CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 9 TEM Transmission Lines

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OUTLINE

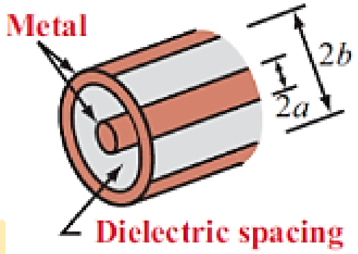
- Overview
  - What is transmission line?
  - Types of TEM TX lines
- Physical Description of TX line propagation
- The TX line Equations

1.1 WHAT IS TRANSMISSION LINES?

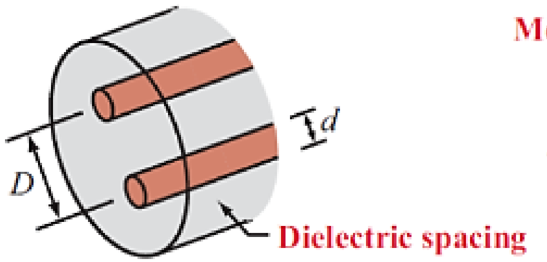
In electrical engineering, a transmission line is a specialised cable or other structure designed to conduct electromagnetic waves in a **contained manner**. They are used for efficient transmission of power and data from the source to the load.

The term applies when the conductors are long enough that the wave nature of the transmission must be considered. This applies especially to radio-frequency engineering because the short wavelengths mean that wave phenomena arise over very short distances (this can be as short as millimetres depending on frequency).

