Uncontrolled rectifier (Single-phase)

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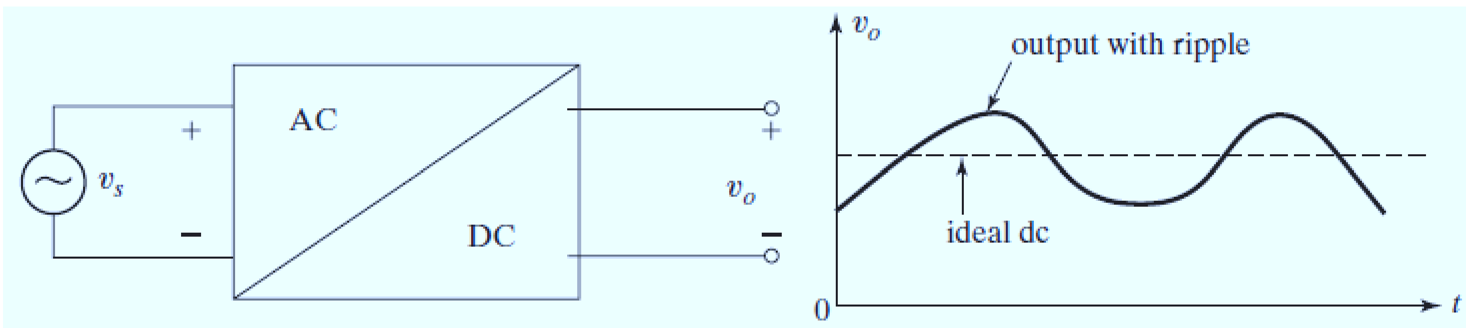
Before we start …

- What is "Rectifier"?
  - A rectifier is an electrical device that converts alternating current (AC), *current that periodically reverses direction*, to direct current (DC), *current that flows in only one direction*.



  - Rectifiers may be made of ***solid state diodes***, vacuum tube diodes, mercury arc valves, and other components.

Semiconductor devices

Most broadly used due to their good performance, small sizes, inexpensive price, and ease for integration

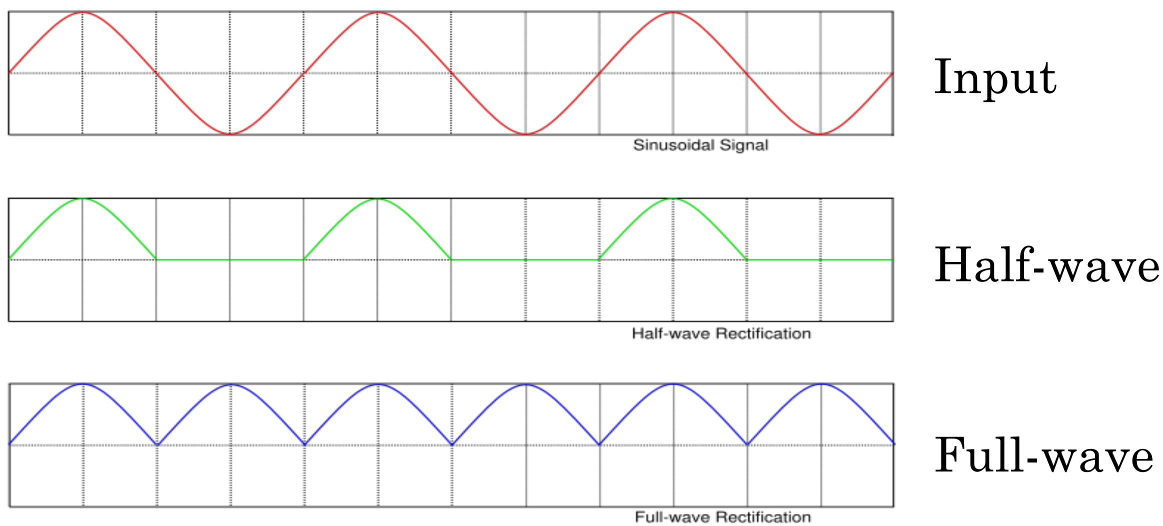
Classification of rectifiers

- Controllability
  - Uncontrolled ➡ Uncontrolled turn-ON & OFF (e.g., diode)
  - Semicontrolled ⇨ Controlled turn-ON and uncontrolled turn-OFF (e.g., SCR)
  - Controlled ⇨ Controlled turn-ON & OFF (GTO, BJT, MOSFET, IGBT)

- Input power (source)
  - Single phase
  - Three phase

Provides larger efficiency, small fluctuations, filtering of unwanted frequency is easier

- Waveform
  - Half-wave
  - Full-wave



Importance of rectifiers

- The primary application of rectifiers is to *derive DC power from an AC supply*.
  - Virtually all electronic devices require DC, so rectifiers find uses inside the power supplies of all electronic equipment;
  - Drive Electromechanical apparatus;
  - Electrolyte.
- Devices
  - Diodes (二极管) – not controllable;
  - Thyristors (晶闸管)
    - Silicon-controlled rectifiers (SCR): on-controllable
    - Gate Turn-off Thyristors (GTO) : on/off-controllable

Outline

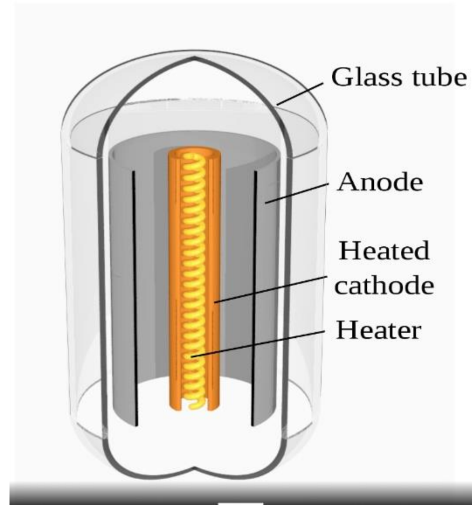
1. Diodes
   - Diodes and power diodes
   - Static & dynamic characteristics
   - Types of power diodes

2. Uncontrolled half-wave rectifier
   - Basic resistive loading
   - Parameters of the rectifier
   - RE load
   - RL load
   - RC load

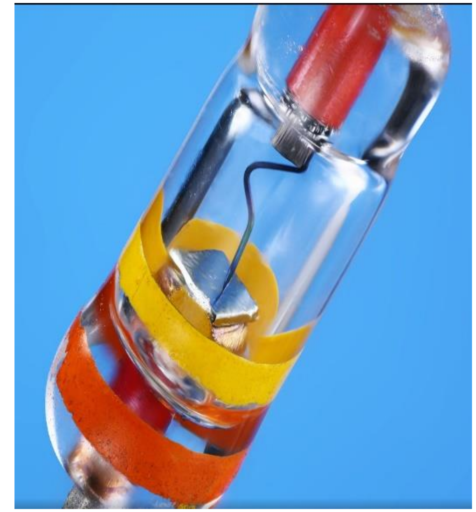
3. Uncontrolled full-wave rectifier
   - Resistive loading
   - RL load
   - RLE load

1.1 Diodes *(Recall EEE109, EEE112, and EEE211)*

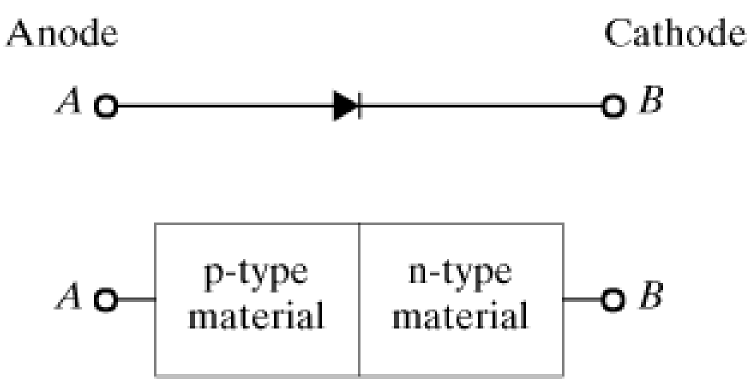
- A diode is a two-terminal, passive, non-linear device;
  - It is constituted by a P-N junction;
  - The P-type side is the anode, and the N-type side is the cathode.

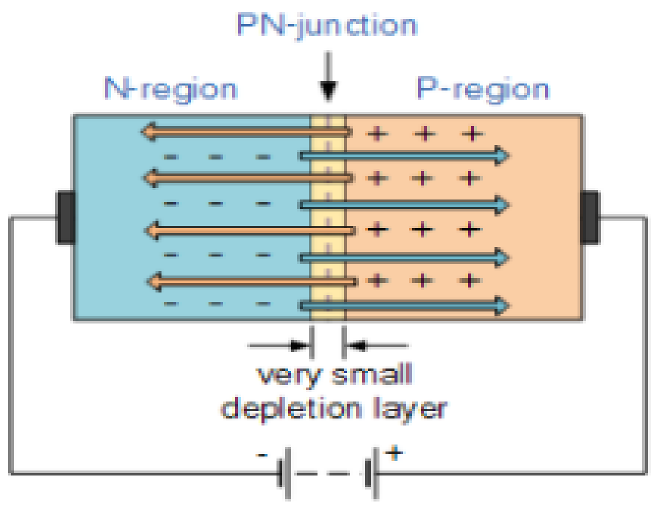


Vacuum tube diode



Point-contact diode

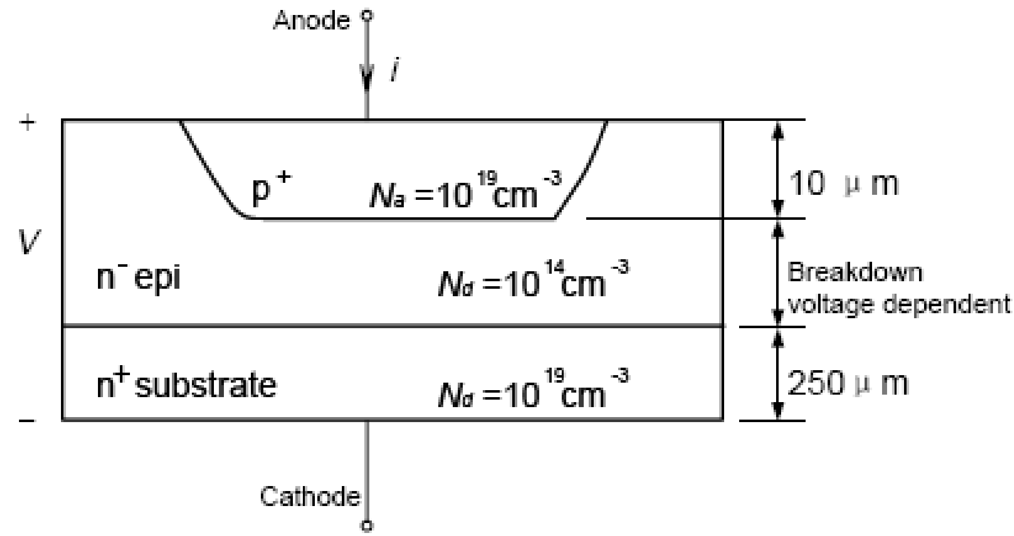


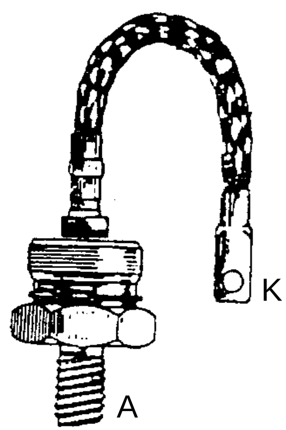


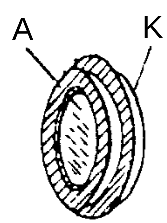
1.1 Power Diodes

- It has 3 layers instead of 2 layers (P+, N-, and N+)
- Features different from low-power (information electronic) diodes
  - Higher voltage / current withstand
  - Low leakage current
  - Low conduction loss

