Xi'an Jiaotong-Liverpool University
西交利物浦大学

MEC208 Instrumentation and Control System

2024-25 Semester 2

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MEC208 office hour: Thursday, 2-4pm

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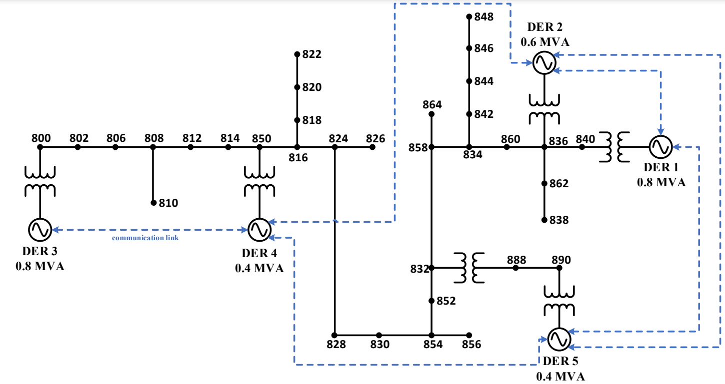
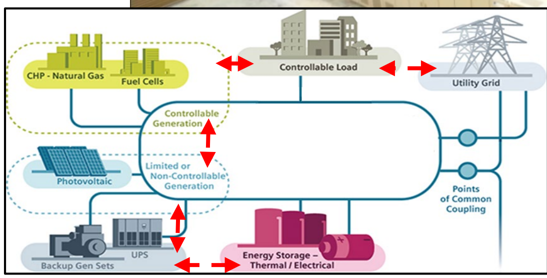
Biography - Chee Shen LIM

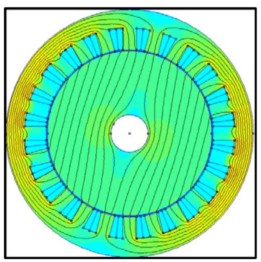
- Obtained my PhD degrees in 2013. I was a research scientist in Singapore, and then an Assistant, then Associate Professor at University of Southampton (Malaysia campus). Joined XJTLU in 2021. My research area in the past 10 years focuses on power electronics, electrical machines and drives, optimal control and energy management (e.g., renewable energies and electric vehicles) and microgrid's hierarchical control.

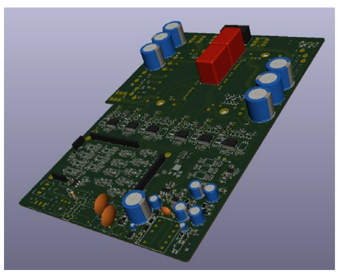
- Currently the Associate Professor and the Programme Director of BEng Electrical Engineering (UEEA). Externally, I am also the
  - Charted Engineer (CEng), Engineering Council (UK)
  - Senior Member, Institute of Electrical and Electronics Engineers (IEEE, US)
  - Member, Institute of Engineering and Technology (IET, UK)
  - Certified Grid PV System Designer, Sustainable Energy Dev. Agency SEDA (Malaysia)
  - Fellow, Advance Higher Education (HEA, UK)

“Visual” of my research area

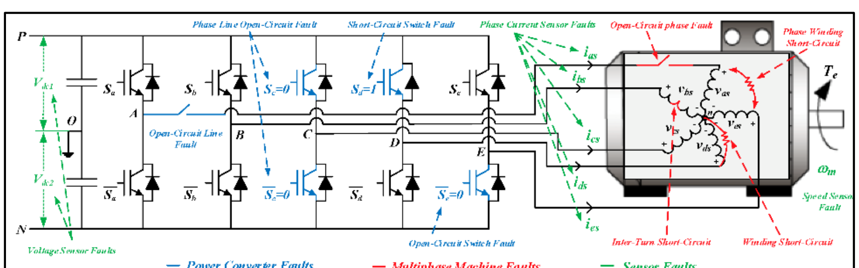








Research area: power electronics, electrical machines and drives, optimal control and energy management (e.g., renewable energies and electric vehicles) and microgrid’s hierarchical control.



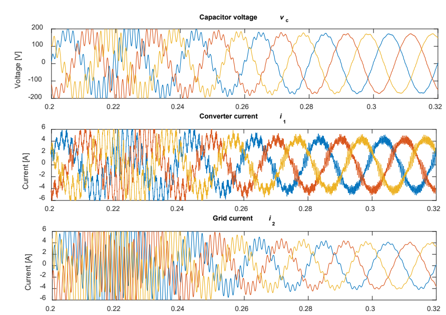



Figure shows the overview of Solar PV (BIPV) at the roof top building.

What content have you covered so far …

- Your learning journey in MEC208:
  - Week 1-3: Instrumentation
  - Week 4: Mathematical model, transfer function, signal flow graph, block diagram
  - Week 5: State-space modelling
  - Week 6: Characteristics of feedback control systems
  - Week 8: Time-domain performance of feedback control systems
  - Week 9: Stability of feedback control systems (basic concept, Routh Hurwitz Stability Criterion, etc.)
  - Week 9/10-11: Root locus method (basic concept, RL plot, PID control design & tuning, etc.)
  - Week 11: Lab sessions on Monday and Wednesday
  - Week 11-12: Frequency response methods (mainly Bode plot, and some about Polar/Nyquist plot)

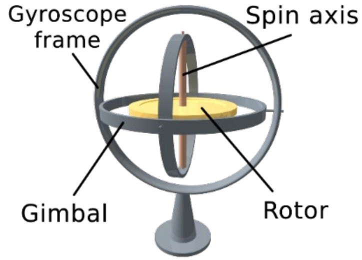
Perhaps its a good time to take a pause, and reflect on this simple question:

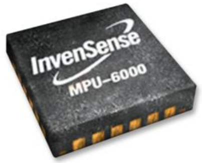
what is control system?

