CAN207 Continuous and Discrete Time Signals and Systems

Lecture-3

Introduction to Signals_Part 1

Zhao Wang

Zhao.wang@xjtlu.edu.cn

Room EE322

Content

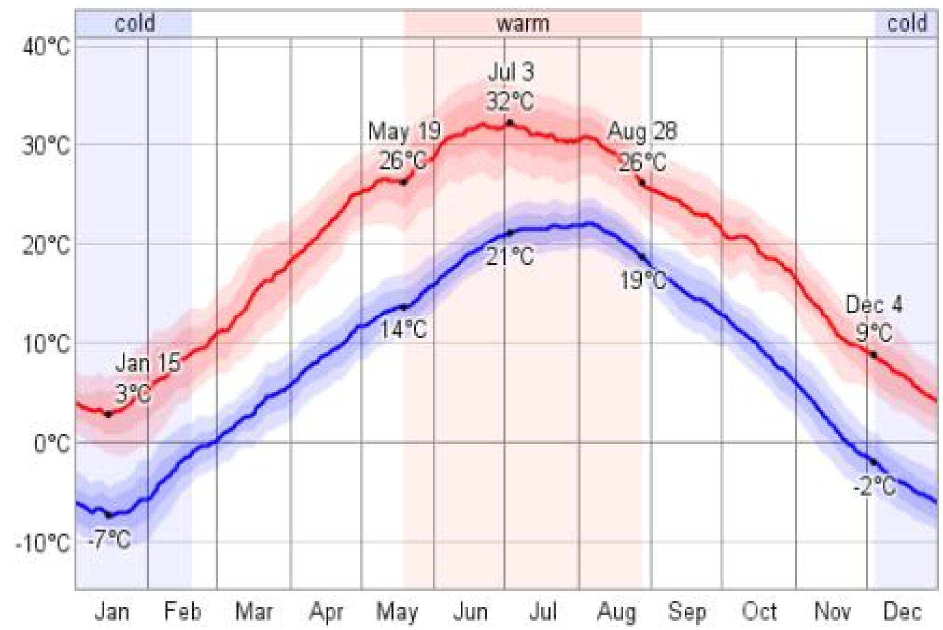
- 1. Introduction
  - signals, signal representation and examples.
- 2. Signal classification (properties)
  - continuity, periodicity, determinacy, symmetry, energy and power.
- 3. Signal operations (time-domain transformation)
  - time shifting, scaling and reversal.
- 4. Elementary signals and sequences
  - unit step, rectangular, signum, ramp, sinusoidal, sinc, exponential and unit impulse functions.

1.1 What are signals?

- ***Signal***
  - can be broadly defined as any quantity that varies as a function of time (and/or space), and has the ability to convey information about a certain plysical phenomenon.
  - In narrow sense, any series of measurements of a physical quantity is a signal (temperature measurements for instance).

http://news.filehippo.com/2013/10/qardio-ecg-monitor-streams-data-iphone/

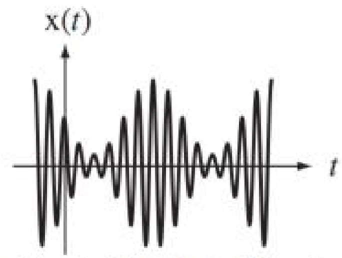
The electrocardiogram (ECG)



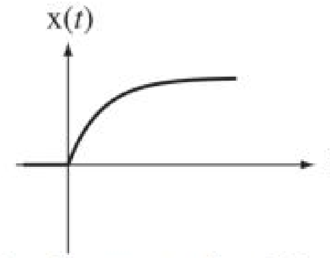
Temperature in Xi'an, China

1.2 Signal representation

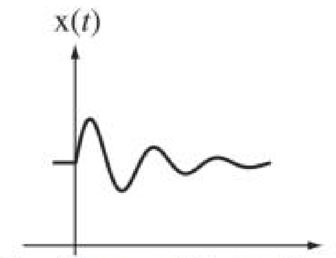
- Signal representation:
  - The most convenient way to represent a signal is via the concept of a function, let us say $x(t)$. In this notation:
    - $x(.)$ represents the dependent variable related to the physical phenomena (e.g., temperature, voltage, pressure, etc.)
    - t represents the independent variable (e.g., time, space, etc.).
  - Roughly speaking, any realizable func -tion can be consider -ed as a signal.



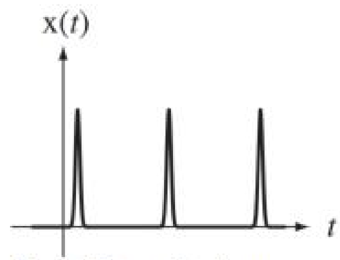
Amplitude-Modulated Carrier in a Communication System



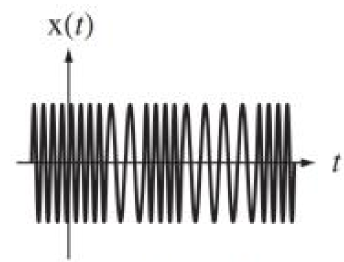
Step Response of an *RC* Lowpass Filter



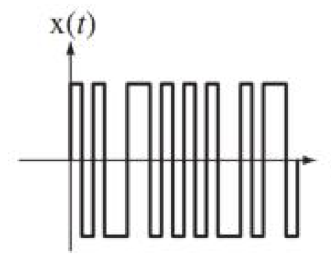
Car Bumper Height after Car Strikes a Speed Bump



