CAN102

Electromagnetism and Electromechanics

2023/24-S2

Lecture 14 Transformers

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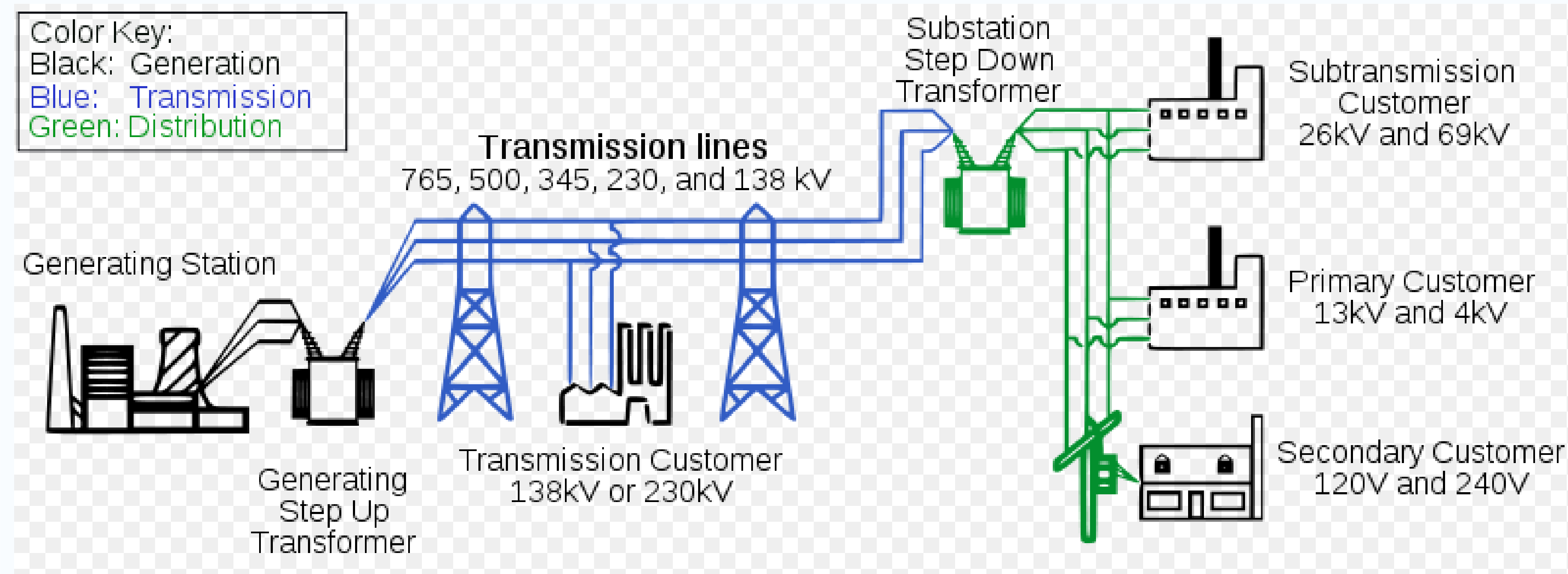
Introduction of Transformers

Introduction of Transformers （变压器）

Electrical Power System



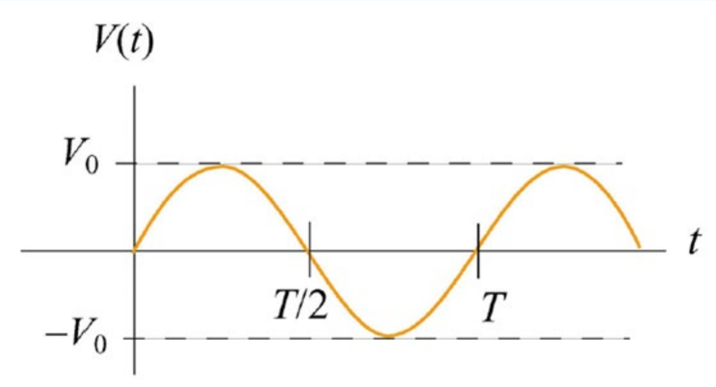
Power Generation (发电) → Power Transmission (输电) → Power Transformation (变电) → Power Distribution (配电) → Power Consumption (用电)

Introduction of Transformers

Definition	• A transformer is a static electrical device that transfers electrical energy between two or more electrically isolated circuits through electromagnetic induction.
Function	• Converting electrical energy (**AC electric power**) from one voltage level to another at the same frequency
Working principle	• A transformer consists of two or more coils wound on the same core or magnetic core. When alternating current passes through the primary coil, it creates an alternating magnetic field in the core. This alternating magnetic field in turn induces alternating current in the coils.

Alternating Current Source (AC Sources)

If a coil rotates in the presence of a magnetic field, the induced emf varies sinusoidally with time and leads to an alternating current (AC), and provides a source of AC power. The symbol for an AC voltage source is



An example of an AC source is:

$V(t) = V_0 sin\omega t$

where the maximum value of V_0 is called the ***amplitude***.

The voltage varies between V_0 and $-V_0$.

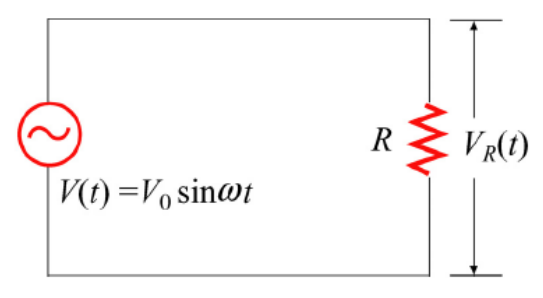
T: the period.

f: the frequency, defined as $f = 1/T$, has the unit of inverse seconds (s^{-1}), or hertz (Hz).

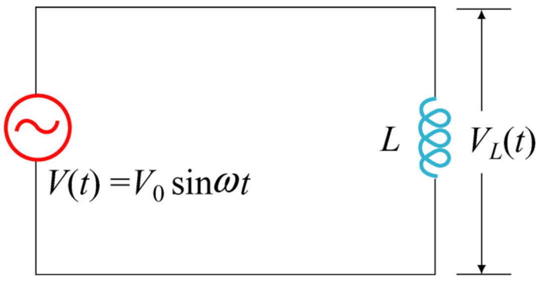
ω: the angular frequency, defined to be $\boldsymbol{\omega} = 2\pi f$.

Simple AC circuits

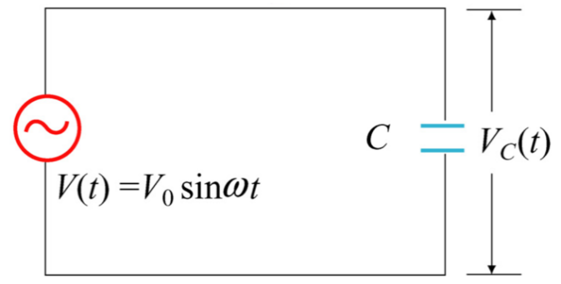
- Purely Resistive Load



- Purely Inductive Load



- Purely Capacitive Load

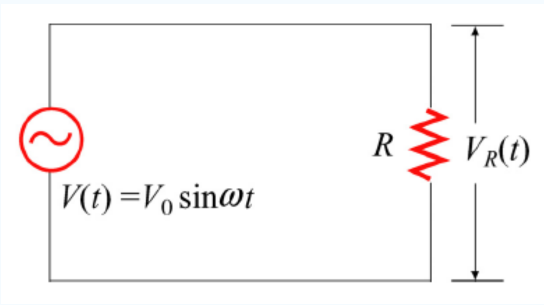


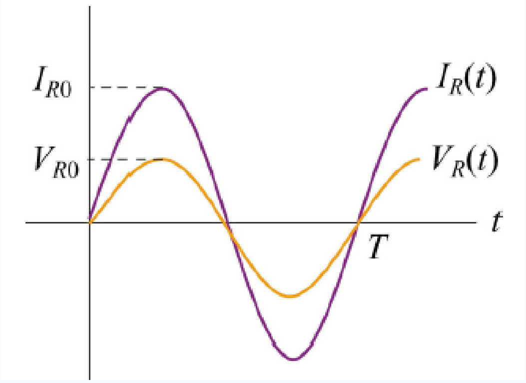
AC source: $V(t) = V_0 sin\omega t$

What is the current *I(t)* in the circuit?

