CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 8-2 EM Wave Propagation

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OUTLINE

- ➢ Electromagnetic (EM) Waves and Spectrum
- ➢ General Wave Equations
  - ✓ Source-free Medium
  - ✓ TEM (Transverse electromagnetic) Waves
  - ✓ Forward and Backward Travelling Waves
- ➢ Plane Wave in Different Media
  - ✓ In Boundless Dielectric Medium
  - ✓ In Free Space

1.1 EM WAVES

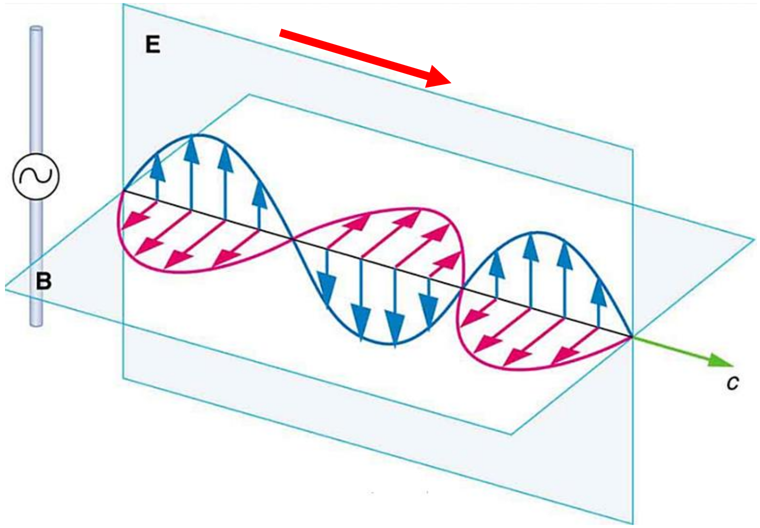
EM wave, *i.e.*, travelling electric and magnetic fields, is one of the most fundamental phenomena of electromagnetism, behaving as waves propagating through space. It is the **consequence** of general Maxwell's equations.

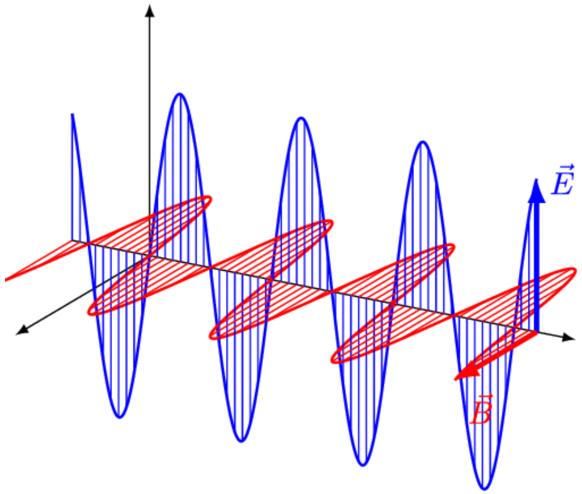
- In vacuum, it propagates at a characteristic speed, the speed of light c, normally in straight lines.
- As an EM wave, it has both ***electric*** and ***magnetic*** field components, which oscillate in a fixed relationship to the other.

