Xi'an Jiaotong-Liverpool University
西交利物浦大学

# MEC208 Instrumentation and Control System

*2024-25 Semester 2*


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Department of Mechatronics and Robotics

School of Advanced Technology

# Final Exam

- Final exam 70% of the total marks
- Include six descriptive questions, all should be answered
- Reasonable derivations of each problem are required

| Module | Module Leader | Date | Day | Student Admission Time | Exam Start Time | Exam Duration | Exam Room |
|---|---|---|---|---|---|---|---|
| MEC208 | Yuqing Chen | 30-May-2025 | Friday | 1:30 pm | 2:00 pm | 3h | GYM-GMG01-Basketball Court-South Campus-SIP |

# Teaching Arrangement

- Instrumentation system

Dr. Yuqing Chen

- Mathematical model – ODE & Transfer function

- Mathematical model – State space

Dr. Bangxiang Chen

- Time domain performance

- Stability of linear system

Dr. Chee Shen Lim

- Root Locus & Frequency response

# Contents Covered in Exam

- Error analysis
- Sensors & amplifiers
- Mathematical modelling of the system
- Transfer function

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- **Error analysis**
- Sensors & amplifiers
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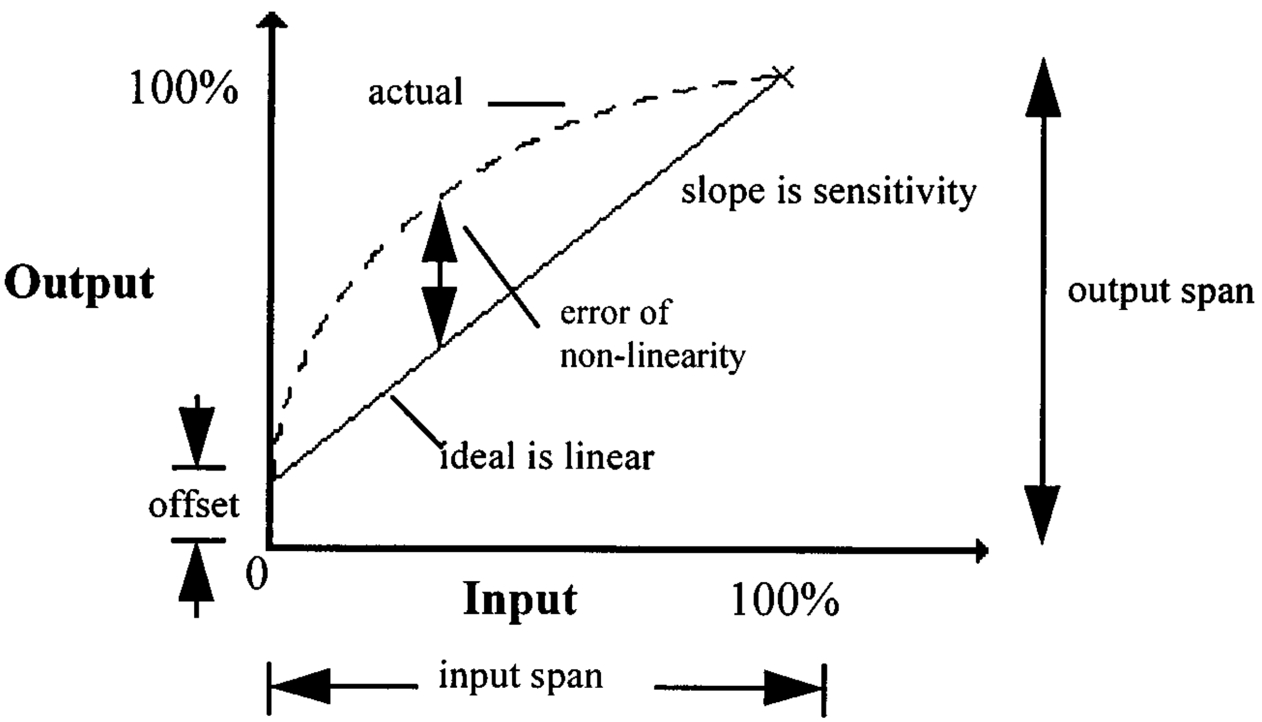
# Static Characteristics of Sensor Performance

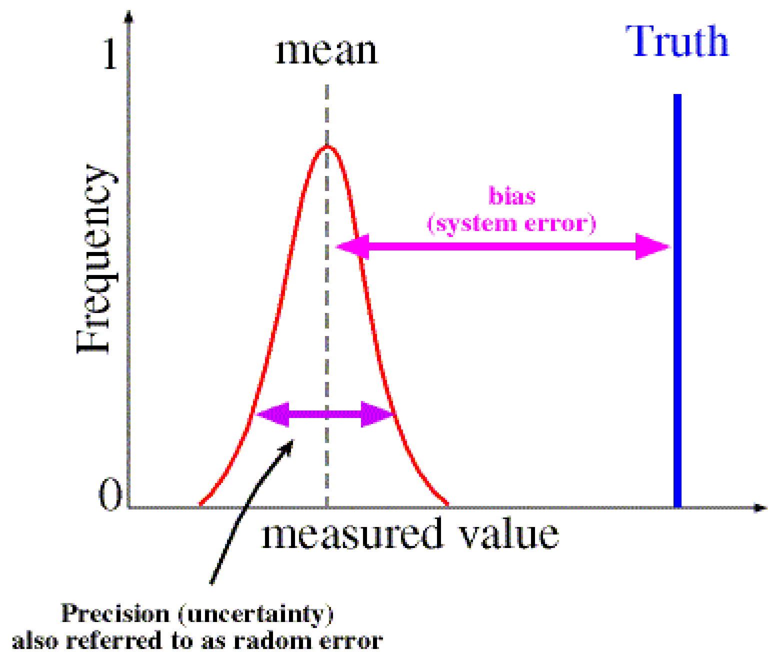



- Range / span
- Error
- Accuracy
- Precision
- Sensitivity
- Resolution
- Hysteresis error
- Non-linearity error
- Repeatability/ reproducibility
- Dead band / time
- Etc.

# Measurement Error: Systematic Error

- **Systematic Error (bias)** sources:
  - usually those that change the input–output response of a sensor resulting in mis-calibration
  - aging, damage or abuse of the sensor
  - measurement process itself changes the intended measurand
  - the transmission path
  - human observers

✓ Systematic error can be corrected by some methods if the error source is known.

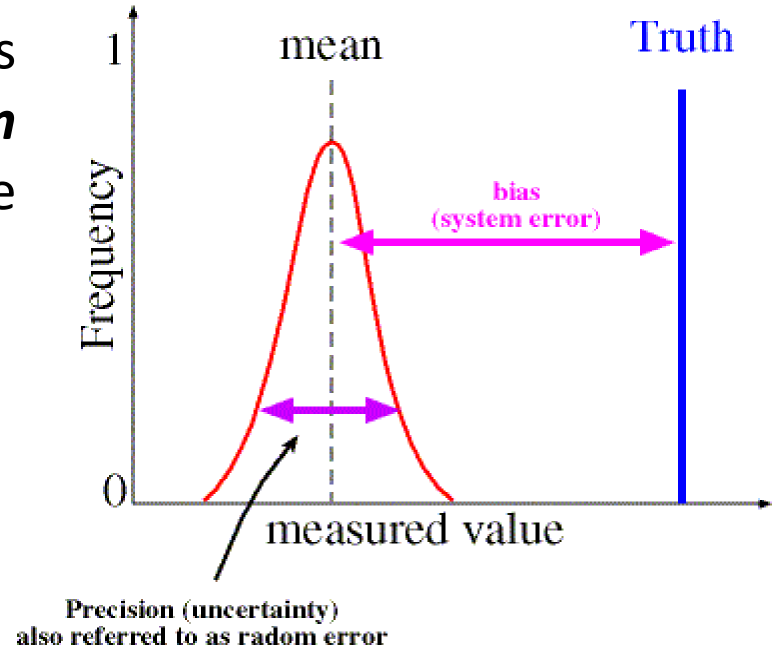


# Measurement Error: Random Error

- **Random Error (noise)** sources:
  - usually unknown and unpredictable changes in the measurement
  - may occur in the measuring instruments or in the environmental conditions

Random error is often referred to as noise, and exhibit a ***Gaussian distribution*** when repeating a large number of measurements.

◆ Random error can NOT be completely eliminated.



