CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 16 Balanced Three-phase Systems

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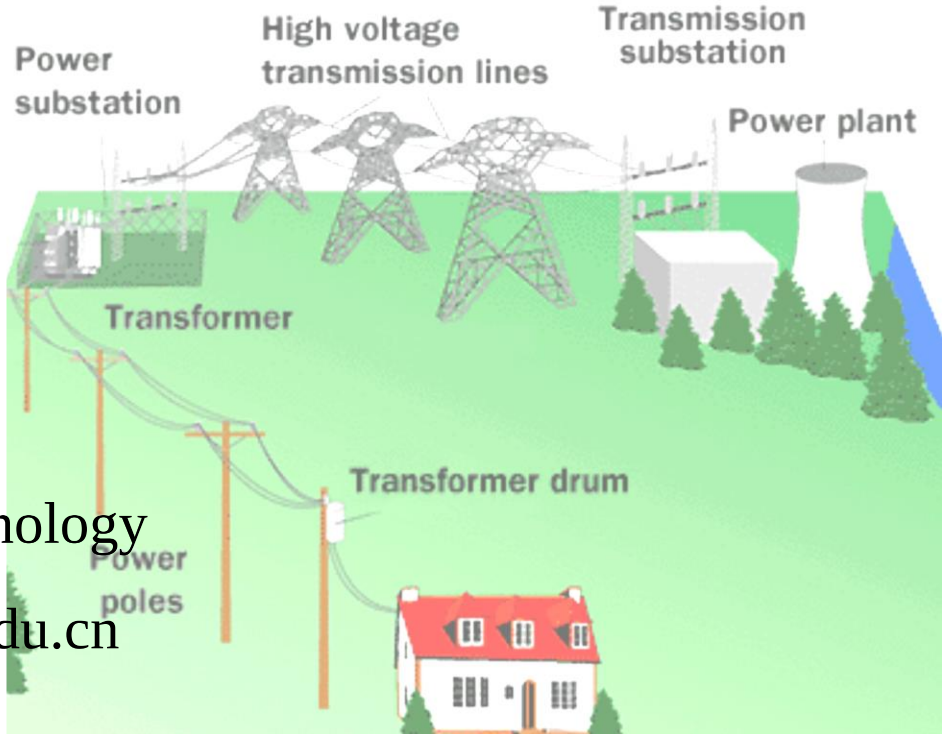


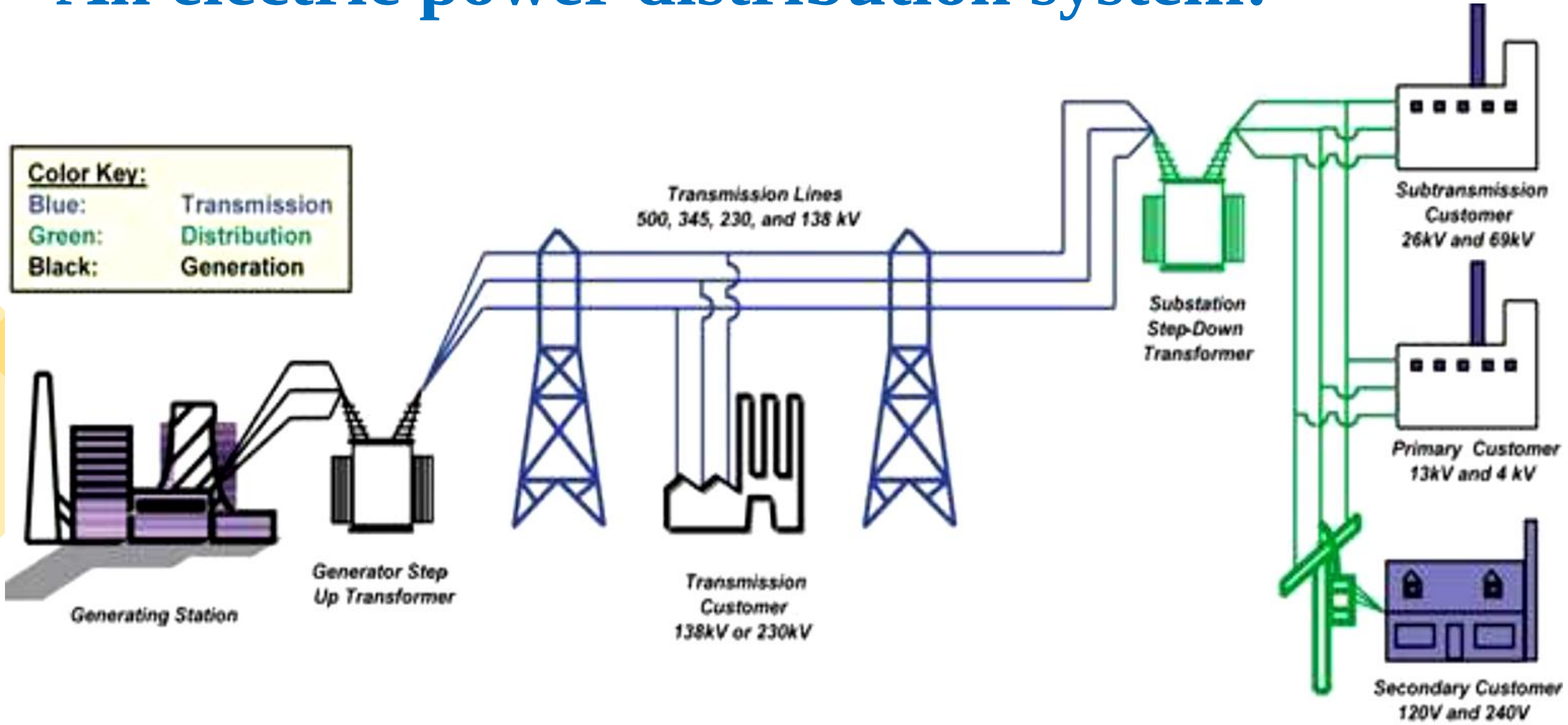
Xi'an Jiaotong-Liverpool University
西交利物浦大學

OUTLINE

- Overview
  - Industrial Applications
  - Single-phase and Three-phase Supply
- Balanced Three-phase Supply and Load
  - Y & Δ Connected Supply
  - Y & Δ Connected Load
- Balanced Three-phase Circuits
  - Four Connections: Y - Y; Δ - Δ; Y - Δ; Δ - Y
- Power Calculations in Balanced Three-phase Circuits

1.1 OVERVIEW

An electric power distribution system:

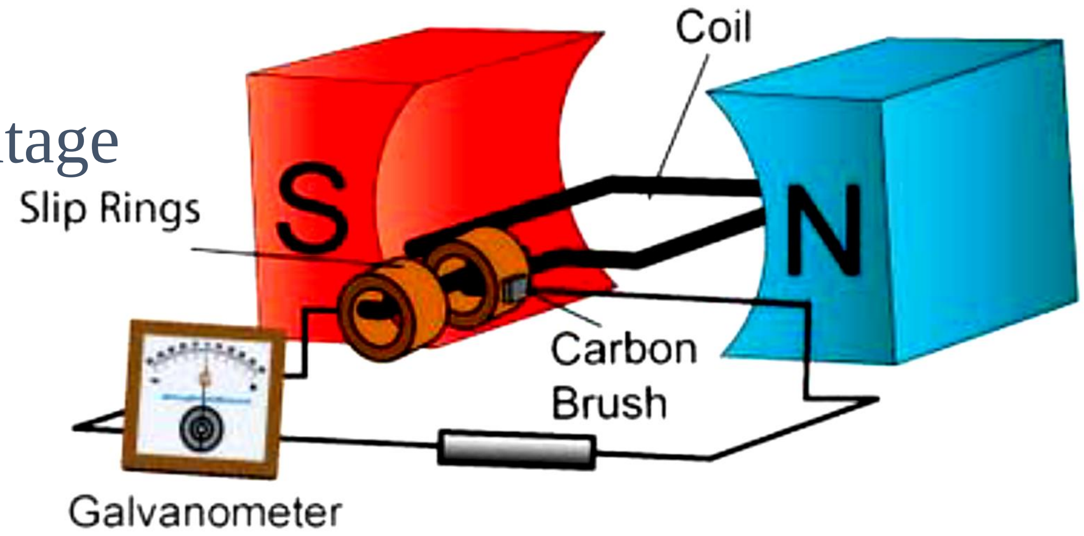


Generation, Transmission, and Distribution

Power transmission uses ‘**balanced three-phase**’ configuration

1.2 SINGLE-PHASE SUPPLY

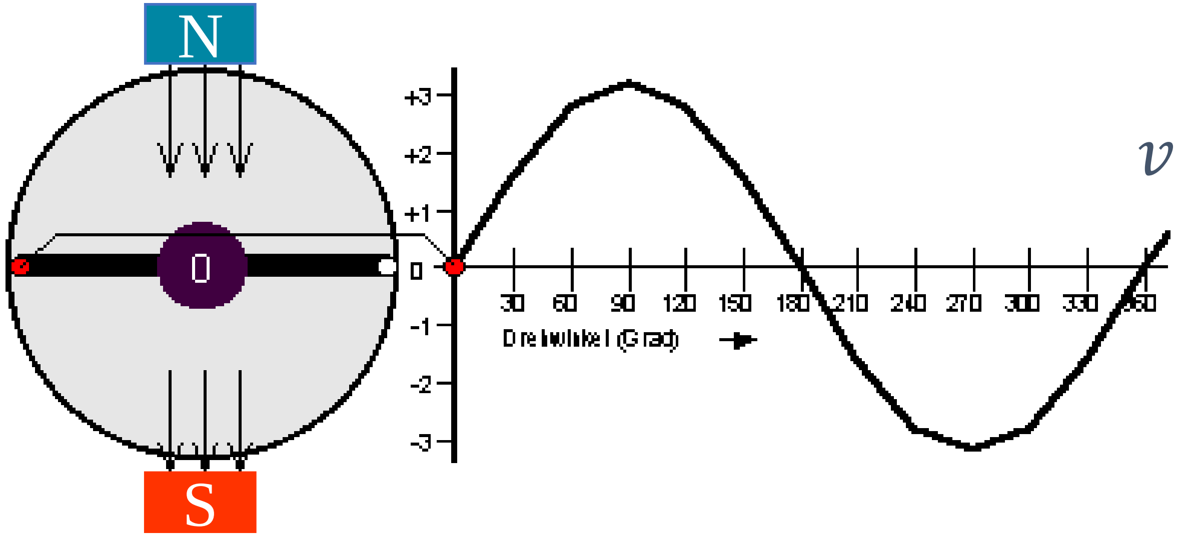
Recall CAN102: One-phase ac generator



Static magnets, **one** rotating coil, **one** output voltage

The voltage induced by a single coil when it rotates in a magnetic field.

