EEE103 ELECTRICAL CIRCUITS

WEEK2-VOLTAGE AND CURRENT LAWS

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CONTENT

- Nodes, Paths, Loops, and Branches
- Kirchhoff's Current Law (KCL)
- Kirchhoff's Voltage Law (KVL)
- The Single -Loop Circuit and Single-Node-Pair Circuit
- Series and Parallel Connected Sources
- Resistors in Series an Parallel
- Voltage and Current Division

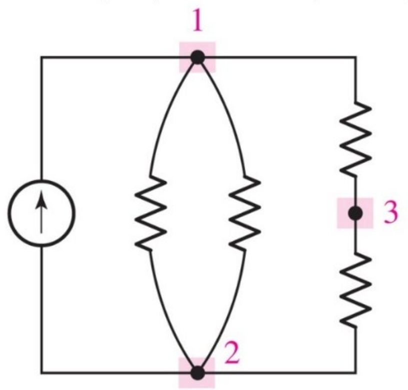
Nodes, Paths, Loops, Branches

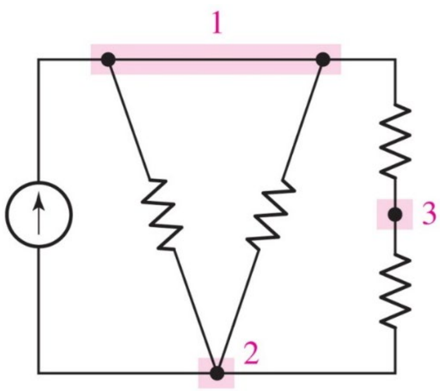
Node: A point at which two or more elements have a common connection.

Path: If no node was encountered more than once, then the set of nodes and elements that we have passed through is defined as a path.

Loop (a closed path): If the node at which we started is the same as the node on which we ended.

Branch:a branch as a single path in a network, composed of one simple element and the node at each end of that element



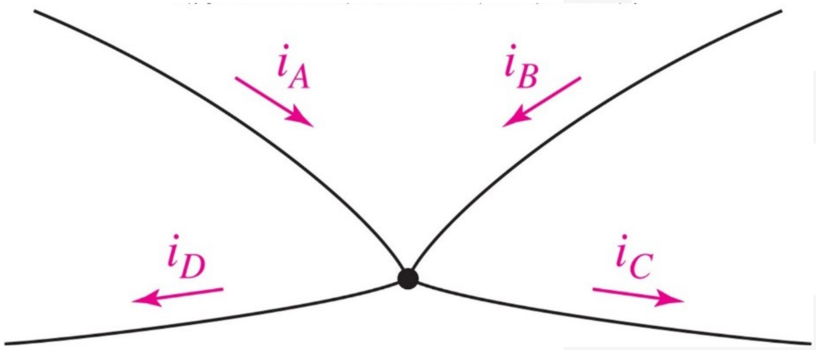


Kirchhoff's Current Law

KCL: Algebraic sum of currents entering any node is zero.

$$i_A + i_B + (-i_C) + (-i_D) = 0$$

KCL: Alternative Forms:



Current *IN* is zero:

$$i_A + i_B + (-i_C) + (-i_D) = 0$$

Current *OUT* is zero:

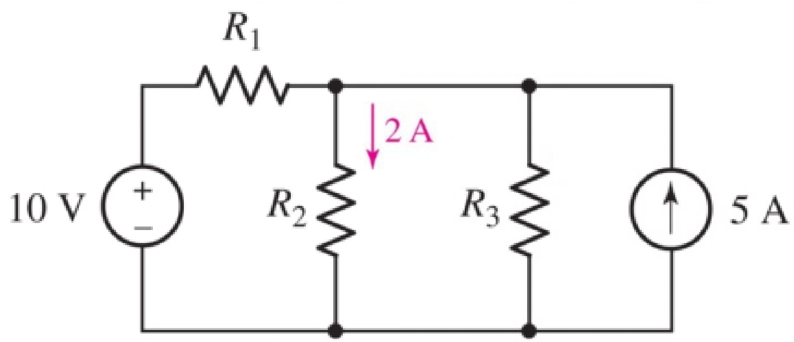
$$(-i_A) + (-i_B) + i_C + i_D = 0$$

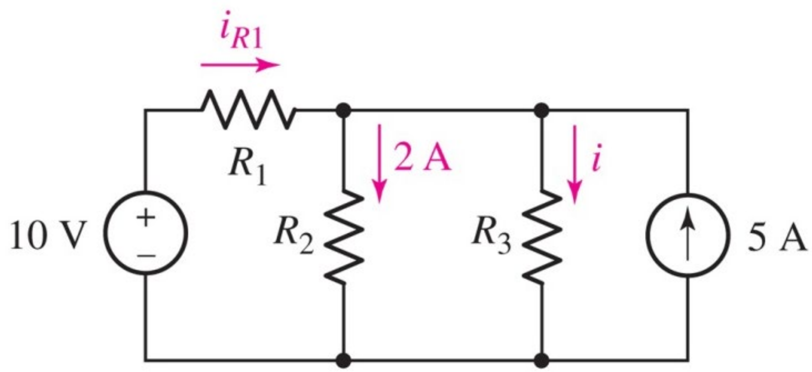
Current *IN* = *OUT*:

$$i_A + i_B = i_C + i_D$$

Example of KCL Application

Find the current through resistor R_3 if it is known that the voltage source supplies a current of 3 A, and the current through R_2 is 2A (as indicated).





Step1: Identify the goal of the problem.

Step2: Collect the known information.

Step3: Devise a plan

Step4: Construct an appropriate set of equation

$$i_{R1}-2-i+5=0$$

Step5: Determine if additional information is required

Step6: Attempt a solution

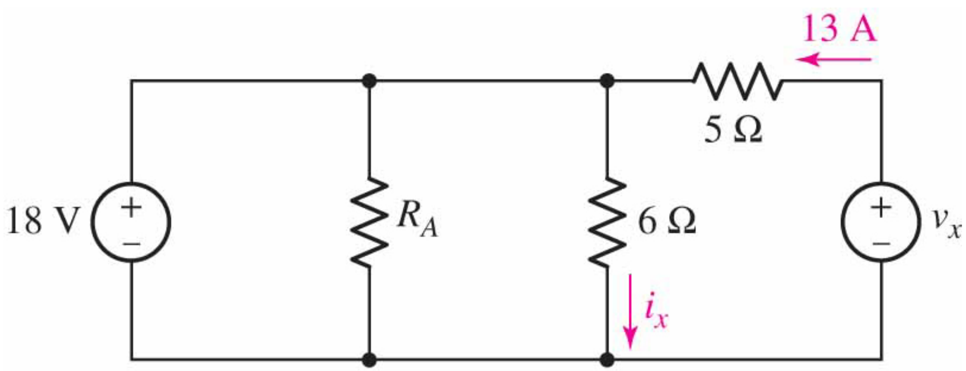
$$i = 3 - 2 + 5 = 6\text{ A}$$

Step 7: Verify the solution. Is it reasonable or expected

Example of KCL Application

(a) Count the number of branches and nodes in the circuit in the circuit blew

(b) If i_x = 3 A and the 18 V source delivers 8 A of current, what is the value of R_A? (Hint: You need Ohm's law as well as KCL.)

