MOSFET (I): Fundamentals

- I - V Characteristics
  - Cutoff Region
  - Linear Region
  - Saturation Region (pinch-off region)
- Switch model of nMOSFETs

Reading: Chapter 3.3

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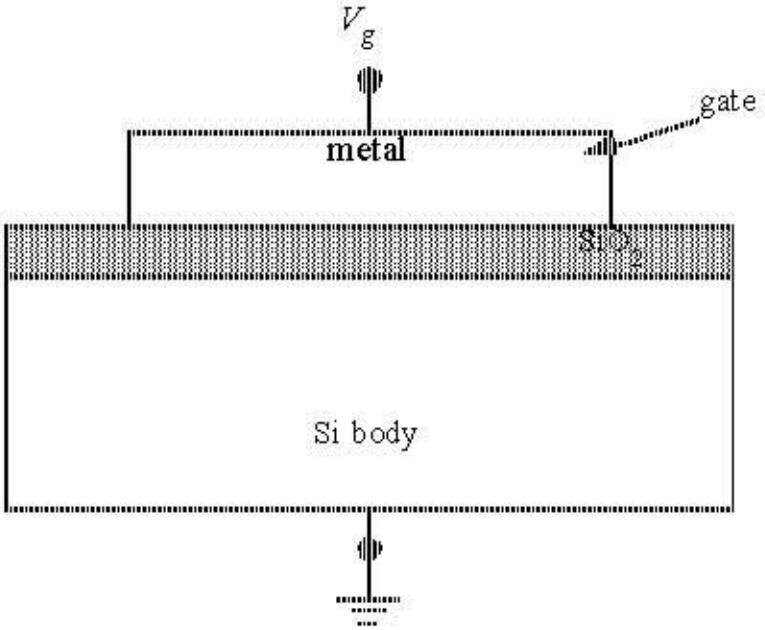
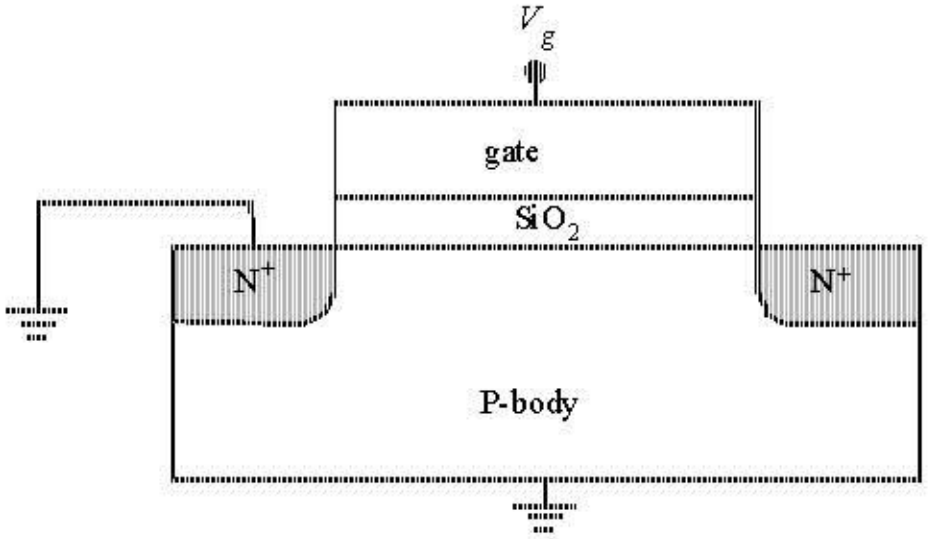
MOS Capacitors

MOS: Metal-Oxide-Semiconductor

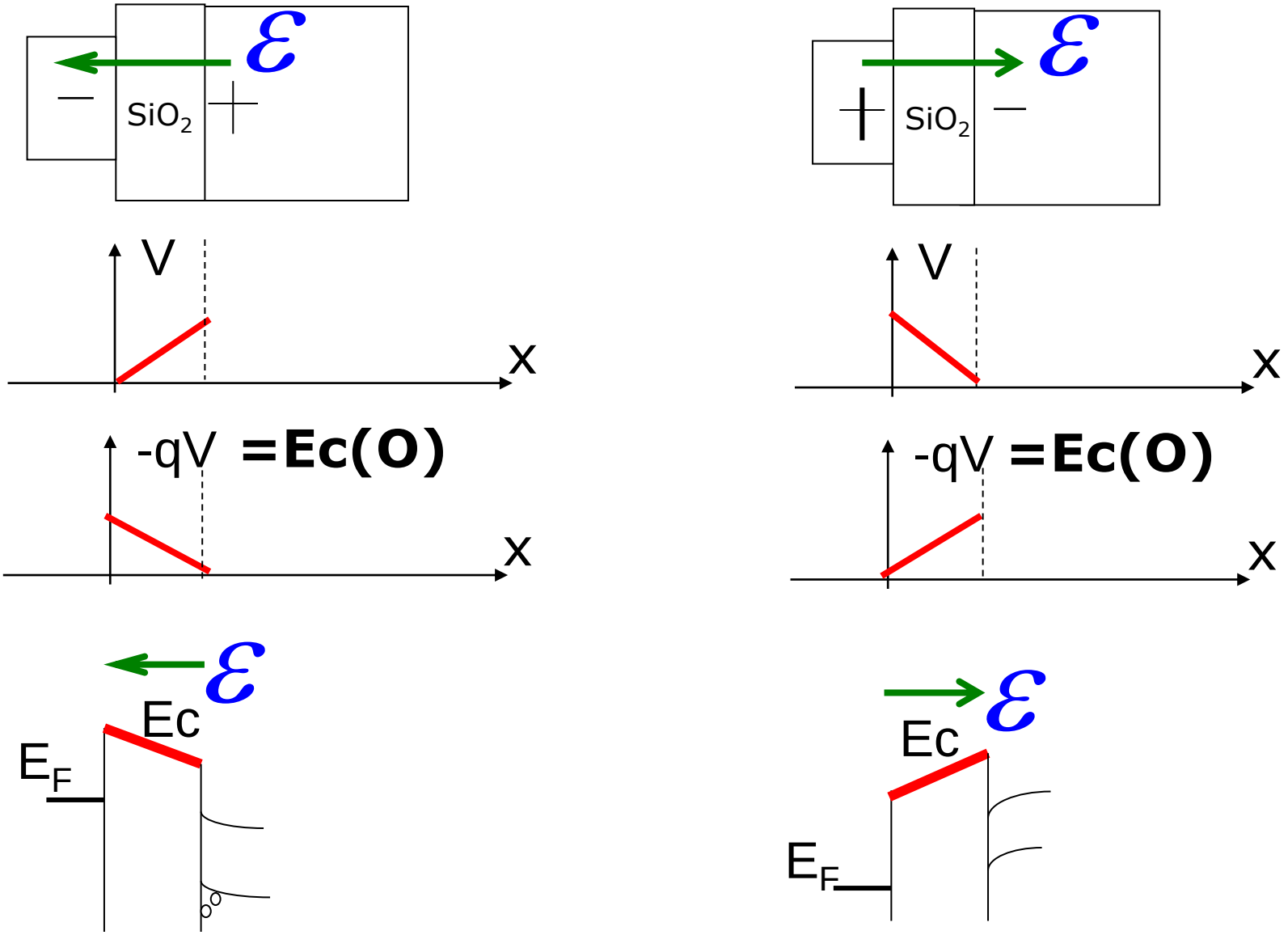


MOS capacitor



MOS transistor

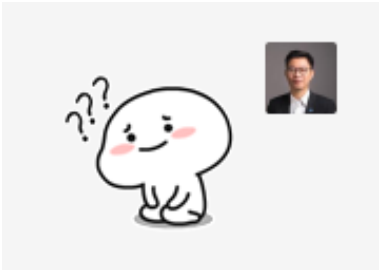
Ec(O) and electric field direction

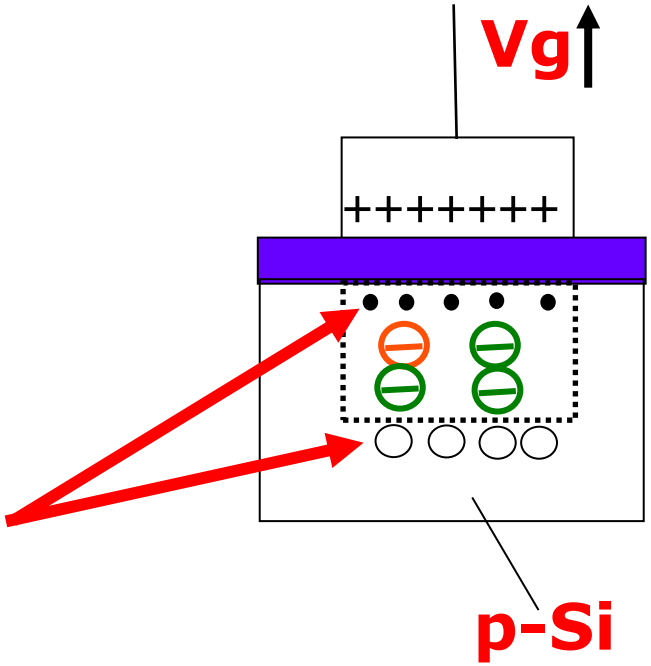


Energy band diagram: $V_g > V_m$

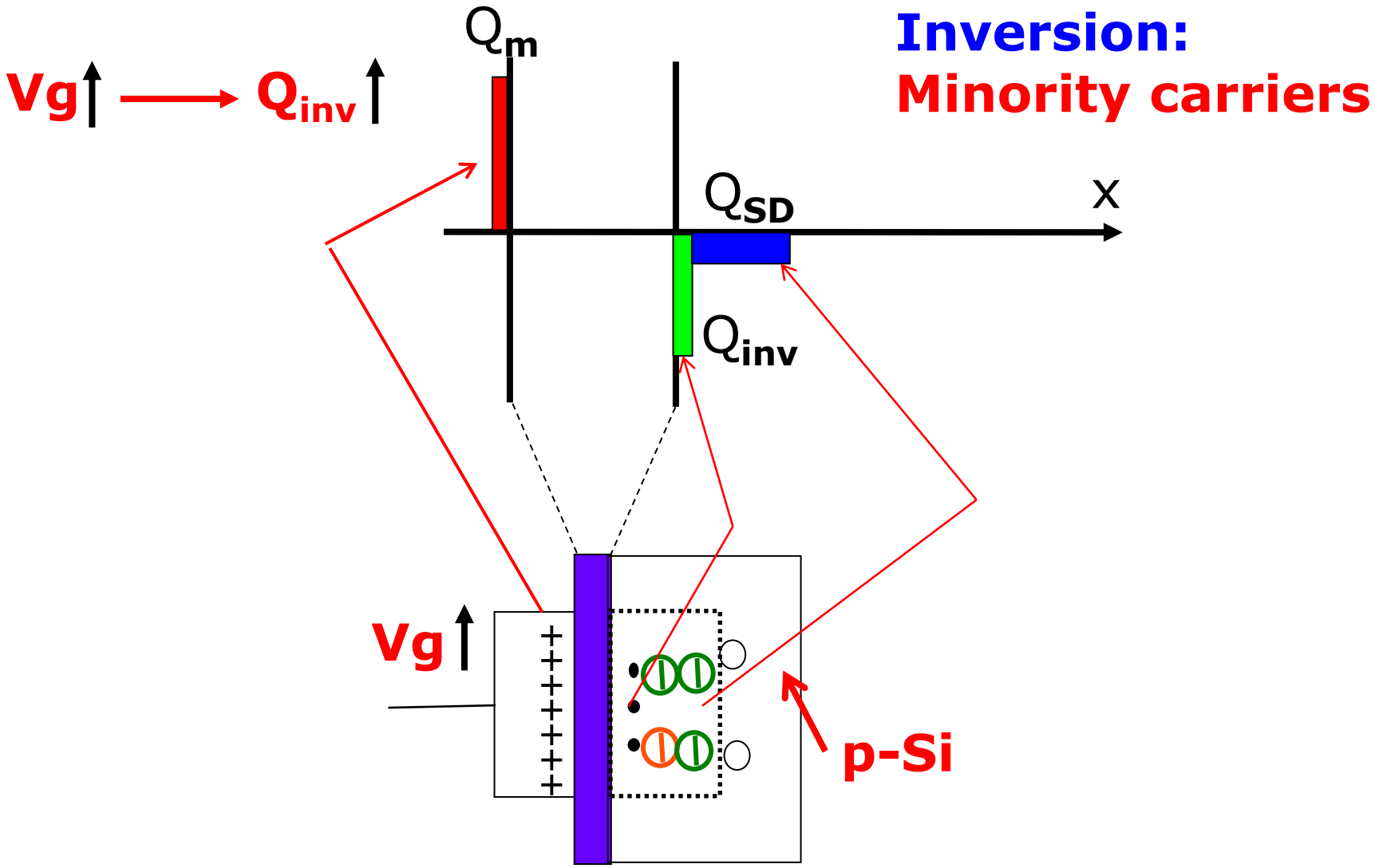
$V_g \uparrow \longrightarrow Q_{inv} \uparrow$