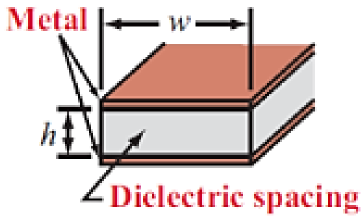
1.2 TYPES OF TEM TX LINES



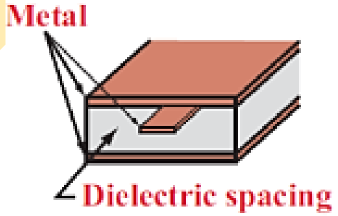
(a) Coaxial line



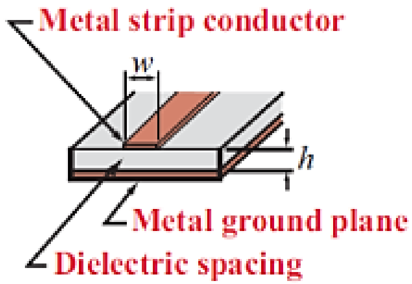
(b) Two-wire line



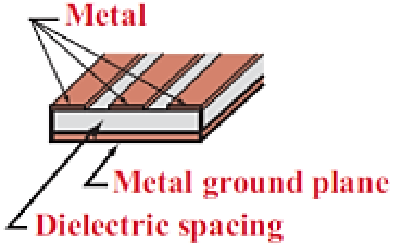
(c) Parallel-plate line



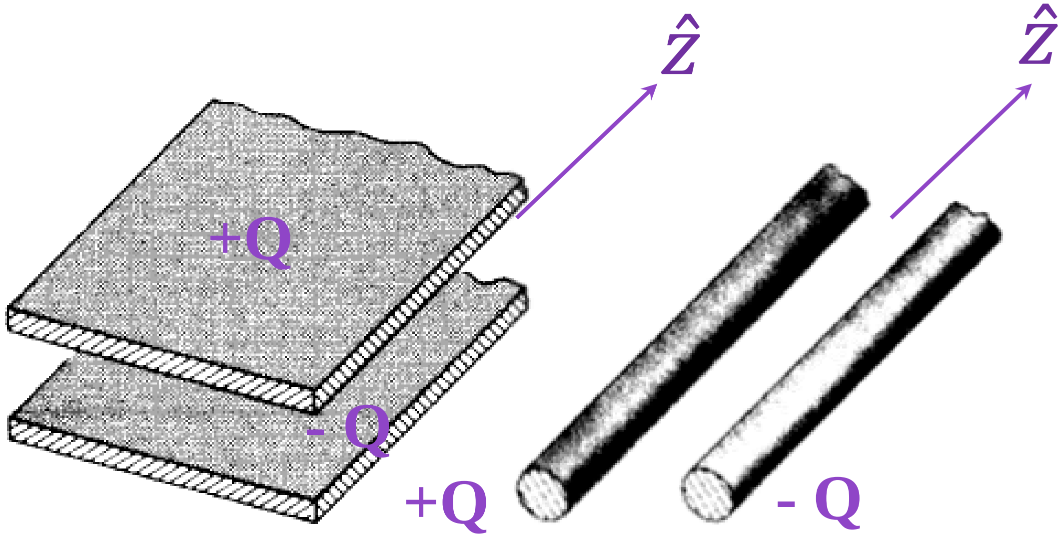
(d) Strip line



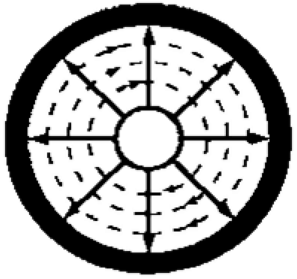
(e) Microstrip line



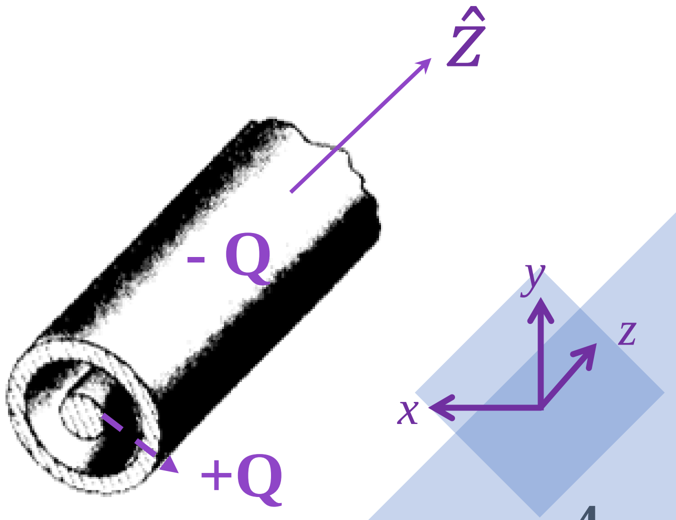
(f) Coplanar waveguide



$$\hat{a}_{propagation} \times \vec{E} = \eta\vec{H}$$



