CAN102 Electromagnetism and Electromechanics

Lecture-1 Introduction and Fundamental Mathematics

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Room EE322

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Module Information

- Module Code: CAN102
- Module Title: Electromagnetism and Electromechanics
- Module Credit: 5 credits
- Module Teachers: Jingchen Wang, Zhao Wang, Zhenzhen Jiang

Week 1-6:	Zhao Wang
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Office hour:	Tue. 13:30-15:30, Thu. 13:30-15:30

Module Information

- Two Coursework:
  - CW1 (15%):
    - ✓Group work (4 students per team)
    - ✓ONE submission per team. ALL team members will receive the same mark.
  - CW2 (15%):
    - ✓Group work (5 students per team)
    - ✓ONE submission per team. ALL team members will receive the same mark.
- Final Exam (3 hours, **70%**): **No** MCQs
  - 60% Electromagnetism + 40% Electromechanics
- Resit Exam (3 hours, **100%**): Same as the Final
  - 60% Electromagnetism + 40% Electromechanics

Continuous Assessment

- CW1:
  - ✓ Release Date: Week 5 Friday
  - ✓ Lab Time: Week 7 Tuesday for D1/1; Week 7 Thursday for D1/2
  - ✓ Assignment Deadline: Week 7 Sunday for D1/1; Week 8 Tuesday for D1/2

  You may form your own team of **exactly 4** people per team. Team information will be collected **before Week 6**.

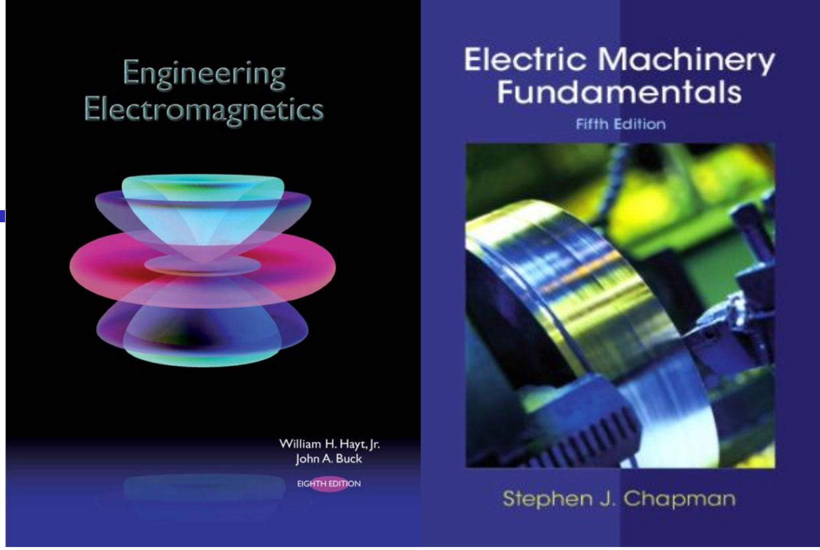
- CW2:
  - ✓ Release Date: Week 8 Friday
  - ✓ Lab Time: Week 10 Tuesday for D1/1; Week 10 Thursday for D1/2
  - ✓ Assignment Deadline: Week 10 Sunday for D1/1; Week 11 Tuesday for D1/2

  You may form your own team of **exactly 5** people per team. Team information will be collected **before Week 9**.

Important Notices

- Recording and broadcasting lectures.
  - The lectures will be recorded, but the recording won't be released every week. They will be supplied only at the end of weeks 6 and 12. Please be noticed that the recording are a resource to help study for the exam and not a substitute for attending class. And of course, since all students should be on-site, there won't be broadcasting link provided.
- Attendance policy.
  - The attendance will be recorded, and there will be consequences. Poor attendance could result in losing the opportunity to resit the module if failed in the final. Since our lectures run in two sessions, please make sure you are attending the correct one. If you have conflict timetabling issue, please contact the Registry to move you to another session.
- Week 7.
  - Week 7 will be treated as a non-teaching week, which mean we will not deliver new knowledge than. A tutorial or revision class might be arranged as optional class (no attendance required).
- Academic Integrity.
  - Presenting someone else's ideas as your own is a serious academic offense with serious consequences. There will be no tolerance, and cases detected will be reported to the department and school exam officers. Records of offences will be kept in system for your whole college life. Please be responsible for your own academic career.

Resources



- Learning Mall (Core):
  - Lecture materials and recorded lectures
  - Quiz, self-practice problems and answers
  - External links to useful resources

- Reference books
  - Engineering Electromagnetics 8th, by W.Hayt and J.Buck
  - Electromagnetic Field Theory Fundamentals 2nd, by B.Guru, H.Hiziroglu
  - Electric Machinery Fundamentals 5th , by Stephen J. Chapman

- Recommended Online Courses:
  - Electricity and Magnetism by Prof. Walter Lewin
  - 电磁场与电磁波 by 马西奎（西安交通大学）

Course outline

- Electromagnetism (Week 1–Week 6)
  - ✓ Vector Analysis
  - ✓ Maxwell's Equations: Static Fields
  - ✓ Nature of Materials
  - ✓ Steady Electric Current
  - ✓ Electromagnetic Induction (Faraday's Law)
  - ✓ Passive Components
- Electromechanics (Week 8–Week 12)
  - ✓ Electromagnetic Induction
  - ✓ Moving Coil Transducers
  - ✓ Linear Actuators
  - ✓ Transformers
  - ✓ DC & AC Rotating Machines

Electromagnetism

- What is Electromagnetism?
- Why do we learn it?

What is Electromagnetism?

- ***Electromagnetics*** (EM) – the subject that deals with the theory and applications of electric & magnetic fields and waves.
  - *Electromagnetism = Electromagnetics n.* 电磁学
- EM is the study of the effects of electric charges ***at rest*** and ***in motion***.
  - There are two kinds of charges: positive and negative, both are sources of electric fields.
  - Moving charges produce a current, which gives rise to a magnetic field.

