CAN102 Electromagnetism and Electromechanics

Lecture-3 Static Electric Fields

(Electrostatics)

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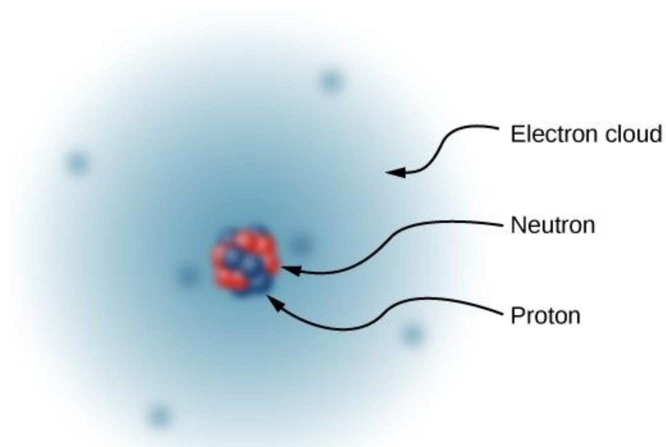
Outline

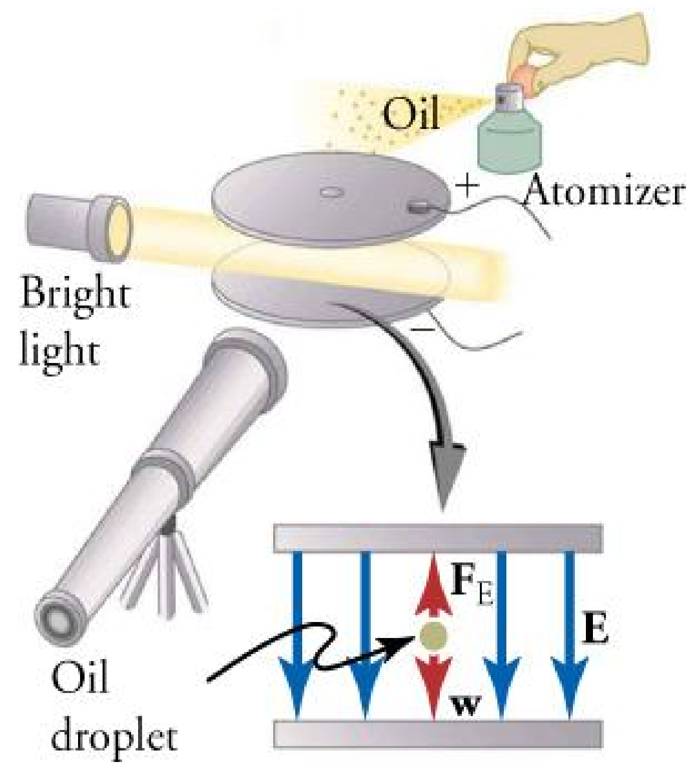
- 1. Electrical charge
  - Conductor and Insulator
- 2. Coulomb's Law
  - Principle of Superposition
- 3. Electric-field and Visualization
  - E-field Intensity $\vec{\boldsymbol{E}}$
  - Visualization - Field lines
- 4. Electric-fields produced by continuous charge distributions
  - using line integral, surface integral and volume integral

1.1 Electrical Charges (电荷)

- ***Electric charge*** is a property of matter that causes objects to attract or repel each other.
  - Two varieties: *positive* and *negative*.
  - Symbol: q or Q
  - Unit: C (reads as "Coulomb")
  - Elecmentary charge: $e = 1.6 \times 10^{-19} C$



The oil-drop experiment involved spraying a fine mist of oil between two metal plates charged with opposite charges. By knowing the mass of the oil droplets and adjusting the electric charge on the plates, the charge on the oil drops can be determined with precision.



1.2 Properties of Electric Charge

- Charge is quantized.
  - Electric charge comes in discrete amounts, as the integer multiple of the elementary charge e.
- The magnitude of the charge is independent of the type.
  - The smallest positive charge is e, and the smallest negative charge is $-e$.
- Charge is conserved.
  - Charge can neither be created nor destroyed; it can only be transferred from place to place, from one object to another.
- Charge is conserved in closed systems.
  - Charges can and do move around, and their effects can and do cancel, but the net charge in a local environment is conserved.
  - This is referred to as the ***law of conservation of charge***.

