CAN102 Electromagnetism and Electromechanics

Revision - Electromagnetism

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Outline

- 1. Information about the final exam
- 2. Go through the syllabus
- 3. Past exam questions

Information about the final exam

1. This is a **closed-book** examination.
   - Date: **May 29th, 2023 (14:00-17:00)**
   - Location: **CG23W, FBG95, SB123, SC169 (North Campus)**
   - Time-allowed: **10 min. for reading + 180 min. for writing**
     - **10 min. reading time** (14:00-14:10, can only read through the paper, but not making notes or drafts)
2. **FOUR** questions in total (100%)
   - 60% for **Electromagnetism** + 40% for **Electromechanics**
   - Each question has several sub-questions
   - **NO** MCQs

Information about the final exam

3. **Only** solutions in **English** are acceptable.
4. Write all the answers in the answer booklet provided.
5. Solutions for each question should start on a **NEW** page (larger text and space are preferred).
6. Only the university approved calculator - **Casio FS82ES/83ES** can be used.
7. Correct answers do not guarantee a full score: mark penalties may be imposed for missing intermediate solution steps or illogical solution processes.

Exam Paper Cover

2nd SEMESTER 2023/24 FINAL EXAMINATIONS

BACHELOR DEGREE – Year 2

Electromagnetism and Electromechanics

Writing Time: 180 minutes

Reading Time: 10 minutes (no writing or annotating allowed anywhere)

TOTAL TIME ALLOWED: 180 minutes writing time + 10 reading time

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INSTRUCTIONS TO ALL CANDIDATES

1、 Total marks available are 100.
2、 The number in the column on the right indicates the approximate marks for each section.
3、 Answer should be written in the answer booklet(s) provided.
4、 Only solutions in English are acceptable.
5、 Answer all questions.
6、 Only the university approved calculator - Casio FS82ES/83ES can be used.
7、 No annotating is allowed in reading time or after the end of writing time.

Tips

- Read the question carefully
  - **Highlight** the key information

- Be careful with the UNITS and SI prefixes

  $m = 10^{-3}$ $\mu = 10^{-6}$ $n = 10^{-9}$ $p = 10^{-12}$

  $K = 10^{3}$ $M = 10^{6}$ $G = 10^{9}$ $T = 10^{12}$

- Write down the equations **CORRECTLY**
  - Majority of equations are provided. Please copy them in the **correct form**!
  - Equation list is provided on LMO

Structure of EM part

- Lecture 01-02 Introduction and Math preparation

- Lecture 03-06 Static Electric Field
- Lecture 07-08 Steady current and R and C

} Q1 (30%)

- Lecture 09-10 Static Magnetic Field
- Lecture 11-12 Electromagnetic Induction and L

} Q2 (30%)

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Lecture 01

- Part 1
  - What is Electromagnetism?
  - Why do we learn it?
- Part 2
  - 1. Scalars and Vector
    - Definition and Representation
    - Vector Algebra
    - Scalar and Vector Fields
  - 2. 2D Coordinate Systems
    - Rectangular CS and Polar CS
    - Conversion between Rect. CS and Polar CS
    - Vector Algebra in 2D CSs

Won't be tested independently, but served as fundamental for following parts.

Lecture 02

- 3D Coordinate Systems
  - Key concepts about a Coordinate System
  - Rectangular, Cylindrical, Spherical CSs
- Vector Analysis
  - Integrals
    - Line/Surface/Volume Integrals
    - Differential Elements in Three CSs
  - Differentials
    - Gradient, Divergence, Curl and Laplacian
  - Theorems
    - Gauss's and Stokes' Theorems

Most important in Lecture 1 & 2.
Cause $\vec{E}$ and $\vec{H}$ analyses are relying on VA.

Lecture 03

- 1. Electrical charge
  - Conductor and Insulator
- 2. Coulomb's Law
  - Principle of Superposition
- 3. Electric-field and Visualization
  - E-field Intensity $\vec{E}$
  - Visualization - Field lines
- 4. Electric-fields produced by continuous charge distributions
  - using line integral, surface integral and volume integral

