CAN102
Electromagnetism and Electromechanics

2023/24-S2

Lecture 16 DC Machinery Fundamentals

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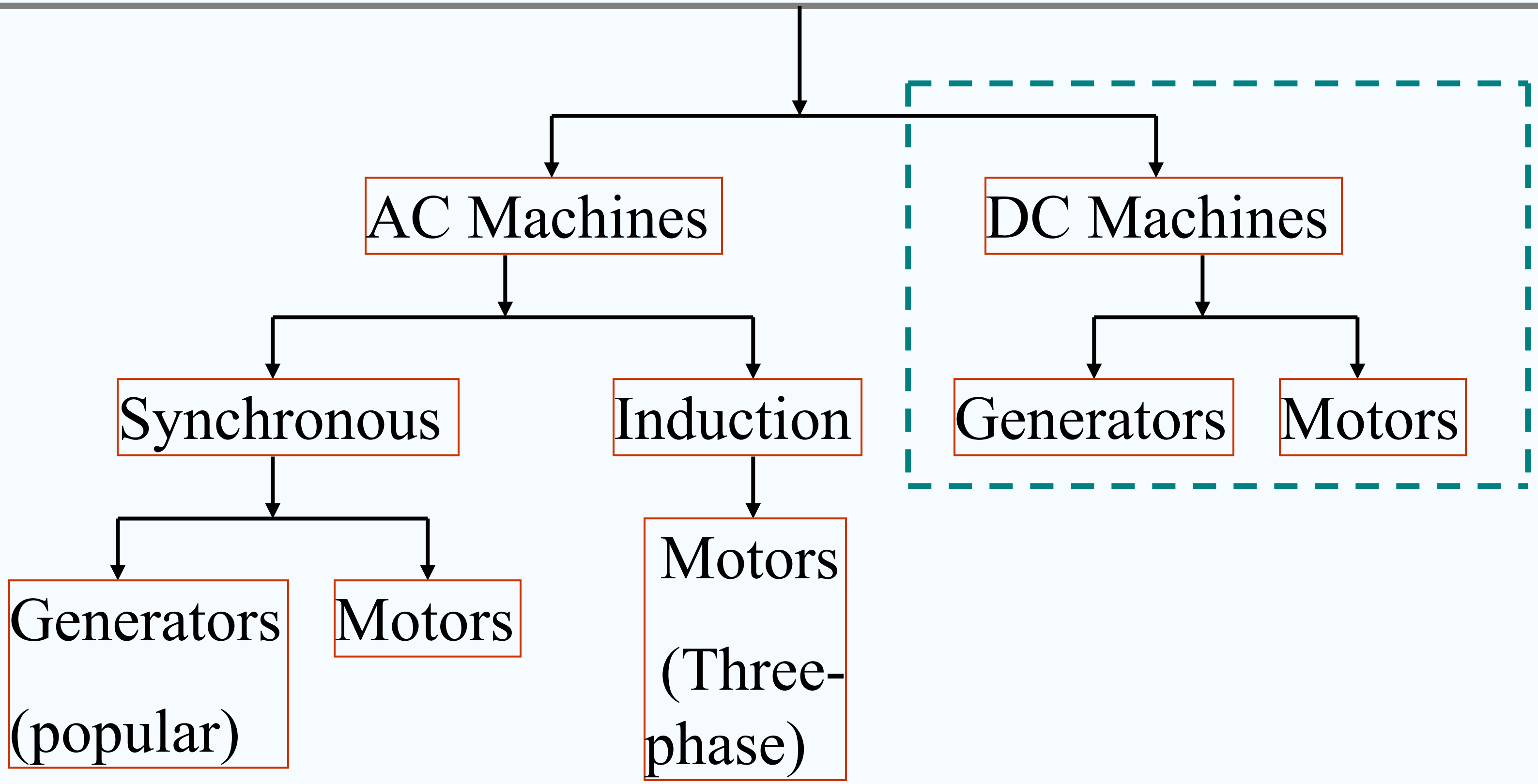
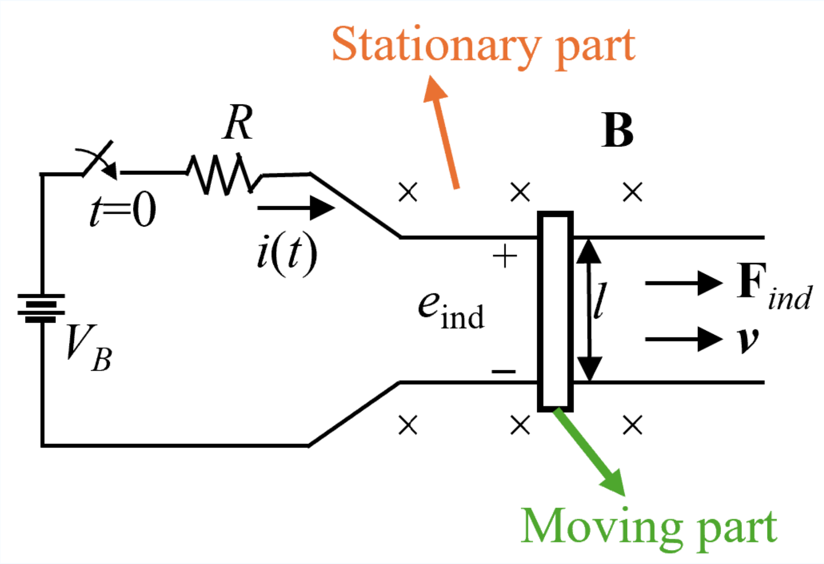
Electrical Machines

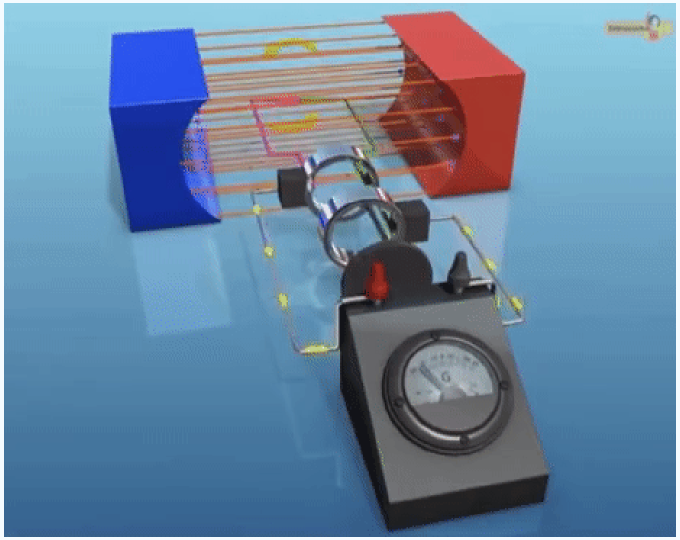


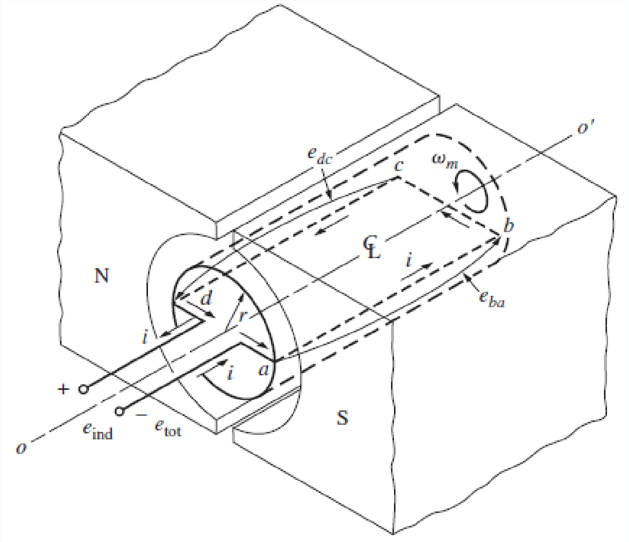
DC Machinery Fundamentals

- ✓ Induced Voltage in Rotating Loop
- ✓ Induced Torque in Rotating Loop

Simple DC Rotating Loop



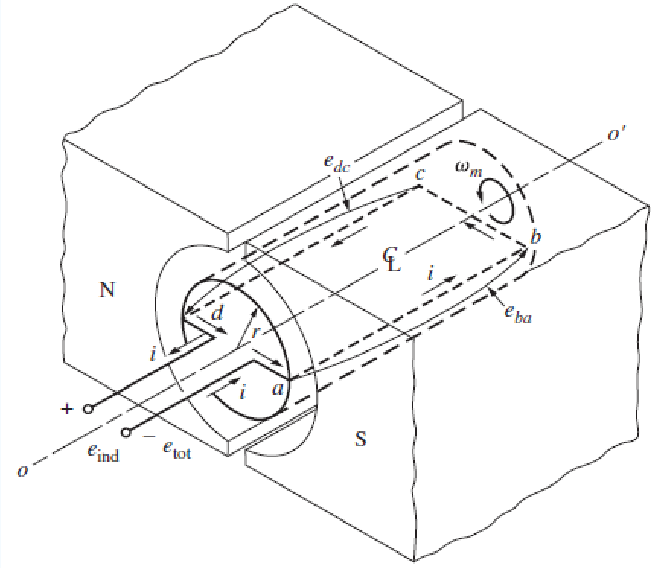


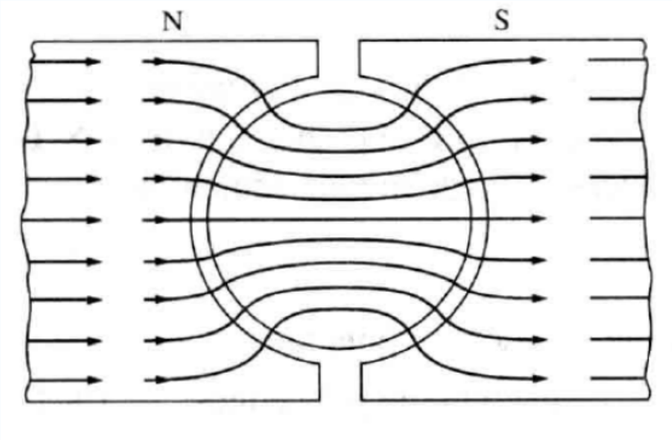


The figure shows a simple rotating loop between curved faces. The rotating part is called the **rotor（转子）**, and the stationary part is called the **stator (静子)**.