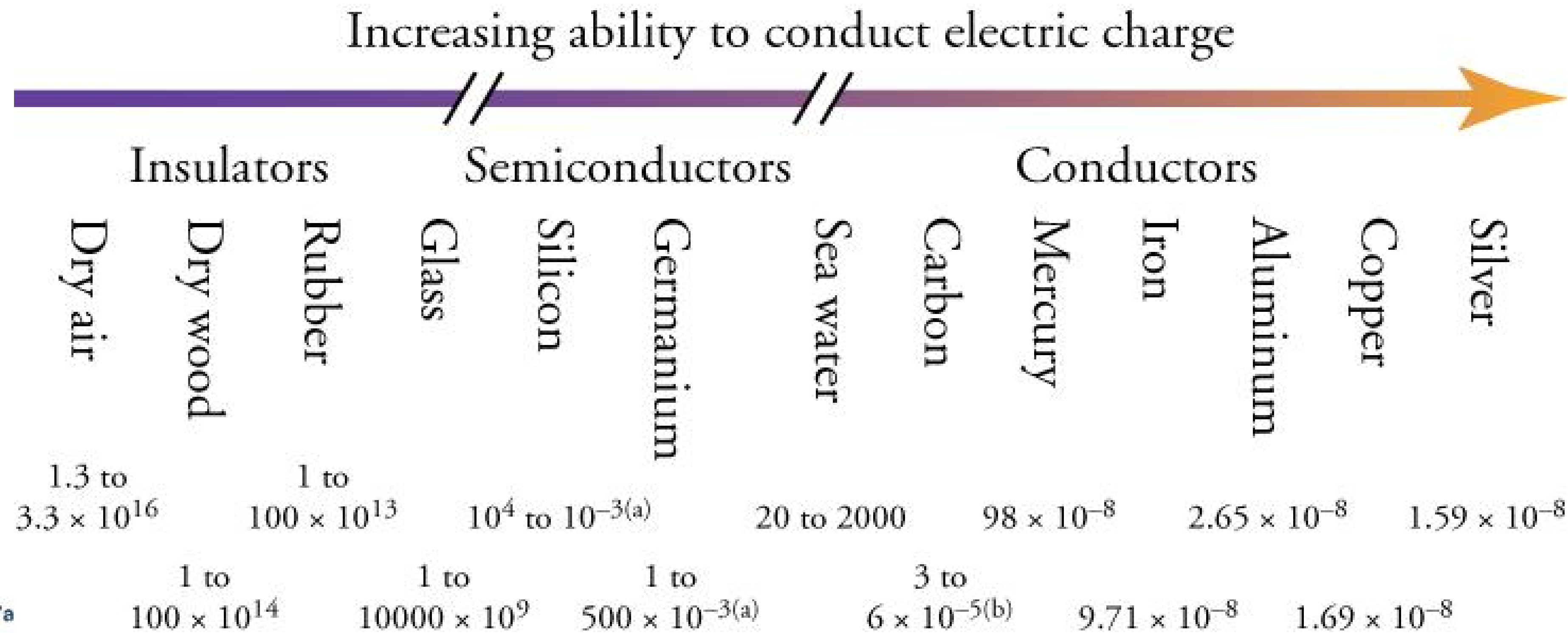
Quiz 1

- Which equation describes conservation of charge?
  a) $q_{initial} + q_{final}$ = constant
  b) $q_{initial} = q_{final} = 0$
  c) $q_{initial} - q_{final} = 0$
  d) $q_{initial}/q_{final}$ = constant

- An isolated system contains two objects with charges q_1 and q_2. If object 1 loses half of its charge, what is the final charge on object 2?
  a) $q_2/2$
  b) $3q_2/2$
  c) $q_2 - q_1/2$
  d) $q_2 + q_1/2$

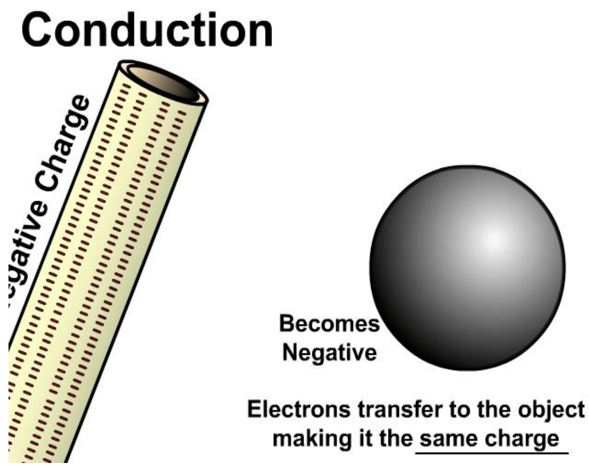
1.3 Conductors and Insulators

- If charge can easily move through a material, such as metals, then these materials are called ***conductors***.
- If charge cannot move through a material, such as rubber, then this material is called an ***insulator***.

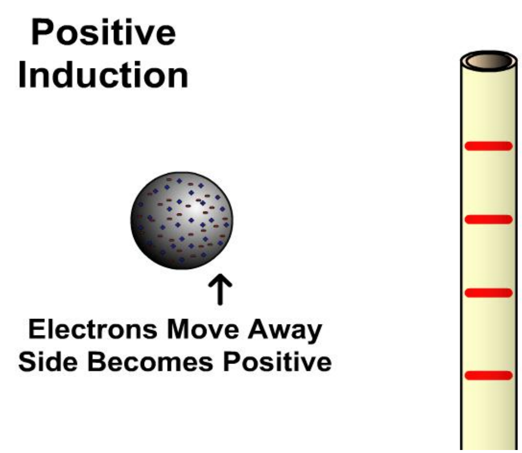


1.4 Charging methods

- Charging by ***conduction***: Conduction occurs on a neutral object when a charged object is in contact with it.



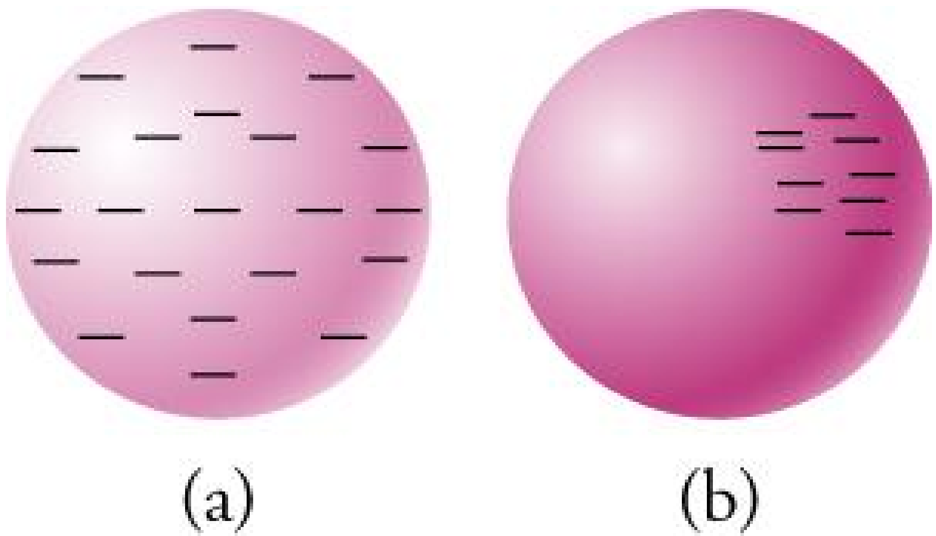
- Charging by ***induction***: Induction creates a temporary and opposite charge in that other object with no contact.



