Ideal Operational Amplifier Circuits

Dr. Xiaoyang Chen

Dept. of Electrical & Electronic Engineering
XJTLU

Part 1: Introduction of the Operational Amplifier

The Operational Amplifier

- An operational amplifier (op-amp) is an integrated circuit that amplifies the **difference** between two input voltages and produces a single output.

- **Versatility -** Op-amps can be used to perform mathematical operations – addition, subtraction, differential and integration – then put together to build analogue computers – which could solve differential equations, etc.

- An op-amp is specially designed to be used with **feedback**. Recall for an amplifier with feedback, the closed loop gain is

$$A_{CL} = \frac{A_{OL}}{1 + \beta A_{OL}}$$

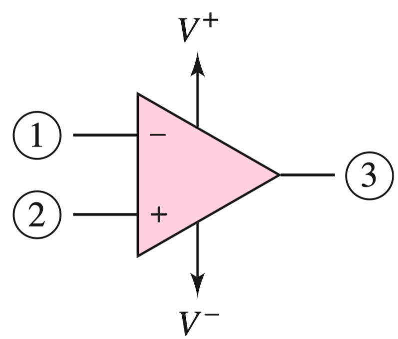
If loop gain >>1 then $A_{CL} \approx 1/\beta$

So to build a precision amplifier (i.e. one with a precise gain) it is sufficient to just buy a **few precision resistors** and a **cheap but high gain** op-amp.

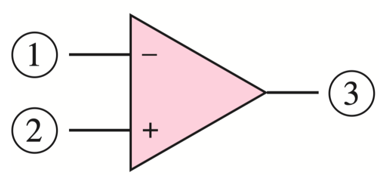
Circuit Representation

- An op-amp is normally made up from 20 to 30 transistors. However, as a typical IC op-amp has parameters that approach the ideal characteristics, **we can treat it as a simple compact device**.

- In most cases, an op-amp requires DC power, so that the internal transistors are biased in the active region.

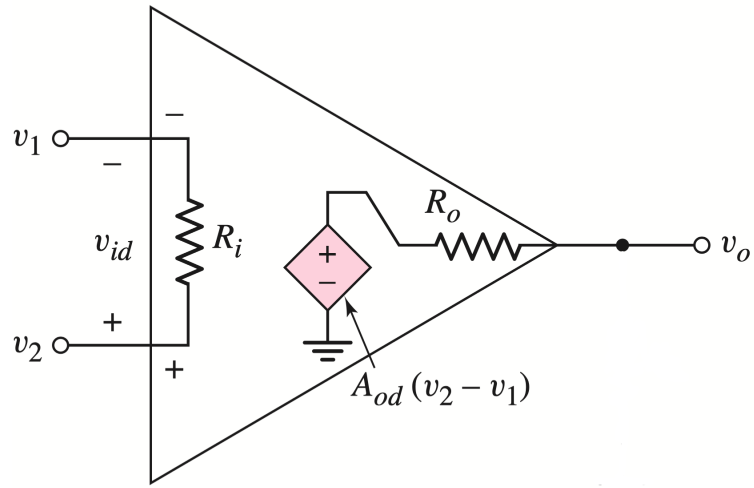


- From a signal point of view, the op-amp has two input terminals and one output terminal. Therefore, we often use a simplified symbol. But keep in mind that the op-amp does require DC input.



Equivalent Circuit

Omitting power supplies, the equivalent circuit for an op-amp is



- The output voltage source is controlled by the ***differential*** input voltage v_{id} so if there is no load, $v_o = A_{od}v_{id} \Longrightarrow$ looks like a reasonable **voltage amplifier**

- An operational amplifier generally has **large** input impedance, **low** output impedance and **very high** voltage gain

Ideal Op-Amp Equivalent Circuit

① Inverting input:

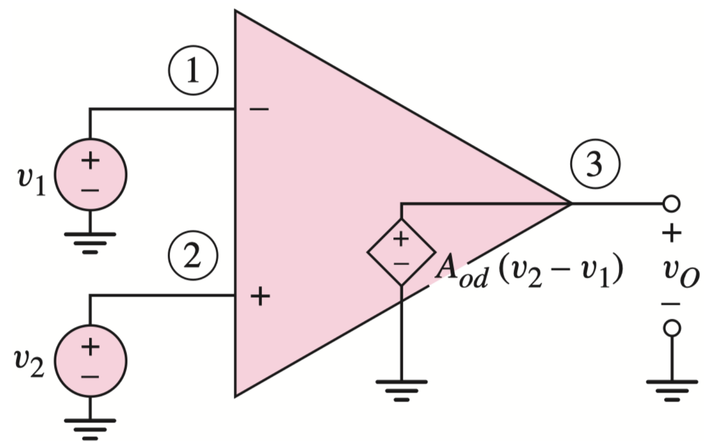
$V_{out} = -A_{od}V_1$

② Non-inverting input:

$V_{out} = A_{od}V_2$

③ Output:

$V_{out} = A_{od}(V_2 - V_1)$



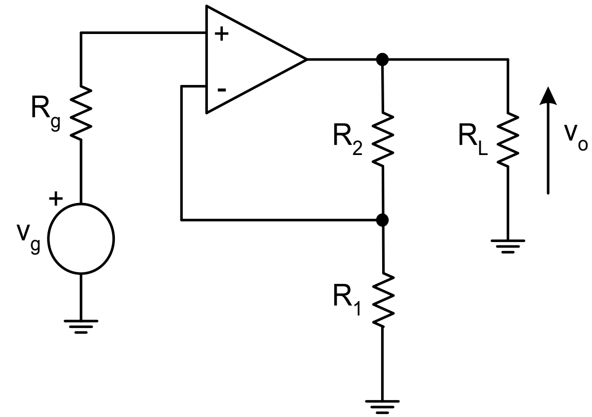
Ideal Parameters:

- the input resistance R_i between terminals 1 and 2 is infinite
- the output terminal of the op-amp acts as an ideal voltage source, i.e., R_o is zero
- the open loop gain A_{od} is very large and approaches infinity

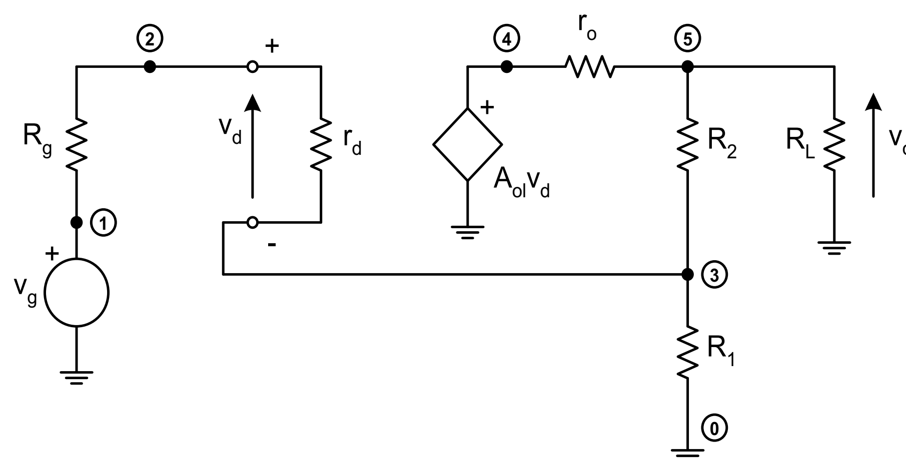
Part 2: Basic Operational Amplifier Circuits