Source-free

Medium-free

Centre-free

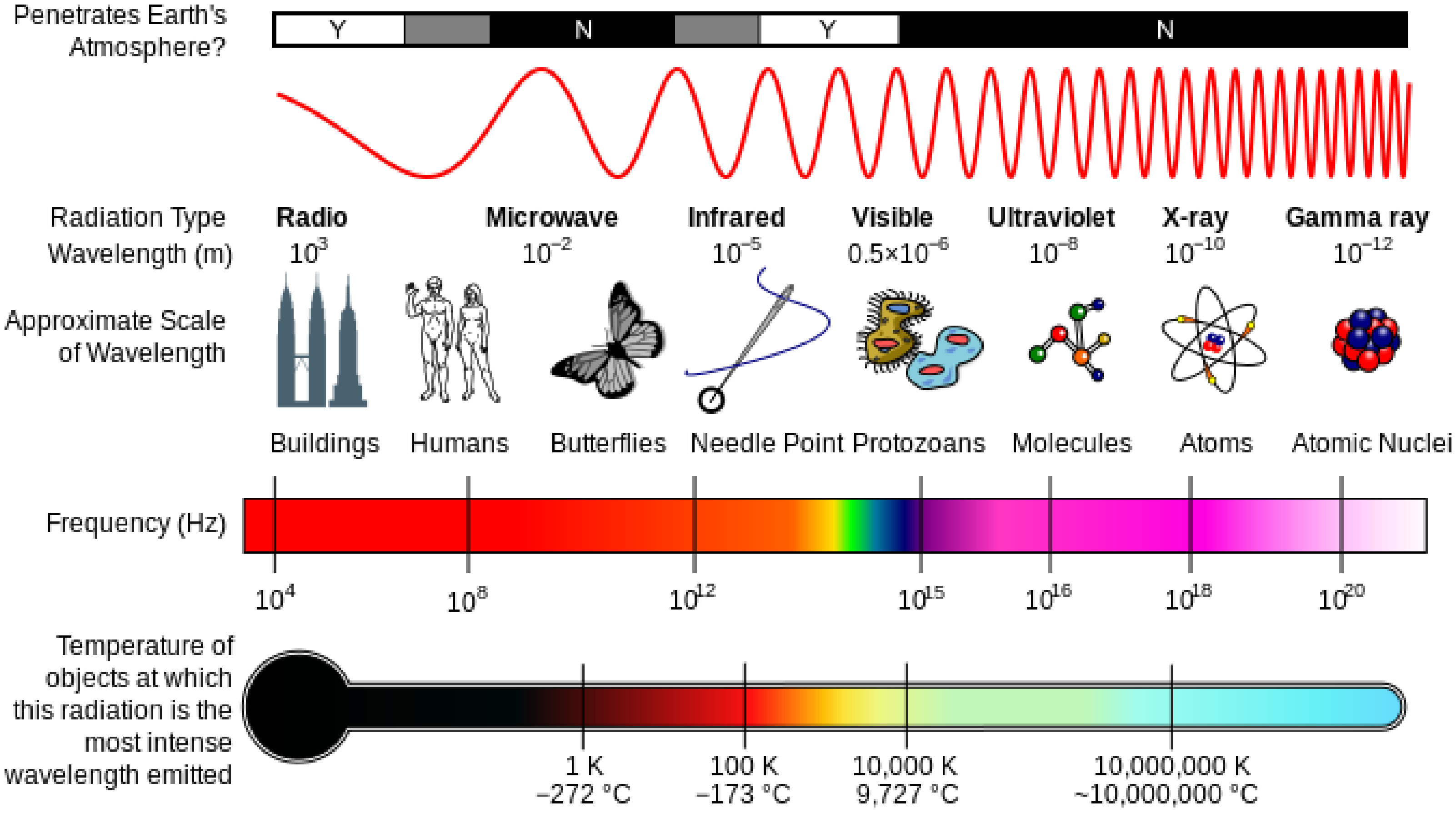




1.2 EM SPECTRUM

An EM wave is characterised by its ***frequency*** or ***wavelength***.

The range of all possible frequencies of electromagnetic radiation is called the **EM spectrum**.



$$\vec{E} = A\cos(\underbrace{\omega}_{2\pi f} t\text{-}\beta z)\ \hat{a}_y$$

In Vacuum:

$$\lambda = \frac{c_0}{f}$$

$c_0 = 3\text{x}10^8$ m/s the speed of light in vacuum

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Differential equations that relate a quantity's 2nd derivative in time to its 2nd derivative in space.

Solutions: Waves

2.1 SOURCE-FREE MEDIUM

$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \boxed{\nabla^2 \vec{A}}$$

Laplacian Operator

Maxwell's equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (1)$$

$$\nabla \times \vec{H} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad (2)$$

$$\nabla \cdot \vec{H} = 0 \qquad (3)$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad (4)$$

These equations are in terms of two variables ($\vec{E}$ and $\vec{H}$).

Consider a **source-free** ($\rho_v = 0$, $\mathbf{J} = 0$) medium having permittivity ε and permeability μ:

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad (2')$$

$$\nabla \cdot \vec{E} = 0 \qquad (4')$$

Take the curl of the eq. (1):

$$\nabla \times \nabla \times \vec{E} = -\nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right) = \nabla(\cancel{\nabla \cdot \vec{E}}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$$

Then,

$$-\nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \vec{H}) = -\mu\left(\sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}\right) = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Similarly:

$$\nabla^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

Derivation is NOT required

The presence of the 1st-order term in the 2nd ODE indicates that the fields decay (lose energy) as they propagate through the medium.

→ a conducting medium is called a ***lossy medium***.

CONTINUING…

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

Both are vector equations & contain 3 components.

This is a set of 6 scalar independent equations.

These equations, called the **homogeneous vector Helmholtz (wave) equations**, represent a set of **six** scalar equations.

The absence of the 1st-order term signifies that the EM fields do not decay as they propagate in a lossless medium.

Perfect dielectric (or lossless medium) is the type of medium with $\sigma = 0$.

SIX EQUATIONS OBTAINED SO FAR

$$\frac{\partial^2 \mathrm{E}_x(x,y,z,t)}{\partial x^2}+\frac{\partial^2 \mathrm{E}_x(x,y,z,t)}{\partial y^2}+\frac{\partial^2 \mathrm{E}_x(x,y,z,t)}{\partial z^2}=\mu\varepsilon\frac{\partial^2 \mathrm{E}_x(x,y,z,t)}{\partial t^2} \qquad (1)$$

$$\frac{\partial^2 \mathrm{E}_y(x,y,z,t)}{\partial x^2}+\frac{\partial^2 \mathrm{E}_y(x,y,z,t)}{\partial y^2}+\frac{\partial^2 \mathrm{E}_y(x,y,z,t)}{\partial z^2}=\mu\varepsilon\frac{\partial^2 \mathrm{E}_y(x,y,z,t)}{\partial t^2} \qquad (2)$$

$$\frac{\partial^2 \mathrm{E}_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 \mathrm{E}_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 \mathrm{E}_z(x,y,z,t)}{\partial z^2}=\mu\varepsilon\frac{\partial^2 \mathrm{E}_z(x,y,z,t)}{\partial t^2} \qquad (3)$$

