Frequency Response of Amplifier Circuits

Dr. Suneel Kommuri

Office: SC431, TEL: +86-(0)-512-8188-3250

Email: Suneel.Kommuri@xjtlu.edu.cn

Dept. Electrical & Electronic Engineering

Xi'an Jiaotong-Liverpool University
西交利物浦大學

Outline

- ✓ Amplifier Frequency Response
- ✓ Frequency Response of Transistor Circuits
- ✓ Expanded Hybrid–π Equivalent Circuit
  - Short-Circuit Current Gain
  - Cutoff Frequency
  - Miller Effect and Miller Cpacitance
- ✓ High-Frequency Response of Transistor Circuits
  - CE, CB, and CC Circuits

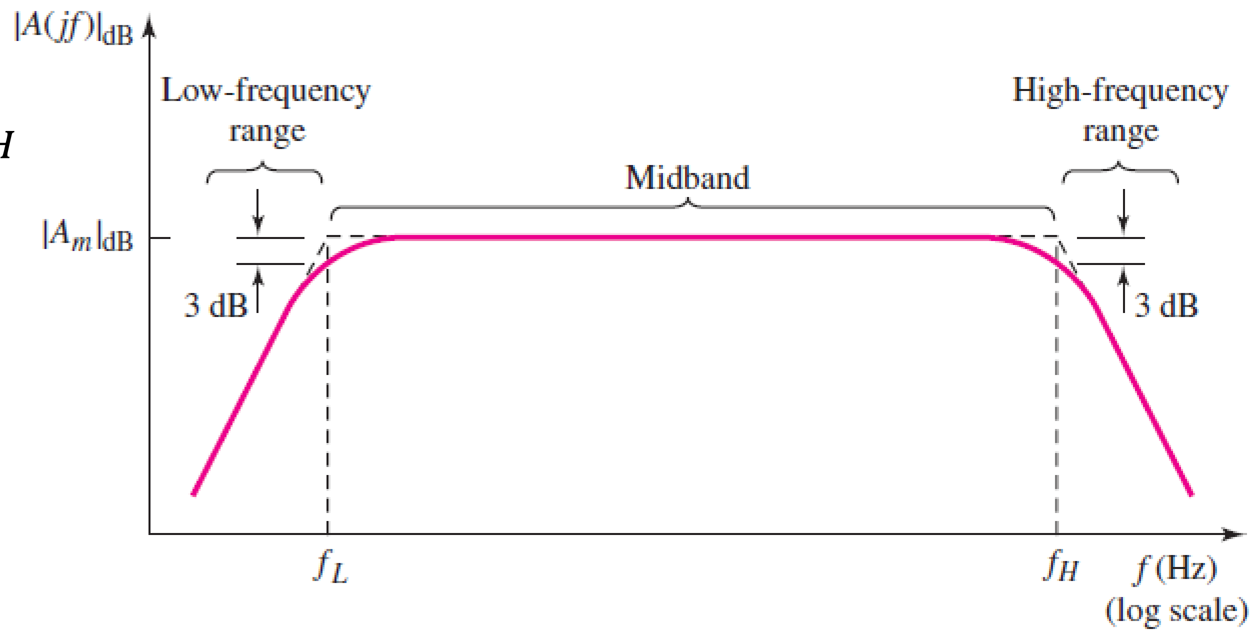
Amplifier Frequency Response

- ✓ Note that all *amplifier gain factors* are functions of signal frequency – so far we have assumed that the signal frequency is
  1) high enough that coupling & bypass capacitors treated as short circuits,
  2) low enough that parasitic, load, & transistor capacitances treated as open circuits.

- ✓ 3 frequency ranges: low ($f < f_L$), midband, and high ($f > f_H$).

The gain at $f = f_L$ & $f = f_H$ is 3 dB less than the maximum midband gain.

Bandwidth, $f_{BW} = f_H - f_L$



Amplifier gain versus frequency

Amplifier Frequency Response

- ✓ In low-frequency range $(f < f_L)$, gain decreases as frequency decreases because of coupling & bypass capacitor effects – they must be included in the equivalent circuit. Stray and transistor capacitances are open circuited. *Use low-frequency equivalent circuit*

- ✓ In high-frequency range $(f > f_H)$, stray & transistor capacitances effect cause the gain to decrease as frequency increase – they must be included in the equivalent circuit. Coupling & bypass capacitors are short circuited. *Use high-frequency equivalent circuit*

- ✓ The midband range is the region where coupling and bypass capacitors act as short circuits, and stray and transistor capacitances act as open circuits. In this frequency range, there are no capacitances in the equivalent circuit.

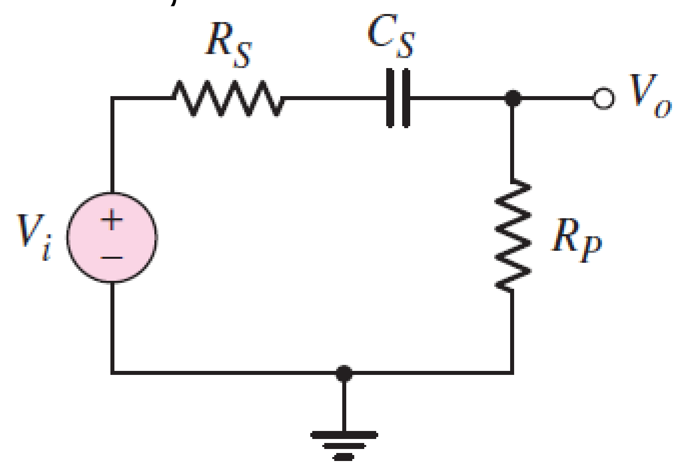
- ✓ The frequency response of a circuit is usually determined by complex frequency (s). Capacitor is represented by complex impedance $(1/sC)$ and inductor is represented by complex impedance (sL). Set $s = j\omega = j2\pi f$.

Frequency Response of Transistor Circuits

Series coupling capacitor circuit

Using voltage divider rule, the voltage transfer function is,

$$\frac{V_o(s)}{V_i(s)} = \frac{R_P}{R_P+R_S+\frac{1}{sC_S}} = \frac{sR_PC_S}{1+s(R_P+R_S)C_S} = \left(\frac{R_P}{R_P+R_S}\right)\left(\frac{s\tau_S}{1+s\tau_S}\right)$$

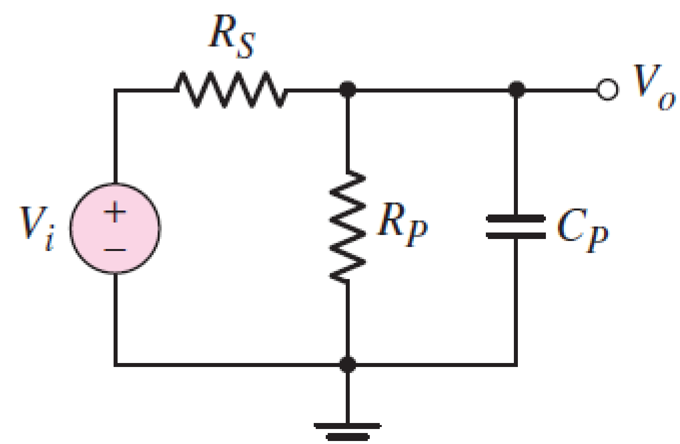


where $\tau_S = (R_P + R_S)C_S$ is a time-constant.

Parallel load capacitor circuit

Writing KCL at output node,

$$\frac{V_o-V_i}{R_S} + \frac{V_o}{R_P} + \frac{V_o}{1/sC_P} = 0 \rightarrow \frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_P+R_S}\right)\left(\frac{1}{1+s\tau_P}\right)$$



where $\tau_P = (R_S||R_P)C_P$ is a time-constant.

