CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 14 Two-port Network

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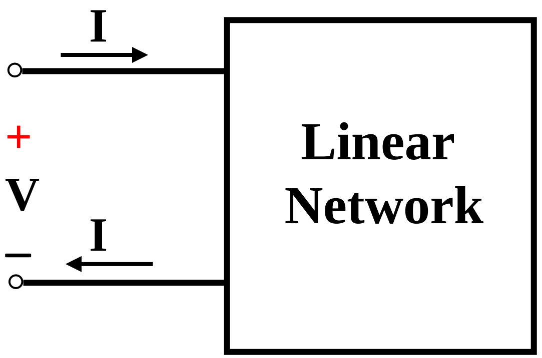
OUTLINE

- Overview
- Impedance Z-parameter
- Admittance Y-parameter
- Relationship between Z- & Y- parameter
- Interconnections
  - ✓ Series Connection
  - ✓ Parallel Connection

1.1 ONE-PORT NETWORK



Port: A pair of terminals through which a current may enter or leave a network.

Port condition: The same current must enter and leave a port

One port network: a network has only <u>one pair of terminals</u>

Two-terminal devices or elements (resistors, capacitors, inductors,…) result in one-port networks

For one-port network:

The current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.

current entering a port = current leaving the port

* MOTIVATION

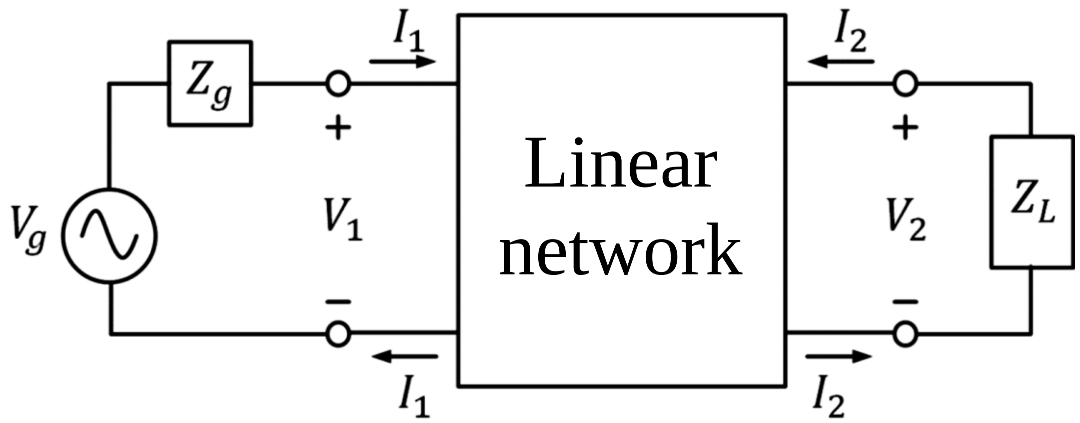
Thévenin and Norton equivalent circuits are used in representing the contribution of a circuit to **one** specific pair of terminals.

Usually, a signal is fed into one pair of terminals (input port), processed by the system, then extracted at a second pair of terminals (output port). This network are useful, typically in communications, control systems, power system and electronics.

It would be convenient to relate the *v*/*i* at one port to the *v*/*i* at the other port without knowing the element values and how they are connected inside the 'black box'. Knowing how to model two-port networks will help in the analysis of large network.

1.2 TWO-PORT NETWORK

Two-port networks (a **four-terminal** network) is an electrical network with two separate ports for input and output



Any linear circuit with four terminals can be transformed into a two-port network if it doesn't contain any independent source and satisfies the port conditions. Each port is not a current source or sink.

The voltages and currents at one port may be expressed as linear combinations of the voltages and currents at the other port, *i.e.*, V_1, V_2, I_1, and I_2.

1.3 TYPES OF PARAMETERS

Impedance **(Z)** parameters

Admittance **(Y)** parameters

Commonly used in the synthesis of filters, and are useful in the design and analysis of impedance-matching networks and power distribution networks

Transmission **(A, B, C, D)** parameters

The A,B,C,D parameters are best used when we want to cascade two networks together

Hybrid **(H)** parameters

H parameters are used almost solely in electronics (in the equivalent circuit of a transistor).

