CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 5-2 Electric Currents

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OUTLINE

- ➢ Types of Current
- ➢ Conduction current & Current Density
  - ✓ Drift Velocity & Mobility
  - ✓ Current Density
  - ✓ Conductivity & Resistivity
- ➢ From Electromagnetics (EM) to Electric Circuits (EC)
  - ✓ Ohm’s law in microscopic and macroscopic views
  - ✓ Continuity and KCL

1.1 TYPE OF CURRENT

Electrostatics – generated by electric **charges at rest**.

Magnetostatics – generated by electric **charges in motion**, which constitute the **currents**.

There are several types of electric currents caused by the *motion of free charges*:

- ***Conduction currents***: **Governed by Ohm's law**

  in conductors are caused by drift motion of conduction electrons;

- ***Convection currents***:

  result from motion of electrons and/or ions in a vacuum; ✗

- ***Electrolytic currents***:

  are the result of migration of positive and negative ions. ✗

1.2 CONDUCTION CURRENT: DEFINITION

An electron which may be considered as not being attached to any particular atom is called a ***free electron***.

- A free electron has the capability of moving through a whole crystal lattice. However, the heavy, positively charged ions are relatively fixed at their regular positions in the crystal lattice and do not contribute to the current in the metal.

Thus, the current in a metal conductor, called ***conduction current***, is simply a flow of electrons.

- The transitory flow of charges comes to a halt in a very short time in an isolated conductor placed in an electric field.
- To maintain a ***steady current*** within a conductor, a continuous supply of electrons at one end and removal at the other is necessary.

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2.1 DRUDE MODEL

In 1897, J. J. Thomson discovered electrons.

In 1905, Einstein interpreted the photoelectric effect

In 1911:

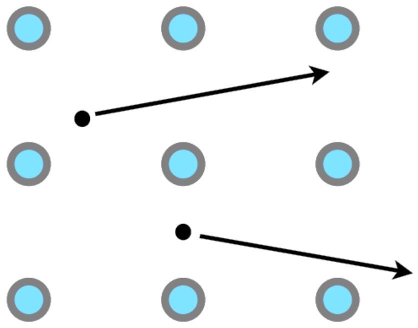
- Rutherford proved that atoms are composed of a point-like positively charged, massive nucleus surrounded by a sea of electrons.

- Physicist Paul Drude constructed the theory of electrical and thermal conduction in metals

Drude Model---- Simple and practical model that has been used even today for rough estimation (*e.g.*, electrical conductivity)

2.1 DRUDE MODEL

Key assumptions



- ✓ Electrons move in straight lines until they collide with immobile ions

  (*i.e.*, no other forces during their travel)

  - No electron-electron interaction (Independent electron approx.)
  - No electron-ion interaction (Free-electron approx.) (* except collisions)
  - Under external fields, electrons move according to Newton’s law of motion

- ✓ Electrons bounce off ion cores instantaneously so that their velocities are abruptly changed

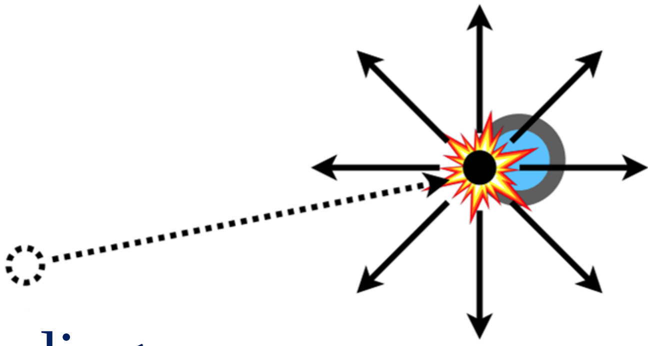
  (*i.e.*, no time delay due to the collision)

- ✓ Mean free time between successive collisions is the relaxation time τ

  - Independent of electron’s position or velocity

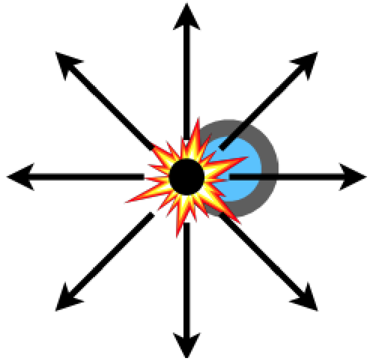
2.1 DRUDE MODEL

Key assumptions



✓ Electrons achieve thermal equilibrium with their surroundings only through collision