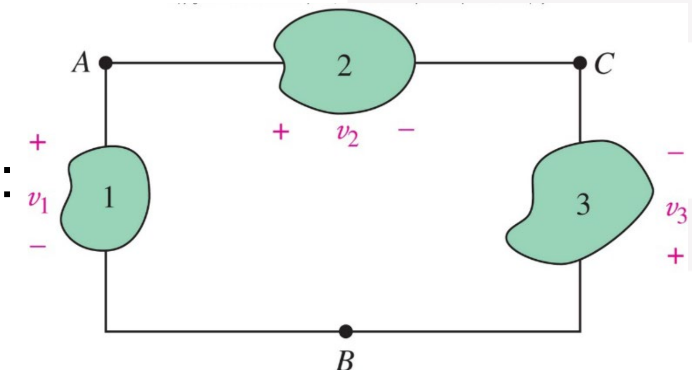


Kirchhoff's Voltage Law

KVL: Algebraic sum of voltages around any closed path is zero.

Sum of *RISES* is zero (clockwise from B):

$$v_1 + (-v_2) + v_3 = 0$$



Sum of *DROPS* is zero (clockwise from B):

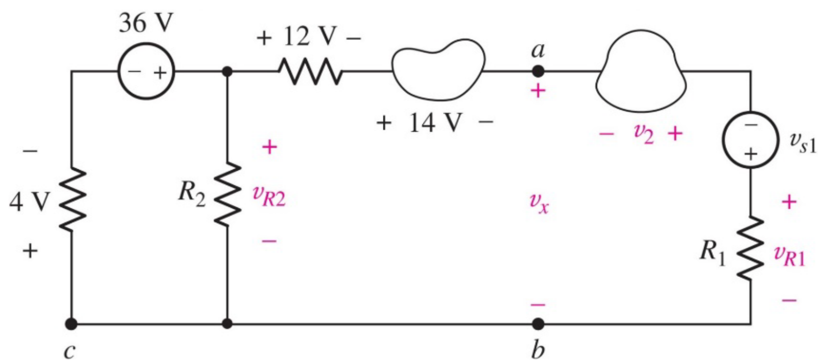
$$(-v_1) + v_2 + (-v_3) = 0$$

Two paths, same voltage (A to B):

$$v_1 = (-v_3) + v_2$$

Example: Applying KVL

Find v_{R2} (the voltage across R_2) and the voltage v_x.



Setp 1: Identify the loop that we can apply KVL

Step 2: Apply KVL to loop 1 (start at point c):

$$4 - 36 + v_{R2} = 0$$

$$v_{R2} = 32V$$

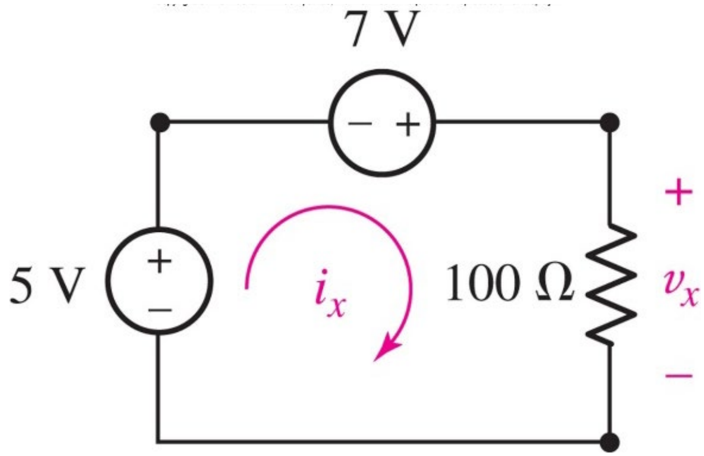
Step 3: Apply KVL to loop 2 (start at point c):

$$4 - 36 + 12 + 14 + v_x = 0$$

$$v_x = 6V$$

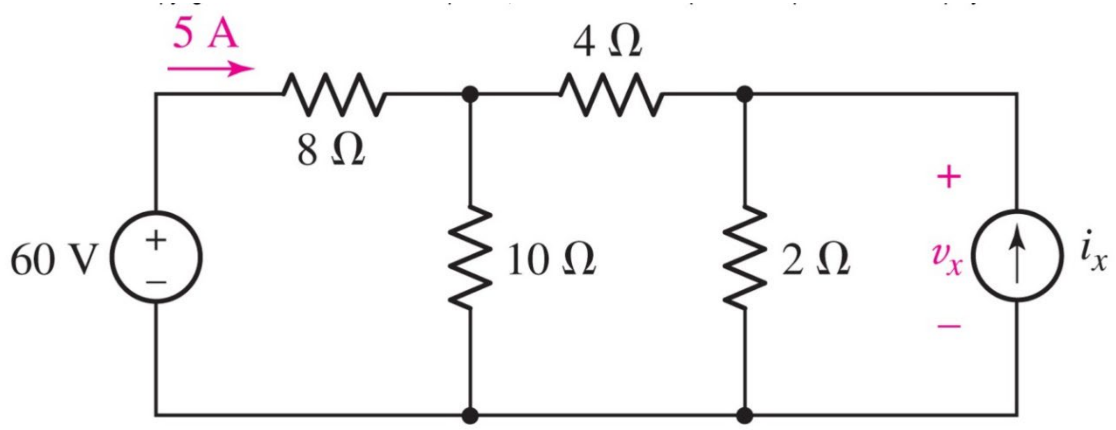
Applying KCL, KVL, Ohm’s Law

Example: find the current i_x and the voltage v_x



Applying KCL, KVL, Ohm's Law

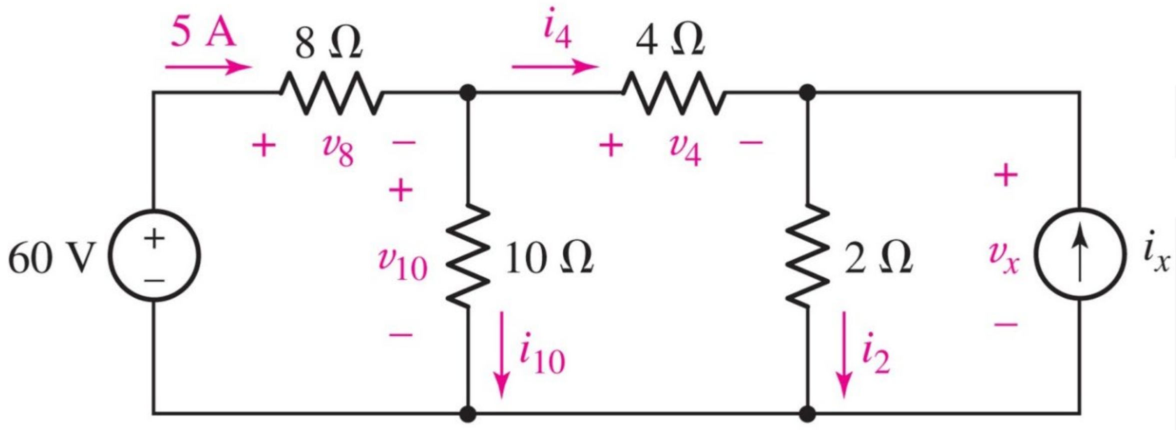
Example: solve for the voltage v_x and and the current i_x



Label all the currents and voltages on the elements in the circuit.

