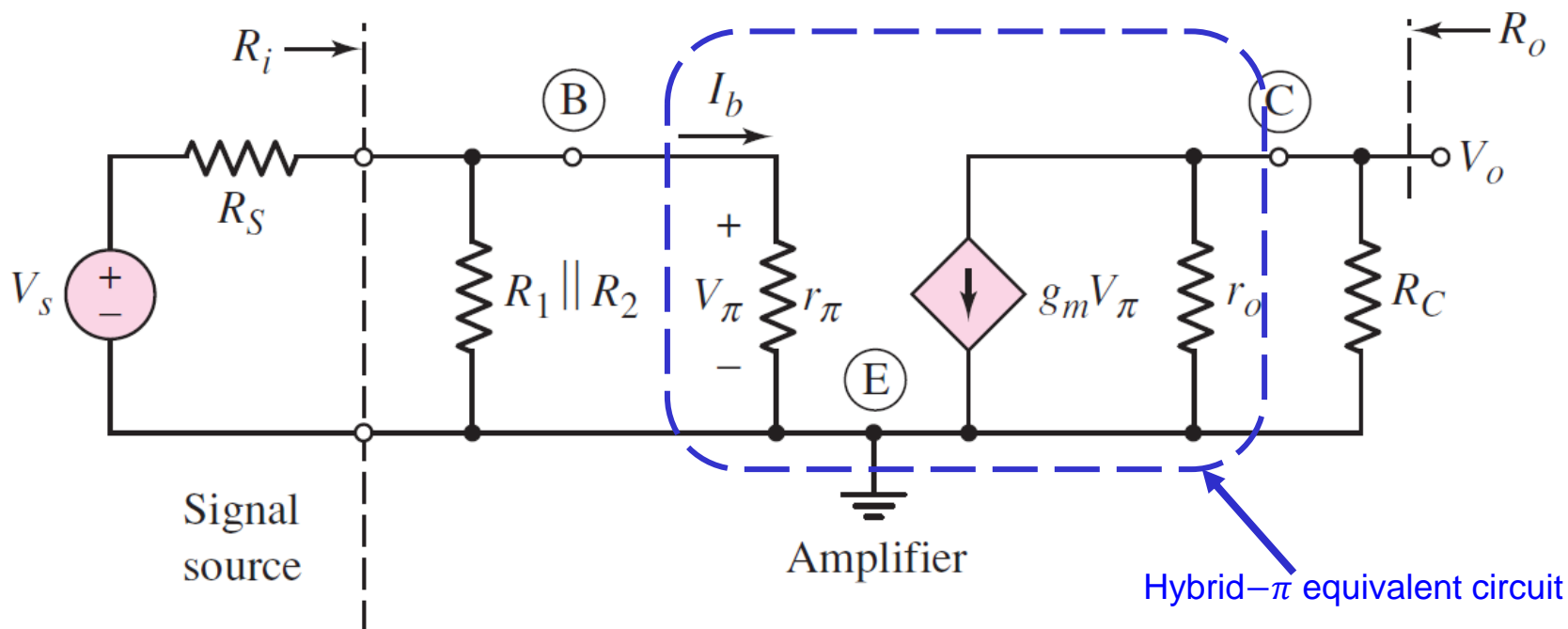


# Common-Emitter – Small-signal Circuit

The output voltage can be written as,  $V_o = -g_m V_\pi (r_o \parallel R_C)$

The control voltage  $V_\pi$  is found to be,  $V_\pi = \frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S} \times V_S$

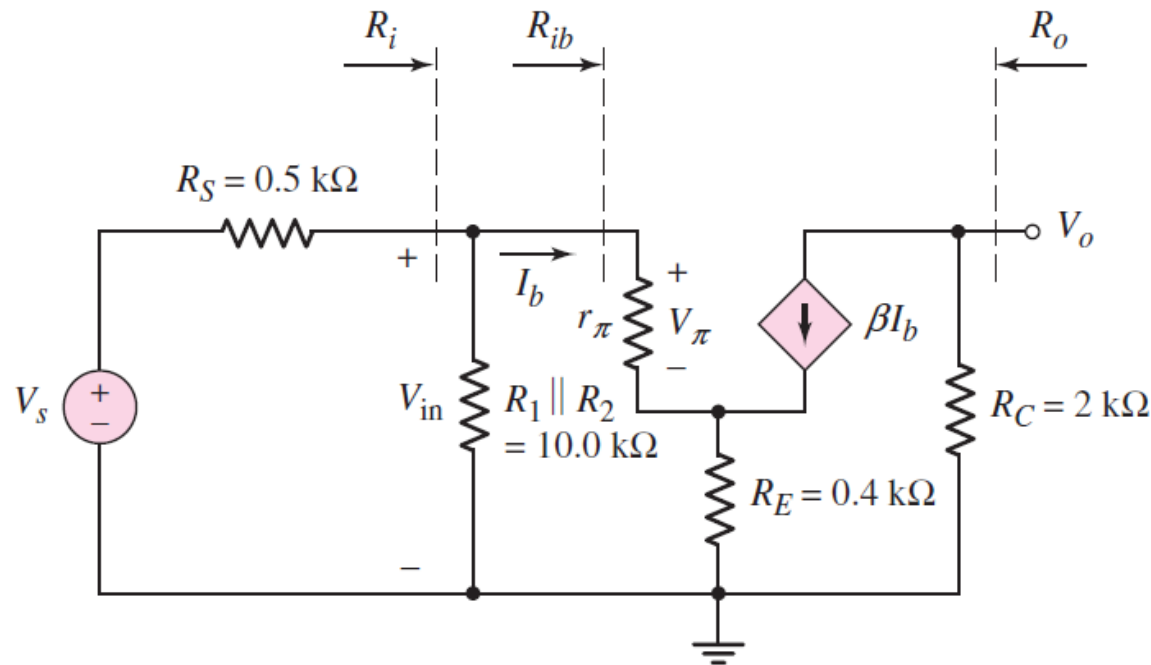
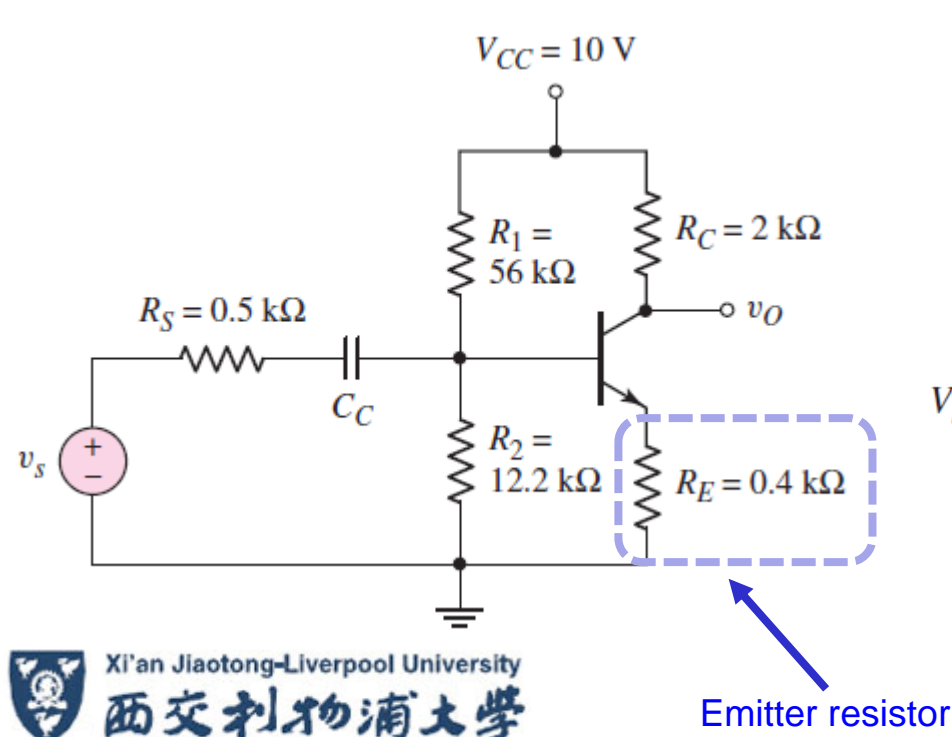
Thus, the small-signal voltage gain is,  $A_v = \frac{V_o}{V_S} = -g_m (r_o \parallel R_C) \frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_S}$



