CAN102 Electromagnetism and Electromechanics

Lecture-8 Resistors and Capacitors

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Room EE322

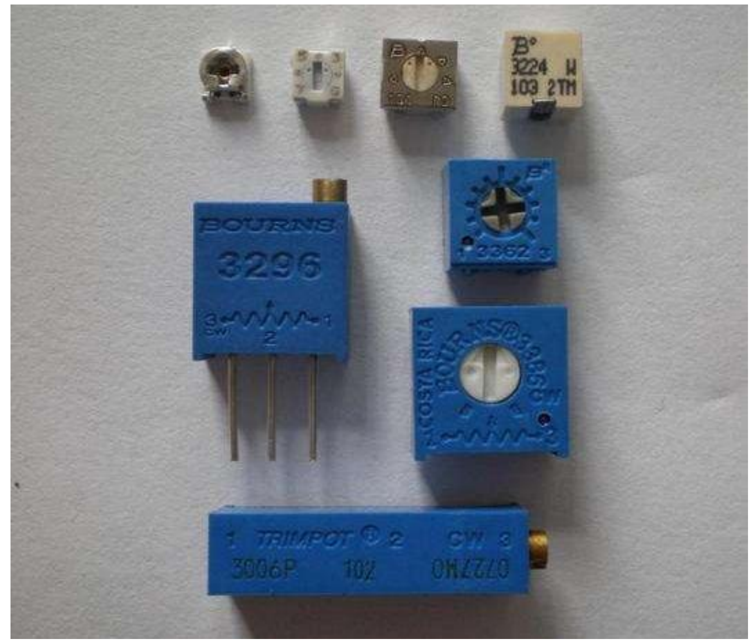
Outline

- Resistors
  - Resistance calculation
  - Resistance, resistivity and conductivity
  - Adminttance
- Capacitors
  - Capacitance calculation
  - Capacitor with dielectrics
  - Parallel and series connection of capacitors
  - Energy stored in capacitors
  - I-V relationship of capacitors

1.1 Resistor

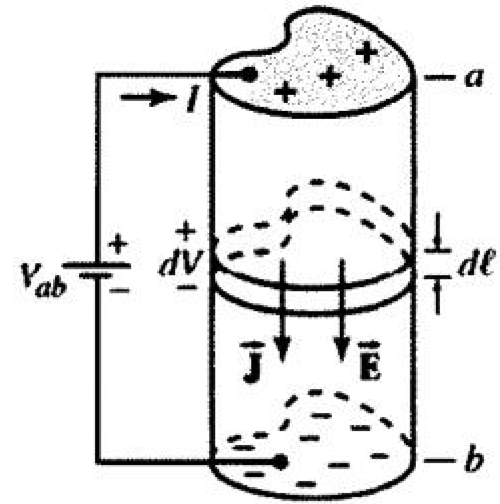
- A resistor is a passive two-terminal electrical component that implements electrical resistance as a circuit element.

Fixed resistors



Variable resistors

1.2 Resistance



- The resistance of a conductor of length dl can be obtained by

$$dR = \frac{dV}{I} = \frac{-\vec{E} \cdot d\vec{l}}{\iint_S \vec{J} \cdot d\vec{s}}$$

  - If we assume that the potential at end a of the conductor is higher than that at end b.
  - The total resistance of the conductor is:

$$R = \int_b^a \frac{-\vec{E} \cdot d\vec{l}}{\iint_S \vec{J} \cdot d\vec{s}}$$

  - This is a general equation to determine the resistance of a conducting medium whose conductivity changes in the direction of the current. For homogeneous medium having constant σ, it reduces to:

$$R = \int_b^a \frac{-\vec{E} \cdot d\vec{l}}{\iint_S \vec{J} \cdot d\vec{s}} = \frac{-\int_b^a \vec{E} \cdot d\vec{l}}{\iint_S \vec{J} \cdot d\vec{s}} = \frac{V_{ab}}{I}$$

1.2 Resistance

How to calculate?

- Simplified model: A potential difference of V_0 is maintained across the two ends of a conducting wire of length l. If A is the cross-sectional area of the wire, obtain an expression for the resistance of the wire.
  - Assume the potential difference between the two ends of the conductor is V_0, the electric field holds:

$$V_0 = -\int_b^a \vec{\boldsymbol{E}} \cdot \boldsymbol{d\vec{l}} = El \Rightarrow E = \frac{V_0}{l}$$

  - If σ is the conductivity of the conducting material, the current density at any cross section of the wire is:

$$J = \sigma E = \frac{\sigma V_0}{l}$$

  - The current through the wire is:

$$I = \iint_S \vec{J} \cdot d\vec{s} = JA = \frac{\sigma V_0 A}{l} = \frac{V_0}{R}$$

