Uncontrolled rectifier

(Three-phase)

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Outline

- Why 3-phase rectifiers?
- Half-bridge rectifiers
- Full-bridge rectifiers
  - With R load
  - With RL load
  - Harmonic analysis

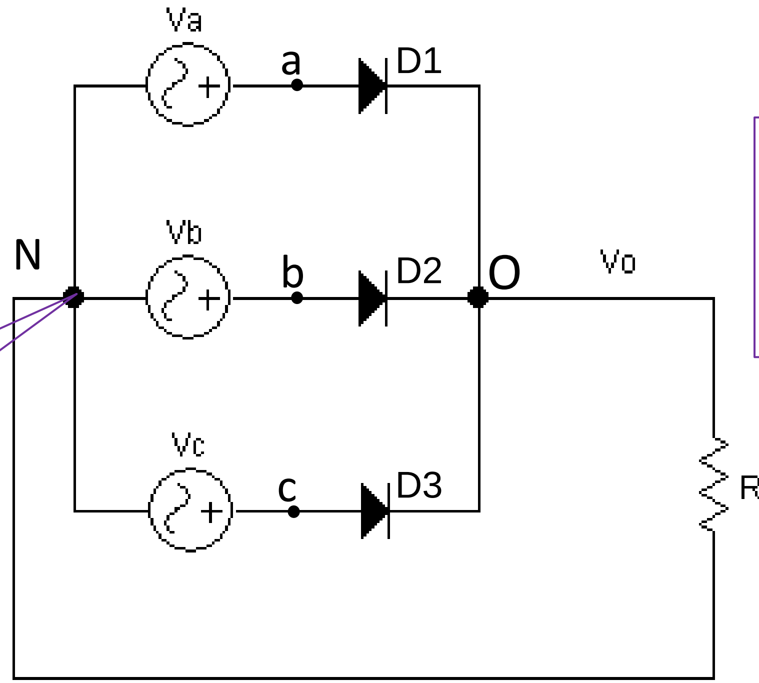
1. Why three-phase rectifier?

- Single phase rectifiers are extensively used in *low-power applications* particularly for power supplies to electronic circuits.
- Single phase rectifiers have several *disadvantages*:
  - Large output voltage and current; form factor;
  - Large low frequency harmonic ripple current causing harmonic power loss and reduced efficiency;
  - Very large filter capacitor for obtaining smooth output dc voltage;
  - Low frequency harmonic current is injected in the input ac line which is difficult to filter. The situation becomes worse with capacitive loads.
- Many of these disadvantages are *mitigated/attenuated* to a large extent using *three-phase rectifiers*.

2. Half-bridge rectifier (Common-Cathode)

Source is star-connected with Va, Vb, Vc (whose magnitudes & frequencies are same with 120 degree phase difference).

Negative ends of the sources are connected together to ground (GND)



Compare v_a v_b and v_c, the highest one will support its corresponding diode to conduct, and stops others from conduction.

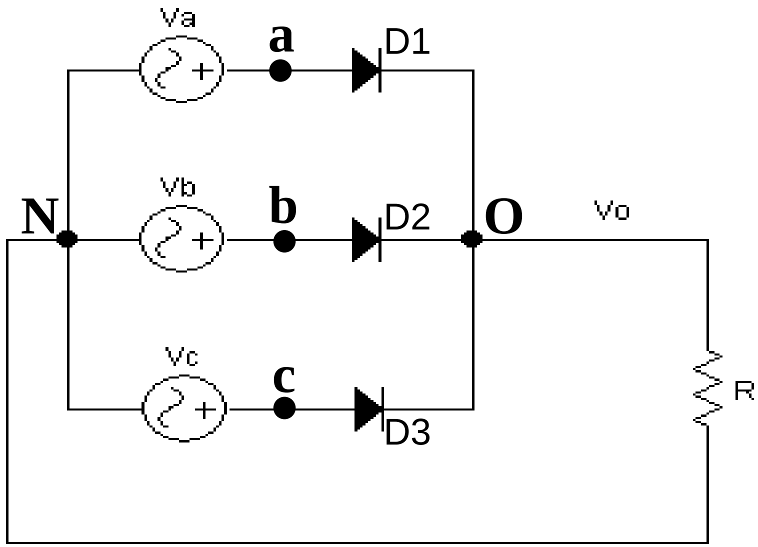
- Questions:
  - What is the waveform for output v_o?
  - What is the PIV (peak inverse voltage) on the diodes?
  - When does each diode conduct? How to determine that?

PIV is defined as the maximum voltage that appears across the diode during non-conduction mode.

Phase voltage (voltage between *a* and N), $v_a = v_{aN} = V_m \sin\omega t$

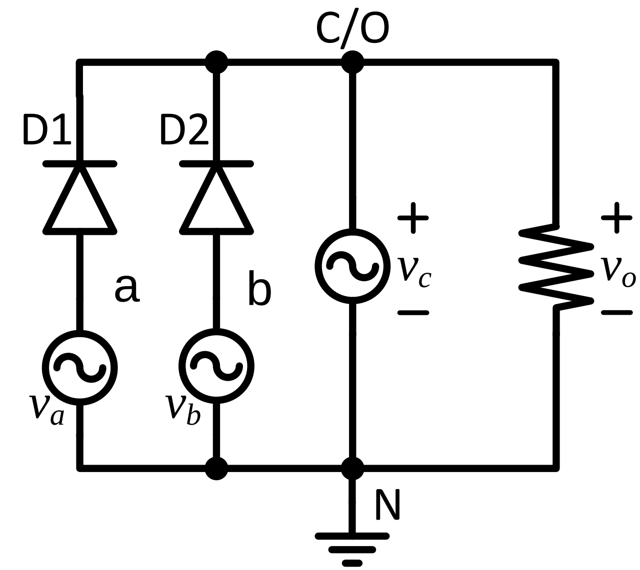
where, V_m is the maximum value of phase voltage.

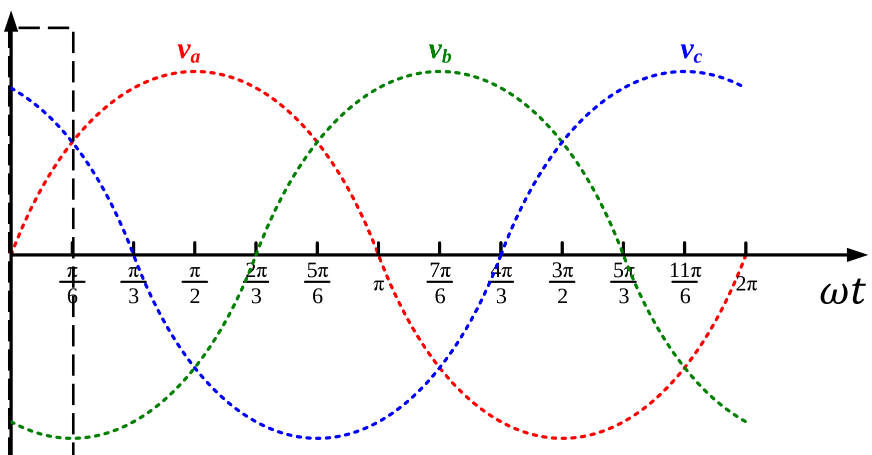
2.1 Operation Principle Analyses



Equivalent circuit

From 0 to π/6



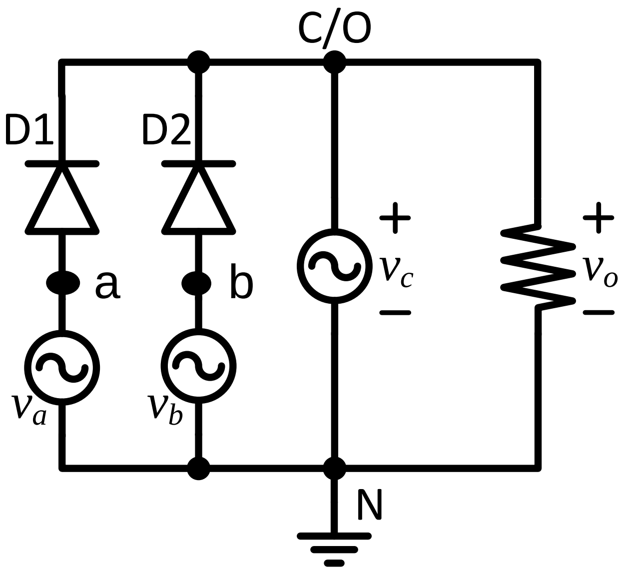


<u>From 0 to 30 degree, π/6:</u>

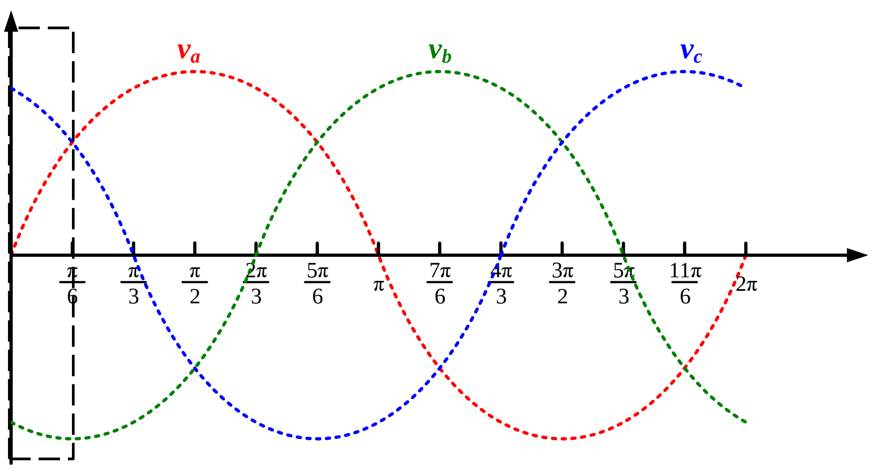
- Blue line (v_c) is the highest one;
- So point 'c' has the highest potential (compared with 'a' and 'b');
- So D3 conducts, D1 and D2 are blocked;

Time-dependent signals of v_a, v_b and v_c.

2.1 Operation Principle Analyses



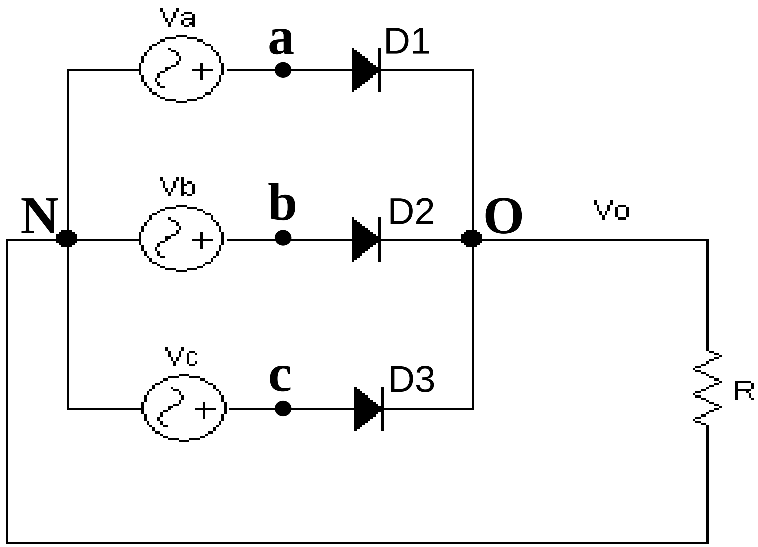
- The potential at point O equals to v_c, so higher than v_a and v_b, so D1 and D2 are reverse-biased and do not conduct.
- During this period, output voltage, v_o equals to v_c.
- Because D3 is conducting, so ideally it can be treated as a wire, i.e,. VD3 is 0;

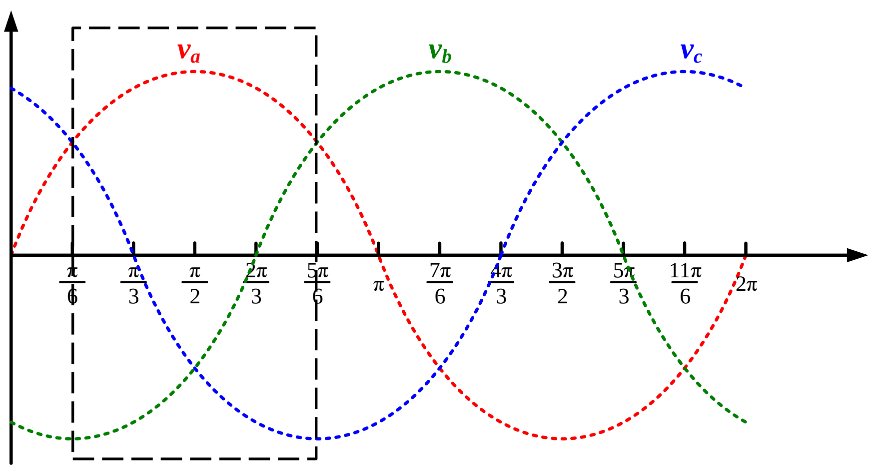


Analysis:

- $v_o = v_c$ (phase voltage);
- Voltage on D3 (V_{D3}) is 0;
- Voltage on D1 (V_{D1}) is $\overrightarrow{V_a} - \overrightarrow{V_c} = \overrightarrow{V_{ac}}$;
- Voltage on D2 (V_{D2}) is $\overrightarrow{V_b} - \overrightarrow{V_c} = \overrightarrow{V_{bc}}$;

2.1 Operation Principle Analyses





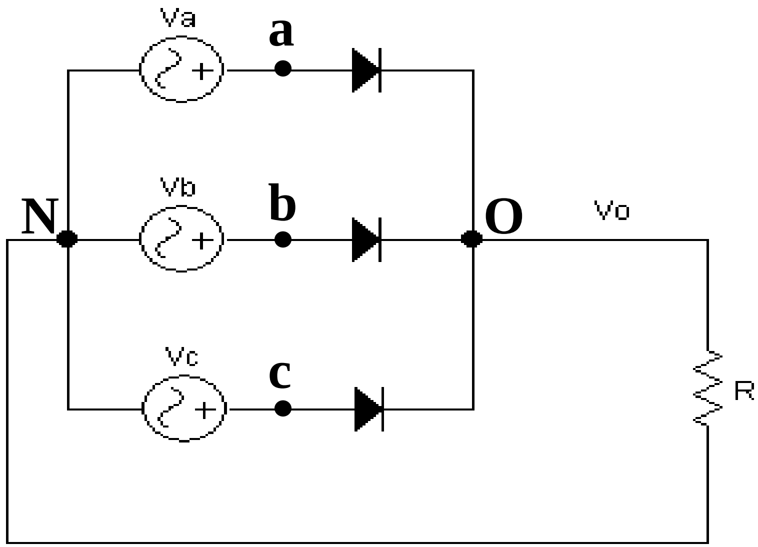
From 30 to 150 degree, π/6 to 5π/6 :

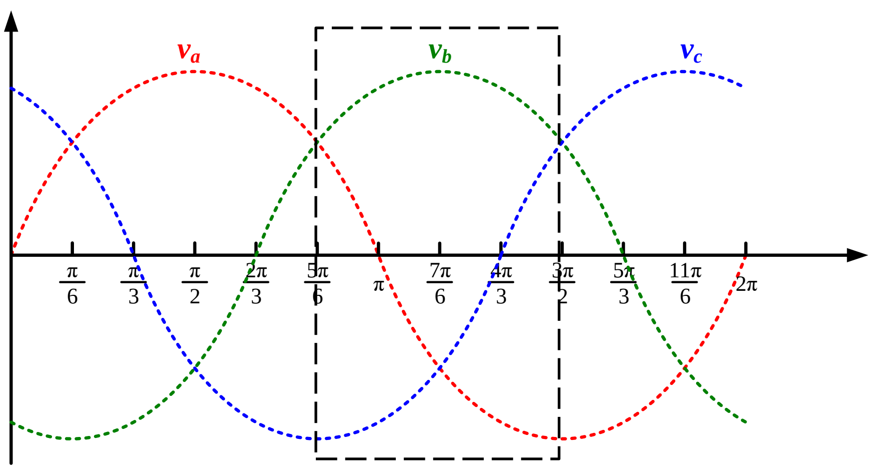
- Red line (v_a) is the highest one;
- So point ‘a’ has the highest potential (compared with ‘b’ and ‘c’);
- So D1 conducts, D2 and D3 are blocked;

Therefore:

- $v_o = v_a$ (phase voltage);
- Voltage on D1 (V_{D1}) is 0;
- Voltage on D2 (V_{D2}) is $\overrightarrow{V_b} - \overrightarrow{V_a} = -\overrightarrow{V_{ab}}$;
- Voltage on D3 (V_{D3}) is $\overrightarrow{V_c} - \overrightarrow{V_a} = -\overrightarrow{V_{ac}}$;

2.1 Operation Principle Analyses





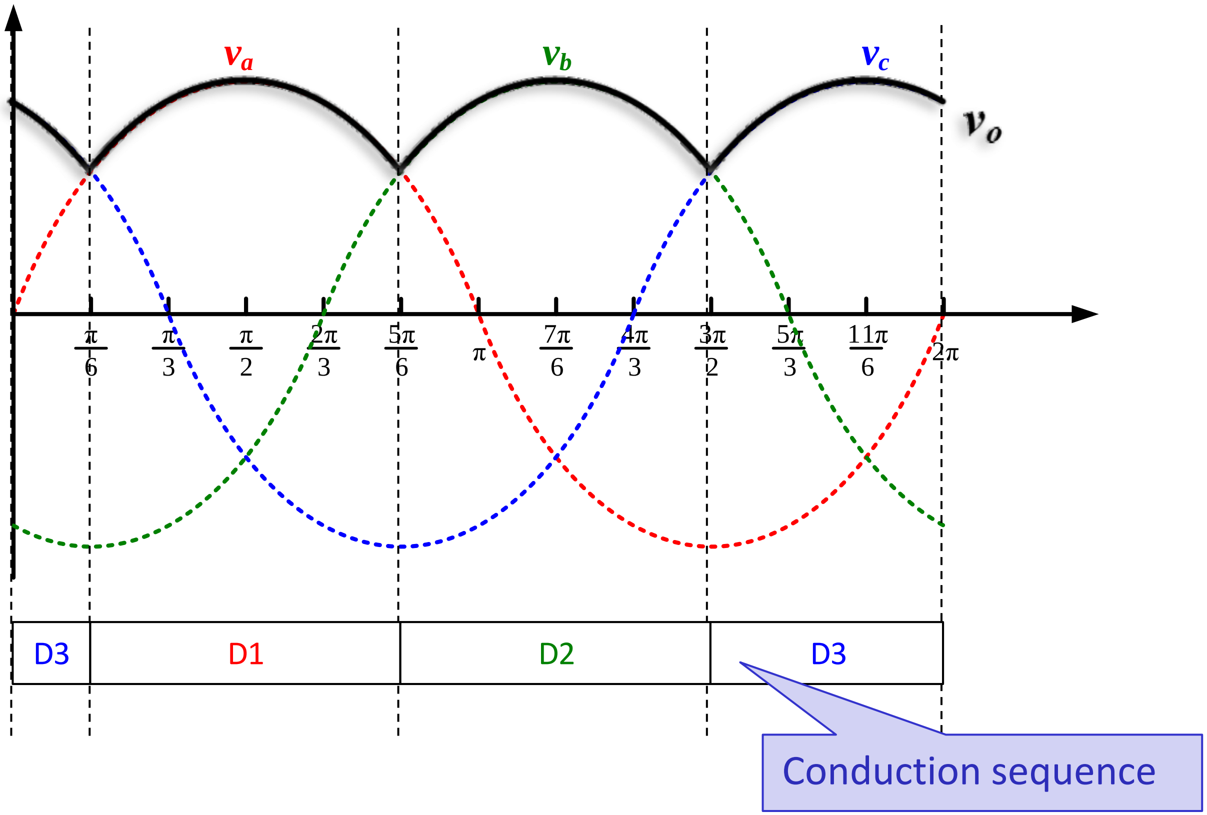
From 150 to 270 degree, 5π/6 to 3π/2 :

- Green line (v_b) is the highest one;
- So point 'b' has the highest potential (compared with 'a' and 'c');
- So D2 conducts, D1 and D3 are blocked;

Therefore:

- $v_o = v_b$ (phase voltage);
- Voltage on D2 (V_{D2}) is 0;
- Voltage on D1 (V_{D1}) is $\overrightarrow{V_a} - \overrightarrow{V_b} = \overrightarrow{V_{ab}}$;
- Voltage on D3 (V_{D3}) is $\overrightarrow{V_c} - \overrightarrow{V_b} = -\overrightarrow{V_{bc}}$;

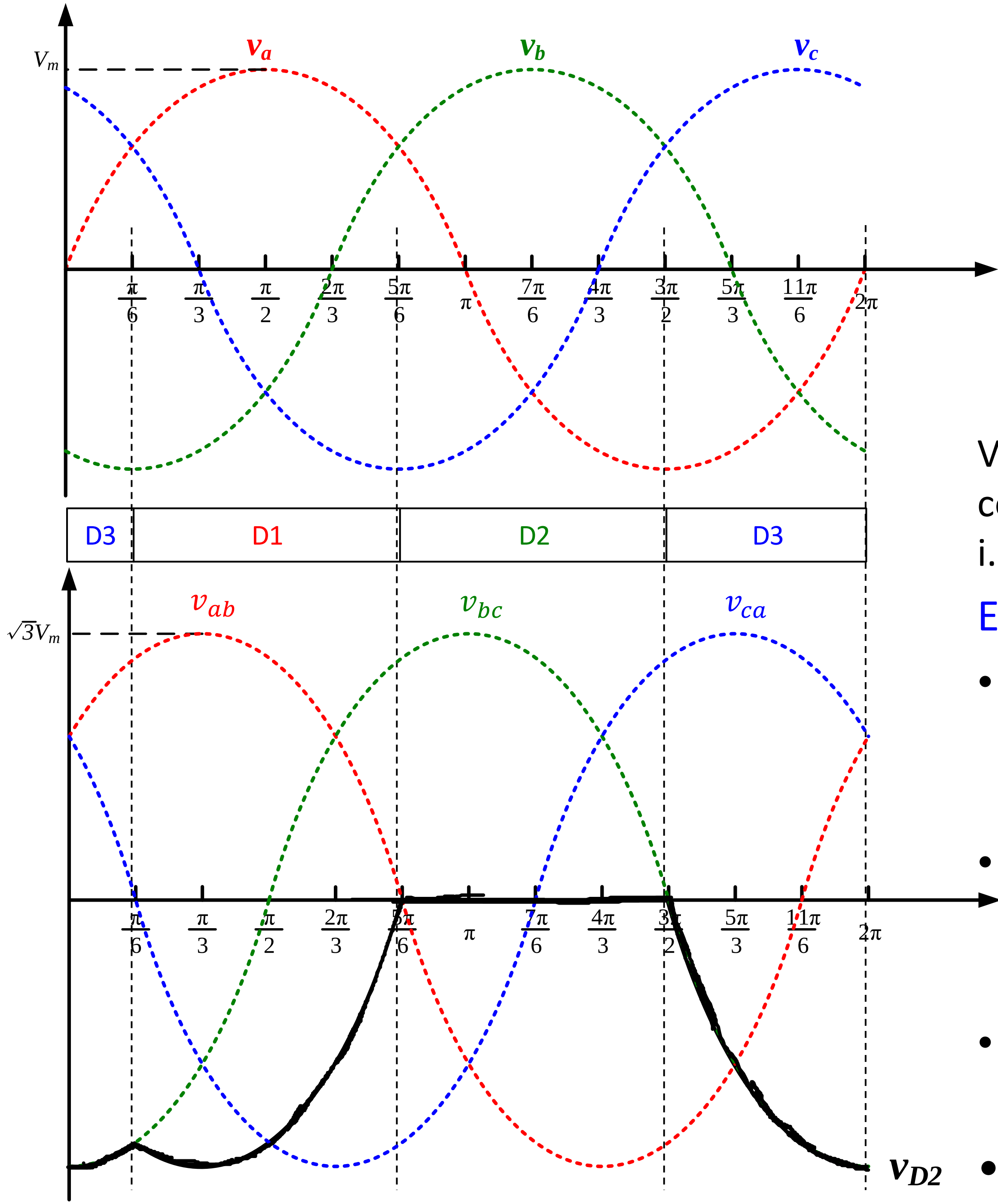
Compare v_a v_b and v_c during the whole period:

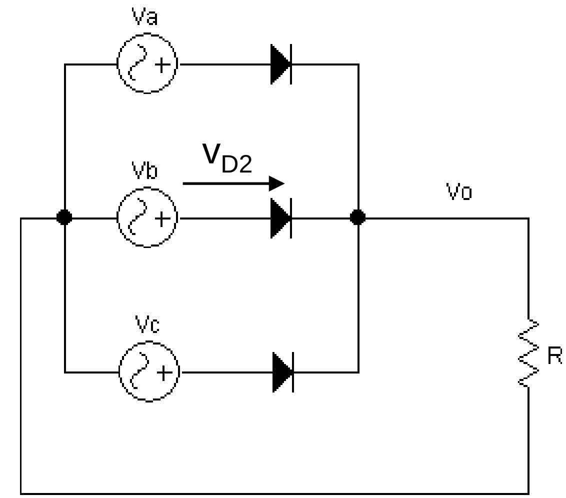


- Summary
  - Each diode conducts 120° in one period.

