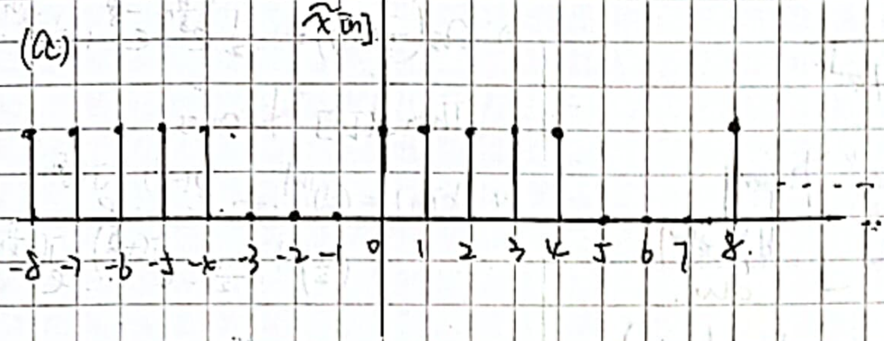
CAN 207 — Assgn 2.

Question 1. (DTFS, DTFT)

(a)



(i) 1° According to the Definition of Periodicity

$\Rightarrow x[n+N] = x[n]$

2° From this diagram, we have:

$\Rightarrow x[0+8] = x[0]$

$\rightarrow$ Thus the period N is $N=8$.

3° We know that $\omega_0 = \frac{2\pi}{N}$

$\Rightarrow \omega_0 = \frac{2\pi}{N} = \frac{2\pi}{8} = \frac{\pi}{4}$

4° Conclusion

$\rightarrow$ In Summary, $\begin{cases} \text{Period } N = 8 \\ \text{Fundamental frequency } \omega = \frac{\pi}{4} \end{cases}$

(ii) Apply Analysis Equation.

$\Rightarrow a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\omega_0 n}$

$\Rightarrow a_k = \frac{1}{8}\sum_{n=0}^{4} e^{-jkn\frac{\pi}{4}}$

$\Rightarrow a_k = \frac{1}{8}\cdot\frac{1-e^{-j\cdot\frac{5}{4}\pi\cdot k}}{1-e^{-j\frac{\pi}{4}k}}$

then, when $k = 0, 1, 2, 3, 4, 5, 6, 7$

we have:

$$\begin{cases} a_0 = \frac{5}{8} \\ a_1 = -\frac{(1+\sqrt{2})}{8}j \\ a_2 = \frac{1}{8} \\ a_3 = \frac{1-\sqrt{2}j}{8} \\ a_4 = \frac{1}{8} \\ a_5 = \frac{\sqrt{2}-1}{8}j \\ a_6 = \frac{1}{8} \\ a_7 = \frac{\sqrt{2}+1}{8}j \end{cases}$$

(b) Find the DTFT of each sequence.

(i) $x[n] = 2^n \cdot u[-n]$.

1° Assume $x_1[n] = (\frac{1}{2})^n \cdot u[n]$

$\Rightarrow$ Apply DTFT: $X_1(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$

$\Rightarrow$ And $x_1[-n] = x[n]$

2° Apply Time-Reversal property

$\Rightarrow x_1[-n] \leftrightarrow X_1(e^{-j\omega})$

$\rightarrow F\{x[n]\} = F\{x_1[-n]\} = \frac{1}{1-\frac{1}{2}e^{+j\omega}}$

(ii) $x[n] = n\cdot(0.7)^n\cdot u[n]$

1° Let $x_1[n] = 0.7^n\cdot u[n]$.

$$\Rightarrow F\{x_1[n]\} = \frac{1}{1-0.7e^{-jw}}$$

2° Apply "Differencing" Property

which is: $n\cdot x[n] \overset{DTFT}{\longleftrightarrow} j\,\frac{dX(e^{jw})}{dw}$.

Then $F\{n\cdot x_1[n]\} = j\cdot \frac{d\left(\frac{1}{1-0.7e^{-jw}}\right)}{dw}$

$$\Rightarrow F\{n\cdot x_1[n]\} = \frac{0.7\,e^{-jw}}{(1-0.7e^{-jw})^2}.$$

3° Since $x[n] = n\cdot x_1[n]$

$$\Rightarrow \text{In Summary, } F\{x[n]\} = \frac{0.7\cdot e^{-jw}}{(1-0.7e^{-jw})^2}$$

(iii) $x[n] = \left(\frac{1}{2}\right)^{|n|}\cdot \cos\left(\frac{\pi}{8}(n-1)\right)$

1° Apply Euler's Formula.

$$\Rightarrow \cos\left(\frac{\pi}{8}(n-1)\right) = \frac{1}{2}e^{j\frac{\pi}{8}n}\cdot e^{-j\frac{\pi}{8}} + \frac{1}{2}e^{-j\frac{\pi}{8}n}\cdot e^{j\frac{\pi}{8}}$$

Then $x[n]$ becomes

$$\Rightarrow x[n] = \left(\tfrac{1}{2}\right)^{|n|}\cdot\tfrac{1}{2}\cdot e^{j(-\frac{\pi}{8})}\cdot e^{j(\frac{\pi}{8}n)} + \left(\tfrac{1}{2}\right)^{|n|}\cdot\tfrac{1}{2}\cdot e^{j(+\frac{\pi}{8})}\cdot e^{j(-\frac{\pi}{8}n)}$$

2° Case 1 : $n \geq 0$

$$\Rightarrow x[n] = \frac{e^{j(-\frac{\pi}{8})}}{2}\cdot\left[\left(\tfrac{1}{2}\right)^n\cdot u[n]\right]\cdot e^{j(\frac{\pi}{8}n)} + \frac{e^{j(-\frac{\pi}{8})}}{2}\left[\left(\tfrac{1}{2}\right)^n\cdot u[n]\right]\cdot e^{j(-\frac{\pi}{8}n)}$$

→ Let $x_1[n] = \left(\frac{1}{2}\right)^n\cdot u[n]$.