All the voltages of the supply vary in unison.



$$v(t) = V_m \cos(\omega t + \theta)$$

$$V = V_m \angle\theta$$

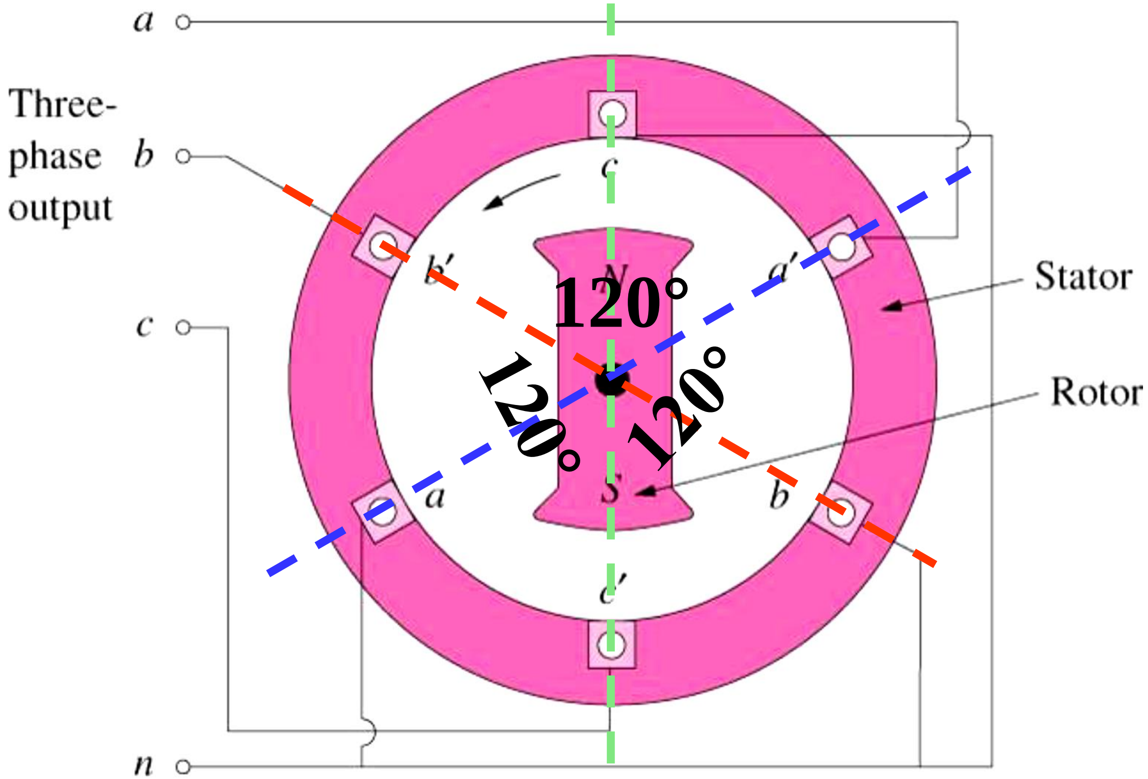
Used mostly for lighting and heating (<10 or 20 kW).

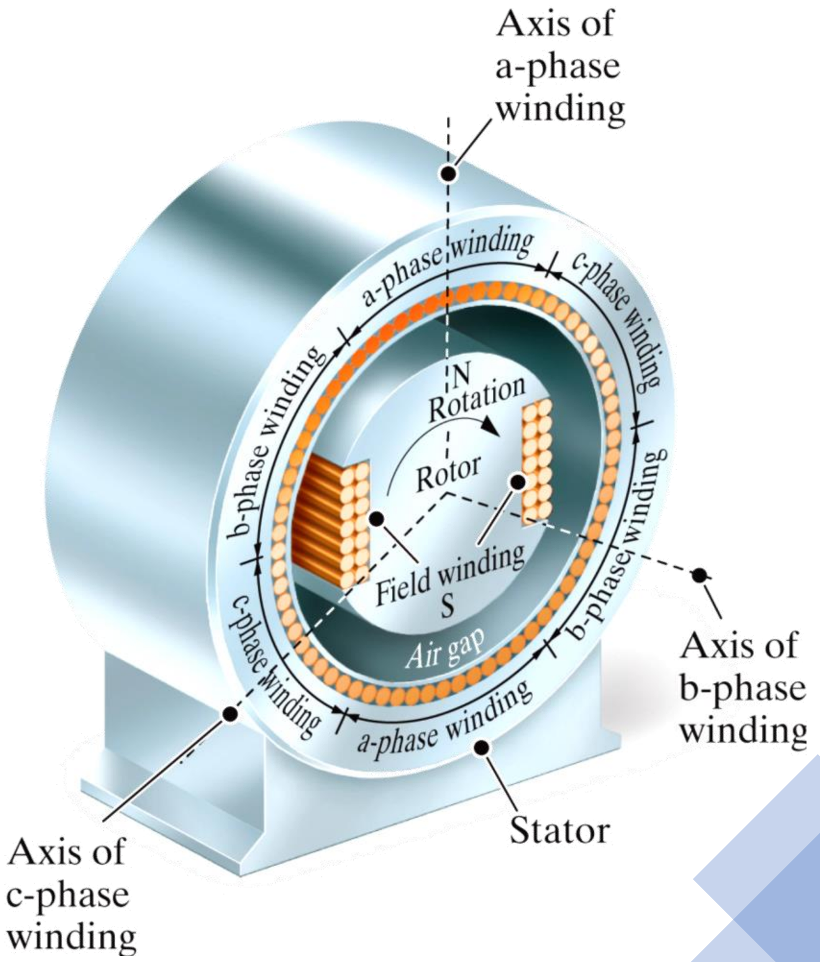
1.3 THREE-PHASE SUPPLY

Polyphase: Circuits or systems where the ac sources operate at the same frequency, but with different phases.

Three-phase ac generator

Static coils, rotating magnets, **three** output voltages





1.3 THREE-PHASE SUPPLY

Three-phase Expression

Time domain:

$$v_a(t) = V_m \cos(\omega t)$$

$$v_b(t) = V_m \cos(\omega t - 120°)$$

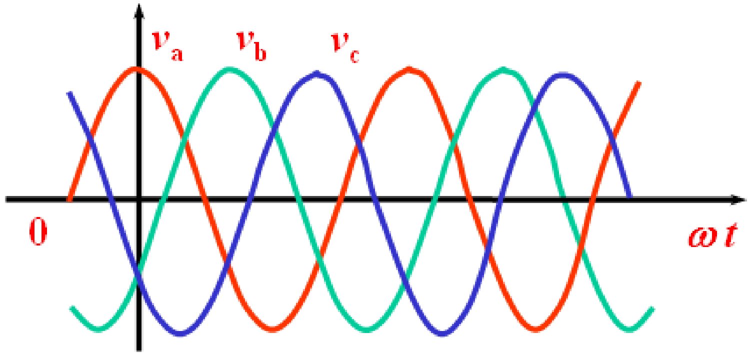
$$v_c(t) = V_m \cos(\omega t - 240°) = V_m \cos(\omega t + 120°)$$

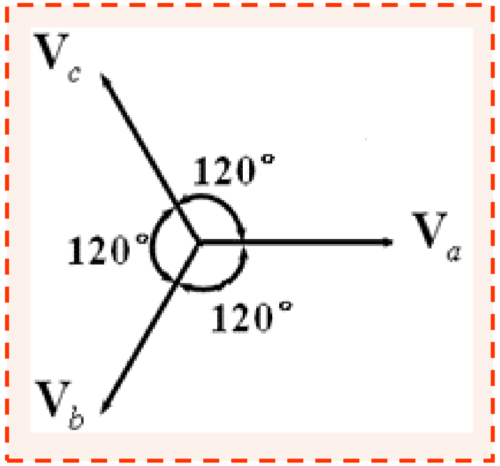
Phasor domain:

$$\mathbf{V}_a = V_m \angle 0°$$

$$\mathbf{V}_b = V_m \angle(-120°)$$

$$\mathbf{V}_c = V_m \angle(-240°) = V_m \angle 120°$$





Phase Sequence

The phase sequence is the time order in which the voltages pass through their respective maximum values.

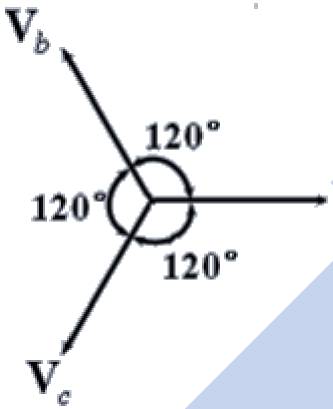
1. abc sequence (positive sequence)

Time-domain: $v_a + v_b + v_c = 0$

Phasor-domain: $\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0$

2. acb sequence (negative sequence)

$$\begin{cases} \mathbf{V}_a = V_m \angle 0° \\ \mathbf{V}_c = V_m \angle(-120°) \\ \mathbf{V}_b = V_m \angle(-240°) \\ \quad = V_m \angle 120° \end{cases}$$



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- Power Calculations in Balanced Three-phase Circuits

OVERVIEW

A three-phase system consists of a **three-phase voltage source** that is used to supply a **three-phase load**.

Both the three-phase voltage source and the three-phase load can be connected in two different ways:

Source Connections

Wye (Y) Connected
(or Star Connected)

Delta (Δ) Connected
(or Mesh Connected)

Load Connections

Wye (Y) Connected
(or Star Connected)

Delta (Δ) Connected
(or Mesh Connected)

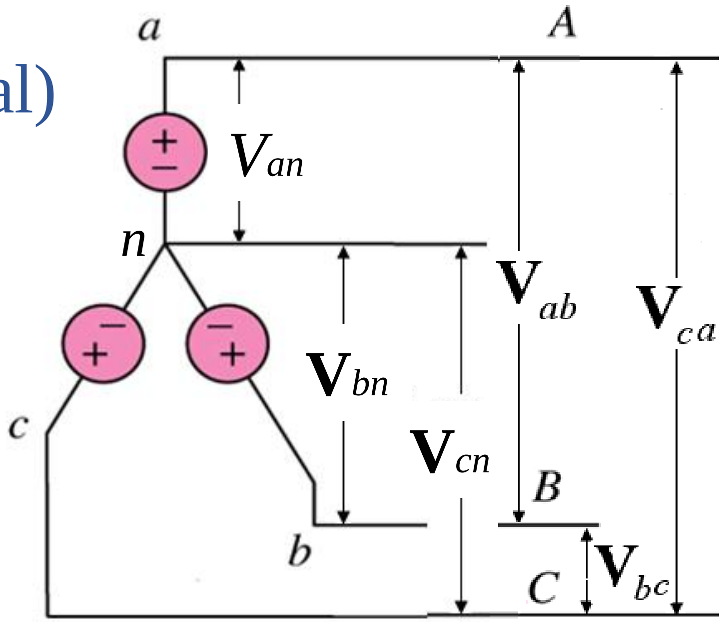
2.1 Y CONNECTED SOURCE: VOLTAGE

The **negative** ends of the three coils connected to form the **Y (star)** point.

The three remaining ends brought out to form the three terminals.

