Overview of the topics covered in Weeks 1-8

- Part 1
  - KCL, KVL, resistor networks, voltage divider, etc.
  - Nodal, mesh analysis, supernode/mesh, etc.
  - Superposition, source transformation, Thevenin/Norton equivalent, MPP, etc.

Overview of the topics covered in Weeks 1-8

- **Part 2**
  - Op-amp rules, positive/negative feedback op-amp, etc.
  - Inverting, non-inverting, voltage follower, summing, difference amplifiers
  - *L*, *C*, and their basic characteristics, etc.
  - *RL*, *RC* circuits (source free, driven), etc.

Reflection: Seems need to remember a lot of things? Is that really the case?

Overview of Part 3

- Homework Assignment 2 (HW2) consists of both Part 2 and Part 3 questions. It will be (or was released) in Week 8 during the end of Part 2 delivery.
  - Will introduce it again in lecture now.

- **Week 9: Sinusoidal Steady-state Analysis (AC)**

- **Week 10: AC Circuit Power Analysis**

- **Week 11-12: Complex Frequency and The Laplace Transform, and its applications**
  - Wednesday 29th Nov. (Lect. Group 2) or Thursday 30th Nov. (Lect. Group 1) of **Week 11**: **In-class Quiz (IQ3)**; 10 questions (some are fill-in-the-blank calculations; some are MCQs), about 40 mins.
  - **End of Week 12**: DDL for HW2. Exact date to be announced in LMC.

Week 13: Revision of all three parts (1 hour each part).

✓ **Then, final exam!**

Sinusoidal Steady-State Analysis

EEE103 ELECTRICAL CIRCUITS I (Part 3)

Week 9

S1, 2023/24

Dr. Chee Shen LIM (CS Lim)

Assoc. Prof.

EEE103 Consultation hours: Thursday 11am to 1pm (SC469)

Figure reference: some figures are obtained from McGraw Hill's Engineering Circuit Analysis (main textbook); some are from own drawings.

Content

- Circuit responses to AC inputs
- Complex AC "Forcing Function"
- Phasor representation
- AC circuit analysis using phasors
- Phasor diagram

IMPORTANT: You must have a scientific calculator when attending Part 3 lecture (and exam!). The calculator should be able to handle complex numbers, solve 2^{nd} and 3^{rd} order polynomials, and change between the units of "degree" and "radian".

Introduction

- Complete response of an electric circuit consists of two parts:
  - Natural responses
  - Forced responses

- Alternatively, there are some other terms that carry somewhat similar but may not be of the same exact meaning, depending on the context:
  - **Engineering:** "transient" and "steady-state" responses; "natural" and "forced" responses.
  - **Mathematics:** "homogeneous" and "inhomogeneous/particular" solutions; "complementary" and "particular integral" functions.

- In electrical and electronic engineering, it is very common to see AC excitation and responses, e.g.:
  - Analogue electronic circuits
  - Antenna, radio transmission, high-frequency electromagnetism
  - Power plant, transmission lines, distribution network (residential houses, factories, etc.)
  - Low-frequency electromagnetism, electric motors/machines (vehicles, trains, aircrafts, more-electric aircrafts)

- In this chapter, we do not focus on transient responses (note: they still exist in reality!) but only deal with and analyze steady-state sinusoidal responses.

Must know the symbols notations

Important: Use voltage signal as an example, can you tell me the differences among these symbol notations?

- v – any variable
- $v(t)$ – time-domain voltage signal
- V or V– voltage amplitude, peak, or RMS
- $V(j\omega)$ – phasor domain voltage signal
- V or $\mathbf{V}$ or $\underline{V}$ – voltage vector (complex); $\mathbf{V}$ may also represent matrices (not in this module)
- $v(s)$ or $V(s)$ – s-domain voltage function

Note: *"s" is a unique symbol in mathematics/engineering. It will be used in Laplace transform-based, or s-domain analyses. Avoid using it to represent any unknown variable.*

Circuit responses to AC inputs

Sinusoidal waveforms

- Consider a sinusoidal voltage:

$$v(t) = V_m \sin(\omega t)$$

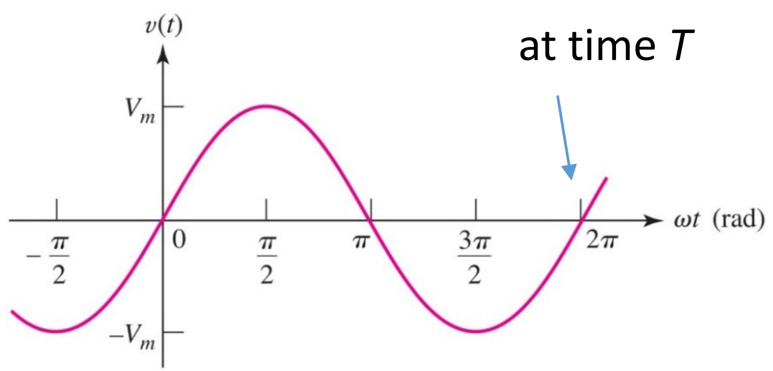
V_m – amplitude (peak) of the sine wave

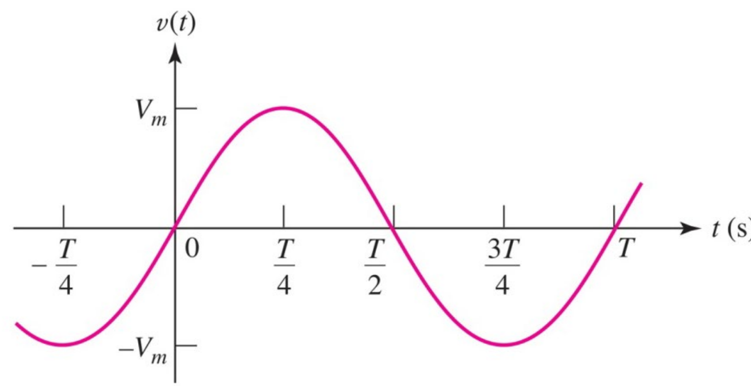
ωt – phase (sometimes known as argument), where ω is the angular frequency

T – time period of the sine wave [s]

f – frequency, = $1/T$ [Hz]

- Two ways to visualize/plot this signal:





voltage v(t) or v(ωt) versus phase ωt

- Waveform period = 2π radians = ωT
- Frequency is f

voltage v(t) versus time t

- Time period is T
- *Frequency is f*

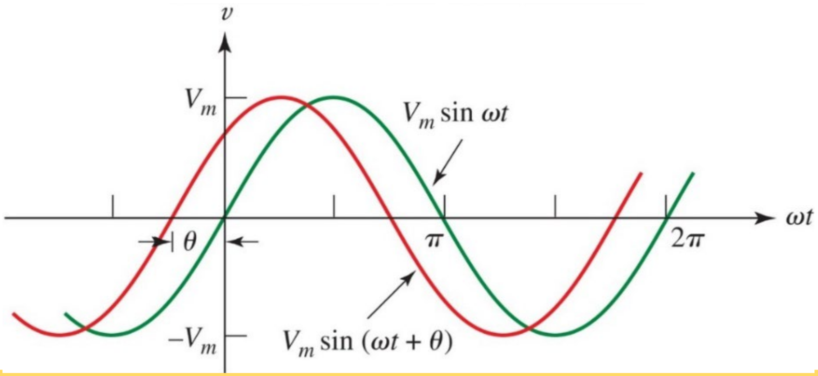
$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

"Leading" and "lagging" terms

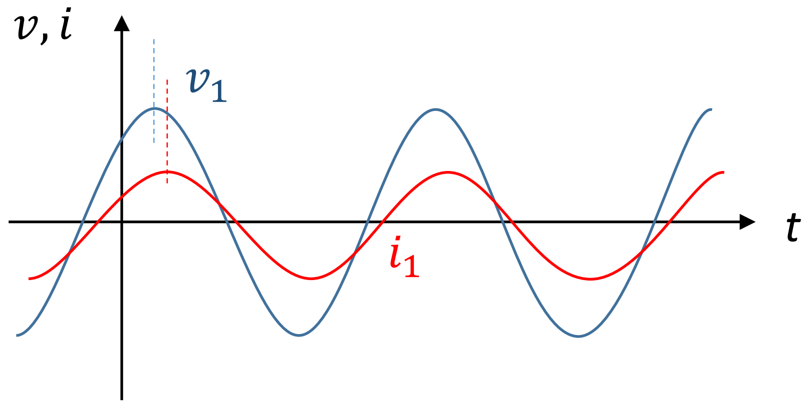
- A more general form of a sine wave includes an additional phase term ϑ

$$v(t) = V_m \sin(\omega t + \theta)$$

- For example: the "red" wave of function $V_m sin(\omega t + \theta)$, assuming θ is a positive value, is said to *lead* the "green" wave of function $V_m sin(\omega t)$ by θ rad.