Understand the difference, especially when it's combined with

① charge distribution { uniformly / conducting

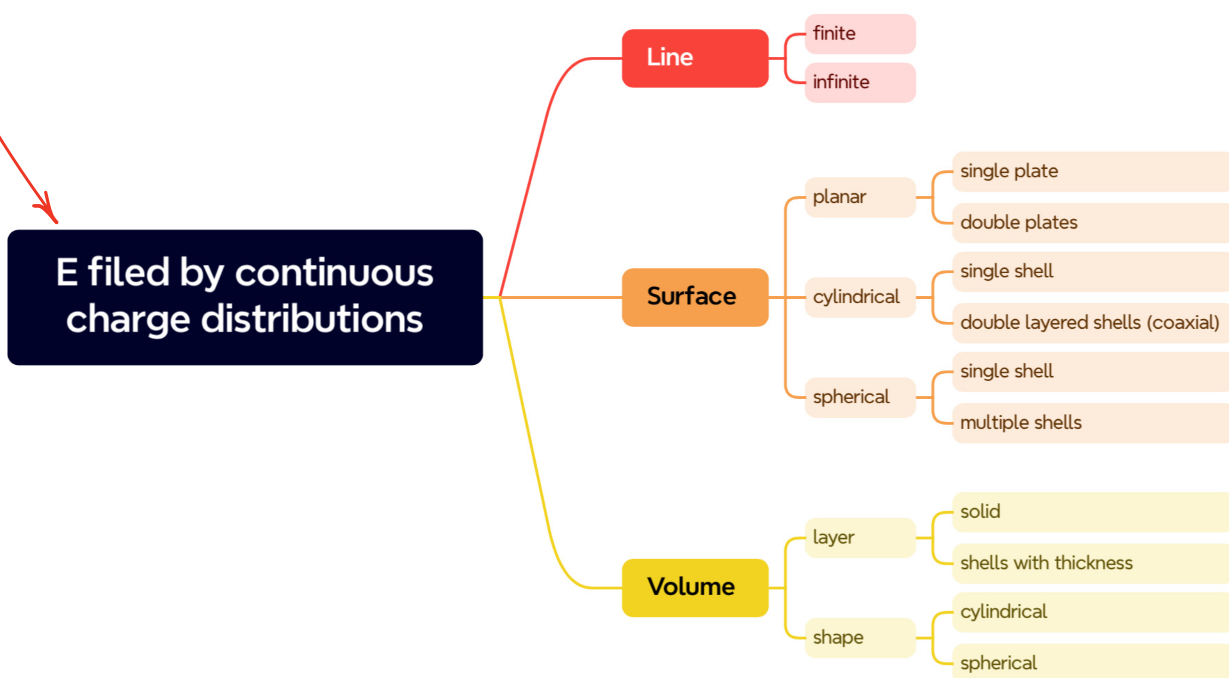
② Equilibrium: induced charges on inner & outter surface

Typical structures

- point charges vs point charges
- point charges vs conducting structures

Lecture 04

- Electric Flux
- Gauss's Law - Integral form
  - Gauss's Law
  - Flux density
  - Calculating E-field using Gauss's Law
- Gauss's Law - Differential form
  - Divergence
  - Divergence Theorem
  - Gauss's Law in differential form



Lecture 05

- Electric Potential
  - Work and energy
  - Potential difference and Potential
  - Potential field due to charges
  - Equipotential lines / surfaces
- E-field Loop Theorem
  - Electric field circulation
  - Conservative fields
  - Gradient
- Poisson's and Laplace's Equations

Linked to Gauss's law
(all the typical structures)

Lecture 06

- Maxwell's equation II - Electric field loop theorem
  - Curl
  - Stoke's Theorem
  - Integral and Differential forms
- Conductors and Dielectrics
  - Ideal conductors
  - Electric Equilibrium
  - Dielectrics and Permittivity
- Boundary Conditions
  - Tangential and normal components of E-field

Lecture 07

- Currents
  - Conduction current
  - Convection current
  - Electrolytic current
- Conduction current and current density
  - Drift Velocity and Mobility
  - Current Density and Current
  - Conductivity and resistivity
- From Electromagnetics (EM) to Electric circuits (EC)
  - Ohm's law in microscopic and macroscopic views
  - Joule's law (Power and Energy)
- Boundary Conditions

Lecture 08

- Resistors
  - Resistance calculation
  - Resistance, resistivity and conductivity
  - Admintance
- Capacitors
  - Capacitance calculation
  - Capacitor with dielectrics
  - Parallel and series connection of capacitors
  - Energy stored in capactors
  - I-V relationship of capacitors

link to L7

link to L5

Lecture 09

- Fundamentals of Magnetic Fields
  - What is a magnetic field
  - Sources of the magnetic fields
- Biot-Savart Law
- Gauss's Law for Magnetic Field
- Magnetic field Loop Theorem – Ampere's Law
  - Integral and Differential forms
  - Application: find magnetic field for given current sources

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Biot-Savart law vs Ampere's law

① Typical structure's method choice

② Solving procedure

③ Comparison to $\begin{cases} \text{Coulomb's law} \\ \text{Gauss's law} \end{cases}$

