

EEE210:
Energy Conversion and Power Systems

One-line diagram and Per-Unit systems

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Per-Unit systems

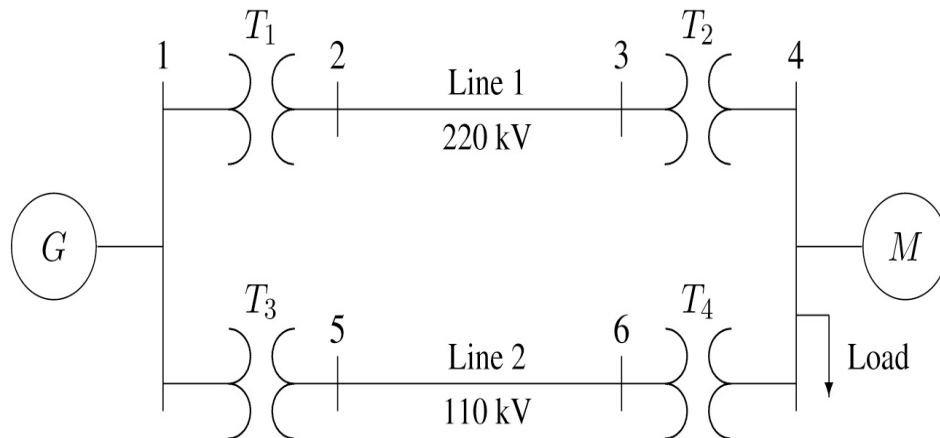


1. Question

The one-line diagram of a three-phase power system is shown in Figure 4. The manufacturer's data for each device is given as follow:

| | | | |
|------------------|----------|-----------|---------|
| G: | 90 MVA | 22 kV | X=18% |
| T ₁ : | 50 MVA | 22/220 kV | X=10% |
| T ₂ : | 40 MVA | 220/11 kV | X=6.0% |
| T ₃ : | 40 MVA | 22/110 kV | X=6.4% |
| T ₄ : | 40 MVA | 110/11 kV | X=8.0% |
| M: | 66.5 MVA | 10.45 kV | X=18.5% |

The three-phase load at bus 4 absorbs 57 MVA, 0.6 power factor lagging at 10.45 kV. Line 1 and line 2 have reactance of 48.4 and 65.43 Ω , respectively.

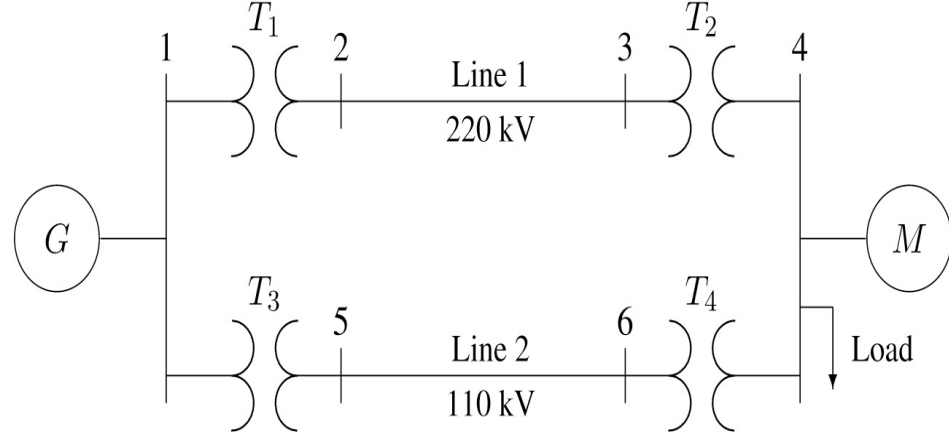


If the motor operates at full-load 0.8 power factor leading at the terminal voltage of 10.45 kV Determine the voltage at Bus 1.

Per-Unit systems

1. Question

Circuit solution?



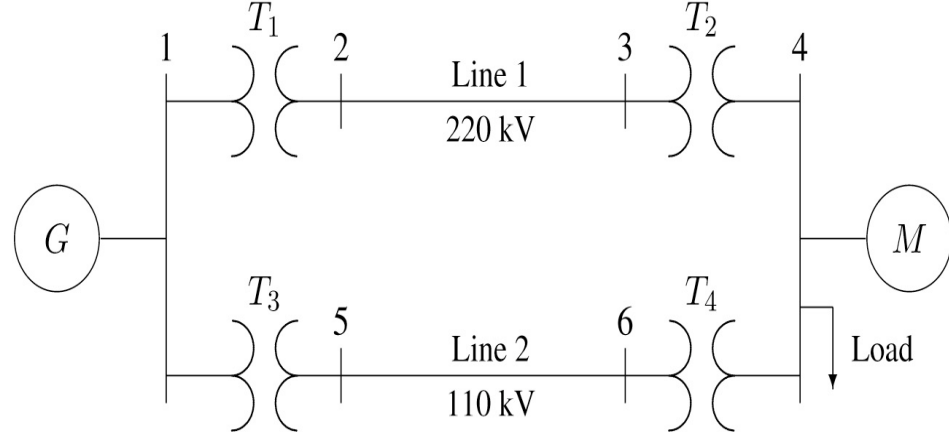
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Per-Unit system

1. Question

1. 3-Phase system unknown three-phase connection
2. 4 rated voltage levels(22kV 220kV 110kV 11kV)
3. Transformer referring and approximation difficulty
4. Current values are difficult to be aligned under different voltage



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Per-Unit systems

2. Details of per-unit systems

To remove the impact of different voltage level and referring calculations:

Power engineers developed the **per-unit systems**.

In definition, it converts the power, voltage, current, and impedance into a decimal fraction or multiples of base quantities.

The general equation is given by:

$$\text{quantity in perunit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

For example:

$$\begin{aligned} S_{pu} &= \frac{S}{S_{base}} \\ V_{pu} &= \frac{V}{V_{base}} \\ I_{pu} &= \frac{I}{I_{base}} \\ Z_{pu} &= \frac{Z}{Z_{base}} \end{aligned}$$

Attention:

- The original number could be complex values or phasor quantities.
- The base values must be real numbers
- 4 base are required to define a power system comprehensively
- The unit of the original number and the base number should stay constant.

Per-Unit systems

2. Details of per-unit systems

$$S_{pu} = \frac{S}{S_{base}}$$
$$V_{pu} = \frac{V}{V_{base}}$$
$$I_{pu} = \frac{I}{I_{base}}$$
$$Z_{pu} = \frac{Z}{Z_{base}}$$

Normally, we select:

- Base apparent power S_{base} and line-to-line base voltage V_{base}
- The unit are set as MVA and kV



The base current and impedance can be derived by:

$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}}$$
$$Z_{base} = \frac{V_{base}/\sqrt{3}}{I_{base}}$$

Or

$$Z_{base} = \frac{V_{base}^2}{S_{base}}$$

If the unit for S_{base} and V_{base} are MVA and kV, the unit for I_{base} and Z_{base} are kA and Ω .

