CAN102
Electromagnetism and Electromechanics

2023/24-S2

Lecture 13 Magnetic Circuits

Jingchen Wang
SAT
jingchen.wang@xjtlu.edu.cn

Module information

Module teacher: Jingchen Wang
Email: Jingchen.wang@xjtlu.edu.cn
Location: EE226
Office hour: Thursday 13:00-15:00
Friday 13:00-15:00

Reference book:
Electric Machinery Fundamentals 5 th , by Stephen J. Chapman

Materials:
Lecture slides and recorded videos
Self-practice questions and solutions
Useful resources from external links

Motivation

- Powering everything from industrial machinery to household appliances, such as electric motors
- Converting mechanical energy into electrical energy, such as generators
- Understanding the interplay of electrical and mechanical systems
- Design and analyzing electromechanical systems, such as robotic actuators and control systems

Module Syllabus

Week 8-12 Electromechanics (40%)

1 Magnetic Circuits	4 DC Machinery Fundamentals	7 AC Machinery Fundamentals
2 Transformers	5 DC motors	8 Synchronous Generators
3 Linear DC machine	6 DC generators	9 Synchronous /Induction Motors
Lecture 13-15	Lecture 16-18	Lecture 19-22
Exam 20%	10%	10%

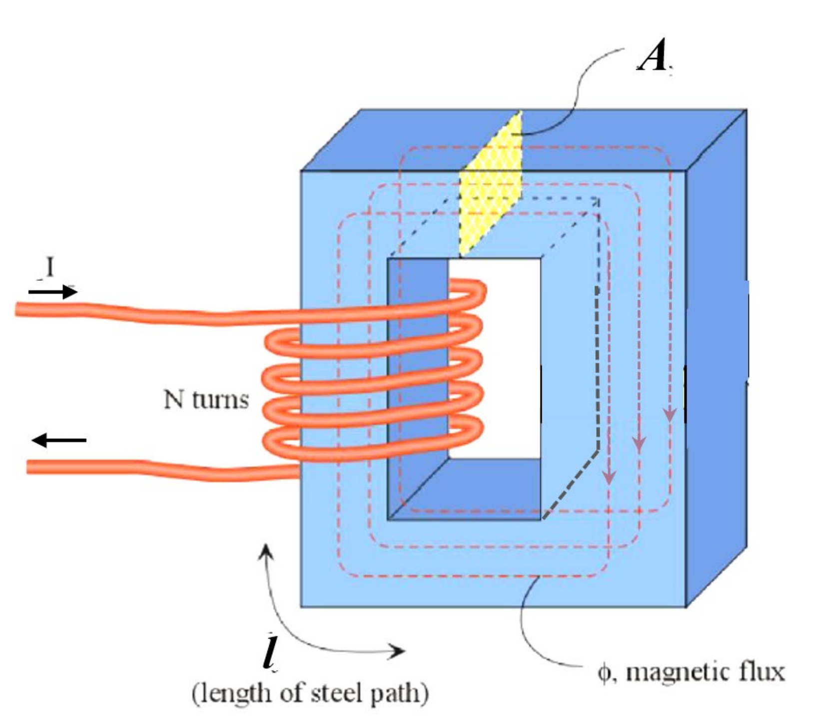
Magnetic Circuits

Consider a coil with N turns, wound onto a core carries a current I

It will generate magnetomotive force (mmf/ $\mathcal{F}$)

$$\boxed{F = NI}$$

SI Unit: Ampere-turns



If l represents the length of steel path, according to Ampere's law (Lecture 9 page 24),

$$F = Hl = NI$$

Where ***H*** is magnetic field intensity

Ampere's circuital law (Ampere's Law):

The line integral of the magnetic field intensity $\vec{H}$ around a closed path equals the current enclosed.