Lecture 10

- Visualisation of Magneitc Fields
  - Magnetic field lines
  - Comparison with electric field lines
- Magnetic Forces
  - on a moving charge
  - on a current-carrying wire
- Magnetic materials
  - Permeability
  - Classification and ferromagnetic materials
- Boundary Conditions

$$\left.\begin{matrix} q\vec{v} \\ I\vec{l} \end{matrix}\right\} \times \vec{B} \rightarrow \vec{F} \rightarrow \vec{a}$$

Lecture 11

- Faraday's Experiments
- Lenz's Law
- Faraday's Law
  - EMF (Electromotive Force)
- Integral and Differential forms
- Motional EMF

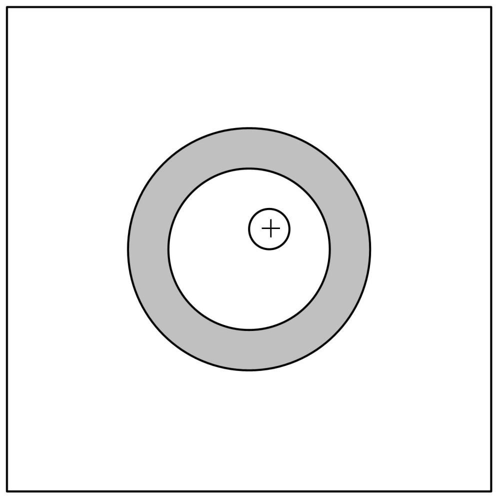
Φ. emf (voltage). current

Lecture 12

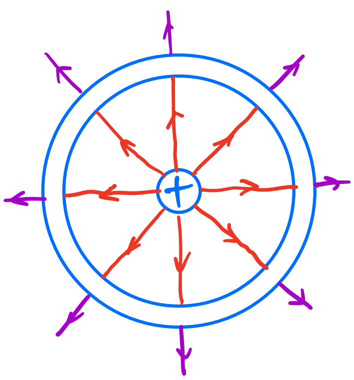
- Inductance
  - Inductors
  - Self-inductance
  - Mutual-inductance
  - Energy stored

Static Electric Field

- A single positive point charge Q is positioned arbitrarily inside an uncharged spherical conducting shell as shown in Figure Q2(a). Draw the electric field lines inside the boxed region on your answer booklet (5 marks)



① Centre



Usually, 8 lines

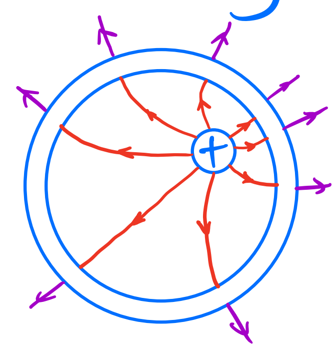
with arrow

starting/ending at $\begin{cases}\text{point charges}\\ \text{conductor edges}\\ \text{infinity}\end{cases}$

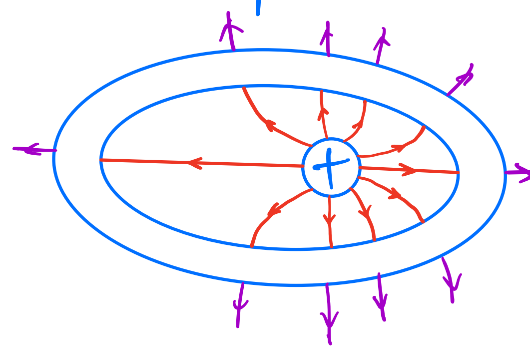
No intersection

Perpendicular to conductor surface

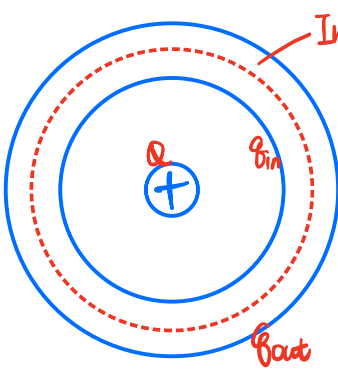
② Arbitrary



③ Ellipsoidal



④ Surface charges (induced)



Inside this conductor (shell)

no electric field $E=0$

$\Rightarrow \oiint \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon} = 0$

