EEE210:
Energy Conversion and Power Systems

Basic principles in power system analysis

Weitao Yao

Email: Weitao.Yao@xjtlu.edu.cn

Office: SC348

Highlights

- **Phasors**
- **Complex power**
- Power in balanced three-phase circuits

Phasors

For both voltage (V) and current (I)

$$x(t) = X_m \cos(\omega(t) + \varphi)$$

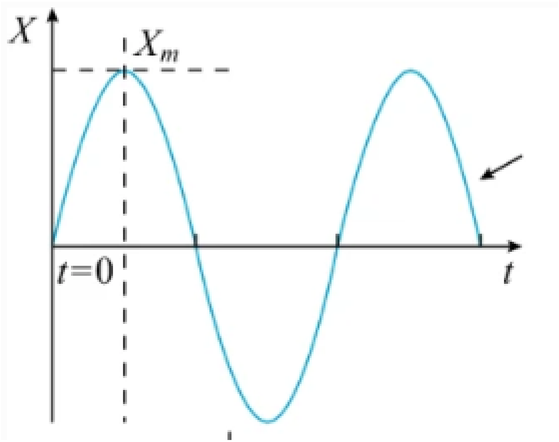
x could be V or I.
X_m is the maximum current or voltage
φ is the phase angle

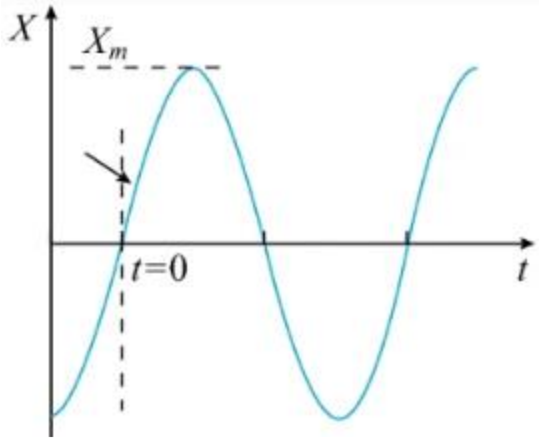
Effective value: $X = \frac{X_m}{\sqrt{2}}$

The rms phasor representation of the voltage is given in three forms—exponential, polar, and rectangular

$$X = X\mathrm{e}^{j\varphi} = X\angle\varphi = X\cos\varphi + jX\sin\varphi$$

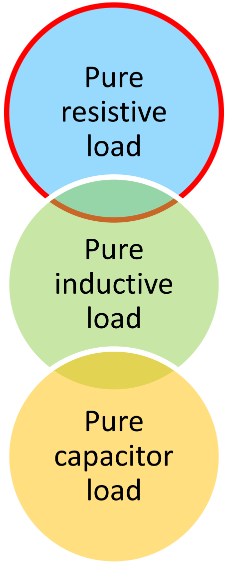
- What changes the phase angle?