- The "green" wave can be said to *lag* the "red" wave by θ rad,
- Though less common, we may also say the "green" wave is *leading* the "red" wave by -θ rad.
- Two waves/sinusoids can be said to be "in phase" when their phase arguments are the same; otherwise, they can be said to be "out of phase".
  - **Question:** can we say the same for two waves with different frequency?



$v_1 = 10\cos(100\pi t - 30^o)\,\mathrm{V}$ $\qquad$ $i_1 = 4\cos(100\pi t - 45^o)\,\mathrm{A}$

What can we say about the phase relationship between v_1 and i_1?

Answer: v_1 leads i_1 by [-30°-(-45°)]=15°

Graphically, the signal that first reaches its "peak" (within 180° from the other signal's peak) is the leading signal.

Example 9.1

- Given the following functions (pay attention to units):
  - $v_1 = 10\cos(100\pi t - 30^o)$ V
  - $i_1 = 4\cos\left(100\pi t - \frac{\pi}{4}\right)$ A
  - $i_2 = 3\sin\left(100\pi t + \frac{\pi}{3}\right)$ A
  - $v_s = -2\cos\left(100\pi t + \frac{\pi}{3}\right)$ V

What can we say about the phase relationship between __ and __?

(a) i_1 and i_2

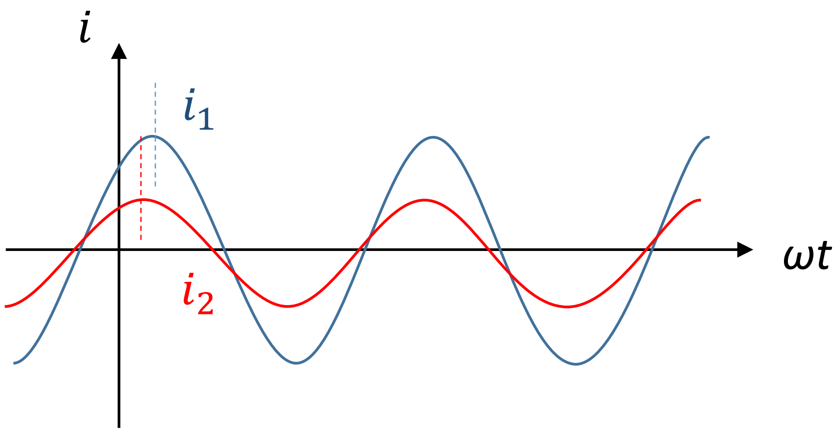
(b) v_1 and v_s

Example 9.1 – sample solutions

$-\sin(\omega t) = \sin(\omega t \pm 180^o)$
$-\cos(\omega t) = \cos(\omega t \pm 180^o)$
$\cos(\omega t + 90^o) = -\sin(\omega t)$
$\sin(\omega t + 90^o) = \cos(\omega t)$
, others to…….

(a) What can we say about the phase relationship between i_1 and i_2?

- $i_1 = 4\cos\left(100\pi t - \frac{\pi}{4}\right) \text{ A} = 4\cos(100\pi t - 45^o) \text{ A}$
- $i_2 = 3\sin\left(100\pi t + \frac{\pi}{3}\right) = 3\cos(100\pi t + 60^o - 90^o) = 3\cos(100\pi t -$



i_1 leads i_2 by [-45°-(-30°)]=-15°

Answer:
i_1 lags i_2 by -15° or
i_2 leads i_1 by 15°

Example 9.1 – sample solutions

(b) What can we say about the phase relationship between v_1 and v_s?

- $v_1 = 10\cos(100\pi t - 30^o)$ V
- $v_s = -2\cos\left(100\pi t + \frac{\pi}{3}\right) = 2\cos(100\pi t + 30^o - 180^o) = 2\cos(100\pi t - 150^o)$ V

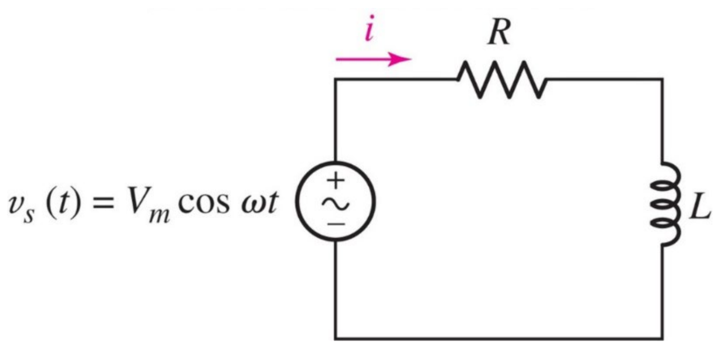
Answer: v_1 leads v_s by [-30^o-(-150^o)]=120^o

Sinusoidal excitation of a circuit

- What will happen in reality?
- What is our focus in this chapter? Why is it relevant?

Forced Response of an RL circuit to a Sinusoidal AC Source (1)

- We can excite an electric circuit using one or many DC or an AC source.
- Let's see what happens when we excite the following RL circuit with a sinusoidal AC source from rest (i.e., assume that the circuit initial condition is zero).



Step (1): Apply KVL:

$$v_s(t) = Ri(t) + L\frac{di(t)}{dt}$$

$$V_m \cos \omega t = Ri(t) + L\frac{di(t)}{dt}$$

Step (2): To proceed, we make an educated guess/assumption of the form of $i(t)$, which is steady-state sinusoidal waveform but with a lagging phase angle of θ_i:

$$i(t) = I_m \cos(\omega t - \theta_i)$$
$$= I_1 \cos \omega t + I_2 \sin \omega t$$

Step (3): Substitute into (1), compare left and right sides:

$$V_m \cos \omega t = R[I_1 \cos \omega t + I_2 \sin \omega t] + L[-I_1 \omega \sin \omega t + I_2 \omega \cos \omega t]$$

Forced Response of an RL circuit to a Sinusoidal AC Source (2)

$$V_m \cos \omega t = (RI_1 + \omega LI_2)\cos \omega t + (RI_2 - \omega LI_1)\sin \omega tL$$

$$V_m = RI_1 + \omega LI_2 \qquad 0 = RI_2 - \omega LI_1$$

Step (4): Solve for I_1 and I_2, and then $i(t)$:

$$I_1 = \frac{RV_m}{R^2 + \omega^2 L^2} \qquad I_2 = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$$

$$i(t) = \frac{RV_m}{R^2 + \omega^2 L^2}\cos \omega t + \frac{\omega L V_m}{R^2 + \omega^2 L^2}\sin \omega t$$

Alternative form:

$$i(t) = I_m \cos(\omega t - \theta_i) = (I_m \cos \theta_i)\cos \omega t + (I_m \sin \theta_i)\sin \omega t$$

$$\left.\begin{aligned} \tan \theta_i &= \frac{\omega L}{R} \to \theta_i = \tan^{-1}\frac{\omega L}{R} \\ I_m &= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \end{aligned}\right\} \qquad i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

Forced Response of an RL circuit to a Sinusoidal AC Source (3)

$$i(t) = \frac{RV_m}{R^2 + \omega^2 L^2} \cos \omega t + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin \omega t$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R} \right)$$

Observation 1: *$|i(t)| \propto V_m$ and depend on the circuit parameters.* $\propto$ means "*proportional*"

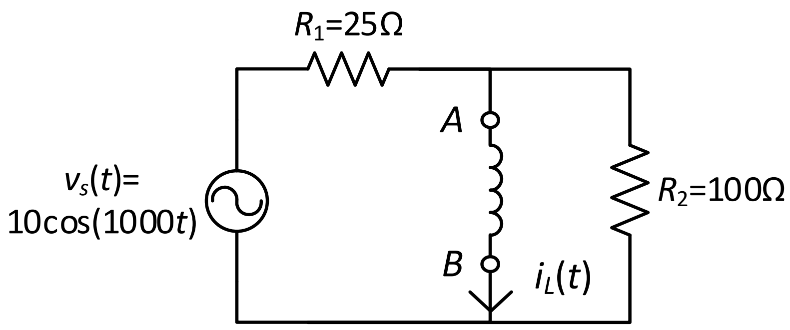
$\angle i(t)$ depends on the circuit parameters and the source frequency.

