Xi'an Jiaotong-Liverpool University
西交利物浦大学

MEC208 Instrumentation and Control System

2024-25 Semester 2

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School of Advanced Technology

Lab 1 Report (15%)

- Deadline: End of **Week 5**.
  - Submission link is available on LMO.
  - Late submission penalty would be imposed.
- The report is **individual** report, everyone should submit your own report.
  - Data/capture could be the same within one group.
- The report should be around 15~20 pages, no more than 25 pages
  - It is important to maintain your report in proper length
  - Reports more than 25 pages would have **penalty in marks**

Lab 1 Report (15%)

- Submit your report in a single PDF file.
- Turnitin would be used to check your report
  - Do not worry about high repeatability: **70~80% is normal**
  - But if your report is **95~99%** overlap with other reports stored, we will check your report carefully
    - Plagiarism check may be imposed on your submission.

Lecture 6

Outline

Control Systems:

Mathematical Models of Systems

- ❑ Differential Equations of Physical Systems
- ❑ Linear Approximation of Physical Systems
- ❑ The Laplace Transform
- ❑ The Transfer Function of Linear Systems
- ❑ Block Diagram Models
- ❑ Signal-Flow Graph Models
- ❑ Simulation Tool

Overview

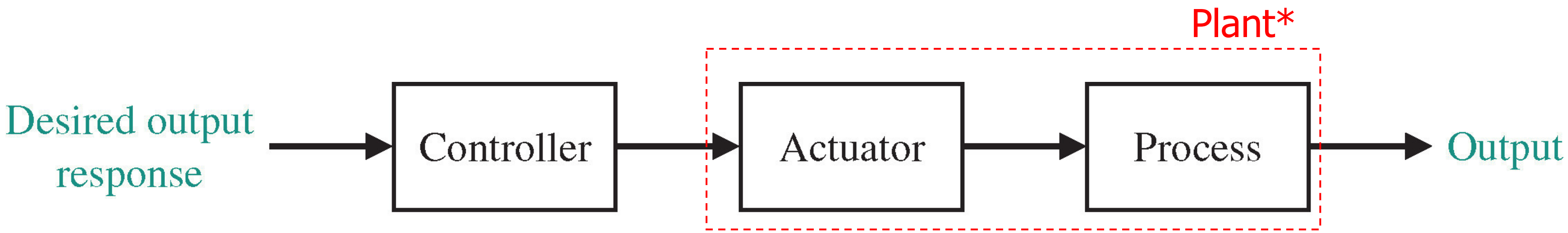
To understand and control complex systems, one must obtain quantitative **mathematical models** of these systems.

Mathematical models of physical systems are key elements in the design and analysis of control systems.

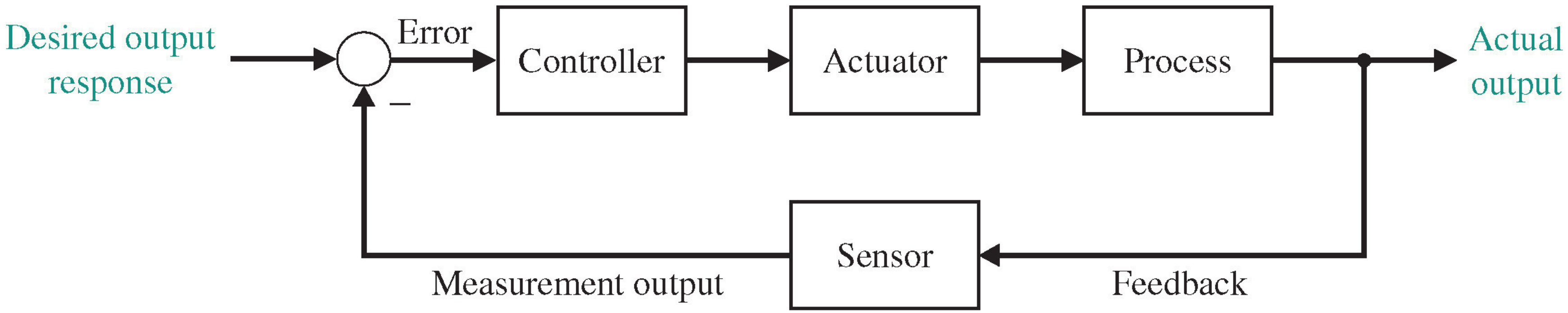
- The dynamic behavior is generally described by **ordinary differential equations (ODEs)**;
- Linearization approximations allow the use of **Laplace transform**;
- **Transfer functions**, which can be transformed into **block diagrams** and **signal-flow graphs**, are very convenient and natural tools for designing and analyzing complicated control systems.

