CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 5-1 Electromagnetic Boundary Conditions

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OUTLINE

- Electromagnetic Boundary Conditions
  - In Electrostatics
    - ✓ for Perfect Dielectric Materials (general case)
    - ✓ for the Conductor-to-Perfect Dielectric Material (special case)
    - ✓ for the Conductor-to-free-space (special case)
  - In Magnetostatics
    - ✓ for Isotropic Homogeneous Linear Materials

1. DEFINITION- BOUNDARY CONDITIONS

In homogeneous media, electromagnetic quantities vary smoothly and continuously. At a boundary between dissimilar media, however, it is possible for electromagnetic quantities to be discontinuous.

Continuities and discontinuities in fields can be described mathematically and used to constrain solutions for fields away from boundaries.

When a field exists in a medium consisting of two different media, the conditions the field must satisfy are called boundary conditions.

Boundary conditions are derived by applying the integral form of Maxwell's equations to an infinitesimal region at an interface of two media.

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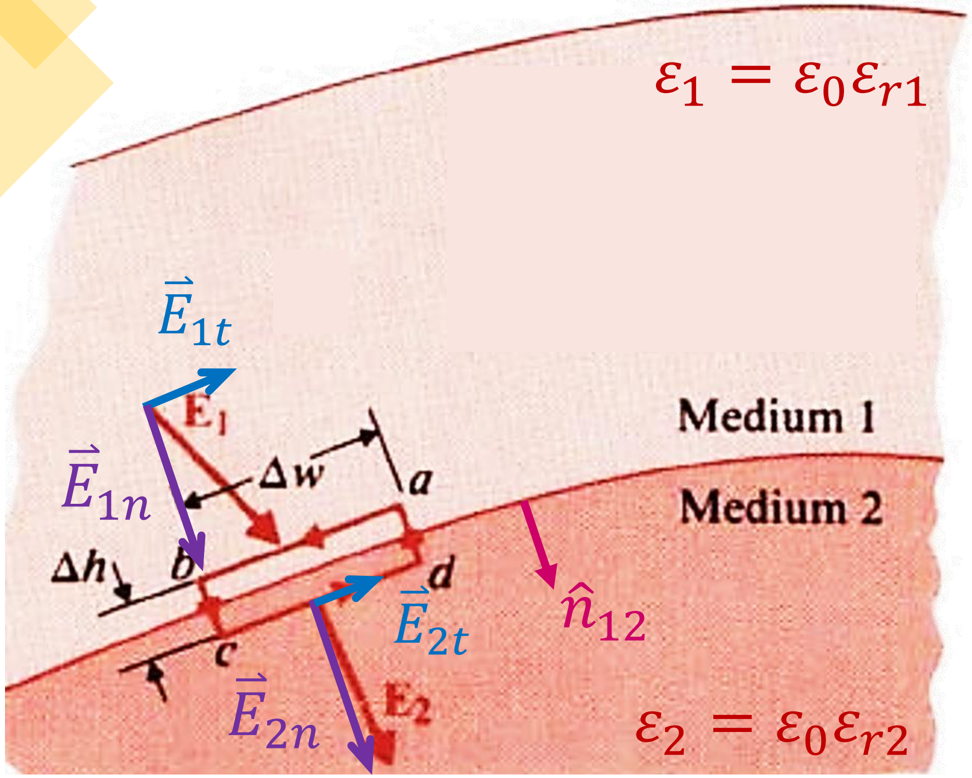
2.1 DIELECTRIC-DIELECTRIC

Consider the boundary between two perfect dielectrics (Medium 1 & Medium 2) with permittivity $\varepsilon_1 = \varepsilon_0\varepsilon_{r1}$ and $\varepsilon_2 = \varepsilon_0\varepsilon_{r2}$

The fields in the two media can be expressed as $\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}$ $\quad \vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$

For the ***tangential*** component:



Construct a small closed path *a-b-c-d-a*:

Sides *ab* & *cd* are parallel to the boundary & equal to *Δw*.