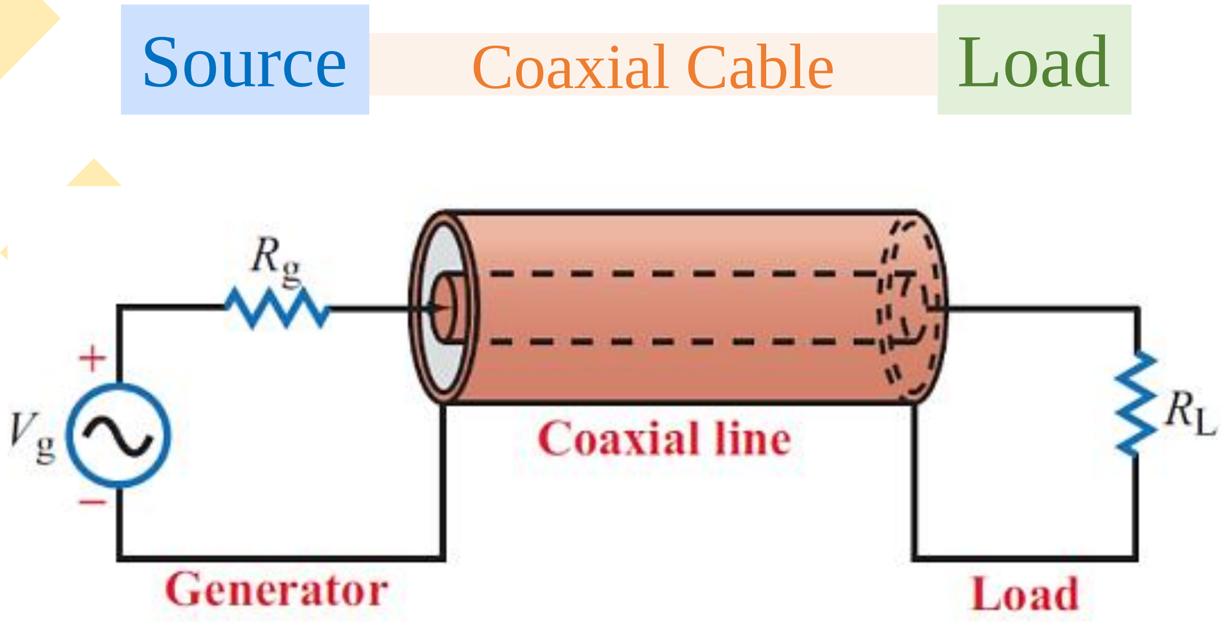
Cross section

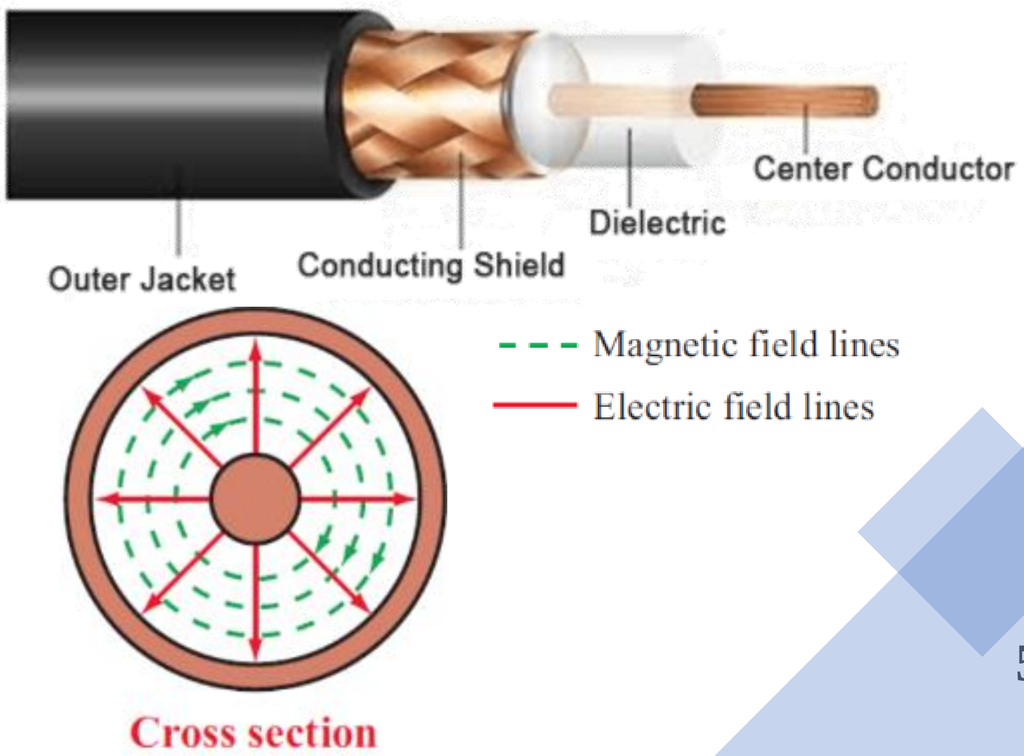


*EXAMPLE OF TEM MODE

A two-conductor cable made of a single conductor surrounded by a braided wire jacket, with a plastic insulating material separating the two.

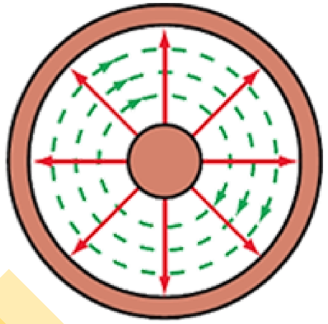
This type of cabling is often used to conduct weak (low-amplitude) voltage signals, due to its excellent ability to shield such signals from external interference.



