EEE210:
Energy Conversion and Power Systems

Transformers

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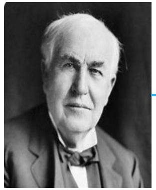
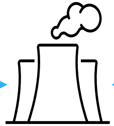
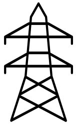
Transformers

1. Industry Significance





1882



120V

Light up the world

- Current value
- Voltage drops
- Energy losses

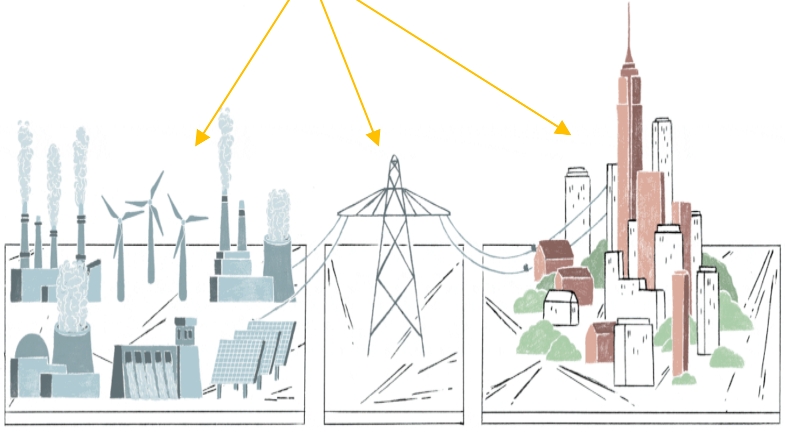
$$I = \frac{V_c}{Z_{load}}$$

$$V_{\mathrm{drop}} = IZ_{line}$$

120V, copper cable:
3% voltage drops =
480 meters

?

- Control voltage
- Adjust current
- Realize long distance power transmitting



Unit transformer

Substation transformer

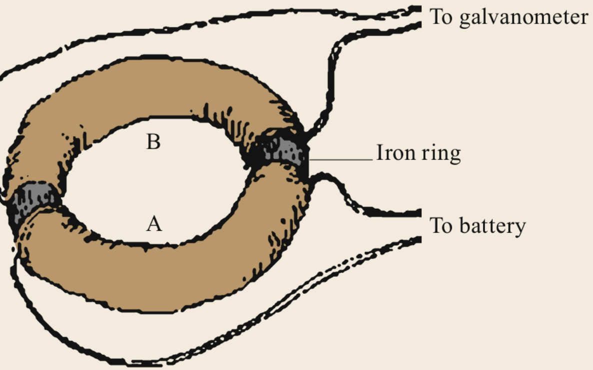
Distribution transformer

Transformers

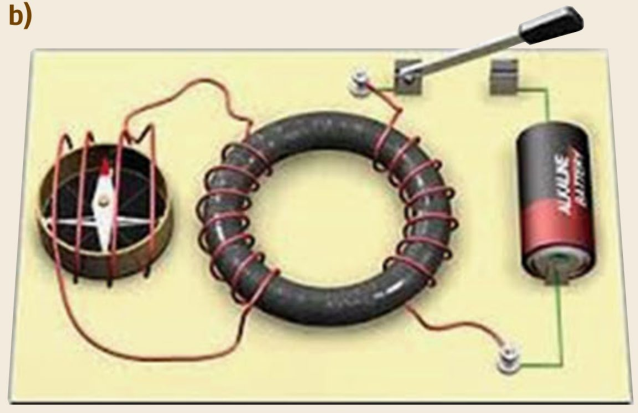
2. History

In 1831, Michael Faraday discovered the principle of induction using a device that presented all of the characteristics of a transformer

a)



b)

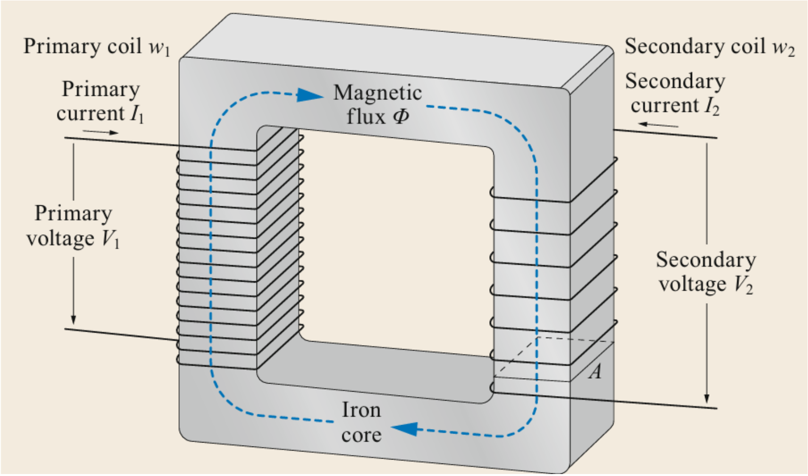


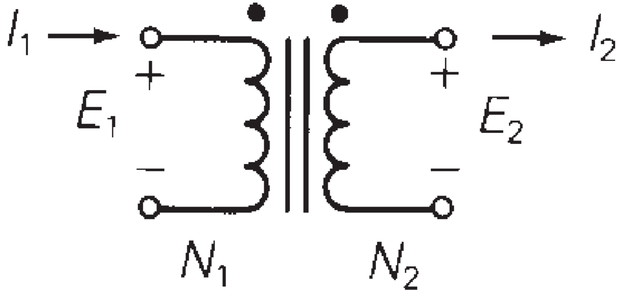
Faraday placed one winding around a magnetic needle and connected the other winding to a battery.
By switching the battery supply on and off, Faraday generated an alternating current and was able to demonstrate magnetic induction in the second coil because the magnetic needle started to move.

Transformers

3. Idea transformer

An ideal transformer is a lossless device with an input winding and an output winding.