Sides *bc* & *da* are vertical to the boundary, *bc* = *da* = *Δh* → 0.

$$\oint_c \vec{E}\cdot d\vec{l} = -E_{1t}\Delta w + E_n\Delta h + E_{2t}\Delta w - E_n\Delta h = (E_{2t} - E_{1t})\Delta w$$

$$\oint_c \vec{E}\cdot d\vec{l} = 0$$

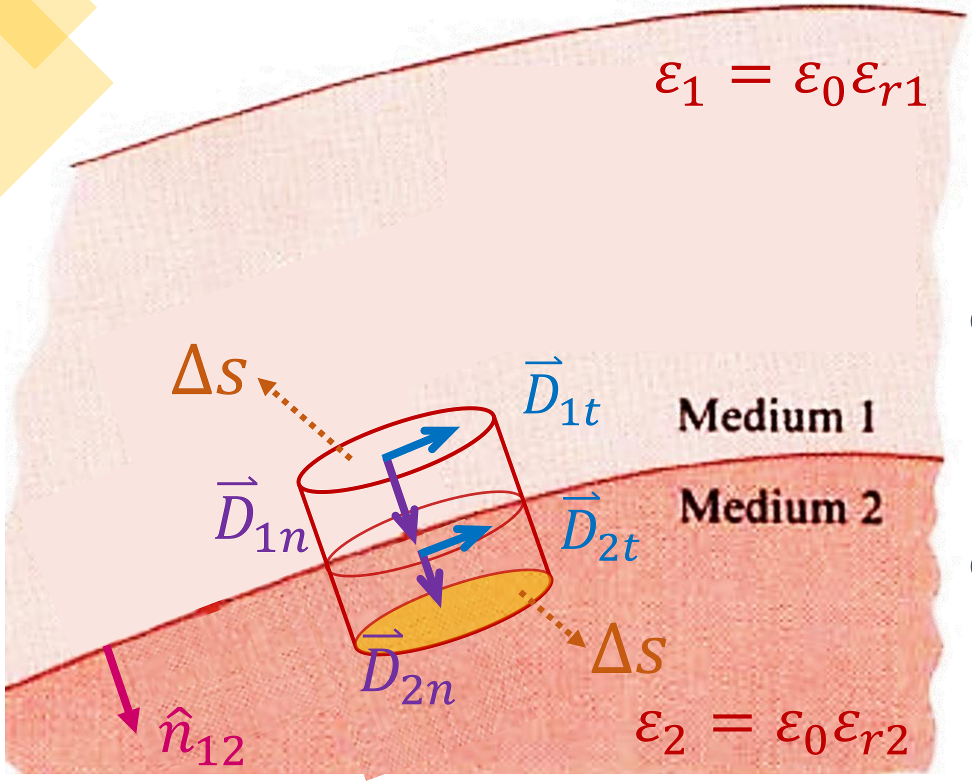
$\therefore E_{2t} = E_{1t}$ or $\left(\vec{E}_2 - \vec{E}_1\right)\times\hat{n}_{12} = 0$

2.1 DIELECTRIC-DIELECTRIC

Consider the boundary between two perfect dielectrics (Medium 1 & Medium 2) with permittivity $\varepsilon_1 = \varepsilon_0\varepsilon_{r1}$ and $\varepsilon_2 = \varepsilon_0\varepsilon_{r2}$

For the ***normal*** component:

Construct a small closed pillbox surface:



Top & bottom faces are parallel to the interface & equal to Δs.

Hight $\Delta h \to 0$.

ρ_s is the surface charge density on the interface.