Analysis Method - Conventional

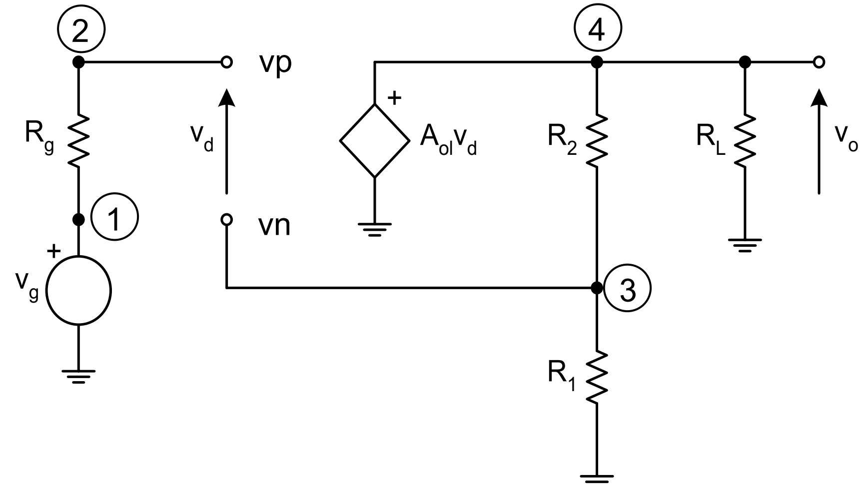
Consider the following <u>non-inverting</u> op-amp circuit (this is not just an op-amp, it also has **feedback resistors)**



Replacing the op-amp by its equivalent circuit gives



Assume the op-amp is ideal, i.e., $r_i \to \infty$ and $r_o \to 0$



Then there are no currents flowing through v_p and v_n terminals, we can write:

<u>For node 2</u> $v_p = v_2 = v_g$

<u>For node 4</u> $\dfrac{v_o - v_n}{R_2} = \dfrac{v_n}{R_1}$

<u>At output</u> $v_o = A_{OL}(v_p - v_n)$

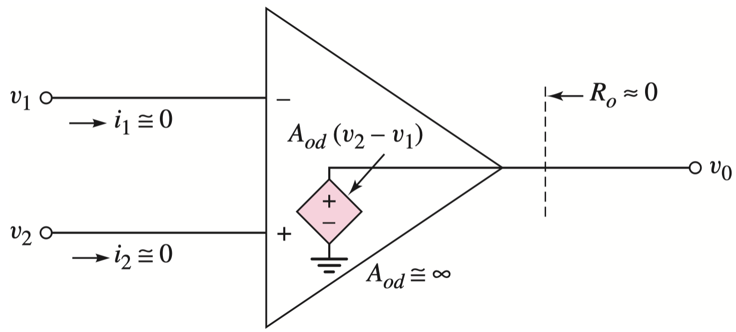
Solving gives:

$$\frac{v_o}{v_g} = \frac{A_{OL}}{1 + A_{OL}\left[\dfrac{R_1}{R_1 + R_2}\right]} = \frac{A_{OL}}{1 + \beta A_{OL}} \quad \text{where} \quad \beta = \frac{R_1}{R_1 + R_2}$$

For $A_{OL} \to \infty$ we have: $$\frac{v_o}{v_g} = \frac{A_{OL}}{1 + A_{OL}\left[\dfrac{R_1}{R_1 + R_2}\right]} = \frac{A_{OL}}{1 + \beta A_{OL}} \approx \frac{1}{\beta} = \frac{R_1 + R_2}{R_1}$$

Analysis Method – Virtual Short Principle ★

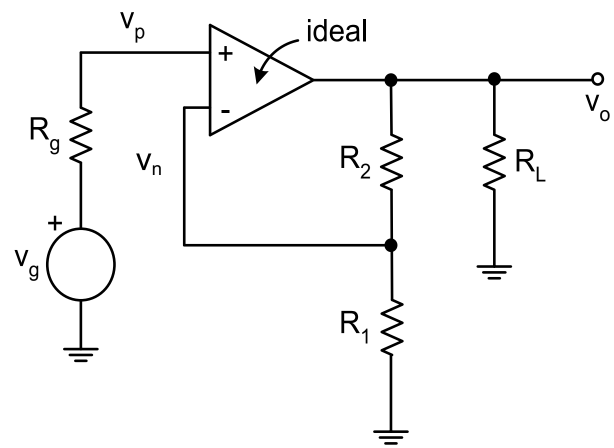
- An Op – amp has a very high gain, so for any reasonable output voltage, the input differential voltage $v_p - v_n$ will be **vanishingly small**
- So if the gain is very large then we can say that $v_p - v_n \approx 0$ or $v_p \approx v_n$
- This is a very useful approximation – a '***Golden Rule***' !! *(But remember – it only applies if the gain is **very** large)*; **We say that v_n tracks v_p**



- This leads to the concept of a **virtual short** – the circuit behaves as though there is a short across the inputs because the voltage difference between v_p and v_n is kept zero, *but it is not actually shorted* . Hence the name '**Virtual** Short'. It greatly simplifies the analysis of op-amp circuits
- To apply virtual short principle, the op-amp must be **ideal**.

Applying Virtual Short Principle to Op-Amp Circuits

1. The Non-Inverting Amplifier



For an ideal op-amp, the input resistance is ∞, then it seems to be an open circuit between v_p and v_n

Then we have: $v_p = v_g$ (virtual open)

According to virtual short principle:

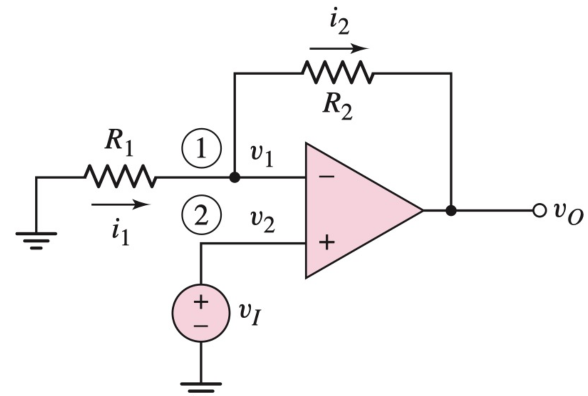
$$v_n = v_p = v_g$$

And we have:

$$\frac{v_n}{R_1} = \frac{v_o - v_n}{R_2} \rightarrow v_n = v_o \frac{R_1}{R_1 + R_2}$$

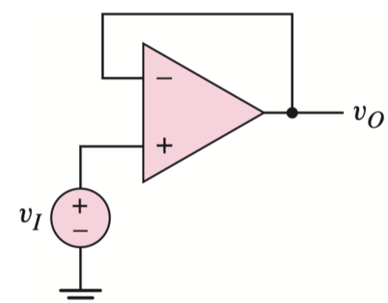
Hence, we can find: $\frac{v_o}{v_g} = \frac{R_1 + R_2}{R_1}$ (non-inverted voltage gain)

An interesting property of the non-inverting op-amp occurs when $R_1 = \infty$, the closed loop gain becomes



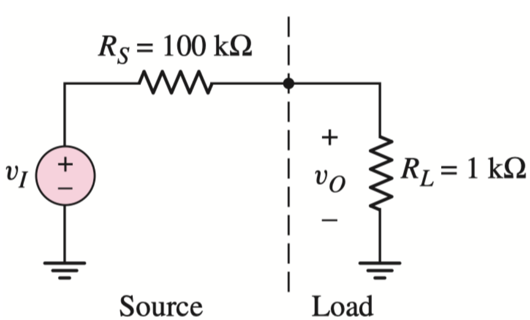
$$A_v = \frac{v_O}{v_I} = \frac{R_1 + R_2}{R_1} = 1$$ (This is a voltage follower!)

