Xi'an Jiaotong-Liverpool University 西交利物浦大学

AC Circuit Power Analysis

EEE103 ELECTRICAL CIRCUITS I (Part 3)

Week 10

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Figure reference: some figures are obtained from McGraw Hill's Engineering Circuit Analysis (main text book); some are from own drawings.

Content

- Instantaneous power
- Average power
- Maximum power transfer
- RMS values
- Apparent power and power factor
- Complex power

Introduction

- Electrical power is an important quantity in our society, which only gets more important due to increasingly concern on energy sustainability. Its value has a wide range:
  - Picowatt: telemetry signal from outer space,
  - Several to tens of watts: audio speaker, mobile phones, TV
  - Hundreds watts to several kilowatts: PC, electric bikes, home appliances
  - Tens to hundreds of kilowatts: large motors in factories, electric cars
  - Megawatts and above: substation transformers, distribution and transmission networks, power plants, locomotives, electric propulsion, future more-electric aircrafts

- Different power quantities will be introduced in this chapter:
  - You may find that some of the previous circuit representation are not suitable to be used to calculate some power quantities;
  - You may also find that the simple concept of average power is insufficient to capture the full picture of energy exchange between AC power sources and reactive loads.

Instantaneous Power

- The instantaneous power delivered to any device is given by

$$p(t) = v(t)i(t)$$

- E.g., the instantaneous power absorbed by single element *R* is

$$p_R(t) = v_R(t)i_R(t) = Ri_R^2(t) = \frac{v_R^2(t)}{R}$$

- E.g., the instantaneous power absorbed by single element *L* is

$$p_L(t) = v_L(t)i_L(t) = L\frac{di_L(t)}{dt} \cdot i_L(t)$$

- E.g., the instantaneous power absorbed by single element *C* is

$$p_C(t) = v_C(t)i_C(t) = v_C(t) \cdot C\frac{dv_C(t)}{dt}$$

Instantaneous Power in an RL circuit – Step DC

- Current response due to the stepped DC voltage V_o (at time $t = 0$ s):

$$i(t) = \frac{V_o}{R}\left(1 - e^{-Rt/L}\right) \cdot u(t)$$

- Instantaneous power supplied by power source is:

$$p_{source} = vi = \frac{V_o^2}{R}\left(1 - e^{-Rt/L}\right) \cdot u(t)$$

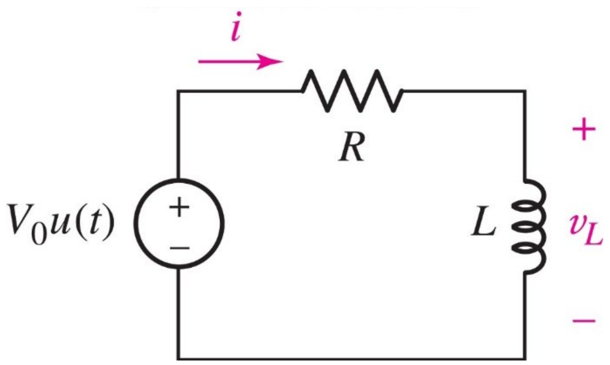
- Instantaneous power absorbed by R is:

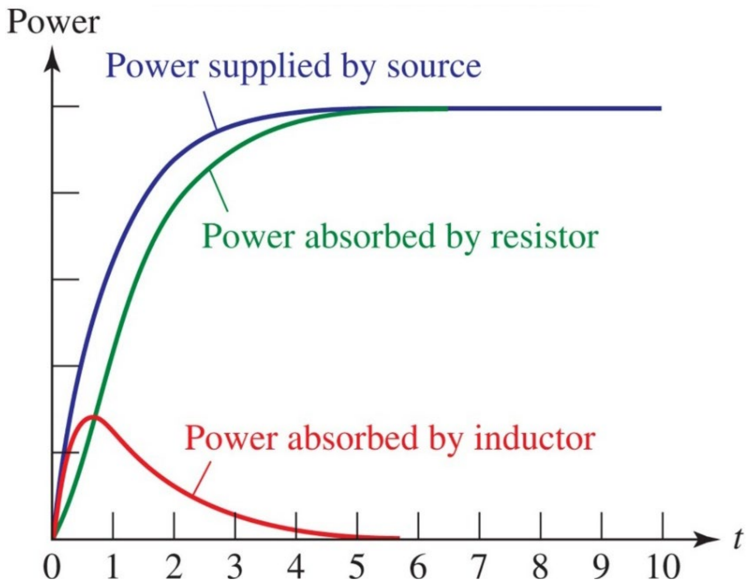
$$p_R = Ri^2 = \frac{V_o^2}{R}\left(1 - e^{-Rt/L}\right)^2 \cdot u(t)$$

- Instantaneous power absorbed by L is:

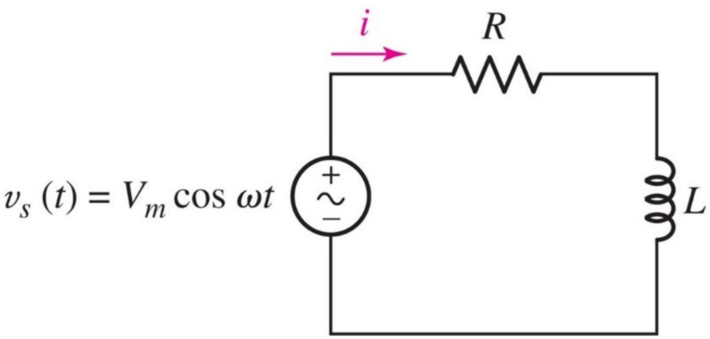
$$p_L = Li\frac{di}{dt} = \frac{V_o^2}{R}e^{-Rt/L}\left(1 - e^{-Rt/L}\right) \cdot u(t)$$

- Power is conserved: $p_{source} = p_R + p_L$





Instantaneous Power in an RL circuit – AC at SS



- Steady-state current response $i(t) = I_m \cos(\omega t + \emptyset)$ to an AC voltage $v_s(t)$:

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \qquad \phi = -\tan^{-1}(\omega L / R)$$

- Instantaneous power supplied by power source at steady state is:

$$\begin{aligned} p_{source} &= vi = V_m I_m \cos(\omega t + \emptyset) \cos \omega t \\ &= \frac{V_m I_m}{2}[\cos(2\omega t + \emptyset) + \cos\emptyset] \\ &= \frac{V_m I_m}{2}\cos\emptyset + \frac{V_m I_m}{2}\cos(2\omega t + \emptyset) \end{aligned}$$

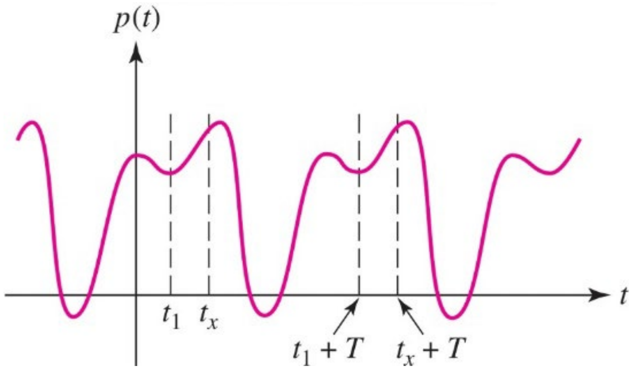
- **Useful observation:**
  - First term: not a function of time, constant (at SS), being the averaged power supplied by the source.
  - Second term: averaged to zero, but with doubled frequency. Not active power, but kind like an "oscillating power" continuously supplied and absorbed by the source.