Light Intensity from a Q-Switched Laser



Frequency-Shift-Keyed Binary Bit Stream

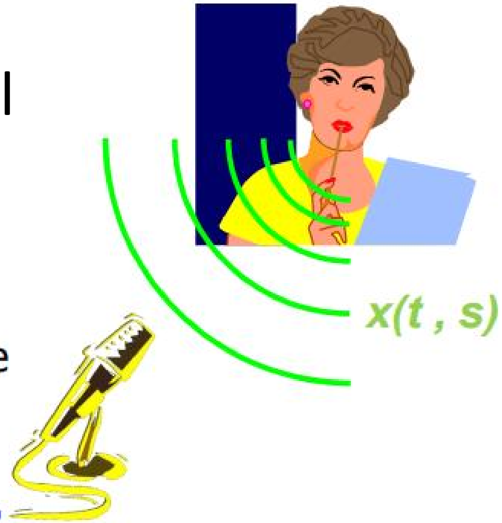


Manchester Encoded Baseband Binary Bit Stream

1.3 Examples of signals

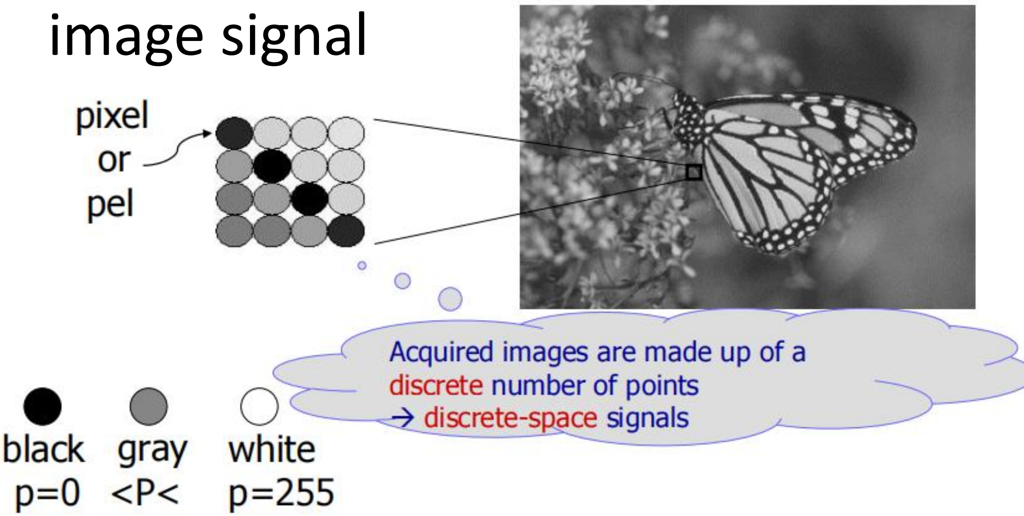
1D signal:
speech signal

microphone

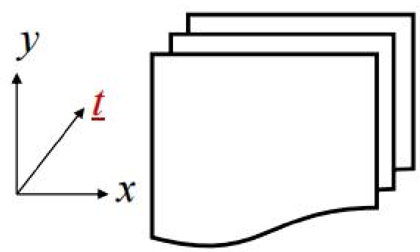


Speech signal
$x(t) = x(t, s0)$

2D signal:
image signal



3D signal:
video signal

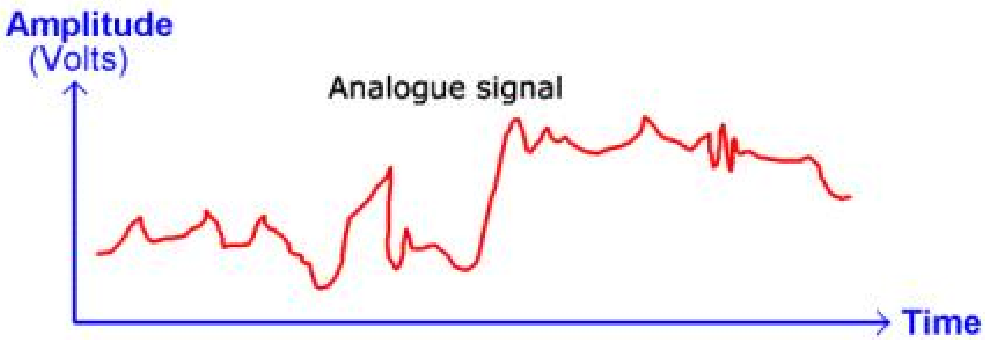


2. Classifications

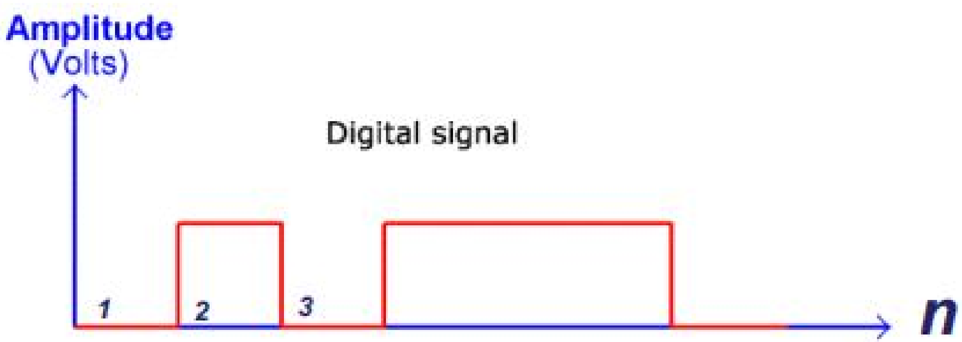
Classification	Elemetary Signals	Operations
• Continuous VS Discrete	• Unit step and rectangular func.	• Elementary operations
• Periodic VS Aperiodic	• Signum and ramp func.	• Time Shifting
• Deterministic VS Random	• Sinusoidal and sinc func.	• Time Scaling
• Symmetric VS Asymmetric	• Real and complex exponential func.	• Time Reversal (folding)
• Energy & Power	• Unit impulse func.	• Combined operations

2.1 Continuous VS Discrete

- Analog (Analogue) VS Digital



Analog Signal on an analog network



Analog Signal on a digital network

2.2 Periodicity - CT (Continuous-Time) signal

- Periodic:
  - A periodic signal is a function of time that repeat itself every certain period of time T ≠ 0:

    $$\text{if } x(t+nT) = x(t) \quad \text{for all } t, n \text{ is an integer}$$

  - The fundamental period is the smallest value of time for which the equation holds true, and it is simply known as the ***period***.