N_p or N_1 : the number of turns on the primary or left side

N_s or N_2 : the number of turns on the secondary or right side

I_p or I_2' : the current flows in the primary or left side

I_s or I_2 : the current flows in the secondary or right side.

According to the faraday's law:

$$e \;= N\frac{d\varphi}{dt} = 2\pi f N \varphi_{\max}\cos(wt + 90°)$$

N is the number of coils;

φ is the average flux per turn of coils on the induced side

For the rms value

$$E_p = E_1 = 4.44 f N_1 \varphi_{\max}$$

$$E_s = E_2 = 4.44 f N_2 \varphi_{\max}$$

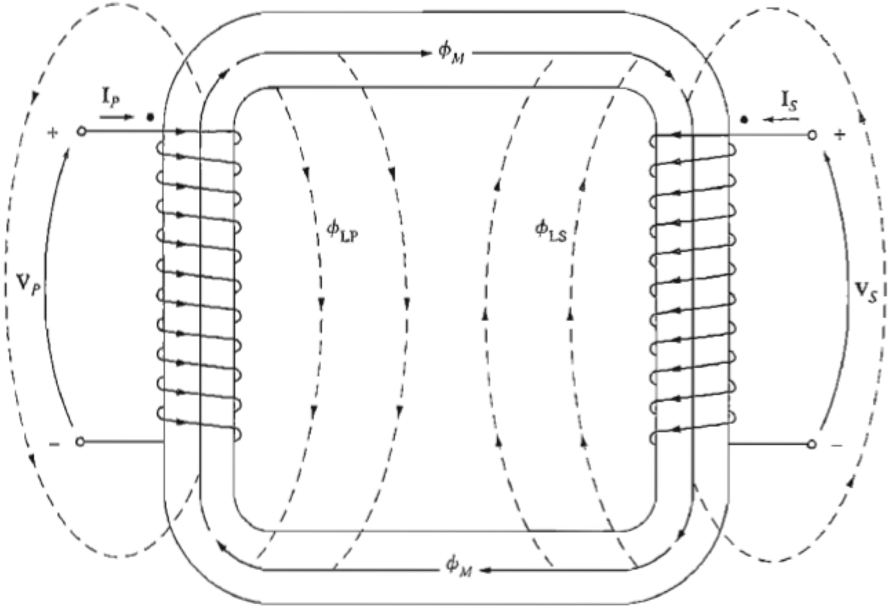
Suppose the core has no losses

$$\frac{N1}{N2} = \frac{E_1}{E_2} = \frac{I_S}{I_P} = \frac{I_2}{I_2'} = \text{a}$$

a is the turns ratio

Transformers

4. Reality



Heating

Core loss (iron loss):

- Eddy current losses
- Hysteresis Losses

Copper loss:

- Current flows on the primary and secondary coils

Flux leakage:

- φ_{lp} : Leakage on the primary side
- φ_{ls} : Leakage on the secondary side

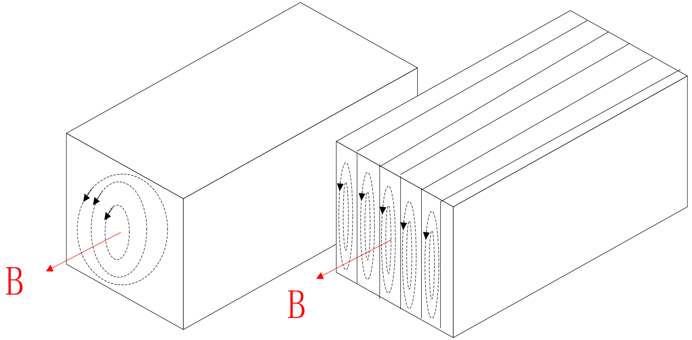
Transformers

4. Reality

Hysteresis: once the core is magnified by induced magnetic flux B. It requires addition negative magnetizing force H to demagnetize.

- B is the magnetic flux density;
- H is the magnetizing force, change with the currents on the coil.

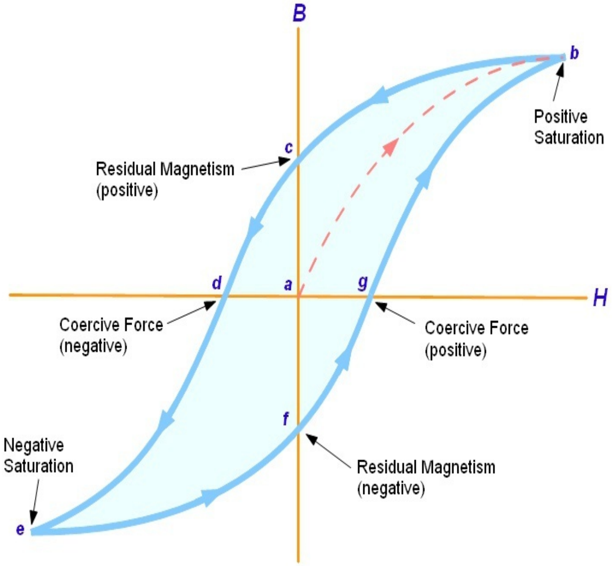
Eddy current losses



$$P_{el} = k_e B^2 f^2 V$$

k_e is a constant value, determined by lab test; B is the maximum flux; f is the frequency; V is the volume of the coil

Hysteresis Losses



a-b:

Current value change from 0-max;

Initial H applies;

Core goes magnetised

b-c:

Current value reduced to 0;

Core stay magnetized

c-d:

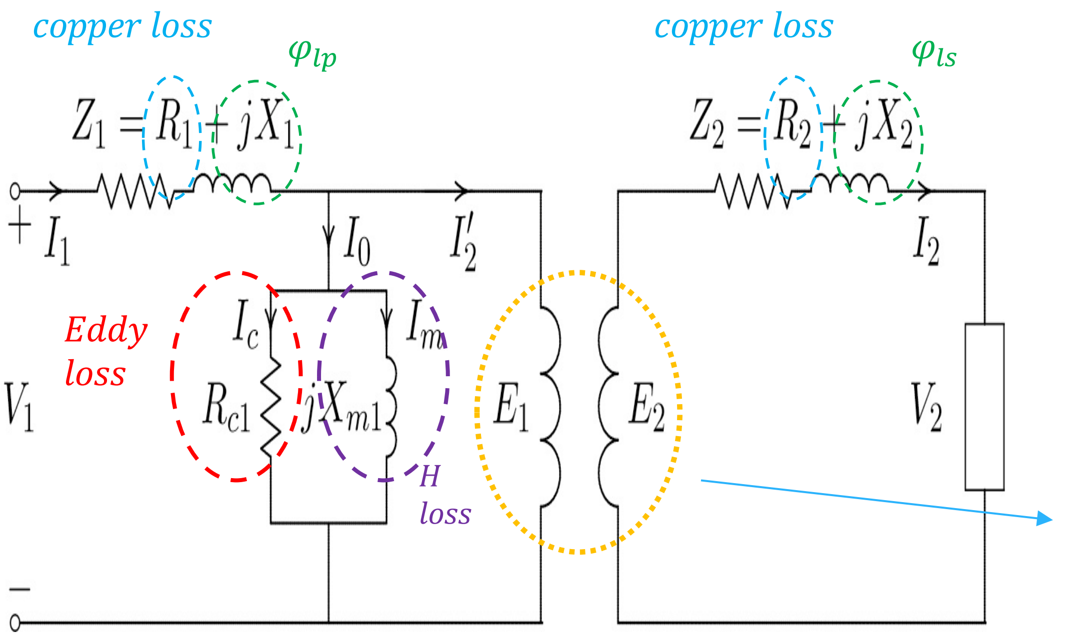
Current reduced to negative value;

Core get demagnetised

Transformers

4. Equivalent Circuit

How can the effects of losses be modeled?



On the primary side:

$$E_1 = V_1 - Z_1 I_1$$

On the secondary side:

$$E_2 = V_2 + Z_2 I_2$$

According to the turns ratio:

$$\frac{E_1}{E_2} = \frac{N1}{N2}$$

- Because of the voltage conversion between P and S side.
- *I,V,E* on P and S sides are in different voltage level

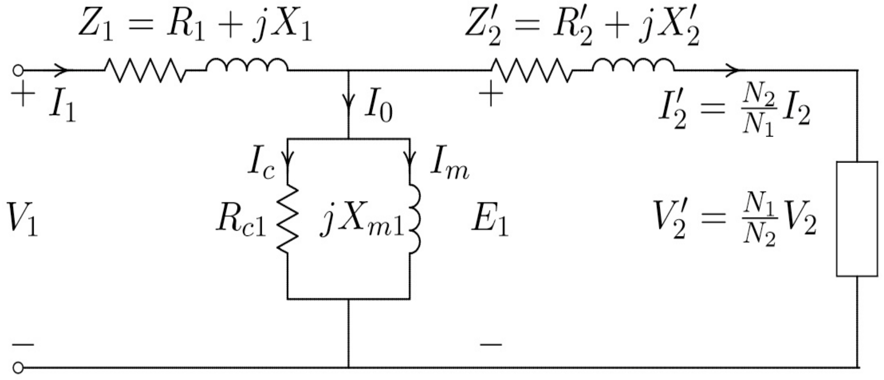
Not a useful equivalent circuits

Transformers

4. Equivalent Circuit

Convert the entire circuit to an equivalent circuit at a single voltage level

Refer to the primary side:



On the secondary side:

$$\frac{N2}{N1} E_1 = E_2 = V_2 + Z_2 I_2$$

$$E_1 = \frac{N1}{N2}(V_2 + Z_2 I_2)$$

Given that $\frac{I_2}{I_2'} = \frac{N1}{N2}$ & $\frac{V_2'}{V_2} = \frac{N1}{N2}$

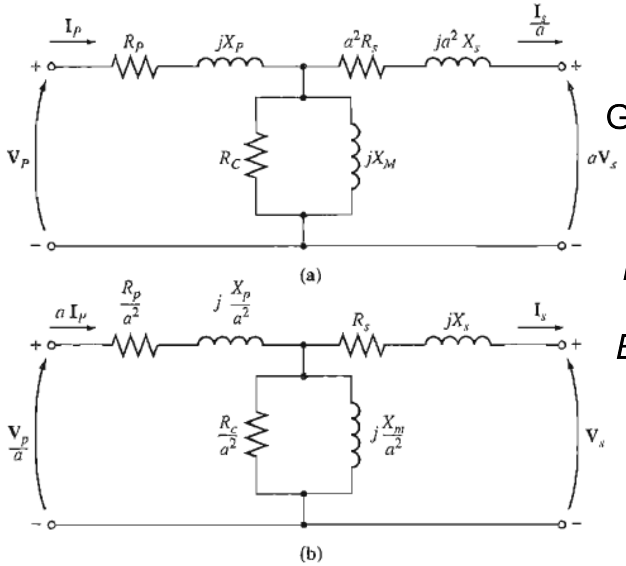
$$E_1 = V_2' + \left(\frac{N1}{N2}\right)^2 Z_2 I_2'$$

$$E_1 = V_2' + Z_2' I_2'$$

where $Z_2' = \left(\frac{N1}{N2}\right)^2 Z_2$

$$= \left(\frac{N1}{N2}\right)^2 R_2 + j\left(\frac{N1}{N2}\right)^2 X_2$$

Comparison between refer to P and S sides:



Given that $\frac{N1}{N2} = \text{a}$

$$E_2 = \frac{E_1}{a}$$

$$E_2 = \frac{V_p - Z_p I_p}{a}$$

$$E_2 = V_p^1 - \frac{Z_p}{a^2} I_p'$$

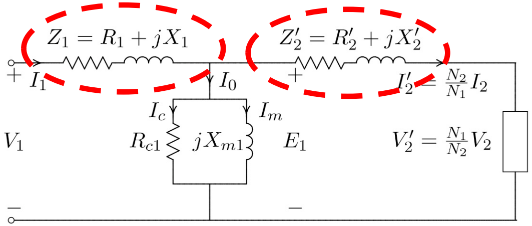
where $I_p' = aI_p$

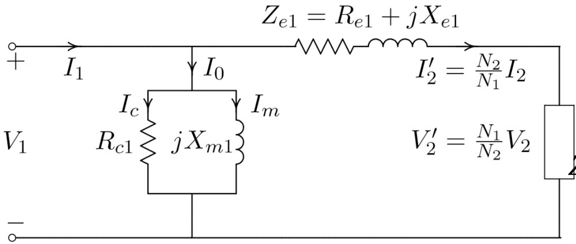
Transformers

4. Approximation

How to simplify the circuit for the reason of engineering simplicity?

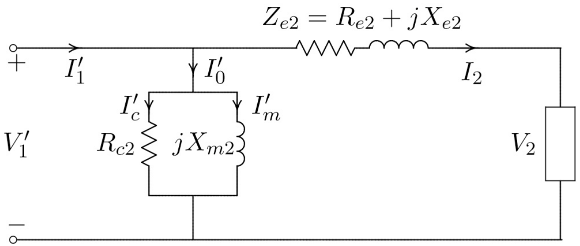
Approximation 1:
The excitation current is only about 2-3% of the full load current for typical power transformers.





Refer to P Side

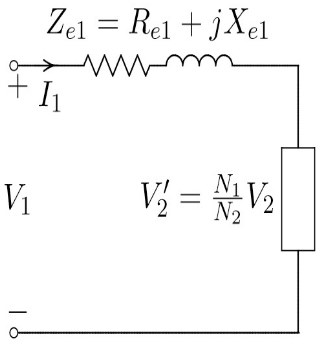
$$Z_{e1} = Z_1 + Z_2'$$
$$Z_{e1} = Z_1 + a^2 Z_2$$



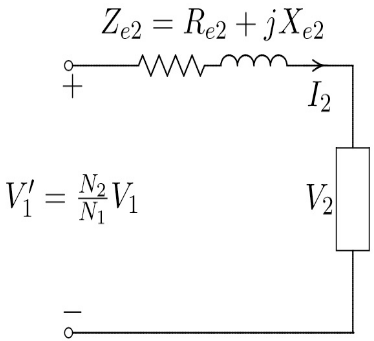
Refer to S side

$$Z_{e2} = Z_1' + Z_2$$
$$Z_{e2} = \frac{Z_1}{a^2} + Z_2$$

Approximation 2:
In fact, general transformers have very high permeability core and very small core resistance



Refer to P Side

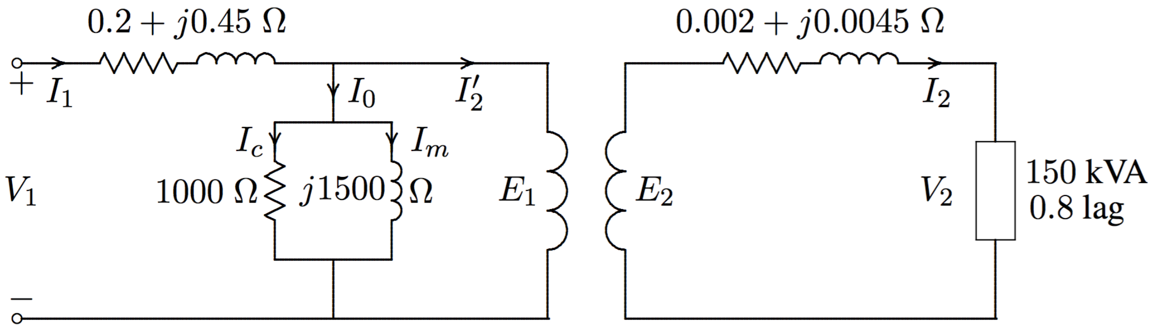


Refer to S side

Transformers

5. Example question

A 150-kVA, 2400/240-V single-phase transformer has the parameters as shown in the following figure.
1. Determine the equivalent circuit referred to the high-voltage side.
2. Find the primary voltage when the transformer is operating at full load 0.8 power factor lagging and 240 V terminal voltage at the secondary side with neglected iron losses.
3. Find the primary voltage when the transformer is operating at full-load 0.8 power factor leading and 240 V terminal voltage at the secondary side with neglected iron losses.



Read the problem:

S = 150 kVA
a = 2400/240 = 10

Transformers

5. Example question

1) Determine the equivalent circuit referred to the high-voltage side.

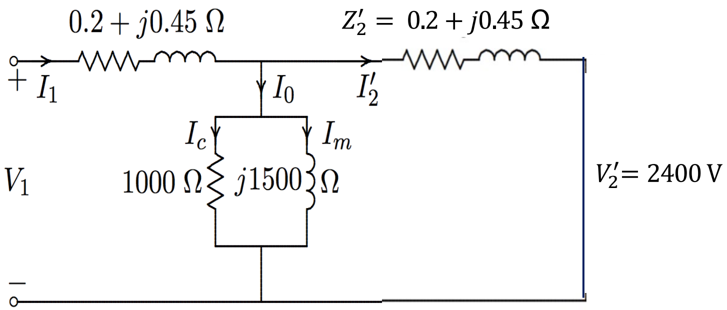
- Recall the equations

$$Z_2' = (a)^2 R_2 + j(a)^2 X_2 \quad \& \quad V_2' = a\,V_2$$

$$Z_2' = 10^2 * 0.002 + j10^2 * 0.0045$$

$$= 0.2 + j0.45\ \Omega$$

$$V_2' = 10 * 240 = 2400\ \text{V}$$



2) Find the primary voltage when transformer is operating at full load 0.8 power factor lagging and 240 V terminal voltage at the secondary side with neglected iron losses