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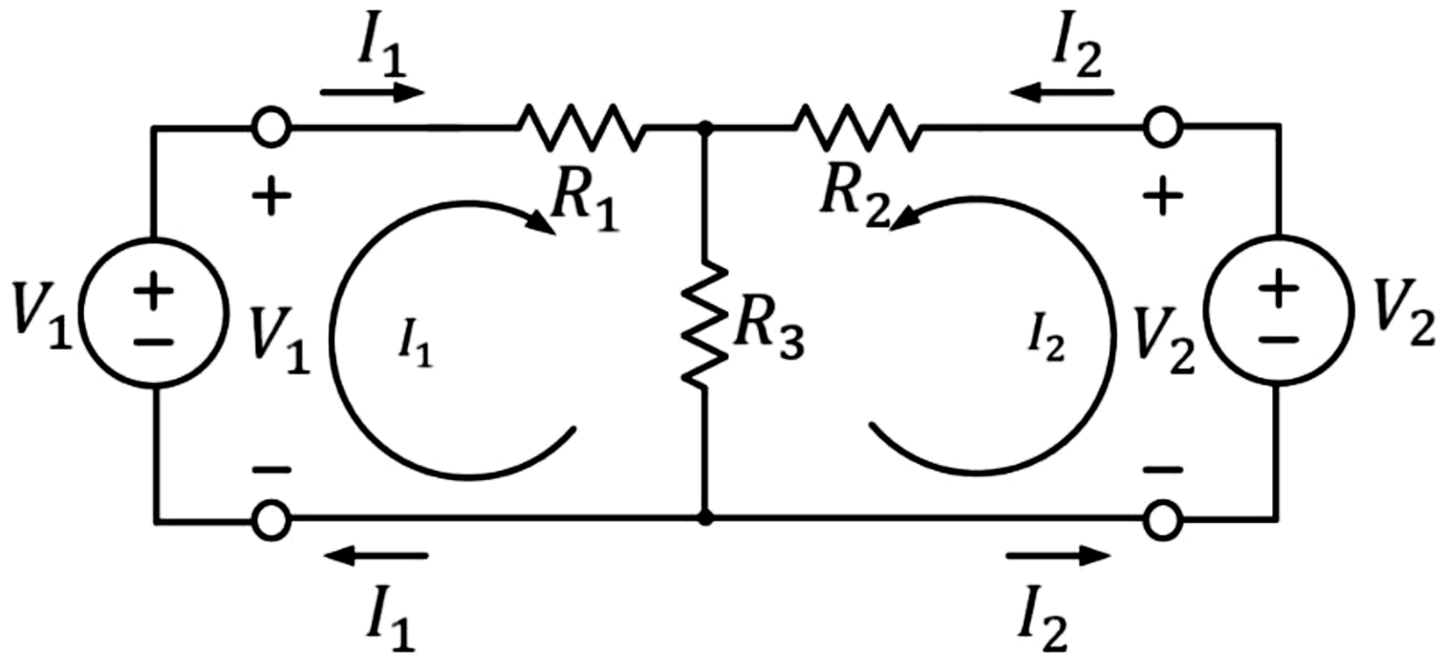
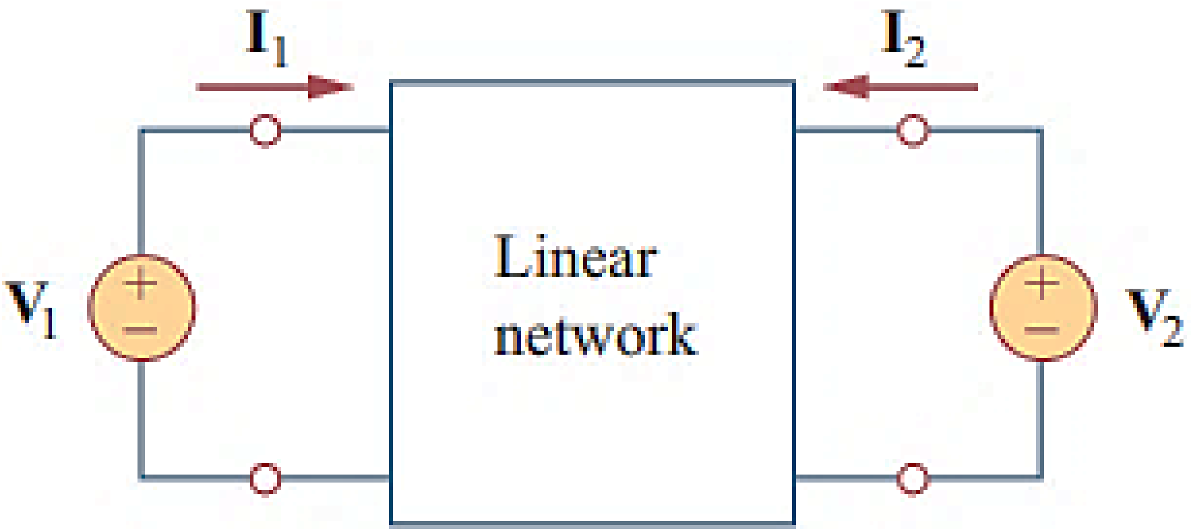
RECALL LOOP ANALYSIS…



Left Loop: $V_1 = (R_1 + R_3)I_1 + R_3 I_2$

Right Loop: $V_2 = R_3 I_1 + (R_2 + R_3)I_2$

2.1 VOLTAGE SOURCE DRIVEN 2-PORT



The terminal voltages can be found by loop analysis:

$$V_1 = z_{11} I_1 + z_{12} I_2$$

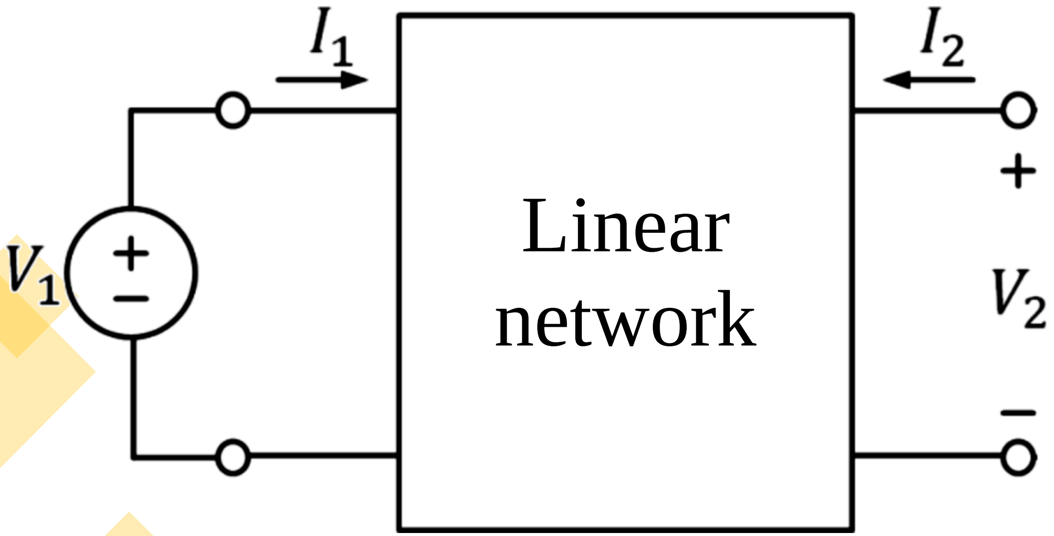
$$V_2 = z_{21} I_1 + z_{22} I_2$$

Matrix Form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Notice: Only two of the four variables (V_1, V_2, I_1, and I_2) are independent. The other two can be found using above equations.

2.2 Z PARAMETERS



$$V_1 = z_{11}I_1 + \cancel{z_{12}I_2}$$

$$V_2 = z_{21}I_1 + \cancel{z_{22}I_2}$$

Open-circuit input impedance:

The impedance seen looking into port 1 when port 2 is open.