→ Then $X_1(e^{jw}) = \frac{1}{1-\frac{1}{2}e^{-jw}}$

→ Apply phase-shifting property

$$\Rightarrow F\{x[n]\} = \frac{e^{j(-\frac{\pi}{8})}}{2-e^{-j(w-\frac{\pi}{8})}} + \frac{e^{j(+\frac{\pi}{8})}}{2-e^{-j(w+\frac{\pi}{8})}}$$

3° Case 2 : $n < 0$

$$\Rightarrow x[n] = \frac{e^{j(-\frac{\pi}{8})}}{2}\cdot 2^n\cdot u[-n-1]\cdot e^{j\frac{\pi}{8}n} + \frac{e^{j(\frac{\pi}{8})}}{2}\cdot 2^n\cdot u[-n-1]\cdot e^{j(-\frac{\pi}{8}n)}$$

Since: $2^n\cdot u[-n-1] \overset{DTFT}{\longleftrightarrow} \frac{-1}{1-2e^{-jw}}$

Then Apply phase-shifting property

we have

$$x[n] = \frac{-e^{j(-\frac{\pi}{8})}}{2-4e^{-j(w-\frac{\pi}{8})}} + \frac{-e^{j(\frac{\pi}{8})}}{2-4e^{-j(w+\frac{\pi}{8})}}$$

4° Summary

→ Add up Case 1 and Case 2

$\Rightarrow F\{x[n]\}$

$$= e^{j(-\frac{\pi}{8})}\cdot\left[\frac{1}{2-e^{-j(w-\frac{\pi}{8})}} - \frac{1}{2-4e^{-j(w-\frac{\pi}{8})}}\right] + e^{j(\frac{\pi}{8})}\cdot\left[\frac{1}{2-e^{-j(w+\frac{\pi}{8})}} - \frac{1}{2-4e^{-j(w+\frac{\pi}{8})}}\right]$$

Q2

(a) To determine whether the signal can be sampled without any loss is to determine whether it is periodic.

(i) $x(t) = u(t) - u(t-3)$

→ Obviously, $x(t)$ is Aperiodic.

→ thus it can not be sampled without information loss.

(ii). $x(t) = \cos(100\pi t) + 2\sin(250\pi t)$

→ Obviously, $x(t)$ is a periodic signal with $\Omega_m = 250\pi$ rad/s, thus $\Omega_s \geq 2\Omega_m = 500$ rad/s is its Nyquist Rate

(iii) $x(t) = \cos(100\pi t) + 2.5\sin(150\pi t)\cdot\cos(200\pi t)$

Apply Trignometric Formula.

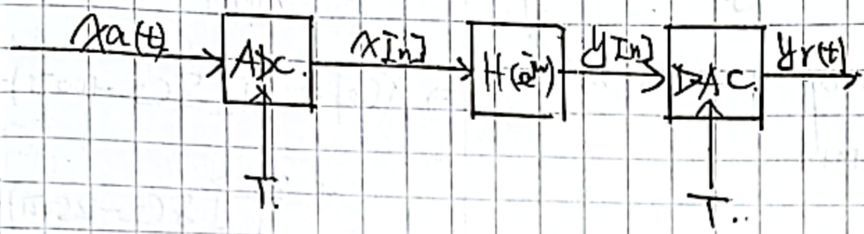
→ then. $x(t) = \cos(100\pi t) + 1.25\sin(350\pi t) - 1.25\sin(50\pi t)$

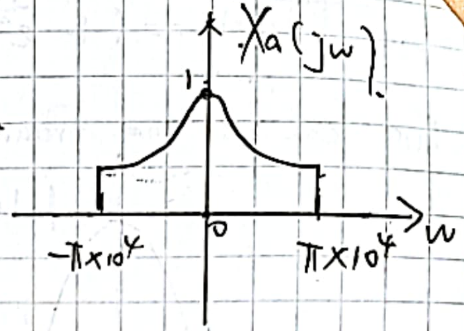
→ Obviously, $x(t)$ is a periodic signal with $\Omega_m = 350\pi$ rad/s,

thus. $\Omega_s \geq 2\Omega_m = 700\pi$ rad/s.

is its Nyquist Rate

Q2 (b)





(i). From the graph of $X_a(j\omega)$

⇒ we have : $\omega_m = \pi\times10^4 = \Omega_m$ rad/s.

To Avoid Aliasing

⇒ we have to satisfy Sampling Period

$\Omega_s \geq 2\Omega_m = 2\pi\times10^4$ rad/s.

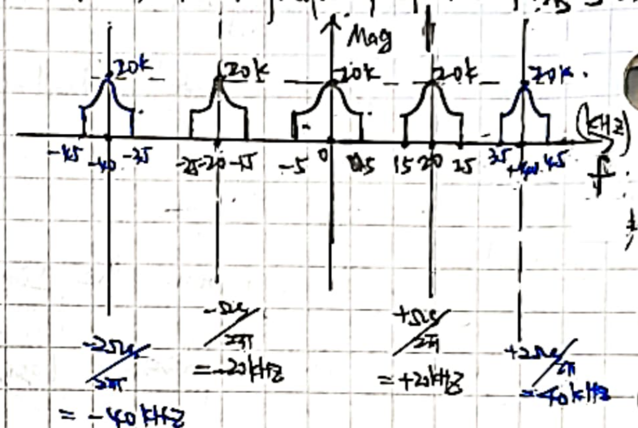
⇒ then, the maximum value of Ts

becomes $T_{s_{max}} = \frac{2\pi}{\Omega_{s_{min}}} = 10^{-4}$ s.

(ii). If $\frac{1}{T} = 20kHz$.

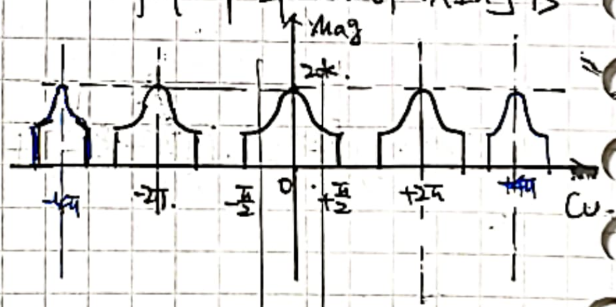
→ then $\Omega_s = 2\pi\cdot\frac{1}{T} = 4\pi\times10^4$ rad/s

1° → the frequency spectrum of $x_s(t)$ is.

