EEE104 – Digital Electronics (I)

Lecture 7

Dr. Ming Xu

Dept of Electrical & Electronic Engineering

XJTLU

In This Session

- Boolean Algebra and Logic Simplification
  - Boolean Operations
  - Laws and Rules
  - DeMorgan’s Theorems
  - Logic Simplification

Boolean Operations

Concepts

- **Boolean algebra** is the mathematics of logic functions.
- A **variable** is a symbol used to represent a logical quantity. It can have a 1 or 0 value.
- The **complement** is the inverse of a variable and is indicated by an overbar, e.g. $\overline{A}$
- A **literal** is a variable or its complement.

Boolean Operations

Boolean Addition

- Equivalent to the OR operation with the basic rules:

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

- A **sum term** is a sum of literals, and is equal to 0 only if each of the literals is 0, e.g. $\bar{A} + B + C + \bar{D}$

Boolean Operations

Boolean Multiplication

- Equivalent to the AND operation with the basic rules:

$$
\begin{aligned}
0 \cdot 0 &= 0 \\
0 \cdot 1 &= 0 \\
1 \cdot 0 &= 0 \\
1 \cdot 1 &= 1
\end{aligned}
$$

- A **product term** is the product of literals. It is equal to 1 only if each of the literals is 1, e.g. $\overline{A}BC\overline{D}$

Laws of Boolean Algebra

Commutative Laws

$$A + B = B + A$$

$$AB = BA$$

Associate Laws

$$A + (B + C) = (A + B) + C$$

$$A(BC) = (AB)C$$

Distributive Laws (or Factoring)

$$A(B + C) = AB + AC$$

Rules of Boolean Algebra

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

- A, B, C can represent a single variable or a combination of variables.
- Rules 10-12 can be derived using simpler rules and the laws.

Rules of Boolean Algebra

Rule 10: $A + AB = A$

$A + AB = A(1 + B)$ Rule 2

$= A$

Rule 11: $A + \overline{A}B = A + B$

$A + \overline{A}B = A + AB + \overline{A}B$ Rule 10 or Rule 2

$= A + (A + \overline{A})B$

$= A + B$ Rule 6

Rules of Boolean Algebra

Rule 12:

$$(A+B)(A+C)=A+BC$$

$$(A+B)(A+C)=AA+AC+AB+BC$$

$$=A+AC+AB+BC \qquad \text{Rule 7}$$

$$=A+AB+BC \qquad \text{Rule 10 or Rule 2}$$

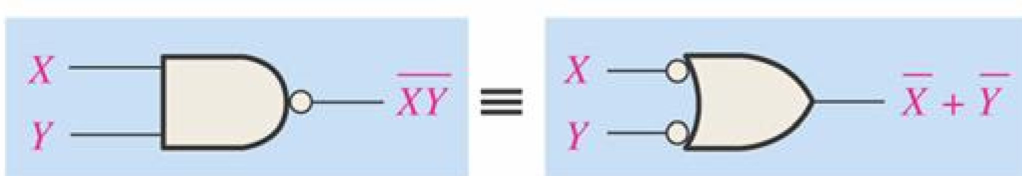
$$=A+BC \qquad \text{Rule 10 or Rule 2}$$

DeMorgan's Theorems (1st)

- The complement of a product of variables is equal to the sum of the complements of variables.

$$\overline{XY} = \overline{X} + \overline{Y}$$

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



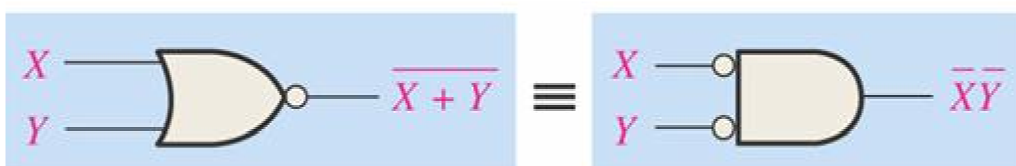
NAND Negative-OR

DeMorgan's Theorems (2nd)

- The complement of a sum of variables is equal to the product of the complements of variables.

$$\overline{X+Y} = \overline{X}\,\overline{Y}$$

Inputs		Output	
X	Y	$\overline{X+Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

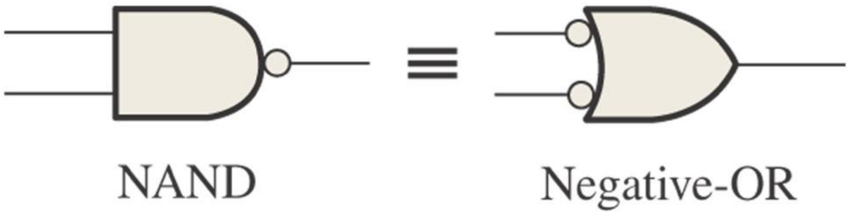


NOR   Negative-AND

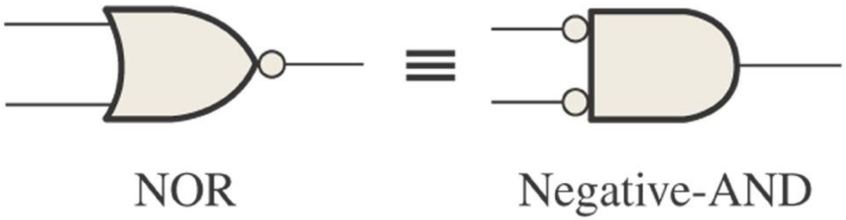
DeMorgan's Theorems

Break the bar, change the sign.

$$\overline{XY} = \overline{X} + \overline{Y}$$



$$\overline{X + Y} = \overline{X}\,\overline{Y}$$



DeMorgan's Theorems

Application Example

- Each variable can also represent a combination of other variables.

$$\overline{(AB + C)(A + BC)} = (\overline{AB + C}) + (\overline{A + BC})$$
$$= (\overline{AB})\overline{C} + \overline{A}(\overline{BC})$$
$$= (\overline{A} + \overline{B})\overline{C} + \overline{A}(\overline{B} + \overline{C})$$

$$\overline{\overline{[A + B\overline{C}]} + [D(\overline{E + \overline{F}})]} = [\overline{\overline{A + B\overline{C}}}][\overline{D(\overline{E + \overline{F}})}]$$
$$= (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$$
$$= (A + B\overline{C})(\overline{D} + E + \overline{F})$$

Simplification Using Boolean Algebra

- To use the fewest gates possible to implement a given expression.

$$AB + A(B + C) + B(B + C)$$
$$= AB + AB + AC + BB + BC$$
$$= AB + AC + B + BC$$
$$= AB + AC + B = B + AC$$