The output voltage v_o:

- From 0 to π/6, blue line (v_c) is the highest one, so D3 conducts, D1 and D2 are stopped.
  - $v_o = v_c$ (phase voltage);
- From π/6 to 5π/6, red line (v_a) is the highest one, so D1 conducts, D2 and D3 are stopped.
  - $v_o = v_a$;
- From 5π/6 to 3π/2, green line (v_b) is the highest one, so D2 conducts, D1 and D3 are stopped.
  - $v_o = v_b$;
- From 3π/2 to 2π, blue line (v_c) is the highest one again, so D3 conducts again, D1 and D2 are stopped.
  - $v_o = v_c$



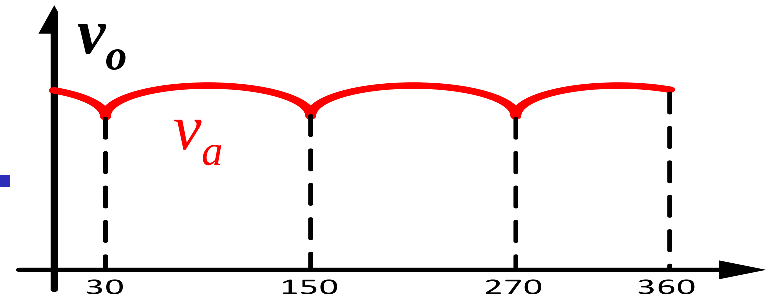


Voltage on diode has 2 possibilities: 1) Diode conducts, i.e., $V_d = 0$ & 2) Not conducting, i.e., $V_d = V_b - V_c$ or $V_b - V_a$) – line voltage

Example – the voltage on diode *D2*

- From 0 to π/6, D3 conducts, D1 and D2 are stopped.
  - $v_{D2} = v_b - v_c = v_{bc}$ (line voltage);
- From π/6 to 5π/6, D1 conducts, D2 and D3 are stopped.
  - $v_{D2} = v_b - v_a = v_{ba} = -v_{ab}$;
- From 5π/6 to 3π/2, D2 conducts.
  - $v_{D2} = 0$;
- PIV is peak value of line voltage $\sqrt{3}V_m$

2.2 Calculation of the key parameters



- 1. Average output voltage

$$V_0 = \frac{1}{T}\int_0^T v_o(t)dt = \frac{1}{2\pi/3} \times \int_{\pi/6}^{5\pi/6} V_m sin\omega t \, d\omega t = 0.83V_m$$

Length of signal repeated in equal intervals of time

- 2. RMS of the output voltage

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v_o^2(t)dt} = \sqrt{\frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} (V_m sin\omega t)^2 d\omega t} = 0.84V_m$$

- 3. Rectification efficiency

$$\eta = \frac{P_0}{P_{RMS}} = \frac{V_0^2}{V_{RMS}^2} \approx 96.7\%$$

Phase voltage,

$$v_a = v_{an} = V_m \sin\omega t$$

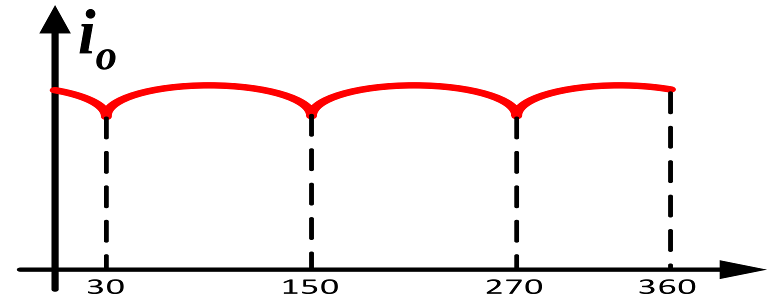
- 4. Ripple factors

$$f_F = \frac{V_{RMS}}{V_0} = 1.01 \quad \Rightarrow \quad f_R = \frac{V_{ac}}{V_0} = \sqrt{f_F^2 - 1} = 0.18$$

2.2 Calculation of the key parameters

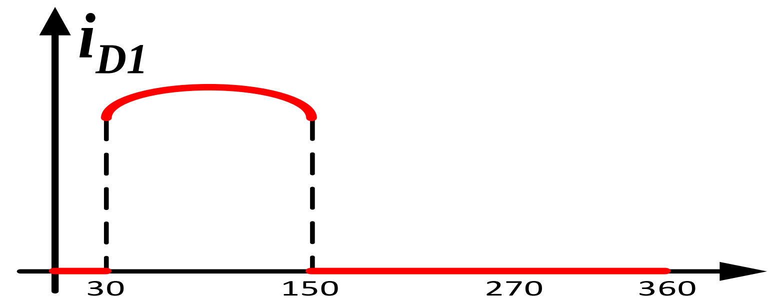
- 5. The average diode current

$$I_{D0} = \frac{I_0}{3} = \frac{V_0}{3R}$$



- 6. The rms output current

$$I_{RMS} = \frac{V_{RMS}}{R}$$



- 7. The rms diode current (line current)
  - The current flowing through single diode
  - also equals to the input current for each phase

$$I_D = I_S = \frac{I_{RMS}}{\sqrt{3}} = \frac{V_{RMS}}{\sqrt{3}R} = 0.48\frac{V_m}{R}$$

2.2 Calculation of the key parameters

- 8. Transformer utilisation factor (TUF)

$$f_T = \frac{P_0}{P_S} = \frac{V_0^2/R}{3V_s I_s} = \frac{V_0^2/R}{3\left(V_m/\sqrt{2}\right)(0.48V_m/R)} = 0.66$$

- 9. Power factor $\boxed{f_P = \frac{V_s I_{s1}}{V_s I_s} cos\phi}$

$$f_P = \frac{P_{RMS}}{P_S} = \frac{V_{RMS}^2/R}{3V_s I_s} = \frac{V_{RMS}^2/R}{3\left(V_m/\sqrt{2}\right)(0.48V_m/R)} = 0.68$$

- 10. Peak inverse voltage of each diode is equal to the peak value of the line voltage, which is

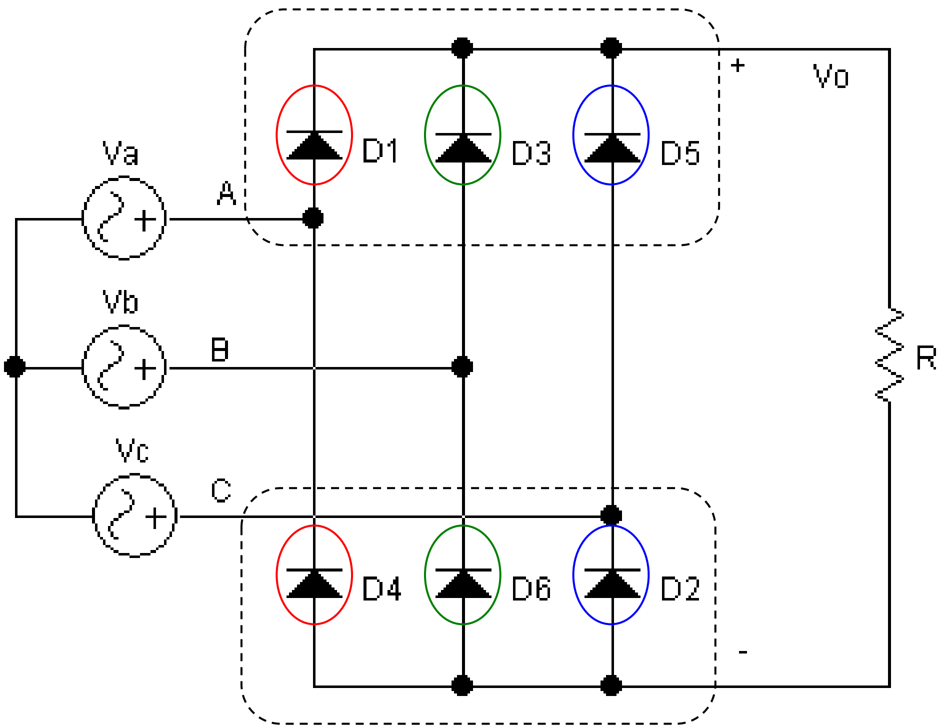
$$PIV = \sqrt{3}V_m$$

Comparison to Single-phase rectifiers

	Single Phase Half-wave	Single Phase Full-wave	Three Phase Half-wave	
V_0	$0.318V_m$	$0.637V_m$	$0.83V_m$	
V_{RMS}	$0.5V_m$	$0.707V_m$	$0.84V_m$	
η	40.5%	81%	96.7%	
f_F	1.57	1.11	1.01	
f_R	1.21	0.482	0.18	
f_C	2	1.414	1.19	
f_P	0.707	1	0.68	
f_T	28.7%	81%	66%	
PIV	V_m	V_m	$\sqrt{3}V_m$	

3.1 Full bridge rectifier

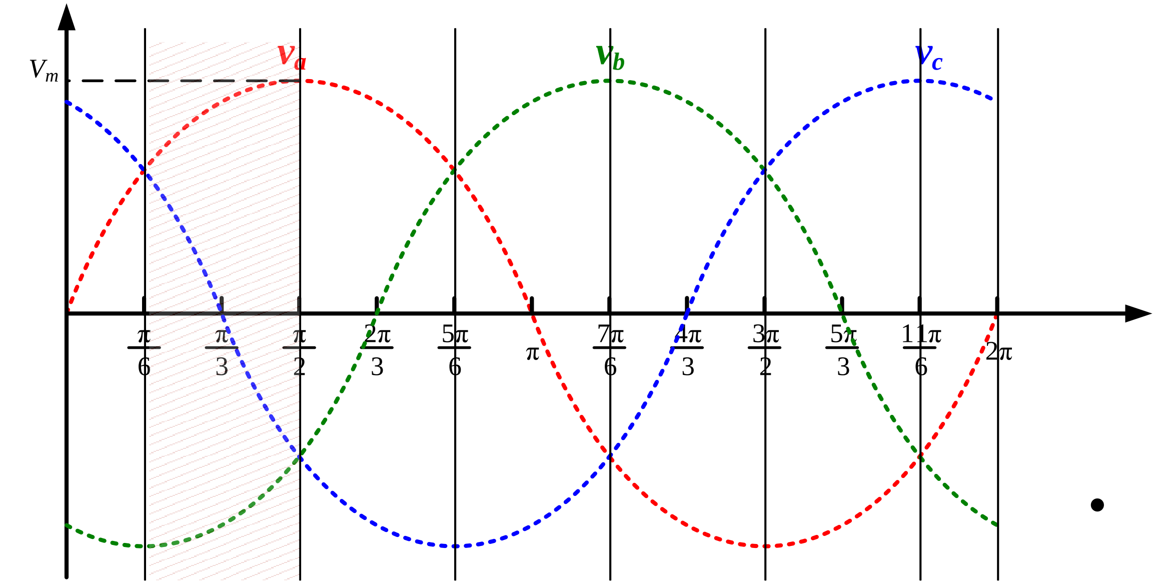
Top/positive group



Bottom/negative group

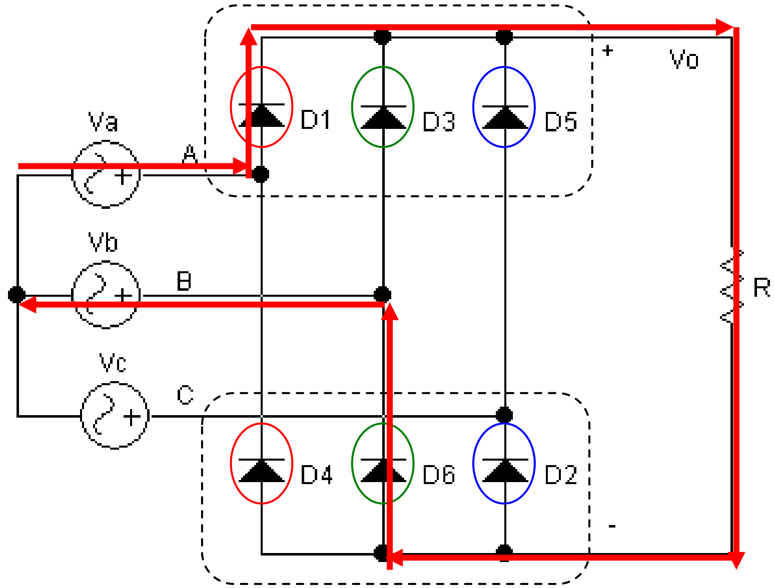
- For full-wave circuit, *6 diodes* are used and arranged in three legs – each leg has two series-connected diodes.
- <u>Top group</u>: Diodes D1, D3, and D4.
- <u>Bottom group</u>: Diodes D4, D6, and D2.
- Note that *diodes are not in the sequence*.
- To form a conduction loop, there always are two diodes conducting, *one from top group and one from bottom group*:
  - Top group: the one has the highest anode voltage will conduct;
  - Bottom group: the one has the lowest cathode voltage will conduct;