Simple AC circuits

- Purely Resistive Load





AC source: $V(t) = V_0 sin\omega t$

$$I_R(t) = \frac{V_R(t)}{R} = \frac{V(t)}{R} = \frac{V_0 \sin\omega t}{R} = I_{R0} \sin\omega t$$

$$\Rightarrow \phi = 0$$

The root-mean-square (rms) current:

$$I_{rms} = I_{R0} / \sqrt{2}$$

The rms voltage:

$$V_{rms} = V_{R0} / \sqrt{2}$$

The power dissipated in the resistor:

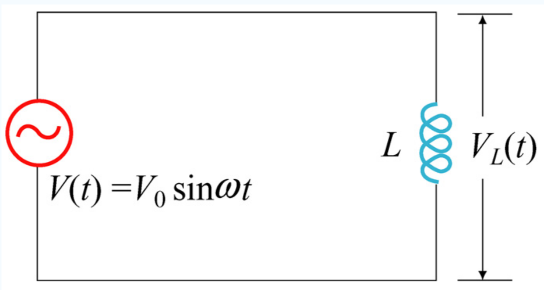
$$P_R(t) = I_R(t) V(t) = I_R^2(t) R$$

The average power dissipated over one period:

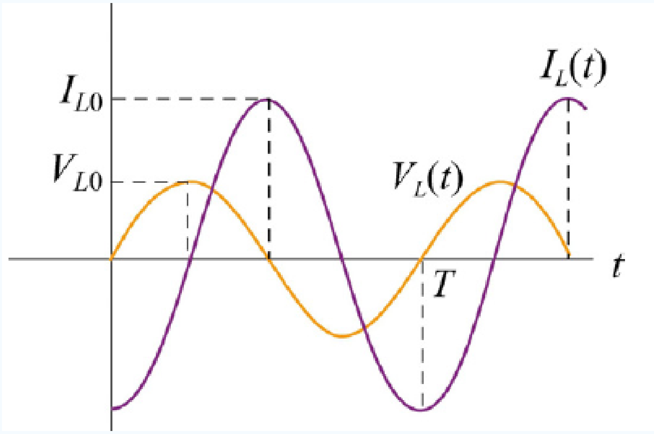
$$P = \frac{1}{2} I_{R0}^2 R = I_{rms}^2 R = I_{rms} V_{rms} = \frac{V_{rms}^2}{R}$$

Simple AC circuits

- Purely Inductive Load



$\boxed{I(t) = I_0 sin\ (\omega t\ \text{-}\ \varphi)}$



AC source: $V(t) = V_0 sin\omega t$

$$V_L(t) = V(t)$$

From Faraday's law (Lecture 12 Page5):

$$V_L(t) = L\frac{dI_L}{dt} \quad \Rightarrow \quad \frac{dI_L}{dt} = \frac{V(t)}{L} = \frac{V_{L0}}{L}\sin\omega t$$

$$I_L = \int dI_L = \frac{V_{L0}}{L}\int \sin\omega t\ dt = -(\frac{V_{L0}}{\omega L})\cos\omega t$$

$$I_L = (\frac{V_{L0}}{\omega L})\sin(\omega t - \pi/2)$$

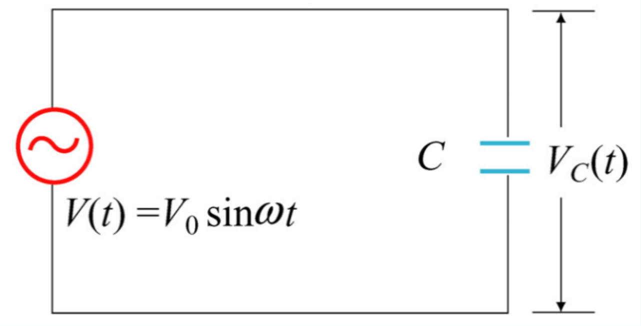
$$\Rightarrow \phi \boxed{= +\pi/2}$$

Where, $X_L = \omega L$ inductive reactance and SI units is ohms Ω

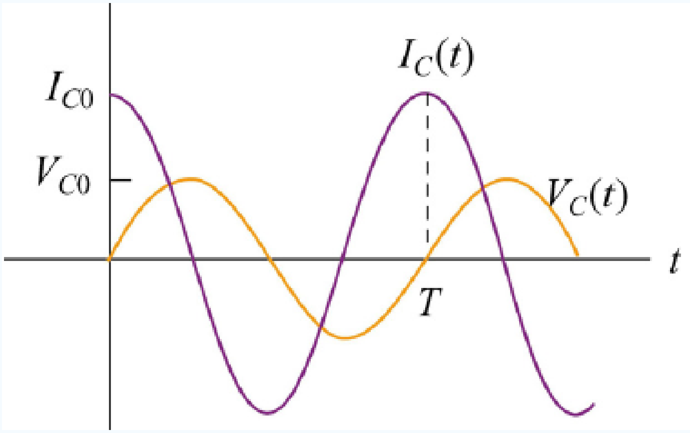
The current lags behind the voltage by π/2

Simple AC circuits

- Purely Capacitive Load



$\boxed{I(t) = I_0 sin\,(\omega t - \varphi)}$



AC source: $V(t) = V_0 sin\omega t$

$$V_C(t) = V(t)$$

$$Q(t) = CV(t) = CV_C(t) = CV_0 \sin \omega t$$

$$I_C(t) = \frac{dQ}{dt} = \omega C V_0 \cos \omega t$$

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$$I_C(t) = \omega C V_0 \sin(\omega t + \frac{\pi}{2})$$

$$\Rightarrow \boxed{\phi = -\pi / 2}$$

Where, $X_c = 1/\omega C$ capacitive reactance and SI units is ohms Ω

The current leads the voltage by $\pi/2$

Power Triangle

Assume the angle between voltage and current is φ, then three types of power is

- **Real Power *P***

the power supplied to the load

$$P = VI\boxed{\cos\varphi} \qquad \text{Watts (W)}$$

Power Factor (PF)

- **Reactive Power *Q***

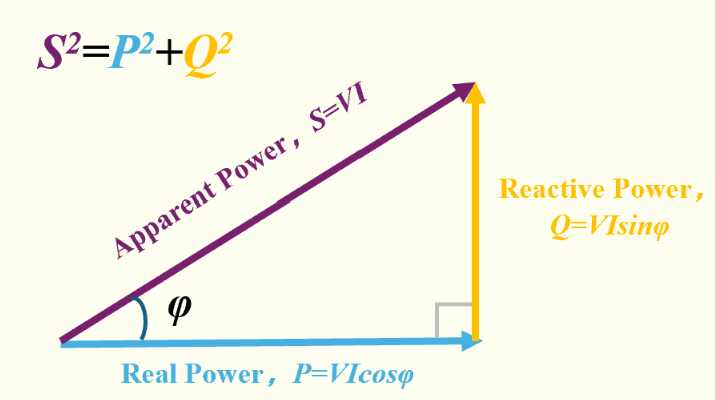
the power exchanges back and forth between a source and a load