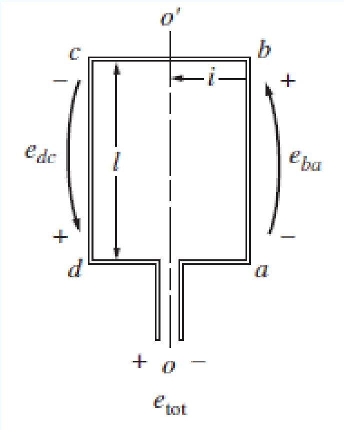
Induced Voltage in a Rotating Loop

- If the rotor of the machine is rotated, a voltage will be induced in the rotating loop.
- The magnetic field **B** is constant and perpendicular to the surface (a cylinder) of the rotor everywhere under the (N/S) poles faces and rapidly falls to zero beyond the pole edges.

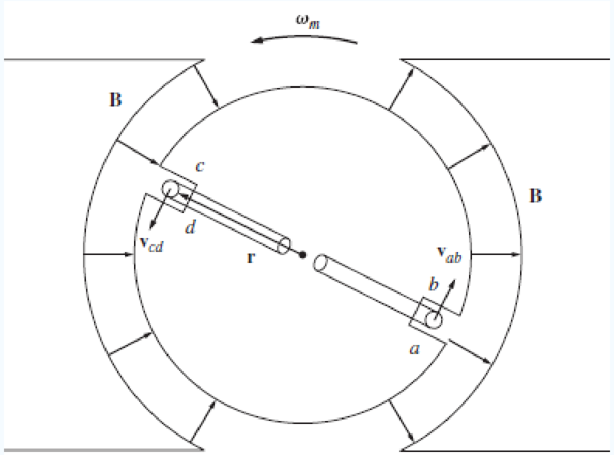




View of field lines



Top View



Front View

Recall

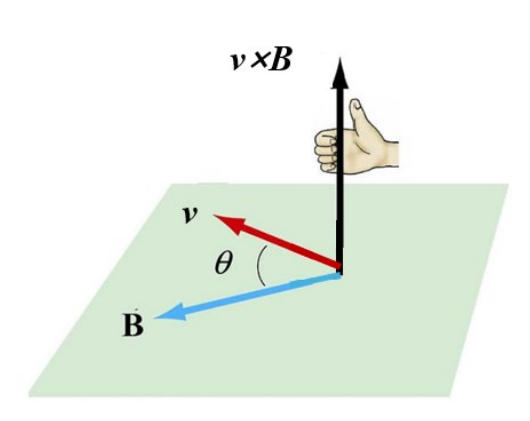
Induced voltage in a moving wire

- The voltage induced in a wire moving in a magnetic field

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$

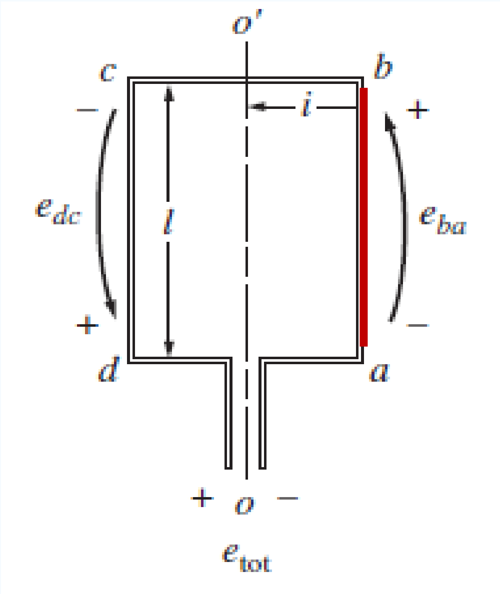
v: velocity of the wire
l: length of conductor in the magnetic field, points along the direction of the wire toward the end making the smallest angle with respect to the vector $\mathbf{v} \times \mathbf{B}$
B: magnetic flux density vector

The positive end of induced voltage in the wire is in the direction of the vector $\mathbf{v} \times \mathbf{B}$



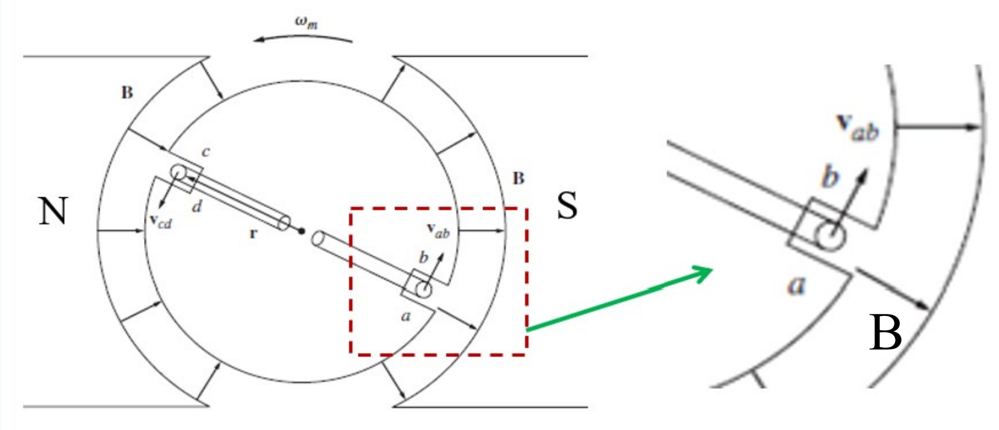
Induced Voltage in a Rotating Loop

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$



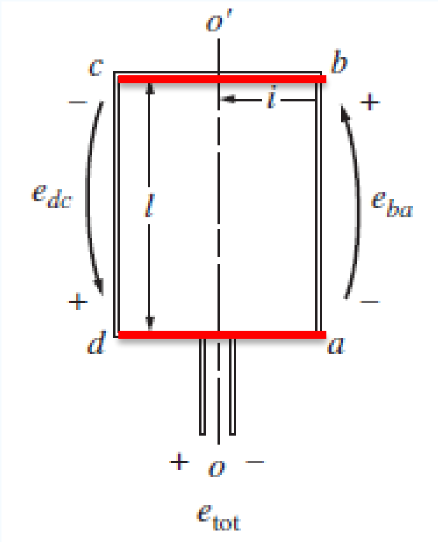
- Segment *ab:*

$$e_{ba} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l} = \begin{cases} vBl & \text{Positive into page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$



Induced Voltage in a Rotating Loop

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$

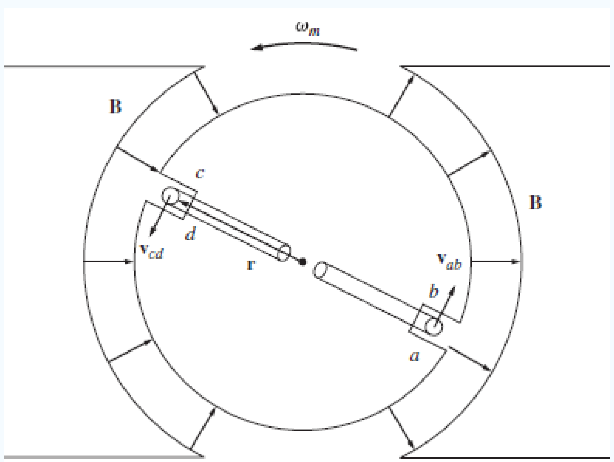


- Segment *bc:*

v × **B** is either into the page (*o'b*) or out of the page (*o'c*).

The length **l** is in the plane of the page.

So **v** × **B** is perpendicular to **l**. Then: $e_{bc} = 0$

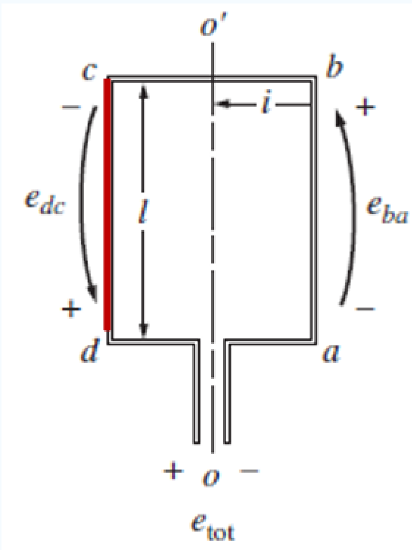


- Segment *da*:

Similar with *bc*:
v x **B** is perpendicular to **l**. So: $e_{da} = 0$

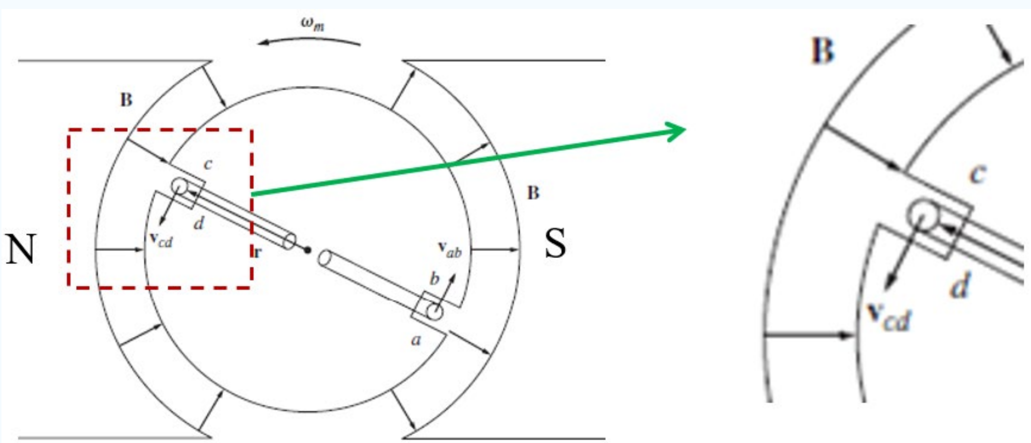
Induced Voltage in a Rotating Loop

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$

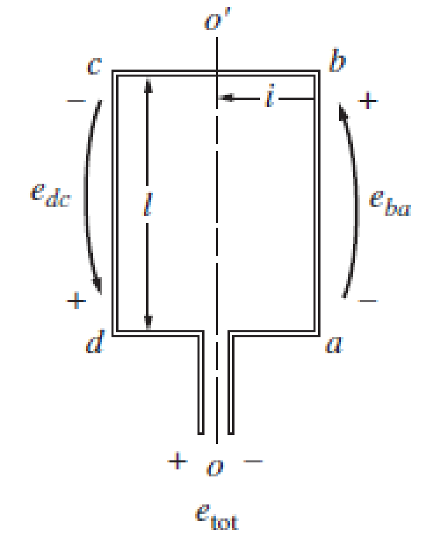


- Segment *cd:*

$$e_{cd} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l} = \begin{cases} vBl & \text{Positive out of page} \quad \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$



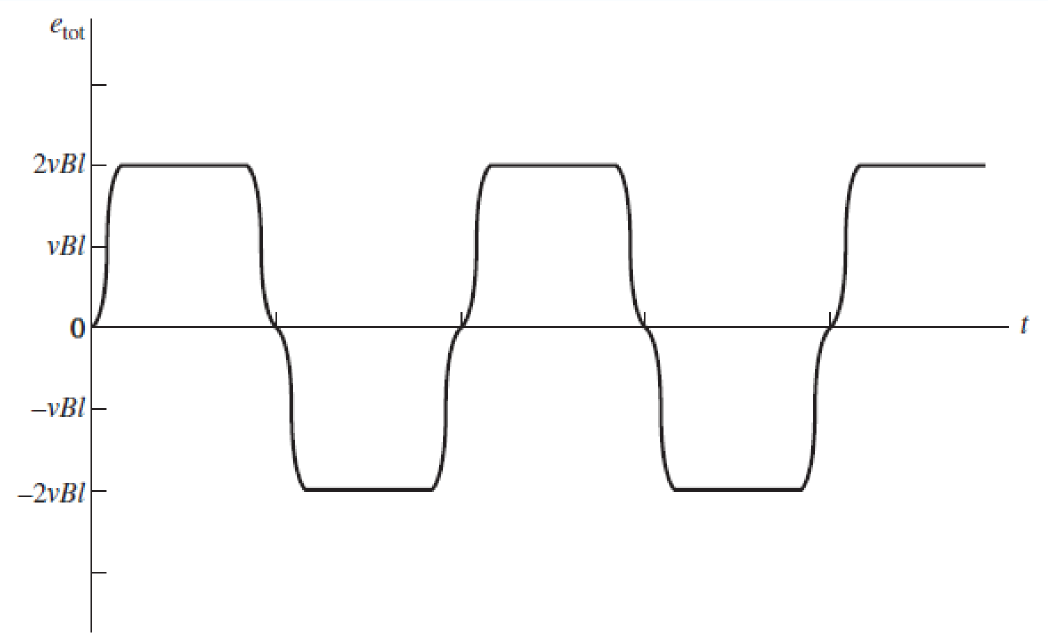
Induced Voltage in a Rotating Loop



The total induced voltage:

$$e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$$\boxed{e_{ind} = \begin{cases} 2vBl & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}}$$



When the loop rotates 180^0:

ab from S to N

dc from N to S

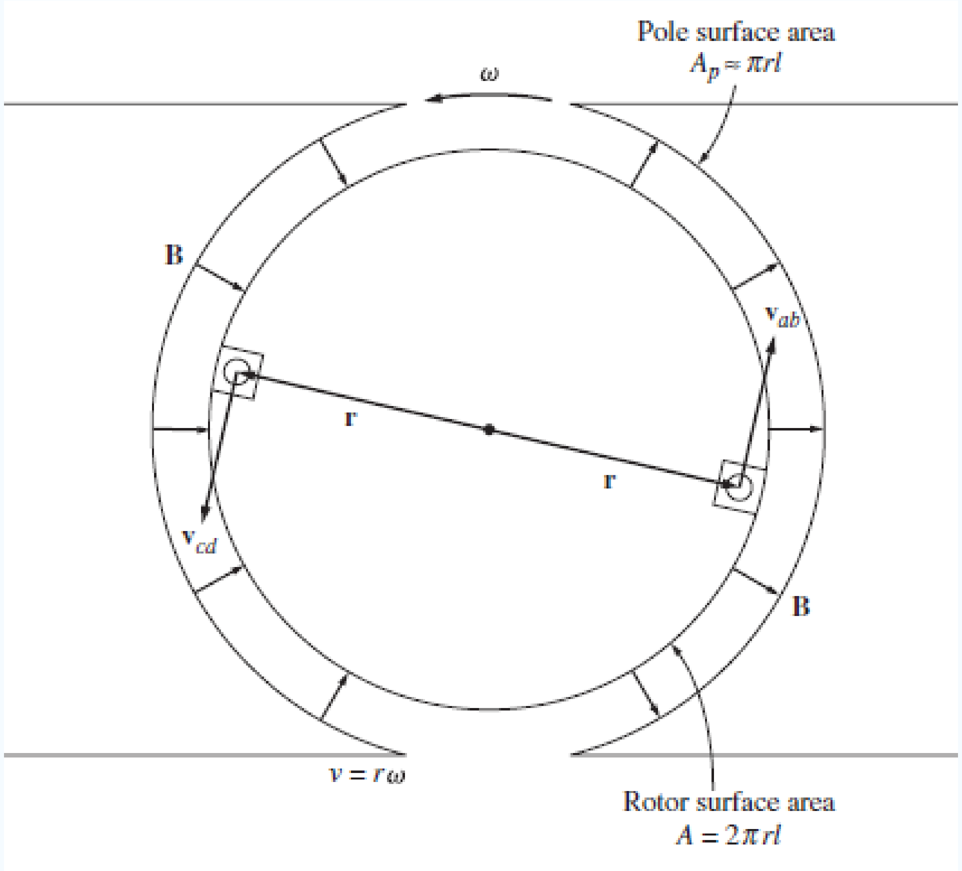
Voltage:

Direction: reverses

Magnitude: same

Induced Voltage in a Rotating Loop

$$e_{ind} = \begin{cases} 2vBl & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$



Alternative expression for real DC machines

Substituting $v = r\omega$ (ω is angular velocity) into the e_{ind} equation:

$$e_{ind} = 2r\omega Bl$$

The rotor surface is a cylinder, so the area of the rotor surface: $A = 2\pi rl$.

Since there are 2 poles, the area under each pole is $A_p = \pi rl$.

$$\boxed{e_{ind} = 2vBl = 2(r\omega)Bl = 2rlB\omega = \frac{2}{\pi} A_P B\omega}$$

Induced Voltage in a Rotating Loop

$$e_{ind} = 2rlB\omega = \frac{2}{\pi} A_P B\omega$$

The magnitude of the flux density **B** is constant everywhere in the air gap under the pole faces, thus, the final form of the voltage equation is:

$$e_{ind} = \begin{cases} \dfrac{2}{\pi}\Phi\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$\text{where } \Phi = A_p B$$

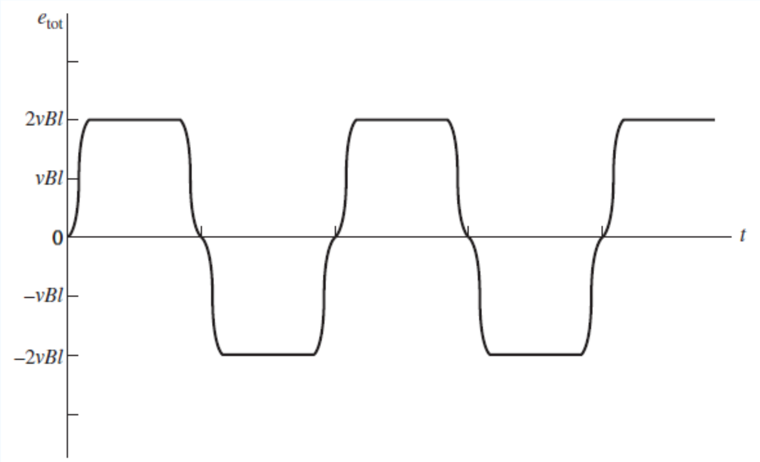
In general, the voltage will depend on the same 3 factors:

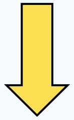
$$e_{ind} = K\Phi\omega$$

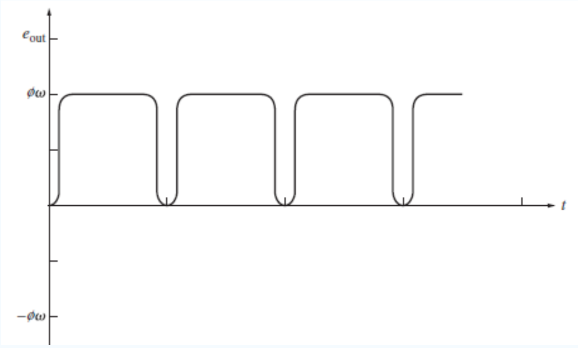
1. the flux in the machine
2. the speed of rotation
3. a constant representing the construction of the machine.

