CAN102 Electromagnetism and Electromechanics

Lecture-10 Static Magnetic Fields II

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Outline

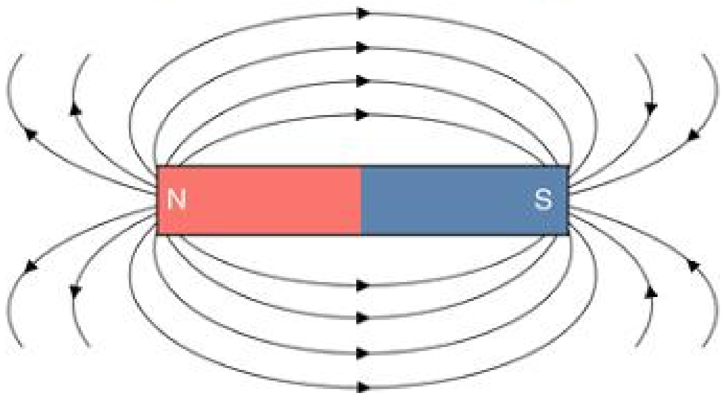
- Visualisation of Magneitc Fields
  - Magnetic field lines
  - Comparison with electric field lines
- Magnetic Forces
  - on a moving charge
  - on a current-carrying wire
- Magnetic materials
  - Permeability
  - Classification and ferromagnetic materials
- Boundary Conditions

1.1 Visualisation - Magnetic field lines

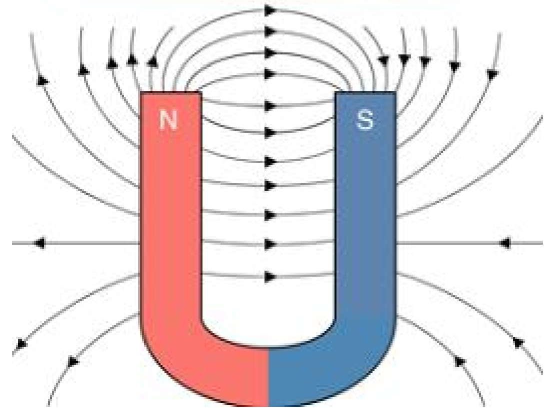
- It is often helpful to visualise the magnetic field in the vicinity of a current source:
  - At each point, the field line is tangent to the $\vec{B}$.
  - The more densely the field lines are packed, the stronger the field is at that point.
  - At each point, the field lines point in the same direction a compass would, so magnetic field lines point away from N poles and toward S poles.
  - Field lines never intersect since the direction of $\vec{B}$ at each point is unique.

