CAN102 Electromagnetism and Electromechanics

Lecture-12 Inductors and Summary of Electromagnetism

Zhao Wang

Zhao.wang@xjtlu.edu.cn

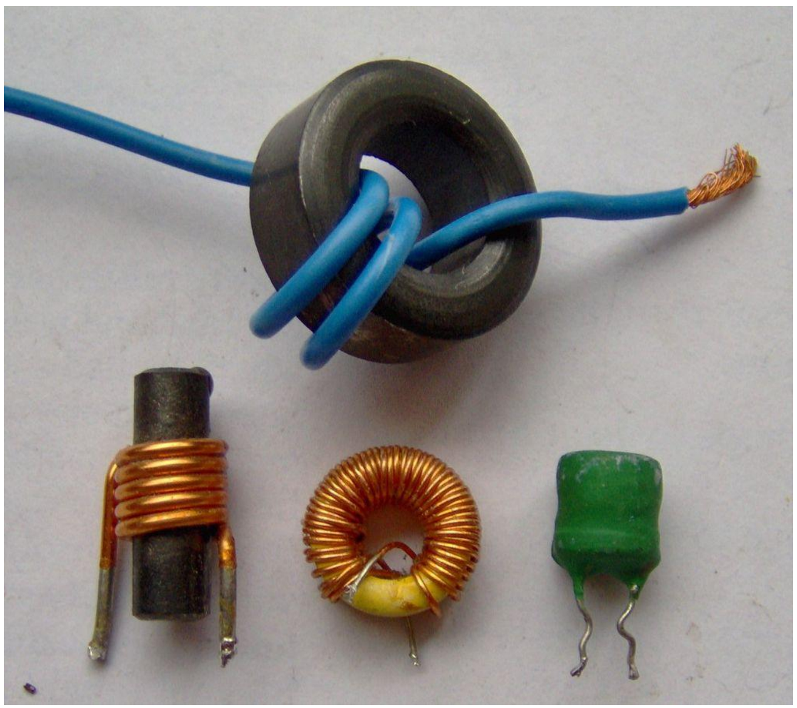
Room EE322

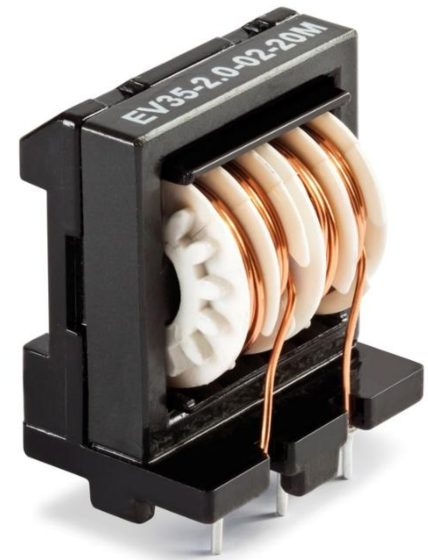
Outline

- 1. Inductance
  - Inductors
  - Self-inductance
  - Mutual-inductance
  - Energy stored
- 2. Summary
  - Relationship to programmes
  - Relationship to other modules
  - Internal relationships

1. Inductors

- An inductor, also called a coil, is a passive two-terminal electrical component that stores energy in a magnetic field when electric current flows through it.





Typically, an inductor is a conducting wire shaped as a coil, the loops helping to create a strong magnetic field inside the coil.

1.1 Self-Inductance

$$emf = -N\frac{d\Phi}{dt}$$

Changing current ⇨ Changing flux ⇨ emf (oppose)

