EEE205 – Digital Electronics (II)

Lecture 9

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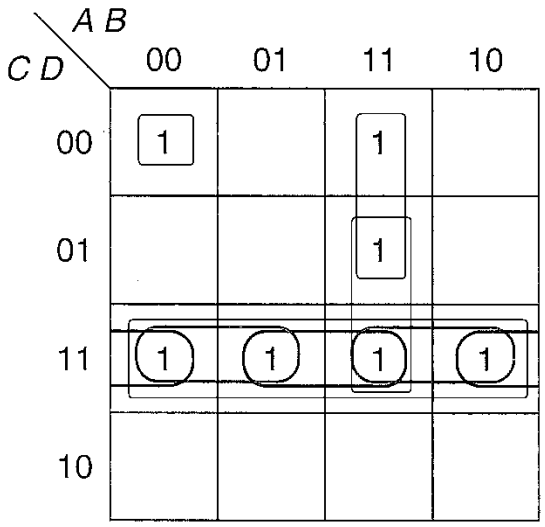
In This Session

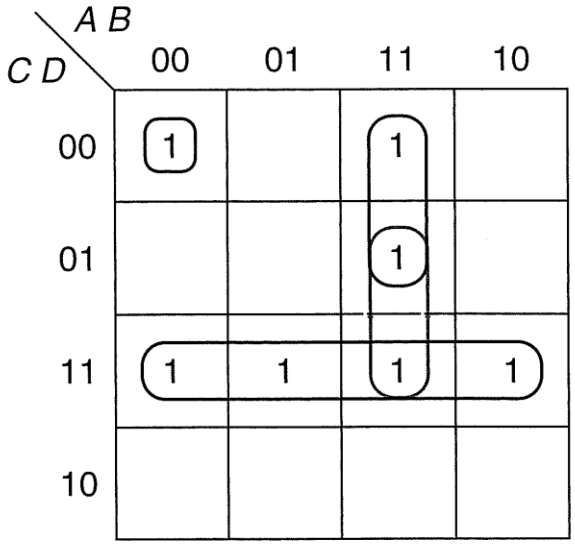
- Quine-McCluskey Method

A Revision of Some Terminology

	Boolean Algebra View	Karnaugh Map View
Minterm	a product term with all variables	a 1 cell
Implicant	a product term	a group of 1's
Prime Implicant	a shortest product term	a group of 1's that is not fully contained in another group of 1's
Essential Prime Implicant	product terms which must stay	has a 1 not included in other group

A Revision of Some Terminology





This K-map contains:

- 7 minterms
- 14 implicants
- 4 prime implicants.
- 3 essential prime implicants.

Quine-McCluskey Method

- The Karnaugh map method is for logic functions with a small number of variables.
- **Quine-McCluskey method** can simplify logic functions with *a large number of variables*.
- The latter is a systematic approach that can be *readily programmed* for a digital computer.

Quine-McCluskey Method

The *procedure* consists of two steps:

1. Finding prime implicants. Eliminate as many as literals as possible from each term by repeatedly applying XY + XY’ = X.

2. Select a minimum set of prime implicants by using a prime implicant chart.

Finding Prime Implicants

- We will start from a standard SOP form consisting of minterms only.
- By using the Theorem XY + XY′ = X, two minterms will be combined if they differ in only one variable.

$$AB'CD' + AB'CD = AB'C$$

$$\underbrace{1\ 0\ 1}_{X}\ 0 + \underbrace{1\ 0\ 1}_{X}\ 1 = \underbrace{1\ 0\ 1}_{X}\ -$$

(the dash indicates a missing variable)

$$A'BC'D + A'BCD' \quad \text{(will not combine)}$$

$$0\ 1\ 0\ 1 + 0\ 1\ 1\ 0 \quad \text{(will not combine)}$$

Finding Prime Implicants

$$f(a, b, c, d) = \Sigma\, m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$$

- The binary minterms are sorted into groups according to the number of 1's.
- This is to reduce the unnecessary comparisons.

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Finding Prime Implicants

- Only terms in adjacent groups must be compared.
- Terms in non-adjacent groups differ in at least two variables.
- Two terms within a group differ in at least two variables.

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Finding Prime Implicants

	Column I			Column II	
group 0	0	0000	✓	0, 1	000–
group 1	1	0001	✓	0, 2	00–0
	2	0010	✓	0, 8	–000
	8	1000	✓	1, 5	0–01
group 2	5	0101	✓	1, 9	–001
	6	0110	✓	2, 6	0–10
	9	1001	✓	2, 10	–010
	10	1010	✓	8, 9	100–
group 3	7	0111	✓	8, 10	10–0
	14	1110	✓	5, 7	01–1
				6, 7	011–
				6, 14	–110
				10, 14	1–10

- Terms 0000 and 0001 can be combined to yield 000- (a'b'c'), and so on.
- The outcomes are listed in a new column.
- The terms which can be combined are checked off.
- The remaining ones are prime implicants.