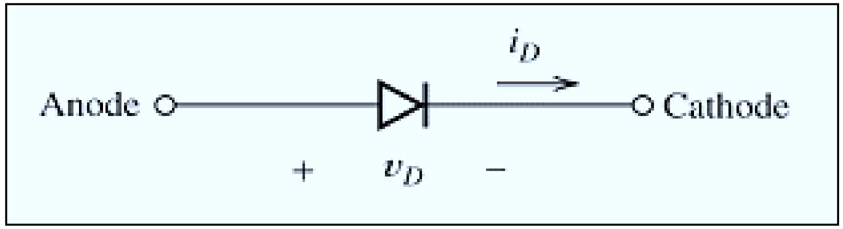
  - Larger size
  - Low speed







1.2 Static characteristics (I-V Relation)



- i_D = current through diode, from anode (P region) to cathode (N region)
- v_D = bias voltage
  – negative for reverse bias
  – positive for forward bias
- The Shockley ideal diode equation:

$$i_D = I_s\left[e^{\frac{v_D}{\eta \cdot v_T}} - 1\right]$$

Reverse saturation current
10^{-18} to 10^{-12} A

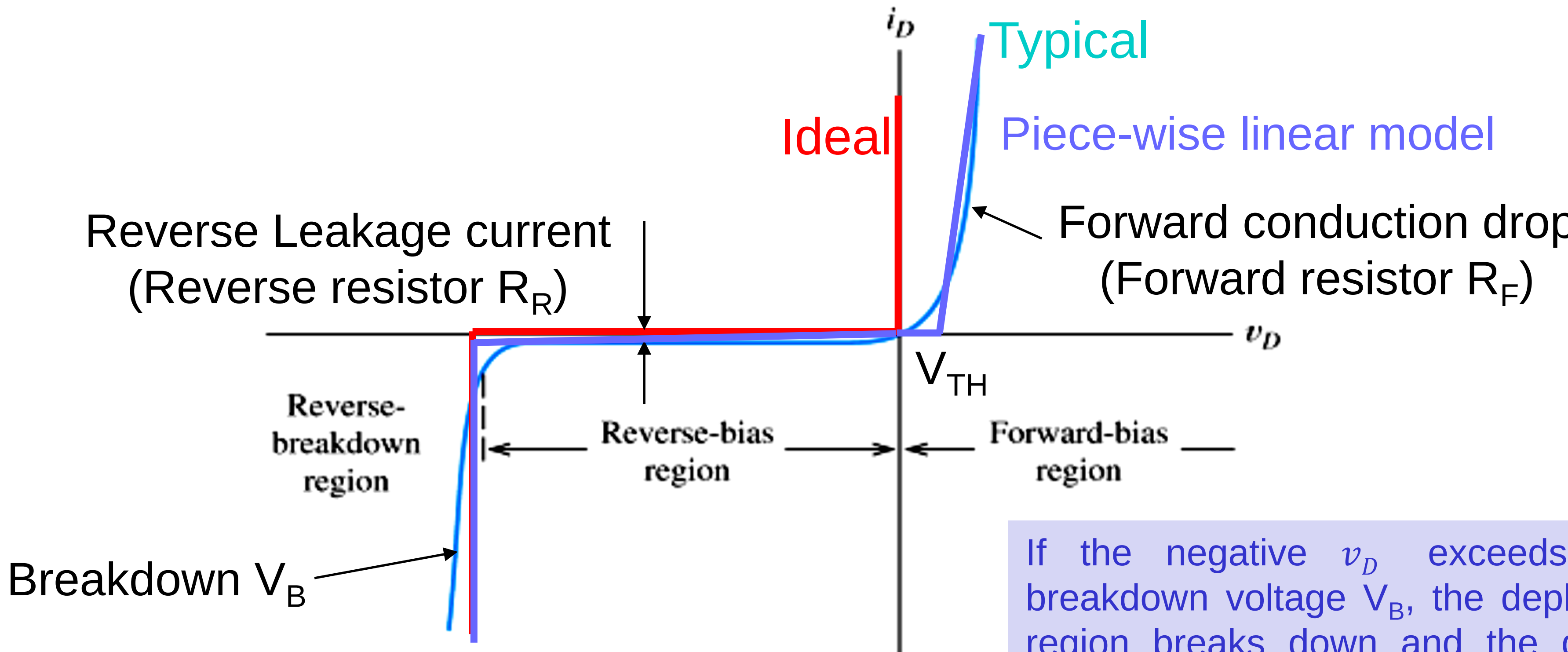
Emission coefficient
Ranging 1~2

Thermal equivalent voltage
$v_T = kT/q$

Boltzmann's constant

Electron charge

1.2 Static characteristics (I-V Relation)



If the negative v_D exceeds the breakdown voltage V_B, the depletion region breaks down and the diode performs like a conductor, allowing electrons flow through it without any resistance.

Requirement of diodes to have better performance:

– Forward resistor R_F ⬇
– Threshold voltage V_{TH} (0.7V for Si, 0.3V for Ge)
– Breakdown voltage V_B ⬆
– Reverse resistor R_R ⬆

1.2 I-V Relation

- Forward-biased region: a conductor (ON)
  - In forward bias, a diode has a threshold voltage V_{TH} (门槛电压), at which current flow becomes significant and increases exponentially: $V_{TH} = 0.3V$ for Ge, 0.7V for Si.

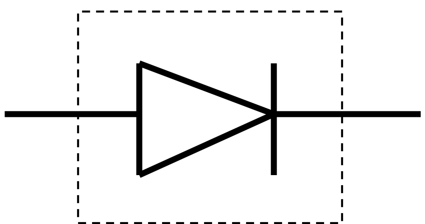
- Reverse-biased region: an insulator (OFF)
  - In reverse bias, there is a negligible reverse saturation current I_s.

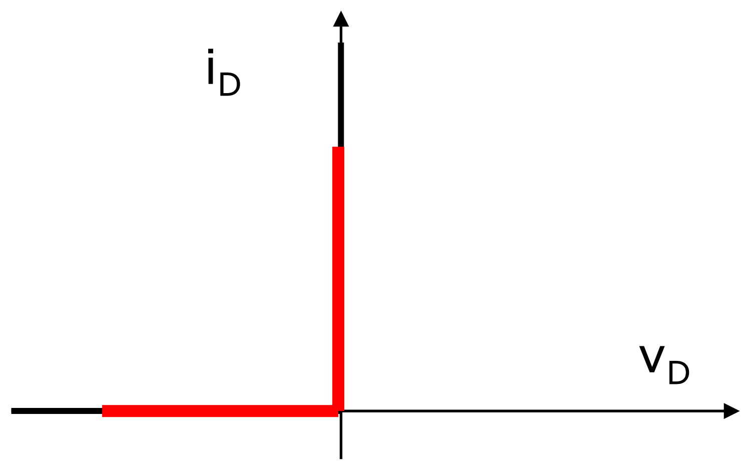
- Breakdown region:
  - If reverse bias is increased sufficiently to a breakdown voltage V_B, there is a sudden increase in reverse current, which is called breakdown.

Equivalent Circuit Representation I

- The ideal diode model
  - A perfect conductor or short circuited, if forward biased;
  - A perfect insulator or open circuited, if reverse biased.

<u>The Ideal Diode Model</u>

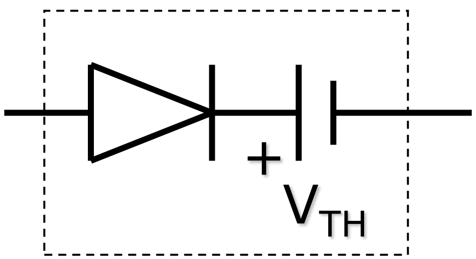


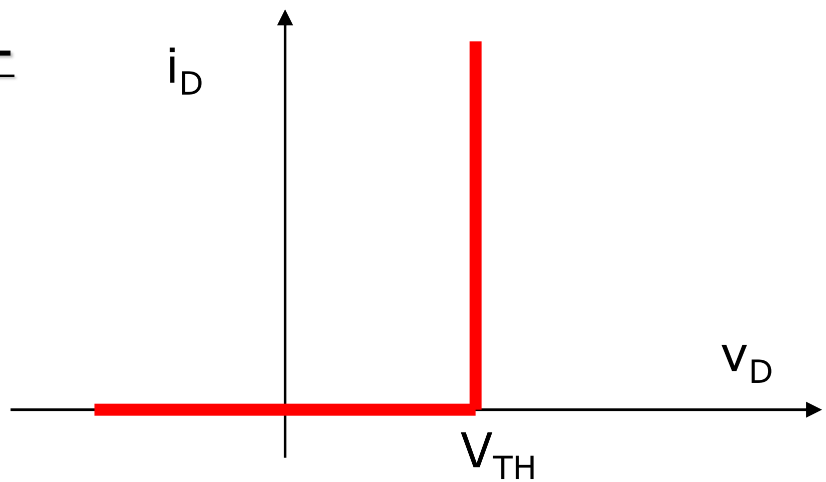


Equivalent Circuit Representation II

- The Constant-Voltage-Drop Model
  - A *modified model* taken the *threshold voltage* into consideration
  - A perfect conductor or short circuited, if forward bias voltage exceed the threshold voltage V_{TH}
  - A perfect insulator or open circuited, if reverse biased.

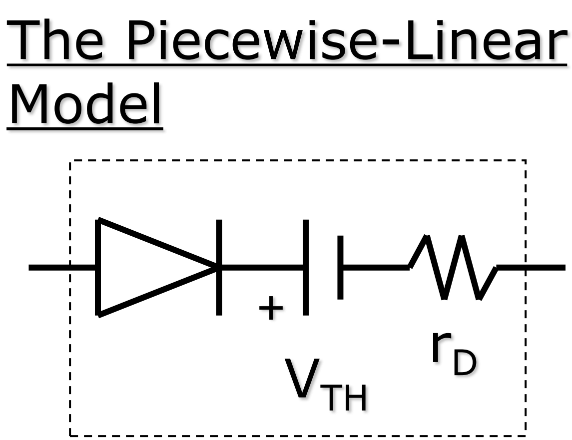
<u>The Constant-Voltage-Drop Model</u>

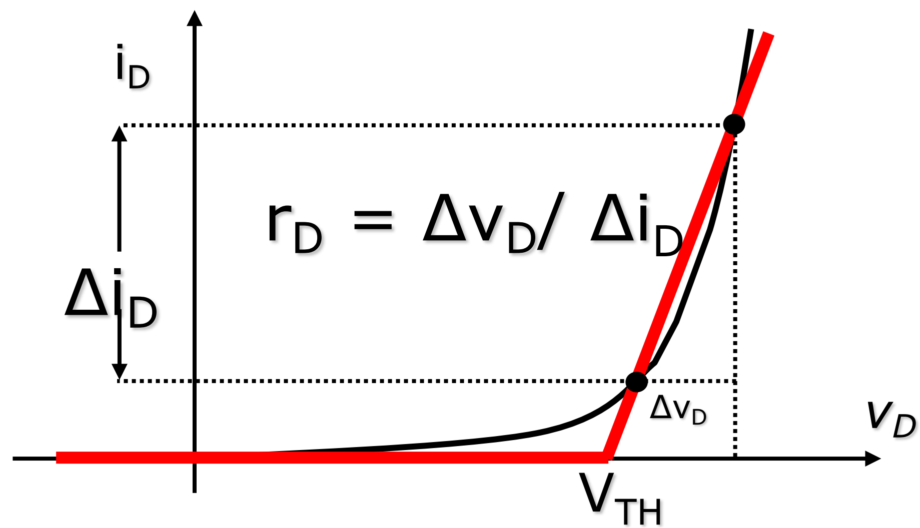




Equivalent Circuit Representation III

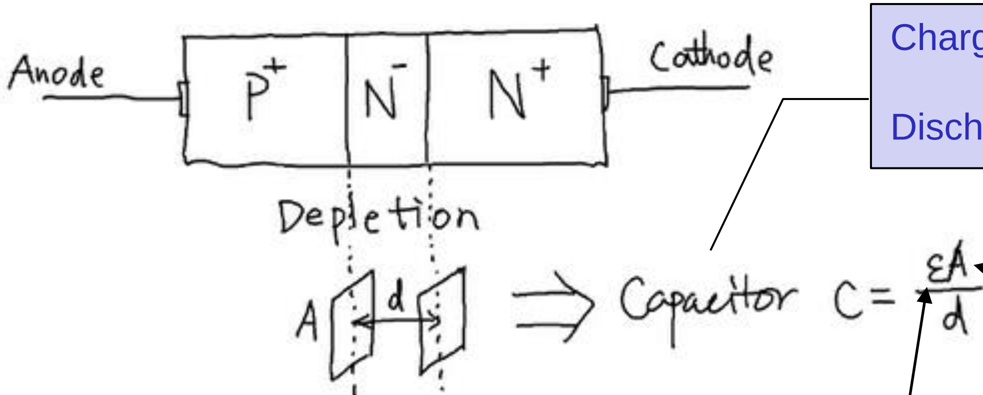
- The Piecewise-Linear Model
  - A resistive conductor, if forward bias voltage exceeds the threshold voltage V_{TH}
  - A perfect insulator or open circuited, if reverse biased.