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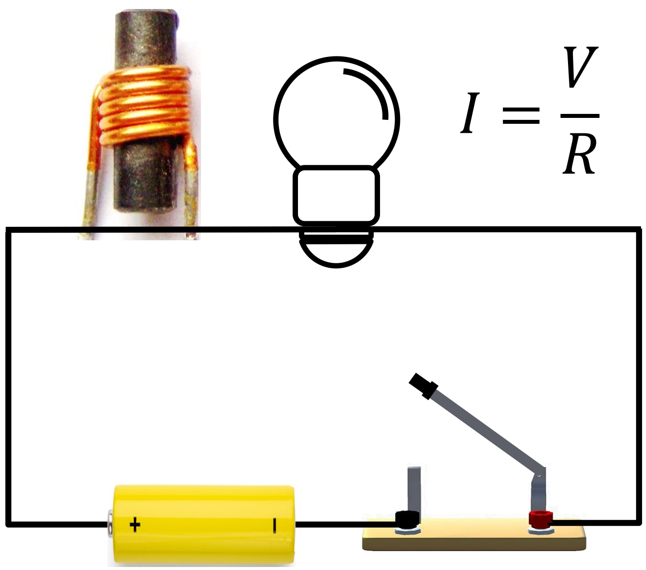


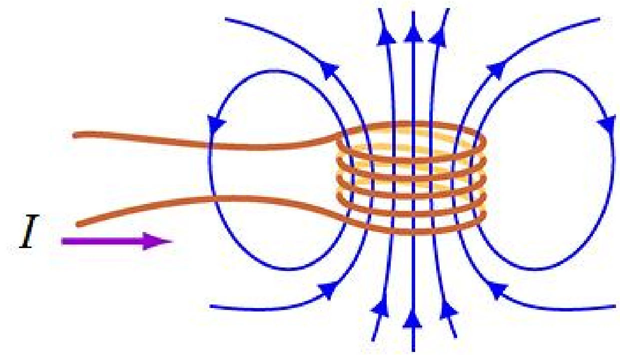
$$I = \frac{V}{R}$$

$$emf = -N\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

$$L = N\frac{\Phi}{I} \quad \mathrm{V\cdot s/A} = \mathrm{H\ (Henry)}$$

1.1 Self-inductance

- Consider a coil consisting of N turns and carrying current I. If current is steady, magnetic flux through the loop remains constant. If I changes with time, then an induced emf arises to oppose the change. The property of the loop in which its own magnetic field opposes any change in current is called "***self-inductance***" and the emf generated is called the ***self-induced emf*** or ***back emf***.

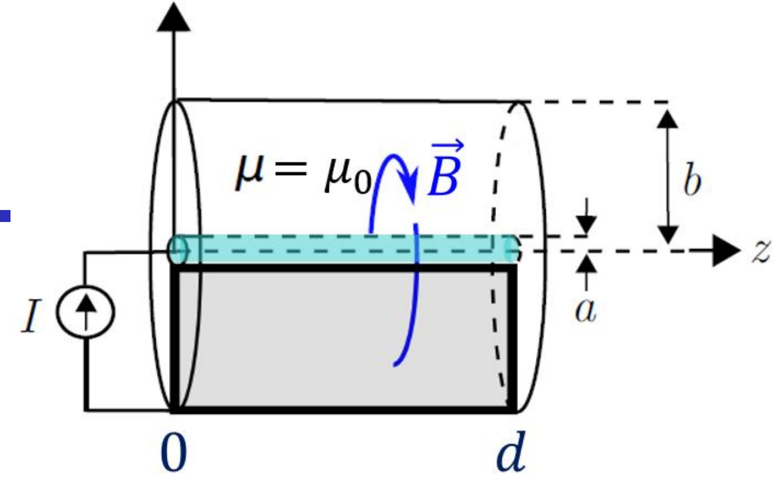


- From Faraday's law: $$emf_L = -N\frac{d\Phi_B}{dt}$$
- The self-induced *emf*: $$L\frac{dI}{dt} = N\frac{d\Phi_B}{dt}$$
- So, the self-inductance: $$\boxed{L = N\frac{\Phi_B}{I}}$$

the ratio of the total flux linkages to the current

SI unit: H (Henry) = 1 Wb/A = 1 V·s/A

Example 1 - Coaxial cable



- Calculate the inductance per meter length of a coaxial cable of inner radius a and outer radius b.
  - The magnetic flux density is:

$$B_\varphi = \frac{\mu_0 I}{2\pi r} \quad (a < r < b)$$

  - The magnetic flux contained between the conductors for length d:

$$\Phi_B = \iint \vec{B}\, d\vec{s} = \mu_0 \int_0^d \int_a^b \frac{I}{2\pi r} dr dz\, \hat{\varphi} \cdot \hat{\varphi} = \frac{\mu_0 I d}{2\pi} \ln\left(\frac{b}{a}\right)$$