Average Power

- The general average power expression (over an arbitrary time interval from t_1 to t_2) is

$$\langle p(t)\rangle = P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)\cdot dt$$

- When the power is periodic with period T (e.g., figure), the average power over any one period (or more integer periods) from t_1 or t_x is:



$$\langle p\rangle = P = \frac{1}{T}\int_{t_1}^{t_1+T} p(t)\cdot dt = \frac{1}{T}\int_{t_x}^{t_x+T} p(t)\cdot dt$$

Average Power: Sinusoidal Steady State

- Given $v(t) = V_m \cos(\omega t + \theta)$ and $i(t) = I_m \cos(\omega t + \phi)$,
  - Instantaneous power $p(t) = V_m I_m \cos(\omega t + \theta)\cos(\omega t + \phi)$

$$= \frac{V_m I_m}{2}\cos(\theta - \phi) + \frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)$$

  - Average power is

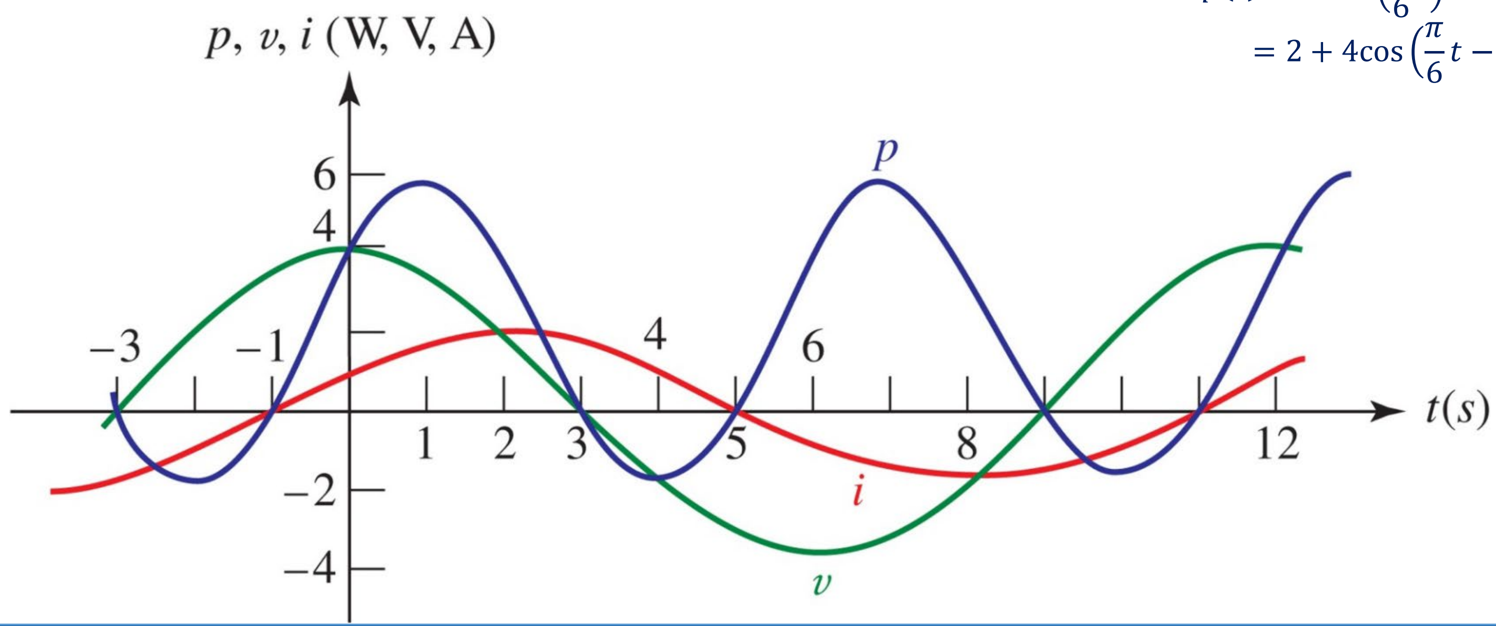
$$\langle p \rangle = P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} Re\{\mathbf{VI}^*\}$$

$v(t) = 4\cos\left(\frac{\pi}{6}t\right)$ A

$i(t) = 2\cos\left(\frac{\pi}{6}t - 60^o\right)$ A

$p(t) = 8\cos\left(\frac{\pi}{6}t\right)\cos\left(\frac{\pi}{6}t - 60^o\right)$

$= 2 + 4\cos\left(\frac{\pi}{6}t - 60^o\right)$ W



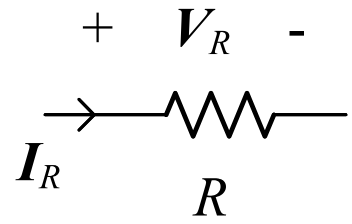
$\mathbf{VI}^* = V_m e^{j\theta} \cdot I_m e^{-j\phi} = V_m I_m e^{j(\theta - \phi)}$

Average Power for *R*, *L*, and *C*

- The **average power** absorbed by a resistor *R* (with the amplitudes of the voltage across and current through it being V_R and I_R) is

$$\boldsymbol{V}_R = V_R \angle 0^o \quad \mathrm{V_{peak}}$$
$$\boldsymbol{I}_R = I_R \angle 0^o \quad \mathrm{A_{peak}}$$

$$P_R = \frac{1}{2} V_R I_R \cos 0 = \frac{1}{2} I_R^2 R = \frac{V_R^2}{2R}$$



- The **average power** absorbed by a purely **reactive** element(s) is zero, since the current and voltage are 90 degrees out of phase:

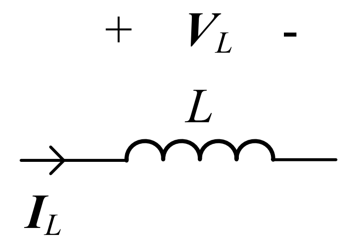
$$\boldsymbol{V}_L = V_L \angle 0^o \quad \mathrm{V_{peak}}$$
$$\boldsymbol{I}_L = I_L \angle -90^o \quad \mathrm{A_{peak}}$$

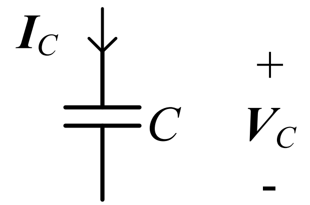
$$P_L = \frac{1}{2} V_L I_L \cos(90^o) = 0$$

$$\boldsymbol{V}_C = V_C \angle 0^o \quad \mathrm{V_{peak}}$$
$$\boldsymbol{I}_C = I_C \angle 90^o \quad \mathrm{A_{peak}}$$

$$P_C = \frac{1}{2} V_C I_C \cos(-90^o) = 0$$

$$\left. \begin{array}{l} P_L = 0 \\ P_C = 0 \end{array} \right.$$





Example 10.1: Average Power 1

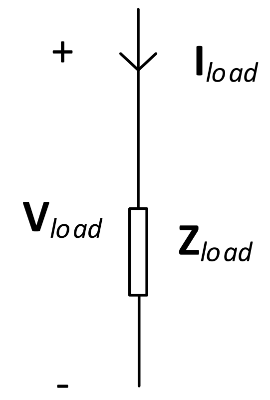
- Find the **average power** being delivered to an impedance $\mathbf{Z}_L = 8 - j11\ \Omega$ by a current $\mathbf{I} = 5e^{j20^o}\,\mathrm{A}$.
  - Analysis: only the 8 Ω resistance is relevant to **average power** calculation because reactive element *j11* Ω absorb zero average power.