Inversion:
Minority carriers

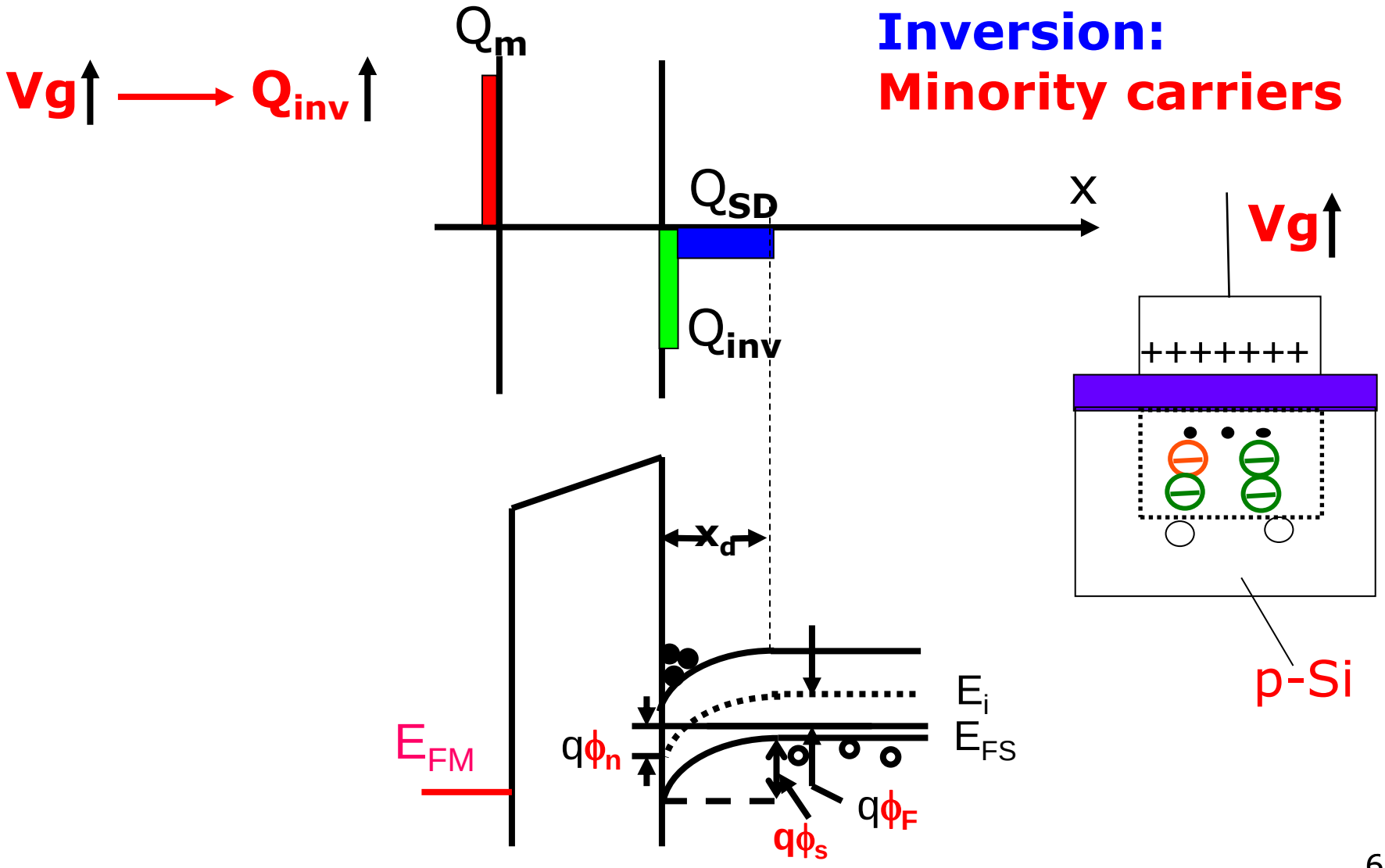




Energy band diagram: $Vg > V_m$



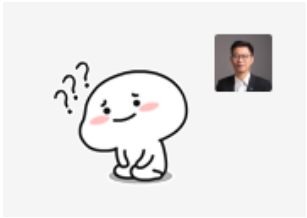
Energy band diagram: Vg>V_m



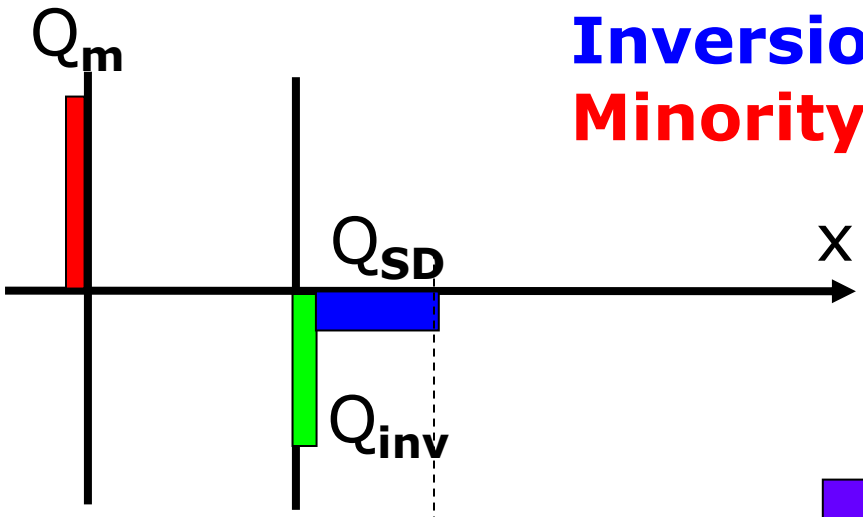
Energy band diagram: Vg>V_m

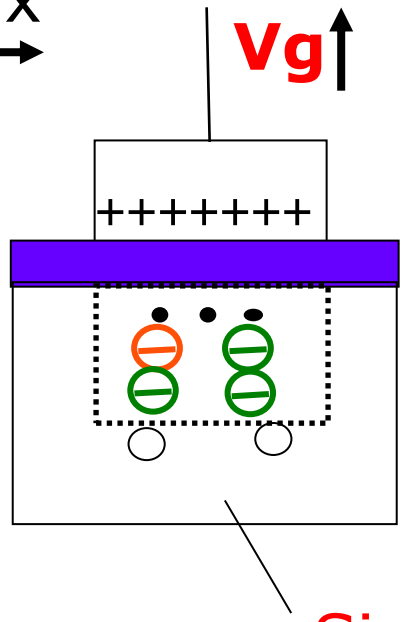
Vg↑ ⟶ Q_{inv}↑

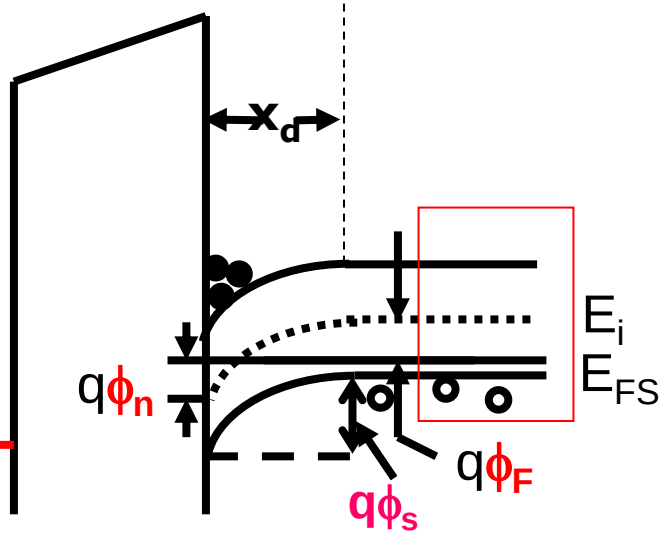
As Vg>Vm, E_{FS} is at the up-half of the bandgap



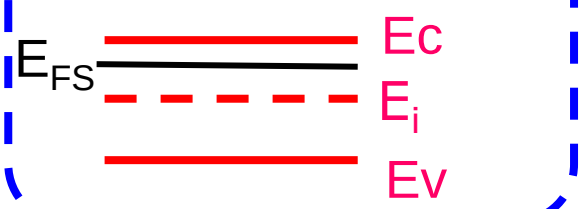
Inversion: Minority carriers







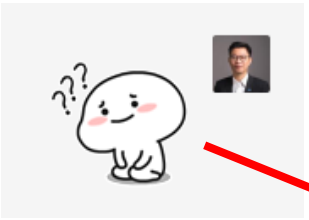
At the interface



Energy band diagram: Vg>V_m

Vg↑ ⟶ Q_{inv}↑

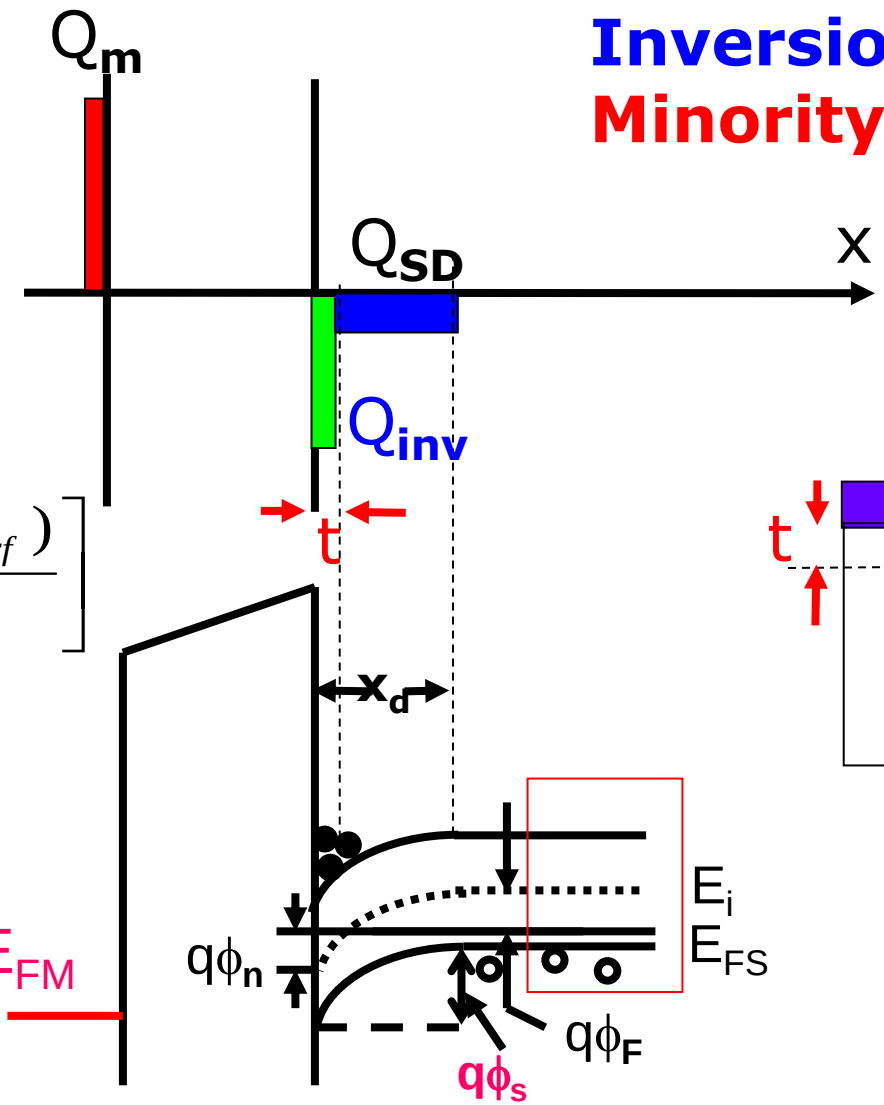
C/cm²



$$n_s = n_i \exp\left[\frac{(E_F - E_{i,surf})}{kT}\right]$$

electrons/cm³

$$Q_{inv} = q\, n_s\, t$$



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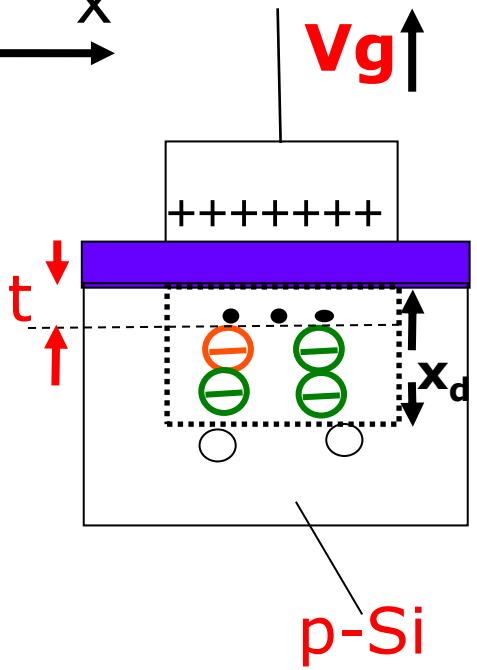