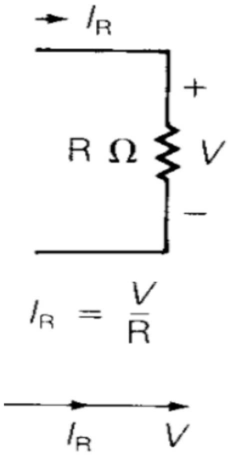


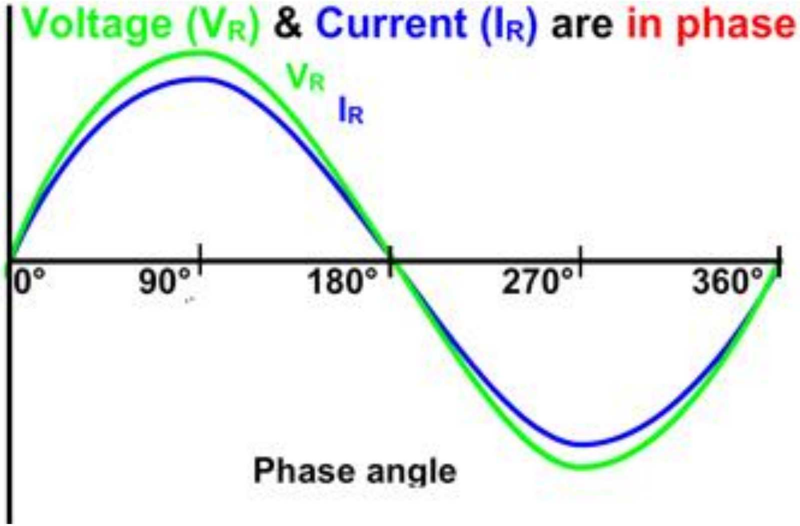
Phasors

On the demand side



Resistive load





$$v(t) = V_m \cos(\omega(t) + \varphi)$$
$$i_R(t) = I_{R,max} \cos(wt + \varphi)$$

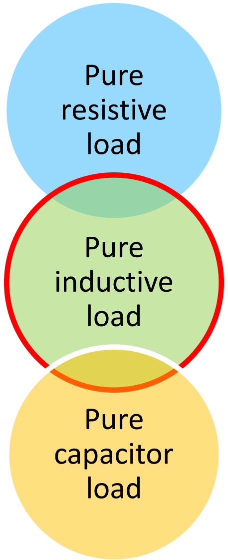
$$p(t) = v(t) i_R(t) = \frac{1}{2} V_{max} I_{R,max} \{1 + \cos[2(wt + \varphi)]\}$$

Here $V = \frac{V_{max}}{\sqrt{2}}$ & $I = \frac{I_{max}}{\sqrt{2}}$

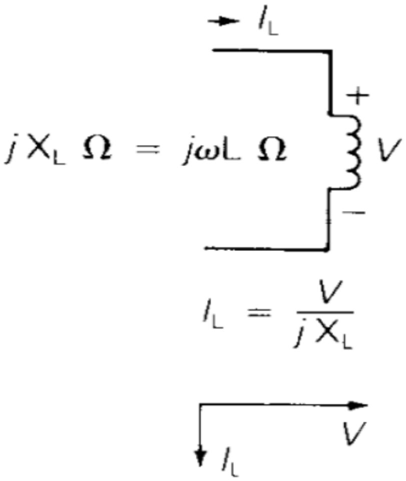
$\therefore p(t) = V \;\; I_R \{1 + \cos[2(wt + \varphi)]\}$ (W, kW, MW)