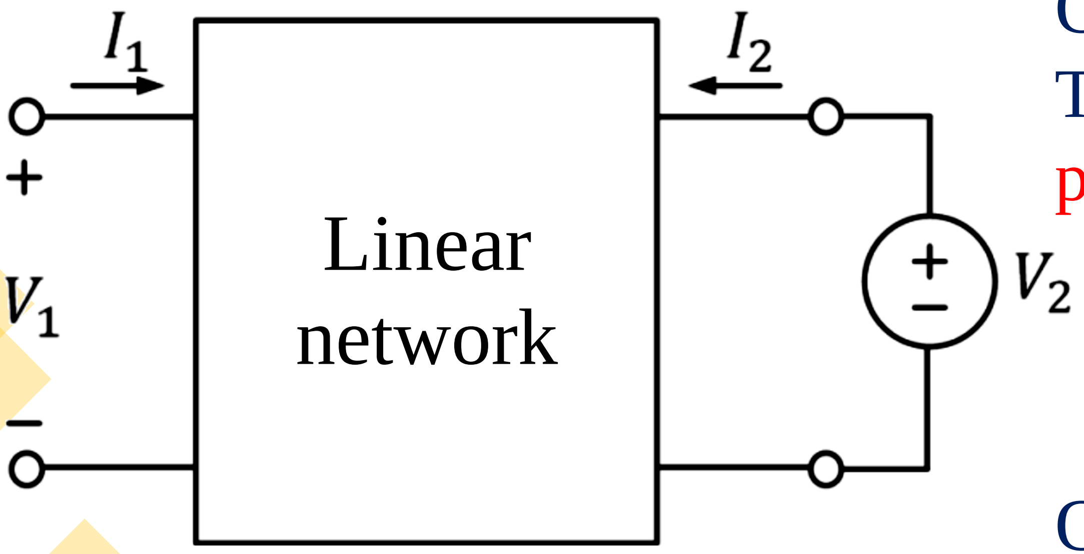
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

Open-circuit transfer impedance:

The ratio of the voltage at port 2 to the current at port 1 when port 2 is open.

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

2.2 Z PARAMETERS



$$V_1 = \cancel{z_{11} I_1} + z_{12} I_2$$

$$V_2 = \cancel{z_{21} I_1} + z_{22} I_2$$

Open-circuit output impedance:

The impedance seen looking into port 2 when port 1 is open.

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Open-circuit transfer impedance:

The ratio of the voltage at port 1 to the current at port 1 when port 1 is open.

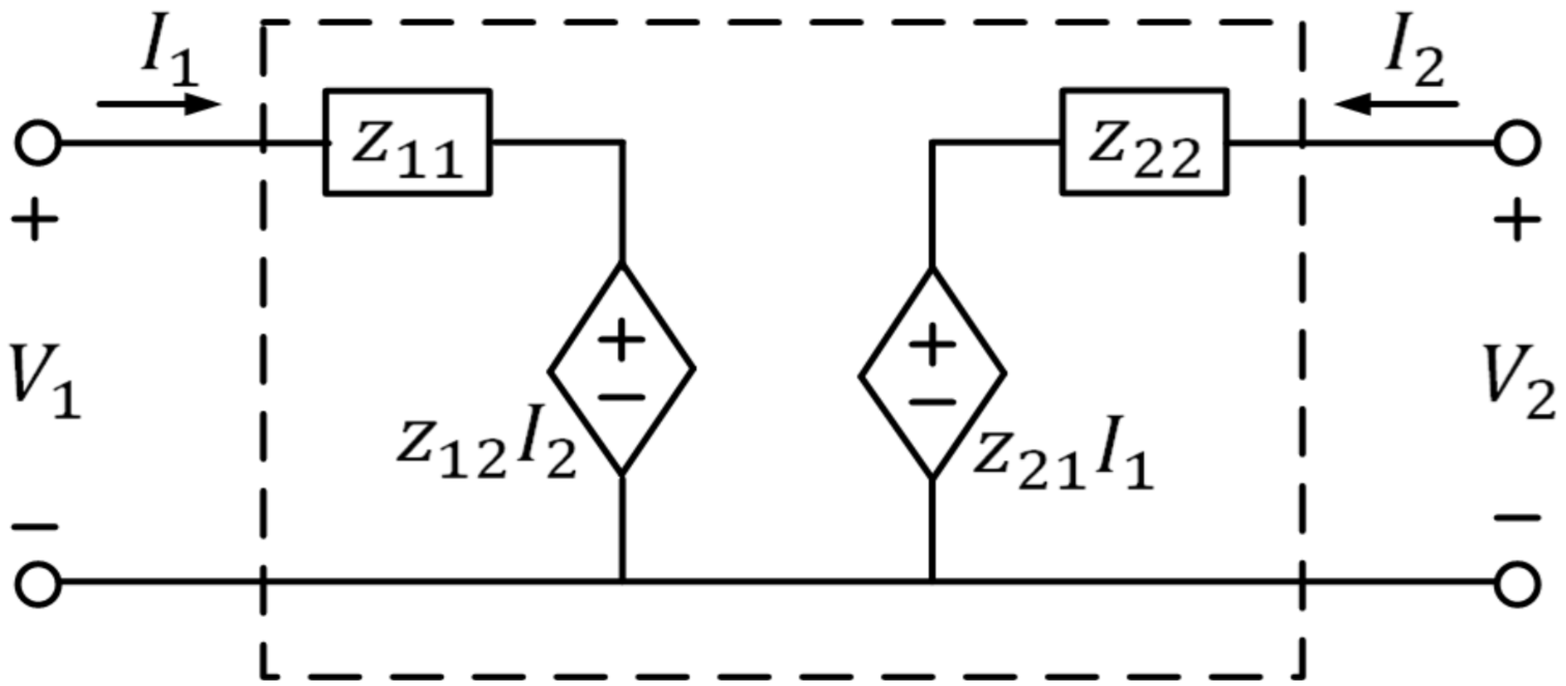
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

2.3 EQUIVALENT CIRCUIT

Z-parameters – Equivalent Circuit

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$



2.4 CHARACTERISTICS

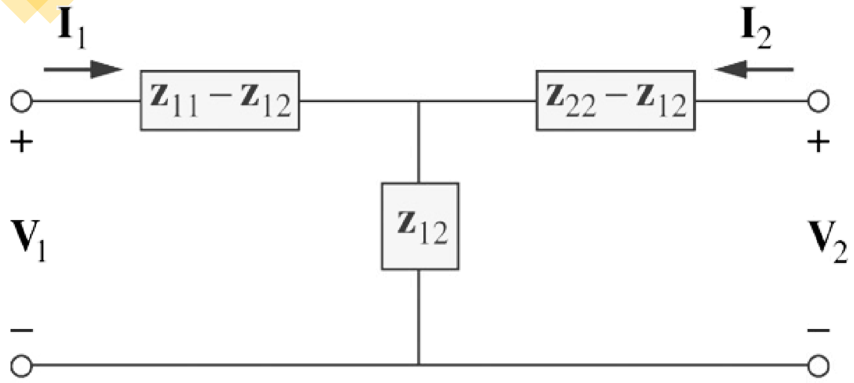
Reciprocal

The two-port network is **linear** and has **no dependent sources**:

$$z_{21} = z_{12}$$

It is said to be reciprocal.

Any two-port made **entirely** of resistors, capacitors and inductors must be reciprocal.