Induced Voltage in a Rotating Loop

How can this machine be made to produce a DC voltage instead of the AC voltage?

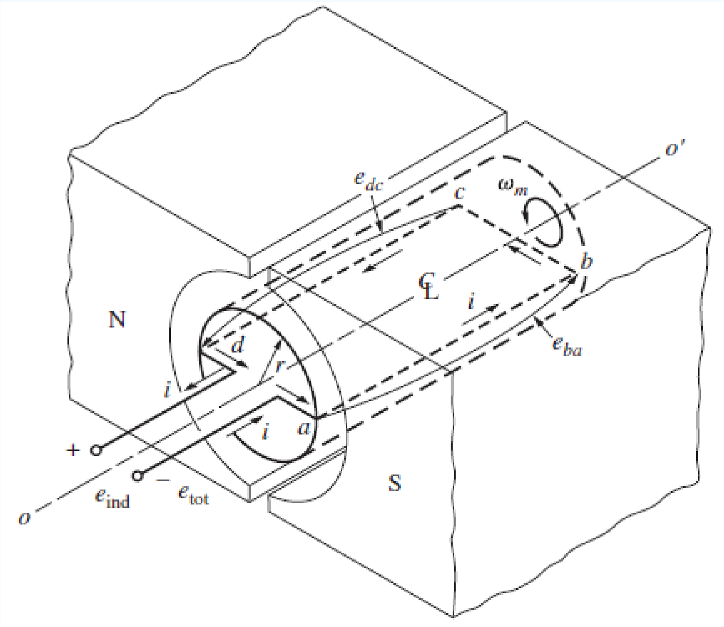


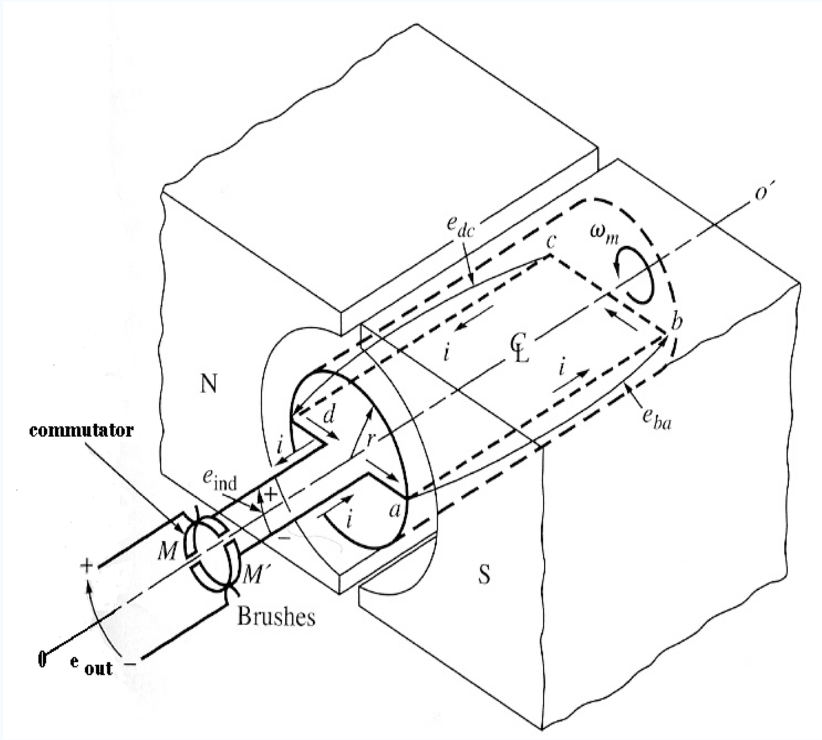




Induced Voltage in a Rotating Loop

Getting DC Voltage out of the Loop

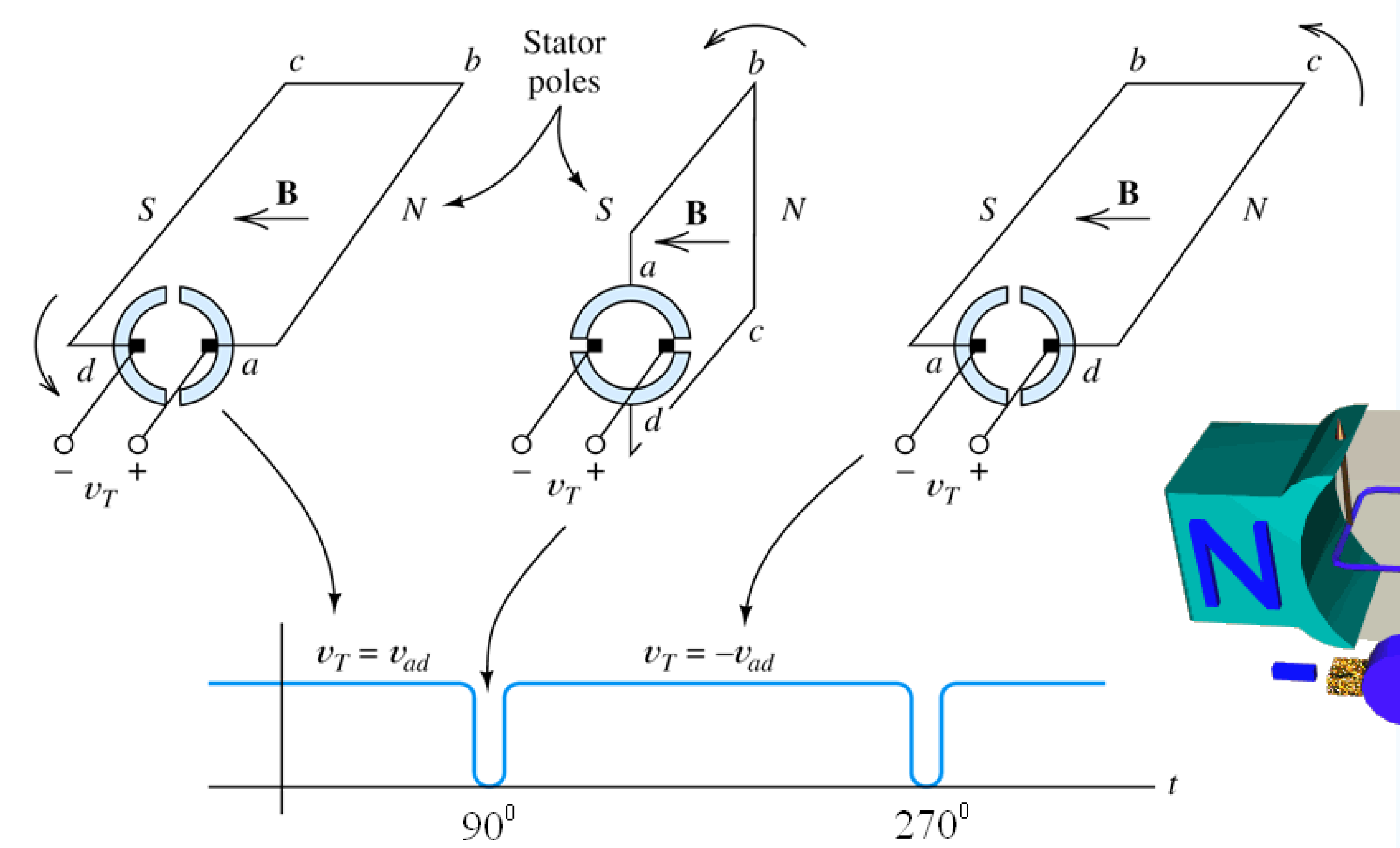


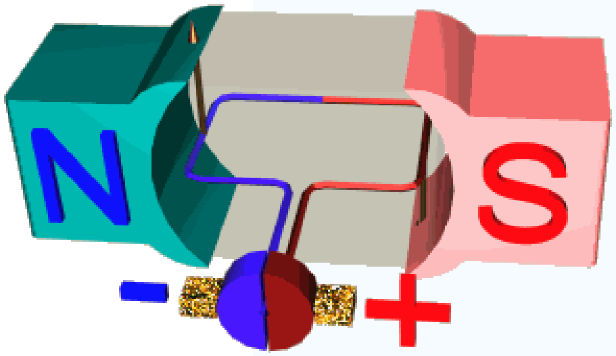


To produce a DC voltage, a mechanism called commutator (换向器) and brushes (电刷) is used.

