# EEE103 ELECTRICAL CIRCUITS

## WEEK3-BASIC NODAL AND MESH ANALYSIS

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# CONTENT

- Nodal Analysis
- The Supernode
- Mesh Analysis
- The Supermesh
- NODAL VS. MESH ANALYSIS: A COMPARISON

# Circuit Analysis

As circuits get more complicated, we need an organized method of applying KVL, KCL, and Ohm's Law

Nodal analysis assigns voltages to each node, and then we apply KCL

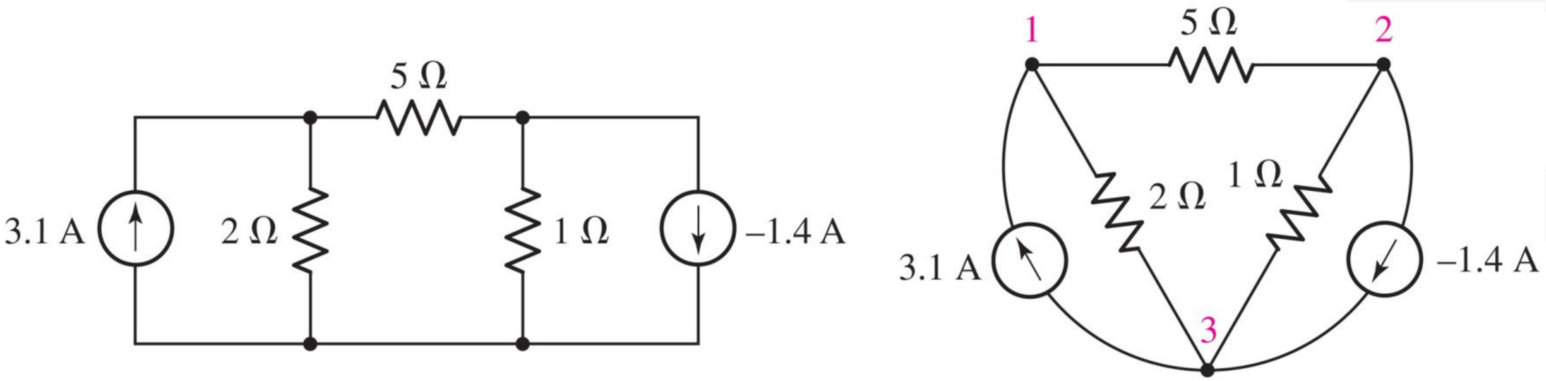
Mesh analysis assigns currents to each mesh, and then we apply KVL

Noted:

Step 1: Make an assumption. (set the **polarity** of the voltage and the **direction** of the current)
Step 2: Analyze the circuit based on the assumption.

# The Nodal Analysis Method

Assign voltages to every node relative to a reference node




(*a*) (*b*)

In this example, there are three nodes, only current sources are present.

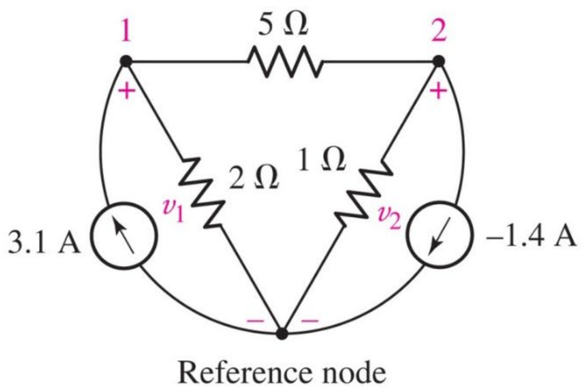
# Choosing the Reference Node

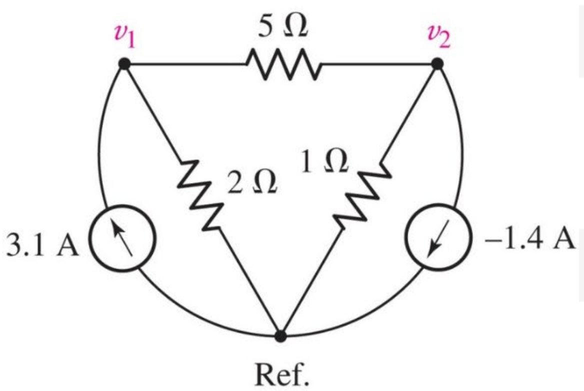
How to chose reference node?

As the **bottom node**, or

As the **ground connection**, if there is one, or

A node with **many connections**



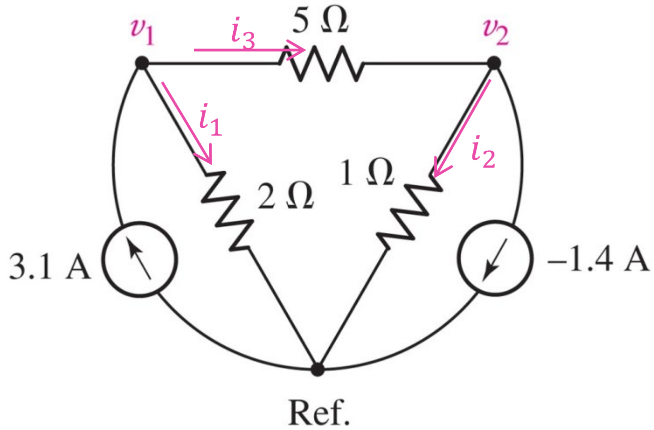

## Assign voltages relative to reference

( N-node circuit will need (N−1) voltages and (N−1) equations)

# Apply KCL to Find Voltages

Apply KCL to node 1 ( Σ out = 0) : $i_1 + i_3 - 3.1 = 0$

Apply Ohm's law to each resistor: $\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1$