1.3 Junction capacitor (结电容)

- The positive and negative charge in the depletion region is variable with the changing of external voltage, *creating C_J*
  - Junction capacitor C_J



Charging when it is forward-biased

Discharging when reverse-biased

Cross-section area of PN junction

Absolute permittivity

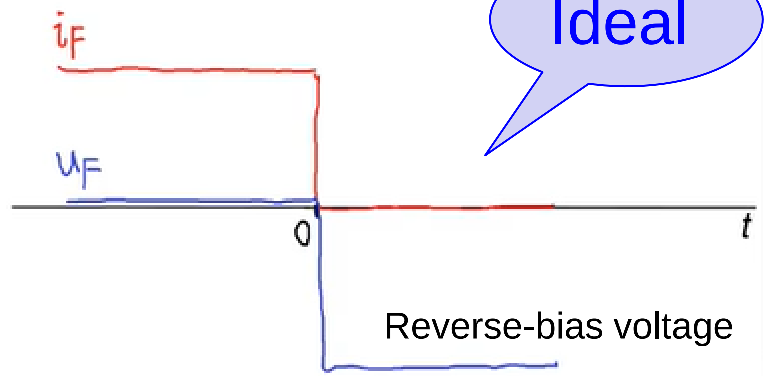
    - Potential barrier capacitor C_B (势垒电容)
    - Diffusion capacitor C_D (扩散电容)

- Junction capacitor influences the switching characteristics of diodes, especially the switching frequency.

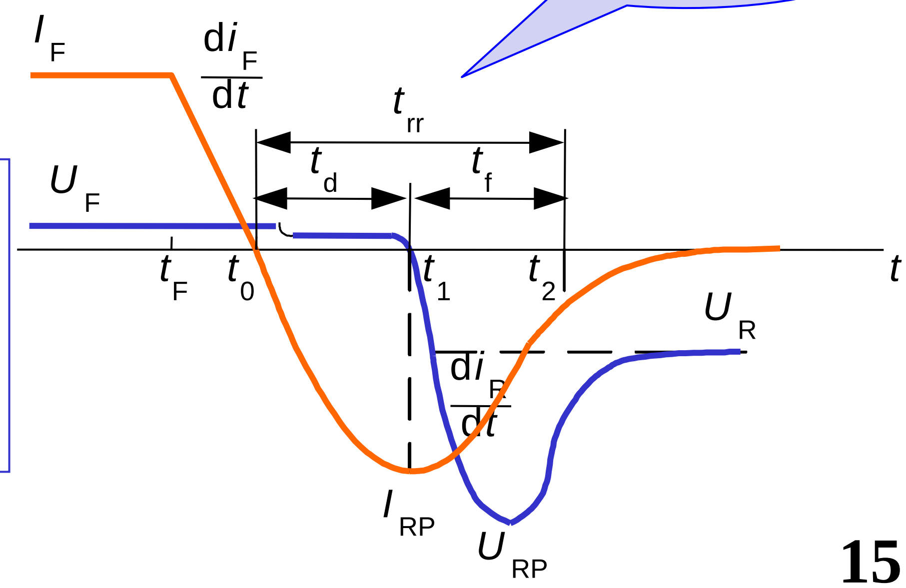
1.3 Switching (dynamic) characteristics I

- Try to turn-OFF diode – which was originally ON, with high forward current i_F & low voltage drop U_F.
- Ideally, we want the *current change* from a steady conduction to unconduction *instantaneously*, and the voltage on the diode also jumps from 0 to the reverse–bias voltage *without any delay*.
- Practically, current in forward-biased junction diode is due to net effect of majority and minority carriers. The *diode continues to conduct due to minority carriers* that remain stored in the PN-junction, and results *reverse-recovery time (t_{rr}).*

Ideal



Practical

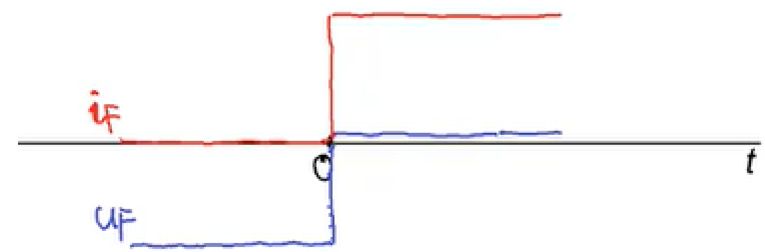


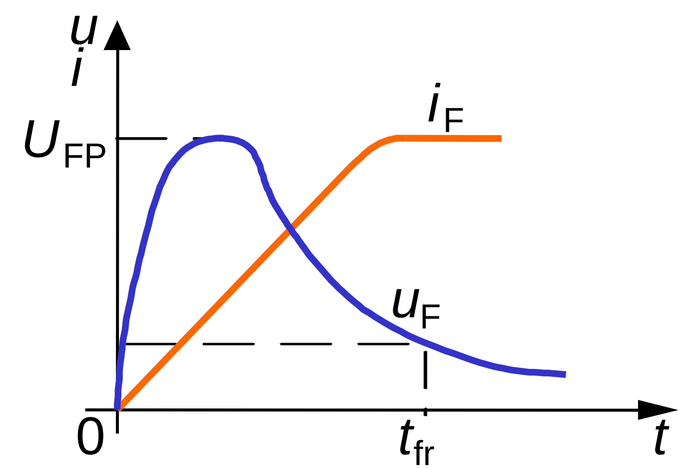
- Reverse recovery time： $t_{rr} = t_d + t_f$
- Delay time, $t_d = t_1 - t_0$; falling time, $t_f = t_2 - t_1$
- Soft factor： $S_r = t_f / t_d$
- Peak reverse recovery current: $I_{RP} = t_d \dfrac{di_F}{dt}$

1.3 Switching (dynamic) characteristics II

- Turn-on transient – forward-recovery process:

> - Forward voltage drop will firstly peak at U_{FP}, then gradually reduce to some value close to steady state voltage drop.
> - Forward recovery time t_{fr}
> - If the rate of rise of the forward current is higher, the U_{FP} is higher.





- Summary
  - Overshot voltage U_{FP};
  - Forward recovery time t_{fr}.

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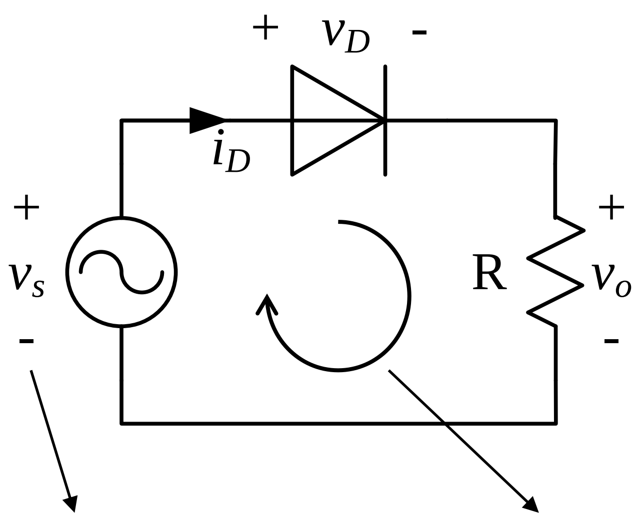
1.4 Types of power diodes

- General purpose diode (rectifier diode):
  - Standard recovery (usually have higher voltage & current withstand
- Fast recovery diode (FRD)
  - Reverse recovery time and charge specified. t_{rr} is usually less than 1μs, for many less than 100 ns —— ultra-fast recovery diode.
- Schottky diode (Schottky barrier diode-SBD)
  - Allows higher switching speeds & better system efficiency
  - Essentially no recovered overshot voltage, and lower forward voltage drop.
  - Restricted to low voltage (less than 200V)

2.1 Single phase uncontrolled half-wave rectifier

- A half-wave rectifier is the simplest rectifier.
  - It consists of only one diode.
  - It is not normally used in industrial applications, but it is very useful in understanding the principle of rectification.

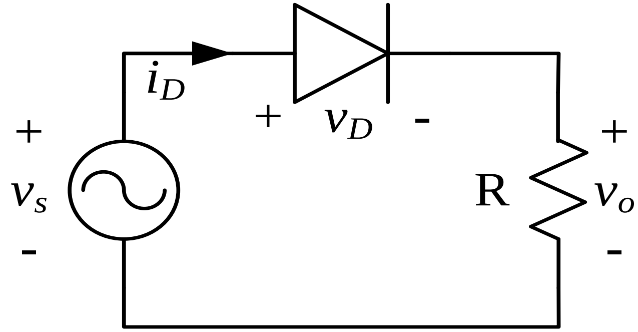


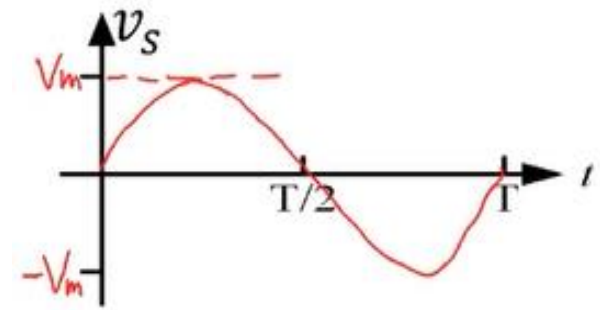
$$v_s = V_m sin\omega t$$

$$v_s = v_D + v_o$$

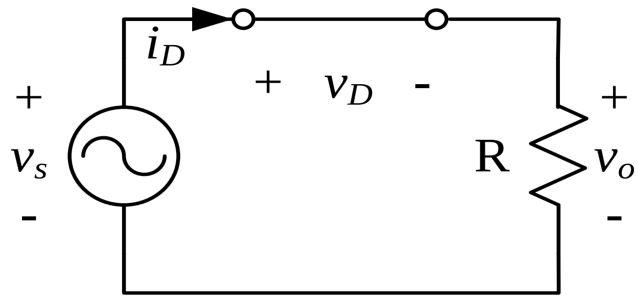
The output voltage and current of this rectifier are strongly influenced by the type of the load.