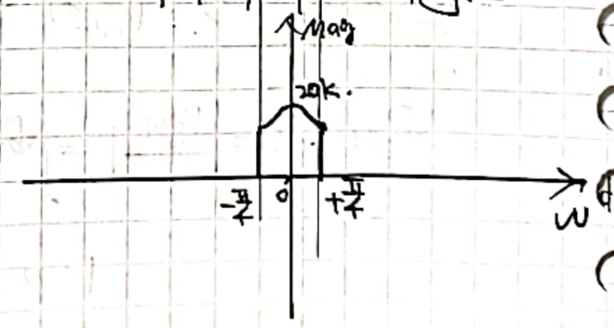


with { x-axis unit : kHz
y-axis magnitude

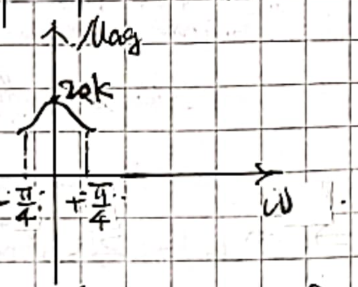
2° The frequency spectrum of $x[n]$ is



3° The frequency spectrum of $y[n]$ is

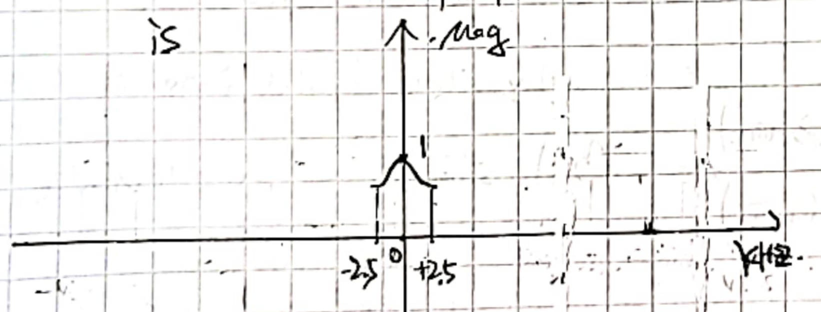


(iii). From (ii), we know than the frequency spectrum of $y[n]$ is



If we apply $(T_s)^{-1} = 20kHz = f_s$.

then, the frequency spectrum of $y_r(t)$ is



With X-axis unit KHZ

Q3. (a) $x[n] = \{1, 2, 3, 4\}$, $0 \le n \le 3$.

(i). Apply DTFT

$$\rightarrow X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n] \cdot e^{-jwn}$$

$$x[n] = \{1, 2, 3, 4\}$$

$$\rightarrow X(e^{jw}) = x[0] + x[1] \cdot e^{-jw} + x[2] \cdot e^{-2jw} + x[3] \cdot e^{-3jw}$$

$$\rightarrow X(e^{jw}) = 1 + 2e^{-jw} + 3e^{-2jw} + 4e^{-3jw}$$

When $w = \pi$.

$$\rightarrow X(e^{j\pi}) = 1 - 2 + 3 - 4 = -2.$$

(ii). Apply Inverse DTFT

$$\rightarrow x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{jw}) \cdot e^{jwn} dw$$

$$\rightarrow \int_{-\pi}^{+\pi} X(e^{jw}) dw = 2\pi \cdot x[0]$$

$$x[0] = 1$$

$$\Rightarrow \int_{-\pi}^{+\pi} X(e^{jw}) dw = 2\pi$$

(iii). Apply Parseval Theorem

$$\rightarrow \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} |X(e^{jw})|^2 dw$$

$$\rightarrow \int_{-\pi}^{+\pi} |X(e^{jw})|^2 dw = 2\pi \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

$$x[n] = \{1, 2, 3, 4\}$$

$$\Rightarrow \int_{-\pi}^{+\pi} |X(e^{jw})|^2 dw = 2\pi (1^2 + 2^2 + 3^2 + 4^2)$$

$$\Rightarrow \int_{-\pi}^{+\pi} |X(e^{jw})|^2 dw = 2\pi \cdot 30 = 60\pi.$$

(iv) Apply Differetiation Property

$$\rightarrow \frac{1}{j} n \cdot x[n] \overset{F}{\longleftrightarrow} \frac{dX(e^{jw})}{dw}.$$

Apply Parseval Theorem

$$\rightarrow \int_{-\pi}^{+\pi} \left| \frac{dX(e^{jw})}{dw} \right|^2 dw = 2\pi \sum_{n=-\infty}^{+\infty} \left| \frac{n \cdot x[n]}{j} \right|^2$$

$$= 2\pi \sum_{n=0}^{3} |n \cdot x[n]|^2$$

$$= 2\pi \left[(x[1])^2 + 4(x[2])^2 + 9(x[3])^2 \right]$$

$$= 2\pi \left[4 + 4 \times 9 + 9 \times 16 \right]$$

$$= 368\pi.$$

Q3 (b). $H(e^{jw}) = \frac{1}{1-\frac{1}{2}e^{-jw}+\frac{1}{4}e^{-2jw}}$

(i). To find the LCCDE of the system, we should replace $H(e^{jw})$ to $\frac{Y(e^{jw})}{X(e^{jw})}$

→ that is $\frac{Y(e^{jw})}{X(e^{jw})} = \frac{1}{1-\frac{1}{2}e^{-jw}+\frac{1}{4}e^{-2jw}}$.

→ then, $X(e^{jw}) = Y(e^{jw}) - \frac{1}{2}e^{-jw}Y(e^{jw}) + \frac{1}{4}e^{-2jw}Y(e^{jw})$

→ Apply Inverse DTFT and Time-Shifting Property

→ we have : $x[n] = y[n] - \frac{1}{2}y[n-1] + \frac{1}{4}y[n-2]$

→ Thus, LCCDE of the system

is : $y[n] - \frac{1}{2}y[n-1] + \frac{1}{4}y[n-2] = x[n]$

(ii). To find the response of the system, we should multiply $X(e^{jw}), H(e^{jw})$

⇒ which is $Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw}) = \frac{2-e^{-jw}}{1+\frac{1}{2}e^{-jw}} \cdot \frac{1}{1-\frac{1}{2}e^{-jw}+\frac{1}{4}e^{-2jw}}$

→ then, $Y(e^{jw}) = \frac{2-e^{-jw}}{1+\frac{1}{8}e^{-3jw}}$ and let $m = e^{-jw}$.