$$Q = VI\sin\varphi \qquad \text{Volt amperes reactive (Var)}$$

- **Apparent Power *S***

The product of the voltage and current without considering of the phase angle

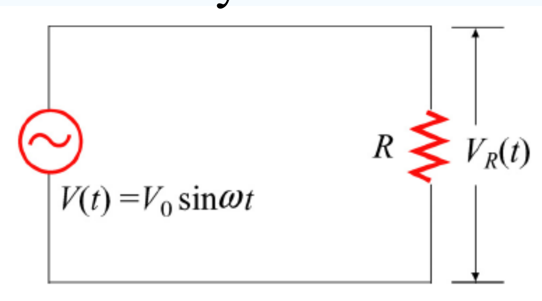
$$S = VI \qquad \text{Volt amperes (V A)}$$



$$S^2 = P^2 + Q^2$$

Simple AC circuits

- Purely Resistive Load



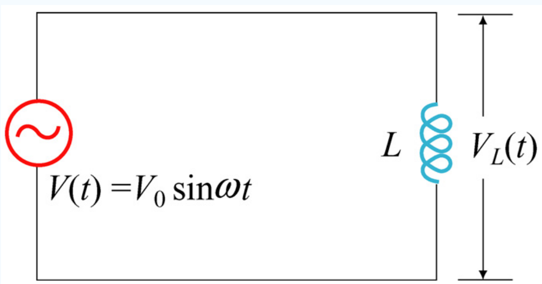
AC source: $V(t) = V_0 sin\omega t$

$$I_R(t) = I_{R0} \sin \omega t$$

Power dissipated in the resistor

$$P_R = I_R^2 R$$

- Purely Inductive Load

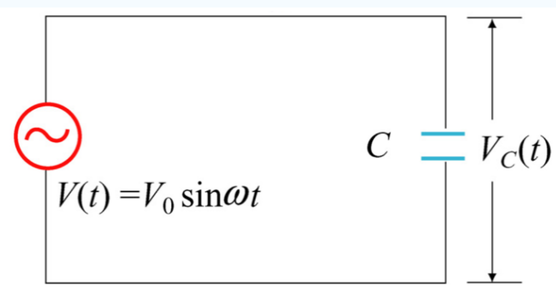


$$I_L = \left(\frac{V_{L0}}{\omega L}\right) \sin\left(\omega t \boxed{-\frac{\pi}{2}}\right)$$

Inductive reactive power (positive power)

Phase difference

- Purely Capacitive Load

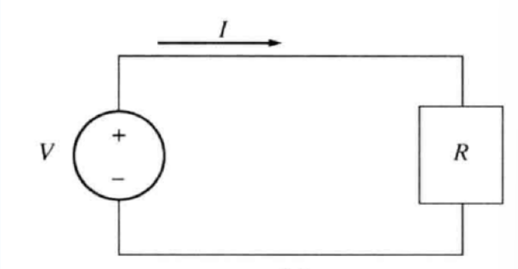


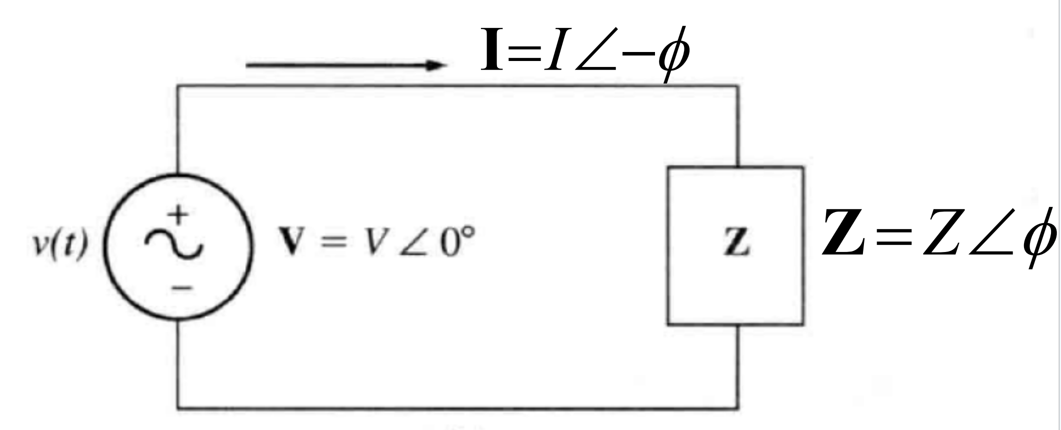
$$I_C(t) = \omega C V_0 \sin\left(\omega t \boxed{+\frac{\pi}{2}}\right)$$

Capacitive reactive power (negative power)

Phase Difference in AC circuits

For DC circuits, the relation between voltage and current is linear





Due to a phase difference between voltage and current, the situation is more complex in AC circuits.

$$\mathbf{Z}=\frac{\mathbf{V}}{\mathbf{I}}=\frac{V\angle 0°}{I\angle-\phi}=Z\angle\phi$$

Phase Difference in AC circuits

Because frequency of voltage and current in AC circuits is the same, we only have a concern on amplitude and phase

- Transformation between

  $A\angle\phi$ and $X+jY$

$$A=\sqrt{X^2+Y^2}, \phi=\tan^{-1}\frac{Y}{X}$$

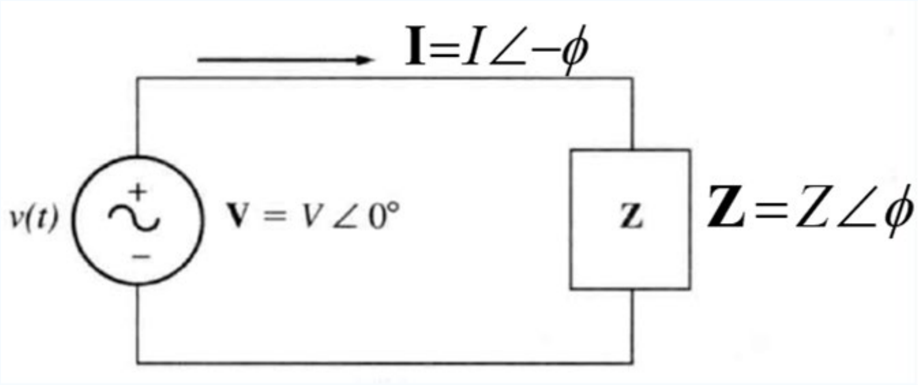
$$X=A\cos\phi, Y=A\sin\phi$$

$$A\angle\phi=A\cos\phi+jA\sin\phi$$

- Product

$$(A\angle\phi)(B\angle\theta)=AB\angle(\phi+\theta)$$

$$\frac{A\angle\phi}{B\angle\theta}=\frac{A}{B}\angle(\phi-\theta)$$



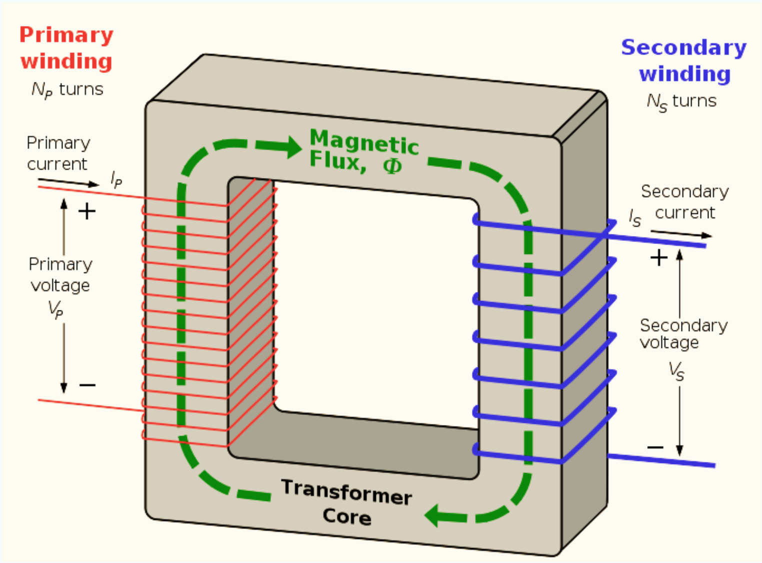
Ideal Transformers

Definition – a lossless device with an input winding and an output winding:

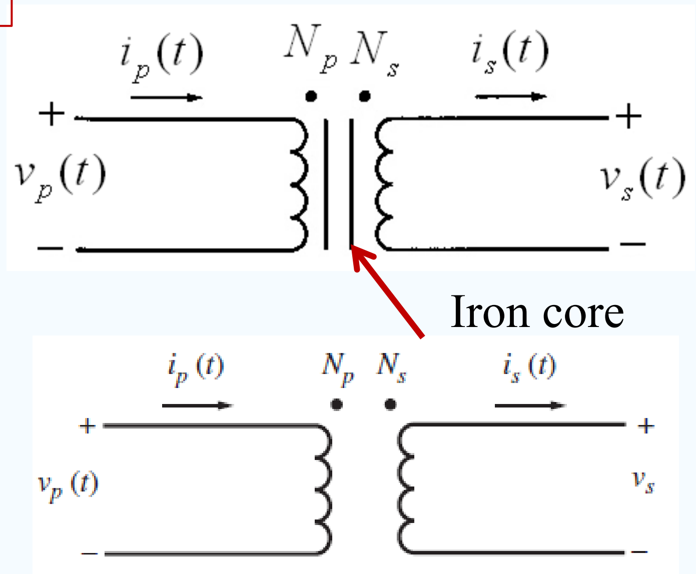
- the windings have no resistance,
- loss-less magnetic core: no hysteresis or eddy currents,
- reluctance of the core is zero.

Subscriptions of 1 or *p*: primary side

Subscriptions of 2 or *s*: secondary side



Sketch of an ideal transformer

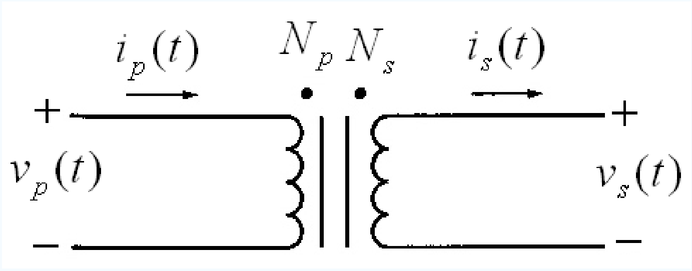


Schematic symbol of a transformer

Ideal Transformers-Voltage Relation

Changing the current in the primary coil changes the magnetic flux that is developed.

The changing magnetic flux induces a voltage in the secondary coil.



Magnitude of induced voltage

$$\left.\begin{aligned} v_p(t) &= N_p \frac{d\Phi(t)}{dt} \\ v_s(t) &= N_s \frac{d\Phi(t)}{dt} \end{aligned}\right\} \Rightarrow \frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

$$\text{where} \quad a = \frac{N_p}{N_s}$$

a (n): turns ratio,

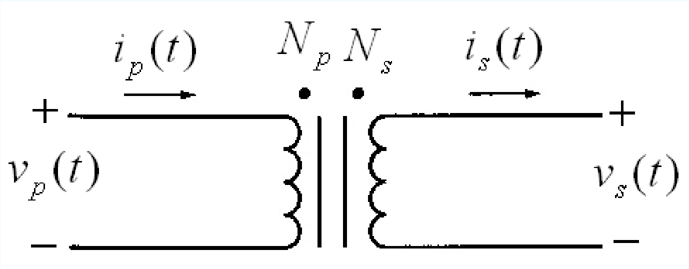
voltage ratio,

ratio of transformation

In terms of RMS quantities :

$$\boxed{\frac{V_p}{V_s} = \frac{V_{p,\max}/\sqrt{2}}{V_{s,\max}/\sqrt{2}} = \frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a}$$

Ideal Transformers-Current Relation



a (n): turns ratio,

voltage ratio,

ratio of transformation

As no loss, so

$$v_p(t)i_p(t) = v_s(t)i_s(t) \Rightarrow \frac{i_p(t)}{i_s(t)} = \frac{v_s(t)}{v_p(t)} = \frac{1}{a}$$

In terms of RMS quantities : $\boxed{\frac{I_p}{I_s} = \frac{i_p(t)}{i_s(t)} = \frac{1}{a}}$

Ideal Transformers-Power Relation

The power supplied to the transformer by the primay side:

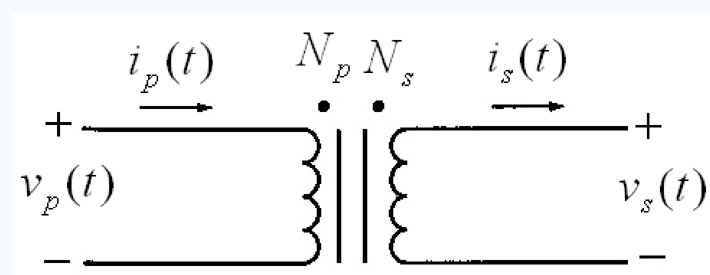
$P_{in} = V_p I_p \cos\theta_p \quad \theta_p$ is the angle between V_p and I_p

The power supplied by the second side to it's load:

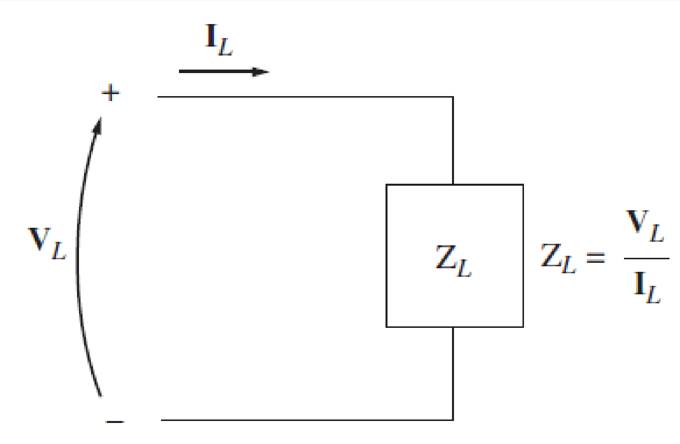
$P_{out} = V_s I_s \cos\theta_s \quad \theta_s$ is the angle between V_s and I_s

$$\left.\begin{aligned} V_p I_p \cos\theta_p &= V_s I_s \cos\theta_s && (1) \\ V_p / V_s &= a && (2) \\ I_p / I_s &= 1/a && (3) \end{aligned}\right\} \Rightarrow \cos\theta_p = \cos\theta_s = \cos\theta$$

$$V_p I_p = V_s I_s \quad \Rightarrow \quad \boxed{P_{in} = V_p I_p \cos\theta = P_{out}}$$



Ideal Transformers-Impedance Relation



The impedance of a device/element is defined as the ratio of the phasor voltage across it to the phasor current flowing through it: $Z_L = \dfrac{\mathbf{V}_L}{\mathbf{I}_L}$

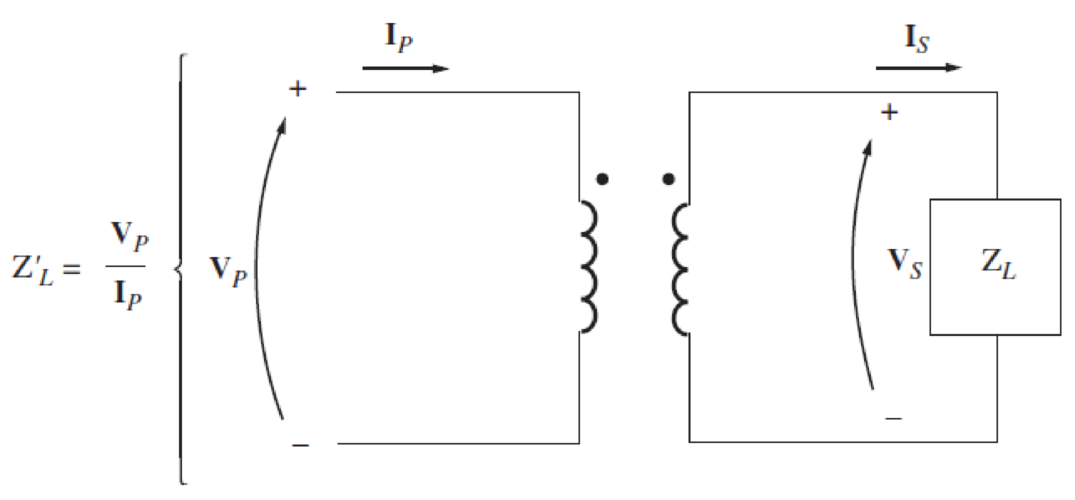
Phasors are the complex representatives of sinusoidal voltages and currents.