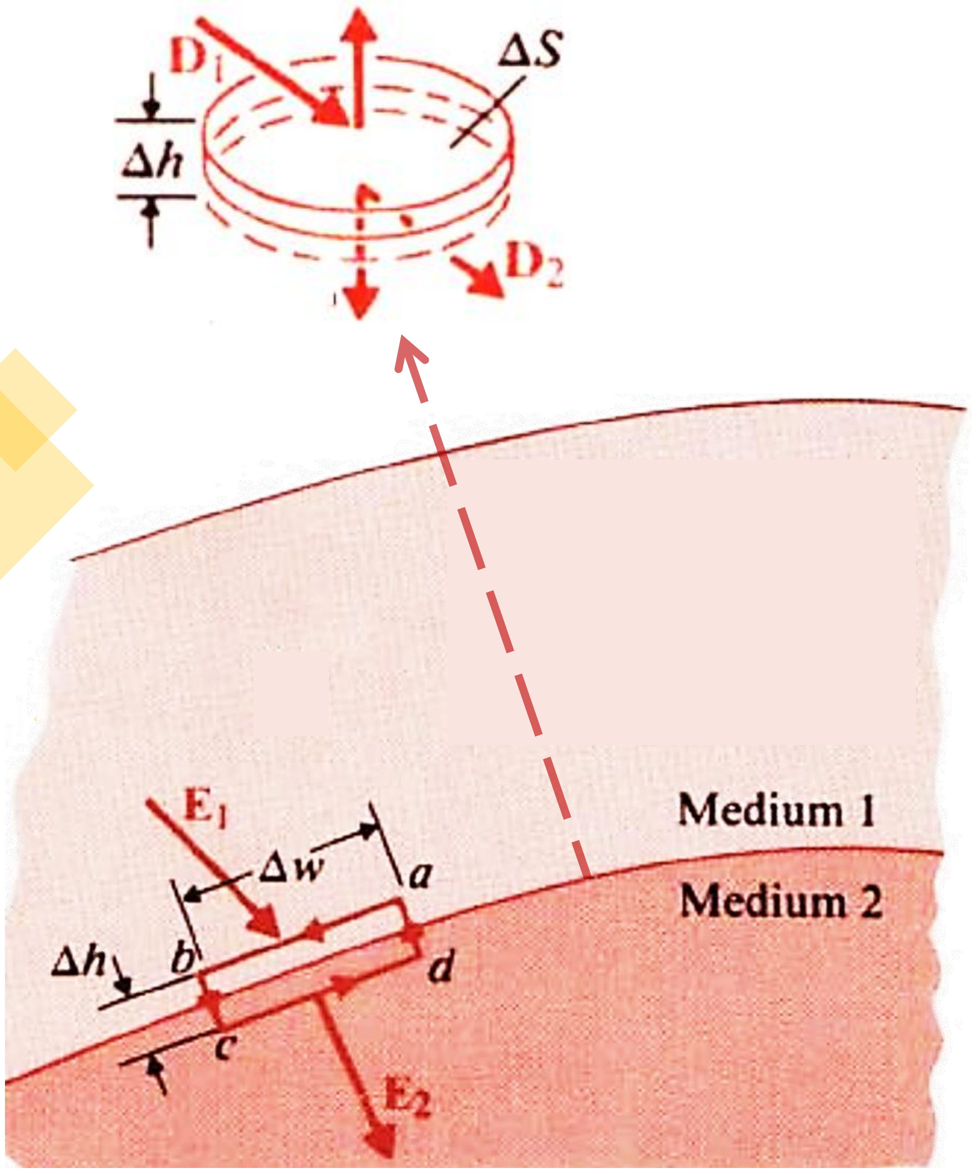
$$\oiint_S \vec{D}\cdot d\vec{s} = \oiint_{top}\vec{D}_1\cdot d\vec{s} + \oiint_{bottom}\vec{D}_2\cdot d\vec{s} + \oiint_{side}\vec{D}\cdot d\vec{s}$$

$$= -D_{1n}\Delta s + D_{2n}\Delta s = (D_{2n} - D_{1n})\Delta s$$

$$\oiint_S \vec{D}\cdot d\vec{s} = Q_{encl} = \rho_s\Delta s$$

$\therefore D_{2n} - D_{1n} = \rho_s$ or $\left(\vec{D}_2 - \vec{D}_1\right)\cdot\hat{n}_{12} = \rho_s$

2.2 CONDUCTOR-DIELECTRIC



If medium 1 is conductor:

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n} = 0$$

$$\therefore E_{2t} = E_{1t} = 0$$

$$D_{2n} - D_{1n} = \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \varepsilon_0 \varepsilon_{r2} E_{2n}$$

$$= \rho_s \quad (\rho_s \text{ is outside the conductor})$$

2.2 CONDUCTORS

Basic Properties

1. The **static** electric field intensity inside a conductor is **zero**.
2. The **static** electric field intensity at the surface of a conductor is everywhere directed **normal** to that surface.
3. Conductor's surface is an **equipotential** surface (等势面). The tangential component of the static electric field intensity on the surface is zero.
4. Net charge can **only** reside on the surface when reaches electrostatic equilibrium.