Solution:

$$P = \frac{1}{2}(5)^2(8) = 100\ \mathrm{W}$$

Example 10.2: Average Power 2

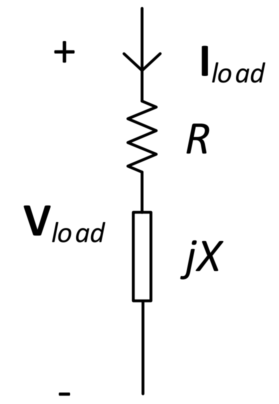
- Find the average power delivered to an impedance $\mathbf{Z}_{load} = 3 - j4\ \Omega$ when the voltage across it is $\mathbf{V}_{load} = 10\angle 20^o \mathrm{V}$.



Solution:

Analysis - Average power across *jX* (imaginary part, + or -) is zero. Only need to calculate the average power absorbed by *R*.

Steps - To calculate P_R, we need either voltage across *R*, or current through *R*.



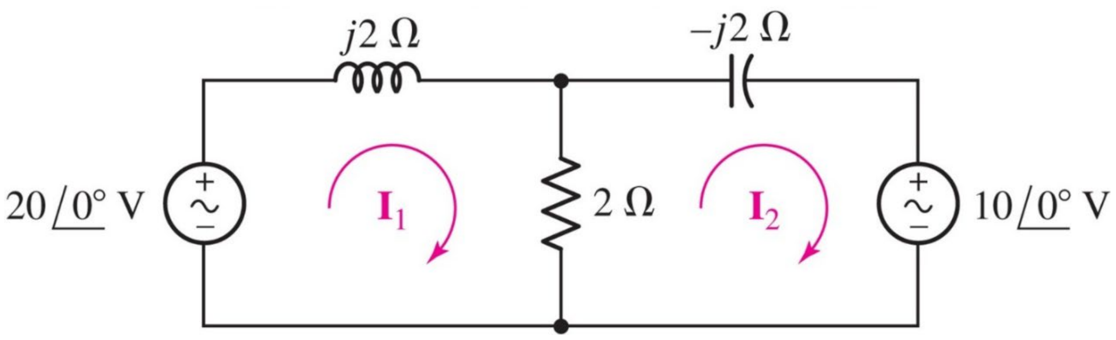
$$\mathbf{I}_{load} = \frac{\mathbf{V}_{load}}{\mathbf{Z}_{load}} = \frac{10\angle 20^o}{3 - j4}$$

$$= \frac{10\angle 20^o}{5\angle 53.13^o} = 2\angle -33.13^o \text{ A}$$

$$P_{Z_{load}} = P_R = \frac{1}{2}(2)^2(3) = 6 \text{ W}$$

Example 10.3: Average Power 3

- Find the average power absorbed by each element (including sources).



Solution:

Analysis - Average powers across all *jX* are zero, but still need to calculate the average power absorbed by *R*. This means the currents across *R* (being $\mathbf{I}_1$-$\mathbf{I}_2$) and the sources are to be known.

"Circuit analysis tools" that can be used are:

- Mesh analysis – seems "most fit"
- Nodal analysis
- Principle of superposition
- Others

Example 10.3: Average Power 3

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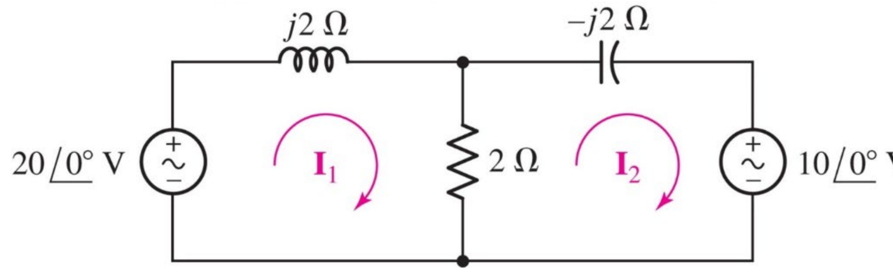
Step (1): Mesh analysis, start by applying KVL to each loop:



$$\text{Loop 1:}\quad 20\angle 0^o - j2\mathbf{I}_1 - 2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$\text{Loop 2:}\quad 2(\mathbf{I}_2 - \mathbf{I}_1) + (-j2)\mathbf{I}_2 = -10\angle 0^o$$

Step (2): Solve for $\mathbf{I}_1$ and $\mathbf{I}_2$:

$$(2+j2)\mathbf{I}_1 - 2\mathbf{I}_2 = 20 + j0$$

$$2(1+j)\mathbf{I}_1 + (-2+j2)(1+j)\mathbf{I}_2 = (10+j0)(1+j)$$

$$\mathbf{I}_2 = \frac{-10+j10}{(-2+j2)(1+j)+2} = 5 - j5 = 7.071\angle -45^o \text{ A}$$

$$\mathbf{I}_1 = \frac{20+2\mathbf{I}_2}{2+j2} = 5 - j10 = 11.18\angle -63.43^o \text{ A}$$

Step (3): Solve for P_R, $P_{s,left}$, and $P_{s,right}$:

$$P_{2\,\Omega} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2 R = \frac{1}{2}|-j5|^2(2) = 25 \text{ W}$$

$$P_{j2\Omega} = P_{-j2\Omega} = 0 \text{ W}$$

$$P_{s,left,\text{supply}} = \frac{1}{2}V_{s,left}|\mathbf{I}_1|\cos(0^o - \angle\mathbf{I}_1) = \frac{1}{2}(20)(11.18)\cos(0^o + 63.43^o) = 50 \text{ W}$$

$$P_{s,right,absorb} = \frac{1}{2}V_{s,right}|\mathbf{I}_2|\cos(0^o - \angle\mathbf{I}_2) = \frac{1}{2}(10)(7.071)\cos(0^o + 45^o) = 25 \text{ W}$$