The impedance of the load:

$$Z_L = \frac{\mathbf{V}_S}{\mathbf{I}_S}$$

The load impedance seen from primary side:

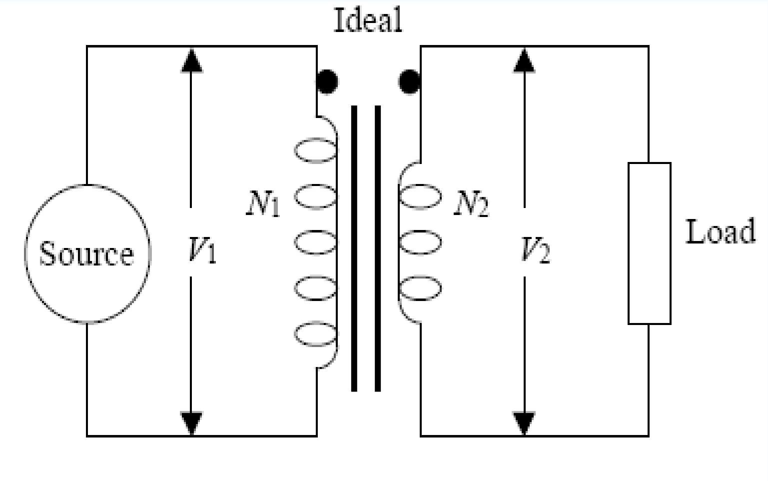
$$\boxed{Z_L^{'} = \frac{\mathbf{V}_P}{\mathbf{I}_P} = \frac{a\mathbf{V}_S}{\mathbf{I}_S / a} = a^2 \frac{\mathbf{V}_S}{\mathbf{I}_S} = a^2 Z_L}$$



Ideal Transformers

Example

Consider an ideal, 2400 V - 240 V transformer. The primary is connected to a 2200 V source and the secondary is connected to an impedance of $2\angle 36.9^0\ \Omega$



Find:

a) the secondary output current and voltage
b) the primary input current
c) the input and output reactive powers
d) the load impedance as seen from the primary side

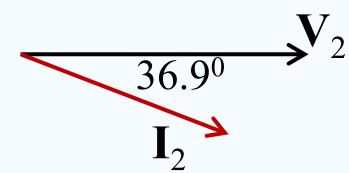
Ideal Transformers

Solution

a) The secondary output current and voltage

$$a = \frac{2400}{240} = 10 \Rightarrow V_2 = V_1 / a = 2200/10 = 220 \quad \text{V}$$

Take V_2 as a reference phase: $\mathbf{V}_2 = 220\angle 0^0 \quad \text{V}$

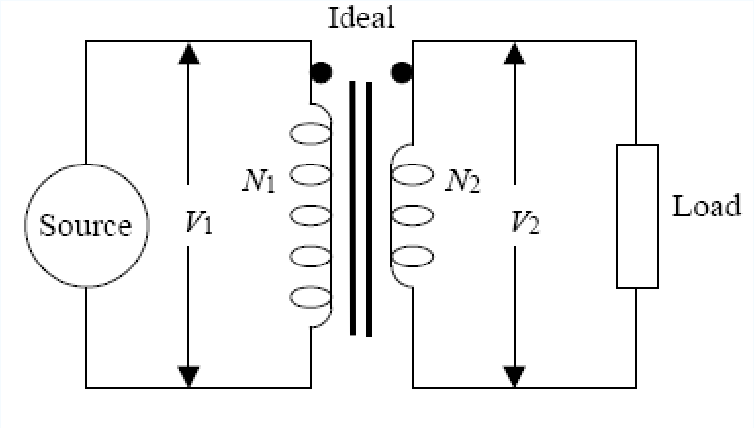


$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{220\angle 0^0}{2\angle 36.9^0} = 110\angle(-36.9^0) \ \text{A}$$

$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2} = a, \quad \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{1}{a} \quad \text{where } \mathbf{V} \text{ and } \mathbf{I}\text{: phasor}}$$

The turns ratio of the ideal transformer affects the magnitudes of the voltages and currents, but not their angles.

Ideal Transformers



b) The primary input current

$$\mathbf{I}_1 = \mathbf{I}_2 / a = \frac{110\angle(-36.9^0)}{10} = 11\angle(-36.9^0) \text{ A}$$

c) The input and output reactive powers

Input:

$$Q_1 = V_1 I_1 \sin\theta = 2200 \times 11 \times \sin 36.9^0 = 14.53\,\text{kVAr}$$

Output:

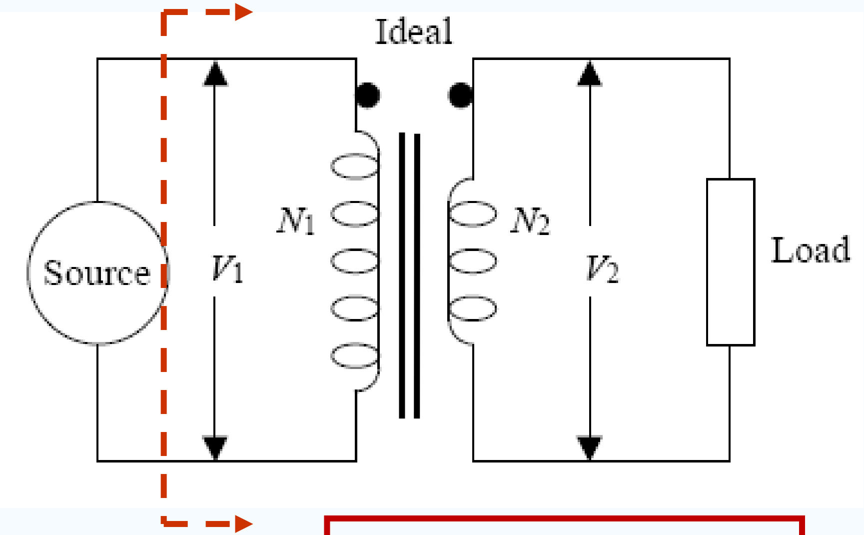
$$Q_2 = V_2 I_2 \sin\theta = 220 \times 110 \times \sin 36.9^0 = 14.53\,\text{kVAr}$$

Ideal Transformers

d) The load impedance as seen from the primary side

$$Z_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{2200\angle 0^0}{11\angle(-36.9^0)}$$

$$= 200\angle 36.9^0 \ \ \Omega$$



$$\frac{V_P}{V_S} = \frac{v_s(t)}{v_p(t)} = \frac{N_P}{N_S} = a$$

$$\frac{I_P}{I_S} = \frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

$$P_{in} = P_{out}$$

$$\frac{Z_P}{Z_S} = a^2$$

Or

Since the load impedance $2\angle 36.9^0 \ \ \Omega$

$$\mathbf{Z}_{in} = a^2\mathbf{Z}_{load} = 10^2 \times 2\angle 36.9^0 = 200\angle 36.9^0 \ \ \Omega$$