  - So the resistance of the piece of the conducting material is

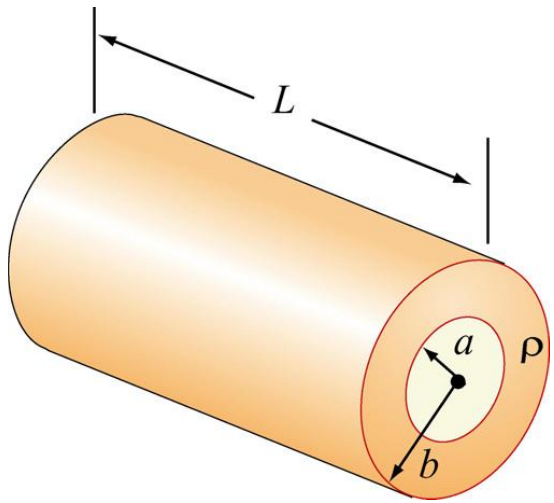
$$R = \frac{V_0}{I} = \frac{l}{\sigma A}$$

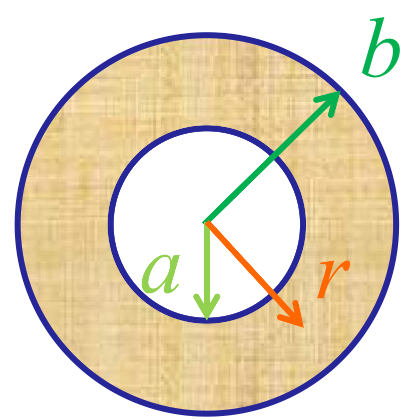
1.3 Comparison

	Unit	Expression	Physical meaning
Resistance	Ω	$R = \frac{l}{\sigma A} = \frac{\rho l}{A}$	Resistance is property of an object, depends on geometry (shape and size) as well as resistivity.
Resistivity	$\Omega \cdot \mathrm{m}$	$\rho = \frac{1}{\sigma} = \frac{\vec{E}}{\vec{J}}$	Resistivity is property of a substance. Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature, not on its shape or size.

Example 1

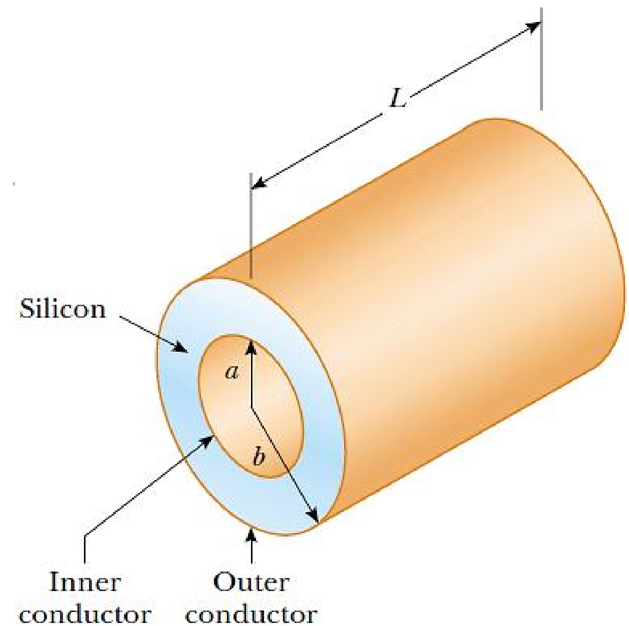
- Consider a hollow cylinder of length L and inner radius a and outer radius b. The material has resistivity ρ.
  - Suppose a potential difference is applied between the ends of the cylinder and produces a current flowing parallel to the axis. What is the measured resistance?
  - If the potential difference is applied between the inner and outer conducting surfaces so that current flows radially outward, what is the measured resistance?





Quiz 1

- A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is filled with silicon. a = 0.5 cm, b = 1.75 cm, L = 15.0 cm. The resistivity of silicon is 640 Ω·m.
  - Calculate the resistance of the silicon between the two conductors.
  - Assuming the inner conductor is made of copper with ρ = 17 nΩ·m, determine the resistance of the inner conductor.



1.4 Admittance (导纳)

- Admittance is defined as a measure of how easily a circuit or device will allow current to flow through it.
  - Symbol: Y
  - Unit: S (siemens)

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}}$$

- If the impendance $\boldsymbol{Z}$ of a component or device is complex:

$$\boldsymbol{Z} = R + jX$$

- Then the admittance $\boldsymbol{Y}$ is:

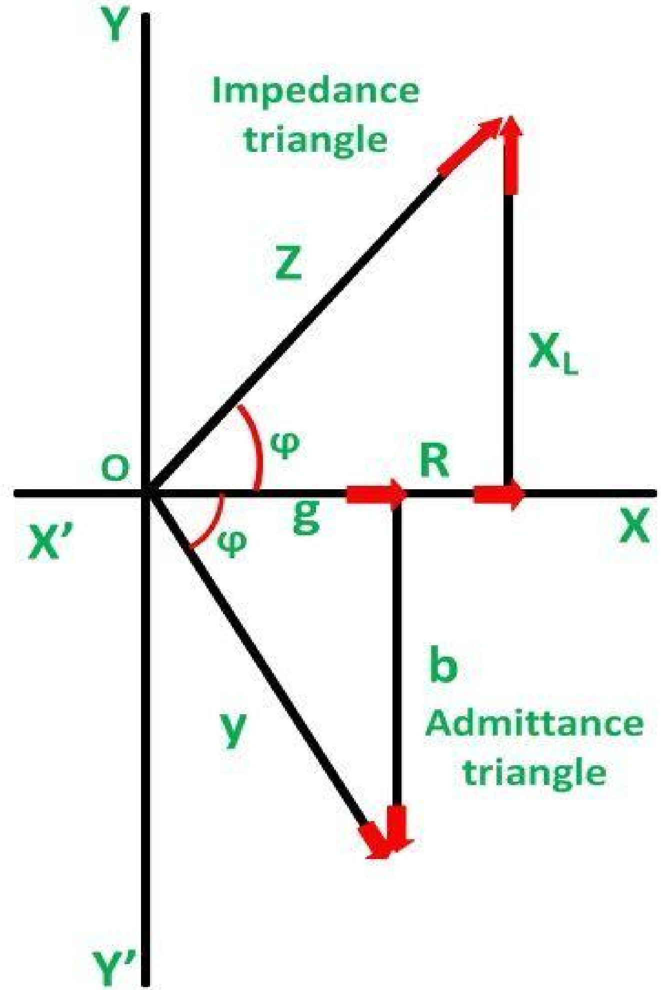
$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{1}{R + jX} = \underbrace{\boxed{\frac{R}{R^2 + X^2}}}_{\text{conductance}} - j\underbrace{\boxed{\frac{X}{R^2 + X^2}}}_{\text{Susceptance}}$$

1.4 Admittance (电导)

- Impedance Triangle and Admittance Triangle

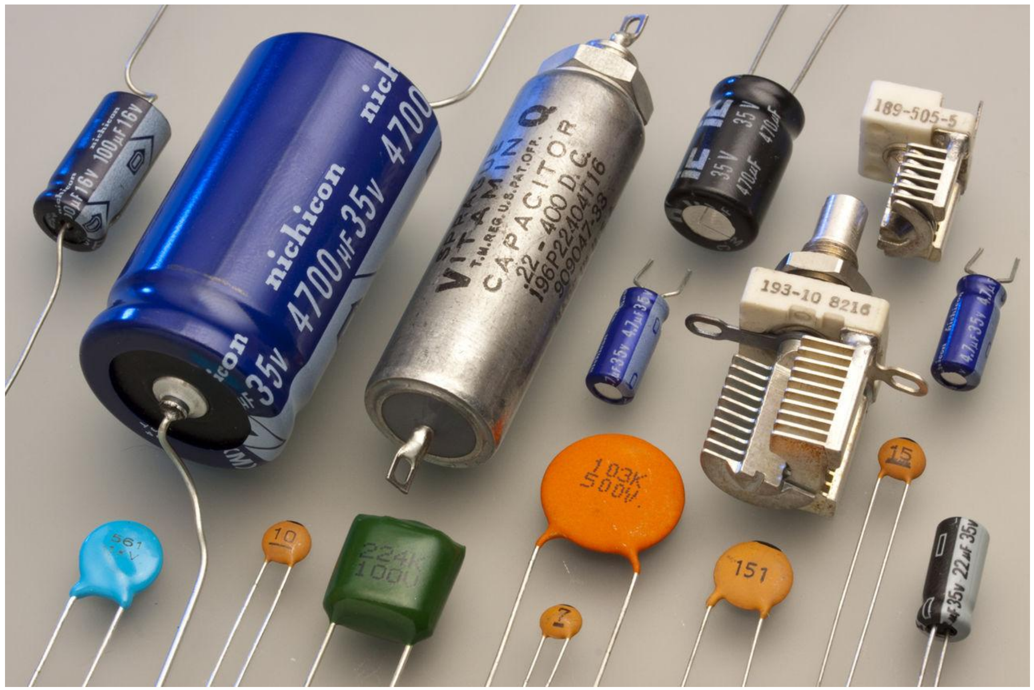
- Real part only:

$$\boldsymbol{Y} = G = \frac{1}{R} = \sigma\frac{A}{l} = \frac{1}{\rho}\frac{A}{l}$$



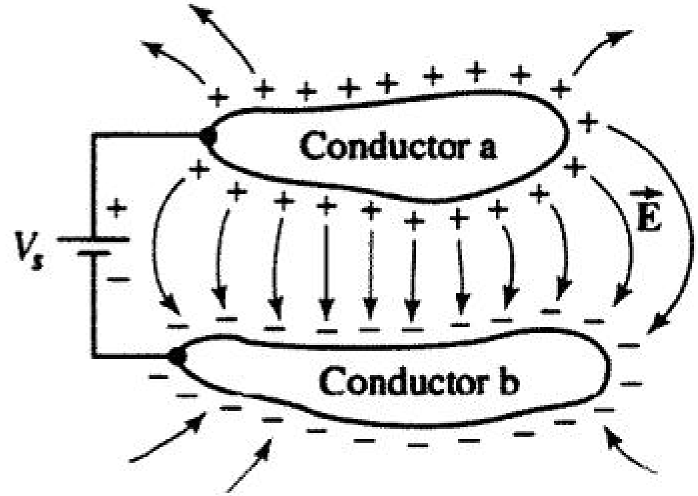
2.1 Capacitors

- A capacitor is also a two-terminal passive device which stores electric charge.