The closed-loop gain is independent of R_2 (except for $R_2 = \infty$), so we can set $R_2 = 0$ to create a short circuit.

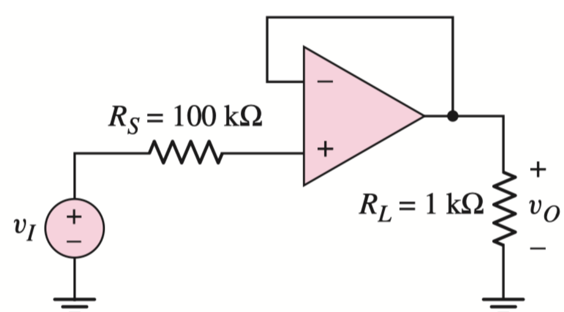


For a ideal o-amp, the input impedance is essentially infinite, and the output impedance is essentially zero (a perfect buffer)

<u>For example:</u>



(a)



(b)

2. The Inverting Amplifier

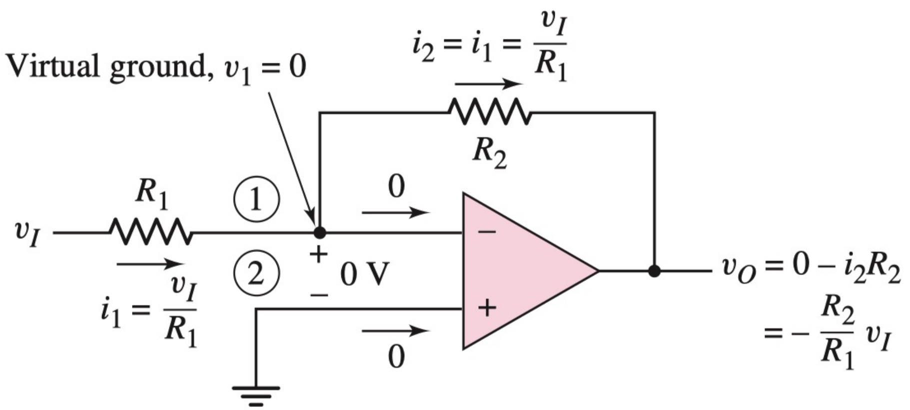
From the virtual short principle, we have:

$$v_1 \approx v_2 = 0$$

Since the current flowing into the op-amp is assumed to be zero, we have:

$$i_1 = i_2$$

where $i_1 = \dfrac{v_I - v_1}{R_1}$ and $i_2 = \dfrac{v_I - v_O}{R_2}$



Then we have:

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} \quad \text{(inverted voltage gain)}$$

We also can find the input resistance by:

$$R_i = \frac{v_I}{i_1} = R_1$$

3. The Inverting Amplifier with a T-Network

Assuming an inverting op-amp is to be designed having a closed-loop voltage gain of $A_v = -100$ and an input resistance of $R_i = R_1 = 50\ k\Omega$, the feedback resistor R_2 would have to be $5\ M\Omega$. However, this value is too large in practice. Instead, we can apply a **T-Network**.

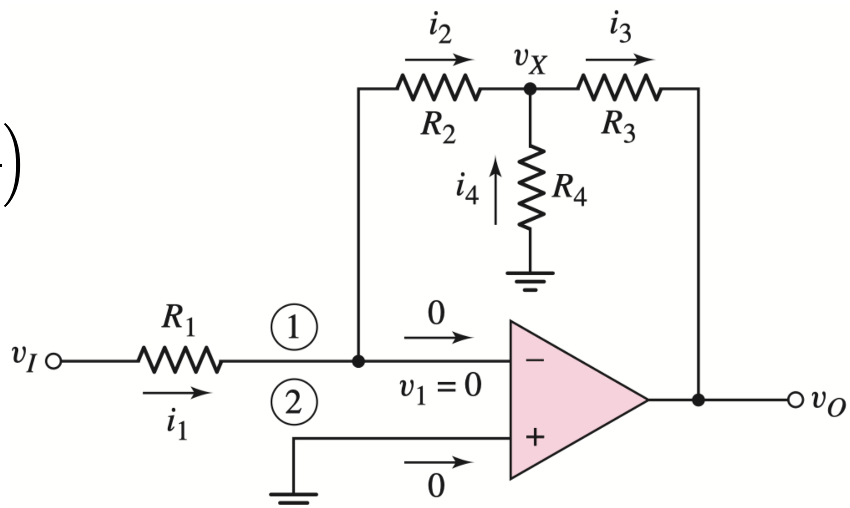
At the input, we have $i_1 = \dfrac{v_I}{R_1} = i_2$

We can also write that $v_X = 0 - i_2 R_2 = -v_I \left(\dfrac{R_2}{R_1}\right)$

Applying a KCL at node v_X, we have

$i_2 + i_4 = i_3$

which can be written as $-\dfrac{v_X}{R_2} - \dfrac{v_X}{R_4} = \dfrac{v_X - v_O}{R_3}$



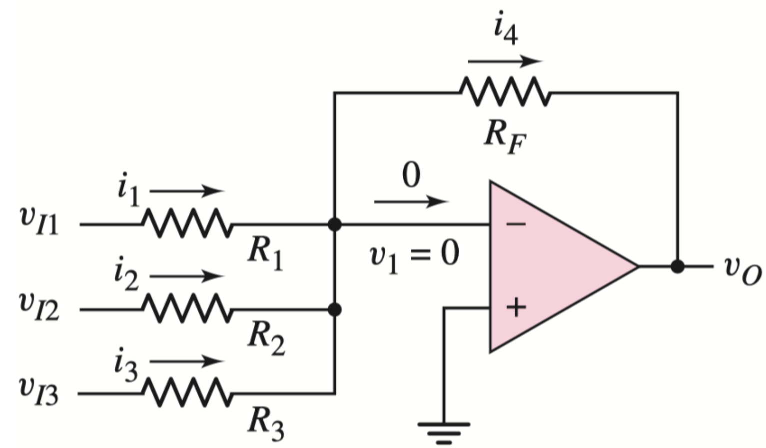
Finally, we can find

$$A_v = \frac{v_O}{v_I} = -\frac{R_2}{R_1}\left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2}\right)$$

One possible solution to the above question would be $R_1 = 50\ k\Omega$, $R_2 = R_3 = 400\ k\Omega$, and $R_4 = 38.1\ k\Omega$

4. The Summing Amplifier

This example has 3 inputs, but could be as many as you want.