Lawrence Sperry, 1914 – Gyroscopic stabilizer enabled autopilot

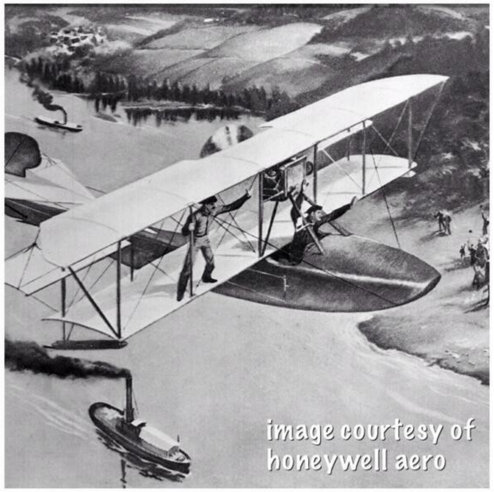
Onlookers surround the Curtiss C-2 in which 21-year-old Lawrence Sperry demonstrated his Sperry gyroscopic stabilizer to win the Concours de la Securité en Aéroplane on June 18, 1914.

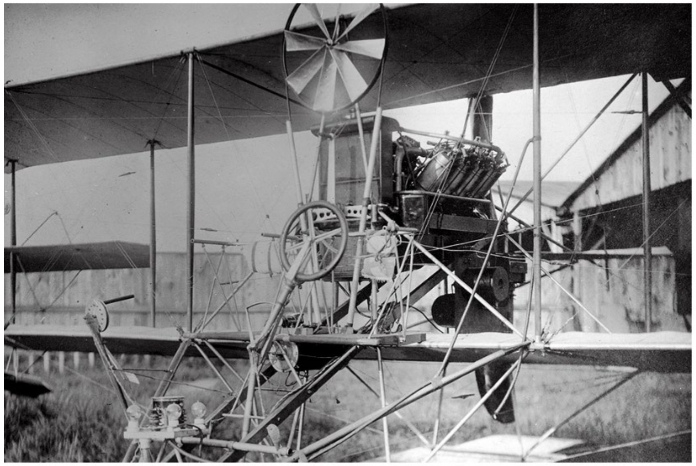
Gyroscope (mechanical vs IC)





The MPU-6050 device combines a 3-axis gyroscope and a 3-axis accelerometer on the same silicon die together with an onboard Digital Motion Processor (DMP) capable of processing complex 9-axis MotionFusion algorithms. The parts' integrated 9-axis MotionFusion algorithms access external magnetometers or other sensors through an auxiliary master I2C bus, allowing the devices to gather a full set of sensor data without intervention from the system processor.

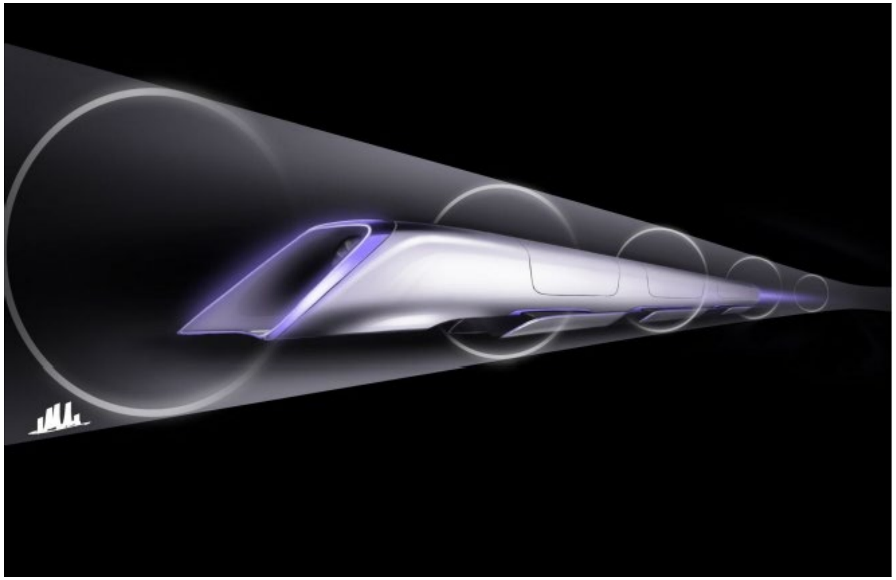




A view of the Sperry autopilot installed on an early Curtiss biplane at Hammondsport, N.Y., where Lawrence received his pilot's license in 1913. (Glenn H. Curtiss Museum)

Pictures from internet

Hyperloop (Elon Musk?), 1799/2013: Airgap control through e.g. maglev linear motor or air bearing