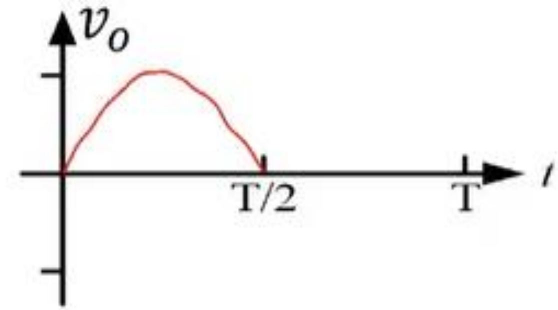
1. Ideal diode model



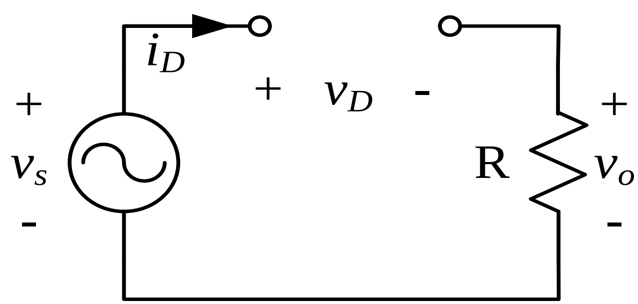


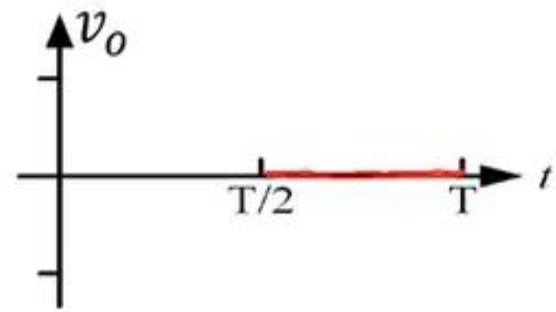
- First half $v_s > 0$:



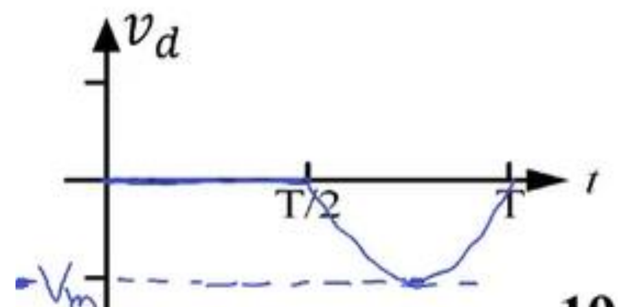


- Second half $v_s < 0$:

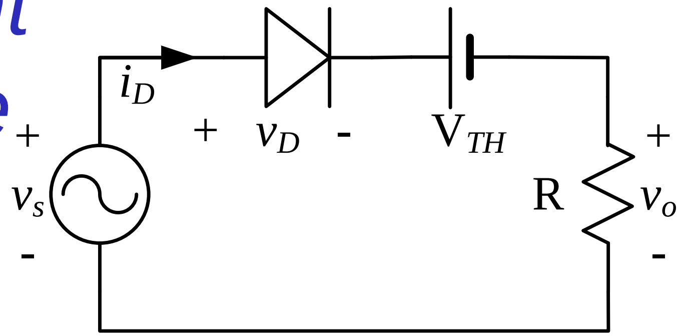


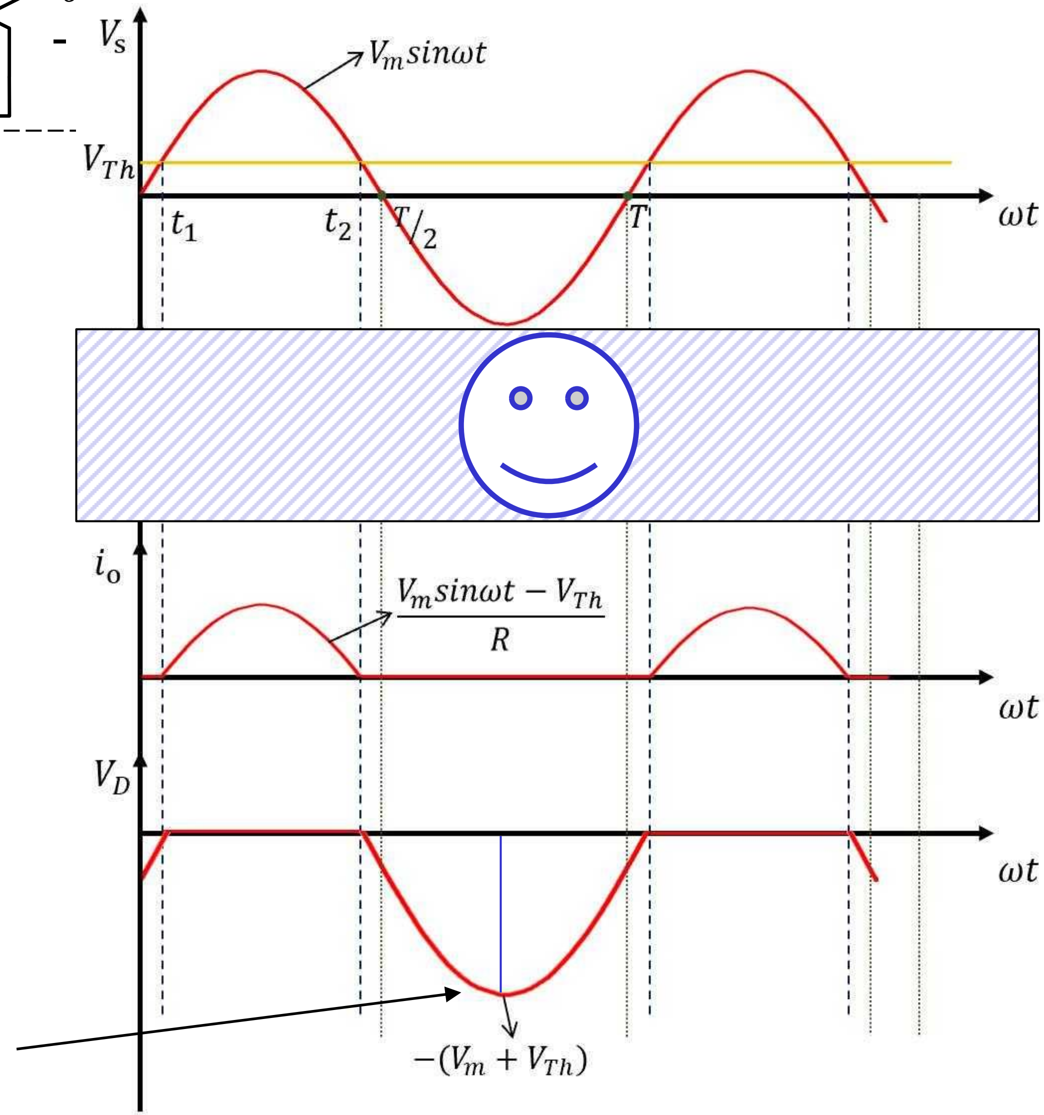


- Voltage on the diode v_d:

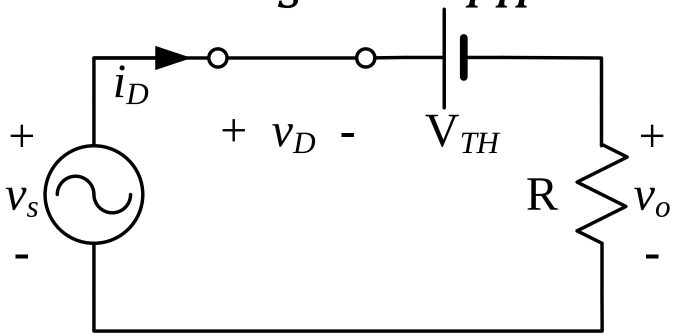


2. Constant voltage-drop model

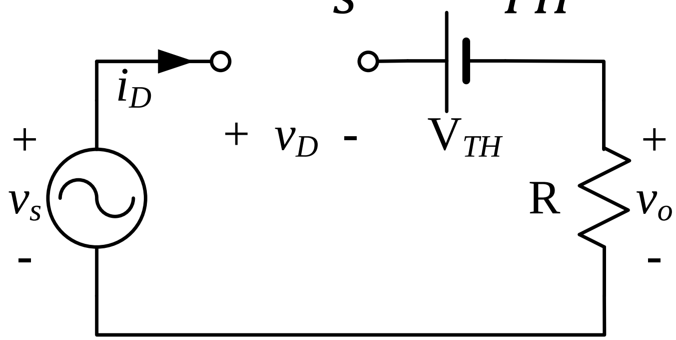




- First half $v_s > V_{TH}$:

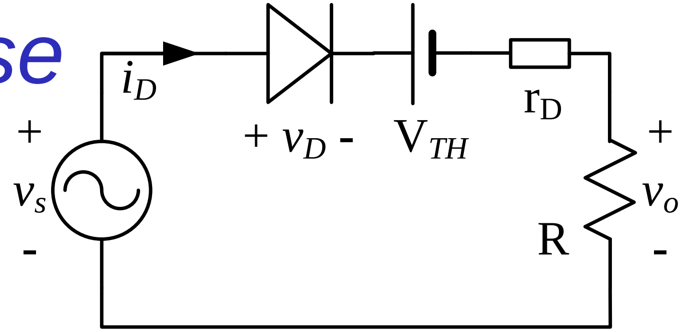


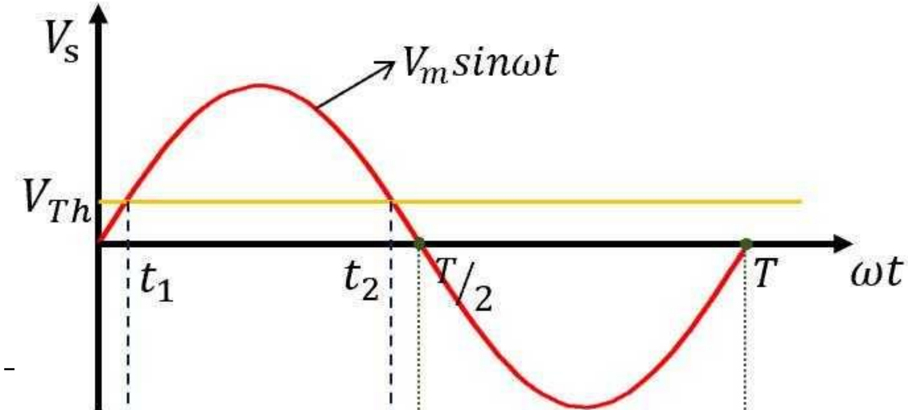
- Second half $v_s < V_{TH}$:



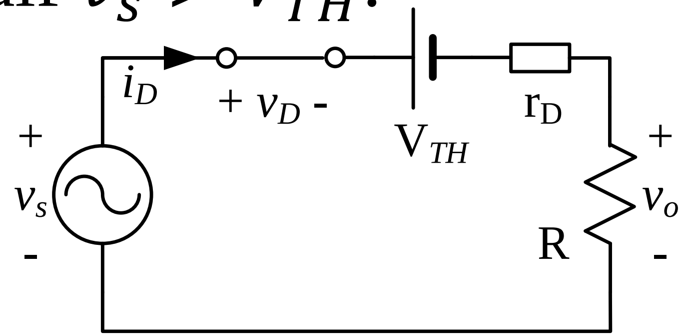
PIV= $-V_m - V_{TH} = -(V_m + V_{TH})$

3. Piecewise linear model

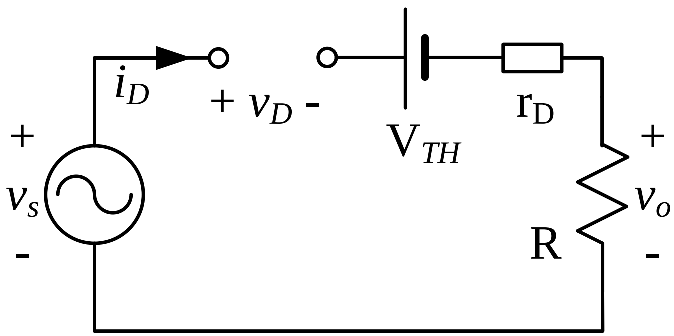




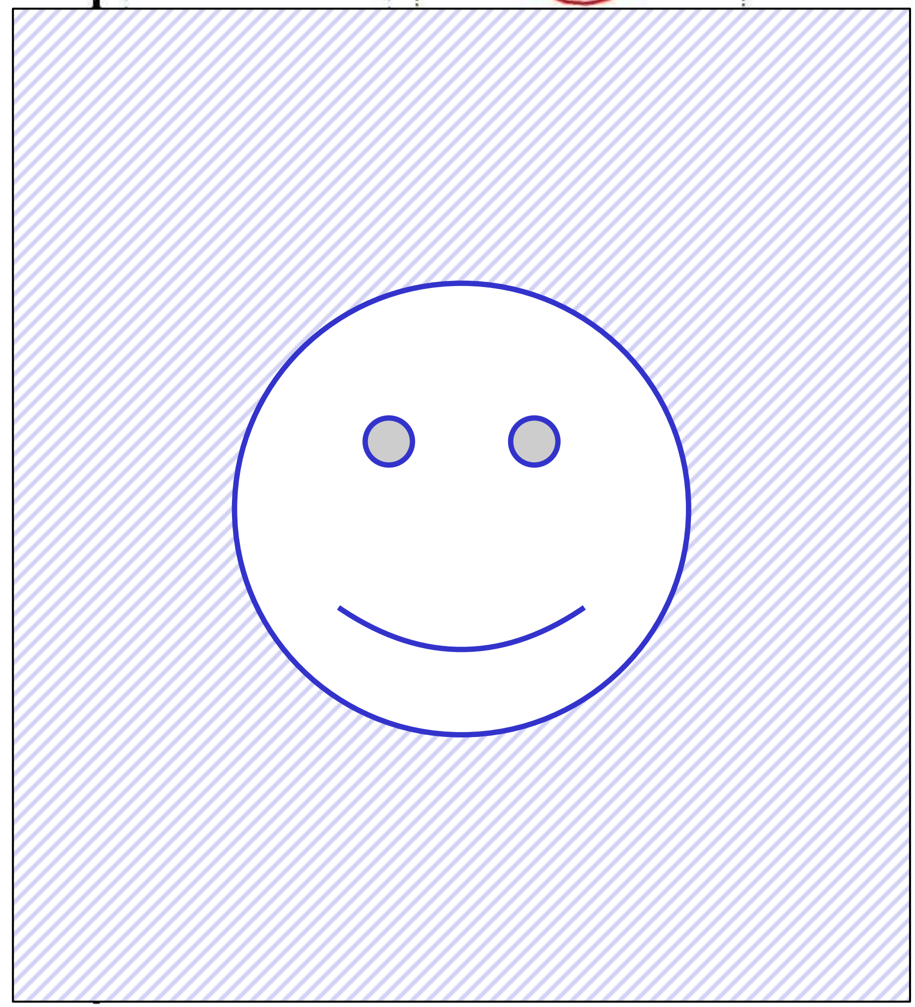
- First half $v_s > V_{TH}$:



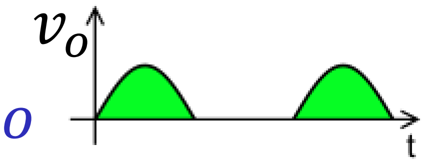
- Second half $v_s < V_{TH}$:



- Voltage on the diode v_D:



About output voltage, v_o



- v_o is a DC signal, always positive (it's not the flat constant DC we knew!!!).
- Discontinuous, which means discontinuous power / energy.
- A periodic waveform can be described with *Fourier series* as,

$$\boxed{v_o = V_o + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)}$$

$V_o = \frac{a_o}{2} \rightarrow$ average value of v_o

Appendix E, M. H. Rashid

Page. 45, D. W. Hart

$$V_o = \frac{1}{T}\int_0^T v_o(t)dt = \frac{1}{T}\int_0^{T/2} V_m \sin \omega t dt = \frac{V_m}{\pi}$$

$$a_n = \frac{2}{T}\int_0^T v_o(t)\cos n\,\omega t dt = \frac{2}{T}\int_0^{T/2} V_m \sin \omega\, t \cos n\,\omega t dt = \begin{cases} \frac{2V_m}{\pi(1-n^2)} & n = 2,4,6\ldots \\ 0 & n = 1,3,5\ldots \end{cases}$$

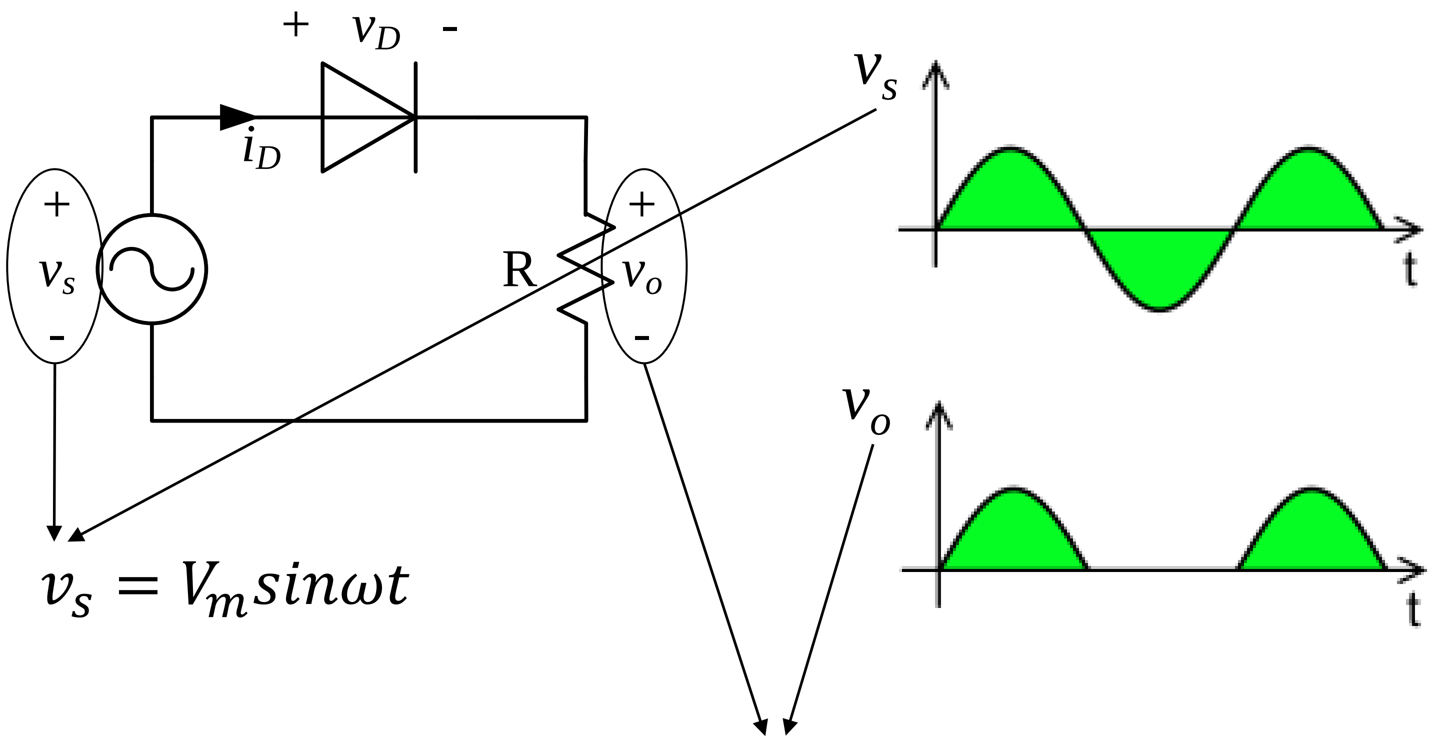
$$b_n = \frac{2}{T}\int_0^T v_o(t)\sin n\,\omega t dt = \frac{2}{T}\int_0^{T/2} V_m \sin \omega\, t \sin n\,\omega t dt = \begin{cases} \frac{V_m}{2} & n = 1 \\ 0 & n = 2,3,4\ldots \end{cases}$$

$$\Rightarrow \boxed{v_o = \frac{V_m}{\pi} + \frac{V_m}{2}\sin \omega t - \frac{2V_m}{3\pi}\cos 2\omega t - \frac{2V_m}{15\pi}\cos 4\omega t - \ldots\ldots}$$

$T = 2\pi$

2.2 Evaluation parameters of an example

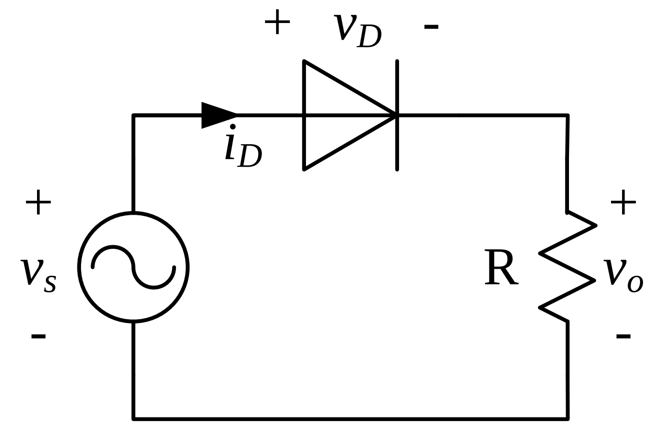
- Calculate all the parameters for the shown circuit.



$$v_s = V_m sin\omega t$$

$$v_o = \frac{V_m}{\pi} + \frac{V_m}{2} sin\omega t - \frac{2V_m}{3\pi} cos2\omega t - \frac{2V_m}{15\pi} cos4\omega t - \cdots$$

2.2 Parameters of rectifiers I



- The average (DC) output voltage

$$V_0 = \frac{1}{T}\int_0^T v_0(t)dt$$

$$V_0 = \frac{1}{T}\int_0^{T/2} V_m sin\omega t dt = \frac{V_m}{\pi}$$
$$\approx 0.318V_m$$

- The average (DC) output current

$$I_0 = \frac{1}{T}\int_0^T i_0(t)dt$$

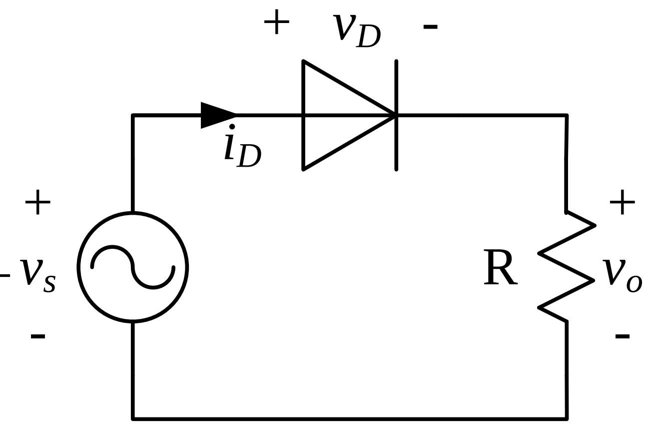
$$I_0 = \frac{V_0}{R} = \frac{V_m}{\pi R} \approx \frac{0.318V_m}{R}$$