Apply KCL to node 2 ( Σ out =0)：

$i_2 + (-1.4) - i_3 = 0$

=> $i_2 - i_3 = 1.4$

ApplyOhm's law to each resistor:

$$v_2 + \frac{v_2 - v_1}{5} = 1.4$$

2 unknow variabls
2 equations

$$[v_1 = 5\text{V and } v_2 = 2\text{V}]$$

# Apply KCL to Find Voltages

Node 1 top branch :

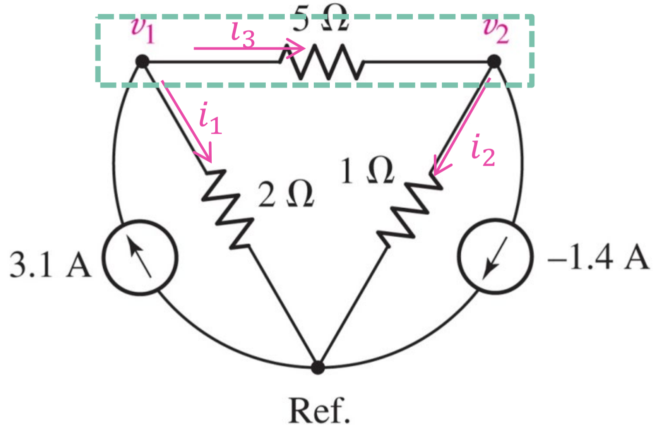
Current out: $i_3, \dfrac{v_1 - v_2}{5}$

Current in: $-i_3, \frac{v_2 - v_1}{5}$

Node 2 top branch :

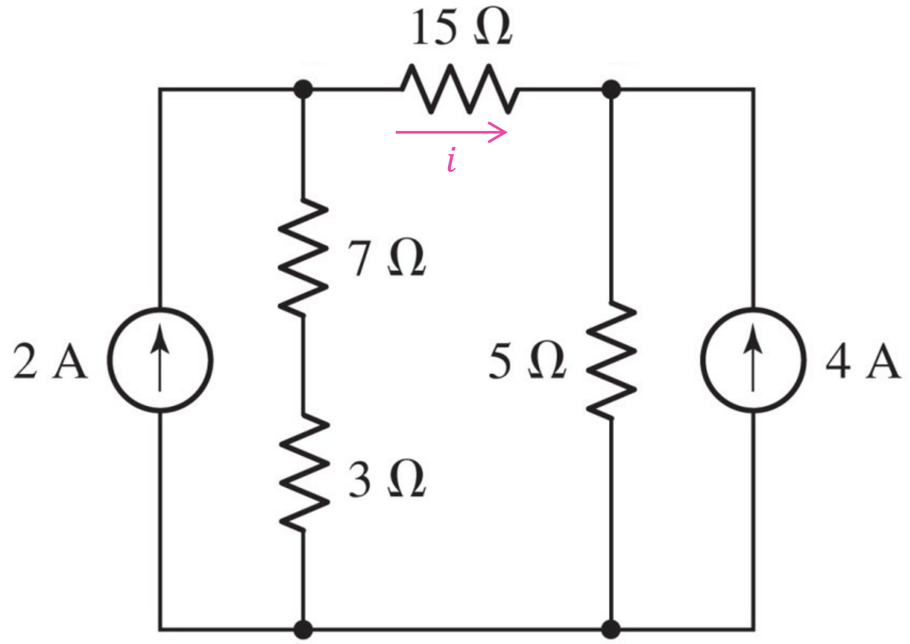
Current out: $-i_3, \frac{v_2 - v_1}{5}$

Current in: $i_3, \frac{v_1 - v_2}{5}$



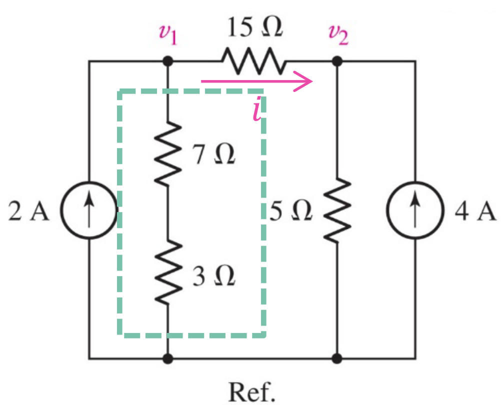
# Example: Nodal Analysis

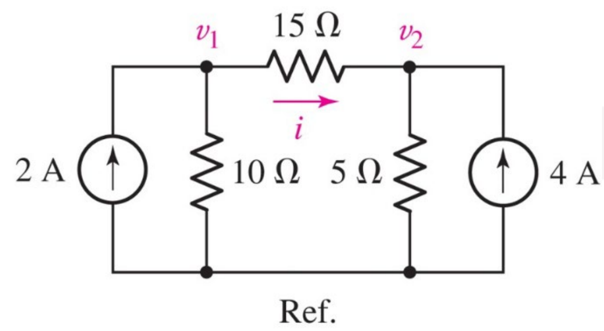
Find the current *i* in the circuit.



# Example: Nodal Analysis

Find the current *i* in the circuit.