Electrostatics [hide]

Electric charge • Static electricity
Electric field • Conductor
Insulator • Triboelectricity
Electrostatic discharge • Induction
Coulomb's law • Gauss's law
Electric flux / potential energy
Electric dipole moment
Polarization density

Magnetostatics [hide]

Ampère's law • Magnetic field
Magnetization • Magnetic flux
Biot–Savart law
Magnetic dipole moment
Gauss's law for magnetism

Electrodynamics [hide]

Lorentz force law
Electromagnetic induction
Faraday's law
Lenz's law • Displacement current
Maxwell's equations
Electromagnetic field
Electromagnetic radiation
Maxwell tensor • Poynting vector
Liénard–Wiechert potential
Jefimenko's equations
Eddy current

Maxwell's Equations

- "A Dynamical Theory of the Electromagnetic Field"
  - By J. C. Maxwell, in 1864.
- Maxwell's equations are a set of **four** partial differential equations that describe the properties of the electric and magnetic fields and relate them to their sources, charge density and current density.

① Gauss' law for E-fields $\nabla \cdot \vec{D} = \rho$

② Gauss' law for H-fields $\nabla \cdot \vec{B} = 0$

③ Faraday's law of induction $\nabla \times \vec{E} = 0 - \dfrac{\partial \vec{B}}{\partial t}$

④ Ampere's law $\nabla \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

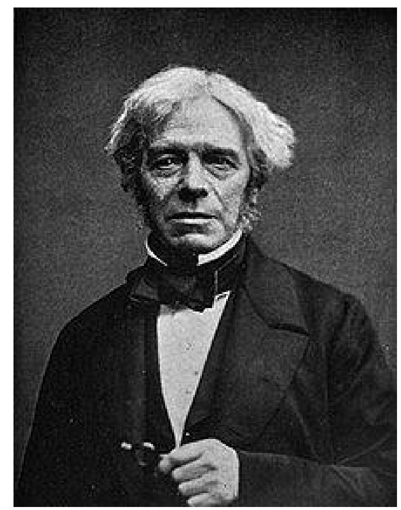
Important Scientists

Hendrik Lorentz

James C. Maxwell

André-Marie Ampère

Hermann von Helmholtz



Michael Faraday

Why do we study EM?

- Do we know the answers to these questions?
  - What is a field? What is the nature of a field?
    - Is it a scalar field or a vector field? Is it a continuous or a rotational field?
  - What is electric monopole? What is electric dipole?
  - What is magnetic dipole? Does magnetic monopole exist?
  - What are the properties of materials?
  - What is EM shielding?
  - How does free electrons move inside a conductor?
  - How is the magnetic field produced by current-carrying coils?
  - How does a capacitor/inductor store energy?

Why do we study EM?

- Electromagnetic principles and laws **govern all electrical and computer engineering systems**.

Electromagnetics is everywhere!

Applications

Part 2 Mathematic Review

- 1. Scalars and Vector
  - Definition and Representation
  - Vector Algebra
  - Scalar and Vector Fields

- 2. 2D Coordinate Systems
  - Rectangular CS and Polar CS
  - Conversion between Rect. CS and Polar CS
  - Vector Algebra in 2D CSs

1. Scalars and Vectors

1.1 Definitions of Scalar and Vector

1.2 Vector Representation

1.3 Vector Algebra

1.4 Scalar and Vector Fields

1.1 Definition - Scalar (标量)

- A scalar is **completely** specified by its magnitude

- Require **two** things:
  - A value (positive or negative)
  - Appropriate unit

- Examples:
  - Distance: 65 km, 0.05 mm.
  - Speed: 100 km/h
  - Mass: 5 kg, 1.53 ton.
  - Temperature: 20° C, -32° F
  - Voltage: 10 kV, -3 mV.

1.1 Definition - Vector (矢量)

- A vector is specified by both its magnitude (with unit) and a direction.

- Require **three** things:
  - A value (positive or negative)
  - Appropriate unit
  - A direction!

- Examples:
  - Displacement: 5 meters backward
  - Velocity: 5 mph west
  - Force: 10 N up

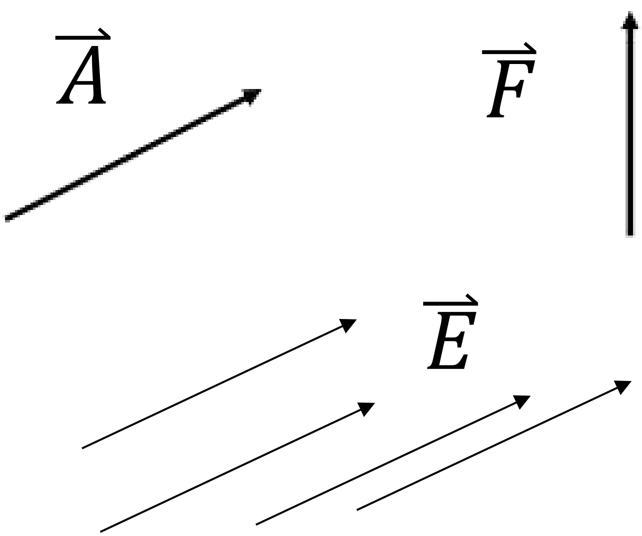
- Notations:
  - Most widely used in books: bold **A**
  - Also written as: $\vec{A}$, $\widehat{A}$, $\bar{A}$, $\underline{\mathrm{A}}$
  - The magnitude of a vector $\vec{A}$: $|\vec{A}|$ or $|\boldsymbol{A}|$

1.2 Vector Representation

- Vectors are represented graphically or quantitatively:

Graphically:

Through arrows with the orientation representing the direction and length representing the magnitude