- The output DC power

$$P_0 = V_0 I_0$$

$$P_0 = V_0 I_0 = \frac{V_m^2}{\pi^2 R} \approx \frac{0.101V_m^2}{R}$$

2.2 Parameters of rectifiers II



$$V_S = \frac{V_m}{\sqrt{2}}$$

- The RMS/effective output voltage

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v_o^2(t)dt}$$

- The RMS output current

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i_o^2(t)dt}$$

- The output AC power

$$P_{RMS} = V_{RMS} I_{RMS}$$

- The input AC power

$$P_S = V_S I_S$$

(V_S is effective value of source) – Try to Check

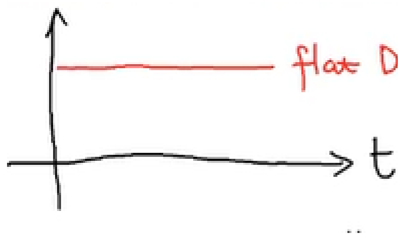
$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^{T/2} (V_m sin\omega t)^2 dt} = 0.5V_m$$

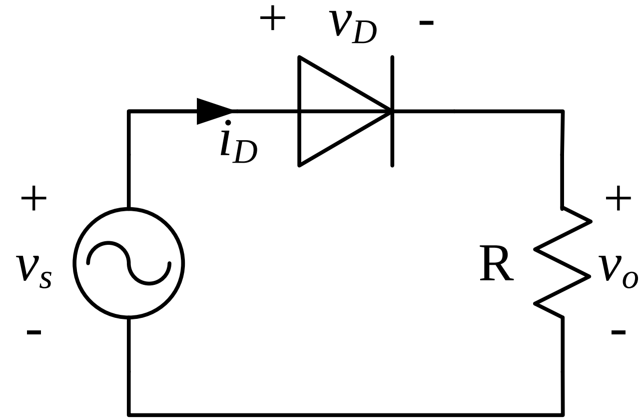
$$I_{RMS} = \frac{V_{RMS}}{R} = \frac{V_m}{2R} = \frac{0.5V_m}{R}$$

$$P_{RMS} = V_{RMS} I_{RMS} = \frac{V_m^2}{2^2 R} = \frac{0.25V_m^2}{R}$$

$$P_S = \frac{V_m}{\sqrt{2}} \times I_{RMS} = \frac{V_m^2}{2\sqrt{2}R} = \frac{0.439V_m^2}{R}$$

2.2 Parameters of rectifiers III





- Rectification efficiency (figure of merit)

$$\eta = \frac{P_0}{P_{RMS}} \qquad \eta = \frac{V_0^2/R}{V_{RMS}^2/R} = \frac{V_0^2}{V_{RMS}^2} = \left(\frac{0.318}{0.5}\right)^2 \approx 40.5\%$$

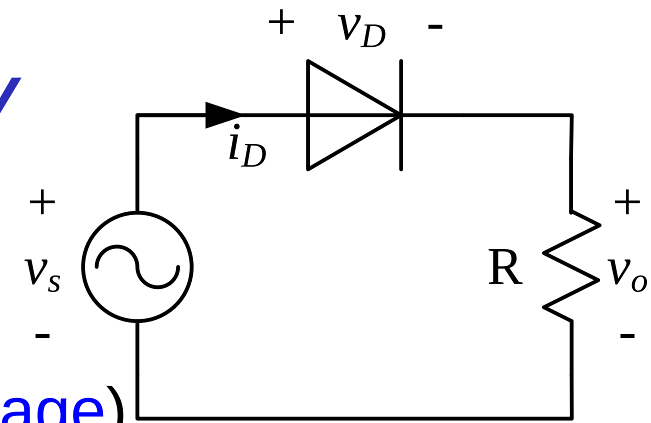
- The transformer utilization factor (i.e. power conversion efficiency)

$$f_T = \frac{P_0}{P_S} \qquad f_T = \frac{V_0 I_0}{V_S I_S} = \frac{0.101 V_m^2/R}{0.439 V_m^2/R} = 0.287$$

- The effective (RMS) value of the AC component of the output voltage

$$V_{ac} = \sqrt{V_{RMS}^2 - V_0^2} \qquad V_{ac} = V_m\sqrt{0.5^2 - 0.318^2} = 0.386 V_m$$

2.2 Parameters of rectifiers IV



- Form factor (measure of shape of output voltage)

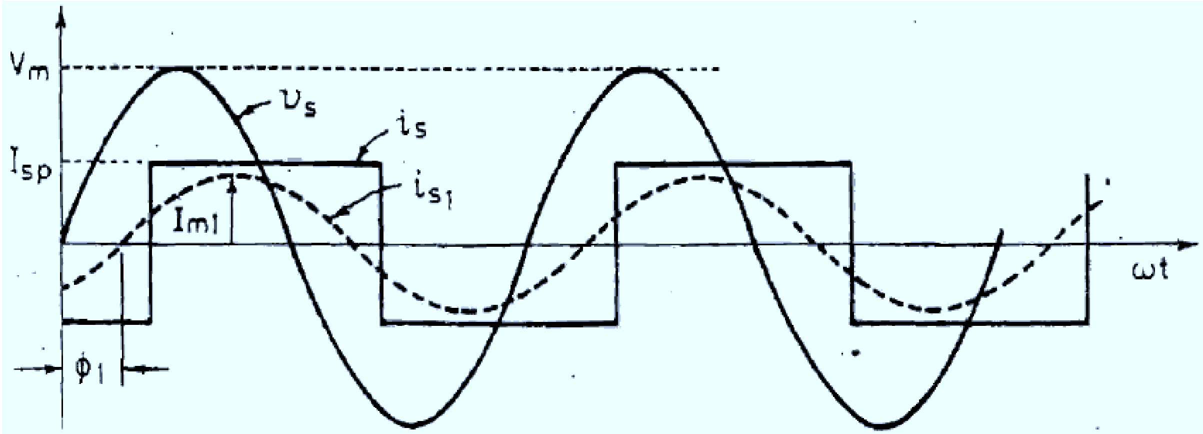
$$f_F = \frac{V_{RMS}}{V_0} \qquad f_F = \frac{V_{RMS}}{V_0} = \frac{0.5}{0.318} = 1.57$$

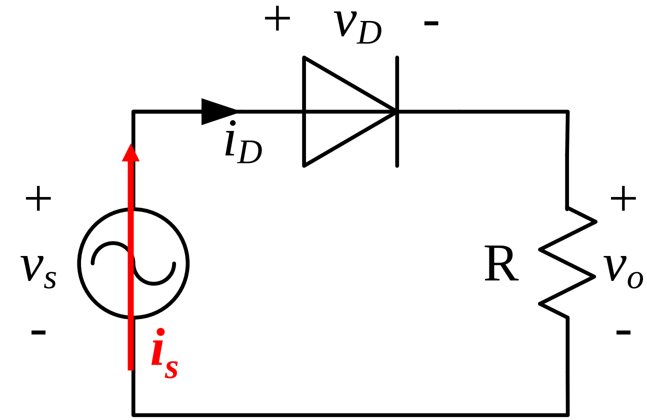
- Ripple factor (measure of the ripple content)

$$f_R = \frac{V_{ac}}{V_0} = \sqrt{f_F^2 - 1} \qquad f_R = \frac{V_{ac}}{V_0} = \sqrt{f_F^2 - 1} = 1.21$$

- Crest factor (measure of the peak ripple)

$$f_c = \frac{I_{s(peak)}}{I_{RMS}} \qquad f_c = \frac{I_{s(peak)}}{I_{RMS}} = \frac{V_m/R}{0.5V_m/R} = 2$$





- The total harmonic distortion (THD) of input current

$$THD = \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1} \times 100\% \qquad THD = \sqrt{\left(\frac{0.5V_m/R}{0.5V_m/\sqrt{2}R}\right)^2 - 1} = 1 = 100\%$$

  – where I_s and I_{s1} are the effective values of the *total* and *fundamental frequency* of the input current i_s.

- Input power factor

$$f_P = \frac{\text{real power}}{\text{apparent power}}$$

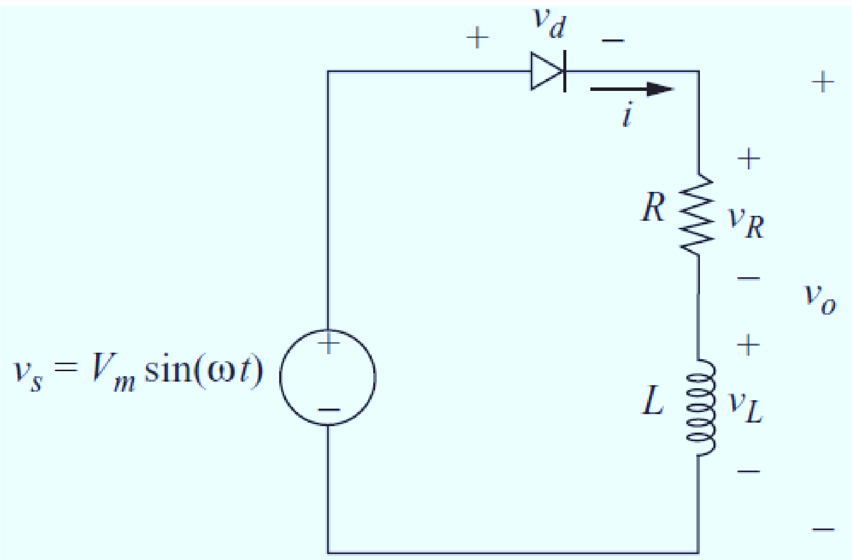
Displacement factor (DF)

$$= \frac{V_s I_{s1}}{V_s I_s} cos\phi = \frac{I_{s1}}{I_s} cos\phi_1 \qquad f_P = \frac{I_{s1}}{I_s} cos\phi = \frac{0.5V_m/\sqrt{2}R}{0.5V_m/R} = 0.707$$

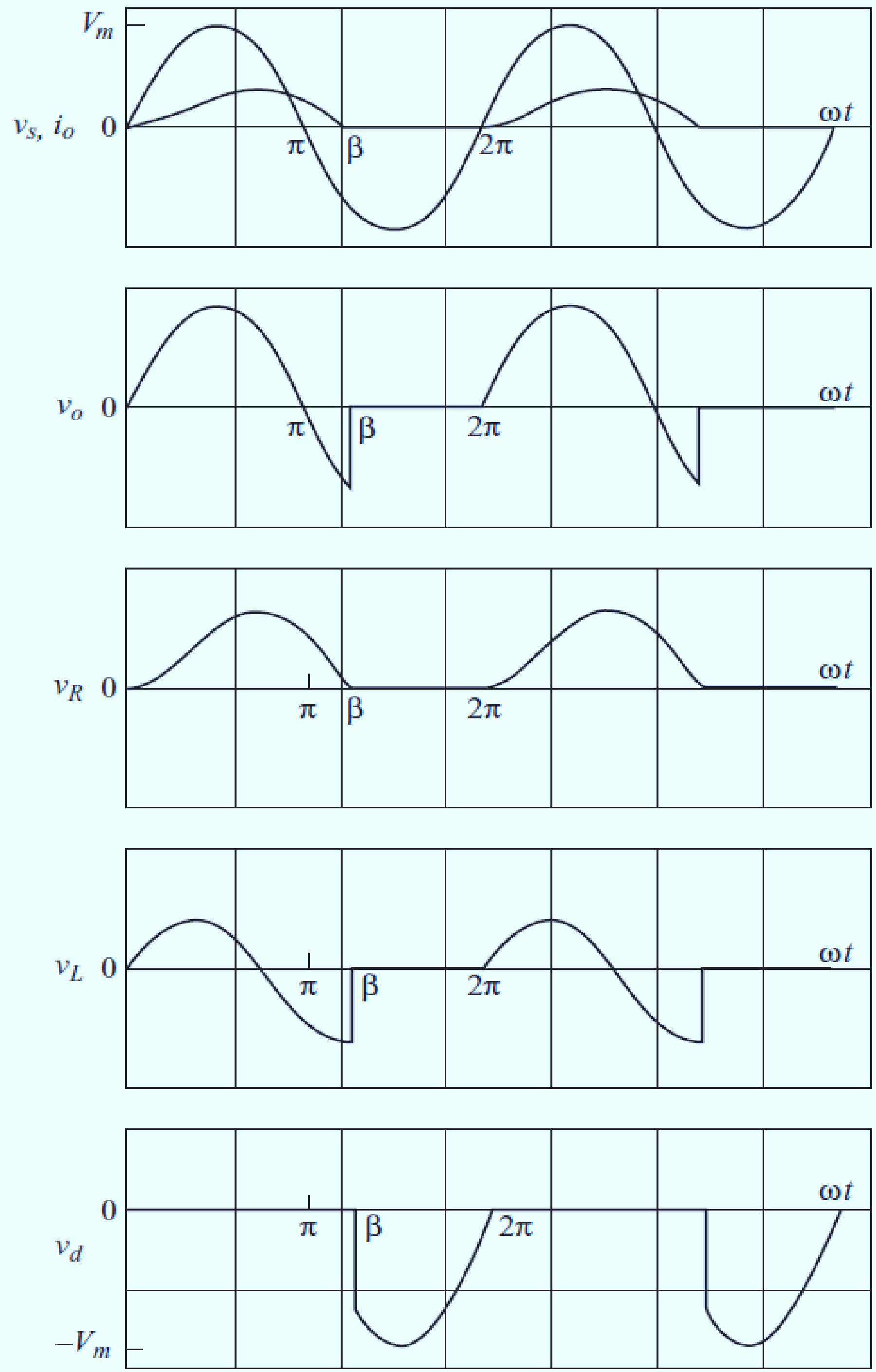
  – Input displacement angle ϕ_1 is defined as the phase angle between sinusoidal supply voltage V_s and fundamental component I_{s1} of supply current.