Applying KCL, KVL, Ohm's Law

Example: solve for the voltage *vx* and and the current *ix*



Step 1-3：Identify the goal of the problem. Collect the known information. and Devise a plan

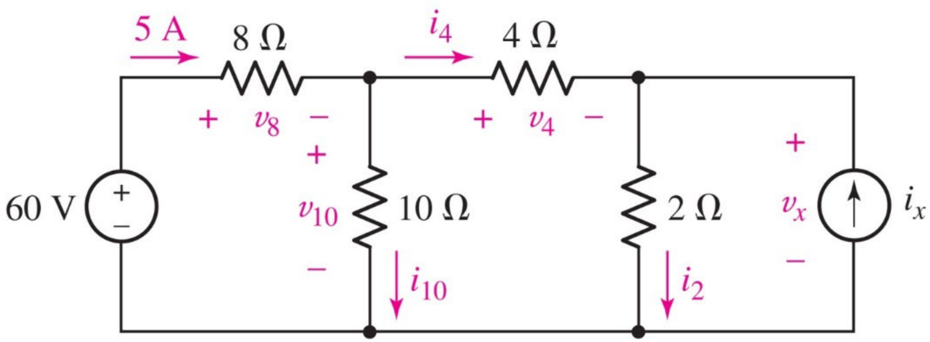
Apply KVL to loop 1:$-60 + v_8 + v_{10} = 0$

Apply KVL to loop 2 : $-v_{10} + v_4 + v_x = 0$

4 unknow variabls
2 equations

Applying KCL, KVL, Ohm's Law

Example: solve for the voltage *vx* and and the current *ix*



$$-60 + v_8 + v_{10} = 0 \quad (1)$$
$$-v_{10} + v_4 + v_x = 0 \quad (2)$$

v_8: Apply Ohm's Law: $v_8 = 5A * 8\Omega = 40V$

v_{10}: From eq (1): $v_{10} = 20V$

Eq(2) reduced to : $v_x = 20 - v_4$

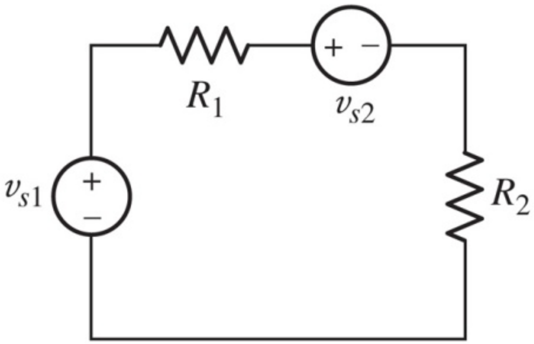
v_4: Apply KCL: $i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10} = 3A$

$$v_4 = 4 * 3 = 12V$$

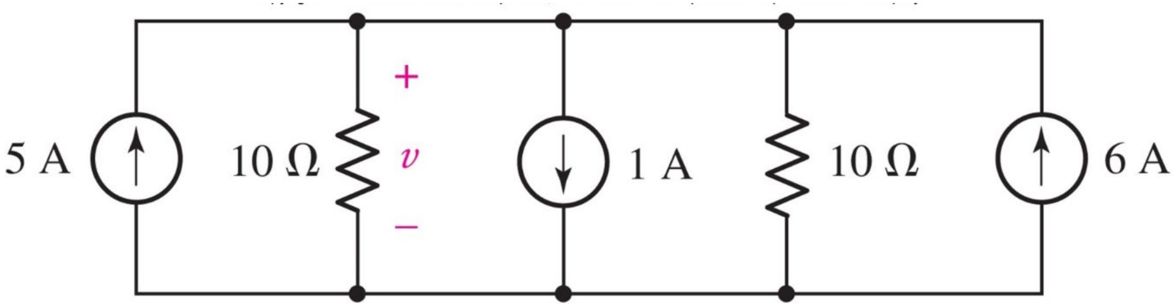
v_x : $v_x = 20 - 12 = 8V$

Series & Parallel Connections

All of the elements in a circuit that carry the same current are said to be connected in **series.**

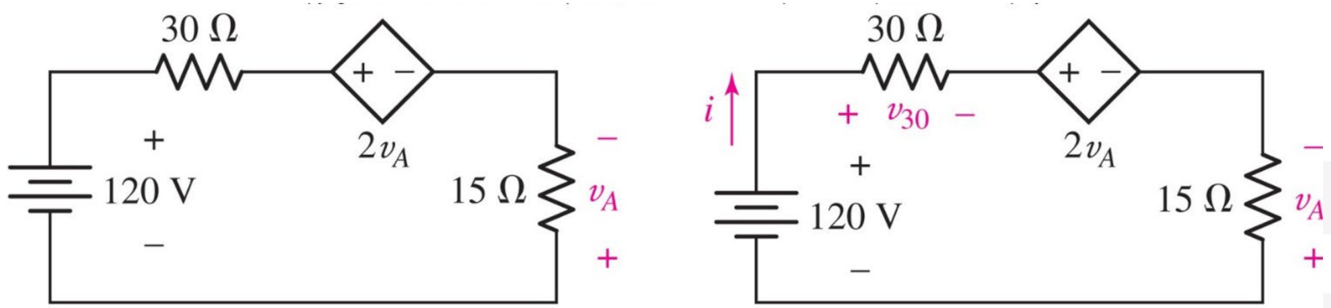


Elements in a circuit having a common voltage across them are said to be connected in **parallel.**



Example: Single Loop Circuit

Calculate the power *absorbed* by each circuit element.



Assign a reference direction for current i and a reference polarity for the voltage v_{30} (the current in the loop are the same everywhere)

Apply KVL: $-120 + v_{30} + 2v_A - v_A = 0$

Using Ohm's law to introduce the known resistor values:

$$v_{30} = 30i \quad and \quad v_A = -15i$$

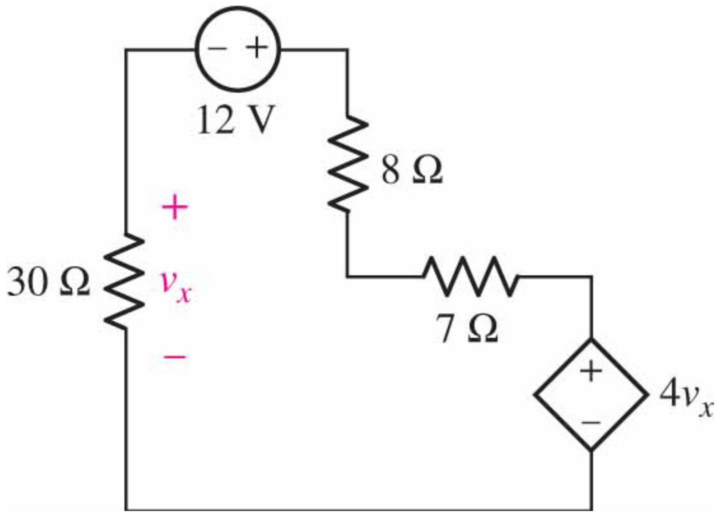
$$-120 + 30i - 30i + 15i = 0 \quad (i{=}8A)$$