- They emerge after collision at a random direction with speed appropriate to the temperature of the region where collision happened
- The hotter the region; the higher the speed of the emerging electrons



$\boldsymbol{v}$: velocity of the electron right after a collision

It is a pre-quantum mechanical semi-classical model – still roughly applicable for simple alkaline metals

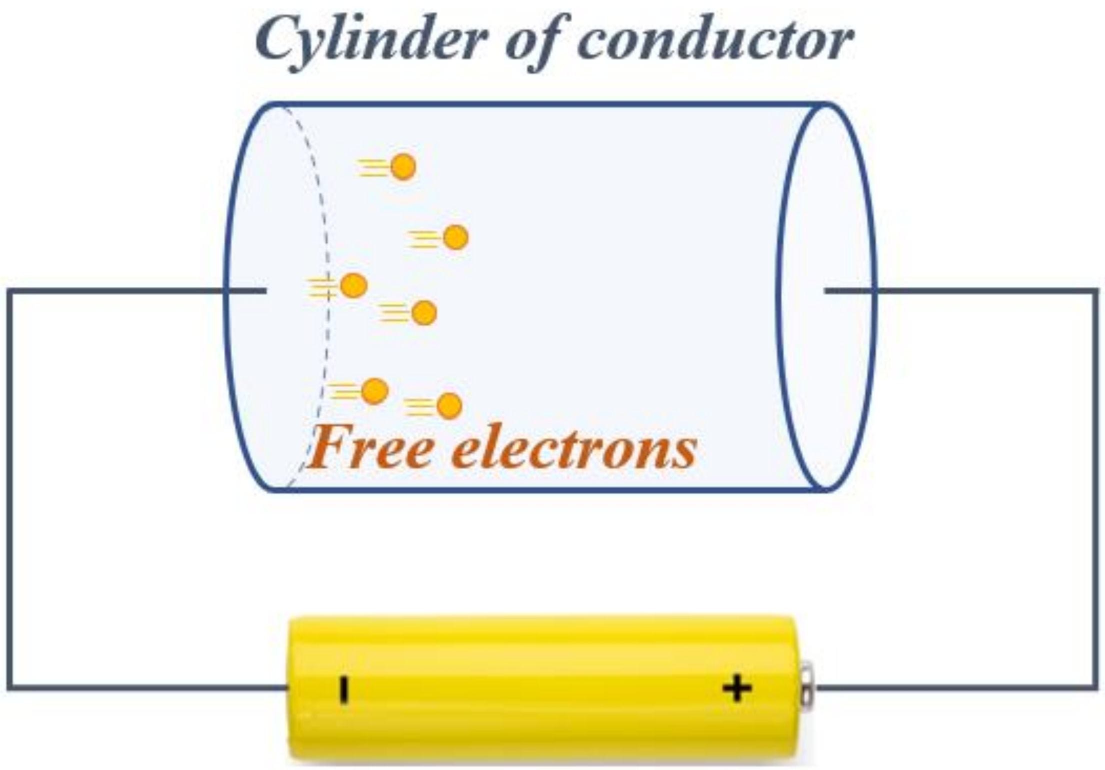
➢ Used to explain electrical and thermal conduction of a metal

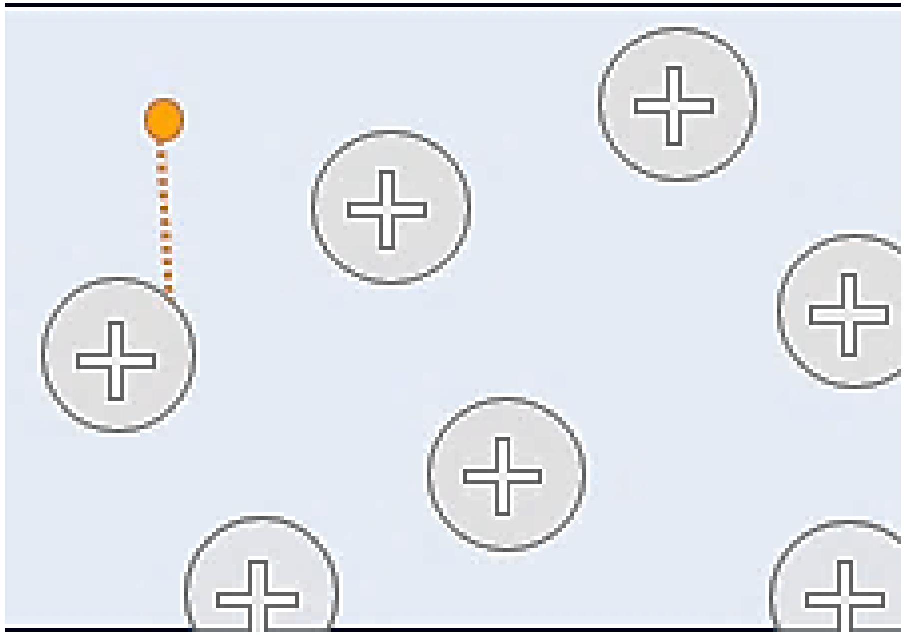
➢ Applied a kinetic theory of gases to electrons in a metal