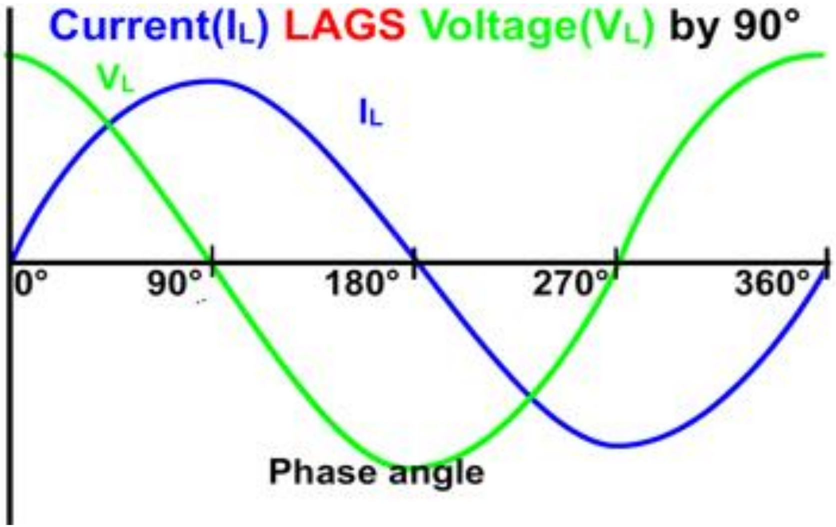
Phasors

On the demand side



Inductive load





$$v(t) = V_m \cos(\omega(t) + \varphi)$$

$$i_L(t) = I_{L,max} \cos(wt + \varphi - 90)$$

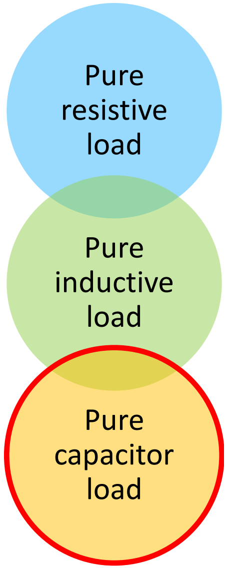
$$p(t) = v(t) i_L(t) = \frac{1}{2} V_{max} I_{L,max} \cos[2(wt + \varphi) - 90]$$

Here $V = \frac{V_{max}}{\sqrt{2}}$ & $I = \frac{I_{max}}{\sqrt{2}}$

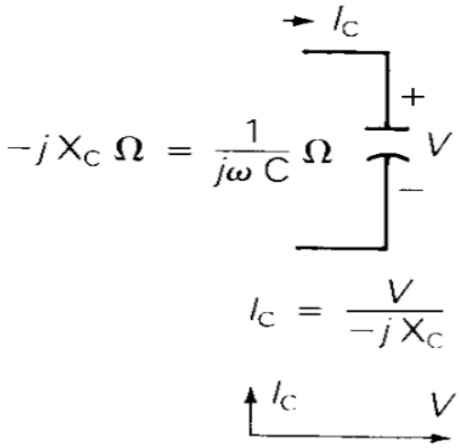
$\therefore p(t) = V\ \ I_L \sin[2(wt + \varphi)]$ (W, kW, MW)

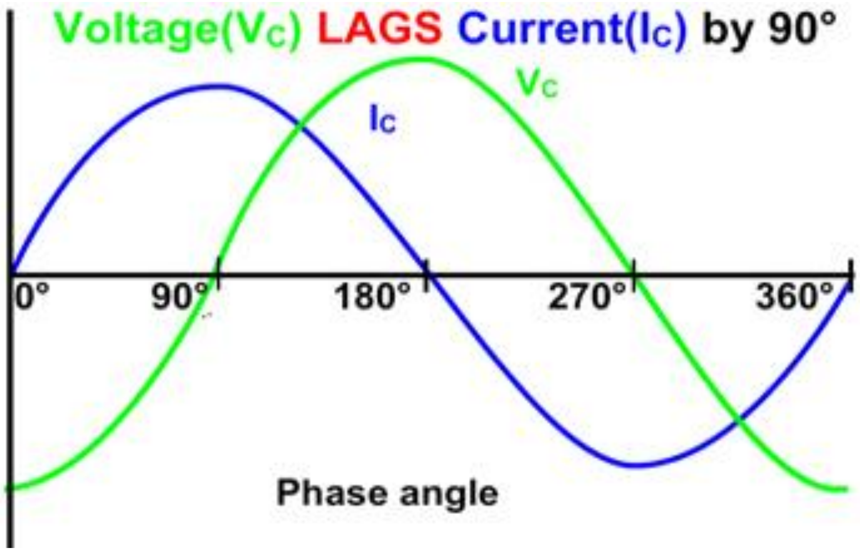
Phasors

On the demand side



Capacitive load





$$v(t) = V_m \cos(\omega(t) + \varphi)$$

$$i_C(t) = I_{C,max} \cos(wt + \varphi + 90)$$

$$p(t) = v(t) i_C(t) = \frac{1}{2} V_{max} I_{C,max} \cos[2(wt + \varphi) + 90]$$