Frequency Response of Transistor Circuits

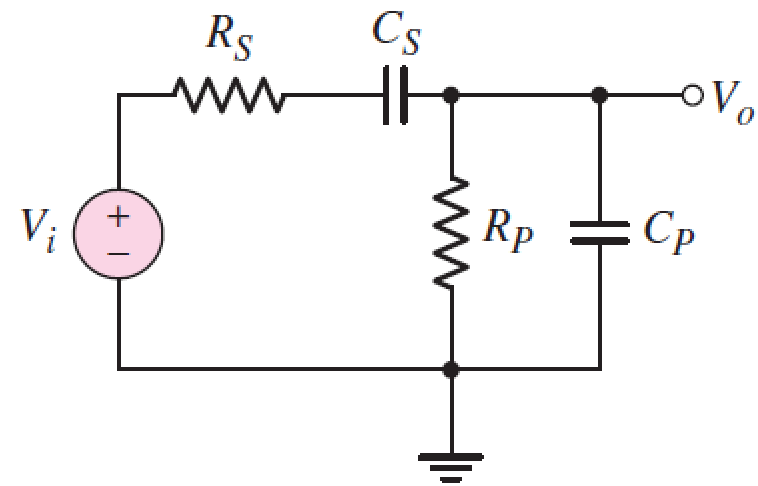
Now consider a circuit that contains both coupling & parallel load capacitors.

Using KCL at output node, the result is,

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_P+R_S}\right) \times \frac{1}{\left[1+\left(\frac{R_P}{R_S+R_P}\right)\left(\frac{C_P}{C_S}\right)+\frac{1}{s\tau_S}+s\tau_P\right]}$$

where τ_S & τ_P are time-constants.

Note that C_S affects low-frequency response and C_P affects low-frequency response.



Circuit with both series coupling and a parallel load capacitor

✓ At low-frequencies, we can treat C_P as an open-circuit, therefore, $\tau_S = (R_P + R_S)C_S$ is called an open-circuit time-constant.

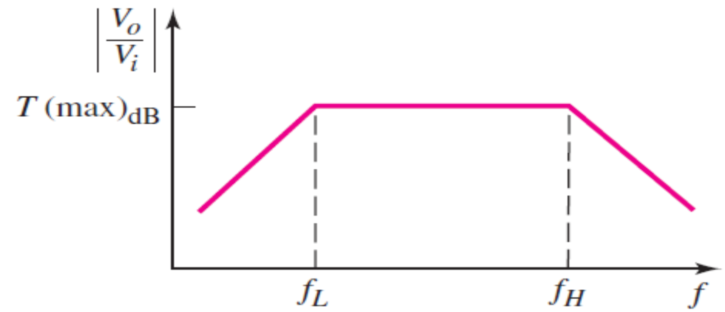
✓ At high-frequencies, we can treat C_S as a short-circuit, therefore, $\tau_P = (R_P||R_S)C_P$ is called a short-circuit time-constant.

Frequency Response of Transistor Circuits

See the Bode plot of voltage transfer function magnitude for the earlier circuit.

Low corner or 3-dB frequency, $f_L = \frac{1}{2\pi\tau_S}$

Upper corner or 3-dB frequency, $f_H = \frac{1}{2\pi\tau_P}$



Exercise–1: Consider the circuit parameters: $R_S = 1\ k\boldsymbol{\Omega}, R_P = 10\ k\boldsymbol{\Omega}, C_S = 1\ \mu F$, and $C_P = 3\ \mu F$. Determine corner frequencies and bandwidth.

Solution

$$\tau_S = (R_P + R_S)C_S = 1.1 \times 10^{-12} s \quad \& \quad \tau_P = (R_P || R_S)C_P = 2.73 \times 10^{-9} s$$

Corner frequencies: $f_L = \frac{1}{2\pi\tau_S} = 14.5\ Hz \quad \& \quad f_H = \frac{1}{2\pi\tau_P} = 58.3\ MHz$

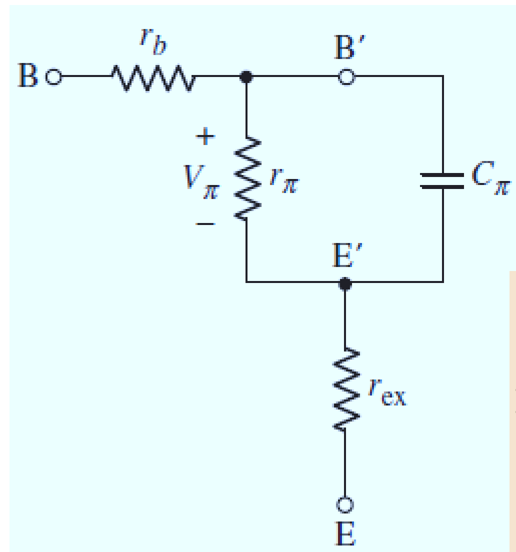
Bandwidth, $f_{BW} = f_H - f_L = 58.3\ MHz - 14.5\ Hz \cong 58.3\ MHz$

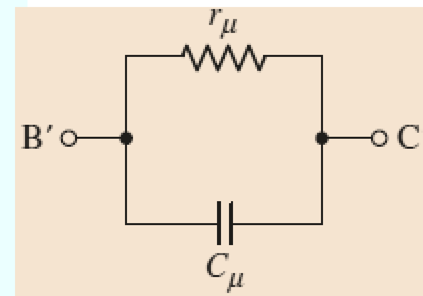
Expanded Hybrid−π equivalent Circuit

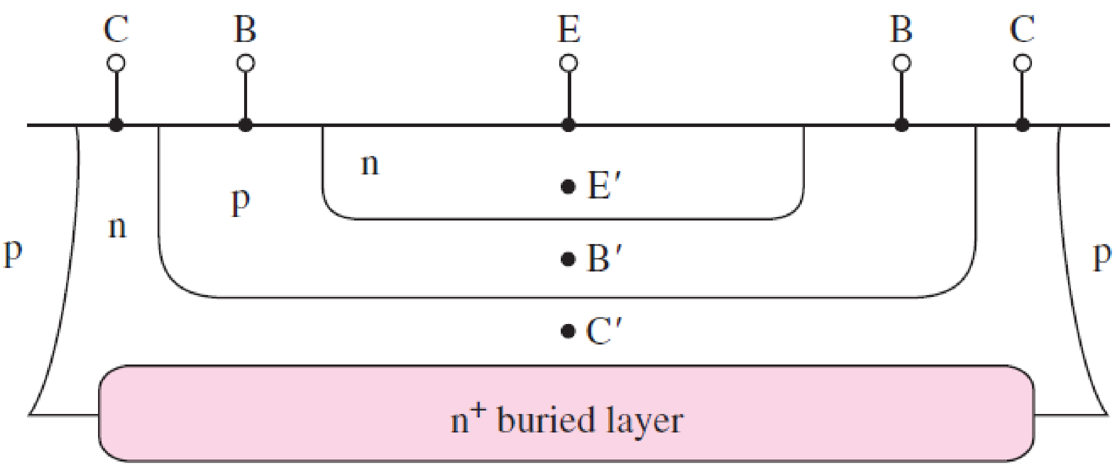
See the cross section of an npn bipolar transistor for hybrid−π model.

✓ C, B, and E terminals are external connections to the transistor, and C', B' & E' are idealized internal collector, base, and emitter regions.

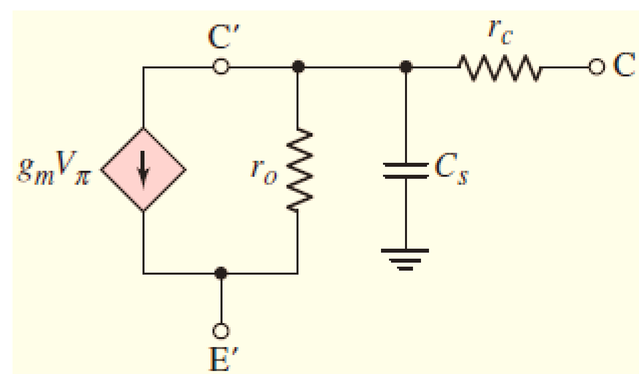
r_b is base series resistance between B & B'.
C_π is forward biased junction capacitance.
r_π is forward biased diffusion resistance.
r_{ex} is emitter series resistance E' & E.







