CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 15 Magnetically-Coupled Circuits

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OUTLINE

- Introduction
- Mutual Inductance
  - Dot Convention
  - Coupling Coefficient
  - Energy in a Coupled Circuit
- Transformers
  - Linear Transformers
  - Ideal Transformers
  - Auto Transformers

1 INTRODUCTION

- When two loops (with or without contacts between them) affect each other through the magnetic field generated by one of them, they are said to be magnetically coupled.
- The transformer is an electrical device designed based on the concept of magnetic coupling. It uses magnetically coupled coils to transfer energy from one circuit to another.
- The transformers are used in power systems for stepping up or stepping down ac voltages or currents.
- The transformers are used in electronic circuits such as radio and television receivers for such purposes as impedance matching, isolating one part of a circuit from another, and for stepping up or down ac voltages and currents.

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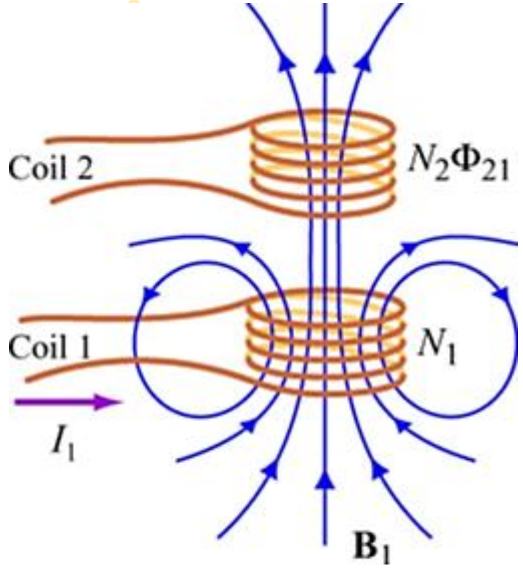
2.1 DEFINITION

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as mutual inductance.

Mutual inductance (M) is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H). It is always a **positive** quantity.

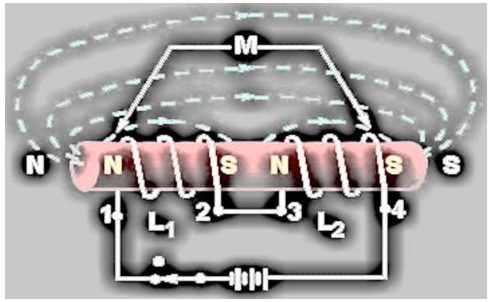


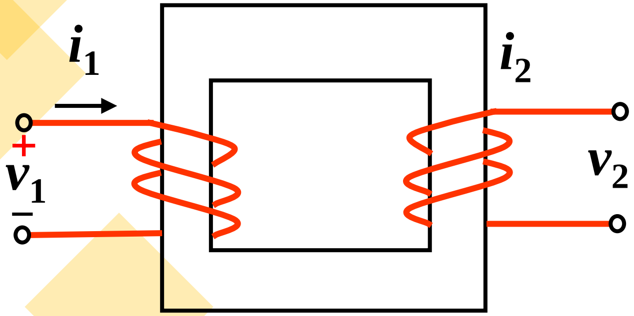
M_{21} defines the mutual inductance of coil 2 with respect to coil 1, which relates the induced voltage in coil 2 to the current in coil 1.

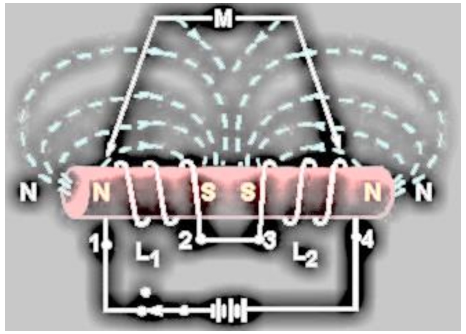
M_{12} defines the mutual inductance of coil 1 with respect to coil 2, which relates the induced voltage in coil 1 to the current in coil 2.

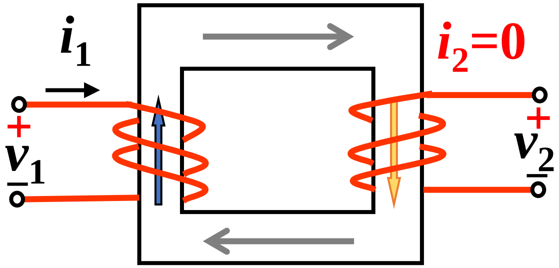
$$M_{21} = M_{12} \equiv M$$

EXAMPLES



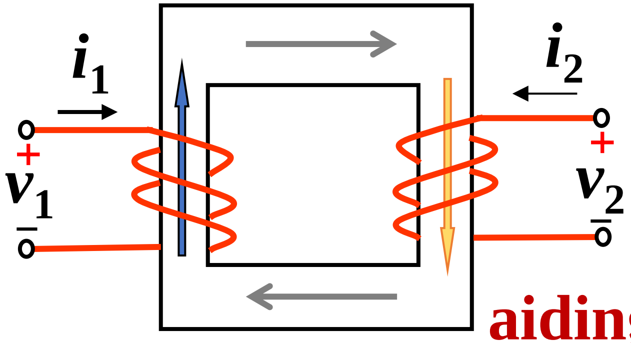






$$v_1(t) = L_1 \frac{di_1(t)}{dt}$$

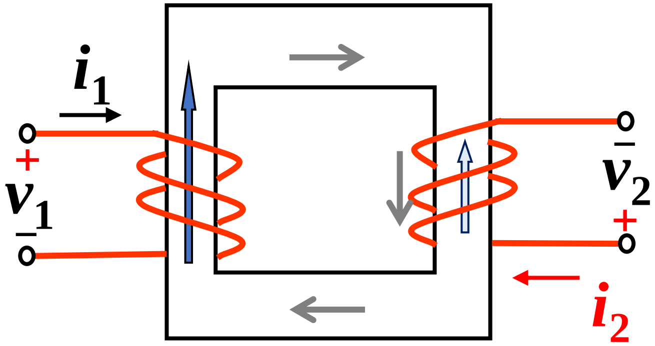
$$v_2(t) = M \frac{di_1(t)}{dt}$$



aiding fields

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$



opposing fields

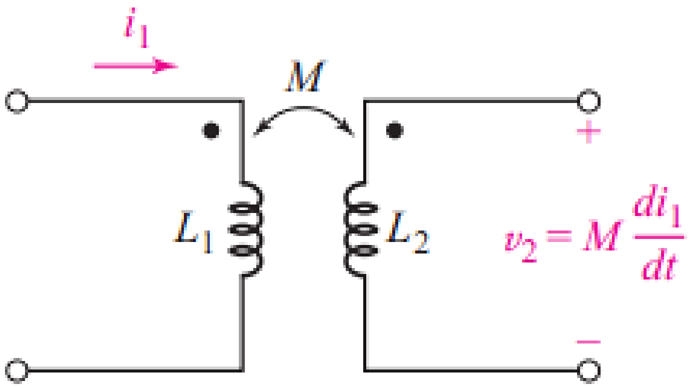
$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$

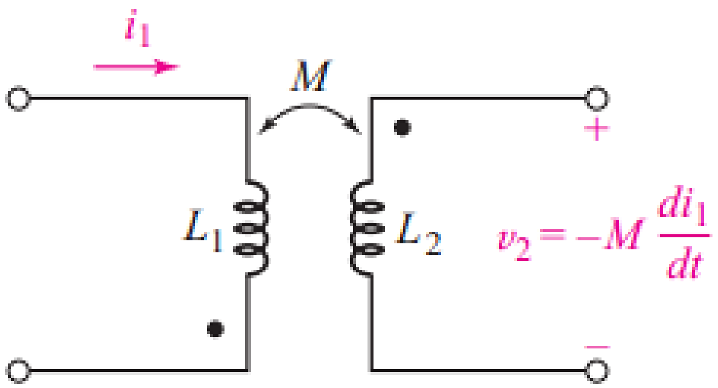
$$v_2(t) = L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$$

2.2 DOT CONVENTION

Although mutual inductance M is always a positive quantity, the mutual voltage Mdi/dt may be negative or positive.

It is inconvenient to show the construction details of coils on a circuit schematic all the time, so the dot convention is applied for circuit analysis.





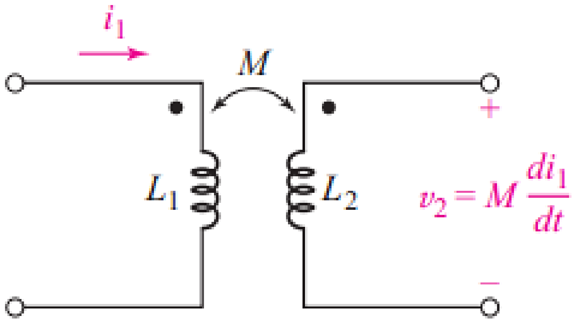
Dot Convention is used to establish the polarity for the mutually induced voltages in coupled circuits.