$p_{120V} = 120(-8) = -960W$ $\qquad$ $p_{30\Omega} = (8)^2 * 30 = 1920W$

$p_{dep} = (2v_A) * 8 = -1920W$ $\qquad$ $p_{15\Omega} = (8)^2 * 15 = 960W$

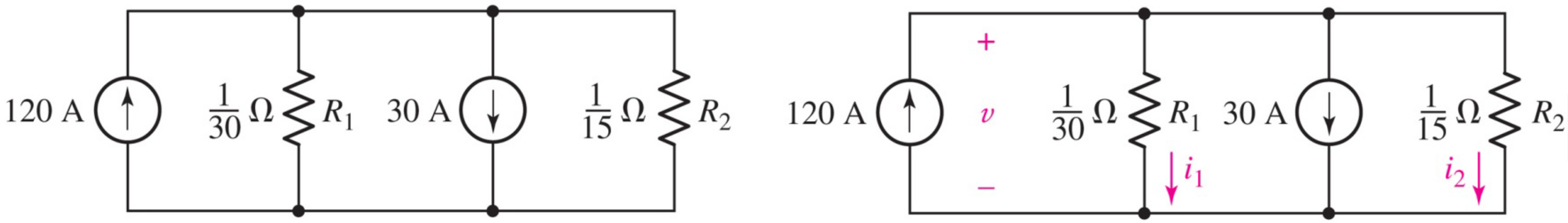
Example: Single Loop Circuit

Calculate the power absorbed by each circuit element.



Example: Single-Node-Pair 1

Find the voltage, current, and power associated with each element in the circuit (a).



(a) (b)

Assign a reference direction for current i_1 and i_2 a reference polarity for the voltage v, see fig.(b) (the voltageacross the same node pair are the same)

Apply KCL to the upper node: $-120 + i_1 + 30 + i_2 = 0$

Using Ohm's law :$i_1 = 30v \quad and \quad i_2 = 15v$

$$-120 + 30v + 30 + 15v = 0 \quad (v{=}2V)$$

$$i_1 = 60A \quad and \quad i_2 = 30A$$

$p_{R1} = (2)^2 * 30 = 120W$

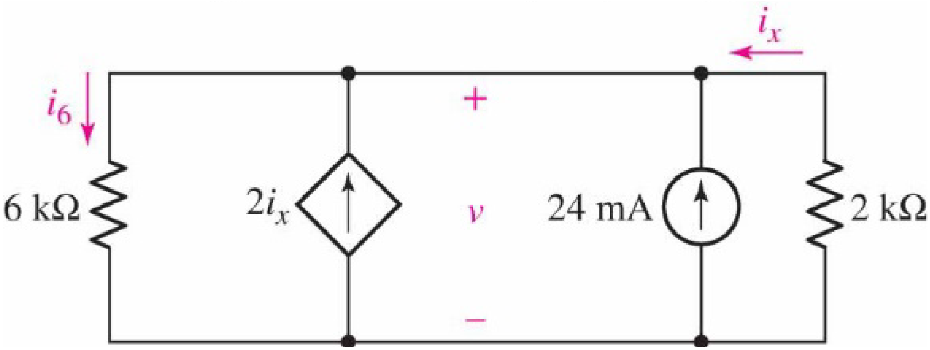
$p_{R2} = (2)^2 * 15 = 60W$

$p_{120A} = (-2) * 120 = -240W$

$p_{30A} = 2 * 30 = 60W$

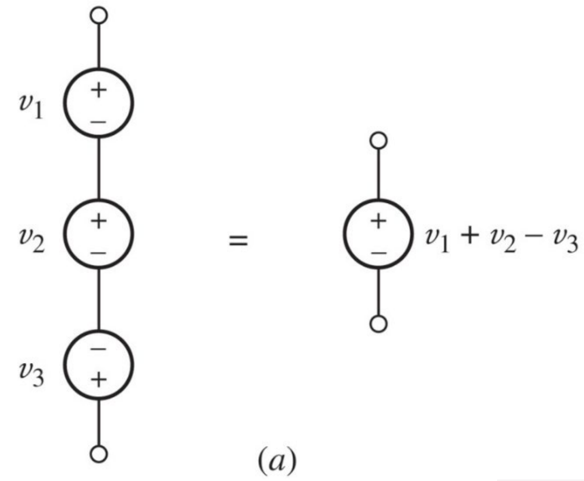
Example: Single-Node-Pair 2

Determine the value of *v* and the power supplied by the independent current source.

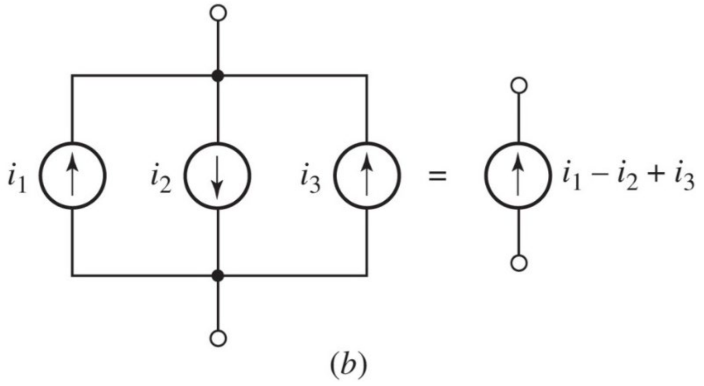


Series and Parallel Sources

Voltage sources connected in series can be combined into an equivalent voltage source:

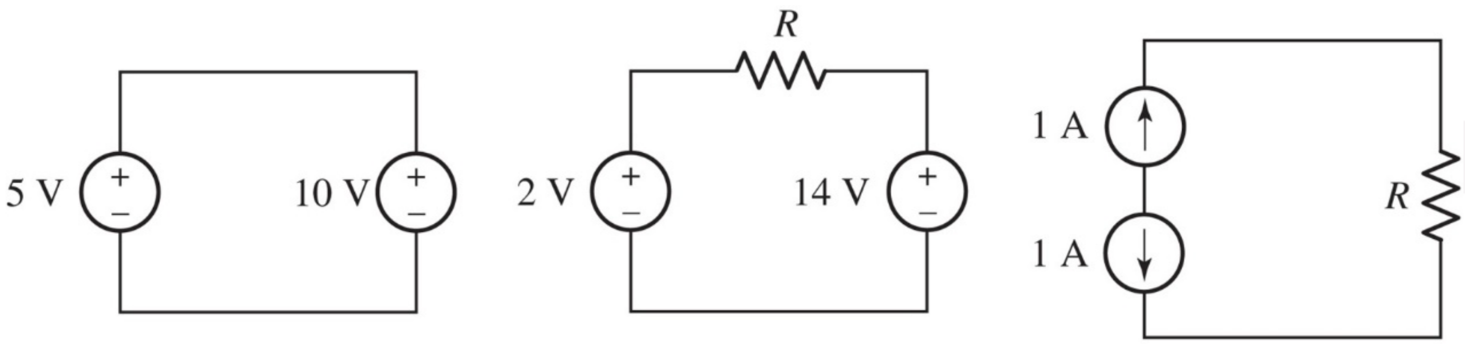


Current sources connected in parallel can be combined into an equivalent current source:



Impossible Circuits

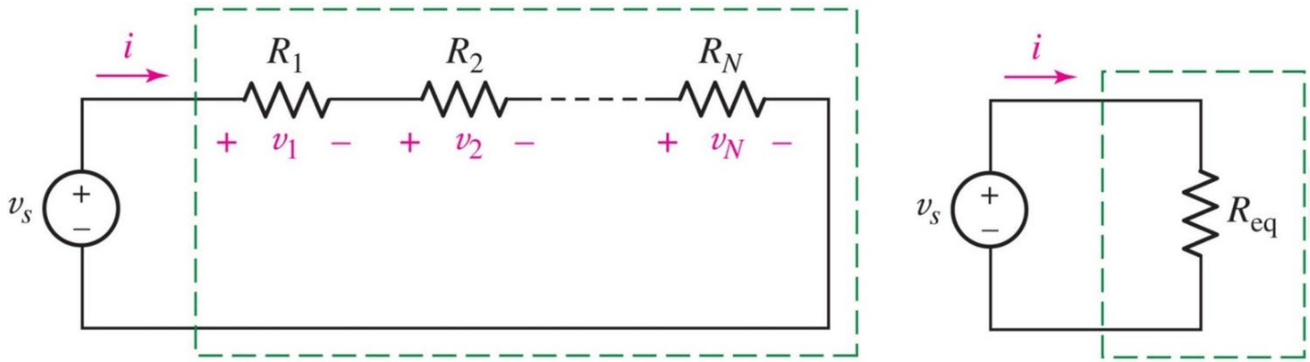
Our circuit models are idealizations that can lead to apparent physical absurdities:



(a) (b) (c)

V_s in parallel (a) and I_s in series (c) can lead to “impossible circuits”

Resistors in Series



(a) (b)

Using KVL : $v_s = v_1 + v_2 + \ldots + v_N$

Using Ohm's Law: $v_s = iR_1 + iR_2 + \ldots + iR_N$

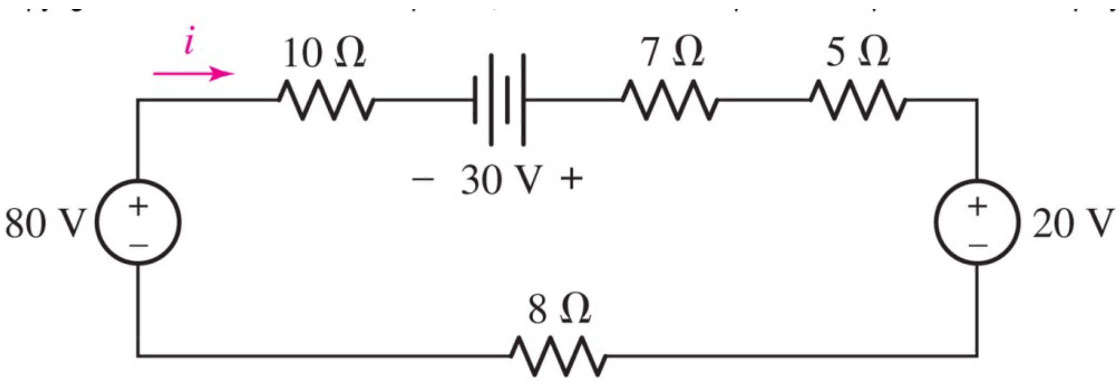
$$= i(R_1 + R_2 + \ldots + R_N)$$

$$\boxed{R_{eq} = R_1 + R_2 + \cdots + R_N}$$

$$v_s = iR_{eq}$$

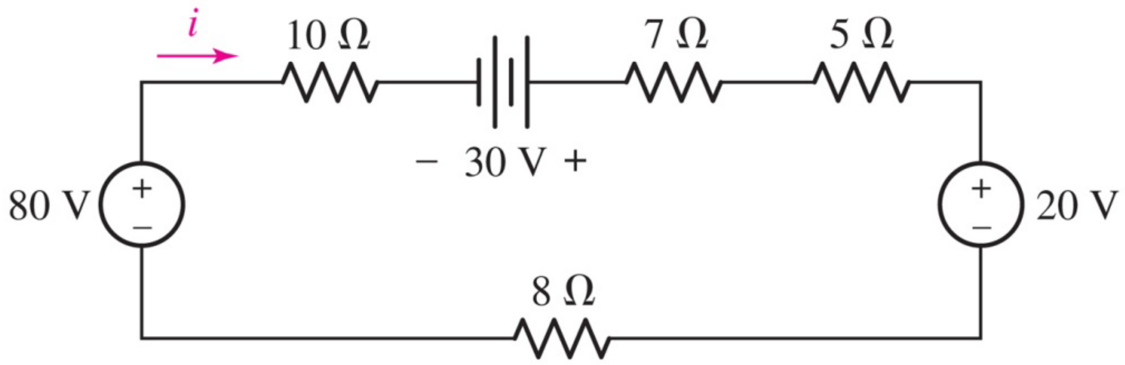
Example: Circuit Simplifying

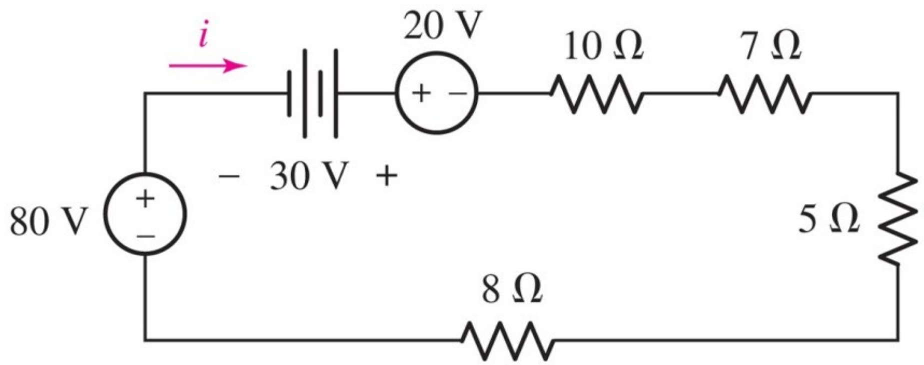
Find *i* and the power supplied by the 80 V source.

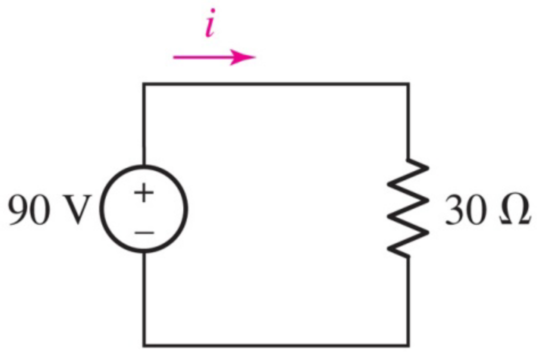


Example: Circuit Simplifying

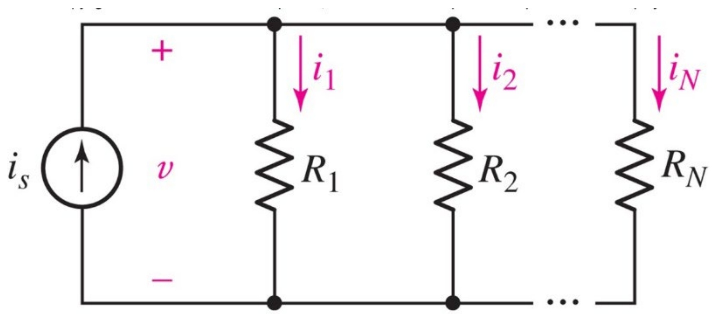
Find *i* and the power supplied by the 80 V source.