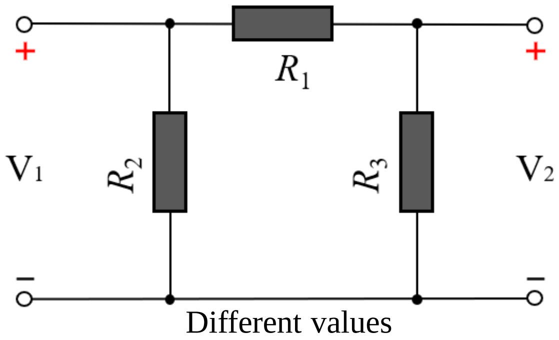


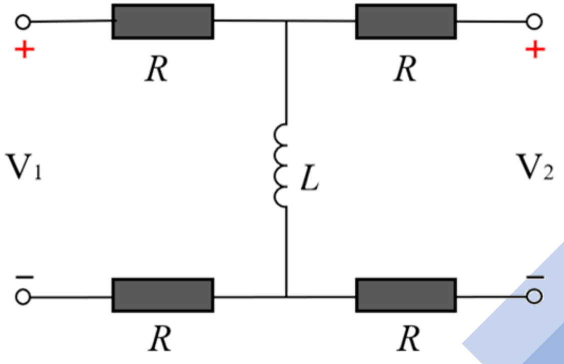
Symmetrical

The network has mirror-like symmetry about the centre line.

$$z_{11} = z_{22}$$

If a two-port network is **reciprocal** and **symmetrical**, only 2 parameters need to be determined.





reciprocal ?

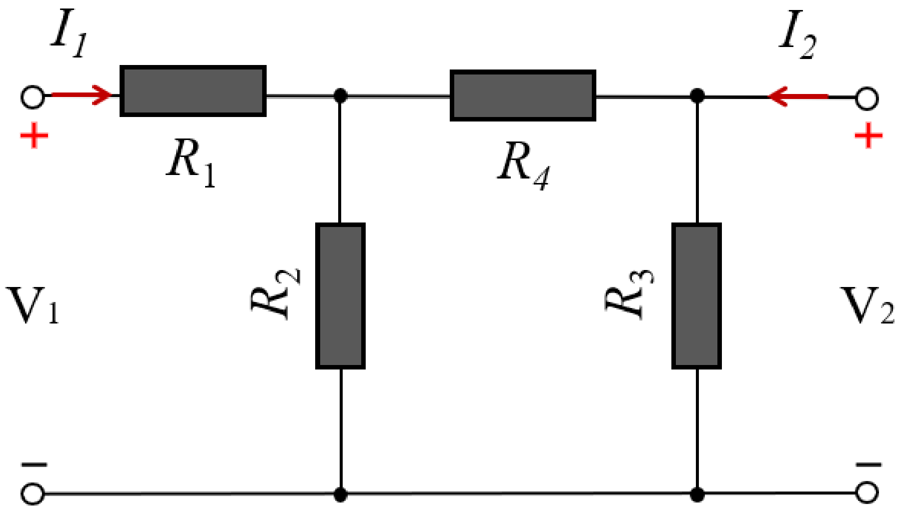
symmetrical ?

reciprocal ?

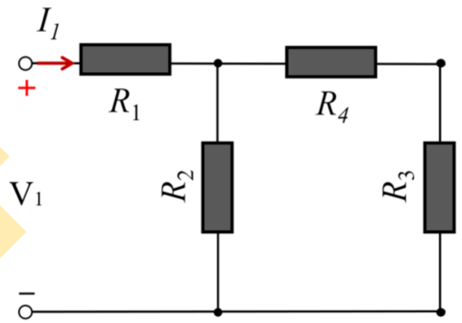
symmetrical ?

QUIZ 2.1

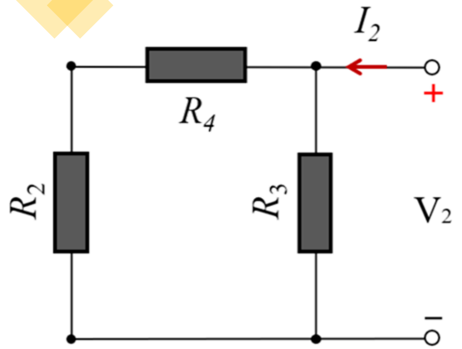
Determine the **Z** parameters for the given circuit.



When port 2 is open:

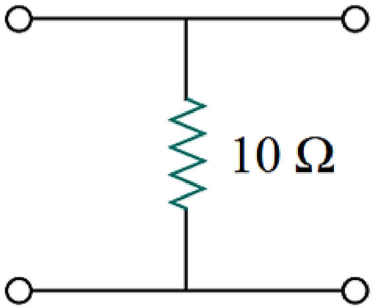


When port 1 is open:

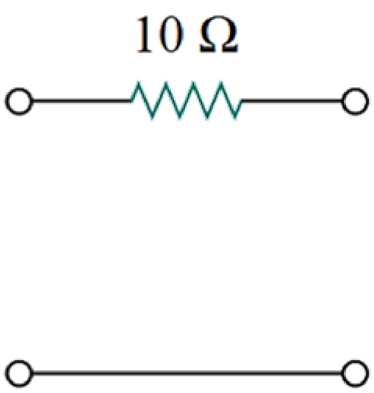


QUIZ 2.2

1. For the single-element two-port network in figure (a), find the related z-parameters.



(a)



(b)

2. If the circuit is changed to figure (b),

z_{11} is __________, z_{12} is __________.

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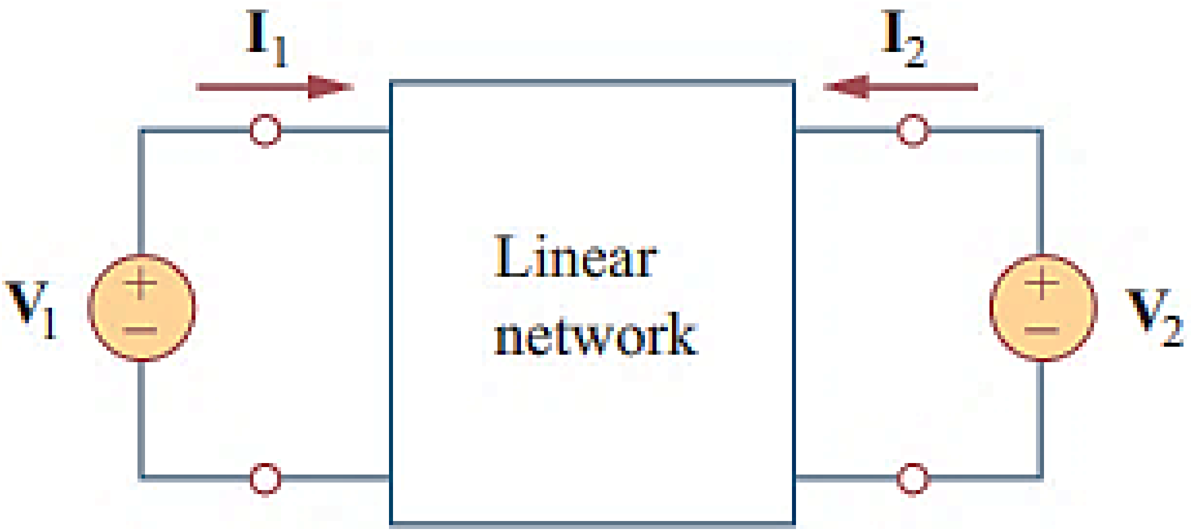
3.1 VOLTAGE SOURCE DRIVEN 2-PORT