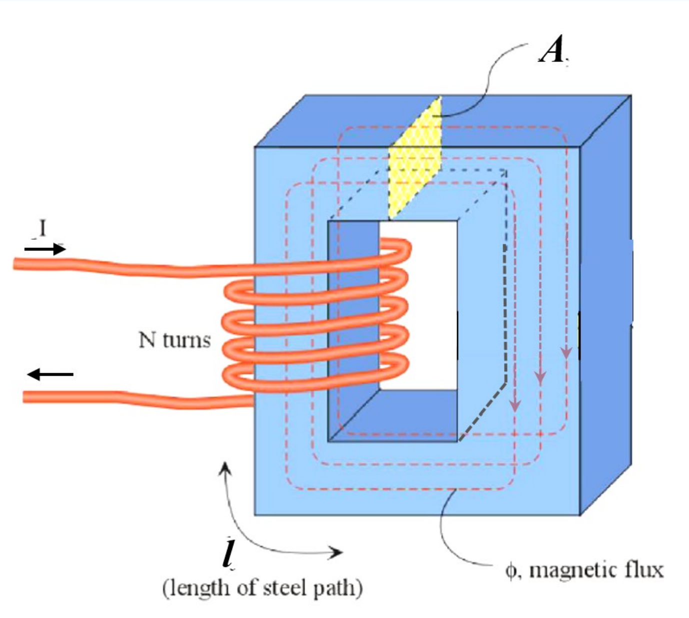
Magnetic Circuits

Since magnetic flux density ***B*** is defined as

$$B = \mu H = \mu_r \mu_0 H$$

Where μ_r is the relative permeability of material

$$\mu_0 = 4\pi \times 10^{-7} \;\; H/m$$



The total flux Φ in a given area *A* is provided as $\Phi = BA$

$$F = Hl = \frac{B}{\mu_r \mu_0} l = \frac{\Phi}{\mu_r \mu_0 A} l = \Phi \frac{l}{\mu_r \mu_0 A} = \Phi \Re$$

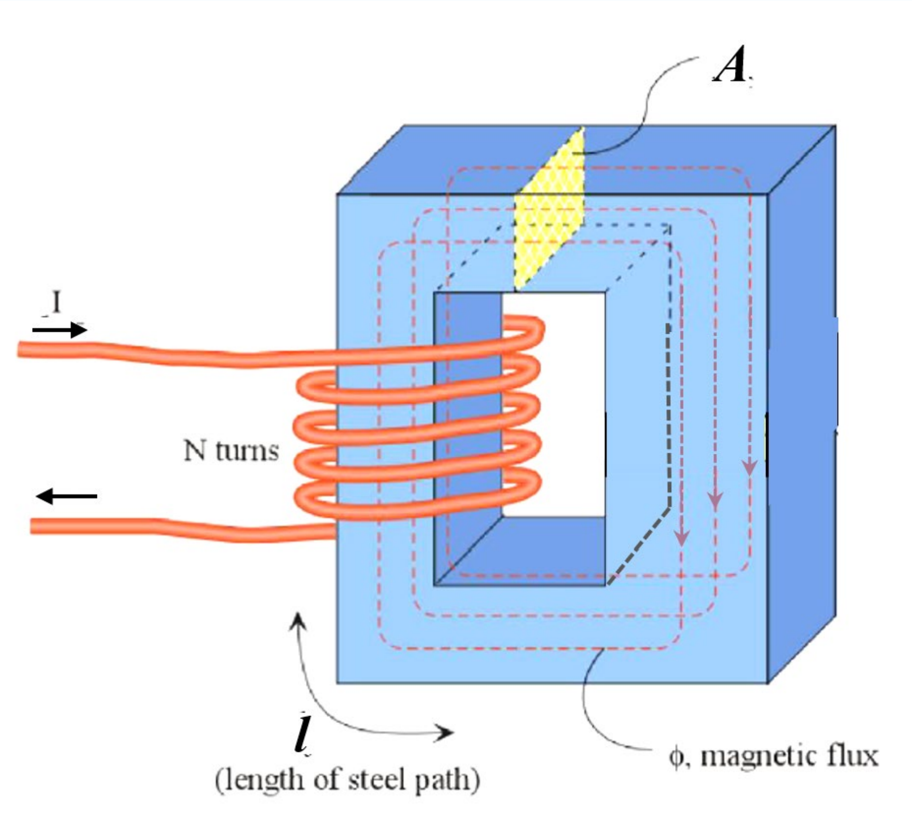
Where $\Re$ is the reluctance of the magnetic circuit