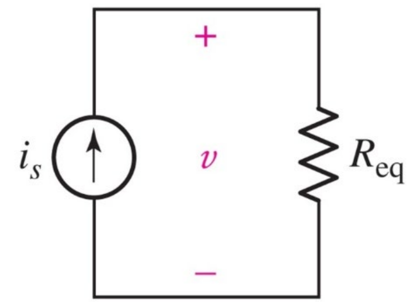






Resistors in Parallel





Using KCL : $i_s = i_1 + i_2 + \ldots + i_N$

Using Ohm's Law: $i_s = v/R_1 + v/R_2 + \ldots + v/R_N$

$$= v(1/R_1 + 1/R_2 + \ldots + 1/R_N)$$

$$\boxed{\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

$$i_s = v/R_{eq}$$

Two Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = R_1 \parallel R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

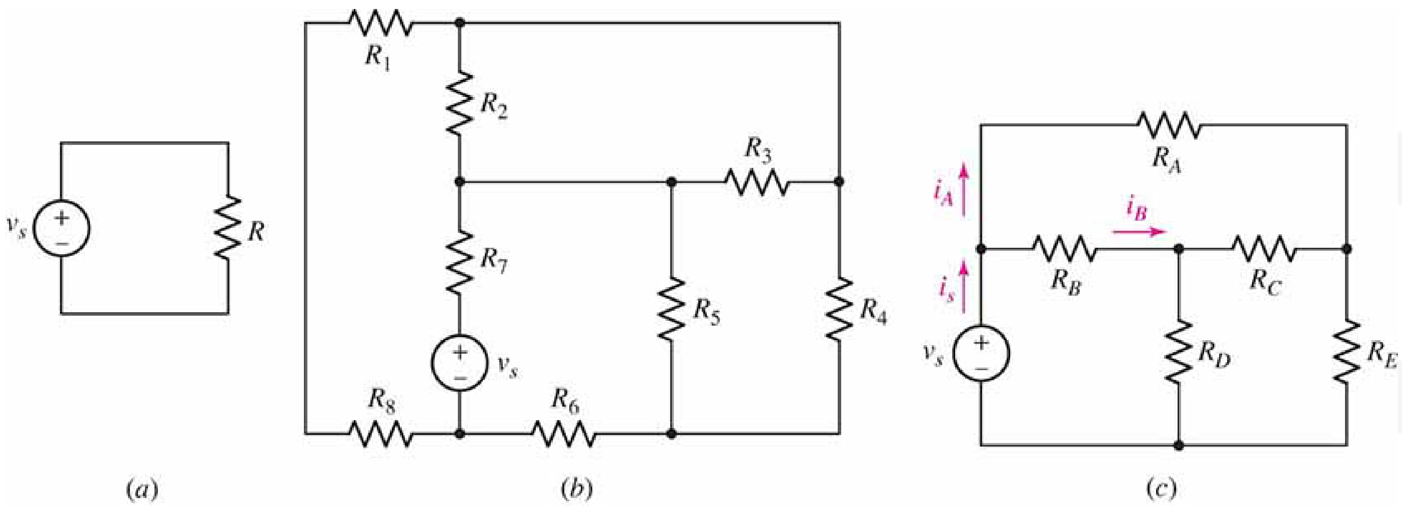
Two resistors in parallel can be combined using the

product / sum

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Connecting resistors in parallel makes the result *smaller*

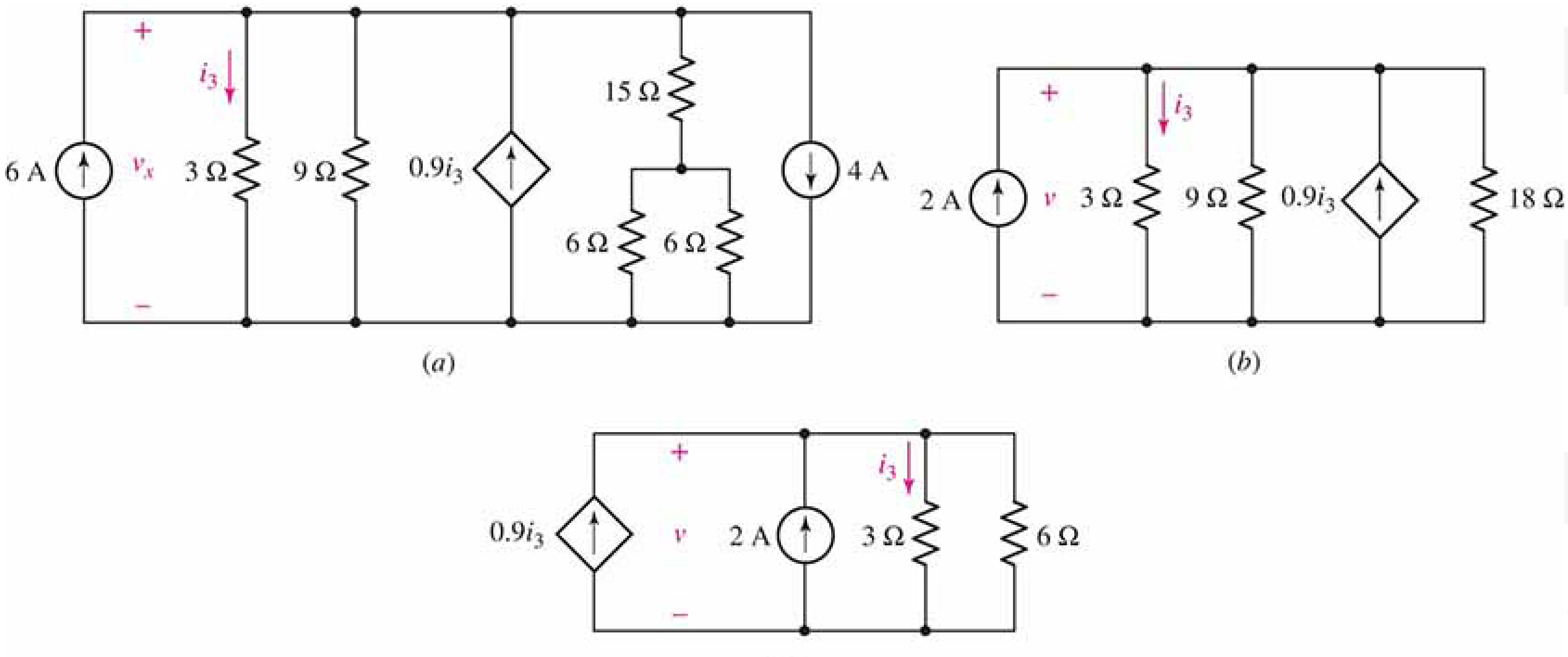
Example: Resistor in Series/Parallel



(a) These two circuit elements are both in series and in parallel.
(b) R_2 and R_3 are in parallel, and R_1 and R_8 are in series.
(c) There are no circuit elements either in series or in parallel with one another.