    $$x(t+T) = x(t) \quad \text{for all } t$$

  - The fundamental frequency of the periodic signal is

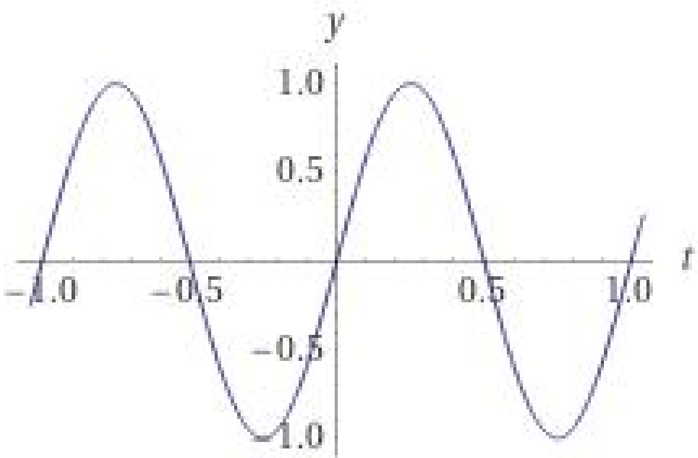
    $$f = 1/T$$

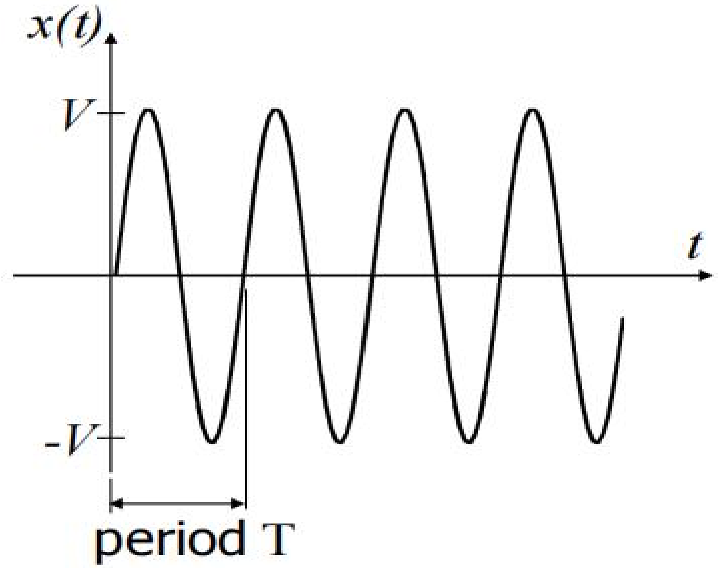
- Aperiodic (non-periodic) signal:

  $$\text{if } x(t+T) \neq x(t) \quad \text{for whatever } T \neq 0$$

2.2 Periodicity - CT Example

- Sine (Cosine) signals (also called *sinusoidal signals*)





$$x(t) = \sin(2\pi t)$$

$$x(t) = V\sin(\Omega t + \theta)$$

$$x(t) = V\sin(2\pi F t + \theta)$$

sinusoid

amplitude

frequency

phase

Units:

– Period: T [s (second)]

– Frequency: F [1/second = Hz(hertz)]

Ω [radians]

– Phase: [radians]

Quiz 1

- Are the following CT signals periodic or non-periodic? Determine their periods if periodic.
  - 1. $\log(|t|)$
  - 2. $\sin(\sqrt{2}t)$
  - 3. $\sin(t) + \sin(\pi t)$
  - 4. $\sin(t^2)$
  - 5. $e^{j(2t+7)}$
  - 6. $5\cos(2\pi 1.5t) + 3\cos(2\pi 2.5t)$

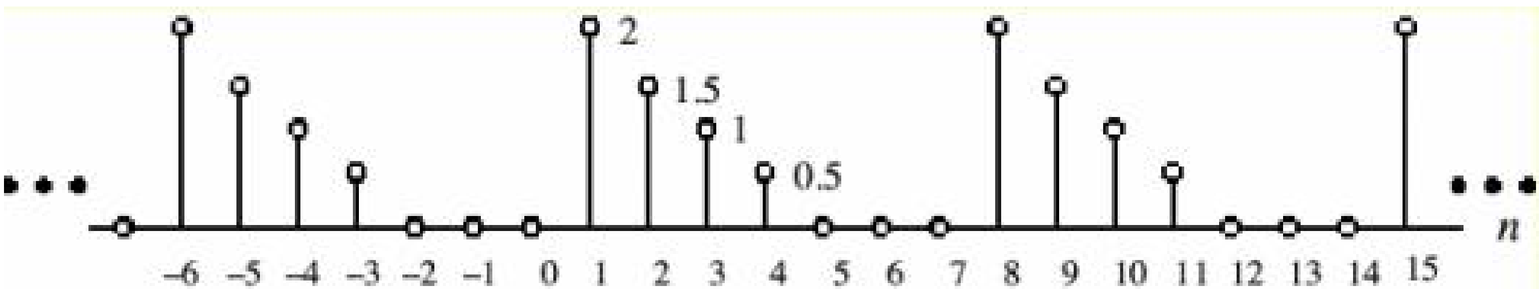
https://www.wolframalpha.com/

2.2 Periodicity - DT (Discrete-Time) signal

- For DT (Discrete-Time) signals
  - A signal is periodic with period N ($N \in \mathbb{Z}$ and $N > 0$):

$$\text{if } x[n + kN] = x[n] \quad \text{for all } n$$

  - The smallest value of N for which the above condition holds is called the (fundamental) ***period***.



- A signal not satisfying the periodicity condition is called ***nonperiodic*** or ***aperiodic***.

Xi'an Jiaotong-Liverpool University
西交利物浦大学

2.2 Periodicity - DT (Discrete-Time) signal

- Sinusoidal sequences

$$x[n] = \cos(2\pi f n); \ \ \forall n \in \mathbb{Z}, f \text{ is digital frequency}$$

- For sinusoidal sequence to be periodic:

$$\cos[2\pi f(n+N)] = \cos(2\pi f n + 2\pi f N) \stackrel{?}{=} \cos(2\pi f n)$$

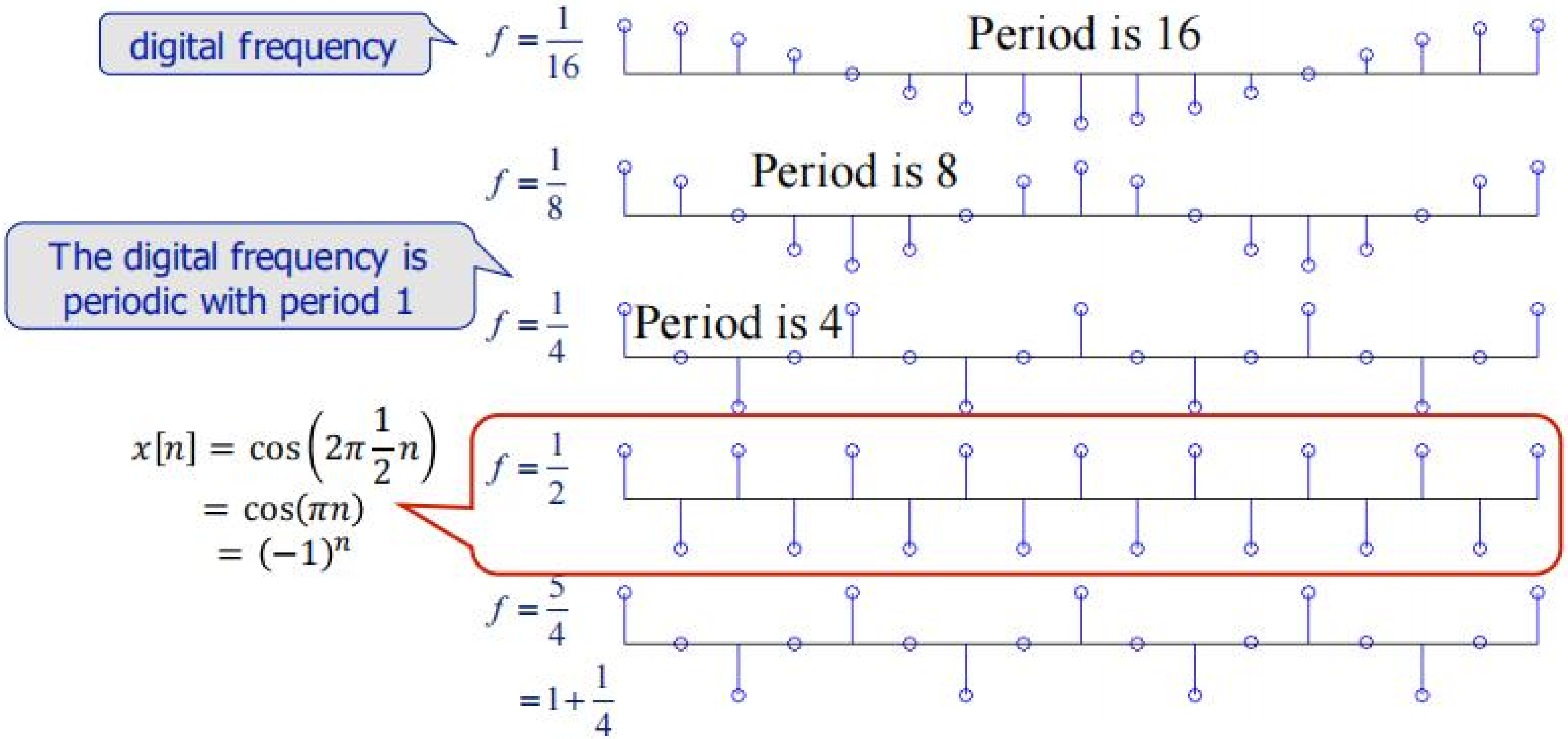
  – if f is a rational number, for some N, fN can be integer, therefore:

$$\cos(2\pi f n + 2\pi f N) = \cos(2\pi f n)$$

- A discrete-time sinusoidal is periodic only if its digital frequency f is a rational number.

2.2 Periodicity - DT sinusoidal sequences

- Example: $x[n] = \cos(2\pi f n);\ \forall n \in \mathbb{Z}$



- The highest rate of oscillation is attained when f=1/2.

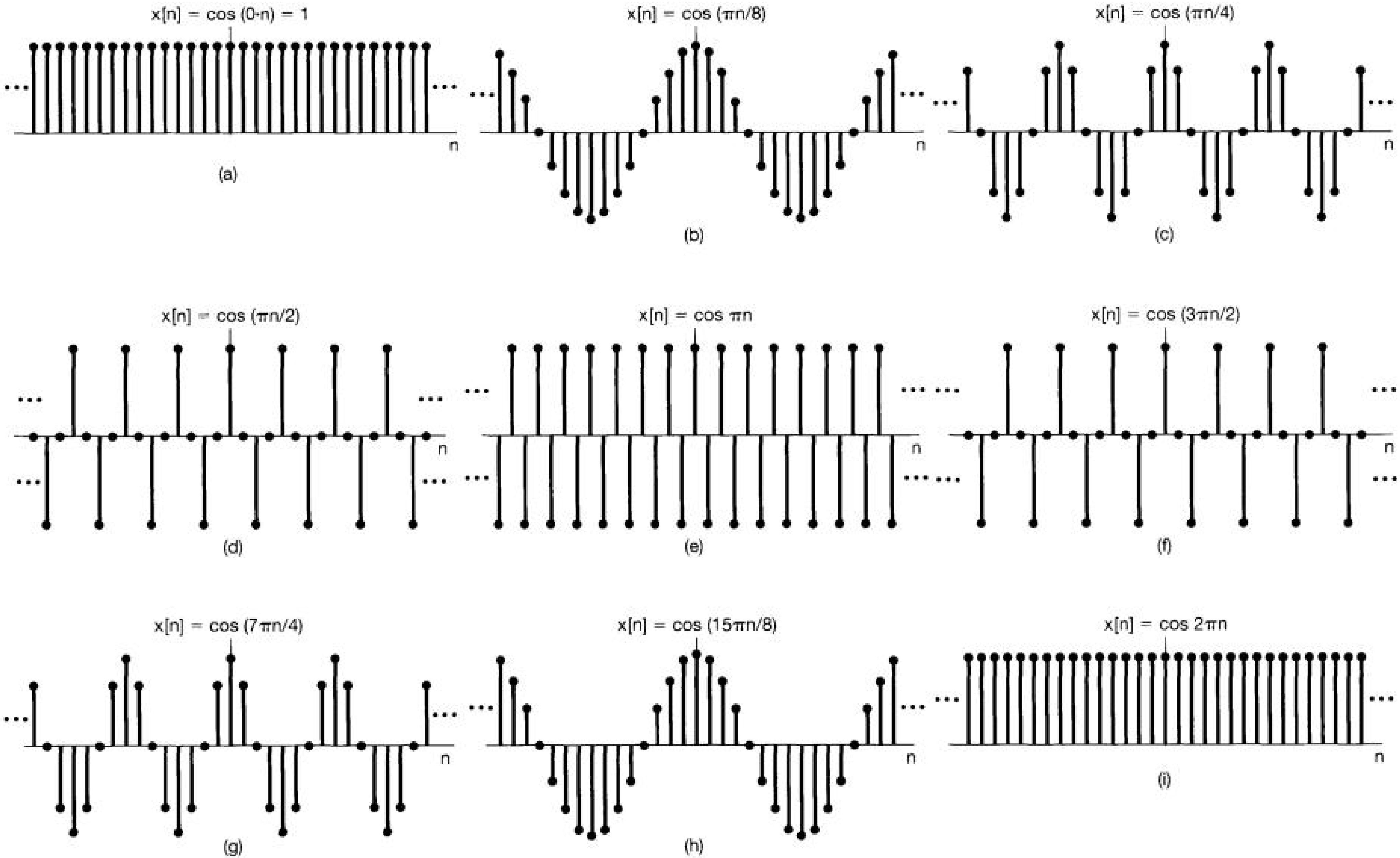


Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

2.2 Periodicity - DT period calcualtion

- For discrete-time sinusoidal sequences, they are not always periodic.
  - To be periodic with period of *N*, must have:

    $$\cos[2\pi f(n+N)] = \cos(2\pi f n)$$

  - i.e. A discrete-time sinusoidal is periodic only if its digital frequency *f* is a rational number.
  - That is to have $2\pi f N = 2\pi m$, or equivalently: $f = \frac{m}{N}$;
    - where $f = \frac{m}{N}$ is a rational number.
  - The fundamental period of the signal: $N = \frac{m}{f} = m\left(\frac{2\pi}{\omega}\right)$
    - Assumes that *m* and *N* are integers without any factors in common.