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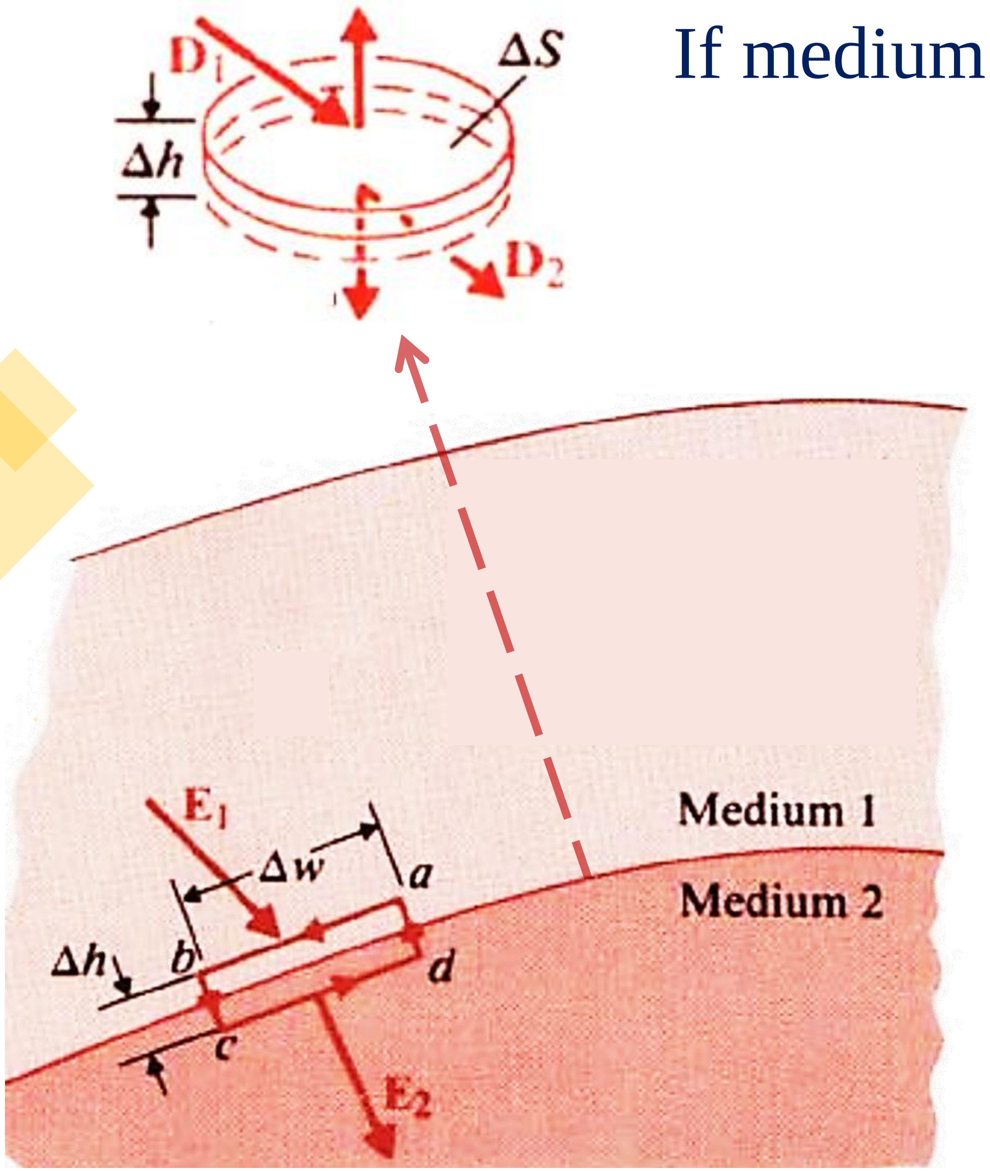
2.3 CONDUCTOR-FREE SPACE



If medium 1 is conductor & medium 2 is free space:

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n} = 0$$

$$\therefore E_{2t} = E_{1t} = 0$$

$$D_{2n} - D_{1n} = \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \varepsilon_0 E_{2n} = \rho_s$$

(ρ_s is outside the conductor)

2.2 CONDUCTORS

Basic Properties

1. The **static** electric field intensity inside a conductor is **zero**.

2. The **static** electric field intensity at the surface of a conductor is everywhere directed **normal** to that surface.

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4. Net charge can **only** reside on the surface when reaches electrostatic equilibrium.