→ So, $Y(m) = \frac{2-m}{1+\frac{1}{8}m^3}$, and let $1+\frac{1}{8}m^3 = 0$, $\begin{cases} m_1 = -1-\sqrt{3}j \\ m_2 = -1+\sqrt{3}j \\ m_3 = (-2) \end{cases}$,

→ then, $Y(m) = \frac{2-m}{(1+\frac{1}{2}m)(-\frac{1+\sqrt{3}j}{2}+\frac{1}{2}m)(-\frac{1-\sqrt{3}j}{2}+\frac{1}{2}m)}$

→ Let $Y(m) = \frac{A}{1+\frac{1}{2}m} + \frac{B}{(-\frac{1+\sqrt{3}j}{2}+\frac{1}{2}m)} + \frac{C}{(\frac{1}{2}m-\frac{1-\sqrt{3}j}{2})}$

→ Apply Partial Fraction Expansion (PFE)

we have $\begin{cases} A = \left.\frac{2-m}{(-\frac{1+\sqrt{3}j}{2}+\frac{1}{2}m)(-\frac{1-\sqrt{3}j}{2}+\frac{1}{2}m)}\right|_{m=-2} = \frac{4}{3} \\ B = \left.\frac{2-m}{(1+\frac{1}{2}m)(\frac{1}{2}m-\frac{1-\sqrt{3}j}{2})}\right|_{m=-1-\sqrt{3}j} = -\frac{2}{3} \\ C = \left.\frac{2-m}{(1+\frac{1}{2}m)(-\frac{1+\sqrt{3}j}{2}+\frac{1}{2}m)}\right|_{m=-1+\sqrt{3}j} = -\frac{2}{3} \end{cases}$

Q3 (b)(ii). Con't d.

→ Substitute m with $e^{j\omega}$; we have

$$\Rightarrow Y(e^{j\omega}) = \frac{\frac{4}{3}}{(1+\frac{1}{2}e^{-j\omega})} - \frac{\frac{2}{3}}{(-\frac{1+\sqrt{3}j}{2}+\frac{1}{2}e^{-j\omega})} - \frac{\frac{2}{3}}{(-\frac{1-\sqrt{3}j}{2}+\frac{1}{2}e^{-j\omega})}$$

$$\rightarrow Y(e^{j\omega}) = \frac{\frac{4}{3}}{1-(-\frac{1}{2})e^{-j\omega}} + \frac{\frac{4}{3}\cdot\frac{1}{(1+\sqrt{3}j)}}{1-\frac{1}{1+\sqrt{3}j}e^{-j\omega}} + \frac{\frac{4}{3}\cdot\frac{1}{(1-\sqrt{3}j)}}{1-\frac{1}{1-\sqrt{3}j}e^{-j\omega}}$$

Since the coefficients of three terms both have absolute value <1 $\left(|-\frac{1}{2}|<1;\ |\frac{+1}{1+\sqrt{3}j}|<1;\ |\frac{+1}{1-\sqrt{3}j}|<1\right)$

→ thus, apply DTFT, $\boxed{y[n] = \frac{4}{3}\cdot(-\frac{1}{2})^n\cdot u[n] + \frac{4}{3}\left(\frac{1}{1+\sqrt{3}j}\right)^{n+1}\cdot u[n] + \frac{4}{3}\cdot\left(\frac{1}{1-\sqrt{3}j}\right)^{n+1}\cdot u[n]}$

Q4. (a) Find $H(z)$ and its ROC, indicate system stable / causal?

(i). $x[n] = (\frac{1}{2})^n \cdot u[n]$, $y[n] = 3\cdot(\frac{1}{2})^n \cdot u[n] + 2\cdot(\frac{3}{4})^n \cdot u[n]$

Apply ~~DTFT~~ Z-transform to $x[n]$, and $y[n]$

we have: $\begin{cases} X(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}}, & |z| > \frac{1}{2}. \\ Y(z) = \dfrac{3}{1-\frac{1}{2}z^{-1}} + \dfrac{2}{1-\frac{3}{4}z^{-1}}, & |z| > \frac{3}{4} \end{cases}$

$\Rightarrow$ then $H(z) = \dfrac{Y(z)}{X(z)} = 3 + \dfrac{2(1-\frac{1}{2}z^{-1})}{1-\frac{3}{4}z^{-1}}$, with ROC $|z| > \frac{3}{4}$.

For Causality, $H(z) = \dfrac{3-\frac{9}{4}z^{-1}+2-z^{-1}}{1-\frac{3}{4}z^{-1}} = \dfrac{5-\frac{13}{4}z^{-1}}{1-\frac{3}{4}z^{-1}} = \dfrac{5z-\frac{13}{4}}{z-\frac{3}{4}}$.

Since the order of the ~~numerator~~ numerator = order of denominator

which means $H(\infty) \neq \infty$, So, it is Causal.

For Stability, Since the ROC of $H(z)$ is $|z|$

which includes the Unit Circle,

thus, it is Stable.

In Summary $\begin{cases} H(z) = \dfrac{5z-\frac{13}{4}}{z-\frac{3}{4}}, \quad |z| > \frac{3}{4} \\ \text{the System is Causal.} \\ \text{the System is Stable.} \end{cases}$

Q4 (a)

(ii). $x[n] = 1.25\,\delta[n] - 0.25 \cdot (0.8)^n \cdot u[n]$, $y[n] = (0.8)^n \cdot u[n]$.

1° For transfer function.