2.2 Periodicity - DT Example

- Q1: Consider the following DT sequence

$$x[n] = 5\cos(\frac{\pi}{2}n)$$

Determine the fundamental period of the signal.

- Solution: $$x[n] = 5\cos(\frac{\pi}{2}n) = 5\cos(2\pi \cdot \frac{1}{4}n)$$
  - The digital frequency $f = \frac{m}{N} = \frac{1}{4}$, so the smallest integer to make *fN* an integer is *N*=4, which is the fundamental period.
- Q2: Consider a dual-frequency DT sequence

$$y[n] = 5\cos(\boxed{\frac{\pi}{2}}n) + 2\sin(\boxed{\frac{\pi}{7}}n)$$

$$\underbrace{N_1 = 4 \qquad\qquad N_2 = 14}_{N = LCM(N_1, N_2) = 28}$$

Xi'an Jiaotong-Liverpool University 西交利物浦大學

Quiz 2

- Determine whether the following sequences are periodic. Find their fundamental periods if they are periodic.
  - 1. $\log(|n|)$
  - 2. $\sin(\sqrt{2}n)$
  - 3. $\sin(n-5)$
  - 4. $\sin(n^2)$
  - 5. $e^{j(2n+7)}$
  - 6. $5\cos(2\pi 1.5n) + 3\cos(2\pi 2.5n)$

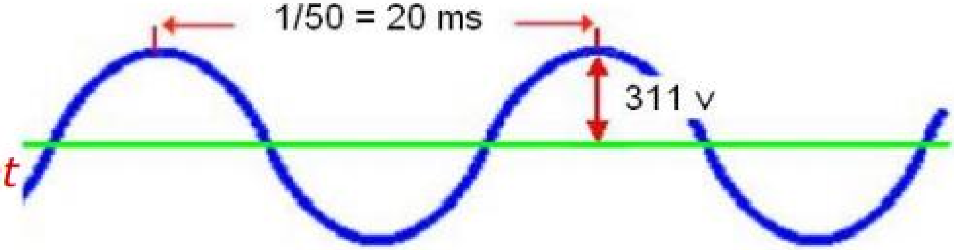
2.3 Deterministic VS Random

- Deterministic and random signals:
  - If the signal can be described by a mathematical equation, it is a deterministic signal;
  - If we know how the signal will behave in future then it is deterministic;
  - Otherwise it is called a random signal

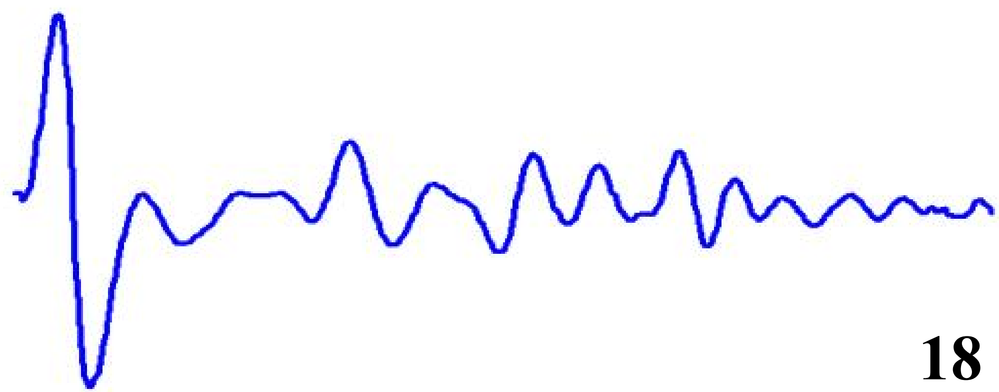
$$x(t) = \sqrt{2}220 \cos(2\pi 50\, t)$$

RMS = ?
Peak-to-peak voltage?

Deterministic
(and periodic)
AC : Alternating current



Random
Is the temperature random or deterministic signal?



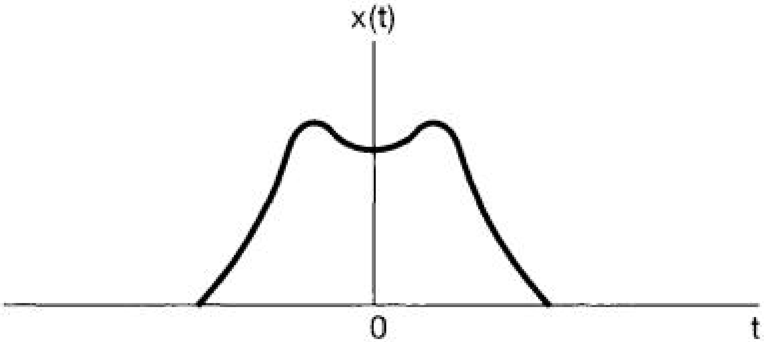
2.4. Symmetry - Odd VS Even

For real-valued signals:

- Even signal: if a signal is identical to its time-reversed counterpart, i.e., with its reflection about the origin:

$$x(t) = x(-t)$$

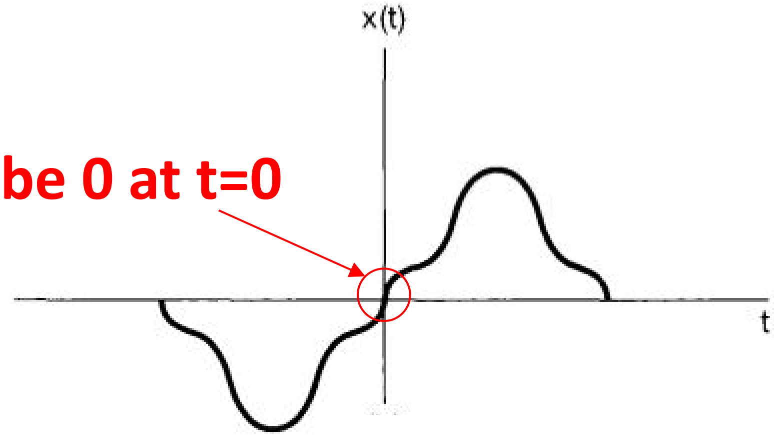
$$x[n] = x[-n]$$



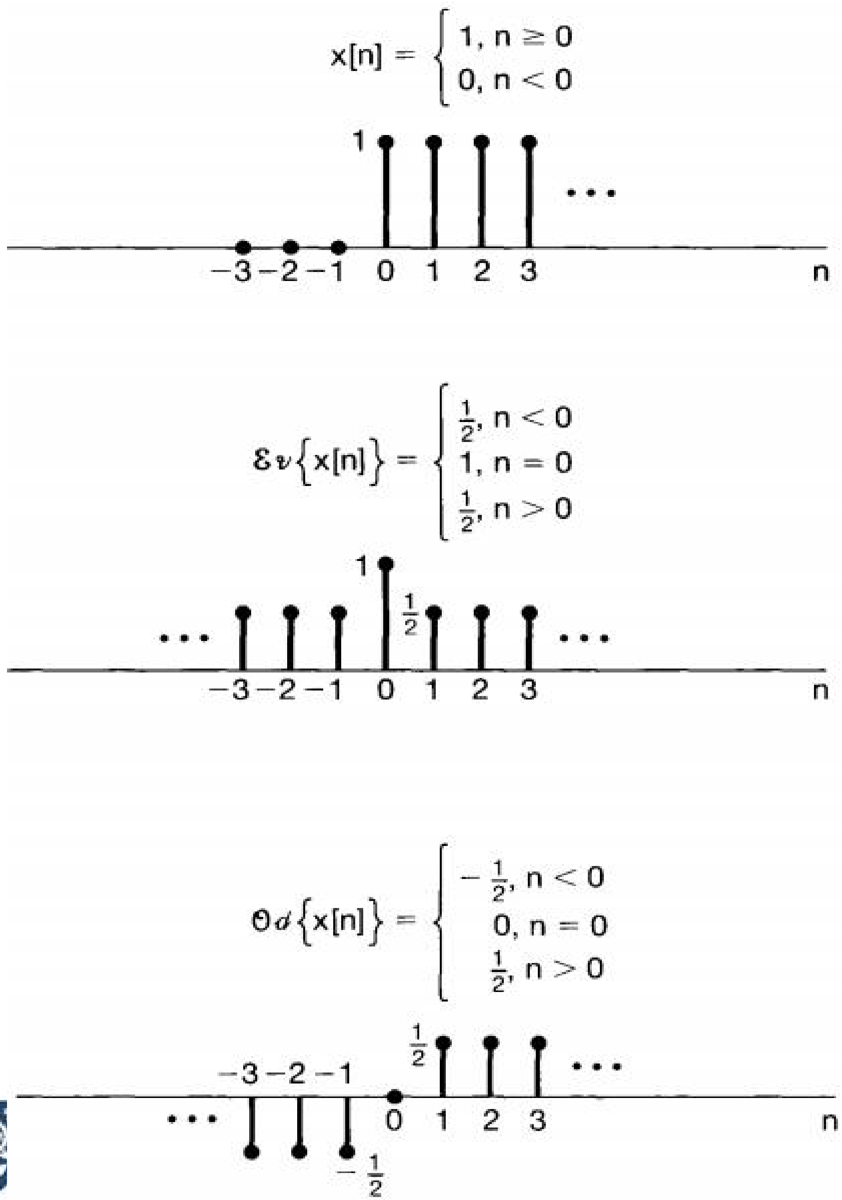
- Odd signal: if a signal is opposite to its time-reversed counterpart:

$$x(t) = -x(-t)$$

$$x[n] = -x[-n]$$



2.4. Symmetry - Sum of odd and even signals



Important fact: any signal can be broken into a sum of two signals, one of which is even and one of which is odd.

$$\mathcal{E}v\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$$

$$\mathcal{E}v\{x[n]\} = \frac{1}{2}[x[n] + x[-n]]$$

$$\mathcal{O}d\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

$$\mathcal{O}d\{x[n]\} = \frac{1}{2}[x[n] - x[-n]]$$

2.4. Symmetry - Conjugate-symmetry

- Conjugate-symmetric sequence: $x(t) = x^*(-t)$ and $x[n] = x^*[-n]$;
  - Real part: even;
  - Imaginary part: odd;
  - If $x(t)$ or $x[n]$ is real, then the symmetric is the same as conjugate-symmetric, and the signal is an even sequence.
- Conjugate-anti-symmetric sequence: $x(t) =- x^*(-t)$ and $x[n] =- x^*[-n]$;
  - Real part: odd;
  - Imaginary part: even;
  - If $x(t)$ or $x[n]$ is real, the signal is called anti-symmetric or odd sequence.

Quiz 3

- 1. What are the even and odd parts of the function

$$g(t) = t(t^2 + 3)$$

- 2. Prove that the integration of a CT odd signal with the range [-T, T] results in a zero value, i.e.

$$\int_{-T}^{T} g_o(t)dt = 0$$

- 3. Prove that the summation of a DT even signal with the range [-N, N] can be simplified as

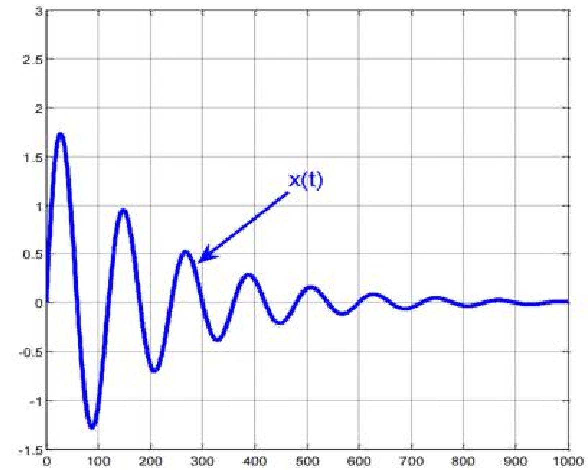
$$\sum_{n=-N}^{N} g_e[n] = g_e[0] + 2\sum_{n=1}^{N} g_e[n]$$