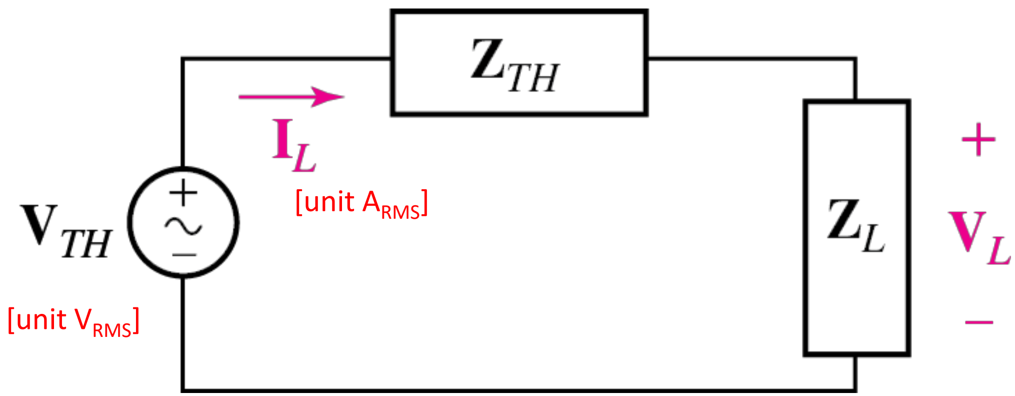
Reverse $\mathbf{I}_2$ direction, $-\mathbf{I}_2 = 7.071\angle 135^o$

$$P_{s,right,\text{supply}} = \frac{1}{2}V_{s,right}|-\mathbf{I}_2|\cos(0^o - \angle -\mathbf{I}_2)$$

$$= \frac{1}{2}(10)(7.071)\cos(0^o - (180^o - 45^o)) = -25 \text{ W}$$

Maximum Power Transfer

- The concept of "maximum power transfer" is useful in ensuring e.g. a maximum torque is achieved in an induction motor, maximum power is transferred across a transmission lines (both power and signals), etc.
- Given a AC circuit comprised of a Thévenin circuit ($\mathbf{V}_{Th}$ source in series with $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$) connected in SERIES to a general load $\mathbf{Z}_L = R_L + jX_L$:
  - *Maximum power, or maximum average power,* is delivered to the $\mathbf{Z}_L$ load IF AND ONLY IF the load impedance $\mathbf{Z}_L$ is equal to the conjugate of $\mathbf{Z}_{Th}$, *which means* $\mathbf{Z}_L = \mathbf{Z}_{Th}^*$.



Maximum Power Transfer Derivation (1)

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- To prove, obtain the expression for the average load power P_L:

Derivation of these formulae are NOT REQUIRED in EXAMS!

Load current phasor $\mathbf{I}_L$:

$$\mathbf{I}_L = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} = \frac{\mathbf{V}_{Th}}{(R_{Th} + R_L) + j(X_{Th} + X_L)}$$

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_{Th}|}{\sqrt{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}} \qquad \angle \mathbf{I}_L = \angle \mathbf{V}_{Th} - \tan^{-1}\left(\frac{X_{Th} + X_L}{R_{Th} + R_L}\right)$$

Load voltage phasor $\mathbf{V}_L$:

$$\mathbf{V}_L = \mathbf{V}_{Th} \frac{\mathbf{Z}_L}{\mathbf{Z}_{Th} + \mathbf{Z}_L} = \mathbf{V}_{Th} \frac{R_L + jX_L}{(R_{Th} + R_L) + j(X_{Th} + X_L)}$$

$$|\mathbf{V}_L| = \frac{|\mathbf{V}_{Th}|\sqrt{R_L^2 + X_L^2}}{\sqrt{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}} \qquad \angle \mathbf{V}_L = \angle \mathbf{V}_{Th} + \tan^{-1}\left(\frac{X_L}{R_L}\right) - \tan^{-1}\left(\frac{X_{Th} + X_L}{R_{Th} + R_L}\right)$$

Average load power P_L:

$$P_L = \frac{1}{2}|\mathbf{V}_L||\mathbf{I}_L|\cos(\angle \mathbf{V}_L - \angle \mathbf{I}_L) = \frac{|\mathbf{V}_{Th}|^2\sqrt{R_L^2 + X_L^2}}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}\cos\left[\tan^{-1}\left(\frac{X_L}{R_L}\right)\right]$$

Maximum Power Transfer Derivation (2)

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$$P_L = \frac{|\mathbf{V}_{Th}|^2 \sqrt{R_L^2 + X_L^2}}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2} \cos\left[\tan^{-1}\left(\frac{X_L}{R_L}\right)\right]$$

$$= \frac{|\mathbf{V}_{Th}|^2 \sqrt{R_L^2 + X_L^2}}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2} \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$$

$$= \frac{|\mathbf{V}_{Th}|^2 R_L}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

NOTE: Derivation is not required for exam, but you should be able to interpret the problem and decide whether to use this "MPT" condition.

Condition to achieve MPT for steady-state AC network (must know!):

$$\boxed{\left.\begin{aligned} R_L &= R_{Th} \\ X_L &= -X_{Th} \end{aligned}\right\} \quad \mathbf{Z}_L = \mathbf{Z}_{Th}^*}$$

- Firstly, it can be recognized that any non-zero value of $(X_L + X_{Th})$ only makes P_L smaller. Hence, X_L should be opposite sign of X_{Th} , i.e., $X_L = -X_{Th}$, to ensure largest possible power transfer.
- Secondly, solve for $dP_L/dR_L = 0$ to know what R_L can give maximum power transfer.

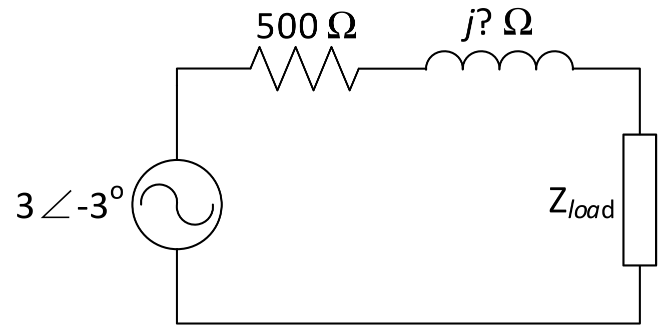
$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{|\mathbf{V}_{Th}|^2 R_L}{(R_{Th}+R_L)^2}\right] = \frac{(R_{Th}+R_L)^2|\mathbf{V}_{Th}|^2 - |\mathbf{V}_{Th}|^2 R_L \cdot 2(R_{Th}+R_L)}{(R_{Th}+R_L)^4} = 0$$

$$(R_{Th}+R_L) - R_L \cdot 2 = 0$$

$$R_L = R_{Th}$$

Example 10.4: Applying the MPT concept

- This circuit is supplied by a sinusoidal AC voltage source of $3\cos(100t - 3^o)\,\mathrm{V}$, a 500 Ω, 0.03 H, and an unknown load impedance.
  - What should be the load impedance value to ensure that maximum power is delivered to it?



Analysis

- Calculate the reactance
- Identify the supply-load structure (with Thévenin's equivalent circuit at the supply side).
- Apply MPT concept: $\mathbf{Z}_L = \mathbf{Z}_{Th}^*$

Answer:

500 – j3

Effective Values of Current and Voltage

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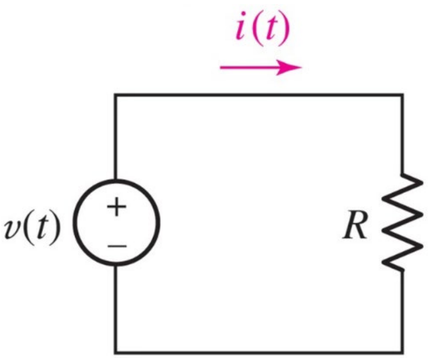
- In **China**, the rated (single-phase) household voltage supply is **220 V 50 Hz**; in **UK**, the value is **230 V 50 Hz**; in **US**, the value is **120 V 60 Hz**.
- These voltage values are the RMS values of the supply voltage, sometimes known as "effective values".