For traditional complex power:

$$S = 3V_p I_p^* \quad \& \quad |V_p| = \frac{|V_l|}{\sqrt{3}} \text{ or } |I_p| = \frac{|I_l|}{\sqrt{3}}$$

In per-unit systems, the circuit law directly applies, no requirement to consider phase and line conversion.



And

$$S_{pu} = V_{pu} I_{pu}^*$$

$$V_{pu} = Z_{pu} I_{pu}$$

For load:

$$Z_{load,pu} = \frac{V_{pu}^2}{S_{load,pu}^*}$$

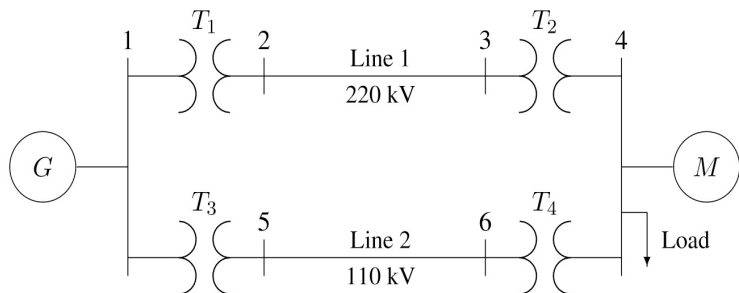


Per-Unit systems

3. Change of Base

The manufacturer's data for each device is given as follow:

| | | | |
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The impedance of individual generators and transformers are generally given by manufacturer in terms of percent or per-unit quantities based on their own ratings.

Example:

If rated capacity for the generator is 90MVA and rated voltage is 22 kV, $X = 18\%$ means the per-unit impedance for the 90MVA base capacity & 22kV base voltage is 0.18

If system base capacity is 100 MVA, what is the per-unit reactance for those devices?

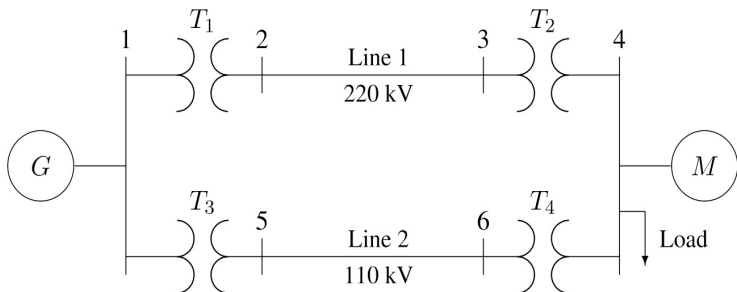
If the rated voltage is different from the base voltage, what is the per-unit reactance for those devices?

Per-Unit systems

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Change of base to solve rated value and base value conflict.

$$Z_{pu}^{old} = \frac{Z}{Z_{base}^{old}} = Z \frac{S_{base}^{old}}{(V_{base}^{old})^2}$$

$$Z_{pu}^{new} = \frac{Z}{Z_{base}^{new}} = Z \frac{S_{base}^{new}}{(V_{base}^{new})^2}$$



$$\frac{Z_{pu}^{old}}{Z_{pu}^{new}} = \frac{S_{base}^{old}}{(V_{base}^{old})^2} \frac{(V_{base}^{new})^2}{S_{base}^{new}}$$



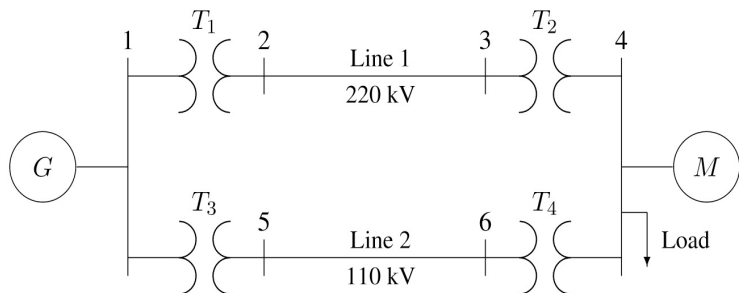
$$Z_{pu}^{new} = Z_{pu}^{old} \frac{S_{base}^{new}}{(V_{base}^{new})^2} \frac{(V_{base}^{old})^2}{S_{base}^{old}}$$

Per-Unit systems

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Once a voltage base has been selected for a point in a system, **the other voltage bases are determined by the transformer turns ratios**. In usual case, the voltage base similar to nominal values is selected. Example:

If turns ratio = 10; the voltage base is 10kV at the primary side, the secondary voltage base = $\frac{10 \text{ kv}}{10} = 1 \text{ kV}$

Set $V_b = 22 \text{ kV}$ at the generator side, $S_b = 100 \text{ MVA}$, thus:

$$V_{b2-3} = 220 \text{ kV};$$

$$V_{b5-6} = 110 \text{ kV}$$

$$V_{b4} = 11 \text{ kV}$$

$$X_{G1p} = X_{G1} * \frac{S_b}{S_{G1}} * \left(\frac{V_{G1}}{V_b} \right)^2 = 0.18 * \frac{100}{90} * \left(\frac{22}{22} \right)^2 = 0.2 \text{ pu}$$

$$X_{T1p} = X_{T1} * \frac{S_b}{S_{T1}} * \left(\frac{V_{T1}}{V_b} \right)^2 = 0.1 * \frac{100}{50} * \left(\frac{22}{22} \right)^2 = 0.2 \text{ pu}$$

$$X_{T2p} = X_{T2} * \frac{S_b}{S_{T2}} * \left(\frac{V_{T2}}{V_{b4}} \right)^2 = 0.06 * \frac{100}{40} * \left(\frac{11}{11} \right)^2 = 0.15 \text{ pu}$$

$$X_{T3p} = X_{T3} * \frac{S_b}{S_{T3}} * \left(\frac{V_{T3}}{V_b} \right)^2 = 0.064 * \frac{100}{40} * \left(\frac{22}{22} \right)^2 = 0.16 \text{ pu}$$

$$X_{T4p} = X_{T4} * \frac{S_b}{S_{T4}} * \left(\frac{V_{T4}}{V_{b4}} \right)^2 = 0.08 * \frac{100}{40} * \left(\frac{11}{11} \right)^2 = 0.2 \text{ pu}$$

Note:

$$X_{Mp} = X_M * \frac{S_b}{S_M} * \left(\frac{V_M}{V_{b4}} \right)^2 = 0.185 * \frac{100}{66.5} * \left(\frac{10.45}{11} \right)^2 = 0.25 \text{ pu}$$