Cross section of npn transistor for hybrid−π model



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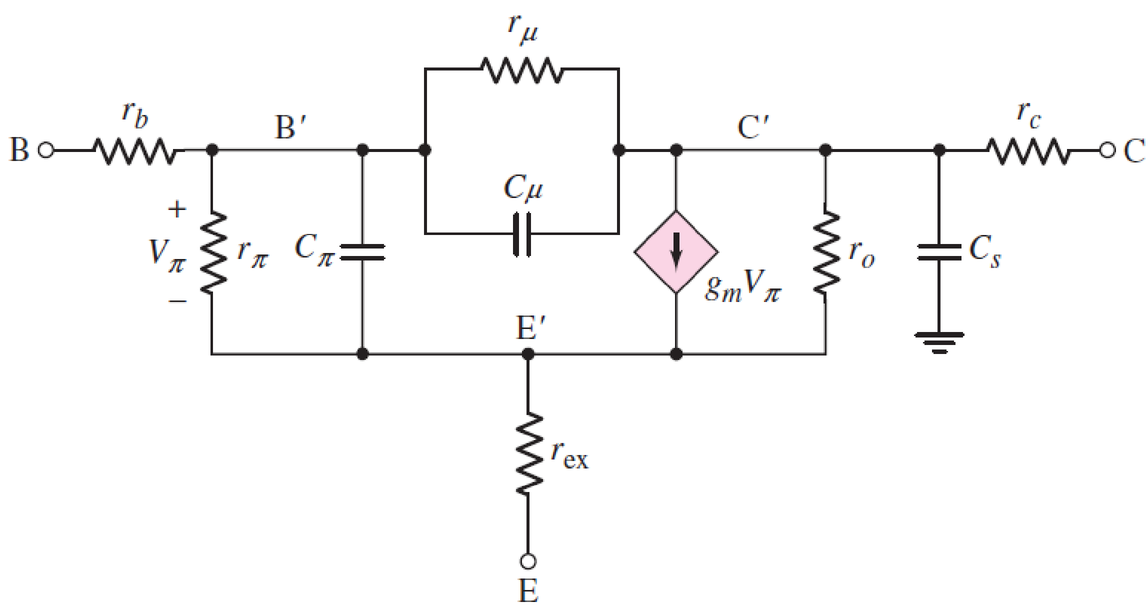
Expanded Hybrid–π equivalent Circuit

r_c is collector series resistance between C & C'.
C_s is junction capacitance of reverse-biased collector-substrate junction.
$g_m V_\pi$ is collector current controlled by internal base-emitter voltage.
r_o is inverse of output conductance g_o and is due primarily to Early effect.

r_μ is reverse-biased diffusion resistance (range of megohms & neglected).
C_μ is reverse-biased junction capacitance (normally < C_π, however, *cannot be neglected due to Miller effect*).



Short-Circuit Current Gain

Neglect the parasitic resistances r_b, r_c, r_{ex}, & r_μ, and substrate capacitance C_s.

Write KCL at the input node, $I_b = \frac{V_\pi}{r_\pi} + \frac{V_\pi}{\frac{1}{j\omega C_\pi}} + \frac{V_\pi}{\frac{1}{j\omega C_\mu}} = V_\pi\left[\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)\right]$

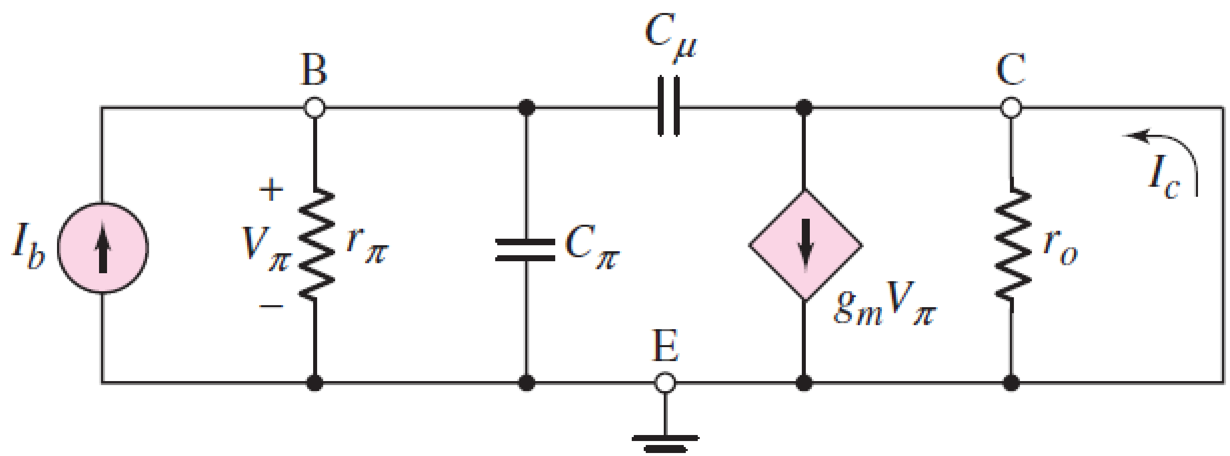
Note that $V_\pi \neq I_b r_\pi$, since a portion of I_b is now shunted through C_π & C_μ. From KCL at the output node,

$$\frac{V_\pi}{\frac{1}{j\omega C_\mu}} + I_c = g_m V_\pi \;\rightarrow\; I_c = V_\pi(g_m - j\omega C_\mu) \;\rightarrow\; V_\pi = \frac{I_c}{(g_m - j\omega C_\mu)}$$

Substitute V_π in I_b results as,

$$I_b = I_c \times \frac{\left[\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)\right]}{g_m - j\omega C_\mu}$$

$$\therefore\; A_i = \frac{I_c}{I_b} = \frac{(g_m - j\omega C_\mu)}{\left[\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)\right]}$$



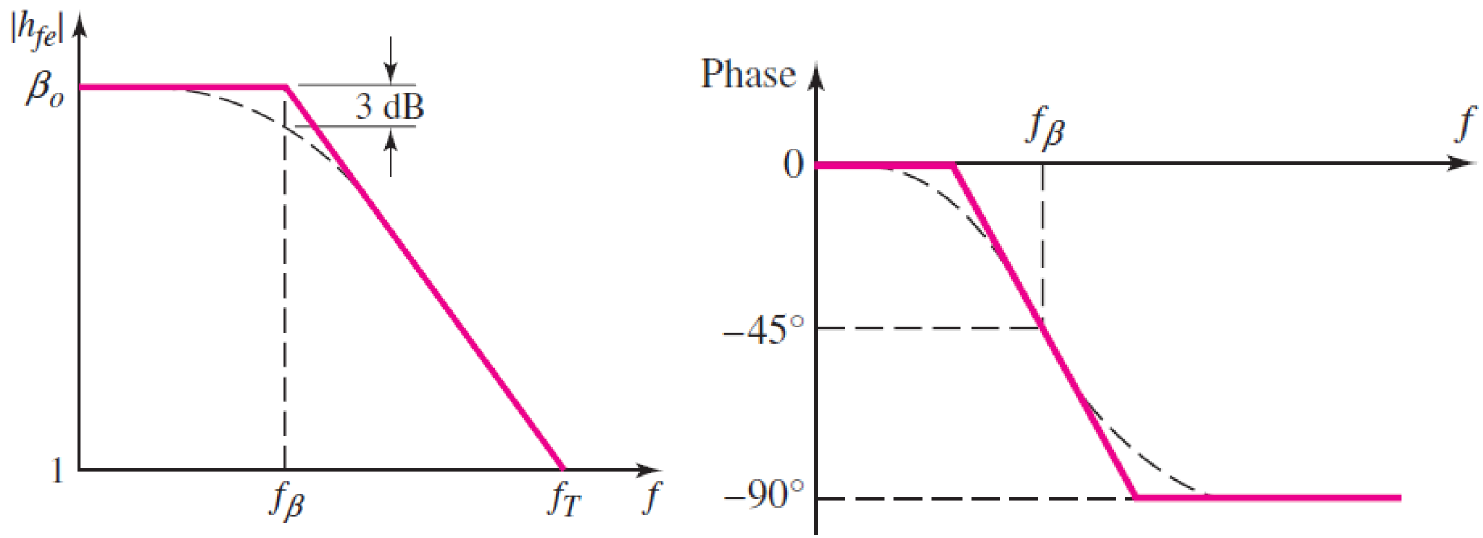
Simplified hybrid−π equivalent circuit

Short-Circuit Current Gain

$$A_i = h_{fe} = \frac{I_c}{I_b} = \frac{(g_m - j\omega C_\mu)}{\left[\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)\right]} \cong \frac{g_m r_\pi}{1 + j\omega r_\pi(C_\pi + C_\mu)} \quad \text{if } \omega C_\mu \ll g_m$$