  - So the inductance for a length d is:

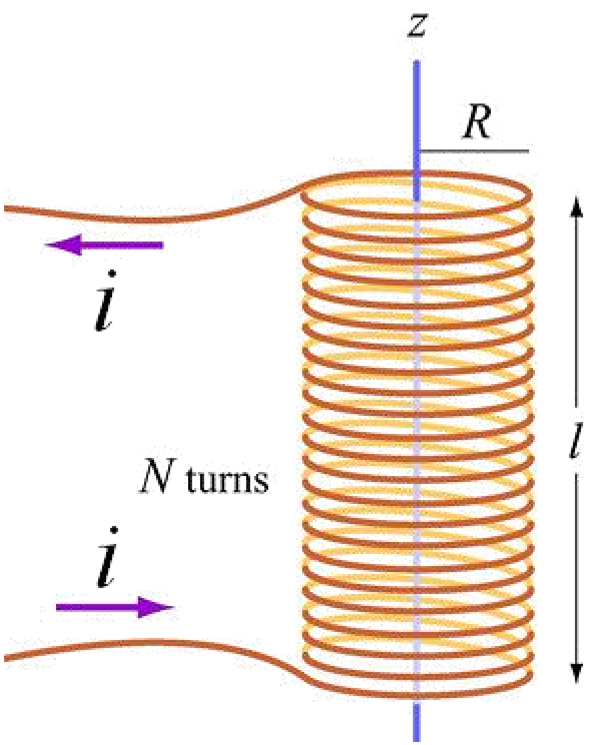
$$L = N\frac{\Phi_B}{I} = \frac{\mu_0 d}{2\pi} \ln\left(\frac{b}{a}\right)$$

On a per-meter basis:

$$L_l = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \ \text{H/m}$$

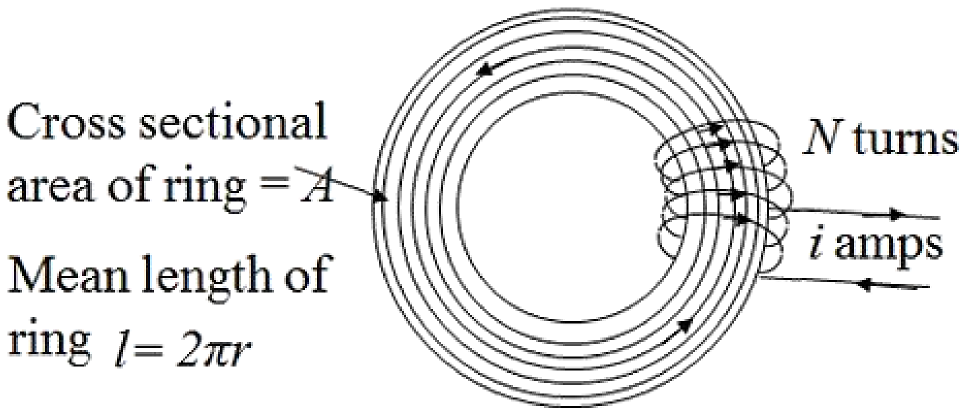
Quiz 1 Solenoid

- Calculate the inductance of a solenoid with N turns, length l, and radius R with a current i flowing through each turn.



