CAN102 Electromagnetism and Electromechanics

Lecture-11 Electromagnetic Induction

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Outline

- 1. Faraday's Experiments
- 2. Lenz's Law
- 3. Faraday's Law
  - EMF (Electromotive Force)
- 4. Integral and Differential forms
- 5. Motional EMF

Recall: Static E and M Fields

Fundamental Relations	Electrostatic Model	Magnetostatic Model
Governing equations	$\nabla \times \mathbf{E} = 0$ $\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J}$
Constitutive relations (linear and isotropic media)	$\mathbf{D} = \epsilon\mathbf{E}$	$\mathbf{H} = \frac{1}{\mu}\mathbf{B}$

- The electric fields produced by stationary charges.
- The magnetic fields produced by moving charges (currents).

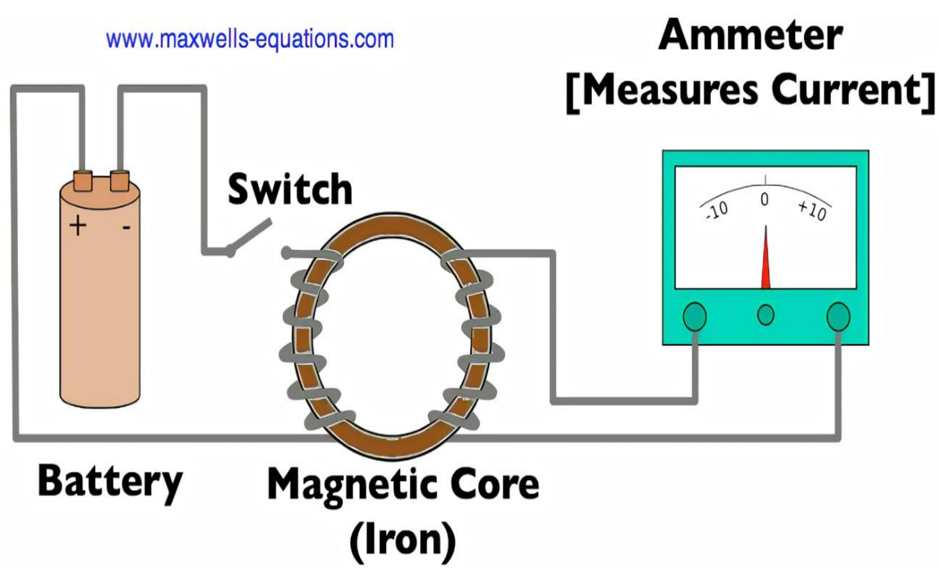
Recall: Static E and M Fields (cont.)

- Imposing an electric field on a conductor gives rise to a current which in turn generates a magnetic field.
  - Oersted's experiment
  - Faraday's modified experiment
  - Ampere's Law

- Whether or not an electric field could be produced by a magnetic field?

1.1 Faraday’s Experiment 1 (current loops)

Experiment 1



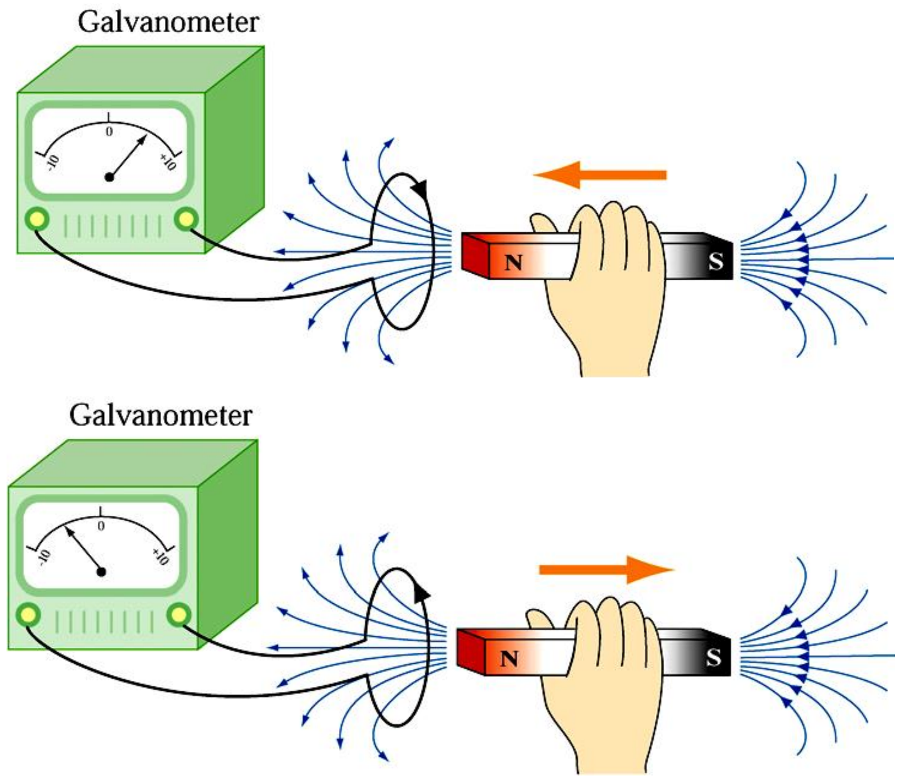
Induced voltage can be detected in coil 2 at the time of opening or closing the switch *s*.