Inversion

- **Weak Inversion**: $0<\phi_n<\phi_F$ ($\phi_F < \phi_S < 2\phi_F$)
- **Strong Inversion:** $\phi_n \geq \phi_F$ ($\phi_S \geq 2\phi_F$), electron density at the interface ≥ hole density in Si bulk.
- Vg for strong inversion: V_T 'threshold voltage'.

$$Vg = V_T \rightarrow \phi_s = 2\phi_F \rightarrow \phi_n = \phi_F \rightarrow n_s = p_b$$

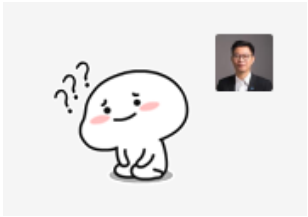
$$n_s = n_i \exp\left[\frac{(E_F - E_{i,surf})}{kT}\right]$$

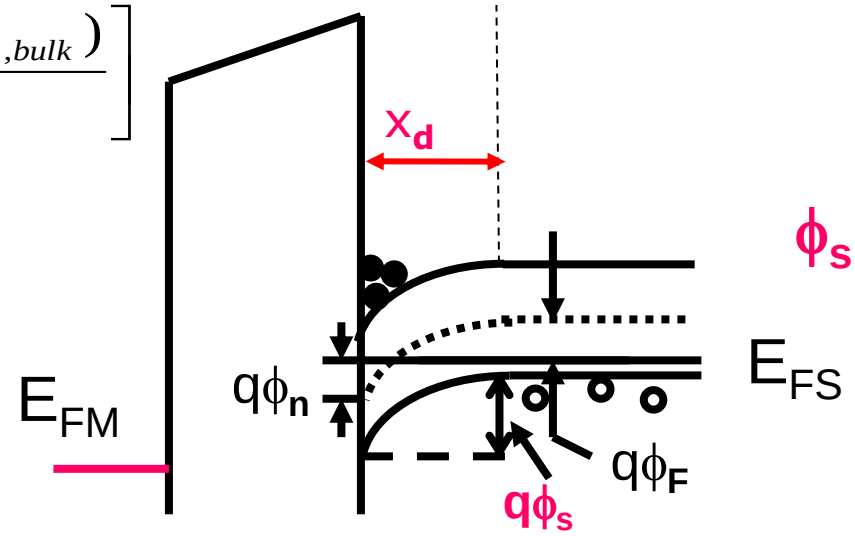
$$p_b = n_i \exp\left[\frac{-(E_F - E_{i,bulk})}{kT}\right]$$

$$= N_A$$

$$p_s \approx n_i^2 / n_s$$

$$n_b = n_i^2 / p_b$$





$$\phi_s = \phi_F + \phi_n$$

Voltage drops in a MOS system

$$V_G = V_{FB} + V_{ox} + \phi_s$$

$$V_T = V_{FB} + V_{ox} + 2\phi_F$$

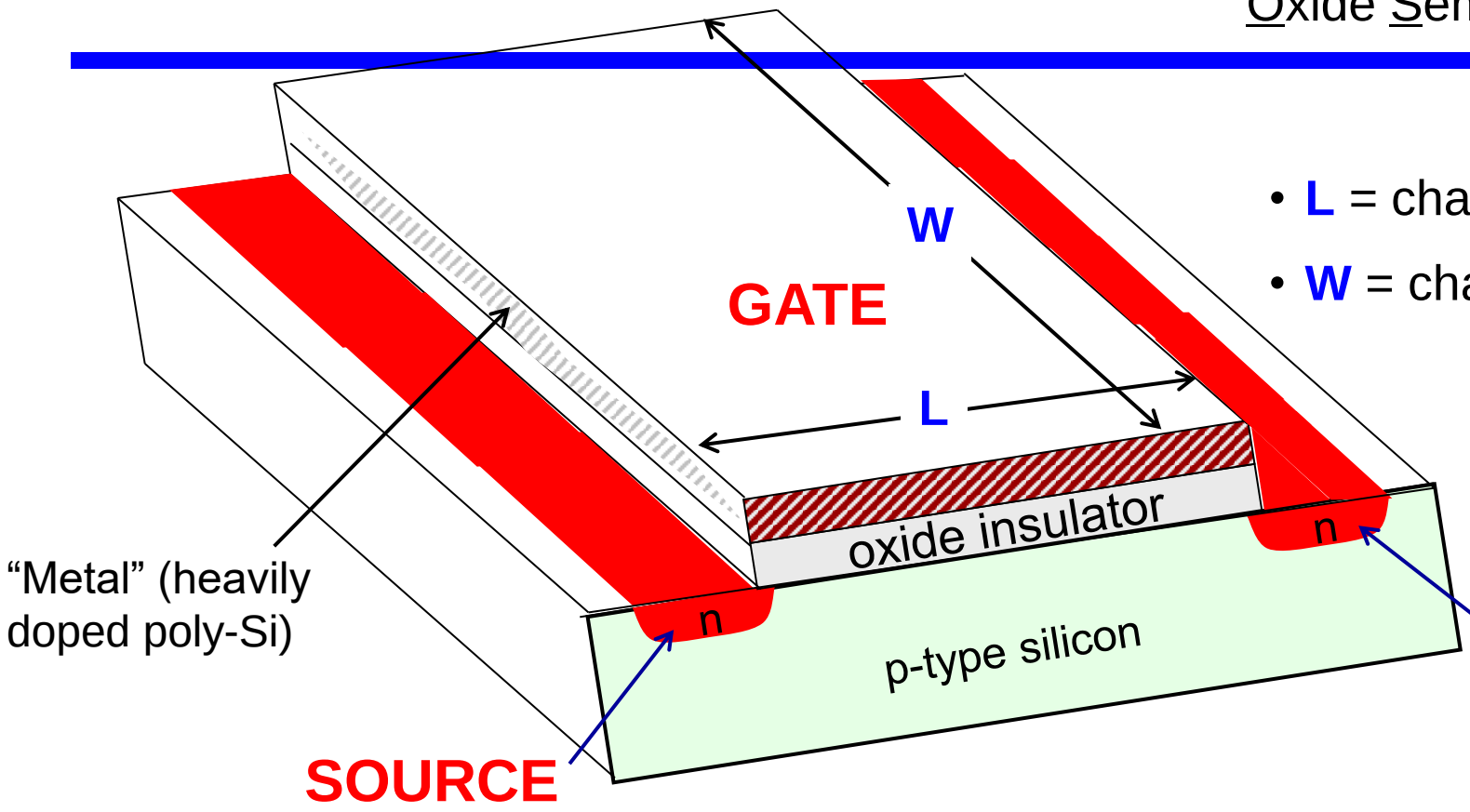
$Vg=V_T \rightarrow$
$\phi_s = 2\phi_F$

$$V_{ox} = +\frac{\sqrt{2qN_A\varepsilon_{Si}(2\phi_F)}}{C_{ox}}$$ for p-Si sub.

$$V_{ox} = -\frac{\sqrt{2qN_D\varepsilon_{Si}|2\phi_F|}}{C_{ox}}$$ for n-Si sub.

nMOSFET

- **NMOS:** N-channel Metal Oxide Semiconductor

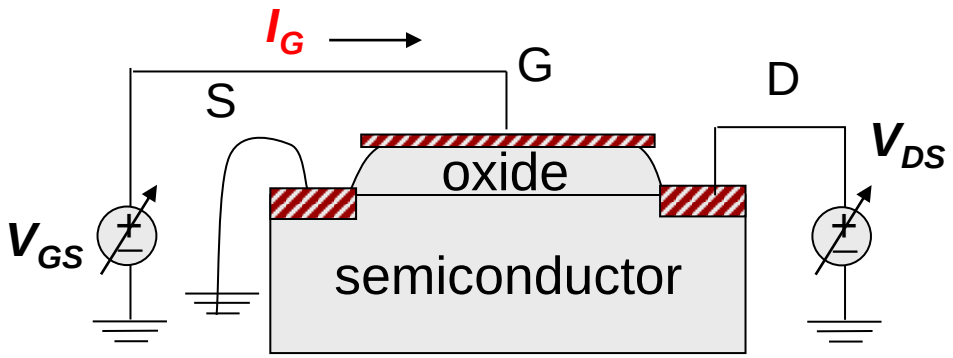


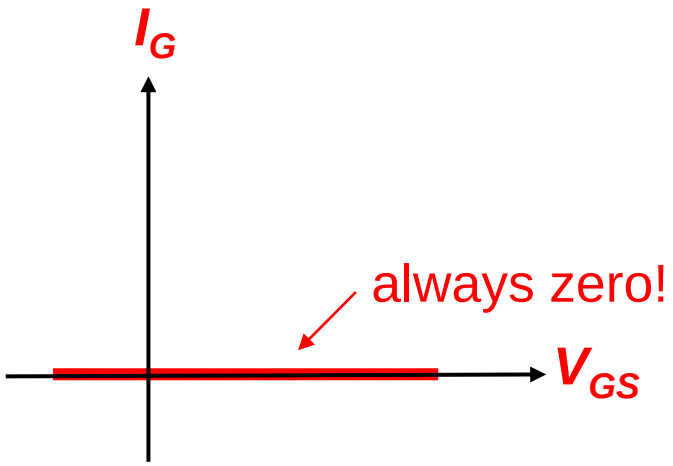
- **L** = channel length
- **W** = channel width

- A GATE electrode is placed above (electrically insulated from) the silicon surface, and is used to control the resistance between the SOURCE and DRAIN regions

Gate oxide

Consider the current I_G (flowing into **G**) versus V_{GS} :





The gate is insulated from the semiconductor, so there is **no** significant (steady) gate current.

OUTLINE

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  - **Cutoff Region** ←
  - Linear Region
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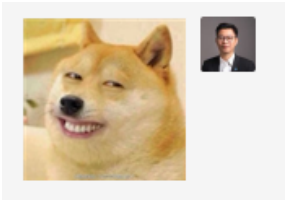
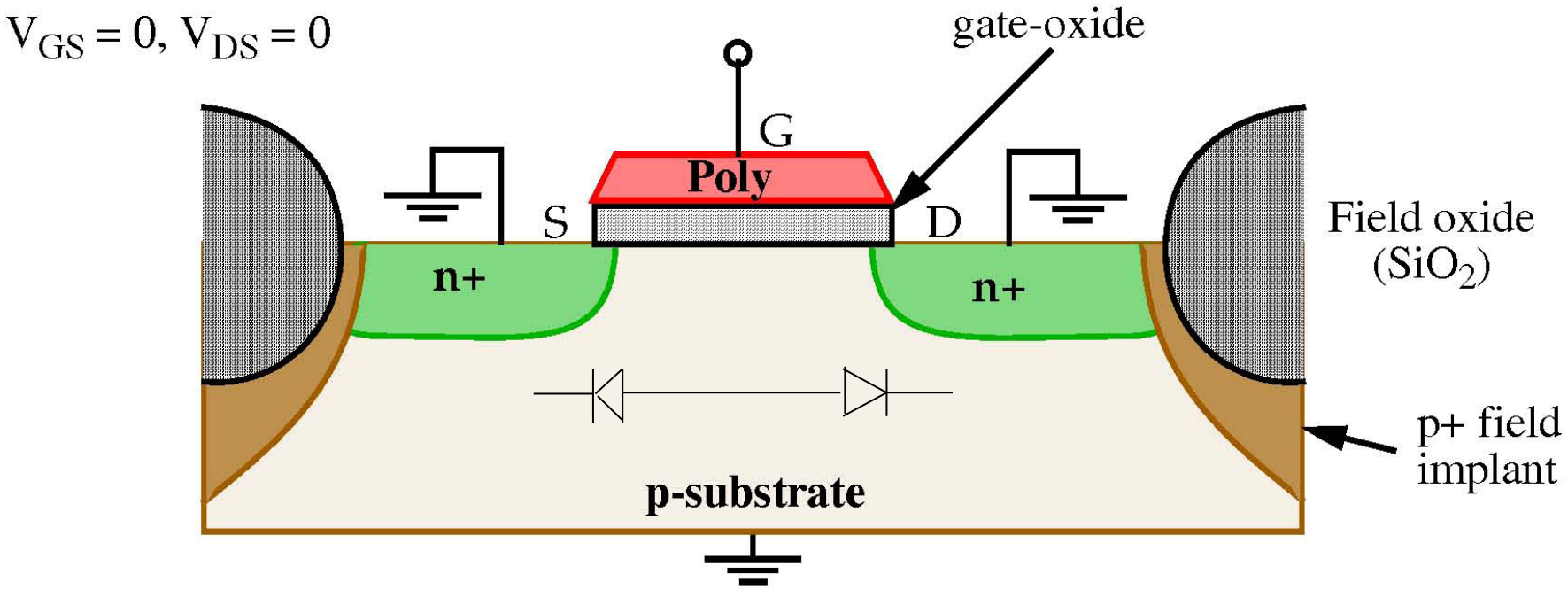
- **Switch model of nMOSFETs**



nMOSFET: $V_{GS}<V_T$



Under **zero** bias, two back-to-back *pn*-junctions create a very high resistive path between source and drain.