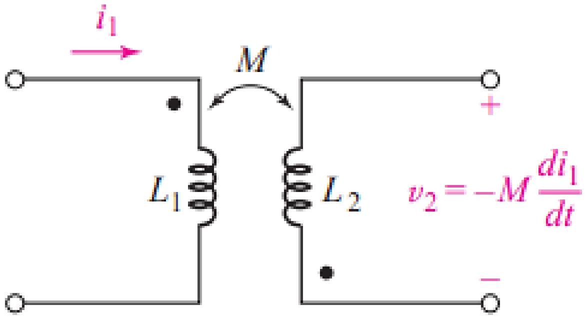
The dots are used along with the dot convention to determine the polarity of the mutual voltages.

2.2 DOT CONVENTION

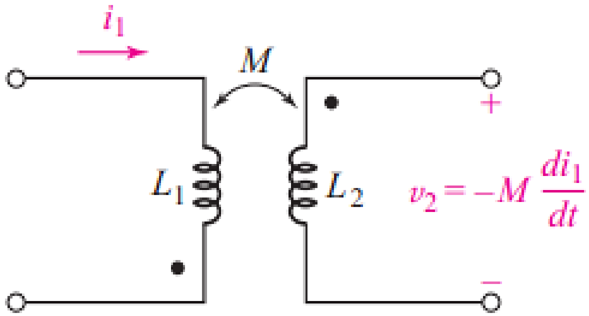
A dot is placed in the circuit at one end of each of the two magnetically coupled coils to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil.

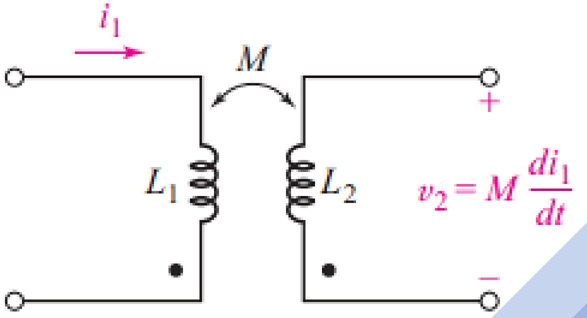
If a current **enters** the dotted terminal of one coil, the **reference polarity** of the mutual voltage in the **second** coil is **positive** at the dotted terminal of the second coil.





If a current **leaves** the dotted terminal of one coil, the **reference polarity** of the mutual voltage in the **second** coil is **negative** at the dotted terminal of the second coil.

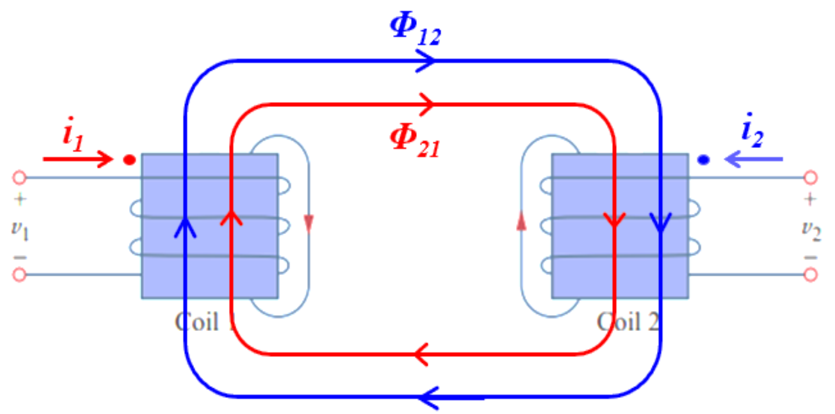




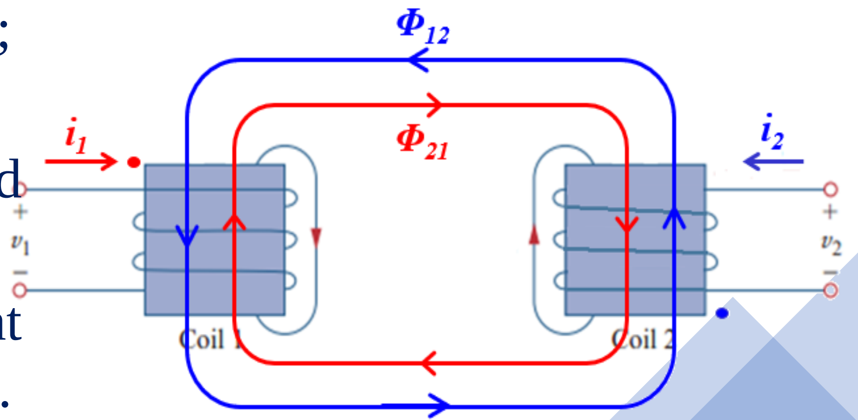
Thus, the reference polarity of the mutual voltage depends on the **reference direction** of the inducing current and **the dots on the coupled coils**.

2.3 DOT POSITION

1. Assume a dot at arbitrary terminal of coil 1, currents i_1 enters the dot;
2. Select one arbitrary terminal of coil 2, currents i_2 enters the terminal;
3. Analyse the direction of Φ_{21} and Φ_{12} based on the direction of i_1 and i_2 and the winding of coil 1 and 2;
4. If Φ_{21} and Φ_{12} have the same direction, then the assumed dots' positions are correct; if not, change the position of one of the dots.



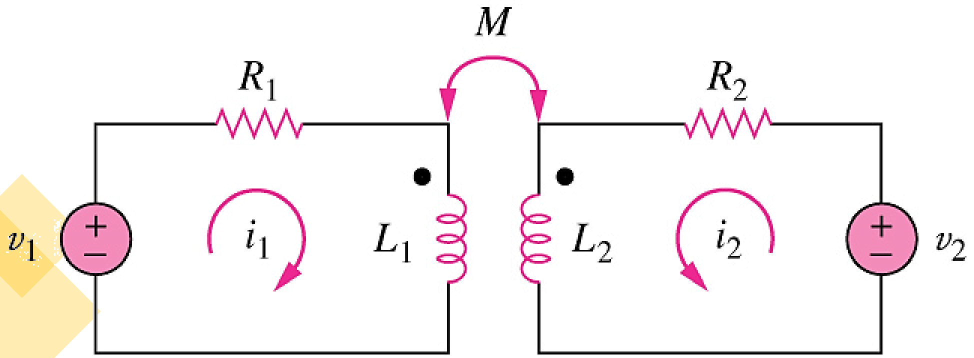
1. Assume a dot at arbitrary terminal of coil 1, currents i_1 enters the dot;
2. Select one arbitrary terminal of coil 2, currents i_2 enters the terminal;
3. Analyse the direction of Φ_{21} and Φ_{12} based on the direction of i_1 and i_2 and the winding of coil 1 and 2;
4. If Φ_{21} and Φ_{12} have the same direction, then place the dot of coil 2 at the selected terminal; if not, place the dot of coil 2 at the other terminal.



Current entering each dotted terminal produces magnetic flux in the same direction

EXAMPLES

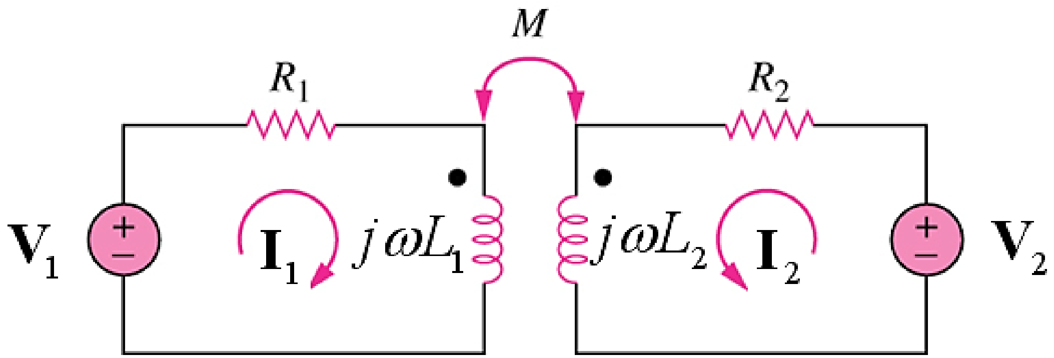
Time Domain



$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Frequency Domain ☆



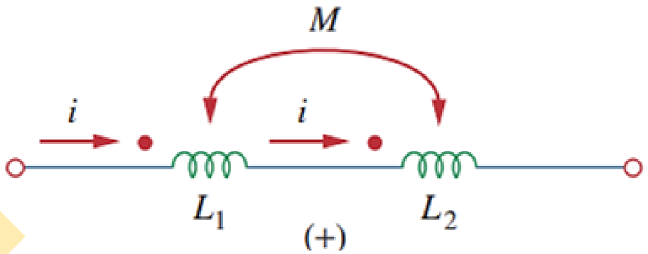
$$V_1 = (R_1 + j\omega L_1)I_1 + j\omega M I_2$$

$$V_2 = (R_2 + j\omega L_2)I_2 + j\omega M I_1$$

EXAMPLES

Some examples to see how we apply the dot convention:

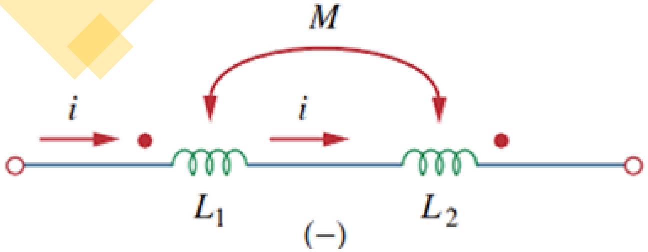
Series-aiding connection:



$$V = 2j\omega MI + j\omega(L_1 + L_2)I$$

$$L_{aidding} = L_1 + L_2 + 2M$$

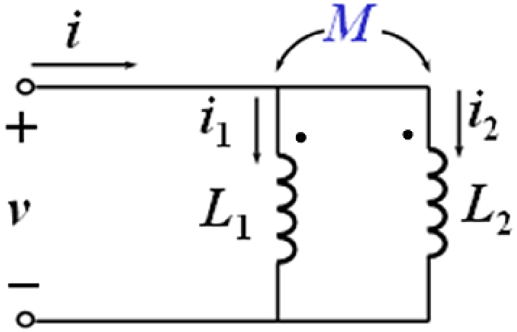
Series-opposing connection:



$$V = -2j\omega MI + j\omega(L_1 + L_2)I$$

$$L_{opposing} = L_1 + L_2 - 2M$$

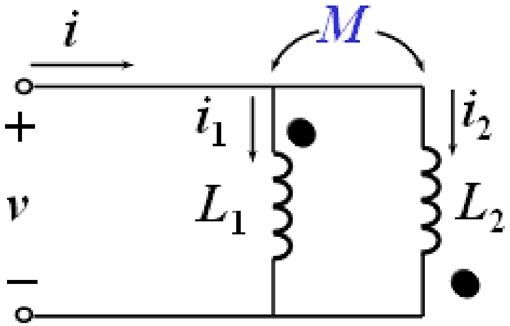
Parallel connections:



$$V = j\omega L_1 I_1 + j\omega M I_2 = j\omega L_2 I_2 + j\omega M I_1$$

$$I = I_1 + I_2$$

$$V = \frac{j\omega\,(L_1 L_2 - M^2)}{L_1 + L_2 - 2M} I \quad L = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 - 2M}$$



$$V = j\omega L_1 I_1 - j\omega M I_2 = j\omega L_2 I_2 - j\omega M I_1$$

$$I = I_1 + I_2$$

$$V = \frac{j\omega\,(L_1 L_2 - M^2)}{L_1 + L_2 + 2M} I \quad L = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 + 2M}$$

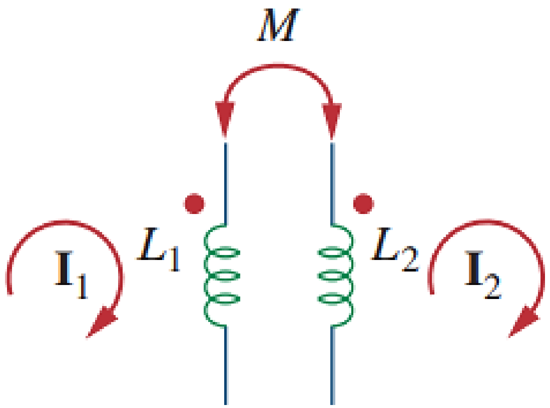
2.4 SUMMARY

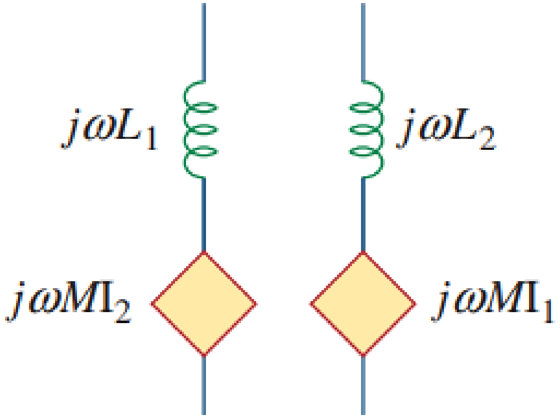
1. Determine the value of the induced voltages and then the appropriate signs.

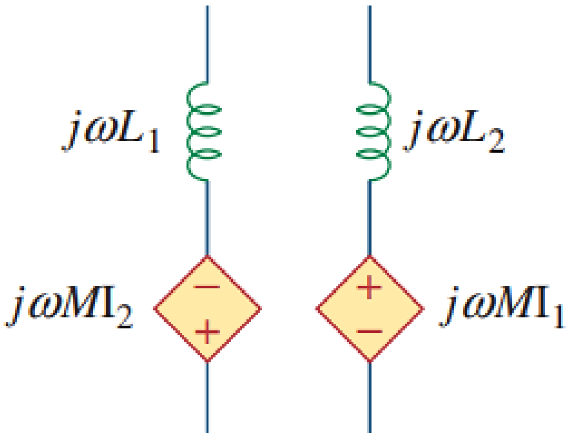
2. $\mathbf{I}_1$ induces a voltage within the second coil represented by the value $j\boldsymbol{\omega M}\mathbf{I}_1$ and $\mathbf{I}_2$ induces a voltage of $j\boldsymbol{\omega M}\mathbf{I}_2$ in the first coil.

3. Since $\mathbf{I}_1$ enters L_1 at the dotted end, it induces a voltage in L_2 that tries to force a current out of the dotted end of L_2 which means that the source must have a plus on top.

4. $\mathbf{I}_2$ leaves the dotted end of L_2 which means that it induces a voltage in L_1 which tries to force a current into the dotted end of L_1 requiring a dependent source that has a plus on the bottom.

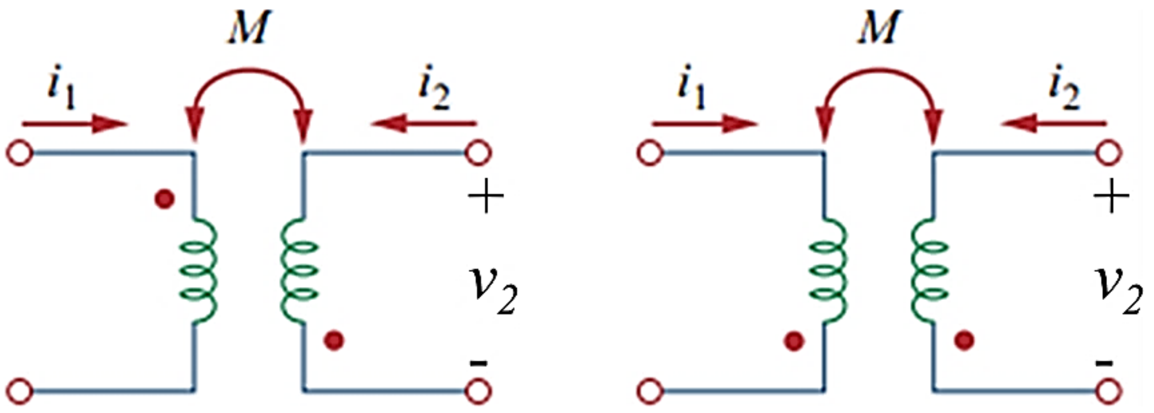






QUIZ 2.1

1. Referring to the two pairs of magnetically coupled coils and current i_1 is flowing through the first coil, the polarity of the mutual voltage in the second coil for both cases are:



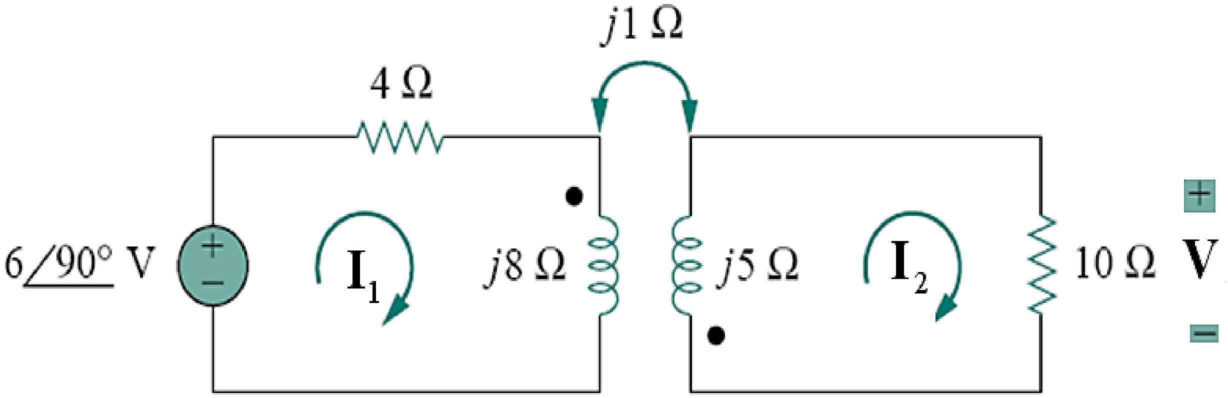
(a) negative; positive

(b) negative; negative

(c) positive; positive

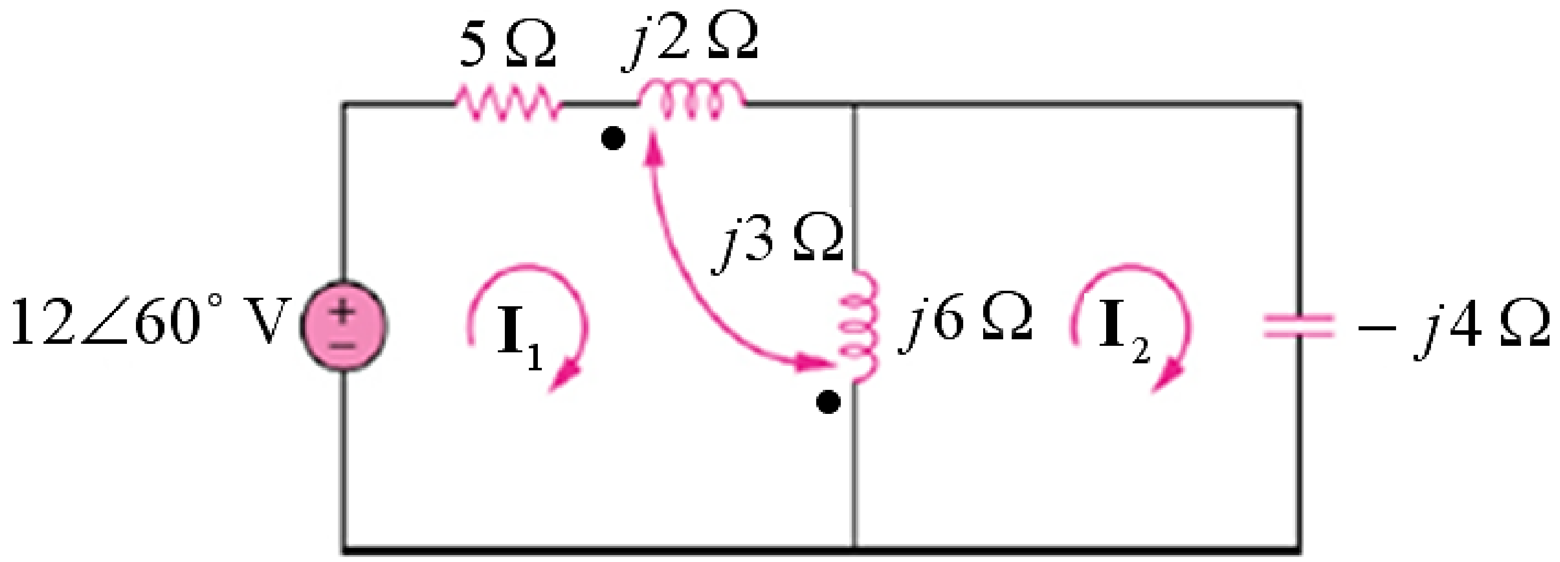
(d) positive; negative

2. Determine the voltage V and the current I_2 in the figure.



QUIZ 2.2

Find I_1 and I_2 for the figure below.



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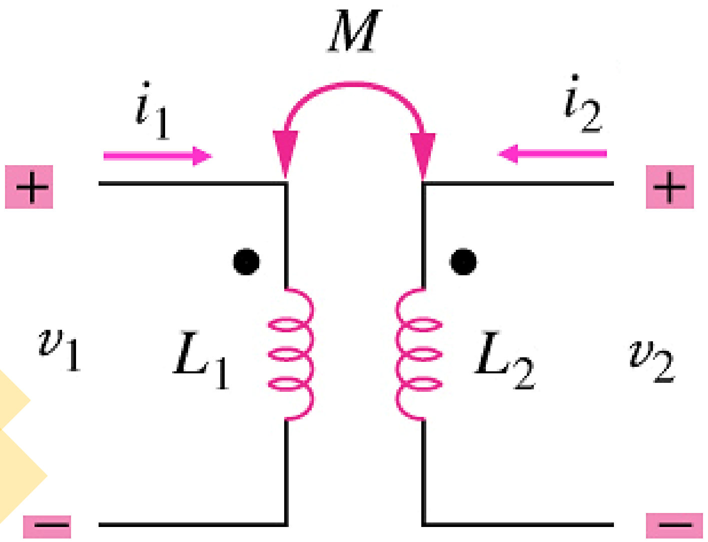
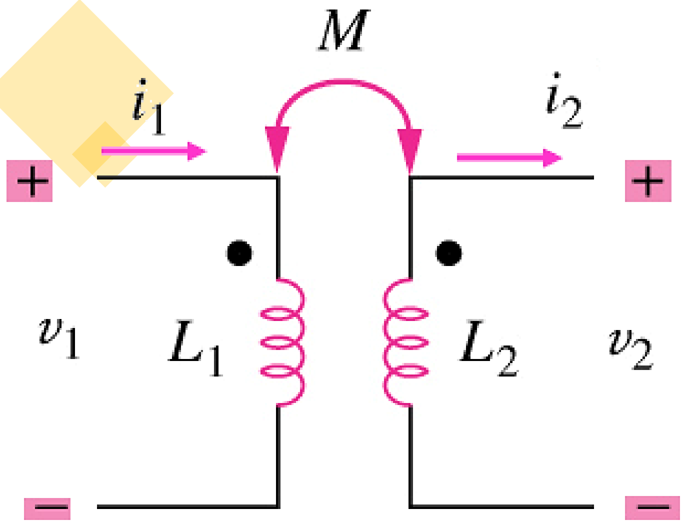
2.3 ENERGY





The total energy w stored in a mutually coupled inductor is:

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm i_1 i_2 M$$

If both currents ENTER or LEAVE the dotted terminals

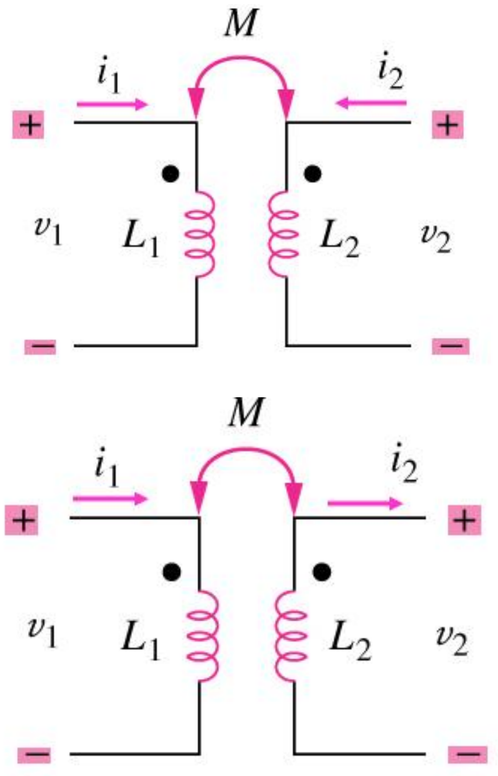
$\longrightarrow$ Positive sign

Otherwise

$\longrightarrow$ Negative sign

2.3 ENERGY

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm i_1 i_2 M$$



Since the energy stored in a passive network **cannot** be negative

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - i_1 i_2 M \geq 0$$

Then

$$w = \frac{1}{2}\left(i_1\sqrt{L_1} - i_2\sqrt{L_2}\right)^2 + i_1 i_2\left(\sqrt{L_1 L_2} - M\right) \geq 0$$

The squared term is never negative (≥ 0), then the 2nd term should be

$$\sqrt{L_1 L_2} - M \geq 0$$

The **mutual inductance** cannot be greater than the geometric mean of the self-inductances of the coils.

$$0 \leq M \leq \sqrt{L_1 L_2}$$

2.4 COUPLING COEFFICIENT

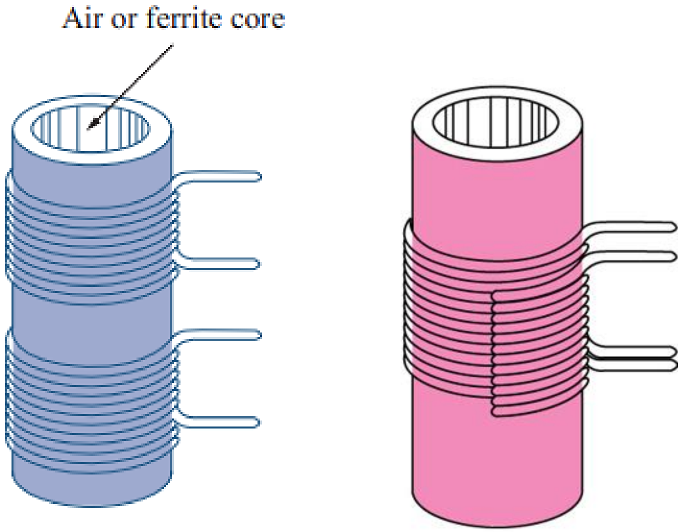
$$0 \le M \le \sqrt{L_1 L_2}$$

Coupling coefficient k: a measure of the magnetic coupling between two coils.

$$0 \le k = \frac{M}{\sqrt{L_1 L_2}} \le 1$$

or

$$M = k\sqrt{L_1 L_2}$$



k: depends on the closeness of the two coils, their core, their orientation, and their windings.