3.1.1 Operation Analyses



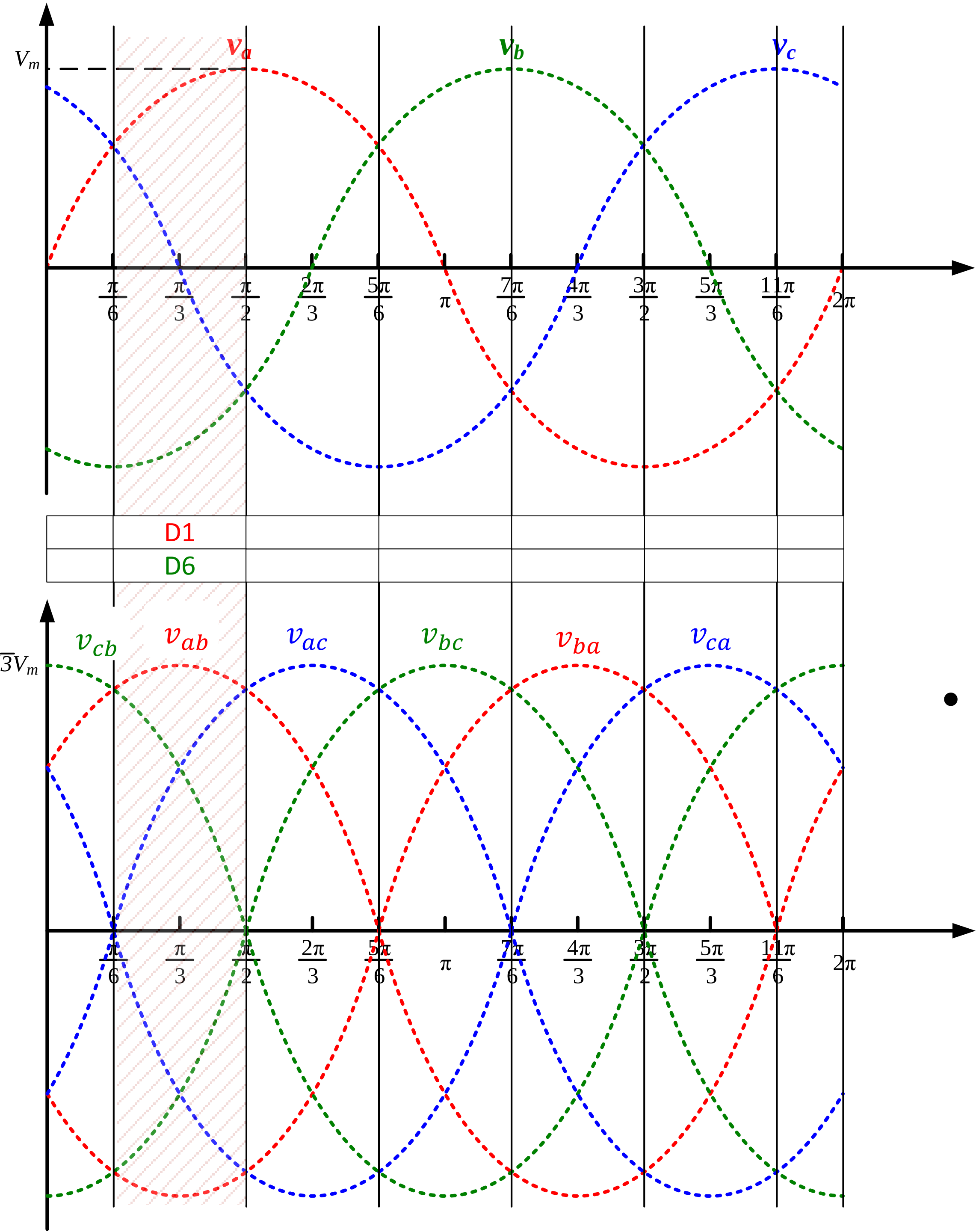
TOP		D1					
BOT		D6					

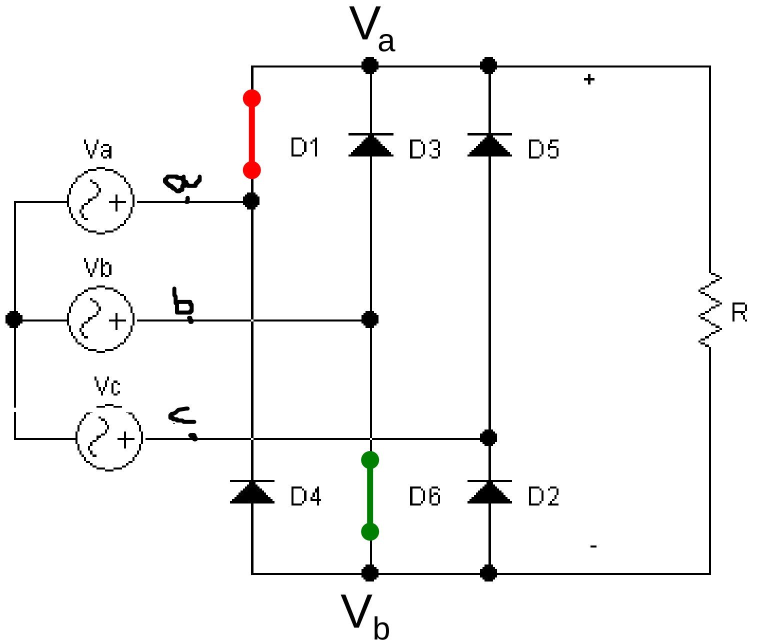
Conduction sequence table



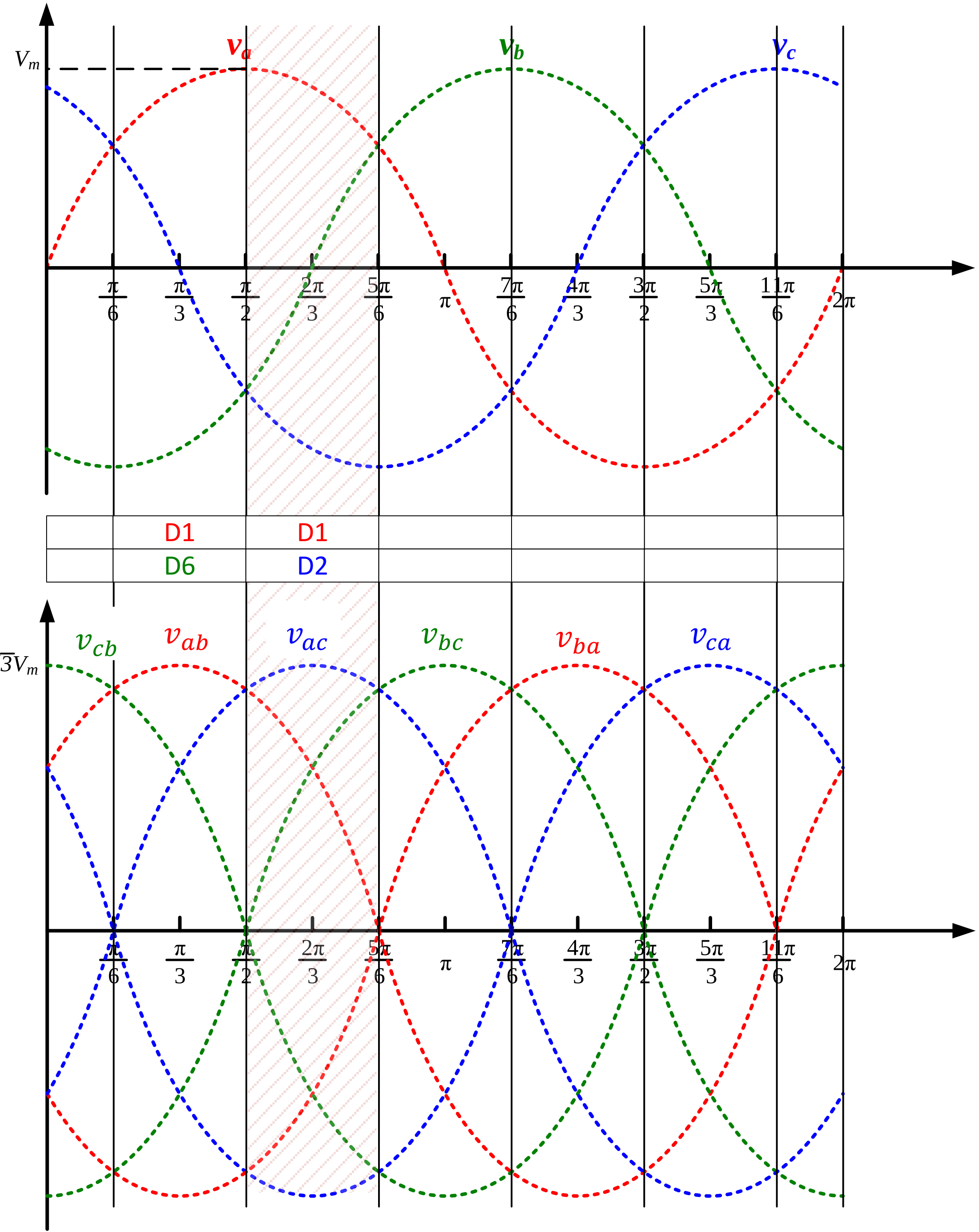
- From π/6 to π/2, v_a is more positive and v_b is more negative.
  - So the top-group diode connected to v_a (D1) and the bottom-group diode connected to v_b (D6) are conducting;

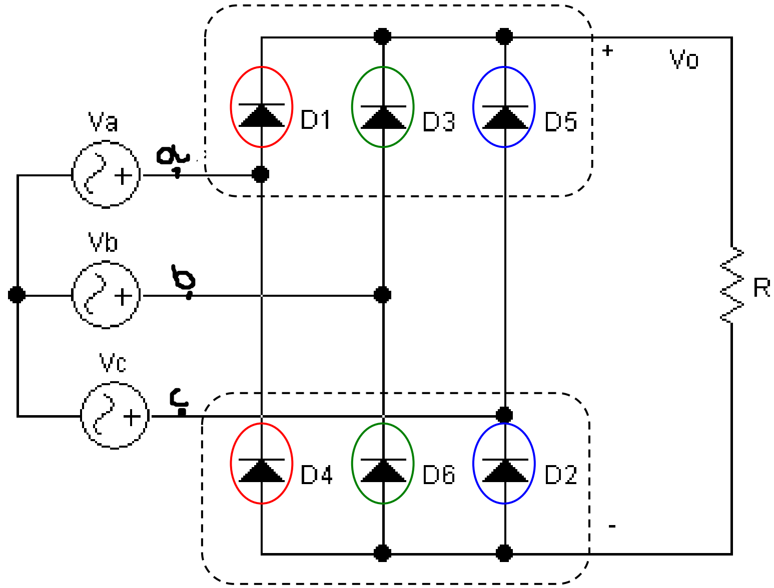
Any crossing point means the exchange of highest or lowest voltage.