Appling a **positive bias** (V_{GS}) to the gate, creates a **depletion region** under the gate (repels mobile holes). The depletion region is **same** as the one occurring in a *MOS Capacitor*.

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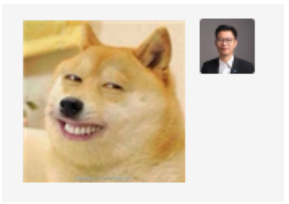
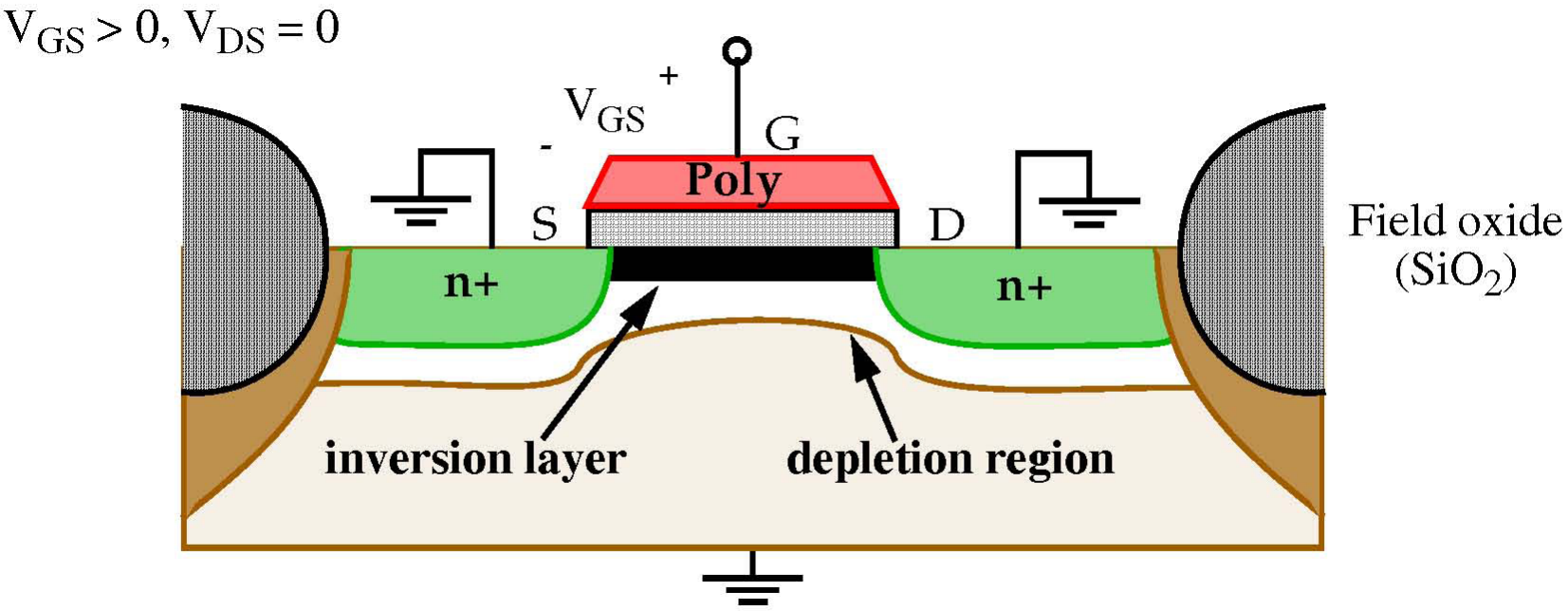


nMOSFET: $V_{GS}>V_T$



Inversion layer charge expressions are (the surface potential ϕ_s is $2\phi_F$):

Inversion layer charge $$Q_{inv} = -C_{ox}(V_G - V_T)$$

$V_G \uparrow \rightarrow Q_{inv} \uparrow$

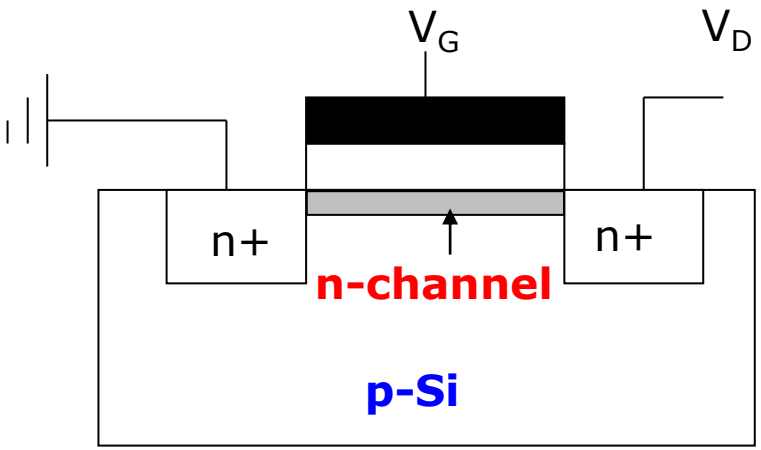
Threshold voltage $$V_T = V_{FB} + 2\phi_F + \frac{\sqrt{2qN_A\varepsilon_{Si}(2\phi_F)}}{C_{ox}}$$

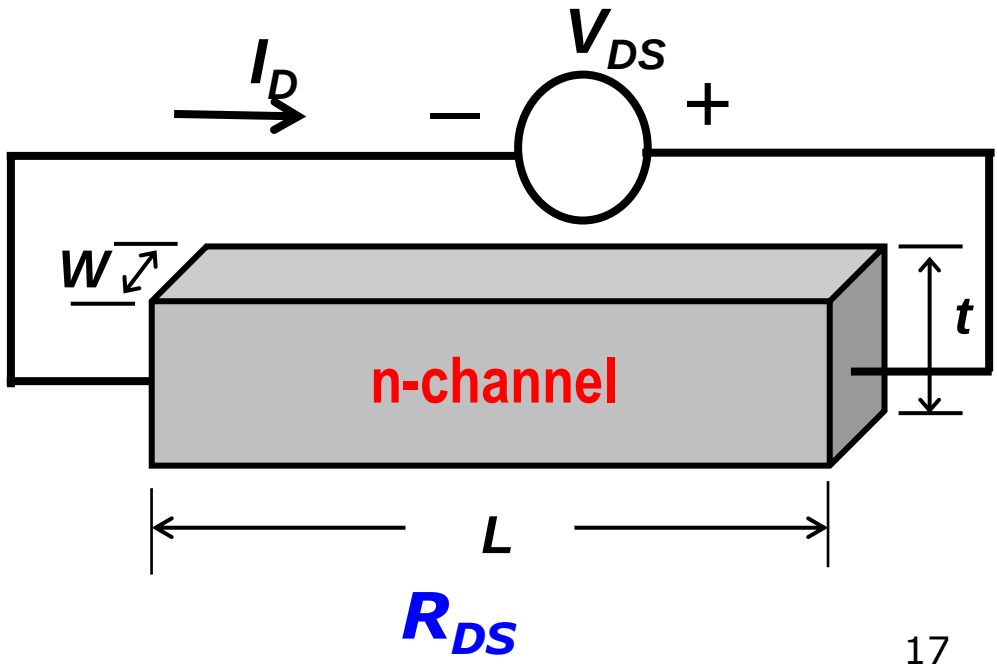
Inversion layer

$$Q_{inv} \times W \times L = q \times n \times t \times W \times L$$
$$\rightarrow Q_{inv} = q\,n\,t$$

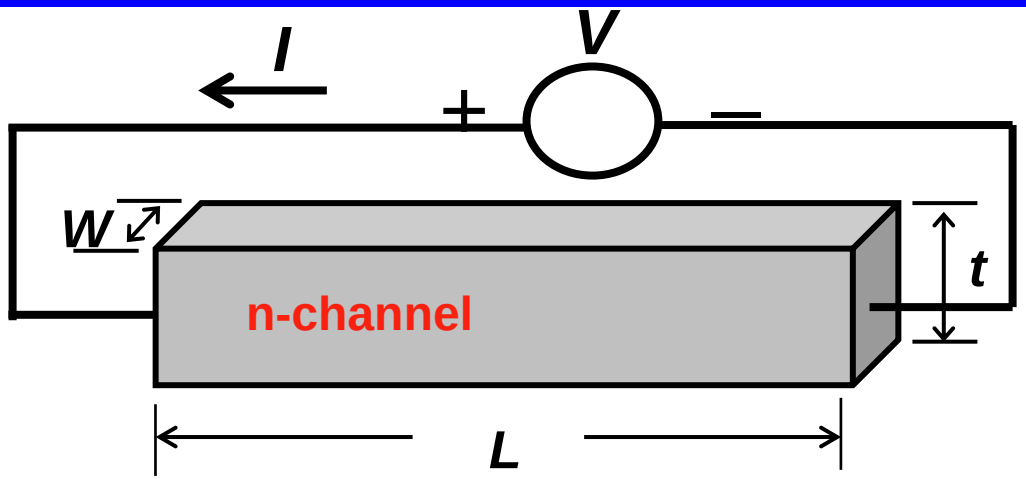
- **Without gate bias**, MOSFET is **off** because two diodes are "back-to-back". One of them will be reversely biased. To switch on, the interfacial region is inverted by applying a gate bias.

- Above a certain gate-to-source voltage (***threshold voltage V_T***), **a conducting layer** of mobile electrons is formed at the Si surface beneath the oxide. These electrons can carry current between the source and drain.





Electrical Resistance



Resistance $R \equiv \frac{V}{I} = \rho\frac{L}{A} = \frac{\rho L}{tW} = \left(\frac{\rho}{t}\right)\left(\frac{L}{W}\right)$ (Unit: ohms)

where ρ is the resistivity (Ω•cm)

R_S

Electrical Conductivity σ

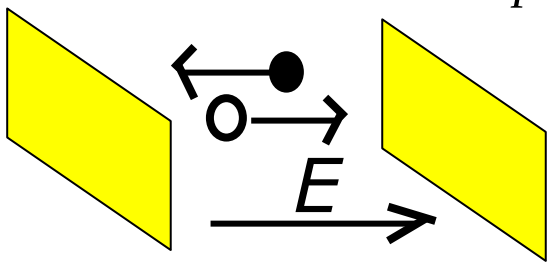
Negatively charged electron

Direction of electron drift

When an electric field is applied, current flows due to drift of mobile electrons and holes:

electron current density: $J_n = (-q)nv_e = qn\mu_n E$

hole current density: $J_p = (+q)pv_h = qp\mu_p E$



total current density: $J = J_n + J_p = (qn\mu_n + qp\mu_p)E$

$J = \sigma E$

★ *conductivity*

$$\sigma \equiv qn\mu_n + qp\mu_p$$

Units: $(\Omega \cdot cm)^{-1}$

Electrical Resistivity ρ

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$$

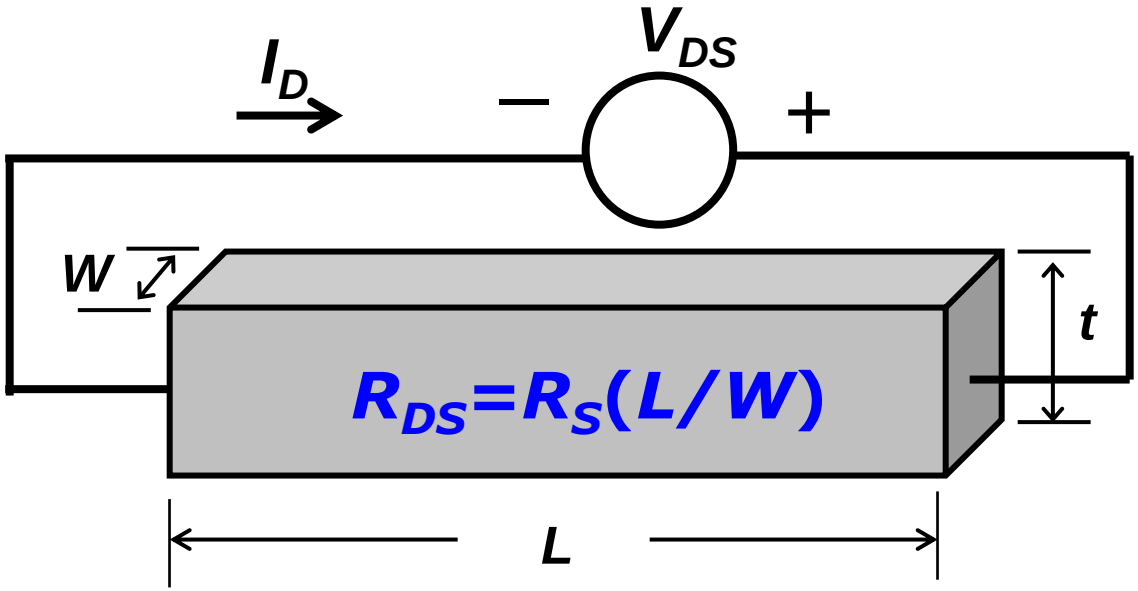
$$\rho \cong \frac{1}{qn\mu_n}$$ for n-type material

$$\rho \cong \frac{1}{qp\mu_p}$$ for p-type material

(Units: ohm•cm)