$\Rightarrow Q_{enc} = 0$

$\quad\; \|$

$Q + q_{in} = 0 \Rightarrow q_{in} = -Q$

$q_{in} + q_{out} = 0 \Rightarrow q_{out} = Q$

⑤ $q_{in} = -Q$

$q_{out} + q_{in} = Q_2$

$\Rightarrow q_{out} = Q_2 + Q$

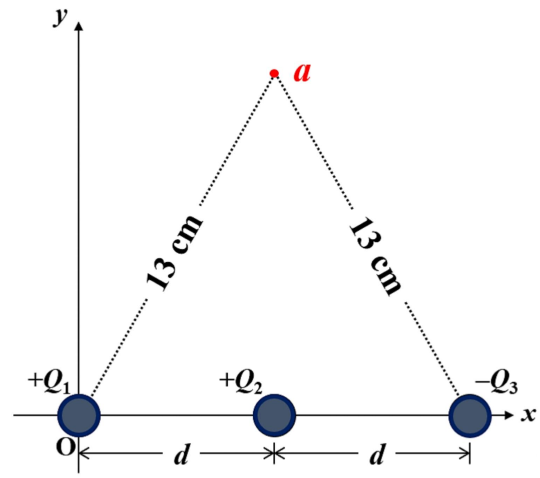
Static Electric Field

- Two positive charges +Q1 and +Q2, and a negative charge –Q3 are arranged in a line as shown in Figure Q1(a). In this arrangement $\boxed{\text{+Q1 is in equilibrium}}$ while +Q2 and –Q3 are fixed. Gravitational forces are negligible.

i) Plot and label **all** the forces exerted on the charge $+Q_1$ and determine their magnitudes.

ii) Determine the relationship between charge Q_2 and charge Q_3 which ensures the equilibrium status of $+Q_1$.

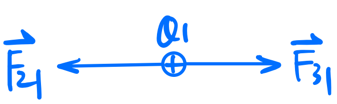
iii) If the charge $+Q_2$ is now removed and $+Q_1$ and $-Q_3$ have charges of +12 nC and –12 nC respectively, determine the electric field intensity and the electric potential V_a at the point a with the distance $2d$ = 10 cm.



① $\vec{F}_{21} = k\dfrac{Q_1 Q_2}{d^2}(-\hat{a}_x)$

exerted on Q_1 by Q_2

$\vec{F}_{31} = k\dfrac{Q_1 Q_3}{(2d)^2}\hat{a}_x$

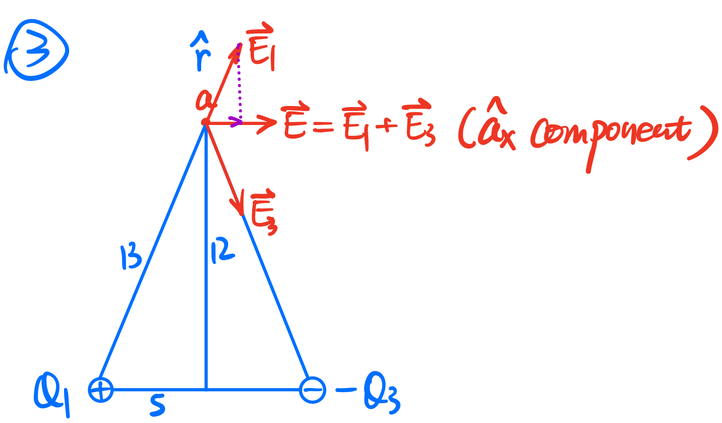


② Q_1 is in equillibrium means

$\vec{F}_{21} + \vec{F}_{31} = 0$

$\Rightarrow \cancel{k}\dfrac{\cancel{Q_1}Q_2}{\cancel{d^2}} = \cancel{k}\dfrac{\cancel{Q_1}Q_3}{4\cancel{d^2}} \Rightarrow Q_3 = 4Q_2$

③



Coulomb's law

$\vec{E}_1 = \dfrac{1}{4\pi\varepsilon}\dfrac{Q_1}{r^2}\hat{r} = \dfrac{12 \times 10^{-9}}{4\pi \times \frac{1}{36\pi} \times 10^{-9}} \times \dfrac{1}{0.13^2}\hat{r} = 6.39\,\hat{r}\ (\mathrm{kV/m})$

But in $\hat{a}_x$ direction: $\vec{E}_x = |\vec{E}_1| \times \frac{5}{13} = 2.46\,\hat{x}\ (\mathrm{kV/m})$

Take $\vec{E}_3$ into consideration: $\vec{E} = 2\vec{E}_{1x} = 4.92\,\hat{x}\ (\mathrm{kV/m})$

Potential

$V_1 = k\dfrac{Q_1}{r}$

$V_3 = k\dfrac{-Q_3}{r}$

$\Big\}$ Same amount opposite sign.