2.3 With RL load

- The ripple factor of output current can be reduced by connecting an inductor in series with the load resistance.

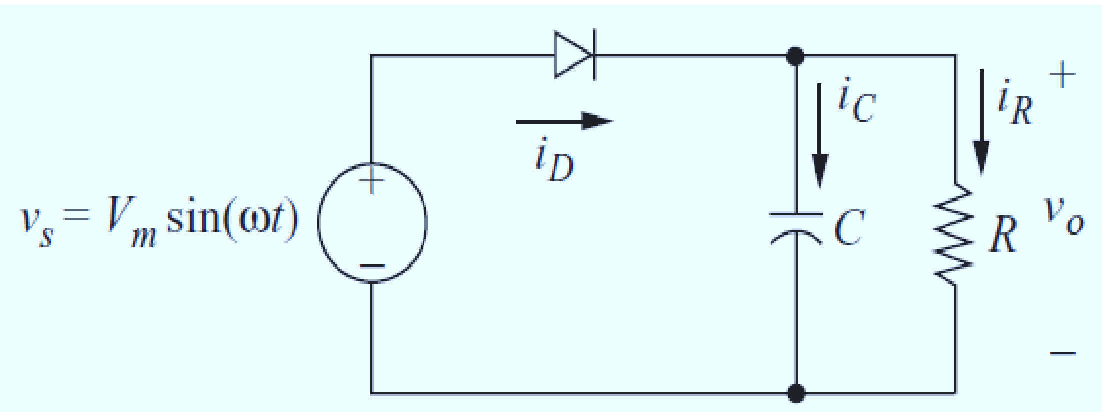


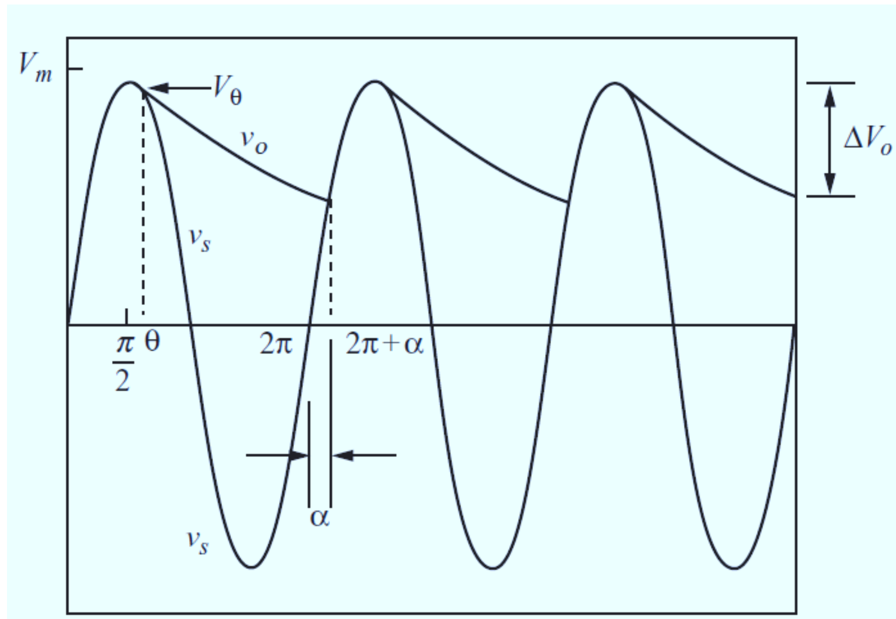
- Current continues to flow for a while even after the input voltage has gone negative.



2.4 With RC load

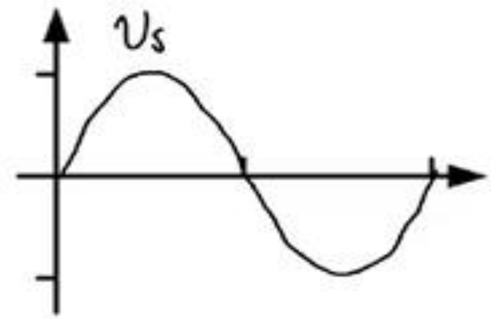
- The problem of poor ripple factor of the output voltage can also be solved by connecting a capacitor across the load resistance.
- Assuming the capacitor is initially uncharged and the circuit is energized at $\omega t = 0$, the diode becomes forward-biased as the source becomes positive.
- With the *diode ON*, the v_o is same as the v_s, and the *capacitor charges*. The capacitor is charged to V_m until the input voltage reaches its positive peak at $\omega t = \pi/2$.
- As *v_s decreases after $\omega t = \pi/2$, the capacitor decreases* into the load resistor. The v_o is a *decaying exponential* with *time constant RC while diode is OFF*.

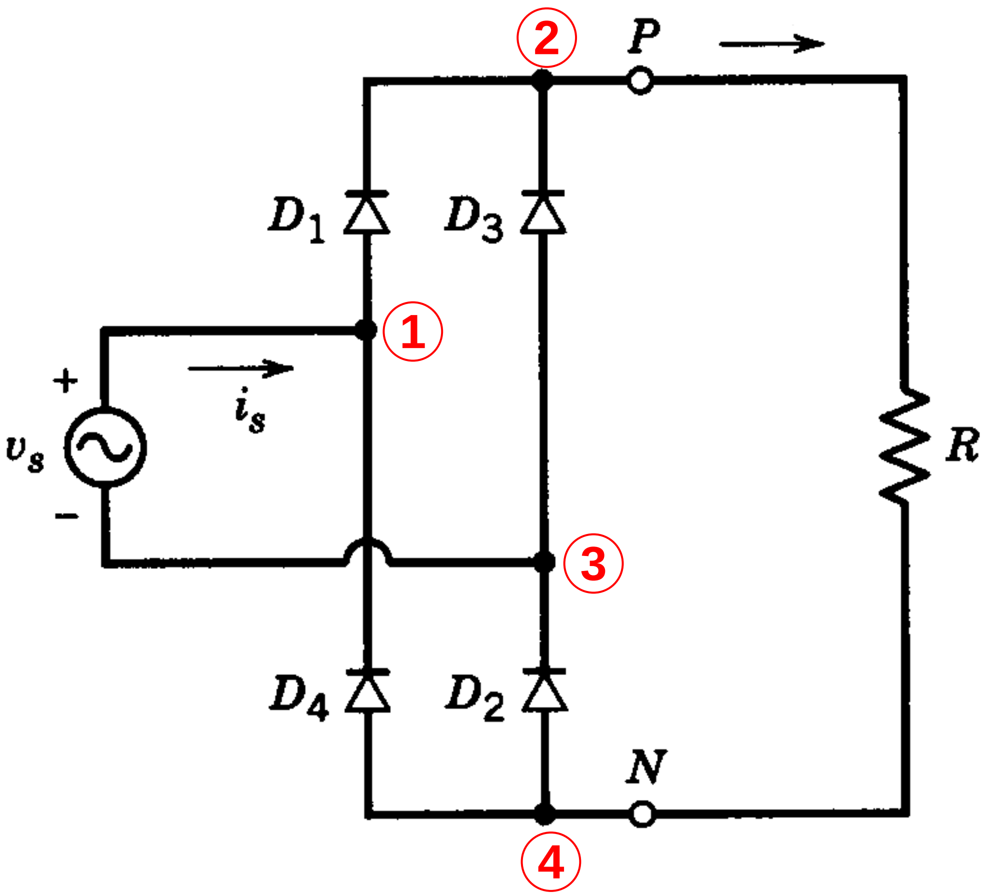


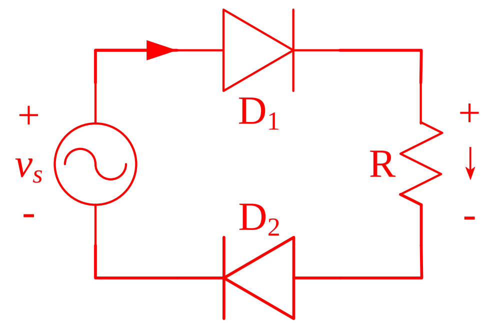


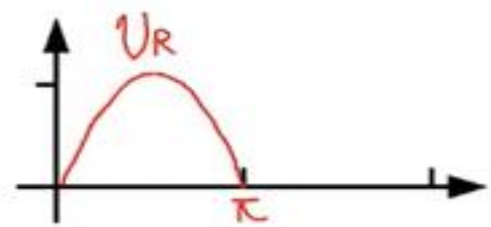
3.1 Single phase uncontrolled full-wave rectifier

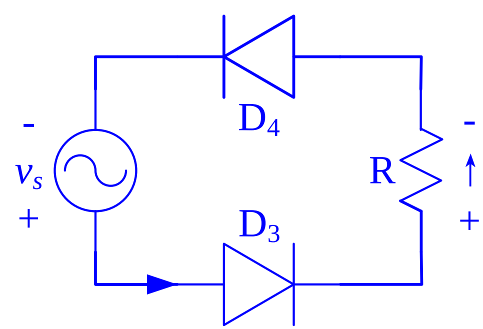
- Bridge rectifier (using 4 diodes)
  - The positive half-cycle goes through D_1 and D_2
  - The negative half-cycle goes through D_3 and D_4

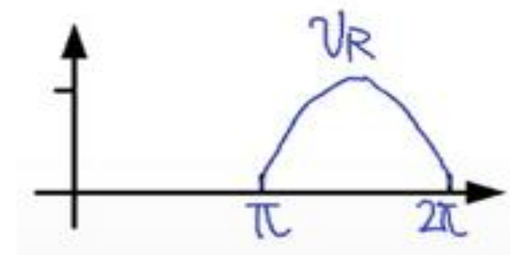












3.1 Single phase uncontrolled full-wave rectifier

- Average output voltage: $V_0 = \frac{2}{T}\int_0^{T/2} V_m sin\omega t dt = \frac{2V_m}{\pi} \approx 0.637V_m$
- RMS output voltage: $V_{RMS} = \sqrt{\frac{2}{T}\int_0^{T/2} (V_m sin\omega t)^2 dt} = 0.707V_m$
- Rectification efficiency: $\eta = \frac{V_0^2}{V_{RMS}^2} \approx 81\%$
- Form factor: $f_F = \frac{V_{RMS}}{V_0} = 1.11$
- Ripple factor: $f_R = \sqrt{f_F^2 - 1} = 0.482$
- Fourier Series: $v_o = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi} cos2\omega t - \frac{4V_m}{15\pi} cos4\omega t - \cdots$