Phase voltage $\boldsymbol{V_p}$: voltage across a single phase (line to neutral)

Line voltage $\boldsymbol{V_L}$: voltage across any pair of lines (line to line)



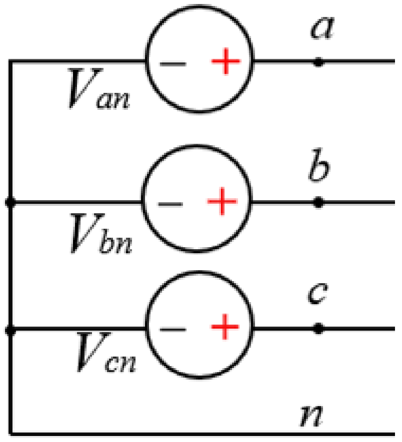
For a **balanced** Y-connected source, the magnitude of line voltage is $\sqrt{3}$ times the magnitude of phase voltage:

$$V_L = \sqrt{3} V_p$$

where

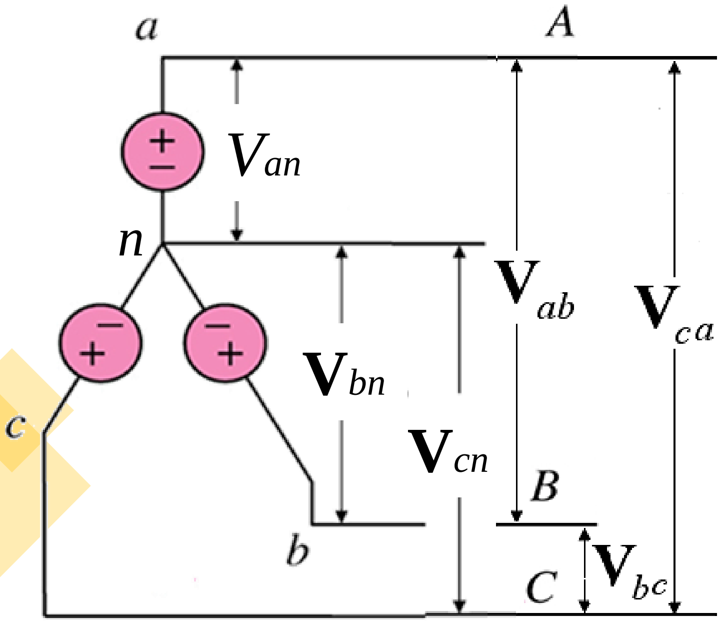
$$V_p = |\vec{V}_{an}| = |\vec{V}_{bn}| = |\vec{V}_{cn}|$$

$$V_L = |\vec{V}_{ab}| = |\vec{V}_{bc}| = |\vec{V}_{ca}|$$

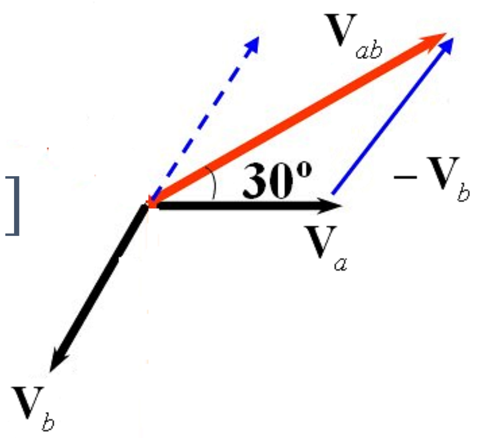


$$\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn} = 0$$

* VERIFICATION



$$\vec{V}_{ab} = \vec{V}_{an} - \vec{V}_{bn}$$
$$= V_m\angle 0° - V_m\angle(-120°)$$
$$= V_m - [V_m cos(-120°) + jV_m sin(-120°)]$$
$$= V_m\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)$$
$$= \sqrt{3}V_m\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)$$
$$= \sqrt{3}V_m[cos(30°) + jsin(30°)]$$
$$= \sqrt{3}V_m\angle 30°$$



the magnitude of line voltage is $\sqrt{3}$ times the magnitude of phase voltage

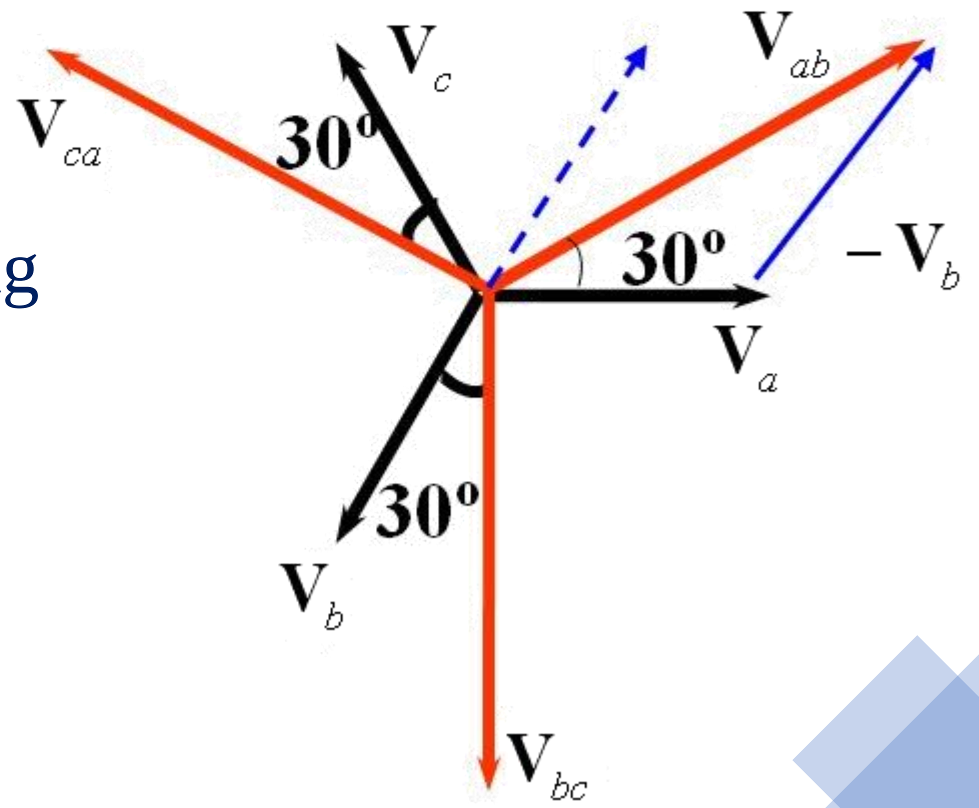
Phase voltages:

$$\vec{V}_{an} = V_m\angle 0°$$
$$\vec{V}_{bn} = V_m\angle(-120°)$$
$$\vec{V}_{cn} = V_m\angle(-240°)$$
$$= V_m\angle 120°$$

SUMMARY: VOLTAGE

For any balanced Y-connected sources, the magnitude of line voltage is $\sqrt{3}$ times the magnitude of phase voltage:

$$V_L = \sqrt{3}V_p$$



Each line-to-line voltage leads its corresponding phase voltage by 30°.

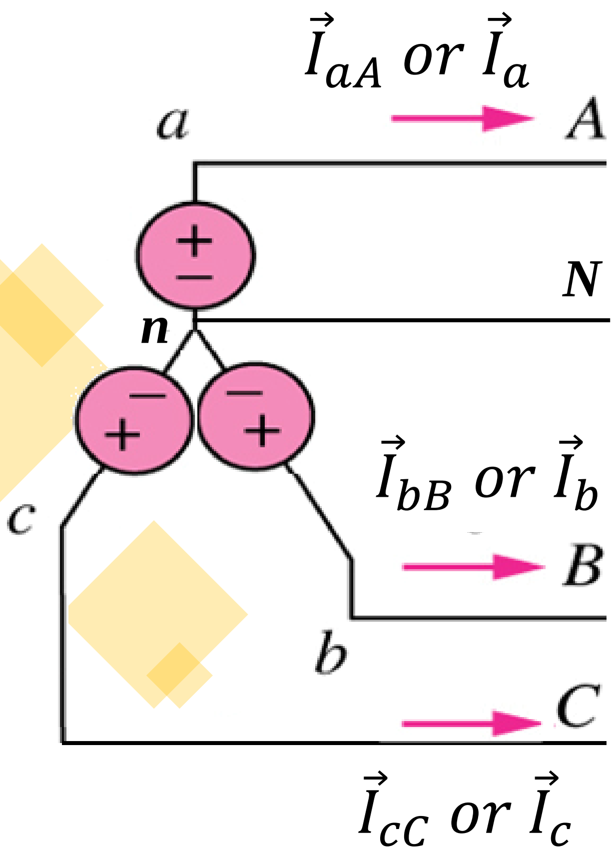
Phase voltages:

$$\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn} = 0$$

Line voltages:

$$\vec{V}_{ab} + \vec{V}_{bc} + \vec{V}_{ca} = 0$$

2.1 Y CONNECTED SOURCE: CURRENT



In a system with **balanced** Y-connected sources, **line** currents are the same as their related **phase** currents:

$$I_L = I_p$$

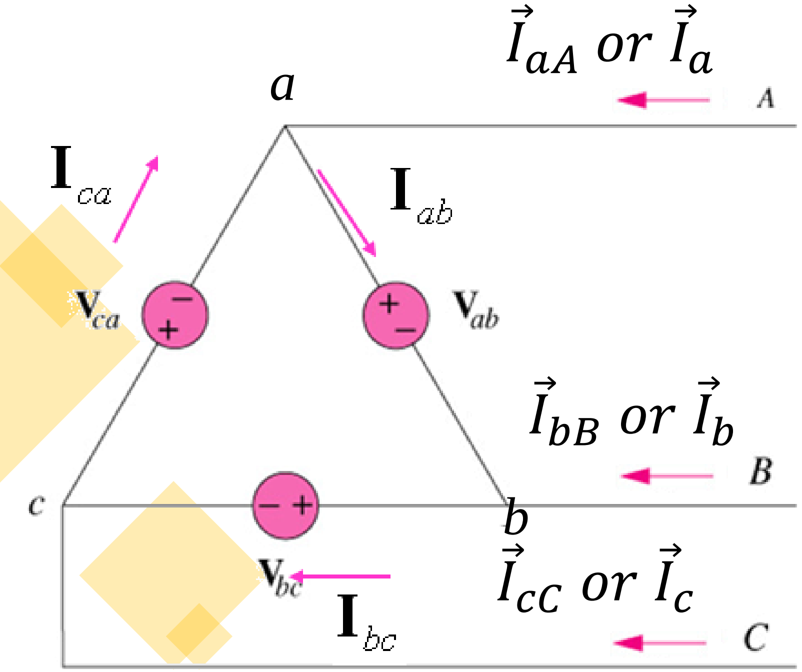
If the load is balanced as well, all line currents form a balanced set:

$$\vec{I}_{aA} = \vec{I}_{an} = \frac{\vec{V}_{an}}{Z} \qquad \vec{I}_{bB} = \vec{I}_{bn} = \frac{\vec{V}_{bn}}{Z} \qquad \vec{I}_{cC} = \vec{I}_{cn} = \frac{\vec{V}_{cn}}{Z}$$

If one current is known, the other 5 currents can be determined by inspection.

2.2 Δ CONNECTED SOURCE: VOLTAGE

The <u>end of one coil</u> is connected to the <u>start of the next coil</u> to form the loop.