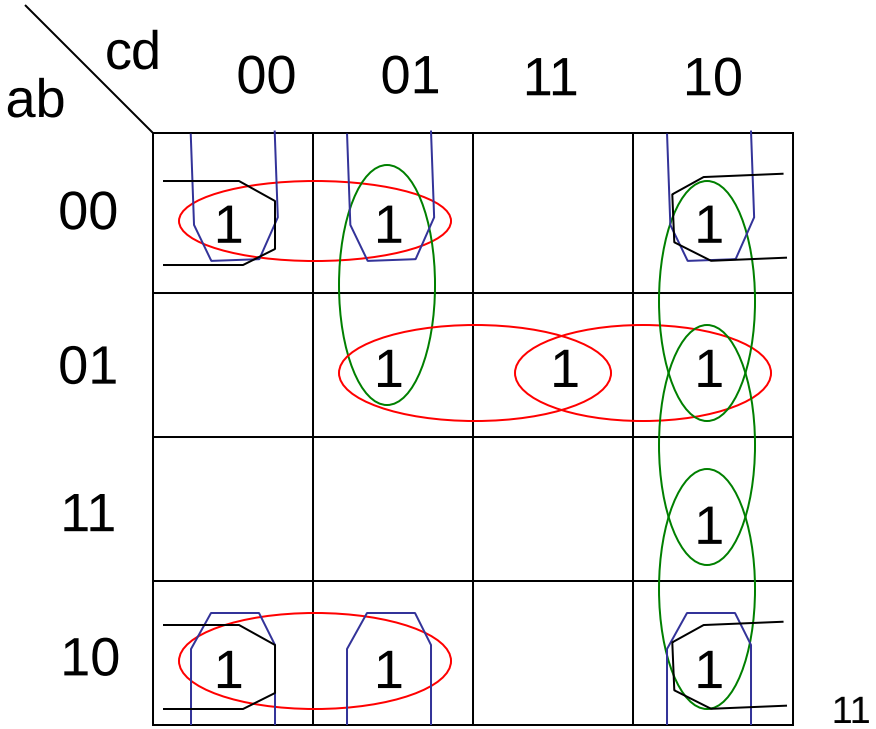
Finding Prime Implicants

Karnaugh Map's View

$$f(a, b, c, d) = \Sigma\, m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$$

- To find groups of 2.
- If a 1 is not in any group of 2, it is a prime implicant.

ab \ cd	00	01	11	10
00	1	1		1
01		1	1	1
11				1
10	1	1		1



Finding Prime Implicants

	Column I			Column II	
group 0	0	0000	✓	0, 1	000–
group 1	1	0001	✓	0, 2	00–0
	2	0010	✓	0, 8	–000
	8	1000	✓	1, 5	0–01
group 2	5	0101	✓	1, 9	–001
	6	0110	✓	2, 6	0–10
	9	1001	✓	2, 10	–010
	10	1010	✓	8, 9	100–
group 3	7	0111	✓	8, 10	10–0
	14	1110	✓	5, 7	01–1
				6, 7	011–
				6, 14	–110
				10, 14	1–10

- Terms in the new column are grouped according to the number of 1's.
- Terms which have dashes in the same place and which differ in only one variable can be combined.
- Terms 000- (a'b'c') and 100- (ab'c') yield -00- (b'c').

Finding Prime Implicants

- Find and delete the duplicate terms.

	Column I			Column II			Column III	
group 0	0	0000	✓	0, 1	000–	✓	0, 1, 8, 9	–00–
group 1	1	0001	✓	0, 2	00–0	✓	0, 2, 8, 10	–0–0
	2	0010	✓	0, 8	–000	✓	~~0, 8, 1, 9~~	~~=00=~~
	8	1000	✓	1, 5	0–01		~~0, 8, 2,10~~	~~=0=0~~
group 2	5	0101	✓	1, 9	–001	✓	2, 6, 10, 14	--10
	6	0110	✓	2, 6	0–10	✓	~~2, 10, 6, 14~~	~~--10~~
	9	1001	✓	2, 10	–010	✓		
	10	1010	✓	8, 9	100–	✓		
group 3	7	0111	✓	8, 10	10–0	✓		
	14	1110	✓	5, 7	01–1			
				6, 7	011–			
				6, 14	–110	✓		
				10, 14	1–10	✓		

Finding Prime Implicants

- Keep comparing terms and forming new groups of terms until no more terms could be combined.
- Terms which have not been checked off are prime implicants.

