EEE104 – Digital Electronics (I)

Lecture 4

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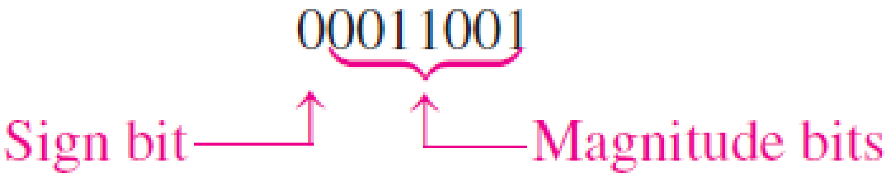
In This Session

- Signed Numbers.
- Arithmetic Operations with Signed Numbers

Signed Numbers

- The left-most bit in a signed binary number is the **sign bit**.

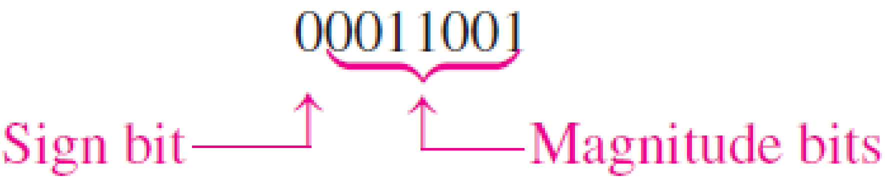


- The sign bit is 0 is for a positive number, and is 1 is for a negative number.
- Digital systems such as computers usually use **2’s complement system** to represent signed numbers.
- The 2’s complement of a number is calculated by inverting its bits and adding 1.

Signed Numbers

In the 2's complement system

- A positive number is represented as a zero sign bit followed by true binary magnitude bits, e.g. +25 is



- Negative numbers are the 2's complements of the corresponding positive numbers, e.g. -25 is 11100111.

original	00011001 (+25)
1's complement	11100110
2's complement	11100111 (-25)

Signed Numbers

In the 2’s complement system

- Positive numbers are the 2’s complements of the corresponding negative numbers.

```
-(-25) = +25

original          11100111 (-25)

1's complement    00011000

2's complement    00011001 (+25)
```

Signed Numbers

- We can add an infinite number of 0’s to the left of a positive number and will not change its value, e.g. 011 (+3) = 00011 (+3).
- We can add an infinite number of 1’s to the left of a negative number and will not change its value, e.g. 101 (-3) = 11101 (-3).

`original`	`101 (-3)`	`11101 (-3)`
`1’s complement`	`010`	`00010`
`2’s complement`	`011 (+3)`	`00011 (+3)`

Signed Numbers

The decimal value of signed binary numbers

- It is determined by summing the weights in all bit positions where there are 1s.
- The weight of the sign bit is calculated as that of a magnitude bit but given a *negative* value.

$$-2^7 \quad 2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$$

Signed Numbers

The decimal value of signed binary numbers

Positive number

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	0	1	1	0

$$64 + 16 + 4 + 2 = \mathbf{+86}$$

Negative number

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	0	1	0	1	0

$$-128 + 32 + 8 + 2 = \mathbf{-86}$$

Signed Numbers

Range of signed integer numbers

- The number of different combinations of n bits is

$$\boxed{\text{Total combinations} = 2^n}$$

  e.g. 8 bits for 256 numbers.

- The range of values for n-bit numbers is

$$\boxed{\text{Range} = -(2^{n-1}) \text{ to } +(2^{n-1} - 1)}$$

  e.g. 8 bits for -128 to +127.

Arithmetic Operations with Signed Numbers

Addition

- Add the two numbers and **discard any final carry bit**.