Since $v_1 = v_2 = 0$ (from virtual short principle) and KCL gives $i_1 + i_2 + i_3 = i_4$, we have

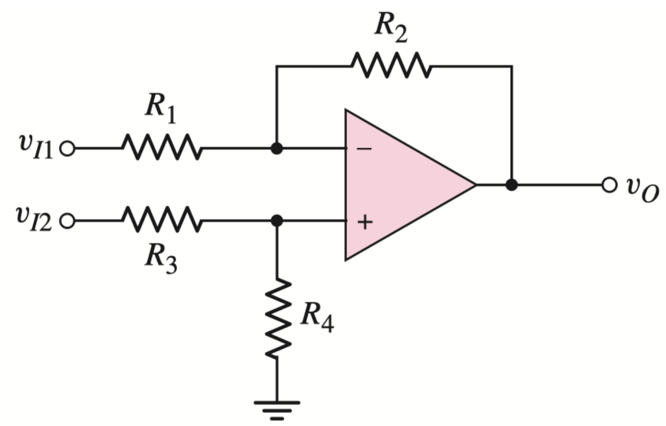
$$\frac{v_{I1}}{R_1} + \frac{v_{I2}}{R_2} + \frac{v_{I3}}{R_3} = -\frac{v_O}{R_F}$$

Therefore, we can find $v_O = -\left(\frac{R_F}{R_1} v_{I1} + \frac{R_F}{R_2} v_{I2} + \frac{R_F}{R_3} v_{I3}\right)$

The output voltage is the sum of the three input voltages, with different weighting factors. A special case occurs when the three input resistances are equal, when $R_1 = R_2 = R_3 = R$, then

$$v_O = -\frac{R_F}{R_1}(v_{I1} + v_{I2} + v_{I3})$$

5. The Difference Amplifier



Use **superposition**:

First v_{I2} is turned down to zero, leaving only its internal resistance in the circuit (0 Ω for a perfect voltage source)

Then the contribution to v_O due to v_{I_1} alone is

$$v_{O1} = -\frac{R_2}{R_1} v_{I1}$$

Now restore v_{I2} and turn v_{I1} down to zero instead, at the non-inverting terminal, we have

$$\frac{v_p}{v_{I2}} = \frac{R_4}{R_3 + R_4}$$

Then the contribution to v_O due to v_{I2} alone is

$$v_{O2} = \frac{R_1 + R_2}{R_1} v_p = \frac{R_1 + R_2}{R_1} \frac{R_4}{R_3 + R_4} v_{I2}$$

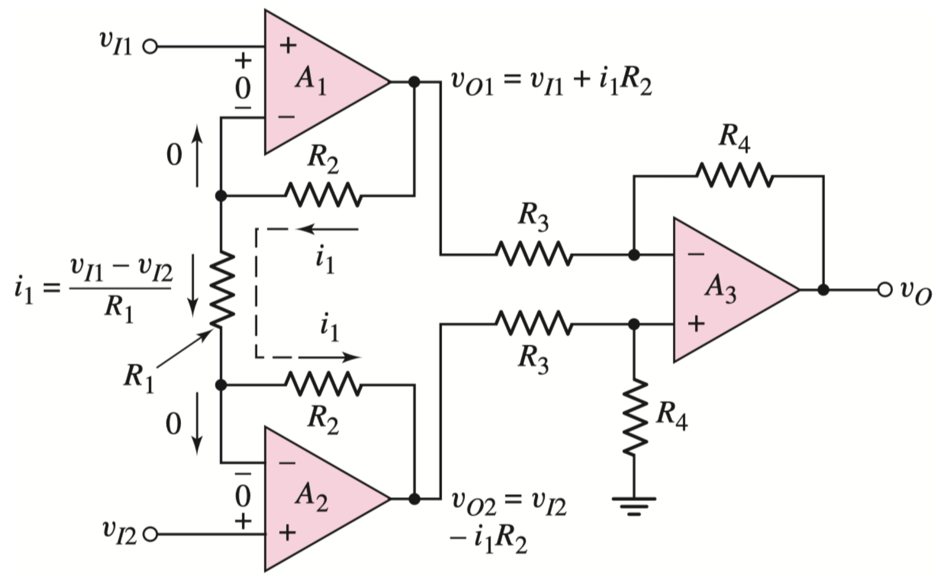
So $v_O = v_{O1} + v_{O2}$ gives $v_o = \frac{R_2}{R_1}\left[\left(\frac{1 + \frac{R_1}{R_2}}{1 + \frac{R_3}{R_4}}\right) v_{I2} - v_{I1}\right]$

To make a difference amplifier we require $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, giving

$$v_O = \frac{R_2}{R_1}(v_{I2} - v_{I1})$$

6. The Instrumentation Amplifier

For a difference amplifier, it is difficult to obtain a high input impedance and a high gain with reasonable resistor values. Optimally, we would like to be able to change the gain by changing only a single resistance value. This is achieved by an **instrumentation amplifier**.



The current in R_1 can be found by

$$i_1 = \frac{v_{I1} - v_{I2}}{R_1}$$

The current in R_1 is also i_1, and the output voltages of op-amps A_1 and A_2 are

$$v_{O1} = v_{I1} + i_1 R_2 = \left(1 + \frac{R_2}{R_1}\right) v_{I1} - \frac{R_2}{R_1} v_{I2}$$

and

$$v_{O2} = v_{I2} - i_1 R_2 = \left(1 + \frac{R_2}{R_1}\right) v_{I2} - \frac{R_2}{R_1} v_{I1}$$

Since the output of the difference amplifier is given as

$$v_O = \frac{R_4}{R_3}(v_{O2} - v_{O1})$$

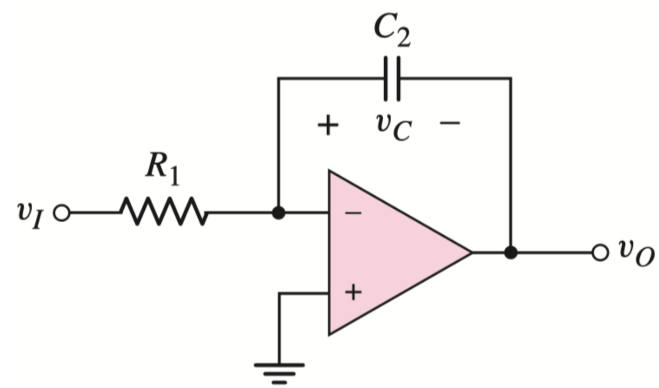
The output voltage can be found as

$$v_O = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right)(v_{I2} - v_{I1})$$

By changing R_1, we can adjust the voltage gain easily.