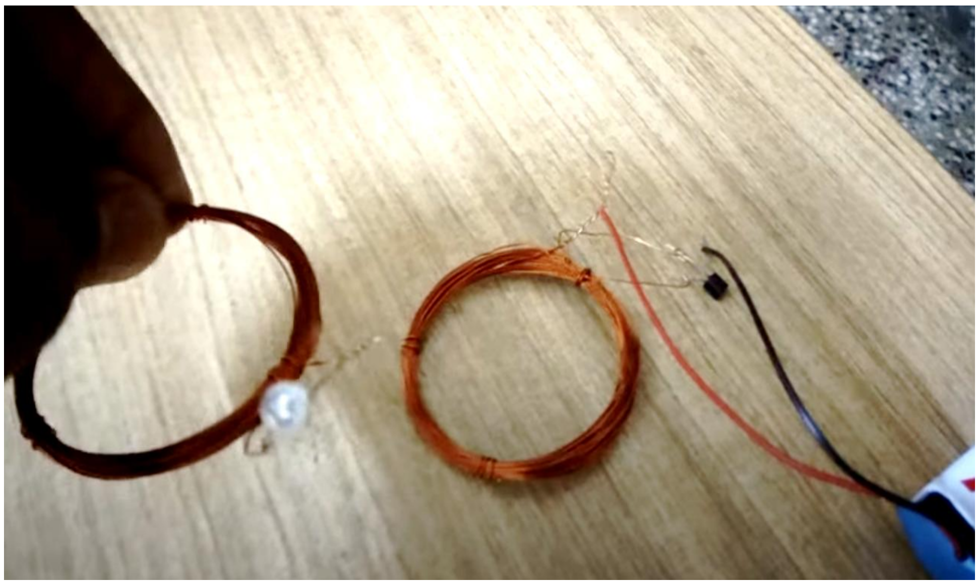
Quiz 2 Toroid

- Consider a toroid with high μ material. Find its self-inductance.



1.2 Mutual Inductance

- Mutual inductance is the effect of Faraday's law of induction for one device upon another, such as the primary coil in transmitting energy to the secondary in a transformer.



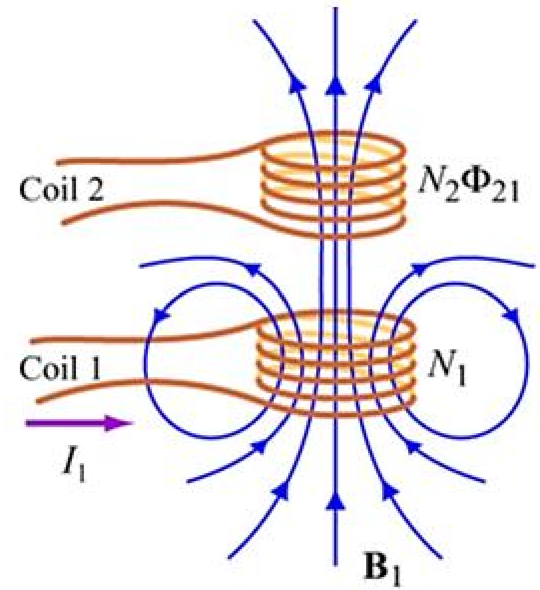
1.2 Mutual inductance

- Suppose two coils are placed near each other.
- Some of the magnetic field lines through coil 1 will also pass coil 2.
  - Let Φ_{21} denote the magnetic flux through one turn of coil 2 due to I_1.
  - By varying I_1 with t, there will be an induced *emf* associated with the changing magnetic flux in coil 2:

$$v_2 = N_2 \frac{d\Phi_{21}}{dt}$$

  - The rate of change of Φ_{21} in coil 2 is proportional to the time rate of the change of the current in coil 1:

$$N_2 \frac{d\Phi_{21}}{dt} = N_2 \frac{d\Phi_{21}}{dI_1} \cdot \frac{dI_1}{dt} = M_{21} \cdot \frac{dI_1}{dt}$$



1.2 Mutual inductance

- Similarly, the induced emf in coil 1 due to current change in coil 2:

$$v_1 = N_1 \frac{d\Phi_{12}}{dt}$$

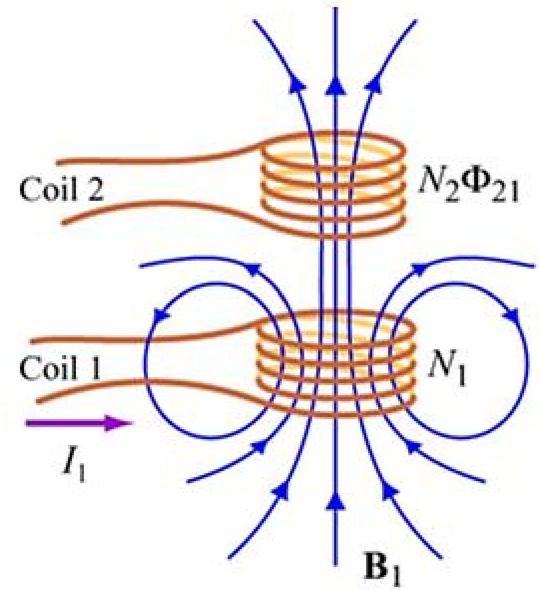
- This changing flux is also proportional to the changing current in coil 2:

$$N_1 \frac{d\Phi_{12}}{dt} = N_1 \frac{d\Phi_{12}}{dI_2} \cdot \frac{dI_2}{dt} = M_{12} \cdot \frac{dI_2}{dt}$$