Actual Transformers

Ideal Transformers
(Lossless)

- No copper losses
  - the windings have no resistance,
- loss-less magnetic core:
  - no hysteresis or eddy currents,
- reluctance of the core is zero.
  - Permeability is infinite

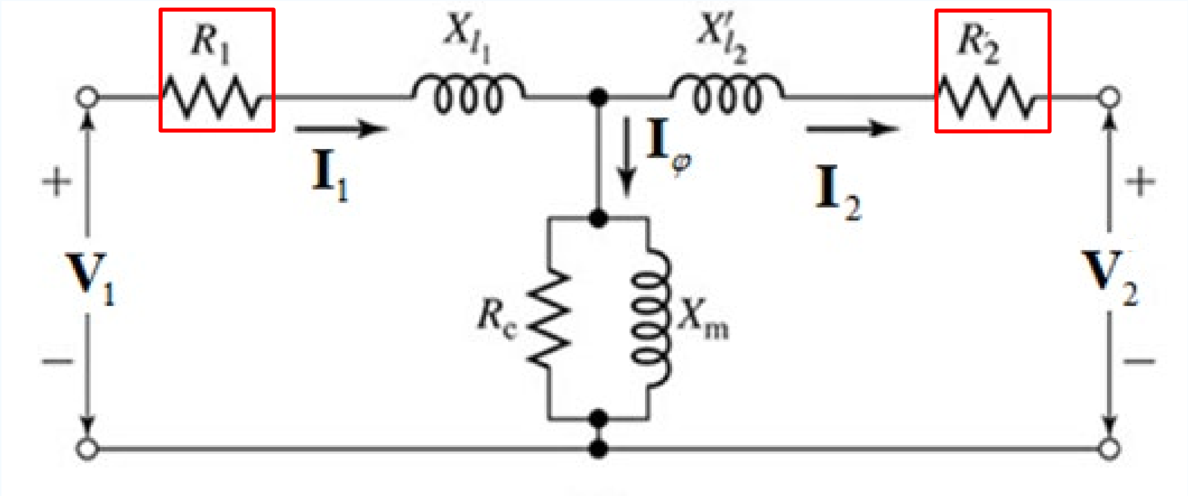
Actual Transformers
(Lossy)

- Copper losses
  - Have resistance in the windings
- Core losses
  - Hysteresis losses
  - Eddy currents losses
- Leakage flux
  - Magnetic core has finite permeability

Actual Transformers-Losses

Copper Losses

- Resistive heating losses in the primary and secondary windings of the transformer.

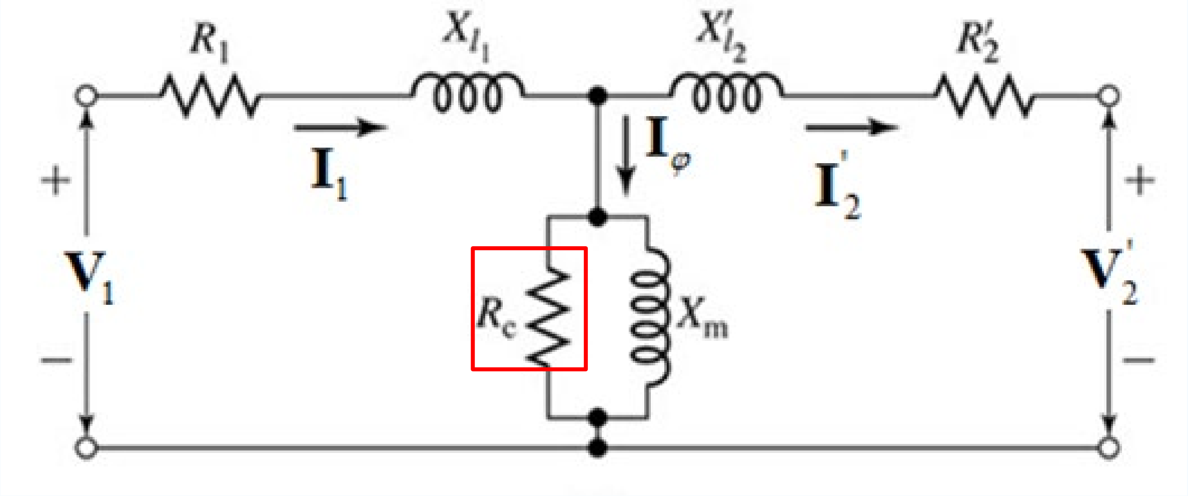


$$\boxed{I_1^{\ 2}R_1 + I_2^{\ 2}R_2}$$

Actual Transformers-Losses

◻ Core Losses

- Eddy current losses: resistive heating losses in the core of the transformer.

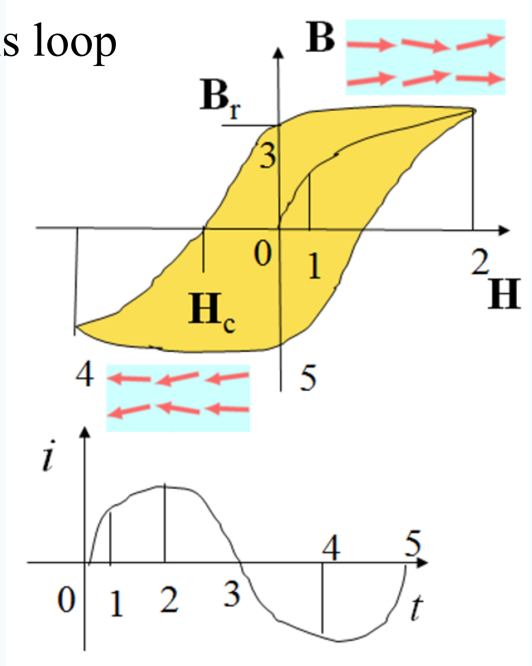


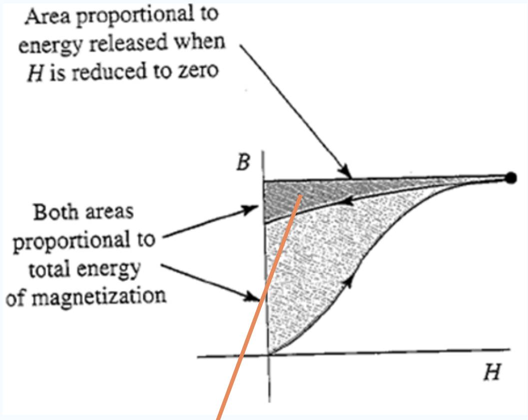
When a conductor (iron core) exposed to a changing magnetic field in the body of the magnetic core, a circulating current in it is induced, which is termed as eddy current.