7. The Integrator

For an ideal op-amp, we assume no current flowing into it, in the circuit, we have



$i_{R1} = i_{C2}$ and $v_C = -v_O$

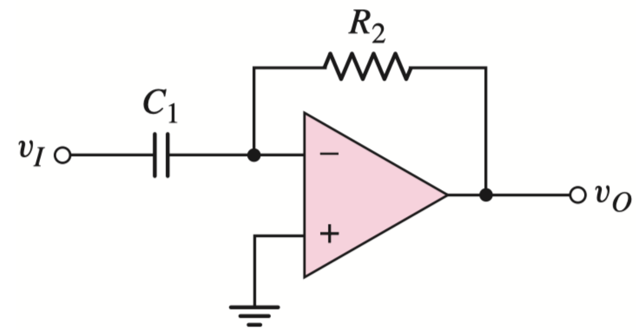
where $i_{R1} = \dfrac{v_I(t) - 0}{R_1}$ and $i_{C2} = C_2 \dfrac{-dv_O}{d_t}$

Hence $\dfrac{dv_O}{d_t} = -\dfrac{1}{R_1 C_2} v_I(t)$

$$\Longrightarrow v_O = -\frac{1}{R_1 C_2} \int v_I(t)dt$$

Output voltage is proportional to the time integral of the input

8. The Differentiator



We have $i_{C1} = i_{R2}$

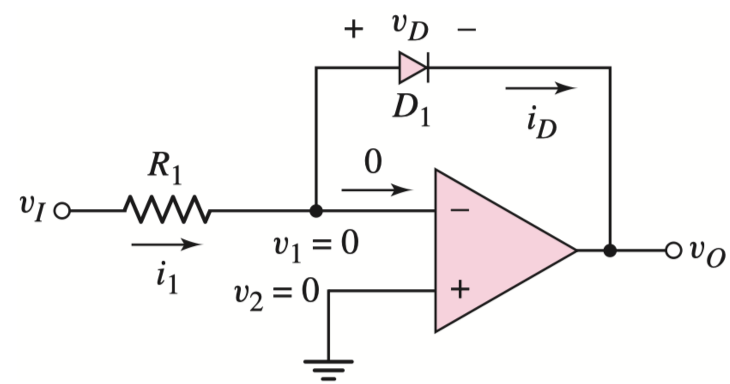
where $i_{C1} = C_1 \frac{d(v_I - 0)}{d_t}$ and $i_{R2} = \frac{0 - v_O(t)}{R_2}$

Hence

$$v_o(t) = -R_2 C_1 \frac{dv_I}{dt}$$

Output voltage is proportional to the time derivative of the input

9. The Log Amplifier



The diode current is $i_D \cong I_s \left(e^{\frac{v_D}{VT}}\right)$

The input current can be written as

$$i_1 = \frac{v_I}{R_1}$$

and the output voltage, since v_1 is virtual ground, is given by

$$v_O = -v_D$$

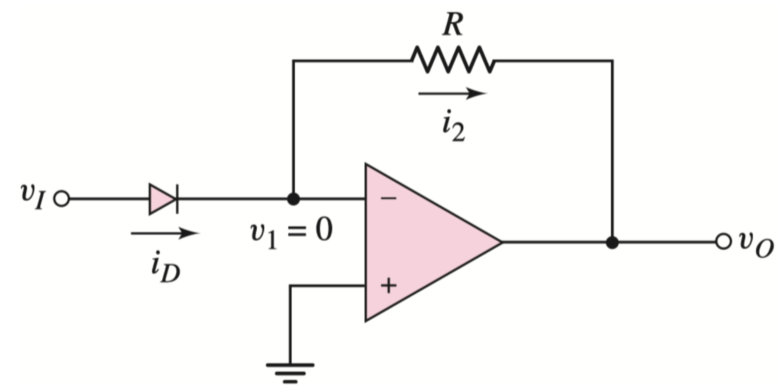
Noting that $i_1 = i_D$, we can write

$$i_1 = \frac{v_I}{R_1} = i_D = I_S e^{-\frac{v_O}{V_T}}$$

If we take the natural log of both sides of this equation, we obtain

$$\ln\left(\frac{v_I}{I_S R_1}\right) = -\frac{v_O}{V_T} \qquad \text{or} \qquad v_O = -V_T \ln\left(\frac{v_I}{I_S R_1}\right)$$

10. The Exponential Amplifier



The complement, or inverse function of the log amplifier is the exponential amplifier. Since v_1 is at virtual ground, we can write

$$i_D \cong I_s\left(e^{\frac{v_I}{VT}}\right)$$

and

$$v_O = -i_2 R = -i_D R$$

or

$$v_O = -I_S R \cdot e^{\frac{v_I}{V_T}}$$

or

A Practical Application – the Analogue Computer

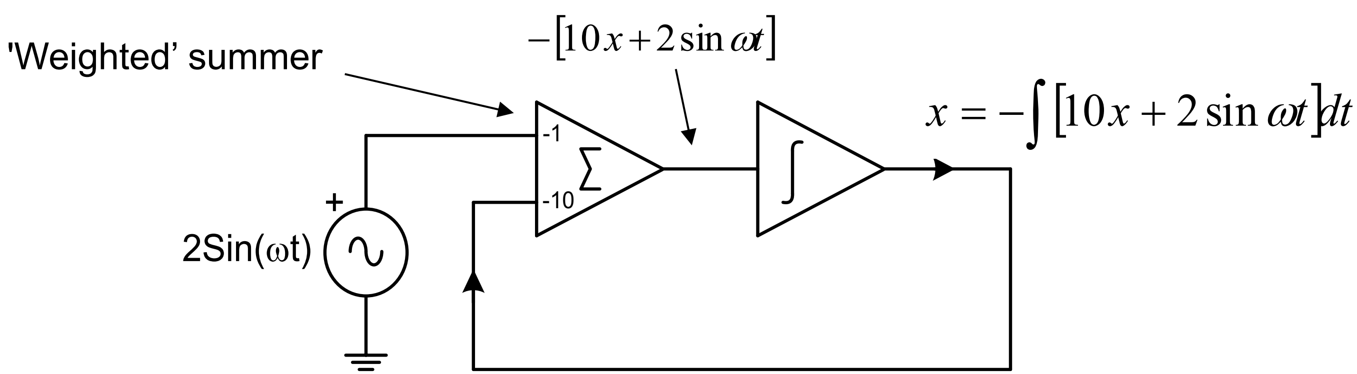
Widely used in the 1960's before the digital revolution – still found in some special applications

Suppose we want to solve the differential equation $\frac{dx}{dt}+10x=-2\sin(\omega t)$

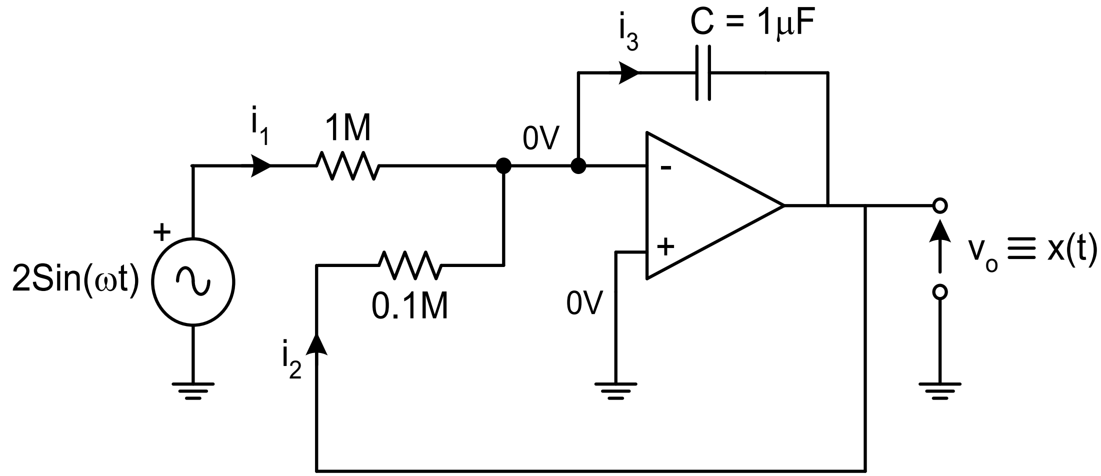
Then $$\frac{dx}{dt}=-10x-2\sin(\omega t)=-[10x+2\sin\omega t]$$

$$x=-\int[10x+2\sin\omega t]dt$$

So we need an <u>integrator and a summing amplifier</u>



Circuit Design



$$i_1 + i_2 = i_3$$

$$\therefore \frac{2\sin(\omega t)}{1M} + \frac{v_0}{0.1M} = -(1\mu F)\frac{dv_0}{dt}$$