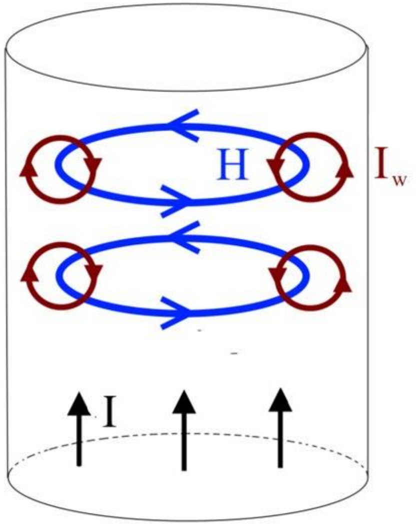
Here $V = \frac{V_{max}}{\sqrt{2}}$ & $I = \frac{I_{max}}{\sqrt{2}}$

$\therefore p(t) = -V\ I_C \sin[2(wt + \varphi)]$ (W, kW, MW)

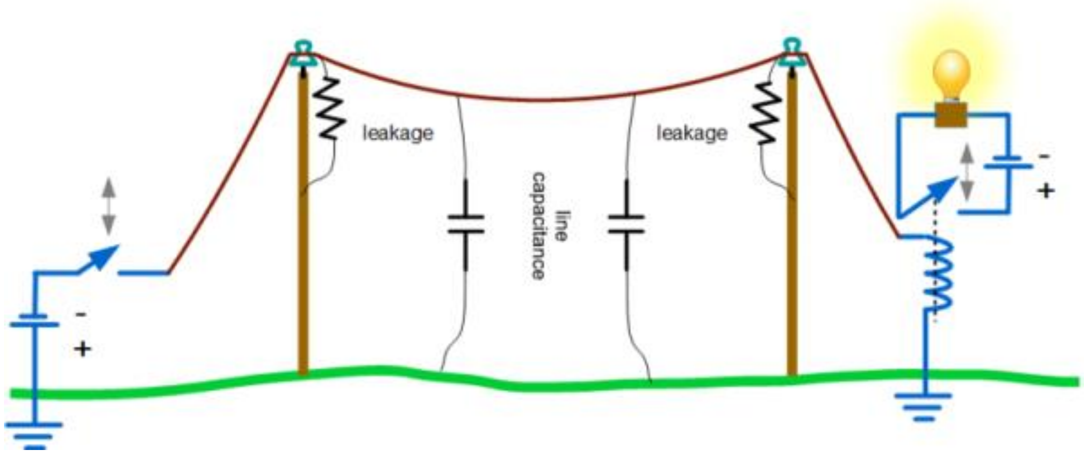
Phasors

On the delivery side

Inductance



Capacitance



Phasors

For general RLC load

If:

$$v(t) = V_{max}\cos(wt+\varphi) \;\; \&\& \;\; i(t) = I_{max}\cos(wt+\beta)$$

The instantaneous power should be:

$$p(t) = v(t) * i(t) = V_{max}I_{max}\cos(wt+\varphi)\cos(wt+\beta)$$

$$= \frac{1}{2}V_{max}I_{max}\{\cos(\varphi-\beta) + \cos[(2wt+\varphi+\beta)]\}$$

Here

$$V = \frac{V_{max}}{\sqrt{2}} \,\&\, I = \frac{I_{max}}{\sqrt{2}}$$

Phasors

For general RLC load

$$p(t) = \frac{1}{2} V_{max} I_{max} \{\cos(\varphi - \beta) + \cos[(2wt + \varphi + \beta)]\}$$

Active power is the **average value** of the instantaneous product of voltage and current over one cycle, measured in watts (W):

$$P \quad = \quad VI\cos(\varphi - \beta)$$

Reactive power is the product of the orthogonal components of voltage and current, measured in volt-amperes reactive (Var).

$$Q \quad = \quad VI\sin(\varphi - \beta)$$

The apparent power is the product of the RMS values of voltage and current, measured in volt-amperes (VA) :

$$S \quad = \quad VI\cos(\varphi - \beta) + jVI\sin(\varphi - \beta)$$

The apparent power is a complex number, not a phasor

Phasors

For general RLC load

	S	P	Q	
Apparaent power $\lvert S\rvert$	The Complex Power VA	The Active Power W	The Reactive Power var	Power factor $= \cos(\varphi - \beta)$

The apparent power (VA): the total power consumed by an electrical circuit or device.
The active power (W): the actual power consumed by a device to perform useful work, such as lighting a bulb or powering a motor.
Reactive power (Var): does not perform any useful work, but it is necessary to maintain the operation of certain types of equipment, such as motors and transformers. These devices require reactive power to create and maintain the magnetic fields that are needed for their operation.