Cut from the EM class of Prof. Lewin (MIT)

Changes of the magnetic field induced current in the secondary loop.

1.2 Faraday's Experiment 2 (loop and magnet)

Experiment 2



Induced voltage and current can be detected in the coil when moving the magnet towards or away from the coil.

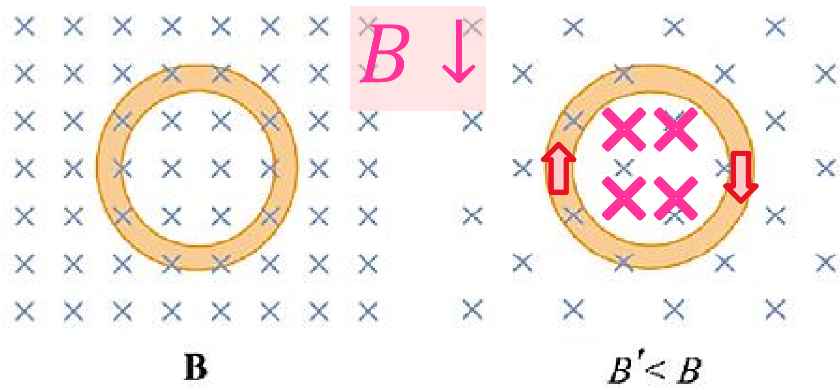
Cut from the EM class of Prof. Lewin (MIT)

Lenz's Law - The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

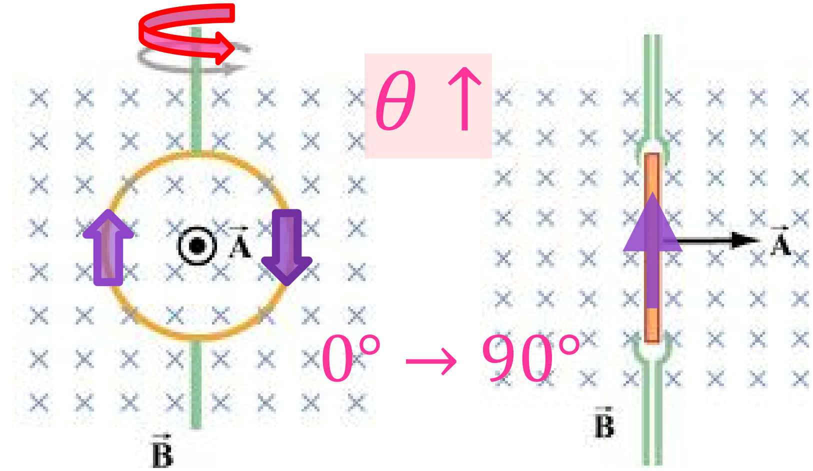
- Most "human" law;
- Determine the "direction of induced current".

2. Lenz's Law

The induced magnetic field **opposes** the changes in magnetic flux



Vary the magnitude of B with time



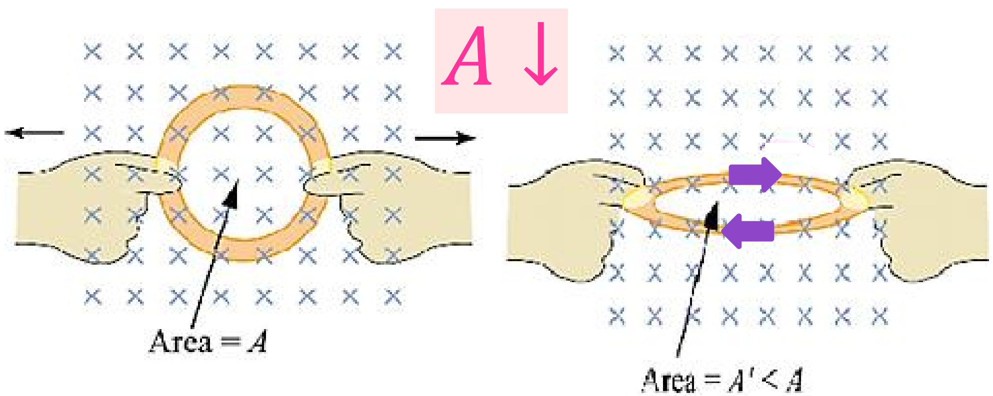
Vary the angle between B&A with time

$$\Phi_B \downarrow = \iint \vec{B} \cdot d\vec{s}$$

$$= \vec{B} \cdot \vec{A}$$

$$= BA\cos\theta \downarrow$$

$$\Phi_B \downarrow = B A \downarrow \cos\theta$$



Vary the magnitude of A with time

3.1 Faraday's more experiments

- The process of inducing a voltage in a coil (also called a loop) by placing it in a time-varying magnetic field is now commonly referred to as an ***electromagnetic induction***.