Think ...

- Example 1

What happens if an excess negative charge is placed on conducting/ insulating objects?



These two spheres, which one is conducting, which one is insulating?

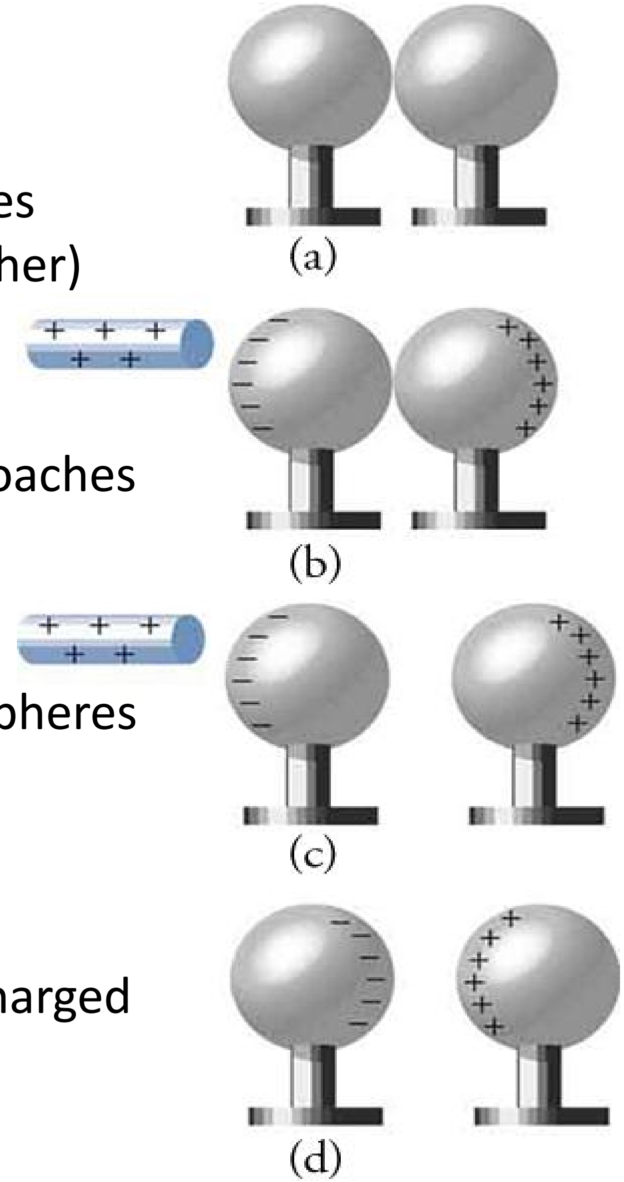
- Example 2

(a) Two neutral conducting spheres (touching each other)

(b) A positively charged rod approaches

(c) Separate the spheres

(d) Remove the charged rod



Quiz 2

- In a physics lab, you charge up three metal spheres, two with +3nC and one with −5nC. When you bring all three spheres together so that they all touch one another, what is the total charge on the three spheres?
  a) +1nC
  b) +3nC
  c) -2nC
  d) +11nC

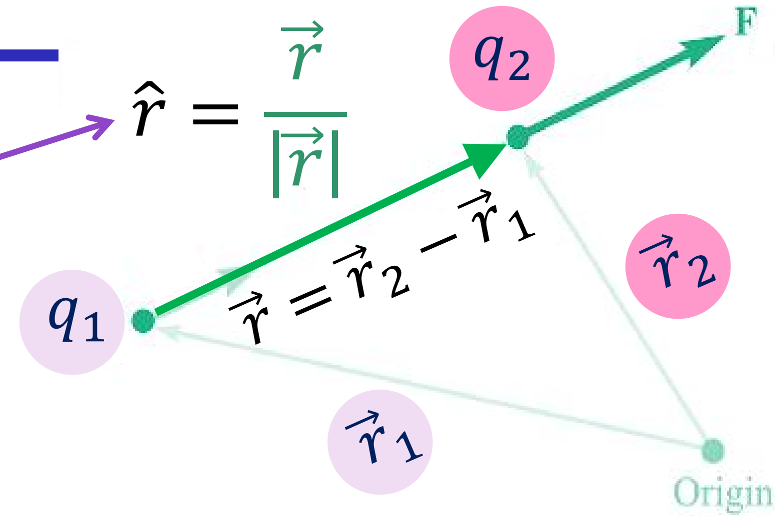
- [Ture or False] Charging an object by ***induction*** requires touching it with an object carrying excess charge.

2.1 Coulomb's Law & Electric Forces

- Vector form of Coulomb's law:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}\hat{r}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$



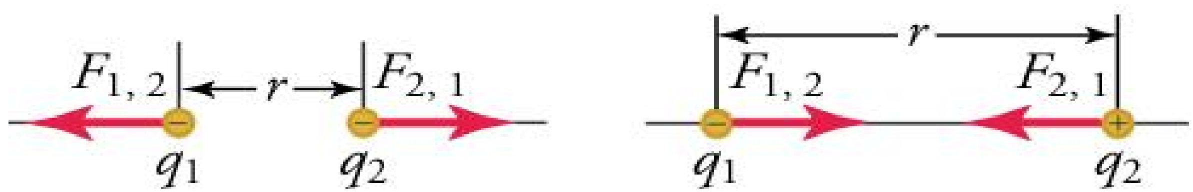
- where
  - Coulomb constant

$$k = \frac{1}{4\pi\varepsilon_0} = 8.9875 \times 10^9 N \cdot m^2/C^2$$

  - Free space permittivity

$$\varepsilon_0 = 8.85 \times 10^{-12} C^2/N \cdot m^2 \approx \frac{1}{36\pi} \times 10^{-9} C^2/N \cdot m^2$$

  - Unit vector $\hat{r}$ is from q_1 to q_2.