☐ Core Losses

- Hysteresis losses: these are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are complex, non-linear function of the voltage applied to the transformer.

hysteresis loop





Core losses

= hysteresis losses + eddy current losses

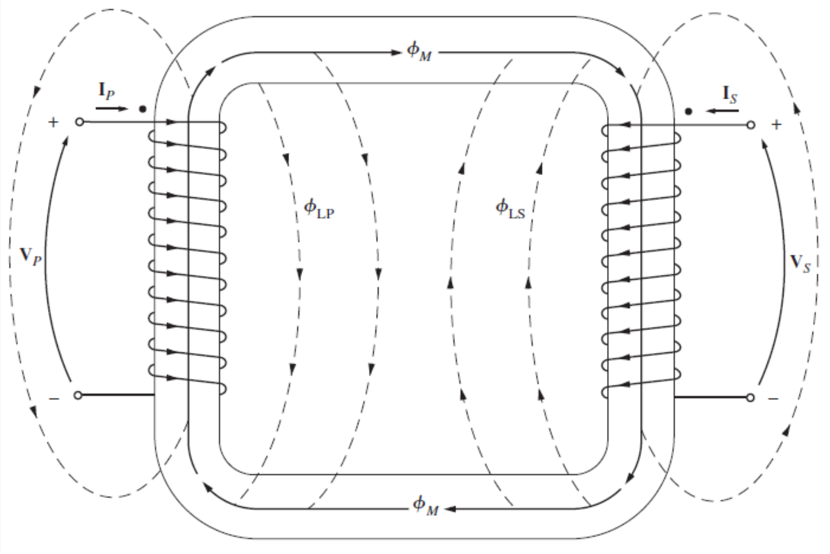
Actual Transformers-Losses

☐ Leakage flux

- The flux Φ_{LP} *and* Φ_{LS} which escape the core and pass through only one of the transformer windings

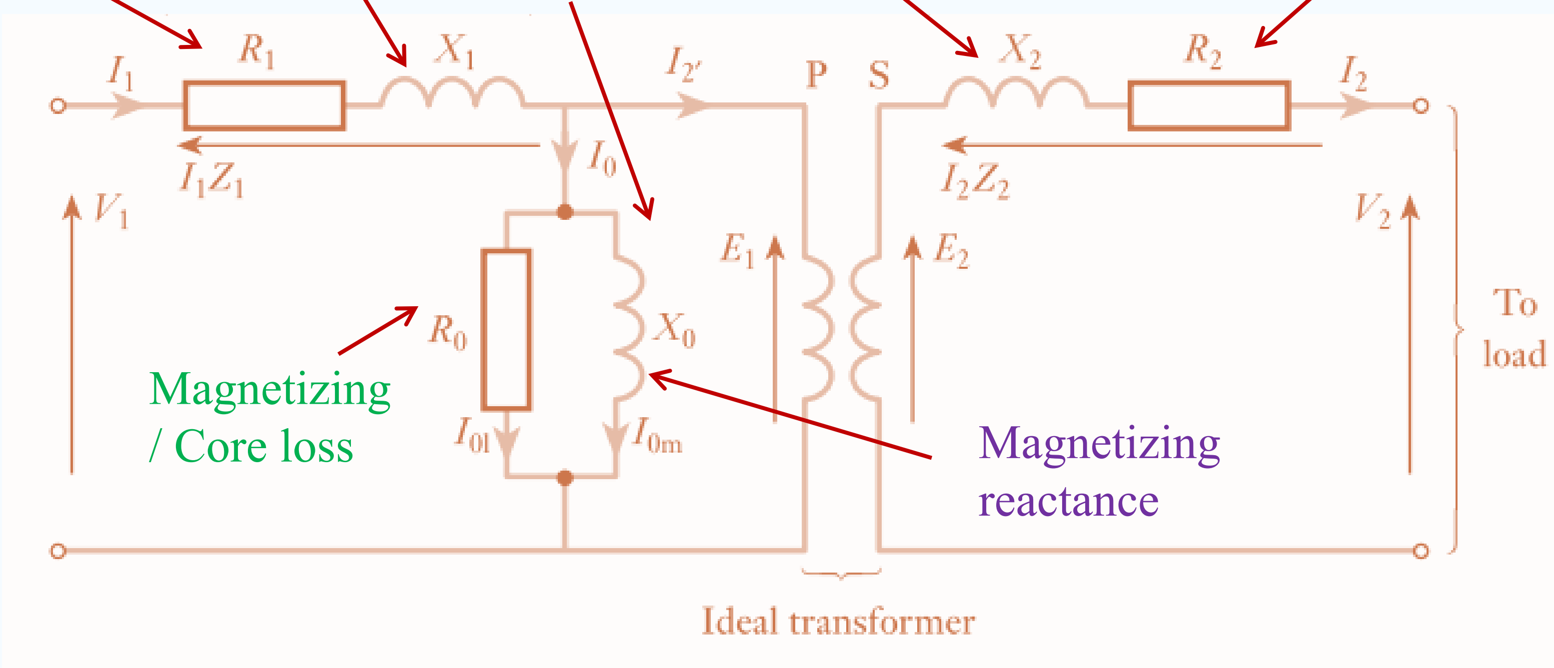
The flux that links both windings: mutual flux

Leakage flux is that which links one winding only and does not contribute to the energy transfer between windings.

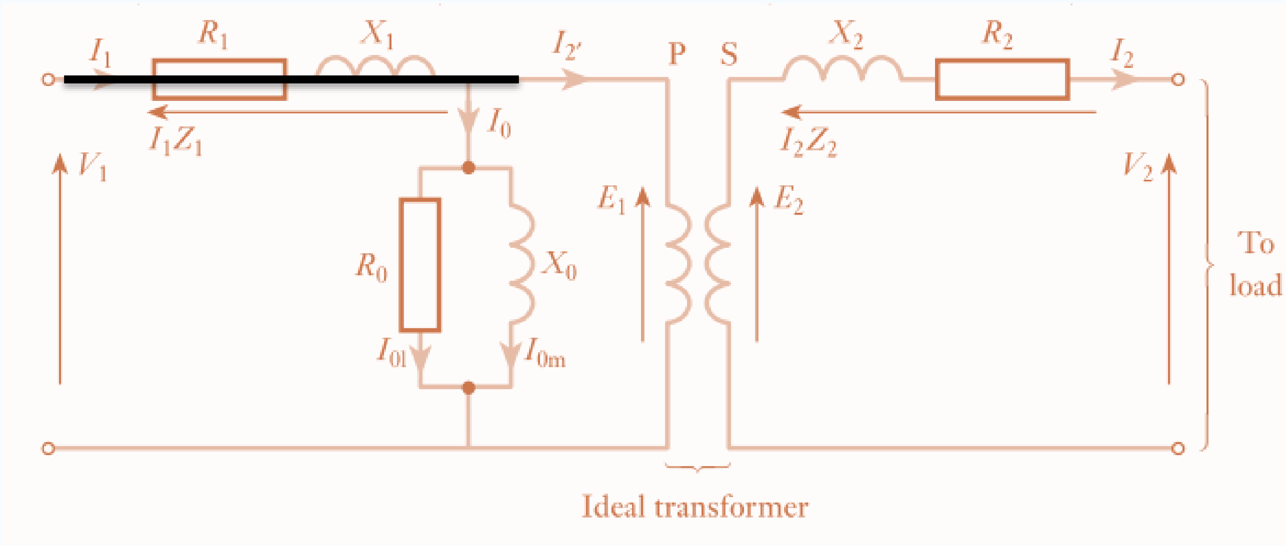


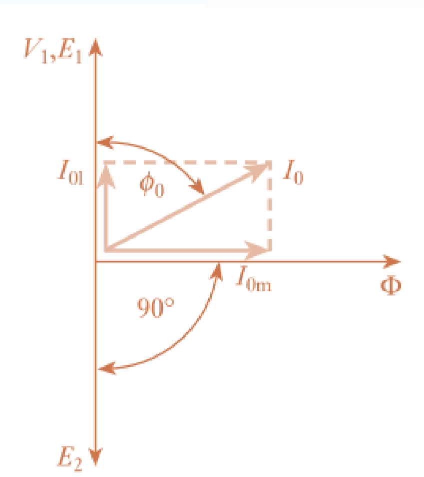
Actual Transformers-Equivalent Circuits

Equivalent-T Circuit



Phasor Diagram of Transformers-no load





Phase diagram

If copper losses in the primary winding is negligible, the no-load current $\boldsymbol{I_0}$ consists of:

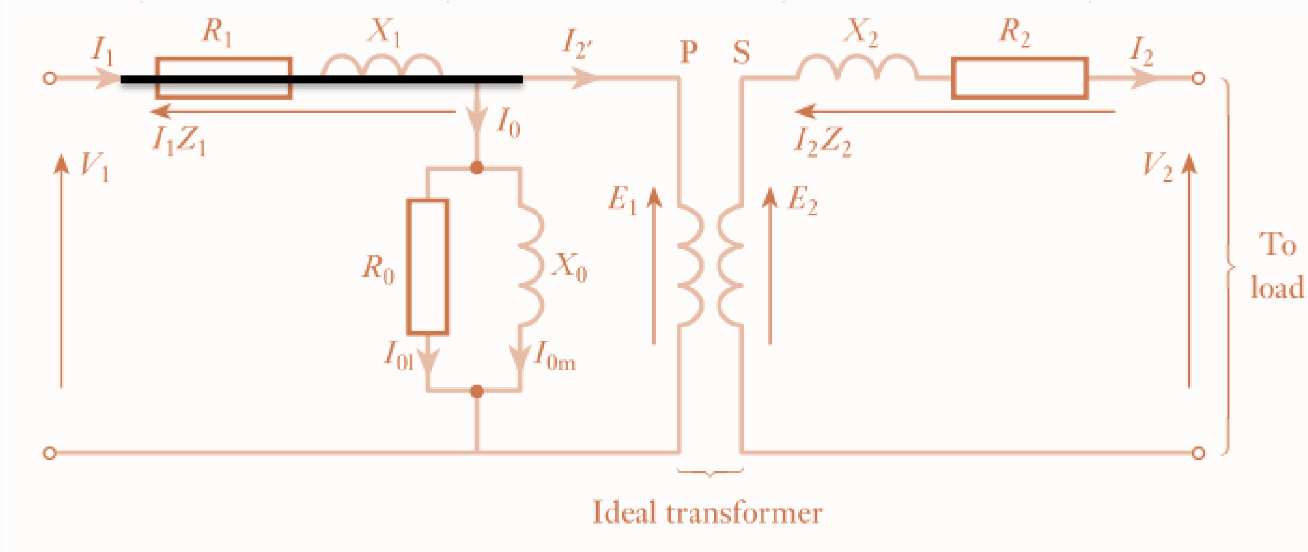
1) a reactive or magnetizing component $\boldsymbol{I_{0m}}$, producing the flux;

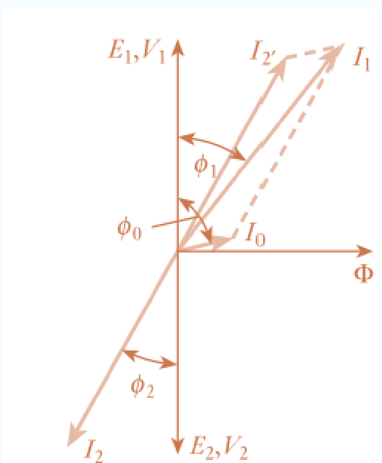
2) an active or power component, $\boldsymbol{I_{01}}$, supplying the hysteresis and eddy current losses in the core, core loss= $I_{01}V_1$

$$\boxed{I_0 = \sqrt{I_{01}^{\ 2} + I_{0m}^{\ 2}}}$$

$$\boxed{\cos\phi_0 = I_{01}V_1 / I_0V_1 = I_{01} / I_0}$$

Phasor Diagram of Transformers-full load





Phase diagram

If the full-load efficiency of a transformer is nearly 100%,

$$\boxed{I_1 V_1 \cos\phi_1 \simeq I_2 V_2 \cos\phi_2}$$

$$\boxed{\begin{aligned} I_1 \cos\phi_1 &= I_0 \cos\phi_0 + I_{2'} \cos\phi_2 \\ I_1 \sin\phi_1 &= I_0 \sin\phi_0 + I_{2'} \sin\phi_2 \end{aligned}}$$

Voltage Regulation

The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because a real transformer has series impedance within it. Full load Voltage Regulation is a quantity that compares the output voltage at no load ($V_{2,nl}$) with the output voltage at full load ($V_{2,fl}$), defined by :

$$VR = \frac{V_{2,nl} - V_{2,fl}}{V_{2,fl}} \times 100\%$$

Ideal transformer, VR = 0%.

Performance Characteristics of Actual Transformers

Efficiency

It is a quantity that compares the output power with input power, defined by :

$$\eta=\frac{P_{out}}{P_{in}}\times100\%=\frac{P_{out}}{P_{out}+\underline{P_{losses}}}\times100\%=\frac{P_{in}-P_{losses}}{P_{in}}\times100\%$$

1) Copper Losses
2) Core losses=Hysteresis losses + Eddy current losses

The efficiency of a transformer is maximum at a load for which the copper loss is equal to the core loss.

Summary

- A transformer converts AC power at one voltage level to AC power of the same frequency at another voltage level.
- Operation Principles: Faraday's induction law
- Ideal transformers: a lossless device with an input winding and an output winding:
  - the windings have no resistance,
  - loss-less magnetic core,
  - reluctance of the core is zero.

$$\frac{V_p}{V_s} = a \qquad \frac{I_p}{I_s} = \frac{1}{a} \qquad \frac{Z_p}{Z_s} = a^2$$

$$P_{in} = P_{out}$$

Actual Transformers

- Copper (I^2R) Losses
- Core Losses:
  - Eddy Current Losses
  - Hysteresis Losses
- Leakage Flux

- Two important performance characteristics:

Voltage regulation:

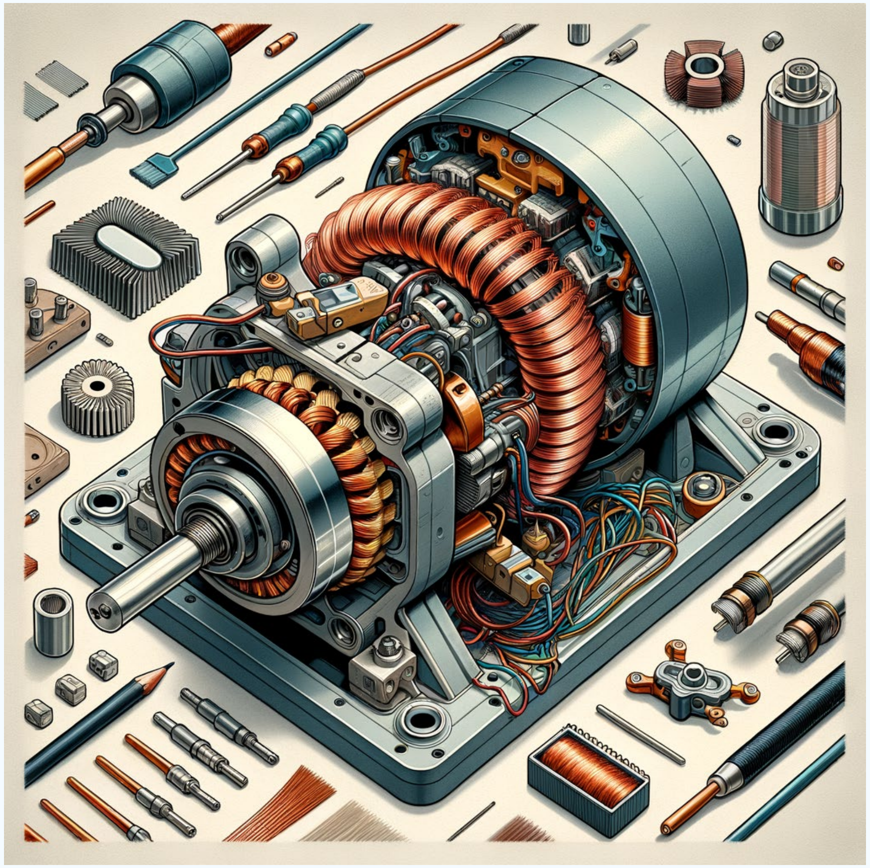
$$VR = \frac{V_{2,nl} - V_{2,fl}}{V_{2,fl}} \times 100\%$$

Efficiency

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

- The maximum efficiency: when the copper loss is equal to the core loss

Next



Linear DC machine

Thanks for your attention