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

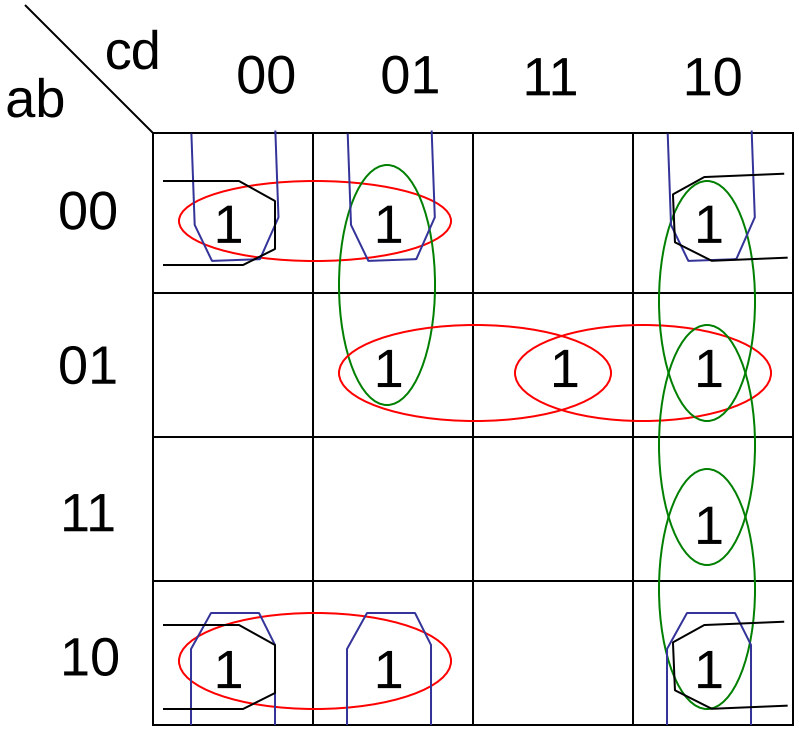
$$(1, 5) \quad (5, 7) \quad (6,7) \quad (0, 1, 8, 9) \quad (0, 2, 8, 10) \quad (2, 6, 10, 14)$$

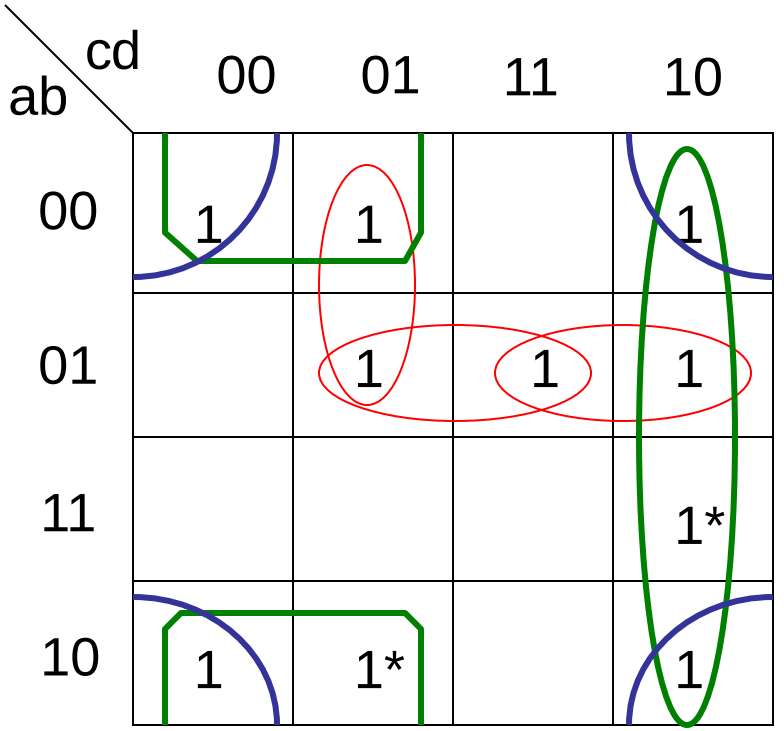
- Each term has a minimum number of literals, but *the number of terms is not minimum*. This is left to the **prime implicant chart**.

Finding Prime Implicants

Karnaugh Map's View

- To find groups of 4.
- If a group of 2 is not in any group of 4, it is a prime implicant.





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The Prime Implicant Chart

- The minterms are listed across the top.
- The prime implicants are listed down the side.
- If a prime implicant covers a given minterm, an X is placed at the intersection.