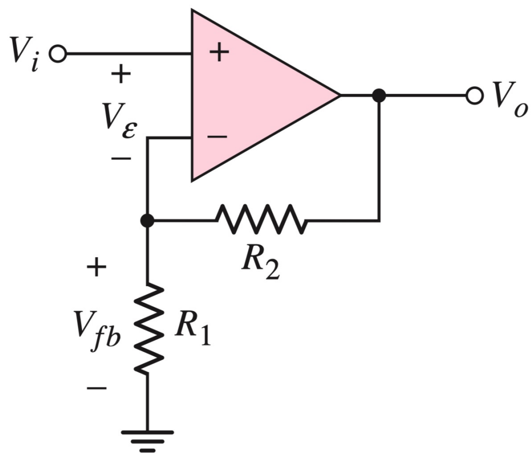
$$\frac{dv_0}{dt} = \frac{-v_0}{10^5 \times 10^{-6}} - \frac{2\sin(\omega t)}{10^6 \times 10^{-6}}$$

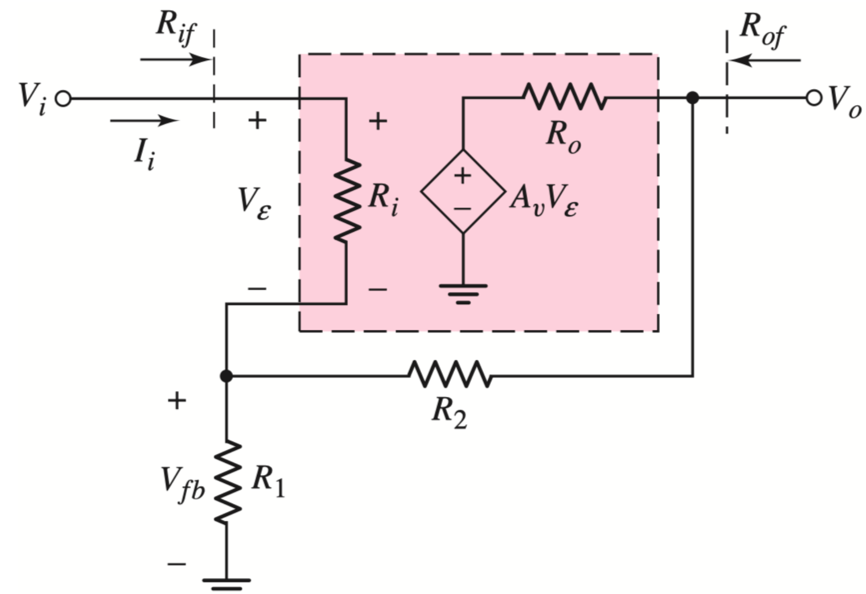
$$\frac{dv_O}{dt} = -\frac{v_0}{0.1} - \frac{2\sin(\omega t)}{1}$$

$$\therefore \frac{dv_0}{dt} + 10\,v_0 = -2\sin(\omega t)$$ as required

Part 3: Op-Amp Representation of of Feedback Amplifiers

1. Voltage Amplifier





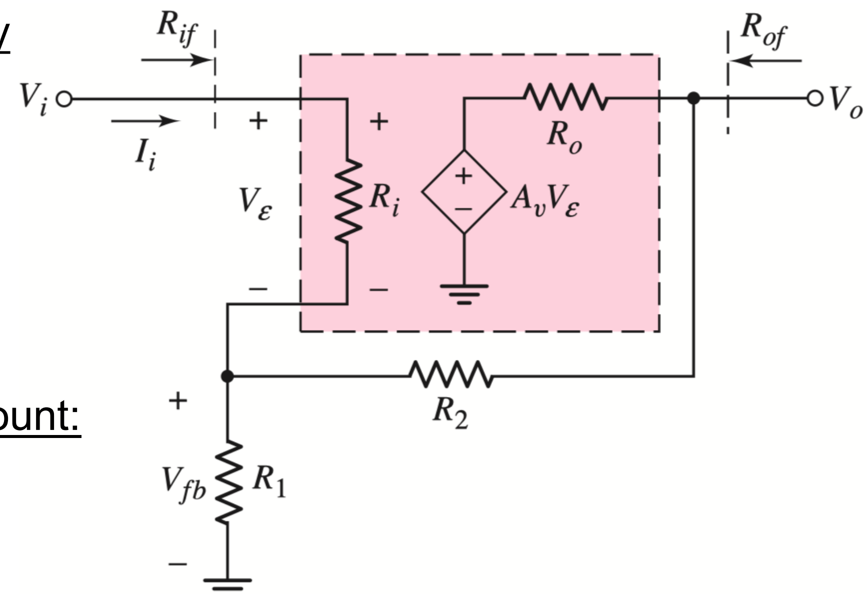
The non-inverting op-amp is an example of the voltage amplifier:

- The input signal is the input voltage V_i
- The error signal is the terminal voltage difference
- In this case, the feedback voltage is taken at R_1

For an ideal non-inverting op-amp (A_v very large), we have

$$A_{vf} = \frac{V_o}{V_i} = \frac{R_1 + R_2}{R_1}$$

Therefore $\beta_v = \dfrac{R_1}{R_1 + R_2}$



We can also take a finite amplifier gain into account:

For $R_o \approx 0$, we have $V_o = A_v V_\varepsilon$

and $V_\varepsilon = V_i - V_{fb}$

Therefore $V_o = A_v(V_i - V_{fb})$

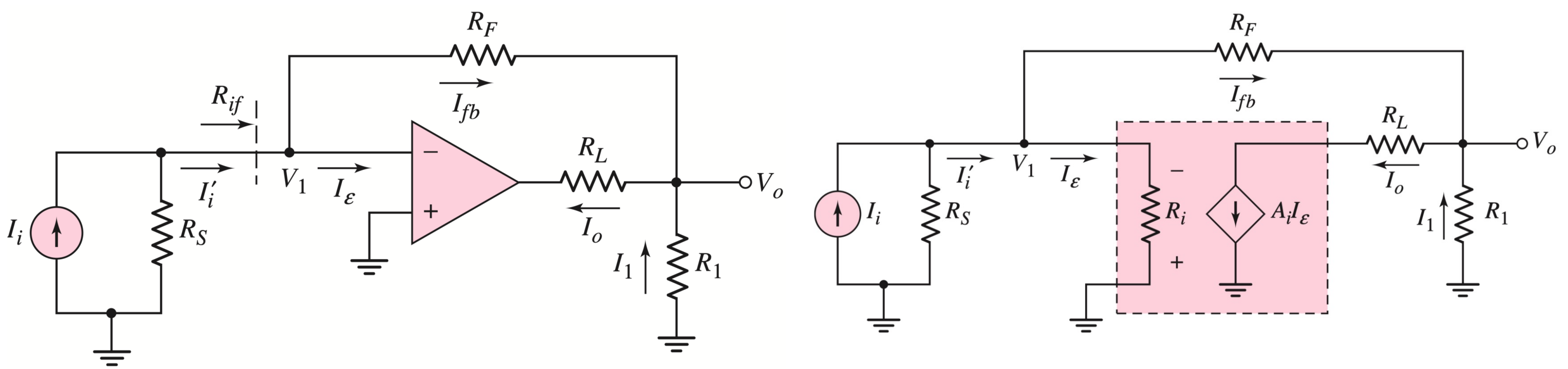
Assuming the input resistance is very large, the feedback voltage is given by