Small-signal equivalent circuit assuming coupling capacitor is a short circuit

# Common-Emitter with Emitter Resistor

- ✓ The earlier CE circuit is not very practical – voltage across  $R_2$  provides the base-emitter voltage to bias the transistor in the forward-active region.
- ✓ A slight variation in the resistor value or in the transistor characteristics may cause the transistor to be biased in cutoff or saturation.
- ✓ *Although the emitter is not at ground potential, it is still called as CE circuit.*



Small-signal equivalent circuit

# Common-Emitter with Emitter Resistor

Note that current gain  $\beta$  is used in the equivalent circuit & assume that Early voltage is infinite so the transistor output resistance ( $r_o$ ) can be neglected.

**Input resistance  $R_{ib}$ :** It is the input resistance looking into the base

Use KVL for the loop,  $V_{in} = I_b r_\pi + (I_b + \beta I_b) R_E \rightarrow R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1 + \beta) R_E$

**Voltage gain  $A_v$ :**

The output voltage is,  $V_o = -(\beta I_b) R_C$

The input resistance to the amplifier,  $R_i = R_1 || R_2 || R_{ib}$

Moreover,  $V_{in} = \left( \frac{R_i}{R_i + R_S} \right) V_S$

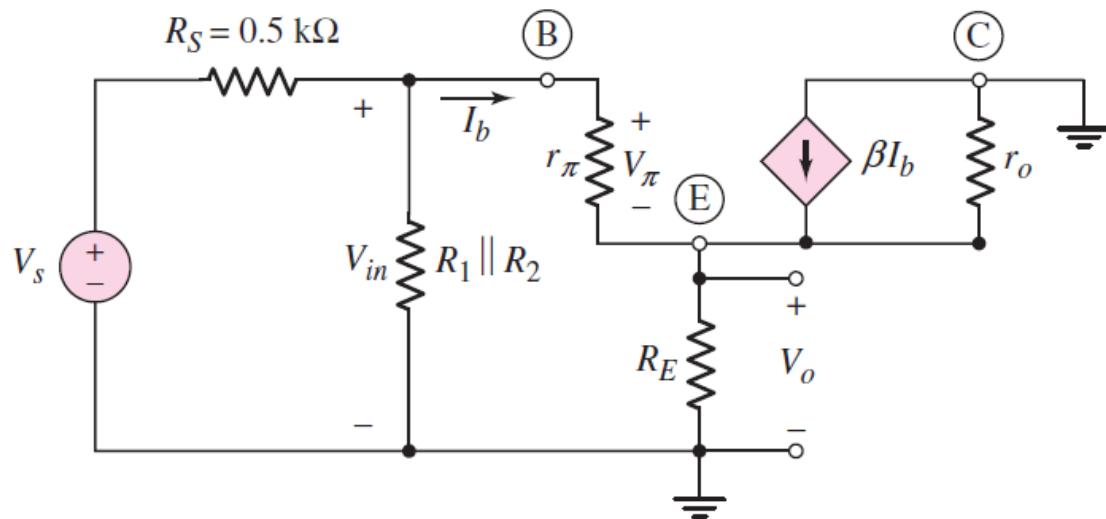
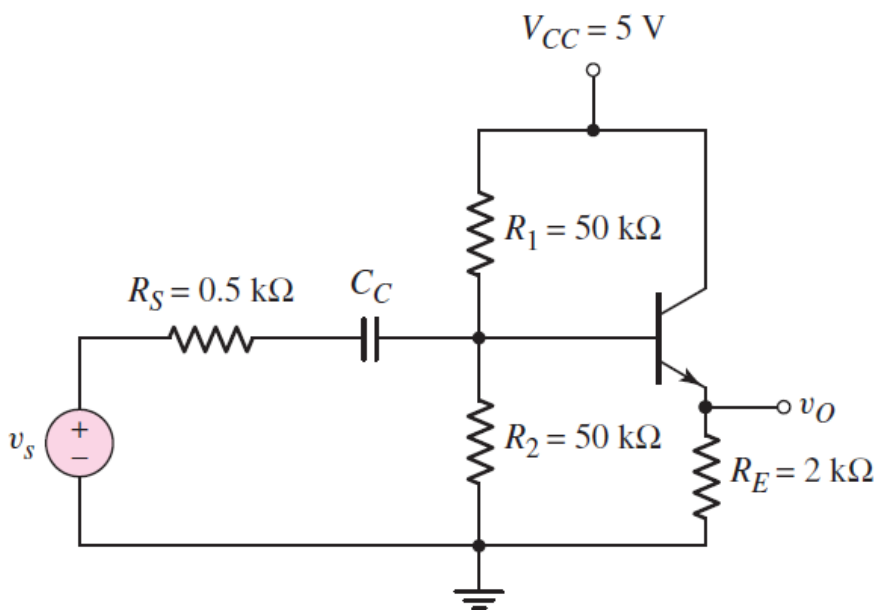
Therefore,  $A_v = \frac{V_o}{V_S} = \frac{-(\beta I_b) R_C}{V_S} = -\beta R_C \left( \frac{V_{in}}{R_{ib}} \right) \left( \frac{1}{V_S} \right) = \frac{-\beta R_C}{r_\pi + (1 + \beta) R_E} \left( \frac{R_i}{R_i + R_S} \right)$