2.5 Energy and power of signals - Energy

- The idea of the "size" of a signal is crucial to many applications. The first concept to be introduced is the "***energy***" of a signal.
  - 1. The *instantaneous power* for real-valued signal:
    - at $t = t_0$, for CT signal x(t): $p_{ins} = x^2(t_0)$;
    - at $n = n_0$, for DT signal x[n]: $p_{ins} = x^2[n_0]$.
  - 2. The *instantaneous power* for complex-valued signal:
    - at $t = t_0$, for CT signal x(t): $p_{ins} = |x(t_0)|^2$;
    - at $n = n_0$, for DT signal x[n]: $p_{ins} = |x[n_0]|^2$.
  - 3. The *energy* within a given time interval:
    - for CT signal: $E_{[T_1,T_2]} = \int_{T_1}^{T_2} |x(t)|^2 dt$ in interval $T_1 \le t \le T_2$;
    - for DT signal: $E_{[N_1,N_2]} = \sum_{n=N_1}^{N_2} |x[n]|^2$ in interval $N_1 \le n \le N_2$.
  - 4. The *total energy* of a signal is calculated over $(-\infty, \infty)$:
    - for CT signal: $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$;
    - for DT signal: $E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$.

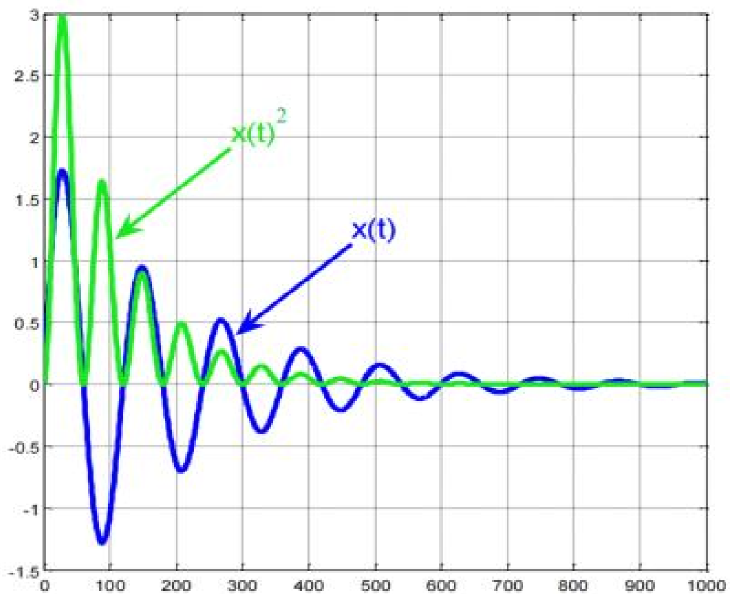
2.5 Energy - example

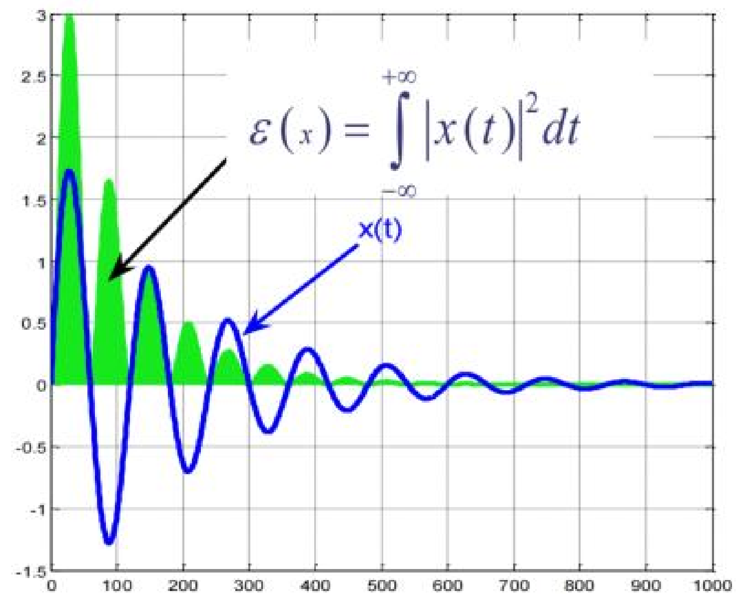
- Evaluate the energy of the signal x(t):



$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- Solve:



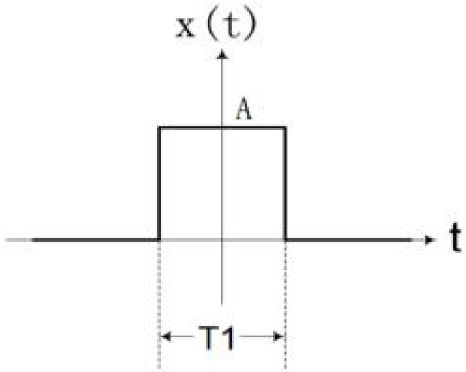


2.5 Energy and power of signals - Power

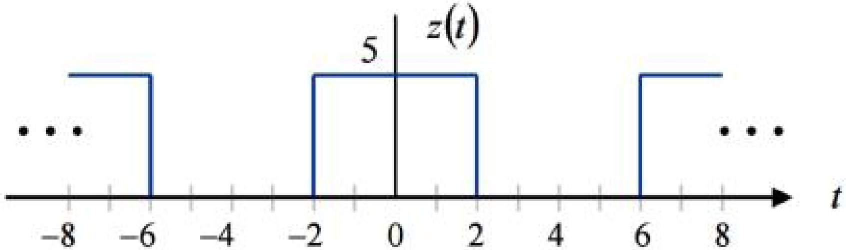
- *Power* is defined as energy per unit time.
  - 1. the *average power* over the interval $(-\infty, \infty)$ is expressed:
    - for CT signal: $P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$;
    - for DT signal: $P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$.
  - 2. for **periodic signals**, the *average power* can be calculated over one period of the signal:
    - for CT signal: $P_x = \frac{1}{T_0} \int_{\langle T_0 \rangle} |x(t)|^2 dt = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} |x(t)|^2 dt$;
    - for DT signal: $P_x = \frac{1}{N_0} \sum_{\langle N_0 \rangle} |x[n]|^2 = \frac{1}{N_0} \sum_{n_1}^{n_1+N_0-1} |x[n]|^2$.
    - where t_1 is an arbitrary real number and n_1 is an arbitrary integer.