$$V_{fb} \cong \left(\frac{R_1}{R_1 + R_2}\right) V_o$$

Thus we obtain

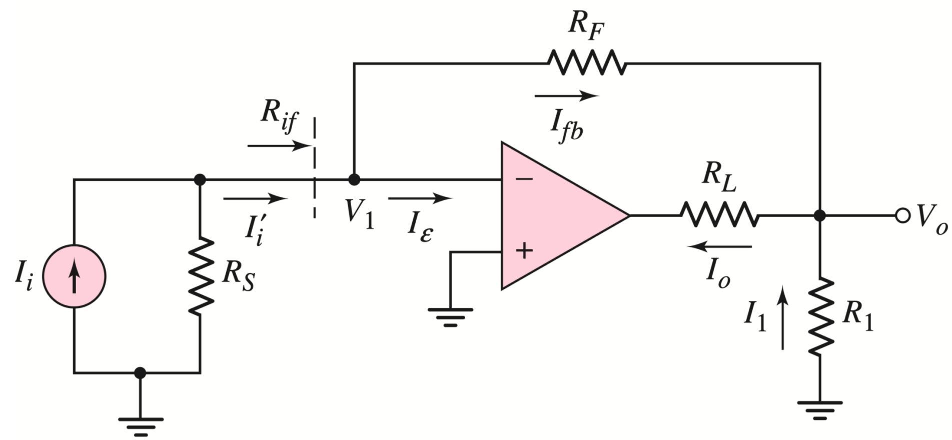
$$A_{vf} = \frac{V_o}{V_i} = \frac{A_v}{1 + \dfrac{A_v}{\dfrac{R_1 + R_2}{R_1}}} = \frac{A_v}{1 + \beta_v A_v}$$

2. Current Amplifier



The inverting op-amp with load resistor is an example of the current amplifier:

- The input signal is the current I_i' from the Norton equivalent source of I_i and R_S
- The feedback signal is I_{fb}
- The output current is taken at R_L
- The error signal is the current I_ε



For an ideal op-amp, we have

$I_\varepsilon = 0$

Then $I_i \cong I_i' = I_{fb}$

(R_S is normally very large for a current source)

Since V_1 is at virtual ground:

$V_o = -I_{fb}R_F = -I_iR_F$

And current I_1 is $I_1 = -V_o/\ R_1$

Hence $\beta_i = \dfrac{1}{1+\dfrac{R_F}{R_1}}$

Then the output current can be expressed by

$$I_o = I_{fb} + I_1 = I_i + \left(-\frac{1}{R_1}\right)(-I_iR_F) = I_i\left(1+\frac{R_F}{R_1}\right)$$

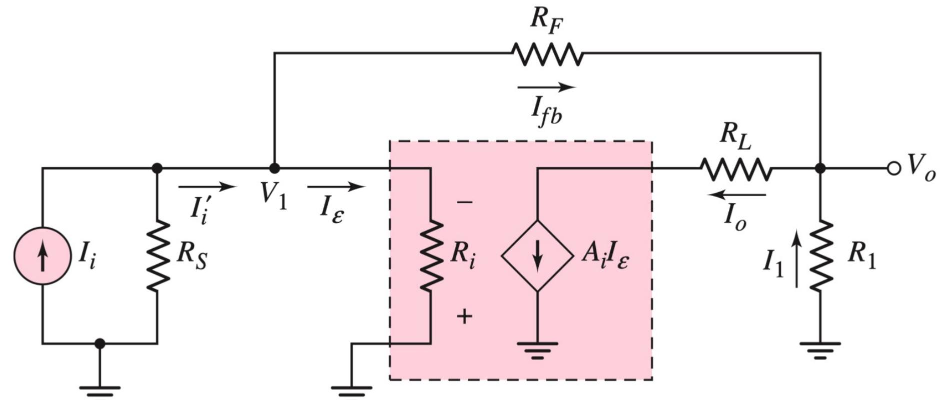
Therefore, the ideal current gain is $\dfrac{I_o}{I_i} = 1 + \dfrac{R_F}{R_1}$

Again we can take a finite amplifier gain into account:

We have $I_o = A_i I_\varepsilon$

and $I_\varepsilon = I_i' - I_{fb} \cong I_i - I_{fb}$

therefore, $I_o = A_i(I_i - I_{fb})$



If we assume V_1 is virtual short, we have

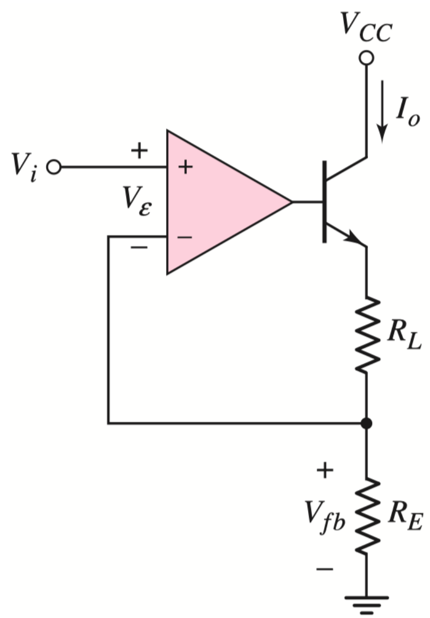
$$V_o = -I_{fb} R_F$$

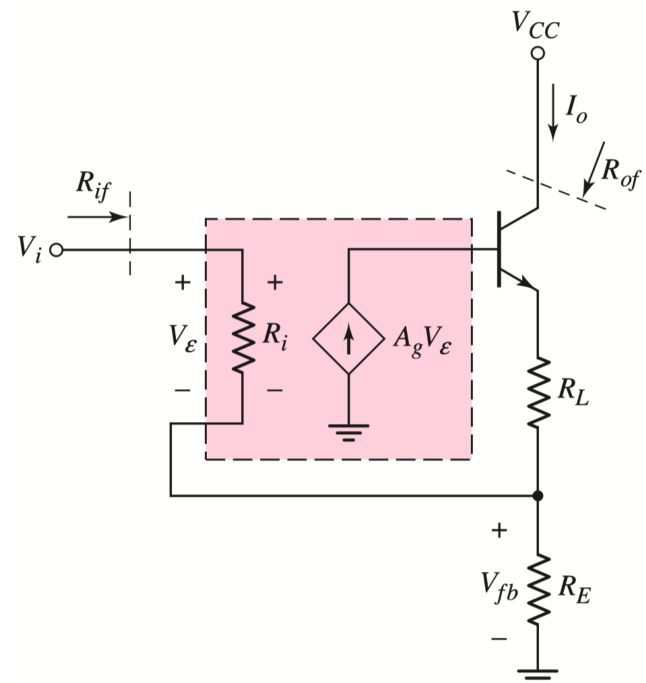
We can then write $I_1 = -\dfrac{V_o}{R_1} = -\left(\dfrac{1}{R_1}\right)\left(-I_{fb}R_F\right) = I_{fb}\left(\dfrac{R_F}{R_1}\right)$