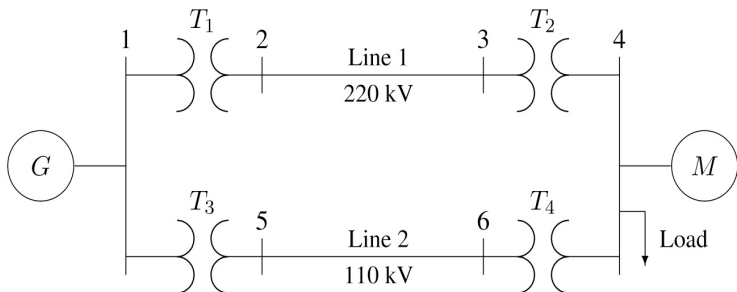


Per-Unit systems

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Derive the impedance in per-unit systems

Since Line 1(2-3) and Line 2(5-6) have different voltages, the base impedance would be:

$$Z_{b2-3} = \frac{V_{b2-3}^2}{S_b} = \frac{220^2}{100} = 484 \, \Omega$$

$$Z_{b5-6} = \frac{V_{b5-6}^2}{S_b} = \frac{110^2}{100} = 121 \, \Omega$$

The per-unit line reactance would be:

$$X_{line1p} = \frac{X_{line1}}{Z_{b2-3}} = \frac{48.4}{484} = 0.1 \, \text{pu}$$

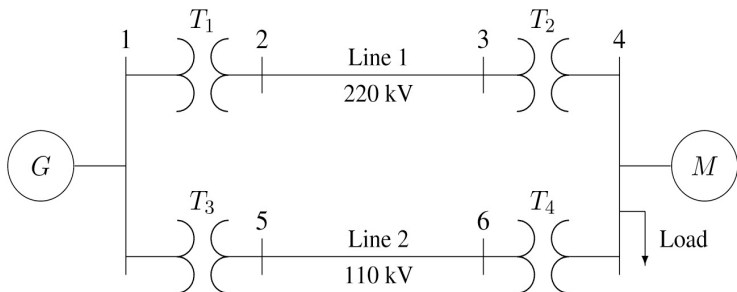
$$X_{line2p} = \frac{X_{line2}}{Z_{b5-6}} = \frac{65.43}{121} = 0.54 \, \text{pu}$$

Per-Unit systems

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Derive the impedance in per-unit systems

For the load, $\theta = \arccos(0.6) = 53.13$, lagging

$$S_{Load} = |S_{load}| \cos(53.13) + j |S_{load}| \sin(53.13) \\ = 34.2 + j45.6 \text{ MVA}$$

Thus, the load impedance would be:

$$Z_{load} = \frac{V_{Load}^2}{S_{load}^*} = \frac{10.45^2}{34.2 - j45.6} = 1.145 + j1.533 \Omega$$

The base impedance for load would be:

$$Z_{b4} = \frac{V_{b4}^2}{S_b} = \frac{11^2}{100} = 1.21 \Omega$$

Thus, the per unit load impedance is:

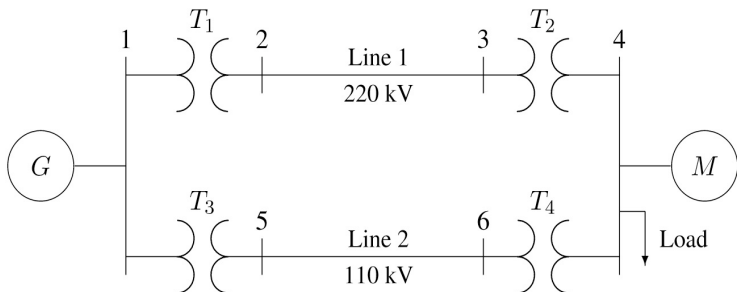
$$Z_{loadp} = \frac{Z_{load}}{Z_{b4}} = 0.95 + j1.2667 \text{ pu}$$

Per-Unit systems

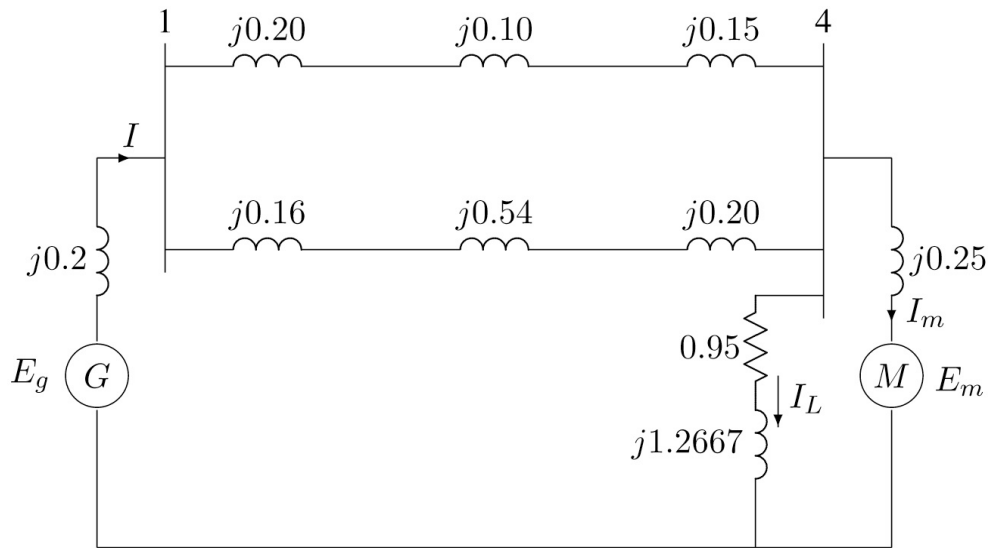
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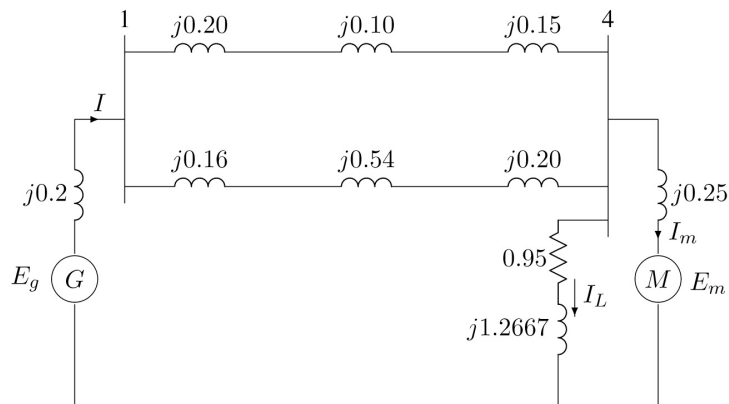
The impedance diagram is:



Per-Unit systems

3. Change of Base

If the motor operates at full-load 0.8 power factor leading at the terminal voltage of 10.45 kV Determine the voltage at Bus 1.