Both numbers positive

$$
\begin{array}{r}
00000111 \\
+\ 00000100 \\
\hline
00001011
\end{array}
\qquad
\begin{array}{r}
7 \\
+\ 4 \\
\hline
11
\end{array}
$$

Positive number with magnitude larger than negative number

$$
\begin{array}{rr}
 & 00001111 \\
 & +\ 11111010 \\
\hline
\text{Discard carry} \longrightarrow 1 & 00001001
\end{array}
\qquad
\begin{array}{r}
15 \\
+\ -6 \\
\hline
9
\end{array}
$$

Arithmetic Operations with Signed Numbers

Addition

Positive number with magnitude larger than negative number

$$\begin{array}{rr} 00010000 & 16 \\ +\ 11101000 & +\ -24 \\ \hline 11111000 & -8 \end{array}$$

Both numbers negative

$$\begin{array}{lrr} & 11111011 & -5 \\ & +\ 11110111 & +\ -9 \\ \hline \text{Discard carry} \longrightarrow & 1\ \ 11110010 & -14 \end{array}$$

Arithmetic Operations with Signed Numbers

Addition

- **Overflow**: when two numbers are added, the number of bits required to represent the sum exceeds the number of bits in the two numbers.
- It occurs only when both numbers are positive or negative.

$$\begin{array}{r r} 01111101 & 125 \\ +\,00111010 & +\ \ 58 \\ \hline 1\underbrace{0110111} & 183 \end{array}$$

Sign incorrect

Magnitude incorrect

Arithmetic Operations with Signed Numbers

Subtraction

- Subtraction is implemented through addition.
- Change the sign of the subtrahend and add to the minuend, e.g. subtracting +6 is equivalent to adding -6.
- The sign of a binary number is changed by taking its 2's complement.

$$\begin{array}{rl} & \text{minuend} \\ - & \text{subtrahend} \\ \hline & \text{difference} \end{array}$$

Arithmetic Operations with Signed Numbers

Subtraction

00001000 − 00000011

In this case, $8 - 3 = 8 + (-3) = 5$.

	00001000	Minuend (+8)
	+ 11111101	2's complement of subtrahend (−3)
Discard carry →	**1 00000101**	Difference (+5)

00001100 − 11110111

In this case, $12 - (-9) = 12 + 9 = 21$.

00001100	Minuend (+12)
+ 00001001	2's complement of subtrahend (+9)
00010101	Difference (+21)

Arithmetic Operations with Signed Numbers

Multiplication — Partial Products Method

1. Compute the magnitude product of corresponding positive numbers.
2. Attach a 0 sign bit. If the signs of the two numbers are different (negative product), take the 2's complement of the outcome.

Arithmetic Operations with Signed Numbers

Multiplication

Multiply the signed number 01010011 (+83) and 11000101 (-59).

Step 1

11000101 ⟶ 00111011

Step 2

1010011	Multiplicand
× 0111011	Multiplier
1010011	1st partial product
+ 1010011	2nd partial product
11111001	Sum of 1st and 2nd
+ 0000000	3rd partial product
011111001	Sum
+ 1010011	4th partial product
1110010001	Sum
+ 1010011	5th partial product
100011000001	Sum
+ 1010011	6th partial product
1001100100001	Sum
+ 0000000	7th partial product
1001100100001	Final product

Step 3

0 1001100100001 (+4897)

↓

1 0110011011111 (-4897)

Arithmetic Operations with Signed Numbers

Division - accomplished using subtraction in computers:

1. Initialize the quotient to zero.
2. Subtract the divisor from the dividend or previous partial remainder. If the partial remainder is:
   - positive, add 1 to the quotient and repeat.
   - zero, add 1 to the quotient and finish.
   - negative, finish.
3. Determine the sign of the quotient by checking the signs of the dividend and divisor.

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

Arithmetic Operations with Signed Numbers

Division

Divide 01100100 (+100)

by 00011001 (+25)

The result is

00000100 (+4)

	01100100	Dividend
+	11100111	2's complement of divisor
	01001011	Positive 1st partial remainder
	01001011	1st partial remainder
+	11100111	2's complement of divisor
	00110010	Positive 2nd partial remainder
	00110010	2nd partial remainder
+	11100111	2's complement of divisor
	00011001	Positive 3rd partial remainder
	00011001	3rd partial remainder
+	11100111	2's complement of divisor
	00000000	Zero remainder