$$\frac{\partial^2 H_x(x,y,z,t)}{\partial x^2}+\frac{\partial^2 H_x(x,y,z,t)}{\partial y^2}+\frac{\partial^2 H_x(x,y,z,t)}{\partial z^2}=\mu\varepsilon\frac{\partial^2 H_x(x,y,z,t)}{\partial t^2} \qquad (4)$$

$$\frac{\partial^2 H_y(x,y,z,t)}{\partial x^2}+\frac{\partial^2 H_y(x,y,z,t)}{\partial y^2}+\frac{\partial^2 H_y(x,y,z,t)}{\partial z^2}=\mu\varepsilon\frac{\partial^2 H_y(x,y,z,t)}{\partial t^2} \qquad (5)$$

$$\frac{\partial^2 H_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 H_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 H_z(x,y,z,t)}{\partial z^2}=\mu\varepsilon\frac{\partial^2 H_z(x,y,z,t)}{\partial t^2} \qquad (6)$$

2.2 UNIFORM PLANE WAVE (均匀平面波)

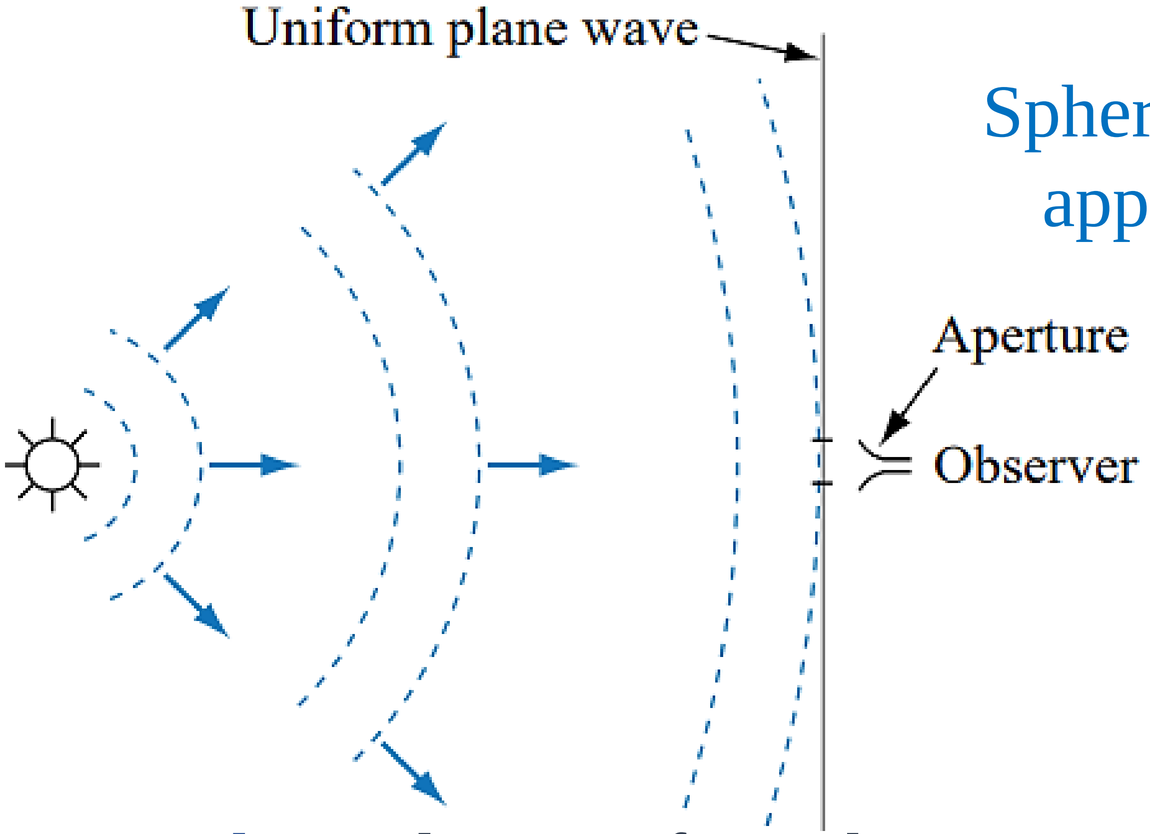
- An electromagnetic wave originates from a point in free space, spreads out uniformly in all directions, and it forms a spherical wave front.
- An observer at a large distance from the source is able to observe only a small part of the wave and the wave appears to him as a plane wave.
- For such a wave the electric field $\vec{E}$ and the magnetic field $\vec{H}$ are **perpendicular** to each other and to the direction of propagation.
- A uniform plane wave is one in which $\vec{E}$ and $\vec{H}$ lie in a plane and have the **same** value everywhere in that plane at any fixed instant.

What kind of plane wave can be considered as "uniform"?

Planar wavefront and uniform (constant) distributions of fields over every plane perpendicular to the direction of wave propagation (与波传播方向垂直的平面上，电场与磁场强度方向、振幅、相位都不变).

2.2 UNIFORM PLANE WAVE

- An electromagnetic wave originates from a point in free space, spreads out uniformly in all directions, and it forms a spherical wave front.