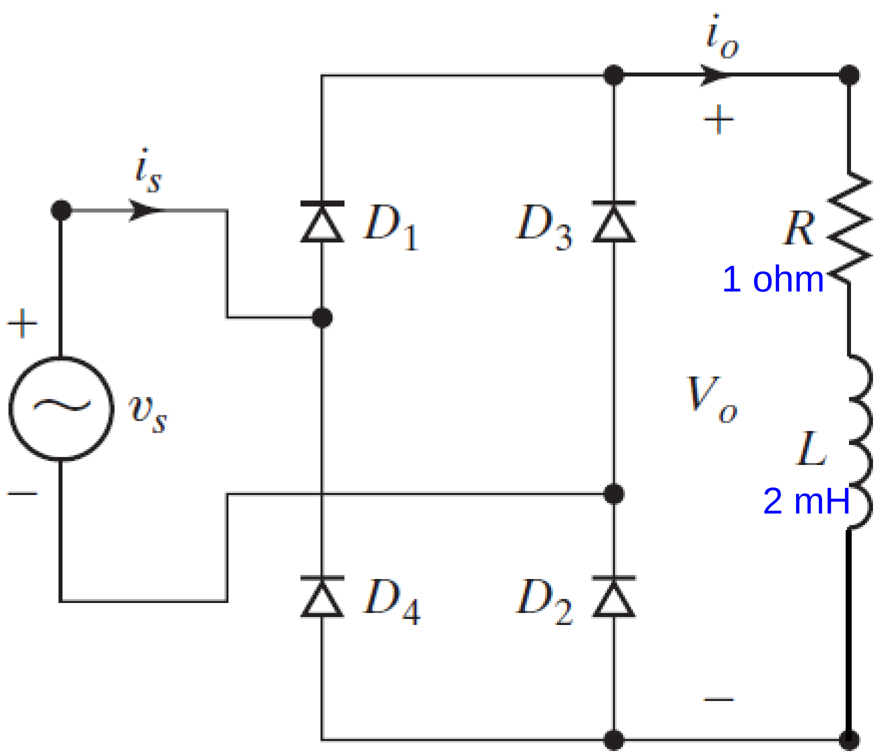
Comparison to half-wave rectifier

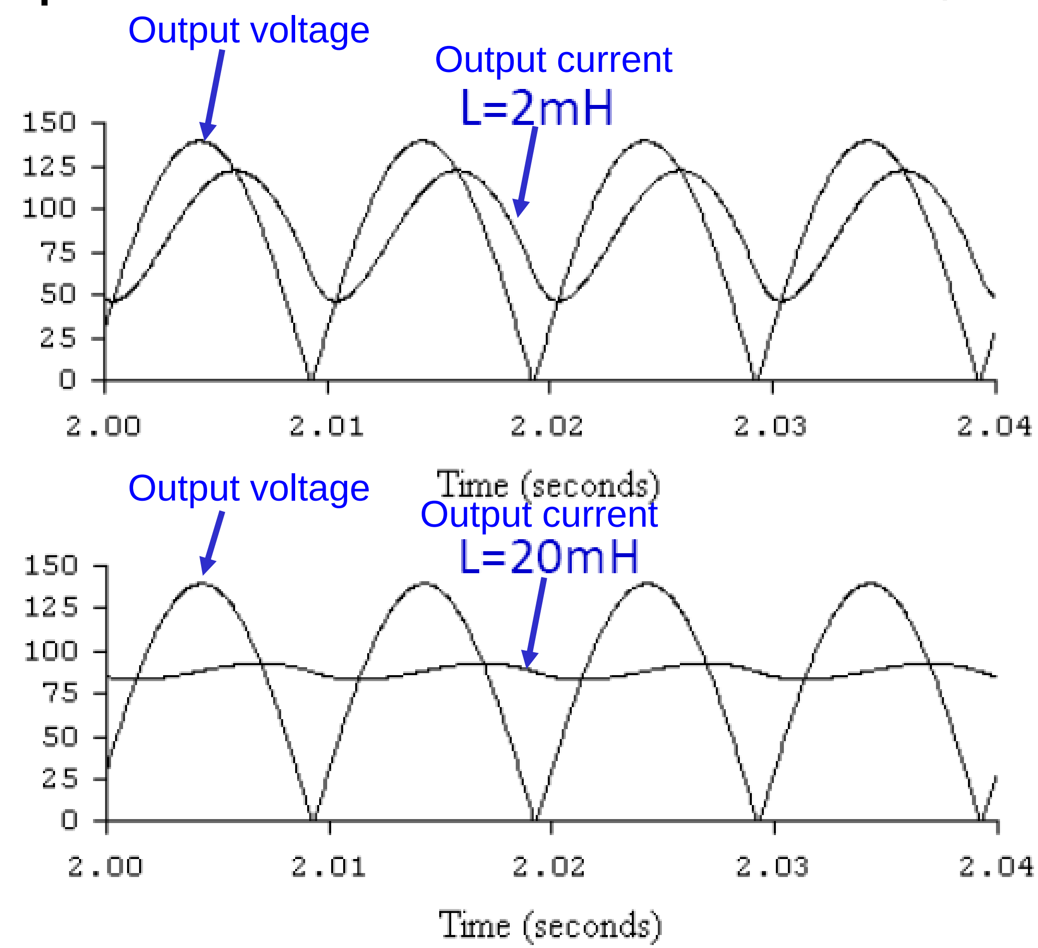
Parameters	Single Phase Half-wave	Single Phase Full-wave		
V_0	$0.318V_m$	$0.637V_m$		
V_{RMS}	$0.5V_m$	$0.707V_m$		
η	40.5%	81%		
f_T	0.287	0.81		
f_F	1.57	1.11		
f_R	1.21	0.482		
f_C	2	1.414		
f_P	0.707	1		
THD	100%	0%		

Have you found anything strange in the table?

3.2 With RL load

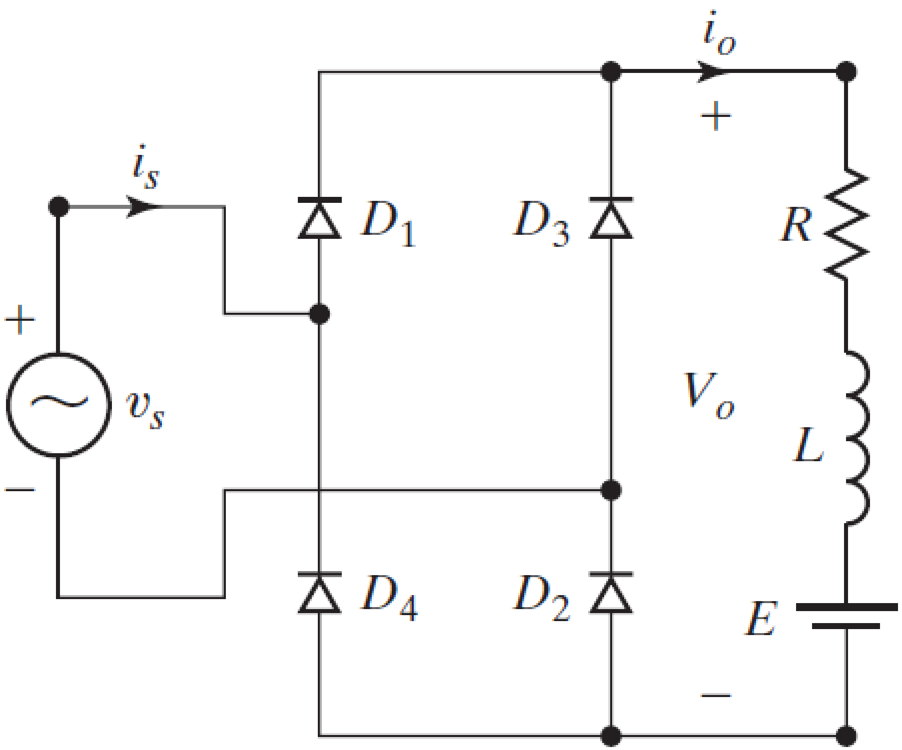
- An inductor can be used to reduce the ripples.
- Larger the inductor, the output current is much smoother, almost flat DC.





3.3 With RLE load

In this circuit, an inductor is used to regulate the output. If the inductor is very small, the charging current is discontinuous; if the inductor is large enough, the charging current is continuous, and almost stable → *much suitable for charging battery*.



Analyze Yourself

Summary:-

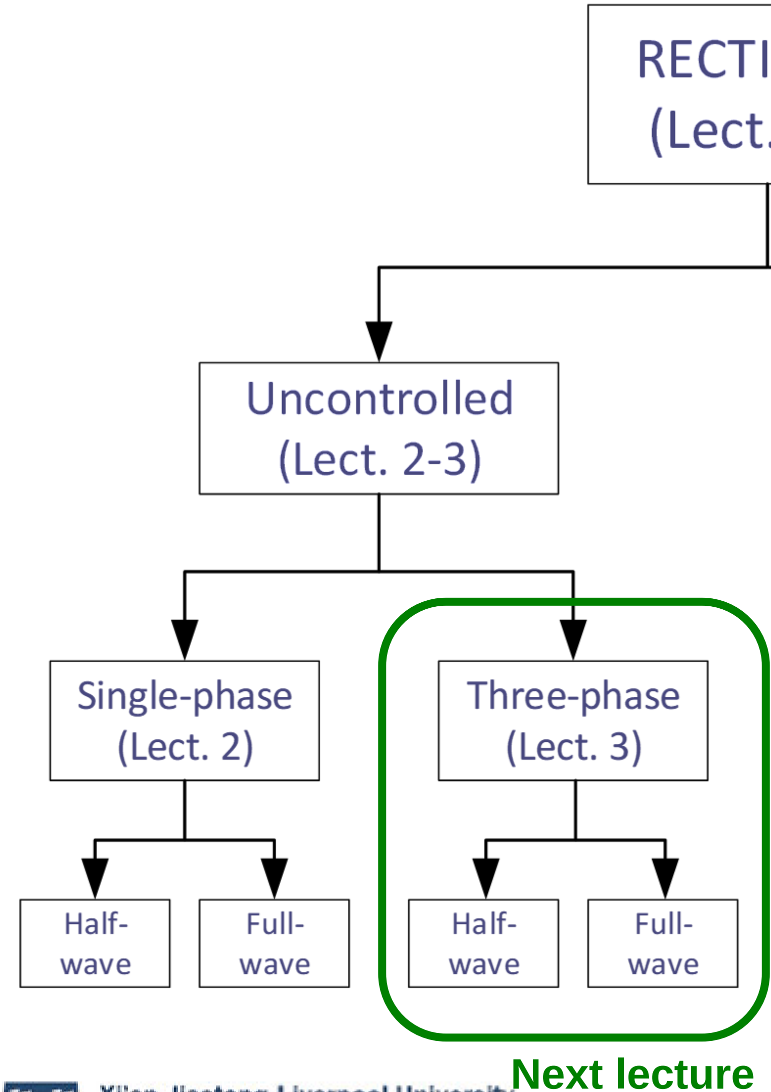
- **Rectifier** – converts alternating current (AC), *current that periodically reverses direction*, to direct current (DC), *current that flows in only one direction*.

- The output voltage v_o is a DC signal, always positive (it's not the flat constant DC we knew!!!).

- The *displacement angle* ϕ is defined to be the phase difference between the fundamental components of the input current and voltage.

- Full-wave rectifier has larger average and RMS output voltages, better efficiency and transform utilization factor, smaller ripples, and much better THD – however, the circuit is a bit expensive due to the usage of 4 diodes.

See you in the next class (March 03rd)



The End