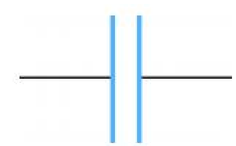
2.2 Capacitance

- Capacitor
  - A capacitor is a device which stores electric charge.
  - Its basic configuration is two conductors carrying equal but opposite charges

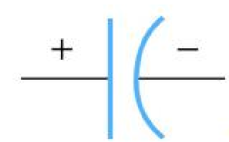


- Capacitance
  - measures the capability of energy storage in electrical devices.
  - the amount of charge Q stored in a capacitor is linearly proportional to the electric potential difference V between the two conductors:

$$\frac{Q}{V} = constant = C$$



  - Unit: 1 F (farad) = 1 C / V (coulomb/volt)

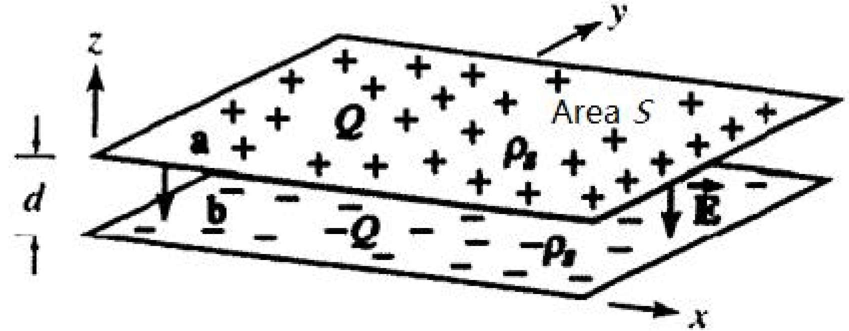


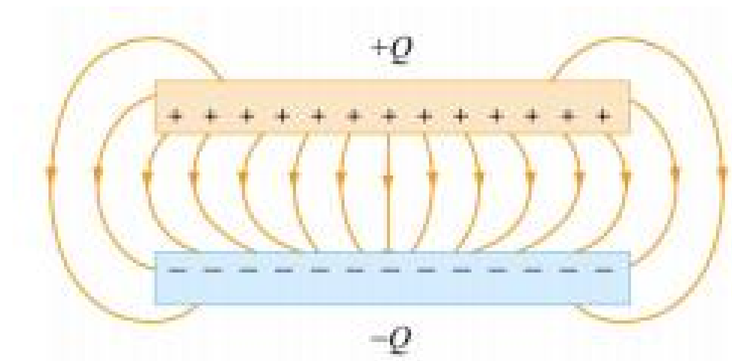
2.3 Capacitor Examples

Parallel Plates

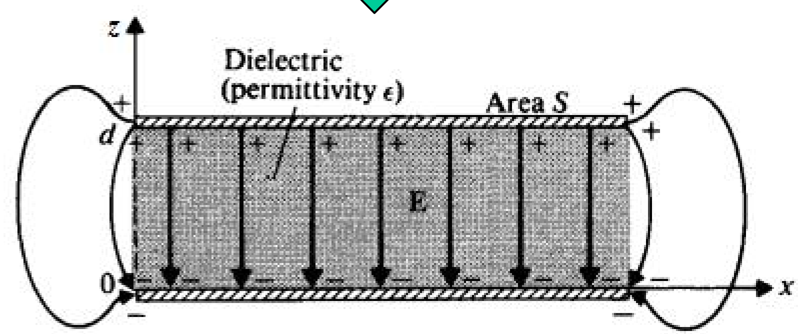
- Two parallel conducting plates, each of area S, and separated by a distance d, form a parallel-plate capacitor. The total charge on the top plate is $+Q$ and that on the other plate is $-Q$.
  - What is its capacitance?

- Solution:
  - Edge effects: The electric field lines at the edge of the plates are not straight lines, and the field is not contained entirely between the plates.
  - Fringing fields: The non-uniform fields near the edge.