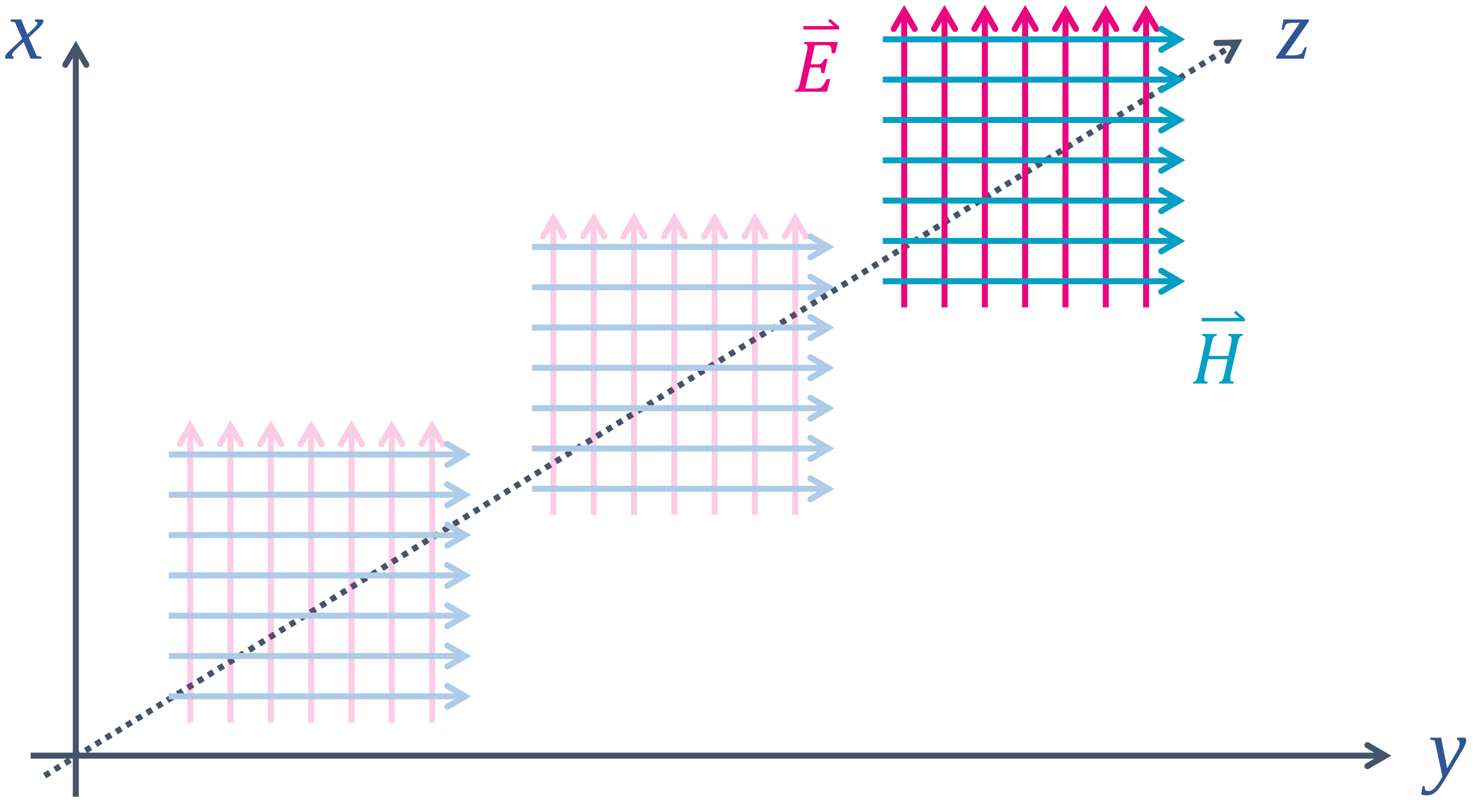


Spherically radiated wave appears locally planar

- An observer at a large distance from the source is able to observe only a small part of the wave and the wave appears to him as a plane wave.

2.2 UNIFORM PLANE WAVE

➢ For such a wave the electric field $\vec{E}$ and the magnetic field $\vec{H}$ are **perpendicular** to each other and to the direction of propagation.



➢ A uniform plane wave is one in which $\vec{E}$ and $\vec{H}$ lie in a plane and have the **same** value everywhere in that plane at any fixed instant.

2.2 UNIFORM PLANE WAVE

- For a uniform plane wave travelling in the z direction, the **space variations** of $\vec{E}$ and $\vec{H}$ are **zero** over a z = constant plane. It means $\vec{E}$ and $\vec{H}$ fields have **no** components in the **longitudinal direction** (the direction of wave propagation).

- The related fields (wave travels along the z direction) have neither x nor y dependence. It means $\vec{E}$ and $\vec{H}$ are not functions of x & y.

$$\frac{\partial \vec{E}}{\partial x} = 0 \qquad \frac{\partial \vec{E}}{\partial y} = 0 \qquad \frac{\partial \vec{H}}{\partial x} = 0 \qquad \frac{\partial \vec{H}}{\partial y} = 0$$

Examples:

$$\vec{E} = E_0 e^{-jz} \hat{a}_y \qquad \vec{E} = E_0(x, y) e^{-jz} \hat{a}_y$$

Such a wave is one kind of ***TEM (transverse electromagnetic)*** waves.

