# Assignment 2: DT Signals and Systems

Deadline: Dec. 9th, 9:00 a.m.

Submission: Submit the electronic version to Learning Mall.

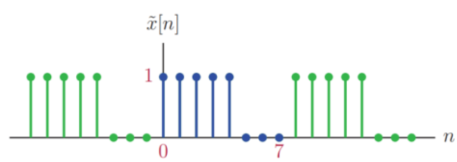
Information: This assignment takes 15% in the total mark.

Late submission: 5% each day, less than 1 day is counted as 1 day.

Submissions later than 5 working days won't be accepted.

## Question 1 (DTFS, DTFT) 18 marks

(a) For the discrete-time signal shown in below figure: 8




i) Determine the period and fundamental frequency of the sequence; (4')

ii) Determine the DTFS coefficients for the sequence. (4')

(b) Find the DTFT of each sequences given below. Make use of properties of DTFT where possible. 10

i) $x[n] = 2^n u[-n]$; (3')

ii) $x[n] = n(0.7)^n u[n]$; (3')

iii) $x[n] = \left(\frac{1}{2}\right)^{|n|} \cos(\frac{\pi}{8}(n-1))$. (4')

## Question 2 (Sampling, Filtering) 20 marks

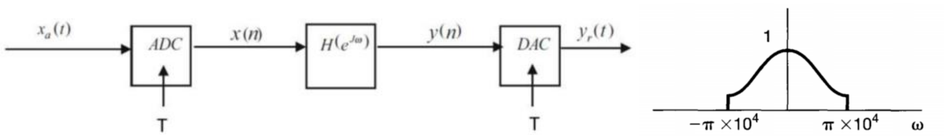
(a) For each of the below signals, determine if it can be sampled without any information loss. If yes, determine the minimum sampling rate that can be used. 8

i) $x(t) = u(t) - u(t-3)$; (2')

ii) $x(t) = \cos(100\pi t) + 2\sin(250\pi t)$ ; (3')

iii) $x(t) = \cos(100\pi t) + 2.5\sin(150\pi t)\cos(200\pi t)$. (3')

(b) Below figure (a) shows a system for filtering continuous-time signal using a discrete-time filter. The input signal $x_a(t)$ is first converted to discrete-time signal $x[n]$ using an ideal ADC with sampling rate $F_s = \frac{1}{T}$. $H(e^{j\omega})$ is a lowpass filter with cutoff frequency of $\frac{\pi}{4}$. The filter $y[n]$ is then reconstructed to analogue signal $y_r(t)$ using an ideal DAC. Assume the spectrum $X_a(j\omega)$ of $x_a(t)$ is given in (b). 12




(a) (b)

i) Find the maximum value of sampling period $T$ to avoid aliasing in the ADC; (3')

ii) If $\frac{1}{T} = 20kHz$, sketch the spectrum of $x[n]$ and $y[n]$; (6')

iii) Using sampling frequency in ii), determine the spectrum of reconstructed signal $y_r(t)$. (3')

## Question 3 (DTFT and LTID system) 20 marks

(a) If $X(e^{j\omega})$ is the DTFT of the sequence 8

$$x[n] = \{1,2,3,4\}, 0 \le n \le 3$$

Evaluate the values of following expressions:

i) $X(e^{j\pi})$; (2')

ii) $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$; (2')

iii) $\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega$; (2')

iv) $\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega$. (2')

(b) Consider a casual LTI system with below frequency response 12

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega} + \frac{1}{4}e^{-j2\omega}}$$

i) Determine the LCCDE of the system; (4')

ii) Find the response of the system to the input with the following Fourier transform: (8')

$$X(e^{j\omega}) = \frac{2 - e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}$$

## Question 4 (Z-transform and LTID system) 25 marks

(a) Consider the input and output pairs listed below. For each case, determine the system transfer function $H(z)$ along with its ROC, and indicate if the system considered is stable and/or causal. 6

i) $x[n] = \left(\frac{1}{2}\right)^n u[n], \quad y[n] = 3\left(\frac{1}{2}\right)^n u[n] + 2\left(\frac{3}{4}\right)^n u[n]$; (3')

ii) $x[n] = 1.25\delta[n] - 0.25(0.8)^n u[n], \quad y[n] = (0.8)^n u[n]$. (3')

(b) Consider a causal LTI system described by the following LCCDE: 9

$$y[n] + 0.2y[n-1] - 0.24y[n-2] = x[n] + x[n-1]$$

i) Obtain the system transfer function $H(z)$; (3')

ii) Discuss the ROC and stability of the system; (2')

iii) Find the system output to a unit step function $u[n]$. (4')

(c) A discrete-time LTI system has the system transfer function 10

$$H(z) = \frac{z - 0.4}{z^3 - 1.4z^2 + 0.85z}$$

i) Draw a zero-pole plot of the system; (2')

ii) Roughly sketch the magnitude plot of the system response, specify the magnitudes at $\omega = 0, \frac{\pi}{6}, \frac{\pi}{4}$ and $\pi$ respectively; determine if the system has a lowpass, highpass, bandpass or bandstop characteristic; (4')

iii) Assume the system is consisting of two LTI systems connected in cascade, i.e., $H(z) = H_1(z)H_2(z)$, draw a block diagram of the system. (4')

## Question 5 (DFT) 17 marks

(a) For below sequences, determine the specified circularly shifted versions: (6') 6

i) $x[n] = \{4,3,2,1\}, 0 \le n \le 3$, find $x[n-2]_{\text{mod } 4}$

ii) $x[n] = \{1,3,2,4,-1,-3\}, 0 \le n \le 5$, find $x[-n]_{\text{mod } 3}$

iii) $x[n] = \{1,4,2,3,1,-2,-5,1\}, 0 \le n \le 3$, find $x[-n+2]_{\text{mod } 8}$

(b) Determine the linear and circular convolution of the below two sequences: (6') 6

$x[n] = \{1,3,2,-4,6\}, -1 \le n \le 3; h[n] = \{5,4,3,2,1\}, -2 \le n \le 2$

(c) The DTF of a length-6 signal is given by 5

$$x[n] = \{(2+j3), (1+j5), (-2+j4), (-1-j3), (2), (3+j)\}$$

Without computing $x[n]$ first, determine the DFT of the real part and DFT of the imaginary part of $x[n]$. (5')