The flux Φ_B is confined inside the solenoid, which is irrelevant to loop 2;

- So the induced current is irrelevant to the size and shape of loop 2.
- But the induced current is proportional to the turns of loop 2.

3.2 Faraday's Law

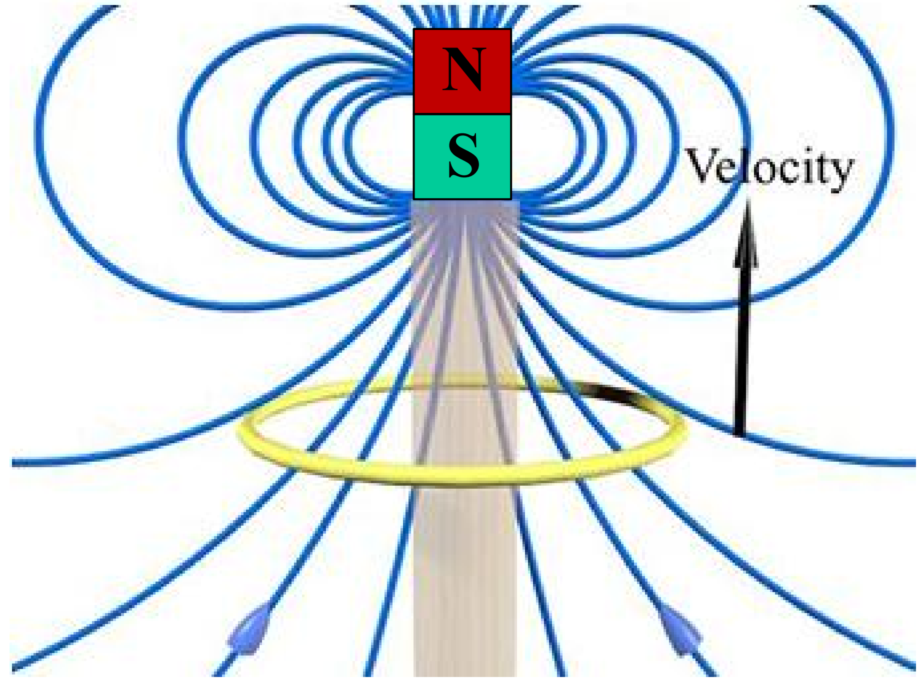
- Faraday's Law - a time-varying magnetic field produces an **electromotive force (*emf*)** that may establish a current in a suitable closed circuit.
  - An electromotive force is merely a voltage that arises from conductors moving in a magnetic field or changing magnetic fields

$$emf = -N\frac{d\Phi}{dt} \quad (V)$$

  - The minus sign is again, from Lenz's Law.
- Lenz's Law – an indication that the *emf* is in such a direction as to produce a current whose flux, if added to the original flux, would reduce the magnitude of the *emf*.

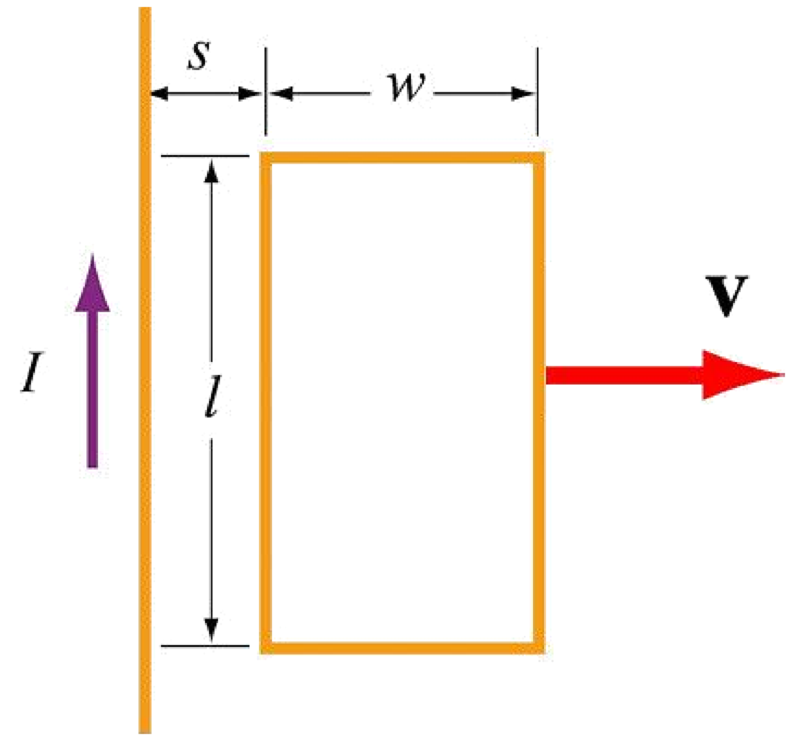
Quiz 1

- A coil moves up from underneath a magnet whose north pole pointing upward. The current in the coil and the net force on the coil are:
  a) Current clockwise; force up
  b) Current counterclockwise; force up
  c) Current clockwise; force down
  d) Current counterclockwise; force down



Quiz 2

- A rectangular loop of wire is pulled away from a long wire carrying current I in the direction shown in the sketch. The induced current in the rectangular circuit is:
  a) Clockwise;
  b) Counterclockwise;
  c) Neither, the current is zero.