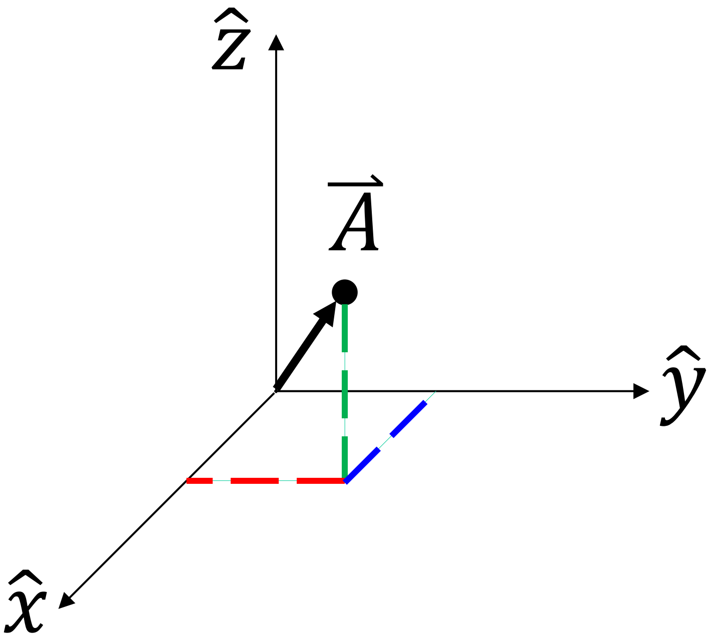
Quantitatively:

A <u>right-hand</u> coordinate system having orthogonal axes is usually chosen:

- ➢ Cartesian / Rectangular
- ➢ Cylindrical
- ➢ Spherical

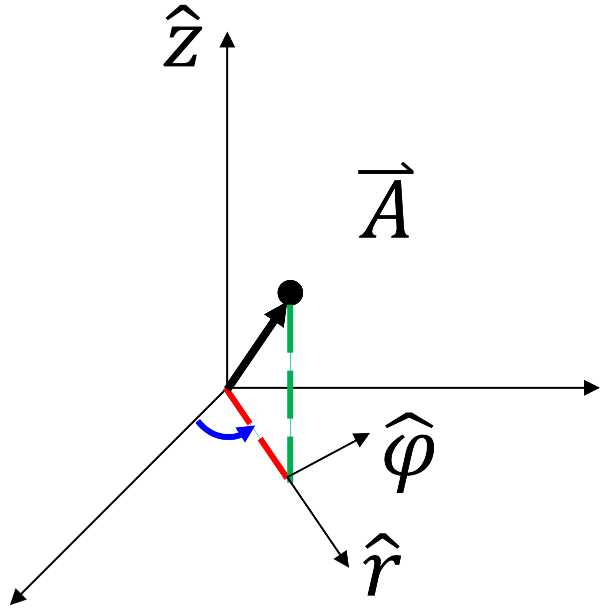
1.2 Vector Representation

$\vec{A} = (x,\ y,\ z)$



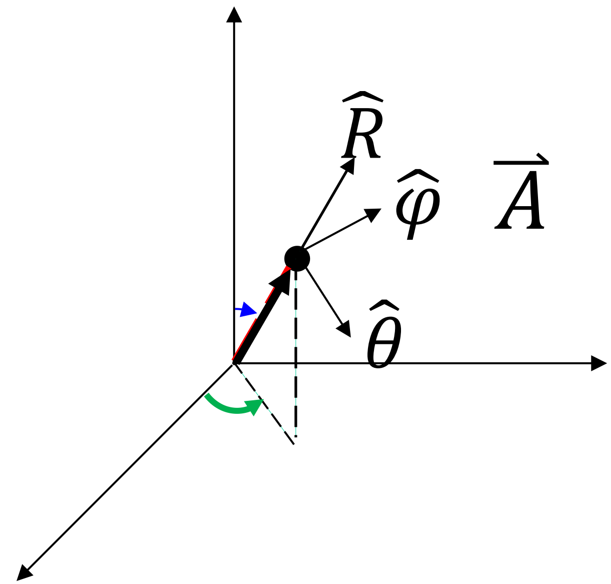
Rectangular C.S.

$\vec{A} = (r, \varphi,\ z)$



Cylindrical C.S.

$\vec{A} = (R, \theta, \varphi)$



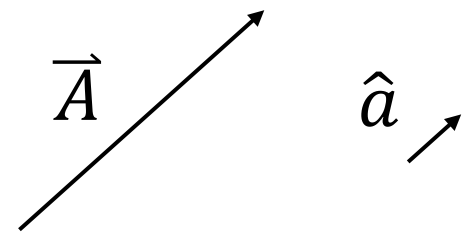
Spherical C.S.

$$\vec{A} = (1,\ 1,\ 1)$$

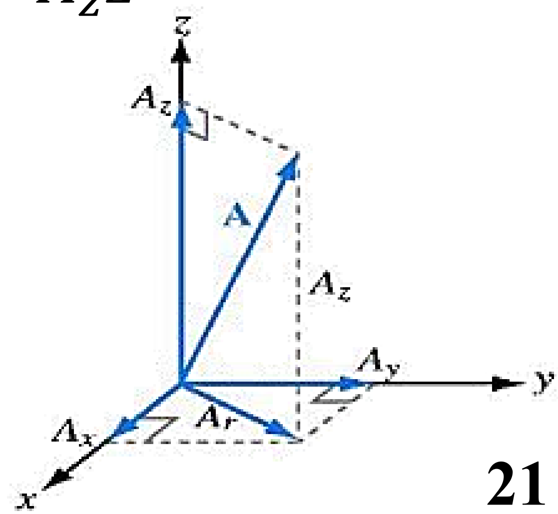
1.2 Vector Representation

- Represent a vector $\vec{A}$ quantitatively by the magnitude $|\vec{A}|$ and the direction:

$$\vec{A} = \hat{a}|\vec{A}| = \hat{a}A, \text{ where } \hat{a} = \frac{\vec{A}}{|\vec{A}|}$$



  - $\hat{a}$ is called unit vector, with the same direction as $\vec{A}$, but magnitude is 1.
  - In Cartesian coordinates: (if) $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$



$$\therefore |\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$$

$$\hat{a} = \frac{A_x}{|\vec{A}|}\hat{x} + \frac{A_y}{|\vec{A}|}\hat{y} + \frac{A_z}{|\vec{A}|}\hat{z}$$

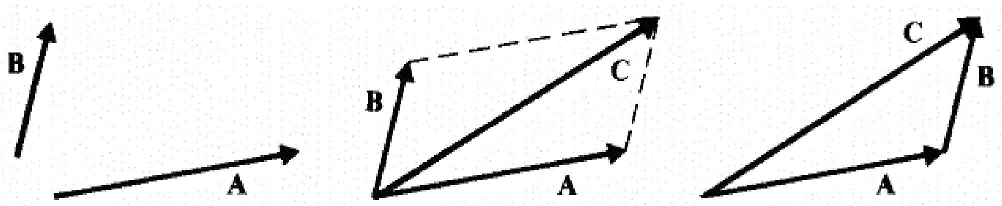
1.3 Vector Algebra - Addition and Subtraction

- Addition

$$\vec{C} = \vec{A} + \vec{B}:$$

  – 1. Parallelogram rule

  – 2. Head-to-tail rule



(a) Two vectors, **A** and **B**. (b) Parallelogram rule. (c) Head-to-tail rule.