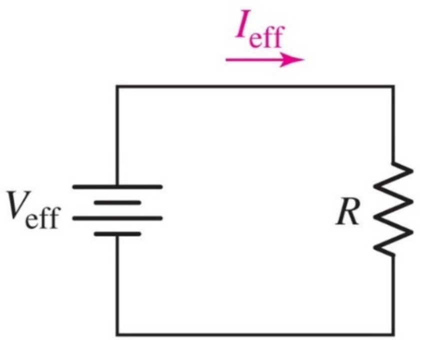


$$P = \frac{R}{T}\int_0^T i^2 \cdot dt$$

<u>For an arbitrary periodic voltage:</u>

- By definition, subject a periodic voltage of period *T* and, separately, a DC voltage to a resistance *R*, the periodic voltage is said to be as effective as the DC voltage when they produce the same resistive power.
  - The above can be assessed by analyzing the currents:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 \cdot dt}$$



$$P_{DC} = I_{eff}^2 R$$

Effective Values of Current and Voltage

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Special case for sinusoidal waveform:

- For a sinusoidal current of period $T = \frac{2\pi}{\omega}$:

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_{eff} = \sqrt{\frac{1}{2\pi/\omega}\int_0^{2\pi/\omega} I_m^2 \cos^2(\omega t + \phi)\cdot dt}$$

$$= I_m \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left[\frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\right]\cdot dt}$$

$$= I_m \sqrt{\frac{\omega}{4\pi}\left[t\right]_0^{2\pi/\omega}}$$

$$= \frac{I_m}{\sqrt{2}} \approx 0.7071 I_m$$

For single-frequency sinusoidal waveform

- The effective value is often referred to as the root-mean-square or RMS value.

Current and Voltage RMS

- The average power delivered by a sinusoidal current of magnitude I_m to R is:

$$P = \frac{1}{2} I_m^2 R = I_{eff}^2 R$$

- Similarly, re-writing the average power supplied by sinusoidal voltage of magnitude V_m across R:

$$P = \frac{V_m^2}{2R} = \frac{V_{eff}^2}{R} \qquad V_{eff} = \frac{V_m}{\sqrt{2}} \approx 0.7071 V_m$$

- Also, in a more general case, the average power supplied by sinusoidal voltage of phasor $V_m\angle\theta$ to $\mathbf{Z}_L$ (resulting in a current phasor $I_m\angle\phi$ flowing across) can be rewritten to:

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$$

Effective Values for Multiple-frequency circuits (FYI)

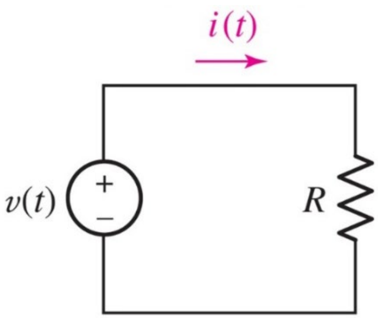
- If the current (and voltage too) is a *R*-circuit contains more than one frequency component, the effective/equivalent average power dissipated in *R* and the effective current through *R* are:

$$P = \left(I_{1eff}^2 + I_{2eff}^2 + \ldots + I_{Neff}^2\right)R$$

$$I_{eff} = \sqrt{I_{1eff}^2 + I_{2eff}^2 + \ldots + I_{Neff}^2}$$

Effective Values for Multiple-frequency circuits (FYI)

If $R = 10$ and $v(t) = 2\sqrt{2}\cos[2\pi(50)t] + \sqrt{2}\cos[2\pi(150)t]$, the total power P and effective current rms I_{eff} can be calculated as:



$$I_{eff,50Hz} = \frac{2}{10} = 0.2\ \mathrm{A_{rms}}$$

$$I_{eff,150Hz} = \frac{1}{10} = 0.1\ \mathrm{A_{rms}}$$

$$I_{effz} = \sqrt{0.1^2 + 0.2^2} = 0.224\ \mathrm{A_{rms}}$$

$$P = \left(0.1^2 + 0.2^2\right)(10) = 0.5\ \mathrm{W}$$

Apparent Power & Power Factor

- The concepts of "apparent power" and "power factor" are normally relevant to power system and power industry:
  - Energy cost calculation
  - Power system transmission/distribution design calculation
  - AC electrical machines

- Given $v(t) = V_m\cos(\omega t + \theta)$ and $i(t) = I_m\cos(\omega t + \phi)$

$$\boxed{P = \frac{V_m I_m}{2}\cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)}$$

  - The apparent power S is defined as $S = V_{eff}I_{eff}$ and has the unit of VA (volt-ampere)
  - Power factor *PF* is $PF = \cos(\theta - \phi)$, where $(\theta - \phi)$ is know as the power factor angle.
  - It can be deduced from the above that:

$$PF = \frac{average\ power}{apparent\ power} = \frac{P}{V_{eff}I_{eff}}$$

Apparent Power & Power Factor

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- Based on the obtained PF expression:

$$PF = \cos(\theta - \phi)$$

  - Case *R*: $(\theta - \phi) = 0, \quad PF = 1 \quad (a.k.a.\,unity)$
  - Case *L*: $(\theta - \phi) = 90^o, \quad PF = 0\ (a.k.a.\,zero)$
  - Case *C*: $(\theta - \phi) = -90^o, \quad PF = 0\ (a.k.a.\,zero)$
  - General **Z**: $0 \leq PF \leq 1$

- It can be observed from the above *PF* expression that the information as to whether current leads or lags voltage is lost. To resolve this issue, an adjective is added PF term.
  - An inductive load has a *lagging PF*.
  - A capacitive load has a *leading PF*.

Complex Power

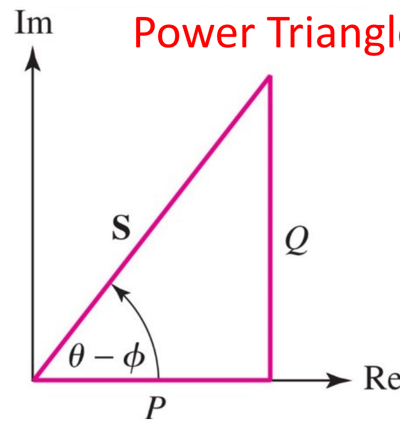
- Given the sinusoidal waveforms of $v(t) = V_m\cos(\omega t + \theta)$ and $i(t) = I_m\cos(\omega t + \phi)$, we simplify the phasors using
  - Amplitude values: $\mathbf{V} = V_m\angle\theta$ and $\mathbf{I} = I_m\angle\emptyset$, or
  - RMS values: $\mathbf{V}_{\text{rms}} = V_{rms}\angle\theta$ and $\mathbf{I}_{\text{rms}} = V_{rms}\angle\emptyset$, or
  - "Effective" values: $\mathbf{V}_{eff} = V_{eff}\angle\theta$ and $\mathbf{I}_{eff} = V_{eff}\angle\emptyset$. (Practically the same as "RMS")