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SUMMARY

The field directed from medium 1 to medium 2 MUST be applied accordingly.

Boundary conditions state:

1. The tangential component of the electric field intensity is continuous across an interface;

2. The normal component of electric flux density is discontinuous across an interface where a surface charge exists

– the amount of discontinuity equals the surface charge density

Scalar Form	Vector Form
$E_{2t} = E_{1t}$	$(\vec{E}_2 - \vec{E}_1) \times \hat{n}_{12} = 0$
$D_{2n} - D_{1n} = \rho_s$	$(\vec{D}_2 - \vec{D}_1) \cdot \hat{n}_{12} = \rho_s$

QUIZ 2.1

Two extensive homogeneous isotropic dielectrics ($\varepsilon_{r1} = 2, \varepsilon_{r2} = 5$) meet on the plane $z = 0$. If the electric flux density in medium 2 is:

$$\vec{D}_2 = 2\hat{x} + 5\hat{y} - 3\hat{z}\ nC/m^2$$

$$E_{2t} = E_{1t}$$

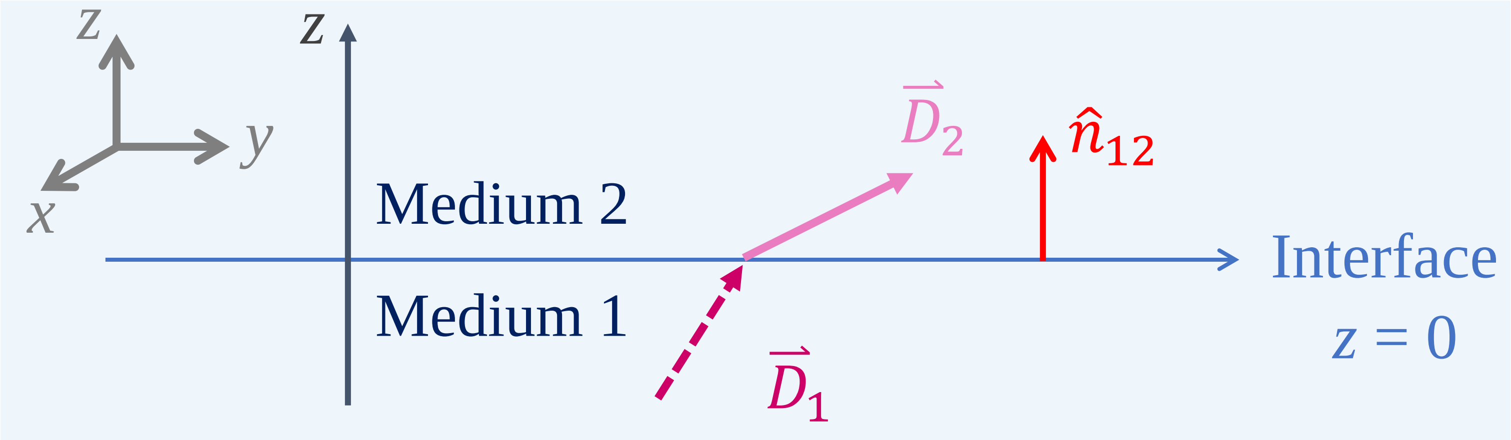
$$D_{2n} - D_{1n} = \rho_s$$

Determine the following:

a) the tangential component of $\vec{D}_1$

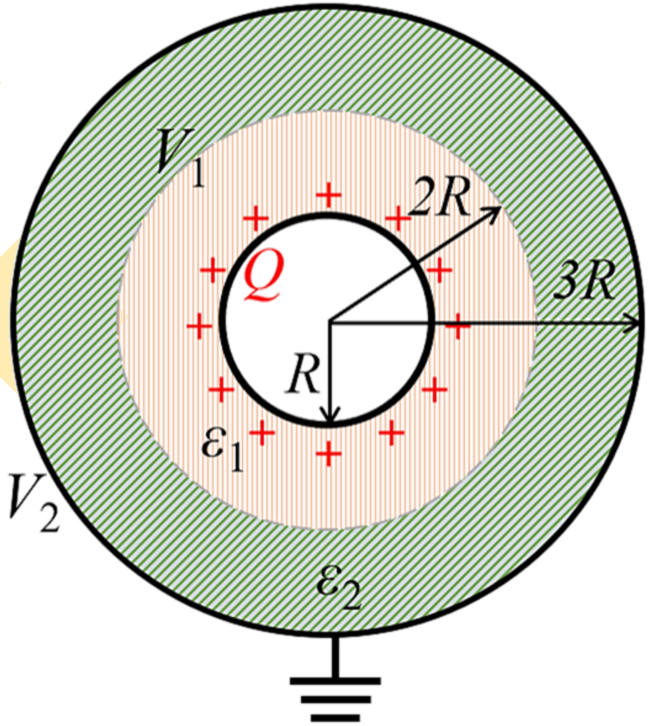
b) the normal component of $\vec{D}_1$

c) the expression of $\vec{D}_1$



QUIZ 2.2

Consider a conducting spherical shell with an inner radius R and outer radius $3R$. The space between the two conducting surfaces is filled with two different dielectric materials so that material 1 with permittivity ε_1 is filled in the concentric layer between R and $2R$, and material 2 with permittivity ε_2 is filled in the concentric layer between $2R$ and $3R$. **Using boundary conditions**, determine the electric field intensity from R to $3R$.



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3.1 NORMAL COMPONENT

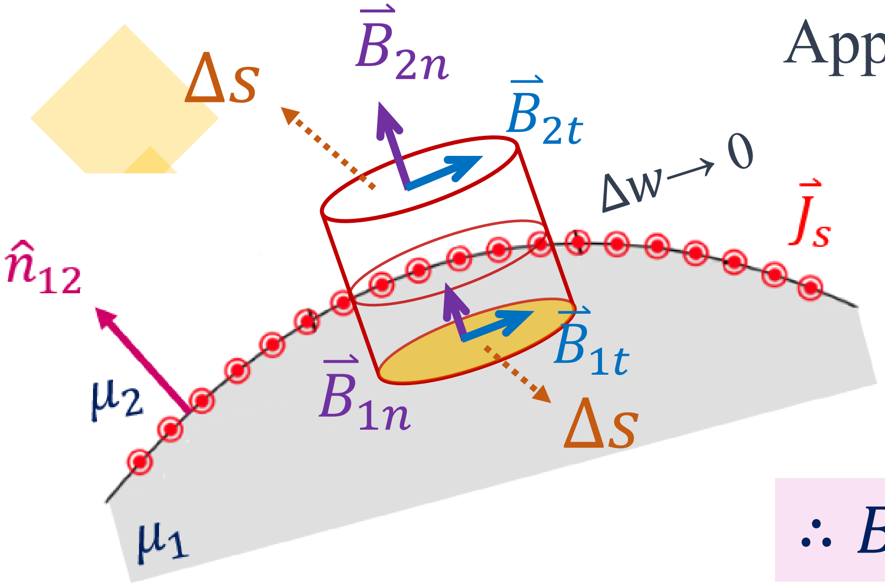
Consider the boundary between two isotropic homogeneous linear materials (Medium 1 & Medium 2) with permeabilities $\mu_1 = \mu_0\mu_{r1}$ and $\mu_2 = \mu_0\mu_{r2}$

The fields in the two media can be expressed as $\vec{H}_1 = \vec{H}_{1t} + \vec{H}_{1n}$ $\vec{H}_2 = \vec{H}_{2t} + \vec{H}_{2n}$

For the ***normal*** component:

Construct a small closed pillbox surface with vanishing thickness:

Top & bottom faces are parallel to the interface & equal to Δs. Hight $\Delta w \to 0$.



Apply Gauss's law:

$$\left.\begin{aligned}\oiint_S \vec{B}\cdot d\vec{s} &= \iint_{top} \vec{B}_2\cdot d\vec{s} + \iint_{bottom} \vec{B}_1\cdot d\vec{s} + \cancel{\iint_{side} \vec{B}\cdot d\vec{s}} \\ &= +B_{2n}\Delta s - B_{1n}\Delta s = (B_{2n} - B_{1n})\Delta s \\ \oiint_S \vec{B}\cdot d\vec{s} &= 0\end{aligned}\right\}$$

$\therefore B_{2n} - B_{1n} = 0$ or $(\vec{B}_2 - \vec{B}_1)\cdot\hat{n}_{12} = 0$