Magnetic Circuits

Reluctance of the magnetic circuit

$$\Re = \frac{l}{\mu_r \mu_0 A}$$

SI unit: $\mathrm{A \cdot turns/Wb}$



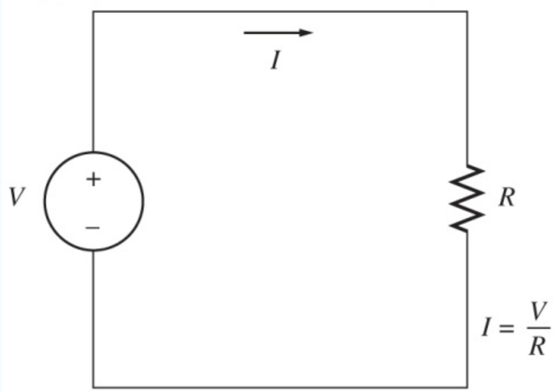
l is the mean path length

μ_r is the relative permeability of material

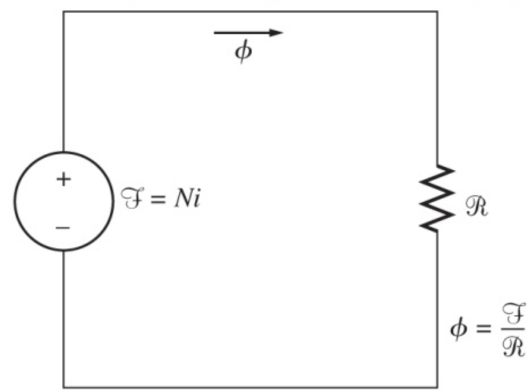
A is the perpendicular area to flux density

Comparison between electric and magnetic circuit

Electric Circuit



Magnetic Circuit



Voltage	V	(V)	mmf	$\mathcal{F}$	(A-t)
Current	I	(A)	Flux	ϕ	(Wb)
Resistance	R	(Ω)	Reluctance	$\mathcal{R}$	(H^{-1})
Electric field intensity	E	(V/m)	Magnetic field intensity	H	(A-t/m)
Current density	J	(A/m^2)	Flux density	B	(T)
Conductivity	σ	(S/m)	Permeability	μ	(H/m)