If  $R_i \gg R_S$  &  $(1 + \beta) R_E \gg r_\pi \rightarrow A_v \cong \frac{-\beta R_C}{(1 + \beta) R_E} \cong \frac{-R_C}{R_E}$



# Common-Collector (CC) Amplifier

- ✓ The output signal is taken off of the emitter with respect to ground and the collector is connected directly to  $V_{CC}$ . Since  $V_{CC}$  is at signal ground in the ac equivalent circuit (see) – named as common-collector (*Emitter follower*).
- ✓ Equivalent circuit – assume the coupling capacitor  $C_C$  acts as a short circuit. Collector terminal is at signal ground & the transistor output resistance  $r_o$  is in parallel with the dependent current source.



# Common-Collector (CC) Amplifier

From the equivalent circuit,

$$I_o = (1 + \beta)I_b$$

$$V_o = I_b(1 + \beta)(r_o || R_E)$$

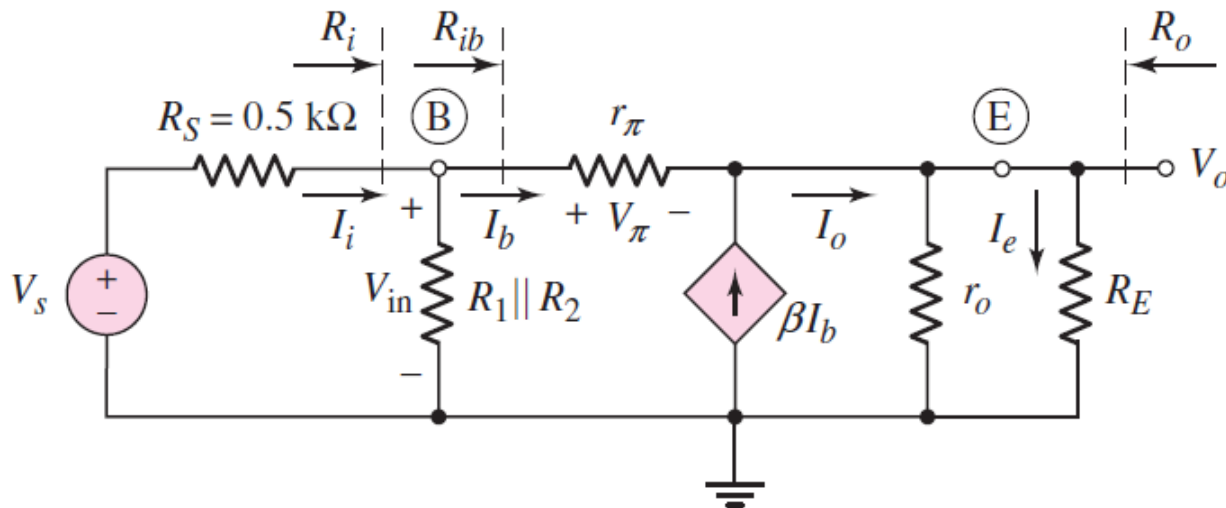
KVL for base-emitter loop,

$$V_{in} = I_b[r_\pi + (1 + \beta)(r_o || R_E)]$$

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1 + \beta)(r_o || R_E) \Rightarrow \text{Input resistance looking into the base}$$

We also write,  $V_{in} = \left(\frac{R_i}{R_i + R_S}\right) V_S$ ; where,  $R_i = R_1 || R_2 || R_{ib}$ .

$$\text{Small-signal voltage gain, } A_v = \frac{V_o}{V_S} = \frac{(1 + \beta)(r_o || R_E)}{r_\pi + (1 + \beta)(r_o || R_E)} \left(\frac{R_i}{R_i + R_S}\right)$$



Small-signal equivalent circuit with all signal grounds connected together

# Common-Collector (CC) Amplifier

Small-signal current gain,  $A_i = \frac{I_e}{I_i}$

Using current divider rule,  $I_b = \left( \frac{R_1 || R_2}{R_1 || R_2 + R_{ib}} \right) I_i$

Since,  $g_m V_\pi = \beta I_b$ , then,  $I_o = (1 + \beta) I_b = (1 + \beta) \left( \frac{R_1 || R_2}{R_1 || R_2 + R_{ib}} \right) I_i$

Write the load current in terms of  $I_o$  produces,  $I_e = \left( \frac{r_o}{r_o + R_E} \right) I_o$

Therefore, small-signal current gain,  $A_i = \frac{I_e}{I_i} = (1 + \beta) \left( \frac{R_1 || R_2}{R_1 || R_2 + R_{ib}} \right) \left( \frac{r_o}{r_o + R_E} \right)$

If we assume  $R_1 || R_2 \gg R_{ib}$  and  $r_o \gg R_E$ , then  $A_i \cong (1 + \beta)$