See the Bode plot of short-circuit current gain magnitude. The corner frequency, beta cutoff frequency (f_β), is given by,

$$f_\beta = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$$



Magnitude plot　　Phase plot

Cutoff Frequency

Note that the magnitude of current gain decreases with increasing frequency and reaches to 1 at f_T (cutoff frequency). We can write A_i in the below form:

$$A_i = h_{fe} = \frac{\beta_o}{1 + j\left(\frac{f}{f_\beta}\right)} \rightarrow \left|h_{fe}\right| = \frac{\beta_o}{\sqrt{1 + \left(\frac{f}{f_\beta}\right)^2}}$$

At f_T, $\left|h_{fe}\right| = 1$ and normally $\beta_o \gg 1, f_T \gg f_\beta$. Therefore, equation becomes

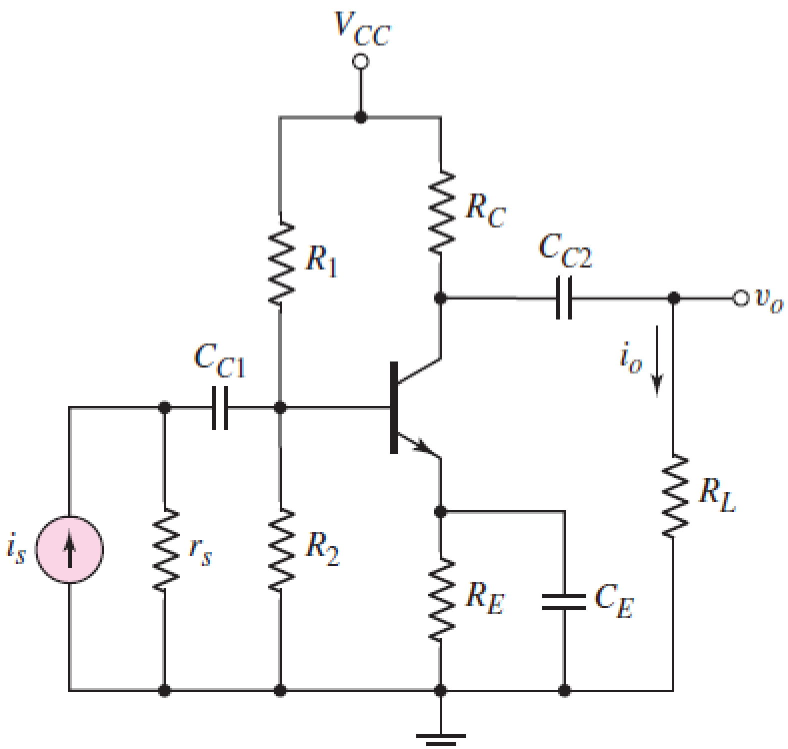
$$1 \cong \frac{\beta_o}{\sqrt{\left(\frac{f}{f_\beta}\right)^2}} = \frac{\beta_o f_\beta}{f_T} \rightarrow f_T = \beta_o f_\beta = \beta_o \frac{1}{2\pi r_\pi (C_\pi + C_\mu)} = \frac{g_m}{2\pi (C_\pi + C_\mu)}$$

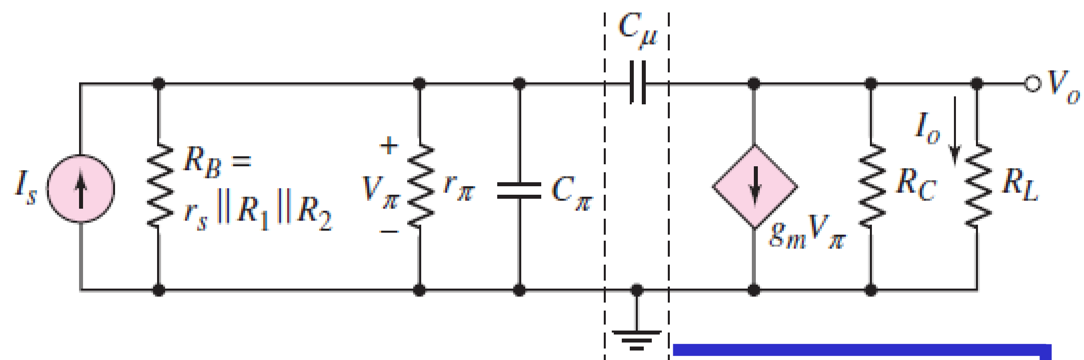
Frequency f_β is called the bandwidth of transistor, therefore, f_T is more commonly called as unity-gain bandwidth (or) gain bandwidth product.

Miller Effect and Miller Capacitance

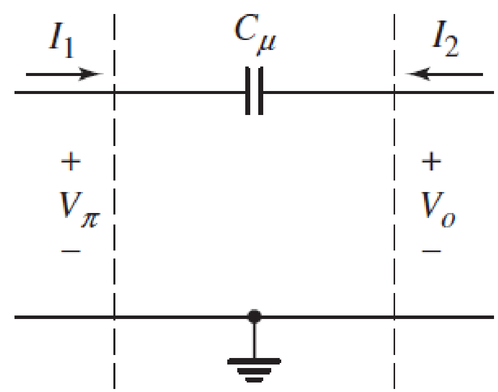
As mentioned earlier, the capacitance C_μ cannot really be ignored – *Miller effect, or feedback effect*, is a multiplication effect of C_μ in circuit applications.

✓ Assume the frequency is sufficiently high for the coupling and bypass capacitors to act as short circuits – C_μ connects output back to the input.





Small-signal hybrid−π equivalent circuit

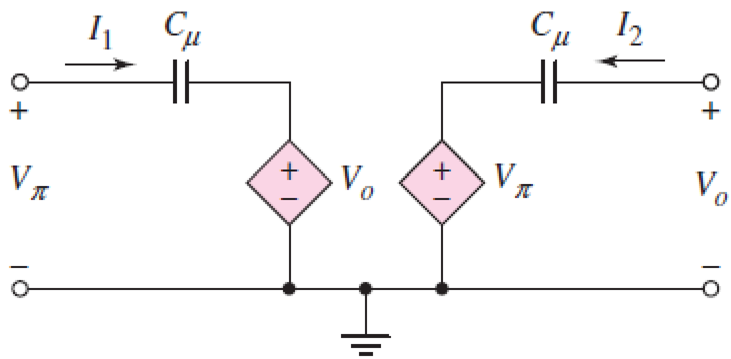


$$V_\pi = I_1\left(\frac{1}{j\omega C_\mu}\right) + V_o$$

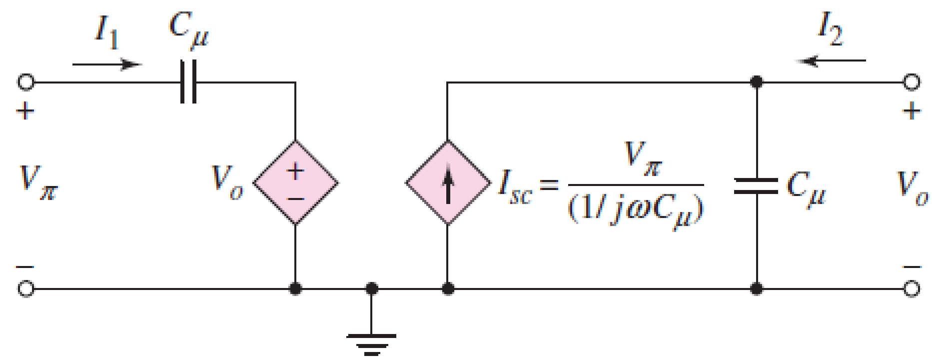
$$V_o = I_2\left(\frac{1}{j\omega C_\mu}\right) + V_\pi$$

Miller Effect and Miller Capacitance

- ✓ Now form a two-port equivalent circuit and then convert the Thevenin equivalent circuit on the output side to Norton equivalent circuit.