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	×	×					×	⊗		
(0, 2, 8, 10)	$b'd'$	×		×				×		×	
(2, 6, 10, 14)	cd'			×		×				×	⊗
(1, 5)	$a'c'd$		×		×						
(5, 7)	$a'bd$				×		×				
(6, 7)	$a'bc$					×	×				

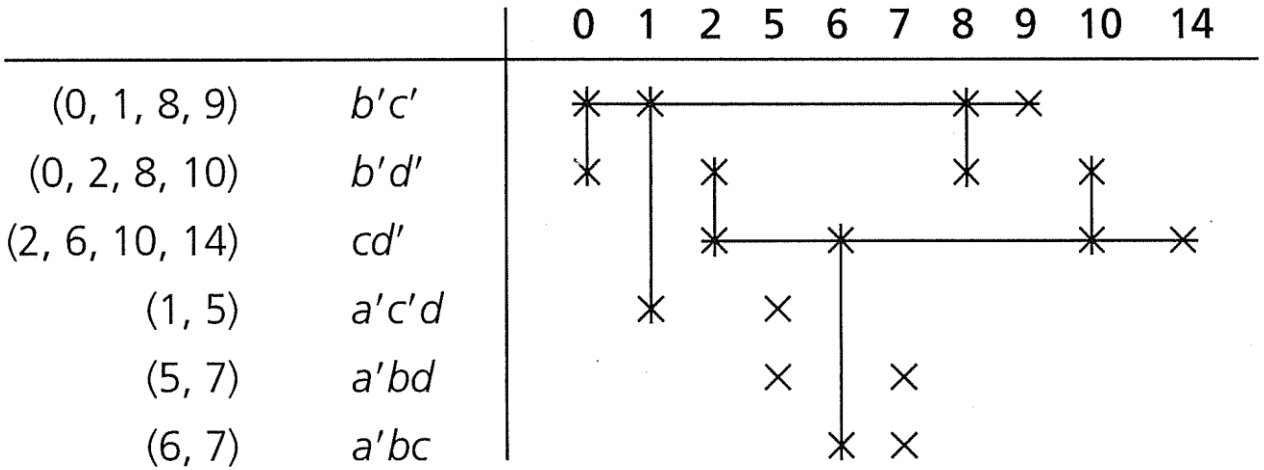
The Prime Implicant Chart

- If a minterm is covered by only one prime implicant (a column contains only one X), the prime implicant is an *essential prime implicant* and the X is circled.
- Essential prime implicants must be included in the minimum sum of products.

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	×	×					×	⊗		
(0, 2, 8, 10)	$b'd'$	×		×				×		×	
(2, 6, 10, 14)	cd'			×		×				×	⊗
(1, 5)	$a'c'd$		×		×						
(5, 7)	$a'bd$				×		×				
(6, 7)	$a'bc$					×	×				

The Prime Implicant Chart

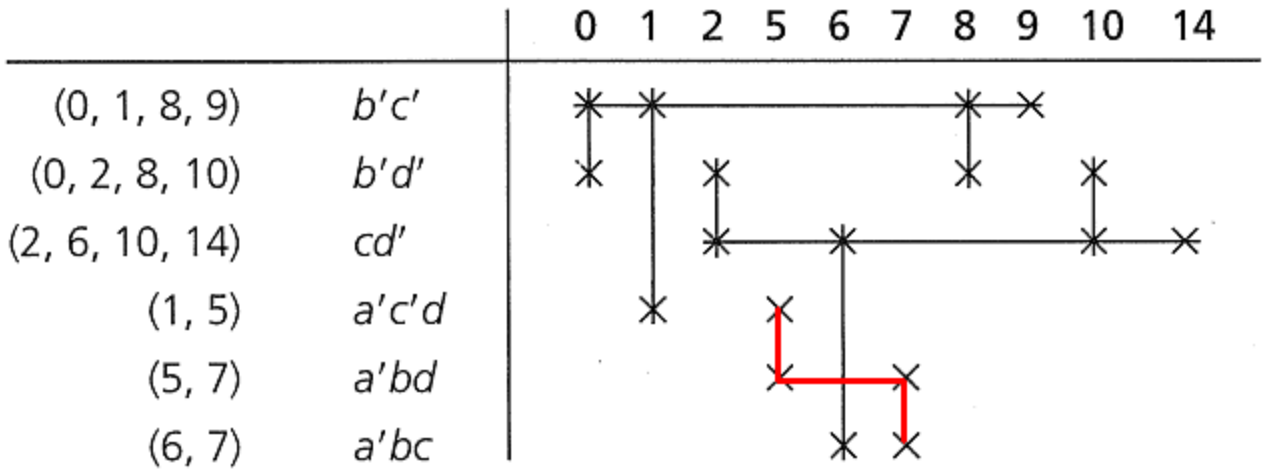
- Each time a prime implicant is selected for inclusion in the minimum sum, cross out:
  - The corresponding row
  - The columns which correspond to all minterms covered by that prime implicant.