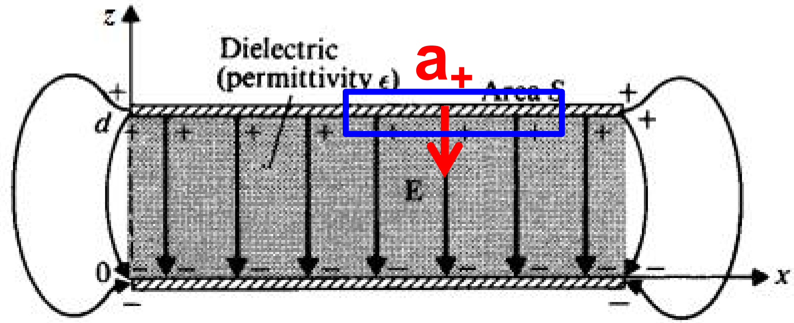




$d \ll \sqrt{S}$



2.3 Capacitor Examples



- Solution:
  - The surface charge density is:

    $$\rho_s = Q/S$$

  - Based on Gauss's Law:

    $$\oiint_{S'} \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\varepsilon_0} \xrightarrow[\textbf{S' is a unit area}]{} \vec{E} = -\hat{a}_z \frac{\rho_s}{\varepsilon_0} = -\hat{a}_z \frac{Q}{\varepsilon_0 S}$$

  - The potential V is:

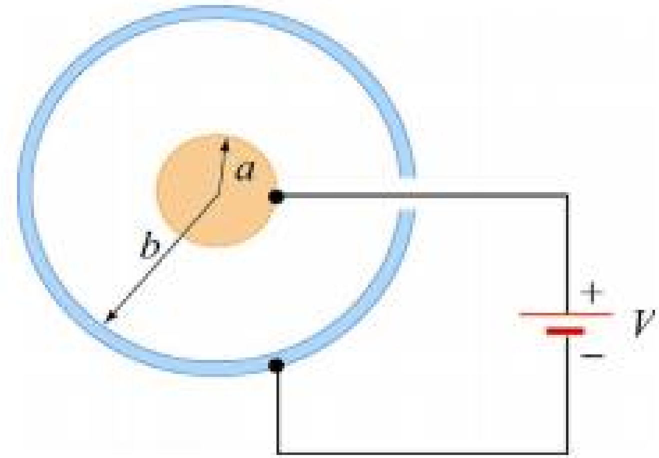
    $$V = -\int_{z=0}^{z=d} \vec{E} \cdot d\vec{l} = -\int_0^d \left(-\hat{a}_z \frac{Q}{\varepsilon_0 S}\right) \cdot (\hat{a}_z dz) = \frac{Q}{\varepsilon_0 S} d$$

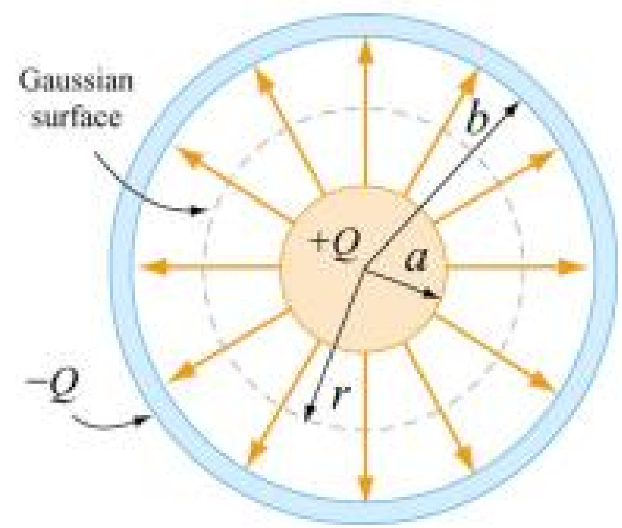
  - Therefore, the capacitance of a parallel – plate is:

    $$C = \frac{Q}{V} = \frac{\varepsilon_0 S}{d}$$

Example 2

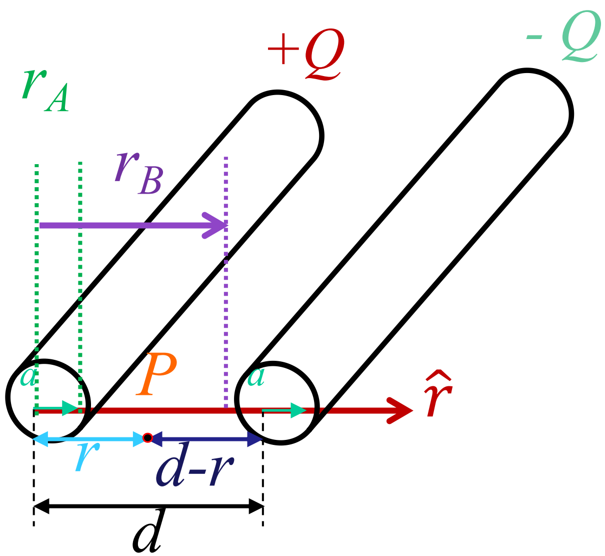
- A spherical capacitor is formed by two concentric spherical shells of radii a and b, between which is air-filled.
- The inner shell has a charge $+Q$ uniformly distributed over its surface, and the outer shell an equal but opposite charge $-Q$.
- Find the capacitor of the structure.