$$V = IR$$

$$J = \sigma E$$

$$R = \frac{\ell}{\sigma A} = \frac{1}{G}$$

Conductance

$$\mathcal{F} = \phi \mathcal{R}$$

$$B = \mu H$$

$$\mathcal{R} = \frac{\ell}{\mu A} = \frac{1}{\mathcal{P}}$$

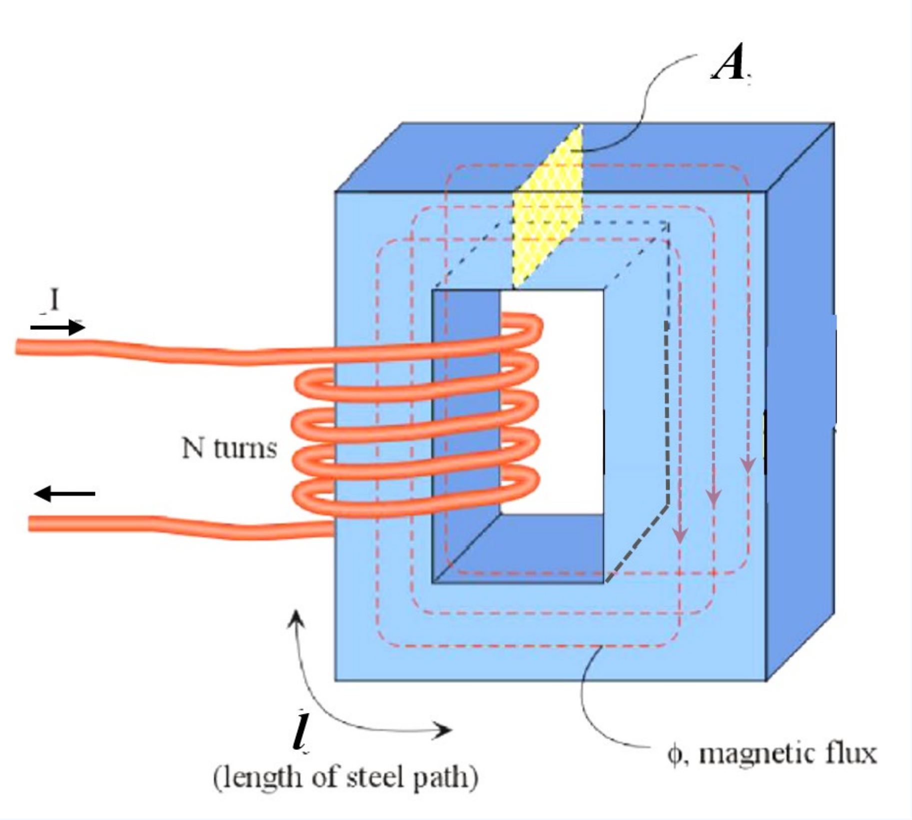
permeance

It is a magnetic analog of conductance

Magnetic Circuits

Example

Consider a square-shaped core of cross-sectional area 5×10^{-4} m^2 and length 0.5m, made of steel with relative permeability of 3,500. It is wound with a coil of 250 turns that carries 2 amps. Find the flux in the core.



Example

Consider a square-shaped core of cross-sectional area 5 x 10^{-4} m^2 and length 0.5m, made of steel with relative permeability of 3,500. It is wound with a coil of 250 turns that carries 2 amps. Find the flux in the core.

Solution using first principles:

Ampere's Law: $H \cdot l = N \cdot I$

$$H = \frac{250 \times 2}{0.5} = 1000 \quad \text{A/m}$$

$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 3500 \times 1000$$
$$= 4.4 \quad \text{Wb/m}^2$$

So the flux in the circuit is

$$\Phi = B \times A = 4.4 \times 5 \times 10^{-4}$$
$$= 2.2 \times 10^{-3} \quad \text{Wb}$$

Magnetic Circuits

Example

Consider a square-shaped core of cross-sectional area 5 x 10^{-4} m^2 and length 0.5m, made of steel with relative permeability of 3,500. It is wound with a coil of 250 turns that carries 2 amps. Find the flux in the core.

Solution using circuit reluctance

$F = NI = 250 \times 2 = 500$ ampere-turns

The reluctance of the circuit: $\boxed{\Re = \dfrac{l}{\mu_0 \mu_r A}}$

$$\frac{0.5}{4\pi \times 10^{-7} \times 3500 \times 5 \times 10^{-4}} = 2.3 \times 10^5 \quad \text{A} \cdot \text{turns/Wb}$$

So the flux in the circuit:

$$\Phi = \frac{F}{\Re} = \frac{500}{2.3 \times 10^5} = 2.2 \times 10^{-3} \quad \text{Wb}$$

Solution using first principles:

Ampere's Law: $H \cdot l = N \cdot I$

$$H = \frac{250 \times 2}{0.5} = 1000 \quad \text{A/m}$$

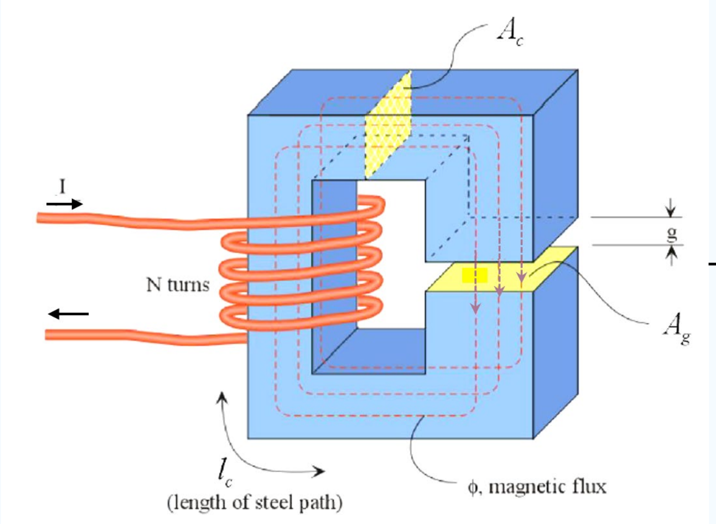
$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 3500 \times 1000$$
$$= 4.4 \quad \text{Wb/m}^2$$

So the flux in the circuit is

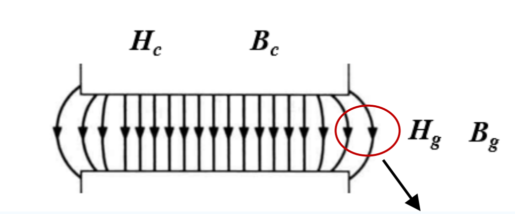
$$\Phi = B \times A = 4.4 \times 5 \times 10^{-4}$$
$$= 2.2 \times 10^{-3} \quad \text{Wb}$$

Magnetic Circuit with Air Gap

$$\boxed{\begin{array}{ll} \text{air gap:} & H_{g},\ B_{g},\ \Re_{g},\ l_{g},\ A_g \\ \text{core:} & H_{c},\ B_{c},\ \Re_{c},\ l_{c},\ A_c \end{array}}$$



Gap detail



Fringing fields

The field in the gap: $H_g = B_g/\mu_0$

The field in the core: $H_c = B_c/\mu = B_c/(\mu_r\mu_0)$

If fringing is considered as neglected and

the flux in core and air gap are same $\Phi = BA$

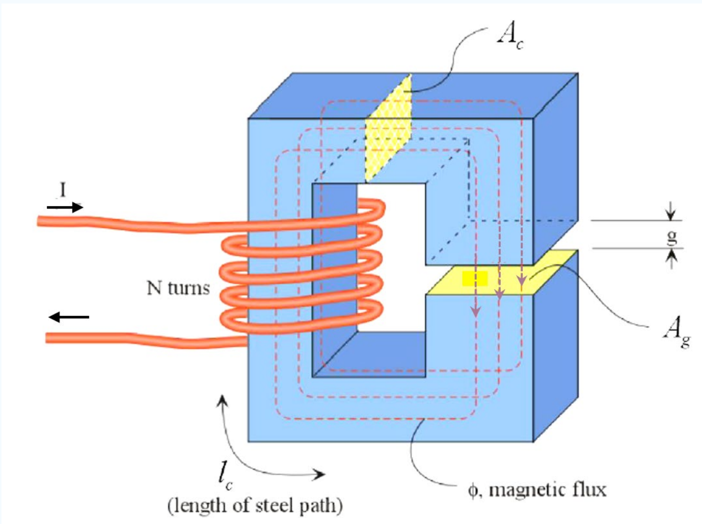
By continuity of the magnetic flux density:

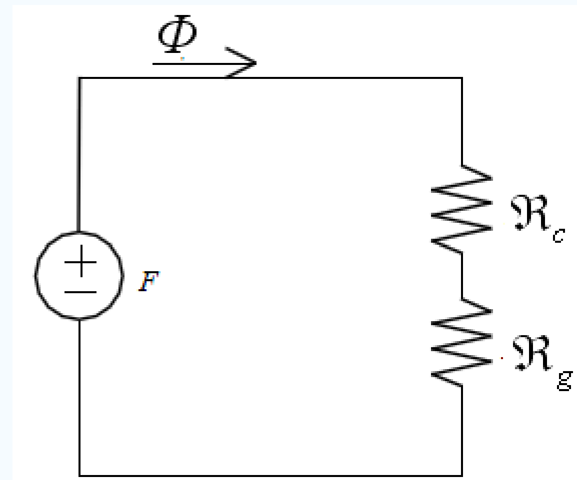
$B_c = B_g = B$ with assuming of the gap being small,
So:

$$H_c = \frac{B}{\mu} = \frac{B}{\mu_r\mu_0} = \frac{H_g}{\mu_r} \quad \Rightarrow \quad \frac{H_g}{H_c} = \mu_r$$