Recall the equation:

$$V_1 = Z_1 I_1 + Z_2' I_2' + V_2'$$

Power factor is 0.8 lagging

$$S = |S|cos(\theta) + j|S|sin(\theta) = 120.32 + j89.58\ \text{kVA}$$

$$I_2' = \frac{S^*}{V_2'^*} = \frac{120.32 - j89.58}{240 * 10} = 50.1 - j37.3\text{A}$$

$$V_1 = V_2' + I_2' * (Z_1 + Z_2') = 2400 + (50.1 - j37.3) * (0.4 + j0.9)$$

$$V_1 = 2453.933\angle 0.7°\text{V}$$

3) Find the primary voltage when transformer is operating at full load 0.8 power factor leading and 240 V terminal voltage at the secondary side with neglected iron losses

Power factor is 0.8 leading

$$S = |S|cos(\theta) - j|S|sin(\theta) = 120.32 - j89.58\ \text{kVA}$$

$$I_2' = \frac{S^*}{V_2'^*} = \frac{120.32 + j89.58}{240 * 10} = 50.1 + j37.3\text{A}$$

$$V_1 = V_2' + I_2' * (Z_1 + Z_2') = 2400 + (50.1 - j37.3) * (0.4 + j0.9)$$

$$V_1 = 2387.004\angle 1.44°\text{V}$$

Transformers

5. Transformer performance

- Transformer efficiency

Determine the percentage of energy that can be converted from P side to S side.

The larger the better (no more than 100%)

Power transformer efficiencies vary from 95% to 99%, the higher efficiencies being obtained from transformers with greater ratings.

The actual efficiency of a transformer in percent is given by:

$$\eta = \frac{output\ power}{input\ power}$$

Considering the copper loss is determined by loading levels, η is given by:

$$\eta = \frac{n * S * pf}{n * S * pf + n^2 * P_{copper} + P_{iron}}$$

n is the loading level, such as 70%.

- Voltage regulation

Determine the percentage of output voltage drops from no-load to full load status, if input voltage is constant.

Voltage regulation indicates the percentage of output voltage drops from no-load to full load status, if input voltage is constant

$$Regulation = \frac{|V_{2,noload}| - |V_{2,fuload}|}{|V_{2,fuload}|} * 100\%$$

If refer to the P side:

$$Regulation = \frac{|V_1| - |V'_{2,fuload}|}{|V'_{2,fuload}|} * 100\%$$

If refer to the S side:

$$Regulation = \frac{|V'_1| - |V_{2,fuload}|}{|V_{2,fuload}|} * 100\%$$

Synchronous Generators

6. Important theory

What indeed carries electricity energy ?

[The Big Misconception About Electricity]

Transformers

7. How to obtain the impedance parameters

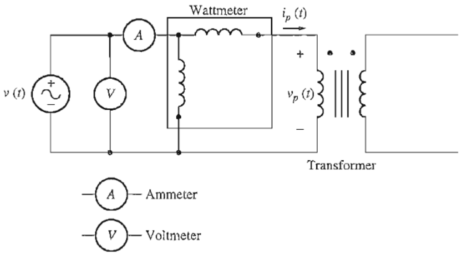
Normally, manufacture will label them on the transformer.

Two methods are normally used to determine the impedance parameters for transformers:

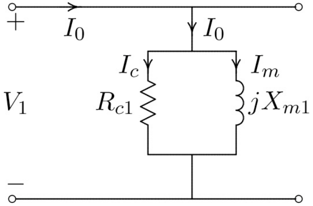
- **Open circuit tests**
  The shunt elements R_c and X_m
- **Short circuit tests**
  The copper impedance Z_e

Open circuit tests

In the open-circuit test, rated voltage is applied at the terminals of one winding while the other winding terminals are open-circuit.



Test circuits



Simplified circuits

The shunt elements R_c and X_m can be determined from the relations:

$$R_{c1} = \frac{V_1^2}{P_0}$$

The no-load input power P_0 gives the transformer core loss commonly referred to as ***iron loss.***

$$I_C = \frac{V_1}{R_{c1}}$$

And

$I_0 = I_c + I_m j \quad |I_0| = \sqrt{{I_c}^2 + {I_m}^2}$ if we have the magnitude of I_0:

$I_m = \sqrt{{I_0}^2 - {I_c}^2}$, therefore $X_{m1} = \frac{V_1}{I_m}$

Transformers

7. How to obtain the impedance parameters

I_0 is minimal

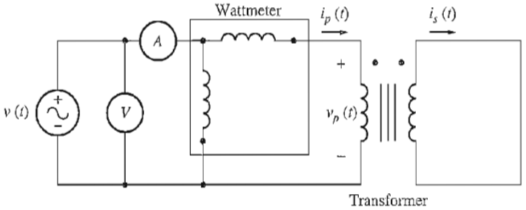
Normally, manufacture will label them on the transformer.

Two methods are normally used to determine the impedance parameters for transformers:

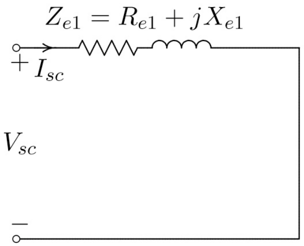
- **Open circuit tests**
  The shunt elements R_c and X_m
- **Short circuit tests**
  The copper impedance Z_e

Short circuit tests

In the short-circuit test, a reduced voltage Vsc is applied at the terminals of one winding while the other terminals are short-circuited.



Test circuits



Simplified circuits

The series elements R_{e1} and X_{e1} may then be determined from the relations

$$Z_{e1} = \frac{V_{sc}}{I_{sc}}$$

And

$$R_{e1} = \frac{P_{sc}}{I_{sc}^2}$$

$$X_{e1} = \sqrt{{Z_{e1}}^2 - {R_{e1}}^2}$$

There is no easy way to split the series impedance into primary and secondary components. Fortunately, such separation is not necessary to solve normal problems. (we have the approximation circuits)

Transformers

7. How to obtain the impedance parameters

Normally, manufacture will label them on the transformer.