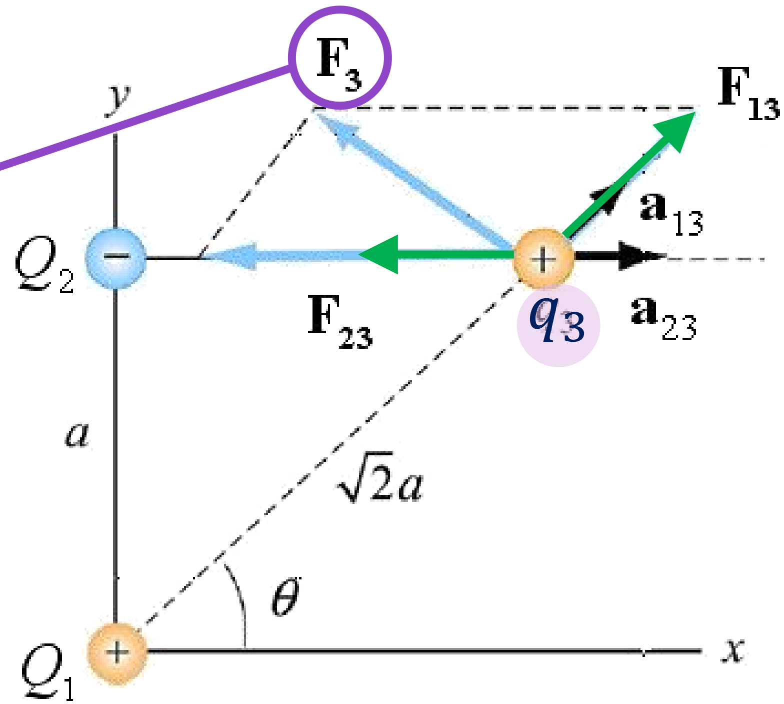


(a) (b)

2.2 Principle of Superposition

- Coulomb's Law applies to any pair of point charges:
  - When more than two charges are present, the net force on any one charge is the vector sum of the forces from other charges.

$$\frac{\vec{F}_3}{q_3} = \vec{E}$$



A system of three point charges

$$\vec{F}_3 = \vec{F}_{1\,on\,3} + \vec{F}_{2\,on\,3}$$

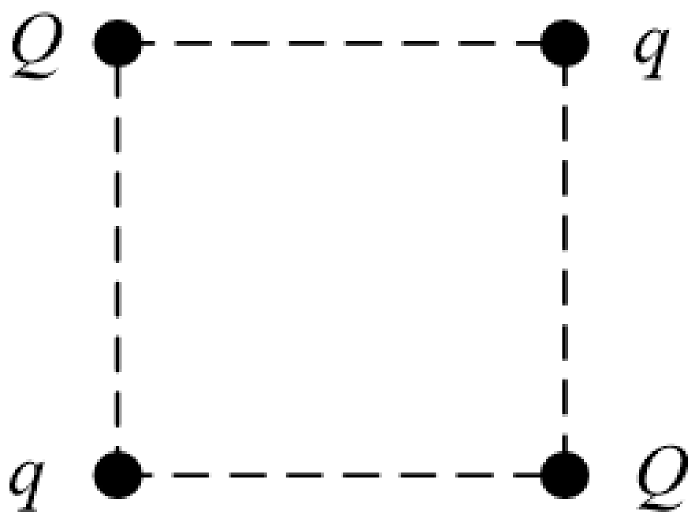
$$\therefore \vec{F}_j = \sum_{i=1, j\neq i}^{N} \vec{F}_{ij}$$

Principle of Superposition

$$\vec{F}(r) = \frac{1}{4\pi\epsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i^2 .$$

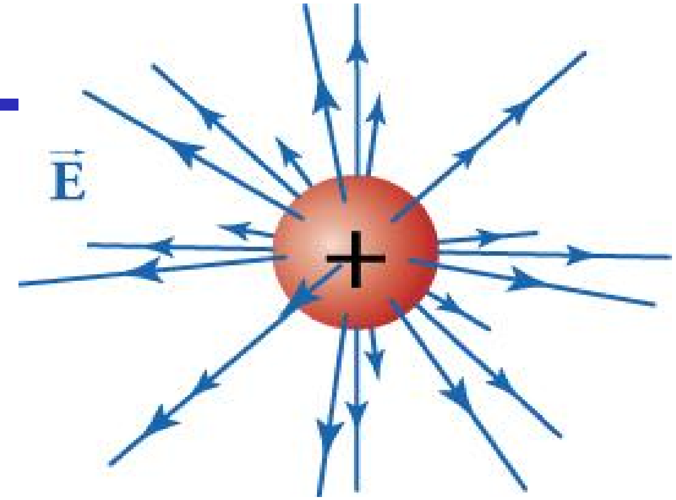
Quiz 3

- Two charges of size Q are placed on the diagonal corners of a square, and another two charges of size q are placed on the other two diagonal corners as shown below. If the total force on each charge Q is zero, find the relationship between Q and q.



3.1 E-field Intensity

- $\vec{E}$: Electric field intensity
- What is an electric field?
  - Michael Faraday: field of force;
  - James Maxwell: the space around an electrified object – a space in which electric forces act.
- A brief definition: the electric field at any location is the number of newtons of electrical force exerted on each coulomb of charge at that location.