Example: Circuit Simplifying

Calculate the power and voltage of the dependent source



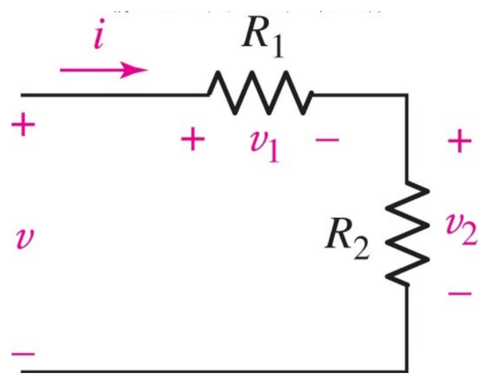
(a)

(b)

(c)

Voltage Division

Resistors in series "share" the applied voltage.



$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$i = \frac{v}{R_1 + R_2}$$

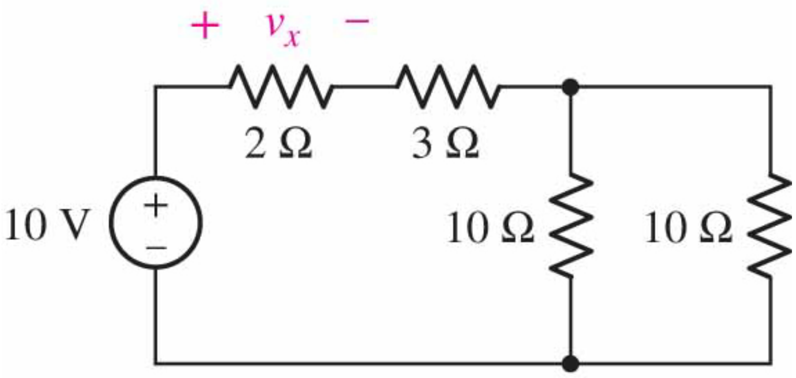
$$v_1 = iR_1 \ , \qquad v_2 = iR_2$$

$$\boxed{v_1 = \frac{R_1}{R_1 + R_2} v}$$

$$\boxed{v_2 = \frac{R_2}{R_1 + R_2} v}$$

Example: Voltage Division

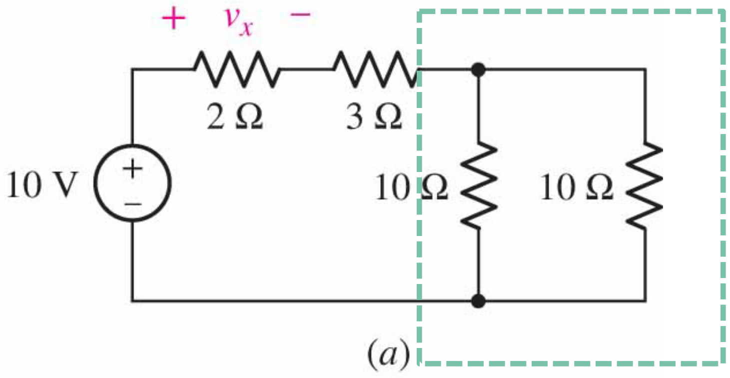
Find v_x

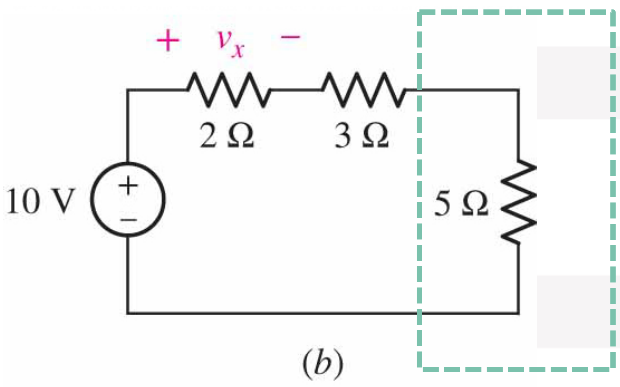


(a)

Example: Voltage Division

Find v_x





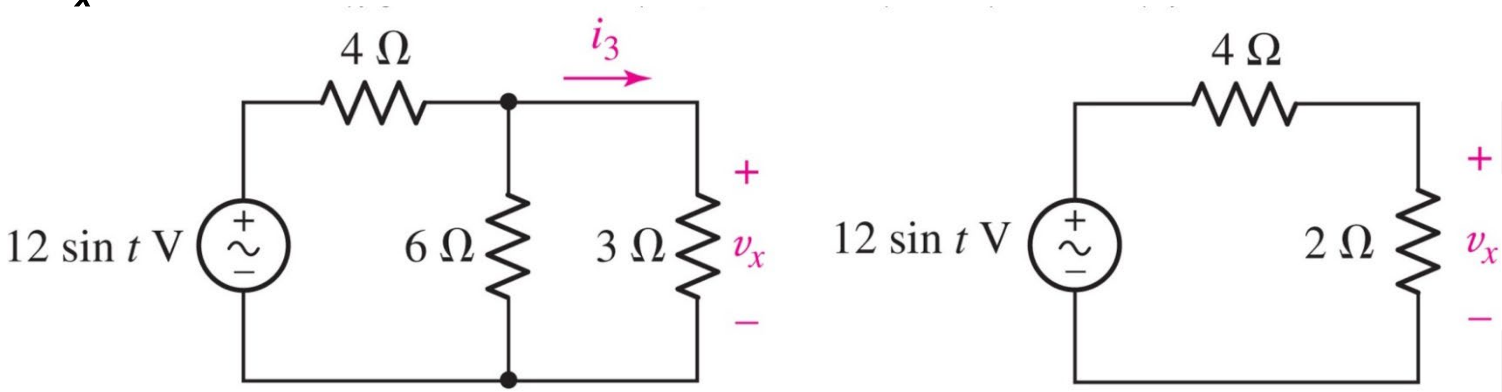
1. Calculate Parallel Resistors $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

2. Apply voltage division：

$$v_x = 10\frac{2}{2+3+5} = 2V$$

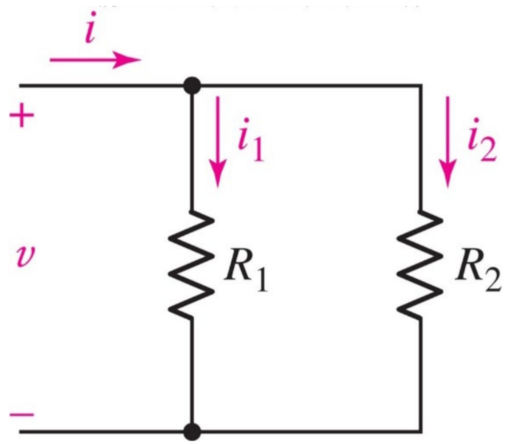
Example: Voltage Division

Find v_x



Current Division

Resistors in parallel “share” current through them.