Commutative law: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

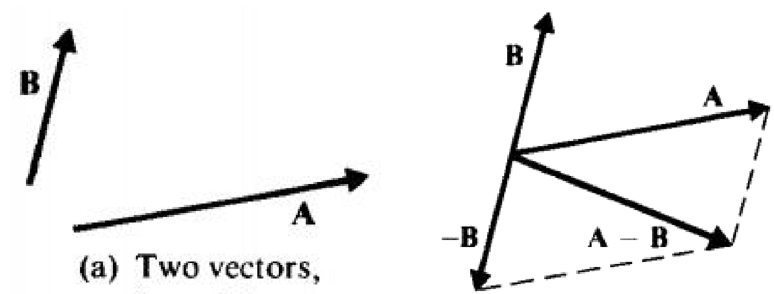
Associative law: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

- Subtraction

$$\vec{C} = \vec{A} - \vec{B}:$$

- Vector subtraction is defined as:

$$\vec{C} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



(a) Two vectors, **A** and **B**.

(b) Subtraction of vectors, **A** − **B**.

- In Cartesian coordinates:

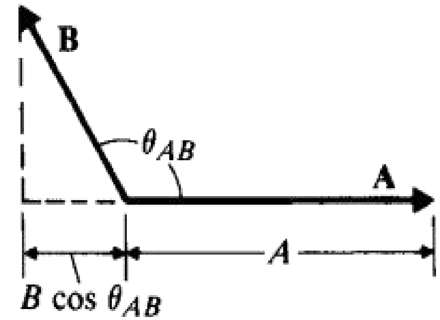
If $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ and $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$

$\rightarrow \vec{A} \pm \vec{B} = (A_x \pm B_x)\hat{x} + (A_y \pm B_y)\hat{y} + (A_z \pm B_z)\hat{z}$

1.3 Vector Algebra - Dot Product

- Dot product
  - Results: scalar
  - Operator: "dot"
  - θ_{AB} ($< 180°$): the smaller angle between $\vec{A}$ & $\vec{B}$

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos(\theta_{AB})}$$



Commutative: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Distributive: $\vec{A} \cdot (\vec{B} + \vec{C}) = (\vec{A} \cdot \vec{B}) + (\vec{A} \cdot \vec{C})$

$\vec{A} \cdot \vec{A} = A^2$

$\vec{A} \perp \vec{B} <=> \vec{A} \cdot \vec{B} = 0$

$=> \hat{x} \cdot \hat{y} = 0$

$=> \hat{x} \cdot \hat{z} = 0$

$=> \hat{y} \cdot \hat{z} = 0$

- In Cartesian coordinates:

If $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ & $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$

$=> \vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

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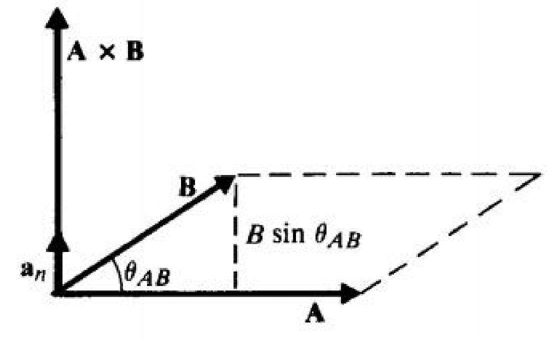
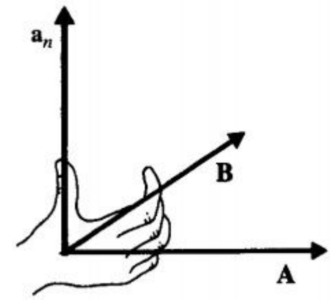
1.3 Vector Algebra - Cross Product-1

- Cross product
  - Result: vector
  - Operator: "cross"
  - θ_{AB}: the angle rotating from $\vec{A}$ to $\vec{B}$

$$\boxed{\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin(\theta_{AB})}$$



(a) $\mathbf{A} \times \mathbf{B} = \mathbf{a}_n|AB \sin\theta_{AB}|$.



(b) The right-hand rule.

- Not commutative: $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
- Distributive: $\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$
- Not associative: $\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$

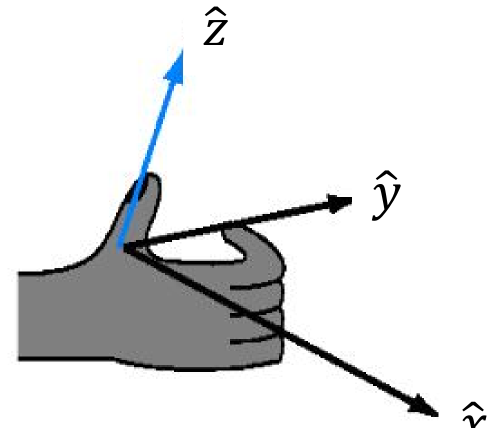
$$\vec{A} \times \vec{A} = 0$$
$$\vec{A}//\vec{B} <=> \vec{A} \times \vec{B} = 0$$

$$\hat{x} \times \hat{x} = 0$$
$$\hat{y} \times \hat{y} = 0$$
$$\hat{z} \times \hat{z} = 0$$

$$\hat{x} \times \hat{y} = \hat{z}$$
$$\hat{y} \times \hat{z} = \hat{x}$$
$$\hat{z} \times \hat{x} = \hat{y}$$