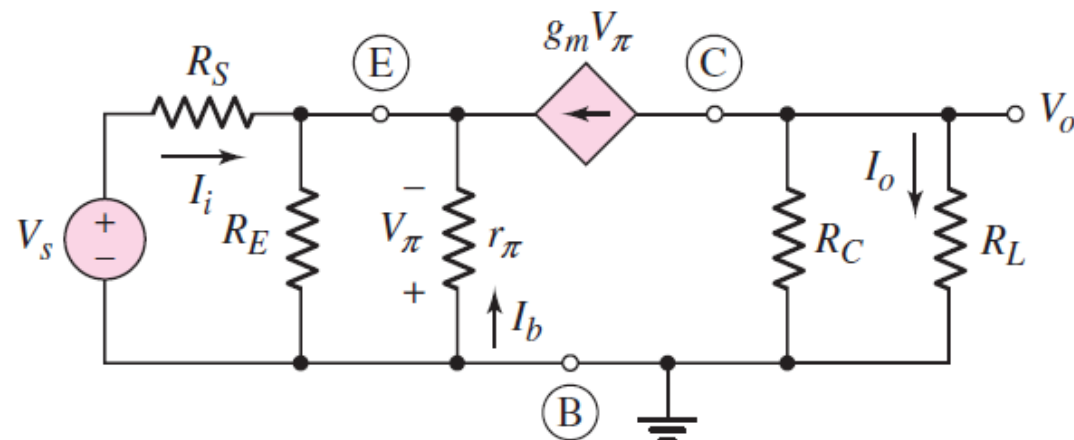
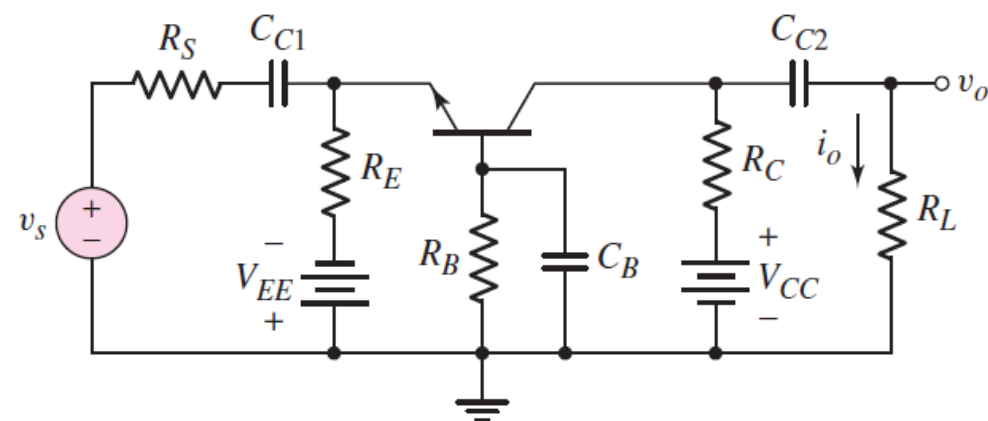
*Although small-signal voltage gain of CC amplifier is slightly less than 1, the small-signal current gain is normally greater than 1.*

# Common-Base (CB) Amplifier

- ✓ Base is at signal ground & input signal is applied to emitter – *Common-Base*
- ✓ Assume the load is connected to the output through coupling capacitor  $C_{C2}$ .
- ✓ Assume output resistance  $r_o$  to be infinite. The small-signal equivalent circuit of a CB configuration with hybrid- $\pi$  model is complex.

Small-signal output voltage,  $V_o = -(g_m V_\pi)(R_C || R_L)$

KCL at the emitter node gives,  $g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_s - (-V_\pi)}{R_S} = 0$



Small-signal equivalent circuit

# Common-Base (CB) Amplifier

Since  $\beta = g_m r_\pi$ , the above equation is  $V_\pi \left( \frac{1+\beta}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right) = -\frac{V_S}{R_S}$

Then,  $V_\pi = -\frac{V_S}{R_S} \left[ \left( \frac{r_\pi}{1+\beta} \right) || R_E || R_S \right]$

Substitute the control voltage  $V_\pi$  in the output voltage equation, which results

Small-signal voltage gain,  $A_v = \frac{V_o}{V_S} = g_m \left( \frac{R_C || R_L}{R_S} \right) \left[ \left( \frac{r_\pi}{1+\beta} \right) || R_E || R_S \right]$

If  $R_S \rightarrow 0$ , the voltage gain becomes,

$$A_v = g_m (R_C || R_L)$$

For CB circuit, the small-signal voltage gain is usually greater than 1.



# Common-Base (CB) Amplifier

Small-signal current gain,  $A_i = \frac{I_o}{I_i}$

Write KCL at the emitter node, we get,  $I_i + \frac{V_\pi}{r_\pi} + g_m V_\pi + \frac{V_\pi}{R_E} = 0$

Solving for  $V_\pi$  gives,  $V_\pi = -I_i \left[ \left( \frac{r_\pi}{1+\beta} \right) || R_E \right]$

The load current,  $I_o = -(g_m V_\pi) \left( \frac{R_C}{R_C + R_L} \right)$

Therefore, the small-signal current gain can be written as

$$A_i = \frac{I_o}{I_i} = g_m \left( \frac{R_C}{R_C + R_L} \right) \left[ \left( \frac{r_\pi}{1 + \beta} \right) || R_E \right]$$

$$A_i = \frac{I_o}{I_i} = \frac{g_m r_\pi}{1 + \beta} = \frac{\beta}{1 + \beta} \quad \text{if } R_E \rightarrow \infty \text{ \& } R_L \rightarrow 0$$



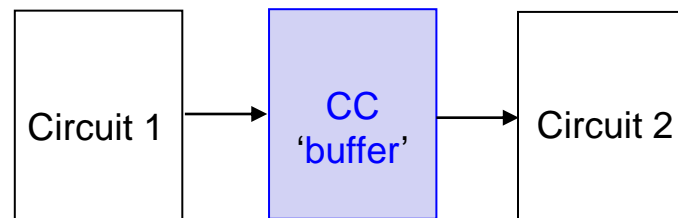
# Comparison of Three Amplifiers

Characteristics of the three BJT amplifier configurations

Configuration	Voltage gain	Current gain	Input resistance	Output resistance
Common emitter	$A_v > 1$	$A_i > 1$	Moderate	Moderate to high
Emitter follower	$A_v \cong 1$	$A_i > 1$	High	Low
Common base	$A_v > 1$	$A_i \cong 1$	Low	Moderate to high

*CC circuit* has very high input resistance, low output resistance, and  $A_v \cong 1$ .

- ✓ It is often used to isolate two circuits from each other, so circuit 2 does not draw current from circuit 1 – useful as *buffer*.



**High input impedance** means it draws very little current from circuit 1 & is also able to drive circuit 2 easily from its **low output impedance & high current gain**. A voltage gain of nearly unity means the signal from circuit 1 is passed onto circuit 2 unchanged.