Quiz 2

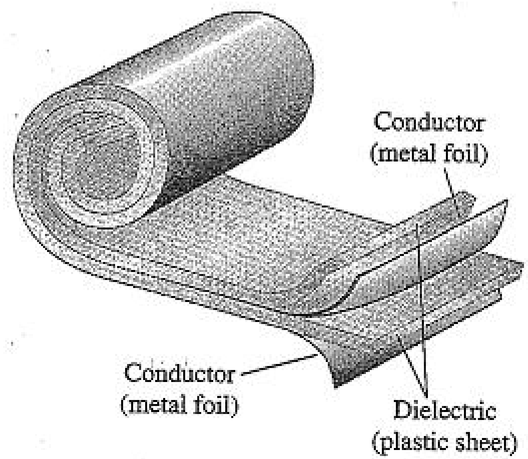
- The conductors are infinite in length and have a uniform charge distribution ρ_l C/m^2 along their length and around their peripheries. Assuming the ratio of *d/a* is large enough, determine the capacitance per unit length for the two-wire line.



2.4 Capacitor with dielectrics

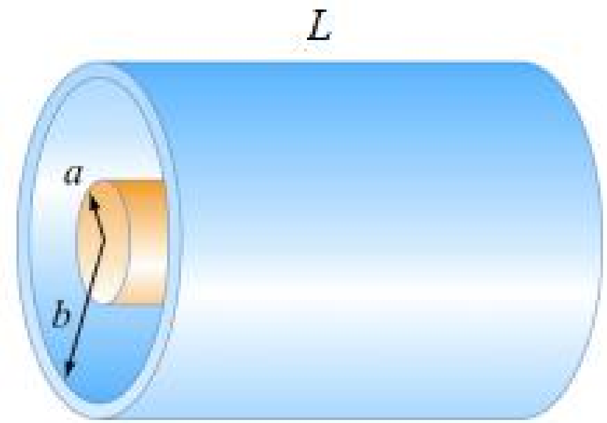
- Most capacitors have an insulating material, such as paper, plastic or ceramic, between their conducting plates.
- Reasons:
  - To maintain a physical separation of the plates;
  - Increase the maximum possible potential difference between the conducting plates;
  - Capacitance increases when the space between the conductors is filled with dielectrics.

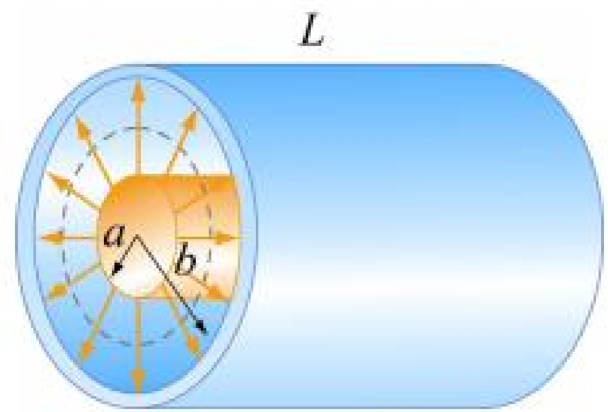
$$C = \frac{Q}{V} = \frac{\varepsilon S}{d}$$



Quiz 3

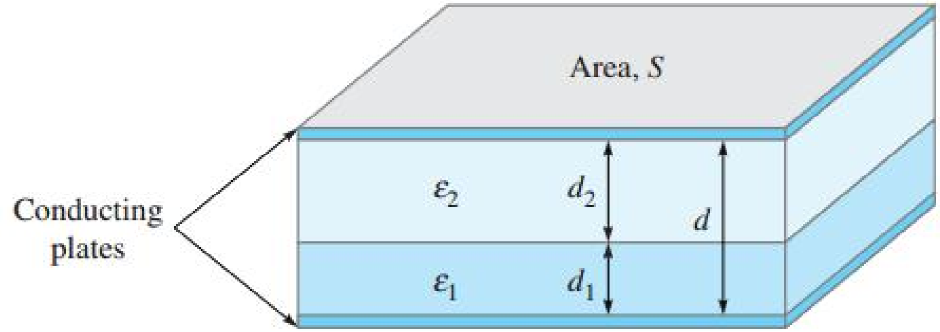
- A cylindrical conductor with inner radius a surrounded by a coaxial cylindrical shell of inner radius b. Filled with dielectrics with ε. The length of both cylinders is L.
- The capacitor is charged so that the inner cylinder has charge $+Q$ while the outer shell has a charge $-Q$.
- Find the capacitor of the structure.





Example 3

- A parallel-plate capacitor containing two dielectrics with the dielectric interface parallel to the plates.
- What is its capacitance?



- Solution 1:
  - It can be considered as two serially connected parallel-plate capacitors.
  - So the total capacitance i $C = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}$
  - where $C_1 = \epsilon_1 S/d_1$ $\quad C_2 = \epsilon_2 S/d_2$

This is the correct result, but let's try to obtain it using less intuition and a more basic approach (from the definition).