Elon Musk's Hyperloop concept

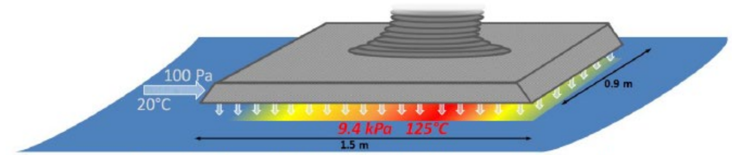
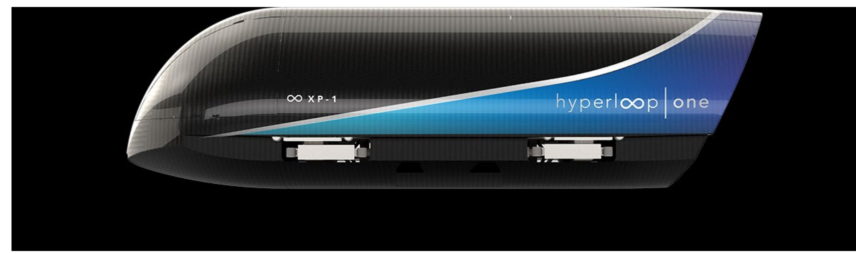
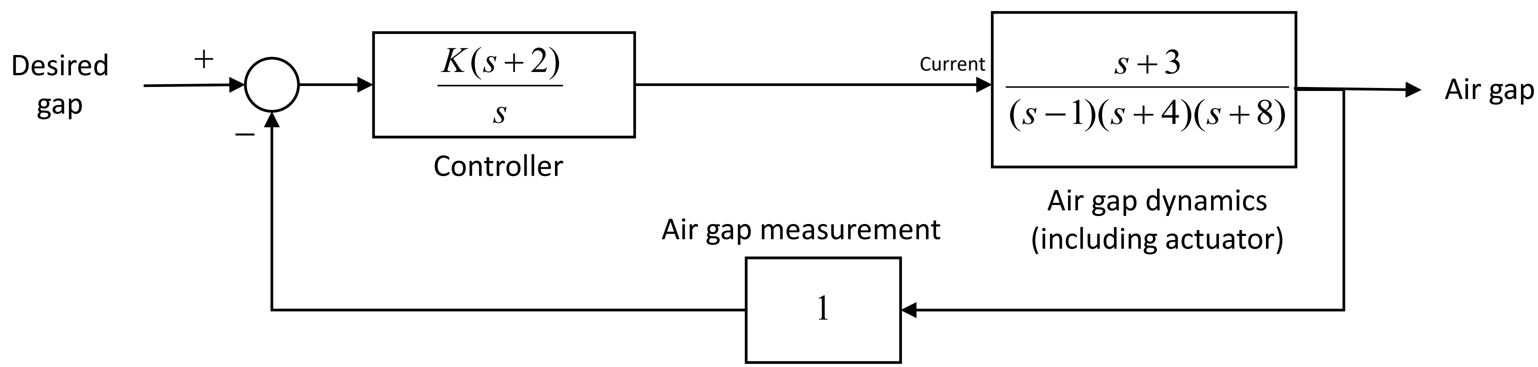


Figure 12: Schematic of air bearing skis that support the capsule.

Train-track suspension [from Hyperloop whitepaper]



Hyperloop One prototype [from website https://hyperloop-one.com/]



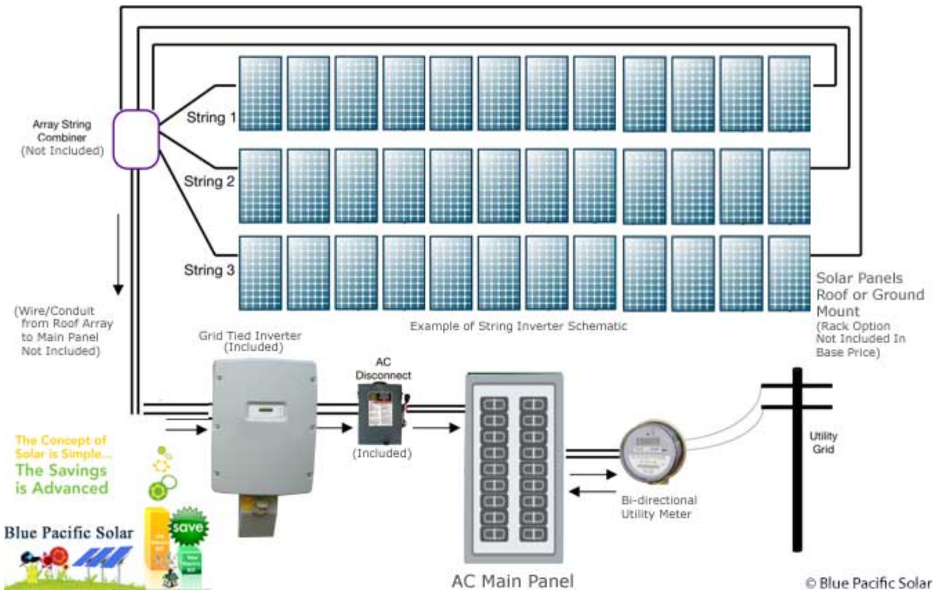
Fictitious example: magnetic levitation, or "air-bearing" in linear motor

University of Zurich, 2021: Advanced model predictive tracking control

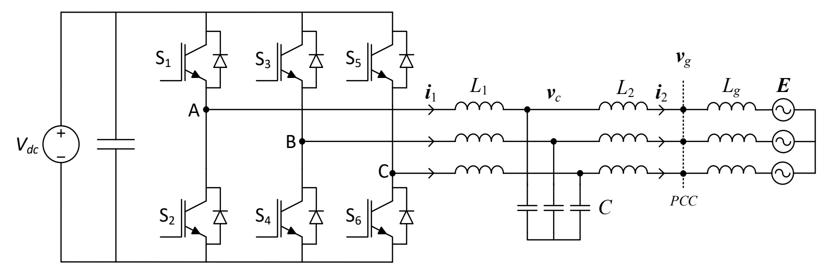
Now/future?