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# Frequency Response of Amplifier Circuits (Lecture 03)

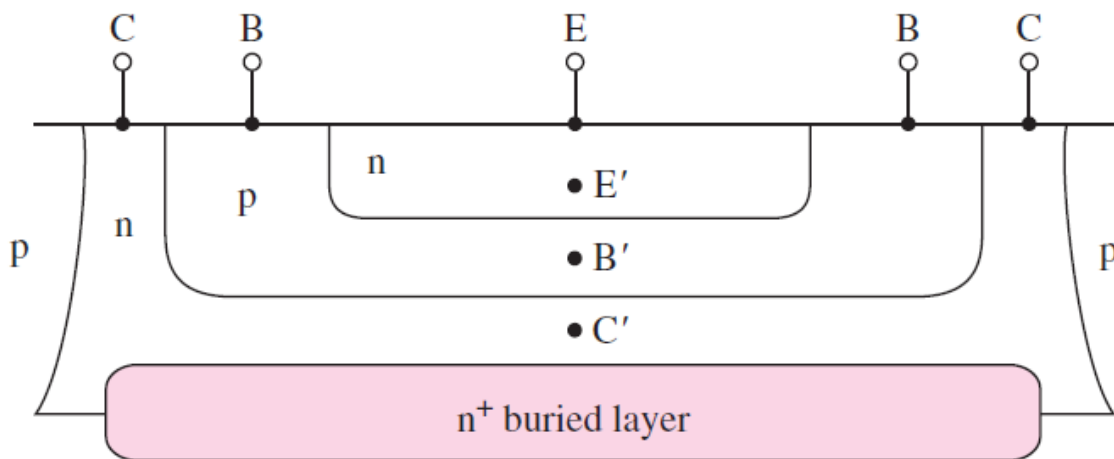


# Expanded Hybrid- $\pi$ equivalent Circuit

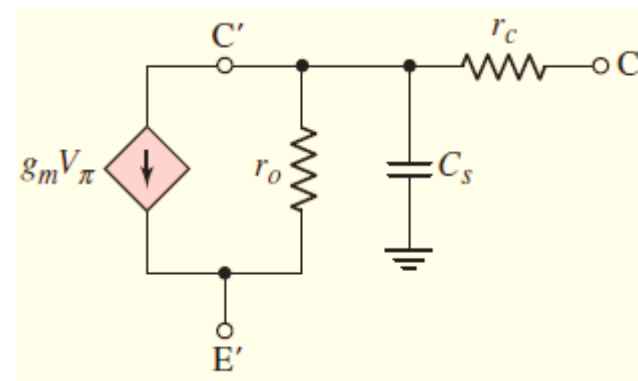
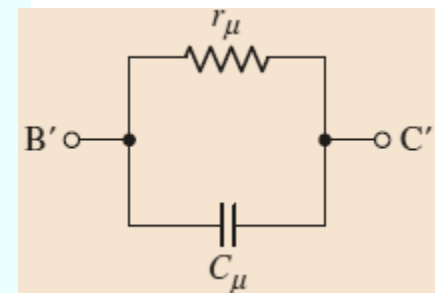
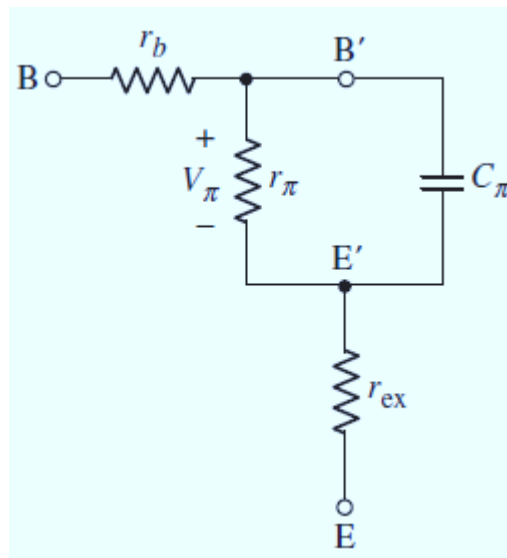
See the cross section of an npn bipolar transistor for hybrid- $\pi$  model.

- ✓  $C, B$ , and  $E$  terminals are external connections to the transistor, and  $C', B'$  &  $E'$  are idealized internal collector, base, and emitter regions.

$r_b$  is base series resistance between  $B$  &  $B'$ .  
 $C_\pi$  is forward biased junction capacitance.  
 $r_\pi$  is forward biased diffusion resistance.  
 $r_{ex}$  is emitter series resistance  $E'$  &  $E$ .



Cross section of npn transistor for hybrid- $\pi$  model



# Expanded Hybrid- $\pi$ equivalent Circuit

$r_c$  is collector series resistance between  $C$  &  $C'$ .

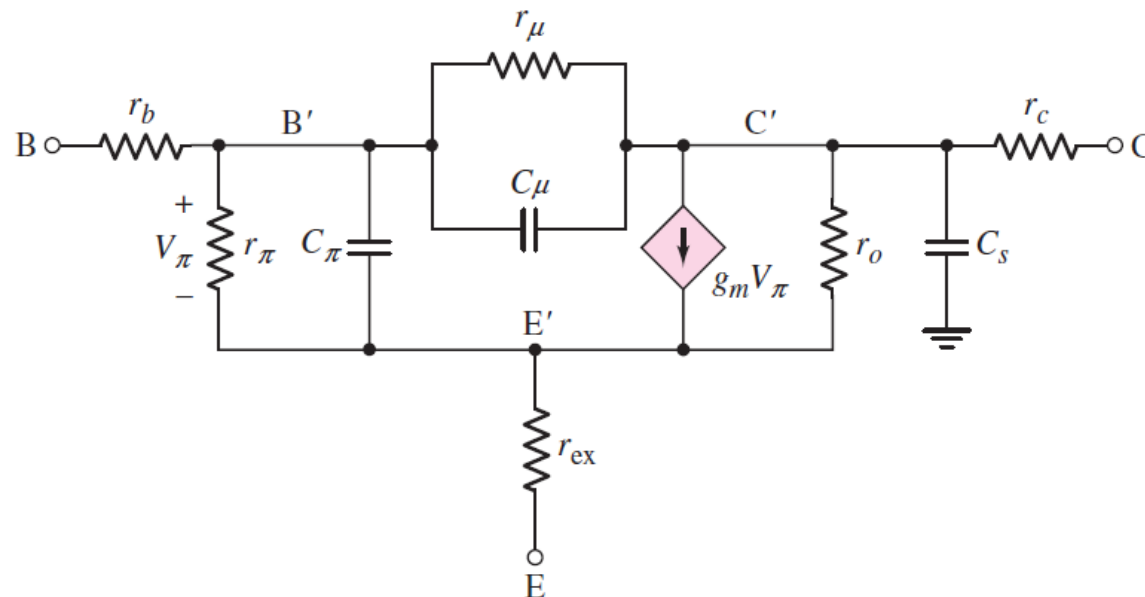
$C_s$  is junction capacitance of reverse-biased collector-substrate junction.