Inversion layer as a resistor

Consider an n-channel:



$$R_s = \frac{\rho}{t} = \frac{1}{\sigma t} = \frac{1}{q\mu_n n_s t} = \frac{1}{\mu_n Q_{inv}}$$

$V_G \uparrow \rightarrow Q_{inv} \uparrow$, $R_s \downarrow$

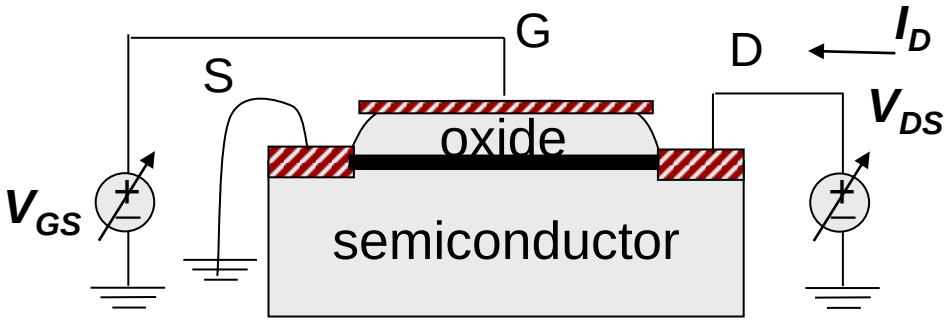
where Q_{inv} is the charge per unit area.

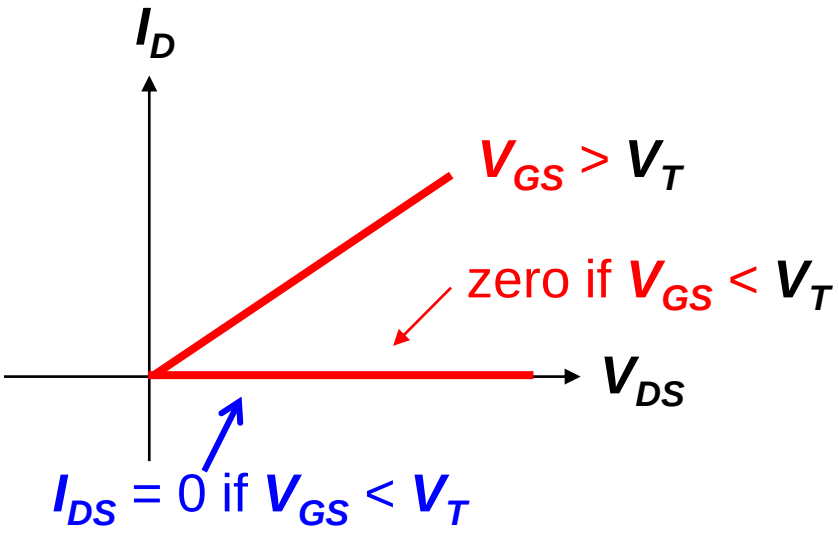
$$Q_{inv} = q\, n_s\, t$$

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$$

nMOSFET I_D vs. V_{DS} Characteristics

Next consider I_D (flowing into **D**) versus V_{DS}, as V_{GS} is varied:





"Cutoff" region: $V_{GS} < V_T$

Above "threshold" ($V_{GS} > V_T$):
"inversion layer" of electrons appears, so conduction between **S** and **D** is possible

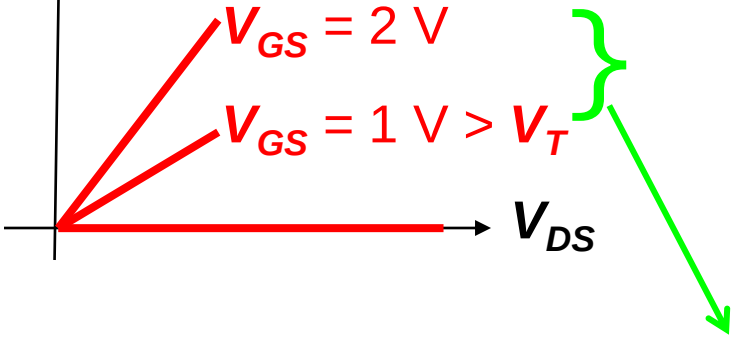
Below "threshold" ($V_{GS} < V_T$):
no charge → no conduction

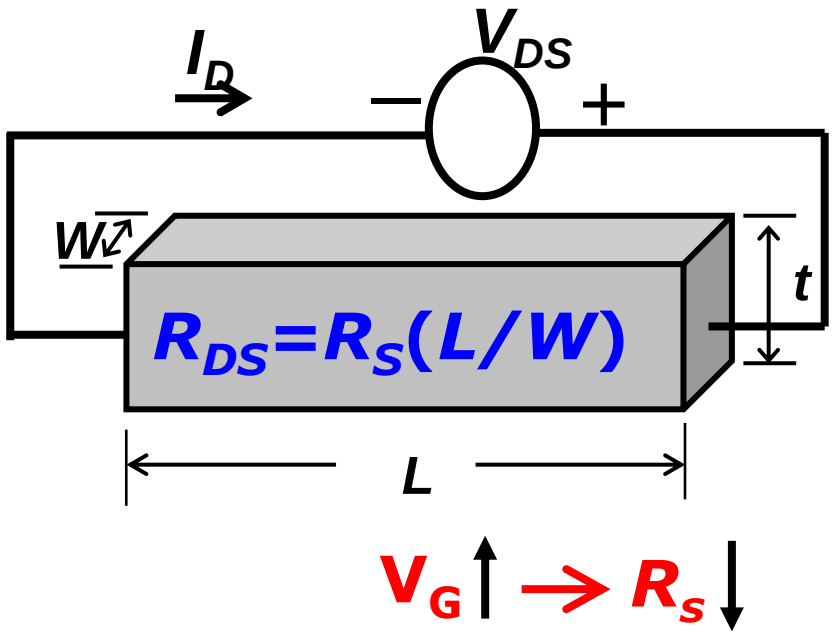
The MOSFET as a Controlled Resistor

- The MOSFET behaves as a **resistor** when V_{DS} is low:
  - Drain current I_D increases linearly with V_{DS}
  - Resistance R_{DS} between SOURCE & DRAIN depends on V_{GS}
    - R_{DS} is lowered as V_{GS} increases above V_T

NMOSFET Example:

$$I_D = \frac{V_{DS}}{R_{DS}}$$

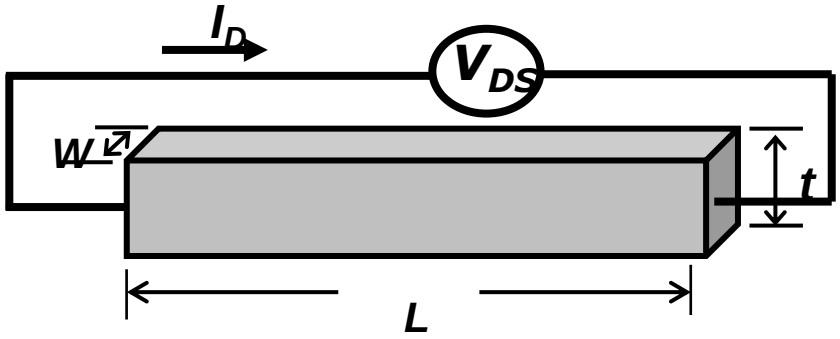




Linear or Resistive or ohmic or "Triode" Region: $0 < V_{DS} < V_{GS} - V_T$

MOSFET as a Controlled Resistor (cont'd)

Let's deduce I_D from R_{DS}



$$I_D = \frac{V_{DS}}{R_{DS}} \quad \& \quad R_{DS} = R_s(L/W) = \frac{L/W}{\mu_n Q_{inv}} = \frac{L/W}{\mu_n C_{ox}(V_{GS} - V_T - \frac{V_{DS}}{2})}$$

average value of $V(x)$

$$\rightarrow \quad I_D = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T - \frac{V_{DS}}{2})V_{DS}$$

We can make R_{DS} **low** by

- applying a **large** "gate drive" ($V_{GS} - V_T$)
- making W **large** and/or L **small**

$$R_s = \frac{1}{\mu_n Q_{inv}}$$

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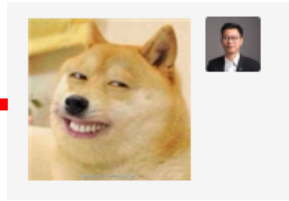
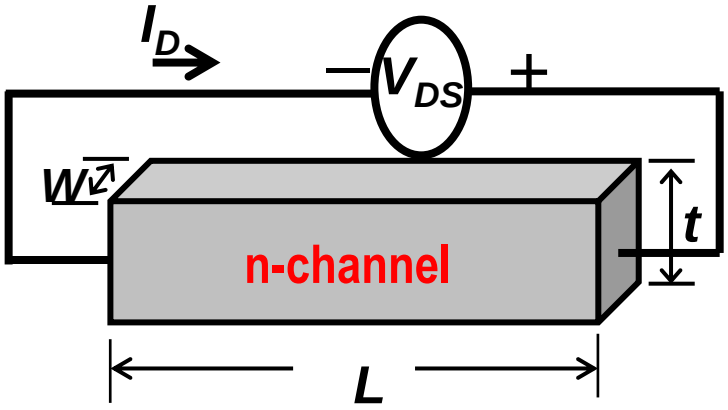


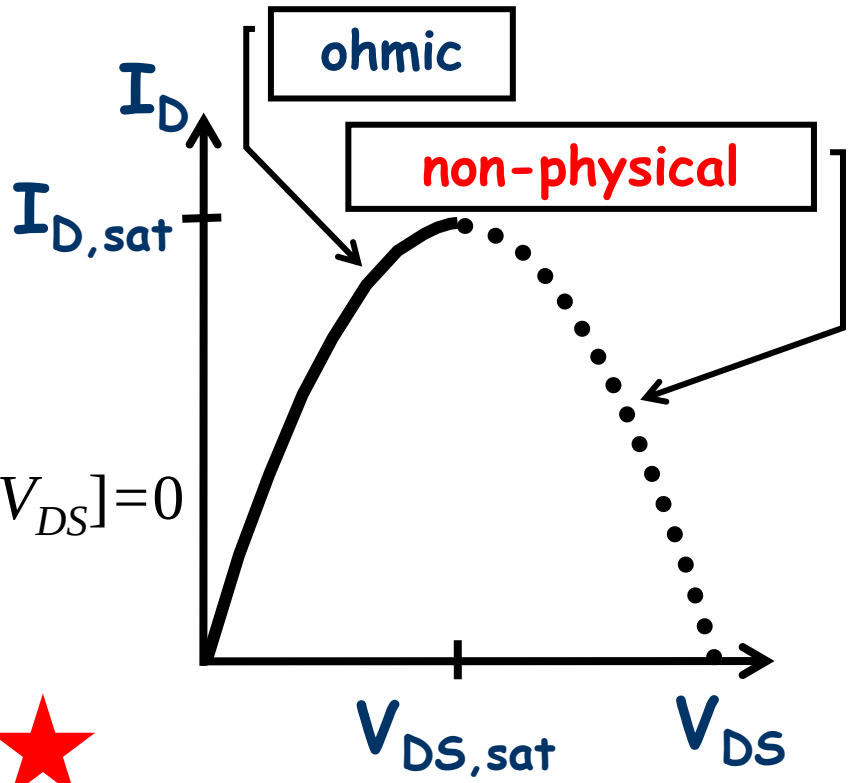
MOSFET as a Controlled Resistor (cont'd)

$$I_D = \mu C_{ox}(W/L)[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2] \quad (1)$$

★ **for a given V_{GS}**

it applies only for the condition $V_{DS} < (V_{GS} - V_T)$ (This is called the 'below pinch-off condition.')