Apply Z-transform to both $x[n]$ and $y[n]$.

$$\Rightarrow \begin{cases} X(z) = 1.25 - 0.25 \cdot \dfrac{1}{1-0.8z^{-1}} , & |z| > 0.8 . \\ Y(z) = \dfrac{1}{1-0.8z^{-1}} , & |z| > 0.8 . \end{cases}$$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{1}{1-0.8z^{-1}}}{1.25 - 0.25 \frac{1}{1-0.8z^{-1}}} = \frac{1}{1-z^{-1}} , \quad |z| > 1 .$$

2° For Causality

Since $H(z) = \dfrac{1}{1-z^{-1}} = \dfrac{z}{z-1}$, $|z| > 1$

that is : the order of numerator = the order of denominator.

$\Rightarrow$ thus, it is ~~a~~ causal.

3° For Stability

Since the ROC of $H(z)$: $|z| > 1$ does not include the unit circle, then it is unstable.

4° In Summary

$$\begin{cases} H(z) = \dfrac{1}{1-z^{-1}} , \quad |z| > 1 \\ \text{the system is unstable} \\ \text{the system is causal.} \end{cases}$$

Q4 (b). Causal System with LCCDE:

$y[n] + 0.2\,y[n-1] - 0.24\,y[n-2] = x[n] + x[n-1]$

(i). Apply Z-Transform to Both LHS & RHS

$\Rightarrow \begin{cases} LHS: Y(z) + 0.2z^{-1}Y(z) - 0.24z^{-2}Y(z) \\ RHS: X(z) + z^{-1}X(z). \end{cases}$

$\Rightarrow$ Let LHS = RHS

$\Rightarrow$ we have: $(1+0.2z^{-1}-0.24z^{-2})Y(z) = (1+z^{-1})\cdot X(z)$

$\Rightarrow$ that is: $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1+z^{-1}}{1+0.2z^{-1}-0.24z^{-2}} = \dfrac{z(z+1)}{(z-0.4)(z+0.6)}$

(ii). 1° For ROC

$\Rightarrow H(z) = \dfrac{1+z^{-1}}{1+0.2z^{-1}-0.24z^{-2}} = \dfrac{z^2+z}{z^2+0.2z-0.24} = \dfrac{z(z+1)}{(z-0.4)(z+0.6)}$

$\Rightarrow$ we have two poles: $\begin{cases} p_1 = -0.6 \\ p_2 = +0.4 \end{cases}$

$\Rightarrow$ Since, it is a causal LTI system

$\Rightarrow$ then its ROC should be $|z| > +0.6$

2° For Stability

$\Rightarrow$. Since ROC: $|z| > 0.6$ includes the unit circle.

$\Rightarrow$ then this system is stable.

(iii) If $x[n] = u[n]$, then its Z-transform is $X(z) = \dfrac{1}{1-z^{-1}}$

$\Rightarrow Y(z) = X(z)\cdot H(z) = \dfrac{z^2(z+1)}{(z-1)(z-0.4)(z+0.6)}$

$\Rightarrow$ Apply Partial Fraction Expansion (PFE) of $\dfrac{Y(z)}{z} = \dfrac{z(z+1)}{(z-1)(z-0.4)(z+0.6)}$

$\Rightarrow$ then $\dfrac{Y(z)}{z} = \dfrac{A}{z-1} + \dfrac{B}{z+0.4} + \dfrac{C}{z-0.6} = \dfrac{Az^{-1}}{1-z^{-1}} + \dfrac{Bz^{-1}}{1-(-0.4)z^{-1}} + \dfrac{Cz^{-1}}{1-0.6z^{-1}}$

$\Rightarrow$ thus: $A = \frac{z(z+1)}{(z-0.4)(z+0.6)}\Big|_{z=1} = \frac{1\times 2}{0.6\times 1.6} = \frac{25}{12}$

$B = \frac{z(z+1)}{(z-1)(z+0.6)}\Big|_{z=+0.4} = \frac{0.4\times 1.4}{-0.6\times 1} = \frac{-14}{15}$

$C = \frac{z(z+1)}{(z-1)(z-0.4)}\Big|_{z=-0.6} = \frac{(-0.6)\times 0.4}{(-1.6)\times(-1)} = -0.15$

$\Rightarrow$ That is $Y(z) = \frac{25}{12}\cdot\frac{1}{1-z^{-1}} - \frac{14}{15}\cdot\frac{z^{-1}}{1-0.4z^{-1}} - 0.15\cdot\frac{z^{-1}}{1+0.6z^{-1}}$

$\Rightarrow$ Apply Inverse Z-transform based on Z-transform pairs

$\Rightarrow$ thus $y[n] = \frac{25}{12}\cdot u[n] - \frac{14}{15}\cdot(0.4)^n\cdot u[n] - 0.15(-0.6)^n\cdot u[n]$

$\Rightarrow$ In Summary, the system output to a unit step function $u[n]$

is $y[n] = \frac{25}{12} u[n] - \frac{14}{15}\cdot(0.4)^n\cdot u[n] - 0.15\cdot(-0.6)^n\cdot u[n]$