$g_m V_\pi$  is collector current controlled by internal base-emitter voltage.

$r_o$  is inverse of output conductance  $g_o$  and is due primarily to Early effect.

$r_\mu$  is reverse-biased diffusion resistance (range of megohms & neglected).

$C_\mu$  is reverse-biased junction capacitance (normally  $< C_\pi$ , however, *cannot be neglected due to Miller effect*).



# Short-Circuit Current Gain

Neglect the parasitic resistances  $r_b, r_c, r_{ex}$ , &  $r_\mu$ , and substrate capacitance  $C_s$ .

Write KCL at the input node,  $I_b = \frac{V_\pi}{r_\pi} + \frac{V_\pi}{\frac{1}{j\omega C_\pi}} + \frac{V_\pi}{\frac{1}{j\omega C_\mu}} = V_\pi \left[ \frac{1}{r_\pi} + j\omega(C_\pi + C_\mu) \right]$

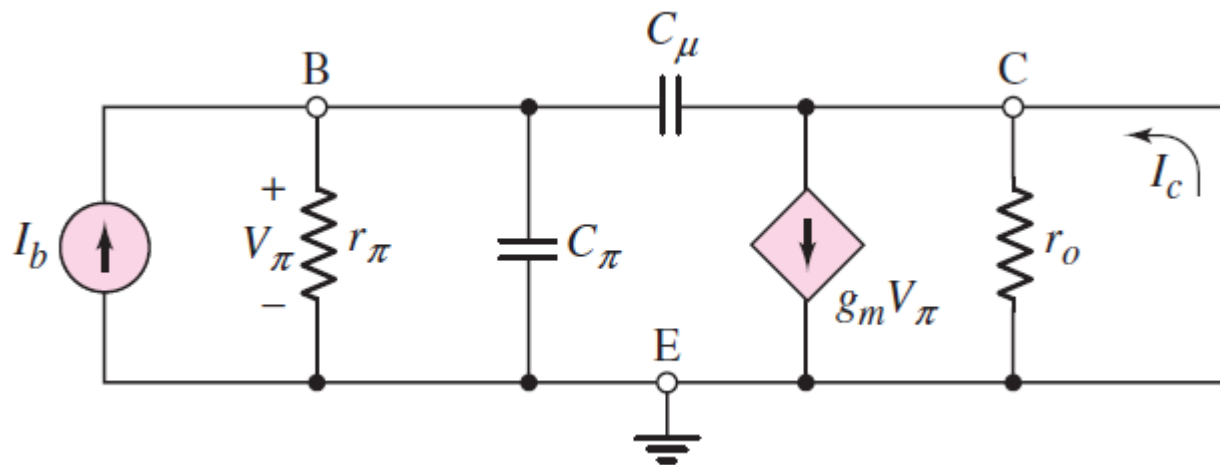
Note that  $V_\pi \neq I_b r_\pi$ , since a portion of  $I_b$  is now shunted through  $C_\pi$  &  $C_\mu$ . From KCL at the output node,

$$\frac{V_\pi}{\frac{1}{j\omega C_\mu}} + I_c = g_m V_\pi \rightarrow I_c = V_\pi (g_m - j\omega C_\mu) \rightarrow V_\pi = \frac{I_c}{(g_m - j\omega C_\mu)}$$

Substitute  $V_\pi$  in  $I_b$  results as,

$$I_b = I_c \times \frac{\left[ \frac{1}{r_\pi} + j\omega(C_\pi + C_\mu) \right]}{g_m - j\omega C_\mu}$$

$$\therefore A_i = \frac{I_c}{I_b} = \frac{(g_m - j\omega C_\mu)}{\left[ \frac{1}{r_\pi} + j\omega(C_\pi + C_\mu) \right]}$$



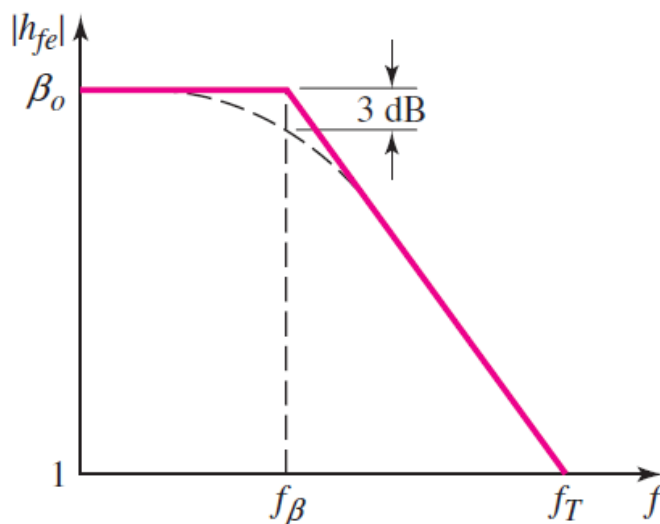
Simplified hybrid- $\pi$  equivalent circuit

# Short-Circuit Current Gain

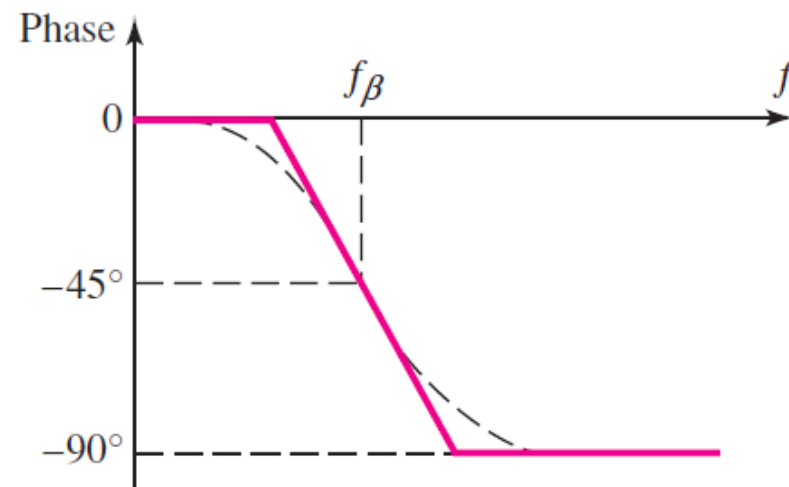
$$A_i = h_{fe} = \frac{I_c}{I_b} = \frac{(g_m - j\omega C_\mu)}{\left[\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)\right]} \cong \frac{g_m r_\pi}{1 + j\omega r_\pi(C_\pi + C_\mu)} \quad \text{if } \omega C_\mu \ll g_m$$