Complex power

The power factor angle

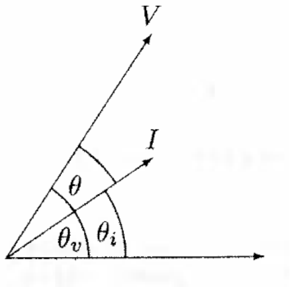
In short: Power factor = $\cos(\varphi - \beta) = \cos(\theta)$

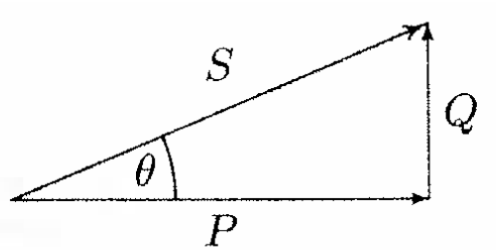
$$S = VI' = |V||I|\angle(\varphi - \beta) = |V||I|\angle\theta = V\frac{V'}{Z'} = \frac{|V|^2}{Z'}$$

*The reason for taking the conjugate, please refer to vector multiplication

The active power $P = |V||I|\cos(\theta)$ & the reactive power $Q = |V||I|\sin(\theta)$

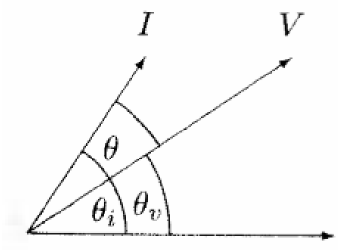
For inductive load ($\varphi > \beta$)

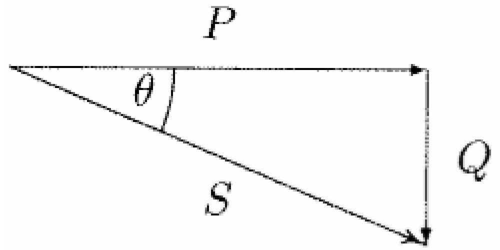




Q is positive

For capacitive load ($\varphi < \beta$)





Q is negative

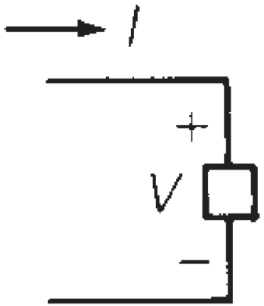
Could you describe the scenario where Q is negative?

Complex power

Complex power analysis:

Load convention

Current enters the positive terminal of circuit element



The complex power:

Resistor: $S_R = VI_R' = [V\angle\varphi]\left[\frac{V}{R}\angle-\varphi\right] = \frac{V^2}{R}$

Inductor: $S_L = VI_L' = [V\angle\varphi]\left[\frac{V}{-jX_L}\angle-\varphi\right] = j\frac{V^2}{X_L}$

$\left(I_L = \frac{V}{jX_L}\angle\varphi\right)$

Capacitor: $S_C = VI_C' = [V\angle\varphi]\left[\frac{V}{jX_C}\angle-\varphi\right] = -j\frac{V^2}{X_C}$

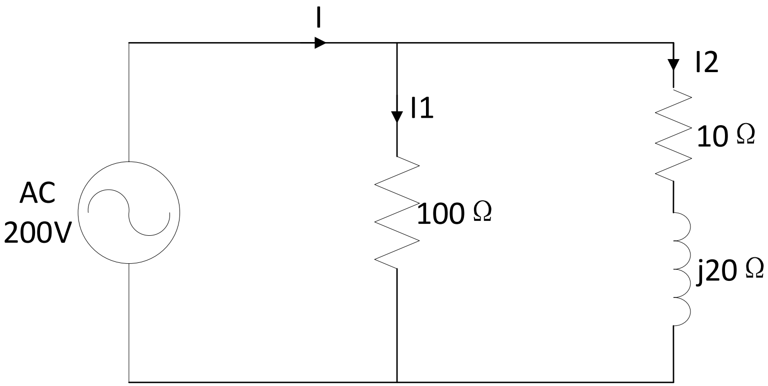
$\left(I_C = \frac{V}{-jX_C}\angle\varphi\right)$