SIX EQUATIONS OBTAINED SO FAR

$$\cancel{\frac{\partial^2 \mathrm{E}_x(x,y,z,t)}{\partial x^2}} + \cancel{\frac{\partial^2 \mathrm{E}_x(x,y,z,t)}{\partial y^2}} + \frac{\partial^2 \mathrm{E}_x(x,y,z,t)}{\partial z^2} = \mu\varepsilon \frac{\partial^2 \mathrm{E}_x(x,y,z,t)}{\partial t^2} \quad (1)$$

$$\cancel{\frac{\partial^2 \mathrm{E}_y(x,y,z,t)}{\partial x^2}} + \cancel{\frac{\partial^2 \mathrm{E}_y(x,y,z,t)}{\partial y^2}} + \frac{\partial^2 \mathrm{E}_y(x,y,z,t)}{\partial z^2} = \mu\varepsilon \frac{\partial^2 \mathrm{E}_y(x,y,z,t)}{\partial t^2} \quad (2)$$

$$\frac{\partial^2 \mathrm{E}_z(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \mathrm{E}_z(x,y,z,t)}{\partial y^2} + \frac{\partial^2 \mathrm{E}_z(x,y,z,t)}{\partial z^2} = \mu\varepsilon \frac{\partial^2 \mathrm{E}_z(x,y,z,t)}{\partial t^2} \quad (3)$$

$$\cancel{\frac{\partial^2 H_x(x,y,z,t)}{\partial x^2}} + \cancel{\frac{\partial^2 H_x(x,y,z,t)}{\partial y^2}} + \frac{\partial^2 H_x(x,y,z,t)}{\partial z^2} = \mu\varepsilon \frac{\partial^2 H_x(x,y,z,t)}{\partial t^2} \quad (4)$$

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$$\frac{\partial^2 H_z(x,y,z,t)}{\partial x^2} + \frac{\partial^2 H_z(x,y,z,t)}{\partial y^2} + \frac{\partial^2 H_z(x,y,z,t)}{\partial z^2} = \mu\varepsilon \frac{\partial^2 H_z(x,y,z,t)}{\partial t^2} \quad (6)$$

2.2 SINUSOIDAL TIME VARIATIONS

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0 \qquad \frac{\partial^2 H_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 H_x}{\partial t^2} = 0$$

$$\frac{\partial^2 E_y}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} = 0 \qquad \frac{\partial^2 H_y}{\partial z^2} - \mu\varepsilon \frac{\partial^2 H_y}{\partial t^2} = 0$$

- ✓ The field components are functions of *z* (direction of propagation) and *t* (time) only.
- ✓ These equations are similar → solutions are also similar.
- ✓ Each one is a 2nd ODE with two possible solutions.

where E_x, E_y, H_x and H_y are the ***transverse components*** of $\vec{E}$ and $\vec{H}$.

It is of particular interest to consider the time-harmonic fields, the time variation of which takes the form of a sinusoidal function.

As all time-harmonic functions involve the common factor $e^{j\omega t}$ in their phasor form expressions, we can eliminate this factor when dealing with the Maxwell's equations.

2.2 SINUSOIDAL TIME VARIATIONS

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0 \qquad \frac{\partial^2 H_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 H_x}{\partial t^2} = 0$$

$$\frac{\partial^2 E_y}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} = 0 \qquad \frac{\partial^2 H_y}{\partial z^2} - \mu\varepsilon \frac{\partial^2 H_y}{\partial t^2} = 0$$

- ✓ The field components are functions of z (direction of propagation) and t (time) only.
- ✓ These equations are similar → solutions are also similar.
- ✓ Each one is a 2nd ODE with two possible solutions.

The wave equations can now be put in phasor form such as:

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0$$

$$\frac{\partial^2 H_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 H_x}{\partial t^2} = 0$$

General Homogeneous Wave equations

$$\frac{\partial}{\partial t} = j\omega$$

$$\frac{d^2 \tilde{E}_x}{dz^2} + \omega^2 \mu\varepsilon \tilde{E}_x = 0$$

$$\frac{d^2 \tilde{H}_x}{dz^2} + \omega^2 \mu\varepsilon \tilde{H}_x = 0$$

Homogeneous Wave equations in **complex time harmonic form**

2.2 SINUSOIDAL TIME VARIATIONS

$$\frac{d^2\tilde{E}_x}{dz^2} + \omega^2\mu\varepsilon\tilde{E}_x = 0$$

2nd order homogenous differential equation (二阶齐次线性微分方程)

For a monochromatic (single frequency) wave propagating in a uniform medium, $\omega^2\mu\varepsilon$ is a constant, define propagation constant (wavenumber) $\beta = \omega\sqrt{\mu\varepsilon}\ rad/m$, then we can rewrite the wave equation:

$$\frac{d^2\tilde{E}_x}{dz^2} + \beta^2\tilde{E}_x = 0$$

There are two solutions for the x component of the E-field:

$$\tilde{E}_x(z) = \tilde{E}_{xf}e^{-j\beta z} \quad \text{and} \quad \tilde{E}_x(z) = \tilde{E}_{xb}e^{j\beta z}$$