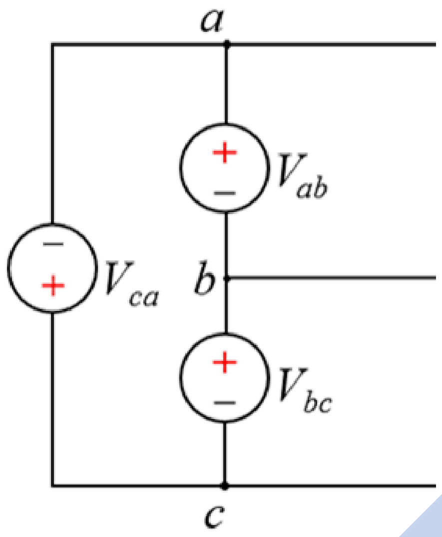
$$\vec{V}_{AB} = \vec{V}_{ab}$$
$$\vec{V}_{BC} = \vec{V}_{bc}$$
$$\vec{V}_{CA} = \vec{V}_{ca}$$

In a system with **balanced Δ**-connected sources, **line** voltage is the <u>same</u> as its related **phase** voltage:

$$V_L = V_p$$

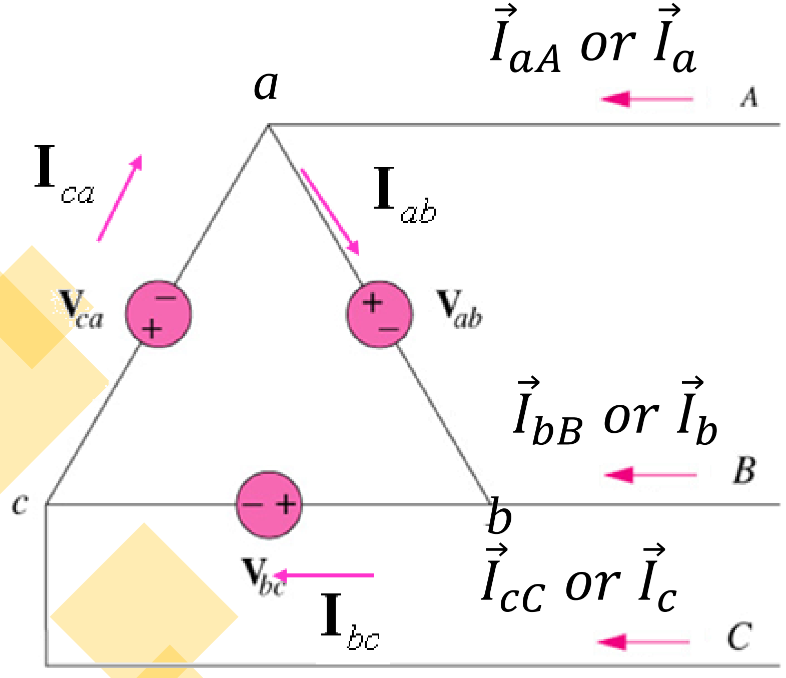
where

$$V_p = \left|\vec{V}_{ab}\right| = \left|\vec{V}_{bc}\right| = \left|\vec{V}_{ca}\right|$$
$$V_L = \left|\vec{V}_{AB}\right| = \left|\vec{V}_{BC}\right| = \left|\vec{V}_{CA}\right|$$



2.2 Δ CONNECTED SOURCE: CURRENT

The end of one coil is connected to the start of the next coil to form the loop.

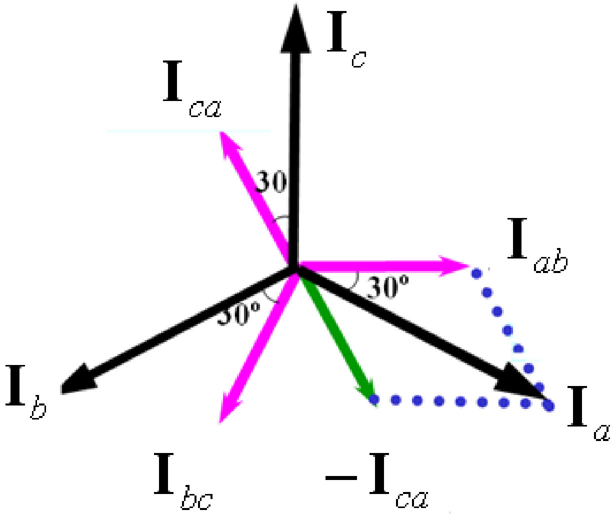


Line currents:

$$\vec{I}_{aA} = \vec{I}_{ab} - \vec{I}_{ca} = \sqrt{3}\vec{I}_{ab}\angle -30^\circ$$

$$\vec{I}_{bB} = \vec{I}_{bc} - \vec{I}_{ab} = \sqrt{3}\vec{I}_{bc}\angle -30^\circ$$

$$\vec{I}_{cC} = \vec{I}_{ca} - \vec{I}_{bc} = \sqrt{3}\vec{I}_{ca}\angle -30^\circ$$



For any balanced Δ-connected sources, the magnitude of line current is $\sqrt{3}$ times the magnitude of current voltage:

$$I_L = \sqrt{3} I_p$$

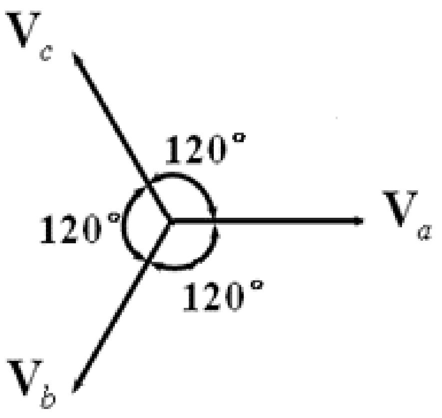
where

$$I_p = |\vec{I}_{ab}| = |\vec{I}_{bc}| = |\vec{I}_{ca}| \qquad I_L = |\vec{I}_{aA}| = |\vec{I}_{bB}| = |\vec{I}_{cC}|$$

QUIZ 2.1

1. Which of the following expression is true for an "abc" sequenced three-phase motor with $\vec{V}_{an}$ = 220∠-100°V?

(a) $\vec{V}_{bn}$ = 220∠140° V (b) $\vec{V}_{cn}$ = 220∠(-20°) V



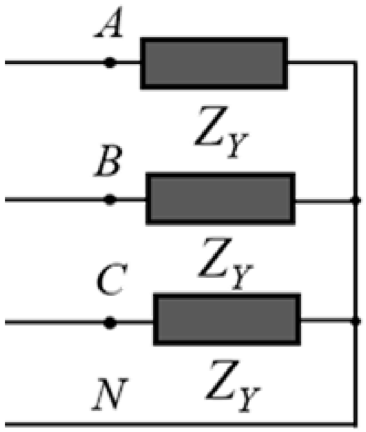
2. For a three-phase supply with the "abc" sequence, if $\vec{V}_{bn}$ = 220∠140°V, which is the correct expression of $\vec{V}_{ca}$?

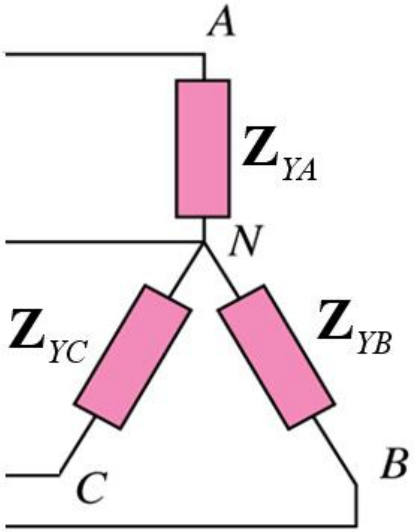
(a) 380∠50° V (b) 380∠(-130°) V

(c) 380∠130° V (d) 380∠170° V

2.3 LOAD CONNECTIONS

Y Connection



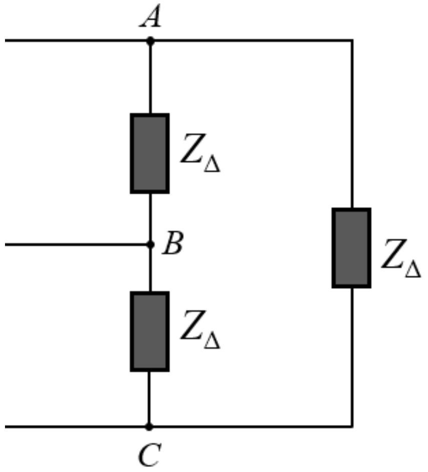


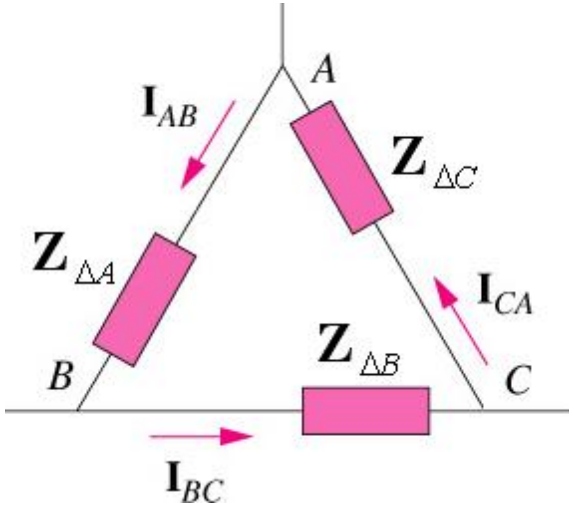
$$Z_Y = Z_{YA} = Z_{YB} = Z_{YC}$$

$$Z_\Delta = 3Z_Y$$

$$Z_Y = \frac{Z_\Delta}{3}$$

Δ Connection





$$Z_\Delta = Z_{\Delta A} = Z_{\Delta B} = Z_{\Delta C}$$

2.4 COMBINATIONS

A three-phase system consists of a **three-phase voltage source** that is used to supply a **three-phase load**. Both the three-phase voltage source and the three-phase load can be connected in two different ways:

Source Connections	**Load Connections**
Wye (Y) Connected (or star Connected)	Wye (Y) Connected (or star Connected)
Delta (Δ) Connected (or Mesh Connected)	Delta (Δ) Connected (or Mesh Connected)

4 possible connections:

1. Y-Y connection
2. Δ-Δ connection
3. Y-Δ connection
4. Δ-Y connection

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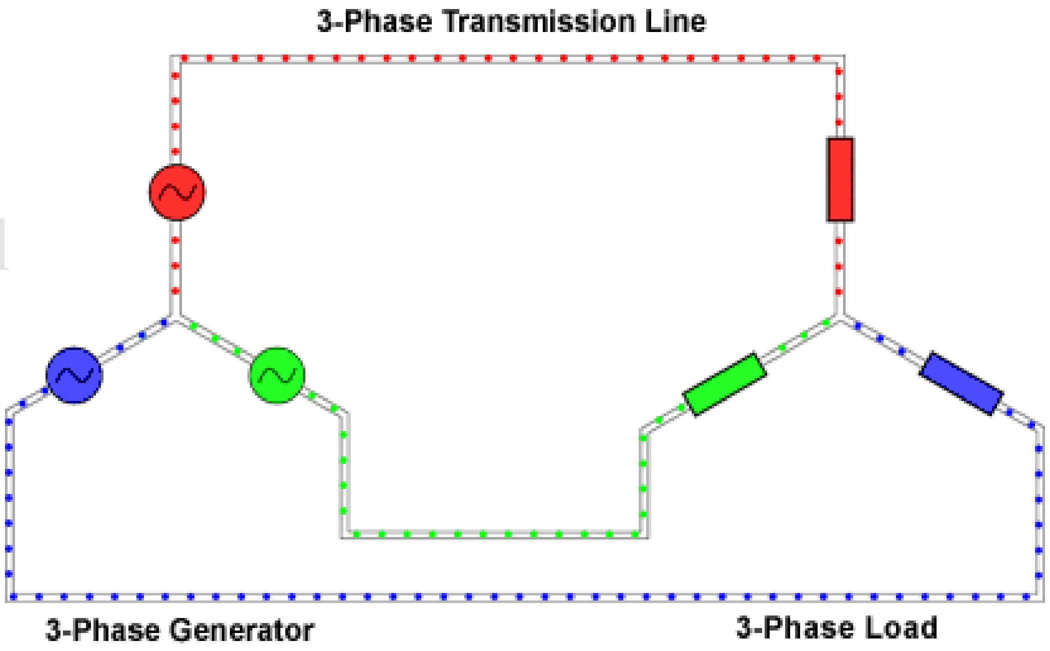
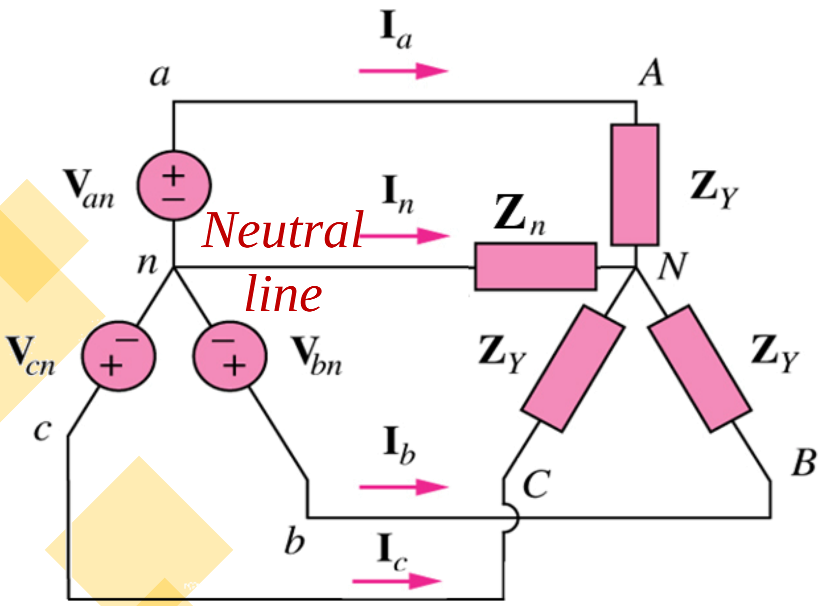


3.1 Y - Y CONNECTIONS

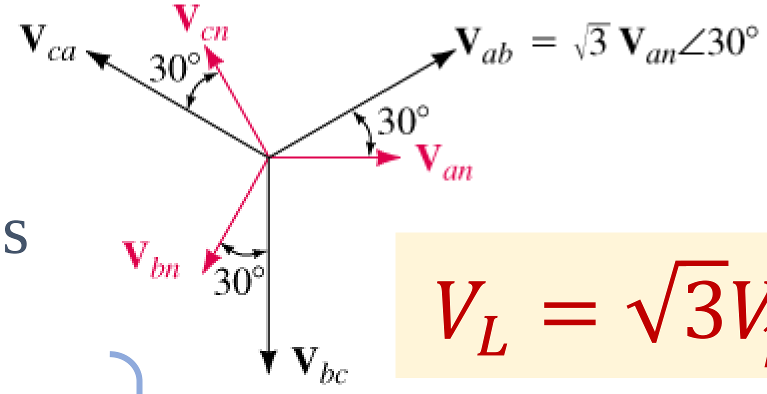
A **balanced** Y-Y system is a three-phase system with a **balanced** Y-connected source and a **balanced** Y-connected load.



$$\mathbf{I}_{Nn} = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

$$\mathbf{V}_{Nn} = \mathbf{Z}_n \mathbf{I}_{Nn} = 0$$

$$\left.\begin{aligned} \mathbf{V}_{an} &= V_p\angle 0^\circ \\ \mathbf{V}_{bn} &= V_p\angle(-120^\circ) \\ \mathbf{V}_{cn} &= V_p\angle(-240^\circ) \end{aligned}\right\} \text{Phase voltages}$$



$$\boxed{V_L = \sqrt{3}V_p}$$

$$\left.\begin{aligned} \mathbf{V}_{ab} &= \mathbf{V}_{an} - \mathbf{V}_{bn} = \sqrt{3}V_p\angle 30^\circ \\ \mathbf{V}_{bc} &= \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle(-90^\circ) \\ \mathbf{V}_{ca} &= \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle(-210^\circ) \end{aligned}\right\} \text{Line voltages}$$

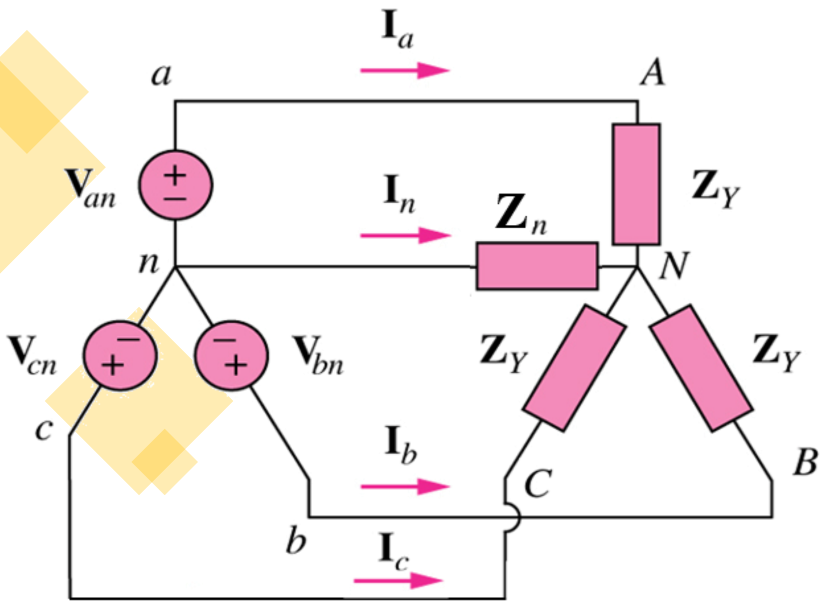
$$\boxed{I_L = I_p}$$

$$\left.\begin{aligned} \mathbf{I}_a &= \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} \\ \mathbf{I}_b &= \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle(-120^\circ)}{\mathbf{Z}_Y} = \mathbf{I}_a\angle(-120^\circ) \\ \mathbf{I}_c &= \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle(-240^\circ)}{\mathbf{Z}_Y} = \mathbf{I}_a\angle(-240^\circ) \end{aligned}\right\} \begin{array}{c}\text{Line currents} \\ \text{Phase currents} \\ \text{on source \& load}\end{array}$$

QUIZ 3.1

For a balanced Y-Y connection three-phase circuit, line voltage $\boldsymbol{V}_{ab} = 380\angle 30°$ V and load Z_Y=100∠30° Ω. The line impedance is zero.

Find expressions of all line currents.



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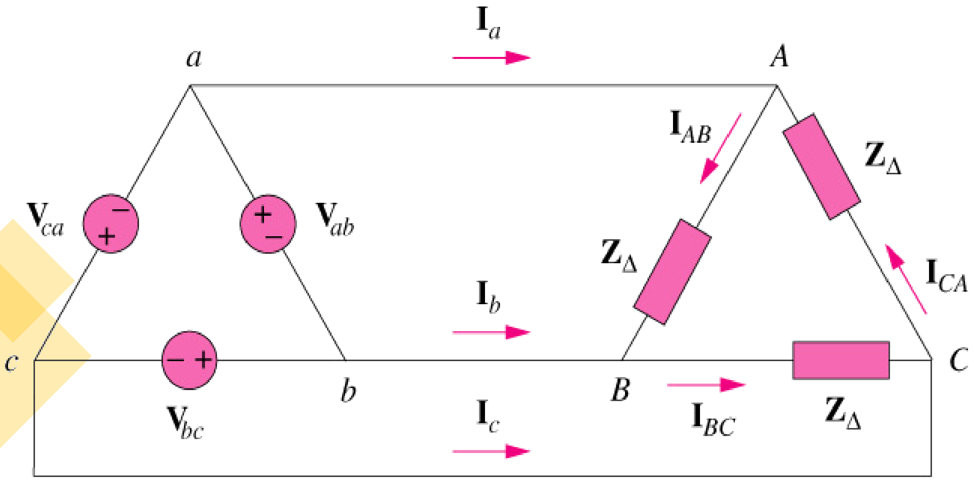
3.2 Δ - Δ CONNECTIONS

A **balanced** Δ -Δ system is a three-phase system with a balanced Δ-connected source and a balanced Δ-connected load.