The terminal voltages are related to the terminal currents as:

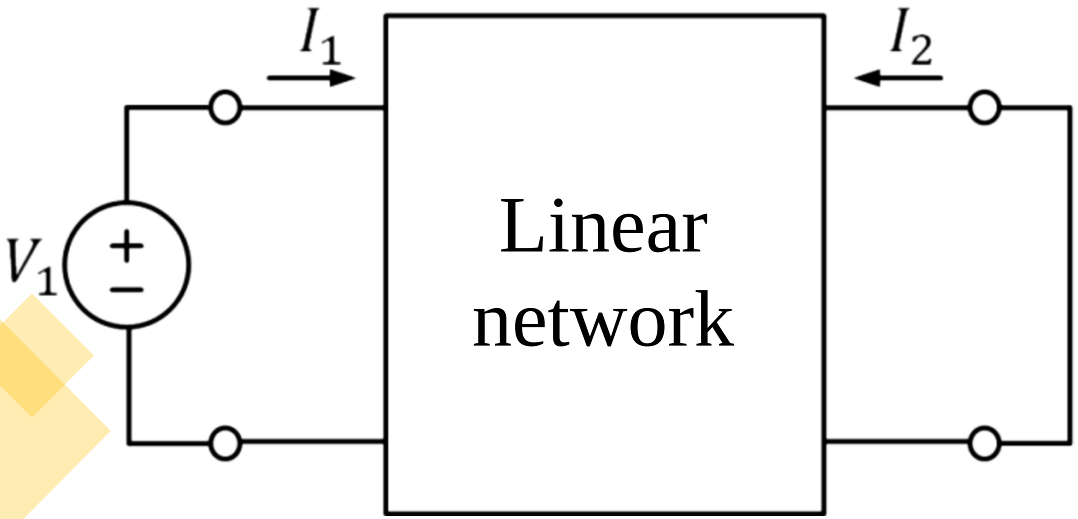
$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Matrix Form

3.2 Y PARAMETERS



$$I_1 = y_{11}V_1 + \cancel{y_{12}V_2}$$

$$I_2 = y_{21}V_1 + \cancel{y_{22}V_2}$$

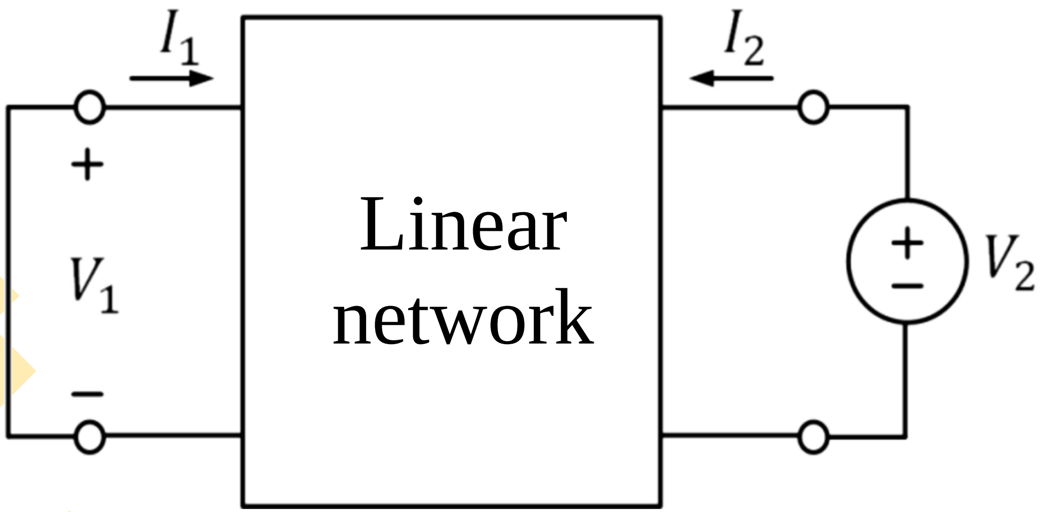
Short-circuit input impedance:
The admittance seen looking into port 1 when port 2 is shorted.

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

Short-circuit transfer admittance:
It is the ratio of the current at port 2 to the voltage at port 1 when port 2 is shorted.

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

3.2 Y PARAMETERS



$$I_1 = \cancel{y_{11} V_1} + y_{12} V_2$$

$$I_2 = \cancel{y_{21} V_1} + y_{22} V_2$$

Short-circuit output impedance:
The admittance seen looking into port 2 when port 1 is shorted.

$$y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1 = 0}$$

Short-circuit transfer admittance:
It is the ratio of the current at port 1 to the voltage at port 2 when port 1 is shorted.

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1 = 0}$$

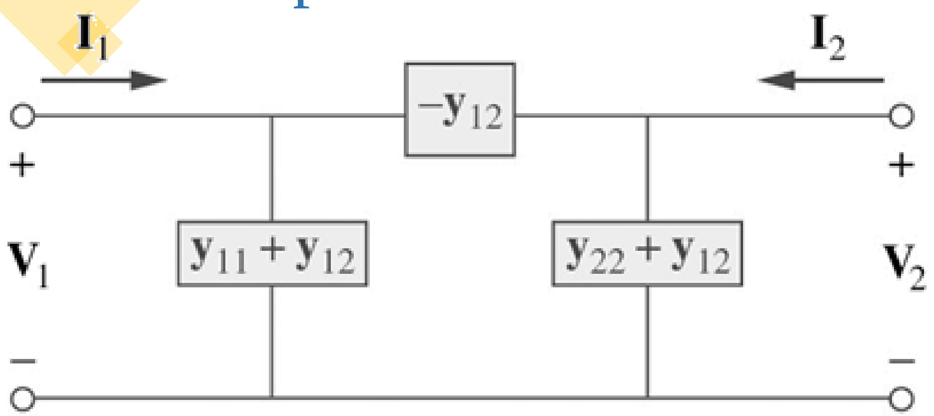
3.2 Y PARAMETERS

Reciprocal

The two-port network is **linear** and has **no dependent sources**.

$$y_{21} = y_{12}$$

Any two-port made **entirely** of resistors, capacitors and inductors must be reciprocal.