Complex power

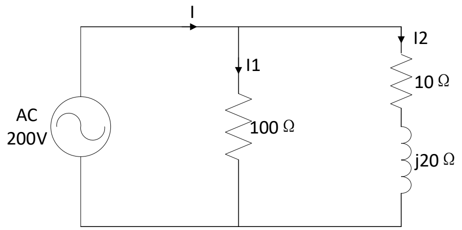
Example questions

The loads $Z_1 = 100 + j0\Omega$ and $Z_2 = 10 + j20\Omega$ are connected across a 200 V 60 Hz source.

- Find the total real and reactive power, the power factor at the source and the total current.
- To improve the overall power factor to 0.8 lagging, find the capacitance of the capacitor connected across the.



Complex power



Example questions

- Find the total real and reactive power, the power factor at the source and the total current.

Set the voltage at the source side as the base voltage (of 0 phase angle)

$$I_1 = \frac{V}{Z_1} = \frac{200\angle 0^\circ}{100 + j0\Omega} = 2 + j0 \ \ A$$

$$I_2 = \frac{V}{Z_2} = \frac{200\angle 0^\circ}{10 + j20\Omega} = 4 - j8 \ \ A$$

Thus, the complex power for demands would be:

$$S_1 = VI_1' = 200\angle 0^\circ * (2 - j0) = 400\ W + j0\ var$$

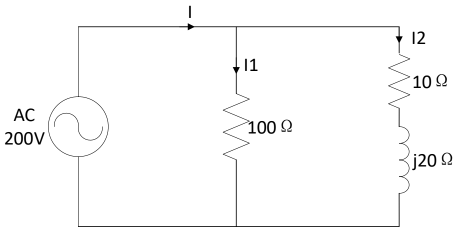
$$S_2 = VI_2' = 200\angle 0^\circ * (4 + j8) = 800\ W + j1600\ var$$

The total complex power and currents are

I = I_1 + I_2 = 2 + 4 – j8 = 6-j8 A = 10 $\angle -53.13°$ $\beta = -53.13°;\ \theta = \ \varphi\ - \beta$ =53.13°

S = S_1+ S_2 = 400 + 800 + j1600 = 1200 W+ j1600 var = 2000 ∠53.13° VA $\theta = 53.13°$

Complex power



Example questions

- Find the total real and reactive power, the power factor at the source and the total current.

S = S_1+ S_2 = 400 + 800 + j1600 = 1200 + j1600 W = 2000 $\angle 53.13°$

The power factor:

$$PF = \cos(53.13°) = 0.6 \ \ lagging$$

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what determines the capacity for power delivery?

Ampacity

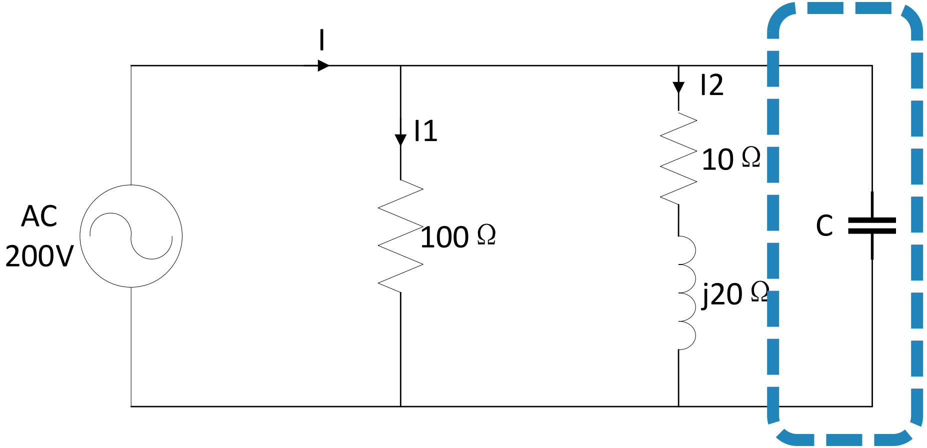
The maximum amount of electrical current that a conductor or electrical wire can safely carry without overheating or causing damage