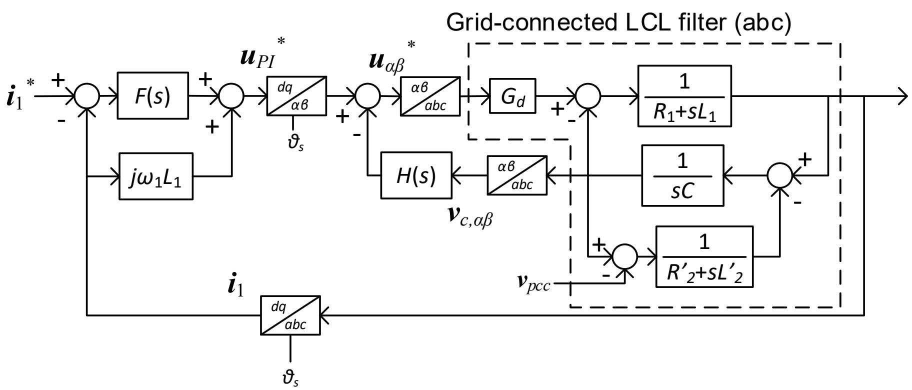
Pictures from Tesla video

Solar energy conversion - 3ph current/power control using power electronic-based grid inverter



Example: German's SMA inverter





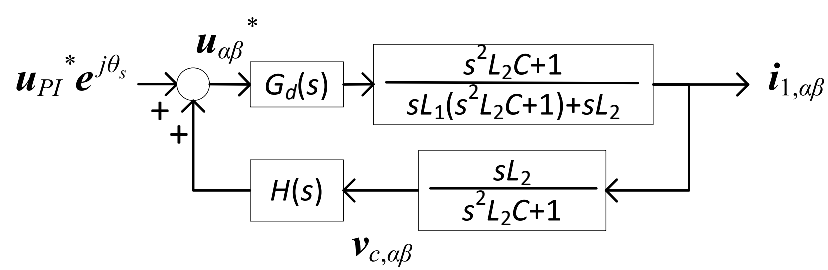


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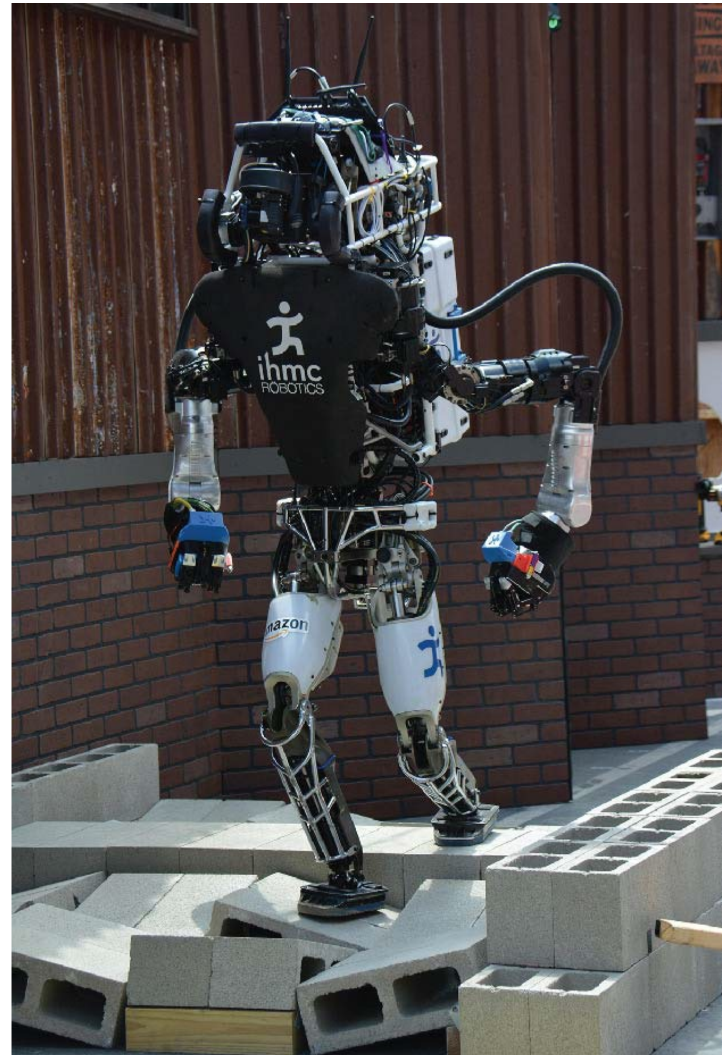
Figure 6.3 Vigorous exercising on the 12th floor likely led to mechanical resonance of the building triggering a two-day evacuation. [Seoul, Korea]