# Measurement Uncertainty

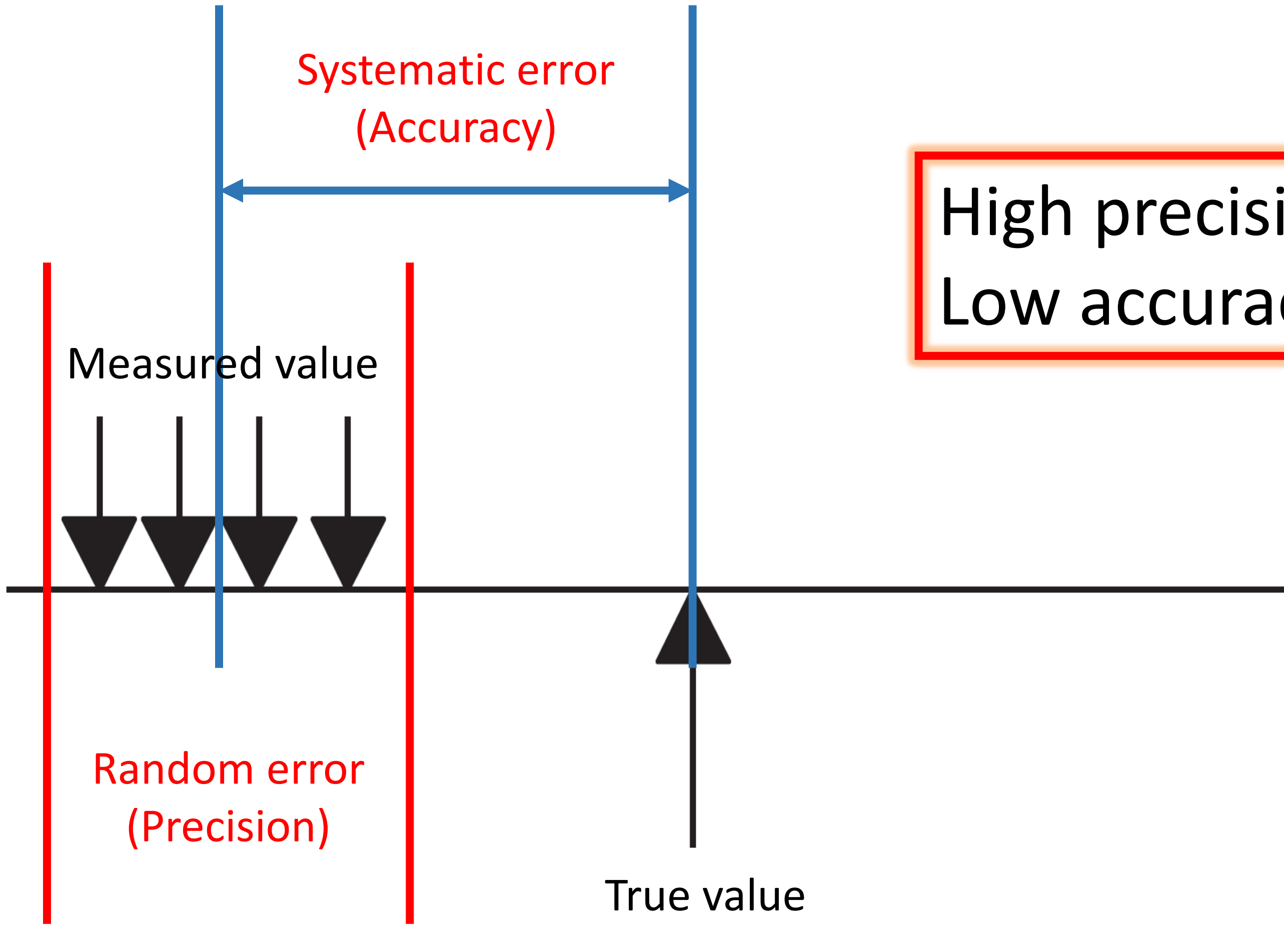
**Accuracy**:

- defined as the difference between the true value of the measurand and the measured value indicated by the instrument;
- determined by systematic error;
- usually expressed as a percentage of the full-scale deflection (FSD) of the transducer or system.

**Precision**:

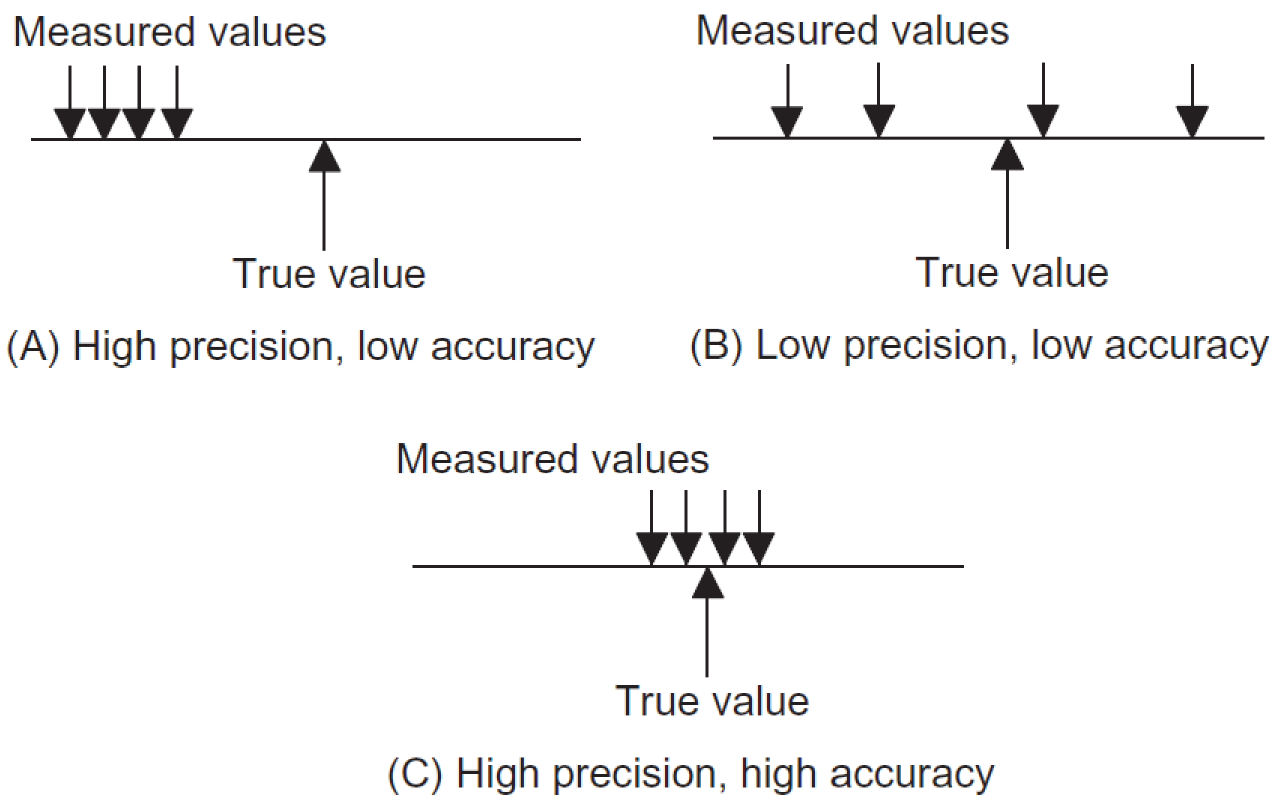
- a term that describes an instrument's degree of freedom from random errors;
- normally quantified by the standard deviation δ that indicates the width of the Gaussian distribution;
- The smaller the standard deviation, the more precise the measurement.

# Target Analogy of Measurement – 1D




High precision
Low accuracy

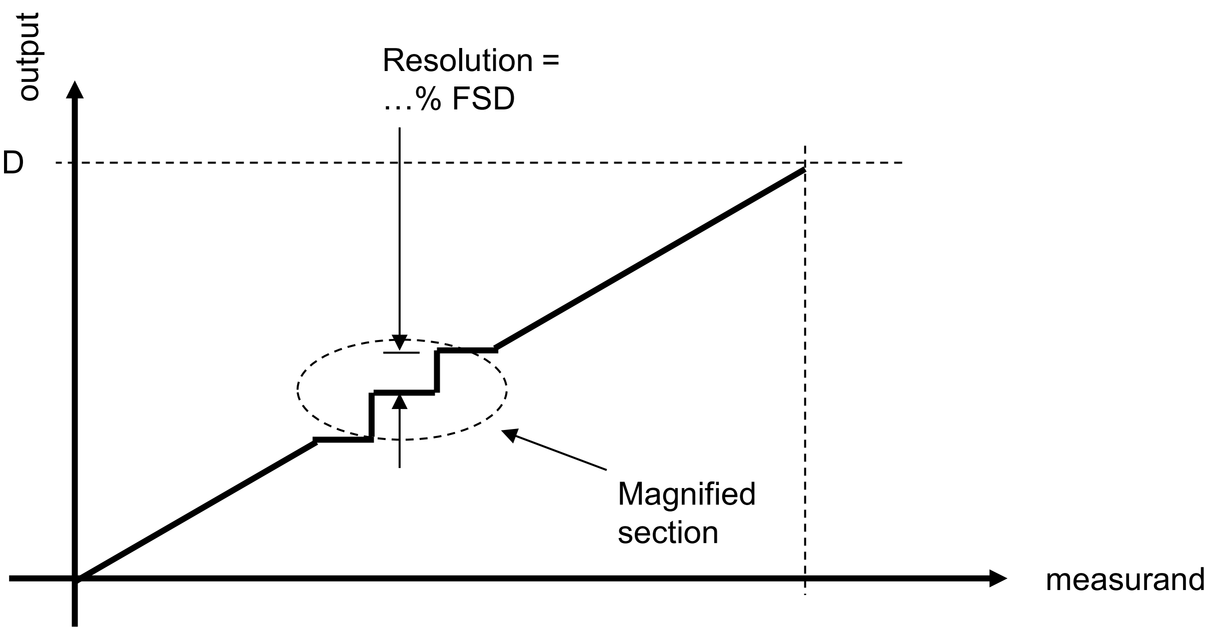
# Accuracy vs. Precision – 1D



# Resolution

**Resolution**:

- is the smallest change in the measurand that can be measured;
- the resulting maximum error is **half** the resolution of the measurement.