Observation 2:

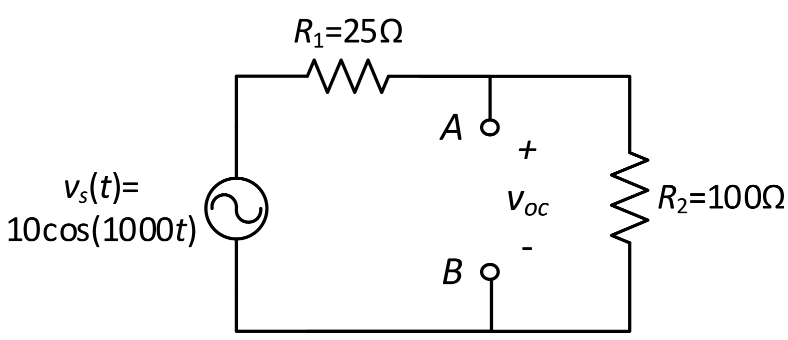
- R is the resistance, which is a quantity that measures the "opposition" imposed by the resistor to the passage of both AC and DC current.
- ωL is the inductance or inductive reactance, which is a quantity that measures the "opposition" imposed by the inductor to the passage of AC current.

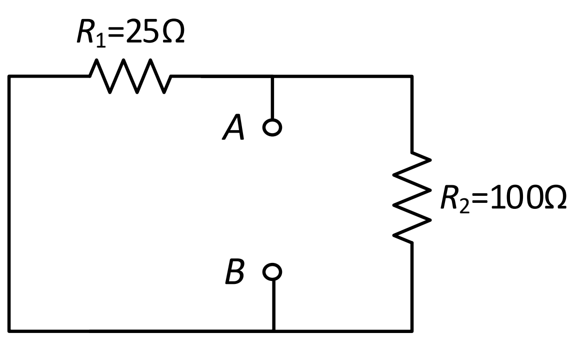
Example 9.2: Solve for steady-state current response in time domain (1)

- Given the following circuit, find the steady-state current $i_L(t)$.



Step (1): If we straightaway apply KCL/KVL to find i_L, the calculation process is somewhat tedious. Alternatively, we can find Thévenin equivalent circuit across the inductor (because we are interested in i_L), being terminals A-B.

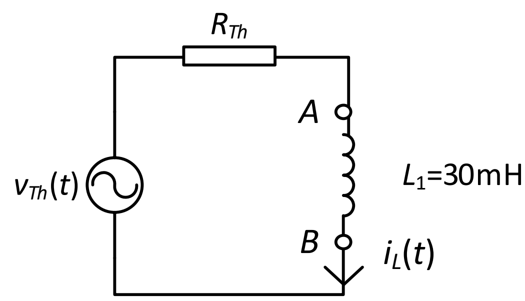




$$v_{Th} = v_{oc} = R_2 \cdot \frac{v_s}{R_1 + R_2} = 8\cos(1000t)$$

$$R_{Th} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2} = 20$$

Example 9.2: Solve for steady-state current response in time domain (2)



$$v_{Th} = 8\cos(1000t)$$

$$R_{Th} = 20$$

Step 2: Recall the steady-state solution for an AC-excited (with the source being a simple cosine function with amplitude V_m) *RL* circuit.

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$i(t) = \frac{8}{\sqrt{20^2 + 1000^2 \times 0.03^2}} \cos\left(1000t - \tan^{-1}\frac{1000 \times 0.03}{20}\right)$$

$$i(t) = 0.222\cos\left(1000t - 56.3^o\right)\text{A} = 222\cos\left(1000t - 0.983\text{rad}\right)\text{mA}$$

Complex AC source or Forcing Function

Complex AC source or Forcing Function

- We live in a "real" world! All the "signals" we see are real, tangible, sometimes "touchable", etc. Also, have you ever wondered why you must learn "complex" number in engineering mathematics?
- In many engineering applications, such as electrical and electronic, mechanical, civil, etc., whenever the analysis involves AC signals, e.g., electric AC power sources, antenna signals, mechanical and building vibration, fluid flow, etc., "complex" number can help to simplify the analysis. We will see that now in the context of AC circuits.
- Recall the Euler's identity, i.e., the trigonometric equivalent expression of the ex

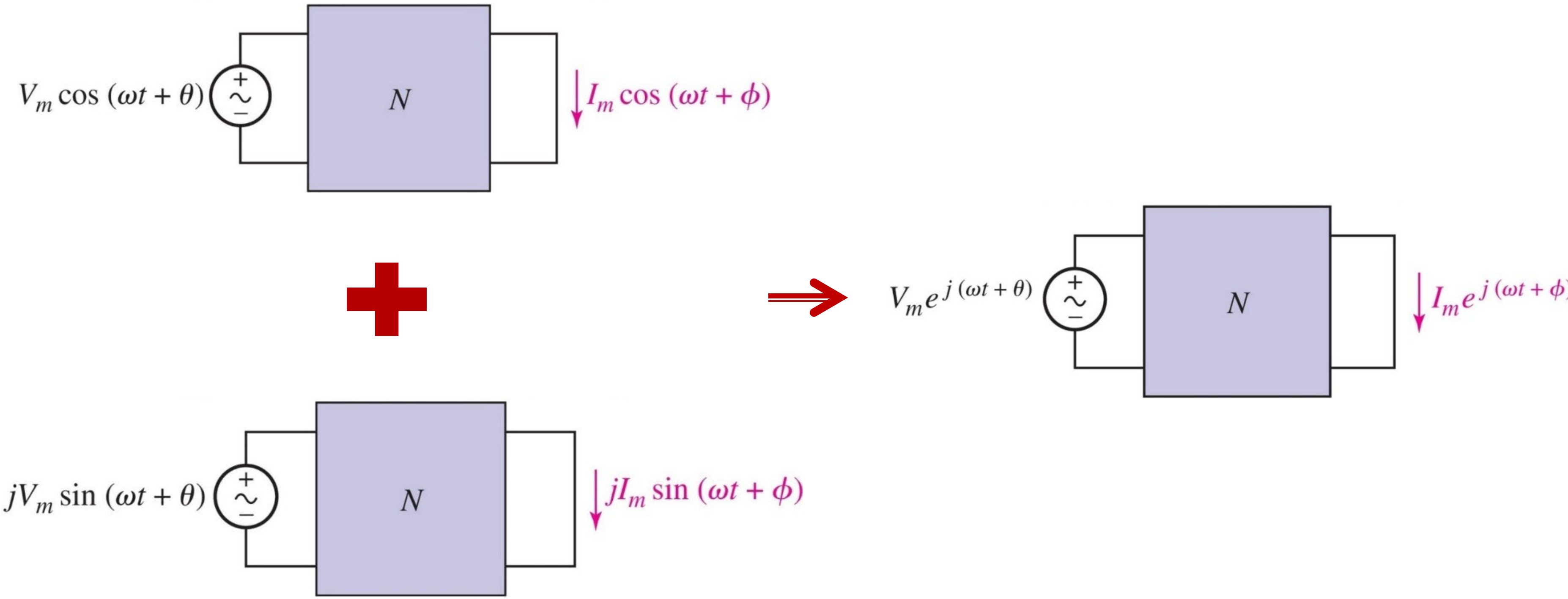
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

- Assume that there is an AC source of $V_m e^{j(\omega t+\theta)}$:

$$V_m e^{j(\omega t+\theta)} = \underbrace{\boxed{V_m \cos(\omega t+\theta)}}_{\text{"real" AC source}} + \underbrace{\boxed{jV_m \sin(\omega t+\theta)}}_{\text{"imaginary" AC source}}$$

Complex AC source or Forcing Function

- Recall the "Principle of superposition" for a linear circuit: the overall response due to multiple independent sources onto a circuit can be obtained by summing the individual responses of the independent sources onto the circuit.