$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{\vec{F}}{q_0} \qquad \vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}$$

Unit: N/C = $Jm^{-1}C^{-1}$ = V/m

For a point charge source $+Q$, the E-field intensity is:

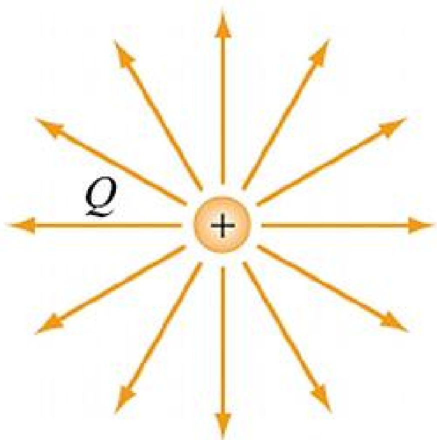
$$\vec{E} = \frac{\vec{F}}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

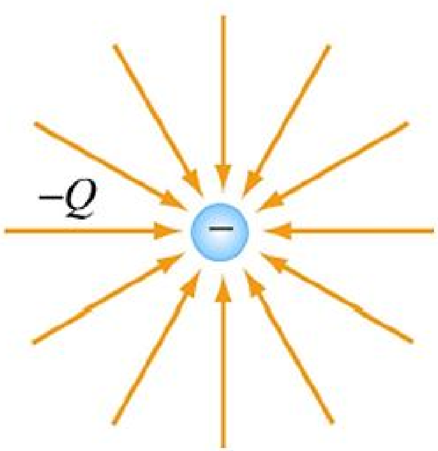
Quiz 4

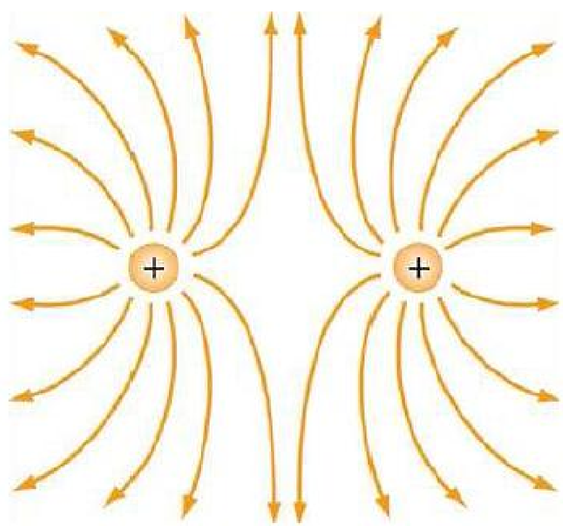
- In free space, a positive point charge, $Q_1 = 5\mu C$, is located at (0, 0, 2) m, and a negative point charge, $Q_2 = -10\ \mu C$, is located at (0, 4, 0) m in a rectangular coordinate system.
- Determine the electric field intensity at (0, 4, 2) m.

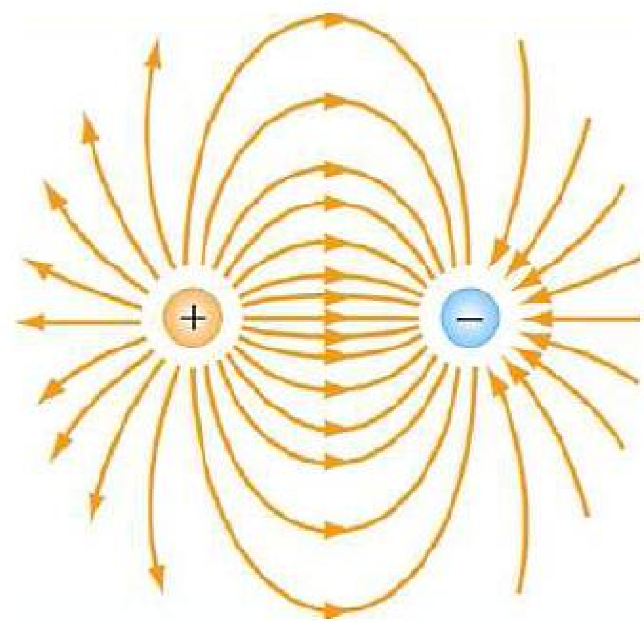
3.2 Visualization of E-field - Field lines

- It is often helpful to be able to visualise the electric field in the vicinity of a charged object.







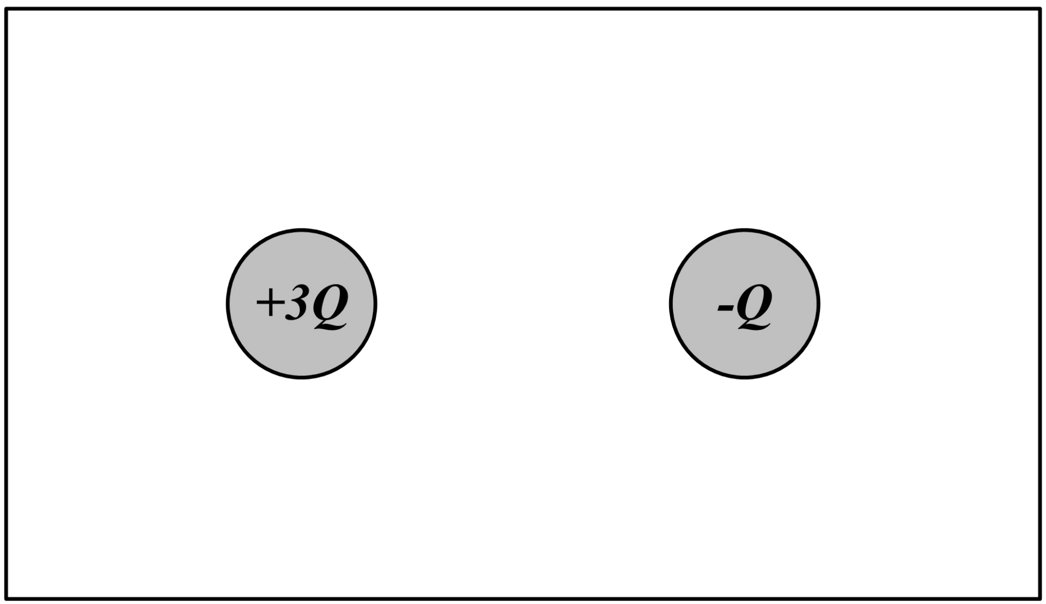


3.2 Visualization of E-field - Field lines

- The field lines **must** begin on positive charges (or at infinity) and terminate on negative charges (or at infinity).
- The direction of the electric field at any point is **tangent** to the field lines at that point.
- The number of lines per unit area through a surface **perpendicular** to the line is devised to be proportional to the magnitude of the E-field in a given region.
- **No** field lines can cross each other.