So, the general solution is $\tilde{E}_x(z) = \tilde{E}_{xf}e^{-j\beta z} + \tilde{E}_{xb}e^{j\beta z}$

where $\tilde{E}_{xf}$ and $\tilde{E}_{xb}$ are two complex constant:

$$\tilde{E}_{xf} = E_{xf}e^{j\theta_{xf}}, \qquad \tilde{E}_{xb} = E_{xb}e^{j\theta_{xb}}$$

Then

$$\tilde{E}_x(z) = E_{xf}e^{-j(\beta z - \theta_{xf})} + E_{xb}e^{j(\beta z + \theta_{xb})}$$

or

$$E_x(z,t) = E_{xf}\cos(\omega t - \beta z + \theta_{xf}) + E_{xb}\cos(\omega t + \beta z + \theta_{xb})$$

FURTHER DISCUSSION…

$$\lambda = \frac{\lambda_0}{\sqrt{\mu_r \varepsilon_r}}$$ wavelength in medium

Propagation constant (wavenumber):

It has unit of rad/m and is equal to the number of wavelengths in a distance of 2π meters.

free space wavelength

Temporal Domain	Spatial Domain
$\omega = \frac{2\pi}{T}$	$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda_0}\sqrt{\mu_r \varepsilon_r}$
Unit: rad/s	Unit: rad/m
T: temporal period	λ: wavelength (or period in spatial domain)
The number of revolutions in a period of 2π seconds, referred to as angular frequency	The number of wavelengths in a distance of 2π meters, referred to as wavenumber (or spatial frequency)

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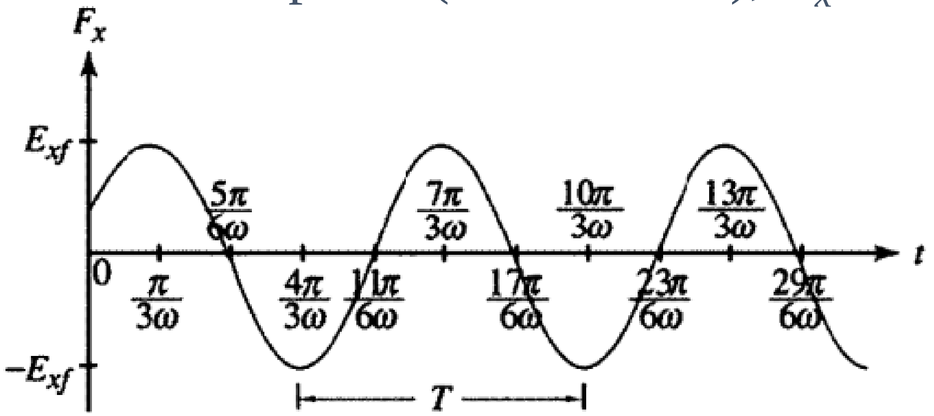
2.3 FORWARD TRAVELLING WAVE

$$E_x(z,t) = E_{xf}\cos(\omega t - \beta z + \theta_{xf}) + E_{xb}\cos(\omega t + \beta z + \theta_{xb})$$

Let's examine the first term $F_x = E_{xf}\cos(\omega t - \beta z + \theta_{xf})$ or $F_x = E_{xf}e^{-j(\beta z - \theta_{xf})}$

At any given point in a transverse plane (z = constant), F_x varies sinusoidally in time.



Travel in the + z direction → **forward** travelling wave

The function F_x also varies with z.

As time progresses, each point on the function moves to the right (forward direction)

→ this term represents a ***forward travelling wave***.