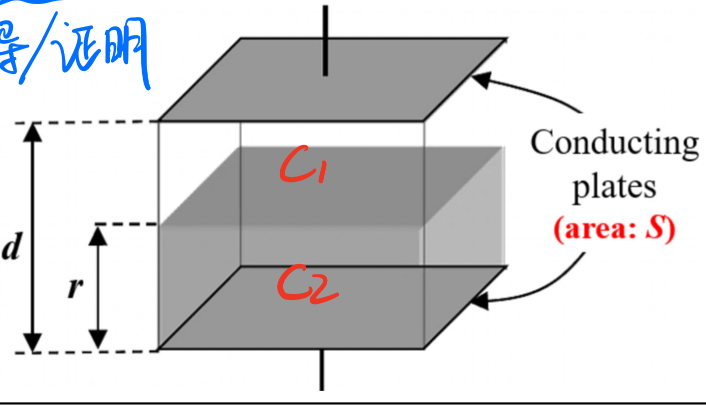
$\Rightarrow V_a = 0\ (\mathrm{V})$

Static Electric Field

- Jerry plans to measure the depth r of water in a cubic tank of volume 1.0×1.0×1.0 m3 as shown in Figure Q1(b). He places conducting plates (with the area of S) on the top and bottom surfaces of the tank and measures the capacitance between them as a function of water depth. The water is non-conducting with the permittivity λ. Assume the tank walls do not contribute to the capacitance and the fringing fields can be neglected. Show that the capacitance of this system is:

推导/证明

$$C = \frac{\varepsilon_0 S}{d - r(1 - \frac{\varepsilon_0}{\lambda})}$$



Capacitance — series connection

$$C_1 = \frac{\varepsilon_1 S}{d-r} = \frac{\varepsilon_0 S}{d-r}$$

$$C_2 = \frac{\varepsilon_2 S}{r} = \frac{\lambda S}{r}$$

$$\Rightarrow C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\frac{\varepsilon_0 S}{d-r} \cdot \frac{\lambda S}{r}}{\frac{\varepsilon_0 S}{d-r} + \frac{\lambda S}{r}} = \frac{\frac{\varepsilon_0 S^2 \lambda}{\cancel{(d-r)r}}}{\frac{r\varepsilon_0 S + (d-r)\lambda S}{\cancel{(d-r)r}}}$$

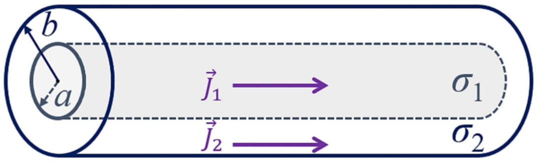
$$= \frac{\varepsilon_0 S}{\frac{r\varepsilon_0 + (d-r)\lambda}{\lambda}} = \frac{\varepsilon_0 S}{d - r + \frac{r\varepsilon_0}{\lambda}}$$

Steady Current and Resistors

- A solid wire with conductivity σ1 and radius a has a jacket of material with conductivity σ2, and its inner radius is a and outer radius is b as shown.

a) Calculate the resistances in both regions in terms of the radii and conductivities.

b) Determine if the ratio of the current densities in the two materials is independent of radius a and radius b and explain the reason with calculations.



a) $R_1 = \frac{l}{\sigma_1 S_1} = \frac{l}{\sigma_1 \pi a^2}$

$R_2 = \frac{l}{\sigma_2 S_2} = \frac{l}{\sigma_2 \pi (b^2 - a^2)}$

b) $\left.\begin{array}{l} J_1 = \frac{I_1}{S_1} \\ I_1 = \frac{V}{R_1} \end{array}\right\} \Rightarrow J_1 = \frac{\frac{V}{l}\cdot \sigma_1 S_1}{S_1} = \frac{V}{l}\sigma_1$

Similarly

$J_2 = \frac{I_2}{S_2} = \frac{\frac{V}{R_2}}{S_2} = \frac{\frac{V}{l}\cdot \sigma_2 S_2}{S_2} = \frac{V}{l}\sigma_2$

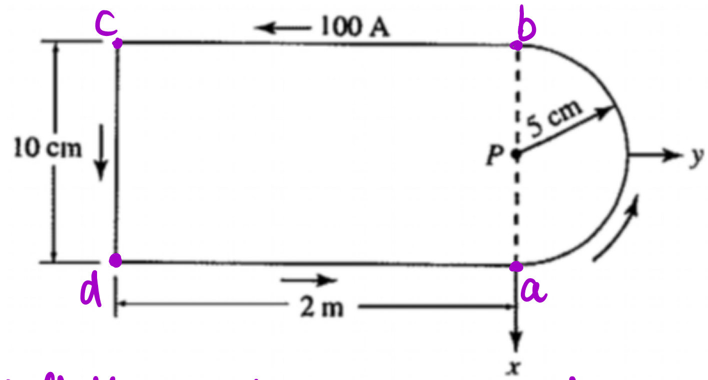
$\Rightarrow \frac{J_1}{J_2} = \frac{\sigma_1}{\sigma_2}$

independent of a & b.