Examples

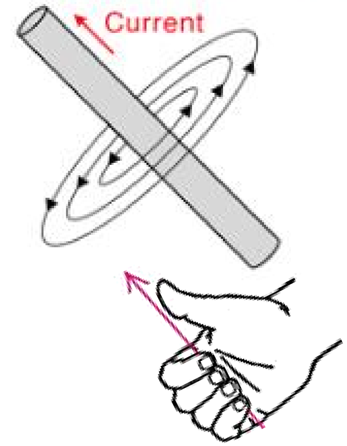
1. Bar Magnet



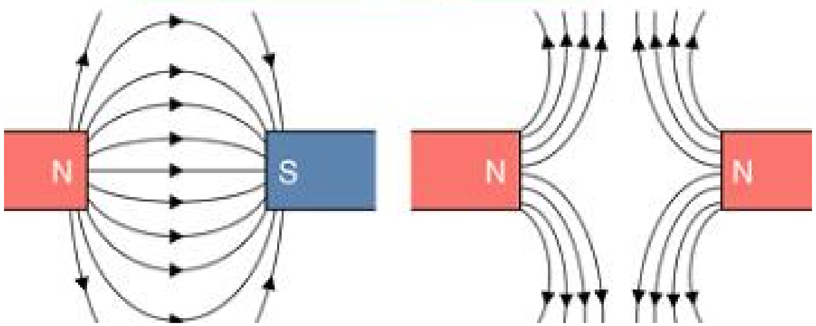
2. Horseshoe Magnet



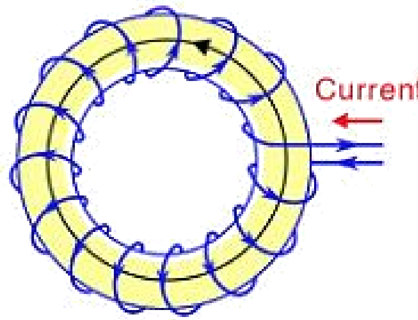
4. Current-carrying wire



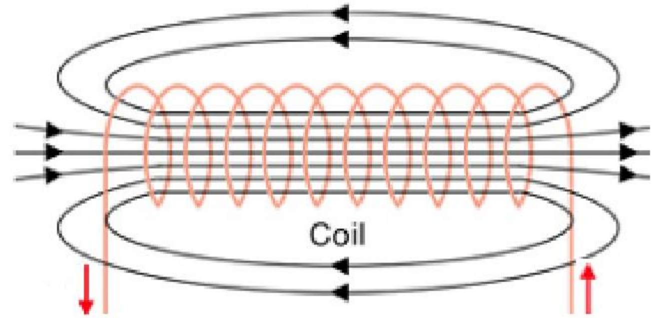
3. Two Bar Magnets



5. Toroid



6. Solenoid



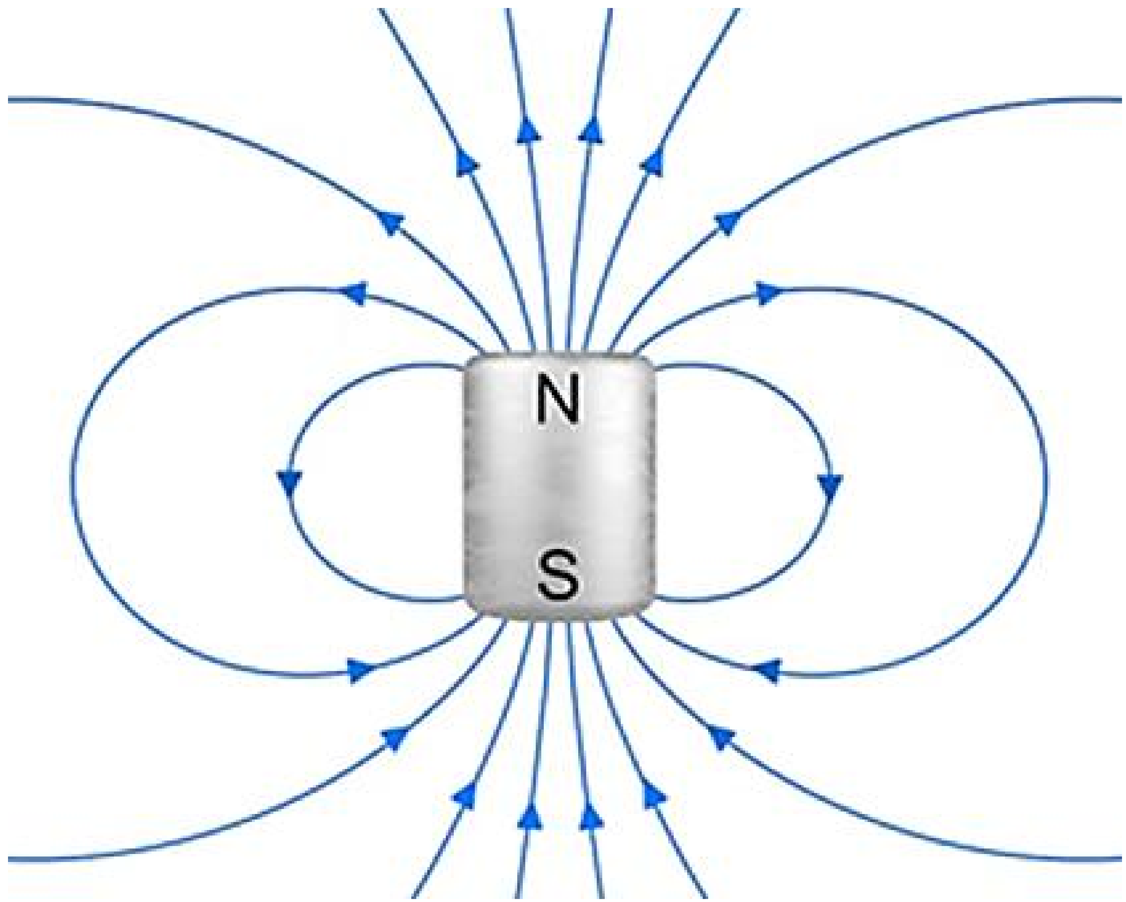
1.2 Comparison

- **Properties of electric field lines**
  - The direction of the electric field at any point is tangent to the field lines at that point.
  - The number of lines per unit area through a surface perpendicular to the line is proportional to the magnitude of the electric field in a given region.
  - The field lines must begin on positive charges (or at infinity) and then terminate on negative charges (or at infinity).
  - No two field lines can cross each other.

- **Properties of magnetic field lines**
  - At each point, the field line is tangent to the magnetic flux density.
  - The more densely the field lines are packed, the stronger the field is at that point.
  - At each point, the field lines point in the same direction a compass would, therefore, magnetic field lines point away from N poles and toward S poles.
  - Because the direction of magnetic flux density at each point is unique, field lines never intersect.

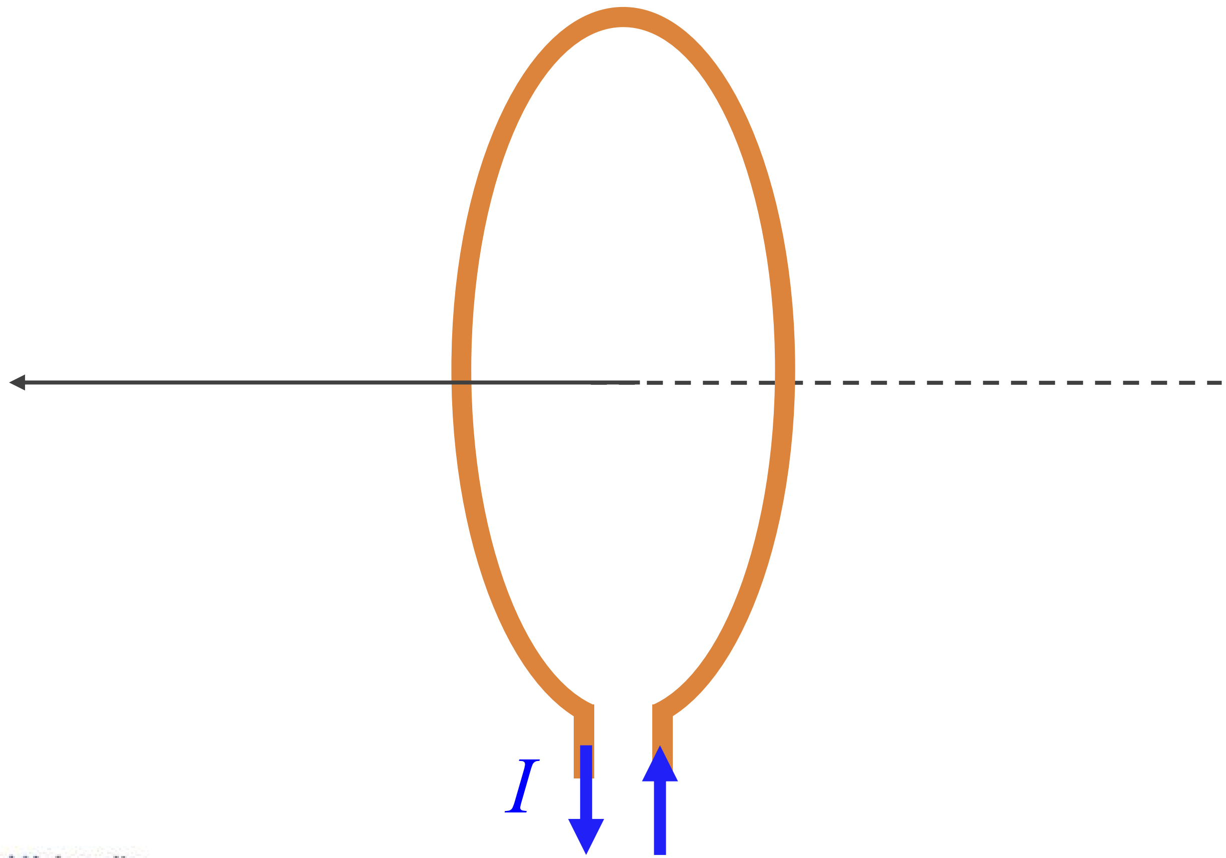
Example 1

- The picture shows the field lines outside a permanent magnet. The field lines inside the bar magnet is:



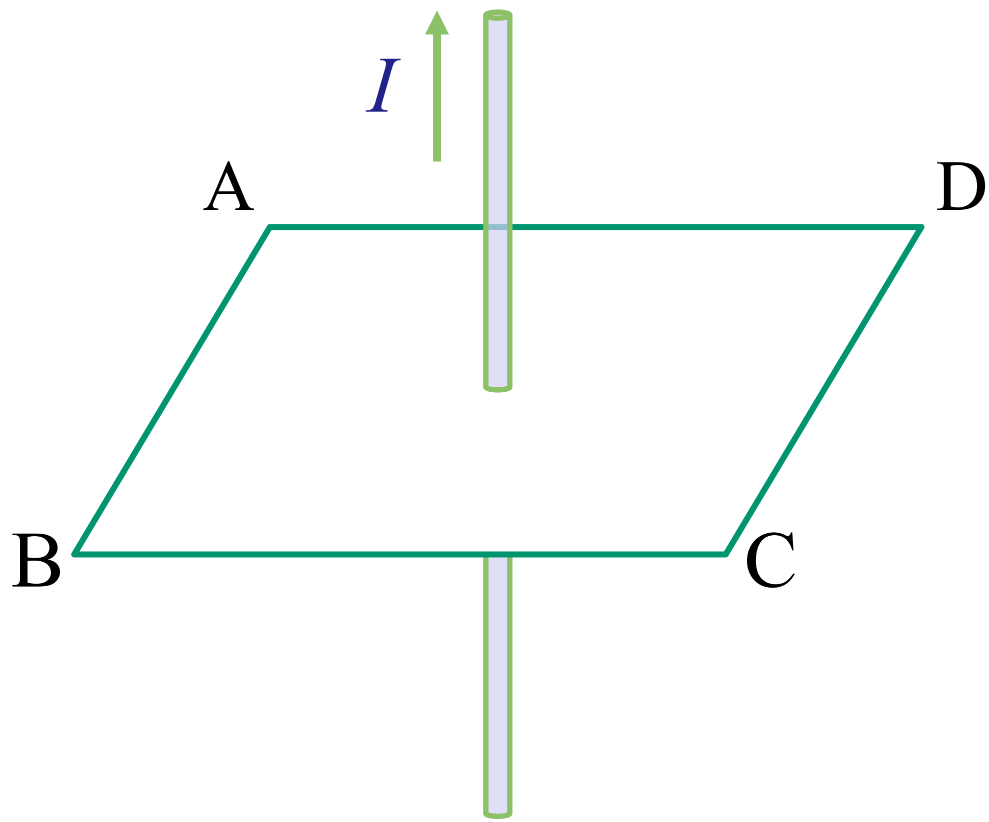
Example 2

- Plot the magnetic field lines of the given flat circular current-carrying loop.



Quiz 2

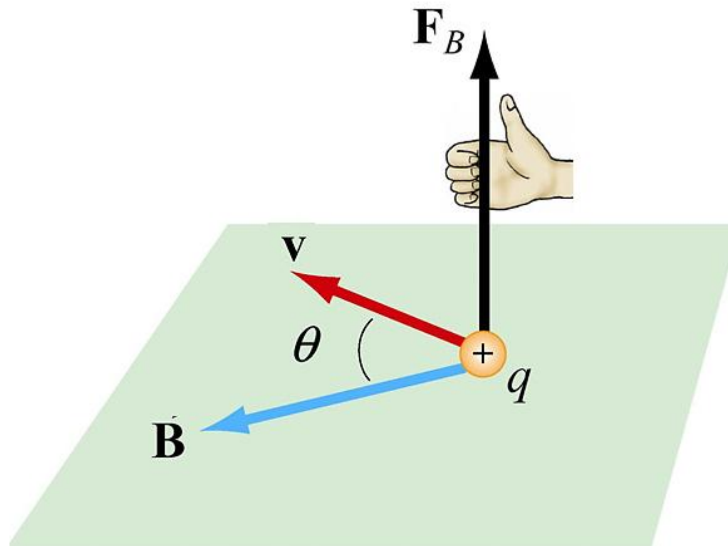
- Plot the magnetic field lines of the given current-carrying wire in the horizontal plane ABCD.



2.1 Magnetic force on a *moving charge*

- Experiment shows:
  - ✓ $\vec{F}_B$ is perpendicular to $\vec{v}$ and $\vec{B}$
  - ✓ F_B is proportional to q, v and B

Magnetic Force: $\vec{F}_B = q(\vec{v} \times \vec{B})$



- **Right-hand rule** for the direction of magnetic force on a **positive** charge moving in a magnetic field:
  - Place the vectors tail to tail
  - Imagine turning toward in the plane (through the smaller angle)
  - The force acts along a line perpendicular to the plane.
  - Do right-hand as shown. The thumb points the direction of the force.

2.1.1 Key Characteristics

- **Four** key characteristics:
  - 1. The magnitude of the force is proportional to the magnitude of the charge

    The bigger the charge, the greater the magnetic force on it
  - 2. The magnitude of the force is proportional to the magnitude, or "density" of the field

    Double the magnitude of the field, the force doubles
  - 3. The magnitude of the force also depends on the particle's velocity

    A rest charge experiences no magnetic force. Different from the electric force
  - 4. The direction of the magnetic force is perpendicular to both magnetic field and the charge velocity.

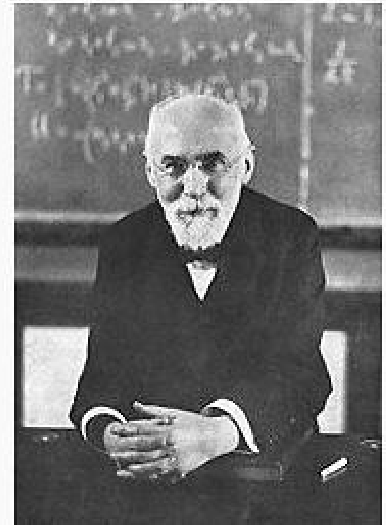
    By experiment observation

2.1.2 Lorentz' Force

- When a charged particle moves in a **mixed** electric field and magnetic field, it experiences a **total** force *Lorentz Force* $\vec{F}_L$ exerted by both fields:

$$\vec{F}_L = \vec{F}_E + \vec{F}_B = q(\vec{E} + \vec{v} \times \vec{B})$$

Hendrik Antoon Lorentz



Electric force

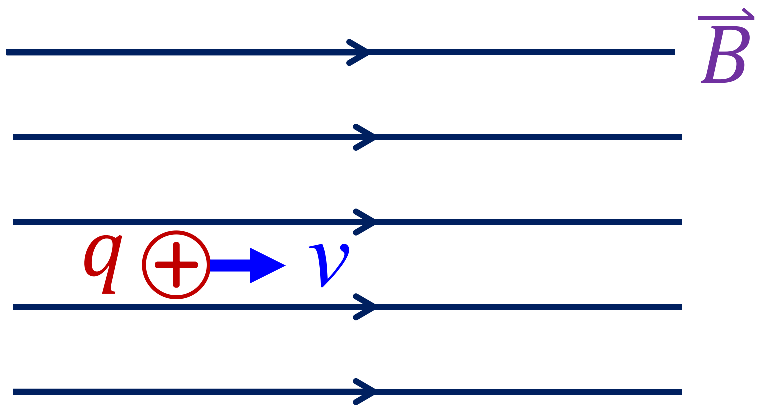
- ➢ always in the direction of the E-field
- ➢ acts on a charged particle no matter it is moving or not
- ➢ expends energy in displacing a charged particle