Quiz 4

- 1. Find the total energy of this rectangular pulse:

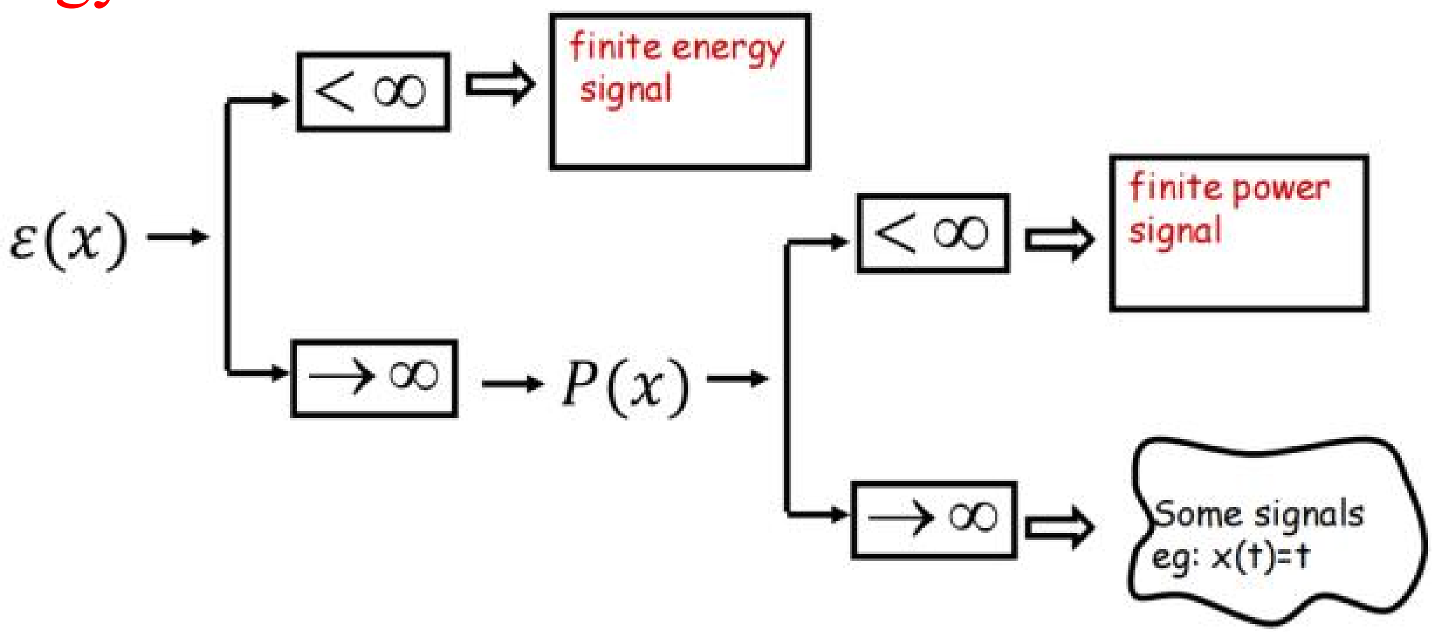


- 2. Find the average power of this periodic signal



2.5 Energy and Power Signals

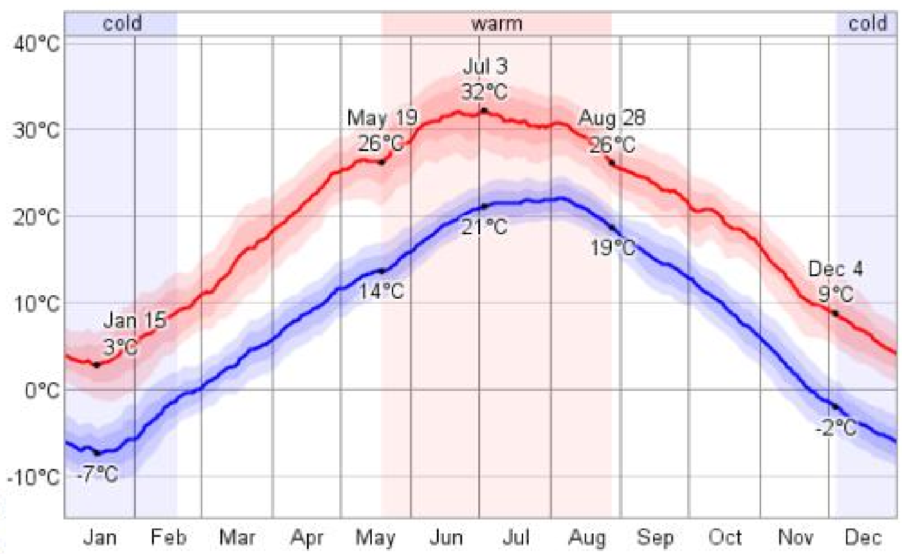
- Energy vs. Power
  - "Energy signals" have finite energy → zero average power.
  - "Power signals" have finite and non-zero power → infinite energy.

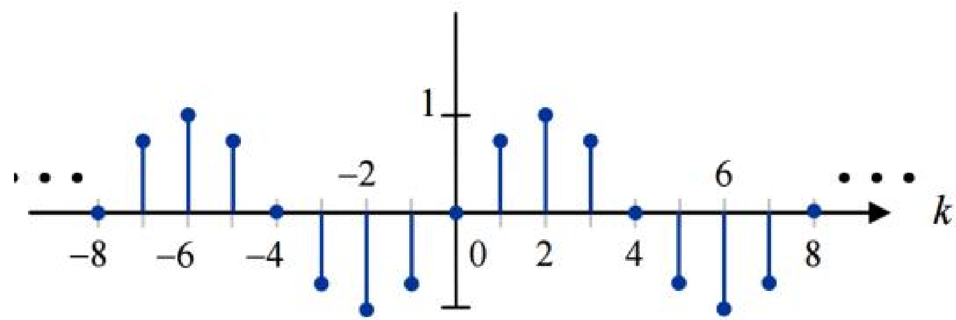


2.6 Summary

- A signal can usually be described by one word from each row from the following:

Continuous (Analogue)	⟷	Discrete (Digital)
Periodic	⟷	Aperiodic
Deterministic	⟷	Random
Finite energy	⟷	Finite power
Symmetric (Odd/Even)	⟷	Asymmetric





Next ...

- Introduction to Signals
  - 3. Signal operations (time-domain transformation)
    - time shifting, scaling and reversal.
  - 4. Elementary signals and sequences
    - unit step, rectangular, signum, ramp, sinusoidal, sinc, exponential and unit impulse functions.