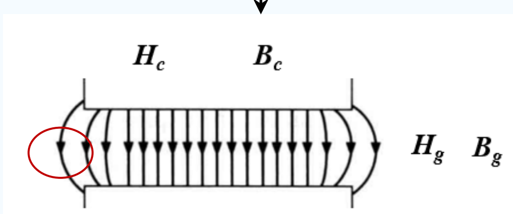
Magnetic Circuit with Air Gap

Let a narrow air gap of thickness *g* be cut in the core.





Gap detail



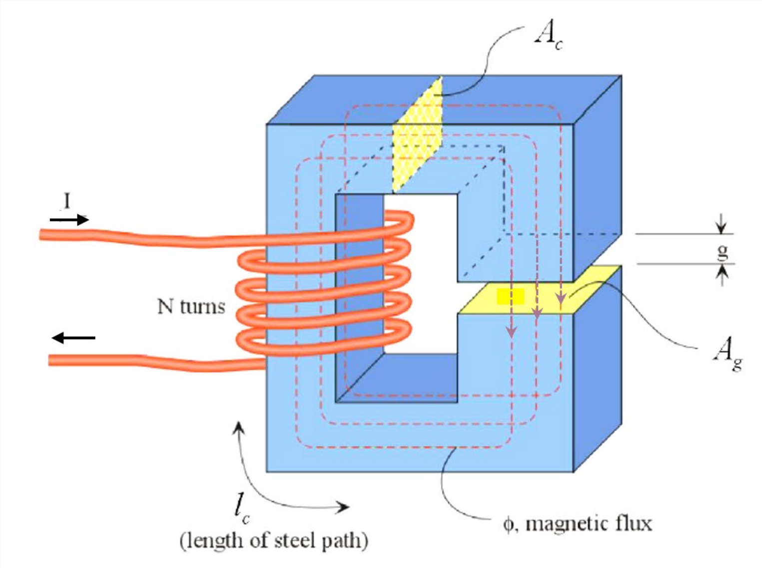
Fringing fields

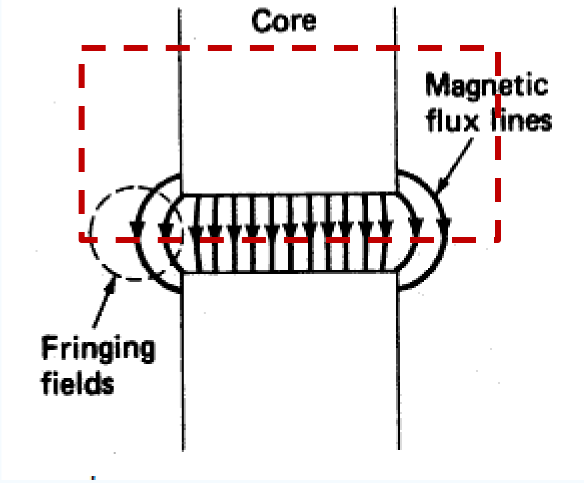
If **magnetic leakage** were neglected, it can be equivalent to a series magnetic circuit.

air gap: $H_g, B_g, \Re_g, l_g, A_g$

core: $H_c, B_c, \Re_c, l_c, A_c$

Magnetic Circuital Laws





By Ampere's law

$$F = NI = H_c l_c + H_g l_g$$

where $H_c l_c = \dfrac{B_c}{\mu_c} l_c = \dfrac{\Phi_c}{\mu_c A_c} l_c = \Phi_c \Re_c$

and $H_g l_g = \dfrac{B_g}{\mu_0} l_g = \dfrac{\Phi_g}{\mu_0 A_g} l_g = \Phi_g \Re_g$