In this example I = 6-j8 = 10 $\angle -53.13°$A

Only 60% delivery efficiency

Complex power

Power factor correction/ reactive power compensation



To power companies' interest, it is better all loads will have a power factor as close to 1. Therefore, network assets could be efficiently use.

Parallel connected capacitors are commonly adopted

To improve the overall power factor to 0.8 lagging, find the capacitance of the capacitor connected.

S = $S_1 + S_2$ = 1200 W + j1600 var

= 2000 ∠53.13° VA

PF = 0.6 lagging; To improve PF to 0.8:

The desired power factor angle should be

$$\theta_{new} = \arccos(PF_{new}) = \arccos(0.8) = 36.87°$$

$$Q_{new} = P\tan(\theta_{new}) = 1200 * \tan(36.87) = 900\ var$$

Therefore, the required reactive power is:

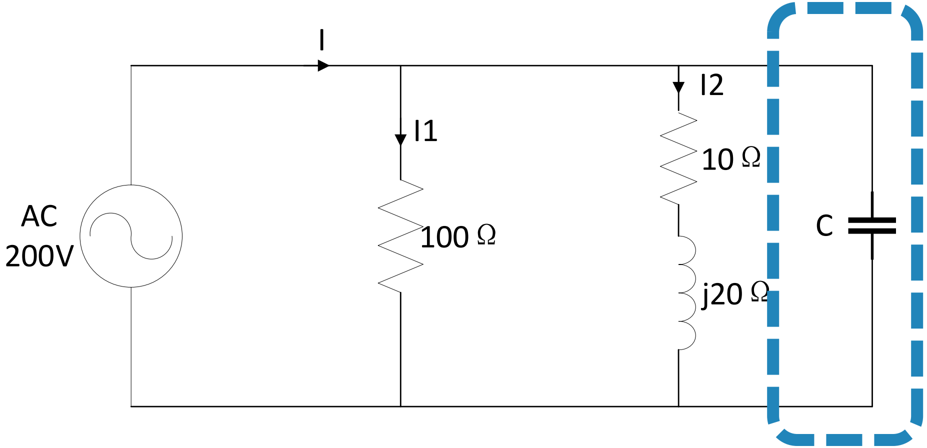
$$Q_c = 1600(ex\ ante) - 900(ex\ post) = 700\ var$$

$$S_C = -j700\ var = \frac{|V|^2}{Z_c'}$$

$$\therefore Z_C = \frac{|V|^2}{S_c'} = \frac{200^2}{j700} = -j57.14\ \Omega$$

Complex power

Power factor correction/ reactive power compensation



To improve the overall power factor to 0.8 lagging, find the capacitance of the capacitor connected.

$$\therefore Z_C = \frac{|V|^2}{S_c'} = \frac{200^2}{j700} = -j57.14\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)(57.14)} = 4.64 * 10^{-5} F$$

To power companies' interest, it is better all loads will have a power factor as close to 1. Therefore, network assets could be efficiently use.