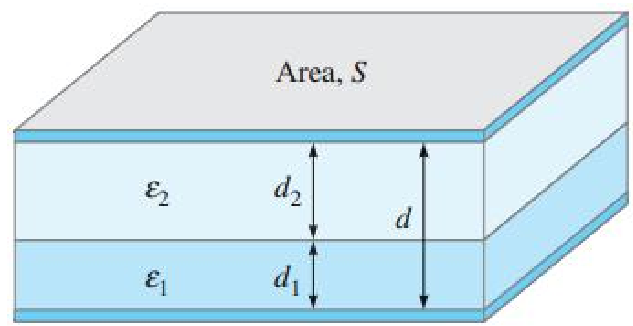
Example 3

- Solution 2:
  - Suppose we assume a potential difference V_0 between the plates. The electric field intensities in the two regions, E_2 and E_1, are both uniform, and $V = E_1 d_1 + E_2 d_2$
  - At the dielectric interface, ***E*** is normal to the interface, and our boundary condition tells us that $D_1 = D_2$, or $\varepsilon_1 E_1 = \varepsilon_2 E_2$
  - The surface charge densities at the conducting plates:

$$\rho_{s1} = D_1 = \varepsilon_1 E_1 = \varepsilon_2 E_2 = \rho_{s2} = \rho_s$$

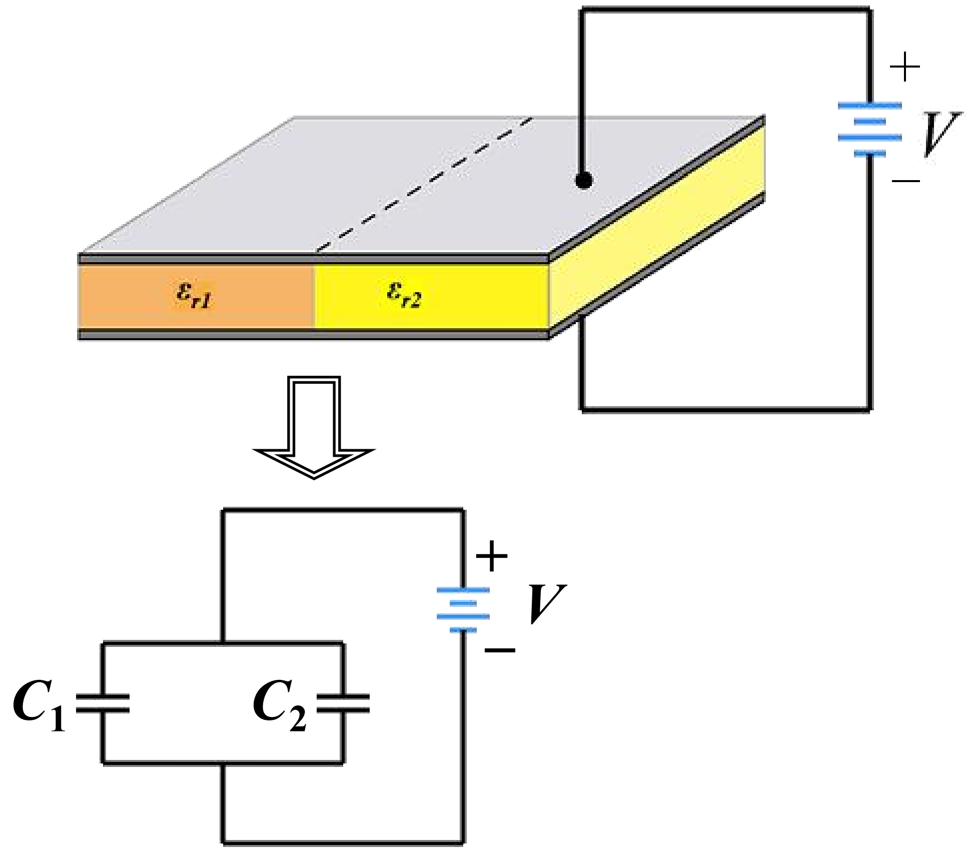
  - So we have:

$$C = \frac{Q}{V} = \frac{\rho_s S}{V} = \frac{1}{\frac{d_1}{\varepsilon_1 S} + \frac{d_2}{\varepsilon_2 S}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$



Example 4

- Two dielectrics with relative permittivity ε_{r1} and ε_{r2} each fill half the space between the plates of a parallel-plate capacitor. Each plate has an area, and the plates are separated by a distance d. Find the capacitance of the system.
  - The potential difference on each half of the capacitor is the same, so the system can be treated as being composed of two capacitors connected in parallel.
  - Thus, the capacitance of the system is $C = C_1 + C_2$