$$dI_D/dV_{DS} = \mu C_{ox}(W/L)[(V_{GS} - V_T) - V_{DS}] = 0$$

so that $V_{DS,sat} = V_{GS} - V_T$

$$I_{D,sat} = \frac{\mu_n C_{Ox}}{2}\frac{W}{L}(V_{GS} - V_T)^2 \quad (2)$$

★

MOSFET as a Controlled Resistor (cont'd)

$$I_D = \frac{\mu_n C_{Ox}}{2}\frac{W}{L}\left(2(V_G - V_T)V_{DS} - V_{DS}^2\right) \qquad (1)$$

for a given V_{GS}

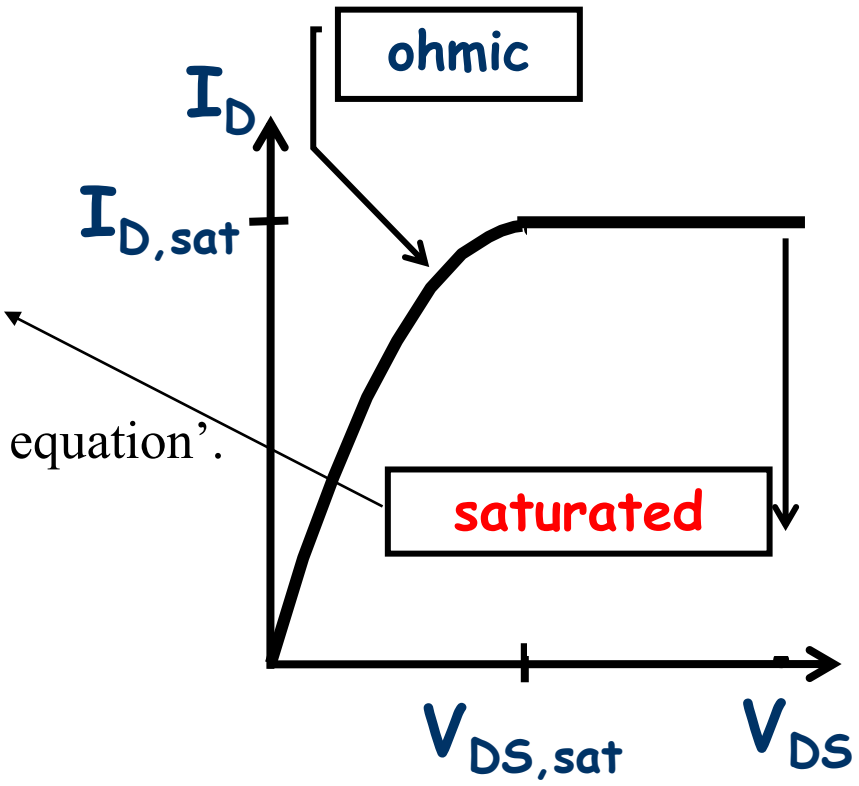
Note for $V_{DS} \geq V_G - V_T = V_{DS,sat}$, result is non-physical.

At $V_{DS,sat}$, $n_{s,x=L} = 0$

assume that channel curr. is const for $V_{DS} \geq V_{DS,sat}$

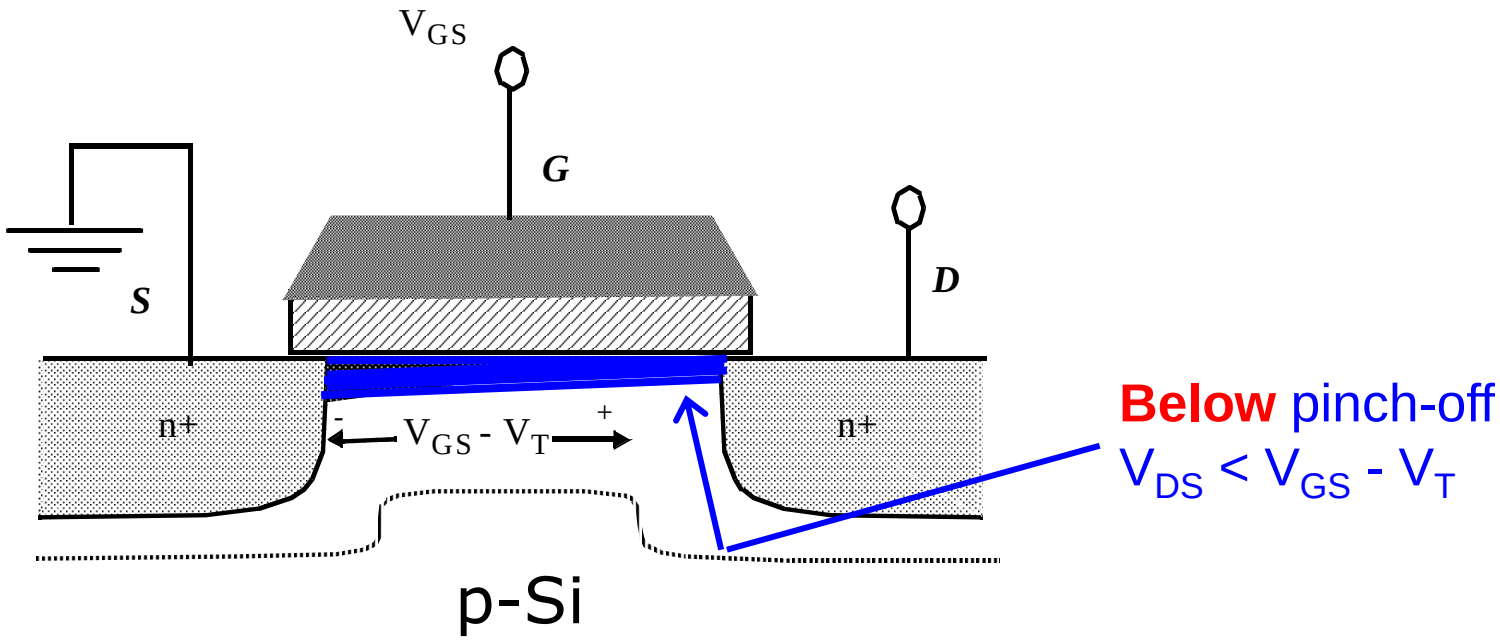
$$I_{D,sat} = \frac{\mu_n C_{Ox}}{2}\frac{W}{L}(V_{GS} - V_T)^2 \qquad (2)$$

which is called the '**above pinch-off** equation'.



What is “Pinch off” ?

- For a given V_{GS}, you have a maximum amount of current you can flow through the channel (regardless of V_{DS}).
- When $V_{DS} = V_{GS} - V_T$ the channel is flowing as much current as it can so we call it pinched off (even though current continues to flow).



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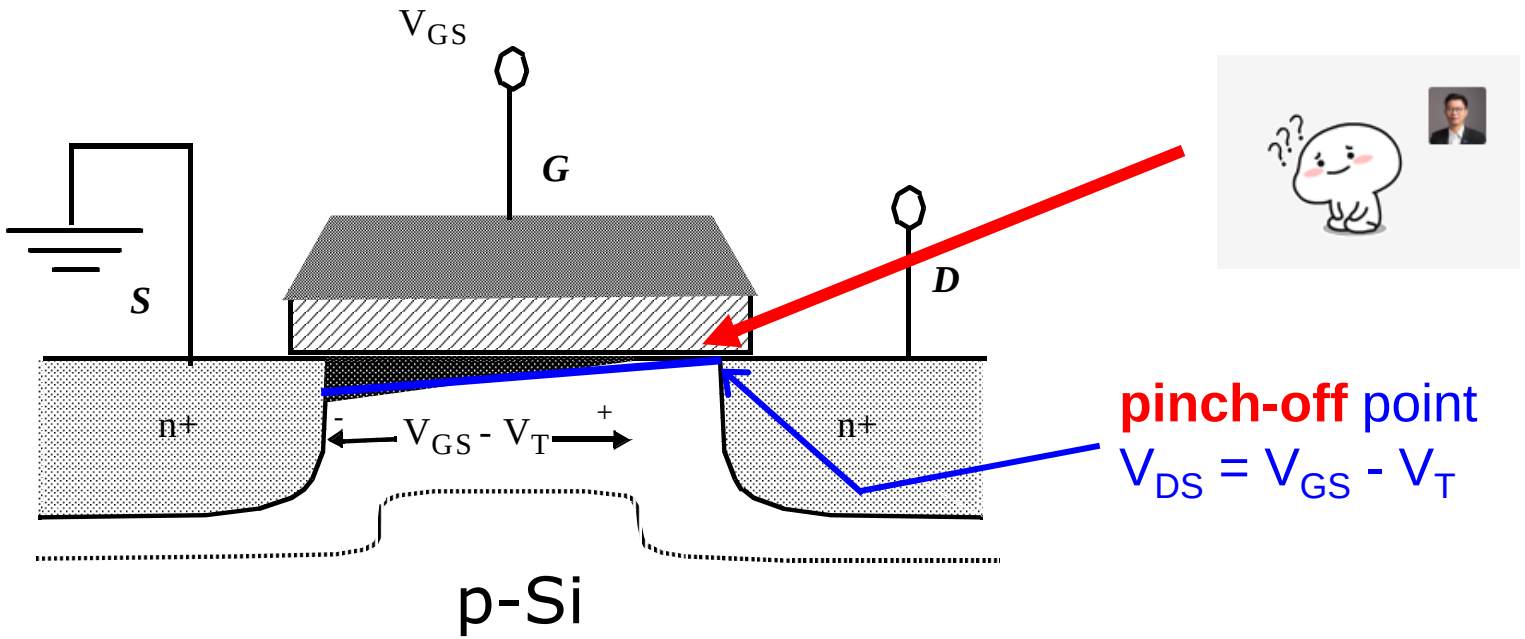

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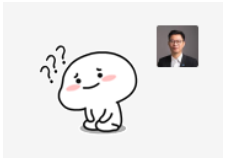

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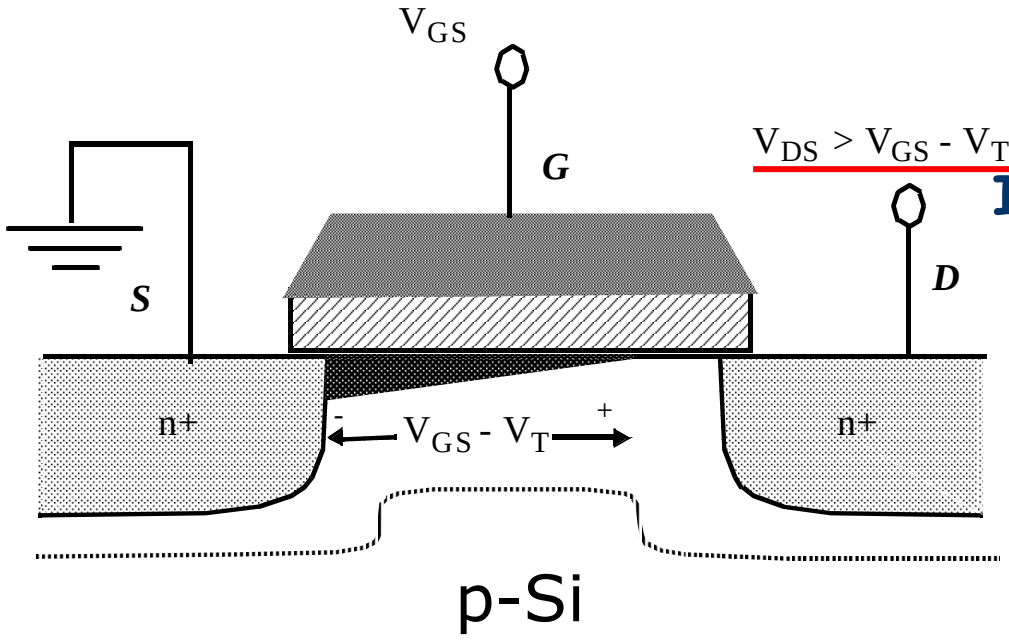
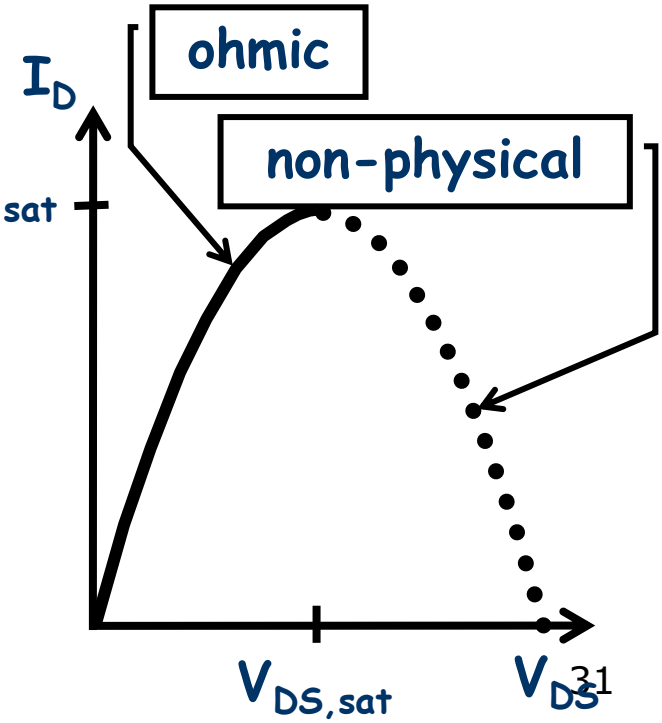
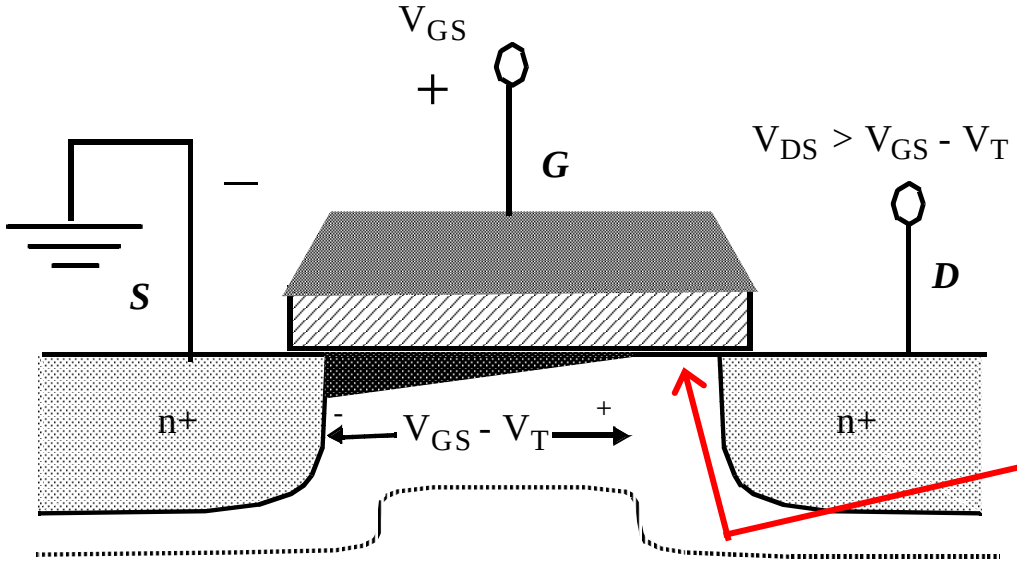




Why “Pinch off” ?