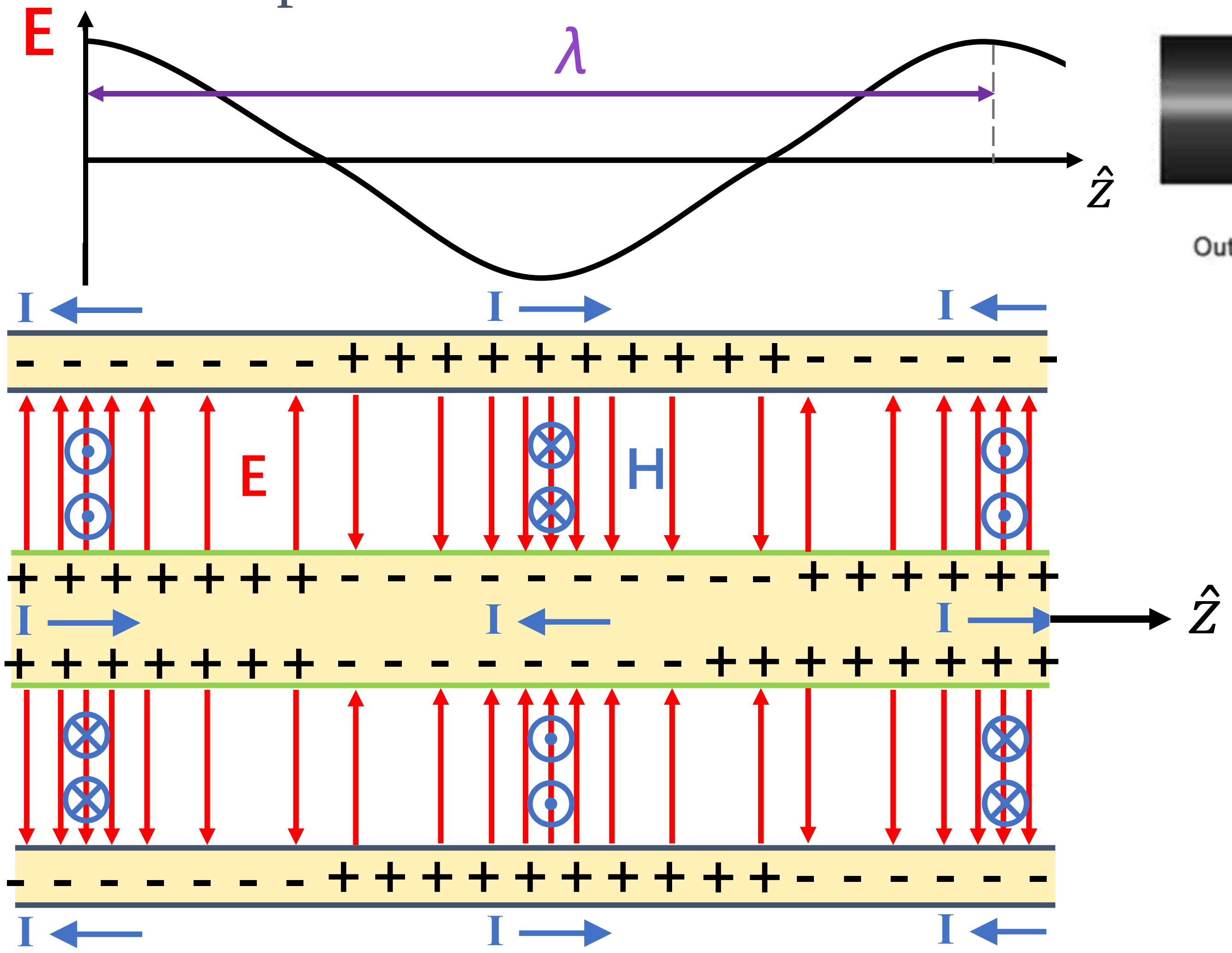


* WAVE PATTERN IN COAXIAL CABLE

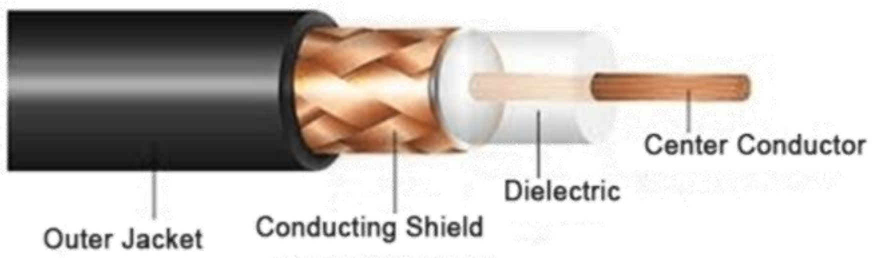
The transverse field pattern for a coaxial line is:



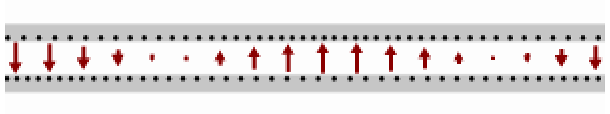
Cross section



Sinusoidal wave travelling from left to right



$$\boxed{\hat{z} \times \vec{E} = \eta \vec{H}}$$



$\vec{E}$ and $\vec{H}$ fields are **orthogonal**

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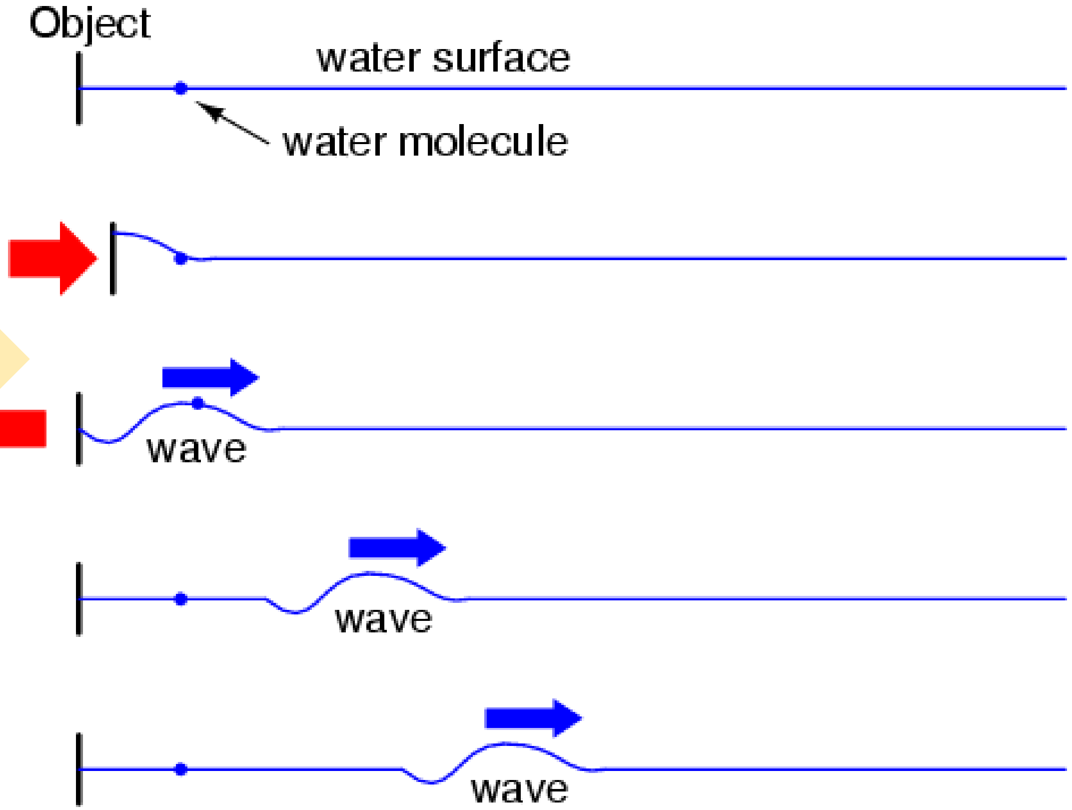
2.1 WHAT IS WAVE?

Imagine the waves in water. Suppose a flat, wall-shaped object is suddenly moved horizontally along the surface of water to produce a wave ahead of it.

The wave will travel as water molecules bump into each other, transferring wave motion along the water's surface.