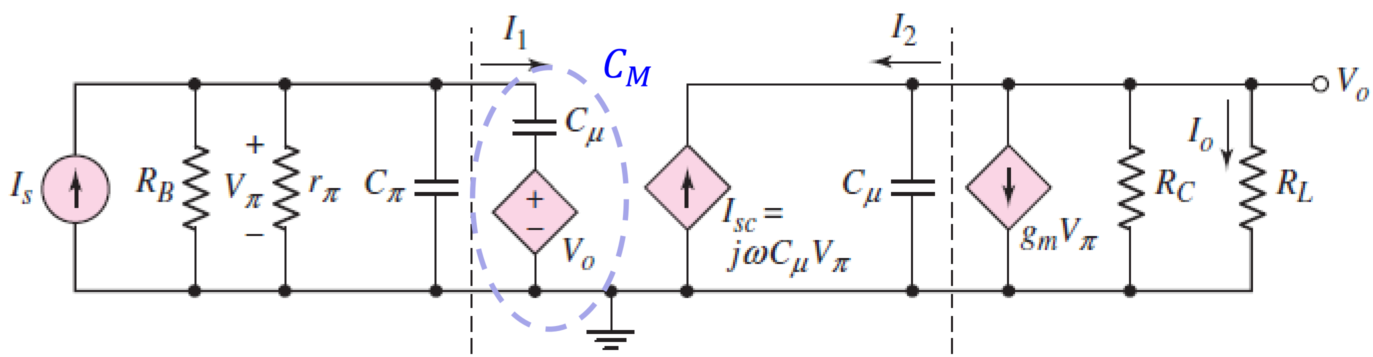


Two-port equivalent circuit of C_μ: Thevenin



Two-port equivalent circuit of C_μ: Norton at output

- ✓ Again, reconsider the original equivalent circuit and now replace the circuit segment between the dotted lines with the above Norton circuit.



Small-signal circuit, including two-port equivalent model of C_μ

Miller Effect and Miller Capacitance

Recall the expression for V_π and obtain the equation for I_1 as follows:

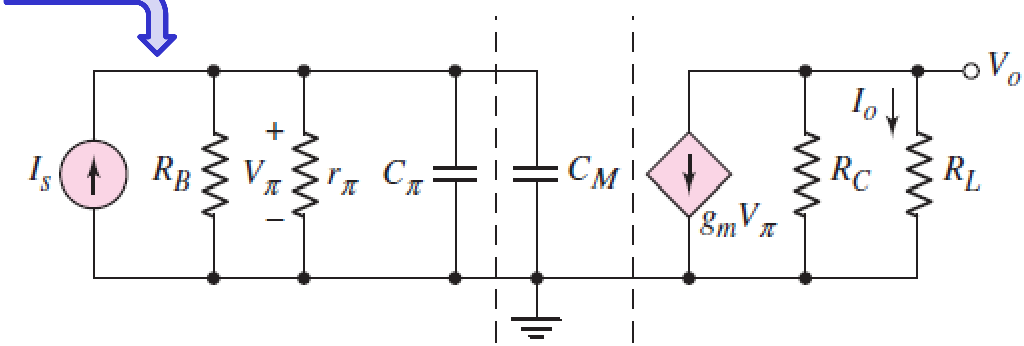
$$I_1 = \frac{V_\pi - V_o}{\frac{1}{j\omega C_\mu}} = j\omega C_\mu (V_\pi - V_o)$$

The output voltage is, $V_o = -g_m V_\pi (R_C||R_L)$

Therefore, I_1 becomes as, $I_1 = j\omega C_\mu [1 + g_m (R_C||R_L)] V_\pi$

The circuit segment between the dotted lines can be replaced by an equivalent capacitance called Miller capacitance as, $C_M = C_\mu [1 + g_m (R_C||R_L)]$
Note that the multiplication effect of C_μ is the Miller effect.

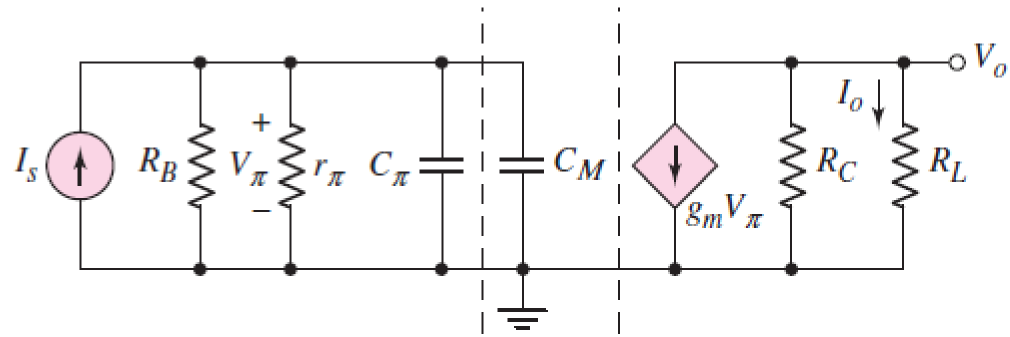
Consider the frequency of operation is very much smaller such that 1) I_{sc} is negligible compared to $g_m V_\pi$ source, 2) C_μ will be much greater than $R_C||R_L$, therefore C_μ can be considered as an open-circuit.



The input capacitance is now $C_\pi + C_M$.

Exercise on Miller Effect

Exercise–2: Consider the small-signal equivalent circuit with the following parameters: $R_C = R_L = 4\ k\boldsymbol{\Omega}, r_\pi = 2.6\ k\boldsymbol{\Omega}, R_B = 200\ k\boldsymbol{\Omega}, C_\pi = 0.8\ pF, C_\mu = 0.05\ pF$, and $g_m = 38.5\ mA/V$. Determine the 3 dB frequency of the current gain with the effect of C_M.



Solution

The output current can be written as,

$$I_o = -(g_m V_\pi)\left(\frac{R_C}{R_C+R_L}\right)$$

The input voltage is, $V_\pi = I_s\left[R_B||r_\pi||\frac{1}{j\omega C_\pi}||\frac{1}{j\omega C_M}||\right] = I_s\left[\frac{R_B||r_\pi}{1+j\omega(R_B||r_\pi)(C_\pi+C_M)}\right]$

Therefore, the current gain is, $A_i = \frac{I_o}{I_s} = -g_m\left(\frac{R_C}{R_C+R_L}\right)\left[\frac{R_B||r_\pi}{1+j\omega(R_B||r_\pi)(C_\pi+C_M)}\right]$

The 3 dB frequency is, $f_{3\ dB} = \frac{1}{2\pi(R_B||r_\pi)(C_\pi+C_M)} = 13.2\ MHz$

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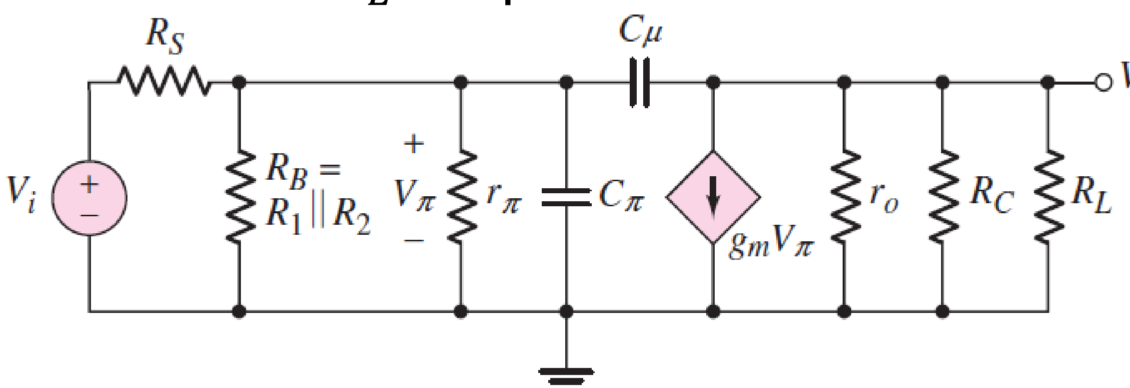
$C_M = C_\mu[1 + g_m(R_C||R_L)] = 3.9\ pF$