Control Systems

Open-loop Control System



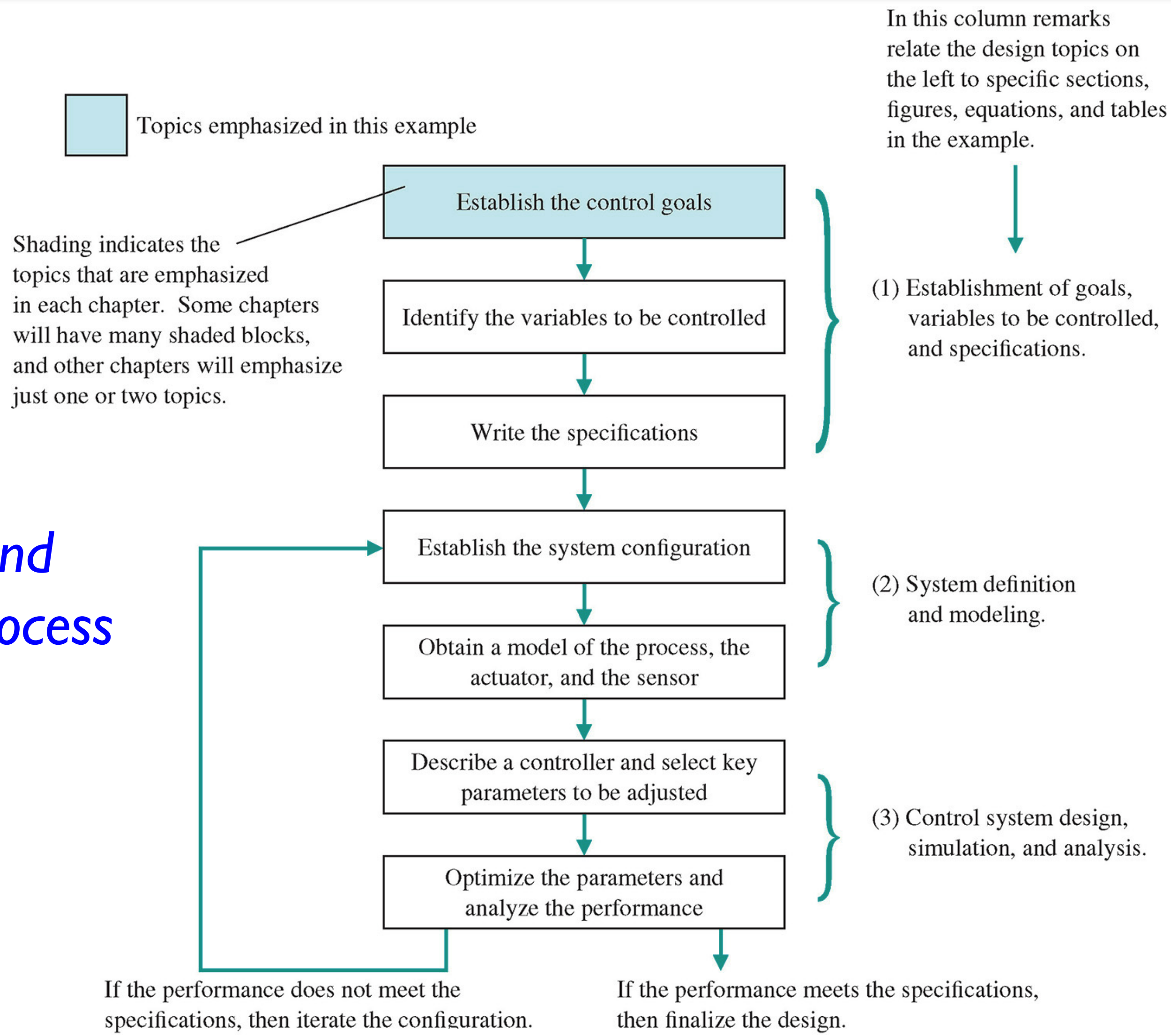
Closed-loop Control System



Six Step Approach to Dynamic System Modeling

1. Define the system and its components
2. Formulate the mathematical model and list the necessary assumptions
3. Write the differential equations describing the model
4. Solve the equations for the desired output variables
5. Examine the solutions and the assumptions
6. If necessary, reanalyze or redesign the system

Control System Design Process



Iterative and Nonlinear Process

Outline

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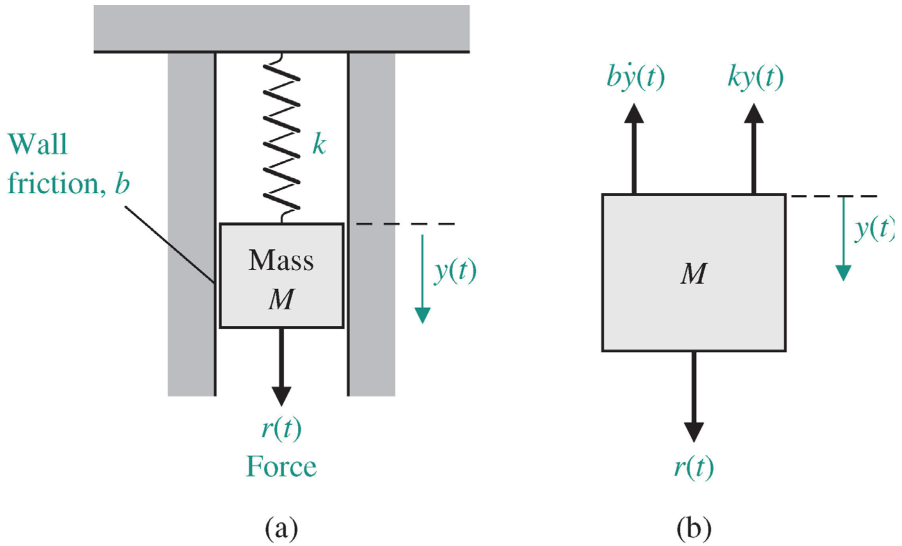
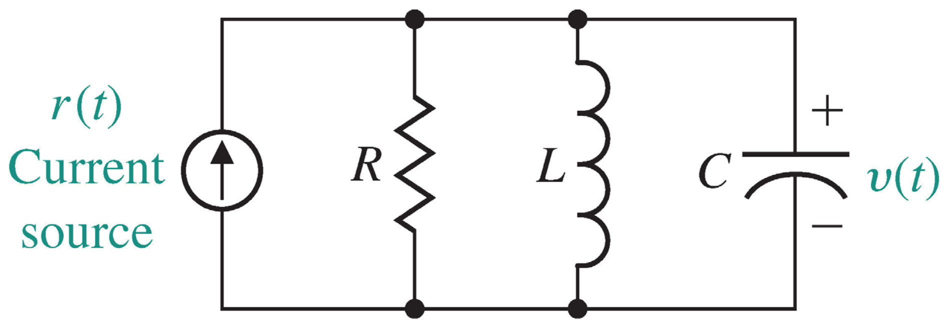
Differential Equation of Physical Systems



- Mechanical System

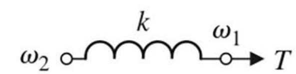
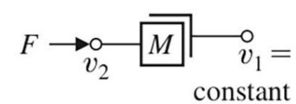
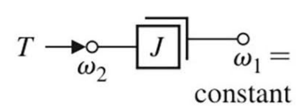
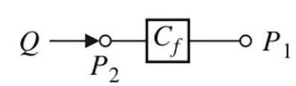
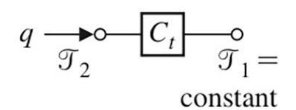
(Spring-mass-damper system)



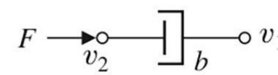
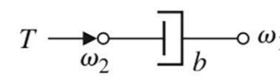
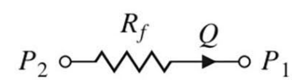
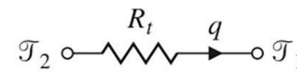
- Electrical System

(RLC circuit)