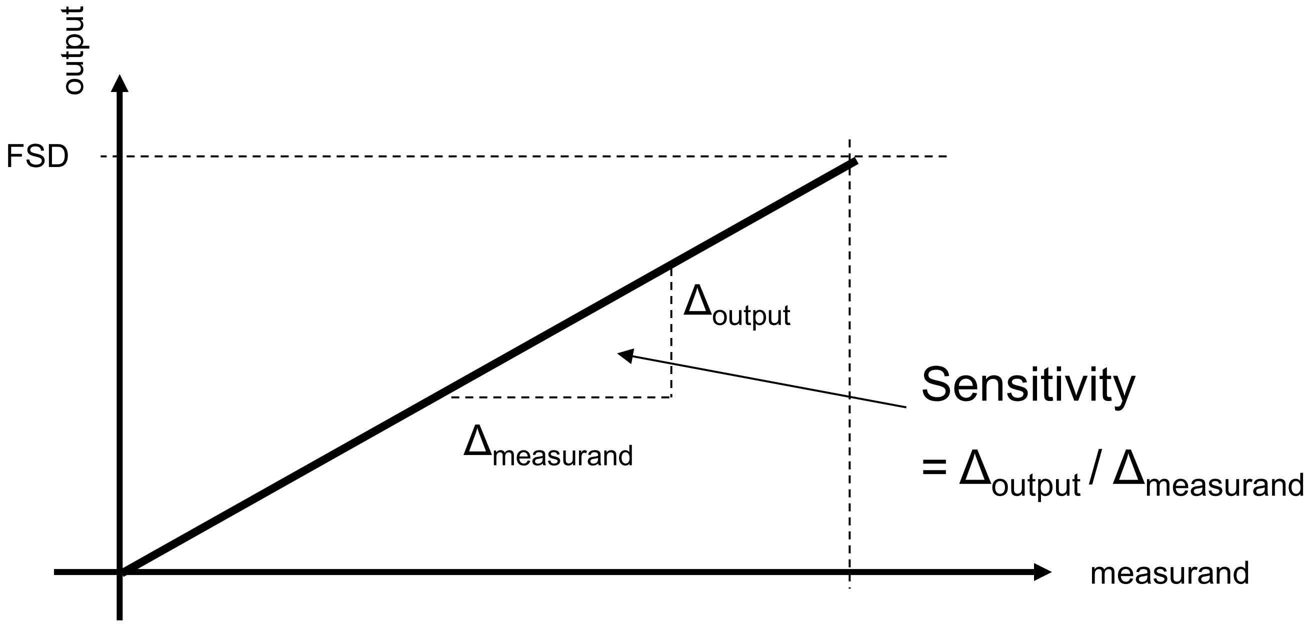



**FSD: Full Scale Deflection**

# Sensitivity

**Sensitivity**:

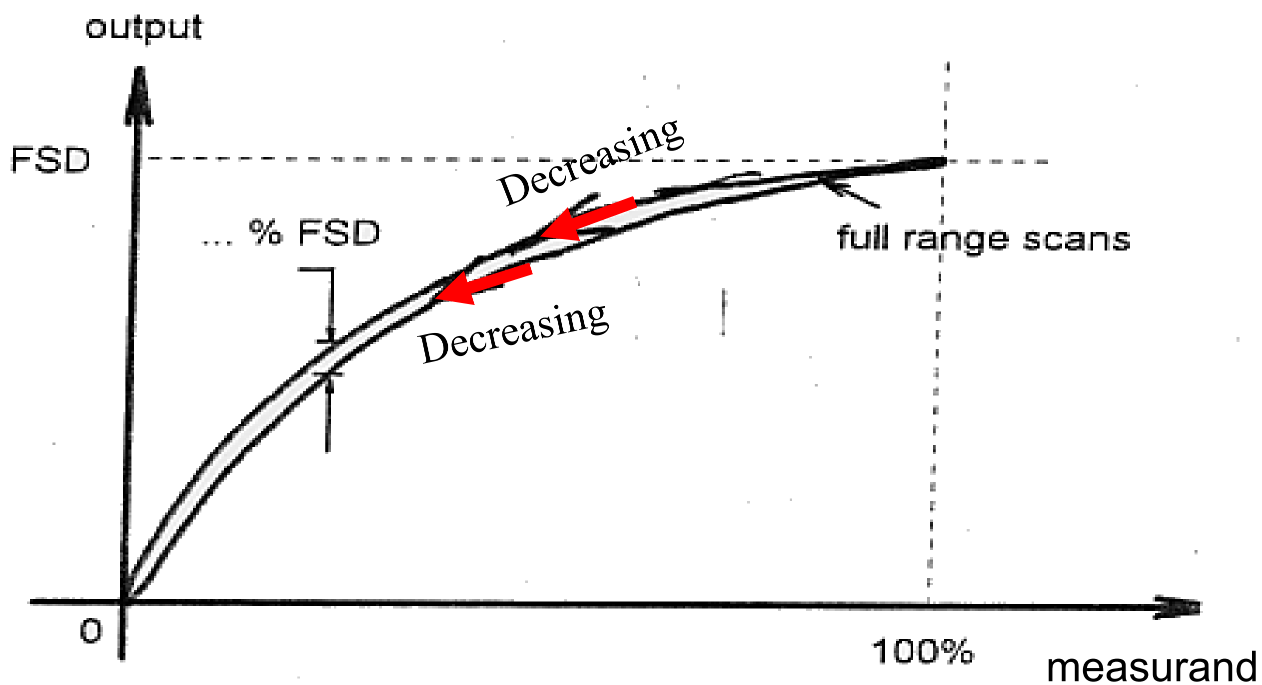
- is the slope of the output vs input (measurand) in the graph;
- note the difference between sensitivity and resolution.



# Repeatability

**Repeatability**:

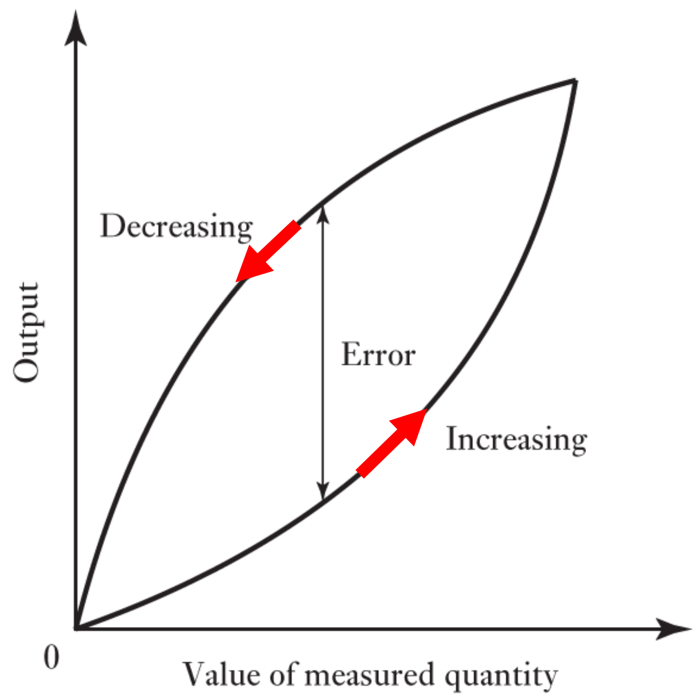
- relates to the maximum difference between any two output values at the same value of measurand taken during full range traverses of the measurand and approached from the same direction.



# Hysteresis Error

**Hysteresis error**:

- the difference in transducer output obtained when any measurement point is approached from different directions during a full range scan of the transducer.




## Example

Thermometer measuring the same temperature

- Reached by warming up to the measured temp;
- Reached by cooling down to the measure temp.

# Non-linearity Error

**Non-linearity error**:

- For many transducers a linear relationship between the input and output is assumed over the working range, i.e., a graph of output plotted against input is assumed to give a straight line. Errors occur as a result of the assumption of linearity.
- Non-linearity is not necessarily a source of error at all. It only becomes one if the transducer is assumed to have a linear output (most are).

The error is defined as the **maximum difference** of the actual input-output curve from a straight line. Various methods are used for numerical expression of the nonlinearity error, depending on how to define the reference straight line.

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- Error analysis
- **Sensors & amplifiers**
- Mathematical modelling of the system
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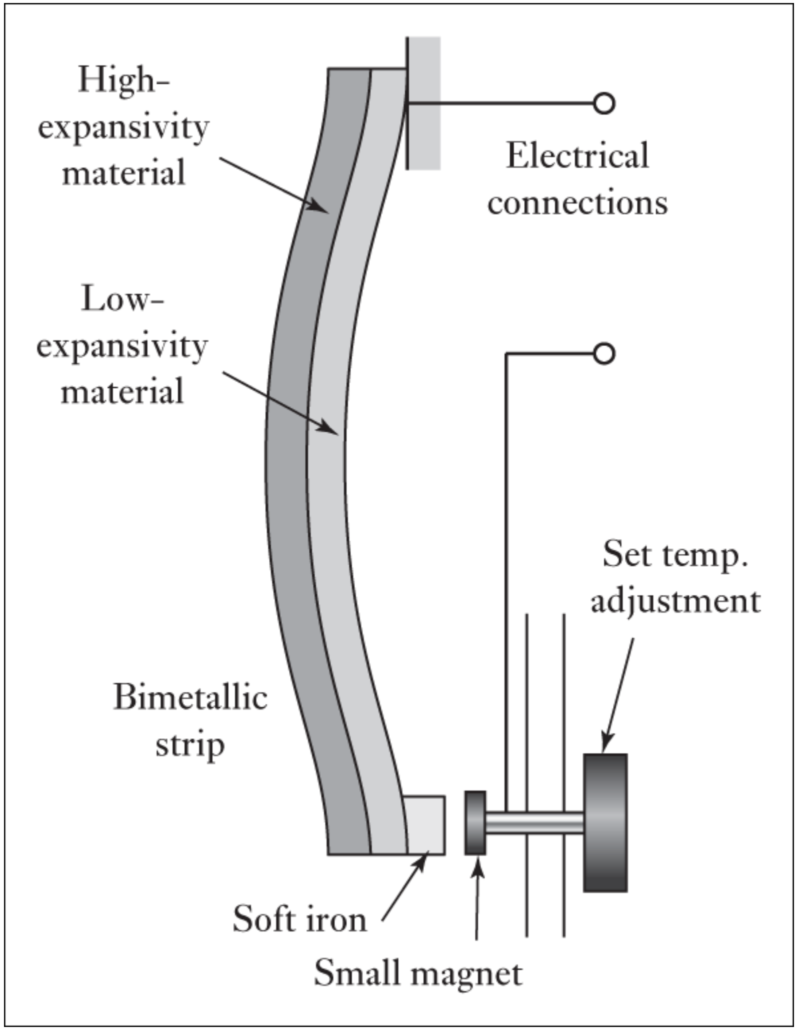
# Bimetallic Strips




- The device consists of two different metal strips bounded together;
- The metals have different coefficients of expansion;
- When the temperature changes, the composite strip bends into a curved strip, with the higher coefficient metal on the outside of the curve;
- This deformation may be used as a temperature-controlled switch.