1.3 Vector Algebra - Cross Product-2

- In Cartesian coordinates:



Right hand rule

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \text{ and } \vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

$$\begin{aligned}\vec{A} \times \vec{B} = {} & \cancel{(\hat{x} \times \hat{x}A_xB_x)} + (\hat{x} \times \hat{y}A_xB_y) + (\hat{x} \times \hat{z}A_xB_z) \\ & + (\hat{y} \times \hat{x}A_yB_x) + \cancel{(\hat{y} \times \hat{y}A_yB_y)} + (\hat{y} \times \hat{z}A_yB_z) \\ & + (\hat{z} \times \hat{x}A_zB_x) + (\hat{z} \times \hat{y}A_zB_y) + \cancel{(\hat{z} \times \hat{z}A_zB_z)}\end{aligned}$$

- Reduce to

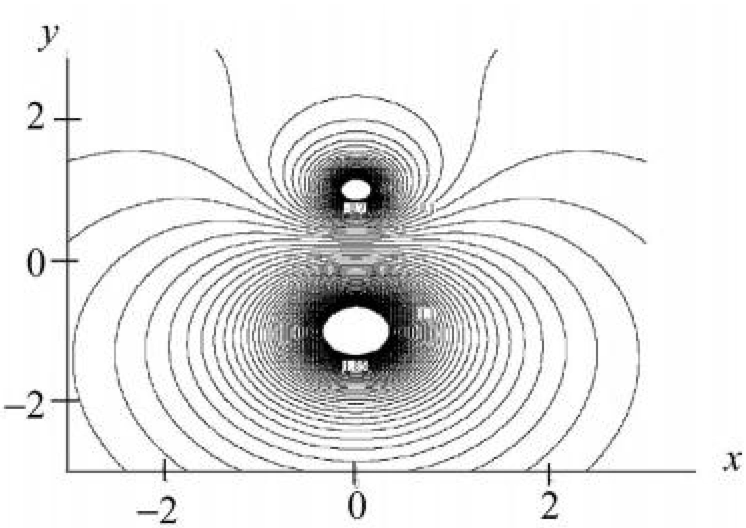
$$\vec{A} \times \vec{B} = \hat{x}(A_yB_z - A_zB_y) + \hat{y}(A_zB_x - A_xB_z) + \hat{z}(A_xB_y - A_yB_x)$$

- Expressed concisely in determinant form as:

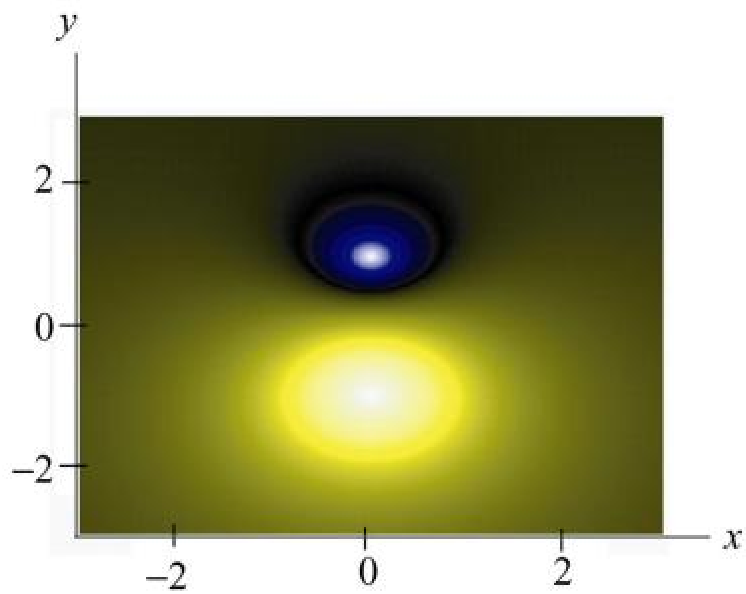
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

1.4 Fields - Scalar Fields

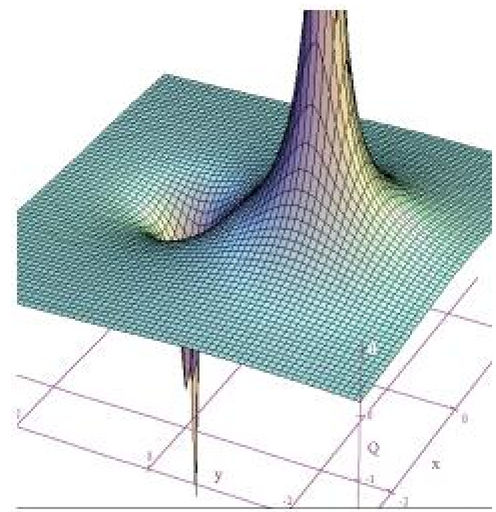
- A ***scalar field*** is a function that gives us a single value of some variable for every point in space (Two dimension or three-dimension).



Contour Map



Color-Coding



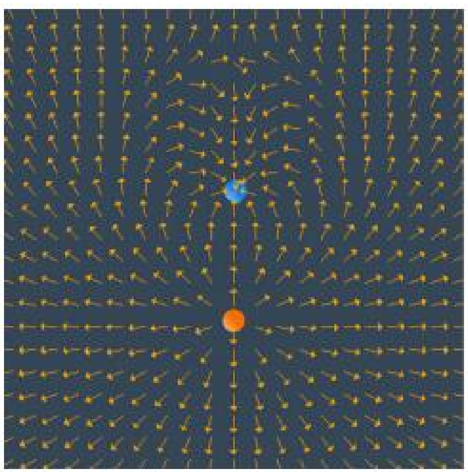
Relief Map

1.4 Fields - Vector Fields

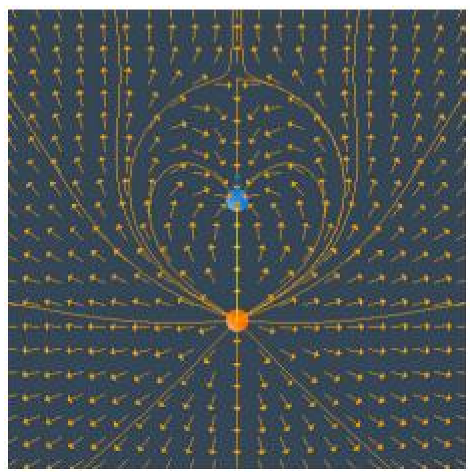
- A ***vector field*** is a construction, which associates a vector to every point in a (locally) Euclidean space.
  – It is uniquely specified by giving its *divergence* and *curl* within a region and its normal component over the boundary.