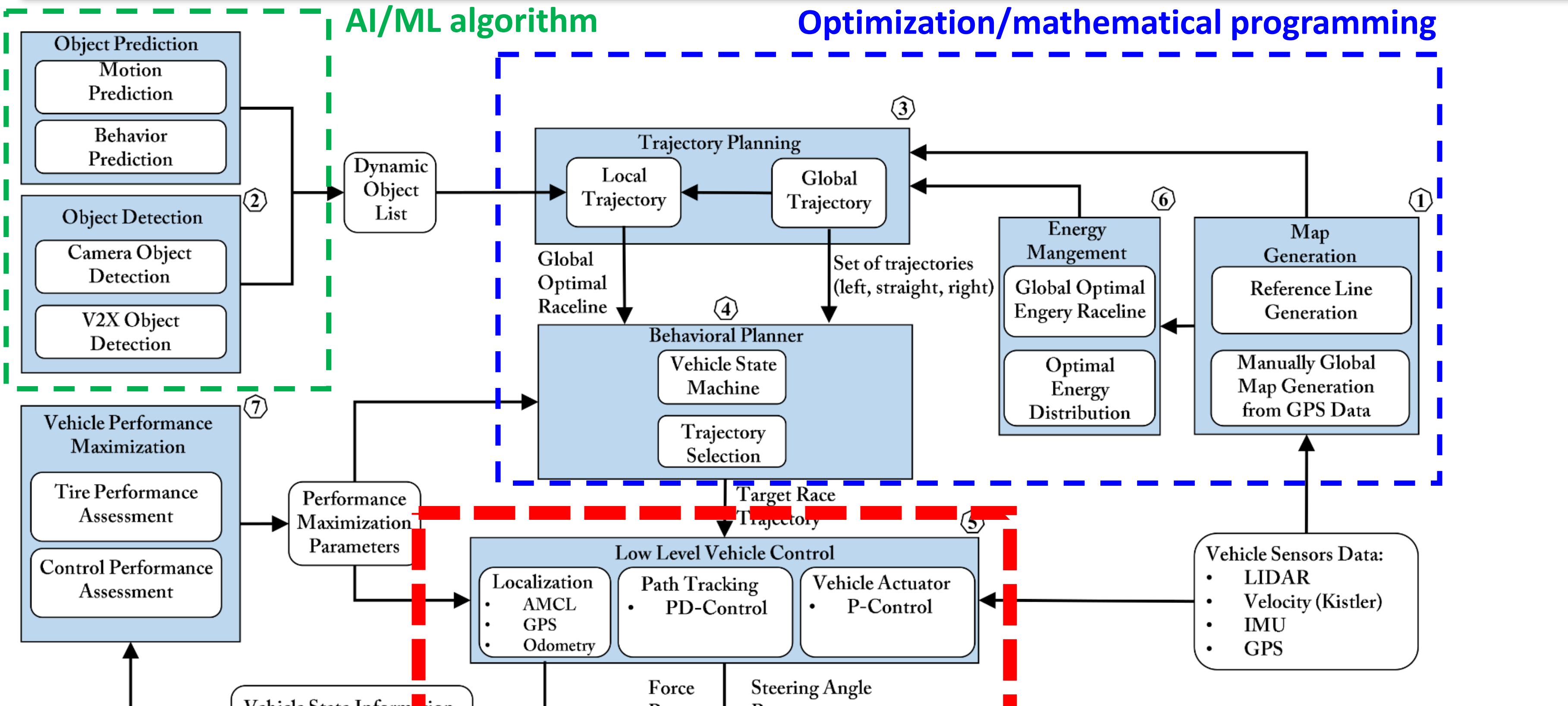
Figure 6.4 Team *IHMC* on the rubble on the first day of the DARPA Robotics Challenge 2015. [US]



What may happen when a control system fails or becomes “unstable”?

Elon Musk's SpaceX, 2014: Engine sensor failure leading to control system failure

Where can you find "control system"? E.g., a "software stack" for autonomous driving (it's a very complex engineering system!)



(a) Devbot 2.0 (b) Devbot with sensors setup

Dynamical control – usually highly dynamics hence requires the most accurate control (means very challengings!)

Johannes Betz, Alexander Heilmeier, Alexander Wischnewski,Tim Stahl and Markus Lienkamp, "Autonomous Driving—A Crash Explained in Detail" Applied Sciences, MDPI

Lecture 15

Outline

Stability of Linear Feedback Systems

- ❑ **The Concept of Stability**
- ❑ **Routh-Hurwitz Stability Criterion**
- ❑ Relative Stability of Feedback Control Systems
- ❑ Stability of State Variable Systems
- ❑ System Stability Using Matlab

Stability

A stable system is a dynamic system with a bounded (limited) response to a bounded input.

❖ When considering the design and analysis of feedback control systems, **stability** is an important feature to be established at the beginning. From a practical point of view, a closed-loop (CL) feedback system that is unstable is usually of less practical value (but not always).

❖ Many physical systems are inherently open-loop unstable. Using feedback, we can stabilize the unstable systems and then with a further judicious design of controller gains, we can adjust the transient performance.

❖ For open-loop stable systems, we can use feedback to adjust the CL performance to meet the design specifications (e.g., percent overshoot, settling time, steady-state error etc.).

❖ Two types of stability criteria:
- **absolute stability** – informs whether the system is stable or not stable
- **relative stability** – informs on the degree of stability

Conditions for A Feedback System To Be Stable

In terms of linear systems, stability requirement may be defined in terms of the location of the poles of the CL transfer function which can be written as

$$T(s) = \frac{p(s)}{q(s)} = \frac{K\prod_{i=1}^{M}(s+z_i)}{s^N \prod_{k=1}^{Q}(s+\sigma_k)\prod_{m=1}^{R}[s^2+2\alpha_m s+(\alpha_m^2+\omega_m^2)]}$$

where $q(s) = 0$ is the characteristic equation, with its roots being the poles of the CL system. The output response for an impulse function input (when $N = 0$) is then

$$y(t) = \sum_{k=1}^{Q} A_k e^{-\sigma_k t} + \sum_{m=1}^{R} B_m(\frac{1}{\omega_m})e^{-\alpha_m t}\sin(\omega_m t+\theta_m)$$

Where A_k, B_m are constants that depend on K, z_i, σ_k, α_m and ω_m.