- As V_{DS} increases, the inversion-layer charge density at the drain end of the channel is reduced; therefore, I_D does not increase linearly with V_{DS}.
- When V_{DS} reaches $V_{GS} - V_T$, the channel is “pinched off” at the drain end, and I_D saturates (*i.e.* it does not increase with further increases in V_{DS}).
- In the pinched-off region: $Q_{inv}(x) = -C_{ox}[V_{GS} - V_T - V_{DS,sat}] = 0$



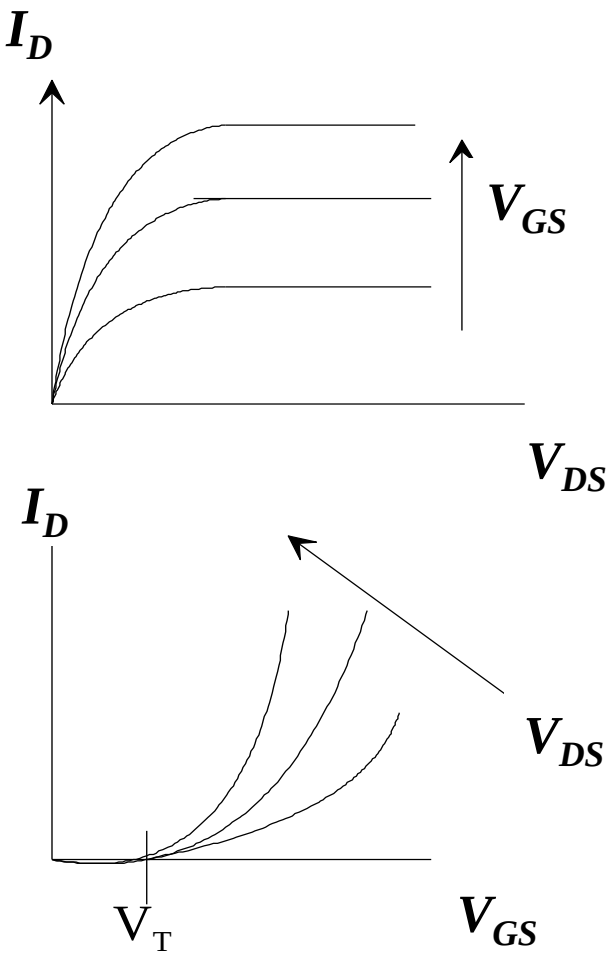
$$I_{D,sat} = \mu_n C_{ox} \frac{W}{2L}(V_{GS} - V_T)^2$$

I_D vs. V_{DS} or V_{GS} Characteristics

$$I_D = \frac{\mu_n C_{Ox}}{2}\frac{W}{L}\left(2\left(V_{GS} - V_T\right)V_{DS} - V_{DS}^2\right) \qquad (1)$$

For $V_{DS} \geq V_{DS,sat}$

$$I_{D,sat} = \frac{\mu_n C_{Ox}}{2}\frac{W}{L}\left(V_{GS} - V_T\right)^2 \qquad (2)$$

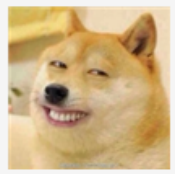


The graphs shows ideal charactersitics. The top graph is the **output characteristic,** the lower one is the **transfer characteristic**. Equations (1) and (2) are the simple form of the design equations.

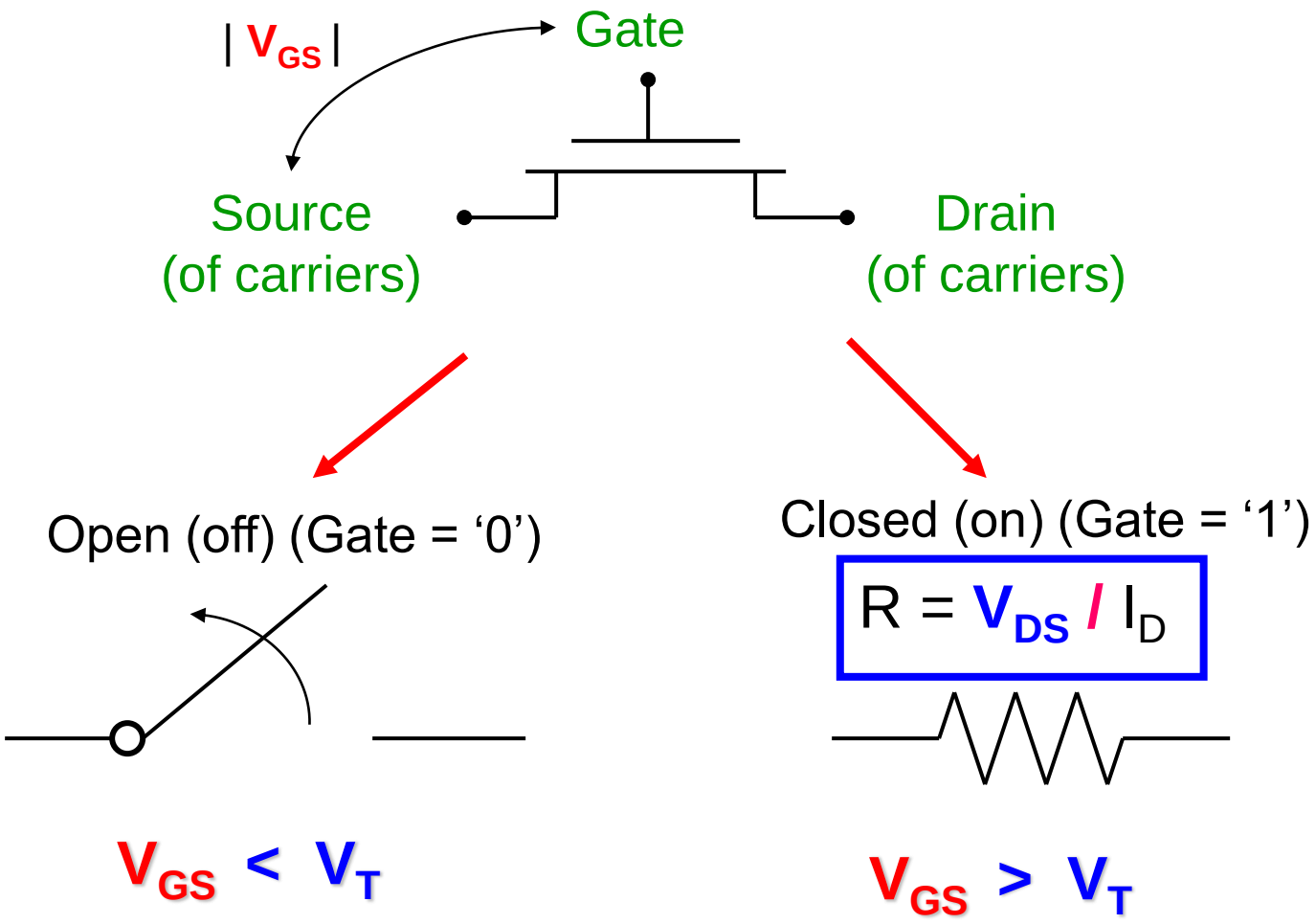
The quantity μC_{ox} (W/L) = β is the device constant. The designer can only vary W/L so as to change β. Other values are fixed during the process development.

OUTLINE

- I - V Characteristics
  - **Cutoff Region**
  - Linear Region
  - **Saturation** Region (pinch-off region)
- **Switch model of nMOSFETs** ⟵