1. Simplify the circuit

2. Apply KCL for node 1: $\frac{v_1}{10} + \frac{v_1 - v_2}{15} = 2$

for node 2: $\frac{v_2}{5} + \frac{v_2 - v_1}{15} = 4$

3. Simplify the equations: $\boxed{\begin{aligned} 5v_1 - 2v_2 &= 60 \\ -v_1 + 4v_2 &= 60 \end{aligned}}$

2 unknow variabls
2 equations

4. Solve: $v_1 = 20\text{V},\ v_2 = 20\text{V},\quad i = (v_1 - v_2)/15 = 0$

# Example: Nodal Analysis

Alternative way to solve $\begin{cases} 5v_1 - 2v_2 = 60 \\ -v_1 + 4v_2 = 60 \end{cases}$

Transfer to matrix format in the form $\mathbf{Av=B}$

$$\begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$

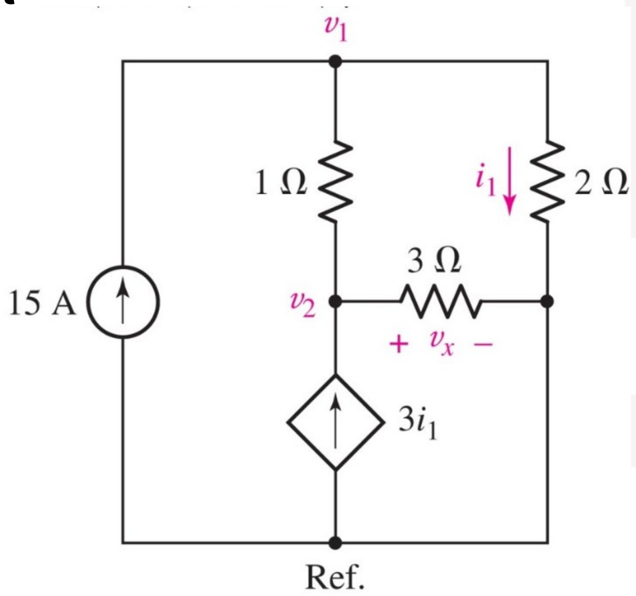
Where $\mathrm{A} = \begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix}$, $\mathrm{B} = \begin{bmatrix} 60 \\ 60 \end{bmatrix}$

Entering the numbers for matrix **A** and vector **B** into a calculator and solving for **v = A$^{-1}$B** yields our final solution

$$\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \end{bmatrix} \mathrm{V}$$

# Nodal Analysis: Dependent Source Example

Determine the power supplied by the dependent source.




At node 1: $\frac{v_1}{2} + \frac{v_1 - v_2}{1} = 15$

At notde 2: $\frac{v_2}{3} + \frac{v_2 - v_1}{1} = 3i_1$

Key step: eliminate $i_1$ from the equations using $v_1 = 2i_1$

Simplify the equations:

$$\begin{cases} 3v_1 - 2v_2 = 30 \\ -15v_1 + 8v_2 = 0 \end{cases} \Rightarrow \begin{cases} v_1 = -40V \\ v_2 = -75V \end{cases}$$

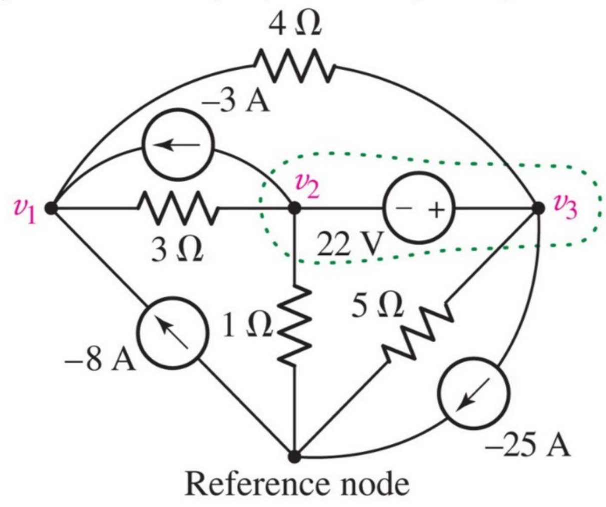
$$i_1 = 0.5v_1 = -20A$$

$$p = -(3i_1)(v_2) = -4.5kW$$

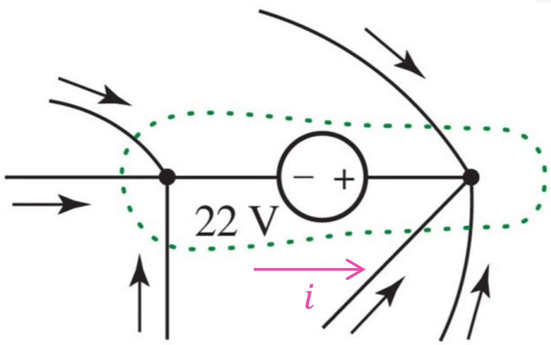
The dependent source absorbs -4.5kW power
The dependent source supplies 4.5kW power

# Voltage Sources and the Supernode

What is the current through a voltage source connected between nodes?




We can eliminate the need for introducing a current variable by applying KCL to the *supernode.*

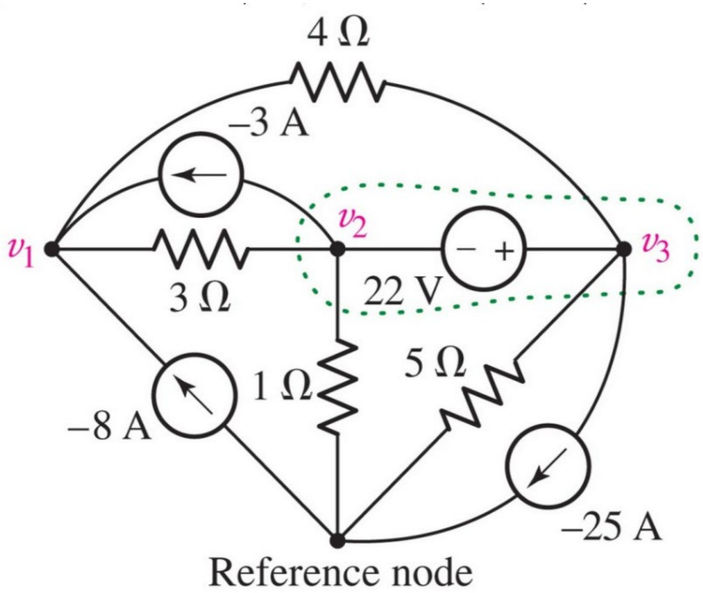



Total current entering the supernode=

total current entering the left node+

total current entering the right node

# The Supernode




Apply KCL at Node 1.( Σ out = Σ in)

$$\frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{3} = -3 - 8$$

Apply KCL at the supernode.