$$\left.\begin{aligned} \mathbf{V}_{ab} &= V_p\angle 0^\circ \\ \mathbf{V}_{bc} &= V_p\angle(-120^\circ) \\ \mathbf{V}_{ca} &= V_p\angle(-240^\circ) \end{aligned}\right\} \text{Phase voltages}$$

$$\left.\begin{aligned} \mathbf{V}_{ab} &= \mathbf{V}_{AB} \\ \mathbf{V}_{bc} &= \mathbf{V}_{BC} \\ \mathbf{V}_{ca} &= \mathbf{V}_{CA} \end{aligned}\right\} \text{Line voltages}$$

$$V_L = V_p$$

$$\left.\begin{aligned} \mathbf{I}_{AB} &= \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} \\ \mathbf{I}_{BC} &= \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_\Delta} \\ \mathbf{I}_{CA} &= \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_\Delta} \end{aligned}\right\} \text{Phase currents on loads}$$

$$I_L = \sqrt{3} I_p$$

$$\left.\begin{aligned} \mathbf{I}_a &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \sqrt{3}\mathbf{I}_{AB}\angle(-30^\circ) \\ \mathbf{I}_b &= \mathbf{I}_{BC} - \mathbf{I}_{AB} = \sqrt{3}\mathbf{I}_{AB}\angle(-150^\circ) \\ \mathbf{I}_c &= \mathbf{I}_{CA} - \mathbf{I}_{BC} = \sqrt{3}\mathbf{I}_{AB}\angle(-270^\circ) \end{aligned}\right\} \text{Line currents}$$

where

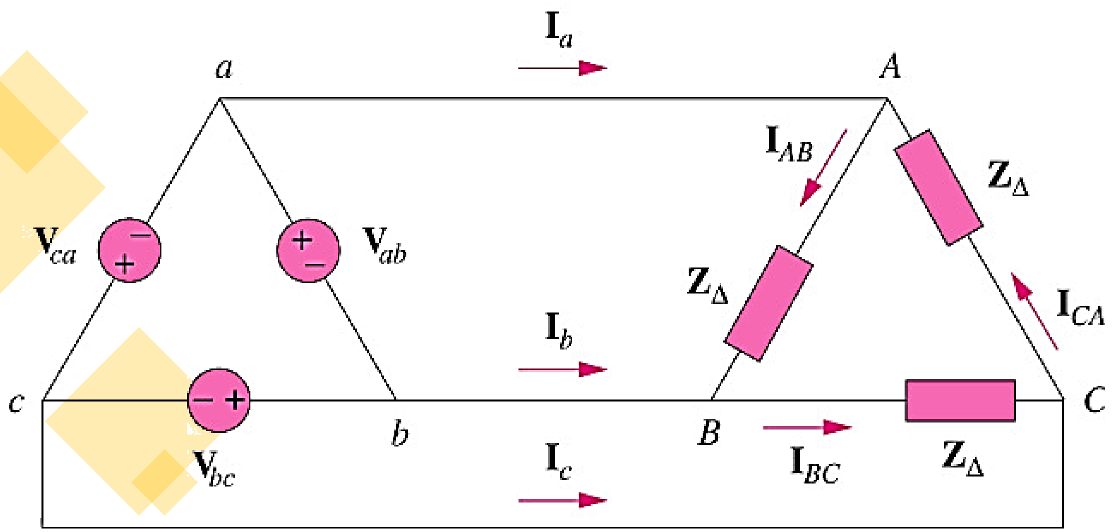
$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

QUIZ 3.2

A balanced Δ-connected load having an impedance $Z_\Delta = 20\text{-}j15\ \Omega$ is connected to a balanced Δ-connected positive-sequence generator having $V_{ab} = 330\angle 0^{o}$V.

Calculate the phase currents of the load and the line currents for the system.



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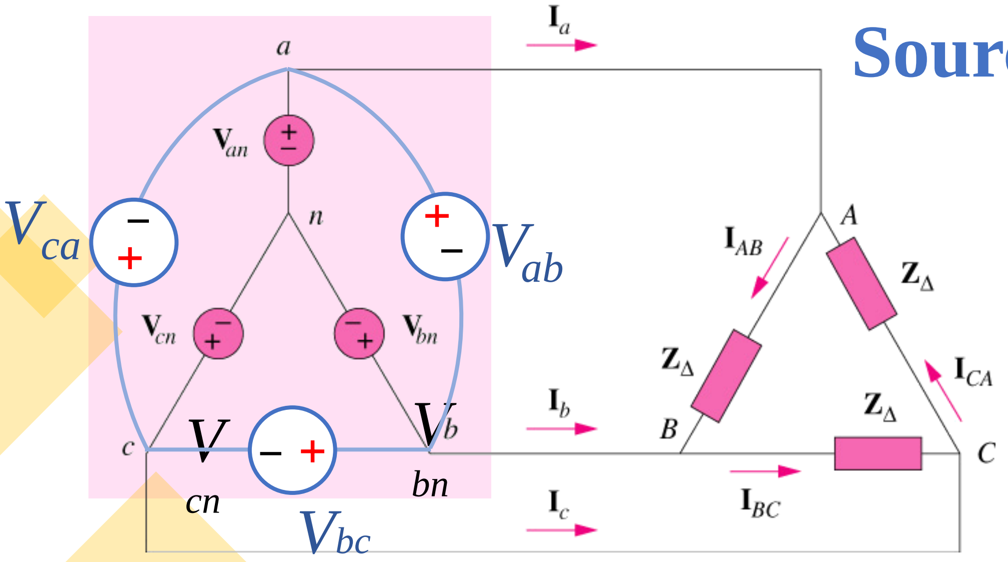
3.3 ONE OPTION

A **balanced** Y-Δ system is a three-phase system with a balanced Y-connected source and a balanced Δ-connected load.



Source Y → Δ:

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}$$
$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle(-90^\circ) = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle(-210^\circ) = \mathbf{V}_{CA}$$

$$\left.\begin{aligned} \mathbf{I}_{AB} &= \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} \\ \mathbf{I}_{BC} &= \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} \\ \mathbf{I}_{CA} &= \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} \end{aligned}\right\} \text{Phase currents on loads}$$

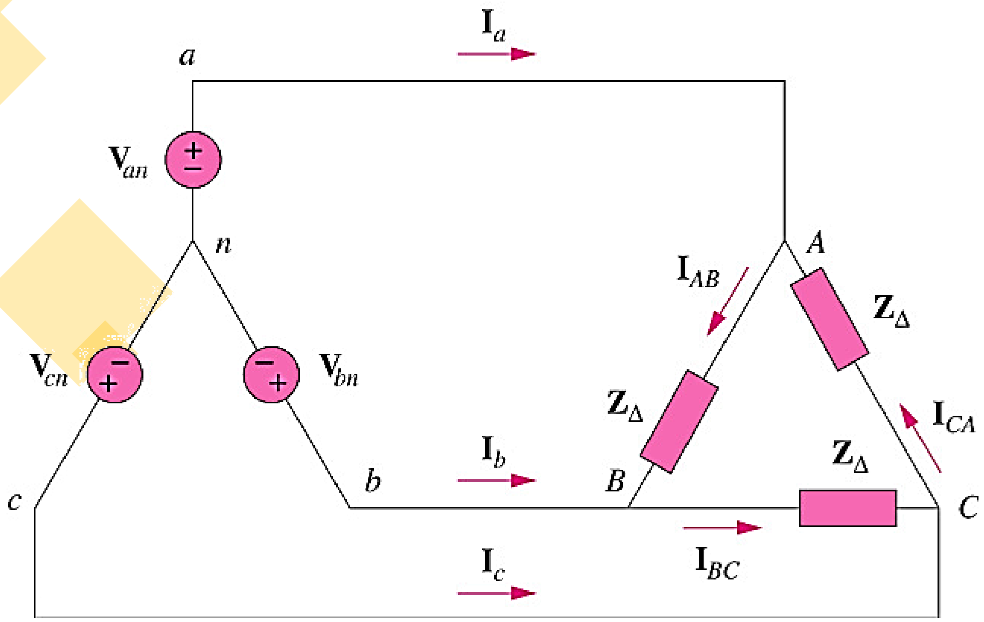
$$I_L = \sqrt{3}I_p$$

$$\left.\begin{aligned} \mathbf{V}_{an} &= V_p\angle 0^\circ \\ \mathbf{V}_{bn} &= V_p\angle(-120^\circ) \\ \mathbf{V}_{cn} &= V_p\angle(-240^\circ) \end{aligned}\right\} \text{Phase voltages}$$

$$\left.\begin{aligned} \mathbf{I}_a &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \sqrt{3}\mathbf{I}_{AB}\angle(-30^\circ) \\ \mathbf{I}_b &= \mathbf{I}_{BC} - \mathbf{I}_{AB} = \sqrt{3}\mathbf{I}_{AB}\angle(-150^\circ) \\ \mathbf{I}_c &= \mathbf{I}_{CA} - \mathbf{I}_{BC} = \sqrt{3}\mathbf{I}_{AB}\angle(-270^\circ) \end{aligned}\right\} \text{Line currents}$$

QUIZ 3.3

A three-phase system with a balanced Y-connected source of line voltage 415 V supplies a balanced *delta*-connected load. The load is purely resistive of 10 Ω resistance per phase. Calculate the <u>magnitude</u> of currents of this system.



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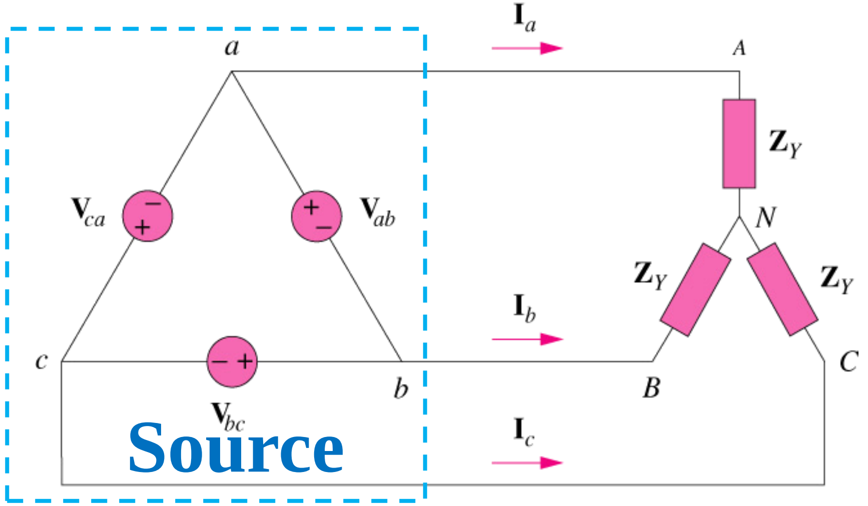
3.4 Δ - Y CONNECTIONS

A **balanced** Δ-Y system is a three-phase system with a balanced Δ-source and a balanced Y-connected connected load.

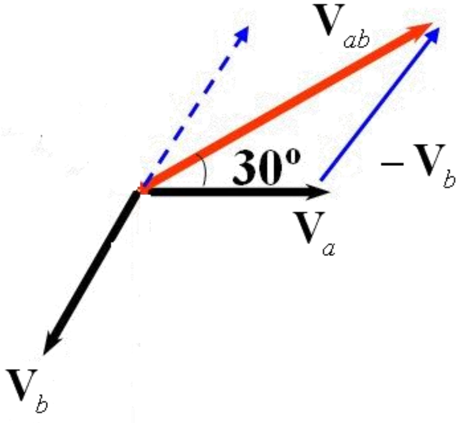


$\Delta \rightarrow \mathbf{Y}$

$$\left.\begin{aligned} \mathbf{V}_{ab} &= V_p\angle 0^\circ \\ \mathbf{V}_{bc} &= V_p\angle(-120^\circ) \\ \mathbf{V}_{ca} &= V_p\angle(-240^\circ) \end{aligned}\right\} \text{Phase voltages}$$

Source Δ →Y:

$$\left\{\begin{aligned} \mathbf{V}_{an} &= \frac{1}{\sqrt{3}}\mathbf{V}_p\angle(-30^\circ) \\ \mathbf{V}_{bn} &= \frac{1}{\sqrt{3}}\mathbf{V}_p\angle(-150^\circ) \\ \mathbf{V}_{cn} &= \frac{1}{\sqrt{3}}\mathbf{V}_p\angle(-270^\circ) \end{aligned}\right.$$