- The proportionality constant M_{12} and M_{21} are equal:

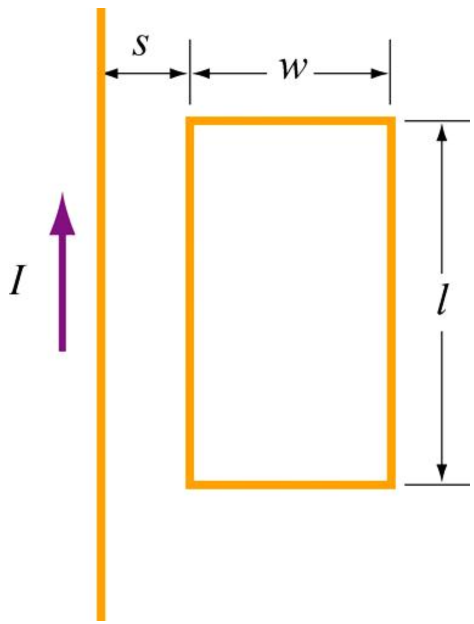
$$\boxed{M_{12} = M_{21} \equiv M}$$

Mutual inductance



Example 2

- An infinite straight wire carrying current I is placed to the left of a rectangular loop of wire with width w and length l. Determine the mutual inductance of the system (N=1).



The total magnetic flux is:

$$\Phi = \int_{s}^{s+w} \frac{\mu_0 I}{2\pi r} l dr = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{s+w}{s}\right)$$

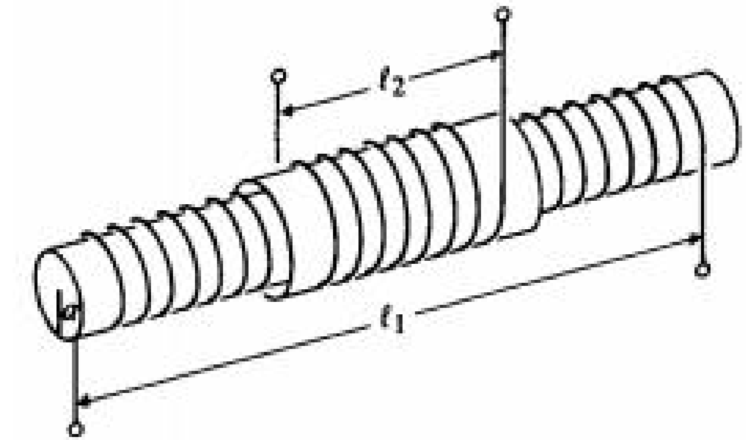
The mutual inductance is:

$$M = \frac{\Phi}{I} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{s+w}{s}\right)$$

M depends only on the geometrical factors of the system (l, s, w) and is independent of the current.

Quiz 3

- Two coils of N_1 and N_2 turns are wound concentrically on a straight cylindrical core of radius a and permeability μ. The windings have lengths l_1 and l_2, respectively.
- Find the mutual inductance between the coils.