For $k < 0.5$, coils are said to be ***loosely coupled***

For $k > 0.5$, they are said to be ***tightly coupled***

If the entire flux produced by one coil links another coil, then $k = 1$, and we have 100% coupling, and the coils are said to be ***perfectly coupled***.

QUIZ 2.3

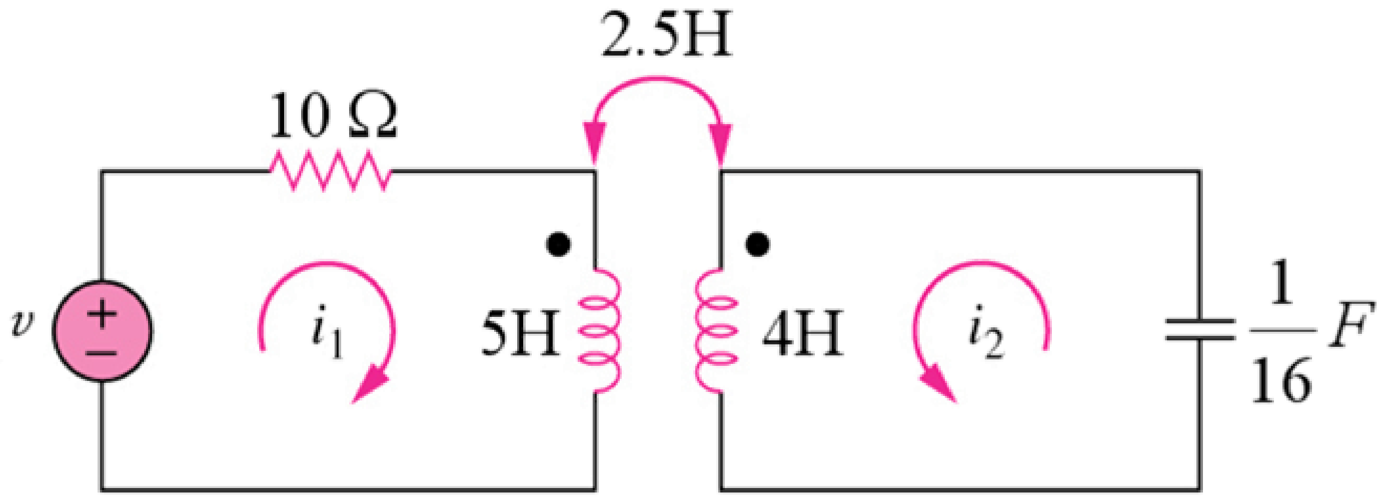
Given that

$$v(t) = 60\cos(4t + 30°)\ \text{V}$$

Determine:

a) the coupling coefficient

b) the stored energy in the coupled coils at $t = 1$ s



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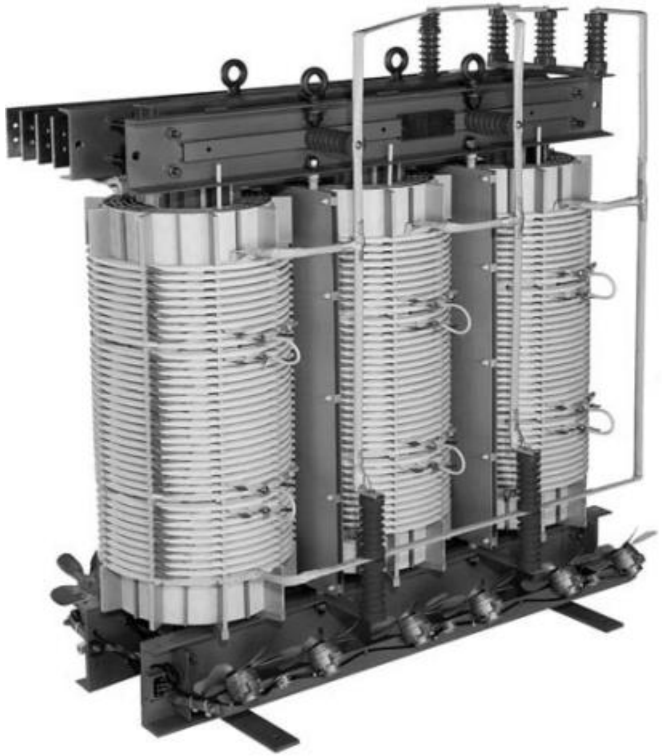
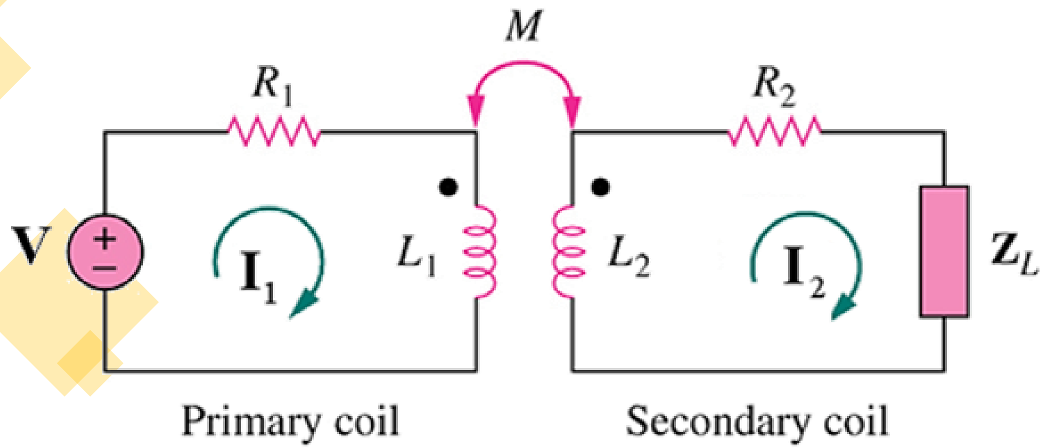


3.1 LINEAR TRANSFORMER

The transformer is LINEAR if the coils are wound on magnetically linear material, such as air - hollow transformers.

Linear transformers: flux is proportional to current in the windings.
Resistances R_1 and R_2 account for losses in the coils.
The coils are named as PRIMARY and SECONDARY.



$$\begin{cases} V = (R_1 + j\omega L_1)\mathrm{I}_1 - j\omega M \mathrm{I}_2 \\ -j\omega M \mathrm{I}_1 + (j\omega L_2 + R_2 + Z_L)\mathrm{I}_2 = 0 \end{cases}$$

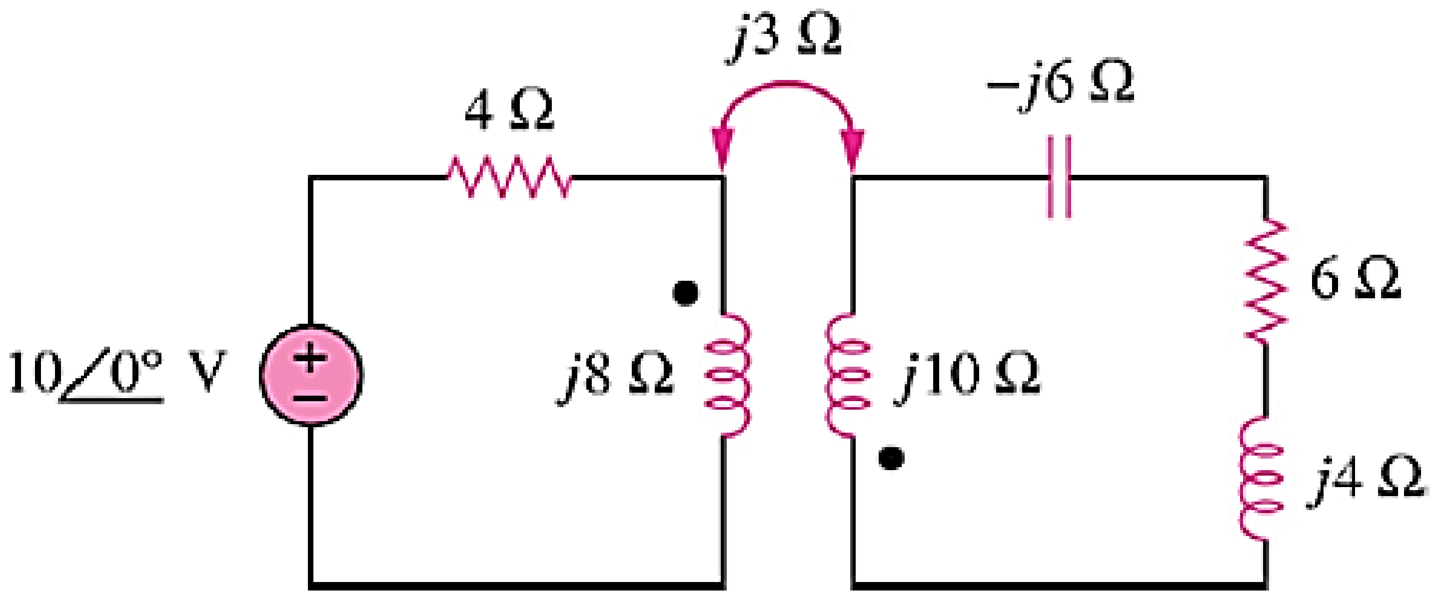
Input impedance offered at the terminals of the primary circuit:

$$Z_{in} = \frac{\mathrm{V}}{\mathrm{I}_1} = Z_{11} - \frac{(j\omega)^2 M^2}{Z_{22}} = Z_{11} + \boxed{\frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}$$

Reflected Impedance $\mathbf{Z}_R$: secondary impedance seen from the primary side

QUIZ 3.1

Find the input impedance and the current.