# Resistance Temperature Detectors (RTDs)

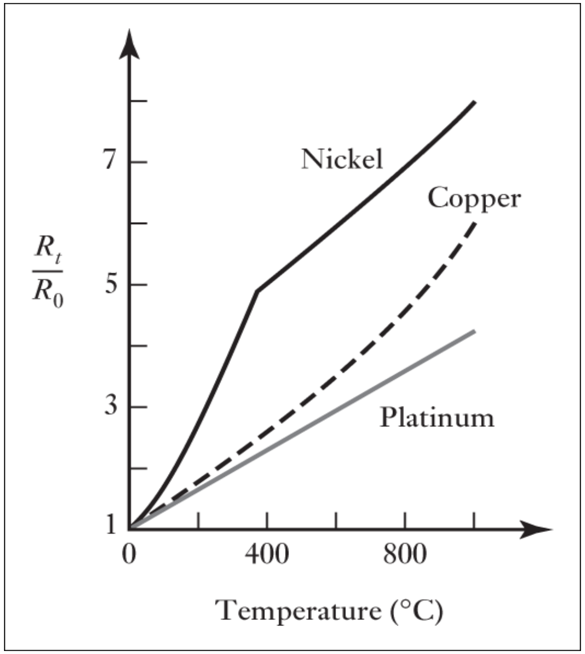



*Fig. Variation of resistance with temperature for metals.*

In practice, the linear equation is only approximately valid over a limited temperature range.

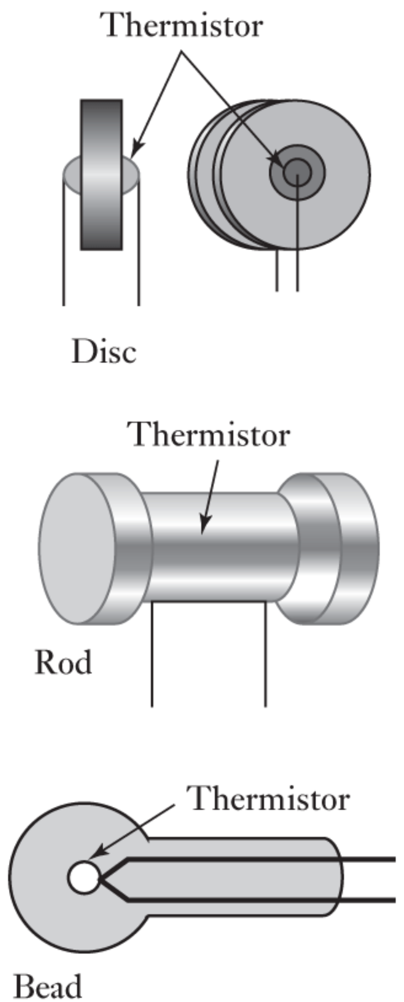
- The resistance of most metals increases, over a limited temperature range, in a reasonably linear way with temperature.
- RTDs are highly stable and give reproducible response over long periods of time; they tend to have response time of the order of 0.5 to 5s or more.
- For such a linear relationship,

$$R_T = R_0(1 + \alpha T)$$

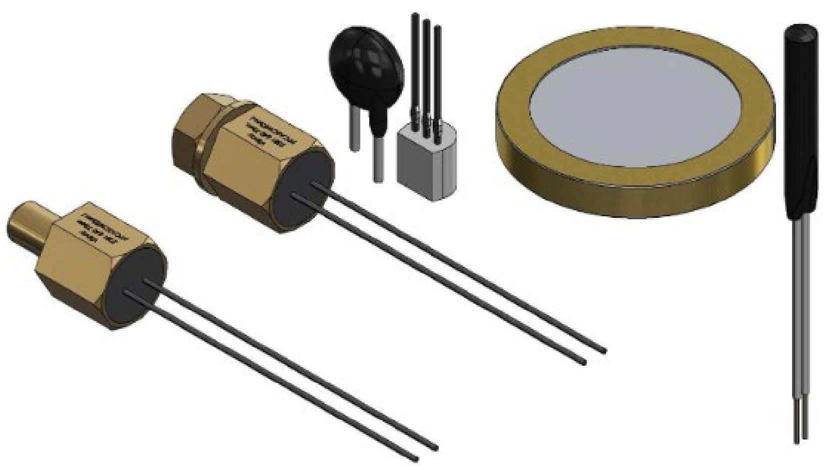
where

- $R_t$ is the resistance at a temperature $t(^oC)$,
- $R_0$ is the resistance at $0^oC$,
- $\alpha$ is a constant for the metal termed the temperature coefficient of resistance.

# Thermistors

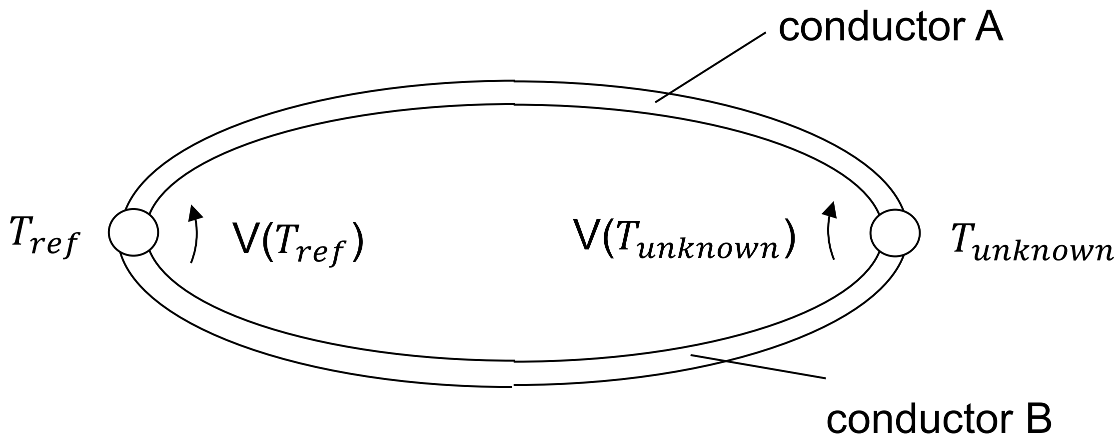



- Thermistors are small pieces of material made from mixtures of metal oxides, such as those of chromium, cobalt, iron, manganese and nickel. These oxides are semiconductors;



# Thermocouples

If both ends of conductors A and B are joined together, two inter-metallic contacts are formed.




No net potential difference (net e.m.f. = 0) will be produced in the circuit provided: both junctions are at the same temperature.

However, if one is at a different temperature than the other ($T_{ref} \neq T_{unknown}$), a potential difference will occur and can be measured.

# Law of Intermediate Temperature

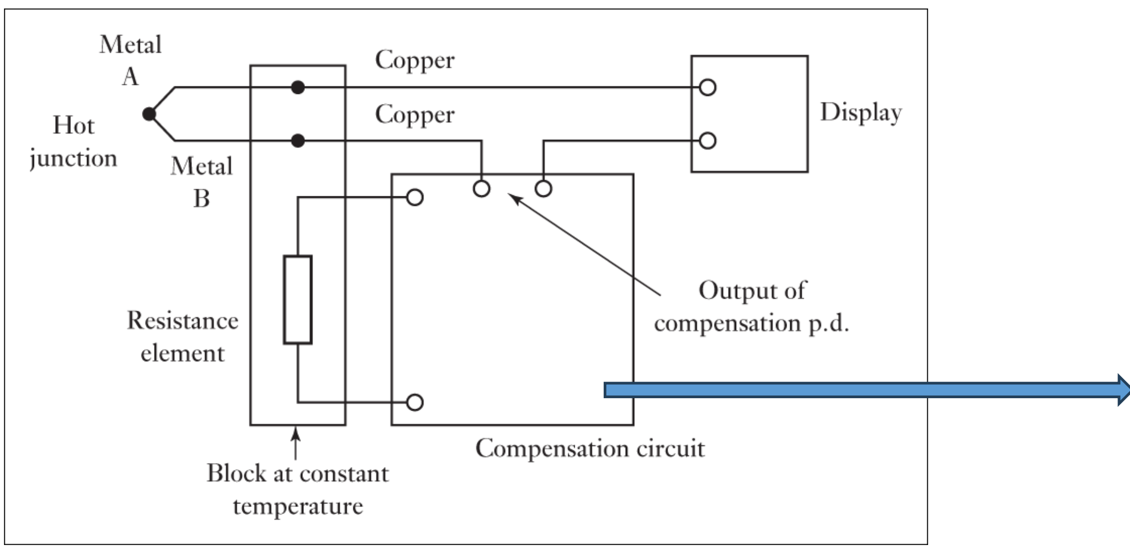
- The standard table assumes that the reference junction is always at 0 °C
  - this is very inconvenient in practice
- When a thermocouple has reference junction not at 0 °C, a correction has to be applied before the tables can be used.
- Law of intermediate temperature:

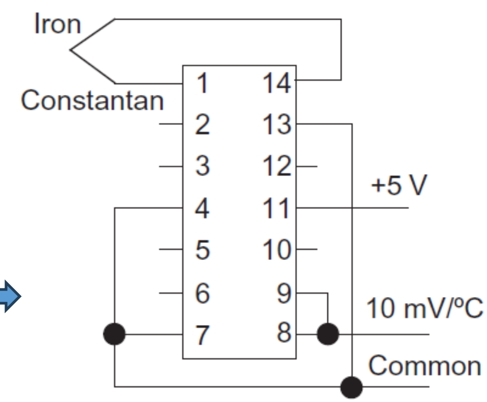
$$E_{T,I} = E_{T,0} - E_{I,0}$$

  - $E_{T,I}$: emf at temperature $T$ when the cold junction is at $I$ °C
  - $E_{T,0}$: emf at temperature $T$ when the cold junction is at 0 °C
  - $E_{I,0}$: emf at temperature $I$ when the cold junction is at 0 °C

# Cold Junction Compensation

- To maintain one junction of a thermocouple at 0ºC, i.e. have it immersed in a mixture of ice and water, is often not convenient.
- A compensation circuit can, however, be used to provide an e.m.f. which varies with the temperature of the cold junction (reference temperature). This is called 'cold-junction compensator‘.
  - In such a way, the thermocouple will always generates a combined e.m.f. which is the same as when the cold junction is held at 0ºC.