Left: $\frac{v_2}{1} + \frac{v_2 - v_1}{3} + (-3)$

Right: $-25 + \frac{v_3}{5} + \frac{v_3 - v_1}{4}$

$$\frac{v_2}{1} + \frac{v_2 - v_1}{3} + \frac{v_3}{5} + \frac{v_3 - v_1}{4} = -(-25) - (-3)$$

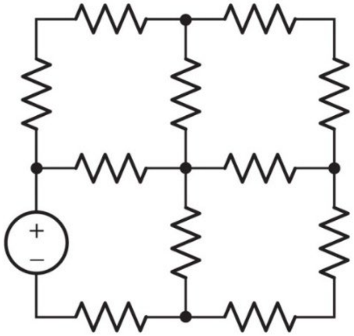
Add the equation for the voltage source inside the supernode.

$$v_3 - v_2 = 22$$

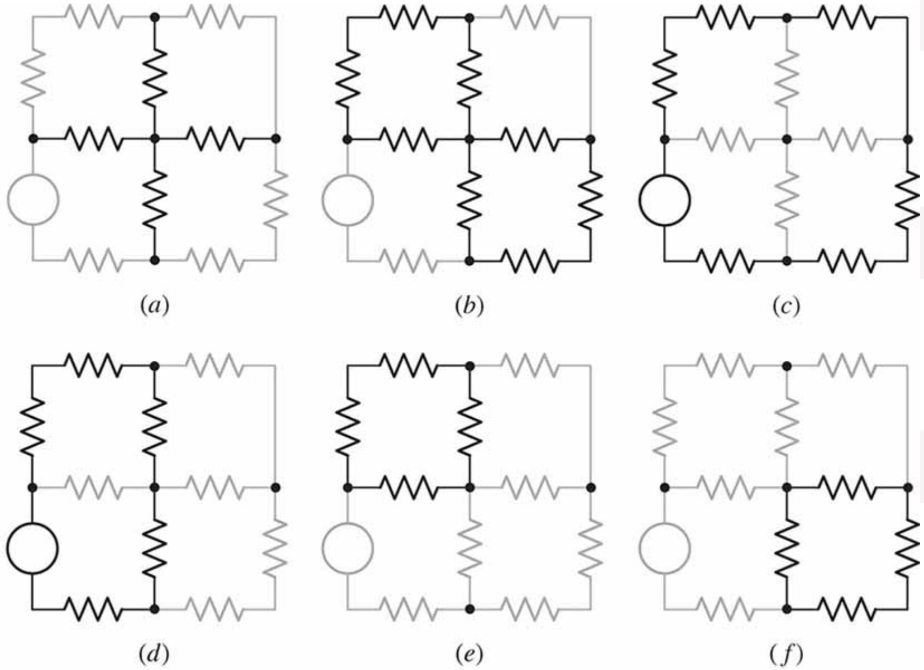
3 unknow variables
3 equations

# Mesh Analysis: Nodal Alternative

A mesh is a loop which does not contain any other loops within it



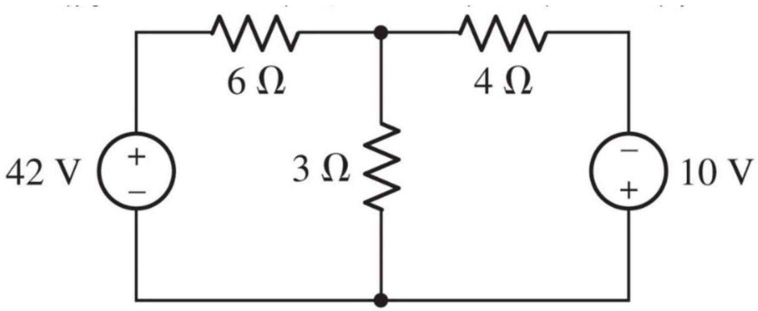

(a)



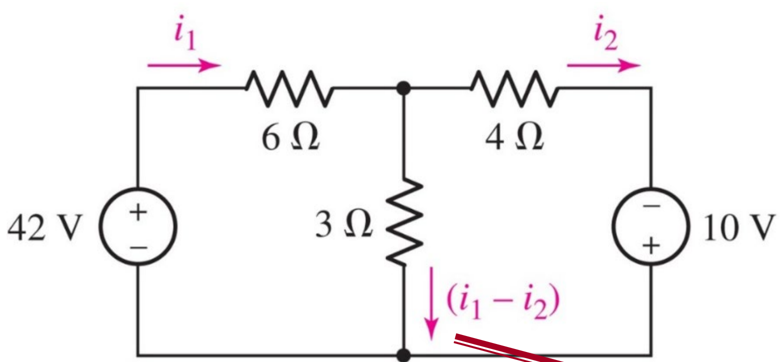
# The Mesh Analysis Method

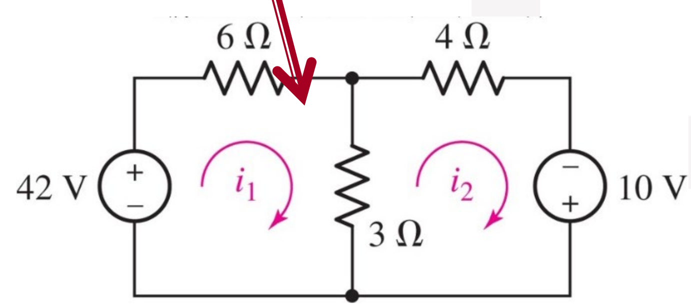
In mesh analysis, we assign currents and solve using KVL