Magnetic force

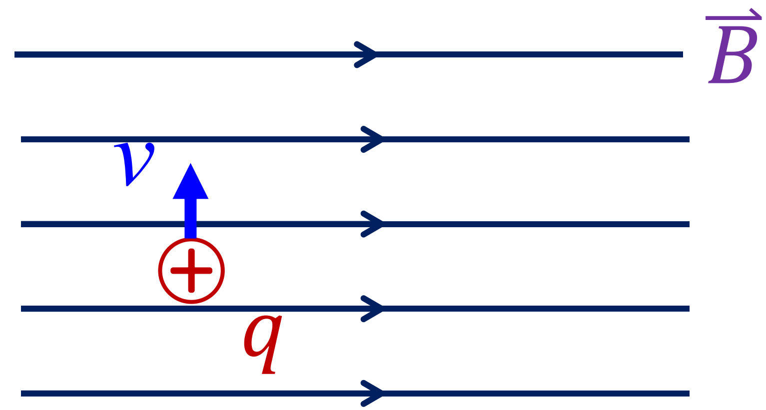
- ➢ always perpendicular to the magnetic field.
- ➢ acts on it only when it is in motion.
- ➢ does no work when a particle is displaced

Example 3

- A **uniform** magnetic field with the strength $B = 0.1$ T is directed toward the right. A positive charge $q = 2$ C moves in the given direction with a constant speed of 8 m/s. Determine the magnetic force on the charge.



(a)



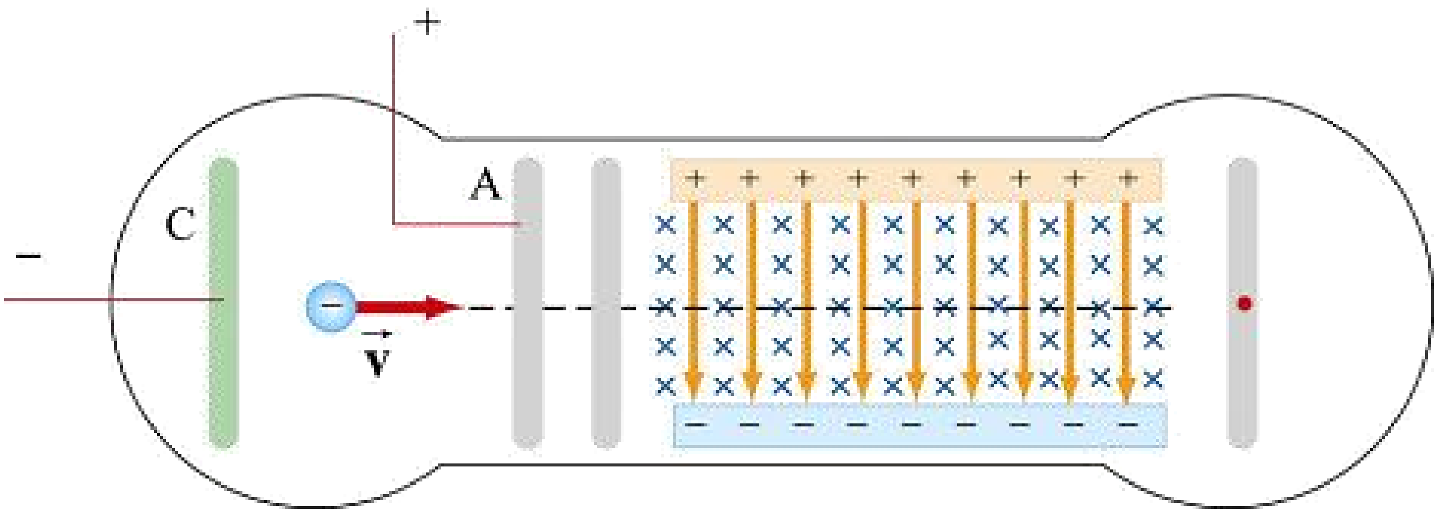
(b)

$$\vec{F}_B = q\vec{v} \times \vec{B}$$
$$= 2(8\hat{y}) \times (0.1\hat{y}) = 0$$

$$\vec{F}_B = q\vec{v} \times \vec{B} = 2(8\hat{z}) \times (0.1\hat{y})$$
$$= 1.6(-\hat{x})N$$

Quiz 3

- In a ***velocity selector***, by combining the two fields, particles which move with a certain velocity can be selected. This was the principle used by J. J. Thomson to measure the charge-to-mass ratio of the electrons. The electrons with negative charge $q = -e$ and mass m are emitted from the cathode C and then accelerated toward slit A. Let the potential difference between A and C be ΔV, and the electric and magnetic field in the apparatus are set as $\boldsymbol{E}$ and $\boldsymbol{B}$. To make the electron moving in **straight line**, what the accelerating voltage ΔV should be?

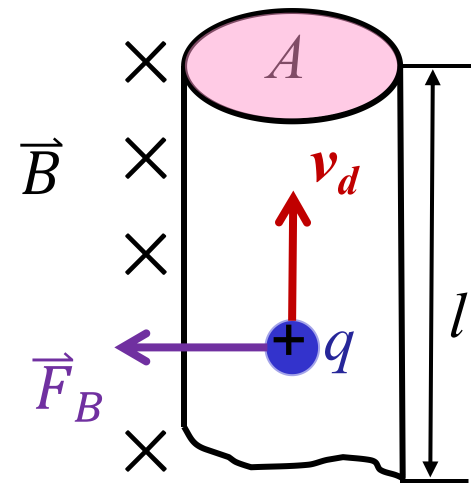


2.2 Magnetic force on a ***current-carrying wire***

- The total amount of charge in the segment:

$$Q_{tot} = q(NAl)$$

  - N is the number of charges per unit volume.



- Then $\vec{F}_B = Q_{tot}\vec{v}_d \times \vec{B} = qNAl\left(\vec{v}_d \times \vec{B}\right)$
  - $\vec{v}_d$ and the length of the current moving l is in the same direction, so $\vec{\boldsymbol{v}}_{\boldsymbol{d}} l = v_d \vec{\boldsymbol{l}}$
  - And the current density $J = Nqv_d$ and current $I = JA$
- Finally, we have

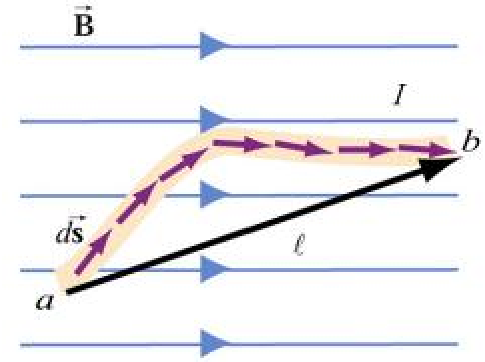
$$\vec{F}_B = (Nqv_dA)\left(\vec{l} \times \vec{B}\right) = I\left(\vec{l} \times \vec{B}\right)$$

  - where $\vec{l}$ is the length vector with a magnitude l and directed along the direction of the current.