4.1 Integral form of the Faraday's Law

- Define the induced *emf* in a conductor in terms of the induced electric field intensity inside the conductor as:

$$emf = \oint_C \vec{E} \cdot \vec{dl}$$

- The total flux enclosed by contour *c* is

$$\Phi = \iint_S \vec{B} \cdot \vec{ds}$$

- Therefore, the Faraday's law can be written as:

$$\oint_C \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \iint_S \vec{B} \cdot \vec{ds}$$

The direction of the surface ***ds*** is defined by the direction of contour *c* and the right-hand rule.

$$= -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} \longrightarrow \boxed{\text{Integral form of Faraday's Law}}$$

4.2 Differential form of the Faraday's Law

$$\boxed{emf = \oint_C \vec{E} \cdot \vec{dl} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}}$$

- Using Stokes' theorem:

$$\oint_C \vec{E} \cdot \vec{dl} = \iint_S (\nabla \times \vec{E}) \cdot \vec{ds} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

- The integrand should be equal on both sides, so

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow \boxed{\text{Differential form of Faraday's Law}}$$

- The electric field intensity in a region of time-varying magnetic flux density is ***nonconservative***.

Example 1

- The rectangular loop of wire in the figure is fixed. Suppose that the current is a function of time with $I(t) = a + bt$, where a and b are positive constants. What is the induced *emf* in the loop and the direction of the induced current?
  - The magnetic flux through an area element $d\vec{s}$ is:

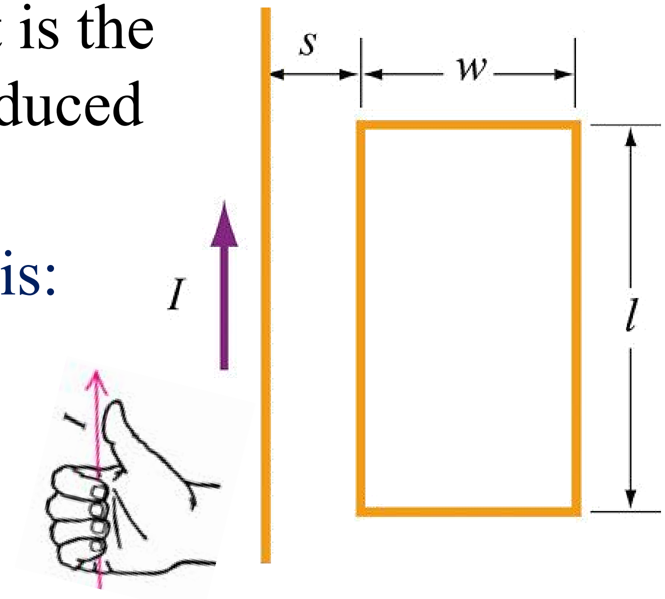
$$d\Phi = \vec{B} \cdot d\vec{s} = BdA = \frac{\mu_0 I}{2\pi r} ldr$$

  - Then the total magnetic flux is:

$$\Phi = \int_s^{s+w} \frac{\mu_0 I}{2\pi r} ldr = \frac{\mu_0 Il}{2\pi} ln\left(\frac{s+w}{s}\right)$$

  - So the induced *emf* is:

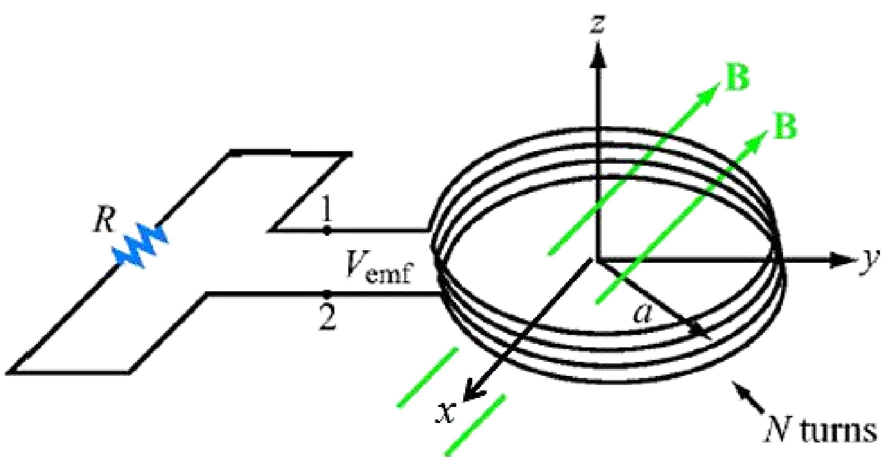
$$emf = -\frac{d\Phi}{dt} = -\frac{\mu_0 bl}{2\pi} \ln\left(\frac{s+w}{s}\right)$$