Try to determine the generator and motor internal emfs

The per-unit voltage at bus 4 is:

$$V_{4p} = \frac{V_{\text{ter}}}{V_{b4}} = \frac{10.45}{11} = 0.95 \angle 0^\circ$$

The moto complex power at 0.8 power factor leading is:

$$S_{mp} = \frac{S_m}{S_{base}} = \frac{66.5 \angle -36.87^\circ}{100} = 0.665 \angle -36.87^\circ$$

The current drawn by the motor is:

$$I_{mp} = \frac{S_{mp}^*}{V_{4p}^*} = \frac{0.665 \angle +36.87^\circ}{0.95 \angle 0^\circ} = 0.56 + j0.42$$

The current drawn by the load is:

$$I_{Lp} = \frac{V_{4p}}{Z_{loadp}} = \frac{0.95 \angle 0^\circ}{0.95 + j1.2667} = 0.36 - j0.48$$

Total current drawn from Bus 4 is:

$$I_{tp} = I_{mp} + I_{Lp} = 0.92 - j0.06$$

The reactance of the parallel branches is:

$$X_{Lt} = \frac{0.45 * 0.9}{0.45 + 0.9} = 0.3$$

The voltage at bus 1 is:

$$\begin{aligned} V_{1p} &= V_{4p} + jX_{Lt}I_{tp} \\ &= 0.95 \angle 0^\circ + j0.3 * (0.92 - j0.06) = 1.0 \angle 15.91^\circ \\ V_1 &= V_{1p} * V_b = 1.0 \angle 15.91^\circ * 22 = 22 \angle 15.91^\circ \text{ kV} \end{aligned}$$



Per-Unit systems

3. Advantages

- It gives **clear magnitudes of V , I , P , and Z** ,
- The **per-unit impedance** of equipment falls in **a narrow range**,
- The per-unit values of Z , V , and I of a transformer are the same **regardless of primary or secondary side**. This is a great advantage since the different voltage levels disappear and the entire system reduces to a system of simple impedance,
- The per-unit systems are **ideal for the computerized analysis** and simulation of complex power system problems
- The circuit laws are valid in per-unit systems, and the **power and voltage equations are simplified since the factors of 3 and $\sqrt{3}$ are eliminated** in the per-unit system



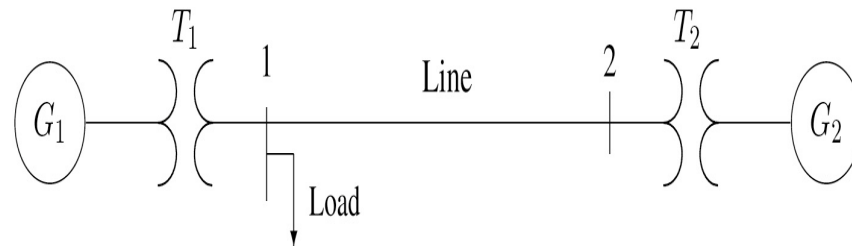
Per-Unit systems



4. Example questions

1. Draw an impedance diagram for the electric power system shown in figure below, showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

| | | | |
|---------|--------|------------------------------------|----------|
| G_1 : | 90 MVA | 20 kV | $X=9\%$ |
| T_1 : | 80 MVA | 20/200 kV | $X=16\%$ |
| T_2 : | 80 MVA | 200/20 kV | $X=20\%$ |
| G_2 : | 90 MVA | 18 kV | $X=9\%$ |
| Line: | 200 kV | $X=120 \Omega$ | |
| Load: | 200 kV | $S=48 \text{ MW}+j64 \text{ Mvar}$ | |

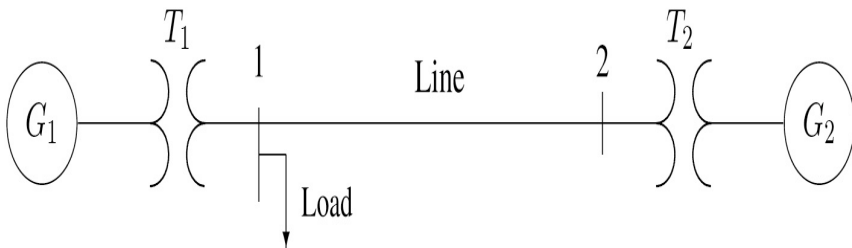


Per-Unit systems

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| Line: | 200 kV | $X=120 \Omega$ | |
| Load: | 200 kV | $S=48 \text{ MW}+j64 \text{ Mvar}$ | |



Read the problem:

$S_b = 100 \text{ MVA}$ $V_b = 20 \text{ kV}$ (at the G_1 side)

Thus, $V_{b1-2} = 200 \text{ kV}$ $V_{b_M} = 20 \text{ kV}$

$S_{G1} = 90 \text{ MVA}$ $V_{G1} = 20 \text{ kV}$ $X_{G1} = 9\%$

$S_{G2} = 90 \text{ MVA}$ $V_{G2} = 18 \text{ kV}$ $X_{G2} = 9\%$

$S_{T1} = 80 \text{ MVA}$ $V_{T1L} = 20 \text{ kV}$ $V_{T1H} = 200 \text{ kV}$ $X_{T1} = 16\%$

$S_{T2} = 80 \text{ MVA}$ $V_{T2L} = 200 \text{ kV}$ $V_{T2H} = 20 \text{ kV}$ $X_{T2} = 20\%$

$V_{Line1-2} = 200 \text{ kV}$ $Z_{Line} = j120 \Omega$

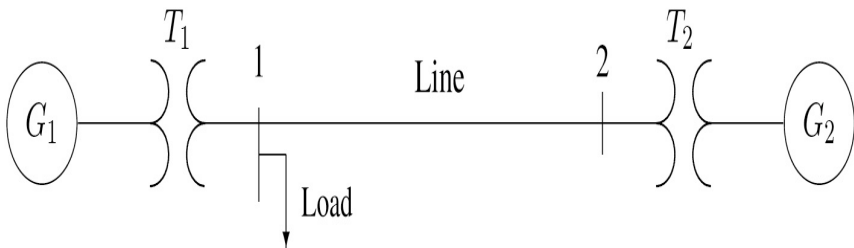
$S_{load} = 48 \text{ MW}+j64 \text{ Mvar}$