2.2 Magnetic force on a ***current-carrying wire***

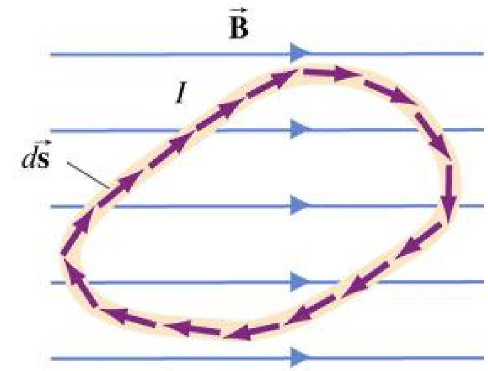
- When a current-carrying conductor is placed in an external magnetic field, the magnetic force exerted on it is:

$$\vec{F}_B = I\left(\int_a^b d\vec{l}\right) \times \vec{B} = I\vec{l} \times \vec{B}$$



- If the wire forms a closed loop of arbitrary shape, in a uniform field, the force on the loop becomes:

$$\vec{F}_B = I\left(\oint d\vec{l}\right) \times \vec{B} = 0$$



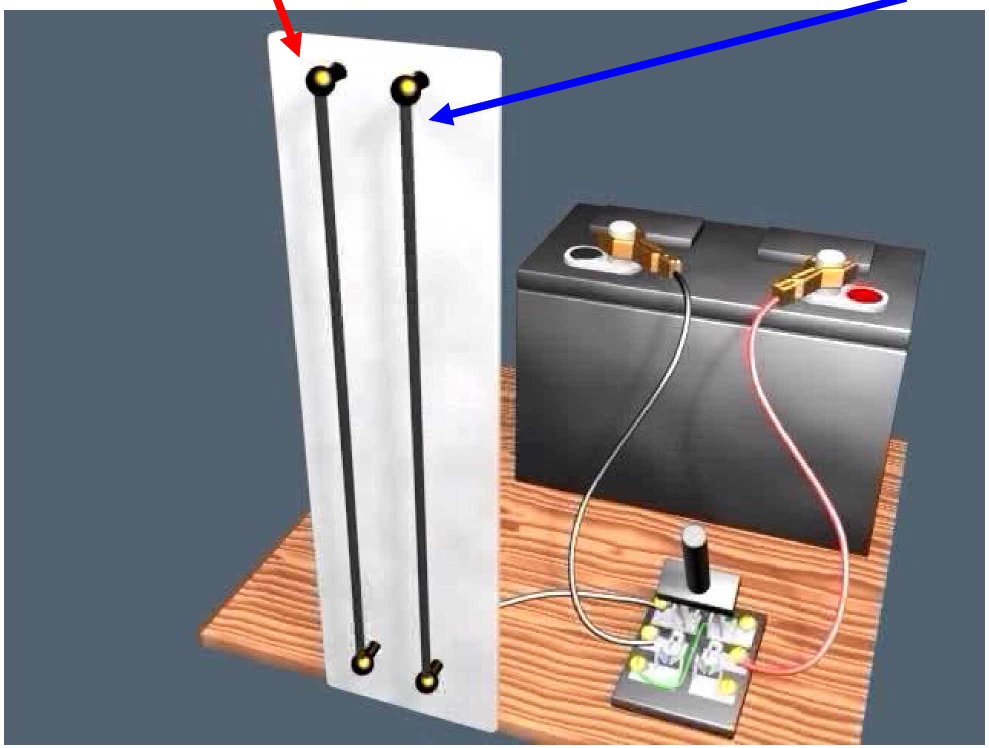
- That means: the magnetic forces exerted on two current carrying wires with the same terminals are always the same.

Recall Ampere's Experiment

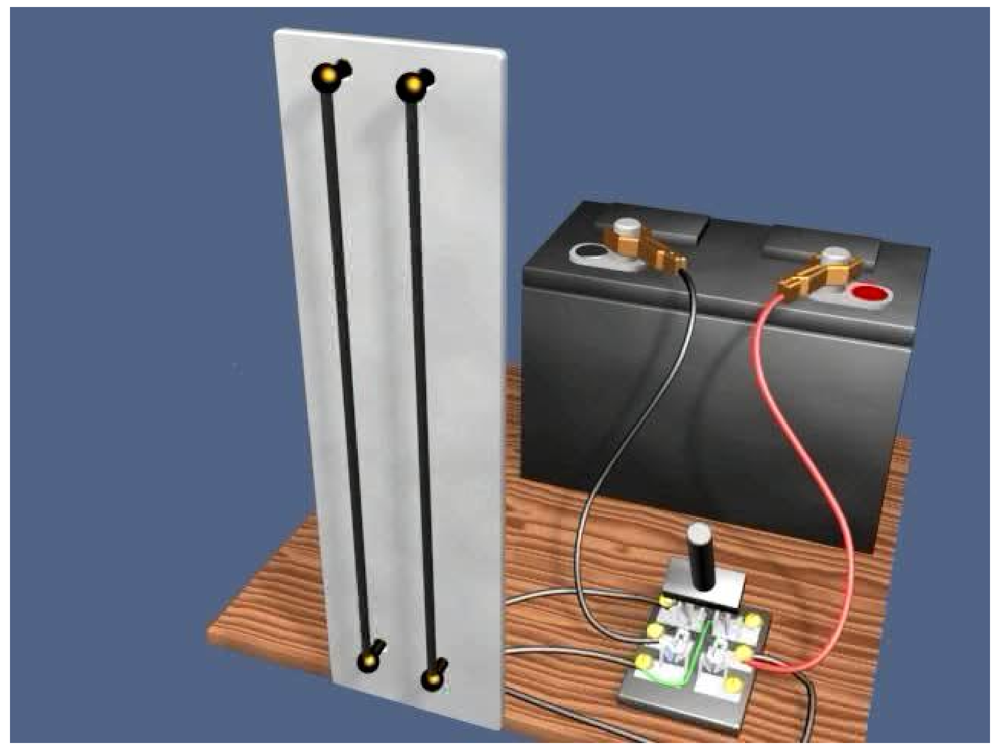
Two current-carrying wires to exert force on each other.

A current-carrying wire produces a magnetic field.

In a magnetic field, a wire carrying a current experiences a net force.



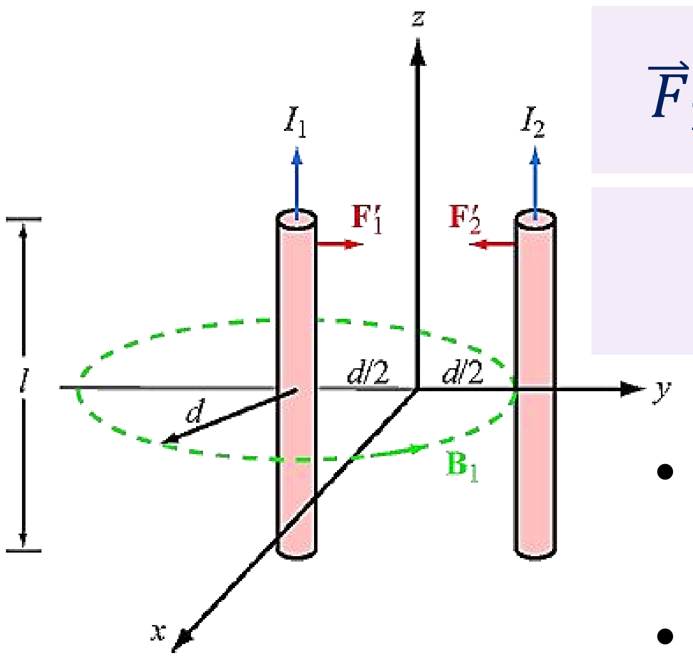
Parallel – current in same direction



Series – current in different direction

2.2 Forces between TWO parallel wires

- The magnetic force $\vec{F}_{21}$, exerted on wire 2 by wire 1 is generated by the magnetic field lines due to I_1.
- At an arbitrary point P on wire 2, we have $\vec{B}_1 = -\left(\frac{\mu_0 I_1}{2\pi d}\right)\hat{x}$, which points in the direction perpendicular to wire 2.