Two methods are normally used to determine the impedance parameters for transformers:

- **Open circuit tests**
  The shunt elements R_c and X_m
- **Short circuit tests**
  The copper impedance Z_e

Attention safety reasons:

- the open-circuit test is usually performed on the low-voltage side of the transformer

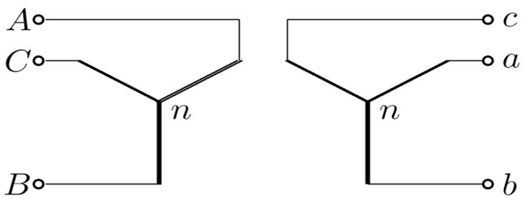
  R_c and X_m refer to the LV side

- the short-circuit test is usually performed on the high voltage side of the transformer

  R_{eq} and X_{eq} refer to the HV side

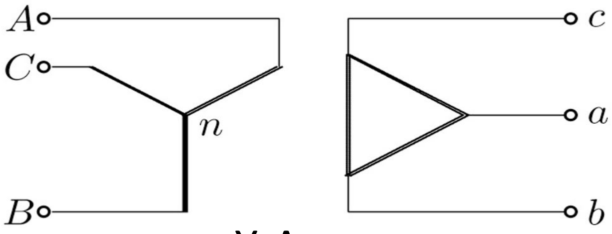
Transformers

8. Three phase transformers



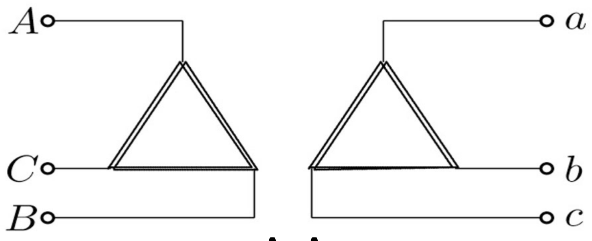
Y-Y

- Low insulation cost
- Available neutral to ground connections
- Rarely used due to 3rd harmonics



Y-Δ

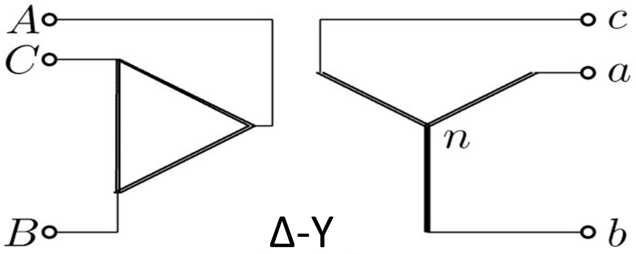
- Often used as a step-down transformer
- Block 3rd harmonics
- Have a neutral-ground connection



Δ-Δ

- Block 3rd harmonics
- No neutral to ground connection
- Normally three single transformers are connected as the Δ-Δ connection:

If required, one single transformer can be disconnected, while the rest two can still provide three phase power conversion at a reduced power.



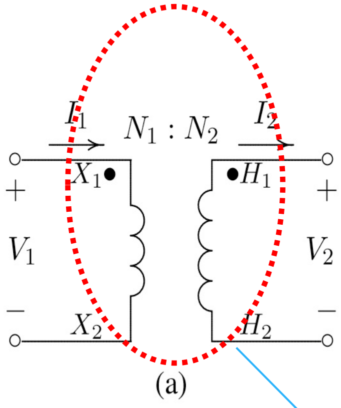
Δ-Y

- Often used as a step-up transformer
- Block 3rd harmonics
- Have a neutral-ground connection

Transformers

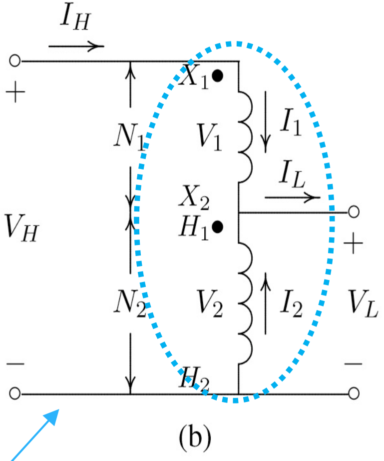
9. Auto transformers

No electrical connection



Normal transformer

P and S side are electrically connected



AT transformer

connecting P and S

Winding name:

X_1 to X_2: series winding

H_1 to H_2: common winding

AT can work as both a step-up and a step-down transformer

- step-up

H_1 to H_2 is the primary winding

- step-down

H_1 to H_2 is the secondary winding

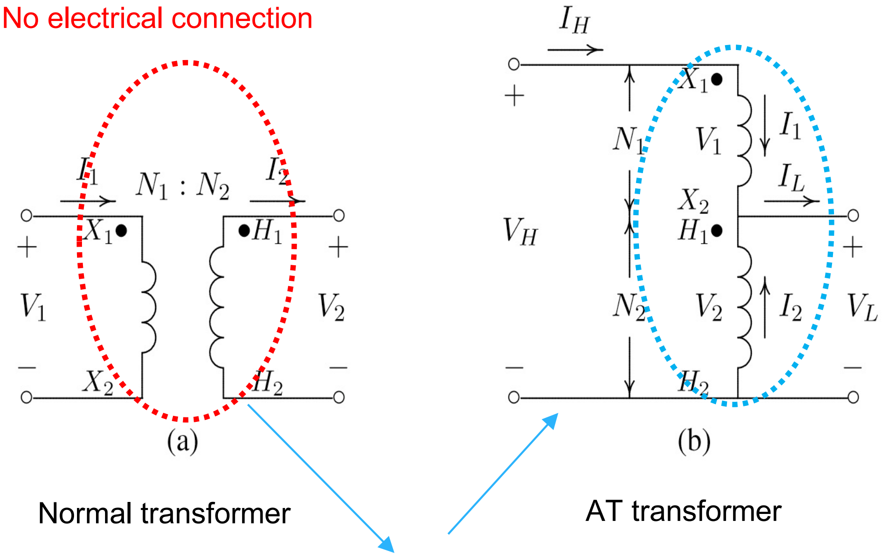
Compared to normal transformer, the ideal AT transformer circuit is used to approximate the power ratings.