The output current is expressed as $I_o = I_{fb} + I_1 = I_{fb} + I_{fb}\left(\dfrac{R_F}{R_1}\right)$

Solving for I_{fb} yields the closed-loop current gain $A_{if} = \dfrac{I_o}{I_i} = \dfrac{A_i}{1 + \dfrac{A_i}{1 + \dfrac{R_F}{R_1}}}$

3) Transconductance Amplifier



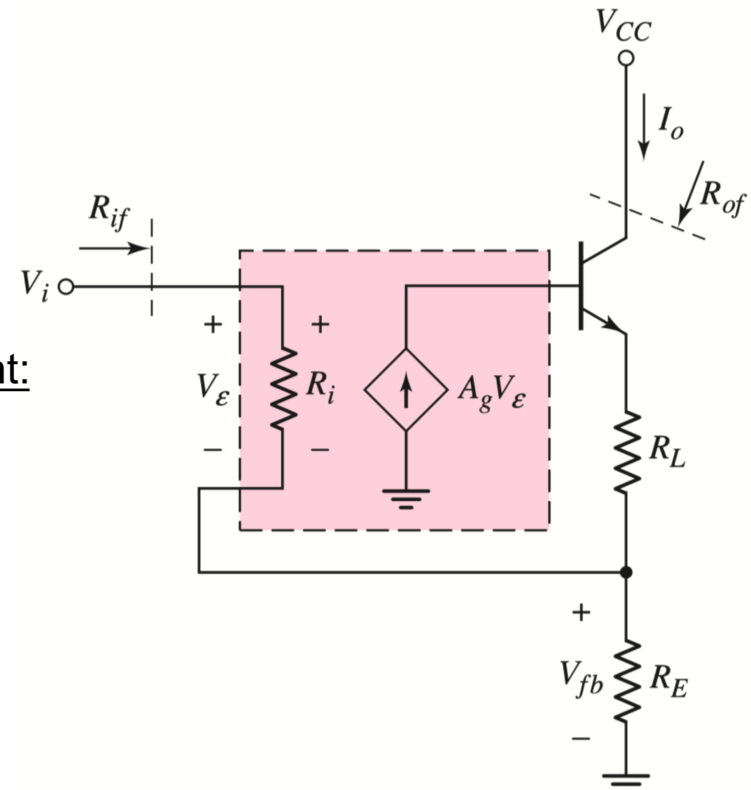


- The input signal is the input voltage V_i
- The output signal is I_o
- The feedback voltage is taken at R_E

Assuming an ideal op-amp and neglecting the transistor base current, we have

$$V_i = V_{fb} = I_o R_E \quad \text{and} \quad A_{gf} = \frac{I_o}{V_i} = \frac{1}{R_E}$$

Therefore $\beta_z = R_E$



Still, we can take a finite amplifier gain into account:

Assuming the collector and emitter currents are nearly equal and R_i is very large, we have

$$I_o = \frac{V_{fb}}{R_E} = h_{FE} I_b = h_{FE} A_g V_\varepsilon$$

where h_{FE} is the current gain of the transistor

Also $V_\varepsilon = V_i - V_{fb} = V_i - I_o R_E$

Therefore

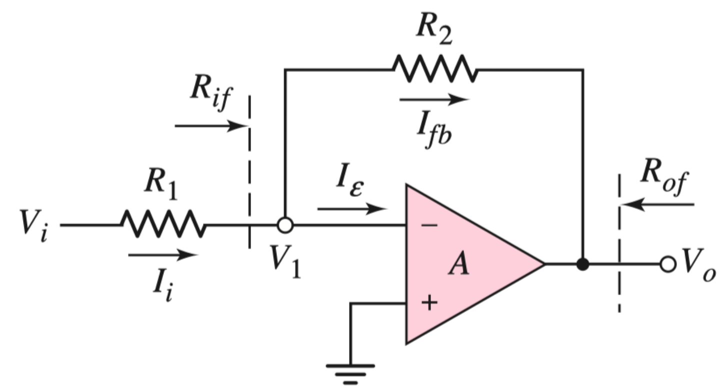
$$I_o = h_{FE} A_g (V_i - I_o R_E)$$

and $$A_{gf} = \frac{I_o}{V_i} = \frac{h_{FE} A_g}{1 + h_{FE} A_g R_E}$$

3) Transresistance Amplifier

An inverting op-amp circuit can also perform like a transresistance amplifier:

- The input signal is I_i
- The feedback current I_{fb}
- The output signal is V_o



For an ideal op-amp,

we have $V_o = -I_{fb}R_2$ and $I_{fb} = I_i$

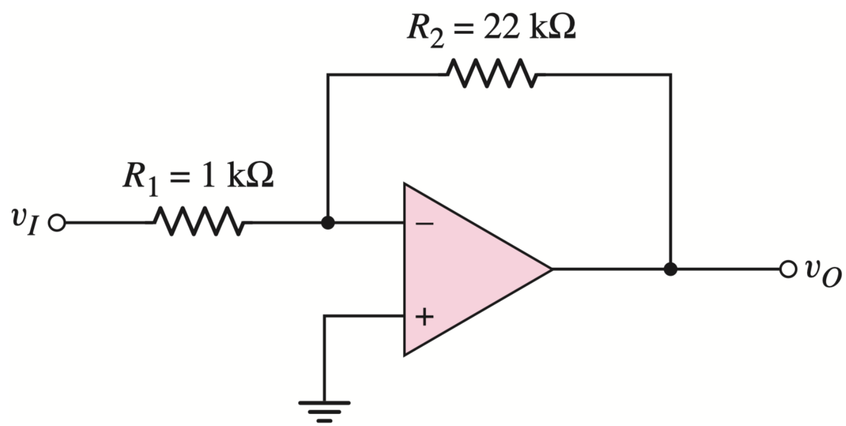
Then $A_{zf} = \frac{V_o}{I_i} = \frac{1}{\beta_g} = -R_2$

So without the need of detailed derivation, if we take a finite amplifier gain into account, we will see:

$$A_{zf} = \frac{V_o}{I_i} = \frac{A_z}{1 + A_z\beta_g}$$

Exercise

Consider the circuit shown below. (a) Determine the ideal output voltage v_O if $v_I = -0.40$ V. (b) Assume the op-amp is ideal except it has a finite open-loop gain. Determine the actual output voltage if the open-loop gain of the op-amp is $A_{od} = 5\times10^3$.



See you in the next lecture...

The End