Governing Differential Equations for Ideal Elements

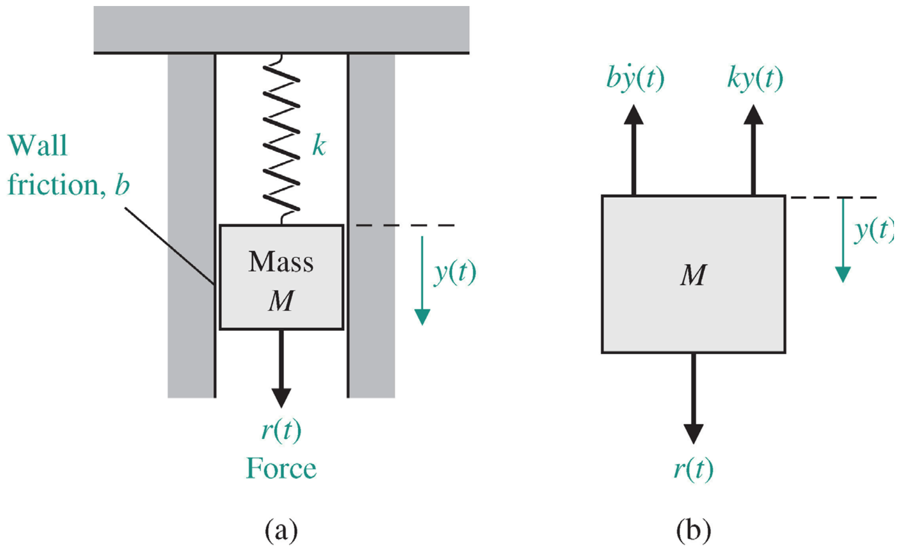
Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	
Capacitive storage	Electrical capacitance	$i = C\frac{dv_{21}}{dt}$	$E = \frac{1}{2}C{v_{21}}^2$	
	Translational mass	$F = M\frac{dv_2}{dt}$	$E = \frac{1}{2}M{v_2}^2$	
	Rotational mass	$T = J\frac{d\omega_2}{dt}$	$E = \frac{1}{2}J{\omega_2}^2$	
	Fluid capacitance	$Q = C_f\frac{dP_{21}}{dt}$	$E = \frac{1}{2}C_f{P_{21}}^2$	
	Thermal capacitance	$q = C_t\frac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	

Governing Differential Equations for Ideal Elements (cont'd)

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Energy dissipators	Electrical resistance	$i = \frac{1}{R} v_{21}$	$\mathcal{P} = \frac{1}{R} {v_{21}}^2$	
	Translational damper	$F = b v_{21}$	$\mathcal{P} = b {v_{21}}^2$	
	Rotational damper	$T = b \omega_{21}$	$\mathcal{P} = b {\omega_{21}}^2$	
	Fluid resistance	$Q = \frac{1}{R_f} P_{21}$	$\mathcal{P} = \frac{1}{R_f} {P_{21}}^2$	
	Thermal resistance	$q = \frac{1}{R_t} \mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t} \mathcal{T}_{21}$	

- *Through-variable:* F = force, T = torque, i = current, Q = fluid volumetric flow rate, q = heat flow rate.
- *Across-variable:* v = translational velocity, ω = angular velocity, v = voltage, P = pressure, $\mathcal{T}$ = temperature.
- *Inductive storage:* L = inductance, $1/k$ = reciprocal translational or rotational stiffness, I = fluid inertance.
- *Capacitive storage:* C = capacitance, M = mass, J = moment of inertia, C_f = fluid capacitance, C_t = thermal capacitance.
- *Energy dissipators:* R = resistance, b = viscous friction, R_f = fluid resistance, R_t = thermal resistance.

ODE for Mechanical System



(a) Spring-mass-damper system.
(b) Free-body diagram.

$$F = ma$$

$$m = M, a = \frac{d^2y}{dt^2}$$

$$F = r - b\frac{dy}{dt} - ky$$

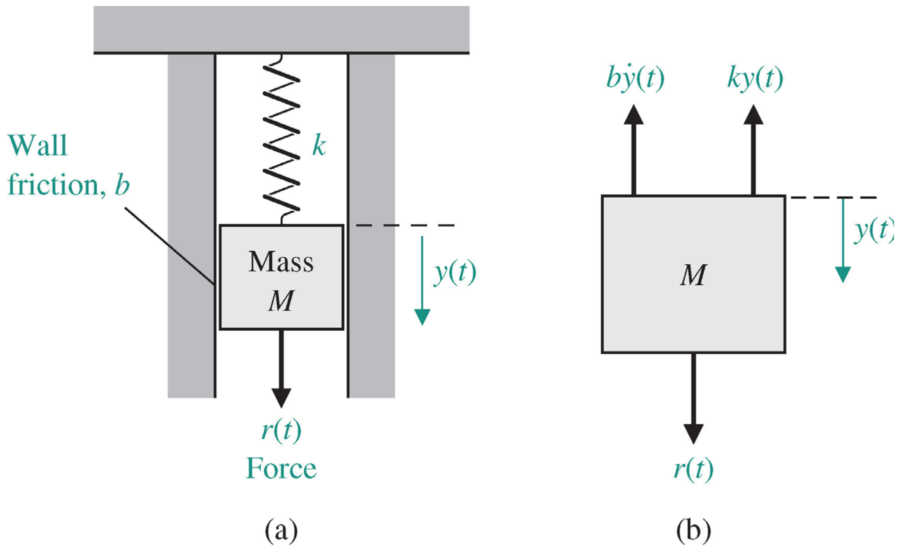
$$M\frac{d^2y}{dt^2} = r - b\frac{dy}{dt} - ky$$

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = r$$

Using Newton's laws:

- Model wall friction as a viscous damper, that is, the friction force is linearly proportional to the velocity of the mass;
- M is the mass; b is the friction constant; k is the spring constant of ideal spring;

ODE for Mechanical System



(a) Spring-mass-damper system.
(b) Free-body diagram.