Per-Unit systems

4. Example questions

1. Draw an impedance diagram for the electric power system shown in figure below, showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

| | | | |
|---------|--------|------------------------------------|----------|
| G_1 : | 90 MVA | 20 kV | $X=9\%$ |
| T_1 : | 80 MVA | 20/200 kV | $X=16\%$ |
| T_2 : | 80 MVA | 200/20 kV | $X=20\%$ |
| G_2 : | 90 MVA | 18 kV | $X=9\%$ |
| Line: | 200 kV | $X=120 \Omega$ | |
| Load: | 200 kV | $S=48 \text{ MW}+j64 \text{ Mvar}$ | |



$$X_{G1P} = X_{G1} * \frac{S_b}{S_{G1}} * \left(\frac{V_{G1}}{V_b} \right)^2 = 0.09 * \frac{100}{90} * \left(\frac{20}{20} \right)^2 = 0.1 \text{ pu}$$

$$X_{G2P} = X_{G2} * \frac{S_b}{S_{G2}} * \left(\frac{V_{G2}}{V_b} \right)^2 = 0.09 * \frac{100}{90} * \left(\frac{18}{20} \right)^2 = 0.081 \text{ pu}$$

$$X_{T1P} = X_{T1} * \frac{S_b}{S_{T1}} = 0.2 \text{ pu}$$

$$X_{T2P} = X_{T2} * \frac{S_b}{S_{T2}} = 0.25 \text{ pu}$$

Derive the base impedance at 200 kV side

$$Z_{bline} = \frac{V_{T1H}^2}{S_b} = \frac{200^2}{100} = 400 \Omega$$

Thus,

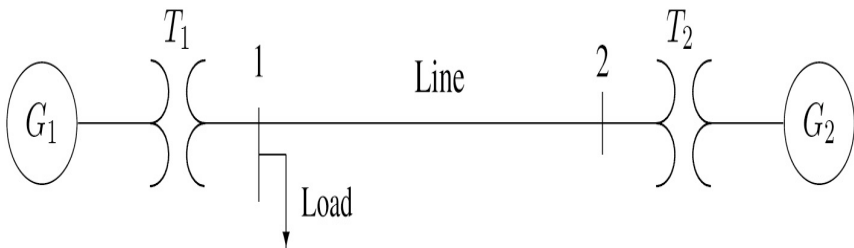
$$X_{Linep} = \frac{X_{Line}}{Z_b} = \frac{120}{400} = 0.3 \text{ pu}$$

Per-Unit systems

4. Example questions

1. Draw an impedance diagram for the electric power system shown in figure below, showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

| | | | |
|---------|--------|-----------|----------------------------------|
| G_1 : | 90 MVA | 20 kV | $X=9\%$ |
| T_1 : | 80 MVA | 20/200 kV | $X=16\%$ |
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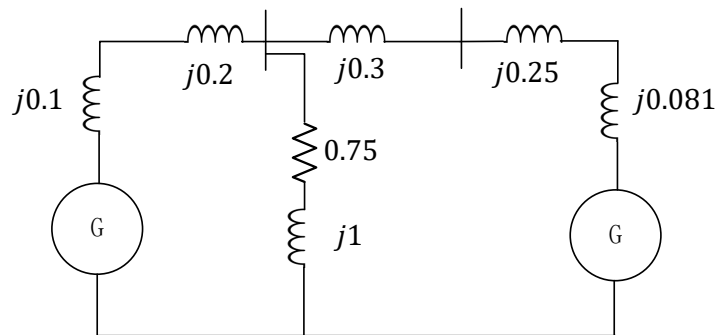
For the load, its impedance is given by:

$$Z_{load} = \frac{V_{T1H}^2}{S_{load}^*} = \frac{200^2}{48 - j64} = 300 + j400\ \Omega$$

Thus,

$$Z_{loadp} = \frac{Z_{load}}{Z_b} = 0.75 + j1\text{ pu}$$

The impedance diagram in per-unit systems:

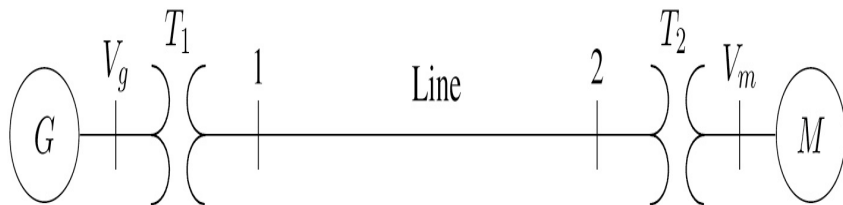


Per-Unit systems

4. Example questions

2. The three-phase power and line-line ratings of the electric power system shown in the figure are given below.

| | | | |
|---------|----------|-----------|---------------------|
| G_1 : | 60 MVA | 20 kV | $X=9\%$ |
| T_1 : | 50 MVA | 20/200 kV | $X=10\%$ |
| T_2 : | 50 MVA | 200/20 kV | $X=10\%$ |
| M : | 43.2 MVA | 18 kV | $X=8\%$ |
| Line: | | 200 kV | $Z=120+j200 \Omega$ |



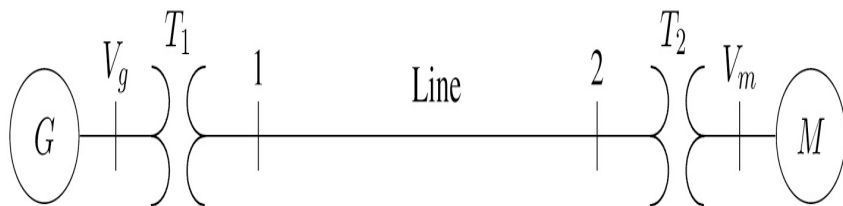
- Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.
- The motor is drawing 45 MVA, 0.8 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

Per-Unit systems

4. Example questions

- a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

| | | | |
|---------|----------|-----------|---------------------|
| G_1 : | 60 MVA | 20 kV | $X=9\%$ |
| T_1 : | 50 MVA | 20/200 kV | $X=10\%$ |
| T_2 : | 50 MVA | 200/20 kV | $X=10\%$ |
| M : | 43.2 MVA | 18 kV | $X=8\%$ |
| Line: | | 200 kV | $Z=120+j200 \Omega$ |



Read the problem:

$S_b = 100 \text{ MVA}$ $V_b = 20 \text{ kV}$ (at the G_1 side)

Thus, $V_{b1-2} = 200 \text{ kV}$ $V_{b_M} = 20 \text{ kV}$

$S_G = 60 \text{ MVA}$ $V_{G1} = 20 \text{ kV}$ $X_{G1} = 9\%$

$S_M = 43.2 \text{ MVA}$ $V_M = 18 \text{ kV}$ $X_M = 8\%$

$S_{T1} = 50 \text{ MVA}$ $V_{T1L} = 20 \text{ kV}$ $V_{T1H} = 200 \text{ kV}$ $X_{T1} = 10\%$

$S_{T2} = 50 \text{ MVA}$ $V_{T2L} = 20 \text{ kV}$ $V_{T2H} = 200 \text{ kV}$ $X_{T2} = 10\%$

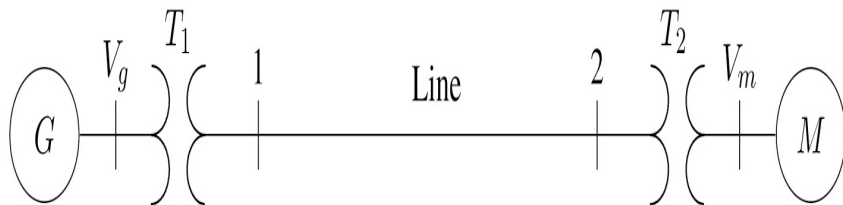
$V_{Line1-2} = 200 \text{ kV}$ $Z_{Line} = 120 + j200 \Omega$