- The complex power $\mathbf{S} = P + jQ$ can be defined as:

$$\mathbf{S} = P + jQ = \mathbf{V}_{eff}\mathbf{I}_{eff}^{*} = V_{eff}I_{eff}e^{j(\theta-\emptyset)}$$

- Effective, it means:
  - Average power $P = V_{eff}I_{eff}\cos(\theta - \emptyset)$
  - Reactive power $Q = V_{eff}I_{eff}\sin(\theta - \emptyset)$

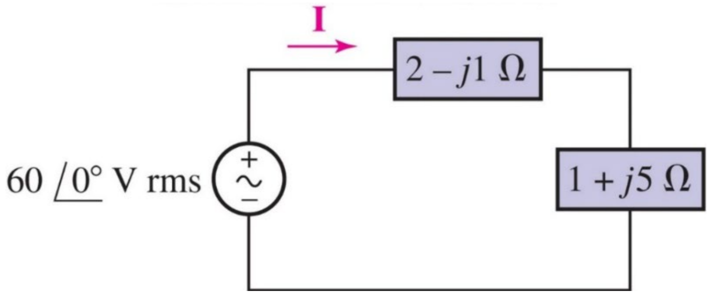
Power Triangle



- **Tip:** You can think of "reactive power" being a form energy that flows "back and forth" from the source (e.g., any AC supply, power plant, solar farm, grid battery, etc.) to the inductive or capacitive elements of the load. Capacitance/inductance are present in all electronics (including microelectronics), electrical machines, transformers, transmission lines (means the whole power system)…..
  - In small electronics, the concern is less on the "power" aspect, but on how the signals are affected by inductive or capacitive elements in the circuit.

Example 10.5: Power quantities

- Find the average, apparent, and complex powers supplied by the source.
- Fine the average, apparent, and complex powers absorbed by the two loads
- Find the power factor of the combined loads.



Solution:

$$\mathbf{I} = \frac{60\angle 0^o}{(2-j)+(1+j5)} = \frac{60\angle 0^o}{3+j4} = 12\angle -53.1^o \;\; \mathrm{A}_{rms}$$

- $\mathbf{S}_s = \mathbf{V}_s\mathbf{I}^* \; = (60\angle 0^o)(12\angle 53.1^o) = 432 + j576\ \mathrm{VA}$

AC Source: Average power = 432 W, Apparent power = $\sqrt{432^2 + 576^2} = 720$ VA, Complex power = $(432 + j576)$ VA.

- $\mathbf{S}_1 = (\mathbf{I}\mathbf{Z}_1)\mathbf{I}^* \; = |\mathbf{I}|^2(2-j) = |12|^2(2-j) = 288 - j144\ \mathrm{VA}$

Load $\mathbf{Z}_1$ = (2-*j*) Ω : Average power = 288 W, Apparent power = $\sqrt{288^2 + (-144)^2} = 323$ VA, Complex power = $(288 - j144)$ VA.

- $\mathbf{S}_2 = (\mathbf{I}\mathbf{Z}_2)\mathbf{I}^* \; = |\mathbf{I}|^2(1+j5) = |12|^2(1+j5) = 144 + j720$
- Load $\mathbf{Z}_2$ = (1+*j*5) Ω : Average power = 144 W, Apparent power = $\sqrt{144^2 + (720)^2} = 734.26$ VA, Complex power = $(144 + j720)$ VA.

$$PF_{loads} = \cos\left(0^o - (-53.1^o)\right) = 0.6$$

Overview of AC Power Quantities

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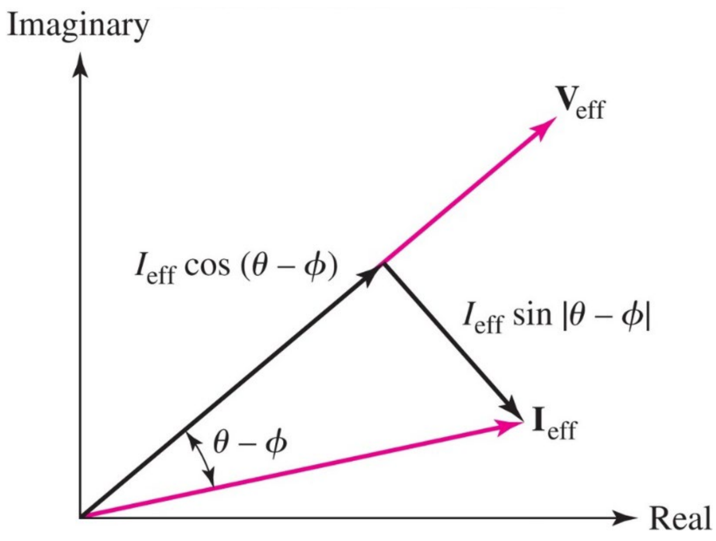
Quantity	Symbol	Formula	Units
Average/Active power	*P*	$\frac{1}{2}V_\mathrm{m}I_\mathrm{m}\cos(\theta-\emptyset)$ $V_{eff}I_{eff}\cos(\theta-\emptyset)$	watt (W)
Reactive power	*Q*	$\frac{1}{2}V_\mathrm{m}I_\mathrm{m}\sin(\theta-\emptyset)$ $V_{eff}I_{eff}\sin(\theta-\emptyset)$	volt-ampere-reactive (VAR)
Complex power	**S**	$P+jQ$ $\mathbf{V}_{eff}\mathbf{I}_{eff}^{*}$ $V_{eff}I_{eff}\angle(\theta-\emptyset)$	volt-ampere (VA)
Apparent power	|**S**|, *S*	$V_{eff}I_{eff}$ $\sqrt{P^2+Q^2}$	volt-ampere (VA)

An Alternative Interpretation of Reactive Power

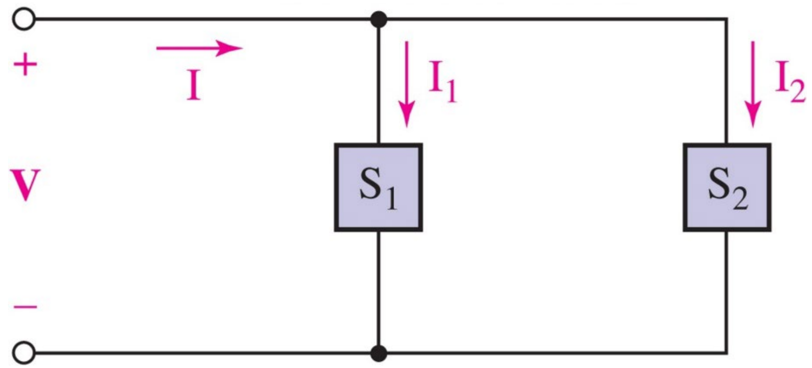
- An alternative way to imagine/view “reactive power” is as follows:
  - Splitting the current phasor $\mathbf{I}_{eff}$ into in-phase component and 90° out-of-phase component with respect to the voltage phasor.