Q4(c) LTID System with $H(z) = \frac{z-0.4}{z^3-1.4z^2+0.85z}$

(i) 1° For Zeroes

$\Rightarrow$ Let $z-0.4=0$

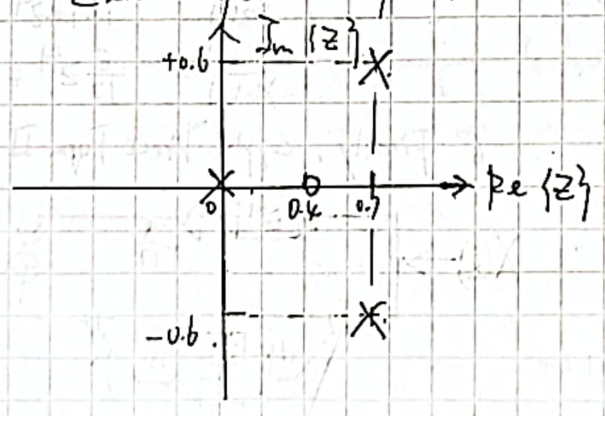
$\Rightarrow$ $z_1 = 0.4$

2° For Poles

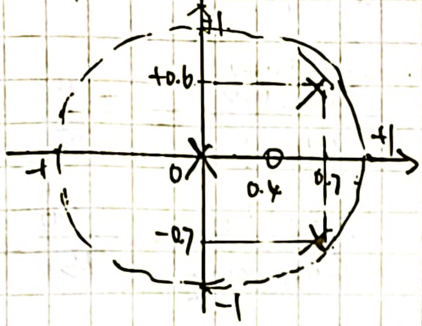
$\Rightarrow$ Let $z^3-1.4z^2+0.85z=0$

$\Rightarrow$ $p_1=0$; $p_2=0.7-0.6j$; $p_3=0.7+0.6j$

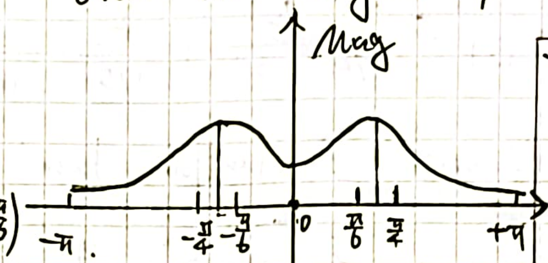
3° Zero-Pole Plot.



(ii). From (i), we have.



By Evaluating the Magnitude of $|H(z)|$ at different w, then, we are able to roughly draw the Magnitude Plot of this system



Plot the graph in MATLAB, we have

$$\begin{cases} |H(0)| = 1.33 \\ |H(\frac{\pi}{6})| = 3.17 \\ |H(\frac{\pi}{4})| = 5.50 \\ |H(\pi)| = 0.43 \end{cases}$$

Note that 1° when one specific w between $(\frac{\pi}{6}, \frac{\pi}{4})$ and the other one on $(-\frac{\pi}{4}, -\frac{\pi}{6})$ will create the local Maximum.

2° when $w = 0$ rad/s, the Magnitude reaches its local Minimum

And the system is a Bandstop filter.

(iii) $H(z) = \dfrac{z-0.4}{z^3-1.4z^2+0.85z}$

1° Since the order of denominator is small

→ then, we apply partial Fraction Expansion to express $H(z)$ as the cascade of two system in parallel.

2° Let $H(z) = \dfrac{A}{z} + \dfrac{Bz+C}{z^2-1.4z+0.85}$

3° After calculations, we have

$$\begin{cases} A = -\frac{8}{17} \\ B = +\frac{8}{17} \\ C = +\frac{29}{85} \end{cases}$$

→ thus $H(z) = H_1(z) + H_2(z)$ where

$$\begin{cases} H_1(z) = -\frac{8}{17} z^{-1} \\ H_2(z) = \dfrac{\frac{8}{17}z^{-1} + \frac{29}{85}z^{-2}}{1-1.4z^{-1}+0.85z^{-2}} \end{cases}$$

4° For System $H_1(z)$.

→ $H_1(z) = \dfrac{Y_1(z)}{W_1(z)} \cdot \dfrac{W_1(z)}{X_1(z)}$.

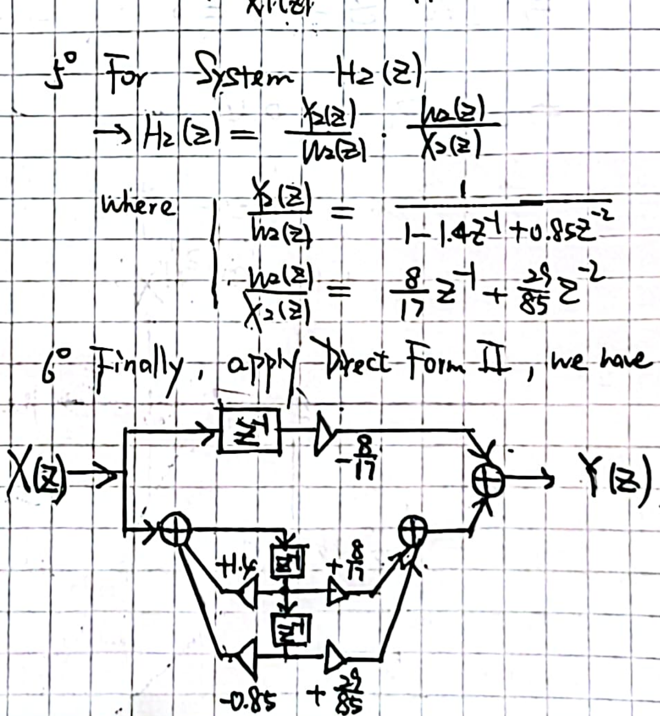
where $\begin{cases} \dfrac{Y_1(z)}{W_1(z)} = 1 \\ \dfrac{W_1(z)}{X_1(z)} = -\frac{8}{17}z^{-1} \end{cases}$

5° For System $H_2(z)$

→ $H_2(z) = \dfrac{Y_2(z)}{W_2(z)} \cdot \dfrac{W_2(z)}{X_2(z)}$

where $\begin{cases} \dfrac{Y_2(z)}{W_2(z)} = \dfrac{1}{1-1.4z^{-1}+0.85z^{-2}} \\ \dfrac{W_2(z)}{X_2(z)} = \frac{8}{17}z^{-1} + \frac{29}{85}z^{-2} \end{cases}$

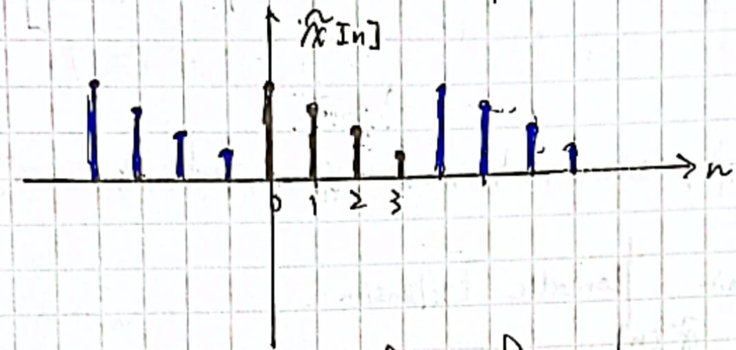
6° Finally, apply Direct Form II, we have.