The field of two point charges, one negative and one positive: $Q^{+} = 3 \times Q^{-}$

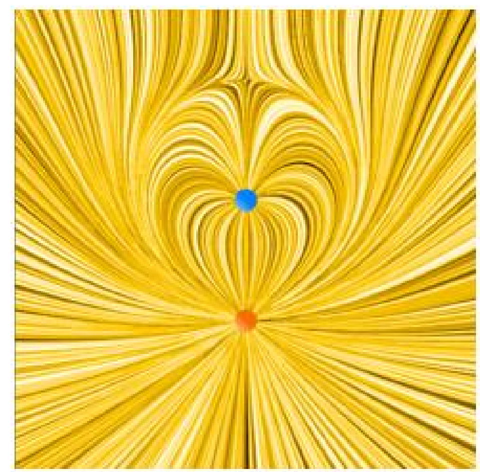
Field vector representation



Field line representation



Grass seeds representation



Quiz 1

1. $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C}$ implies that $\vec{B}$ must always be equal to $\vec{C}$.
   – (a) Ture
   – (b) False

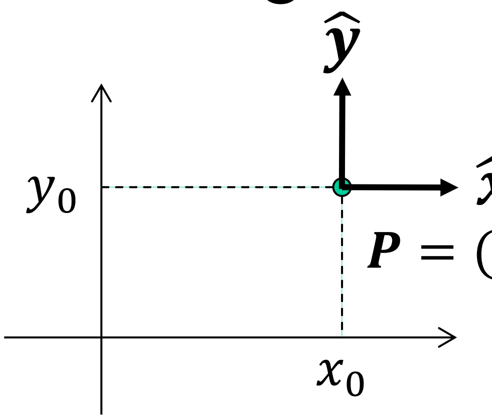
2. Given the vectors
   $\vec{B} = -10\hat{x} + 4\hat{y} - 8\hat{z}$, and $\vec{C} = 8\hat{x} + 7\hat{y} - 2\hat{z}$.
   Find the **unit** vector in the direction of $\vec{A} = 2\vec{C} - \vec{B}$.

2. Coordinate Systems (CS)

- Two-Dimensional (2D) Coordinate Systems (CSs)
  - 2.1 Rectangular CS vs. Polar CS
    - Orthogonality
    - Range of values
    - Cutting lines
  - 2.2 Conversion between the CSs
    - Values
    - Unit vectors
  - 2.3 Vector Algebra in 2D CSs (二维坐标系)
    - Dot and Cross Products

2.1 Rectangular (直角坐标系) vs. Polar (极坐标系)

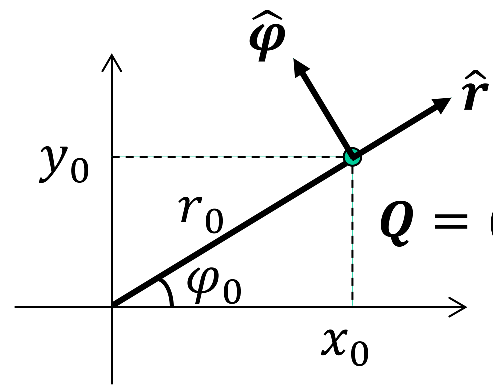
- Rectangular C.S.



$\boldsymbol{P} = (x_0, y_0) = x_0\hat{\boldsymbol{x}} + y_0\hat{\boldsymbol{y}}$

- Unit vectors $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ (or $\boldsymbol{x}$ or $\boldsymbol{a}_x$ or $\boldsymbol{e}_x$)
  - Orthogonal: $\hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{y}} = 0$
  - Unit length: $\hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{x}} = \hat{\boldsymbol{y}} \cdot \hat{\boldsymbol{y}} = 1$
  - Constant unit vectors: no matter how x_0 and y_0 changes, $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are unchanged
    => Cartesian Coordinate Sys.

- Polar C.S.



$\boldsymbol{Q} = (r_0, \varphi_0) = r_0\hat{\boldsymbol{r}} + \varphi_0\hat{\boldsymbol{\varphi}}$

- Unit vectors $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\varphi}}$
  - Orthogonal: $\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{\varphi}} = 0$
  - Unit length: $\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{r}} = \hat{\boldsymbol{\varphi}} \cdot \hat{\boldsymbol{\varphi}} = 1$
  - Not constant unit vectors: with different r_0 and φ_0, $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\varphi}}$ are different everywhere.

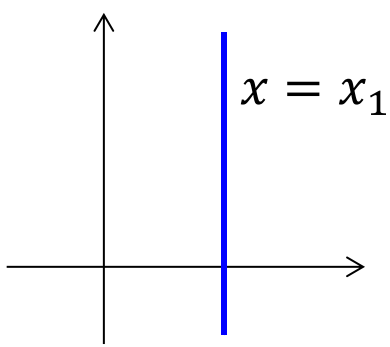
2.1 Rectangular vs. Polar

Range of values

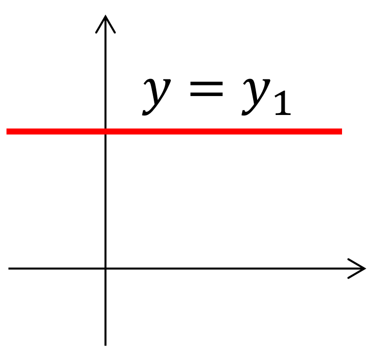
- Rectangular
  - $x_0 \in \mathbb{R}: (-\infty, +\infty)$
  - $y_0 \in \mathbb{R}: (-\infty, +\infty)$
- Polar
  - $r_0 \in [0, +\infty)$
  - $\varphi_0 \in [0, 2\pi)\ or\ (-\infty, +\infty)$