when

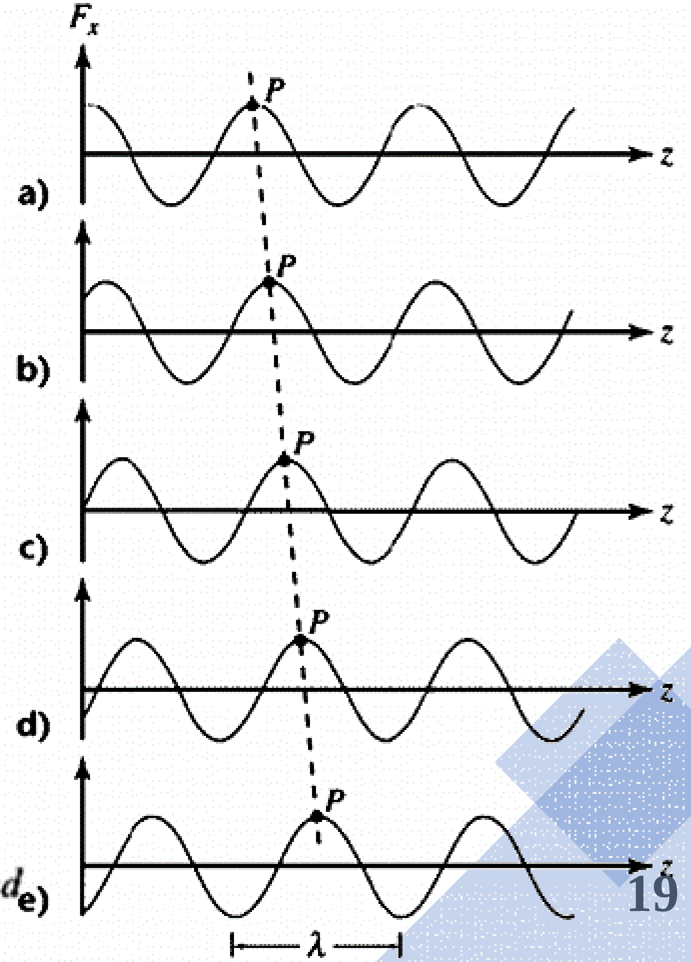
(a) $\omega t = -\theta_{xf}$

(b) $\omega t = \frac{\pi}{4} - \theta_{xf}$

(c) $\omega t = \frac{\pi}{2} - \theta_{xf}$

(d) $\omega t = \frac{3\pi}{4} - \theta_{xf}$ and

(e) $\omega t = \pi - \theta_{xf}$



2.3 FORWARD TRAVELLING WAVE

At any given time (t = constant) the wave returns to its original magnitude and phase when z increases by a wavelength λ:

$$\beta\lambda = 2\pi$$

The **wavelength** is the distance between two planes when the phase difference between them at any given time is 2π radian:

$$\lambda = \frac{2\pi}{\beta}$$

The **phase velocity** (wave speed):

$$\vec{u}_p = \frac{\omega}{\beta}\hat{a}_{propagation} = \frac{c_0}{\sqrt{\mu_r\varepsilon_r}}\hat{a}_{propagation}$$

where ***the speed of light*** $c_0 = 3 \times 10^8 \mathrm{m/s}$

index of refraction $n = \sqrt{\mu_r\varepsilon_r}$

Phase velocity is **independent** of frequency!

2.3 BACKWARD TRAVELLING WAVE

$$E_x(z,t) = E_{xf}\cos\left(\omega t - \beta z + \theta_{xf}\right) + E_{xb}\cos(\omega t + \beta z + \theta_{xb})$$

The second term represents a backward travelling wave, since it moves in the negative z direction as time progresses.

Thus, the wave travels in the **backward** direction with a phase velocity of

$$\vec{u}_p = -\frac{\omega}{\beta}\hat{a}_{propagation}$$

Similarly, we can get a solution for the y component of the $\vec{E}$ field, as

- In phasor form: $\tilde{E}_y(z) = E_{yf}e^{-j\left(\beta z - \theta_{yf}\right)} + E_{yb}e^{j\left(\beta z + \theta_{yb}\right)}$
- In time domain: $E_y(z,t) = E_{yf}\cos\left(\omega t - \beta z + \theta_{yf}\right) + E_{yb}\cos\left(\omega t + \beta z + \theta_{yb}\right)$

The solutions for H_x and H_y are similar.

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3.1 IN BOUNDLESS DIELECTRIC MEDIUM

Assume: i) the dielectric medium is of infinite extend

ii) there is **only one** wave propagating along the (*e.g.*,) *z*-direction

=> only the **forward wave** is propagating.

Then the *x* and *y* components are: $\tilde{E}_x(z) = E_{xf} e^{-j(\beta z - \theta_{xf})}$, $\tilde{E}_y(z) = E_{yf} e^{-j(\beta z - \theta_{yf})}$

Using the Maxwell's eq.(1), get the *x* and *y* of $\vec{H}$ field as:

$$\tilde{H}_x(z) = -\sqrt{\frac{\varepsilon}{\mu}}\tilde{E}_y(z), \quad \tilde{H}_y(z) = \sqrt{\frac{\varepsilon}{\mu}}\tilde{E}_x(z)$$

The $\vec{E}$ and $\vec{H}$ relationship can be written as $\hat{a}_{propagation} \times \vec{E} = \sqrt{\frac{\mu}{\varepsilon}}\vec{H} = \eta\vec{H}$

where the **intrinsic (or wave) impedance**: $\eta = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$

Intrinsic impedance for the wave in free space: $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ \Omega$

3.2 IN FREE SPACE

Free space (or vacuum) is a <u>special case</u> of a dielectric medium in which $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$

$c_0 = 3\text{x}10^8$ m/s: speed of light in vacuum

Phase constant	$\beta_0 = \omega\sqrt{\mu_0\varepsilon_0} = \dfrac{\omega}{c_0}$
Wave speed	$u_p = \dfrac{\omega}{\beta_0} = c_0$
Wavelength	$\lambda_0 = \dfrac{2\pi}{\beta_0} = \dfrac{c_0}{f}$
Intrinsic impedance	$\eta_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 377\ \Omega$

An EM wave propagates in free space travelling with the speed of light.

QUIZ 1

If the electric field intensity as given by

$$\vec{E} = 377\cos(10^9 t - 5y)\,\hat{z}\ V/m$$

It represents a uniform plane wave propagating in the $+y$ direction in a dielectric medium ($\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$).