Assigning mesh currents automatically ensures KCL is followed




Mesh currents




Branch currents

# Mesh: Apply KVL

Apply KVL to mesh 1

( Σ drops = 0 ):

$$-42 + v_6 + v_3 = 0$$

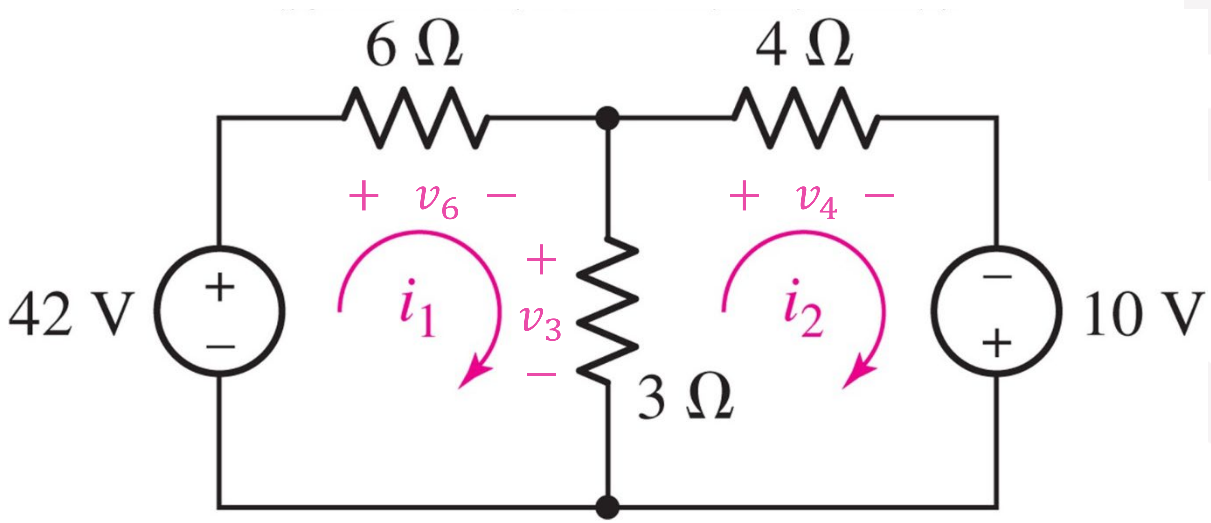
$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

Apply KVL to mesh 2

( Σ drops = 0 ):

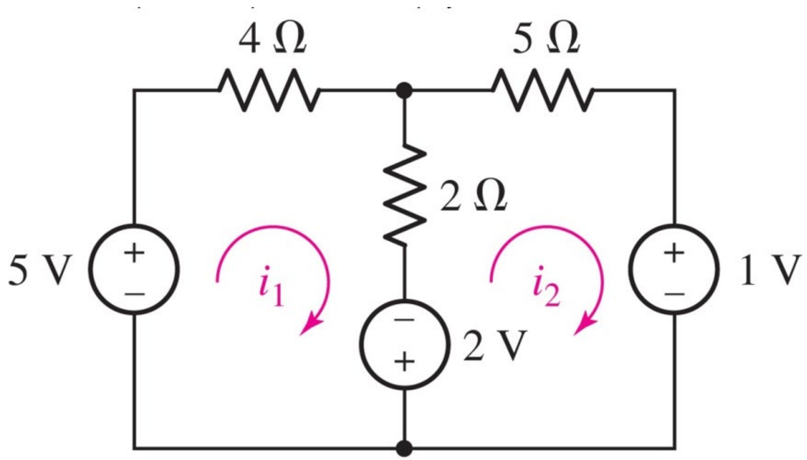
$$-v_3 + v_4 - 10 = 0$$

$$3(i_2 - i_1) + 4i_2 - 10 = 0$$



# Example: Mesh Analysis

Determine the power supplied by the 2 V source.




Applying KVL to the meshes:

$$-5+4i_1+2(i_1-i_2)-2=0$$

$$+2+2(i_2-i_1)+5i_2+1=0$$

$$Solve: \; i_1 = 1.132\text{A}, \qquad i_2 = -0.1053\text{A}$$

$$p = 2(i_2 - i_1) = -2.474W$$

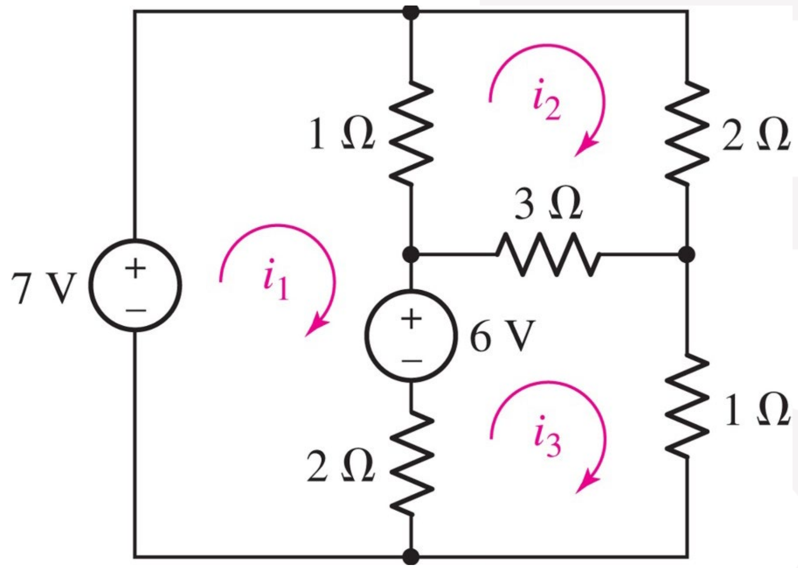
The 2V source absorbs -2.474W power
The 2V source supplies 2.474W power