3.2 TANGENTIAL COMPONENT

Consider the boundary between two isotropic homogeneous linear materials (Medium 1 & Medium 2) with permeabilities $\mu_1 = \mu_0\mu_{r1}$ and $\mu_2 = \mu_0\mu_{r2}$

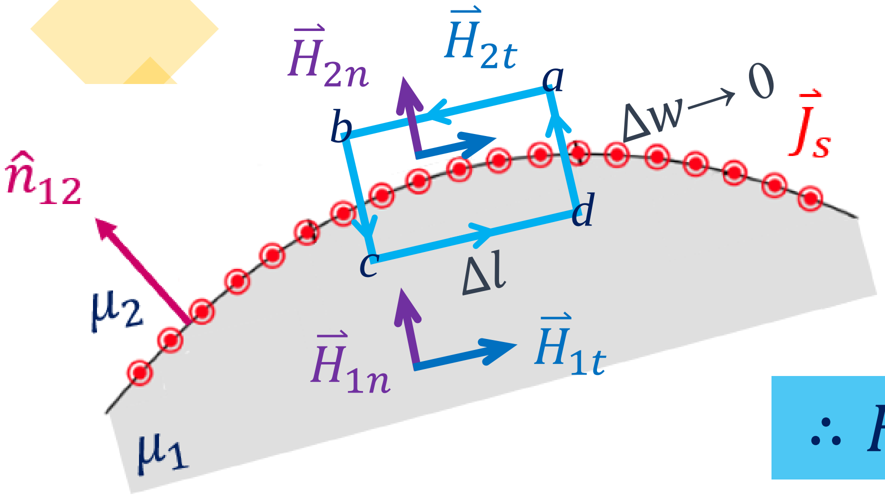
For the ***tangential*** component:

Construct a small closed path *a-b-c-d-a*:

Sides *ab* & *cd* are parallel to the boundary & equal to *Δl*.

Sides *bc* & *da* are vertical to the boundary, *bc* = *da* = *Δw* → 0.

Apply Ampere's law:



$$\left.\begin{aligned}\oint_c \vec{H}\cdot d\vec{l} &= -H_{2t}\Delta l - H_n\Delta w + H_{1t}\Delta l + H_n\Delta w \\ &= (H_{1t} - H_{2t})\Delta l \\ \oint_c \vec{H}\cdot d\vec{l} &= I_{encl} = J_s\Delta l\end{aligned}\right\}$$

$\boxed{\therefore H_{1t} - H_{2t} = J_s}$ or $\boxed{(\vec{H}_1 - \vec{H}_2)\times\hat{n}_{12} = \vec{J}_s}$

SUMMARY

The field directed from medium 1 to medium 2 MUST be applied accordingly.

Boundary conditions state:

1. The normal component of magnetic flux density is continuous across an interface

2. The tangential component of the magnetic field intensity is continuous across the boundary when the surface current density is zero

Scalar Form	Vector Form
$B_{2n} - B_{1n} = 0$	$(\vec{B}_2 - \vec{B}_1) \cdot \hat{n}_{12} = 0$
$H_{1t} - H_{2t} = J_s$	$(\vec{H}_1 - \vec{H}_2) \times \hat{n}_{12} = \vec{J}_s$

QUIZ 3.1

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

$$\left(\vec{B}_2 - \vec{B}_1\right) \cdot \hat{n}_{12} = 0$$

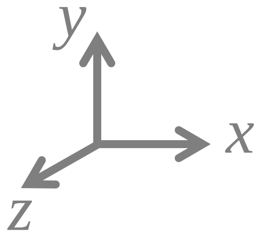
$$\left(\vec{H}_1 - \vec{H}_2\right) \times \hat{n}_{12} = \vec{J}_s$$

Assume that a plane located at $y = 0$ separates 2 mediums.

Medium 1 (M1) is in $y > 0$ with relative permeability $\mu_{r1} = 2$;

Medium 2 (M2) is in $y < 0$ with relative permeability $\mu_{r2} = 1$.

The magnetic field intensity vector in medium 1 near the boundary is



$$\vec{H}_1 = (4\hat{x} - 2\hat{y} + 8\hat{z})\ \ \text{A/m}$$

If the surface current density on the boundary is $3\hat{x}$, find $\vec{H}_2$ in M2 near the boundary.

NEXT…

- Currents