1.3 Energy in a magnetic field

- In an electric field
  - The energy density:

$$w_e = \frac{1}{2}\vec{\mathbf{D}}\cdot\vec{\mathbf{E}}$$

  - The total electric energy stored in a medium:

$$W_e = \frac{1}{2}\int_v \vec{\mathbf{D}}\cdot\vec{\mathbf{E}}\,dv$$

  - In a capacitor

$$W_C = \frac{1}{2}CV^2$$

- In a magnetic field
  - The energy density:

$$w_m = \frac{1}{2}\vec{\mathbf{B}}\cdot\vec{\mathbf{H}}$$

$$= \frac{1}{2}\mu H^2 = \frac{1}{2\mu}B^2$$

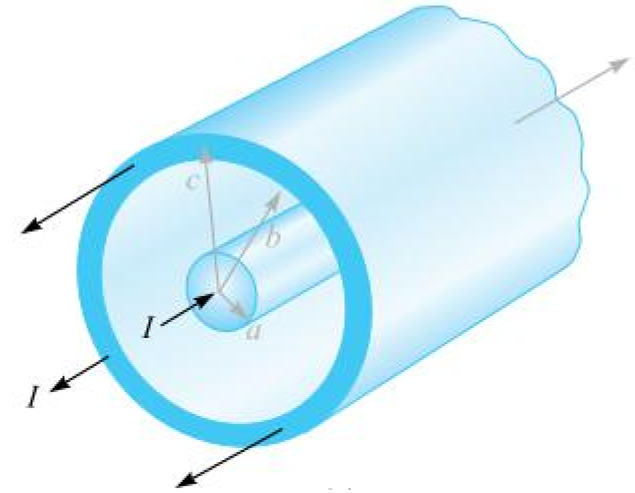
  - The total electric energy stored in a medium:

$$W_m = \int_v w_m\,dv$$

  - In an inductor:

$$W_L = \frac{1}{2}LI^2$$

Example 3



- Calculate the energy stored in a unit-length coaxial cable of inner radius *a* and outer radius *b*
- Solution:
  - The magnetic field intensity should be

$$H_\phi = \frac{I}{2\pi\rho} \quad (a < \rho < b)$$

  - So the total energy stored in this coaxial cable is

$$W_m = \int_v \frac{1}{2}\mu_0 H^2 dv = \frac{\mu_0}{2}\int_0^1 dz \int_0^{2\pi}\int_a^b \left(\frac{I}{2\pi\rho}\right)^2 \rho d\varphi d\rho = \frac{\mu_0 I^2}{4\pi} ln\left(\frac{b}{a}\right) \; (W)$$

  - which agrees with the result calculated from $W_L = \frac{1}{2}LI^2$

1.4 Current – voltage relationship

- Any change in the current through an inductor creates a changing flux, inducing a voltage across the inductor. By Faraday's law of induction, the voltage induced by any change in magnetic flux through the circuit is given by:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- And we know that $\Phi_B = LI$, substitute into the equation above, get:

$$\mathcal{E} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

  – which means "the current on the inductor is always continuous";

  – Also points out that the voltage "induced" on the inductor is proportional to the inductance and the changing rate of the current flowing through the inductor.

2.1 Relationship to programmes

- Telecommunications:
  - Communication Networks
  - Signal Processing
  - Circuits and Embedded systems
  - Microwave and RF engineering
- Electrical Engineerings:
  - Electric Machinery
  - Power Systems
    - Planning and Operation
    - Transient & Relay
  - High Voltage & Insulation
- Electronic Science and Technology:
  - Solid-state Electronics
    - Devices
    - Materials
  - IC Design

2.2 Relationship to other Modules

Year 1		Year 2		Year 3		Year 4	
S1	S2	S3	S4	S5	S6	S7	S8
MTH013	MTH008	MTH101		MTH201			
MTH007			MTH102				
	PHY002		CAN102	CAN209	CAN206	CAN305	CAN306
						CAN307	
		EEE103			EEE210		EEE335
					EEE213		EEE340
		EEE109		EEE211			
			EEE112	EEE201		EEE337	EEE332
			EEE104		EEE205	EEE339	
				CAN207	CAN202	CAN303	CAN309
							CAN308
							CAN310

2.3 Relationships, comparison and analogy

1. Electric vs Magnetic
2. Fields vs Sources
3. Integral vs Differential
4. Charges vs Currents
5. Fields vs Circuits
6. Potential vs Energy
7. Power vs Energy
8. Static vs Time-varying
9. DC vs AC
10. All units

Next ...

- Week 7
  - No lecture
  - Leave for you to review
  - Office hours are still available

- Week 8-12
  - Second half, Electromechanics (电机)

- Week 13
  - Revision class (1 or 2)