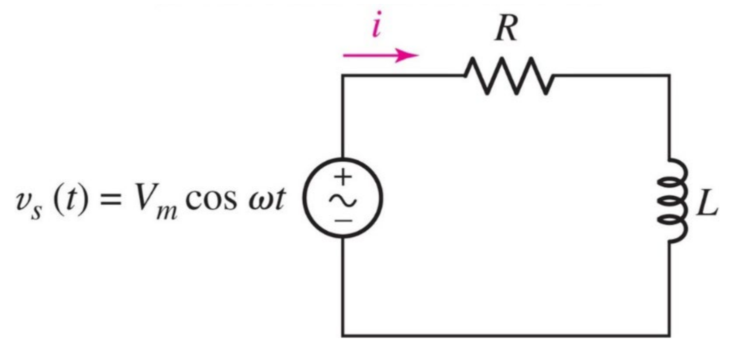


- The concept is best learnt through an example. See next.

Forced Response of an RL circuit - Revisit using Complex Forcing Function (1)

Xi'an Jiaotong-Liverpool University 西交利物浦大学

- Given the following circuit, find the steady-state current $i_L(t)$ using the complex forcing function concept .



$$v_s(t) = Ri(t) + L\frac{di(t)}{dt}$$

Step (1): Write the circuit equation, and express the AC source in the complex form:

$$V_m \cos \omega t = \mathrm{Re}\left\{V_m e^{j\omega t}\right\} = \mathrm{Re}\left\{V_m \cos \omega t + jV_m \sin \omega t\right\}$$

Step (2): Assume the function of current i is in the complex form too, with real amplitude I_m and unknown phase angle Φ:

$$i(t) = I_m \cos(\omega t + \phi) = \mathrm{Re}\left\{I_m e^{j(\omega t + \phi)}\right\}$$

Step (3): Substitute complex (real and imaginary parts) functions and compare:

$$V_m e^{j\omega t} = RI_m e^{j(\omega t + \phi)} + L\frac{d}{dt}\left[I_m e^{j(\omega t + \phi)}\right]$$

$$= RI_m e^{j(\omega t + \phi)} + j\omega L I_m e^{j(\omega t + \phi)}$$

Forced Response of an RL circuit - Revisit using Complex Forcing Function (2)

Step (4): Solve for $i(t)$

$$V_m e^{j\omega t} = (R + j\omega L) I_m e^{j\omega t} e^{j\phi}$$

$$V_m = (R + j\omega L) I_m e^{j\phi}$$

$$I_m e^{j\phi} = \frac{V_m}{R + j\omega L}$$

$$= \frac{V_m}{(\sqrt{R^2 + \omega^2 L^2}) e^{j \tan^{-1}(\omega L / R)}}$$

$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{-j \tan^{-1}(\omega L / R)}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$$

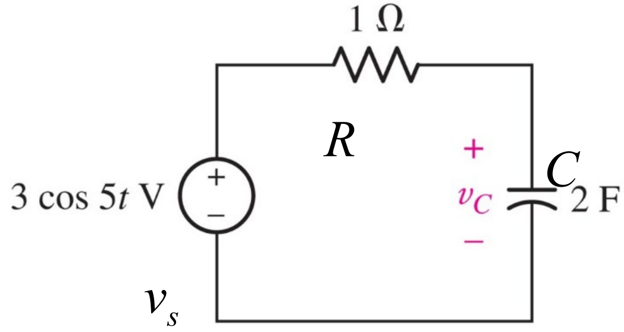
$$\phi = -\tan^{-1}(\omega L / R)$$

Solution for $i(t)$:

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R}\right)$$

Example 9.3: Forced response of an RC circuit to an Sinusoidal AC source (1)

- Find the voltage $v_c(t)$ across the capacitor.



Step (1): Writing KVL of the loop in the right-hand-side (RHS) circuit

$$v_s(t) = 3e^{j5t} = Ri_C(t) + v_C(t)$$

Across the capacitor: $$i_c(t) = C\frac{dv_C(t)}{dt}$$

We have: $$v_s(t) = RC\frac{dv_C(t)}{dt} + v_C(t)$$

Example 9.3: Forced response of an RC circuit to an Sinusoidal AC source (2)

Xi'an Jiaotong-Liverpool University 西交利物浦大学

Step (2): Assume that $v_c(t)$ takes the following form:

$$v_C(t) = \mathrm{Re}\left\{\boldsymbol{V}_m e^{j5t}\right\}$$

NOTE: Here, $\boldsymbol{V}_m$ is a complex value. If you would like a real-value V_m, then the expression should be modified to $V_m e^{j(5t+\varphi)}$ (like previous example).

Step (3): Substitute and manipulate further.

$$3e^{j5t} = j5RC\boldsymbol{V}_m e^{j5t} + \boldsymbol{V}_m e^{j5t} = j5(1)(2)\boldsymbol{V}_m e^{j5t} + \boldsymbol{V}_m e^{j5t}$$

$$\boldsymbol{V}_m = \frac{3}{1+j10} = \frac{3}{\sqrt{101}} \angle -\tan^{-1}(10)$$

e^{j5t} is common to all terms.

Step (4): Time-domain expression for $v_c(t)$:

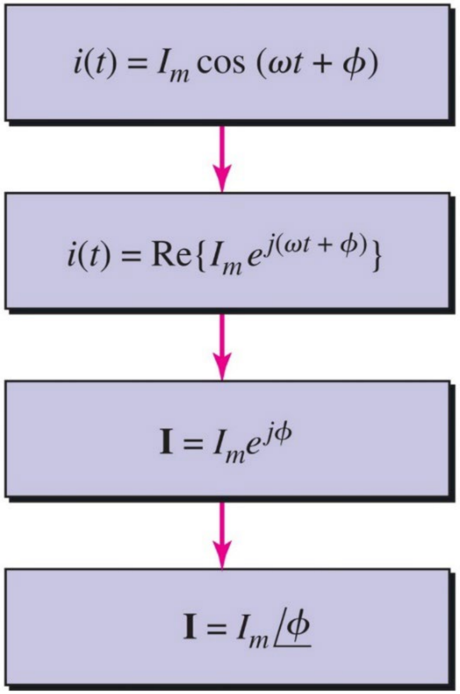
$$v_c(t) = \mathrm{Re}\left\{\frac{3}{\sqrt{101}} e^{-j\tan^{-1}(10)} \cdot e^{j5t}\right\} = \mathrm{Re}\left\{\frac{3}{\sqrt{101}} e^{j\left[5t-\tan^{-1}(10)\right]}\right\}$$

$$v_c(t) = 0.2985\cos(5t - 84.3^o)\ \mathrm{V}$$

Introducing "Phasor" representation (1)

- It can be noticed from previous examples that the term $e^{j\omega t}$ is common to all voltage/current expressions and is cancelled out from the intermediate equations when solving for answer.
- For example, assume that there is only single angle frequency component ω and we are interested in the steady state solutions. We have:

$$
\begin{aligned}
i(t) &= 10\cos(\omega t + 30^o) = \mathrm{Re}\left\{10e^{j(\omega t+\pi/6)}\right\} \\
&= \mathrm{Re}\left\{10e^{j\pi/6}e^{j\omega t}\right\} = \mathrm{Re}\left\{\mathbf{I}e^{j\omega t}\right\}
\end{aligned}
$$



- The phasor of time-domain voltage waveform $i(t)$ of angular frequency ω can be written as (normally in bold):

$$\mathbf{I} = 10e^{j\pi/6} \qquad \text{or} \qquad \mathbf{I} = 10\angle 30^o \ \text{ or } \ 10\angle \pi/6$$

$$\underset{\sim}{I} = 10e^{j\pi/6} = 10\angle \pi/6$$

Introducing "Phasor" representation (2)

Xi'an Jiaotong-Liverpool University 西交利物浦大学

- Given a phasor, to "transform" it back to time domain, unless otherwise specified by the question, we can write the time-domain function in sine or cosine.
- For example, given $\mathbf{V} = 220e^{j2\pi/3}$ .The time-domain function is either

$$v(t) = 220\cos(\omega t + \frac{2\pi}{3}) \quad \text{or} \quad v(t) = 220\sin(\omega t + \frac{2\pi}{3})$$