# A Three Mesh Example

$$-7+1(i_1-i_2)+6+2(i_1-i_3)=0$$
$$1(i_2-i_1)+2i_2+3(i_2-i_3)=0$$
$$2(i_3-i_1)-6+3(i_3-i_2)+1i_3=0$$

Follow each mesh clockwise

$$3i_1-i_2-2i_3=1$$
$$-i_1+6i_2-3i_3=0$$
$$-2i_1-3i_2+6i_3=6$$

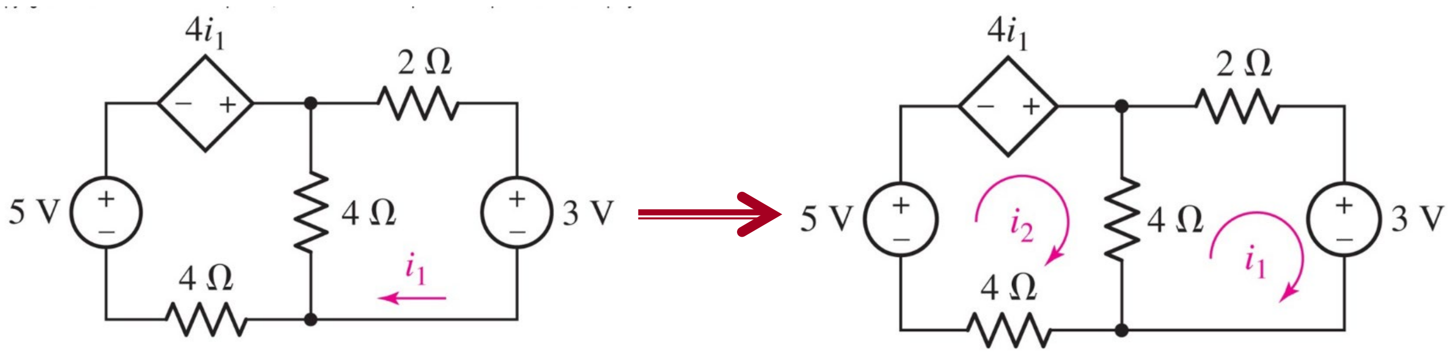



Solve the equations:

$i_1$ = 3 A, $i_2$ = 2 A, and $i_3$ = 3 A.

# Dependent Source Example

Find $i_1$



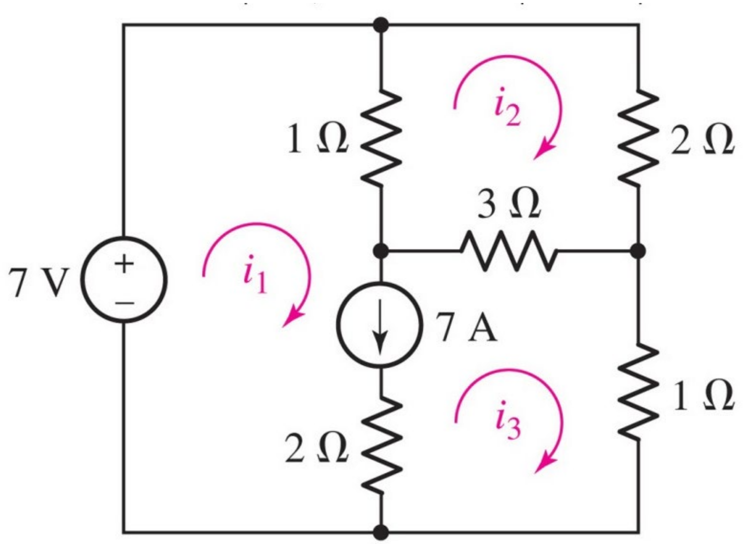

Apply KVL to mesh 1: $4(i_1 - i_2) + 2i_1 + 3 = 0$

Apply KVL to mesh 2: $-5 - 4i_1 + 4(i_2 - i_1) + 4i_2 = 0$

$$\Rightarrow \begin{cases} -8i_1 + 8i_2 = 50 \\ 6i_1 - 4i_2 = -3 \end{cases} \Rightarrow \begin{cases} i_1 = -250 \text{ mA} \\ i_2 = 375 \text{ mA} \end{cases}$$

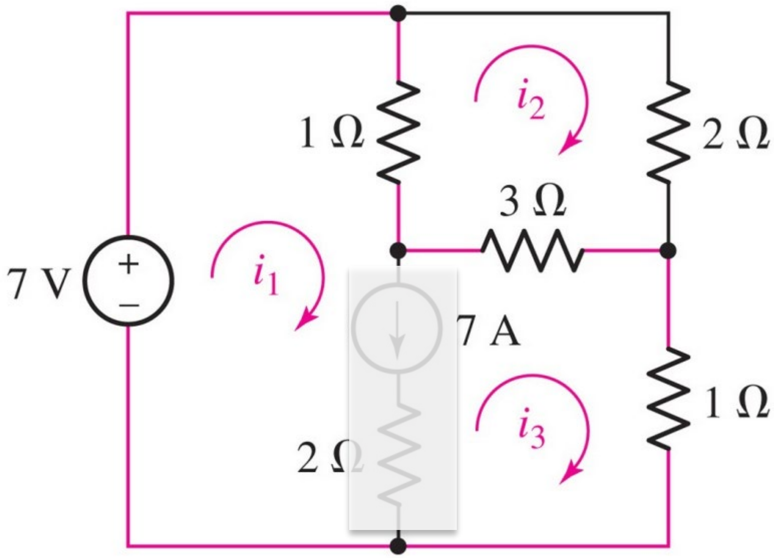
# Current Sources and the Supermesh

What is the voltage across a current source in between two meshes?