The Prime Implicant Chart

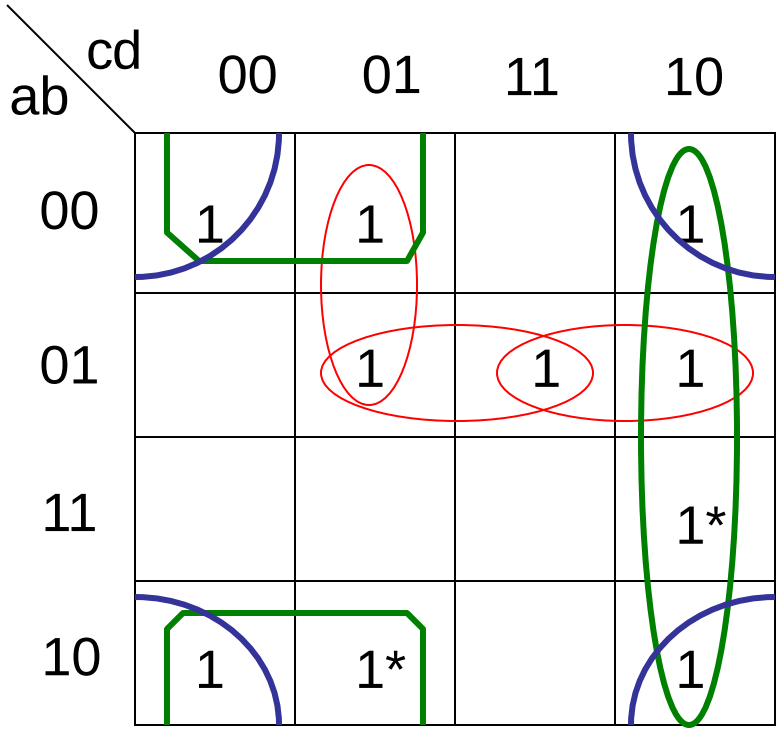
- The essential prime implicants are chosen first.
- Then additional non-essential prime implicants are selected by trial.
- They should cover as many minterms as possible.

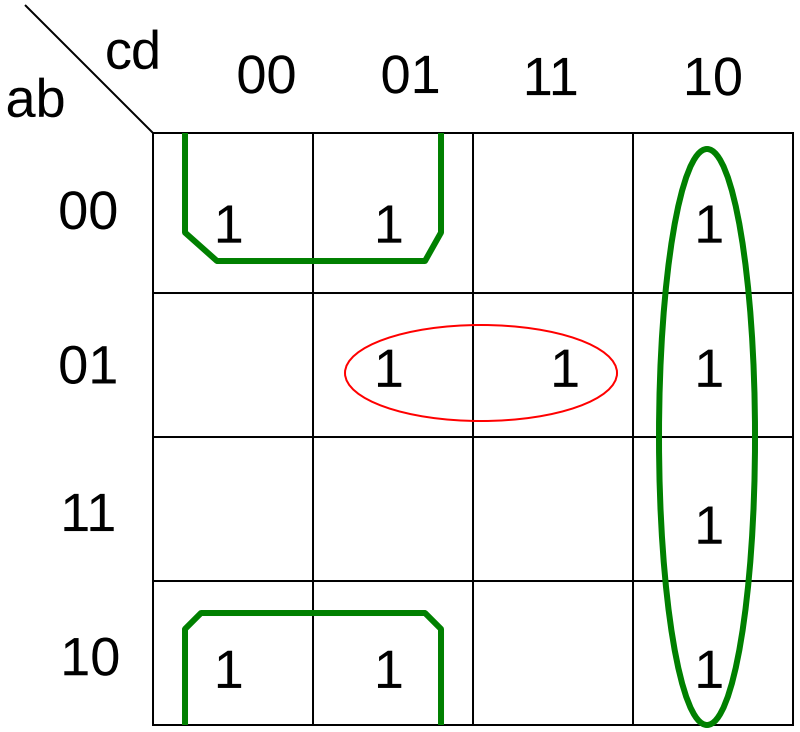
$f = b'c' + cd' + a'bd$ (minimum sum of products)



Finding Prime Implicants

Karnaugh Map's View





The Prime Implicant Chart

This example is to show how to select non-essential prime implicants when alternative solutions exist.

$$F = \Sigma\, m(0, 1, 2, 5, 6, 7)$$

Step 1

0	000	✓	0, 1	00–
1	001	✓	0, 2	0–0
2	010	✓	1, 5	–01
5	101	✓	2, 6	–10
6	110	✓	5, 7	1–1
7	111	✓	6, 7	11–

The Prime Implicant Chart

Step 2

		0	1	2	5	6	7
(0, 1)	$a'b'$	*	*				
(0, 2)	$a'c'$	*		*			
(1, 5)	$b'c$		*		*		
(2, 6)	bc'			*		*	
(5, 7)	ac				*		*
(6, 7)	ab					*	*