Static Magnetic Field

- Find the magnetic flux density at point P:

How about this one ?

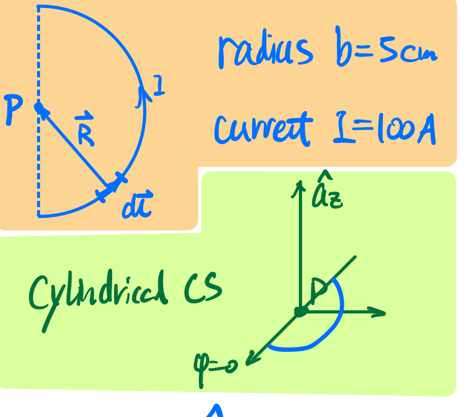


This structure is not infinite, not very symmetric, so we can only use Biot-Savart Law.

Cut it into four sections, calculate the $\vec{B}$ from each section and add them up afterwards.

i) Section a-b is a semi-circle

$$\left.\begin{array}{l}\vec{R} = -b\hat{a}_r \\ d\vec{l} = b\,d\varphi\,\hat{a}_\varphi\end{array}\right\} \Rightarrow d\vec{B} = \frac{\mu_0 I d\vec{l}\times\vec{R}}{4\pi R^3} = \frac{\mu_0 I \cancel{b^2} d\varphi}{4\pi \cancel{b^3}}\hat{a}_z$$

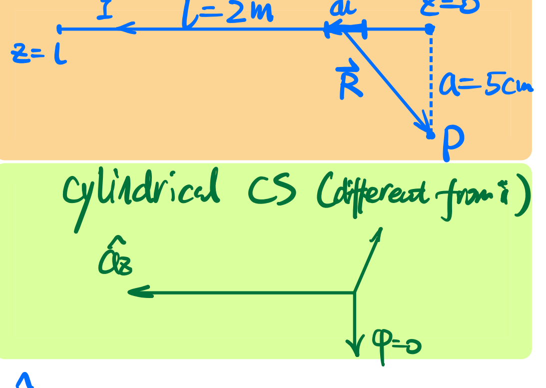


$$\vec{B}_1 = \int_{\varphi=0}^{\pi} d\vec{B} = \frac{\mu_0 I}{4\pi b}\int_0^{\pi} d\varphi\,\hat{a}_z = \frac{\mu_0 I}{4b}\hat{a}_z = 0.628\hat{a}_z\ (\mu T)$$

point out of paper @ P

ii) Section b-c is the horizontal conductor

$$\left.\begin{array}{l}d\vec{l} = dz\,\hat{a}_z \\ \vec{R} = a\cdot\hat{a}_r - z\hat{a}_z\end{array}\right\} \Rightarrow d\vec{B} = \frac{\mu_0 I d\vec{l}\times\vec{R}}{4\pi R^3} = \frac{\mu_0 I a\,dz}{4\pi(a^2+z^2)^{3/2}}\hat{a}_\varphi$$



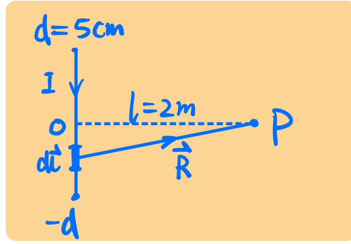
$$\vec{B}_2 = \int_{z=0}^{L} d\vec{B} = \frac{\mu_0 I a}{4\pi}\int_{z=0}^{L}\frac{dz}{(a^2+z^2)^{3/2}}\hat{a}_\varphi = \frac{\mu_0 I \cancel{a}}{4\pi}\cdot\frac{L\hat{a}_\varphi}{a^{\cancel{2}}\sqrt{L^2+a^2}} = 0.2\hat{a}_\varphi\ (\mu T)$$

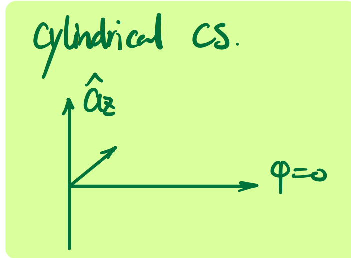
point out of paper @ P

iii) Section c–d is the vertical conductor

$$\left.\begin{aligned} d\vec{l} &= -dz\,\hat{a}_z \\ \vec{R} &= L\hat{a}_r - z\hat{a}_z \end{aligned}\right\} \Rightarrow d\vec{B} = \frac{\mu_0 I d\vec{l}\times\vec{R}}{4\pi R^3}$$