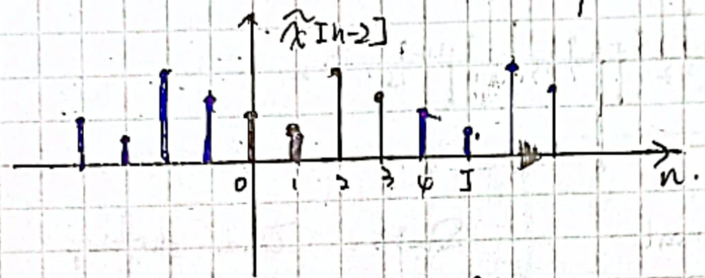
Q5. (DFT)

(a) (i). $x[n] = \{4, 3, 2, 1\}$, $0 \le n \le 3$, $x[n-2]_{\bmod 4}$

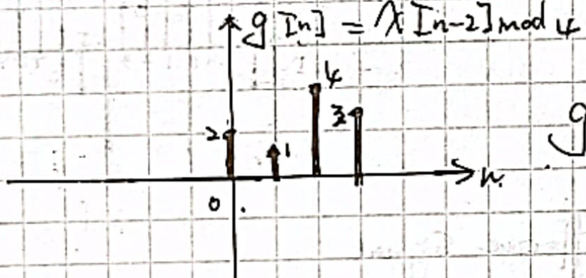
Step 1 : Time Domain Periodic Expansion.



Step 2 : Time-Shifting for periodic signal.



Step 3 : Extract the Effective signal with n from 0 to N-1

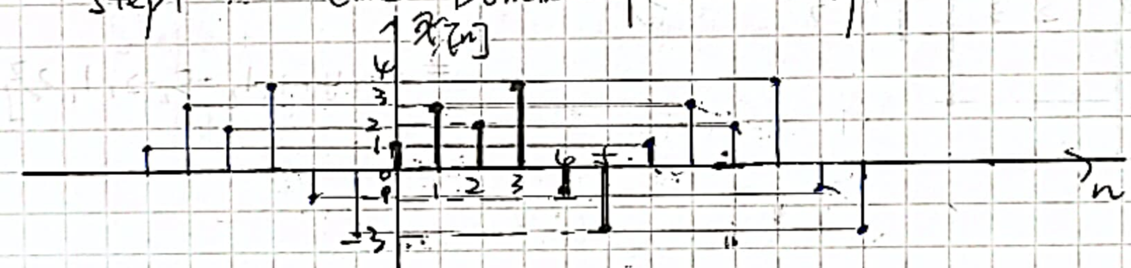


$g[n] = x[n-2]_{\bmod 4} = \{2, 1, 4, 3\}$

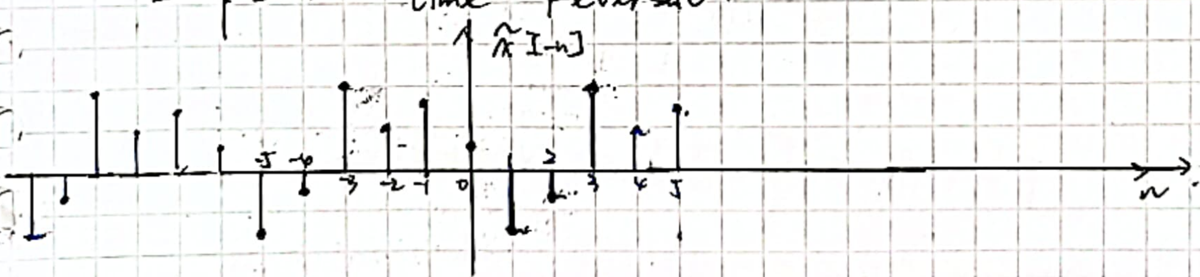
with $n \in [0, 3]$

(ii). $x[n] = \{1, 3, 2, 4, -1, -3\}$, $0 \le n \le 5$, find $x[-n]_{\bmod 3}$.

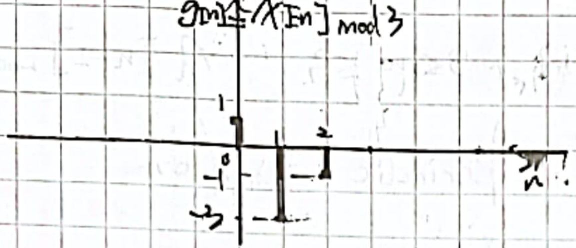
Step 1 : Time Domain Periodic Expansion.



Step 2 : Time Reversal.



Step 3. Extract the efficient signal.



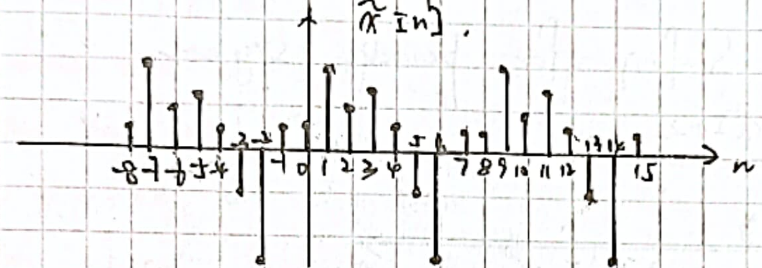
$g[n] = x[-n] \bmod 3$

$= \{1, -3, -1\}$

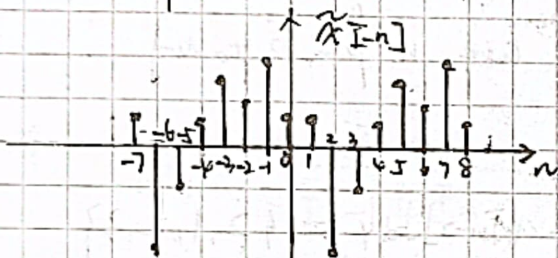
(iii). $x[n] = \{1, 4, 2, 3, 1, -2, -5, 1\}$, $0 \le n \le 3$, find $x[-n+2] \bmod 8$

$-(n-2)$

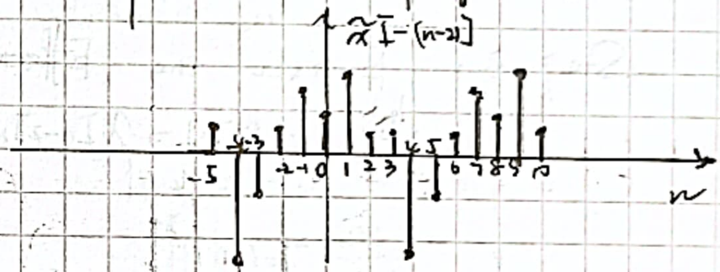
Step 1: Time Domain Periodic Extension.