$$F = a'b' + bc' + ac.$$

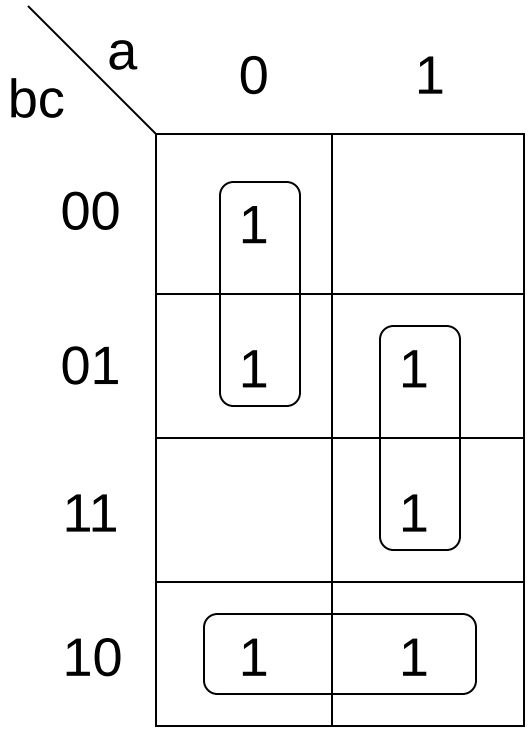
		0	1	2	5	6	7
(0, 1)	$a'b'$	*	*				
(0, 2)	$a'c'$	*		*			
(1, 5)	$b'c$		*		*		
(2, 6)	bc'			*		*	
(5, 7)	ac				*		*
(6, 7)	ab					*	*

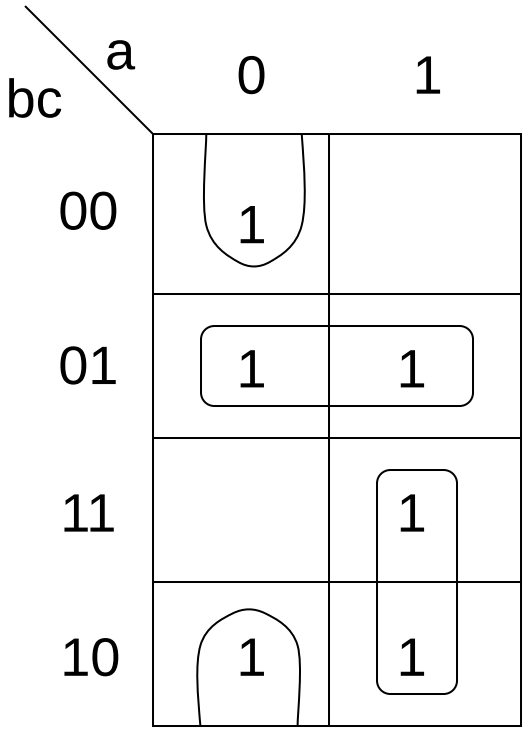
$$F = a'c' + b'c + ab$$

There are two minimum sum-of-products solutions.

The Prime Implicant Chart

Equivalent Karnaugh Maps





$$F = a'b' + bc' + ac.$$

$$F = a'c' + b'c + ab$$

Simplification of Functions with Don't Cares

- In finding the prime implicants, the don't cares are treated as minterms.
- When forming the prime implicant chart, the don't cares are NOT listed at the top.