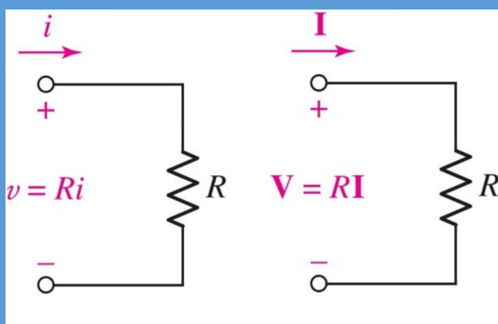
- What is the value of the angular frequency *ω*?
  - We cannot tell from the phasor expression alone.
  - Its value depends on the context of the problem, normally given or can be deduced easily from the problem in hand.
- One must not forget that *v*(*t*) is a sinusoidal function. It means that we can calculate the instantaneous value. For example, assume a cosine function for the above *v*(*t*) and *ω* = 314.16 rad/s, the voltage at time 10 ms is

$$v(0.01) = 220\cos(314.16 \times 0.01 + \frac{2\pi}{3}) = 110\text{V}$$

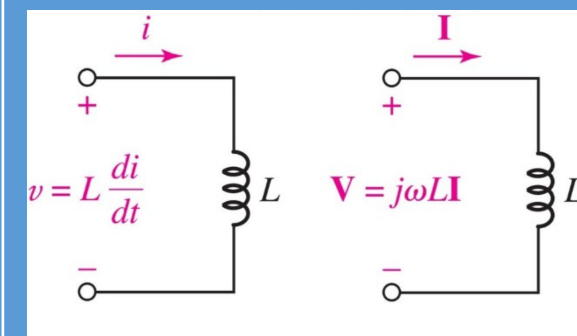
- In some textbooks, phasor is sometimes known to be in "*frequency*" domain. **Try not to use this term!** Use "*phasor*" domain instead. As you learn more advanced circuit analyses, you will revisit this topic with more advanced tools.

Phasors for *R*, *L* and *C* elements

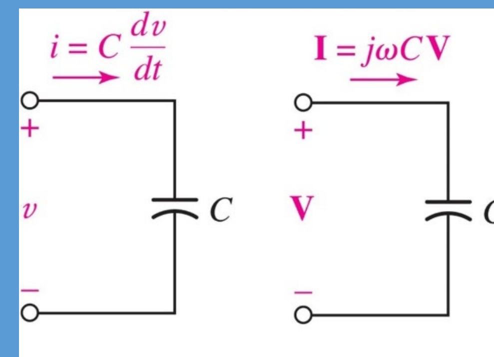
- In phasor representation (sometimes, rather rarely, frequency domain representation), we now see how phasor simplifies the circuit analysis for individual *R*, *L*, and *C* component, start by substituting complex functions into the differential equations.



$$v(t) = Ri(t)$$
$$V_m e^{j(\omega t+\theta)} = RI_m e^{j(\omega t+\phi)}$$
$$V_m e^{j\theta} = RI_m e^{j\phi}$$
$$\mathbf{V} = R\mathbf{I}$$



$$v = L\frac{di}{dt}$$
$$V_m e^{j(\omega t+\theta)} = L\frac{d}{dt}\left(I_m e^{j(\omega t+\phi)}\right)$$
$$V_m e^{j\theta} = j\omega L I_m e^{j\phi}$$
$$\mathbf{V} = j\omega L\mathbf{I}$$



$$i(t) = C\frac{dv(t)}{dt}$$
$$I_m e^{j(\omega t+\phi)} = C\frac{d}{dt}\left(V_m e^{j(\omega t+\theta)}\right)$$
$$I_m e^{j\phi} = j\omega C V_m e^{j\theta}$$
$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$$

Phasor Voltage/Current Relationships

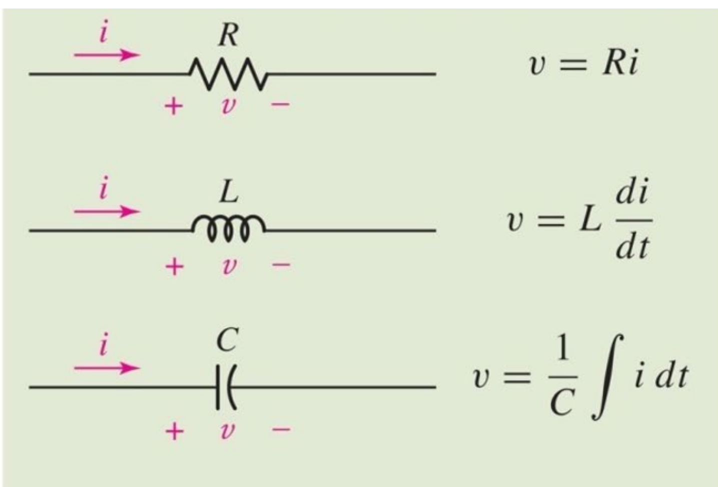
Xi'an Jiaotong-Liverpool University 西交利物浦大学

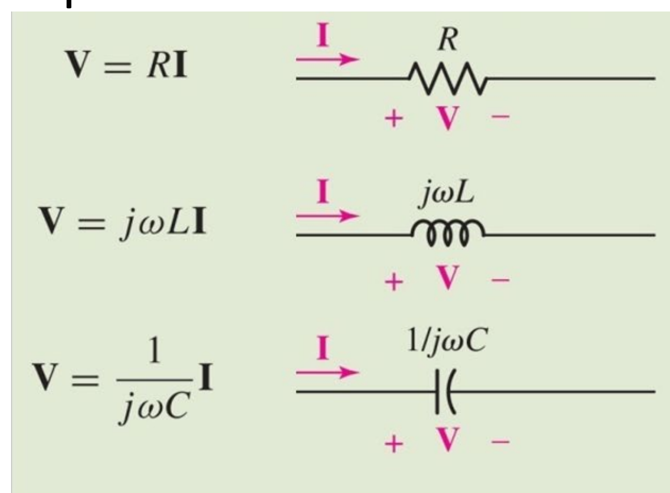
Time domain

- Integrodifferential equation
- Requires ODE to solve
- KVL, KCL (and all other techniques) apply to with integrodifferential equations.

Phasor (a.k.a. frequency domain)

- Algebraic equation
- Only need algebraic operation, means “+,-,x, / ”
- KVL, KCL (and all other techniques) are still applicable to phasors.





Question: Phasor representation provides an alternative to solving circuit equations using ODE/PDE. Can it fully replace the fundamental circuit analysis techniques?

Kirchhoff’s Laws for Phasors

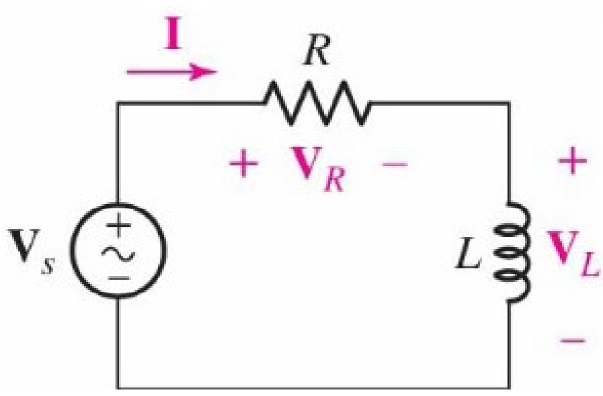
- Applying KVL in phasors: the algebraic sum of the voltage drops around any closed path is zero:

$$\mathbf{V}_1 + \mathbf{V}_2 + \ldots + \mathbf{V}_N = \mathbf{0}$$

- Applying KCL for phasors: the algebraic sum of the currents entering any node is zero.

$$\mathbf{I}_1 + \mathbf{I}_2 + \ldots + \mathbf{I}_N = \mathbf{0}$$

- For example, given $\mathbf{V}_s = V_m \angle 0^o$, find $\mathbf{I}$:



$$\mathbf{V}_s = \mathbf{V}_R + \mathbf{V}_L \quad \text{or} \quad \mathbf{V}_s - \mathbf{V}_R - \mathbf{V}_L = 0$$

$$\mathbf{V}_s = R\mathbf{I} + j\omega L\mathbf{I}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Impedance and Admittance

Impedance

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

- Impedance is **Z** = *R+jX*; *R* is the *resistance*, *X* is the *reactance* (unit ohm Ω)
- Impedance is the ratio of voltage to current.
- It can be seen as an equivalent of resistance in the phasor domain; impedances in series or parallel can be combined using "resistor rules."