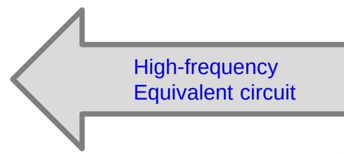
High-Frequency Response of Transistor Circuits

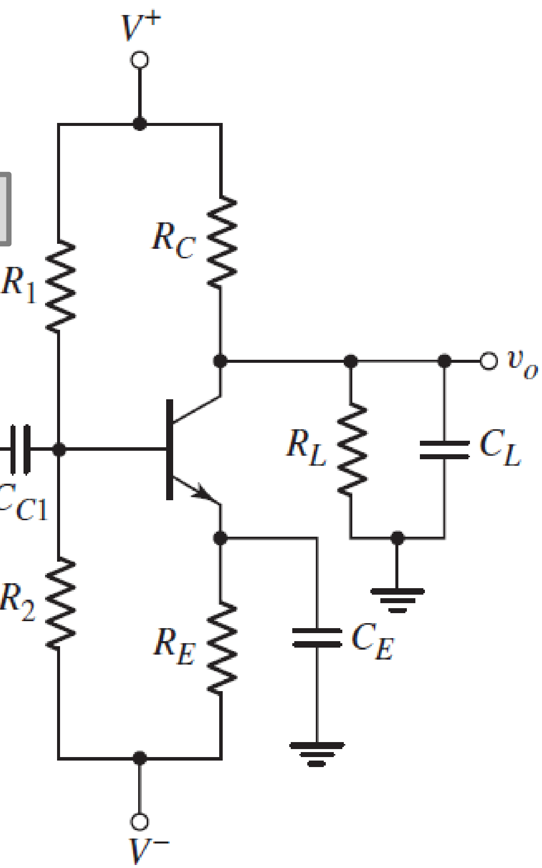
High-Frequency Response of CE Circuit

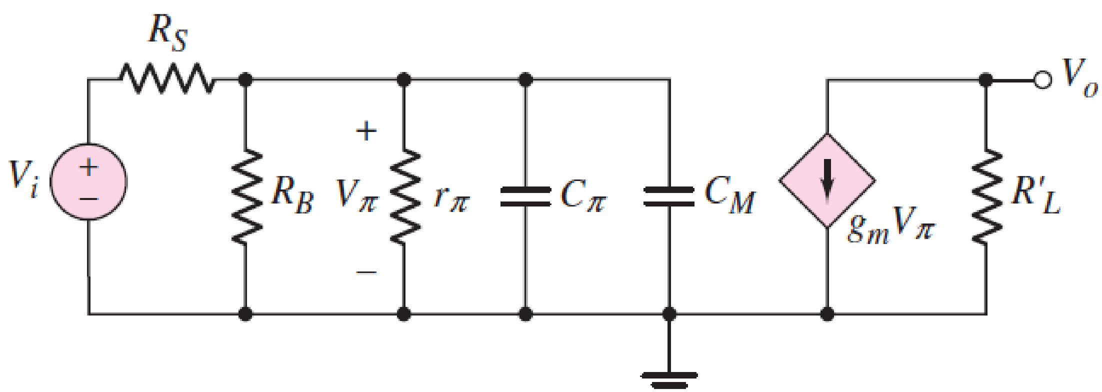
We have developed high-frequency equivalent circuits for bipolar transistors.

Look at the CE circuit, note that the transistor capacitances and load capacitance affect the high-frequency response. Assume C_C & C_E are short circuited and C_L is open-circuited.









High-frequency circuit, including Miller capacitance

CE configuration

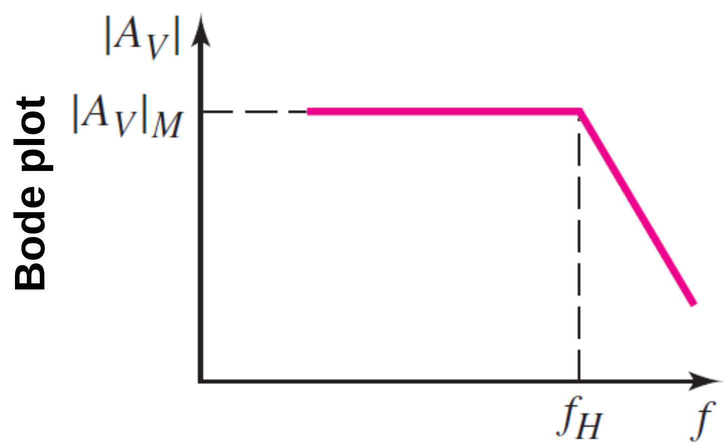
High-Frequency Response of CE Circuit

- ✓ From the previous analysis, $C_M = C_\mu[1 + g_m(r_o||R_C||R_L)]$.
- ✓ The upper 3 dB frequency using time constant technique, $f_H = \frac{1}{2\pi\tau_P}$

  Here, $\tau_P = R_{eq}C_{eq}$, where $C_{eq} = C_\pi + C_M$ and $R_{eq} = r_\pi||R_B||R_S$. Therefore,

  $$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi[r_\pi||R_B||R_S](C_\pi+C_M)}$$

- ✓ We can determine the *midband voltage gain* magnitude by assuming C_π & C_M are open circuits, $|A_v|_M = \left|\frac{V_o}{V_i}\right|_M = g_m(r_o||R_C||R_L)\left[\frac{R_B||r_\pi}{R_B||r_\pi+R_S}\right]$
- ✓ Bandwidth of CE circuit is reduced by Miller effect.



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High-Frequency Response of CE Circuit

Exercise−3: Consider the CE amplifier with the following parameters: $V^+ = 5\ V, V^- = -5\ V, R_S = 0.1\ k\boldsymbol{\Omega}, R_1 = 40\ k\boldsymbol{\Omega}, R_2 = 5.72\ k\boldsymbol{\Omega}, R_E = 0.5\ k\boldsymbol{\Omega}, R_C = 5\ k\boldsymbol{\Omega}$ & $R_L = 10\ k\boldsymbol{\Omega}$. The transistor parameters are: $\beta = 150, V_{BE(on)} = 0.7\ V, V_A = \infty, C_\pi = 35\ pF, C_\mu = 4\ pF$. Determine the upper corner frequency and midband gain of CE circuit.

Solution

From dc analysis, we can find $I_{CQ} = 1.03\ mA$. The small signal parameters are therefore, $g_m = 39.6\ mA/V$ and $r_\pi = 3.79\ k\boldsymbol{\Omega}$.