Cutting lines

- Rectangular

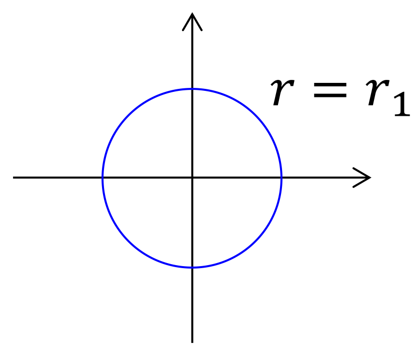


vertical line
perpendicular to x-axis

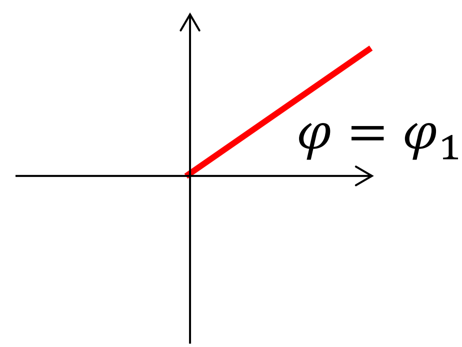


horizontal line
perpendicular to y-axis

- Polar



circle
perpendicular to r-axis



ray
perpendicular to φ-axis

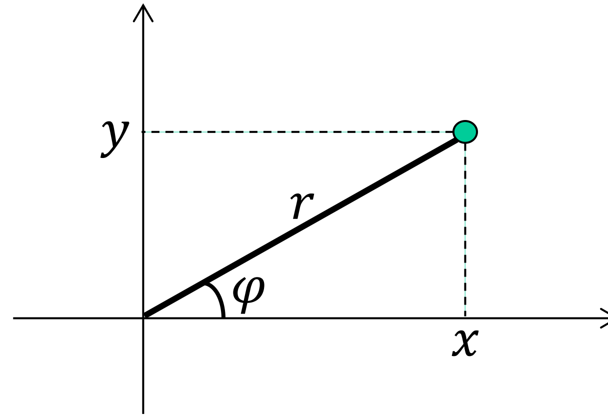
2.2 Conversion between the CSs - Values

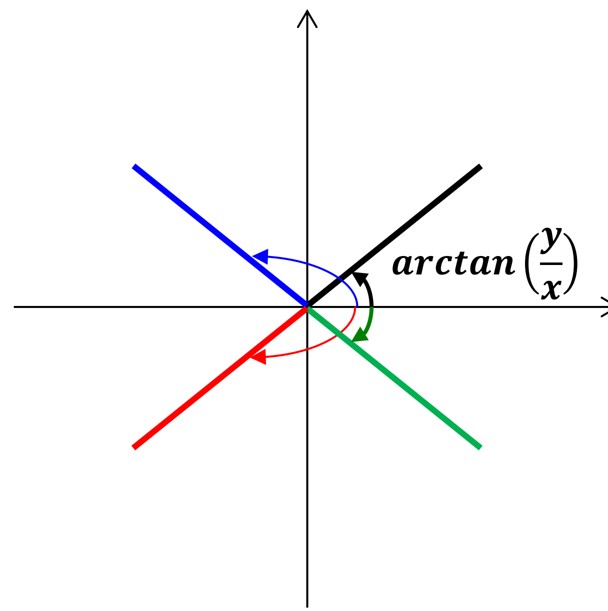
- From Rectangular CS to Polar CS

$$r = \sqrt{x^2 + y^2}$$

$$\varphi = arctan\left(\frac{y}{x}\right)$$

  - $\varphi \in [0, 2\pi)$, but $arctan\left(\frac{y}{x}\right) \in [0, \frac{\pi}{2})$. So further discussion is needed based on the sign of x and y.