Determine the following:

a) the propagation velocity

b) the relative permittivity

c) the intrinsic impedance

d) the wavelength

e) the magnetic field intensity in phasor domain

QUIZ 2

The electric field intensity of a uniform plane wave in free space is given by

$$\vec{E} = 94.25\cos(\omega t + 6z)\,\hat{x}\;V/m$$

Determine:

a) the propagation velocity

b) the wave frequency

c) the wavelength

d) the magnetic field intensity in phasor domain

e) whether this expression satisfies the Helmholtz equation

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon\frac{\partial^2 E_x}{\partial t^2} = 0$$

or

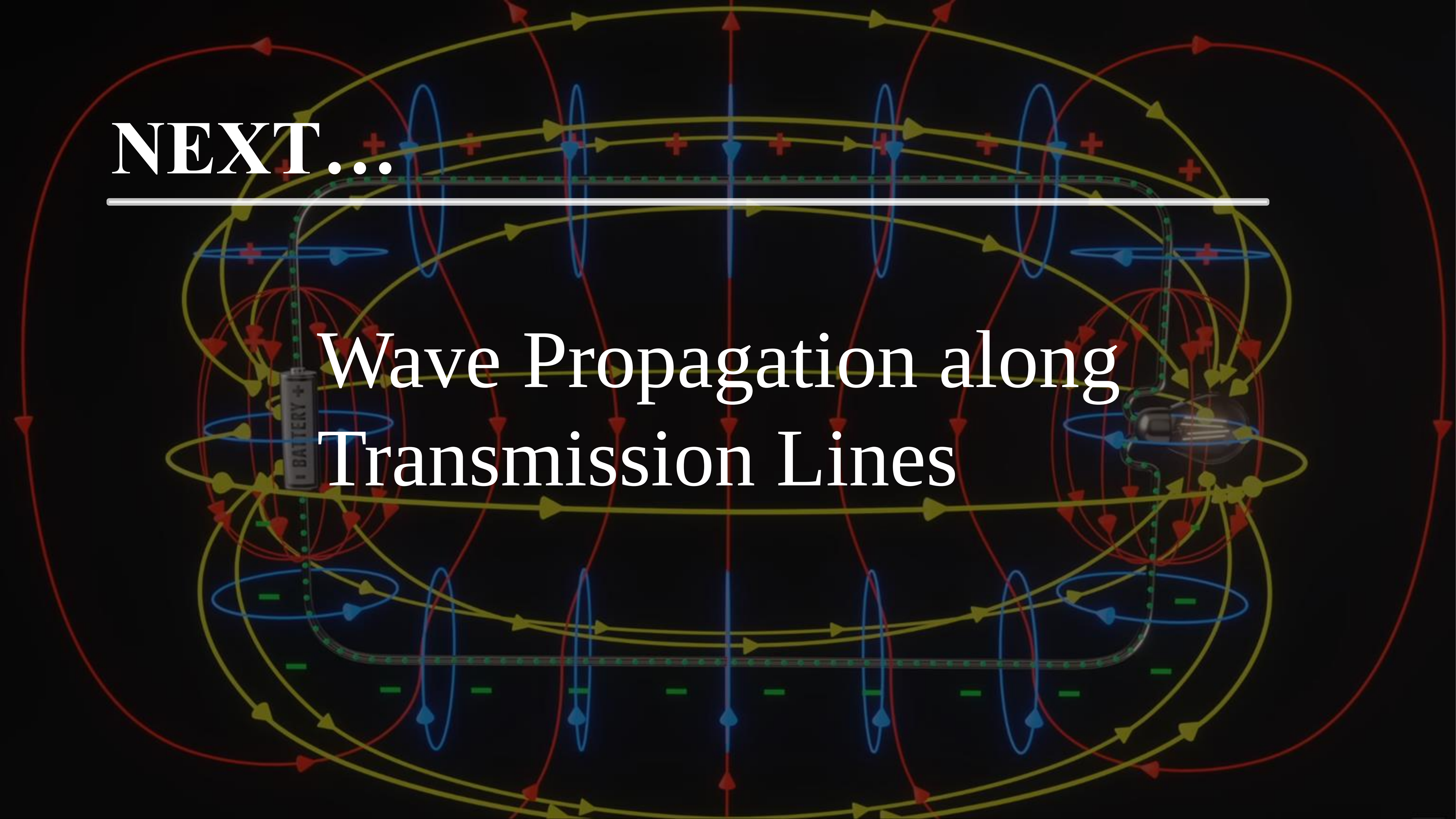
$$\frac{d^2\tilde{E}_x}{dz^2} + \omega^2\mu\varepsilon\tilde{E}_x = 0$$

SUMMARY

- A time-harmonic field is one that varies **periodically** or **sinusoidally** with time.
- Wireless applications are possible because electromagnetic fields can propagate in free space without any guiding structures.
- Plane waves are one example of time-harmonic fields.
- When the electric (E) and magnetic (H) field vectors of a wave are in planes perpendicular to the direction of propagation, say the z-direction, this is called a plane wave.
- Plane waves are good approximations of electromagnetic waves in engineering problems after they propagate a short distance from the source.

* APPENDIX: LAPLACIAN OPERATOR

- Laplacian operator – a 2nd-order differential operator that occurs frequently in the study of field theory.
  - Symbolically written as Δ or ∇^2
  - Defined as the divergence of a gradient of a scalar function.
- In different coordinates:
  - Cartesian: $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$
  - Cylindrical: $\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$
  - Spherical: $\Delta f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2 sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$



NEXT…

Wave Propagation along Transmission Lines