A necessary and sufficient condition for a feedback system to be stable is that all the poles of the system transfer function have negative real parts.
A system is stable if all the poles of the transfer function are in the left-hand side (LHS) of the *s*-plane.

Stability and Root Location

- **Stable:**
  - All the roots of characteristic equation are on the LHS of the s-plane;
  - Output is bounded to bounded input.

- **Marginally Stable:**
  - The characteristic equation has simple roots on the imaginary axis ($j\omega$-axis) with all other roots in the LHS of the s-plane;
  - The steady-state output will be sustained oscillations for a bounded input, unless a sinusoid (which is bounded) whose frequency is equal to the magnitude of the $j\omega$-axis roots. For this case, the system becomes unbounded. (theoretical explanation to follow)

- **Unstable:**
  - The characteristic equation has at least one root on the right-hand side (RHS) of the s-plane, or repeated $j\omega$ roots;
  - The output will become unbounded for any input.

Routh-Hurwitz Stability Criterion

- In the late 1800s, A. Hurwitz and E. J. Routh independently published a method of investigating the stability of a linear system. The method is presently known as "Routh-Hurwitz stability criterion", or simply RHC here.

- RHC is a test that informs (i.e., yes or no) about the stability of a linear time-invariant system without finding the roots of the transfer function. It is a **necessary** and **sufficient condition** for the stability of the system.
  - It is particularly relevant when there are unknowns in the transfer function.
  - Useful prior to the advent use of computer-aided control design.

- RHC states that:

"The number of roots of a characteristic equation with positive real parts is equal to the number of changes in sign of the first column of the Routh array"

- This criterion requires that there be no changes in sign in the first column (of Routh array) for a stable system.

- The key idea is really about constructing the Routh array!

Steps to apply RHC

- Steps of applying RHC:
  - First, order the coefficients
  - Then, arrange them into two rows in a specific manner, then produce the subsequent rows
  - Count the number of sign changes at the first column travelling down the rows. This number is the number of poles at the RHS of the *s*-plane.

Characteristic eq.: $q(s) = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots + a_1 s + a_0 = 0$

Routh Array

s^n	a_n	a_{n-2}	a_{n-4}	…
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	…
s^{n-2}	b_{n-1}	b_{n-3}	b_{n-5}	…
s^{n-3}	c_{n-1}	c_{n-3}	c_{n-5}	…
…	…	…	…	
s^0	h_{n-1}			

Own calculation

$$b_{n-1} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}$$

$$b_{n-3} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}$$

$$c_{n-1} = -\frac{1}{b_{n-1}} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix}$$

…and so on and so forth.

Example 15.1

- Example I: $q(s) = s^3 + s^2 + 2s + 24$
  - Routh array:

s^3	1	2
s^2	1	24
s^1	-22	
s^0	24	

  - Outcome: 2 sign changes → 2 poles at RHS of s-plane → unstable
- Example II: $q(s) = s^3 + 2s^2 + (K-2)s + K$
  - Determine the range of *K* for which the system is stable.

Answer: $k > 4(stable)$

Routh-Hurwitz Stability Criterion – complication

Now, let's try this example:

- $q(s) = s^5 + s^4 + 2s^3 + 2s^2 + s + 1$

- Apparently, some additional rules are needed to handle the complication.

Routh-Hurwitz Stability Criterion – distinct cases

Four distinct (special) cases or configurations of the first column array must be considered, and each must be treated separately and requires suitable modifications of the array calculation procedure:

1) No element in the first column is zero (previous examples);
2) There is a “zero” in the first column, but some other elements of the row containing the “zero” are non-zero;
3) There is a “zero” in the first column, and the other elements of the row containing the “zero” are also zero;
4) As in the third case, and also repeated roots on the $j\omega$-axis.

Case 1: Third-order System (establishing relationship among unknown variables)

The characteristic polynomial of a third-order system is

$$q(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

The Routh array is

$$\begin{array}{c|cc} s^3 & a_3 & a_1 \\ s^2 & a_2 & a_0 \\ s^1 & b_1 & 0 \\ s^0 & c_1 & 0 \end{array}$$

Calculation:

$$b_1 = \frac{a_2 a_1 - a_3 a_0}{a_2} \qquad c_1 = \frac{b_1 a_0}{b_1} = a_0$$

Stability analysis: for the third-order system to be stable, it is necessary and sufficient that coefficients be positive and $a_2 a_1 > a_0 a_3$. The condition when $a_2 a_1 = a_0 a_3$ results in the marginal stability case (which means that a pair of roots are located on the imaginary axis of the *s*-plane).

Case 2

- **Case 2. There is a zero in the first column, but some of other elements of the row containing the zero in the first column are nonzero.**

In this case, if only one zero element in the array is zero, it may be replaced with a small positive number, ε, that is allowed to approach zero after completing the array.