Admittance

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}}$$

- Admittance **Y** = *G+jB; G* is the *conductance*, *B* is the *susceptance*: (unit siemen S)
- Admittance is the reciprocal of impedance, i.e., the ratio of current to voltage.

Important concept: Impedance for individual element

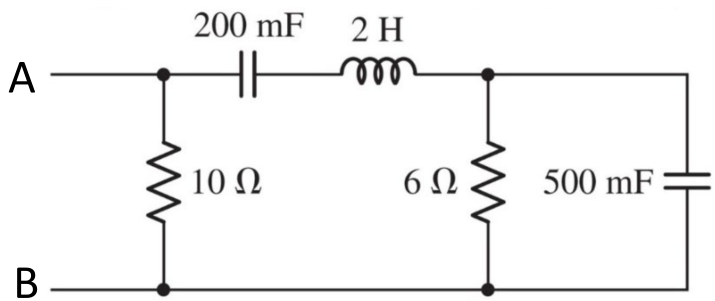
$$\boxed{\mathbf{Z}_R = R \qquad \mathbf{Z}_C = 1/j\omega C \qquad \mathbf{Z}_L = j\omega L}$$

$$\mathbf{Y}_R = \frac{1}{R} \qquad \mathbf{Y}_C = j\omega C \qquad \mathbf{Y}_L = \frac{1}{j\omega L}$$

Example 9.4: Using Phasors to find Equivalent Impedance

- Find the equivalent impedance of the network at 5 rad/s.

$$\mathbf{Z}_{C_1} = \frac{1}{j(5)(0.2)} = -j$$



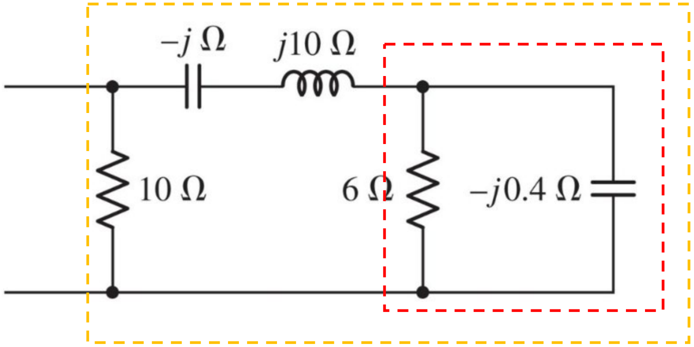
$$\mathbf{Z}_L = j(5)(2) = j10$$

$$\mathbf{Z}_{C_2} = \frac{1}{j(5)(0.5)} = \frac{j}{(-1)2.5} = -j0.4$$

Solution:

Step (1): Express all element as individual impedance (i.e., resistance or reactance) terms.

Step (2): Simplify in several steps to get the equivalent impedance across A-B.



$$\frac{(6)(-j0.4)}{6-j0.4} = 0.02655 - j0.3982$$

$$10 \,||\, (-j + j10 + 0.02655 - j0.3982)$$
$$= 4.255 + j4.929$$

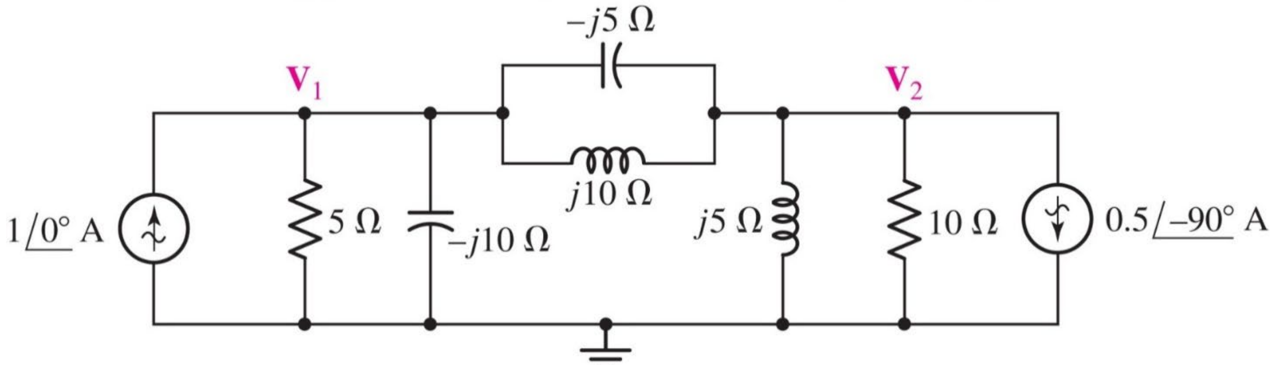
Equivalent impedance as seen from terminals A - B

$$= 4.255 + j4.929 = 6.512\angle 49.2^o \ \ \Omega$$

Example 9.5: Apply Nodal Analysis using Phasors

Xi'an Jiaotong-Liverpool University 西交利物浦大学

- Find the phasor voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.



Step (1): Apply KCL at node of $\mathbf{V}_1$ and $\mathbf{V}_2$:

$$\frac{\mathbf{V}_1}{5}+\frac{\mathbf{V}_1}{-j10}+\frac{\mathbf{V}_1-\mathbf{V}_2}{-j5}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j10}=1\angle 0^o=1+j0$$

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{-j5}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j10}+\frac{\mathbf{V}_2}{j5}+\frac{\mathbf{V}_2}{10}=-0.5\angle -90^o=j0.5$$

Step (2): Simplify and solve for $\mathbf{V}_1$ and $\mathbf{V}_2$:

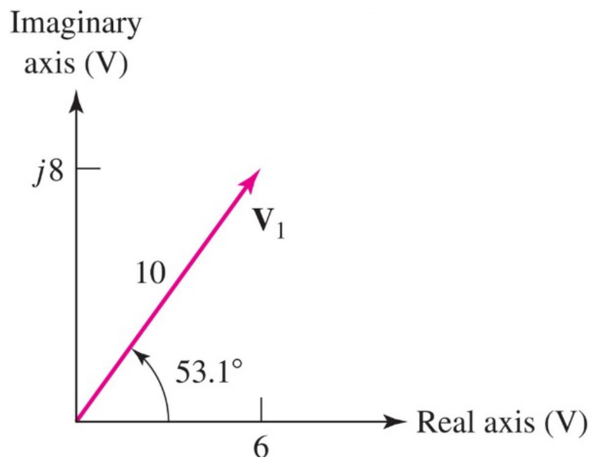
$$(-2+j2)\mathbf{V}_1+\mathbf{V}_2=j10$$
$$\mathbf{V}_1+(1+j)\mathbf{V}_2=-5$$

$$\mathbf{V}_1=1-j2\ \text{V}=2.24\angle -63.4^o\ \text{V}$$
$$\mathbf{V}_2=-2+j4\ \text{V}=4.47\angle 116.6^o\ \text{V}$$

Note: if the frequency info is given, we can continue to get the time-domain function of $v_1(t)$ and $v_2(t)$. They must be of the same frequency.

Phasor Diagrams

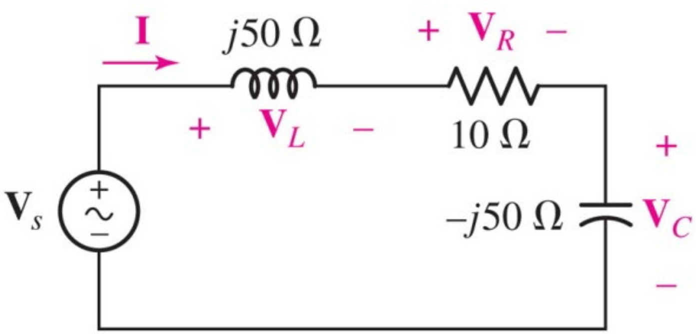
- Phasor Diagram is a complex plane drawing that shows the relationships of the phasor voltages and currents in a circuit.
- An example of phasor diagram showing phasor $\mathbf{V}_1 = 6 + j8$.