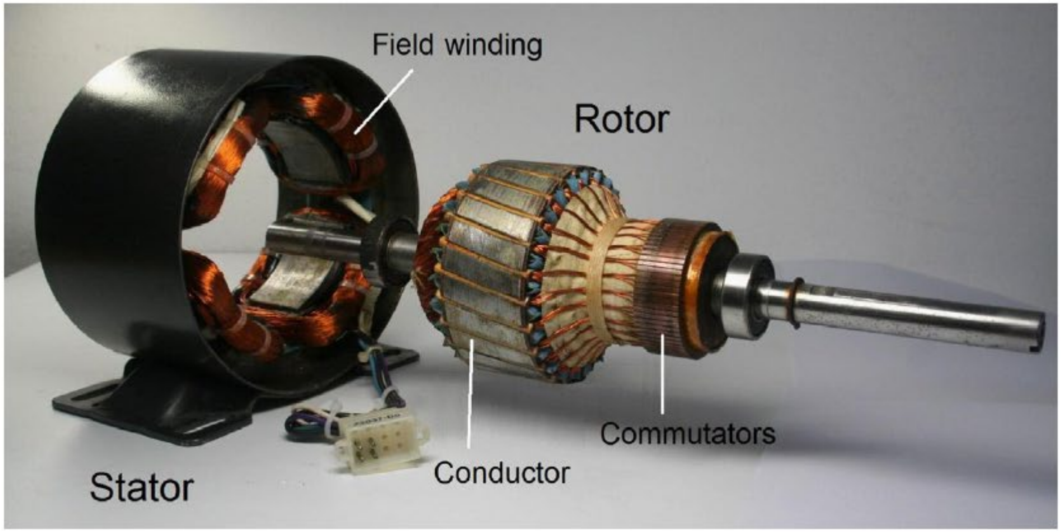
Induced Voltage in a Rotating Loop

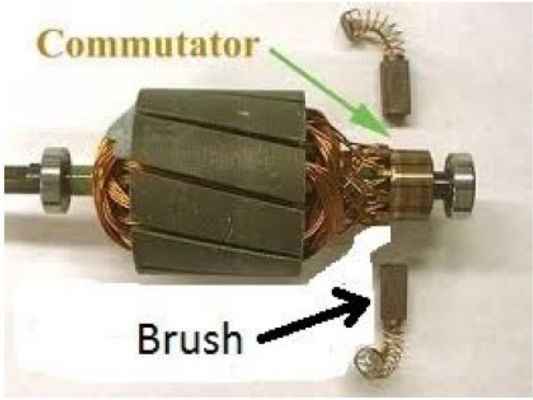
Getting DC Voltage out of the Loop

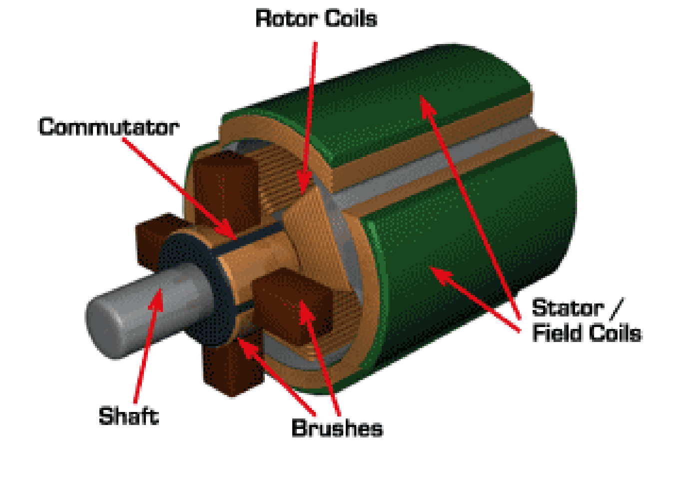


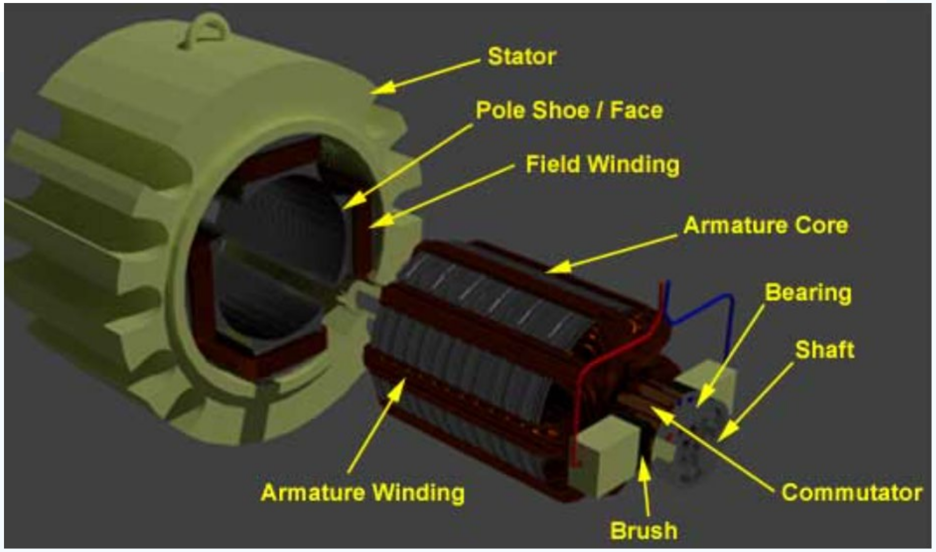


Structure


Field winding
Rotor
Commutators
Conductor
Stator

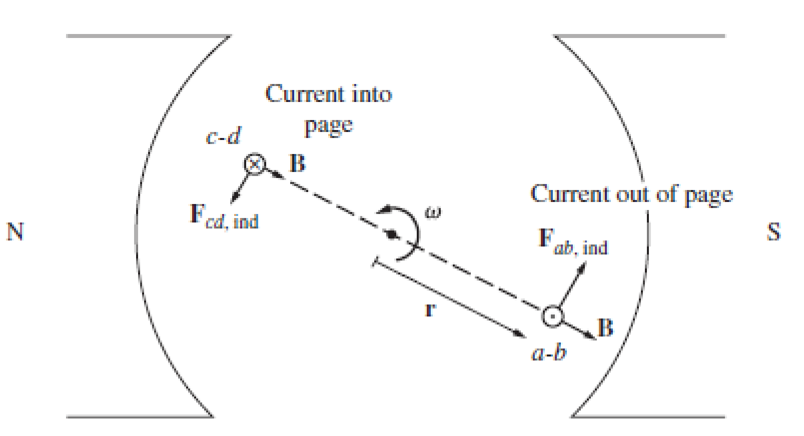

Commutator
Brush

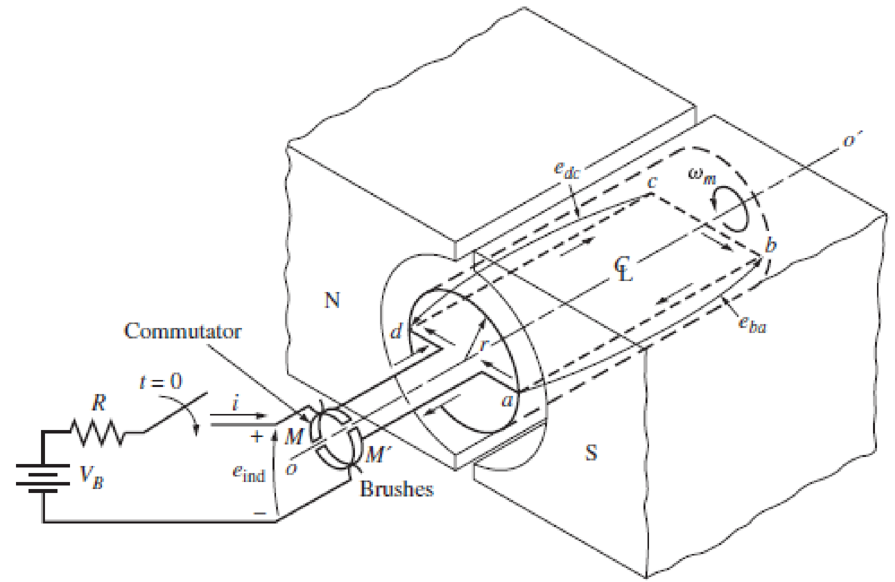

Rotor Coils
Commutator
Stator / Field Coils
Shaft
Brushes


Stator
Pole Shoe / Face
Field Winding
Armature Core
Bearing
Shaft
Armature Winding
Commutator
Brush

The Torque Induced in a Rotating Loop

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$$

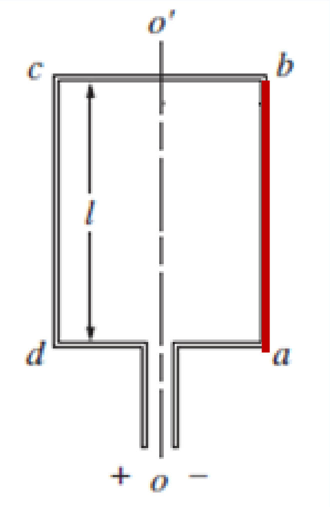




- Segment *ab*:

$$\mathbf{F}_{ab} = i(\mathbf{l} \times \mathbf{B}) = ilB$$ Tangent to direction of motion

$$\tau_{ab} = rF\sin\theta = r(ilB)\sin 90^0 = rilB$$ CCW

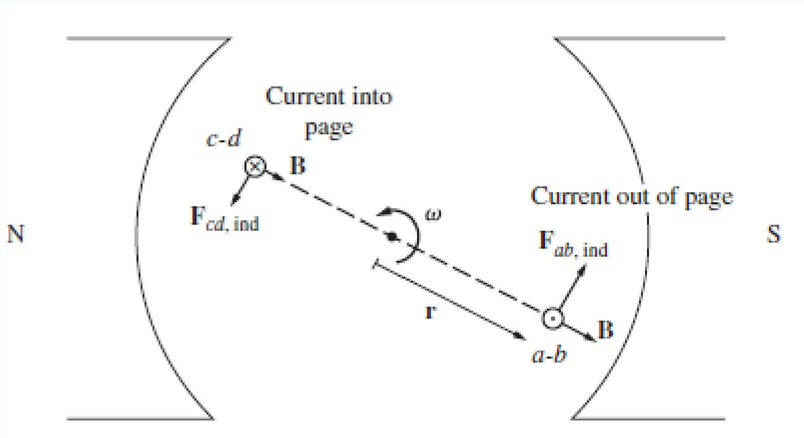


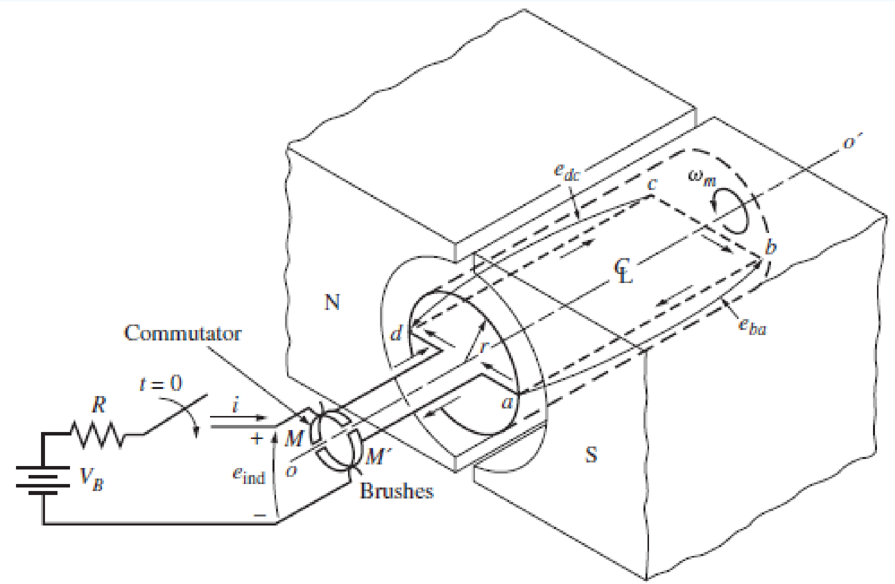
The magnitude of the induced torque:

$$\tau_{ab} = \begin{cases} rilB & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