See the Bode plot of short-circuit current gain magnitude. The corner frequency, **beta cutoff frequency** ( $f_\beta$ ), is given by,

$$f_\beta = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$$



Magnitude plot



Phase plot



# Cutoff Frequency

Note that the magnitude of current gain decreases with increasing frequency and reaches to 1 at  $f_T$  (cutoff frequency). We can write  $A_i$  in the below form:

$$A_i = h_{fe} = \frac{\beta_o}{1 + j\left(\frac{f}{f_\beta}\right)} \rightarrow |h_{fe}| = \frac{\beta_o}{\sqrt{1 + \left(\frac{f}{f_\beta}\right)^2}}$$

At  $f_T$ ,  $|h_{fe}| = 1$  and normally  $\beta_o \gg 1$ ,  $f_T \gg f_\beta$ . Therefore, equation becomes

$$1 \cong \frac{\beta_o}{\sqrt{\left(\frac{f}{f_\beta}\right)^2}} = \frac{\beta_o f_\beta}{f_T} \rightarrow f_T = \beta_o f_\beta = \beta_o \frac{1}{2\pi r_\pi (C_\pi + C_\mu)} = \frac{g_m}{2\pi (C_\pi + C_\mu)}$$

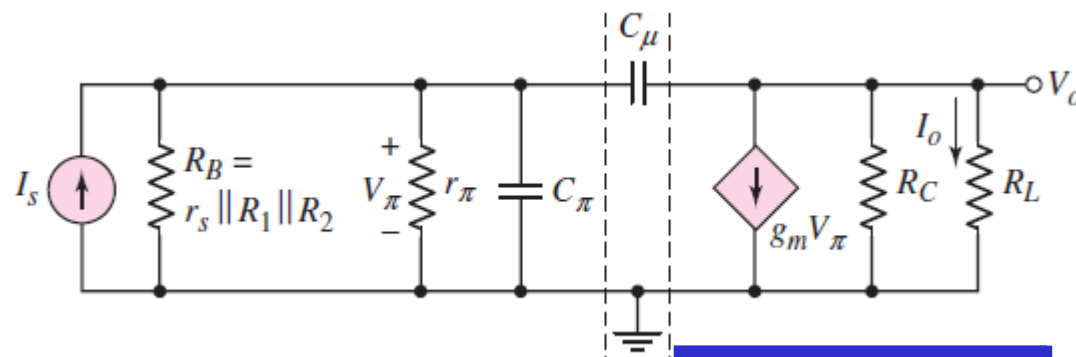
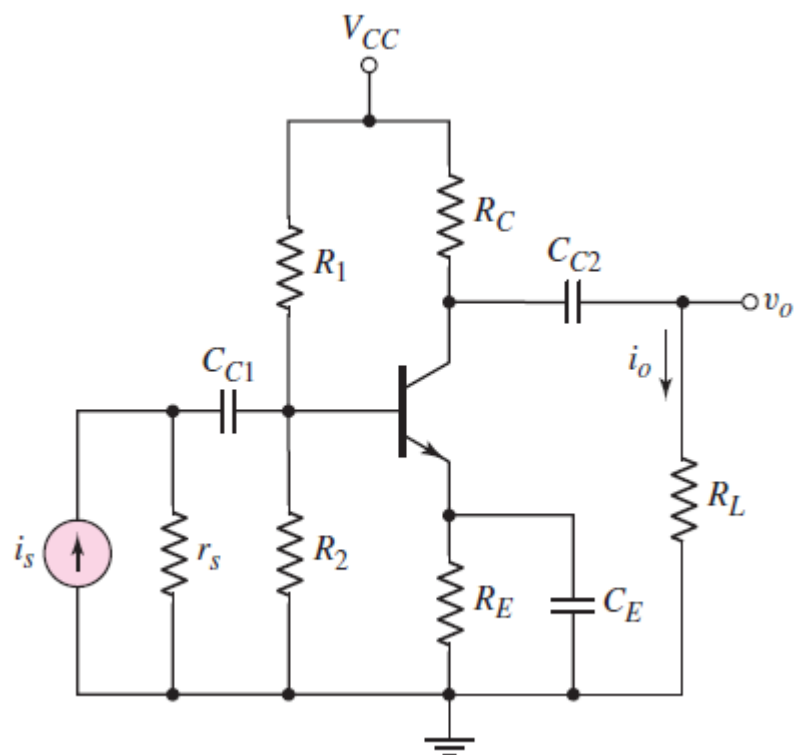
Frequency  $f_\beta$  is called the bandwidth of transistor, therefore,  $f_T$  is more commonly called as unity-gain bandwidth (or) gain bandwidth product.



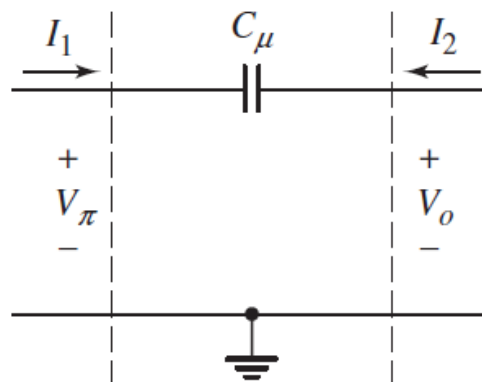
# Miller Effect and Miller Capacitance

As mentioned earlier, the capacitance  $C_\mu$  cannot really be ignored – *Miller effect, or feedback effect*, is a multiplication effect of  $C_\mu$  in circuit applications.

✓ Assume the frequency is sufficiently high for the coupling and bypass capacitors to act as short circuits –  $C_\mu$  connects output back to the input.