Per-Unit systems

4. Example questions

- a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

| | | | |
|---------|----------|-----------|---------------------|
| G_1 : | 60 MVA | 20 kV | $X=9\%$ |
| T_1 : | 50 MVA | 20/200 kV | $X=10\%$ |
| T_2 : | 50 MVA | 200/20 kV | $X=10\%$ |
| M : | 43.2 MVA | 18 kV | $X=8\%$ |
| Line: | | 200 kV | $Z=120+j200 \Omega$ |



$$X_{G1P} = X_{G1} * \frac{S_b}{S_{G1}} * \left(\frac{V_{G1}}{V_b} \right)^2 = 0.09 * \frac{100}{60} * \left(\frac{20}{20} \right)^2 = 0.15 \text{ pu}$$

$$X_{MP} = X_{G2} * \frac{S_b}{S_{G2}} * \left(\frac{V_{G2}}{V_b} \right)^2 = 0.08 * \frac{100}{43.2} * \left(\frac{18}{20} \right)^2 = 0.15 \text{ pu}$$

$$X_{T1P} = X_{T1} * \frac{S_b}{S_{T1}} = 0.2 \text{ pu}$$

$$X_{T2P} = X_{T2} * \frac{S_b}{S_{T2}} = 0.2 \text{ pu}$$

Derive the base impedance at 200 kV side

$$Z_b = \frac{V_{T1H}^2}{S_b} = \frac{200^2}{100} = 400 \Omega$$

Thus,

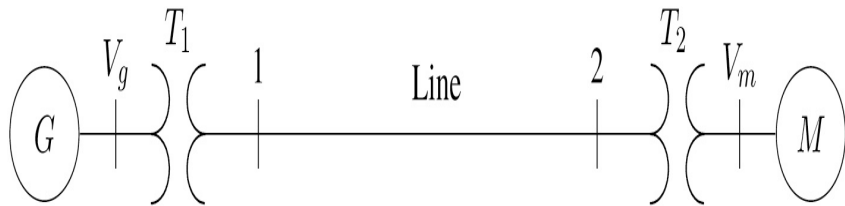
$$Z_{Linep} = \frac{Z_{Line}}{Z_b} = \frac{120 + j200}{400} = 0.3 + j0.5 \text{ pu}$$

Per-Unit systems

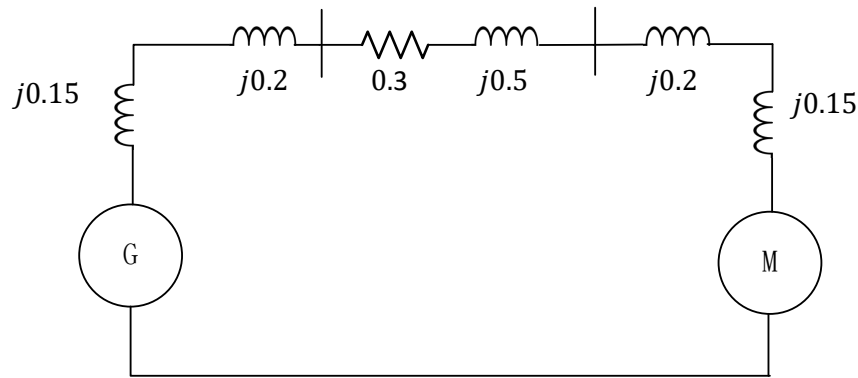
4. Example questions

- a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

| | | | |
|---------|----------|-----------|---------------------|
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| M : | 43.2 MVA | 18 kV | $X=8\%$ |
| Line: | | 200 kV | $Z=120+j200 \Omega$ |



The impedance diagram in per-unit systems:



Per-Unit systems

4. Example questions



b) The motor is drawing 45 MVA, 0.8 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

G_1 : 60 MVA 20 kV $X=9\%$

T_1 : 50 MVA 20/200 kV $X=10\%$

T_2 : 50 MVA 200/20 kV $X=10\%$

M : 43.2 MVA 18 kV $X=8\%$

Line: 200 kV $Z=120+j200 \Omega$

$$|S_{Ma}| = 45 \text{ MVA} \quad pf = 0.8 \quad \theta = \arccos(0.8) = 36.87^\circ$$

$$S_{Ma} = |S_{Ma}| \cos(36.87^\circ) + j |S_{Ma}| \sin(36.87^\circ) \\ = 36.09 + j26.87 \text{ kVA}$$

The per unit apparent power would be:

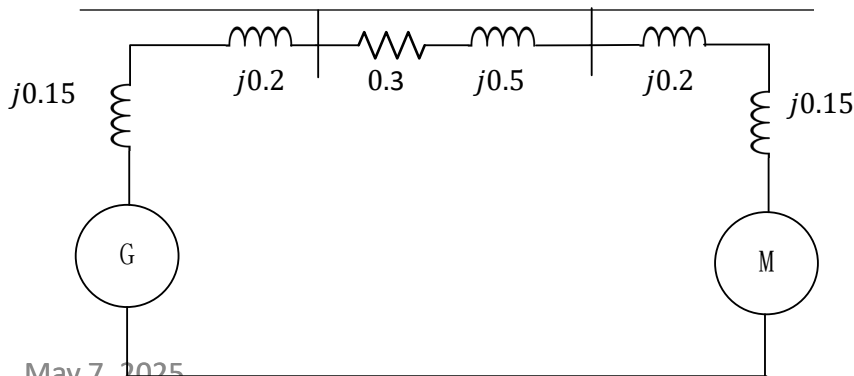
$$S_{Map} = \frac{S_{Ma}}{S_b} = \frac{36.09 + j26.87}{100} = 0.3609 + j0.2687 \text{ pu}$$

Thus, the per unit load current would be:

$$I_{Mp} = \frac{S_{Map}^*}{V_{Mp}^*}$$

$$\text{Considering } V_{Mp} = V_M / V_{b_M} = \frac{18 \angle 0^\circ}{20} = 0.9 \text{ pu;}$$

$$I_{Mp} = (0.3609 - j0.2687) / 0.9 = 0.401 - j0.299 \text{ pu}$$



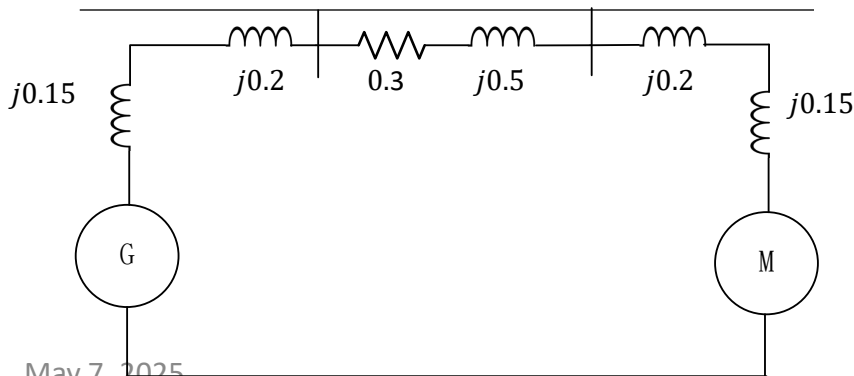
Per-Unit systems

4. Example questions



b) The motor is drawing 45 MVA, 0.8 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

| | | | |
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| M : | 43.2 MVA | 18 kV | $X=8\%$ |
| Line: | | 200 kV | $Z=120+j200 \Omega$ |



$$\begin{aligned}
 V_{drop} &= I_{Mp} * (jX_{T2P} + Z_{Linep} + jX_{T1P}) \\
 &= (0.401 - j0.299) * (j0.2 + 0.3 + j0.5 + j0.2) \\
 &= 0.389 + j0.271 \text{ pu}
 \end{aligned}$$