The Torque Induced in a Rotating Loop

$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$





- Segment *bc*:

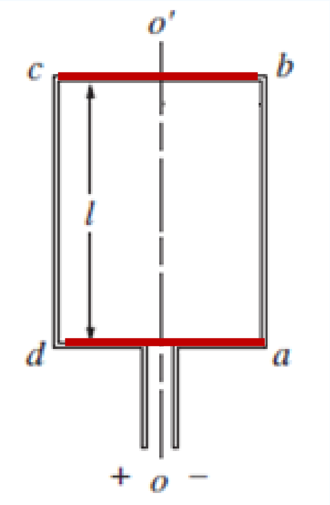
The length **l** is parallel to **B**. Then:

$$\mathbf{F}_{bc} = i(\mathbf{l} \times \mathbf{B}) = 0 \Longrightarrow \tau_{bc} = 0$$

- Segment *da*:

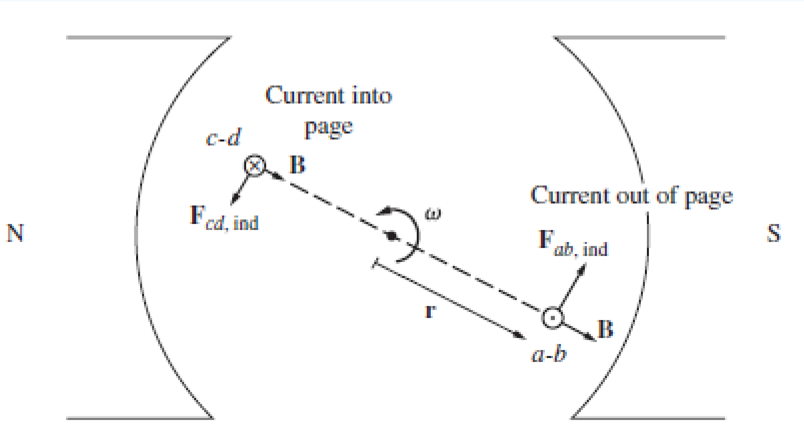
The length **l** is parallel to **B**. Then:

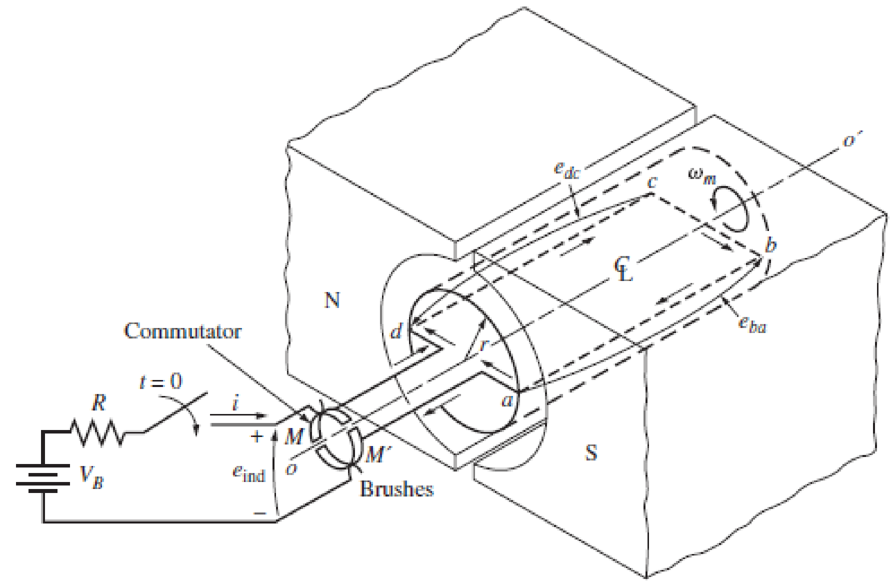
$$\mathbf{F}_{da} = i(\mathbf{l} \times \mathbf{B}) = 0 \Longrightarrow \tau_{da} = 0$$



The Torque Induced in a Rotating Loop

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$$

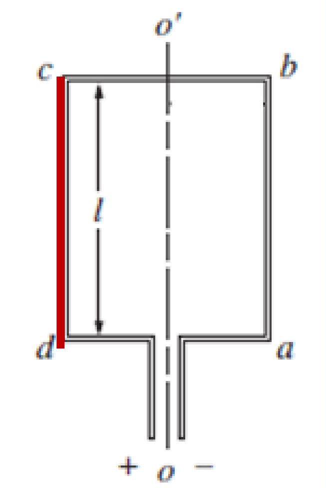




➢ Segment *cd*:

$$\mathbf{F}_{cd} = i(\mathbf{l} \times \mathbf{B}) = ilB$$ Tangent to direction of motion

$$\tau_{cd} = rF\sin\theta = r(ilB)\sin 90^0 = rilB$$ CCW



The magnitude of the induced torque:

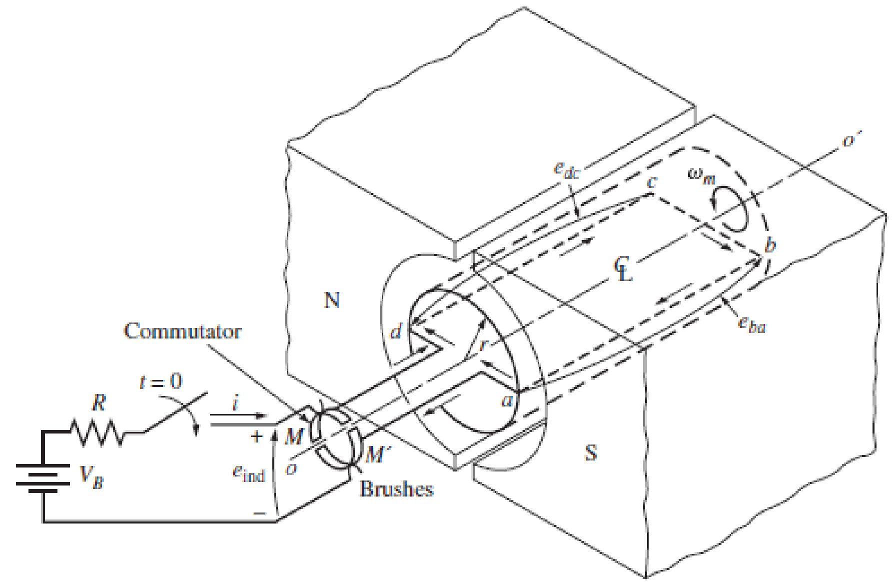
$$\tau_{cd} = \begin{cases} rilB & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

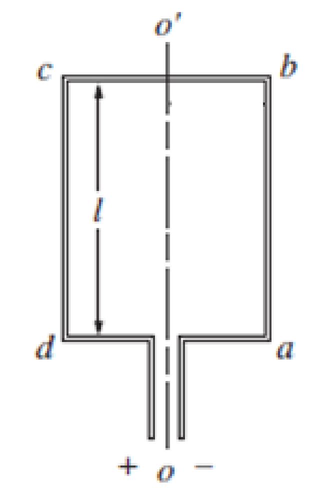
The Torque Induced in a Rotating Loop

The total induced torque on the loop:

$$\tau_{\text{ind}} = \tau_{\text{ab}} + \tau_{\text{bc}} + \tau_{\text{cd}} + \tau_{\text{da}}$$

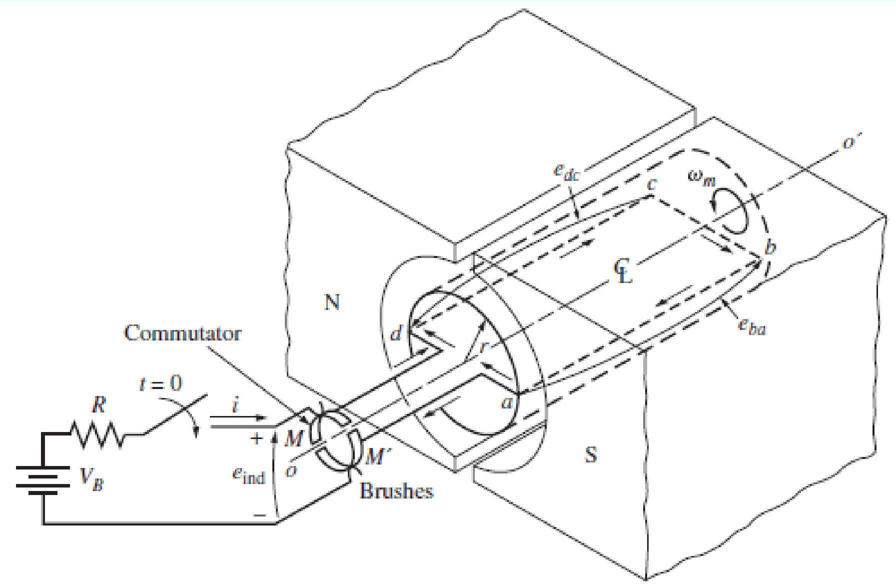
$$\tau_{ind} = \begin{cases} 2rilB & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$