The Miller capacitance is, $C_M = C_\mu[1 + g_m(R_C||R_L)] = 532\ pF$

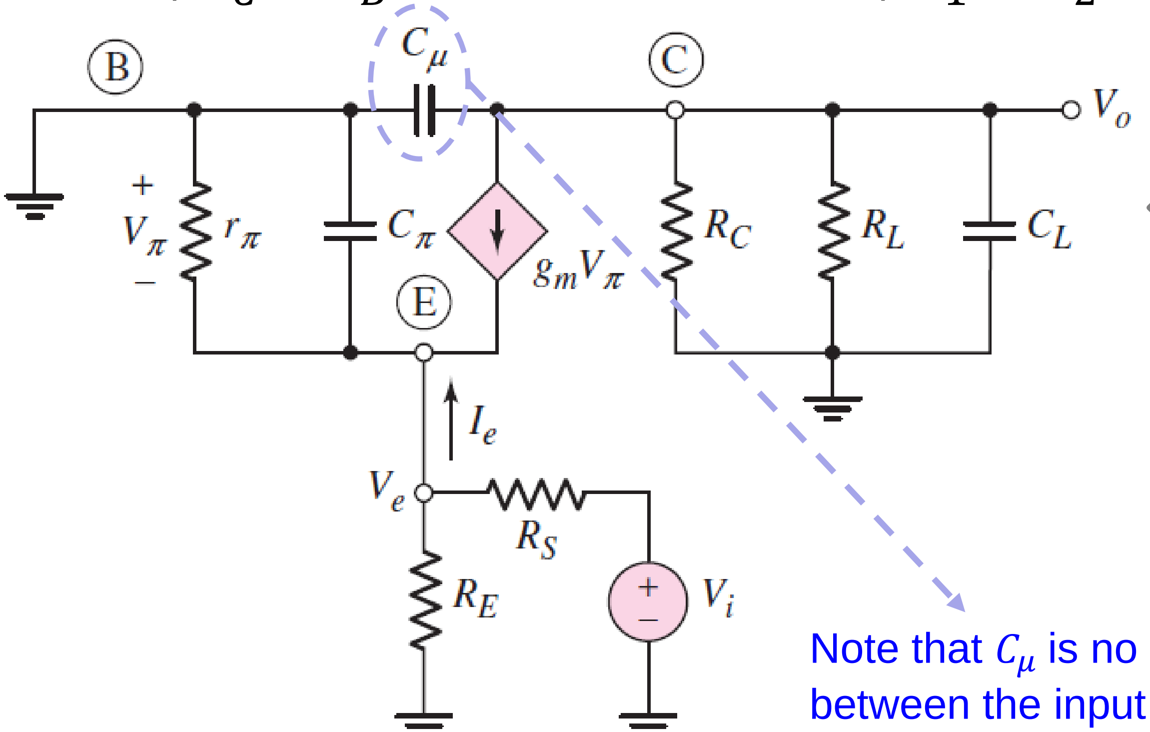
The upper 3 dB frequency is, $f_H = \frac{1}{2\pi[r_\pi||R_B||R_S](C_\pi+C_M)} = 2.94\ MHz$

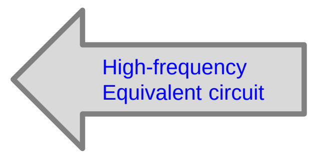
Therefore, the midband gain is, $|A_v|_M = g_m(r_o||R_C||R_L)\left[\frac{R_B||r_\pi}{R_B||r_\pi+R_S}\right] = 126$

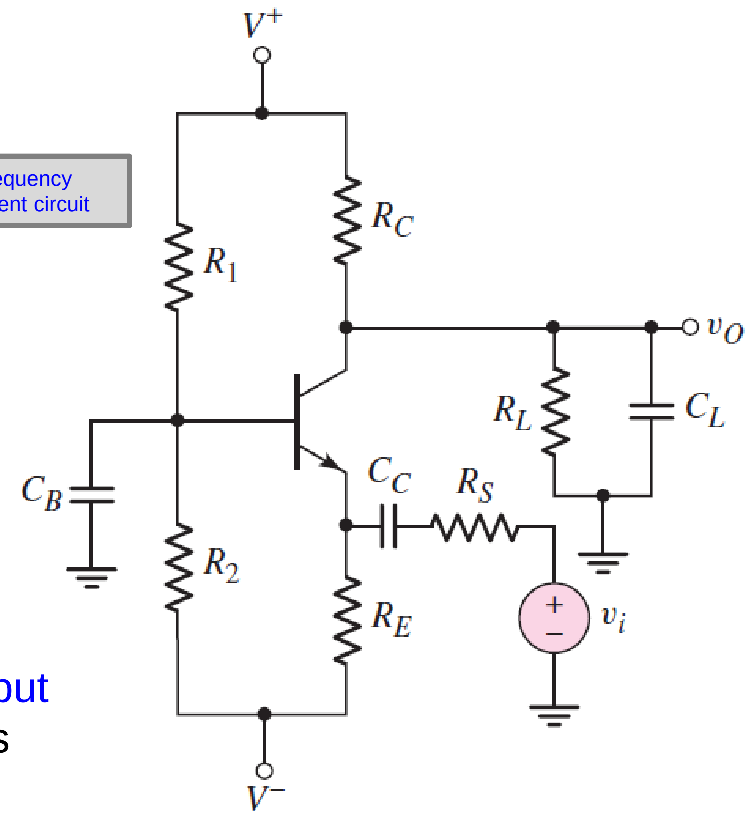
High-Frequency Response of CB Circuit

To increase bandwidth, the Miller effect must be minimized – *Common base*.

The CB circuit is same as earlier, except a bypass capacitor added to the base and the input is capacitively coupled to the emitter. In the equivalent circuit, C_C & C_B are short circuited, R_1 & R_2 short circuited, and $r_o \rightarrow \infty$.







Note that C_μ is no longer between the input and output terminals. One side of C_μ is tied to signal ground – *this eliminates Miller effect*.

CB configuration

High-Frequency Response of CB Circuit

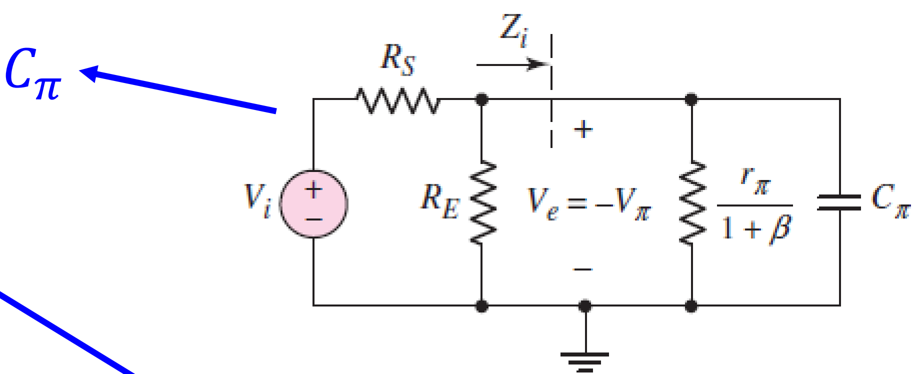
Write KCL at the emitter, $I_e + g_m V_\pi + \frac{V_\pi}{(1/sC_\pi)} + \frac{V_\pi}{r_\pi} = 0$

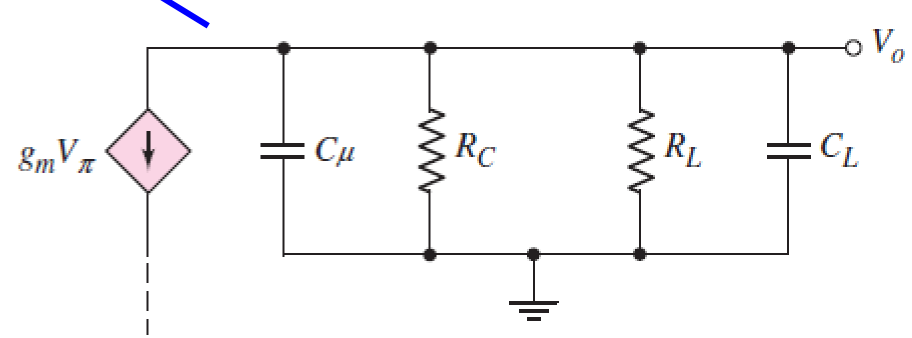
Since $V_\pi = -V_e \quad \rightarrow \quad \frac{I_e}{V_e} = \frac{1}{Z_i} = \frac{1}{r_\pi} + g_m + sC_\pi$, where Z_i is the impedance looking into the emitter. Therefore, $\frac{1}{Z_i} = \frac{1+r_\pi g_m}{r_\pi} + sC_\pi = \frac{1+\beta}{r_\pi} + sC_\pi$

$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}}$, where $\tau_{P\pi} = \left[\left(\frac{r_\pi}{1+\beta}\right)||R_E||R_S\right]C_\pi$

$f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}}$, where $\tau_{P\mu} = [|R_C||R_L]C_\mu$

The factor $\left(\frac{r_\pi}{1+\beta}\right)$ in time constant $\tau_{P\pi}$ is small, therefore, the two time constants are same order of magnitude.