Second-order linear constant-coefficient (time-invariant) system

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = r$$

$$y(0) = y_0$$

$$y(t) = K_1 e^{-\alpha_1 t}\sin(\beta_1 t + \theta_1)$$

Using Newton's laws:

- Model wall friction as a viscous damper, that is, the friction force is linearly proportional to the velocity of the mass;
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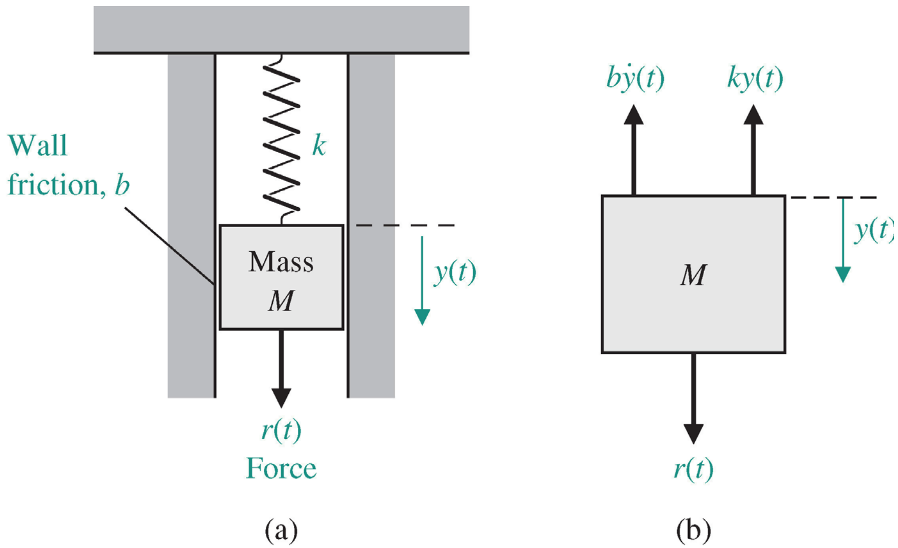
ODE for Mechanical System



(a) Spring-mass-damper system.
(b) Free-body diagram.

In this system,
- what is input?
- what is output?

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- Model wall friction as a viscous damper, that is, the friction force is linearly proportional to the velocity of the mass;
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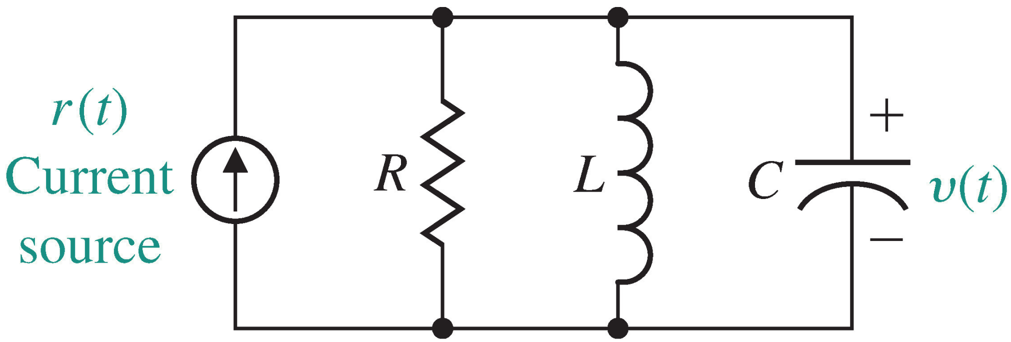
ODE for Electrical System



RLC circuit.

Second-order linear constant-coefficient (time-invariant) system

Using Kirchhoff's laws.

$$\frac{v(t)}{R} + C \cdot \frac{d}{dt} v(t) + \frac{1}{L} \cdot \int_0^t v(t)\, dt = r(t)$$

$$r(0) = r_0$$

$$v(t) = K_2 e^{-\alpha_2 t} \cos(\beta_2 t + \theta_2)$$

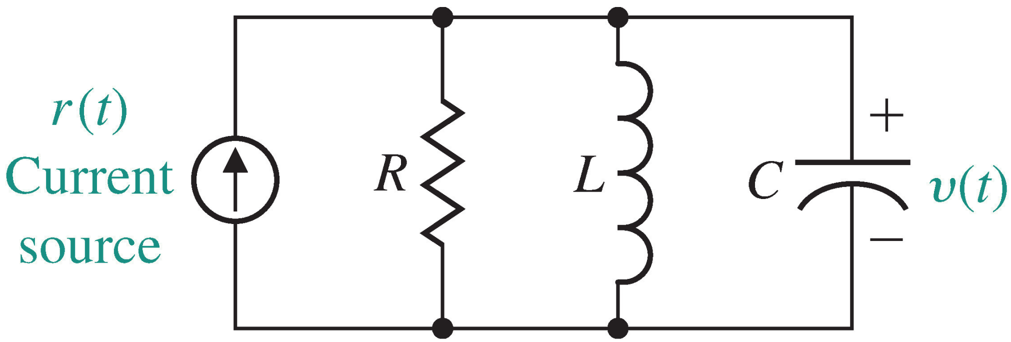
If assume:

$$\int_0^t v(t)dt = y(t)$$

Then we have:

$$\boxed{C \cdot \frac{d^2}{dt^2} y(t) + \frac{1}{R} \frac{d}{dt} y(t) + \frac{1}{L} y(t) = r(t)}$$

ODE for Electrical System



RLC circuit.

In this system,

- what is input?
- what is output?

Using Kirchhoff's laws.

$$\frac{v(t)}{R} + C\cdot\frac{d}{dt}v(t) + \frac{1}{L}\cdot\int_0^t v(t)\,dt = r(t)$$

$$r(0) = r_0$$

$$v(t) = K_2 e^{-\alpha_2 t}\cos(\beta_2 t + \theta_2)$$

If assume:

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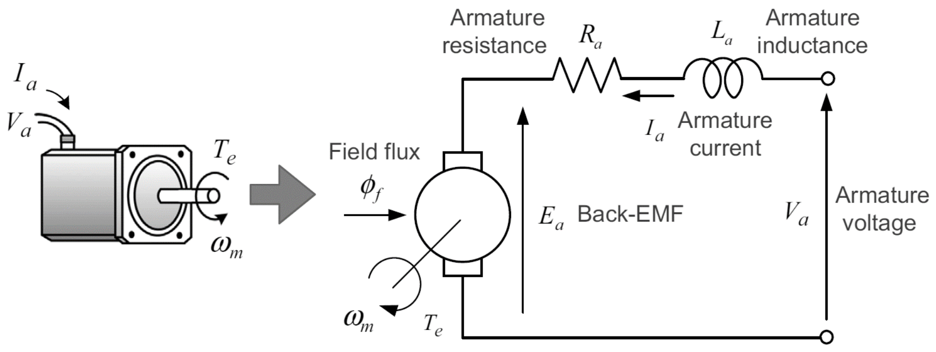
Then we have:

$$\boxed{C\cdot\frac{d^2}{dt^2}y(t) + \frac{1}{R}\frac{d}{dt}y(t) + \frac{1}{L}y(t) = r(t)}$$