The Torque Induced in a Rotating Loop

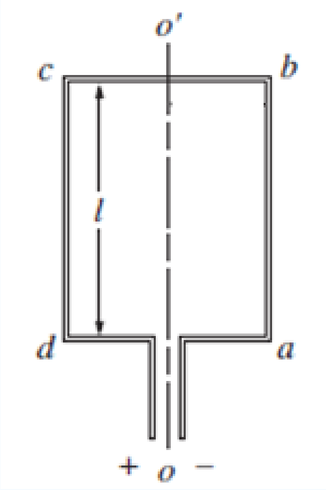
$$\tau_{ind} = \begin{cases} 2rilB & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$



Since there are 2 poles, the area under each pole is $A_p = \pi rl$.

$$\tau_{ind} = \begin{cases} \dfrac{2}{\pi}\Phi i & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

where $\Phi = \pi rlB = A_p B$



The torque in any real machine will depend on the same 3 factors:

$$\tau_{ind} = K\Phi i$$

1. The flux in the machine
2. The current in the machine
3. A constant representing the construction of the machine.

Summary

The total induced voltage

$$e_{ind} = \begin{cases} 2rlB\omega & \text{under the pole face} \\ 0 & \text{beyond the pole edges} \end{cases}$$

$$e_{ind} = \begin{cases} \dfrac{2}{\pi}\Phi\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$

where $\Phi = \pi rlB = A_p B$

The induced voltage in any real machine will depend on the same 3 factors:

1. The flux in the machine
2. The speed of rotation
3. A constant representing the construction of the machine.

$$e_{ind} = K\Phi\omega$$

The total induced torque

$$\tau_{ind} = \begin{cases} 2rlBi & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

$$\tau_{ind} = \begin{cases} \dfrac{2}{\pi}\Phi i & \text{under pole faces} \\ 0 & \text{beyond pole edges} \end{cases}$$

where $\Phi = \pi rlB = A_p B$

The torque in any real machine will depend on the same 3 factors:

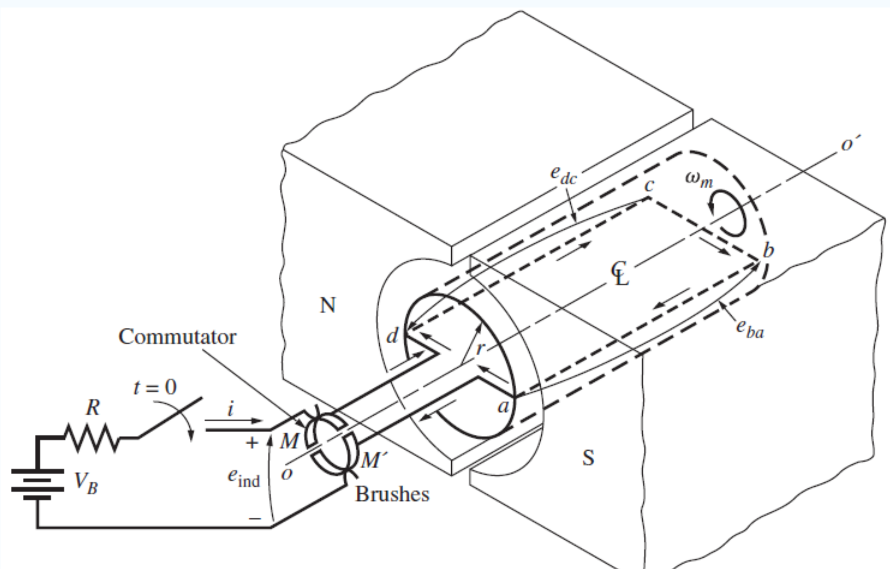
1. The flux in the machine
2. The current in the machine
3. A constant representing the construction of the machine.

$$\tau_{ind} = K\Phi i$$

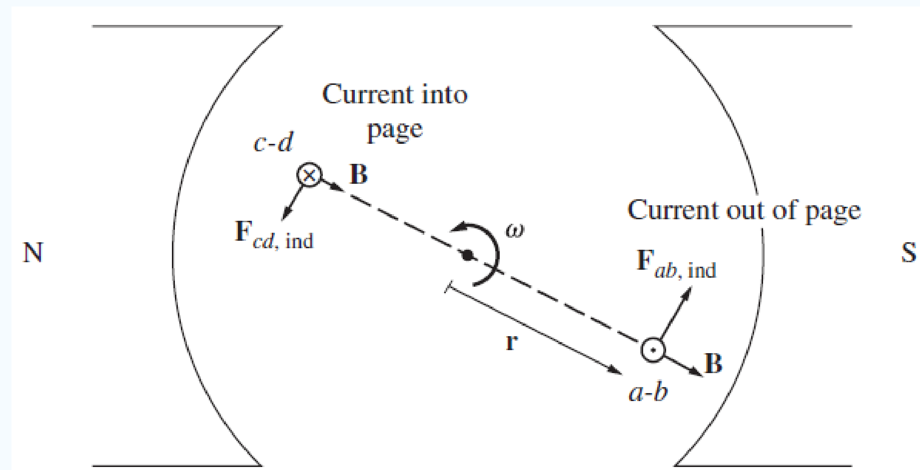
Example

A simple rotating loop between curved pole faces connected to a battery and a resistor through a switch is shown in the figure. The resistor shown models the total resistance of the battery and the wire in the machine. The physical dimensions and characteristics of this machine are in the following:

$$r=0.5\text{ m};\ l=1.0\text{ m};\ R=0.3\ ;\ B=0.25\text{ T};\ V_B=120\text{ V}$$



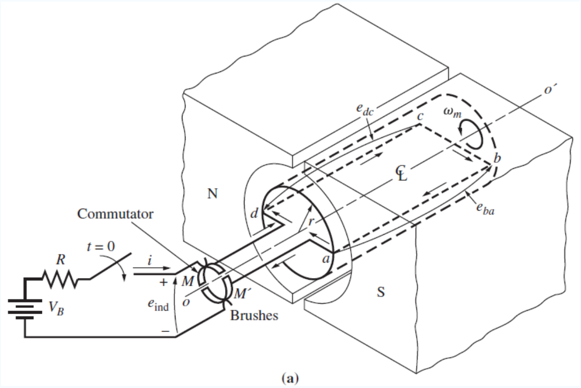
(a)

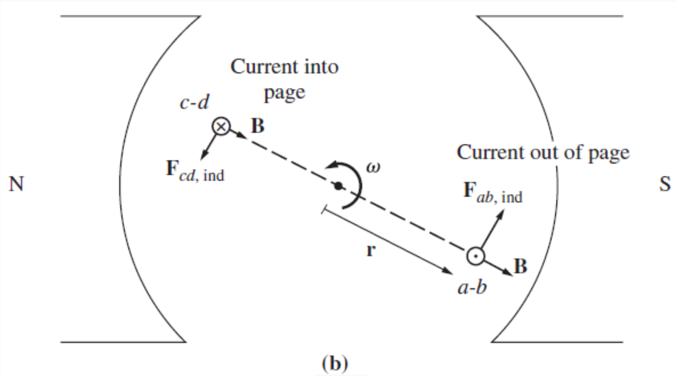


(b)

Example

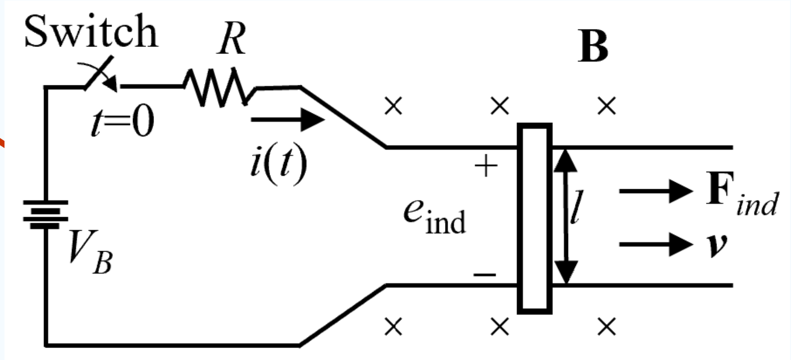
(a) What happens when the switch is closed?
(b) What is the machine's maximum starting current? What is its steady-state angular velocity at no load?
(c) Suppose a load is attached to the loop. And the resulting load torque is 10 N·m. What would the new steady-state speed be? How much power is supposed to the shaft of the machine? How much power is being supplied by the battery? Is this a motor or a generator?
(d) Suppose the machine is again unloaded, and a torque of 7.5 N·m is applied to the shaft in the direction of rotation. What is the new steady-state speed? Is this machine now a motor or a generator?
(e) Suppose the machine is running unloaded. What would the final steady-state speed of the rotor be if the flux density were reduced to 0.2 T?