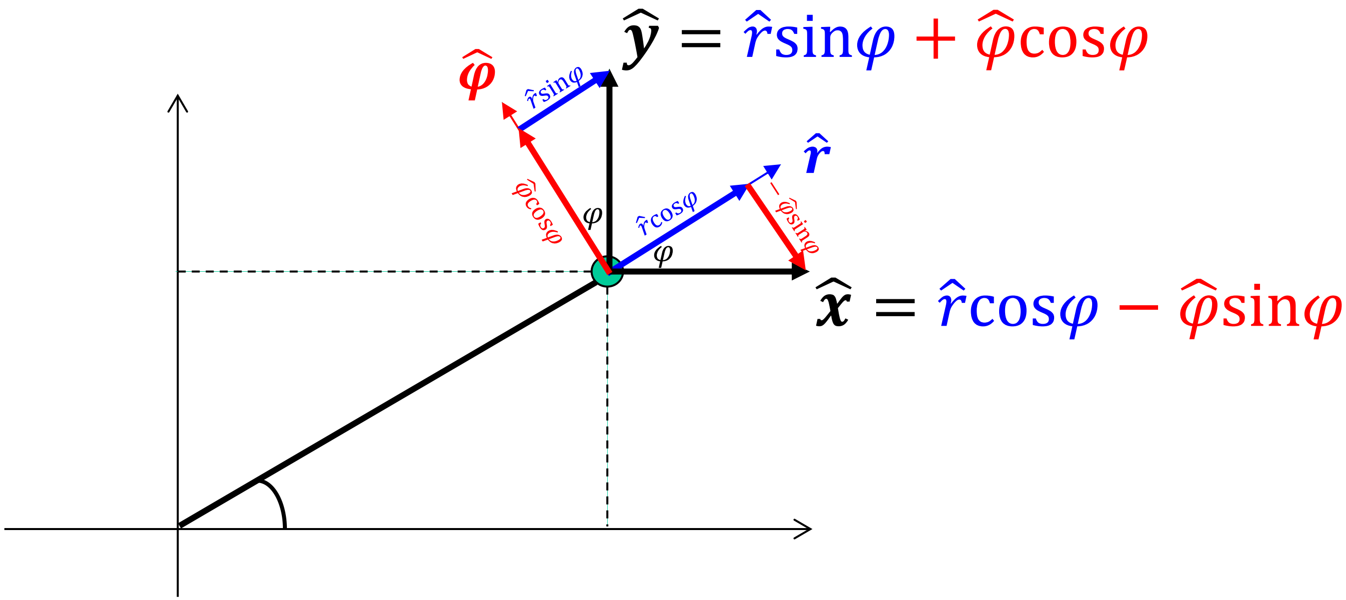
- From Polar CS to Rectangular CS

$$x = r\cos\varphi$$

$$y = r\sin\varphi$$

2.2 Conversion between the CSs - Whole vector

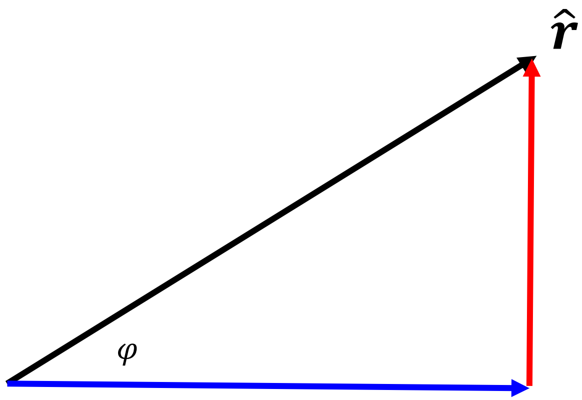
- How to express a vector in Rectangular CS $A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}}$ in Polar CS, like $A_r\hat{\boldsymbol{r}} + A_\varphi\hat{\boldsymbol{\varphi}}$?

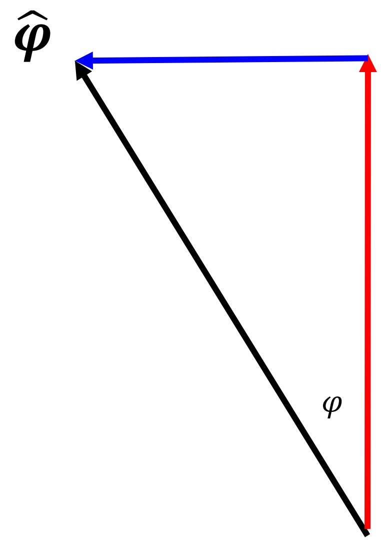


$$\begin{aligned} A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} &= A_x(\hat{r}\cos\varphi - \hat{\varphi}\sin\varphi) + A_y(\hat{r}\sin\varphi + \hat{\varphi}\cos\varphi) \\ &= \hat{\boldsymbol{r}}(A_x\cos\varphi + A_y\sin\varphi) + \hat{\boldsymbol{\varphi}}(A_y\cos\varphi - A_x\sin\varphi) \\ &= \hat{\boldsymbol{r}}A_r + \hat{\boldsymbol{\varphi}}A_\varphi \end{aligned}$$

2.2 Conversion between the CSs - Whole vector

- Similarly, you may try to express $A_r\hat{\boldsymbol{r}} + A_\varphi\hat{\boldsymbol{\varphi}}$ in the Rectangular CS.





- The anwer is:

$$A_r\hat{\boldsymbol{r}} + A_\varphi\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{x}}(A_r\cos\varphi - A_\varphi\sin\varphi) + \hat{\boldsymbol{y}}(A_r\sin\varphi + A_\varphi\cos\varphi)$$

2.3 Vector Algebra in 2D CSs

- Rectangular
  - Dot Product

$$\hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{y}} = 0$$

$$\hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{x}} = \hat{\boldsymbol{y}} \cdot \hat{\boldsymbol{y}} = 1$$

$$(A_x \hat{\boldsymbol{x}} + A_y \hat{\boldsymbol{y}}) \cdot (B_x \hat{\boldsymbol{x}} + B_y \hat{\boldsymbol{y}}) = A_x B_x + A_y B_y$$

  - Cross Product

$$\hat{\boldsymbol{x}} \times \hat{\boldsymbol{y}} = 1 \text{ (perpendicular to x} - \text{y plane)}$$

$$\hat{\boldsymbol{x}} \times \hat{\boldsymbol{x}} = \hat{\boldsymbol{y}} \times \hat{\boldsymbol{y}} = 0$$

$$(A_x \hat{\boldsymbol{x}} + A_y \hat{\boldsymbol{y}}) \times (B_x \hat{\boldsymbol{x}} + B_y \hat{\boldsymbol{y}}) = A_x B_y - A_y B_x$$

- Polar
  - Dot Product

$$\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{\varphi}} = 0$$

$$\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{r}} = \hat{\boldsymbol{\varphi}} \cdot \hat{\boldsymbol{\varphi}} = 1$$

$$(A_r \hat{\boldsymbol{r}} + A_\varphi \hat{\boldsymbol{\varphi}}) \cdot (B_r \hat{\boldsymbol{r}} + B_\varphi \hat{\boldsymbol{\varphi}}) = A_r B_r + A_\varphi B_\varphi$$

  - Cross Product

$$\hat{\boldsymbol{r}} \times \hat{\boldsymbol{\varphi}} = 1 \text{ (perpendicular to r} - \varphi \text{ plane)}$$

$$\hat{\boldsymbol{r}} \times \hat{\boldsymbol{r}} = \hat{\boldsymbol{\varphi}} \times \hat{\boldsymbol{\varphi}} = 0$$

$$(A_r \hat{\boldsymbol{r}} + A_\varphi \hat{\boldsymbol{\varphi}}) \times (B_r \hat{\boldsymbol{r}} + B_\varphi \hat{\boldsymbol{\varphi}}) = A_r B_\varphi - A_\varphi B_r$$

Next ...

- Mathematic Preparation
  - 3D Coordinate Systems
    - Rectangular to 3D Catesian
    - Polar to 3D Cylindrical
    - Spherical CS
  - Vector Analysis
    - Line/Surface/Volume Integrals
    - Differential Elements in Three CSs
    - Operations (gradient, divergence, curl and Laplacian)
    - Theorems (Gaussian and Stokes)