For example, consider the following characteristic polynomial:

$$q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$$

The Routh array is

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 2 & 4 & 10 \\ s^3 & \epsilon & 6 & 0 \\ s^2 & c_1 & 10 & 0 \\ s^1 & d_1 & 0 & 0 \\ s^0 & 10 & 0 & 0 \end{array}$$

Calculation:

$$c_1 = \frac{4\epsilon - 12}{\epsilon} = \frac{-12}{\epsilon}$$

$$d_1 = \frac{6c_1 - 10\epsilon}{c_1} = 6$$

There are two sign changes due to the large negative number in the first column, $c_1 = -12/\varepsilon$. Therefore, the system is unstable, and two roots lie in the right half of the plane.

Example 15.2 (Case 2)

For example, consider the following characteristic polynomial. Obtain the gain K that results in marginally stability.

$$q(s) = s^4 + s^3 + s^2 + s + K$$

The Routh array is

$$\begin{array}{c|ccc} s^4 & 1 & 1 & K \\ s^3 & 1 & 1 & 0 \\ s^2 & \epsilon & K & 0 \\ s^1 & b_1 & 0 & 0 \\ s^0 & K & 0 & 0 \end{array}$$

Calculation:

$$b_1 = \frac{\epsilon - K}{\epsilon} = \frac{-K}{\epsilon}$$

Stability analysis:

Row 3→4: For any value of K greater than zero, the system is unstable.

Row 4→5: Because the last term in the first column is equal to K, a negative value of K will also result in an unstable system.

Conclusion: Consequently, the system is unstable for all values of K<0 and K>0.

[**Additional illustration about the case with *K*=0**: $q(s)$ has a pole at the origin. The system can easily become unstable for any step input or actually any input that is not averaged to perfect zero, so the case with K=0 normally has little practical value and normally not considered further. See Example 6.3 of Textbook]

Case 3

- **Case 3. There is a zero in the first column, and the other elements of the row containing the zero are also zero.**

- Case 3 occurs when all the elements in one row are zero or when the row consists of a single element that is zero.

- This condition occurs when the polynomial contains singularities that are symmetrically located about the origin of the s-plane. Therefore, case 3 occurs when factors such as $(s+\sigma)(s-\sigma)$ or $(s+j\omega)(s-j\omega)$ occur.

- This problem is overcome by further investigating, through RHC, the auxiliary polynomial $U(s)$ that immediately precedes the zero row entry in the Routh array.

- The order of the auxiliary polynomial is always even and it indicates the number of symmetrical, real or imaginary, root pairs.

Example 15.3 (Case 3)

Let us consider a third-order system with the characteristic polynomial where K is an adjustable gain

$$q(s) = s^3 + 2s^2 + 4s + K$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 2 & K \\ s^1 & \frac{8-K}{2} & 0 \\ s^0 & K & 0 \end{array}$$

For a stable system, we require $0 < K < 8$

When $K = 8$, we obtain a row of zeros. In this case we have two roots on the $j\omega$-axis and a marginally stability case.

The auxiliary polynomial is the equation of the row preceding the row of zeros, therefore, in this case, obtained obtain from the s^2-row

$$U(s) = 2s^2 + Ks^0 = 2s^2 + 8 = 2(s + j2)(s - j2)$$

The factors of the characteristic polynomial can be obtained by dividing $q(s)$ by $U(s)$.

$$q(s) = (s + 2)\ (s + j2)(s - j2)$$

Case 4

- **Case 4. Repeated roots of the characteristic equation on the $j\omega$-axis.**

- If the $j\omega$-axis roots are repeated, the system response will be unstable. Routh-Hurwitz criterion's sign change rule could not reveal this form of instability, instead, the judgement is concluded by recognizing ourselves the existence of repeated complex roots.
- If the $j\omega$-axis roots are not repeated, the system is neither stable or unstable. The final/overall stability can be confirmed by further investigating aux. polynomial $U(s)$
  - If aux. polynomial indicates more than a pair of repeated roots on the $j\omega$-axis, the system is not stable;
  - Otherwise, investigate the nature of the remaining roots by analyzing using RHC the quotient polynomial resulted from diving the original polynomial with the aux. polynomial $U(s)$.

Example 15.4 (Case 4)

- Consider the system with a characteristic polynomial:

$$q(s) = s^5 + s^4 + 2s^3 + 2s^2 + s + 1 = \cdots = (s+1)\,(s+j)(s-j)(s+j)(s-j)$$

The Routh array is (with $\varepsilon \to 0$)

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 1 \\ s^4 & 1 & 2 & 1 \\ s^3 & \epsilon & \epsilon & 0 \\ s^2 & 1 & 1 & \\ s^1 & \epsilon & 0 & \\ s^0 & 1 & & \end{array}$$

NO sign change in the first column (i.e., a false sign of being stable), but there are two zero rows, indicating the repeated roots on the $j\omega$-axis. It is an unstable system.

There are two auxiliary polynomials at, respectively, s^2-line (s^2+1) and s^4-line $(s^4+2s^2+1=(s^2+1)^2)$.