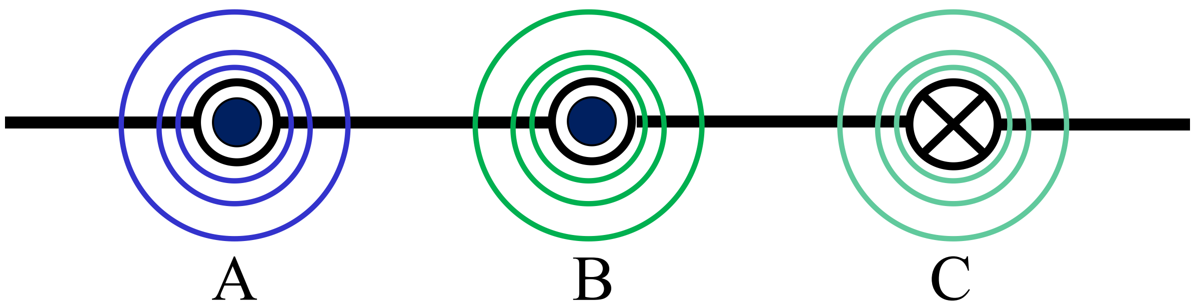
$$\vec{F}_{21} = I_2\vec{l} \times \vec{B}_1 = I_2 l\hat{z} \times \left(-\frac{\mu_0 I_1}{2\pi d}\hat{x}\right) = -\frac{\mu_0 I_1 I_2 l}{2\pi d}\hat{y}$$

$$\vec{F}_{12} = I_1\vec{l} \times \vec{B}_2 = I_1 l\hat{z} \times \left(\frac{\mu_0 I_2}{2\pi d}\hat{x}\right) = \frac{\mu_0 I_1 I_2 l}{2\pi d}\hat{y}$$

- Two parallel wires carrying currents in the same direction will attract each other.
- If the currents flow in opposite directions, the resultant force will be repulsive.

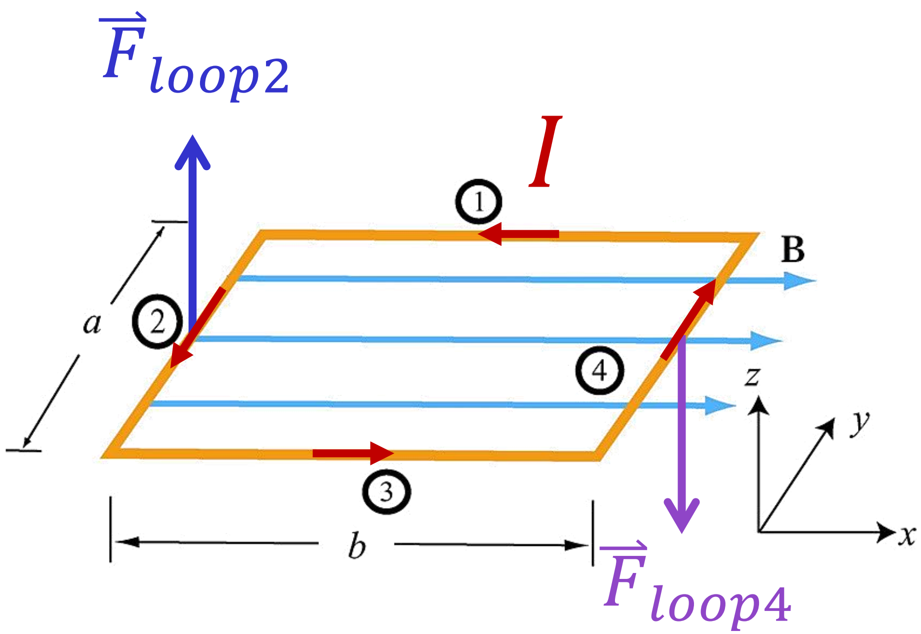
Example 4

- The figure shows three long, straight, parallel, equally spaced wires with identical currents either into or out of the page. **Rank** the wires according to the magnitude of the force on each due to the currents in the other two wires.



Quiz 4

- A rectangular current loop placed in a uniform magnetic field. Determine the force on each side of the loop (1,2,3,4) and the **net force** exerted on this loop.