Thus,

$$V_{Gtp} = V_{drop} + V_{Mp} = 1.289 + j0.271 \text{ pu}$$

$$E_{G_internal_p} = V_{Gtp} + I_{Mp} * jX_{G1p} = 1.334 + j0.332 \text{ pu}$$

$$E_{G_internal} = E_{G_internal_p} * V_b = 26.676 + j6.63 \text{ kV}$$

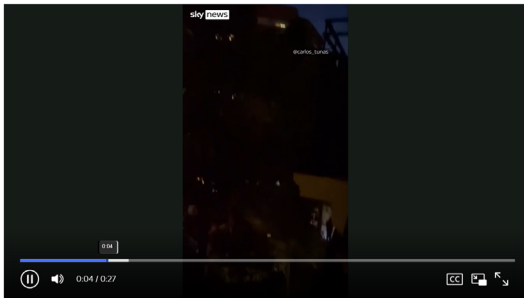
Extensions



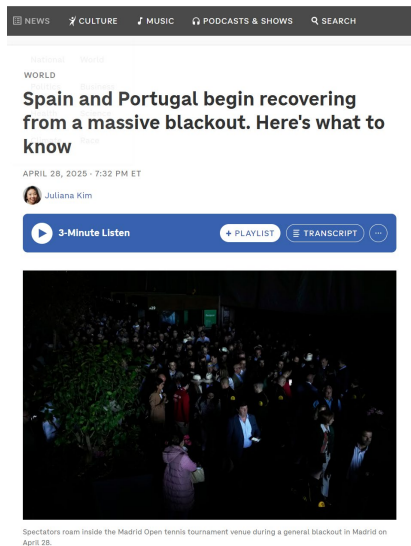
Blackouts latest: Fallout continues as power being restored after tens of millions plunged into darkness in Spain and Portugal

Power is being restored to cities across Spain and Portugal after a major outage. Flights, traffic lights, trains and phone networks have all been affected as the fallout continues. Follow the latest here.

© Tuesday 29 April 2025 06:47, UK



6 hours



18 hours



1 of 20 | A blackout brought much of Spain and Portugal to a standstill Monday, stopping trains, cutting phone service and shutting down traffic lights and ATMs for millions of people across the Iberian Peninsula.

BY JOSEPH WILSON
Updated 1:31 PM GMT+8, April 29, 2025

Share

BARCELONA, Spain (AP) — Power had almost fully returned to Spain early Tuesday morning as many questions remained about what caused one of Europe's [most severe blackouts](#) that grounded flights, paralyzed metro systems, disrupted mobile communications and shut down ATMs across Spain and Portugal.

By 6:30 a.m., more than 99% of energy demand in Spain had been restored, the country's electricity operator Red Eléctrica said.

Power had returned to several regions across Spain and Portugal as the nations reeled from the still-unexplained widespread blackout that had turned airports and train stations into campgrounds for stranded travelers.

The cause of the blackout remains a mystery

At a press conference on Monday evening, Spanish Prime Minister Pedro Sánchez said a nationwide blackout of this scale had never happened before in Spain.

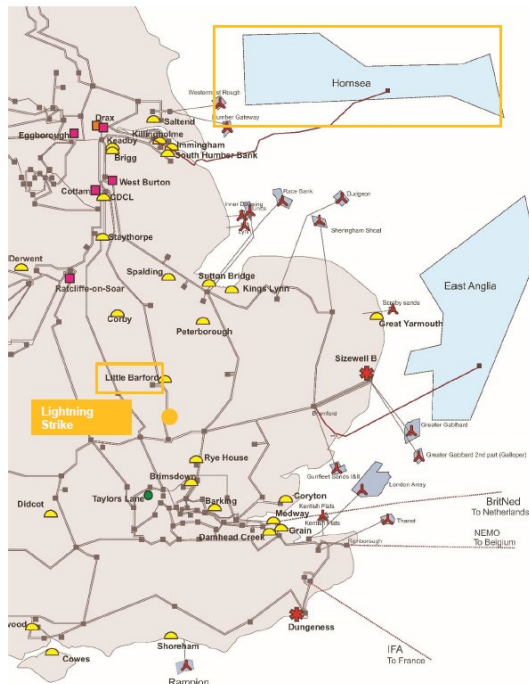
He added that, within a matter of seconds, a large amount of power — equivalent to 60% of Spain's electricity demand — was lost. Sánchez said the underlying cause of the outage remains unclear, and no theory has been ruled out.

May 7, 2025

Extensions

Low Frequency Demand Disconnection (LFDD) following Generator Trips and Frequency Excursion on 9 Aug 2019, UK

| Reporting DNO | | MW of disconnected demand by LFDD | Customers Affected | Final Restoration Time of Demand |
|--|---------------|-----------------------------------|--------------------|----------------------------------|
| Scottish Hydro Electric Power Distribution (SHEPD) | | 0 | | |
| Scottish Power (SP) | | 22 | 23 117 | 16:59 |
| Northern Power Grid (NPG) | North East | 76 | 93 081 | 17:18 |
| | Yorkshire | 14 | 10 571 | 17:12 |
| Electricity North Limited (ENW) | | 52 | 56 613 | 17:17 |
| SP Manweb | | 130 | 74 938 | 17:15 |
| Western Power Distribution (WPD) | East Midlands | 122 | 150 445 | 17:25 |
| | West Midlands | 160 | 187 427 | 17:37 |
| | South Wales | 36 | 29 060 | 17:11 |
| | South West | | 110 273 | 17:22 |
| UK Power Networks (UKPN) | Eastern | 69 | 79 390 | 16:56 |
| | London | 174 | 239 861 | 17:37 |
| | Southern | 69 | 81 358 | 17:15 |
| Scottish Electric Power Distribution (SEPD) | | 7 | 16 744 | 17:07 |
| Totals | | 931 | 1 152 878 | 17:37 |