ODE for Mechatronic System



Voltage equation:

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + \boxed{k_e \phi_f \omega_m}$$

Motion equation

$$\boxed{k_T \phi_f i_a} = J \frac{d\omega_m}{dt} + B\omega_m + T_L$$

Induced Torque

Replace $i_a = \frac{J}{k_T \phi_f} \frac{d\omega_m}{dt} + \frac{B}{k_T \phi_f} \omega_m + \frac{T_L}{k_T \phi_f}$

Assume $T_L = 0$

$$\boxed{V_a = R_a \left(\frac{J}{k_T \phi_f} \frac{d\omega_m}{dt} + \frac{B}{k_T \phi_f} \omega_m \right) + L_a \left(\frac{J}{k_T \phi_f} \frac{d^2\omega_m}{dt^2} + \frac{B}{k_T \phi_f} \frac{d\omega_m}{dt} \right) + k_e \phi_f \omega_m}$$

ODE for Mechatronic System

$$V_a = R_a\left(\frac{J}{k_T\phi_f}\frac{d\omega_m}{dt} + \frac{B}{k_T\phi_f}\omega_m\right) + L_a\left(\frac{J}{k_T\phi_f}\frac{d^2\omega_m}{dt^2} + \frac{B}{k_T\phi_f}\frac{d\omega_m}{dt}\right) + k_e\phi_f\omega_m$$

$$\frac{L_aJ}{k_T\phi_f}\frac{d^2\omega_m}{dt^2} + \left(\frac{R_aJ + L_aB}{k_T\phi_f}\right)\frac{d\omega_m}{dt} + \left(\frac{R_aB}{k_T\phi_f} + k_e\phi_f\right)\omega_m = V_a$$

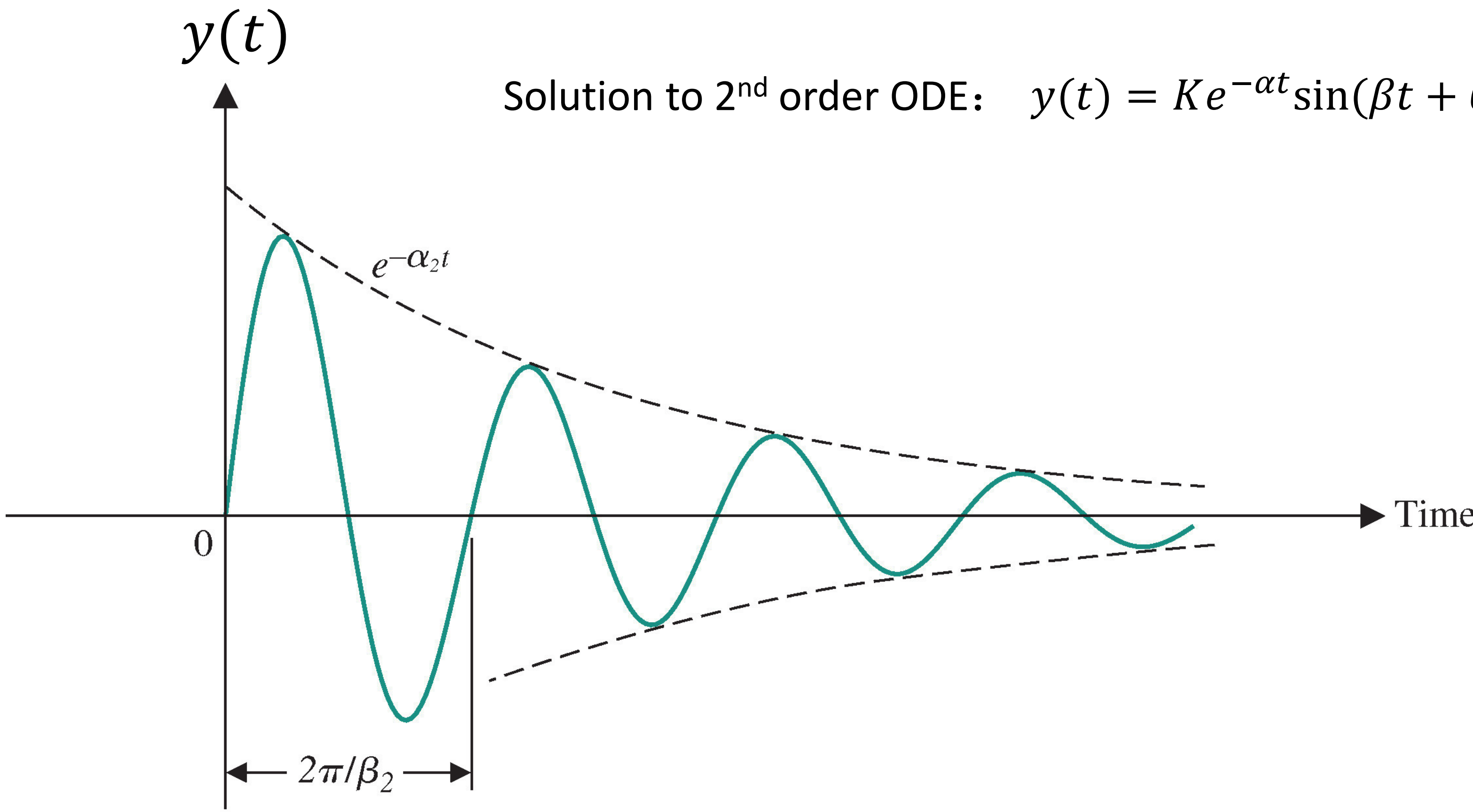
Second-order linear constant-coefficient (time-invariant) system

In this system,

- what is input?
- what is output?