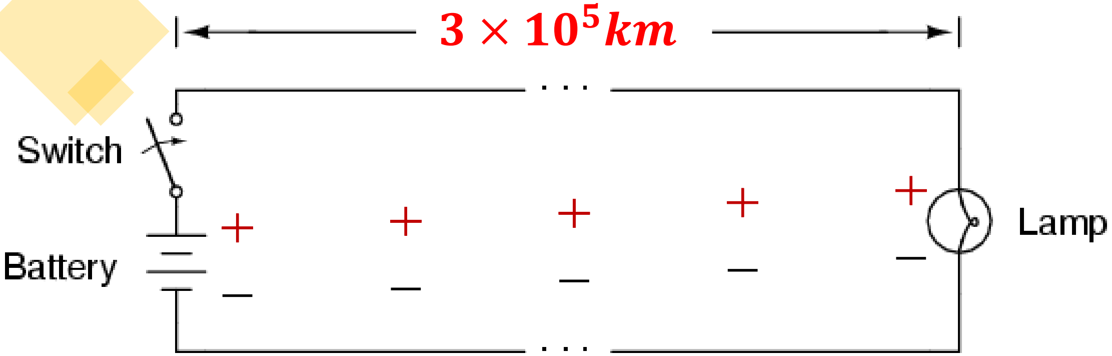


2.1 WHAT IS WAVE IN LONG WIRES?

Suppose a simple circuit:

The overall effect of electrons pushing against each other happens at the speed of light. When the switch is closed, the lamp lights immediately.

Suppose the lamp is connected to the battery by a **long** parallel-wire:

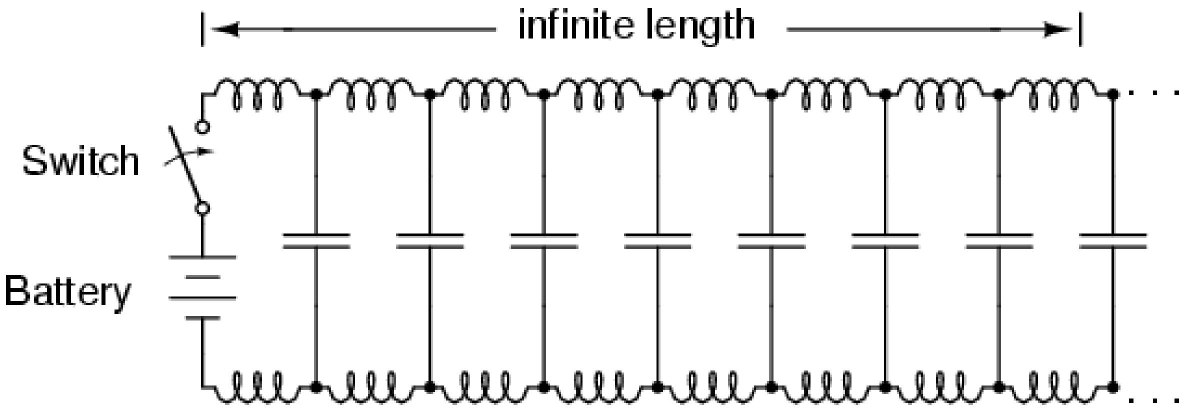


It should introduce a time delay into the circuit, delaying the switch's action on the lamp.

At the speed of light, lamp lights up ~1s

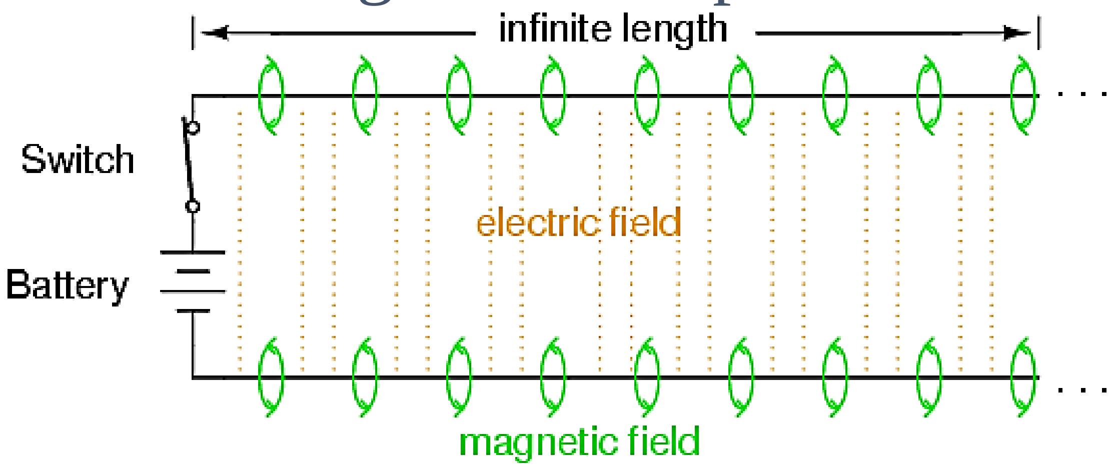
2.2 WAVE PROPAGATION

Equivalent circuit of a lossless transmission line consists of lumped capacitance and inductance:



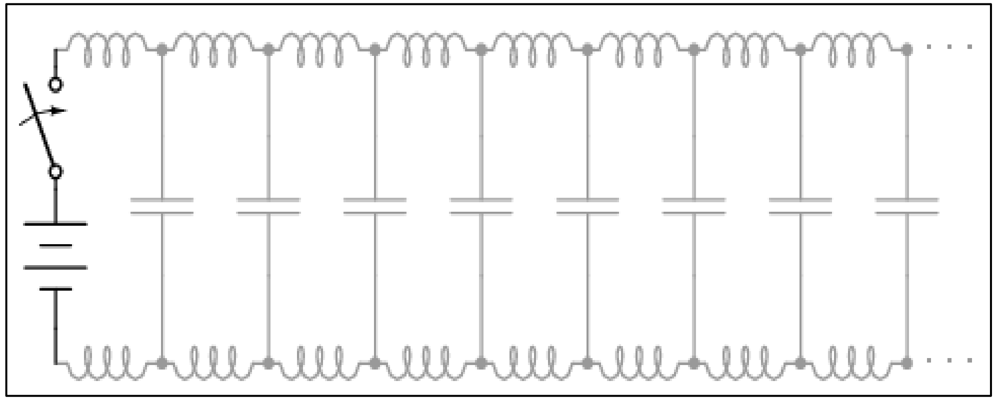
Lossless: all power launched into the line at the input arrives at the output terminal eventually.