Example 15.5 (Case 4)

Consider the characteristic polynomial

$$q(s) = s^5 + s^4 + 4s^3 + 24s^2 + 3s + 63$$

The Routh array is

s^5	1	4	3
s^4	1	24	63
s^3	-20	-60	0
s^2	21	63	0
s^1	0	0	0
s^0	63		

Extract the aux. polynomial from s^2-line:

$$U(s) = 21s^2 + 63 = 21(s + j\sqrt{3})(s - j\sqrt{3})$$

which indicates that two roots are on the imaginary axis. To examine the remaining roots, we divide the original polynomial by the auxiliary polynomial to obtain

$$\frac{q(s)}{s^2 + 3} = s^3 + s^2 + s + 21$$

Note: "21" constant in $21(s^3 + s^2 + s + 21)$ is optional to be included.

$$\frac{q(s)}{s^2+3} = s^3 + s^2 + s + 21$$

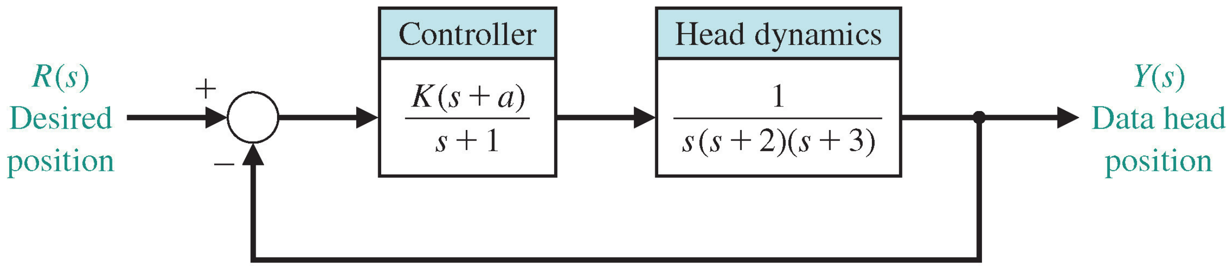
Establishing a second Routh array for this problem:

$$\begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 1 & 21 \\ s^1 & -20 & 0 \\ s^0 & 21 & \end{array}$$

The two changes in sign in the first column indicate the presence of two roots in the RHS of the *s*-plane, and the system is unstable.

Example 15.6: Design for Welding Control

Large welding robots are used in today's auto (car) plants. The welding head is moved to different positions on the auto body, and a rapid, accurate response is required. The diagram is shown as follows. Determine *K* and *a* to make the system stable.



Step 1. Obtain characteristic equation.

$$1 + G(s) = 1 + \frac{K(s+a)}{s(s+1)(s+2)(s+3)} = 0$$

$$q(s) = s^4 + 6s^3 + 11s^2 + (K+6)s + Ka = 0$$

Step 2. Establish the Routh array

$$\begin{array}{c|ccc} s^4 & 1 & 11 & Ka \\ s^3 & 6 & K+6 & \\ s^2 & b_3 & Ka & \\ s^1 & c_3 & & \\ s^0 & Ka & & \end{array}$$

$$b_3 = \frac{60-K}{6} \qquad c_3 = \frac{b_3(K+6) - 6Ka}{b_3}$$

Example 15.6: Design for Welding Control (continue)

Step 3. Examine the first column, and then determine K and a.

$$b_3 > 0 \longrightarrow K < 60$$

$$c_3 > 0 \longrightarrow a < \frac{(60 - K)(K + 6)}{36K}$$ when *a* is positive.

Therefore, if $K = 40$, we require $a \leq 0.639$.

Example 15.7 (in-class)

Determine stability for a system with the following characteristic equation

$$q(s) = s^3 + Ks^2 + (1 + K)s + 6 = 0$$

Flow of thoughts: (1) A characteristic eqn. with an unknown variable K; (2) form Routh array; (3) obtain the leading column expressions/values, subject them to >0 (for stability); (4) solve for overall K range.

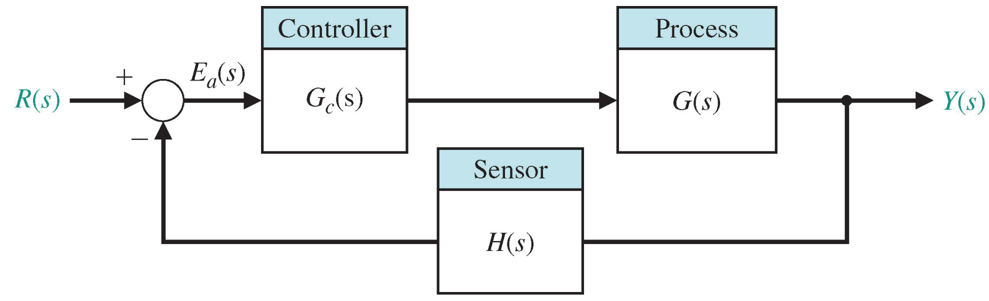
Answer: The system is stable for $K > 2$ (another inequality is $K > 0$, not useful though)

Example 15.8 (in-class)

Consider the following system, determine K, where $K > 0$, when it is marginally stable, and obtain the roots lying on the $j\omega$-axis.

$$G_c(s) = K \qquad G(s) = \frac{s+40}{s(s+10)}$$

$$H(s) = \frac{1}{s+20}$$



Flow of thoughts: (1) Recognize the CL structure, find CL characteristic eqn.; (2) marginally stable means the limit/border of K range; (3) form Routh array, obtain leading column expressions/values, subject them to >0 (for stability); (4) solve for K range, deduce the final answer; (5) Select K, calculate CL poles and extract roots on the $j\omega$-axis.

Answer: $0 < K < 600$; $K = 600$, imaginary poles $\pm j28.28$ (and real pole -30)

Next Lecture

- **In our next lecture:** we will continue to explore about system stability, with state variable system and numerical tool.

- **What you can do from now till the next lecture:** revise the material, further reading, and group study.

- **How to get in touch**: through LMO Module's "*General question and answer forum*" section or during my weekly consultation hour(s).