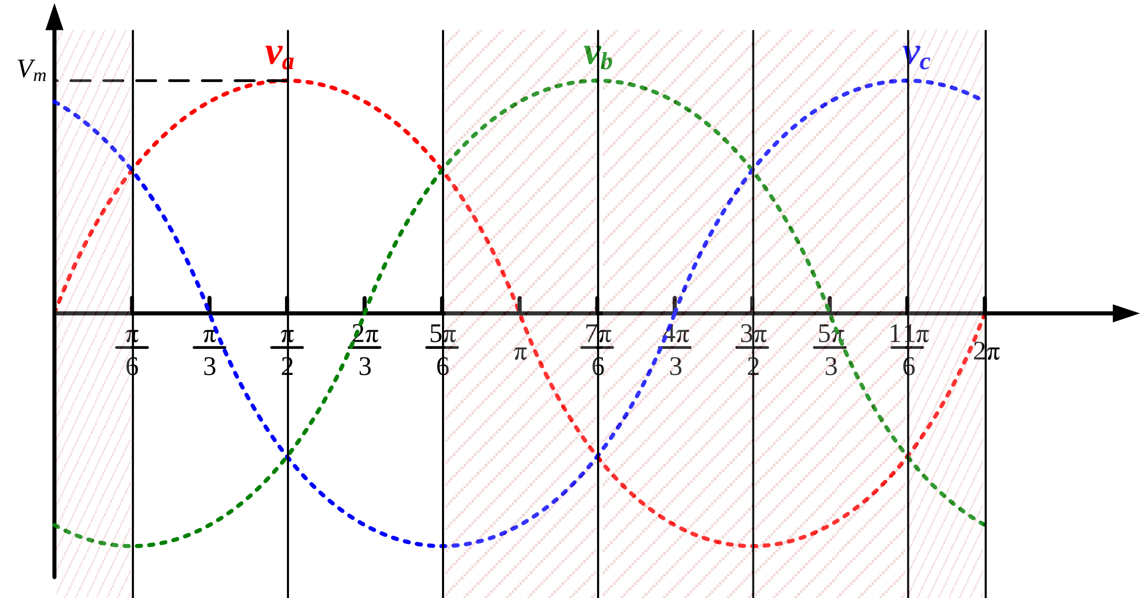


- From π/6 to π/2, v_a is more positive and v_b is more negative.
  - So the top-group diode connected to v_a (D1) and the bottom-group diode connected to v_b (D6) are conducting;
  - The output voltage $v_o = v_a - v_b = v_{ab}$, the line voltage;

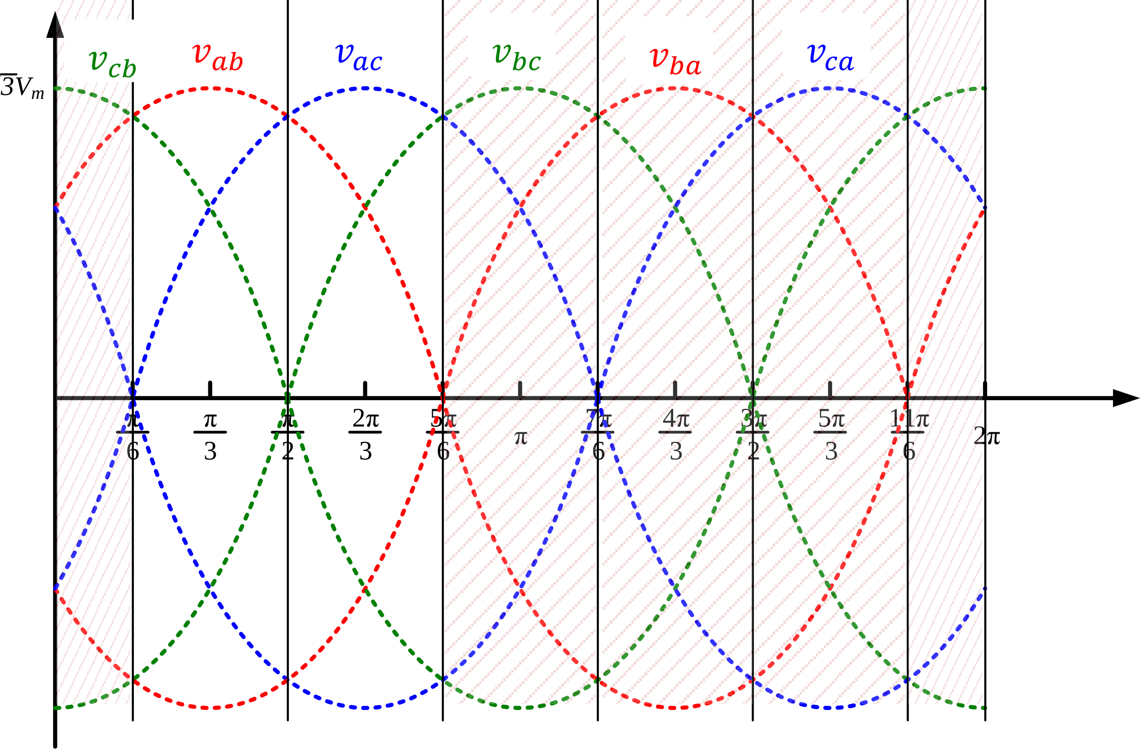


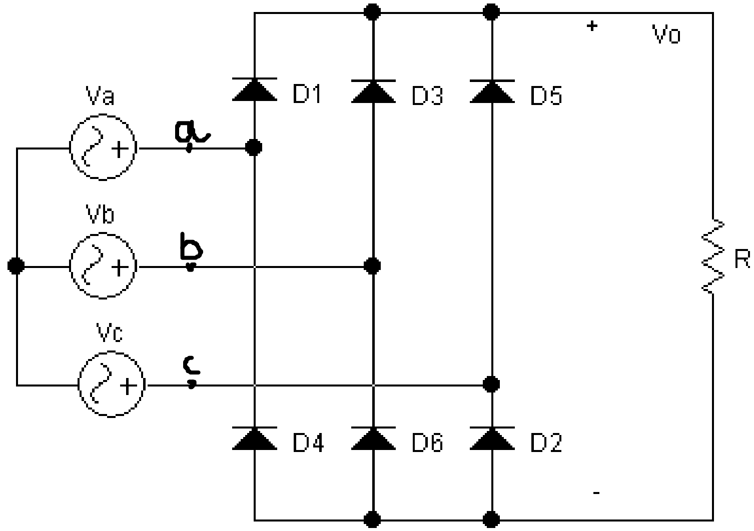


- From π/2 to 5π/6, v_a is still more positive and v_c is the more negative now.
  - So the top-group diode connected to v_a (D1) and the bottom-group diode connected to v_c (D2) are conducting;
  - The output voltage $v_o = v_a - v_c = v_{ac}$, the line voltage;



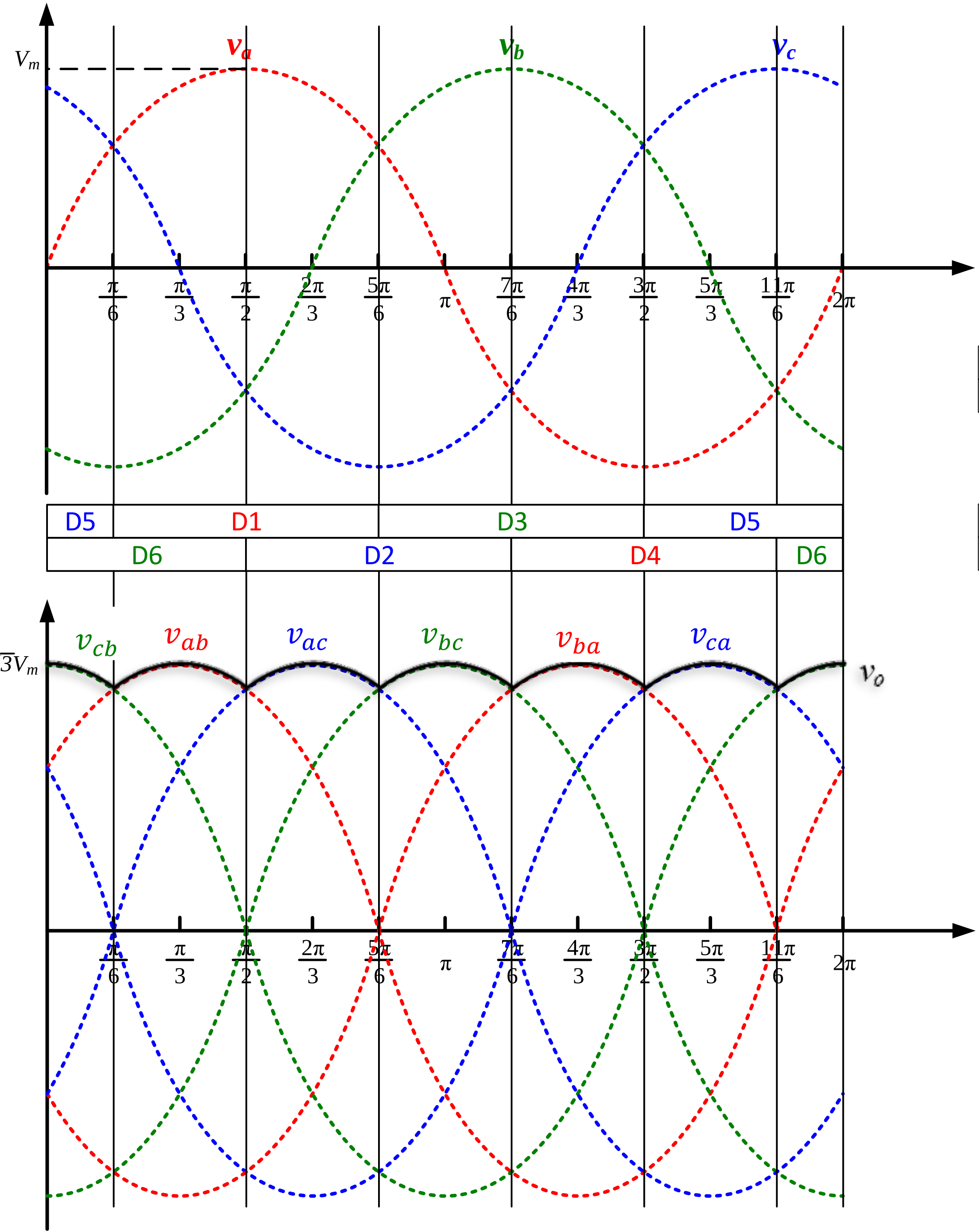
D5	D1	D1	D3	D3	D5	D5
D6	D6	D2	D2	D4	D4	D6





Keep moving the window:

- 5π/6 to 7π/6
  - D3 and D2 conduct, $v_o = v_{bc}$
- 7π/6 to 3π/2
  - D3 and D4 conduct, $v_o = v_{ba}$
- 3π/2 to 11π/6
  - D5 and D4 conduct, $v_o = v_{ca}$
- 11π/6 to 13π/6
  - D5 and D6 conduct, $v_o = v_{cb}$



- Conduction sequence (Sequence of commutation)