$$= -\frac{\mu_0 I L \hat{a}_\varphi}{4\pi R^3}$$





$$\vec{B}_3 = \int_{z=d}^{-d} \frac{\mu_0 I L\,dz}{4\pi(L^2+z^2)^{3/2}}(-\hat{a}_\varphi)$$

$$= \frac{\mu_0 I L}{4\pi}\int_{z=-d}^{d}\frac{dz}{(L^2+z^2)^{3/2}}\hat{a}_\varphi$$

$$= \frac{\mu_0 I L \hat{a}_\varphi}{4\pi}\left.\frac{z}{L^2\sqrt{L^2+z^2}}\right|_{-d}^{d} = \frac{\mu_0 I \cancel{L} \hat{a}_\varphi}{2\cancel{4}\pi}\frac{\cancel{2}d}{L^{\cancel{2}}\sqrt{L^2+d^2}}$$

$$= \frac{\mu_0 I d\,\hat{a}_\varphi}{2\pi L\sqrt{L^2+d^2}} = 250\,\hat{a}_\varphi\ (\mathrm{nT})$$

point out of paper @ P

iv) Section d–a is the other horizontal conductor

So same as ii), $\vec{B}_4 = 0.2\hat{a}_\varphi\ (\mathrm{mT})$

Totally, the magnetic flux density at $\vec{P}$ is:

$$B = (0.628 + 0.2 + 250\times10^{-6} + 0.2)\times10^{-3} \approx 1.028\ (\mathrm{mT})$$

pointing out of the paper.

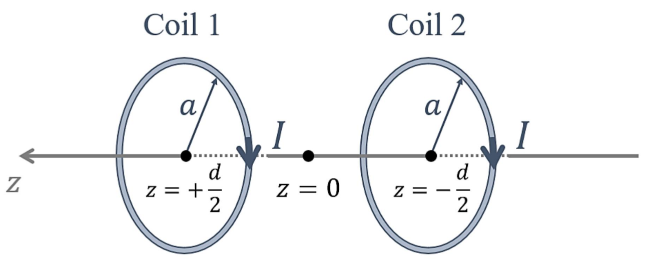
Static Magnetic Field

- Tom has a pair of two identical current-carrying coils (radius a, turn N = 1) carrying the same steady current I. He placed them at a distance d apart from each other as shown. Assume the setup is in free space and the currents carried by the coils are in the same direction.

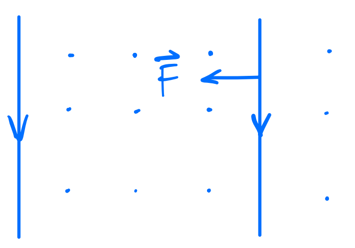
i) Determine whether the two coils are going to attract or repel each other and explain the reason.

ii) With aid of the **cylindrical coordinate system**, determine the magnetic field intensity at any point along the *z*-axis.

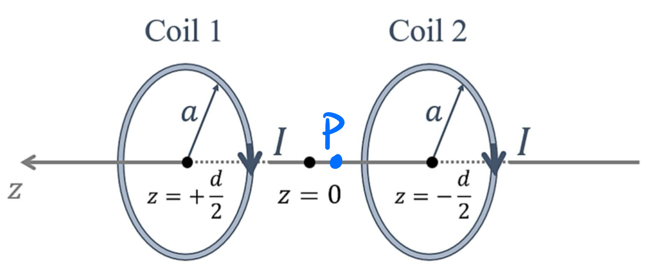
iii) Given that $a = d = 20$ mm, plot the magnetic field lines produced by the whole system.



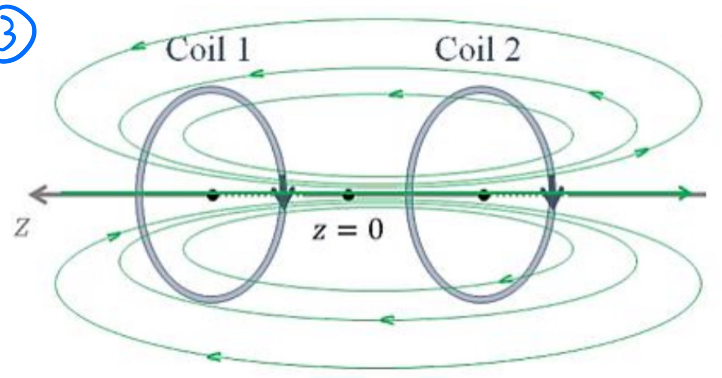
① Attract



② Biot - Savart Law



③



$\vec{P}$ at location z_0

$$d\vec{H}_1 = \frac{1}{4\pi}\frac{I d\vec{l} \times \vec{r}}{r^3}$$

$$d\vec{l} = -a d\varphi \, \hat{a}_\varphi$$

$$\vec{r} = z_0 \hat{a}_z - a\hat{a}_r$$

$$\Rightarrow d\vec{H}_1 = \frac{1}{4\pi}\frac{-I \cdot a d\varphi \cdot (z_0 \hat{a}_r + a\hat{a}_z)}{\left(a^2 + \left(z_0 - \frac{d}{2}\right)^2\right)^{3/2}}$$