Electric field and magnetic field patterns look like the figure below:

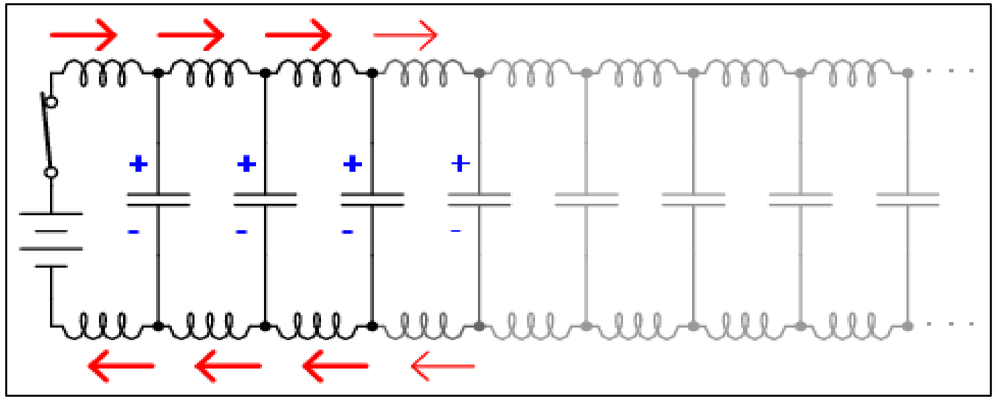


2.2 CONT. ANALYSIS

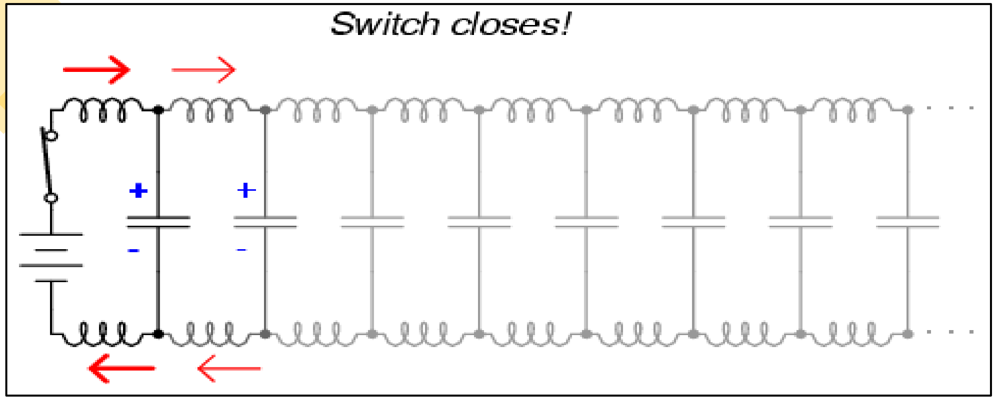
1. Uncharged transmission line



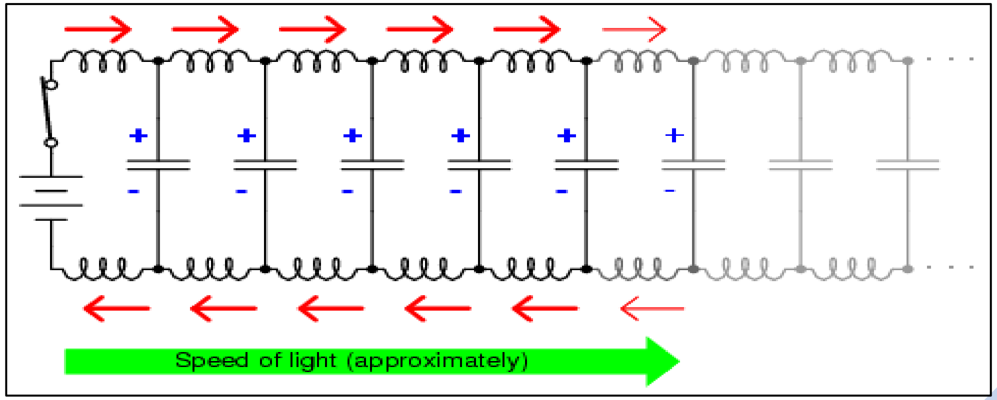
3. Continuing



2. Begin wave propagation



4. Propagate at speed of light

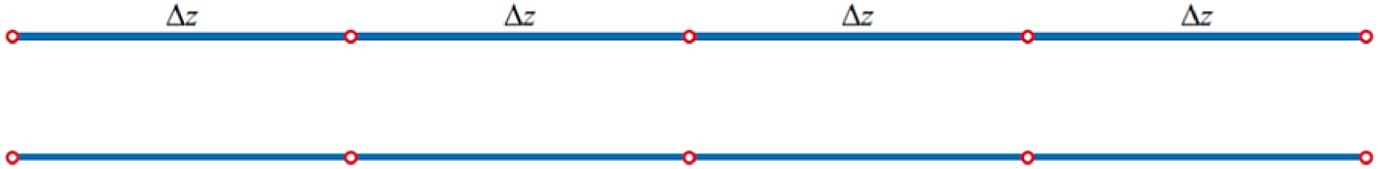


2.3 TRANSMISSION LINE MODEL

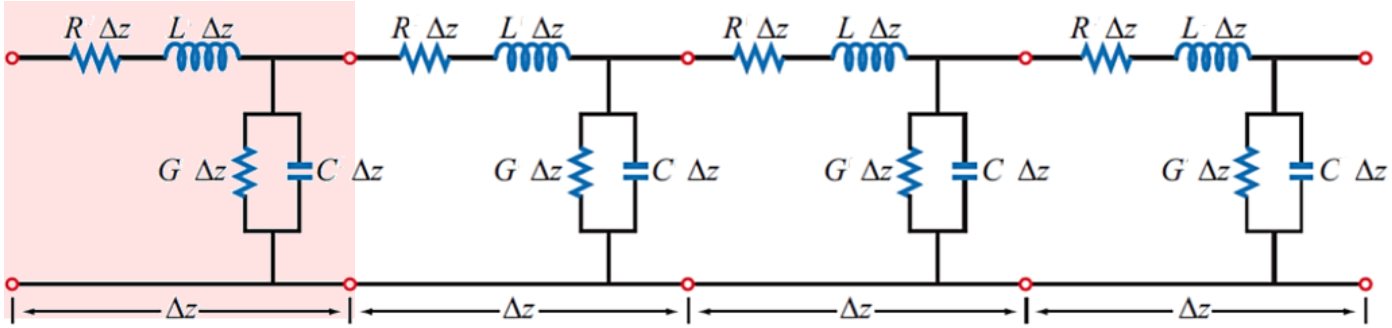
A general structure (lossy) of a transmission line possess not only capacitance and inductance, but also resistance and admittance. These are all expressed on a **per-unit-length** basis. A model using lumped elements can be constructed with all four elements needed to represent it.

(a) Parallel-wire representation

(b) Differential sections with same unit length



(c) Each section is represented by an equivalent circuit



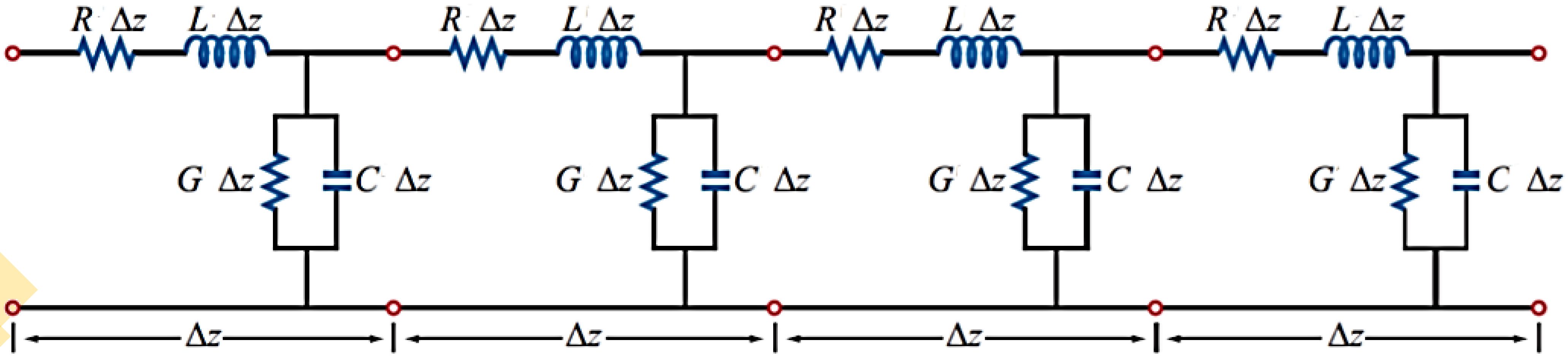
Lumped-element model

2.3 TRANSMISSION LINE MODEL

The circuit model contains the four primary constants:

$$R,\ G,\ L,\ C$$

all of them have values that are specified by *per unit length*.



R = series resistance per unit length for both conductors (Ω/m)

L = series inductance per unit length for both conductors (H/m)

G = shunt conductance per unit length (S/m)

C = shunt capacitance per unit length (F/m)

2.3 TRANSMISSION LINE MODEL

Three basic properties:

Resistance: impacts the flow of current;
controlled by the cross-section area

Inductance: due to magnetic field;
impacted by magnetic object

Capacitance: generally impacted by the grounding

Note that these parameters are very low when the input voltage is DC or operating at low frequency, thus they can be ignored.

R and *G* are responsible for power loss in the transmission process.

i.e., Lossless transmission line has no *R* and *G*.