➢ Conduction electrons move far away from their parent atoms

➢ A gas of conduction electrons move against a background of heavy immobile ions

2.1 DRUDE MODEL





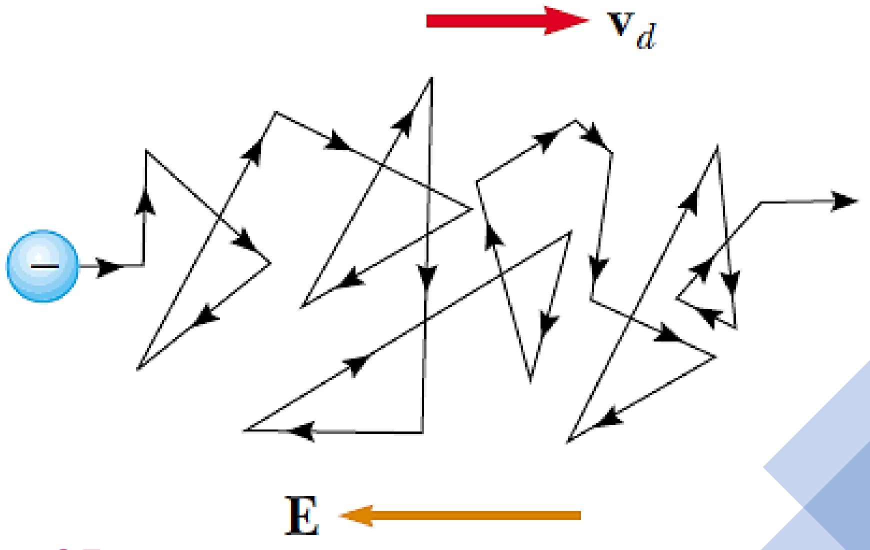
2.2 DRIFT VELOCITY

The speed $\vec{v}_d$ at which the charge carriers are moving is known as the *drift velocity*.

Physically, $\vec{v}_d$ is the **average** speed of the charge carriers inside a conductor when an external electric field is applied.

Imagine: apply a constant electric field $\vec{E}$ to a conductor, for an electron:

- Force applied on an electron: $\vec{F} = -e\vec{E}$
- Acceleration: $\vec{a} = \vec{F}/m_e$
- Drift velocity: $\boxed{\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m_e}\vec{E}}$



$\mu_e = \frac{e\tau}{m_e}$ is the *electron **mobility***.

2.3 MOBILITY

For most conducting materials, the drift velocity $\vec{v}_d$ is directly proportional to the electric field intensity $\vec{E}$.

$$\vec{v}_d = \mu_m \vec{E} = -\mu_e \vec{E} (m/s)$$

where $\mu_m = \frac{q\tau}{m}$ is the ***mobility*** measured in $m^2 \cdot V^{-1} \cdot s^{-1}$.

In a conductor, the free charges are electrons: $\mu_m \to \mu_e$

μ_m and μ_e both are positive.

Materials	μ_e ($cm^2/(V \cdot s)$) at room Temperature (300 K)
Crystalline Silicon	1400
Amorphous Silicon	~1
Metal (Gold, Copper, Silver,..)	30-50

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2.4 CURRENT DENSITY

Consider the steady motion of electrons (each of charge q, negative for electrons)