  - Real (average) power is the product of the current in-phase with the voltage phasor $\mathbf{V}_{eff}$.
  - Reactive power is the product of the current 90° out-of-phase (a.k.a., quadrature component) to the voltage phasor $\mathbf{V}_{eff}$.

Total Complex Power in a Circuit

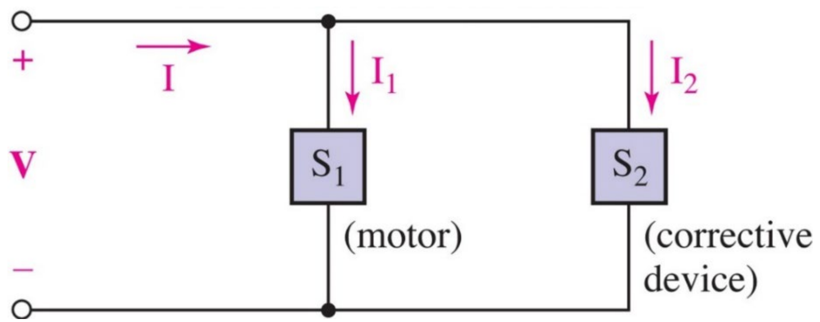
- It can be shown that, in the following circuit, the total complex power is equal to the sum of individual complex powers:



$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = \mathbf{V}(\mathbf{I}_1 + \mathbf{I}_2)^* = \mathbf{V}(\mathbf{I}_1^* + \mathbf{I}_2^*) = \mathbf{S}_1 + \mathbf{S}_2$$

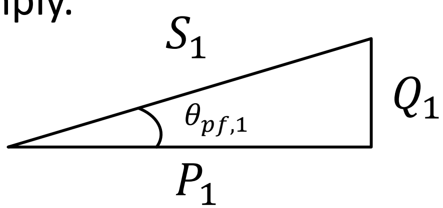
Example 10.6: Power Factor Correction

- An industrial consumer is operating a 50 kW induction motor at a lagging PF of 0.8. The source voltage is 230 V_{rms} 60Hz. In order to obtain lower electrical rates, the customer wishes to raise the PF to 0.95 lagging. Specify a suitable solution.



Solution:

Analysis – find out what power/current/impedance is required at S_2 to comply.

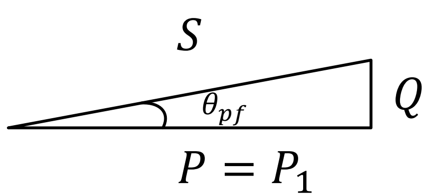


$$\theta_{pf,1} = \cos^{-1}(0.8)$$

- $\mathbf{S}_1 = S_1\angle\theta_{pf,1} = P_1 + jQ_1$, where $P_1 = S_1 \cdot PF_{motor} = 50$ kW

$$S_1 = \frac{50k}{0.8} = 62.5 \text{ kVA}$$

$$Q_1 = \sqrt{S_1^2 - P_1^2} = 37.5 \text{ kVar}$$



- $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = S\angle\theta_{pf} = P + jQ$, where $P = P_1 = S \cdot PF_{supply}$

$$S = \frac{50k}{0.95} = 52.63 \text{ kVA}$$

$$Q = \sqrt{S^2 - P^2} = 16.43 \text{ kVar}$$

$$\theta_{pf} = \cos^{-1}(0.95)$$

Example 10.6: Power Factor Correction

- The corrective device does no supply or consume real/active power.

$$P_2 = 0$$

- The total reactive power of the circuit is

$$Q = Q_1 + Q_2$$
$$Q_2 = 16.43 - 37.5 = -21.07\ \text{kVar}$$

- The total reactive power of the corrective device is

$$\mathbf{S}_2 = \mathbf{V}\mathbf{I}_2^* = -j21070$$
$$\mathbf{I}_2^* = \frac{-j21070}{230\angle 0^o} = -j91.6\ \text{A}$$
$$\mathbf{I}_2 \ = j91.6\ \text{A}$$

Alternatively,

$$\frac{V^2}{X} = -21.07\ \text{k}$$

"-" means capacitor,

$$\frac{V^2}{[1/(2\pi f C)]} = 21.07\ \text{k}$$

- Total impedance of the corrective device is

$$\mathbf{Z}_2 \ = \frac{\mathbf{V}}{\mathbf{I}_2} = \frac{230\angle 0^o}{j91.6} = -j2.51\ \Omega$$

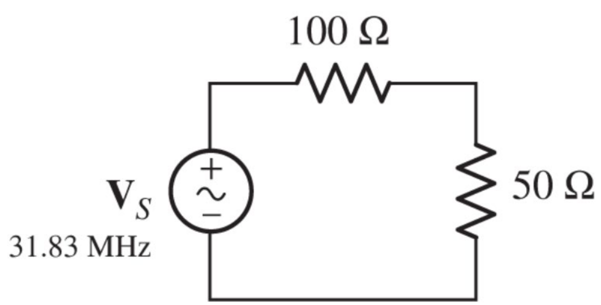
- **Conclusion:** "negative" reactance indicates a capacitive element. With 60 Hz supply, the capacitance C is calculated as:

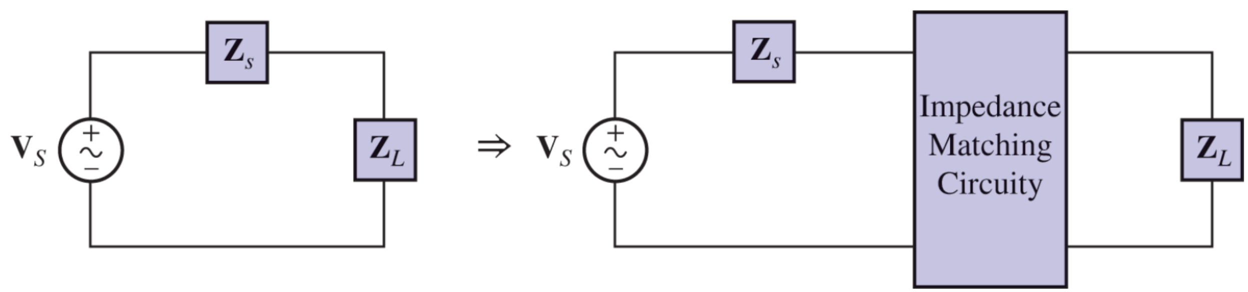
$$\frac{1}{j2\pi(60)C} = -j2.51 \rightarrow C = 1.057mF$$

Example 10.7: Maximum power transfer – Application

- The output resistance of a power amplifier will be different from the load (e.g., a speaker). How can we ensure that maximum power is delivered?
- Impedance matching is particularly important for applications dealing with very weak signals or where power loss should be kept minimum (e.g., to prolong the battery life of your phone).

Consider a power amplifier at a frequency of 31.83 MHz with source's output resistance of 100 Ω. The load is an antenna modelled simply (actual model is actually complicated, is outside the scope of EEE103) as a 50 Ω.