Symmetrical

The network has mirror-like symmetry about the centre line.

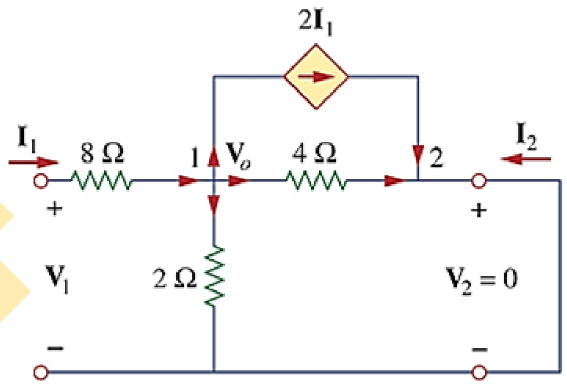
$$y_{11} = y_{22}$$

If the two-port network is **reciprocal** and **symmetrical**, only 2 parameters need to be determined.

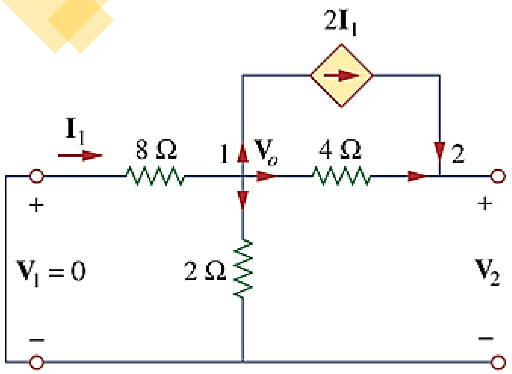
QUIZ 3.1

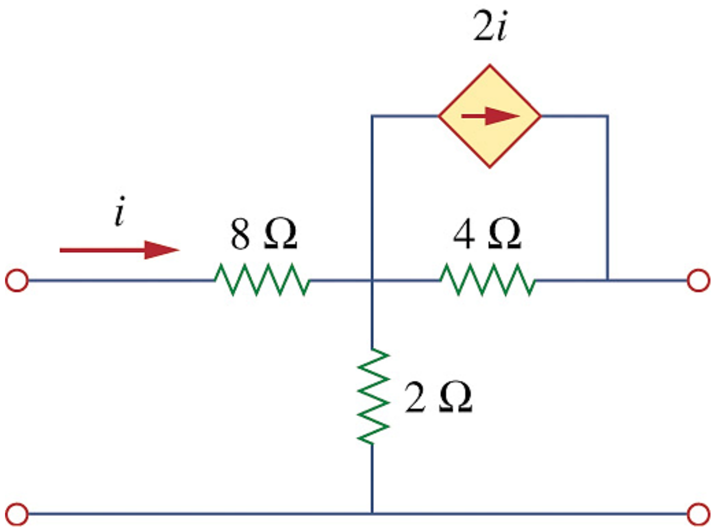
Determine the admittance parameters for the given circuit.

When port 2 is shorted:



When port 1 is shorted:





QUIZ 3.2

When port 1 of a two-port circuit is short-circuited, $I_1 = 4I_2$ and $V_2 = 0.25I_2$. Which of the following is true?

(a) y_{12}=16 (b) y_{21}=16

(c) y_{22}=0.25 (d) y_{11}=4

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4.1 RELATIONS

From the *z* parameter:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[y] = [z]^{-1} = \frac{\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}}{\Delta_z}$$

$$\Delta_z = z_{11} z_{22} - z_{12} z_{21}$$

From the *y* parameter:

$$\begin{bmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \mathrm{V}_1 \\ \mathrm{V}_2 \end{bmatrix} = [y] \begin{bmatrix} \mathrm{V}_1 \\ \mathrm{V}_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \mathrm{V}_1 \\ \mathrm{V}_2 \end{bmatrix} = [y]^{-1} \begin{bmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[z] = [y]^{-1} = \frac{\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}}{\Delta_y}$$

$$\Delta_y = y_{11} y_{22} - y_{12} y_{21}$$

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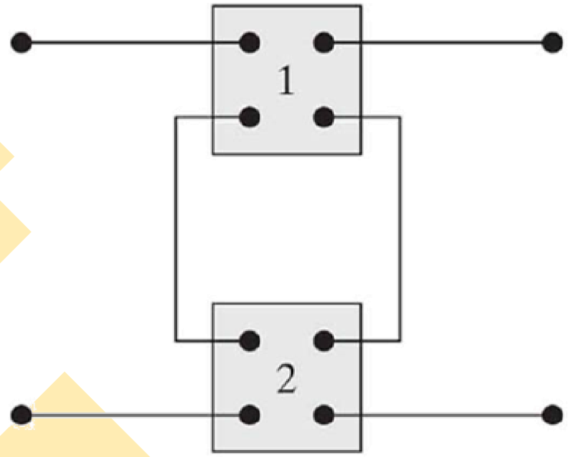
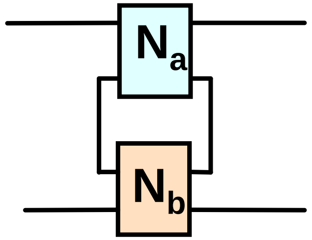
5.1 OVERVIEW