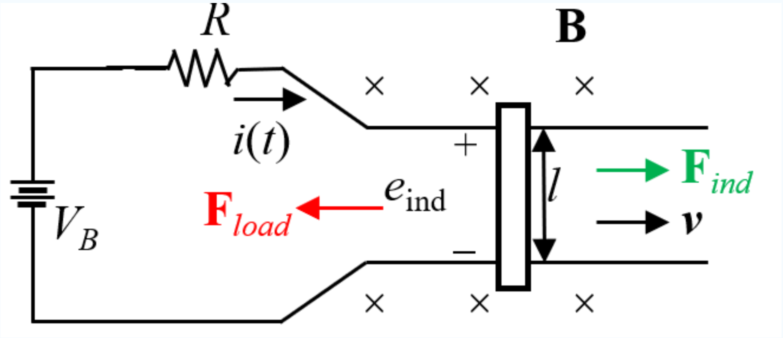


The Linear DC machine –Summary (Recall)

$F = 0$, $e_{ind} = V_B$, $i = 0$, and $v_{nl} = V_B/Bl$.



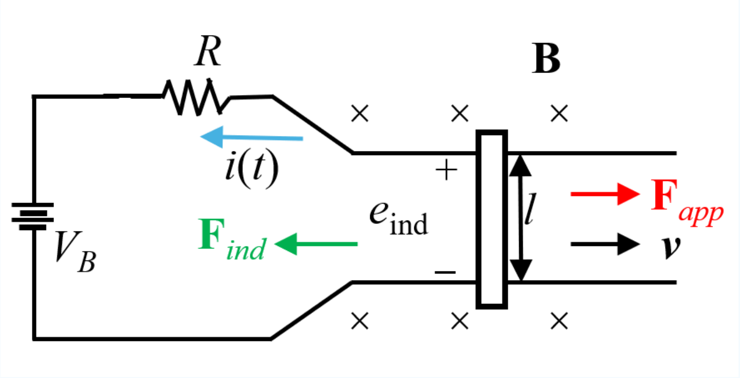
Starting a linear DC machine



The linear DC machine as a motor

$F_{ind} = F_{load}$ (with opposite directions) at a lower speed v_{load}.

Electric power $e_{ind}i$ is being converted to mechanical power $F_{ind}v_{load}$



The linear DC machine as a generator

$F_{ind} = F_{app}$ (with opposite directions) at a higher speed v_{app}.
Mechanical power $F_{ind}v_{app}$ is being converted to electric power $e_{ind}i$

Example

(a) What happens when the switch is closed?

Solution:

① A current will flow in the loop when the switch is closed. Since the loop is initially stationary, e_{ind}=0. Therefore, $i = \dfrac{V_B - e_{ind}}{R} = \dfrac{V_B}{R}$

② This current flows through the rotor loop, producing a torque $\tau_{ind} = \dfrac{2}{\pi}\Phi i$ CCW

③ This induced torque produces an angular acceleration in a CCW direction, so the rotor of the machine begins to turn. But as the rotor begins to turn, an induced voltage is produced in the motor, given by $e_{ind} = \dfrac{2}{\pi}\Phi\omega$

④ So the current falls. As the current falls, the torque decreases, and the machine winds up in steady state with the torque is equal to 0, and the induced voltage should be equal to the batter voltage.

Example

(b) What is the machine's maximum starting current? What is its steady-state angular velocity at no load?

Solution:

At starting conditions: $i = \dfrac{V_B - e_{ind}}{R} = \dfrac{V_B}{R} = \dfrac{120\ V}{0.3\ \Omega} = 400\ \text{A}$

At no-load steady-state conditions: $V_B = e_{ind}$, therefore the speed of the rotor is CCW

$$e_{ind} = 2rlB\omega = V_B$$

$$\omega = \frac{V_B}{2rlB} = \frac{120}{2 \times 0.5 \times 1.0 \times 0.25} = 480\ \text{rad/s}$$

Example

(c) Suppose a load is attached to the loop. And the resulting load torque is 10 Nm. What would the new steady-state speed be? How much power is supposed to the shaft of the machine? How much power is being supplied by the battery? Is this a motor or a generator?

Solution:

If a load torque is applied to the shaft of the machine, it will begin to slow down. But as ω decreases, the induced voltage decreases and the rotor current increases. As the rotor current increases, the induced torque increases too, until the induced torque is equal to the load torque at a lower speed.

At steady state,

$$\tau_{ind} = 2rlBi = \tau_{load}$$

$$i = \frac{\tau_{ind}}{2rlB} = \frac{\tau_{load}}{2rlB} = \frac{10}{2\times 0.5\times 1.0\times 0.25} = 40 \text{ A}$$

By Kirchhoff's voltage law, $e_{ind} = V_B - iR = 120 - 40\times 0.3 = 108 \text{ V}$

Finally, the speed of the shaft $\omega = \dfrac{e_{ind}}{2rlB} = \dfrac{108}{2\times 0.5\times 1.0\times 0.25} = 432 \text{ rad/s}$

The power supplied to the shaft is

$$P_{conv} = P_e = P_m$$

$$P_e = e_{ind} i = 108\times 40 = 4320 \text{ W}$$

$$P_m = \tau\omega = 10\times 432 = 4320 \text{ W}$$

The power out of the battery is $P = V_B i = 120\times 40 = 4800 \text{ W} > P_{conv}$

It is operating as a motor, converting electric power to mechanical power.

Example

(d) Suppose the machine is again unloaded, and a torque of 7.5 N·m is applied to the shaft in the direction of rotation. What is the new steady-state speed? Is this machine now a motor or a generator?

Solution:

If a load torque is applied in the direction of motion, the rotor accelerates. As ω increases, the induced voltage increases and exceeds V_B, so the current flows out of the top of the bar and into the batter. This machine is now *a generator*. This current causes an induced torque opposite to the direction of motion. The induced torque opposes the external applied torque, and eventually the induced torque is equal to the applied torque at a higher speed.

At steady state, $\tau_{ind} = 2rlBi = \tau_{app} = 7.5 \text{ N•M}$

$$i = \frac{\tau_{ind}}{2rlB} = \frac{\tau_{app}}{2rlB} = \frac{7.5}{2\times0.5\times1.0\times0.25} = 30 \text{ A}$$

By Kirchhoff's voltage law, $e_{ind} = V_B + iR = 120 + 30\times0.3 = 129 \text{ V}$

Finally, the speed of the shaft $\omega = \dfrac{e_{ind}}{2rlB} = \dfrac{129}{2\times0.5\times1.0\times0.25} = 516 \text{ rad/s}$

It is operating as a generator, converting mechanical power to electric power.

Example

(e) Suppose the machine is running unloaded. What would the final steady-state speed of the rotor be if the flux density were reduced to 0.2 T?

Solution:

Since the machine is initially unloaded at the original conditions, the speed is . If the flux decreases, there is a transient. However, after the transient is over, the machine much again have zero torque, since there is still no load on its shaft. If the induced toque is equal to 0, then the current in the rotor must be zero, and the induced voltage e_{ind} is equal to the battery voltage V_B.

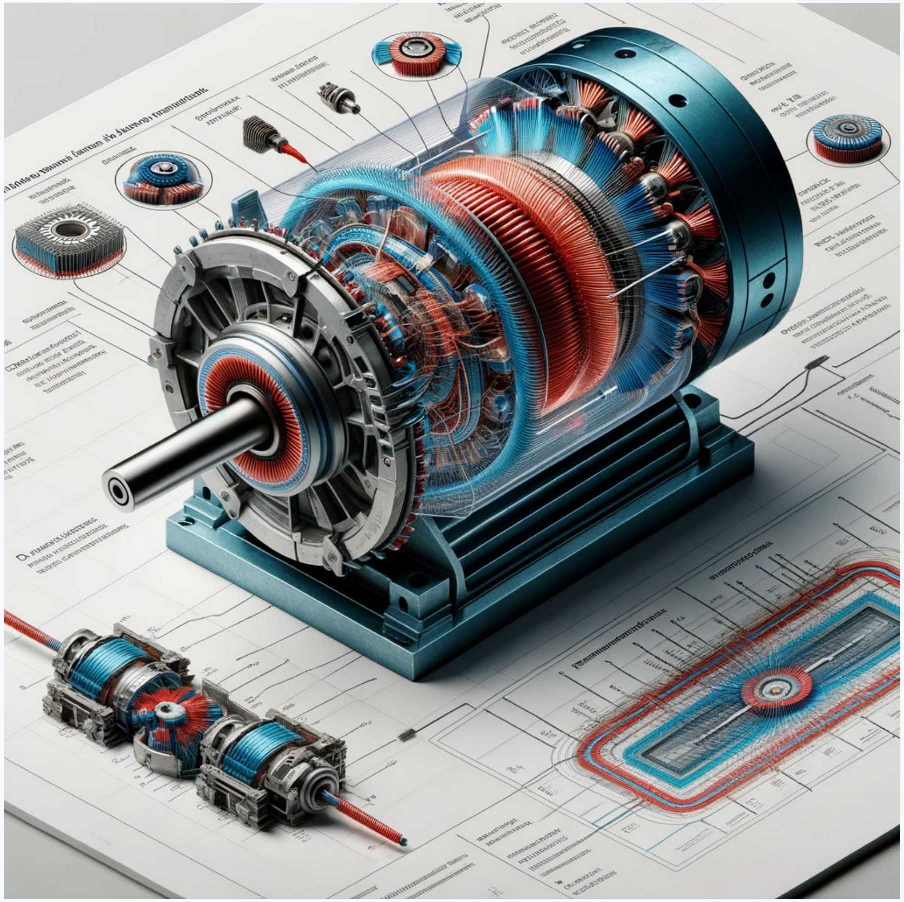
The new steady-state speed at no load is

$$e_{ind} = 2rlB\omega = V_B$$

$$\omega = \frac{V_B}{2rlB} = \frac{120}{2\times 0.5\times 1.0\times 0.2} = 600 \ \text{rad/s}$$

Note that: *When the flux in the machine is decreased, its speed increases.* This is the same behavior seen in the linear machine and the same way for real dc motor behave.

Next



DC Motors

Thanks for your attention