Second-order System Response

Solution to 2nd order ODE： $y(t) = Ke^{-\alpha t}\sin(\beta t + \theta)$



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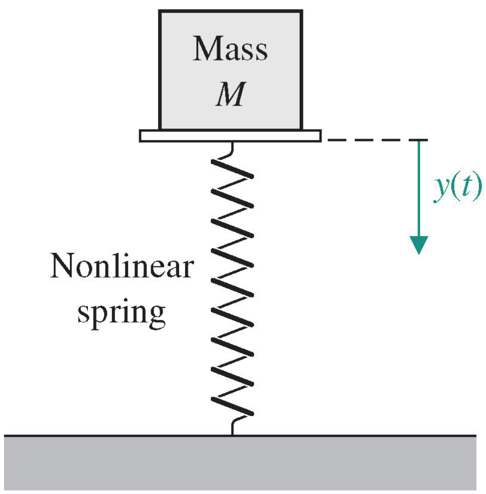
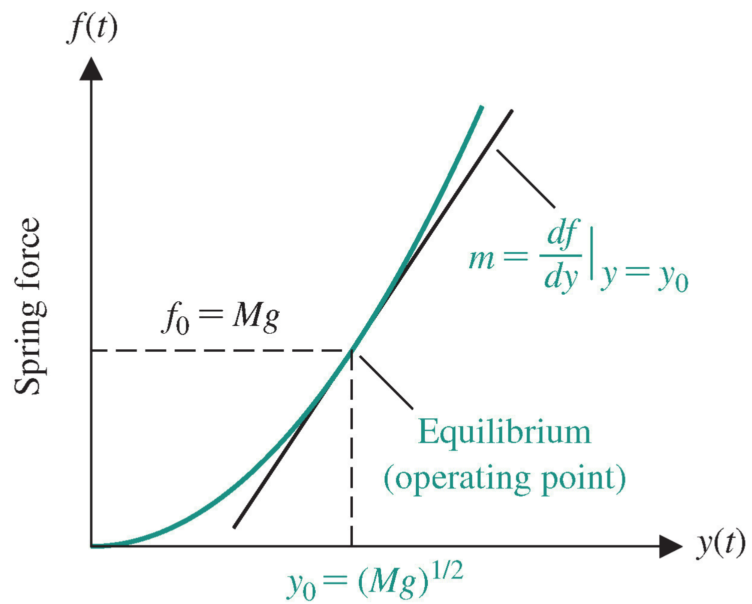
Nonlinear System

A great majority of physical system are linear within some range of the variables. In general, systems ultimately become nonlinear as the variables are increased without limit.

For example, the spring in spring-mass-damper system would behave nonlinearly, eventually overextended and break if the force continually increased.



(a)



(b)

Linear Approximations of Physical Systems

Linear system- Necessary Conditions:

叠加性

Principle of Superposition: when a system at rest is subject to an excitation $x_1(t)$, it provides a response $y_1(t)$; when a system at rest is subject to an excitation $x_2(t)$, it provides a response $y_2(t)$; For a linear system, it's necessary that the excitation $x_1(t)+x_2(t)$ results in a response $y_1(t)+y_2(t)$;

齐次性

Homogeneity: consider a system with an input x(t) that results in y(t); For a linear system, it's necessary that the response to input $\beta x(t)$ must be equal to $\beta y(t)$.

A linear system satisfies the properties of superposition and homogeneity

The linearity of many mechanical and electrical elements can be assumed over a reasonably large range of the variables; but this is not usually the case for thermal and fluid elements, which are more frequently nonlinear in character.

Linear Approximation

- The linearization can be done by assuming small-signal conditions.
- Consider a general element with an excitation $x(t)$ and the response $y(t)$. A general nonlinear relation is

$$y(t) = g\big(x(t)\big)$$ Nonlinear function

- The normal operating point is x_0, when function $g(\cdot)$ is continuous around x_0, we can apply **Taylor Series** to function g

$$y(t) = g\big(x(t)\big)$$

$$= g(x_0) + \boxed{\left.\frac{dg}{dx}\right|_{x=x_0} (x(t) - x_0)} + \boxed{\left.\frac{d^2g}{dx^2}\right|_{x=x_0} \frac{(x(t) - x_0)^2}{2!}} + \cdots$$

First order

Second order

Linear Approximation

- When operation is around x_0 for a small range, we can neglect the second order term, only keep the linear part

$$y(t) = g\big(x(t)\big) = g(x_0) + \boxed{\left.\frac{dg}{dx}\right|_{x=x_0}} (x(t) - x_0)$$

Slope/gradient

$$y(t) = y_0 + m(x(t) - x_0)$$

$$\boxed{y(t) - y_0} = m\boxed{(x(t) - x_0)}$$

$$\Delta y(t) = m\Delta x(t)$$

Linear relation maintaining **Homogeneity**

Example 6.1

Derive the linear approximation between torque on mass and angle θ around equilibrium point $\theta = 0$.

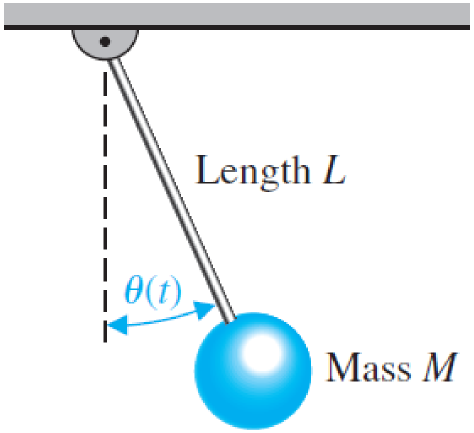
$$T(t) = f\big(\theta(t)\big) = MgL\sin\theta(t)$$

Around $\theta = 0$, we can apply Taylor Expansion

$$T(\theta) = f(0) + \left.\frac{df}{d\theta}\right|_{\theta=0}(\theta - 0) = MgL\cos\theta\Big|_{\theta=0}\theta$$

$$T(t) = MgL\theta(t)$$

The approximation is reasonably accurate for $\theta \in [-\frac{\pi}{4}, \frac{\pi}{4}]$



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The Laplace Transform

The Laplace transform can be used for
linear time-invariant (LTI) systems.

***time-invariant**: coefficients of system don't change with time (they are constants).

Definition:

$$\boxed{L\{f(t)\} = F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt}$$

Inverse Laplace transform:

$$\boxed{f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{+st}ds}$$

For $f(t)$ to be transformable, it is sufficient that $\int_{0^-}^{\infty}|f(t)|e^{-\sigma_1 t}dt < \infty$, $\sigma_1 > 0$.

The Laplace transform method substitute relatively easily solved algebraic equations for the more difficult differential equations. The time-response of a system can be obtained by solving the algebraic equations of variable of interest.