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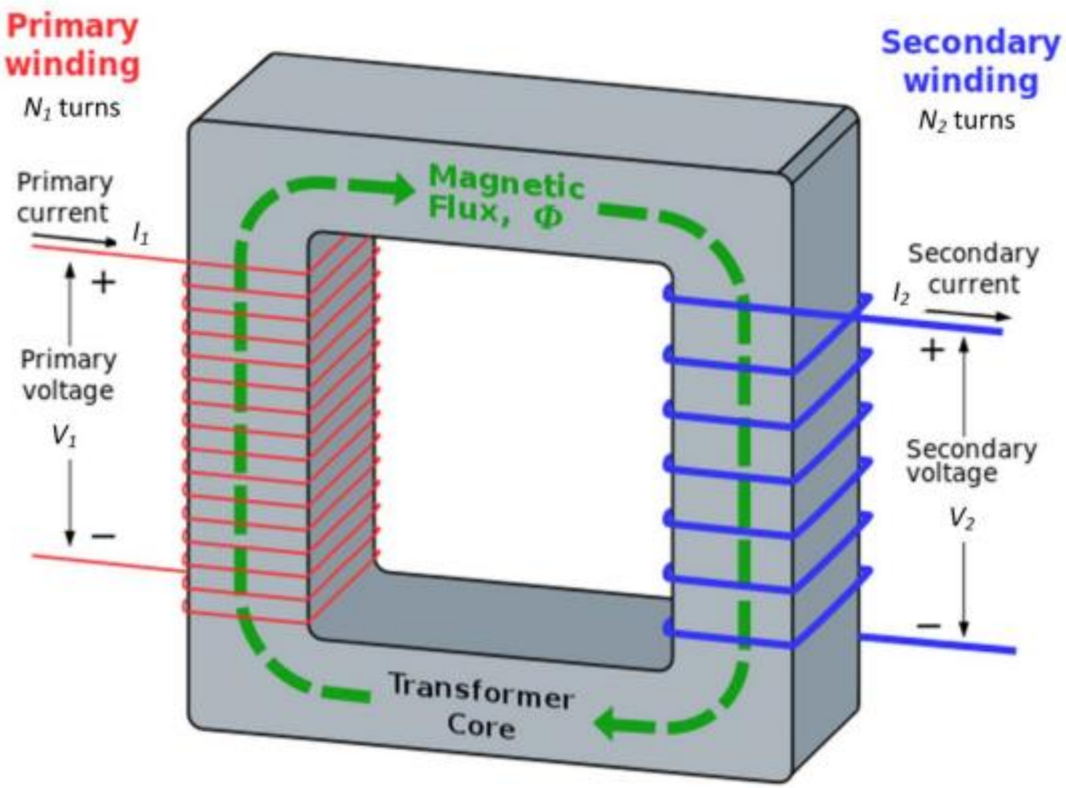
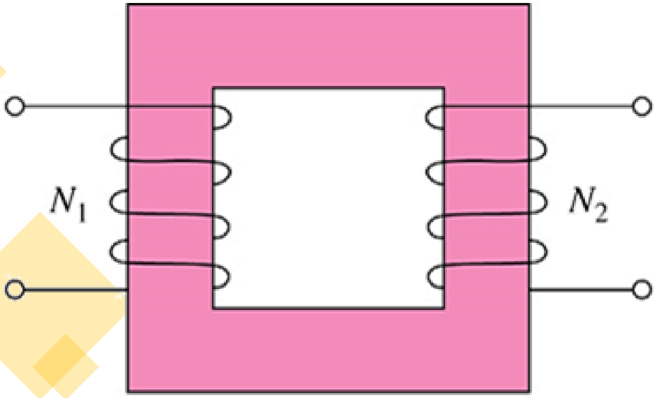


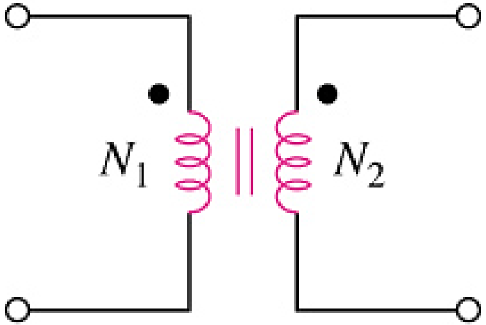
3.2 IDEAL TRANSFORMER

An ideal transformer is one with perfect coupling.

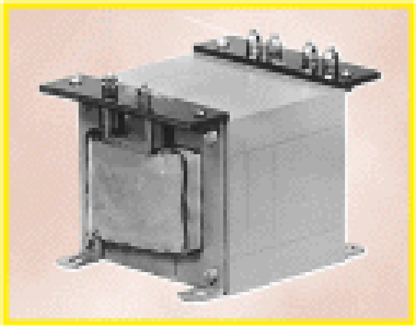
The first mesh, usually containing the source, is called the primary, while the second mesh, usually containing the load, is known as the secondary.



Ideal Transformer



Circuit Symbol



A transformer is ideal if:

- ✓ Large reactance coils
- ✓ Unity coupling $k = 1$
- ✓ Coils are lossless (R_1=R_2=0)

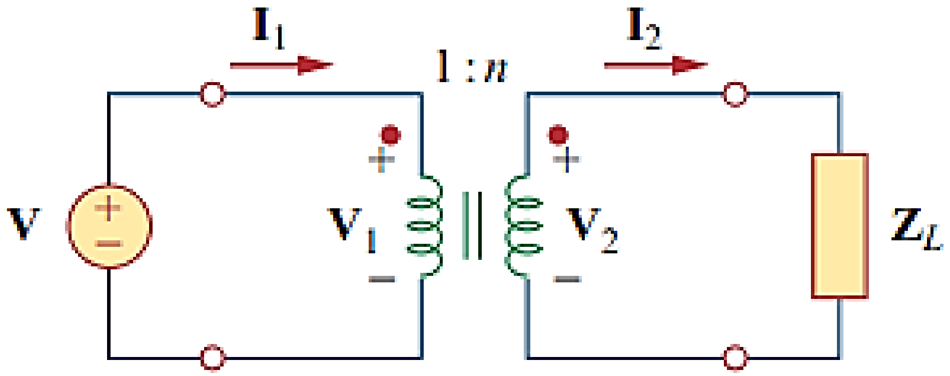
3.2.1 INTRODUCTION

The input and output voltages and currents of an ideal transformer are related only by the turns ratio n.

Step-up transformer if $n > 1$

Step-down transformer if $n < 1$

Isolation transformer if $n = 1$



$$\left.\begin{aligned} V_1 &= j\omega L_1 I_1 - j\omega M I_2 \\ V_2 &= j\omega M I_1 - j\omega L_2 I_2 \end{aligned}\right\} \quad I_1 = \frac{V_1 + j\omega M I_2}{j\omega L_1}, \quad V_2 = j\omega M \frac{V_1 + j\omega M I_2}{j\omega L_1} - j\omega L_2 I_2$$

$$\because M = k\sqrt{L_1 L_2} = \sqrt{L_1 L_2}$$

$$\therefore V_2 = \frac{\sqrt{L_1 L_2} V_1}{L_1} + \frac{j\omega L_1 L_2 I_2}{L_1} - j\omega L_2 I_2 = \sqrt{\frac{L_2}{L_1}} V_1$$

$$\therefore \frac{V_2}{V_1} = \sqrt{\frac{L_2}{L_1}} = n = \frac{N_2}{N_1}$$

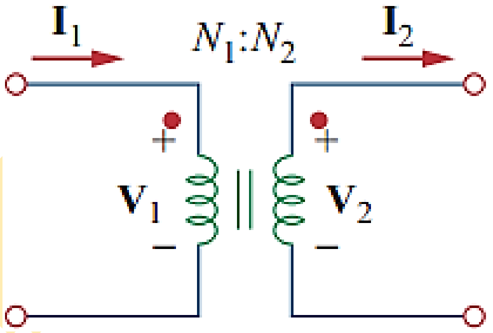
When applying a sinusoidal voltage:

$$\left.\begin{aligned} v_1 &= N_1 \frac{d\Phi}{dt} \\ v_2 &= N_2 \frac{d\Phi}{dt} \end{aligned}\right\} \quad \frac{v_1}{v_2} = \frac{N_1}{N_2} = \frac{1}{n}$$

3.2.2 TRANSFORMER DOT CONVENTION

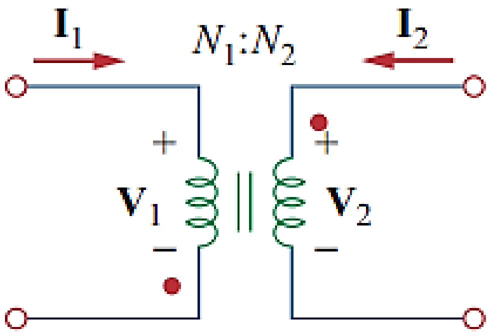
Transformer DOT convention is used to assign the proper polarity of the output variables.