Transformers

9. Auto transformers

No electrical connection

P and S side are electrically connected



Normal transformer

AT transformer

connecting P and S

Like normal transformer:

$$\frac{N1}{N2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} = a$$

However, the difference would be:

$$V_H = V_2 + V_1$$

If refer to V2 side:

$$V_H = V_2 + \frac{N1}{N2} V_2$$

Since $V_2 = V_L$

$$V_H = V_L + \frac{N1}{N2} V_L; \qquad \frac{V_H}{V_L} = 1 + a$$

From Kirchoff's law, $I_2 = I_L - I_1$:

$$I_L = \frac{N1 + N2}{N2} I_1$$

Since $I_1 = I_H$:

$$\frac{I_L}{I_H} = 1 + a = \frac{V_H}{V_L}$$

The ratio of the *S* of an autotransformer to its corresponding two-winding transformer is called the power rating advantage, which is given by:

$$\frac{S_{auto}}{S_{2w}} = \frac{(V_1 + V_2) I_1}{V_1 I_1} = 1 + \frac{1}{a}$$

Transformers

10. Example question

A two-winding transformer is rated at 60-kVA, 240/1200-V, 60-Hz. When operated as a conventional two-winding transformer at rated load, 0.8 power factor, its efficiency is 0.96. This transformer is to be used as a 1440/1200-V step-down autotransformer in a power distribution system.

1. Assuming that the transformer is ideal, find the transformer kVA rating when used as an autotransformer
2. Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

Read the problem:

S_{tw} = 60 kVA

a = 240/1200 = 0.2 as a two winding transformer

pf = 0.8

Efficiency = 0.96 as a two winding transformer

$$a_{auto} = \frac{1400}{1200} = 1.167$$

Transformers

10. Example question

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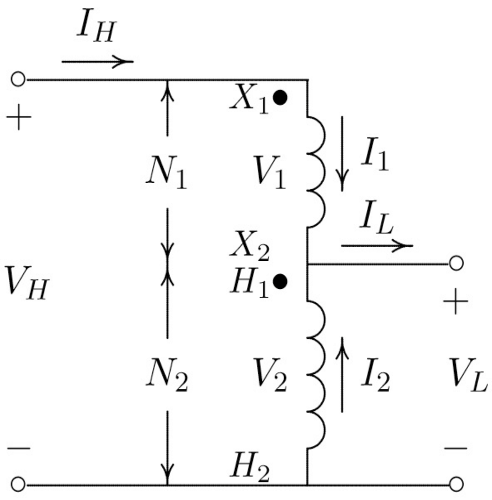
1. Assuming the transformer is ideal, find the transformer kVA rating when used as an autotransformer



(b)

Recall the knowledge of auto transformer:

$$S_{auto} = (V_1 + V_2)I_1 = V_2 I_L$$

$$S_{auto} = 1200*(I_1 + I_2)$$

I_1 and I_2 are derived, suppose the auto transformer is a two-winding one.

$$I_1 = \frac{S_{tw}}{V_1} = \frac{60{,}000}{240} = 250\ \mathrm{A}$$

$$I_2 = \frac{S_{tw}}{V_2} = \frac{60{,}000}{1200} = 50\ \mathrm{A}$$

$$S_{auto} = V_2 I_L = 1200 * (250 + 50) = 360\ \mathrm{kVA}$$

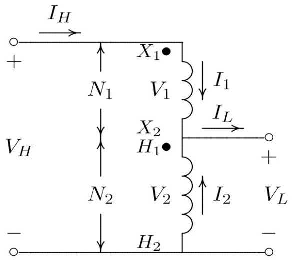
The power advantage = $\frac{S_{auto}}{S_{tw}} = \frac{360}{60} = 6$

Transformers

10. Example question

A two-winding transformer is rated at 60-kVA, 240/1200-V, 60-Hz. When operated as a conventional two-winding transformer at rated load, 0.8 power factor, its efficiency is 0.96. This transformer is to be used as a 1440/1200-V step-down autotransformer in a power distribution system.

2. Find the efficiency with the kVA loading of part (a) and 0.8 power factor



(b)

Recall the efficiency equation at full load:

$$\text{Efficiency} = \frac{|S_{auto}|*pf}{|S_{auto}|*pf+P_{loss}}$$

Remember, when a two winding transformer is connected as an auto transformer, the power loss would not change.

Deem the auto transformer as a two winding one:

$$\text{Efficiency} = \frac{|S_{tw}|*pf}{|S_{tw}|*pf+P_{loss}} = 0.96$$

$$P_{loss} = 2 \text{ kW}$$

Take the P_{loss} into the auto transformer:

$$\text{Efficiency} = \frac{|S_{auto}|*pf}{|S_{auto}|*pf+P_{loss}} = \frac{(360*0.8)}{360*0.8+2} = 0.9931$$

Transformers

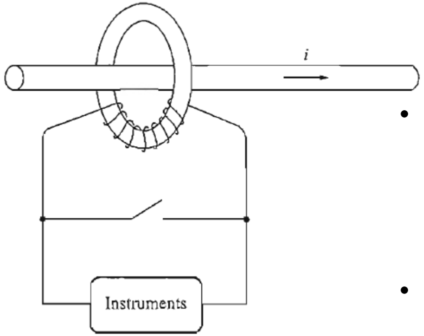
11. Other application

In lab:

- Current meter
- Voltage meter

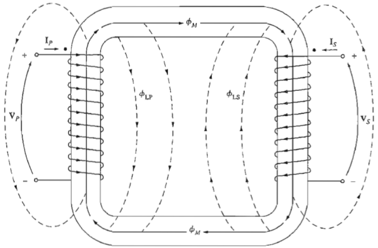
Don't applied to high voltage or high current scenarios

Current transformers:



- Current flows in the conductor would magnetize the **ferromagnetic ring.**
- A small sample of flux flow in the ring will induce voltage and currents on the coils **(a reduced value, e.g. 100 V, 5A)**
- **Loosely coupled** winding covert the original currents into a proportional small one.