Some Important Properties

$f(t)$	$F(s)=\int_0^\infty f(t)e^{-st}\,dt$
$f+g$	$F+G$
$\alpha f\ (\alpha\in\mathbf{R})$	αF
$\dfrac{df}{dt}$	$sF(s)-f(0)$
$\dfrac{d^kf}{dt^k}$	$s^kF(s)-s^{k-1}f(0)-s^{k-2}\dfrac{df}{dt}(0)-\cdots-\dfrac{d^{k-1}f}{dt^{k-1}}(0)$
$g(t)=\int_0^t f(\tau)\,d\tau$	$G(s)=\dfrac{F(s)}{s}$
$f(\alpha t),\ \alpha>0$	$\dfrac{1}{\alpha}F(s/\alpha)$
$e^{at}f(t)$	$F(s-a)$
$tf(t)$	$-\dfrac{dF}{ds}$
$t^kf(t)$	$(-1)^k\dfrac{d^kF(s)}{ds^k}$
$\dfrac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$
$g(t)=\begin{cases}0 & 0\le t<T\\ f(t-T) & t\ge T\end{cases},\ T\ge 0$	$G(s)=e^{-sT}F(s)$

Important Laplace Pairs

$f(t)$	$F(s)$
Step function, $u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
$f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-) - \ldots - f^{(k-1)}(0^-)$
$\frac{1}{a^2+\omega^2} + \frac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at}\sin(\omega t - \phi)$, $\phi = \tan^{-1}\frac{\omega}{-a}$	$\frac{1}{s[(s+a)^2+\omega^2]}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \phi\right)$, $\phi = \cos^{-1}\zeta,\ \zeta < 1$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
$\frac{\alpha}{a^2+\omega^2} + \frac{1}{\omega}\left[\frac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2} e^{-at}\sin(\omega t + \phi)$. $\phi = \tan^{-1}\frac{\omega}{\alpha - a} - \tan^{-1}\frac{\omega}{-a}$	$\frac{s+\alpha}{s[(s+a)^2+\omega^2]}$

$f(t)$	$F(s)$
$\int_{-\infty}^{t} f(t)\,dt$	$\frac{F(s)}{s} + \frac{1}{s}\int_{-\infty}^{0} f(t)\,dt$
Impulse function $\delta(t)$	1
$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{1}{\omega}\left[(\alpha-a)^2+\omega^2\right]^{1/2} e^{-at}\sin(\omega t+\phi)$,	$\frac{s+\alpha}{(s+a)^2+\omega^2}$
$\phi = \tan^{-1}\frac{\omega}{\alpha - a}$	
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t, \zeta < 1$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

The Laplace Transform – Differential Operator

First order differential equations

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

Second order differential equations

$$\mathcal{L}\left\{\frac{d^2 f(t)}{dt^2}\right\} = \mathcal{L}\{f''(t)\} = s^2F(s) - s\,f(0) - f'(0)$$

For the analysis in this module, we always assume $f(0) = f'(0) = \cdots = 0$

The Laplace Transform – Differential Operator

- The Laplace variable s can be considered to be the differential operator

$$s \equiv \frac{d}{dt}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = \mathcal{L}\{f'(t)\} = sF(s)$$

$$\mathcal{L}\left\{\frac{d^2 f(t)}{dt^2}\right\} = \mathcal{L}\{f''(t)\} = s^2 F(s)$$

And also the integral operator

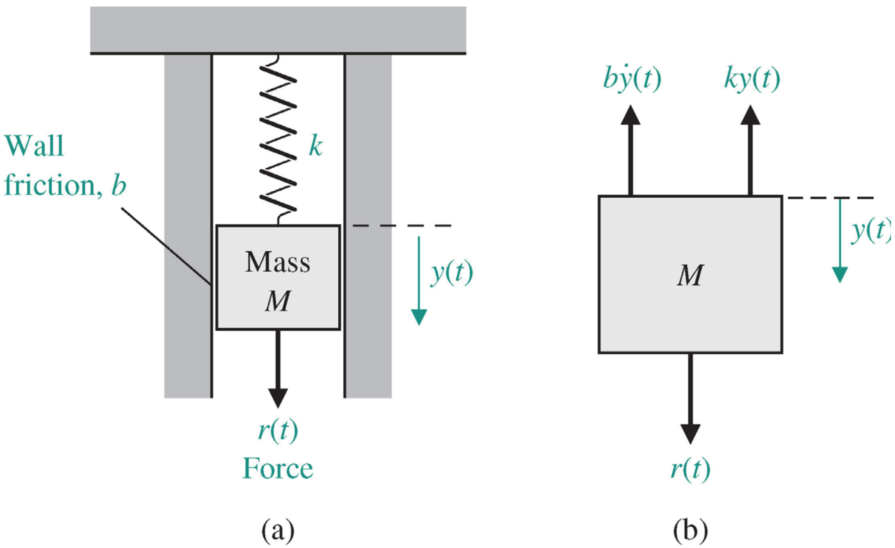
$$\frac{1}{s} \equiv \int_{0^-}^{t} dt$$

- In control theory, Laplace transform s is used to simplify the computation of ODE model.

Example 6.2

Application of Laplace transform when solving ODE:

- Consider the mechanical system considered before



(a) Spring-mass-damper system.
(b) Free-body diagram.

The model is derived as

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = r$$

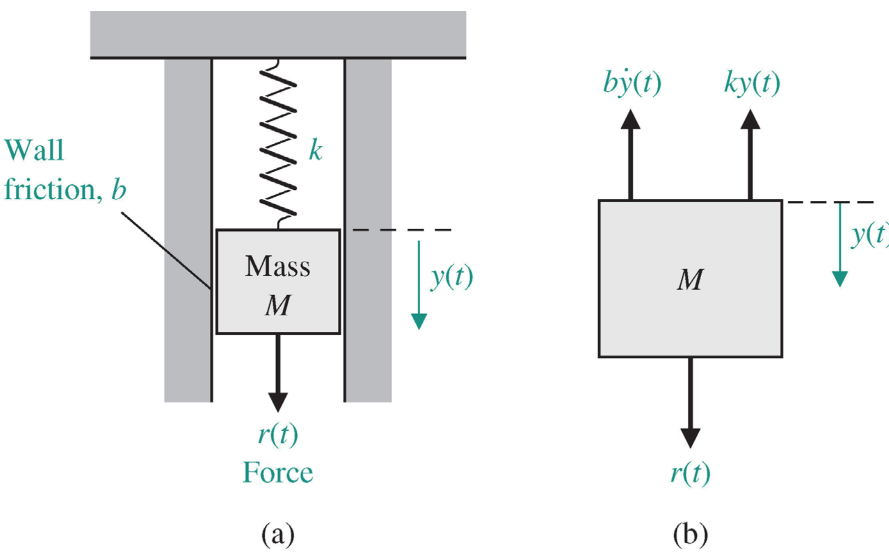
Apply Laplace Transform

$$\mathcal{L}\left(M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky\right) = \mathcal{L}(r(t))$$