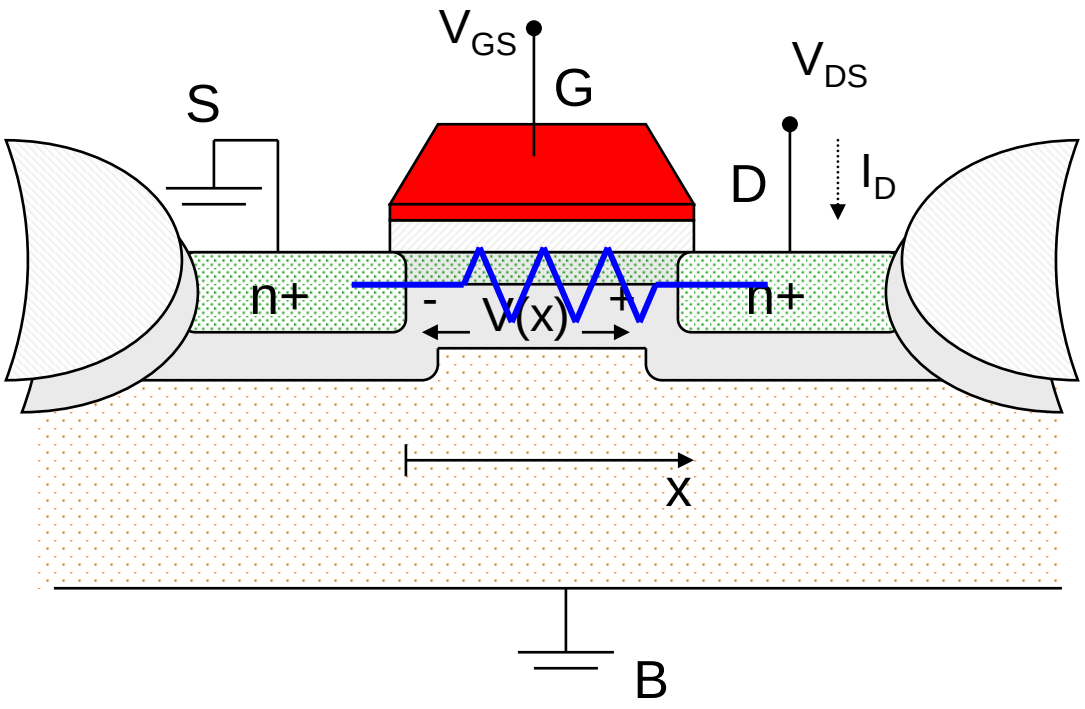


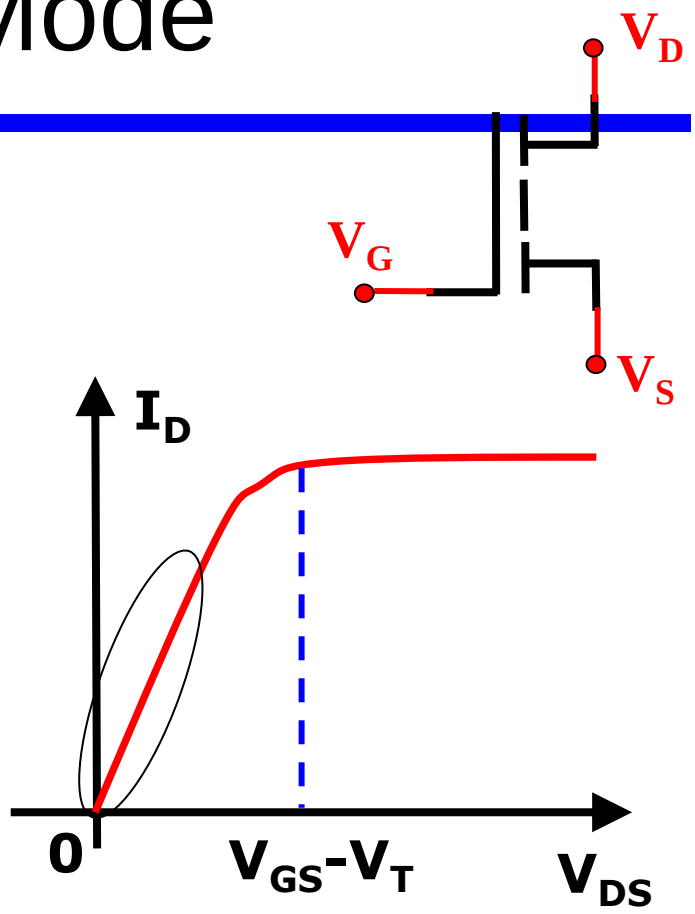
Switch Model of nMOS Transistor



Transistor in **Linear** Mode

Assuming $V_{GS} > V_T$





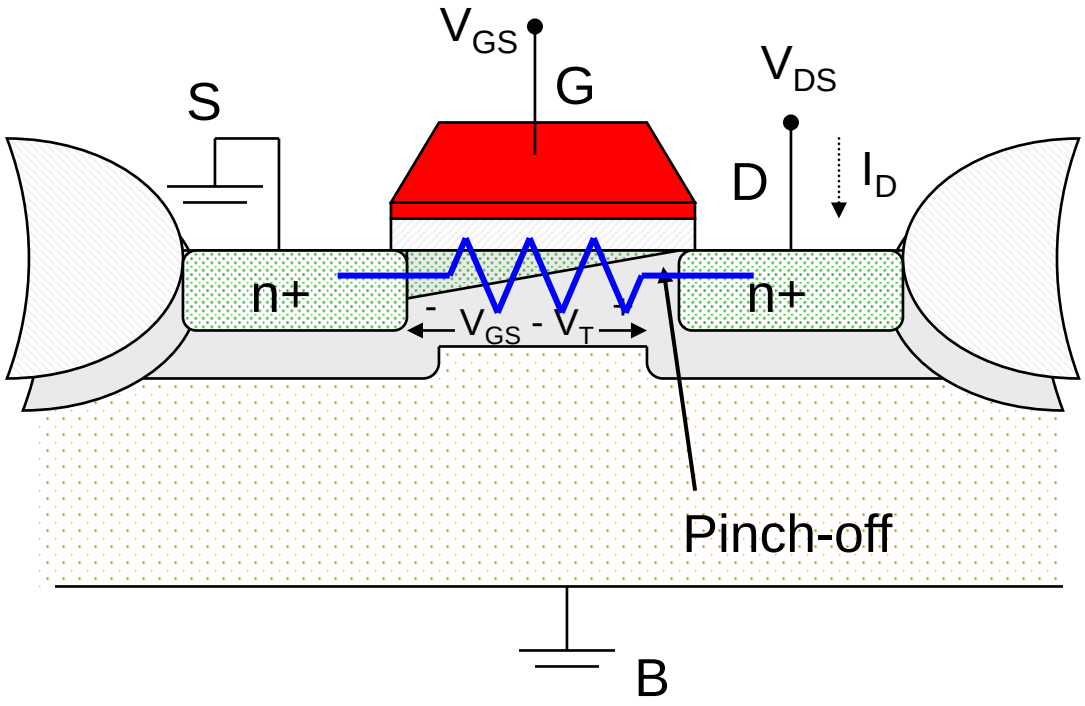
When $V_{DS} \leq V_{GS} - V_T$: $I_D = \beta_0 W/L [(V_{GS} - V_T)V_{DS} - V_{DS}^2/2]$

$\beta_0 = \mu_n C_{ox}$

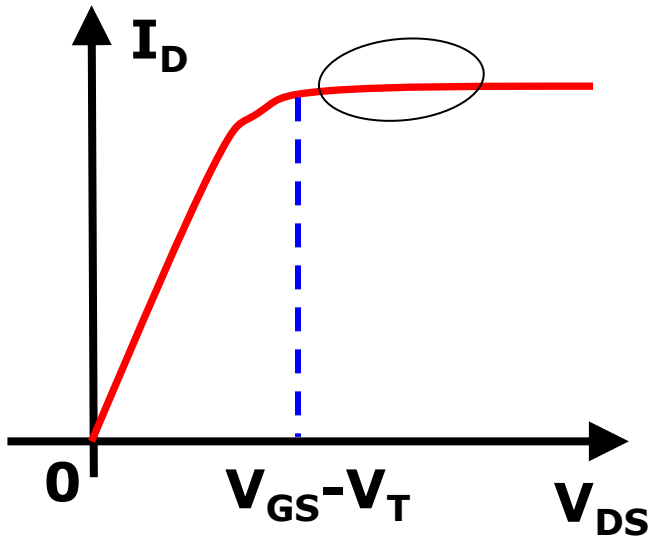
$R = V_{DS} / I_D$

Transistor in **Saturation** Mode

Assuming $V_{GS} > V_T$



$V_{DS} > V_{GS} - V_T$



When $V_{DS} \geq V_{GS} - V_T$: $I_D = (\beta_0/2)\, W/L\, [(V_{GS} - V_T)^2]$

The current remains constant (saturates).

R = V_{DS} / I_D =?

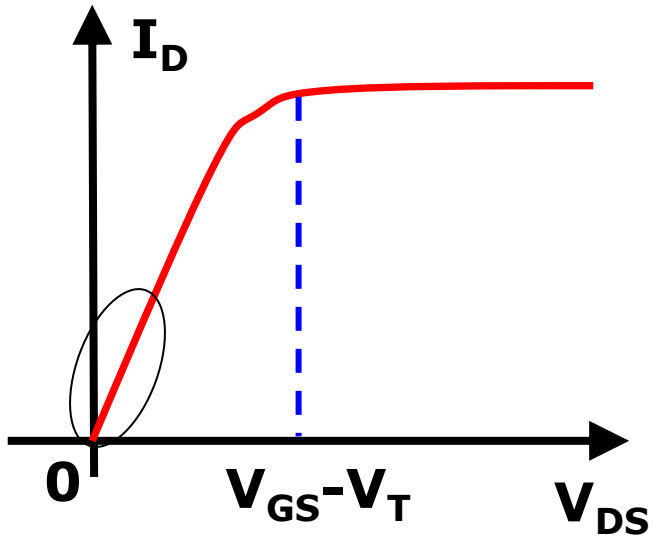
nMOSIC - **MOST as a Linear R_{DS}**

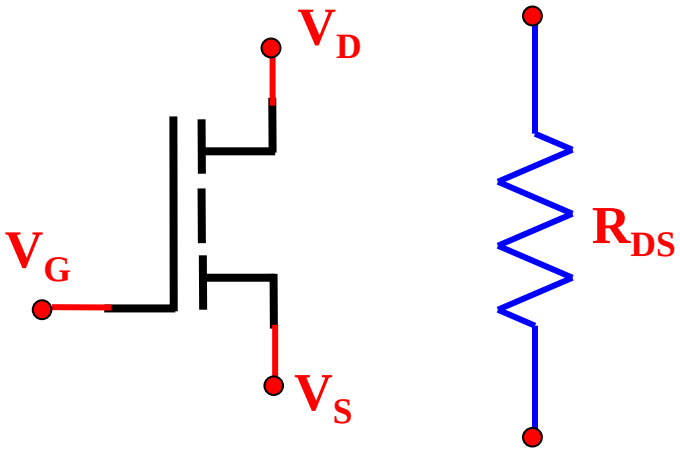
When $V_{DS} \leq V_{GS} - V_T$

$I_D = \beta_0 W/L [(V_{GS} - V_T)V_{DS} - V_{DS}^2/2]$

For **small V_{DS}**, there is a linear dependence between V_{DS} and I_D, hence

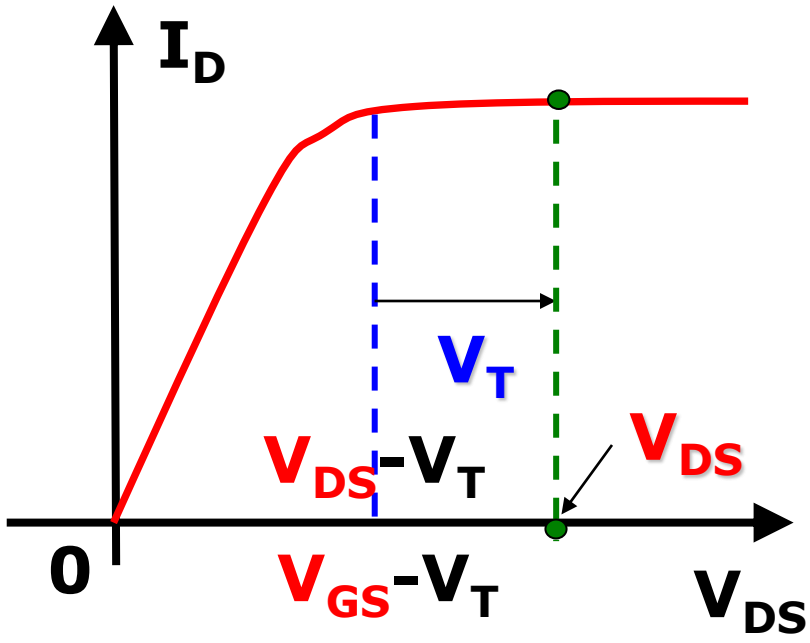
$$1/R_{DS} = I_D/V_{DS}$$
$$= \beta_0 W/L [(V_{GS} - V_T) - V_{DS}/2]$$
$$\approx \beta_0 W/L (V_{GS} - V_T)$$

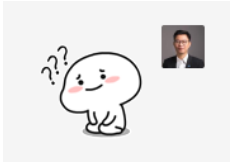


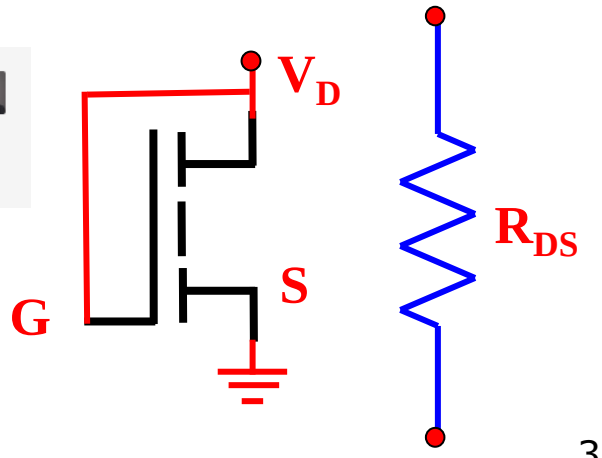


nMOSIC - **MOST as a Load**

- **When $V_{DS} \geq V_{GS} - V_T$:**
  $I_D = (\beta_0/2)\, W/L\, [(V_{GS} - V_T)^2]$
- In this case the gate and the drain are connected together so that $V_{GS}=V_{DS}$. The **pinch off point** coincides with $V_{GS}-V_T=V_{DS}-V_T$.
- The characteristic of the load is shown. It extends along the drain axis by an amount V_T.







nMOSIC - **The MOST as a Load**

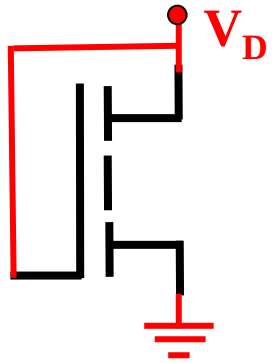
- **Problem**

  Calculate the resistance of a load MOST with an **aspect ratio of 1** when the mobility of the electrons is **$1000 cm^2 V^{-1} sec^{-1}$** and the gate capacitance **per unit area** is **$10^{-2} Fm^{-2}$**. The drain voltage is **V_D=5V** and the threshold voltage **V_T= 0.5V**.

- **Solution**

  The drain current is

$$I_D = \mu_n (W/L) C_{ox} (V_G - V_T)^2/2.$$



but $V_G = V_D$ so that

$$R = V_D/I_D = \ldots$$

- **R=100Ω**

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- **Solution**

  The drain current is the same as at the pinch-off point where

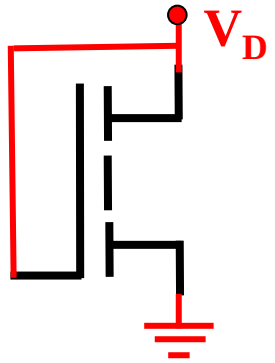
  $$I_D = \mu (W/2L)\, C_{ox}\, (V_G - V_T)^2.$$

  but $V_G = V_D$ so that

  $$I_D = \mu (W/2L)\, C_{ox}\, (V_D - V_T)^2$$

  $$I_D/V_D = 1/R = \mu\, (W/2L)\, C_{ox}\, (V_D - V_T)^2/V_D$$

  $$= 0.1 * 0.5 * 10^{-2} * 4.5^2/5 = 0.2025 * 10^{-2},$$



- **$R = 500\Omega$**