$$i_1 = i\frac{R_2}{R_1 + R_2}$$

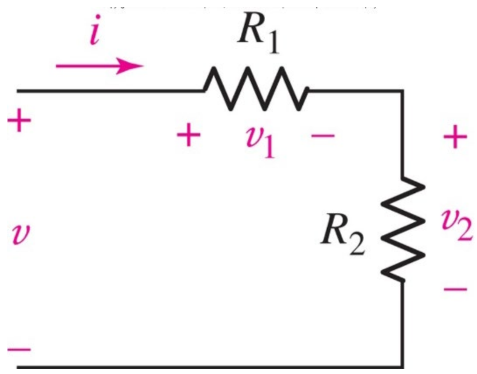
$$i_2 = i\frac{R_1}{R_1 + R_2}$$

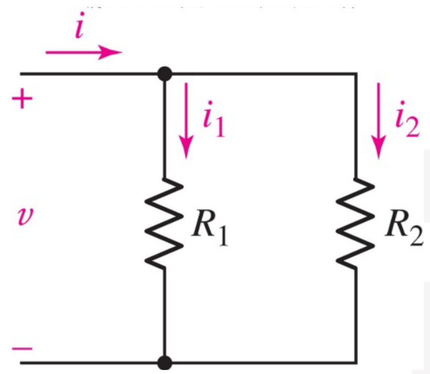
$$i = i_1 + i_2 = v/R_1 + v/R_2$$

$$v = \frac{R_1 R_2}{R_1 + R_2} i$$

$$i_1 = v/R_1 \;, \qquad i_2 = v/R_2$$

Voltage Division vs. Current Division





$$v_1 = \frac{R_1}{R_1 + R_2} v$$

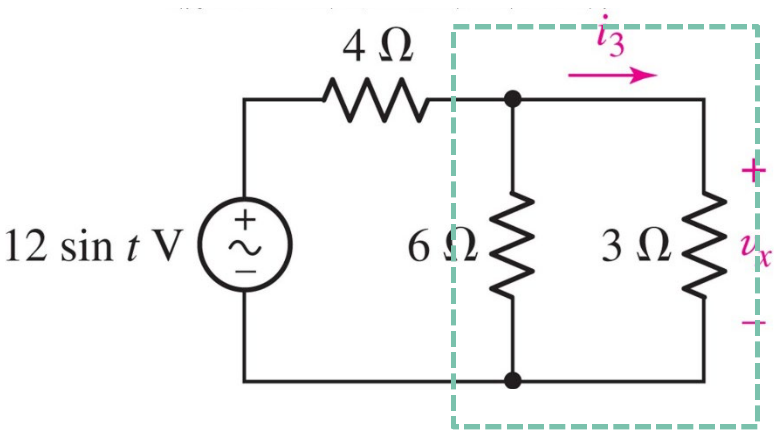
$$i_1 = i \frac{R_2}{R_1 + R_2}$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

$$i_2 = i \frac{R_1}{R_1 + R_2}$$

Example: Current Division

Find $i_3(t)$



1. Calculate Parallel Resistors $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

2. The total current flowing in to the equivalent resistor:

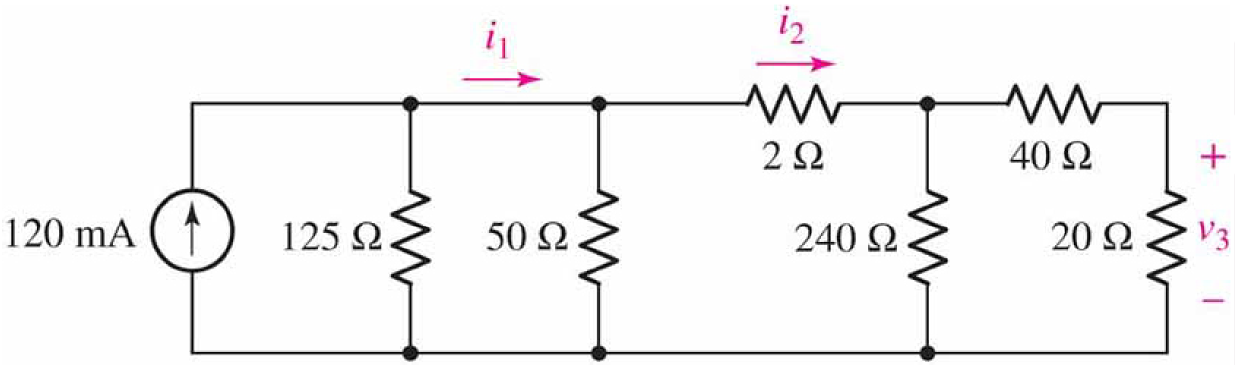
$$i(t) = \frac{12\text{sin}t}{4 + R_{eq}} = 2\text{sin}t\ A$$

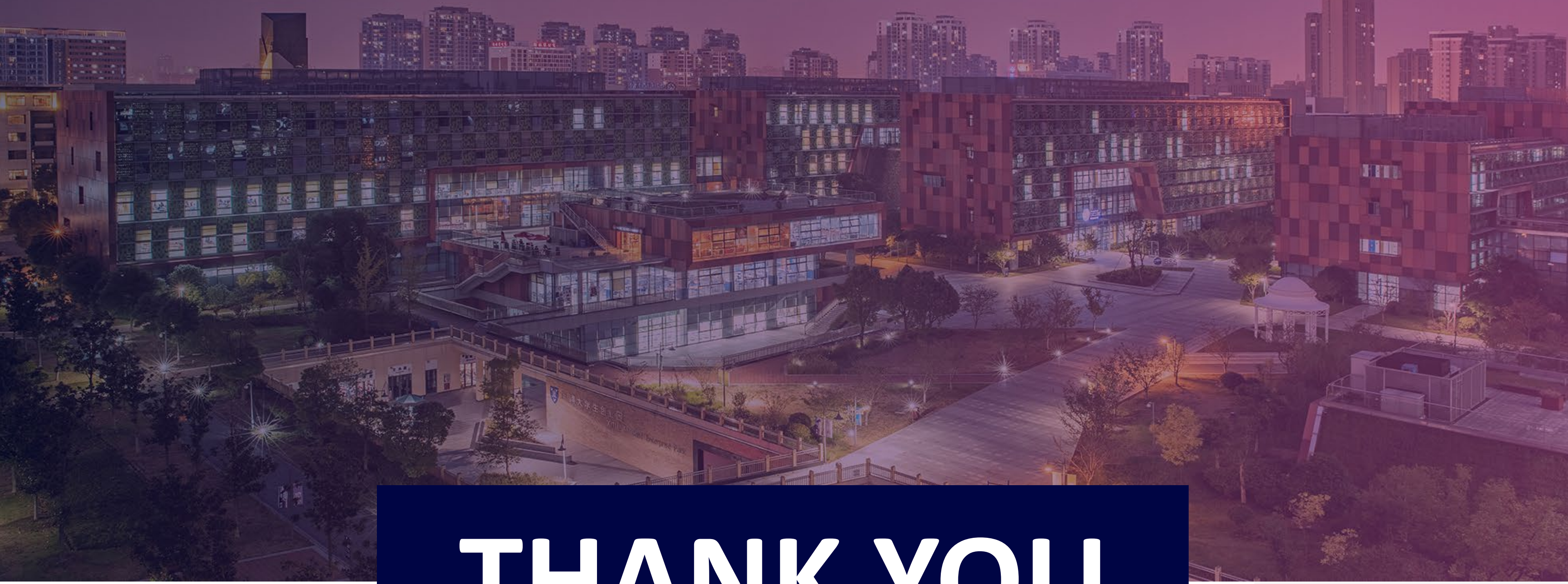
3. Apply current division:

$$i_3 = 2\text{sin}t\ \frac{6}{3+6} = \frac{4}{3}\text{sin}t\ A$$

Example: Circuit Analysis

In the circuit below, use resistance combination methods and current division to find i_1, i_2, and v_3.





THANK YOU

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