Example 6.2

Application of Laplace transform when solving ODE:

- Consider the mechanical system considered before



(a) Spring-mass-damper system.
(b) Free-body diagram.

$$\mathcal{L}\left(M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky\right) = \mathcal{L}(r(t))$$

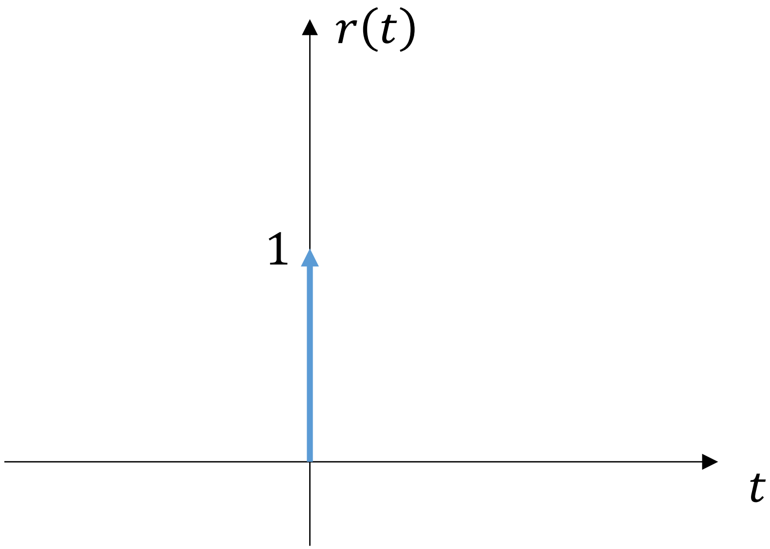
$$Ms^2Y(s) + bsY(s) + kY(s) = R(s)$$

$$Y(s) = \frac{1}{Ms^2 + bs + k}R(s)$$

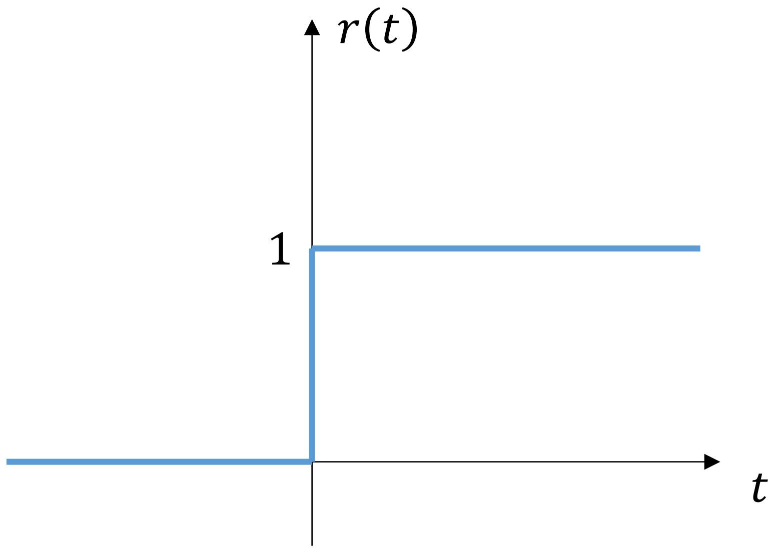
Only algebraic calculation is involved

Example 6.2

- Set $M = 1, b = 3, k = 2$. Consider two kinds of force $r(t)$
  - Impulse function $\sigma(t)$
  - Step function $1(t)$



$$\mathcal{L}(\sigma(t)) = 1$$



$$\mathcal{L}(1(t)) = \frac{1}{s}$$

Example 6.2 – impulse input

- Input as impulse function, $R(s) = 1$
- Output in s-domain is

$$Y(s) = \frac{1}{Ms^2 + bs + k} R(s) = \frac{1}{s^2 + 3s + 2}$$

- Next step is to use **partial fraction expansion**

$$Y(s) = \frac{1}{s^2 + 3s + 2} = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

Example 6.2 – impulse input

- Then use Laplace transformation relation

$$\mathcal{L}(e^{-at}) = \frac{1}{s+a}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$$

- If assume zero initial value condition, then use inverse Laplace

$$\mathcal{L}^{-1}\big(Y(s)\big) = y(t) = \mathcal{L}^{-1}\left(\frac{1}{s+1} - \frac{1}{s+2}\right) = e^{-t} - e^{-2t}$$

- Solution to the ODE is

$$y(t) = e^{-t} - e^{-2t}$$

Example 6.2 – step input

- Input as step function, $R(s) = \frac{1}{s}$
- Output in s-domain is

$$Y(s) = \frac{1}{Ms^2 + bs + k} R(s) = \frac{1}{s(s^2 + 3s + 2)}$$

- Next step is to use **partial fraction expansion**

$$Y(s) = \frac{1}{s(s^2 + 3s + 2)} = \frac{1}{s(s+1)(s+2)} = \frac{1/2}{s} + \frac{-1}{s+1} + \frac{1/2}{s+2}$$

Example 6.2 – step input

- Apply inverse Laplace transformation

$$\mathcal{L}^{-1}\big(Y(s)\big) = y(t) = \mathcal{L}^{-1}\left(\frac{1/2}{s} + \frac{-1}{s+1} + \frac{1/2}{s+2}\right)$$
$$= 0.5 - e^{-t} + 0.5e^{-2t}$$

- What is the steady state value?

$$y(t \to \infty) = \lim_{t\to\infty} y(t) = 0.5$$

Finite Value Theorem

Final Value Theorem:

$$\boxed{\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s),}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{1}{(s+1)(s+2)} = 0.5$$

- The same as our previous analysis
- You can try to compute the steady state of the impulse input

Quiz 6.1

If ODE of a system is: $$\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 3y(t) = 2r(t)$$

with the initial conditions and input: $y(0) = 1, \frac{d}{dt}y(0) = 0,\ r(t) = 1$ (step function)

Compute the time-domain response of the system.

Thank You !