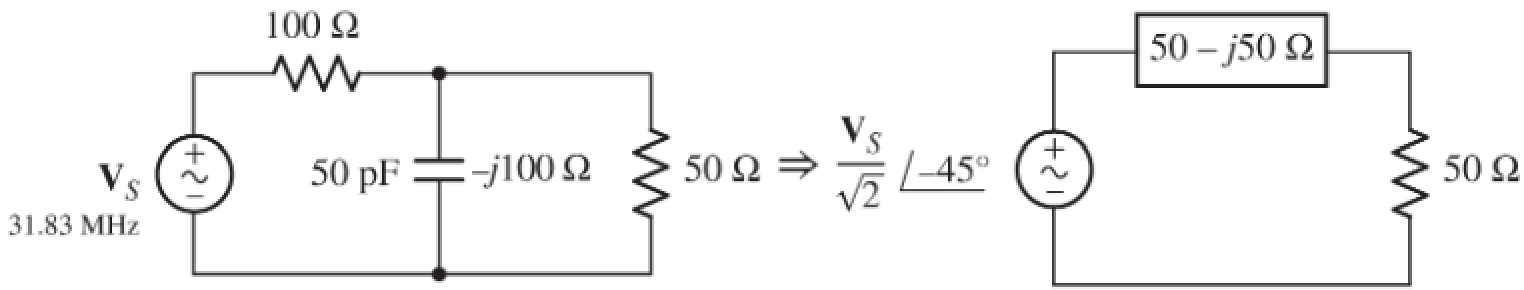




E.g., $|\mathbf{V}_S| = 5\text{ V}$, $|\mathbf{I}_S| = \frac{1}{30}\text{ A}$

$$P = \frac{1}{2} I_m^2 R = \frac{1}{2}\left(\frac{1}{30}\right)^2 (50) = 27.78\text{ mW}$$

Add a capacitor in parallel to match the supply circuit's equivalent resistance



Adding a 50 pF capacitor: $$\boldsymbol{Z}_C = \frac{1}{j(2\pi \times 31.83 \times 10^6)(50 \times 10^{-12})} = -j100\ \Omega$$

Impedance seen from the load (into the supply circuit):

$$\mathbf{Z}_{eq} = \frac{R_s \boldsymbol{Z}_C}{R_s + \boldsymbol{Z}_C} = \frac{(100)(-j100)}{100 - j100} = 50 - j50\ \Omega$$

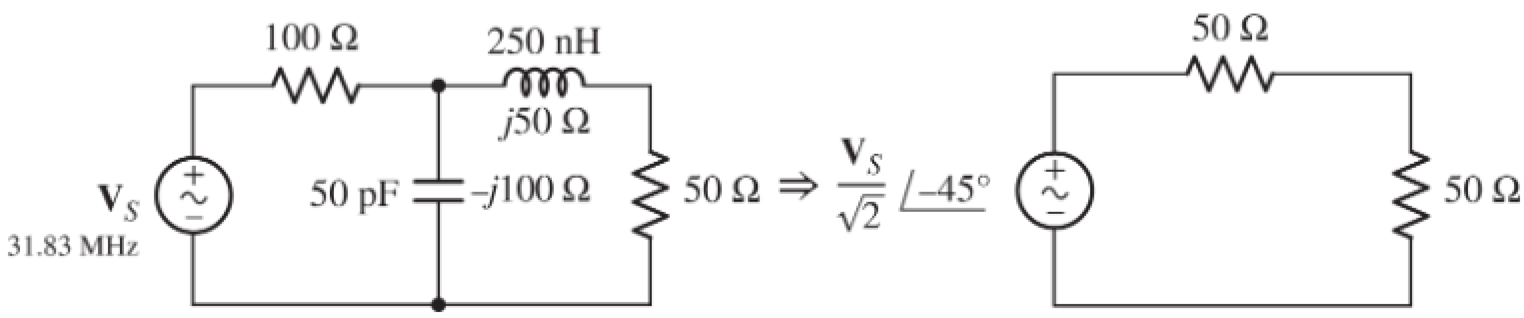
Thevenin network's source: $$\mathbf{V}_{TH} = \frac{(-j100)}{100 - j100}\mathbf{V}_S = \left(\frac{1}{2} - j\frac{1}{2}\right)\mathbf{V}_S = \frac{1}{\sqrt{2}}\mathbf{V}_S \angle -45^o\ \mathrm{V}$$

Current phasor and power:

$$\mathbf{I}_L = \frac{\frac{1}{\sqrt{2}}(5)\angle(0^o - 45^o)}{100 - j50} = 0.03162\angle -18.43^o\ \mathrm{A}$$

$$P = \frac{1}{2}(0.03162)^2(50) = 25.00\ \mathrm{mW}$$

Add a inductor in series (with the load) to ensure MPT



Adding an inductor of *j*50 that can "match" perfectly with the "-*j*50" (to ensure MPT!) → ω*L* = 50@31.83MHz, means

$$L = \frac{50}{j(2\pi \times 31.83 \times 10^6)} = 250 \text{ nH}$$

Current phasor and power:

$$\mathbf{I}_L = \frac{\frac{1}{\sqrt{2}}(5)\angle(0^o - 45^o)}{100} = 0.03536\angle - 45^o \text{ A}$$

$$P = \frac{1}{2}(0.03536)^2(50) = 31.26 \text{ mW}$$

25%↑

We can see that the 50 Ω now receives more power than the original circuit. Ideally, the added *L* and *C* will not consume any real/active power.

Reminder for IQ3 on 29/30th November 2023

Xi'an Jiaotong-Liverpool University 西交利物浦大学

In-class Quiz 3 (IQ3) - Lecture Group 1

Opens: Thursday, 30 November 2023, 10:00 AM
Closes: Thursday, 30 November 2023, 11:00 AM

2. Duration limit: 40 mins

In-class Quiz 3 (IQ3) - Lecture Group 2

Opens: Wednesday, 29 November 2023, 12:00 PM
Closes: Wednesday, 29 November 2023, 1:00 PM

2. Duration limit: 40 mins

- Questions aim to assess your basic understandings on the concepts learnt in Part 3:
  - Mix of MCQ (conceptual) and short/brief calculation (i.e., fill in the blank).
- Onsite, open book, but no discussion. You are reminded about University Academic Integrity policy.
- Different sets of questions for Group 1 and Group 2. No changing of group is allowed.

Tutorial, and some selected questions from the Textbook for self-practices

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- **Week 10**

(1) Please proceed to your tutorial session for some tutorial exercises.

Group 2	(Tutorial*)	(SA136*, SB152*, SB120*)	(continue to 12noon-1pm)

***NOTES:** The tutorial rooms are allocated according to your programmes. Please attend to the assigned session BUT NOT other rooms to avoid overcrowding. **Attendance of tutorials will be taken.**

SA136 – ~~CST and DMT students~~ CST and **EE** students (updated on 13th Sept. 2023)

SB152 – ~~EE and EST students~~ **DMT** and EST students (updated on 13th Sept. 2023)

SB120 – MRS and TE students

(2) After your tutorial, you can also self-practice some questions. For example:

Engineering Circuit Analysis, 9th or 10th ed., Chapter 11

Pg. 462 – 470: 13, 16, 20, 26, 36, 51

If you have extra time, others too (but be selective; you don't have time for all questions!). These questions will be displayed in the tutorial class, and sample solutions will be uploaded to LMC->Tutorials folder for your self-checking.

~ THE END ~