- If V_1 and V_2 are both + or both – at the dotted terminals, then use $+n$
- If I_1 and I_2 both enter or both leave the dotted terminals, then use $-n$



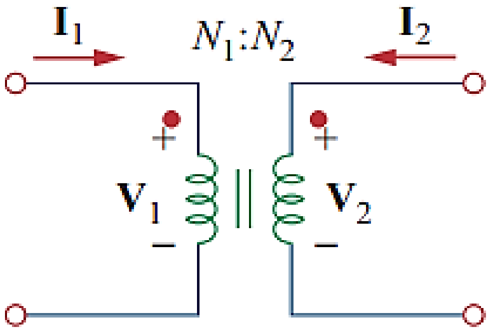
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = n$$



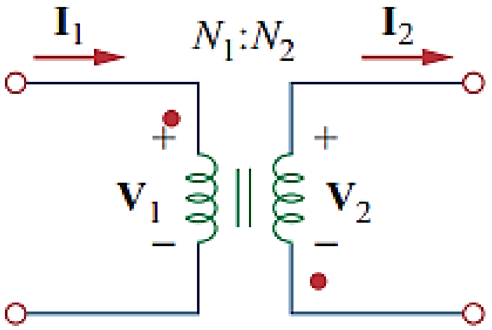
$$\frac{V_2}{V_1} =$$

$$\frac{I_1}{I_2} =$$



$$\frac{V_2}{V_1} =$$

$$\frac{I_1}{I_2} =$$

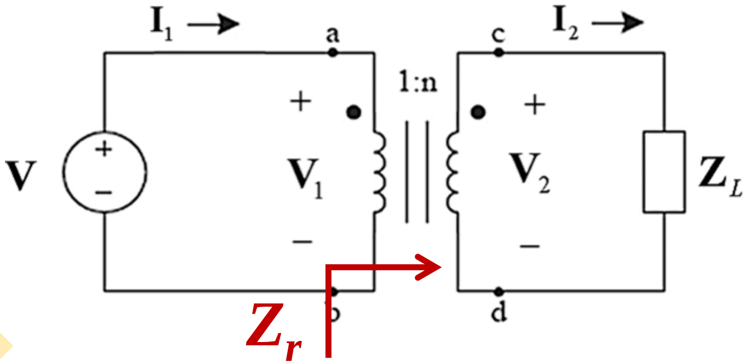


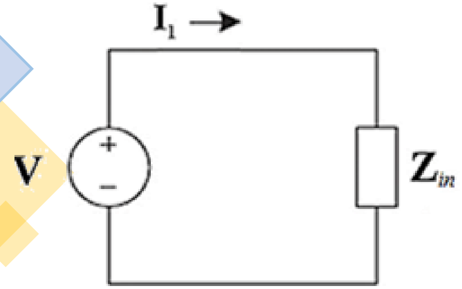
$$\frac{V_2}{V_1} =$$

$$\frac{I_1}{I_2} =$$

3.2.3 REFLECTED IMPEDANCE

Reflected Impedance





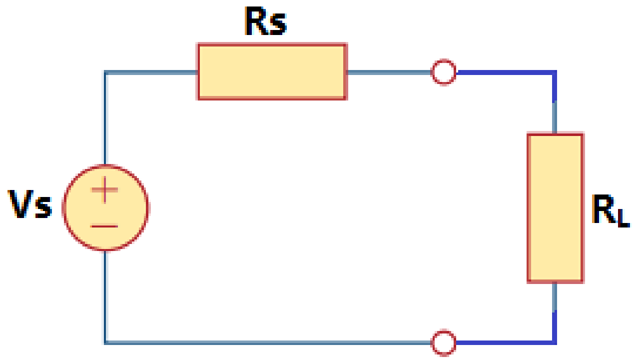
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = n$$

$$Z_r = \left|\frac{V_1}{I_1}\right| = \frac{V_2}{I_2}\frac{1}{n^2} = \frac{Z_L}{n^2}$$

Example:

An ac source V_s with internal resistance R_S is connected to a load.

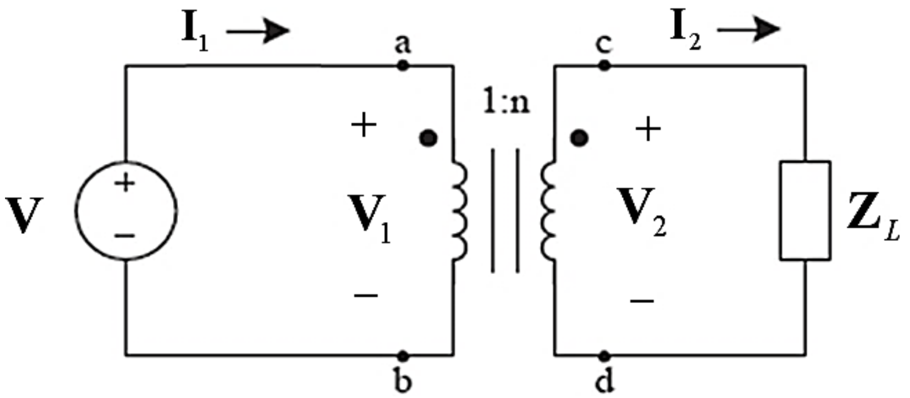


For the resistive load R_L in the figure, the maximum power delivered to R_L happened when $\boldsymbol{R_L = R_s}$.

The ability of an ideal transformer to transform a given impedance to another value is very useful in impedance matching.

QUIZ 3.2

1. Given that the voltage ratio V_1/V_2 of an **ideal** transformer is 220/110 V, if the second grade connect with a resistance with 55 Ω, then the input impedance is:



(a) -13.75 Ω (b) 27.5 Ω

(c) 220 Ω (d) 13.75 Ω

2. An **ideal** transformer has an 800 Ω internal resistor. To provide a maximum power to a resistive load with 8 Ω (the only component in the circuit), turns ratio used in this ideal transformer should be:

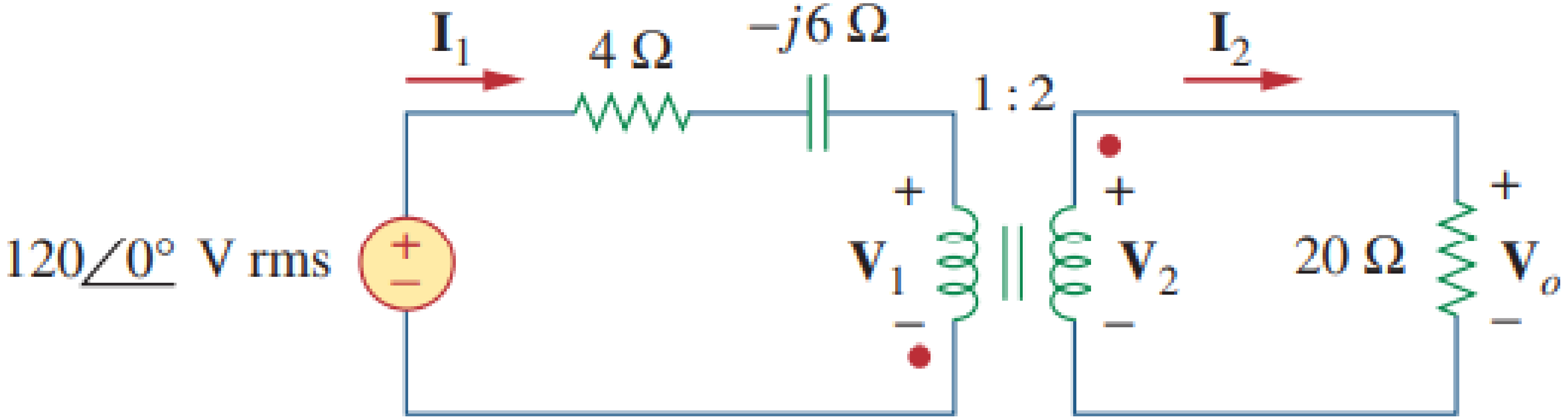
(a) n =10 (b) n = 0.01

(c) n = 0.1 (d) n =100

QUIZ 3.3

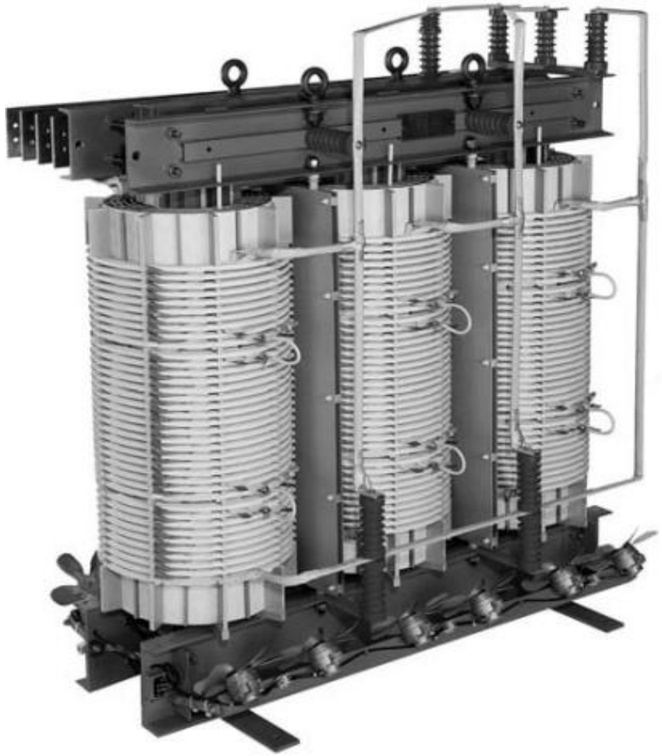
For an ideal transformer shown in the figure below, find:

a) the source current I_1
b) the output voltage V_o
c) the complex power supplied by the source