Quiz 5

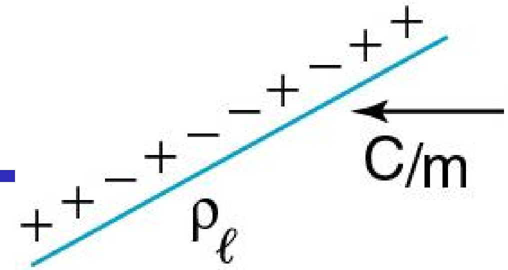
- A positive point charge + 3Q and a negative point charge −Q are placed in free space as shown below.
- Draw the electric field lines inside the boxed region.



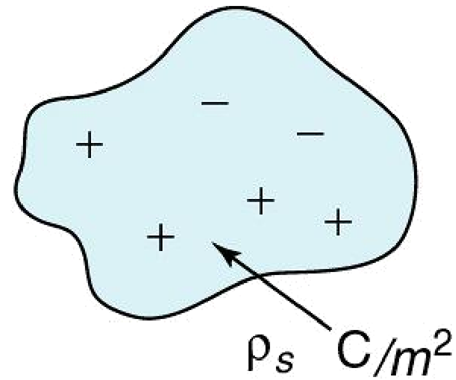
4.1 Charge Density

- The charge of a point charge is considered to reside at an infinitesimally small point.
- Charge is usually distributed as a line charge, a surface charge or a volume charge.

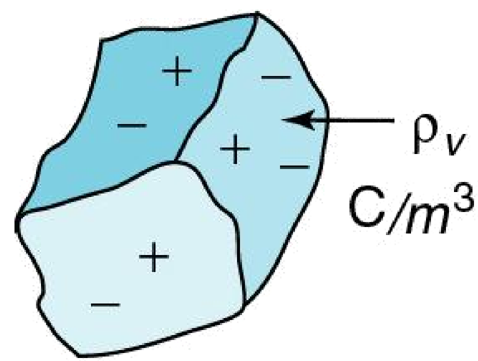
	Unit	Charge Density ρ	Total Amount of Charge Q
Line	C/m	$\rho_l = \dfrac{dQ}{dl}$	$Q = \int_L \rho_l\, dl$
Surface	C/m²	$\rho_S = \dfrac{dQ}{dA}$	$Q = \iint_S \rho_S\, dA$
Volume	C/m³	$\rho_V = \dfrac{dQ}{dV}$	$Q = \iiint_V \rho_V\, dV$



line charge



surface charge



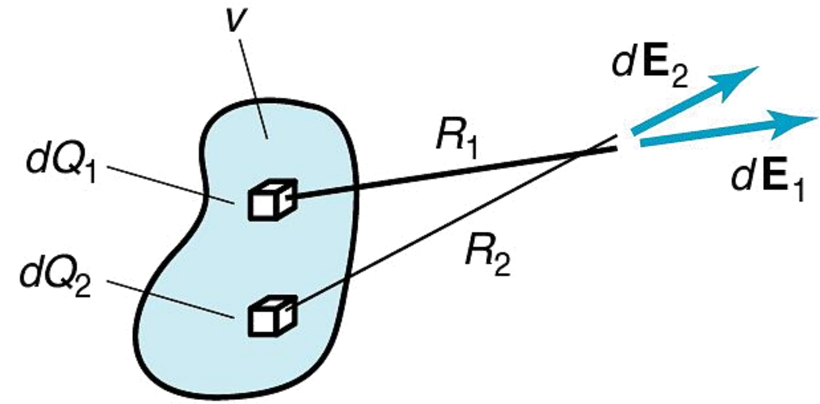
volume charge

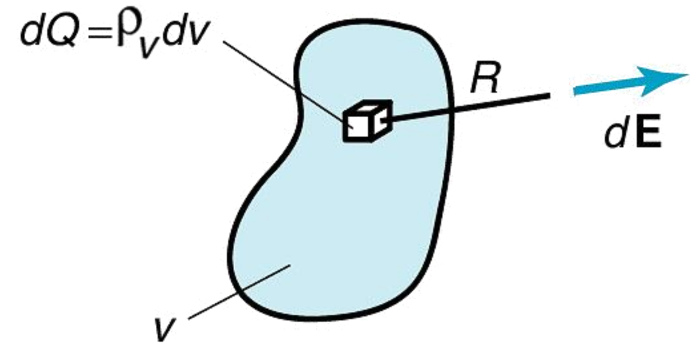
4.2 E-Field due to Charge Density

- When numerous charges are tightly packed within a volume/surface/line, we can take the continuum limit.
- If we can take the continuum limit:
  1. Divide the charge distribution into differential dQ
  2. Use superposition to add up the contributions from all these dQ at the point where we are interested in computing **E**