Equivalent input and output portions

High-Frequency Response of CB Circuit

Exercise−4: Consider the CB amplifier with the following parameters: $V^+ = 5\ V, V^- = -5\ V, R_S = 0.1\ k\boldsymbol{\Omega}, R_1 = 40\ k\boldsymbol{\Omega}, R_2 = 5.72\ k\boldsymbol{\Omega}, R_E = 0.5\ k\boldsymbol{\Omega}, R_C = 5\ k\boldsymbol{\Omega}$ & $R_L = 10\ k\boldsymbol{\Omega}$. The transistor parameters are: $\beta = 150, V_{BE(on)} = 0.7\ V, V_A = \infty, C_\pi = 35\ pF, C_\mu = 4\ pF$. Determine the upper corner frequency and midband gain of CE circuit.

Solution

From dc analysis, we can find $I_{CQ} = 1.03\ mA$. The small signal parameters are therefore, $g_m = 39.6\ mA/V$ and $r_\pi = 3.79\ k\boldsymbol{\Omega}$.

The time constant associated with C_π is, $\tau_{P\pi} = \left[\left(\frac{r_\pi}{1+\beta}\right)||R_E||R_S\right] C_\pi = 0.675\ ns$

The upper 3 dB frequency corresponding to C_π is, $f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} = 236\ MHz$

High-Frequency Response of CB Circuit

The time constant associated with C_μ is, $\tau_{P\mu} = [|R_C||R_L]C_\mu = 13.33\, ns$

The upper 3 dB frequency corresponding to $C_\mu$is, $f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}} = 11.9\, MHz$

So in this case $f_{H\mu}$ is the dominant pole frequency.

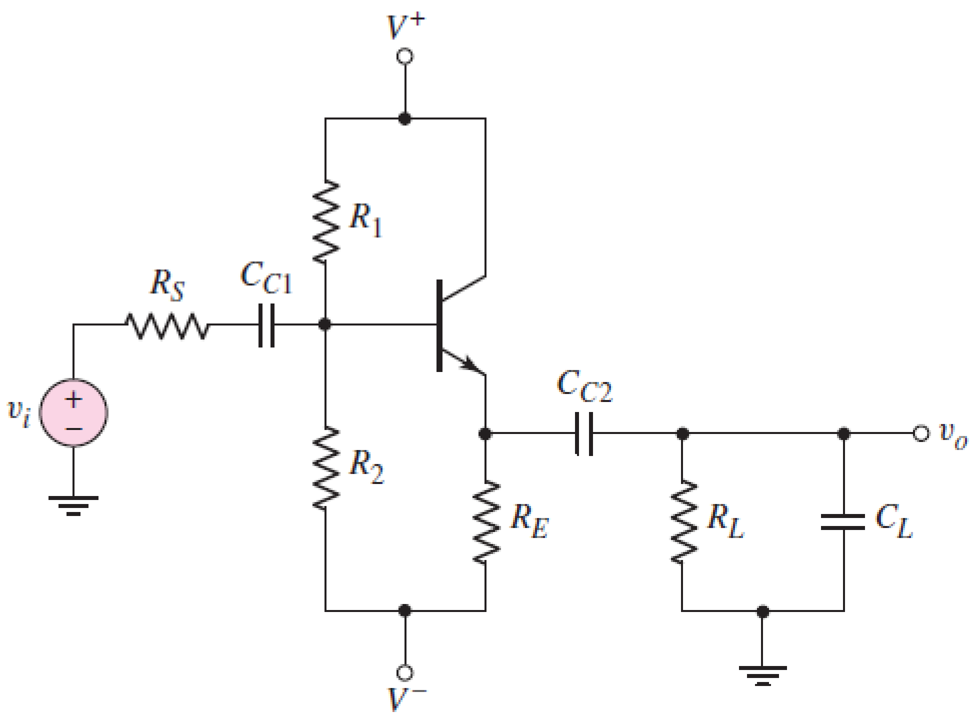
The magnitude of midband voltage gain is,

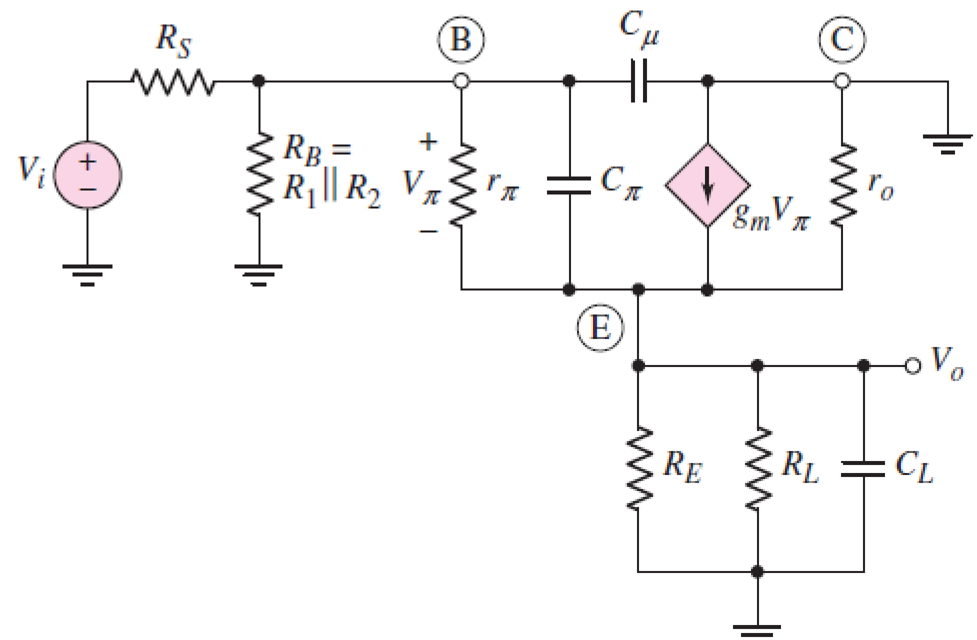
$$|A_v|_M = g_m(R_C||R_L)\left[\frac{R_E||\left(\frac{r_\pi}{1+\beta}\right)}{R_E||\left(\frac{r_\pi}{1+\beta}\right)+R_S}\right] = 25.5$$

The results of this example show that the bandwidth of the CB circuit is limited by the capacitance C_μ in the output portion of the circuit. The bandwidth of this particular circuit is 12 MHz, which is approximately a factor of four greater than the bandwidth of the CE circuit in the previous example.

High-Frequency Response of CC Circuit

Similar to the previous analyses! Since it is not very important, please try to understand the analysis by yourself!





CC configuration (Emitter follower)

Summary:-

- ✓ 3 frequency ranges: low ($f < f_L$), midband, and high ($f > f_H$).
- ✓ C_μ is reverse-biased junction capacitance (normally < C_π, however, *cannot be neglected due to Miller effect*).
- ✓ Shor-circuit current gain: $A_i = \frac{(g_m - j\omega C_\mu)}{\left[\frac{1}{r_\pi} + j\omega(C_\pi + C_\mu)\right]} \cong \frac{g_m r_\pi}{1 + j\omega r_\pi (C_\pi + C_\mu)}$
- ✓ Cutoff frequency: $f_T = \beta_o f_\beta = \beta_o \frac{1}{2\pi r_\pi (C_\pi + C_\mu)} = \frac{g_m}{2\pi (C_\pi + C_\mu)}$

  Frequency f_β is called the bandwidth of transistor.
- ✓ *Miller effect, or feedback effect*, is a multiplication effect of C_μ in circuit applications. Miller capacitance as, $C_M = C_\mu[1 + g_m(R_C || R_L)]$
- ✓ To increase bandwidth, the Miller effect must be *minimized* – *Common base*.

Thanks for your attention and cooperation!

The End