Small-signal hybrid- $\pi$  equivalent circuit

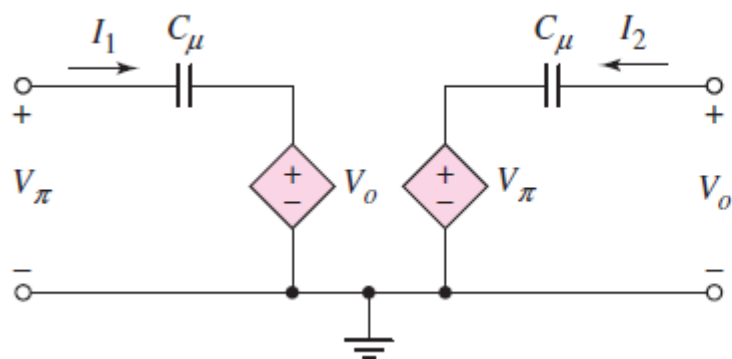


$$V_\pi = I_1 \left( \frac{1}{j\omega C_\mu} \right) + V_o$$

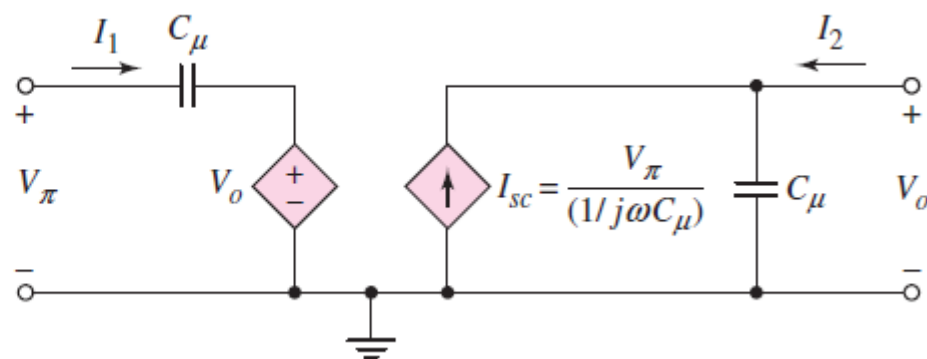
$$V_o = I_2 \left( \frac{1}{j\omega C_\mu} \right) + V_\pi$$

# Miller Effect and Miller Capacitance

- ✓ Now form a two-port equivalent circuit and then convert the Thevenin equivalent circuit on the output side to Norton equivalent circuit.

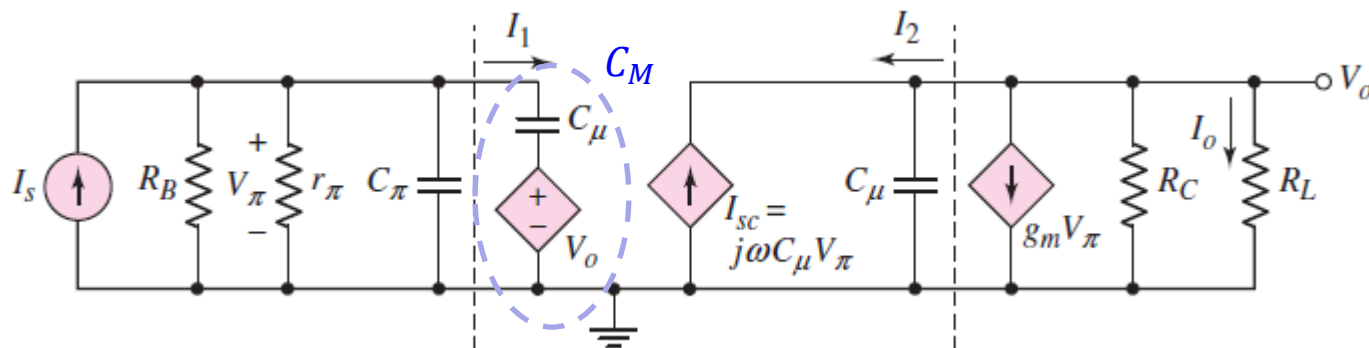


Two-port equivalent circuit of  $C_\mu$ : Thevenin



Two-port equivalent circuit of  $C_\mu$ : Norton at output

- ✓ Again, reconsider the original equivalent circuit and now replace the circuit segment between the dotted lines with the above Norton circuit.



Small-signal circuit, including two-port equivalent model of  $C_\mu$

# Miller Effect and Miller Capacitance

Recall the expression for  $V_\pi$  and obtain the equation for  $I_1$  as follows:

$$I_1 = \frac{V_\pi - V_o}{\frac{1}{j\omega C_\mu}} = j\omega C_\mu (V_\pi - V_o)$$

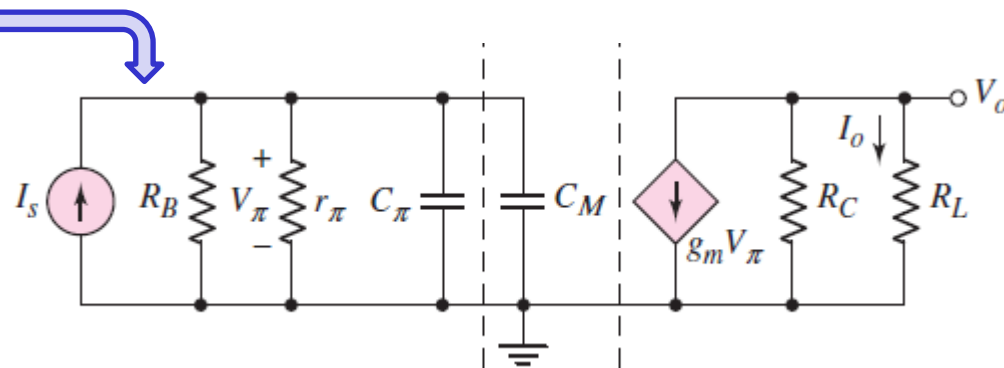
The output voltage is,  $V_o = -g_m V_\pi (R_C || R_L)$

Therefore,  $I_1$  becomes as,  $I_1 = j\omega C_\mu [1 + g_m (R_C || R_L)] V_\pi$

The circuit segment between the dotted lines can be replaced by an equivalent capacitance called **Miller capacitance** as,  $C_M = C_\mu [1 + g_m (R_C || R_L)]$

Note that the multiplication effect of  $C_\mu$  is the Miller effect.

Consider the frequency of operation is very much smaller such that 1)  $I_{sc}$  is negligible compared to  $g_m V_\pi$  source, 2)  $C_\mu$  will be much greater than  $R_C || R_L$ , therefore  $C_\mu$  can be considered as an open-circuit.



The input capacitance is now  $C_\pi + C_M$ .

***See you in the Final Exam (Jan 03, Friday @2 PM)***

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**The End**