Series

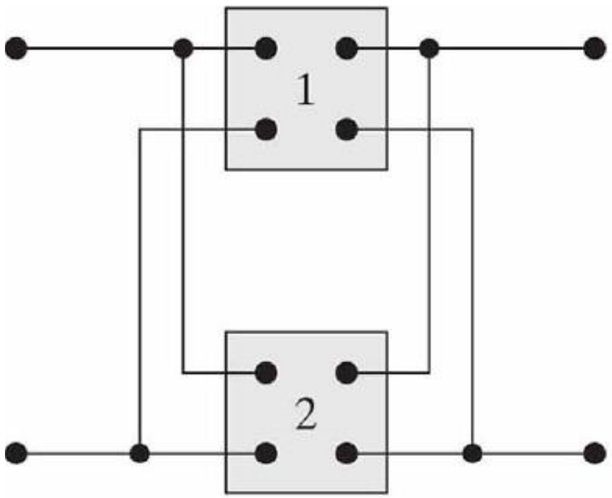


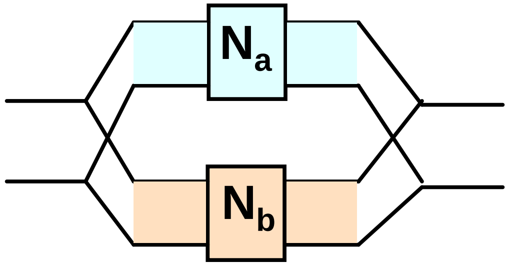


z parameter

$$[z] = [z_a] + [z_b]$$

Parallel





y parameter

$$[y] = [y_a] + [y_b]$$

5.2 SERIES RELATION

For Network N_a:

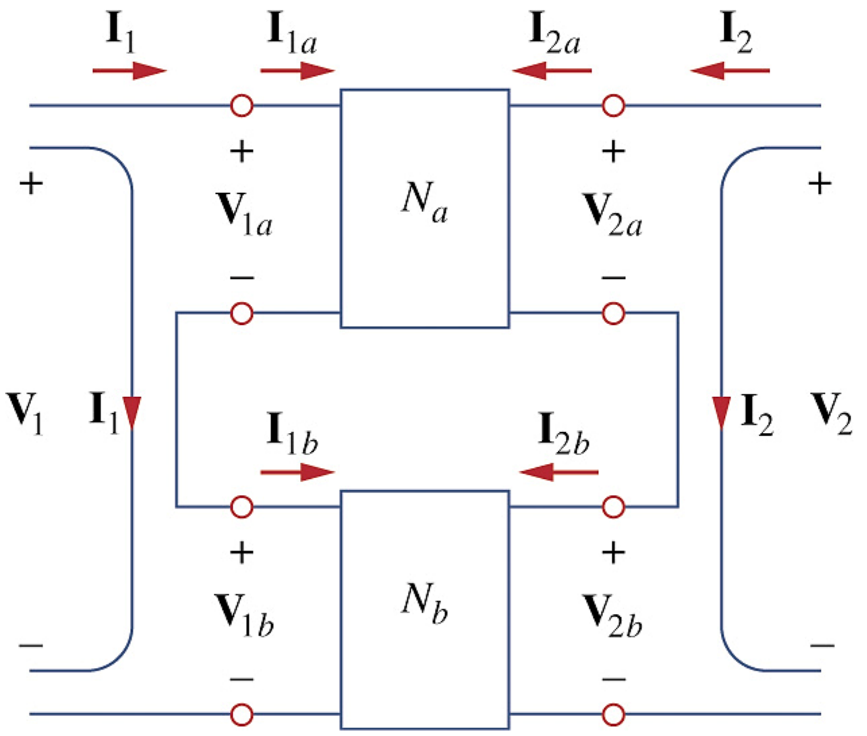
$$V_{1a} = z_{11a}I_{1a} + z_{12a}I_{2a}$$

$$V_{2a} = z_{21a}I_{1a} + z_{22a}I_{2a}$$

For Network N_b:

$$V_{1b} = z_{11b}I_{1b} + z_{12b}I_{2b}$$

$$V_{2b} = z_{21b}I_{1b} + z_{22b}I_{2b}$$



$$\because I_1 = I_{1a} = I_{1b}, I_2 = I_{2a} = I_{2b}$$

$$\therefore V_1 = V_{1a} + V_{1b} = (z_{11a} + z_{11b})I_1 + (z_{12a} + z_{12b})I_2$$

$$V_2 = V_{2a} + V_{2b} = (z_{21a} + z_{21b})I_1 + (z_{22a} + z_{22b})I_2$$

$$[Z] = [z_a] + [z_b]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11a} + z_{11b} & z_{12a} + z_{12b} \\ z_{21a} + z_{21b} & z_{22a} + z_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

This can be extended to *n* networks in series.

QUIZ 5.1

Determine the ratio V_2/V_s for the given circuit.

For Network N_a:

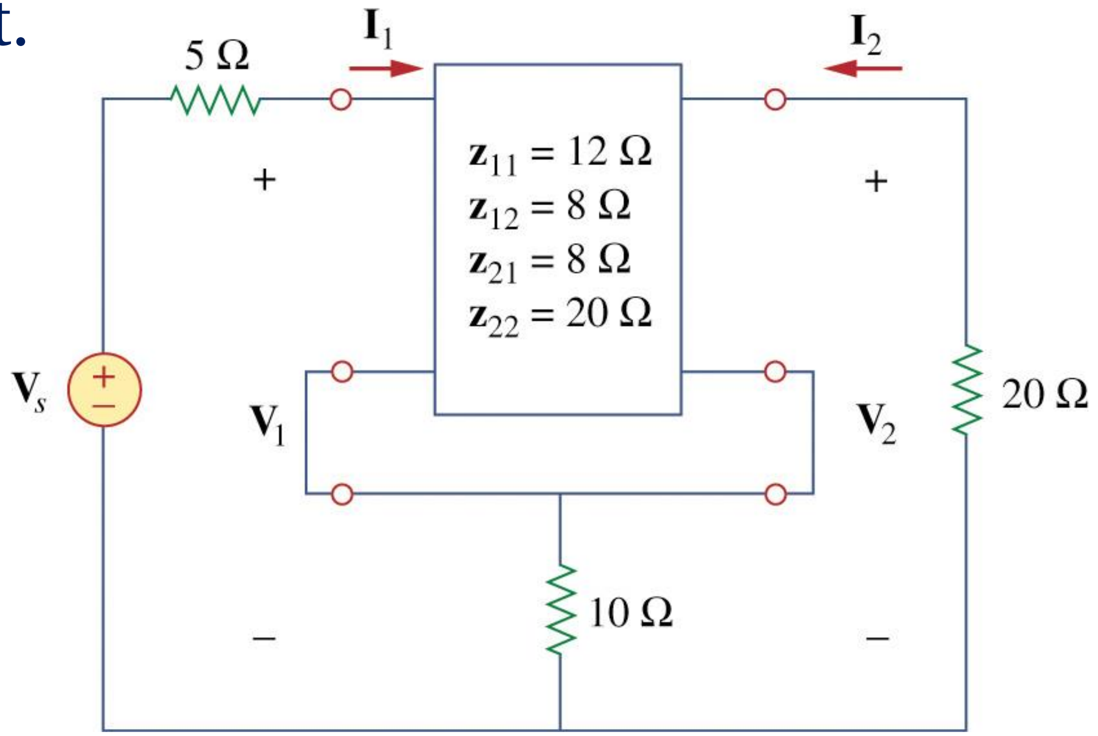
$$[z_a] = \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix}$$