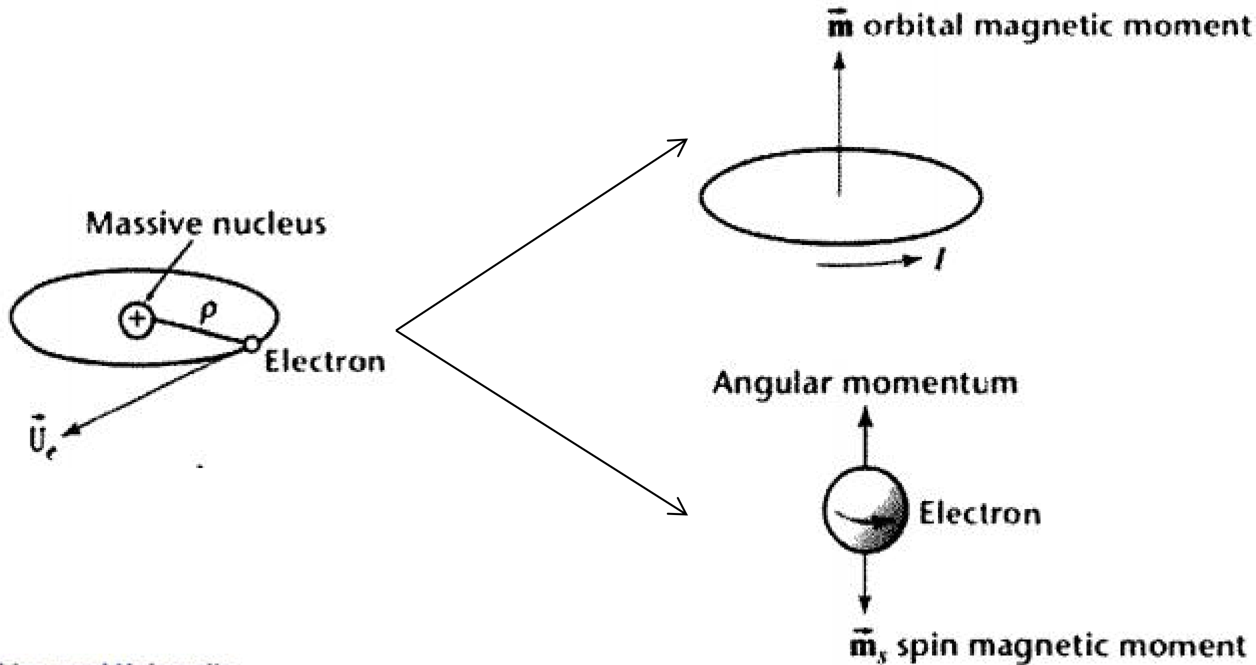


3.1.1 Magnetic materials

Not required

The atomic model

- Although accurate quantitative results can only be predicted through the use of quantum theory, the simple atomic model yields reasonable qualitative results and provides a satisfactory theory
  - The simple atomic model assumes that there is a central positive nucleus surrounded by electrons in various circular orbits



$$\vec{m} = \frac{eU_e\rho}{2}\vec{a}_z$$

$$m_s = 9 \times 10^{-24}\ \mathrm{A \cdot m^2}$$

3.1.2 Magnetic materials

Classification

Not required

- Each atom contains many different component moments, and their combination determines the magnetic characteristics of the material and provides its general magnetic classification:
  - Diamagnetic material (抗磁性)
    - produce a magnetization that **opposes** the magnetic field.
    - B_{in} opposite to B_{appl}.
  - Paramagnetic material （顺磁性）
    - produce a magnetization in the **same direction** as the applied magnetic field.
    - B_{in} parallel to B_{appl}.
  - Ferromagnetic material （铁磁性）
    - can have a magnetization independent of an applied B-field with a complex relationship between the two fields.
    - $B_{in} >> B_{appl}$.

3.2 Permeability

- In the presence of an externally applied magnetic field $\vec{B}_o$, the total magnetic flux density in the material is:

$$\vec{B} = \vec{B}_o + \vec{B}_{in} = \vec{B}_o + \chi_m \vec{B}_o = (1 + \chi_m)\vec{B}_o$$

  – where χ_m is the *susceptibility*.

- In this case:

$$\vec{B} = (1 + \chi_m)\mu_0 \vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

  – $\mu = \mu_0 \mu_r$ is the **permeability** of the medium.

  – μ_r is the **relative permeability**.

For diamagnetic & paramagnetic materials,

$$\mu_r \approx 1$$

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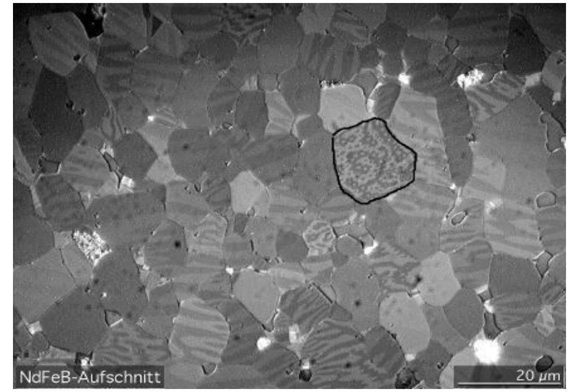
3.3.1 Magnetic materials

Ferromagnetic

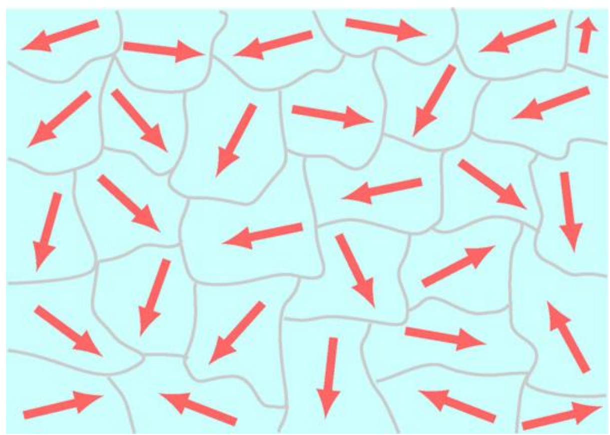
- In ferromagnetic materials, each atom has a relatively large dipole moment, Interatomic forces cause these moments to line up over regions containing a large number of atoms.
  - These regions are called **magnetic domains**.
  - With an external magnetic field applied, those domains which have moments in the direction of the applied field increase their size: $B_{int} >> B_{appl}$
  - Ferromagnetic materials: iron, cobalt, nickel.



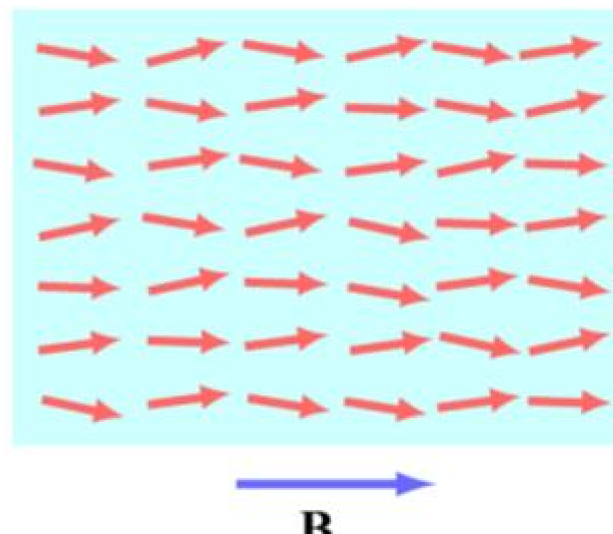
With external field
→Magnetised domains

Without $\vec{B}_{external}$, the domains take on random orientations

→ no net magnetisation



Unmagnetised domains

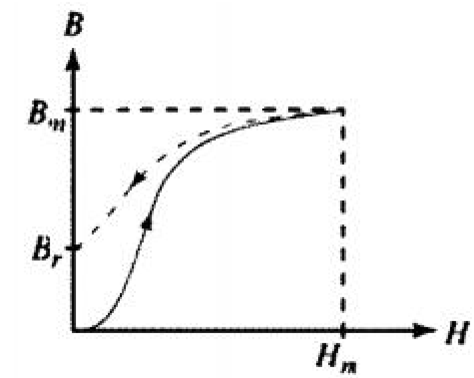


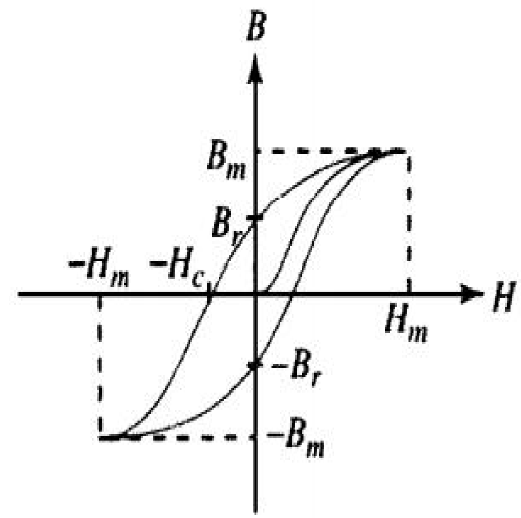
3.3.2 Magnetic materials

Hysteresis

Not required

- Upon application of an external magnetic field, however, those domains which have moments in the direction of the applied field increase their size at the expense of their neighbours, and the internal magnetic field increases greatly over that of the external field alone.
- When the external field is removed, a completely random domain alignment is not usually attained, and a residual dipole field remains in the macroscopic structure. The fact that the magnetic moment of the material is different after the field has been removed, or that the magnetic state of the material is a function of its magnetic history, is called ***hysteresis***