60°

D5	D1	D1	D3	D3	D5	D5
D6	D6	D2	D2	D4	D4	D6

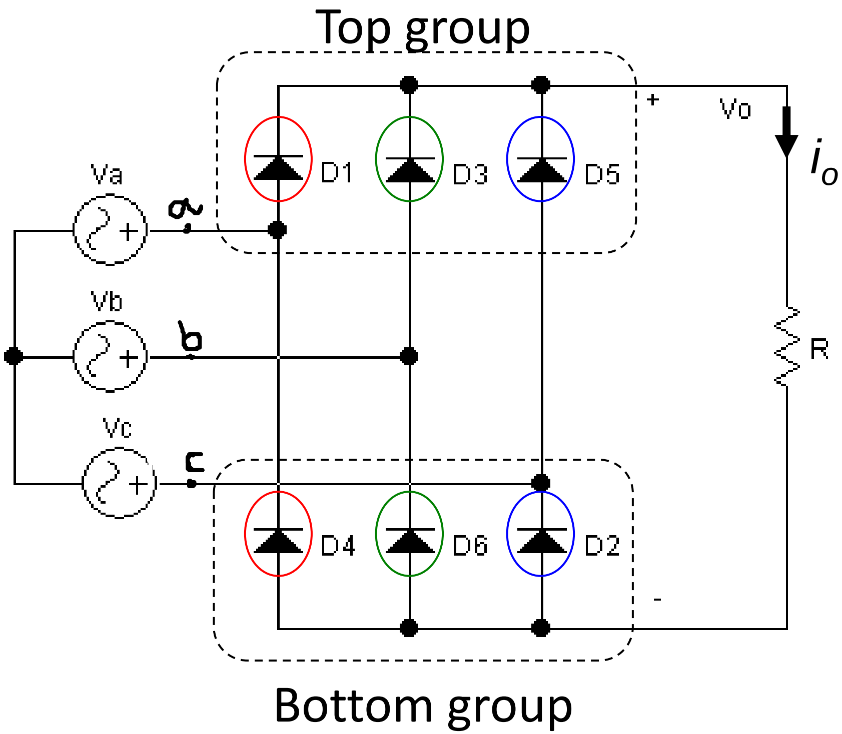
120°

D5	D1		D3		D5	
D6		D2		D4		D6

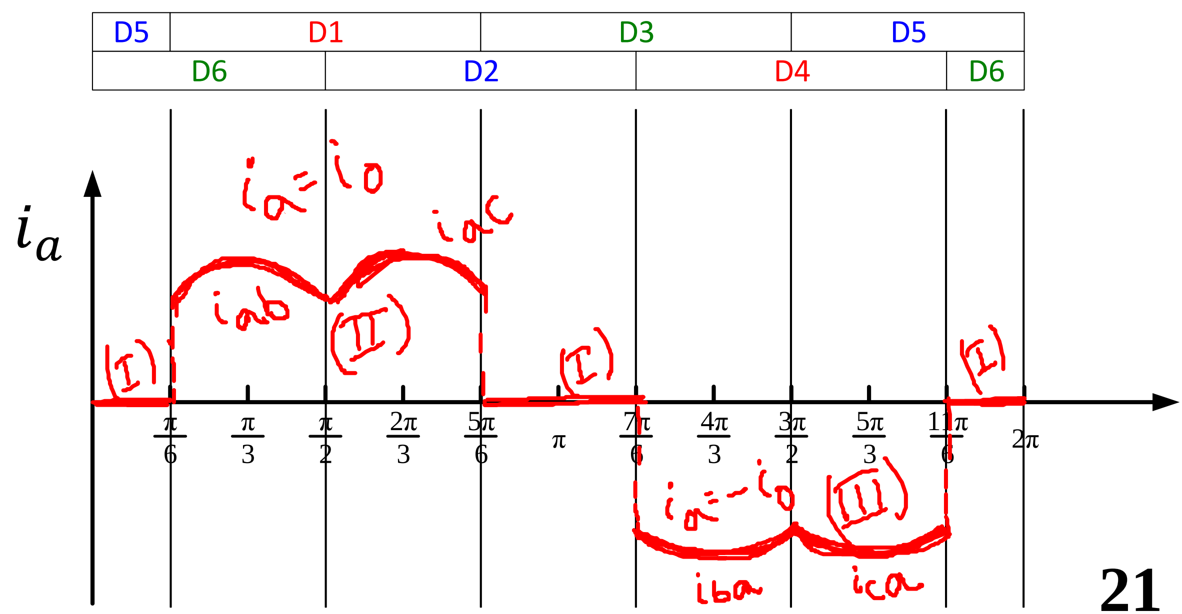
56	16	12	23	34	45	56

- Each diode conducts 120°, but in two slots.
- Output voltage: 6-pulse ripple.

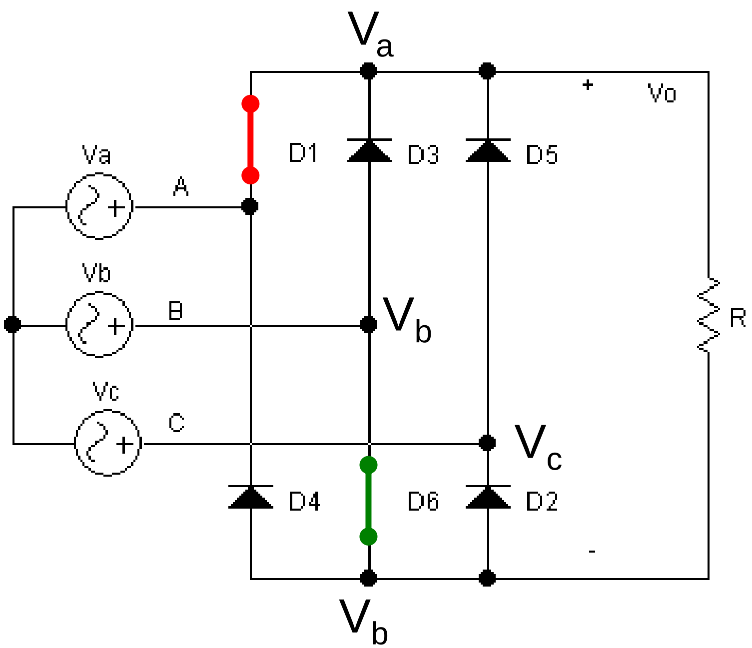
3.1.2 Terminal/line current



- Terminal ‘a’ as an example:
  - D1 and D4 are connected to v_a;
  - When both D1 and D4 are blocked, i_a (I) is zero;
  - (II) When D1 conducts, i_a is the same as i_o;
  - (III) When D4 conducts, i_a equals to $-i_o$.



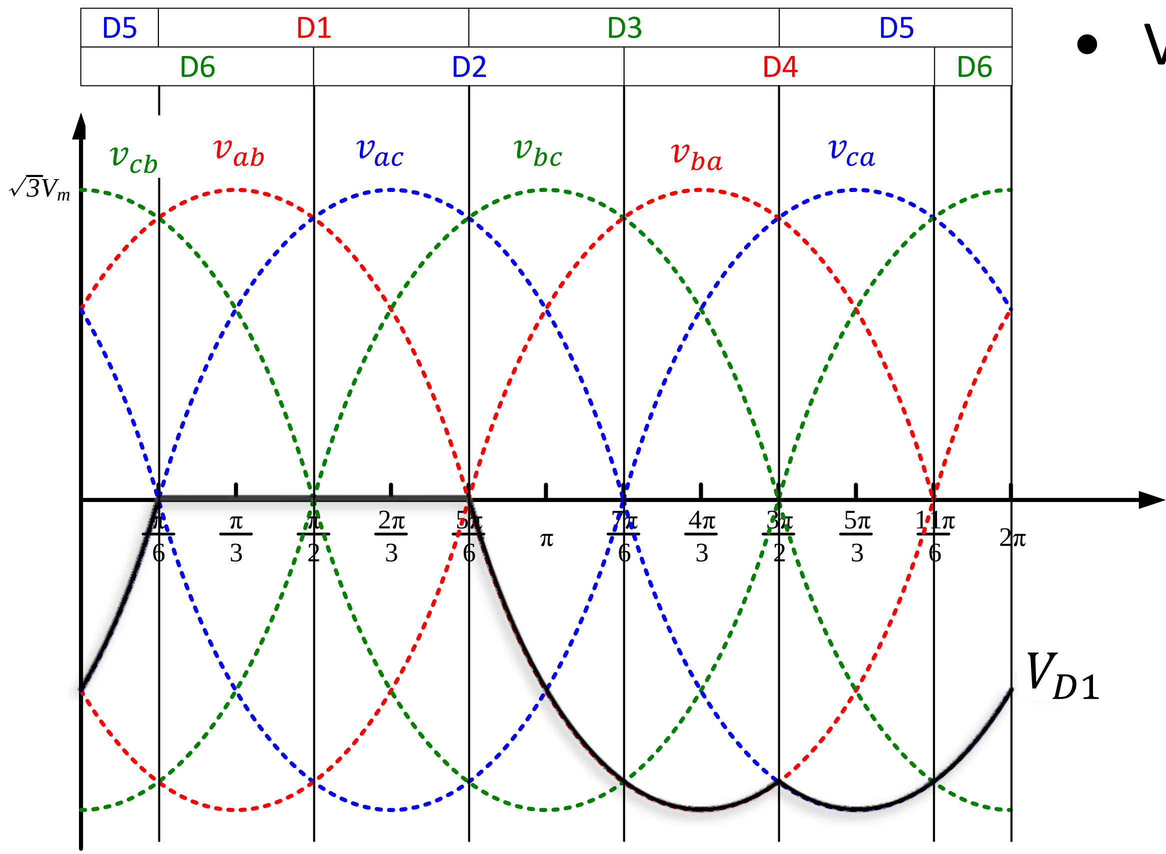
3.1.3 Diode voltage



- $(\pi/6, \pi/2)$, D1 and D6 are conducting;
  - $V_{D1} = 0$ (since D1 is conducting);
  - $V_{D3} = v_b - v_a = v_{ba}$;
  - $V_{D5} = v_c - v_a = v_{ca}$;
  - $V_{D6} = 0$ (since D6 is conducting);
  - $V_{D2} = v_b - v_c = v_{bc}$;
  - $V_{D4} = v_b - v_a = v_{ba}$;

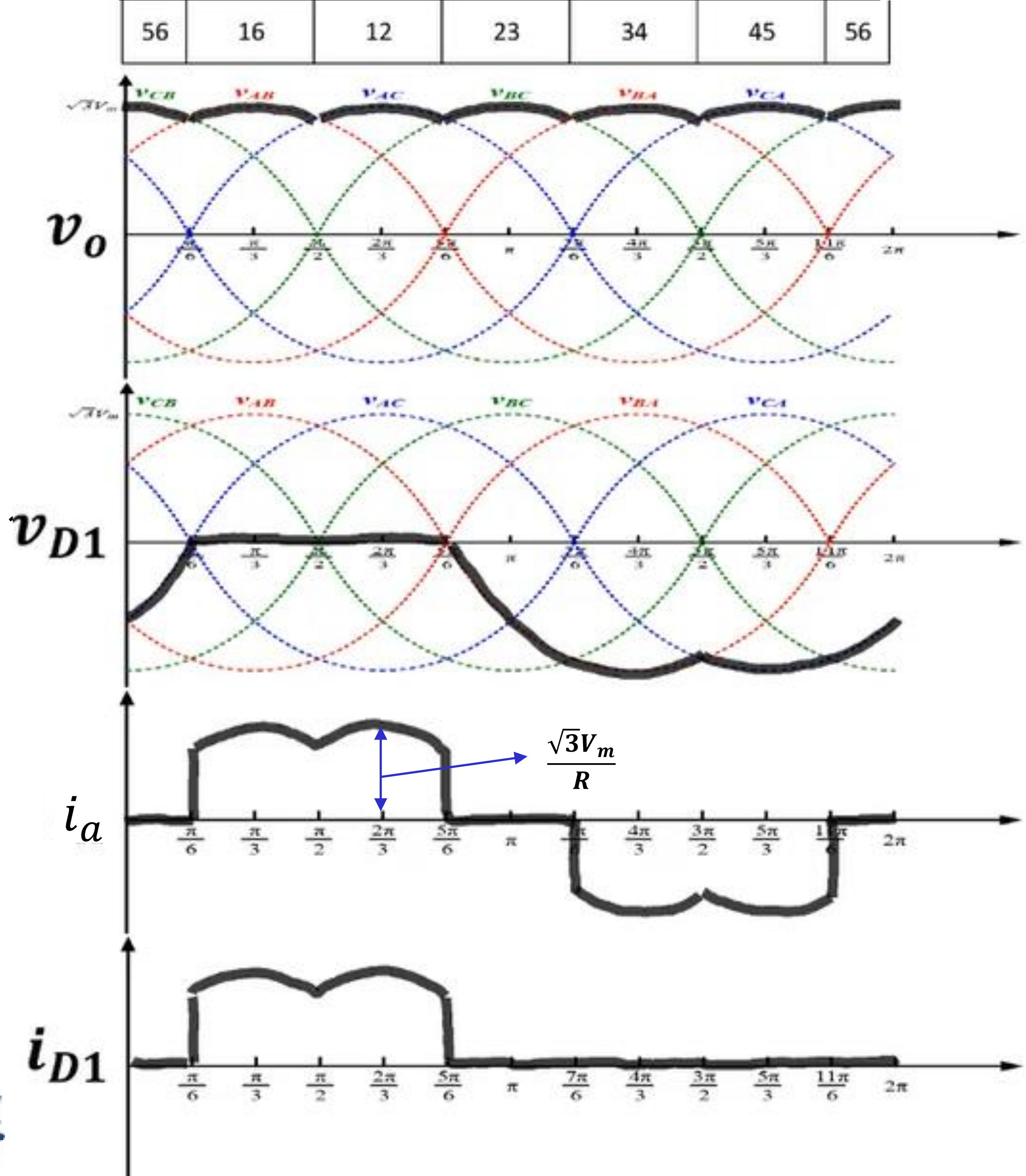
3.1.3 Diode voltage

- Take diode 1 as an example



- V_{D1} on the whole period (0, 2π):
  - (π/6, 5π/6), $V_{D1} = 0$ (since D1 is conducting);
  - (5π/6, 3π/2), D3 conducts, D1 stops. So $V_{D1} = v_a - v_b = v_{ab}$;
  - (3π/2, 2π) and (0, π/6), D5 conducts, D1 stops. So $V_{D1} = v_a - v_c = v_{ac}$;
  - The peak inverse voltage is the peak value of line voltage, so it is $\sqrt{3}V_m$