Potential transformers:



- Similar structure with the normal transformer
- Low rated power (can be deemed as open circuit on the secondary side)
- Lower the voltage to a safe value, e.g. 100V.

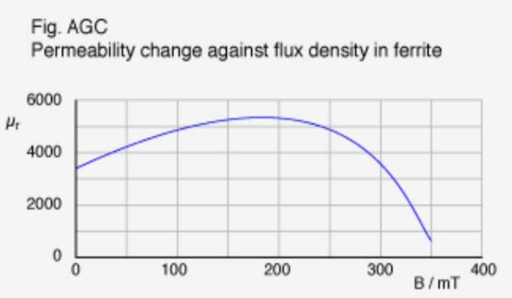
Transformers

12. Extensions

Saturation

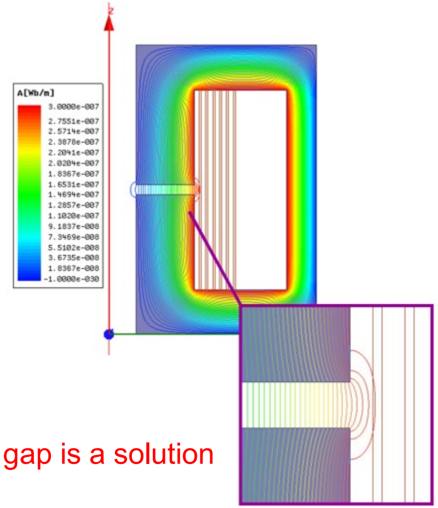
In reality:

- The **applied field strength** and the **flux increase** is not a linear function



Flux

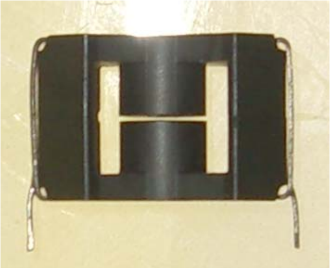
- With the increase of applied field, saturation will reach.

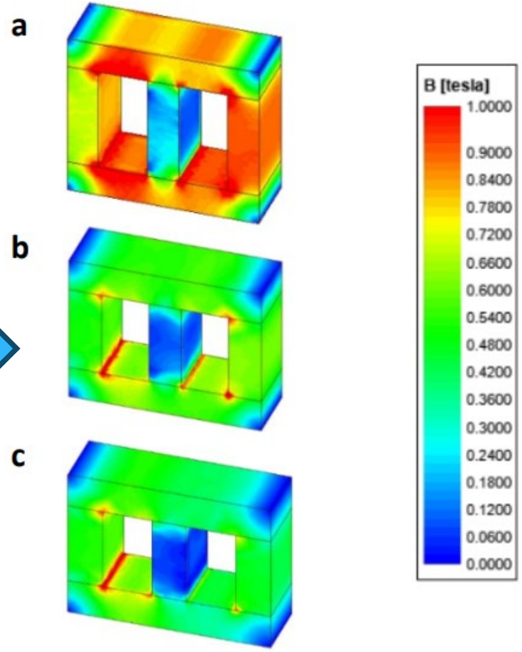


Air gap is a solution

The size of the airgap:

- Smooth conducting path
- Prevent magnetic saturation





Time-dependent distribution of magnetic flux density for **a)** A_c=0.0105 m^2 **b)** A_c=0.0153 m^2 **c)** A_c= 0.018 m^2

Transformers

12. Extensions

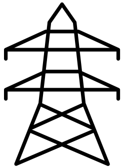
Dynamic rating

Rated capacity:

Estimated lifespan ← Determined — Hot spot temperature

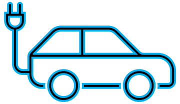
Estimated lifespan → Limited → Assumed constant loads

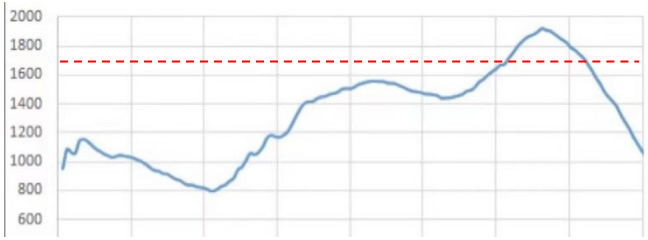
Assumed constant loads → Raise → Hot spot temperature



Household temperature regulation

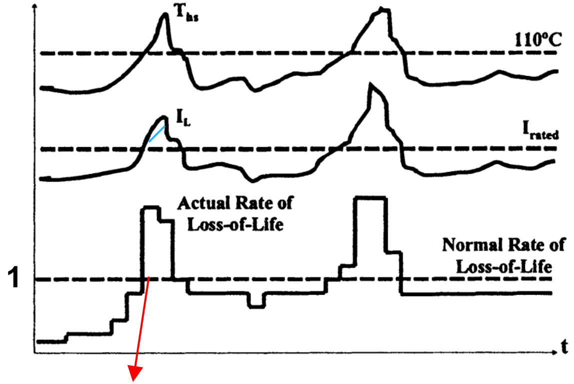
Unexpected loads





Rated capacity at the nameplate

- If RLOL>1 Transformer will have a **lower life** than the rated one.
- If RLOL<1 Transformer will have a **greater life** than the rated one.



Over temperature exacerbate insulation degradation, component aging, etc.