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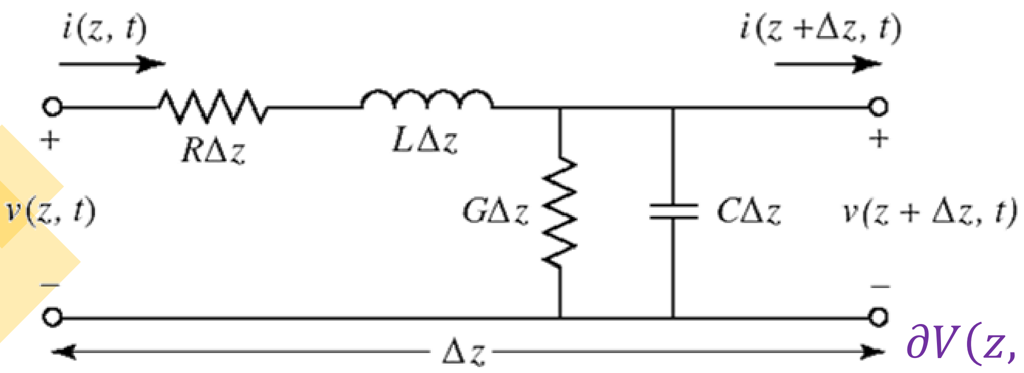
3.1 CIRCUIT MODEL

Consider a line section of length *Δz* containing resistance *RΔz*, inductance *LΔz*, conductance *GΔz*, and capacitance *CΔz*. Since the section of the line looks the same from either end, divide the series elements in half to produce a symmetrical network.



$$V(z,t) - R\Delta z I(z,t) - L\Delta z \frac{\partial I(z,t)}{\partial t} - V(z+\Delta z,t) = 0$$

$$I(z,t) - G\Delta z V(z+\Delta z,t) - C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t} - I(z+\Delta z,t) = 0$$

R = series resistance per unit length for **both** conductors

L = series inductance per unit length for **both** conductors

G = shunt conductance per unit length

C = shunt capacitance per unit length

$$-\frac{\partial V(z,t)}{\partial z} = -\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t} \quad (1)$$

$$-\frac{\partial I(z,t)}{\partial z} = -\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t} \quad (2)$$

Telegrapher Equations

Give the relationship between V & I along a TX

General wave equations for TX:

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG + RC)\frac{\partial V}{\partial t} + RGV \quad (3)$$

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2} + (LG + RC)\frac{\partial I}{\partial t} + RGI \quad (4)$$

CONT.

These equations can be expressed for time harmonic fields, so we can get the ***Time harmonic TX equations***:

(1’) $$\frac{dV}{dz} = -(R + j\omega L)I(z)$$

(2’) $$\frac{dI}{dz} = -(G + j\omega C)V(z)$$

Combine them to solve for *V*(*z*) and *I*(*z*):

$$\left.\begin{aligned}\frac{d^2V(z)}{dz^2} &= \gamma^2 V(z)\\ \frac{d^2I(z)}{dz^2} &= \gamma^2 I(z)\end{aligned}\right\} \quad (5)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

γ – propagation constant

α – attenuation constant (Np m^{-1})

β – phase (lossless propagation) constant (rad m^{-1})

β represents the relative phase angle of the wave as a function of position along the line

Solve (5) we get *V*(*z*) and *I*(*z*) are:

$$V(z) = V^+(z) + V^-(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I^+(z) + I^-(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

where the “+” and “-” superscripts denotes waves travelling in the +z and –z directions.

Wave amplitude is:

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma}$$

The ratio of the voltage and the current for an *infinitely* long line is **independent** of *z*. It is called the line’s *characteristic impedance*.

$$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

3.3 FOR LOSSLESS TX LINE

Lossless line: $R = G = 0$

Propagation constant

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC}$$

Phase velocity

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Characteristic impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}$$

3.4 FOR LOW LOSS TX LINE

Low-loss line: R << ωL, G << ωC

Propagation constant

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC}\sqrt{(1 + \frac{R}{j\omega L})}\sqrt{(1 + \frac{G}{j\omega C})}$$

$$\sqrt{(1+x)} \approx 1 + \frac{x}{2}$$

$$\approx j\omega\sqrt{LC}(1 + \frac{R}{2j\omega L})(1 + \frac{G}{2j\omega C}) \approx j\omega\sqrt{LC}\left[1 + \frac{1}{2j\omega}\left(\frac{R}{L} + \frac{G}{C}\right)\right] = \frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) + j\omega\sqrt{LC}$$

Phase velocity

$$u_p = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{LC}}$$

Characteristic impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}\left(1 + \frac{R}{j\omega L}\right)^{\frac{1}{2}}\left(1 + \frac{G}{j\omega C}\right)^{-\frac{1}{2}} \approx \sqrt{\frac{L}{C}}\left[1 + \frac{1}{2j\omega}\left(\frac{R}{L} - \frac{G}{C}\right)\right] \approx \sqrt{\frac{L}{C}}$$

QUIZ 1

The line parameter of a 0.8m long lossless transmission line operating at 600 MHz are:

$$R = 0, G = 0, L = 0.25\mu H/m, C = 0.1nF/m$$

Find the characteristic impedance, phase constant, and the magnitude of the phase velocity.

QUIZ 2

A uniform transmission line has constants:

$$R = 12m\Omega/m, G = 1.4\mu S/m, L = 1.5\mu H/m, C = 1.4nF/m$$

Find the characteristic impedance at 7000 Hz.

NEXT…

Electric Circuits:

First Order Transient Response

you can do it!