- In almost all E&E subjects, we typically refer phasors to the "frequency-domain" phasor, e.g., $\mathbf{V}_1$ above, without the $exp(j\omega t)$ component. Recall $v_1(t) = \mathrm{Re}(\mathbf{V}_1 e^{j\omega t})$.
  - **NOTE:** in textbook, another time-domain phasor is also introduced, but this is rarely relevant in practice; there a rotating voltage/current quantity in the complex diagram is more commonly known as "vector", instead of "phasor".

Phasors in Series RLC

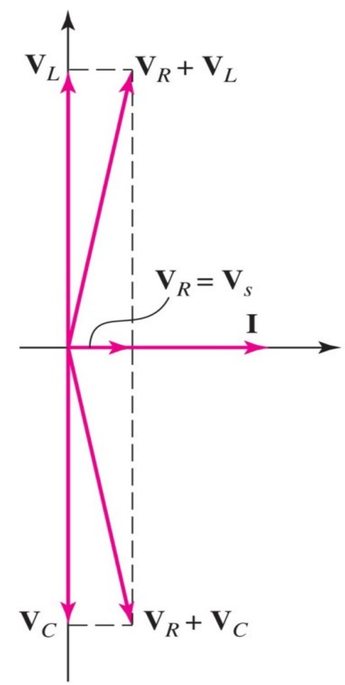
- The series RLC circuit below has several voltage and current phasor quantities:
  - Source voltage $\mathbf{V}_s$
  - Inductor voltage drop $\mathbf{V}_L$
  - Resistor voltage drop $\mathbf{V}_R$
  - Capacitor voltage drop $\mathbf{V}_C$
  - Series circuit current **I**



- If source voltage $v_s(t)$ is known, then its phasor $\mathbf{V}_s$ is a good choice to be the reference (more common in practice), to better match with its nature being the "voltage source".

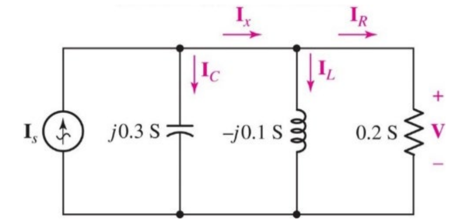
- If source voltage $\mathbf{V}_s$ is not known, then we may select **I** as the reference. We may assume magnitude of 1 (only for diagram sketching purpose), and the rest of the phasor length is scaled accordingly.

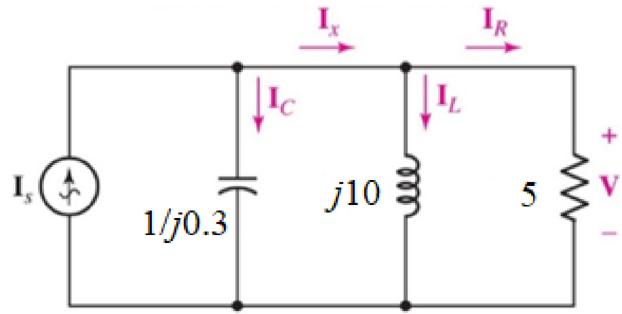
$$\mathbf{I} = I_m \angle 0^o \qquad \mathbf{I} = I_m \angle 0^o = 1\angle 0^o$$



Phasors in Parallel RLC

- The series RLC circuit below has several voltage and current phasor quantities:
  - Source current $\mathbf{I}_s$
  - Inductor current $\mathbf{I}_L$
  - Resistor current $\mathbf{I}_R$
  - Capacitor current $\mathbf{I}_C$
  - Parallel circuit voltage $\mathbf{V}$

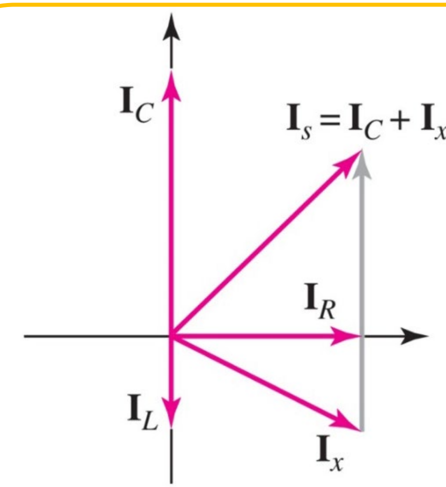




- If source current $\mathbf{I}_s$ is known, then its phasor $\mathbf{I}_s$ is a good choice to be the reference, to match with the "current source" (more common in practice).

- Alternatively, if source voltage $\mathbf{I}_s$ is not known, then we can select $\mathbf{V}$ as the reference and assume a magnitude of 1 (only for diagram sketching purpose), and the rest of the phasor length is scaled accordingly.

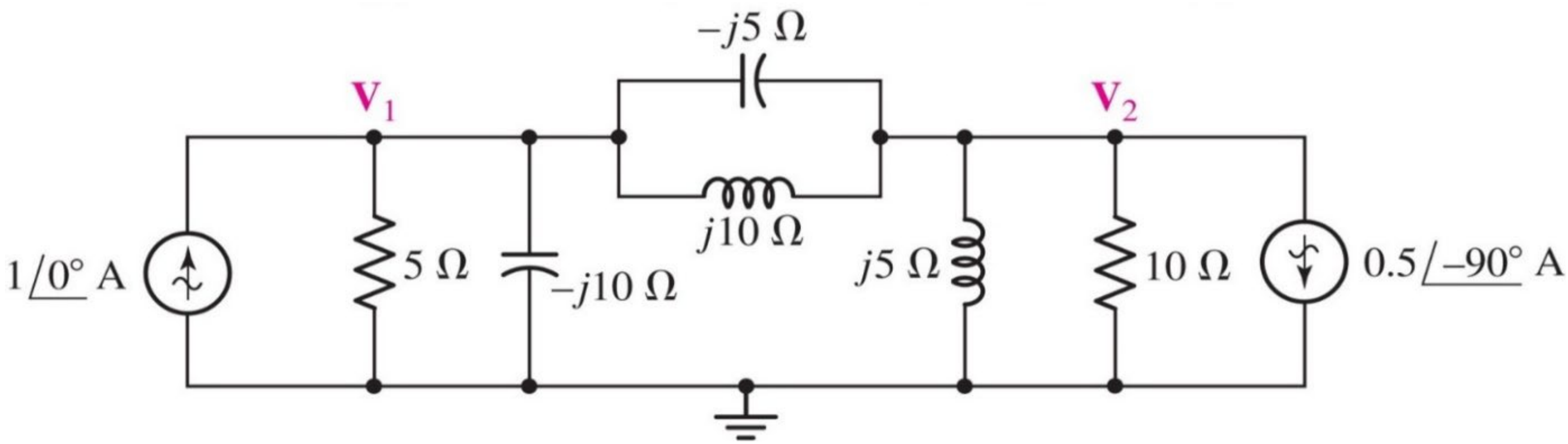
$$\mathbf{V} = V_m \angle 0^o \qquad \mathbf{V} = V_m \angle 0^o = 1\angle 0^o$$



Example 9.6: Apply the Principle of Superposition through Phasors, and Phasor Diagram (1)

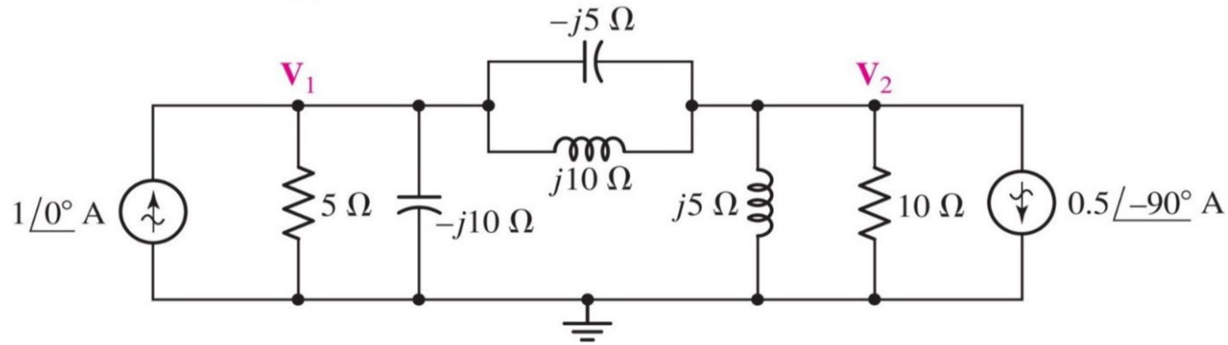
(a) Find the phasor voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.