$$F(A, B, C, D) = \Sigma\, m(2, 3, 7, 9, 11, 13) + \Sigma\, d(1, 10, 15)$$

Simplification of Functions with Don't Cares

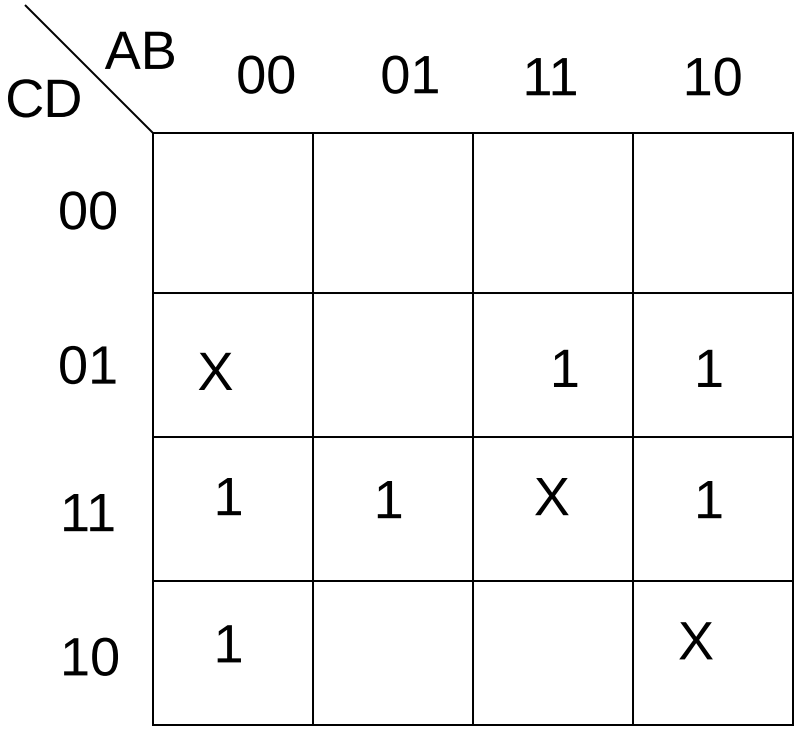
Find Prime Implicants

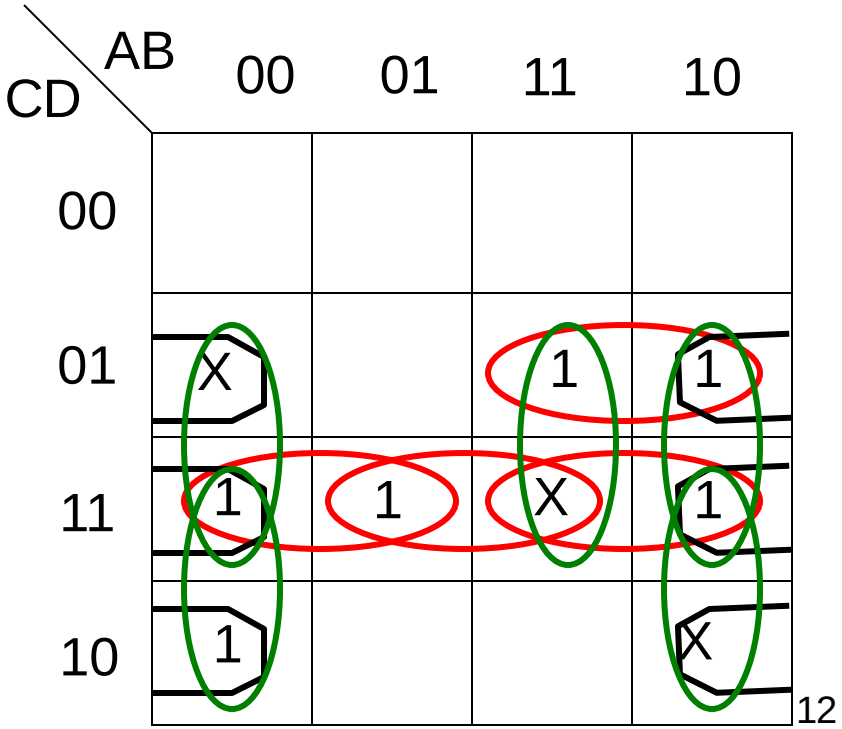
1	0001 ✓
2	0010 ✓
3	0011 ✓
9	1001 ✓
10	1010 ✓
7	0111 ✓
11	1011 ✓
13	1101 ✓
15	1111 ✓

(1, 3)	00–1 ✓
(1, 9)	–001 ✓
(2, 3)	001– ✓
(2, 10)	–010 ✓
(3, 7)	0–11 ✓
(3, 11)	–011 ✓
(9, 11)	10–1 ✓
(9, 13)	1–01 ✓
(10, 11)	101– ✓
(7, 15)	–111 ✓
(11, 15)	1–11 ✓
(13, 15)	11–1 ✓

Simplification of Functions with Don't Cares

Karnaugh Map's View





Simplification of Functions with Don't Cares

Find Prime Implicants

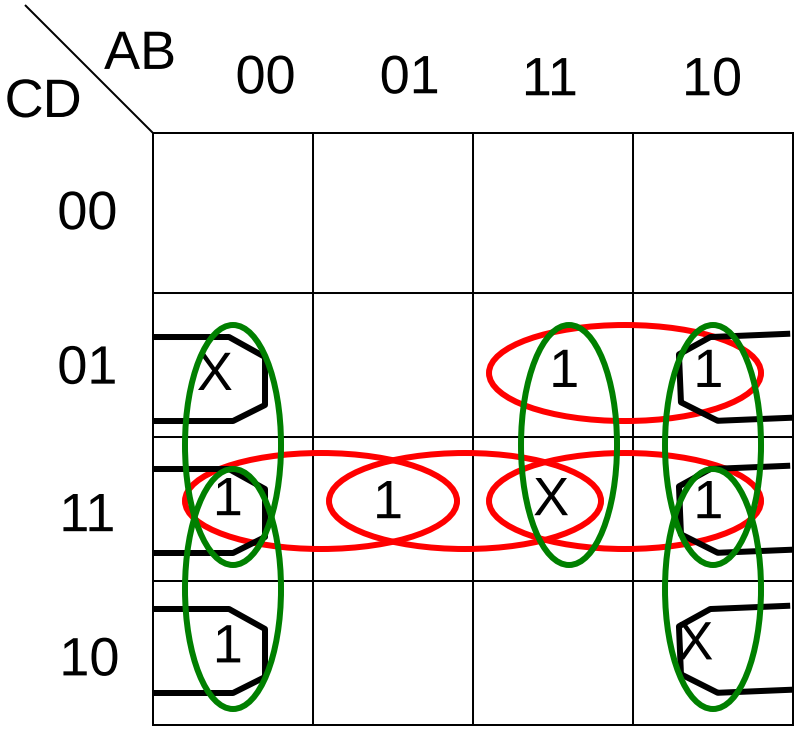
1	0001 ✓
2	0010 ✓
3	0011 ✓
9	1001 ✓
10	1010 ✓
7	0111 ✓
11	1011 ✓
13	1101 ✓
15	1111 ✓