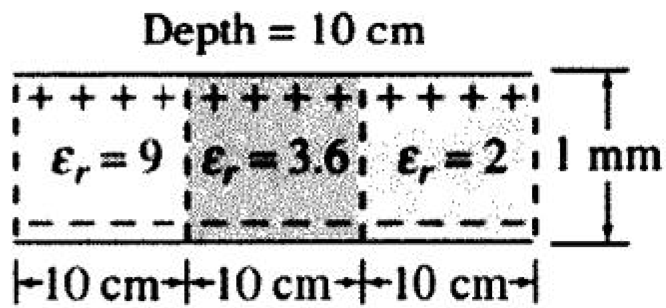


$$C = C_1 + C_2 = \frac{\varepsilon_0 \varepsilon_{r1} A_1}{d} + \frac{\varepsilon_0 \varepsilon_{r2} A_2}{d}$$

$$= \varepsilon_0 \frac{\varepsilon_{r1} A_1 + \varepsilon_{r2} A_2}{d}$$

Quiz 4

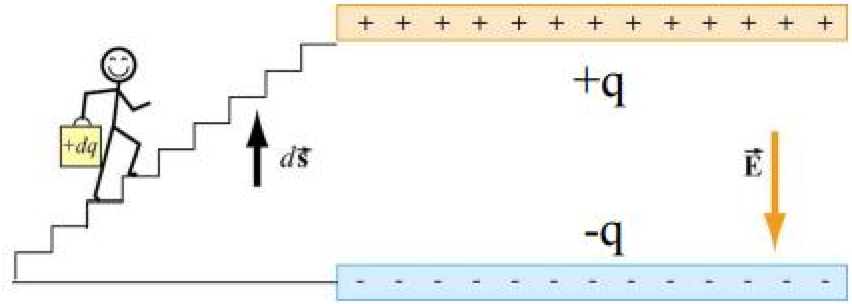
- A parallel-plate capacitor with three dielectric media is shown below. What is the total capacitance of the system?



2.5 Energy stored in a capacitor

- ✓ 1. Capacitor starts uncharged.
- ✓ 2. Carry $+dq$ from bottom to top.

  Now top has charge $q = +dq$, bottom $-dq$
- ✓ 3. Repeat
- ✓ 4. Finish when top has charge $q = +Q$, bottom $-Q$.



- At some point top plate has $+q$, potential difference is: $\Delta\varphi = q/C$
- Work done to lift dq from the bottom to top is: $dW = dq\Delta\varphi = qdq/C$
- So work done to move Q from bottom to top is:

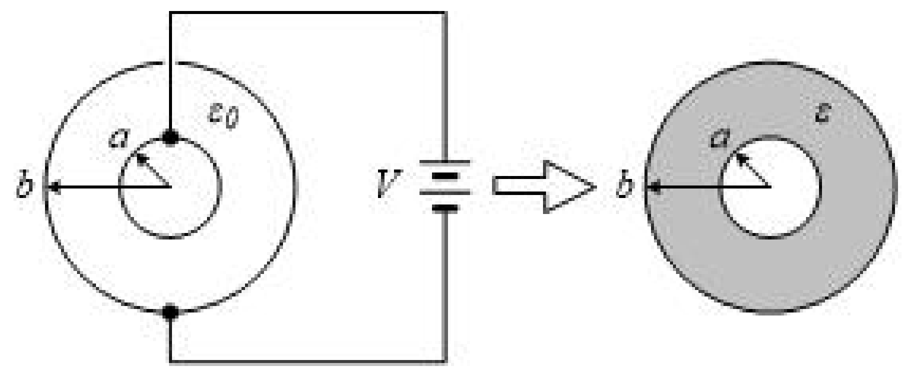
$$W = \int dW = \frac{1}{C}\int_0^Q qdq = \frac{1}{C}\frac{Q^2}{2}$$

- After charging, the total energy stored is:

$$W = \frac{1}{C}\frac{Q^2}{2} = \frac{1}{C}\frac{(CV)^2}{2} = \frac{1}{2}CV^2$$

Quiz 5

- An air-filled spherical capacitor with conductor radii a = 3 cm and b = 15 cm is connected to a source of voltage V = 15 kV as shown in Figure (b).
- After an electrostatic state is established, the source is disconnected. The capacitor is then filled with a liquid dielectric of dielectric constant $\varepsilon_r = 2$ as shown in Figure (c).
- Determine the new voltage between the electrodes of the capacitor.
- Determine the energy stored between the electrodes of the capacitor.



(b) (c)

2.6 Current – Voltage Relationship

- Start from the known relationship:

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C}$$

- In a time-dependent scenario:

$$V(t) = \frac{Q(t)}{C} = \frac{1}{C}\int_{t_0}^{t} I(\tau)d\tau + V(t_0)$$

- Taking the derivative of this and multiplying by *C*, get:

$$I(t) = \frac{dQ(t)}{dt} = C\frac{dV(t)}{dt}$$

  – which means “the voltage on the capacitor is always continuous”;

  – Also points out that the current “flows” through the capacitor is proportional to the capacitance and the changing rate of the voltage on the capacitor.

Next ...

- Magnetic Fields