(b) Then, draw the phasors of the two current sources and $\mathbf{V}_1$ and $\mathbf{V}_2$.



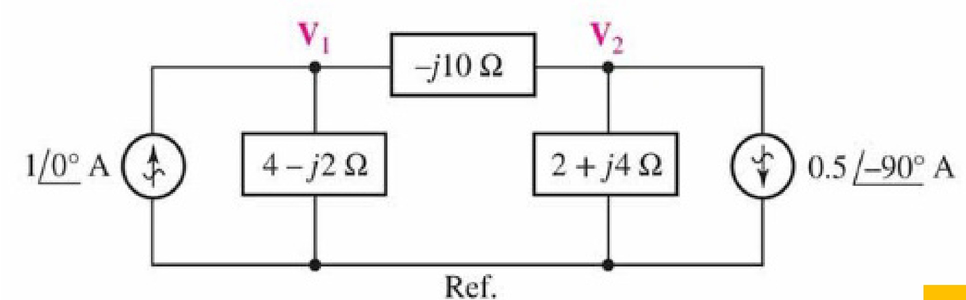
Example 9.6: Apply the Principle of Superposition through Phasors, and Phasor Diagram (1)

(a) Find the phasor voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.



Step (1): Express as individual impedance, and simplify some parts

Step (2): Apply Principle of Superposition: find parts of $\mathbf{V}_1$ response caused by individual independent sources, and sum all to get $\mathbf{V}_1$ (then, repeat for $\mathbf{V}_2$).



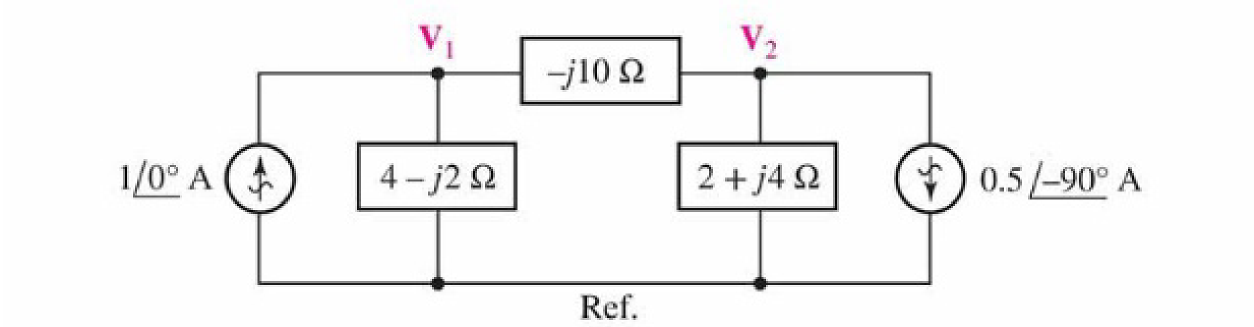
$$\mathbf{V}_{1,S_1} = \left(1\angle 0^o\right)\left[(4-j2)\,||\,(-j10+2+j4)\right]$$
$$= 2 - j2\text{ V}$$

$$\mathbf{V}_{1,S_2} = -\left(0.5\angle -90^o\right)\left[\frac{2+j4}{4-j2-j10+2+j4}\right](4-j2)$$
$$= -1\text{ V}$$

$$\mathbf{V}_1 = \mathbf{V}_{1,S_1} + \mathbf{V}_{1,S_2} = 1 - j2\text{ V} = 2.236\angle\text{-}63.4^{\text{o}}\text{ V}$$

Example 9.6: Apply the Principle of Superposition through Phasors, and Phasor Diagram (2)

- **Step (3):** Apply Principle of Superposition: find parts of $\mathbf{V}_2$ response caused by each independent sources, and sum all to get $\mathbf{V}_2$.



$$\mathbf{V}_{2,S_1} = (1\angle 0^o)\left(\frac{4-j2}{4-j2-j10+2+j4}\right)(2+j4)$$

$$= 0 + j2\text{ V}$$

$$\mathbf{V}_{2,S_2} = -(0.5\angle -90^o)[(2+j4)\,||\,(4-j2-j10)]$$

$$= -2 + j2\text{ V}$$

$$\mathbf{V}_2 = \mathbf{V}_{2,S_1} + \mathbf{V}_{2,S_2} = -2 + j4\text{ V} = 4.472\angle 116.6^{\text{o}}\text{ V}$$

Example 9.6: Apply the Principle of Superposition through Phasors, and Phasor Diagram (3)

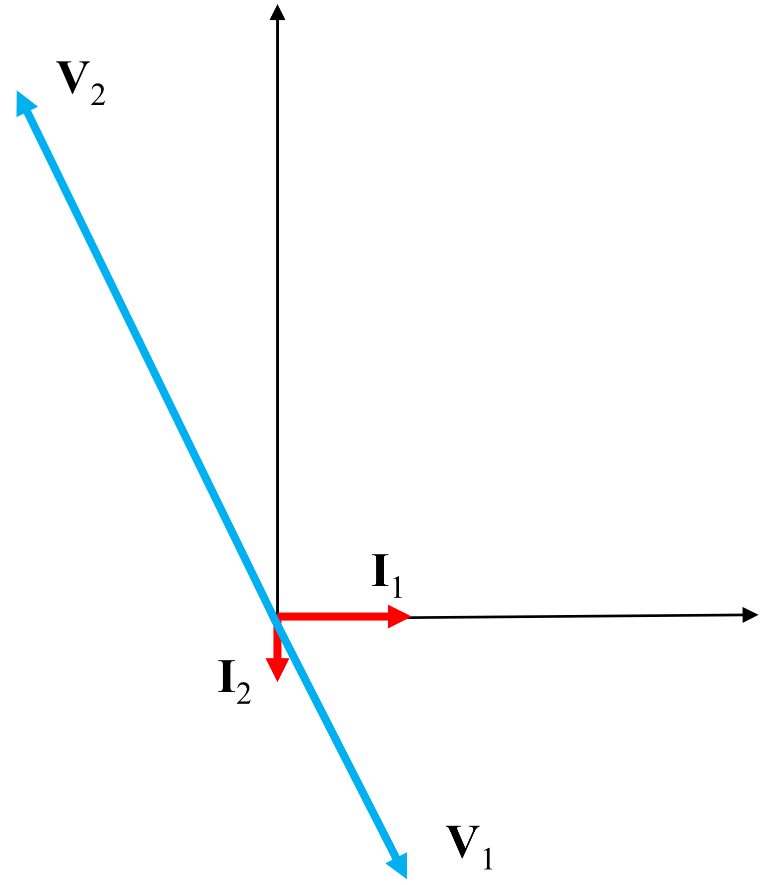
(b) Then, draw the phasors of the two current sources and $\mathbf{V}_1$ and $\mathbf{V}_2$.

$$\mathbf{I}_1 = 1\angle 0^\circ \text{ A}$$

$$\mathbf{I}_2 = 0.5\angle -90^\circ \text{ A}$$

$$\mathbf{V}_1 = 2.236\angle -63.4^\circ \text{ V}$$

$$\mathbf{V}_2 = 4.472\angle 116.6^\circ \text{ V}$$



Tutorial, and some selected questions from the Textbook for self-practices

- **Week 9**

(1) Please proceed to your tutorial session for some tutorial exercises.

Group 2	(Tutorial*)	(SA136*, SB152*, SB120*)	(continue to 12noon-1pm)

***NOTES:** The tutorial rooms are allocated according to your programmes. Please attend to the assigned session BUT NOT other rooms to avoid overcrowding. **Attendance of tutorials will be taken.**

SA136 – ~~CST and DMT students~~ CST and **EE** students (updated on 13th Sept. 2023)

SB152 – ~~EE and EST students~~ **DMT** and EST students (updated on 13th Sept. 2023)

SB120 – MRS and TE students

(2) After your tutorial, you can also self-practice some questions. For example:

Engineering Circuit Analysis, 9th or 10th ed., Chapter 10

Pg. 420 – 430: 35, 44, 50, 62, 76

- If you have extra time, others too (but be selective; you don't have time for all questions!).
- These questions will be displayed in the tutorial class, and sample solutions will be uploaded to LMC->Tutorials folder for your self-checking.

~ THE END ~