- across an element of surface $\vec{A} = A\hat{n}$;
- with a velocity $\vec{v}_d$;
- *N (concentration)* is the number of charge carriers per unit volume;
- In time dt, the amount of charge passing through the elemental surface is:

$$dQ = Nqv_d A\, dt$$

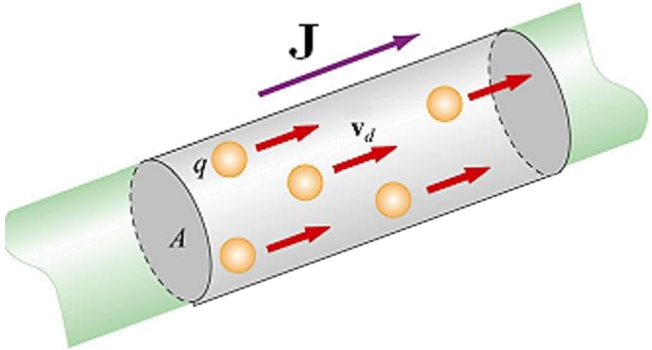
Current is the time rate of change of charges:

$$I = \frac{dQ}{dt} = Nqv_d A = Nq\vec{v}_d \cdot \vec{A}$$

Define the volume current density $\vec{J} = Nq\vec{v}_d$ then

$$I = \vec{J} \cdot \vec{A}$$

The total current I flowing through an arbitrary surface S is then the flux of the $\boldsymbol{J}$ through S:



$$I = \int_S \vec{J} \cdot d\vec{s}$$

The product Nq is the free charge per unit volume, so

$$\vec{J} = Nq\vec{v}_d \ (A/m^2)$$

2.5 CONDUCTIVITY& RESISTIVITY

Conductivity

$$\vec{v}_d = \frac{q\tau}{m_e}\vec{E}$$

$$\vec{J} = Nq\vec{v}_d \ (A/m^2)$$

Substitute v_d in the equation: $I = \boxed{\frac{q^2N\tau}{m_e}}\vec{A}\cdot\vec{E}$

Define

$$\sigma = \frac{q^2N\tau}{m_e}$$

only related to the substance's properties

$$\therefore I = \sigma AE \text{ or } \vec{J} = \sigma\vec{E}$$

where σ is ***conductivity*** (a macroscopic constitutive parameter of the medium).

The conductivity of a material is a measure of **how easily** free charges can travel through the material under the influence of an externally applied E-field.

A *perfect dielectric* is a material with σ = 0;

A *perfect conductor* is a material with σ = ∞.

Conductivity depends **only** on the microscopic properties of the material, not on its shape.

Resistivity

Resistivity is property of a substance. Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature, not on its shape or size.

$$\rho = \frac{1}{\sigma} \quad \Omega\cdot\text{m}$$

The greater the resistivity, the greater the field needed to cause a given current density.

ρ varies with temperature.

QUIZ 2.1

A 10 V voltage source is applied between the ends of one 100 m long copper wire with a diameter of $a = 1$ mm. Given that the electron mobility $\mu_e = 0.004\ m^2V^{-1}s^{-1}$, conductivity of copper is 57 MS/m at T = 290 K and charge density $\rho = 1.4 \times 10^{10}\ Cm^{-3}$, find:

a) the current flowing in the wire.

b) the drift speed of the electrons.

QUIZ 2.2

A silver wire with a radius of $a = 1$ mm carries a steady current of intensity $I = 1$ A. The current is uniformly distributed across the wire cross section. The time in which the electrons drift 1 km along the wire is 2.4×10^{6} s. Find:

a) the **concentration** of conduction electrons (N) in the silver wire.

b) the conductivity of the given silver material if the average collision time of electrons is 2.4×10^{-14}s.

electron mass: $m_e = 1 \times 10^{-30}\ kg$

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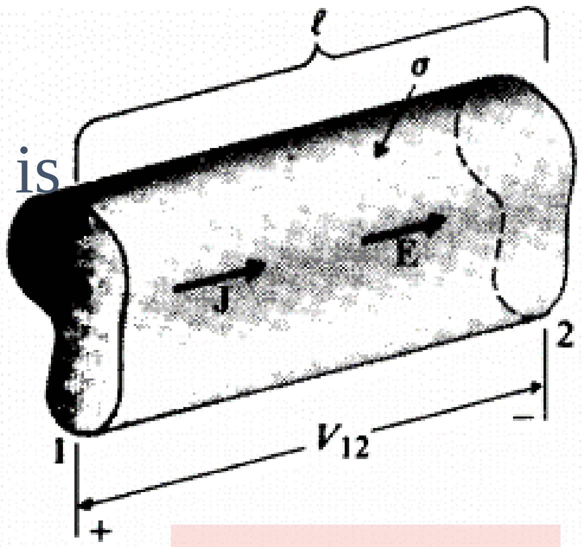
3.1 OHM'S LAW



Within the conducting material, the current through the conductor is given by (Ohm's Law):

$$I = V_{12}/R \qquad V_{12} = El$$

where V_{12} is the potential across ends of block;
R is the resistance of block.