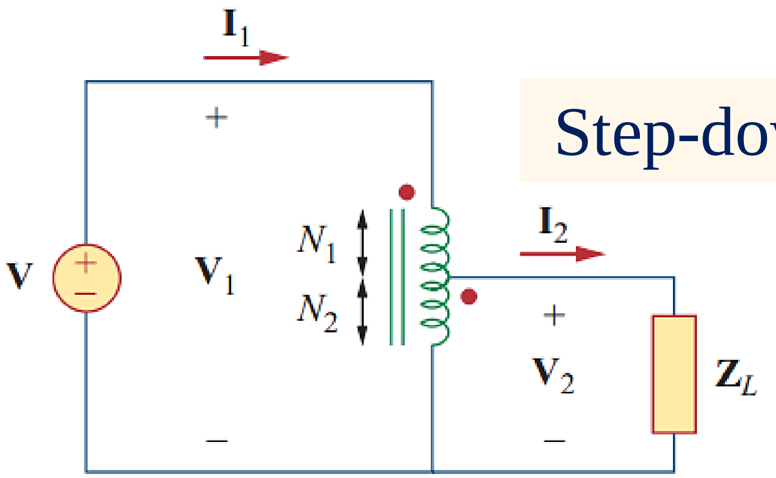
OUTLINE

- Introduction
- Mutual Inductance
  - ✓ Dot Convention
  - ✓ Coupling Coefficient
  - ✓ Energy in a Coupled Circuit
- Transformers
  - ✓ Linear Transformers
  - ✓ Ideal Transformers
  - ✓ Auto Transformers



3.3 AUTO TRANSFORMER

An autotransformer is a transformer in which both the primary and the secondary are in a single winding.



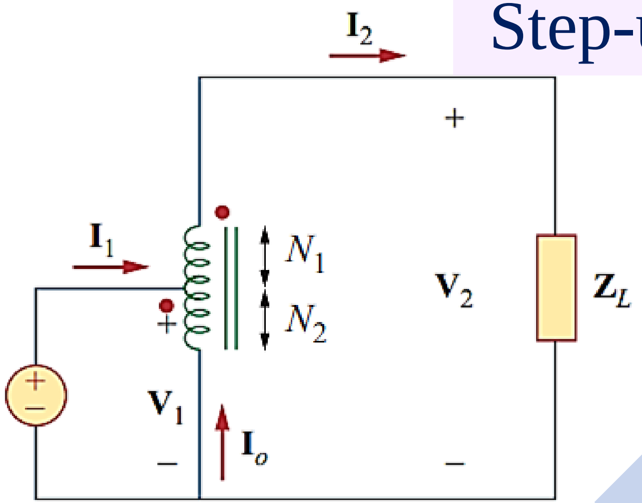
Step-down

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1 + N_2}$$

Advantages:

- ✓ transfer larger apparent power
- ✓ smaller and lighter than an equivalent two-winding transformer

Step-up



$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_1 + N_2}{N_2}$$

Disadvantage:

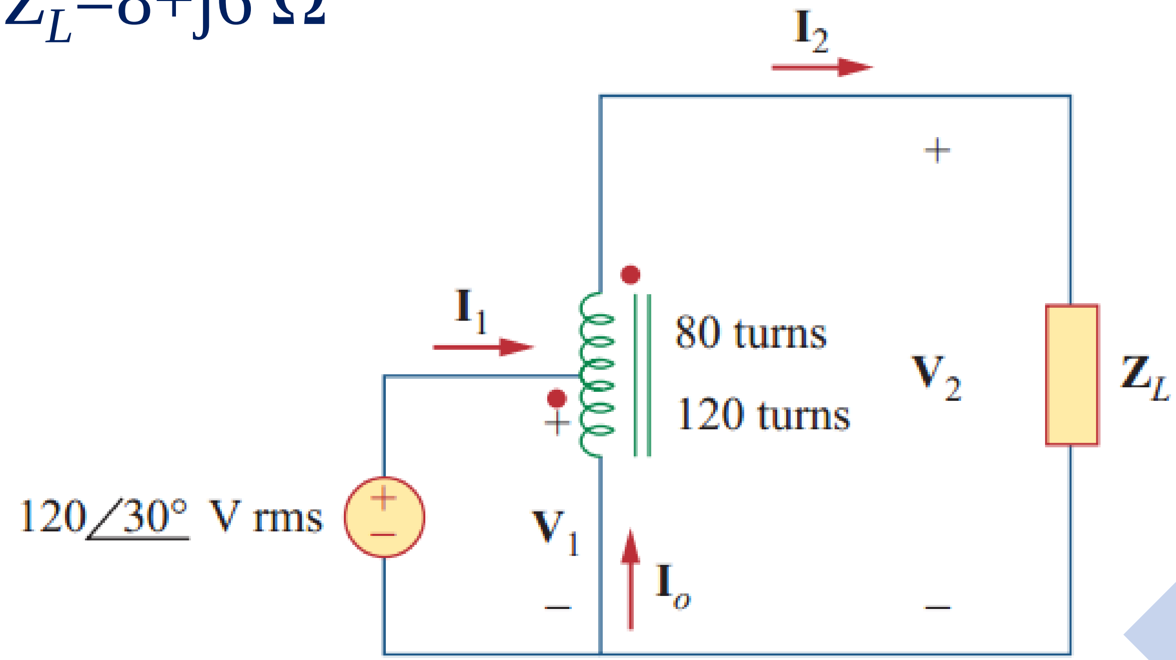
Both the primary and secondary windings are one winding, so electrical isolation is lost (no direct electrical connection)

QUIZ 3.4

For the autotransformer circuit in the figure, calculate the following:

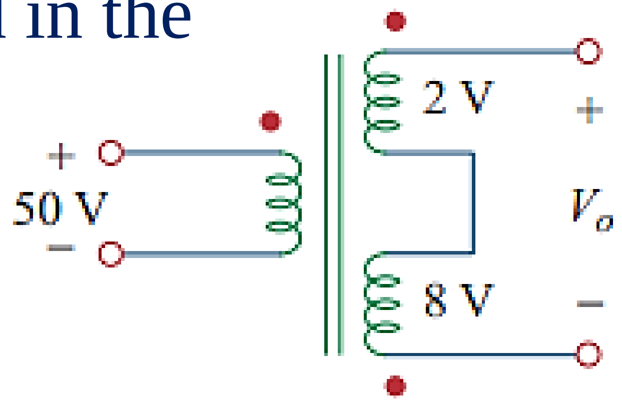
a) I_1, I_2 and I_o if Z_L=8+j6 Ω

b) the average power on the load if Z_L=8+j6 Ω



QUIZ 3.5

1. A three-winding transformer is connected as portrayed in the figure. The value of the output voltage is:



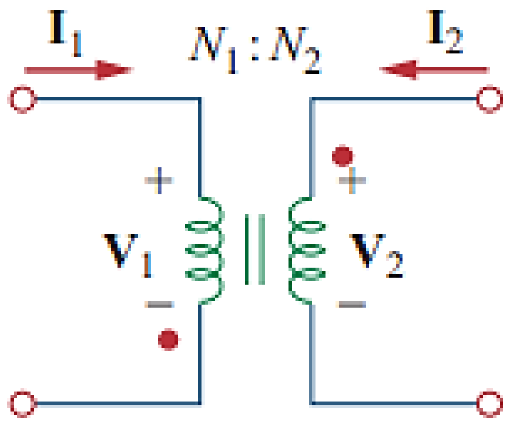
(a) -6 V (b) 10 V

(c) 6 V (d) -10 V

2. The ideal transformer in the figure below has $N_2/N_1 = 10$. The ratio V_2/V_1 is:

(a) -0.1 (b) 10

(c) -10 (d) 0.1



NEXT…

Three-phase Systems