$\mu_r >> 1$

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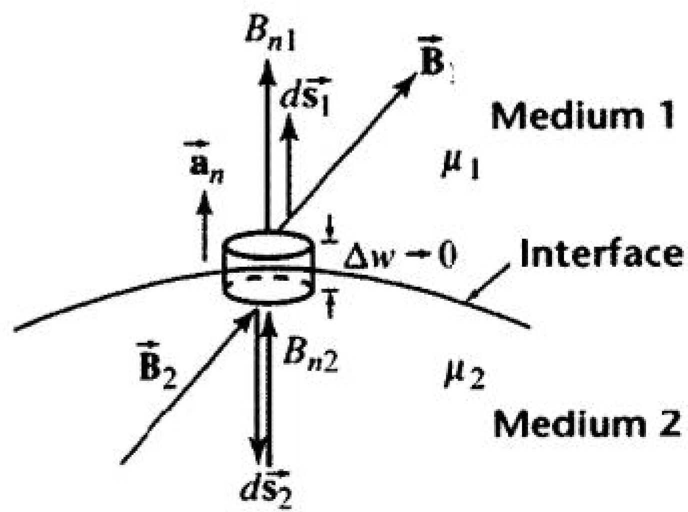
4.1 Boundary Conditions - Normal



- Construct a pillbox shaped Gaussian surface with vanishing thickness
  - Since the magnetic flux lines are continuous, we have

$$\oint_s \vec{\mathbf{B}} \cdot \vec{ds} = 0$$

  - Neglecting the flux flowing through the vanishing side walls of the pillbox, the equation becomes

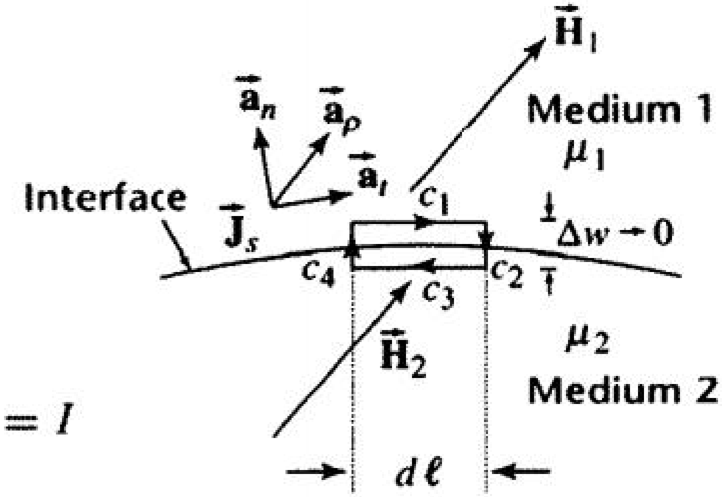
$$\int_{s_1} \vec{\mathbf{B}} \cdot \vec{ds} + \int_{s_2} \vec{\mathbf{B}} \cdot \vec{ds} = 0$$

  - Taking the direction of s_1 and s_2 into consideration, we get

$$\int_{s_1} B_{n1}\, ds_1 - \int_{s_2} B_{n2}\, ds_2 = 0 \quad \Rightarrow \quad \int_s (B_{n1} - B_{n2})\, ds = 0$$

  - Therefore, $B_{n1} = B_{n2}$, or $\vec{\mathbf{a}}_n \cdot (\vec{\mathbf{B}}_1 - \vec{\mathbf{B}}_2) = 0$

4.2 Boundary Conditions - Tangential



- Consider a close path $c_1c_2c_3c_4$, with vanishing sides c_2 and c_4
  - Applying Ampere's law to the close path, we get

$$\oint_c \vec{\mathbf{H}}\cdot\vec{d\ell} = \int_{c_1}\vec{\mathbf{H}}\cdot\vec{d\ell} + \cancel{\int_{c_2}\vec{\mathbf{H}}\cdot\vec{d\ell}} + \int_{c_3}\vec{\mathbf{H}}\cdot\vec{d\ell} + \cancel{\int_{c_4}\vec{\mathbf{H}}\cdot\vec{d\ell}} = I$$

    - where I is the total current enclosed by the loop
  - Neglecting the terms for the vanishing sides c_2 and c_4, the equation becomes

$$\int_{c_1}\vec{\mathbf{H}}\cdot\vec{d\ell} + \int_{c_3}\vec{\mathbf{H}}\cdot\vec{d\ell} = \int_{c_1}(\vec{\mathbf{H}}_1 - \vec{\mathbf{H}}_2)\cdot\vec{\mathbf{a}}_t\,d\ell = I = \int_s \vec{\mathbf{J}}_v\cdot\vec{\mathbf{a}}_\rho\,d\ell\,\Delta w = \int_{c_1}\vec{\mathbf{J}}_s\cdot\vec{\mathbf{a}}_\rho\,d\ell$$

  - So $H_{t1} - H_{t2} = J_s$, or $\vec{\mathbf{a}}_n \times (\vec{\mathbf{H}}_1 - \vec{\mathbf{H}}_2) = \vec{\mathbf{J}}_s$

Quiz 5

- Assume that a plane located at $y = 0$ separates 2 mediums. Medium 1 is in $y > 0$ with relative permeability $\mu_{r1} = 2$ and medium 2 is in $y < 0$ with relative permeability $\mu_{r2} = 1$. The magnetic field intensity vector in medium 1 near the boundary is $\boldsymbol{H_1} = (4\hat{\boldsymbol{x}} - 2\hat{\boldsymbol{y}} + 8\hat{\boldsymbol{z}})\ A/m$.
  - If no free current density exists on the boundary ($J_s = 0$), find the magnetic field intensity vector $\mathbf{H_2}$ in medium 2 near the boundary;
  - If the free current on the boundary is $\boldsymbol{J_s} = 3\hat{\boldsymbol{x}}$, find the magnetic field intensity vector $\mathbf{H_2}$ in medium 2 near the boundary.

Next ...

- Faraday’s Law
- Electromagnetic Induction