$$B = \frac{\mu_0 I}{2\pi r}$$

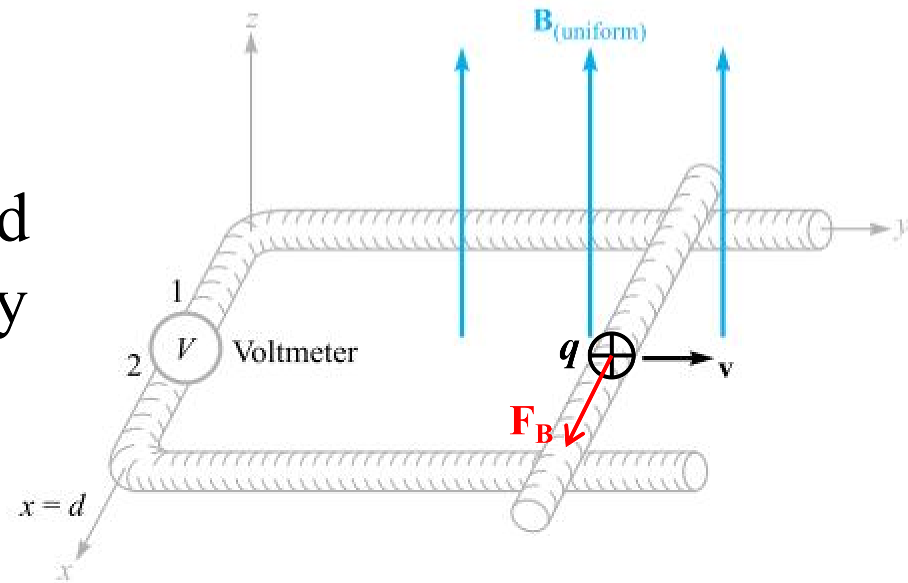
Quiz 3

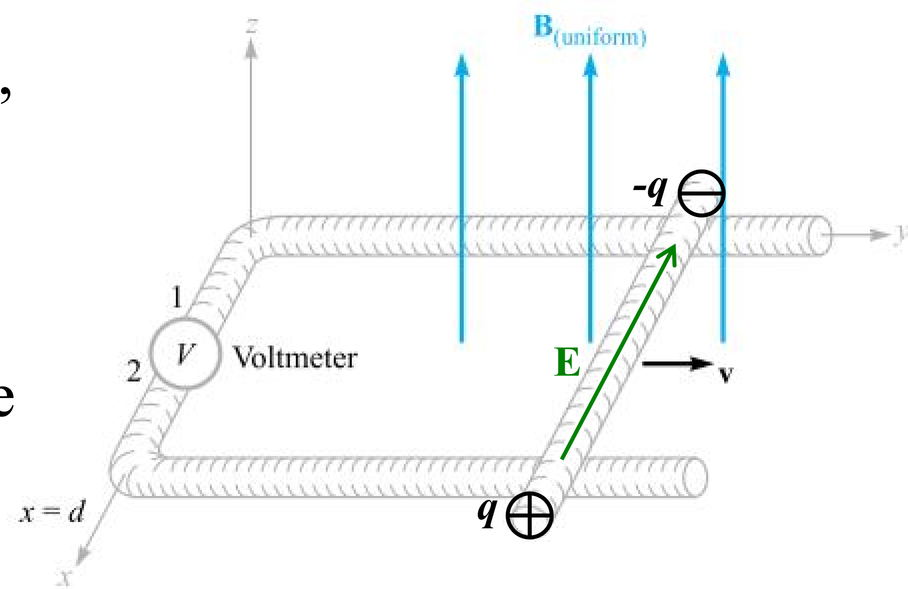
- An **inductor** is formed by winding N turns of a thin conducting wire into a circular loop of radius a. The inductor loop is in the x-y plane with its center at the origin, and connected to a resistor R. In the presence of a magnetic field $\vec{B} = B_0(2\hat{y} + 3\hat{z})\sin(\omega t)$, where ωt is the angular frequency. Find the following:
  - (a) the magnetic flux linking a single turn of the inductor;
  - (b) *emf* ;
  - (c) the **magnitude** of the induced current in the circuit at t=0 (ignore the wire resistance).