We can eliminate the need for introducing a voltage variable by applying KVL to the *supermesh* formed by joining mesh 1 and mesh 3.

# The Supermesh




Apply KVL to mesh 2:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

Apply KVL supermesh 1/3:

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

Add the current source:

$$7 = i_1 - i_3$$

KVL: Algebraic sum of voltages around any closed path is zero.

# Dependent Source Example

Find the currents.

1. $i_1$=15A

2. Relate $i_1$ and $i_3$ to the current from the dependent source using KCL(key step):

$$\frac{v_x}{9} = i_3 - i_1$$

$$\frac{v_x}{3} = i_3 - i_2$$

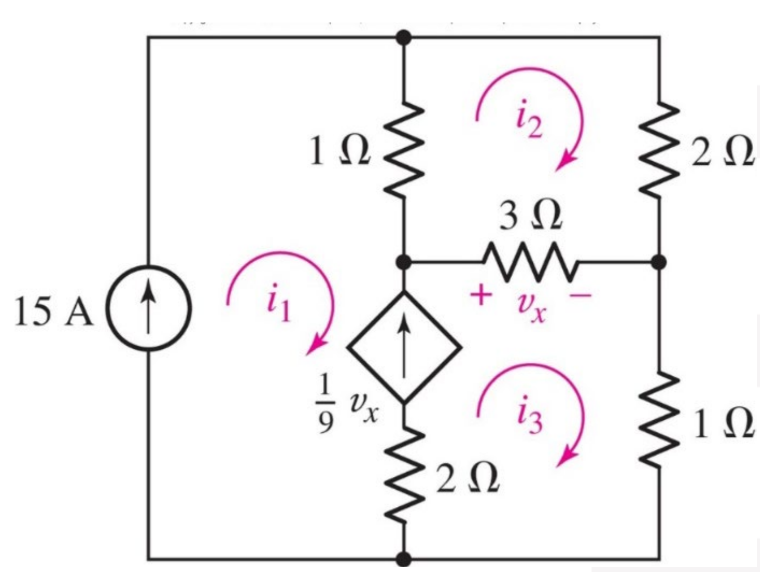
Replace $v_x$

$$\frac{1}{3}i_2 + \frac{2}{3}i_3 = 15 ----(1)$$

KVL equation around mesh 2:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

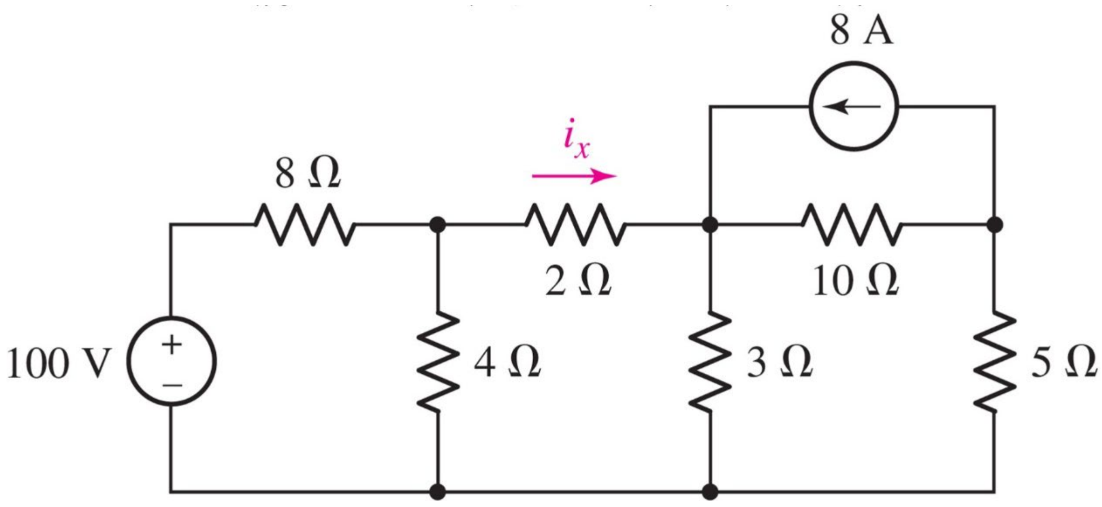
$$6i_2 - 3i_3 = 15 \quad ---(2)$$




$$i_2 = 11A, \quad i_3 = 17A$$

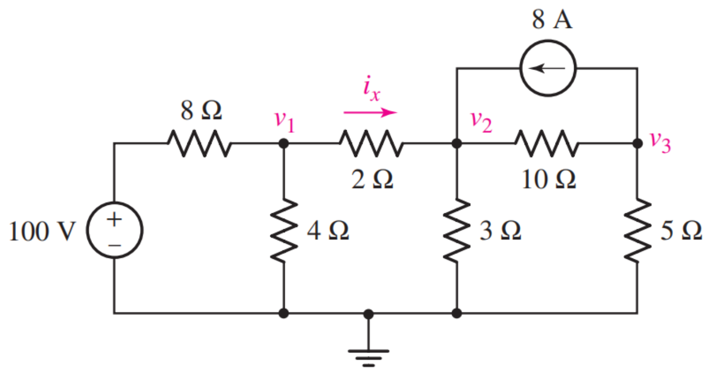
# Node or Mesh: How to Choose?