Going around the loop. $\hat{a}_r$ term cancelled.

$$\Rightarrow \vec{H}_1 = \int_0^{2\pi} \frac{1}{4\pi} \cdot \frac{-I a^2 d\varphi \, \hat{a}_z}{\left(a^2 + \left(z_0 - \frac{d}{2}\right)^2\right)^{3/2}} = \frac{a^2 I (-\hat{a}_z)}{2\left(a^2 + \left(z_0 - \frac{d}{2}\right)^2\right)^{3/2}}$$

For coil 2. Similarly $\vec{H}_2 = \dfrac{a^2 I(-\hat{a}_z)}{2\left(a^2 + \left(z_0 + \frac{d}{2}\right)^2\right)^{3/2}}$

Total field intensity is $\vec{H} = -\hat{a}_z \cdot \dfrac{a^2 I}{2}\left(\dfrac{1}{\left(a^2 + \left(z_0 - \frac{d}{2}\right)^2\right)^{3/2}} + \dfrac{1}{\left(a^2 + \left(z_0 + \frac{d}{2}\right)^2\right)^{3/2}}\right)$

Electromagnetic Induction

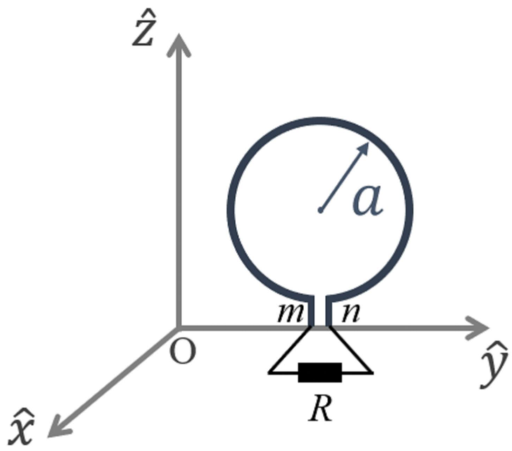
- In a uniform magnetic field, a circular conducting loop with radius a lies in the y-z plane. There is a small gap between points m and n with wires leading to an external circuit of a resistor R situated in the x-y plane as shown. It is given that

$$\vec{B} = B_0\left[2\left(\frac{t}{\tau}\right)^4 - 4\left(\frac{t}{\tau}\right)^2\right]\hat{x}$$

where t is the variable of time, B_0 and τ are constant values.

  a) Derive the expression for the induced emf in the conducting loop;

  b) Given that the resistance of the resistor is $R = 2B_0\ \Omega$, determine an expression for the current flowing through the resistor and the time at which the current through the resistor reverses its direction.



a) $\text{emf} = -\frac{d\Phi}{dt}$

$$\Phi = \iint_S \vec{B}\cdot d\vec{s} = BS = B_0\left[2\left(\frac{t}{\tau}\right)^4 - 4\left(\frac{t}{\tau}\right)^2\right]\cdot \pi a^2$$

$$\Rightarrow \text{emf} = -B_0\pi a^2\left[2\frac{d}{dt}\left(\frac{t}{\tau}\right)^4 - 4\frac{d}{dt}\left(\frac{t}{\tau}\right)^2\right]$$

$$= -B_0\pi a^2\left[\frac{8}{\tau}\left(\frac{t}{\tau}\right)^3 - \frac{8}{\tau}\left(\frac{t}{\tau}\right)\right] = \frac{8B_0\pi a^2 t}{\tau^2}\left(1-\frac{t^2}{\tau^2}\right)$$

b) $I = \frac{\text{emf}}{R} = \frac{4\pi a^2 t}{\tau^2}\left(1-\frac{t^2}{\tau^2}\right)$

Current reverse its direction when I changes its sign.

Since $\frac{4\pi a^2}{\tau^2} > 0$, can be ignored.

let $f_I(t) = t\left(1-\frac{t^2}{\tau^2}\right)$ its roots are the flipping points.

$f_I(t) = t\left(1-\frac{t^2}{\tau^2}\right) = 0 \Rightarrow t_1 = 0,\ t_2 = \tau,\ t_3 = -\tau$, are the roots.

Only $t = \tau$ is larger than zero, so it's the answer.