1.5 Motional EMF

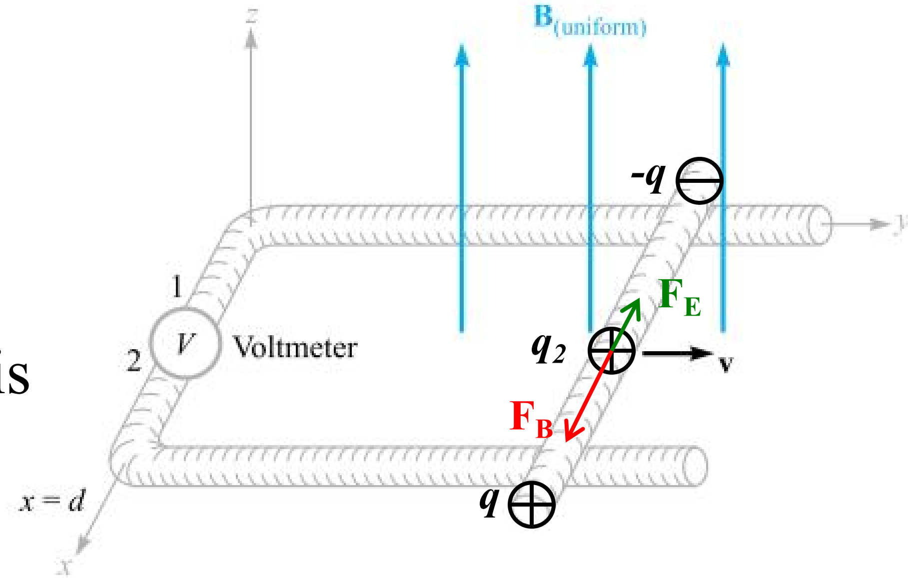
- In the constant magnetic field **B**, the shorting bar moves to the right with a velocity *v*, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter is used to read the emf.
- Analyses:
  - Consider a charge *q* on the conductor, which experiences a Lorentz's force $\mathbf{F_B}$, make it drifted to the lower end (+*x* direction) of the conducting bar.
  - The whole bar is neutral, so a positive and negative charge pair built an internal **E** field inside the bar.

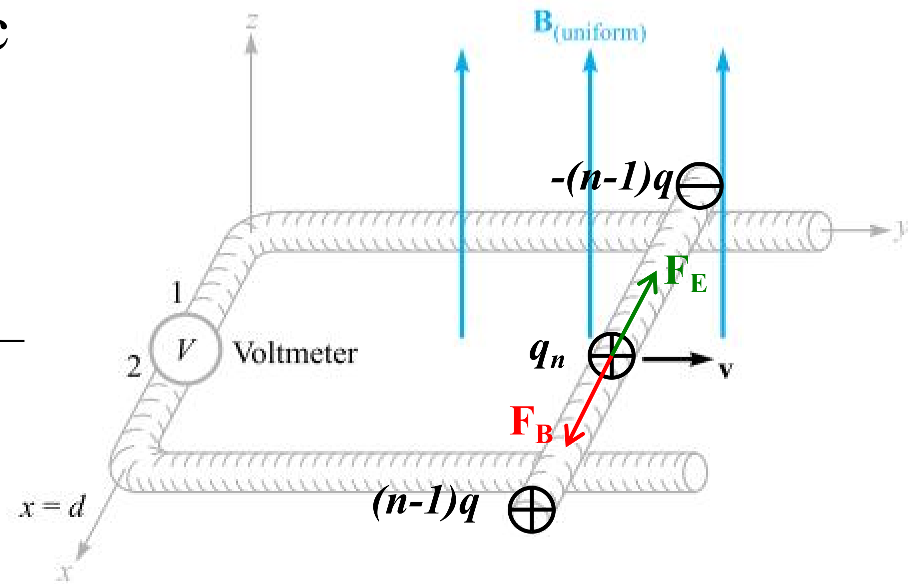




1.5 Motional EMF

- Analyses (continued):
  - Consider a new charge q_2, which experiences two forces, the Lorentz's force due to the field **B** and the electric force due to the field **E**. In this case, $\mathbf{F_B} > \mathbf{F_E}$, so q_2 drifts to $+x$ direction and contributes to field **E**.
  - After a while (very short), the electric field **E** increases to a value large enough to generate the force $\mathbf{F_E} = \mathbf{F_B}$. Now the charges can move in y direction without x direction drifting – equilibrium state.





1.5 Motional EMF

- The force per unit charge is called the motional electric field intensity $\mathbf{E_m}$:

$$\boldsymbol{E}_{\boldsymbol{m}} = \frac{\boldsymbol{F}}{Q} = \boldsymbol{v} \times \boldsymbol{B}$$

- The voltage produced by the induced motional electric field intensity $\mathbf{E_m}$ is then:

$$V_{12} = \oint \boldsymbol{E}_{\boldsymbol{m}} \cdot dL = \oint (\boldsymbol{v} \times \boldsymbol{B}) \cdot d\boldsymbol{L} = \int_{fixed} + \int_{\mathrm{d}}^{0} vBdx = -Bvd$$

  – where *d* is the length of the conducting bar.

- This is referred to as a ***flux cutting emf*** or a ***motional emf***.
  – Obviously, only the part of the circuit that moves in a direction not parallel to the magnetic flux will contribute to V.