Determine the current $i_x$



# Node or Mesh: How to Choose?

Determine the current $i_x$ (Node)




$$\frac{v_1 - 100}{8} + \frac{v_1}{4} + \frac{v_1 - v_2}{2} = 0 \quad \text{or} \quad 0.875\,v_1 - 0.5\,v_2 = 12.5$$

$$\frac{v_2 - v_1}{2} + \frac{v_2}{3} + \frac{v_2 - v_3}{10} - 8 = 0 \quad \text{or} \quad -0.5\,v_1 - 0.9333\,v_2 - 0.1\,v_3 = 8$$

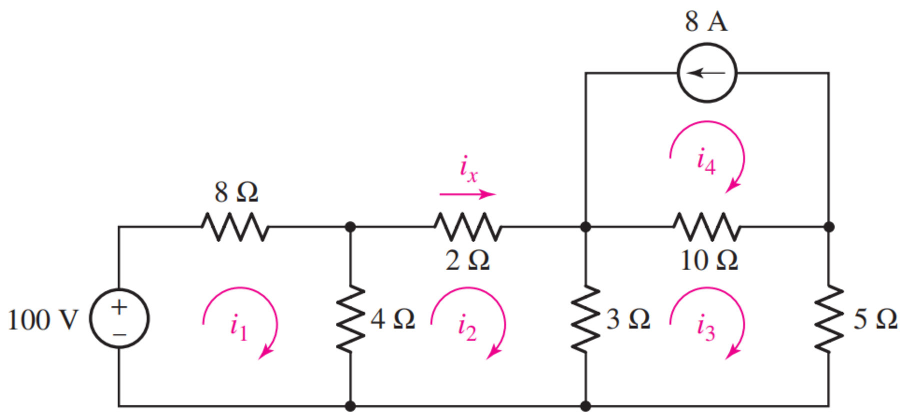
$$\frac{v_3 - v_2}{10} + \frac{v_3}{5} + 8 = 0 \quad \text{or} \quad -0.1\,v_2 + 0.3\,v_3 = -8$$

$$v_1 = 25.89V, \qquad v_2 = 20.31V$$

$$i_x = \frac{v_1 - v_2}{2} = 2.79\ A$$

# Node or Mesh: How to Choose?

Determine the current $i_x$ (Mesh)




$$-100 + 8i_1 + 4(i_1 - i_2) = 0 \quad \text{or} \quad 12i_1 - 4i_2 = 100$$

$$4(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \quad \text{or} \quad -4i_1 + 9i_2 - 3i_3 = 0$$

$$3(i_3 - i_2) + 10(i_3 + 8) + 5i_3 = 0 \quad \text{or} \quad -3i_2 + 18i_3 = -80$$

$$\boxed{i_x = i_2 = 2.79\text{A}}$$

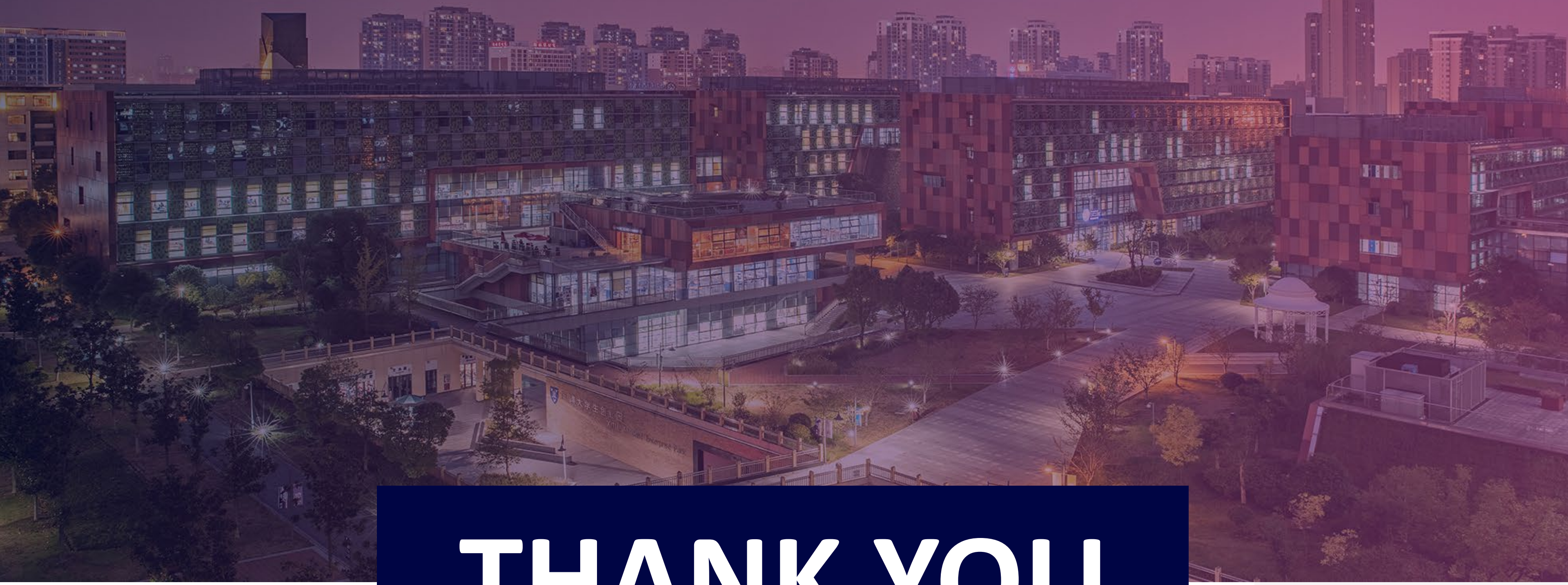
# Node or Mesh: How to Choose?

Use the one with fewer equations, or

Use the method you like best, or

Use both (as a check), or

Use circuit simplifying methods from the next chapter



# THANK YOU

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