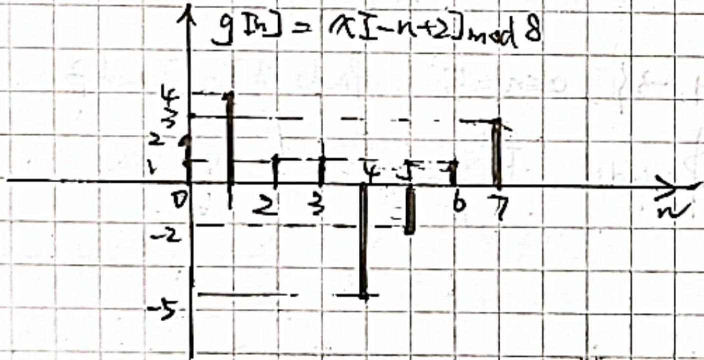
Step 2: Time Reversal



Step 3. Time shifting



Step 4: Extract the effective signal



$g[n] = x[-n+2] \bmod 8$

$= \{2, 4, -1, 1, -5, -2, 1, 2\}$

Q5 (b) $x[n] = \{1, \underline{\underline{3}}, 2, -4, 6\}, \ -1 \le n \le 3$.

$h[n] = \{5, 4, 3, 2, 1\}, \ -2 \le n \le 2$.

Step 1 : Linear Convolution

$x[k] = \{1, \underline{\underline{3}}, 2, -4, 6\}, \ -1 \le n \le 3$

$h[k] = \{5, 4, \underline{\underline{3}}, 2, 1\}, \ -2 \le n \le 2$.

$\Rightarrow$ then

$x[k]$:				1	$\underline{\underline{3}}$	2	-4	6
				5	4	$\underline{\underline{3}}$	2	1.
				1	3	2	-4	6
			2	6	4	-8	12.	
		3	9	6	-12	+18.		
	4	12	8	-16	+24.			
5	15	10	-20	+30				
5	19	25	$\underline{\underline{-1}}$	+27	19	12	8	6

$\Rightarrow x[n] * h[n] = \{5, 19, 25, \underline{\underline{-1}}, 27, 19, 12, 8, 6\}, \ -3 \le n \le 5$

Step 2 : Circular Convolution

$x[n]$ has $N_1 = 5$, $h[n]$ has $N_2 = 5$

$\Rightarrow$ thus $x[n] \circledast h[n]$ has a length $N_3 = \max\{N_1, N_2\} = 5$.

$\Rightarrow$ then $5, 19, 25, \underline{\underline{-1}}, 27, 19, 12, 8, 6.$

$\Rightarrow$ Therefore, $x[n] \circledast h[n] = \{\underline{5}, 27, 24, 31, 33\}$.

Q5(c) $x[n]$'s DFT : $\{2+3j, 1+5j, -2+4j, -1-3j, 2, 3+j\}$
(indices 0, 1, 2, 3, 4, 5)

Find DFT$\{x_r[n]\}$ and DFT$\{x_i[n]\}$

Step 1 : Understand what we are asked to derive.

$$\text{DFT}\{x_r[n]\} = X_E[k] = \frac{X[k] + X^*[-k]_{\text{mod } 6}}{2}$$

$$\text{DFT}\{jx_i[n]\} = X_O[k] = \frac{X[k] - X^*[-k]_{\text{mod } 6}}{2}$$

Step 2 : Find $X^*[-k]_{\text{mod } 6}$.

$\Rightarrow$ $X^*[-k]_{\text{mod } 6}$ needs to discuss in different k.

Case 1 : $k=0$

$X^*[0]_{\text{mod } 6} = 2-3j$

Case 2 : $k=+1$

$X^*[6-1] = X^*[5]$

$X^*[-1]_{\text{mod } 6} = 3-j$

Case 3 : $k=+2$

$X^*[6-2] = X^*[4]$

$X^*[-2]_{\text{mod } 6} = 2$

Case 4 : $k=+3$

$X^*[-3]_{\text{mod } 6} = -1+3j$

Case 5 : $k=+4$

$X^*[2]$

$X^*[-4]_{\text{mod } 6} = -2-4j$

Case 6 : $k=+5$

$X^*[1]$

$X^*[-5]_{\text{mod } 6} = 1-5j$

$\Rightarrow$ Thus, the sequence of $X^*[-k]_{\text{mod } 6}$ is $\{2-3j, 3-j, 2, -1+3j, -2-4j, 1-5j\}$

Step 3 : Find $X[k] \pm X^*[-k]_{\text{mod } 6}$. $\quad X[k]\{2+3j, 1+5j, -2+4j, -1-3j, 2, 3+j\}$

$$\Rightarrow \begin{cases} X[k] + X^*[-k]_{\text{mod } 6} = \{4, 4+4j, 4j, -2, -4j, 4-4j\} \\ X[k] - X^*[-k]_{\text{mod } 6} = \{6j, -2+6j, -4+4j, -6j, 4+4j, 2+6j\} \end{cases}$$

Step 4 : Summary

$$\Rightarrow \text{the Sequence of } \begin{cases} \text{DFT}\{x_r[n]\} = \{2, 2+2j, 2j, -1, -2j, 2-2j\} \\ \text{DFT}\{jx_i[n]\} = \{3j, -1+3j, -2+2j, -3j, 2+2j, 1+3j\} \end{cases}$$

However, what we would like to find is DFT$\{x_i[n]\}$, so, divide j to the second sequence.

$\Rightarrow$ In Summary, the DFT of $x[n]$'s $\begin{cases} \text{Real part is : } \{2, 2+2j, 2j, -1, -2j, 2-2j\} \\ \text{Imaginary part is : } \{3, 3+j, 2+2j, -3, 2-2j, 3-j\} \end{cases}$