$$d\vec{E} = k\frac{dQ}{R^2}\hat{r}$$

  3. Utilise symmetry to simplify the resulting integral



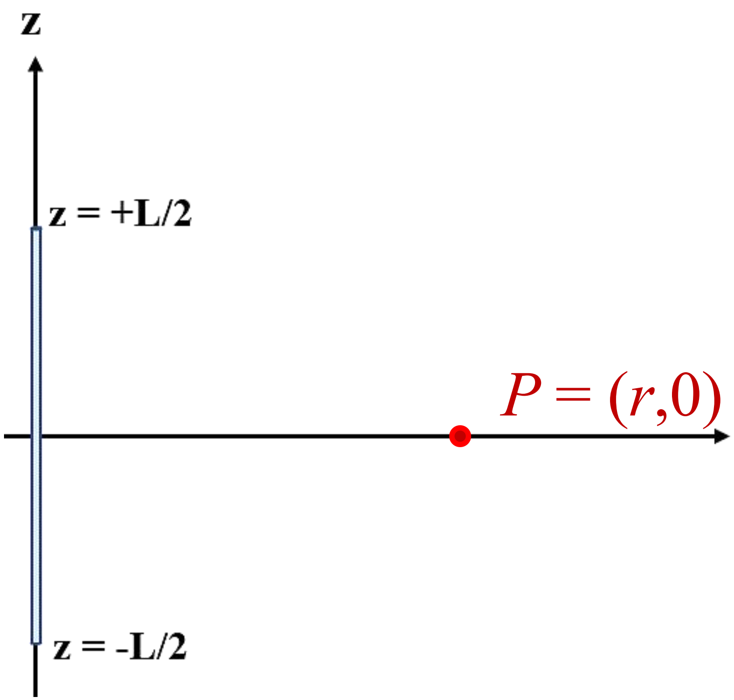


$$\vec{E} = \int d\vec{E} = \frac{k}{R^2}\iiint_V \rho_V\, dV\hat{r}$$

Case 1: Line Charge

$$\int \frac{1}{\left(\sqrt{x^2+a^2}\right)^3}dx = \frac{x}{a^2\sqrt{x^2+a^2}}$$

- A very thin rod of length L with a uniform positive charge density ρ_l is lying along the z-axis. Calculate the electric field at point P.

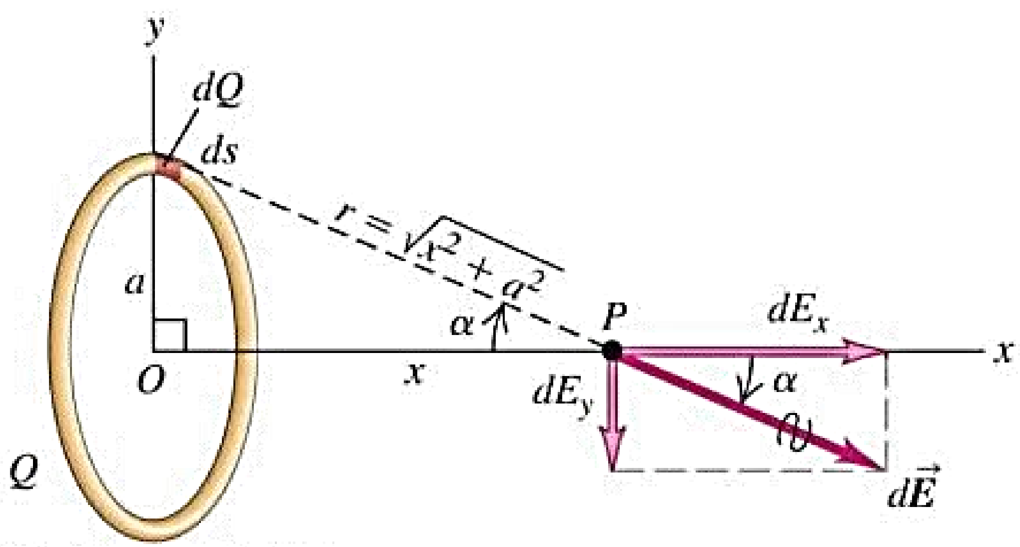


$$\vec{E} = k\frac{\rho_l L}{r\sqrt{r^2+(L/2)^2}}\hat{r}$$

When $L \to \infty$, then $\vec{E} \approx 2k\frac{\rho_l}{r}\hat{r}$

Quiz 6: Ring Charge

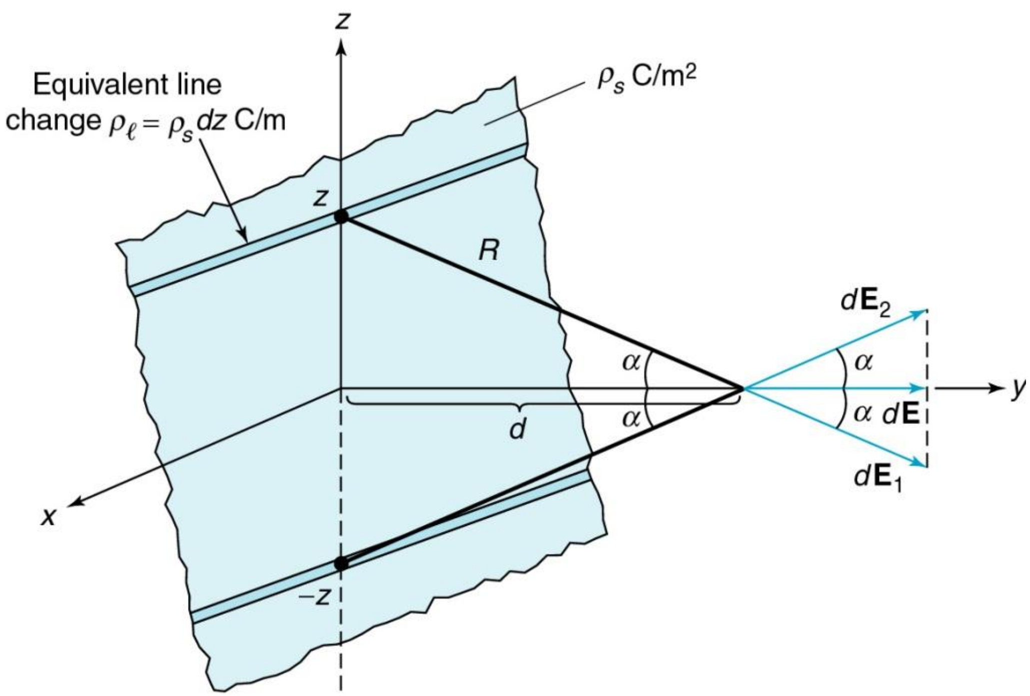
- A ring-sharped conductor with radius a has a uniformly distributed positive charge of Q around it. Find the electric field at point P that lies on the axis of the ring at a distance x from the origin.