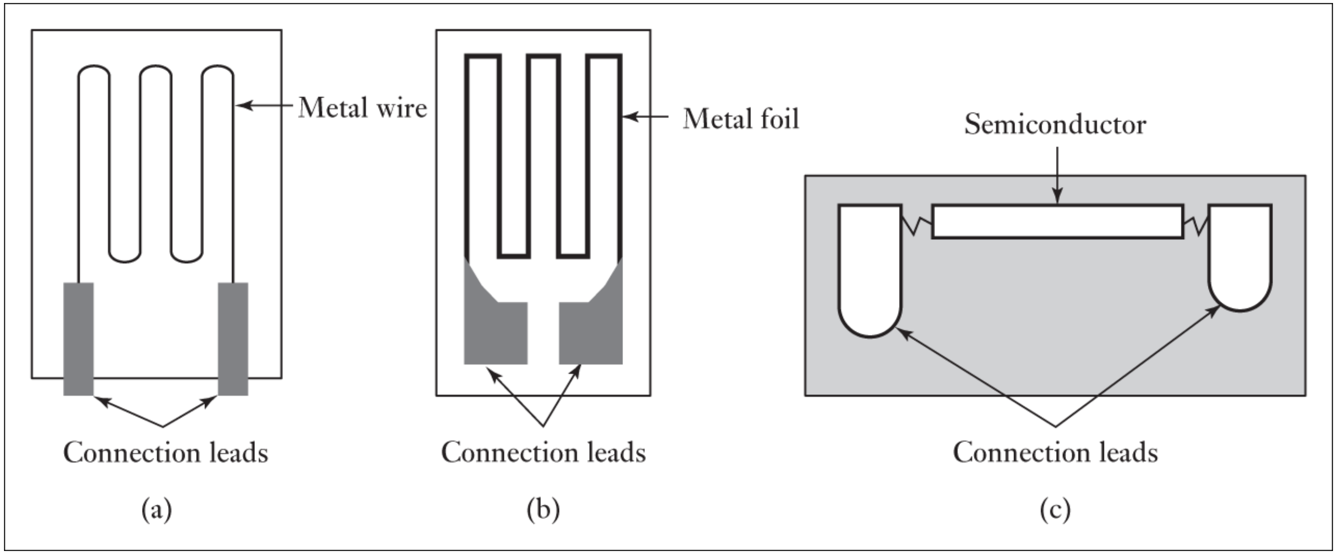



AD594

# Strain Gauge

*Fig. Three types of strain gauges.*




- There are several methods of measuring strain, the most common is with a strain gauge, which is a metal wire, metal foil strip or a strip of semiconductor material which is wafer-like and can be stuck onto surfaces like a postage stamp.

# Strain Gauge (cont'd)

- When subject to strain, resistance of the gauge $R$ changes, the fractional change in resistance $\Delta R/R$ being proportional to the strain $\varepsilon$, i.e.,

$$\frac{\Delta R}{R} = G\varepsilon$$

**Example 3.1**

Consider an electrical resistance strain gauge with a resistance of $100\Omega$ and a gauge factor of 2.0. What is the change in resistance of the gauge when it is subject to a strain of 0.001?

**Solutions:** the fractional change in resistance is

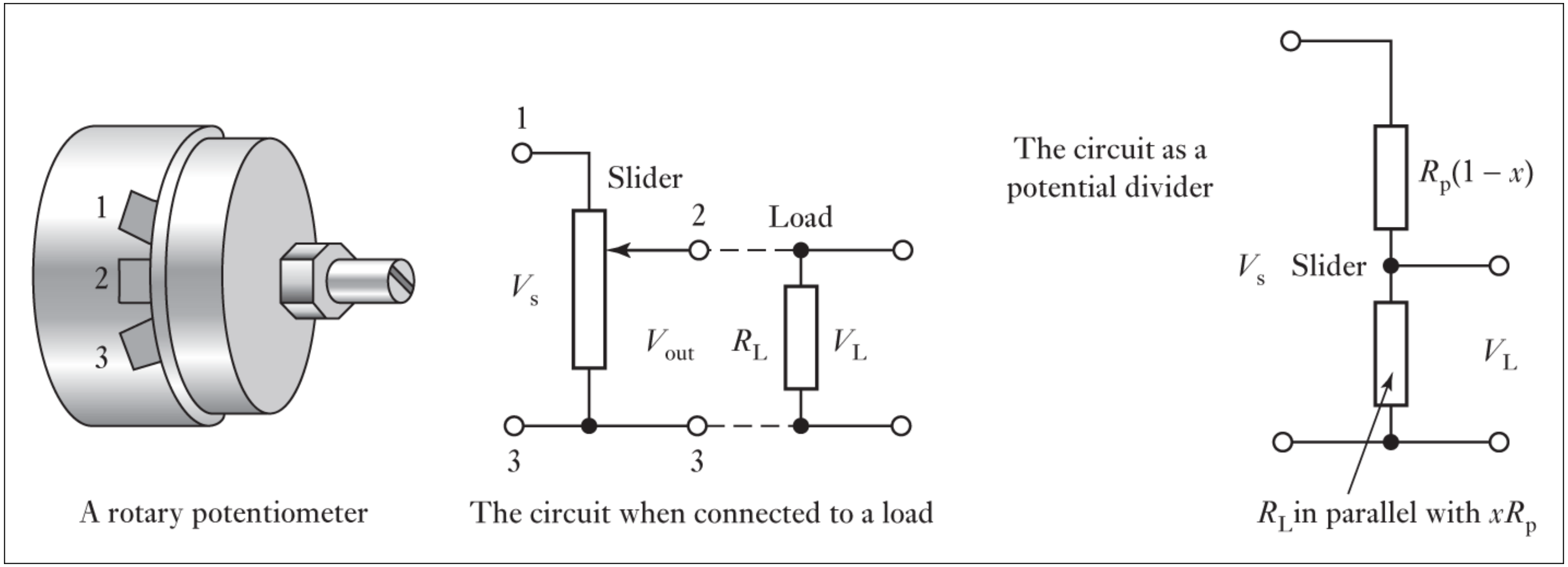
$$\Delta R = RG\varepsilon = 100 \times 2.0 \times 0.001 = 0.2\Omega$$

# Potentiometer Sensor

A **potentiometer** consists of a resistance element with a sliding contact which can be moved over the length of the element.

Such element can be adopted to measure linear, or rotary displacement.

The displacement will be converted into a potential difference.



# Analysis

The resistance $R_L$ is in parallel of the fraction $x$ of the potentiometer.

The total resistance = $R_p(1-x) + xR_pR_L/(xR_p + R_L)$;

Then we have

$$\frac{V_L}{V_s} = \frac{xR_pR_L/(xR_p + R_L)}{R_p(1-x) + xR_pR_L/(xR_p + R_L)} = \frac{x}{(R_p/R_L)\,x(1-x) + 1}$$

Note if $R_L = \infty$, we have

$$V_L = xV_s$$

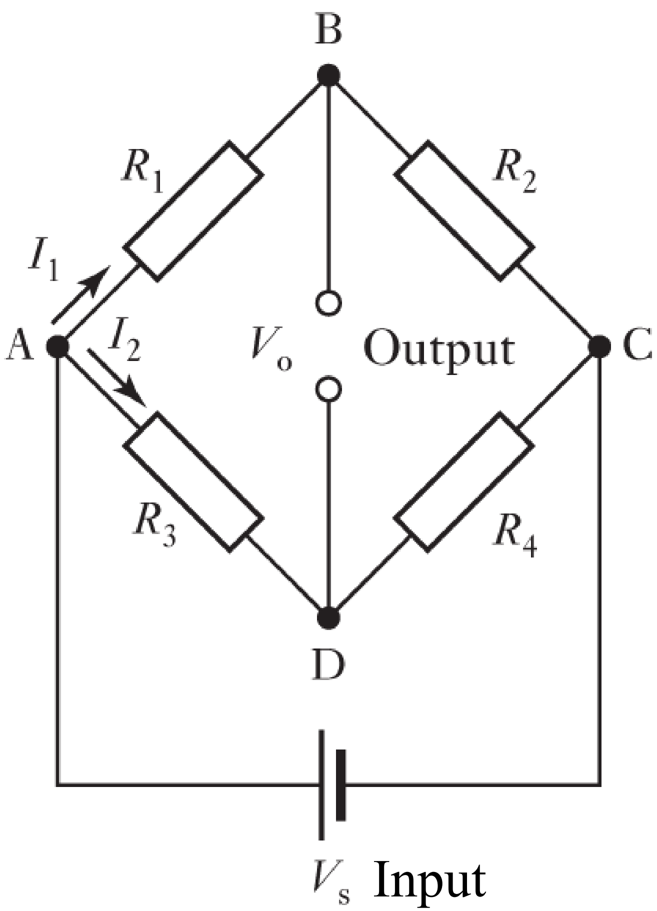
If $R_L \neq \infty$, the error introduced is

$$error = \frac{xV_s}{(R_p/R_L)\,x(1-x) + 1} - xV_S = V_s\frac{R_p(x^3 - x^2)}{R_p(x - x^2) + R_L}$$

Note $0 \le x \le 1$, normally $R_L \gg R_p$, hence the error can be approximated as

$$error \approx V_s\frac{R_p}{R_L}(x^3 - x^2)$$

# Wheatstone Bridge




The **Wheatstone bridge** can be used to convert a resistance change to a voltage change and can detect very small changes in resistance.