Below is the detail of the cumulative losses of infeed

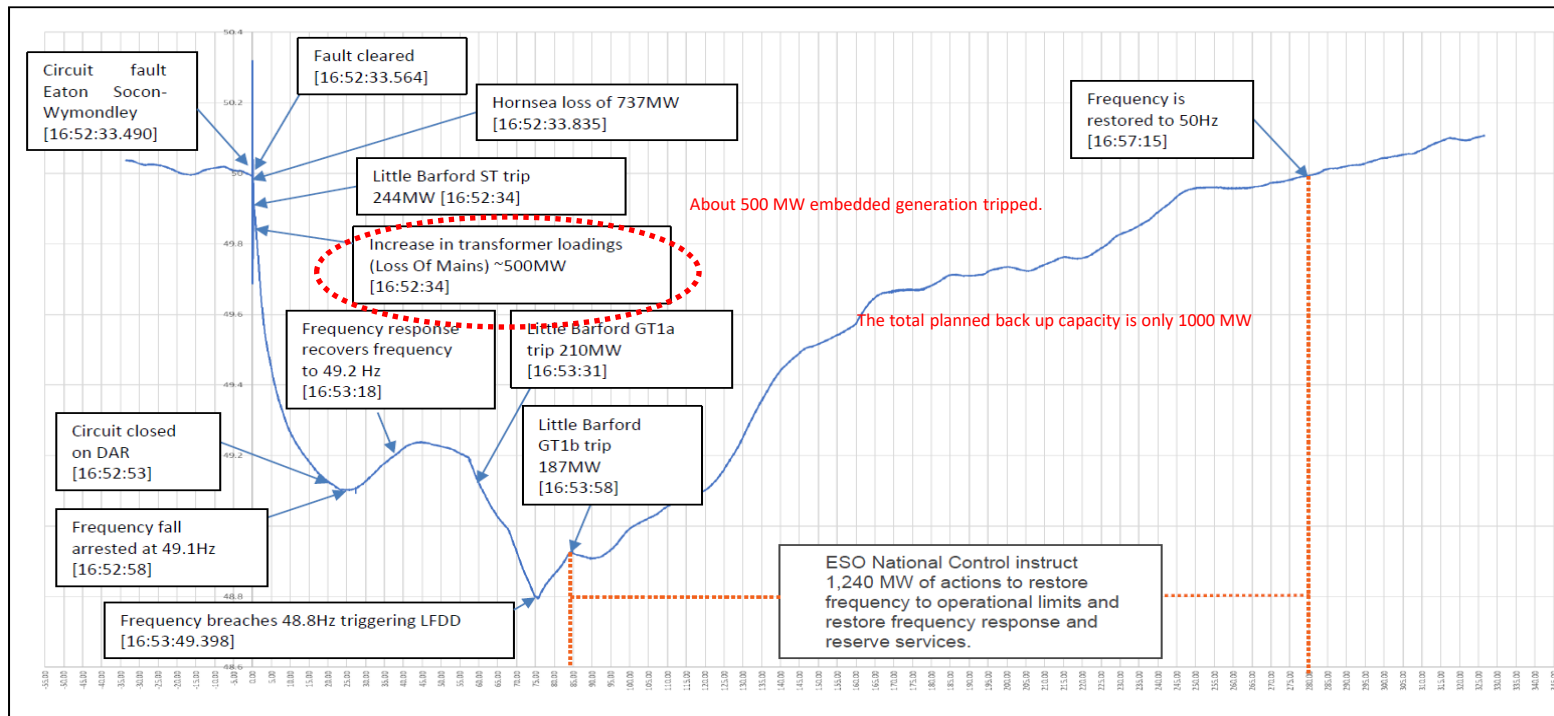
| Generation Unit | Infeed Loss | Cumulative Infeed Loss |
|---|-------------|------------------------|
| Little Barford ST1C | 244 MW | 244 MW |
| Hornsea Offshore Windfarm | 737 MW | 981 MW |
| ESO Security Standards and Planning Required an infeed loss 1,000 MW loss to be covered | | |
| Estimated, Embedded generation infeed loss due to Loss of Mains Protection | ~500 MW | ~1481 MW |
| Little Barford GT1A | 210 MW | ~1691 MW |
| Little Barford GT1B | 187 MW | ~1878 MW |

Table 2: Table of embedded infeed losses

Wind power exceeds over 50% of the generation at that time

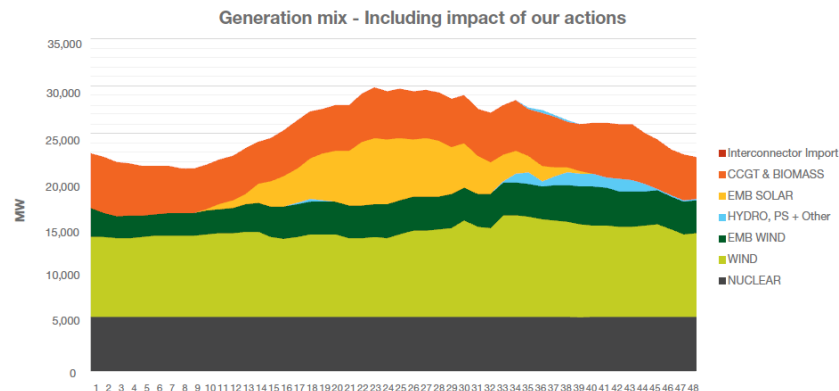
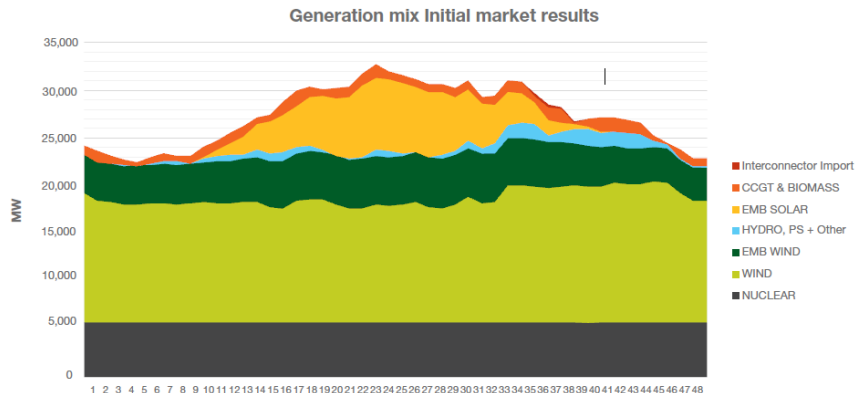
Extensions

Low Frequency Demand Disconnection (LFDD) following Generator Trips and Frequency Excursion on 9 Aug 2019, UK



Extensions

A success in low carbon power systems 23 May 2020



Nearly over 95% zero carbon generation.

Because of technical limitations, the market results cannot be matched.

Over 80% are from zero carbon generations