Case 2: Surface of Charge

$$\int \frac{1}{x^2 + a^2} dx = \frac{\arctan(x/a)}{a}$$

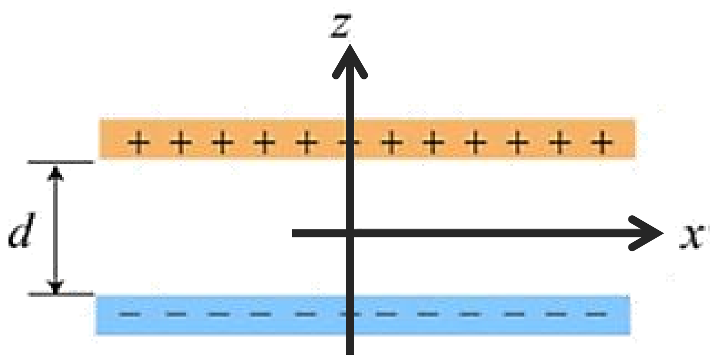
- An **infinite** sheet of charge with uniform charge distribution ρ_s is in the *xz* plane. Determine the electric field at some distance *d* away.

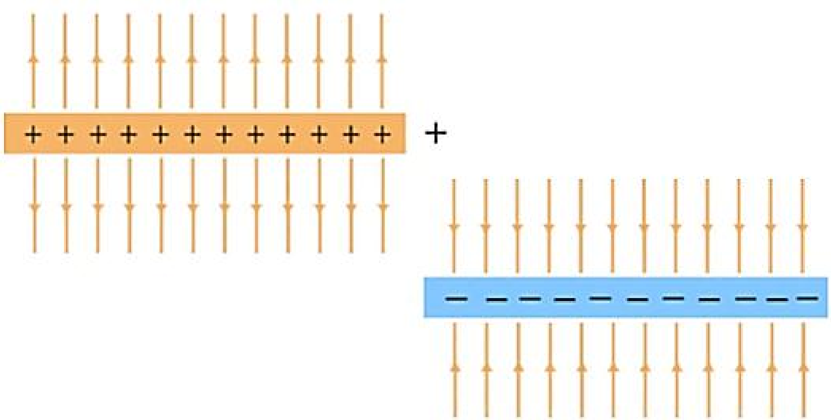


$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{y}$$

Quiz 7: Surface of Charge

- Two parallel **infinite** non-conducting planes lying in the *xy*-plane are separated by a distance. Each plane is uniformly charged with **equal but opposite** surface charge densities. Find the electric field everywhere in space.



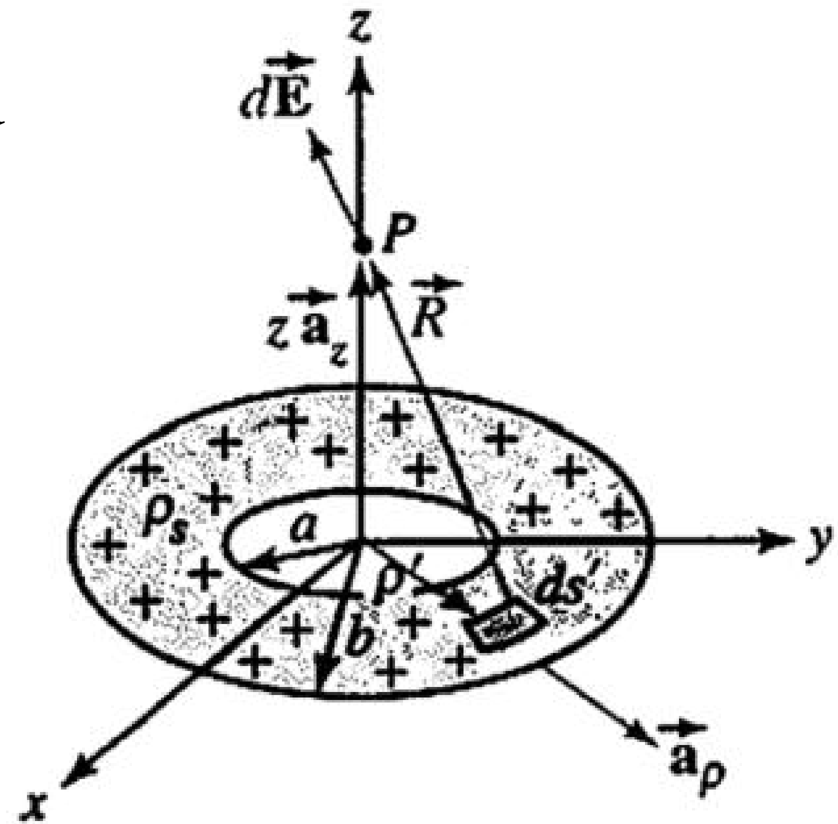


Summary

- Start with $d\vec{E} = k\frac{dQ}{R^2}\hat{r}$
- Rewrite the charge element dQ
- Substitute dQ into the expression for $d\vec{E}$
- Specify an appropriate coordinate system and express the differential element & R in terms of the coordinates
- Rewrite $d\vec{E}$ in terms of the integration variable(s), and apply symmetry argument to identify non-vanishing component(s) of the $\vec{E}$-field
- Complete the integration to obtain the $\vec{E}$-field.

Quiz 9

- A thin annular disc of inner radius a and outer radius b carries a uniform surface charge density ρ_s.
- Determine the electric field intensity at any point on the z-axis when $z \geq 0$.



Quiz 10

- Within a region of free space, charge density is given as

$$\rho_v = \frac{\rho_0 R\cos\theta}{a} \quad C/m^3$$

  where ρ_0 and a are constants.
- Find the total charge lying within:
  a) the sphere, $R \leq a$;
  b) the cone, $R \leq a$, $0 \leq \theta \leq 0.1\pi$;
  c) the region, $R \leq a$, $0 \leq \theta \leq 0.1\pi$, $0 \leq \varphi \leq 0.2\pi$.

Next ...

- Electric Flux and Flux Density
  - Gauss's Law
  - Divergence
  - Maxwell's equation - I

- Electric Potential
  - Energy and Potential
  - The potential field
  - Gradient
  - Maxwell's equation - II