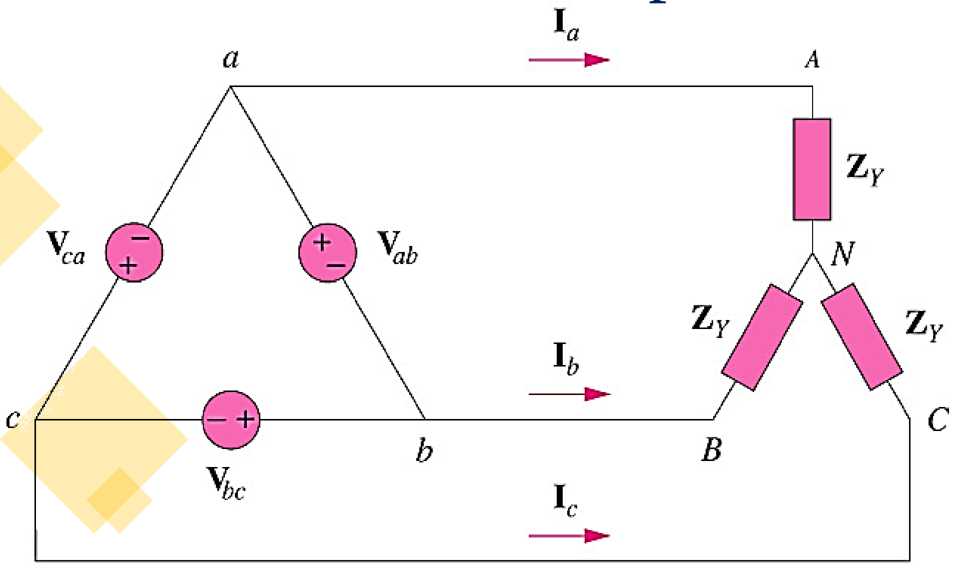


$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$$
$$= \sqrt{3}V_p\angle 30^\circ$$

$$\left.\begin{aligned} \mathbf{I}_a &= \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} \\ \mathbf{I}_b &= \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \mathbf{I}_a\angle(-120^\circ) \\ \mathbf{I}_c &= \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \mathbf{I}_a\angle(-240^\circ) \end{aligned}\right\} \text{Line currents}$$

QUIZ 3.4

A balanced Y-connected load with a phase impedance 40 + j25 Ω is supplied by a balanced, positive-sequence Δ-connected source with a line voltage $\mathbf{V}_{ab}$= 210 V. Calculate the phase currents of loads (use $\mathbf{V}_{ab}$ as reference).



SUMMARY

Connection	Phase Voltage	Phase Current	Line Voltage	Line Current
balanced Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$ $\mathbf{V}_{bn} = V_p\angle(-120°)$ $\mathbf{V}_{cn} = V_p\angle(-240°)$	$I_L = I_p$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle(-120°)$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle 120°$	$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle(-120°)$ $\mathbf{I}_c = \mathbf{I}_a\angle(-240°)$
balanced Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0°$ $\mathbf{V}_{bc} = V_p\angle(-120°)$ $\mathbf{V}_{ca} = V_p(-240°)$	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$V_L = V_p$	$\mathbf{I}_a = \sqrt{3}\mathbf{I}_{AB}\angle(-30°)$ $\mathbf{I}_b = \mathbf{I}_a\angle(-120°)$ $\mathbf{I}_c = \mathbf{I}_a(-240°)$
balanced Y-Δ	$\mathbf{V}_{an} = V_p\angle 0°$ $\mathbf{V}_{bn} = V_p\angle(-120°)$ $\mathbf{V}_{cn} = V_p(-240°)$	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle(-120°)$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle(-240°)$	$\mathbf{I}_a = \sqrt{3}\mathbf{I}_{AB}\angle(-30°)$ $\mathbf{I}_b = \mathbf{I}_a\angle(-120°)$ $\mathbf{I}_c = \mathbf{I}_a\angle(-240°)$
balanced Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0°$ $\mathbf{V}_{bc} = V_p\angle(-120°)$ $\mathbf{V}_{ca} = V_p\angle(-240°)$	$I_L = I_p$	$\mathbf{V}_{ab} = \sqrt{3}\lvert V_{Y-an}\rvert\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle(-120°)$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle(-240°)$	$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle(-120°)$ $\mathbf{I}_c = \mathbf{I}_a\angle(-240°)$

QUIZ 3.5

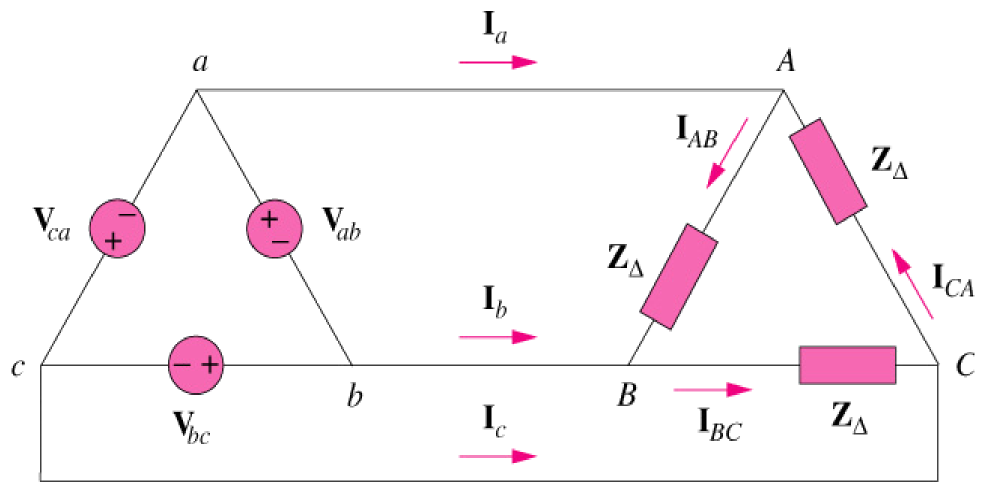
1. In a Y-connected load, the line current and phase current on the load are equal.

(a) true (b) false

2. In a Δ-Δ system, a phase voltage of 100 V produces a line voltage of:

(a) 100 V (b) 141 V

(c) 71 V (d) 173 V



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RECALL EEE103…

Voltage: $v(t) = V_m \cos(\omega t + \theta_v)$

Current: $i(t) = I_m \cos(\omega t + \varphi_i)$

Absorbed Instantaneous Power:

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \varphi_i) \\ &= \frac{1}{2}V_m I_m \cos(\theta_v - \varphi_i) \\ &\quad + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \varphi_i) \end{aligned}$$

Absorbed Average Power:

$$\begin{aligned} P &= \frac{1}{2}V_m I_m \cos(\theta_v - \varphi_i)\frac{1}{T}\int_0^T dt + \frac{1}{T}\int_0^T \left(\frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \varphi_i)\right)dt \\ &= \frac{1}{2}V_m I_m \cos(\theta_v - \varphi_i) = V_{rms} I_{rms} \cos(\theta_v - \varphi_i) \end{aligned}$$

$V_{rms} = \frac{V_m}{\sqrt{2}}$ $I_{rms} = \frac{I_m}{\sqrt{2}}$

Instantaneous Power:

the rate at which an element absorbs energy.

$$p(t) = v(t)i(t)$$

Average Power (Real Power):

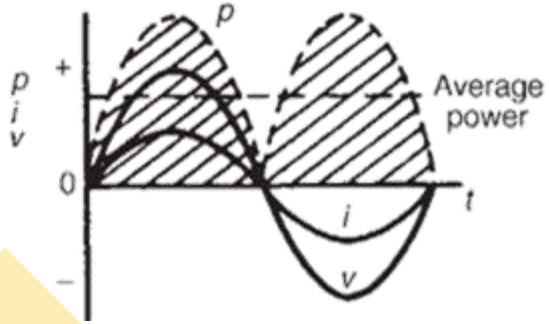
the average of the instantaneous power over one period.

$$P = \frac{1}{T}\int_0^T p(t)dt$$

RECALL EEE103…

$$P = \frac{1}{2}V_m I_m\cos(\theta_v - \varphi_i) = V_{rms}I_{rms}\cos(\theta_v - \varphi_i)$$

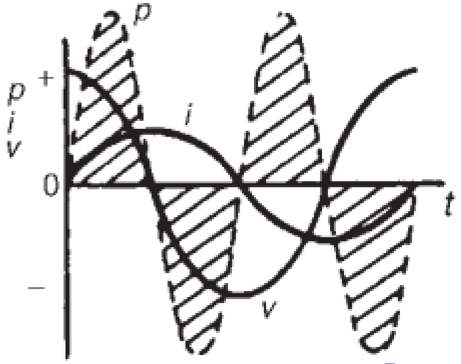
Pure resistive



v, i in phase

$$P = \frac{1}{2}V_m I_m\cos(\theta_v - \varphi_i)$$
$$= \frac{1}{2}V_m I_m$$

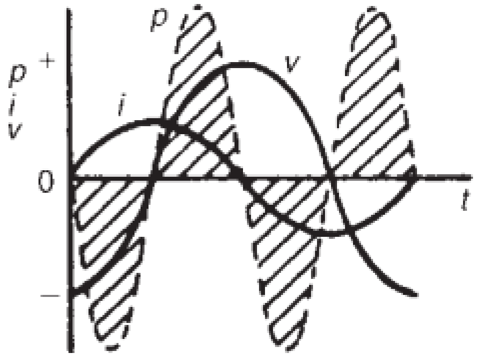
Pure inductive



v leads i

$$P = \frac{1}{2}V_m I_m\cos(\theta_v - \varphi_i)$$
$$= \frac{1}{2}V_m I_m\cos 90° = 0$$

Pure capacitive



i leads v

$$P = \frac{1}{2}V_m I_m\cos(\theta_v - \varphi_i)$$
$$= \frac{1}{2}V_m I_m\cos(-90°) = 0$$

A **pure *reactive*** element absorbs **zero** average power.

RECALL EEE103…

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \varphi_i) = V_{rms} I_{rms} \cos(\theta_v - \varphi_i) = S \cos(\theta_v - \varphi_i)$$

Apparent Power:

the product of the effective (*rms*) values of voltage and current

$$S = V_{rms} I_{rms} = V_{eff} I_{eff} \text{ VA}$$

Power Factor (*pf*): $\cos(\theta_v - \varphi_i)$

Power Factor Angle: $\phi = \theta_v - \varphi_i$

the phase difference between the voltage and current applied across an element, which is the angle of the load as well.

$$pf = \frac{Average\ Power\ P}{Apparent\ Power\ S} = \cos\phi$$

pf can be treated as the factor by which the apparent power must be multiplied to obtain the real power.