The bridge is said to be balanced if:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

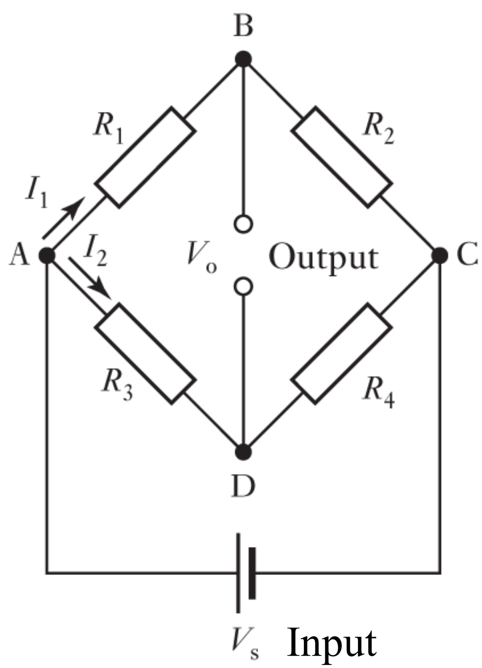
Then the output voltage

$$V_o = V_B - V_D = V_s\left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right)$$

$$= 0.$$

# Wheatstone Bridge Application

Consider $R_2$ to be a sensor which has a resistance change. A change in resistance from $R_2$ to $R_2 + \delta R$ gives a change in output from $V_o$ to $V_o + \delta V_o$:




$$V_o + \delta V_o = V_s\left(\frac{R_2 + \delta R}{R_1 + R_2 + \delta R} - \frac{R_4}{R_3 + R_4}\right)$$

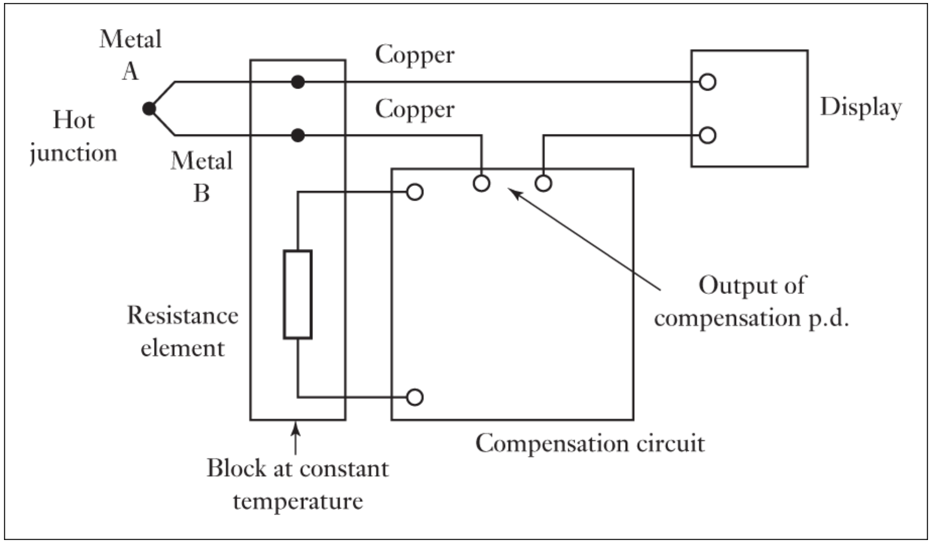
$$\delta V_o = V_s\left(\frac{R_2 + \delta R}{R_1 + R_2 + \delta R} - \frac{R_2}{R_1 + R_2}\right)$$

If $\delta R$ is much smaller than $R_2$, then the above equation approximates to:

$$\delta V_o \approx V_s\left(\frac{\delta R}{R_1 + R_2}\right)$$

With this approximation, the change in output voltage is thus proportional to the changes in the resistance of the sensor (when there is no load resistance across the output. If there is such a resistance then the loading effect has to be considered).

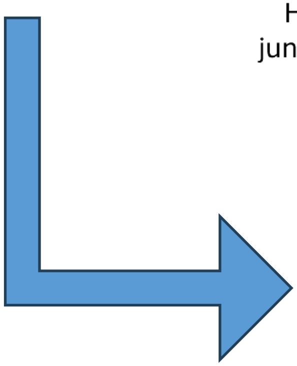
# Cold Junction Compensation

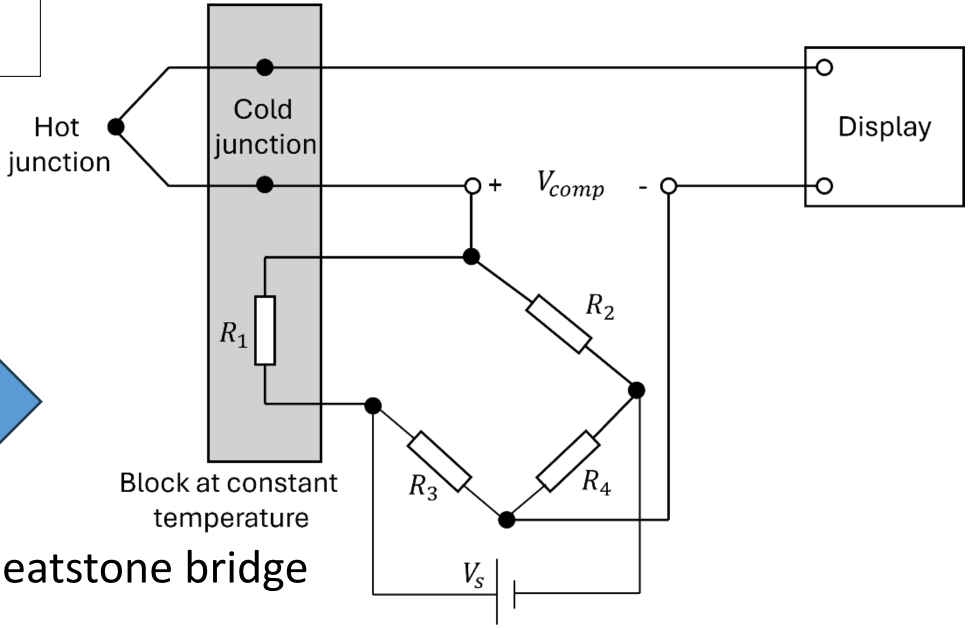



Received by display Compensated by circuit

$$E_{T,0} = E_{T,I} + E_{I,0}$$

Measured from thermocouple





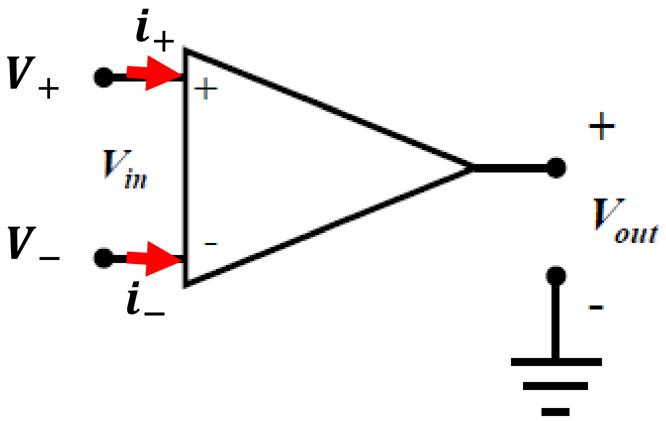

The output of Wheatstone bridge should be $E_{I,0}$

# Ideal Operational Amplifier

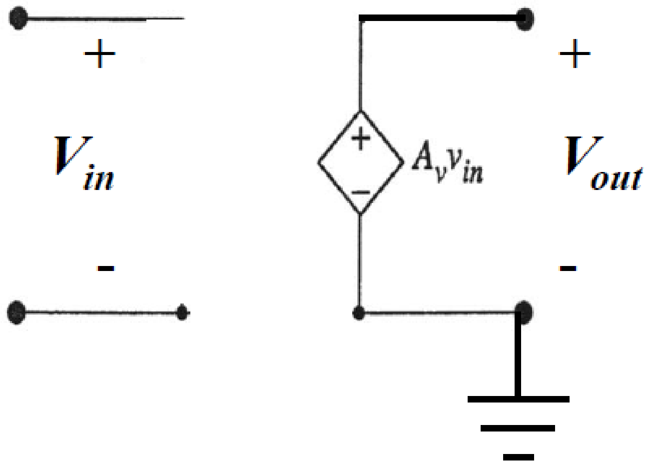
- $R_{in}$ = Infinity;
- Voltage Gain: $A_v$=Infinity at all frequencies;
- $R_{out} = 0$;
- $\boxed{i_+ = i_- = 0;}$
- $\boxed{V_+ = V_-.}$

Main tools to analyze amplifier

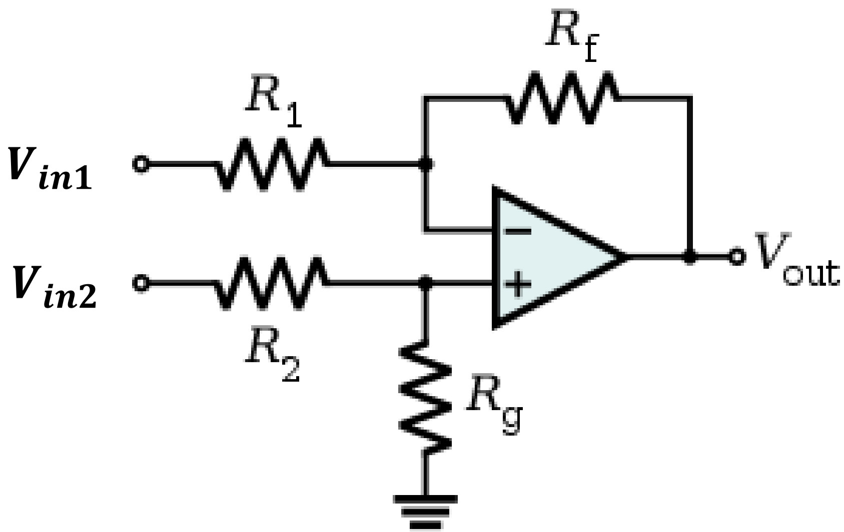
Circuit Symbol




Model



# Difference Amplifier




$$\frac{V_{out} - V_-}{R_f} = \frac{V_- - V_{in1}}{R_1}$$

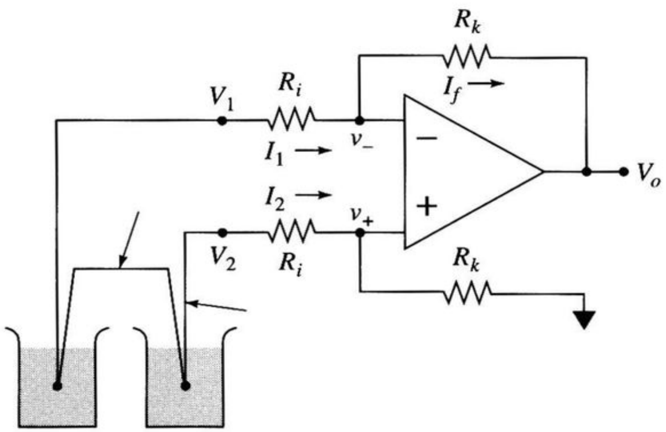
$$V_- = V_+ = V_{in2} \frac{R_g}{R_2 + R_g}$$