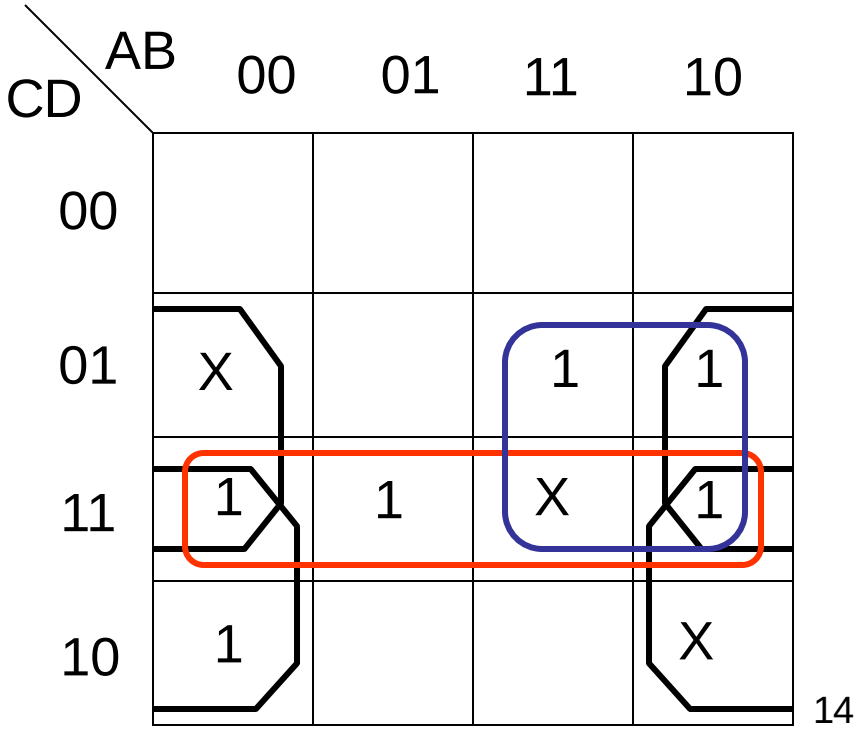
(1, 3)	00–1 ✓
(1, 9)	–001 ✓
(2, 3)	001– ✓
(2, 10)	–010 ✓
(3, 7)	0–11 ✓
(3, 11)	–011 ✓
(9, 11)	10–1 ✓
(9, 13)	1–01 ✓
(10, 11)	101– ✓
(7, 15)	–111 ✓
(11, 15)	1–11 ✓
(13, 15)	11–1 ✓

(1, 3, 9, 11)	–0–1
(2, 3, 10,11)	–01–
(3, 7, 11, 15)	--11
(9, 11, 13, 15)	1--1

Simplification of Functions with Don't Cares

Karnaugh Map's View

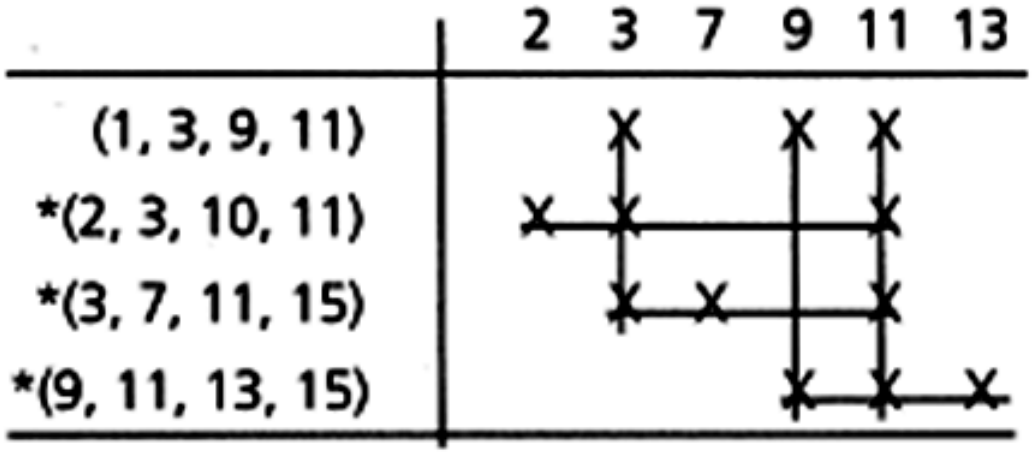




Simplification of Functions with Don't Cares

Prime Implicant Chart

- This is to find the minimum set of prime implicants.
- Always start from essential prime implicants.



*indicates an essential prime implicant.

Simplification of Functions with Don't Cares

Karnaugh Map's View