Example 2

- In a constant magnetic field **B**, a conducting bar moves to the right with a velocity *v*, and the circuit is completed through the two rails and connected by a resistor. If an external force is applied, the bar moves to the right with a constant velocity.
  - The magnetic flux through the closed loop is

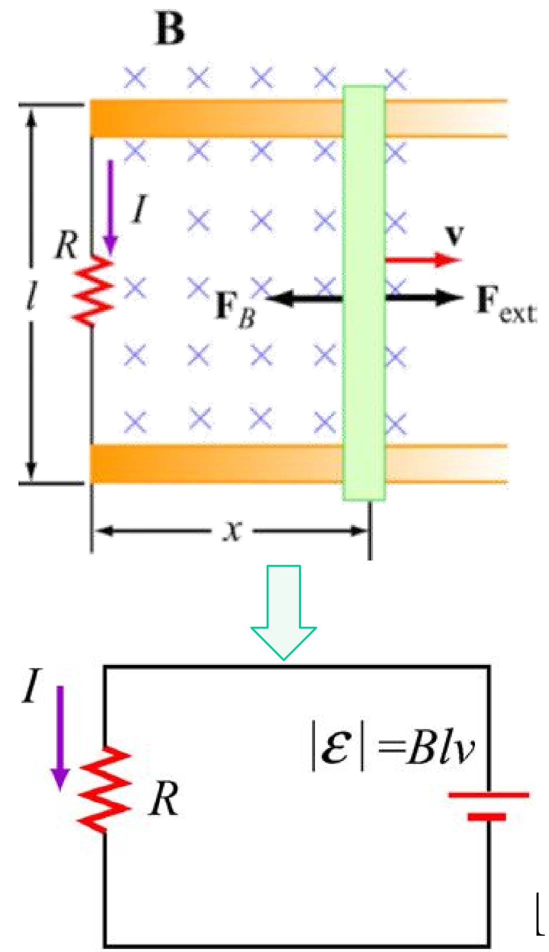
$$\Phi = \iint_S \vec{B}\, d\vec{s} = Blx$$

  - The induced *emf* can be found by Faraday's law

$$emf = -\frac{d\Phi}{dt} = -\frac{d(Blx)}{dt} = -Bl\frac{dx}{dt} = -Blv$$

  - The induced *current* is

$$I = \frac{|emf|}{R} = \frac{Blv}{R}$$



Direction: anti-clockwise

Example 2

- In a constant magnetic field **B**, a conducting bar moves to the right with a velocity *v*, and the circuit is completed through the two rails and connected by a resistor. If an external force is applied, the bar moves to the right with a constant velocity.
  - The magnetic force acting on the bar is

$$\vec{F}_B = I(l\hat{y}) \times (-B\hat{z}) = -\left(\frac{B^2 l^2 v}{R}\right)\hat{x}$$

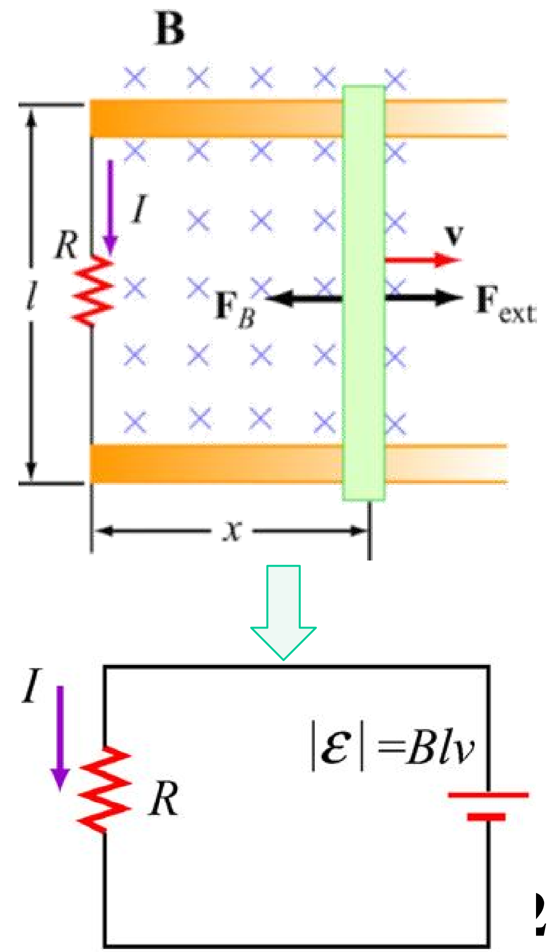
  - The external force is

$$\vec{F}_{ext} = -\vec{F}_B = \left(\frac{B^2 l^2 v}{R}\right)\hat{x}$$

  - The power delivered by the external force is:

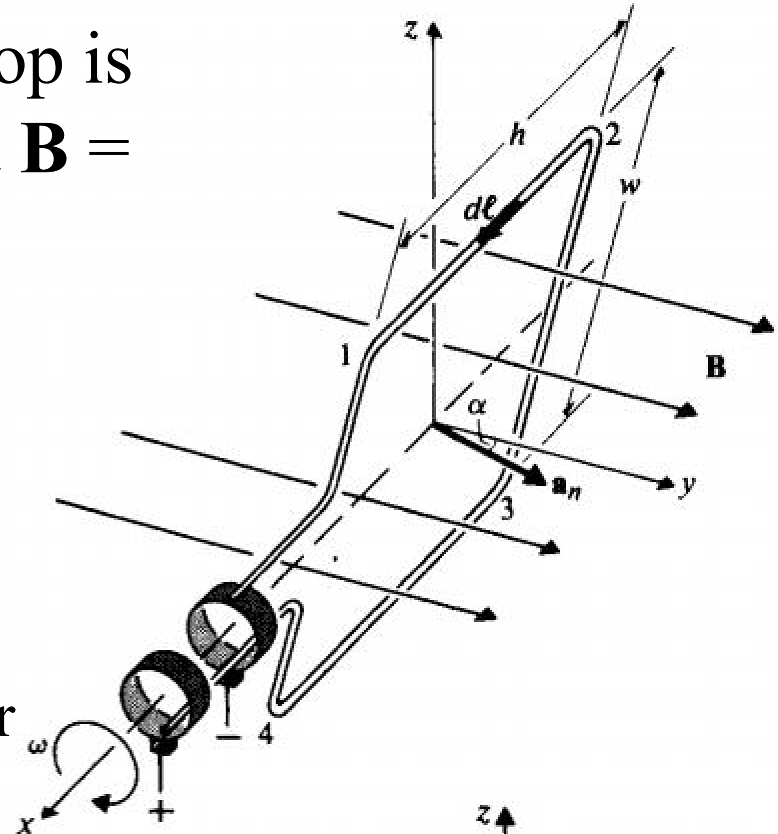
$$P = \vec{F}_{ext}\vec{v} = F_{ext}v = \frac{B^2 l^2 v^2}{R} = \left(\frac{Blv}{R}\right)^2 R = I^2 R$$

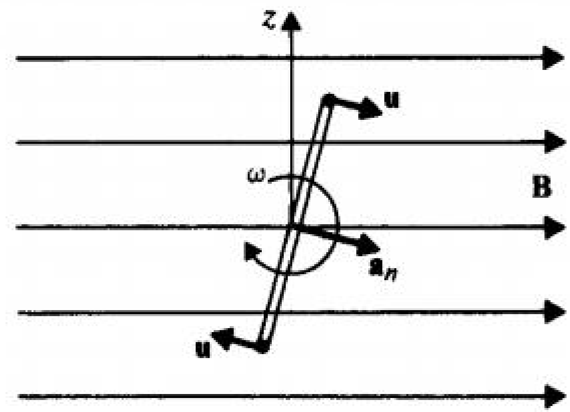
equal to the power dissipated on the resistor



Example 3

- An h by w rectangular conducting loop is situated in a changing magnetic field $\mathbf{B} = \mathbf{a}_y B_0 \sin(\omega t)$. The normal of the loop initially makes an angle α with $\mathbf{a}_y$, as shown in the figure.
- Find the induced emf in the loop:
  - a) when the loop is at rest;
  - b) when the loop rotates with an angular velocity ω about the x-axis.





Summary

MAXWELL'S EQUATIONS – STATIC FIELDS

Law	Integral	Differential	Physical meaning
Gauss's law for E-field	$\oiint_S \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon}$	$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon}$	Electric flux through a closed surface is proportional to the charges enclosed
E-field Loop Theorem	$\oint_C \vec{E}\cdot d\vec{l} = 0$	$\nabla\times\vec{E} = 0$	Work done by moving a charge in the E-field along a closed loop is 0
Gauss's law for H-field	$\oiint_S \vec{B}\cdot d\vec{s} = 0$	$\nabla\cdot\vec{B} = 0$	The total magnetic flux through a closed surface is 0
H-field Loop Theorem	$\oint_C \vec{H}\cdot d\vec{l} = \mathrm{I}$	$\nabla\times\vec{H} = \vec{J}$	The H-field produced by an electric current is proportional to the current

MAXWELL'S EQUATIONS - GENERAL

Law	Integral	Differential	Physical meaning
Gauss's law for E-field	$\oiint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon}$	$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$	Electric flux through a closed surface is proportional to the charges enclosed
Faraday's law	$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Changing magnetic flux produces an E-field
Gauss's law for H-field	$\oiint_S \vec{B} \cdot d\vec{s} = 0$	$\nabla \cdot \vec{B} = 0$	The total magnetic flux through a closed surface is 0
H-field Loop Theorem	$\oint_C \vec{H} \cdot d\vec{l} = I$	$\nabla \times \vec{H} = \vec{J}$	The H-field produced by an electric current is proportional to the current

Next ...

- Inductors
  - Mutual Inductance
  - Self Inductance
  - Commonly used inductors

- Summary of EM part