Important Waveforms



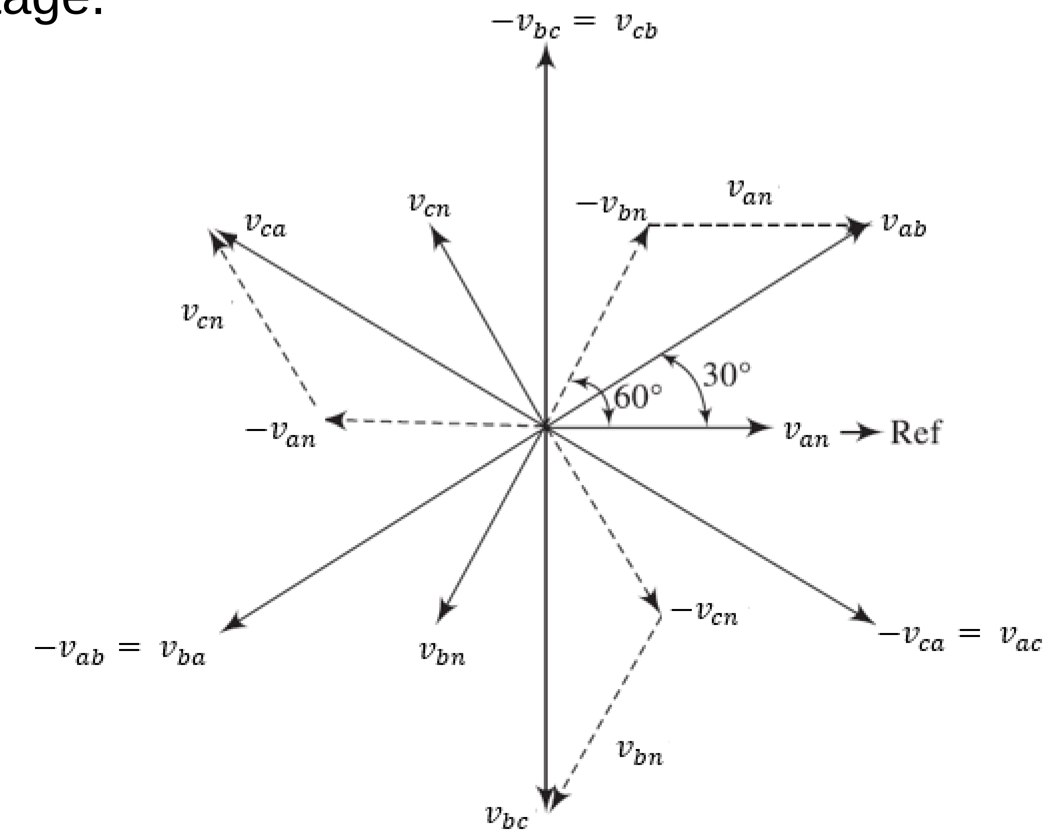
Calculation of the key parameters – Notes

- V_m is the peak value of the phase voltage.

$$v_{an} = V_m \sin\omega t$$
$$v_{bn} = V_m \sin(\omega t - 120^0)$$
$$v_{cn} = V_m \sin(\omega t - 240^0)$$

- Line to line voltages lead the phase voltage by 30^0.

$$v_{ab} = \sqrt{3} V_m \sin(\omega t + 30^0)$$
$$v_{bc} = \sqrt{3} V_m \sin(\omega t - 90^0)$$
$$v_{ca} = \sqrt{3} V_m \sin(\omega t - 210^0)$$
$$\cong V_{ml} \sin(\omega t - 210^0)$$



Calculation of the key parameters I

- 1. Phase A voltage, $v_a = v_{an} = V_m \sin\omega t$
- 2. Line voltage, $v_{ab} = \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6})$
- 3. The average voltage

Maximum value of line voltage = $\sqrt{3}$ × maximum value of phase voltage

$$V_0 = \frac{1}{\pi/3}\int_{\pi/6}^{\pi/2} v_{ab}\, d\omega t = \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3}V_m \sin(\omega t + \frac{\pi}{6})\, d\omega t$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left(cos\frac{\pi}{3} - cos\frac{2\pi}{3}\right)$$

$$\boxed{= \frac{3\sqrt{3}V_m}{\pi}}$$

$$= \frac{3\sqrt{3}\times\sqrt{2}\times V_S}{\pi} \approx 1.654V_m \approx 2.34V_S$$

where V_s is the rms value of per-phase supply voltage;

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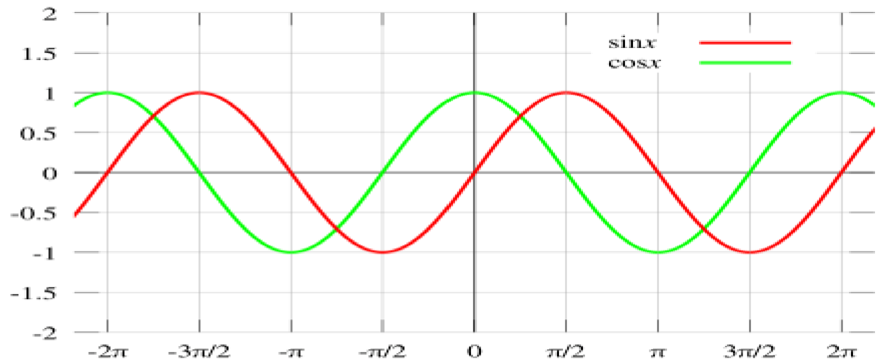
Calculation of the key parameters I



Note: *The average voltage can also be obtained as*

(i) Take any sinusoidal wave and integrate it from $\pi/3$ to $2\pi/3$. It is because the voltage pulse area required extends from $\omega t = \pi/3$ to $2\pi/3$ for the $\sin \omega t$ function.

$$V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} \sqrt{3}V_m \sin \omega t \, d\omega t = \frac{3\sqrt{3}V_m}{\pi}$$

(ii) For a cosine function $\cos \omega t$, voltage pulse of 60^0 duration extends $\pi/6$ to the left of its peak and $\pi/6$ to the right of its peak.

$$V_0 = \frac{1}{\pi/3}\int_{-\pi/6}^{\pi/6} \sqrt{3}V_m \cos \omega t \, d\omega t = \frac{3\sqrt{3}V_m}{\pi}$$

Calculation of the key parameters II

- 4. The RMS voltage

$$V_{RMS} = \sqrt{\frac{3}{\pi}\int_{\pi/6}^{\pi/2} v_{ab}^2 d\omega t} = 3V_m\sqrt{\frac{1}{\pi}\int_{\pi/6}^{\pi/2} sin^2\left(\omega t + \frac{\pi}{6}\right) d\omega t}$$

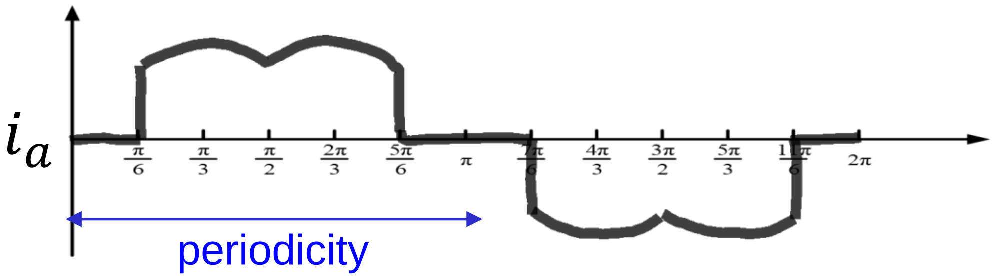
$$= 3V_m\sqrt{\frac{1}{\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} \approx 1.655V_m$$

$$\approx 1.0009V_0$$

  - This is almost the same as V_0.
  - Hence, the efficiency is high.
- 5. The rectification efficiency

$$\eta = \frac{V_0^2/R}{V_{RMS}^2/R} = \frac{V_0^2}{V_{RMS}^2} \approx 99.8\%$$

Calculation of the key parameters III



This current has two pulses, each of 60^0 duration, for each periodicity of π.

- 6. The rms value of the terminal/line current (R load) is

$$I_{L,RMS} = \sqrt{\frac{2}{\pi}\int_{\pi/6}^{\pi/2}\left(\frac{\sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)}{R}\right)^2 d\omega t} = \frac{V_m}{R}\sqrt{\frac{2}{\pi}\int_{\pi/6}^{\pi/2} 3sin^2\left(\omega t + \frac{\pi}{6}\right) d\omega t}$$

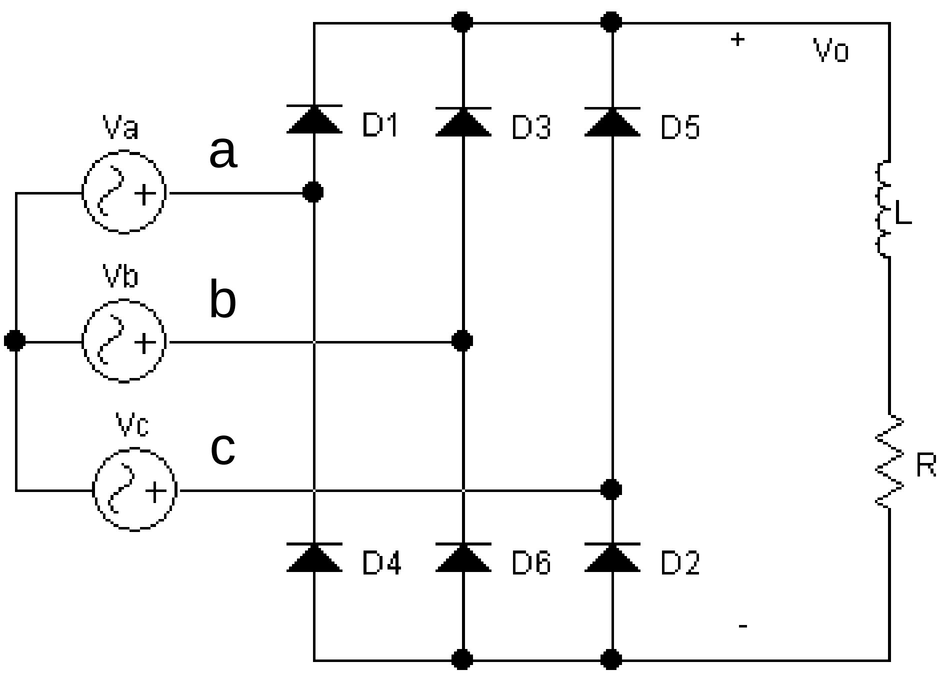
$$= \frac{\sqrt{3}V_m}{R}\sqrt{\frac{2}{\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} = \frac{1.35V_m}{R} \approx 0.78I_{ml}$$

where $I_{ml} = \sqrt{3}V_m/R$ is the peak line current (the same as the peak current through a diode).