The current can also be represented by:

$$I = \int_S \vec{J} \cdot d\vec{s} = JA$$

Since the resistance of the conducting material is: $R = \dfrac{l}{\sigma A}$

Ohm's law at a point

$$J = \frac{I}{A} = \frac{V_{12}}{RA} = \frac{El\sigma A}{lA} = \sigma E$$

Microscopic Ohm's law

$$\vec{J} = \sigma \vec{E}$$

⇩

Macroscopic Ohm's law

$$V = RI$$

3.2 CONTINUITY

***Principle of conservation of charge*:**

In an arbitrary volume *V* bounded by surface *S*, a net charge +*Q* exists within this region. If a net current *I* flows across the surface ***out*** of this region, the charge in the volume must ***decrease*** at a rate that equals the current.

$$I = \oiint_S \vec{J} \cdot d\vec{s} \quad = -\frac{dQ}{dt} = -\frac{d}{dt}\iiint_V \rho\, dv$$

Apply the Gauss's theorem, we have

$$\iiint_V \nabla \cdot \vec{J} dv \quad = -\frac{d}{dt}\iiint_V \rho\, dv \quad = -\iiint_V \frac{\partial \rho}{\partial t} dv$$

The equation must hold regardless of the choice of V, so

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \quad (A/m^3)}$$

Equation of continuity

3.3 KIRCHHOFF'S CURRENT LAW (KCL)

For steady current, charge density does not vary with time, so $\partial\rho/\partial t = 0$, therefore

$$\nabla \cdot \vec{J} = 0$$

Thus, steady electric currents are **divergenceless**.

The integral form:

$$\nabla \cdot \vec{J} = 0 \Rightarrow \oint\!\!\!\oint_S \vec{J} \cdot d\vec{s} = 0 \Rightarrow \sum_j I_j = 0 \longrightarrow \textbf{KCL}$$

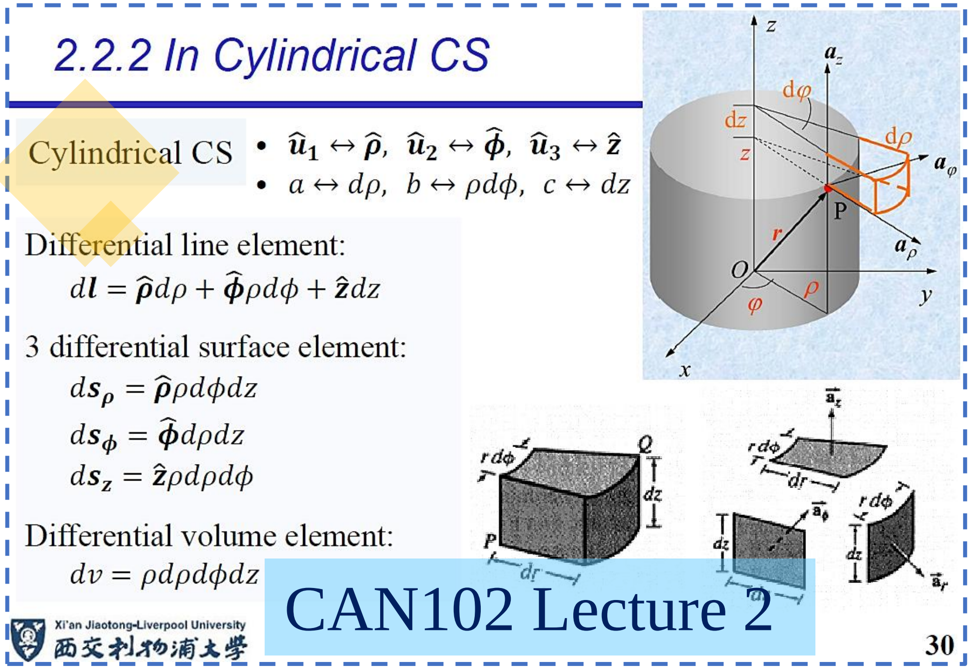
This is the Kirchhoff's current law: the sum of all the currents flowing out of a junction in an electric circuit is zero.

QUIZ 3.1

In a cylindrical coordinate system, the current density is

$$\vec{J} = \begin{cases} -10^6 z^{1.5} \hat{z}\ A/m & 0 \le r \le 20\mu m \\ 0 & r > 20\mu m \end{cases}$$

Find the total current passing through the cross-section area at z = 0.1 m along the +z direction.



Ans: -39.7μA

NEXT…

Passive components:

Resistors, Capacitors