RECALL EEE103…

Considering a complex load $Z = R + jX$, the absorbed power is called the complex power **S** (the product of the *rms* voltage phasor and the complex conjugate of the current phasor):

$$\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}_{rms}^{*} = \boxed{V_{rms}I_{rms}}\angle(\theta_v - \varphi_i) = \boxed{V_{rms}I_{rms}\cos(\theta_v - \varphi_i)} + j\boxed{V_{rms}I_{rms}\sin(\theta_v - \varphi_i)}$$

Complex Power

Apparent Power S (VA)

Average (Real) Power P (W)

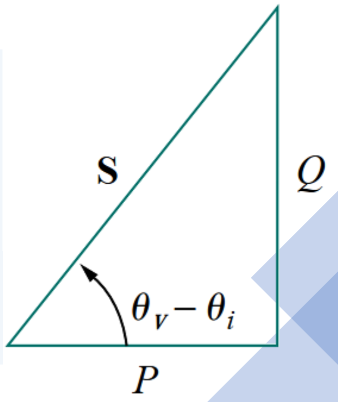
Reactive Power Q (var)

$S = |\mathbf{S}|$ magnitude of complex power

$$\text{Complex Power} = \mathbf{S} = P + jQ = \frac{1}{2}\mathbf{VI}^{*}$$
$$= V_{\text{rms}}I_{\text{rms}}\underline{/\theta_v - \varphi_i})$$
$$\text{Apparent Power} = S = |\mathbf{S}| = V_{\text{rms}}I_{\text{rms}} = \sqrt{P^2 + Q^2}$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \varphi_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \varphi_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \varphi_i)$$

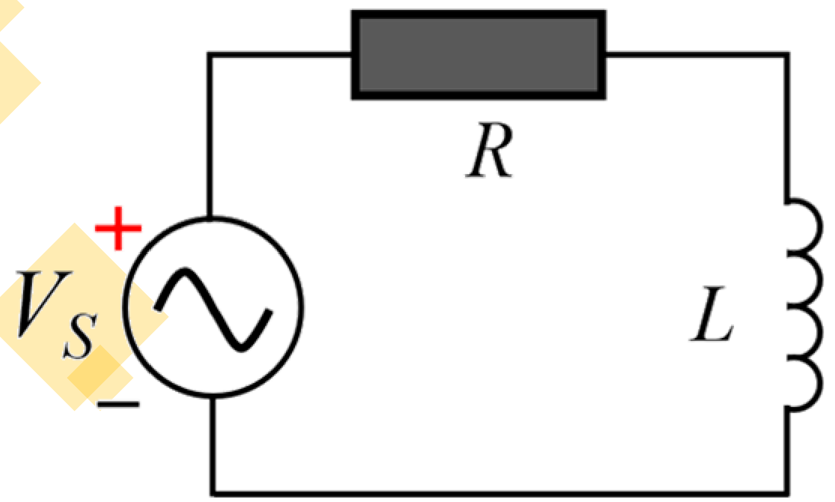
$\because$ the angle of the *pf* equals the angle of the load

$$\therefore \mathbf{S} = \boldsymbol{V}_{rms}\boldsymbol{I}_{rms}^{*} = (Z I_{rms})\mathrm{I}_{rms}^{*}$$
$$= \mathrm{I}_{rms}^{2}Z = \mathrm{I}_{rms}^{2}(R + jX)$$
$$= P + jQ$$



QUIZ 4.1

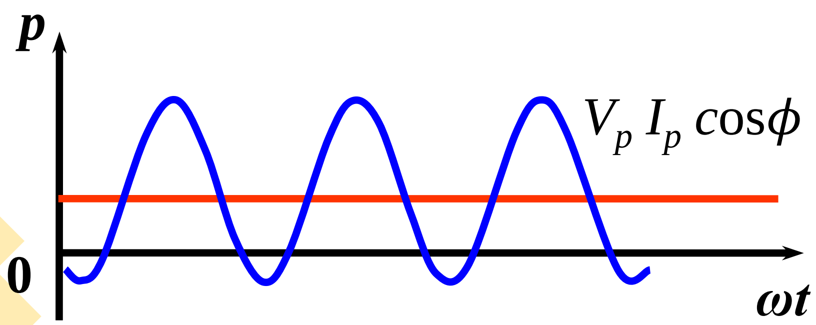
A series circuit of resistance 60 Ω and inductance 75 mH is connected to a 110V, 60 Hz supply. Calculate the power dissipated on the load and the power factor *pf*.



4.1 IN THREE-PHASE SYSTEM

The power in a three-phase system is the **sum** of the power in each phase.

3 kinds of power used by power engineers: Real power, Apparent power & Reactive power



$$\left.\begin{aligned} v_A(t) &= \sqrt{2}V_{rms}\cos\omega t \\ i_A(t) &= \sqrt{2}I_{rms}\cos(\omega t-\phi) \end{aligned}\right\}\ \text{Phase A}$$

$$p_A = v_A i_A = 2V_{rms}I_{rms}\cos\omega t\ \cos(\omega t-\phi)$$

$$= V_{rms}I_{rms}\cos\phi + V_{rms}I_{rms}\cos(2\omega t-\phi)$$

The instantaneous phase power is a function of time

Similarly:

$$p_B = v_B i_B = V_{rms}I_{rms}\cos\phi + V_{rms}I_{rms}\cos[2(\omega t-120°)-\phi]$$

$$p_C = v_C i_C = V_{rms}I_{rms}\cos\phi + V_{rms}I_{rms}\cos[2(\omega t+120°)-\phi]$$

The **total** instantaneous power is ***independent*** of time

Total Instantaneous Power:

$$p = p_A + p_B + p_C = 3V_{rms}I_{rms}\cos\phi = P$$

4.1 IN THREE-PHASE SYSTEM

Average (Real) Power P

Y connected

$$\left.\begin{aligned} V_L &= \sqrt{3}V_p \\ I_L &= I_p \end{aligned}\right\} \quad \begin{aligned} &P_{phase} = V_P I_P cos\phi = \frac{1}{\sqrt{3}} V_L I_L cos\phi \\ &\therefore \boxed{P_t = 3P_{phase} = \sqrt{3} V_L I_L cos\phi} \end{aligned}$$

Δ connected

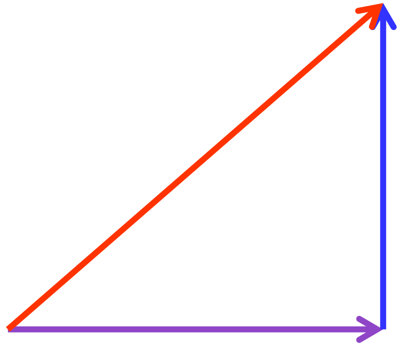
$$\left.\begin{aligned} V_L &= V_p \\ I_L &= \sqrt{3}I_p \end{aligned}\right\} \quad \begin{aligned} &P_{phase} = V_P I_P cos\phi = \frac{1}{\sqrt{3}} V_L I_L cos\phi \\ &\therefore \boxed{P_t = 3P_{phase} = \sqrt{3} V_L I_L cos\phi} \end{aligned}$$

Independent on the connection methods

V_p, I_p: effective values (rms)

Complex Power:

$$S = P + jQ = 3V_p I_p^*$$



$$P = \sqrt{3} V_L I_L \cos\phi = 3V_p I_p \cos\phi$$

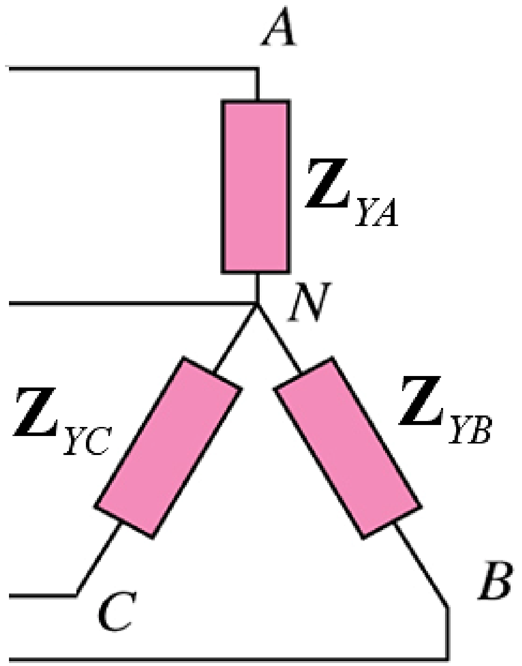
$$Q = \sqrt{3} V_L I_L \sin\phi = 3V_p I_p \sin\phi$$

$$S = \sqrt{3} V_L I_L = 3V_p I_p$$

QUIZ 4.2

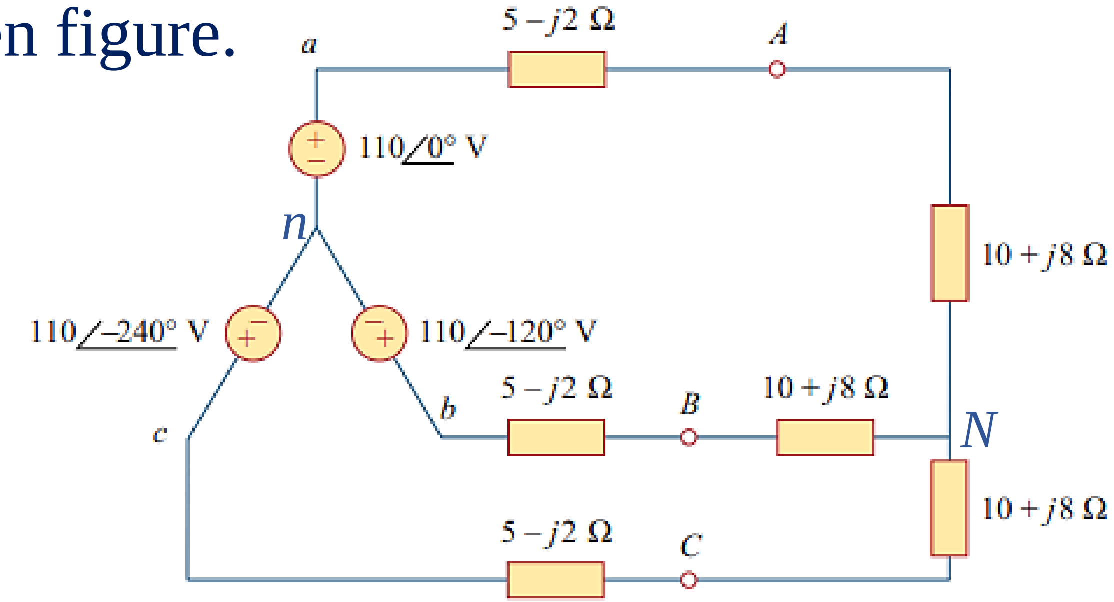
A three-phase motor can be regarded as a **balanced Y-load**. A three-phase motor draws 5.6 kW when the effective value of the line voltage is 220 V*rms* and the effective value of the line current is 18.2 A*rms*.

Find the power factor *pf* of the motor.



QUIZ 4.3

Determine the total average power P, reactive power Q, and complex power $\mathbf{S}$ at the source, at the line, and at the load for the given figure.



*WHY STUDY THREE-PHASE SYSTEM?

- ALL electric power system in the world used 3-phase system to generate, transmit, and distribute
  - ✓ One phase, two phase, or three phase current can be taken from three phase system rather than generated independently.
- Instantaneous power can be constant (not pulsating) – smoother rotation of electrical machines
  - ✓ High power motors prefer a steady torque
- More economical than single phase – less wires for the same power transfer
  - ✓ The amount of wire required for a three-phase system is less than required for an equivalent single-phase system.

NEXT…

Tutorial & Revision