Parallel connected capacitors are commonly adopted

After correcting, the new power and current are:

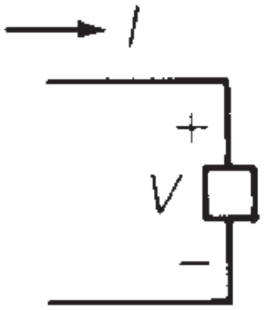
$$S_{new} = P + jQ_{new} = 1200 + j900\ VA$$
$$= 1500\ \angle 36.87°\ \text{VA}$$

$$I_{new} = \frac{S_{new}'}{V'} = \frac{1500\ \angle{-36.87°}}{200} = 7.5\ \angle - 36.87° \text{A}$$

The delivery efficiency improves to 80%

Complex power

Complex power analysis:

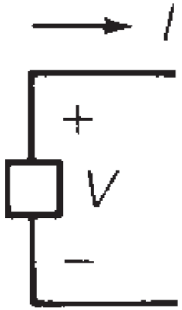


Load convention

Current enters the positive terminal of circuit element

P > 0: positive real power is absorbed

Q > 0: positive reactive power is absorbed



Generator convention

Current leaves the positive terminal of the circuit element

P > 0: positive real power is delivered

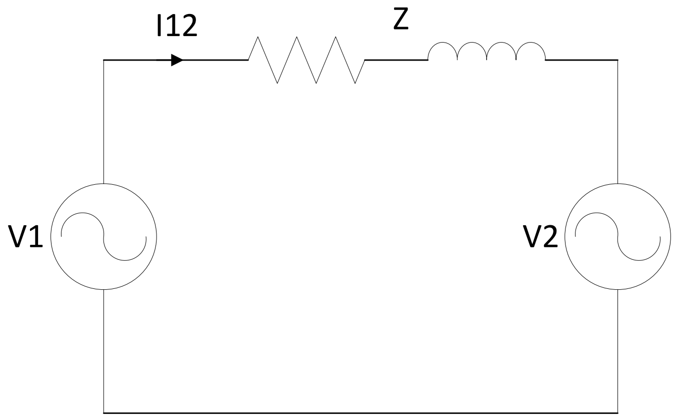
Q > 0: positive reactive power is delivered

Complex power

Complex power flow example questions

Two voltage sources $V_1 = \ 120\angle -5°$V and $V_2 = \ 100\ \angle 0°$V are connected by a short line of impedance $Z = 1 + j7$.

- Determine the real and reactive power supplied or received by each source and the power loss in the line



Complex power

- Determine the real and reactive power supplied or received by each source and the power loss in the line

**Bi-direction analysis

1. Assume currents flow from V1 to V2

$$I_{12} = \frac{\mathrm{V1} - \mathrm{V2}}{Z} = \frac{120\,\angle-5° - 100\,\angle 0°}{1+j7} = 3.135\,\angle-110.02°\ \ \mathrm{A}$$

2. Assume currents flow from V2 to V1

$$I_{21} = \frac{\mathrm{V2} - \mathrm{V1}}{Z} = \frac{100\,\angle 0° - 120\,\angle-5°}{1+j7} = 3.135\,\angle 69.98°\ \ \mathrm{A}$$

Thus,

$$S_{12} = V_1 * I_{12}' = 120\,\angle-5° * 3.135\,\angle 110.02° = 376.2\angle 105.02°$$
$$= -97.5\mathrm{W} + \mathrm{j}363.3\mathrm{var}$$

Source 1 receives 97.5 W active power and generates 363.3 var reactive power

Complex power

- Determine the real and reactive power supplied or received by each source and the power loss in the line

$$S_{21} = V_2 * I'_{21} = 100\,\angle 0° * 3.135\,\angle - 69.98° = 313.5\angle - 69.98°$$
$$= 107.3\mathrm{W} - \mathrm{j}294.5\mathrm{var}$$

Source 2 generates 107.3W active power and receives 294.5 var reactive power.

Important!!!!! Should we consider reactance in calculating line loss?

Power loss $= |I_{12}|^2 * R = 3.135^2 * 1 = 9.8\,W$ (Joule effect)

For the reactance parts, it is commonly named as the reactive power consumption.

reactive power consumption = $|I_{12}|^2 * X = 3.135^2 * 7 = 68.8\,var$

Next Lecture

Basic principles in power system analysis (2)

Thanks for your attendance!