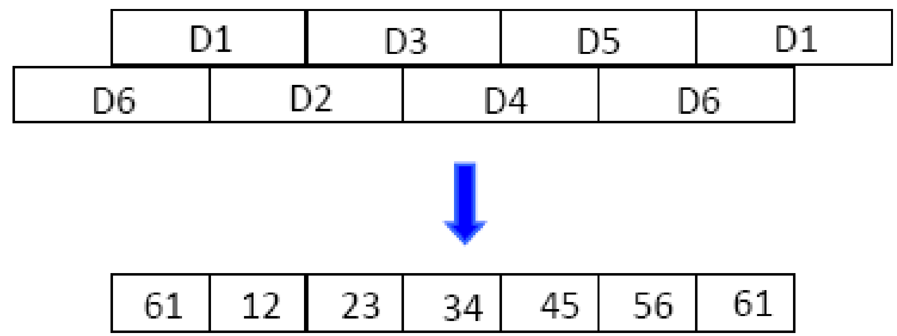
Comparison

	Single Phase Half-wave	Single Phase Full-wave	Three Phase Half-wave	Three Phase Full-wave
V_0	$0.318V_m$	$0.637V_m$	$0.83V_m$	$1.65V_m$
V_{RMS}	$0.5V_m$	$0.707V_m$	$0.84V_m$	$1.66V_m$
η	40.5%	81%	96.7%	99.9%
f_F	1.57	1.11	1.01	1.002
f_R	1.21	0.482	0.18	0.06
f_C	2	1.414	1.19	1.04
f_P	0.707	1	0.68	0.96
f_T	28.7%	81%	66%	94.7%
PIV	V_m	V_m	$\sqrt{3}V_m$	$\sqrt{3}V_m$

3.2 Full bridge rectifier with RL load



- Sequence of commutation

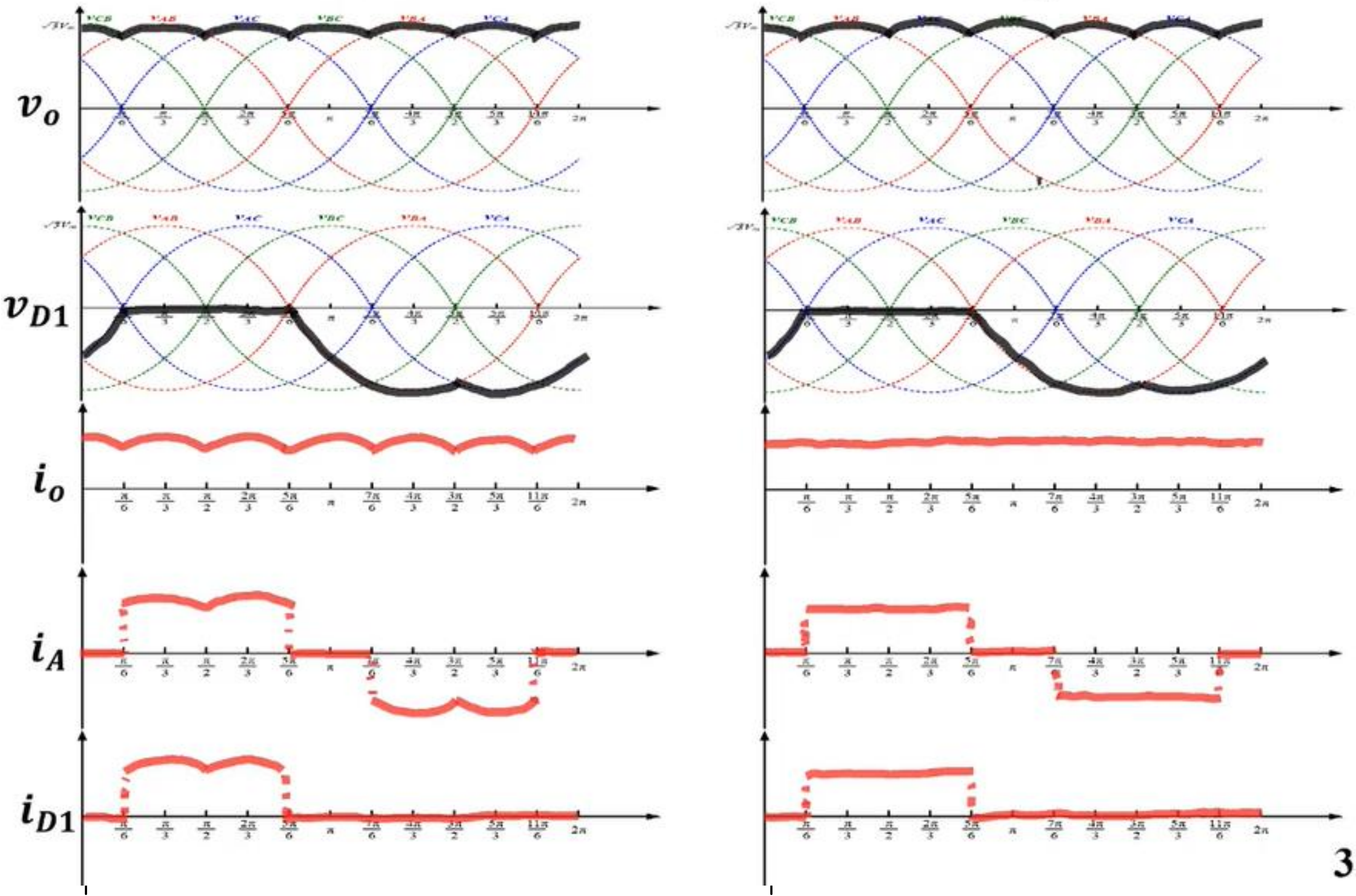


- An inductor is often used to smooth the load current.

- When the inductor is large enough, the current is a straight line.

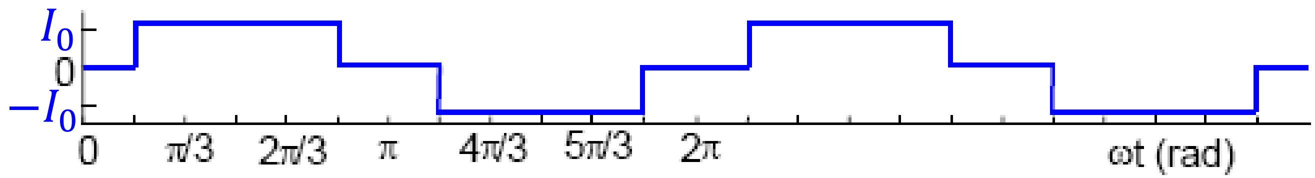
Small L

Large L



3.2.1 Harmonic analysis I

- Line current



- When the inductor is large enough, the ripple in the input current is negligible. The line/source current (phase A) can be described by

$$i_a(t) = \begin{cases} I_0 & \frac{\pi}{6} \le \omega t \le \frac{5\pi}{6} \\ -I_0 & \frac{7\pi}{6} \le \omega t \le \frac{11\pi}{6} \\ 0 & Otherwise \end{cases}$$

- Source/input current in Fourier series as

$$i_s(t) = I_{s,avg} + \sum_{n=1,3,5}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$I_{s,avg}$ is zero as positive & negative half cycles are identical

$$a_n = \frac{2}{2\pi}\left[\int_{\pi/6}^{5\pi/6} I_0(t)\cos n\omega t \, d\omega t + \int_{7\pi/6}^{11\pi/6} -I_0(t)\cos n\omega t \, d\omega t\right]$$

$$b_n = \frac{2}{2\pi}\left[\int_{\pi/6}^{5\pi/6} I_0(t)\sin n\omega t \, d\omega t + \int_{7\pi/6}^{11\pi/6} -I_0(t)\sin n\omega t \, d\omega t\right]$$

$$a_1 = 0, \qquad b_1 = \frac{2\sqrt{3}}{\pi} I_0$$

$$i_s(t) = i_a(t) = \frac{2\sqrt{3}}{\pi} I_0 \left(\sin \omega t - \frac{1}{5}\sin 5\omega t - \frac{1}{7}\sin 7\omega t + \frac{1}{11}\sin 11\omega t + \cdots \right)$$

3.2.1 Harmonic analysis II

- The rms value of the fundamental component, $I_{s1} = \frac{2\sqrt{3}}{\pi} \times \frac{I_0}{\sqrt{2}} = \frac{\sqrt{6}}{\pi} I_0$

  Fundamental component of source current, $i_{s1} = \frac{2\sqrt{3}}{\pi} I_0 \sin \omega t$

- The rms source current, $I_s = \sqrt{\frac{2}{2\pi} \int_{\pi/6}^{5\pi/6} {I_0}^2 \, d\omega t} = \sqrt{\frac{2}{3}} I_0$

- The total harmonic distortion, THD $= \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1} = 0.3106$

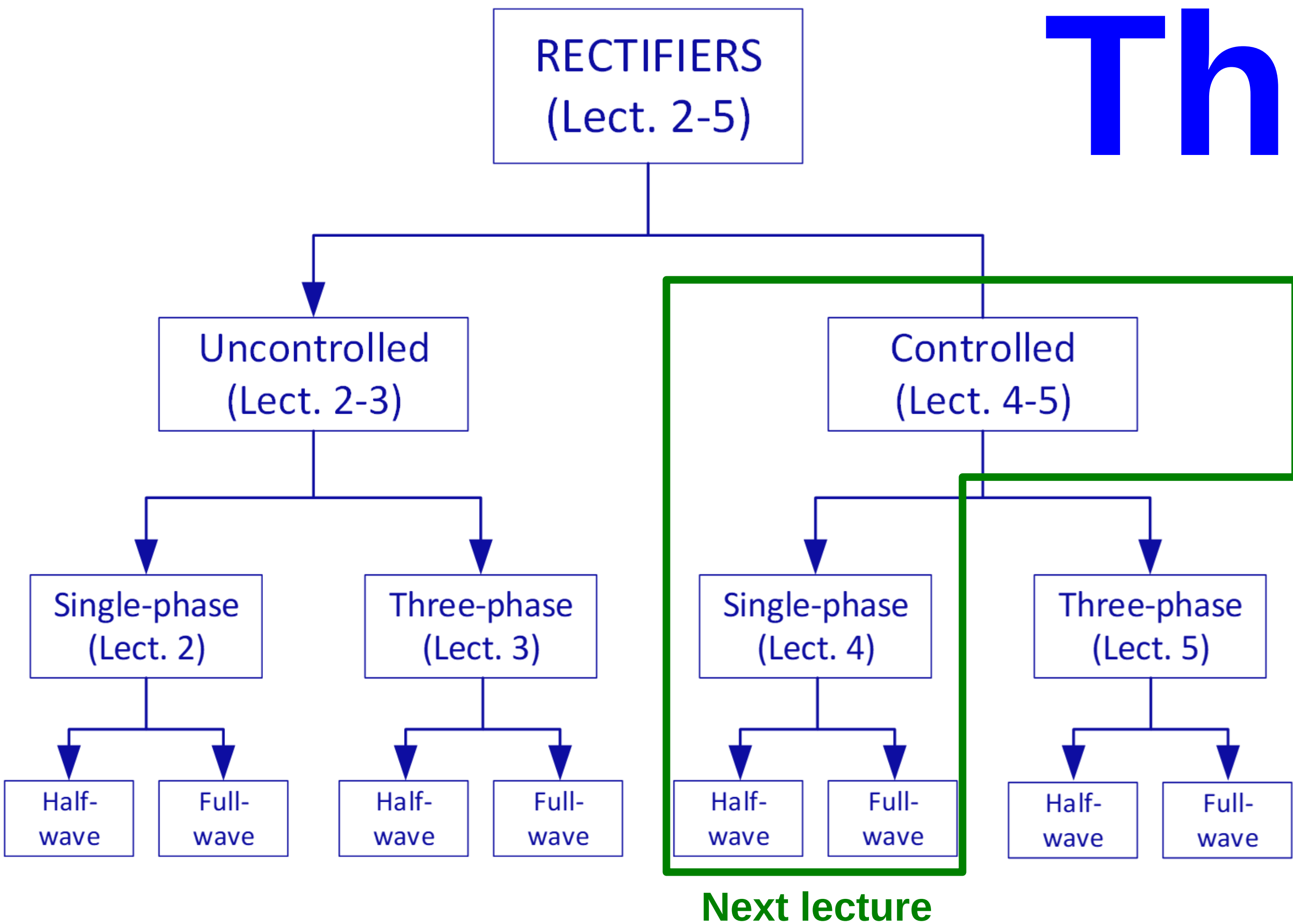
- Since the fundamental component is in phase with the phase voltage, the displacement factor (DF) $\cos \varphi_1$ is 1 and the power factor is

$$\text{PF} = \frac{I_{s1}}{I_s} \cos \varphi_1 = 0.955 \times 1 = 0.9555$$

Summary:-

- Most of the disadvantages in single-phase rectifiers are *mitigated/attenuated* to a large extent using *three-phase rectifiers.*
- Peak Inverse Voltage (PIV) is defined as the maximum voltage that appears across the diode during non-conduction mode.
- Half-wave rectifier – Each diode conducts for 120^o in one period.
- Full-wave rectifier – 6 diodes are used and arranged in three legs. Each leg has two series-connected diodes. To form a conduction loop, there always are two diodes conducting, *one from top group and one from bottom group*:
- The commutation of diodes are not influenced by the *inductor*. No matter L is small or large, the output voltage still follows the peak value of all line voltages.

See you in the next class (March 10^{th})



The End