For Network N_b:

$$[z_b] = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{cases} V_1 = 22I_1 + 18I_2 \\ V_2 = 18I_1 + 30I_2 \end{cases}$$



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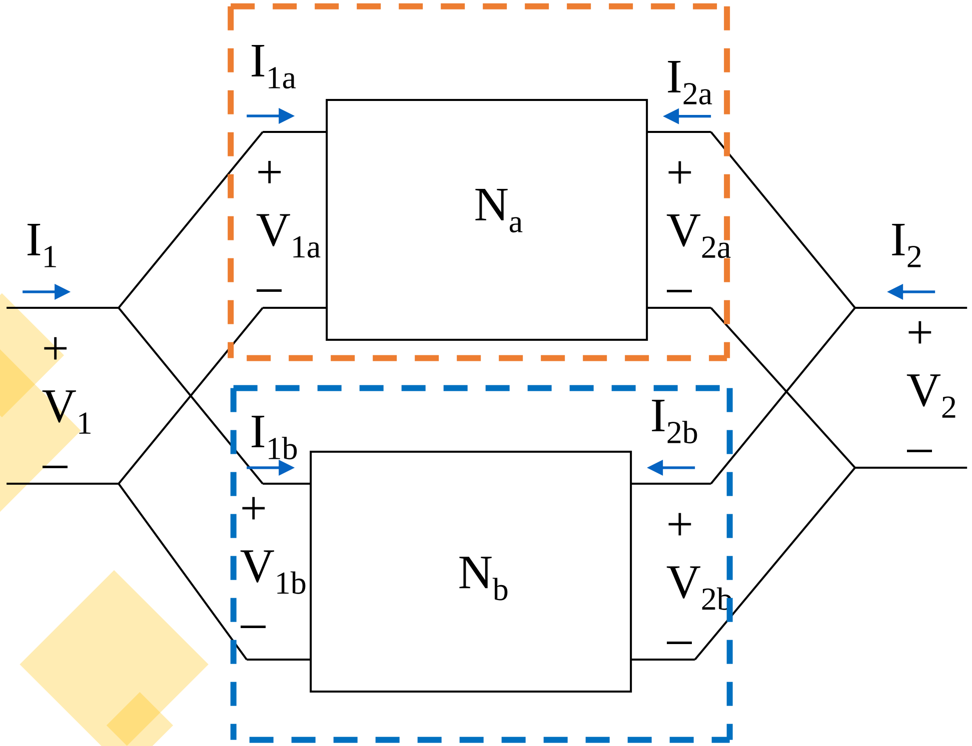
5.3 PARALLEL RELATION



For Network N_a:

$$I_{1a} = y_{11a}V_{1a} + y_{12a}V_{2a}$$

$$I_{2a} = y_{21a}V_{1a} + y_{22a}V_{2a}$$

For Network N_b:

$$I_{1b} = y_{11b}V_{1b} + y_{12B}V_{2b}$$

$$I_{2b} = y_{21b}V_{1b} + y_{22b}V_{2b}$$

$$\because V_1 = V_{1a} = V_{1b}, V_2 = V_{2a} = V_{2b}$$

$$\therefore I_1 = I_{1a} + I_{1b} = (y_{11a} + y_{11b})V_1 + (y_{12a} + y_{12b})V_2$$

$$I_2 = I_{2a} + I_{2b} = (y_{21a} + y_{21b})V_1 + (y_{22a} + y_{22b})V_2$$

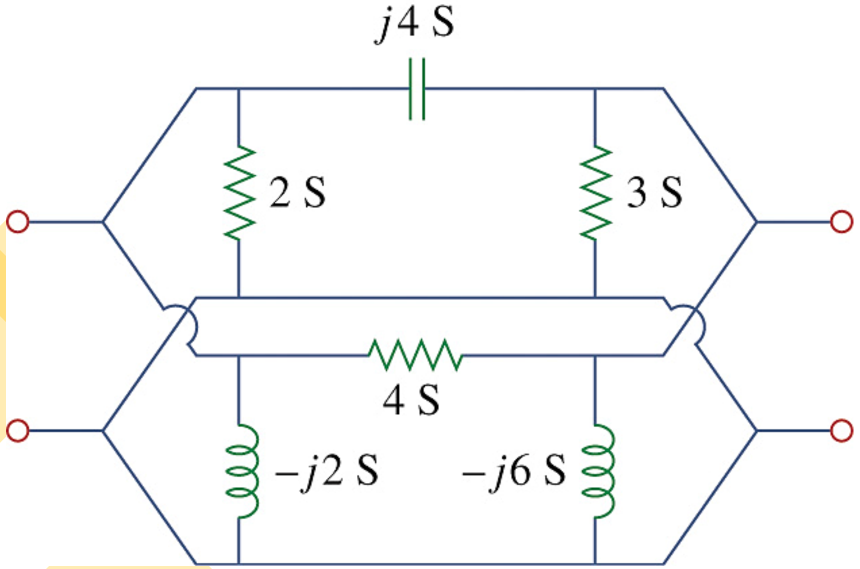
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11a} + y_{11b} & y_{12a} + y_{12b} \\ y_{21a} + y_{21b} & y_{22a} + y_{22b} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[y] = [y_a] + [y_b]$$

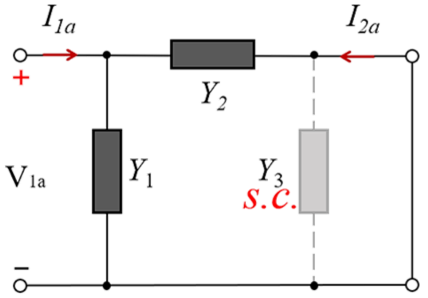
This can be extended to *n* networks in parallel.

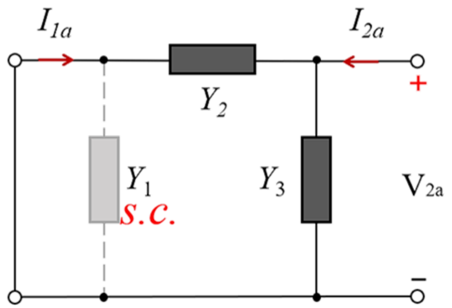
QUIZ 5.2

Determine the admittance parameters for the given network.



For Network N_a:





NEXT…

Magnetically Coupled Circuits