assume: $$\frac{R_1}{R_f} = \frac{R_2}{R_g}$$

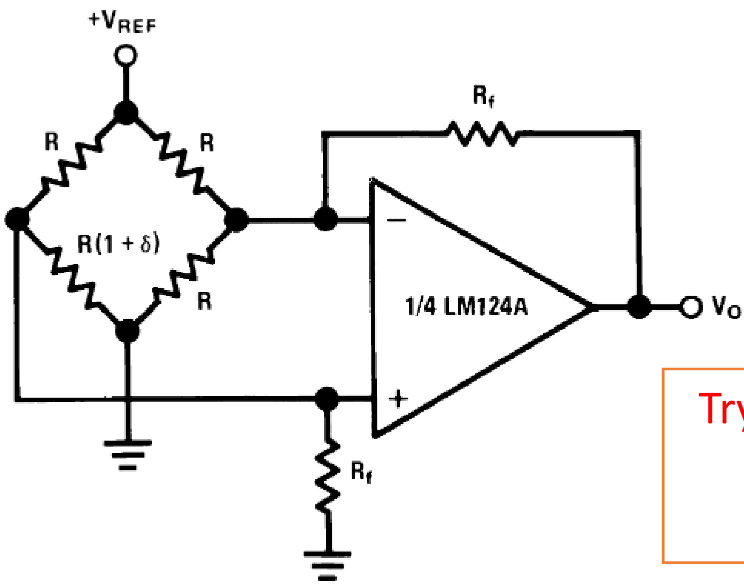
$$V_{out} = \frac{R_f}{R_1}(V_{in2} - V_{in1})$$

# Typical Applications of Difference Amplifiers

- Thermocouple



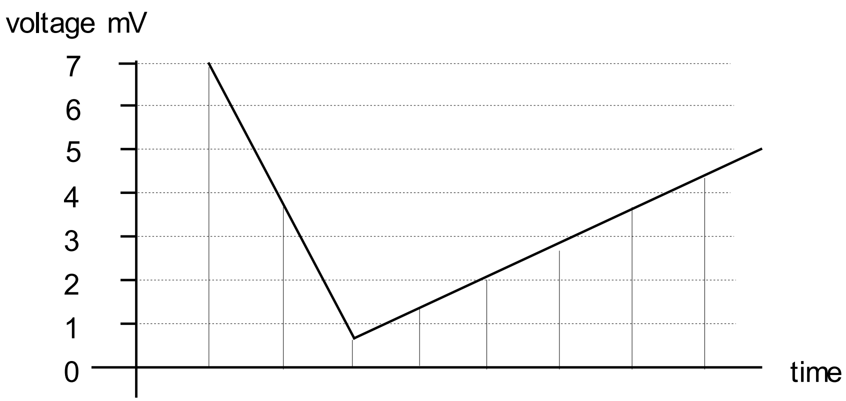

- Wheatstone Bridge




Try to derive the relations by yourself

# Covert to Binary Code

The sampled values are then converted into a binary number using an ADC (analogue to digital converter) so the samples are represented by groups of binary pulses, corresponding to the 0s and 1s.




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sampled voltages | 7 | 3 | 0 | 1 | 2 | 2 | 3 | 4 | (mV) |
| Binary equivalent | 111 | 011 | 000 | 001 | 010 | 010 | 011 | 100 | (3-bit code) |

In practice, a code with more bits would be used. The number of bits (N) used to code the signal determines the **resolution** of the system.

$$Resolution = \frac{Signal\ Range}{2^N - 1}$$



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# Contents Covered in Exam

- Error analysis
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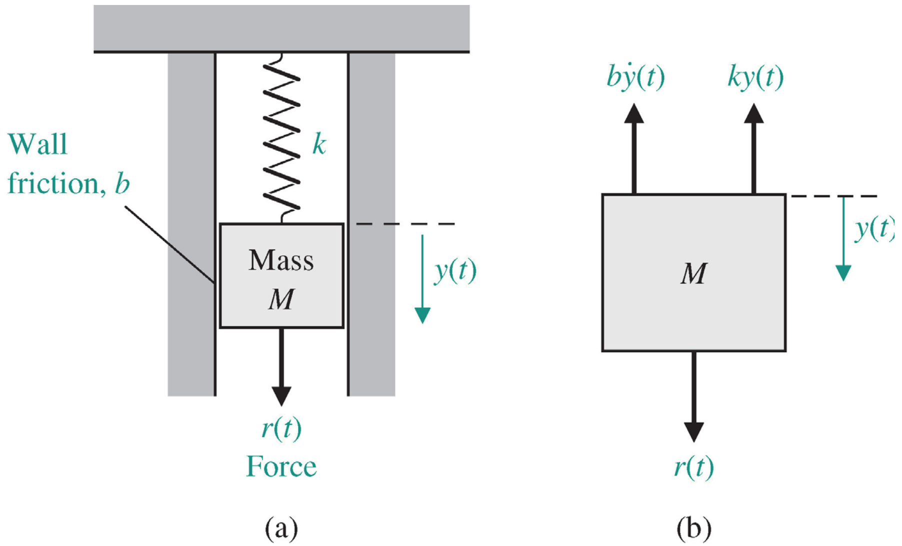
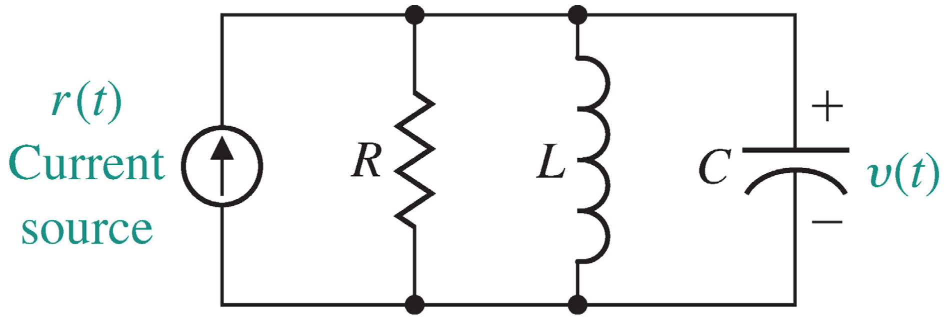
- **Mathematical modelling of the system**
- Transfer function

# Differential Equation of Physical Systems




**- Mechanical System**

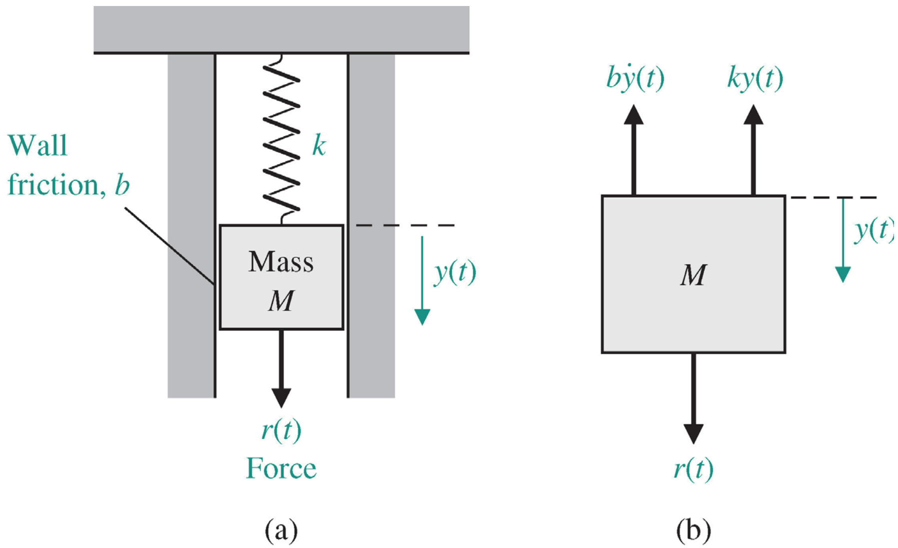
(Spring-mass-damper system)




**- Electrical System**

(RLC circuit)

# ODE for Mechanical System




*(a) Spring-mass-damper system.*
*(b) Free-body diagram.*

$$F = ma$$

$$m = M, a = \frac{d^2y}{dt^2}$$

$$F = r - b\frac{dy}{dt} - ky$$

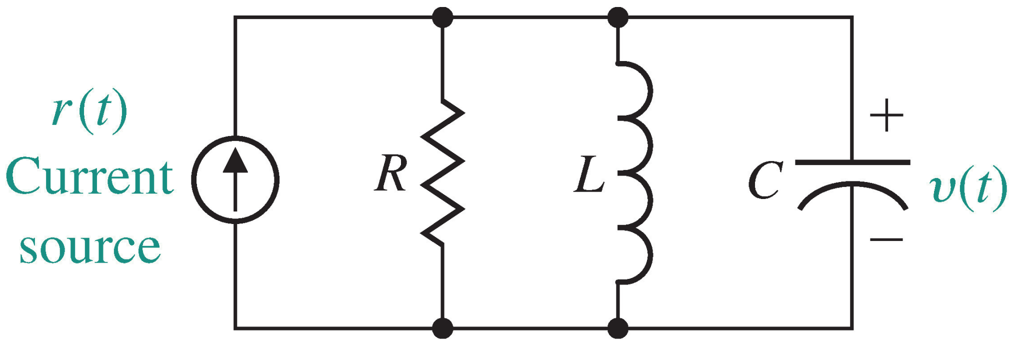
$$M\frac{d^2y}{dt^2} = r - b\frac{dy}{dt} - ky$$

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = r$$

**Using Newton's laws:**

- Model wall friction as a viscous damper, that is, the friction force is linearly proportional to the velocity of the mass;
- M is the mass; b is the friction constant; k is the spring constant of ideal spring;

# ODE for Electrical System




*RLC circuit.*

**Second-order linear constant-coefficient (time-invariant) system**

**Using Kirchhoff's laws.**

$$\frac{v(t)}{R} + C \cdot \frac{d}{dt} v(t) + \frac{1}{L} \cdot \int_0^t v(t)\, dt = r(t)$$

$$r(0) = r_0$$

$$v(t) = K_2 e^{-\alpha_2 t} \cos(\beta_2 t + \theta_2)$$

If assume:

$$\int_0^t v(t) dt = y(t)$$

Then we have:

$$\boxed{C \cdot \frac{d^2}{dt^2} y(t) + \frac{1}{R} \frac{d}{dt} y(t) + \frac{1}{L} y(t) = r(t)}$$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# The Laplace Transform – Differential Operator

- The Laplace variable $s$ can be considered to be the differential operator

$$s \equiv \frac{d}{dt}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \mathcal{L}\{f'(t)\} = sF(s)$$

$$\mathcal{L}\left\{\frac{d^2 f(t)}{dt^2}\right\} = \mathcal{L}\{f''(t)\} = s^2 F(s)$$

And also the integral operator

$$\frac{1}{s} \equiv \int_{0^-}^{t} dt$$

- In control theory, Laplace transform $s$ is used to simplify the computation of ODE model.

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- Error analysis
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# Transfer Function

The **Transfer Function (TF)** of a linear system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input variable, with all initial conditions assumed to be zero.

- TF represents the relationship describing the dynamics of the system under consideration;
- TF may be defined only for linear time-invariant **(LTI)** systems.
  - A time-varying system, whose parameters are time-dependent, and the Laplace transform may not be utilized.
- A transfer function does not describe any information concerning the internal structure of the system and its behavior.
  - A state-space model provides more information about the internal system, which will be introduced later.

# Compute Output in Time Domain

Once we have obtained the transfer function $G(s)$, given the input signal $r(t)$, how to compute the output $y(t)$?

1. Transfer the input signal to $s$ domain $r(t) \rightarrow R(s)$
2. Compute output in $s$ domain
$$Y(s) = G(s)R(s)$$
3. Perform inverse Laplace transform to $Y(s) \rightarrow y(t)$

# Partial Fraction Expansion

- Input as step function, $R(s) = \frac{1}{s}$
- Output in $s$-domain is

$$Y(s) = \frac{1}{Ms^2 + bs + k} R(s) = \frac{1}{s(s^2 + 3s + 2)}$$

- Next step is to use **partial fraction expansion**

$$Y(s) = \frac{1}{s(s^2 + 3s + 2)} = \frac{1}{s(s+1)(s+2)} = \frac{a}{s} + \frac{b}{s+1} + \frac{c}{s+2}$$

- Here the coefficients $a$, $b$, $c$ are called **residues**.
    - ➢How to compute residues?

# Finite Value Theorem

**Final Value Theorem:**

$$\boxed{\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s),}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{1}{(s+1)(s+2)} = 0.5$$

- The same as our previous analysis
- You can try to compute the steady state of the impulse input

# Characteristic Equation

**Characteristic equation & Poles**

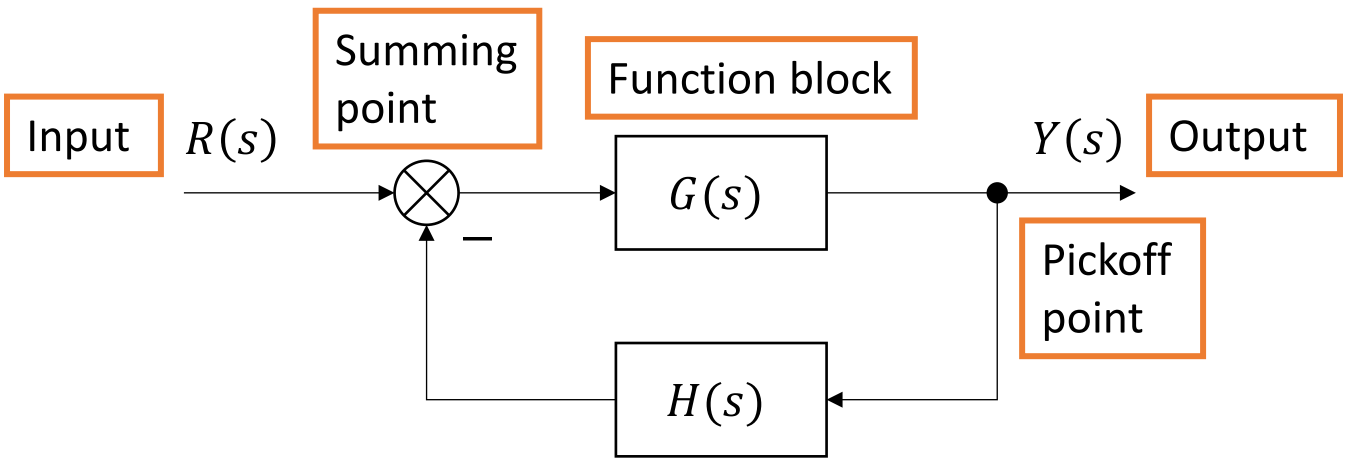
- For systems with transfer function as

$$G(s) = \frac{P(s)}{Q(s)}$$

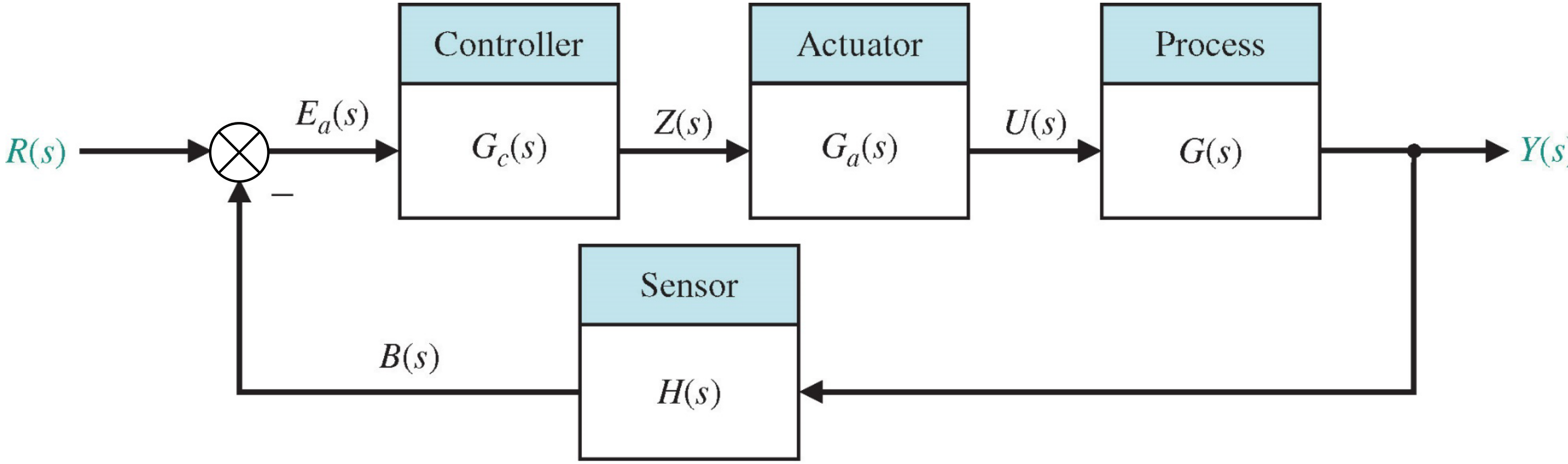
- The denominator $Q(s)$, when set equal to zero, is called the **characteristic equation**.
  - The roots of this equation determine the character of the time response.
  - Order of characteristic equation is the **order of the system**.
- Roots of $Q(s) = 0$ are called **poles**, roots of $P(s) = 0$ are called **zeros**.

# Block Diagram

- A simple negative feedback closed-loop system:



# Transfer Function of A Negative Feedback Control System




The error signal or actuating signal $E_a(s)$ is

$$E_a(s) = R(s) - H(s)Y(s)$$

The output signal $Y(s)$ can be represented as

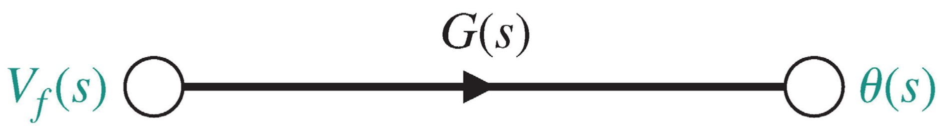
$$Y(s) = E_a(s)G_c(s)G_a(s)G(s)$$

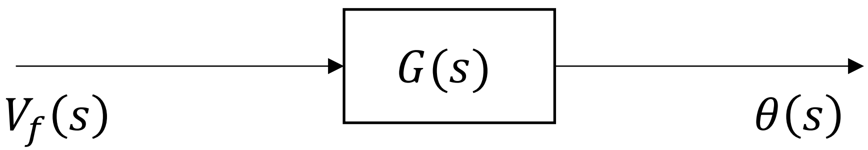
Combining the above two equations, the transfer function of the system is

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_a(s)G(s)}{1 + G_c(s)G_a(s)G(s)H(s)}$$

# Signal-Flow Graph Models

- A **signal-flow graph** is a diagram consisting of **nodes** that are connected by several **directed branches** and is a graphic representation of a set of linear relations;
- Signal-flow graph is particularly useful for feedback control systems because feedback theory is primarily concerned with the flow and processing of signals in the system;



# Thank You !