According to Gauss's law in magnetics:

$$\oint_S \mathbf{B} \bullet d\mathbf{A} = 0$$

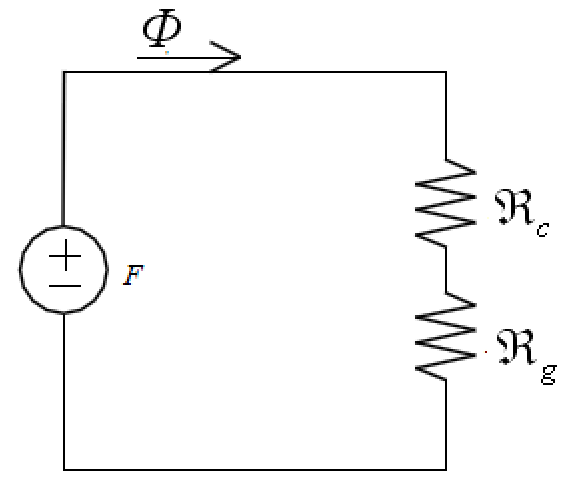
We have: $\Phi_c = \Phi_g = \Phi$

Therefore $\boxed{F = (\Re_c + \Re_g)\Phi}$

Magnetic Circuital Laws

The magnetic circuit with an air gap is analogous to a series electric circuit.

$$\boxed{F = (\Re_c + \Re_g)\Phi}$$



Series magnetic circuit

In an electric circuit, a voltage drives a current I through each resistor.

In a magnetic circuit, the magnetomotive force drives a flux through each reluctance.

The equivalent total reluctance of a number of reluctances in series is just the sum of the individual reluctance:

$$\Re_{eq} = \Re_1 + \Re_2 + \ldots + \Re_n = \sum_{i=1}^{n} \Re_i$$

Magnetic Circuital Laws

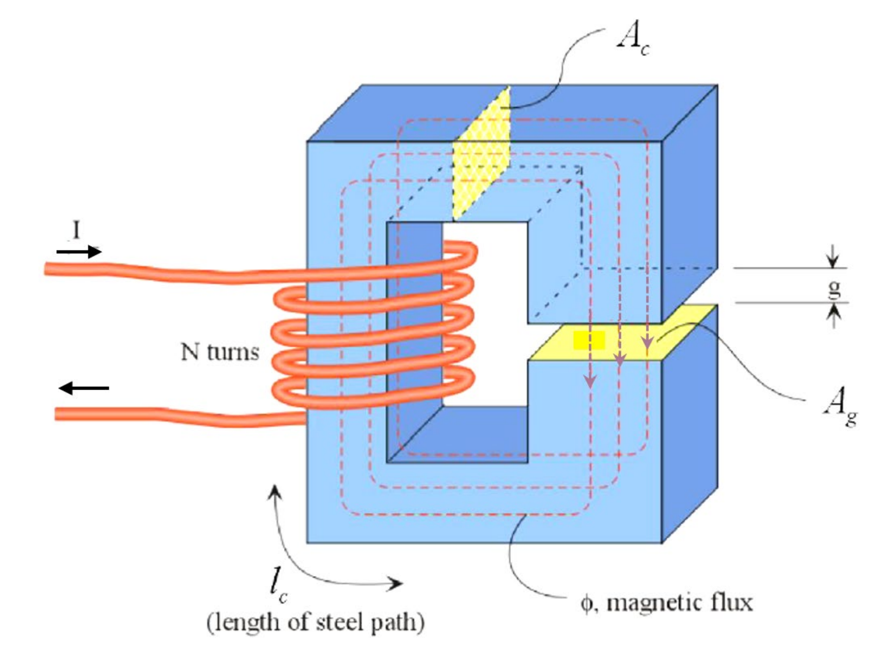
Reluctance in Series Example 1

Dimensions:

$A_c = A_g = 9\ \text{cm}^2$, $g = 0.05\text{cm}$

$l_c = 30$ cm, $N = 500$ turns

$\mu_r = 70{,}000$ for the core and 1 for the gas. The circuit is operating with $B_c = 1.0$ T



Find:

(a) reluctances in the core and in the gap

(b) the flux, and

(c) the current I

Reluctance in Series Example 1

(a) The reluctances: $\Re_{\mathrm{c}} = \dfrac{l_c}{\mu_r \mu_0 A_c} = \dfrac{0.3}{70000(4\pi \times 10^{-7})(9 \times 10^{-4})} = 3.79 \times 10^3 \quad \mathrm{A \cdot turns/Wb}$

$$\Re_{\mathrm{g}} = \frac{l_g}{\mu_r \mu_0 A_c} = \frac{5 \times 10^{-4}}{1(4\pi \times 10^{-7})(9 \times 10^{-4})} = 4.42 \times 10^5 \quad \mathrm{A \cdot turns/Wb}$$

(b) The magnetic flux: $\Phi = B_c A_c = 1.0(9 \times 10^{-4}) = 9 \times 10^{-4} \quad \mathrm{Wb}$

(c) The current: $I = \dfrac{F}{N} = \dfrac{\Phi(\Re_{\mathrm{c}} + \Re_{\mathrm{g}})}{N} = \dfrac{(9 \times 10^{-4})(4.46 \times 10^5)}{500} = 0.8028 \ \mathrm{A} \approx 0.80 \ \mathrm{A}$

Reluctance in Series Example 1

$$\Re_c = 3.79\times10^3 \ \text{A}\cdot\text{turns/Wb} \qquad \frac{\Re_c}{\Re_g} = 0.00857$$

$$\Re_g = 4.42\times10^5 \ \text{A}\cdot\text{turns/Wb}$$

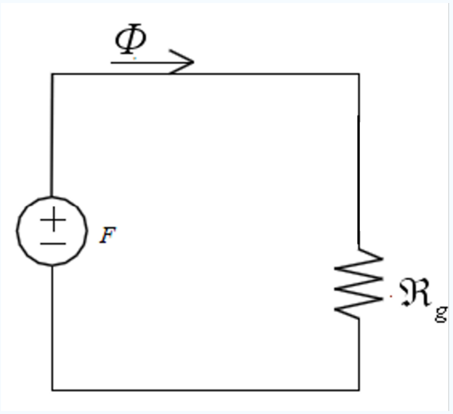
If the reluctance in the core is negligible, then the current

$$I = \frac{F}{N} = \frac{\Phi(\Re_c + \Re_g)}{N} \approx \frac{\Phi\Re_g}{N} = \frac{(9\times10^{-4})(4.42\times10^5)}{500} = 0.7956 \ \text{A} \approx 0.80 \ \text{A}$$

$$\Delta I = \frac{0.8028 - 0.7956}{0.8028}\times100\% = 0.87\%$$

Reluctance in Series

If $\Re_c << \Re_g$, then the reluctance of the core can be neglected. The flux or magnetic filed density B can be obtained from



$$F = (\Re_c + \Re_g)\Phi \approx \Re_g \Phi$$

high material permeability

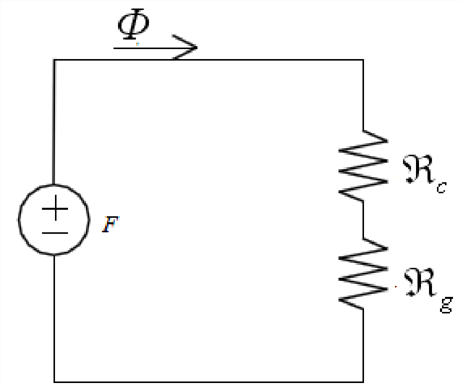
small core reluctance

Not always ture!

low material permeability (2000~6000 times of air)

large core reluctance

$$F = (\Re_c + \Re_g)\Phi$$



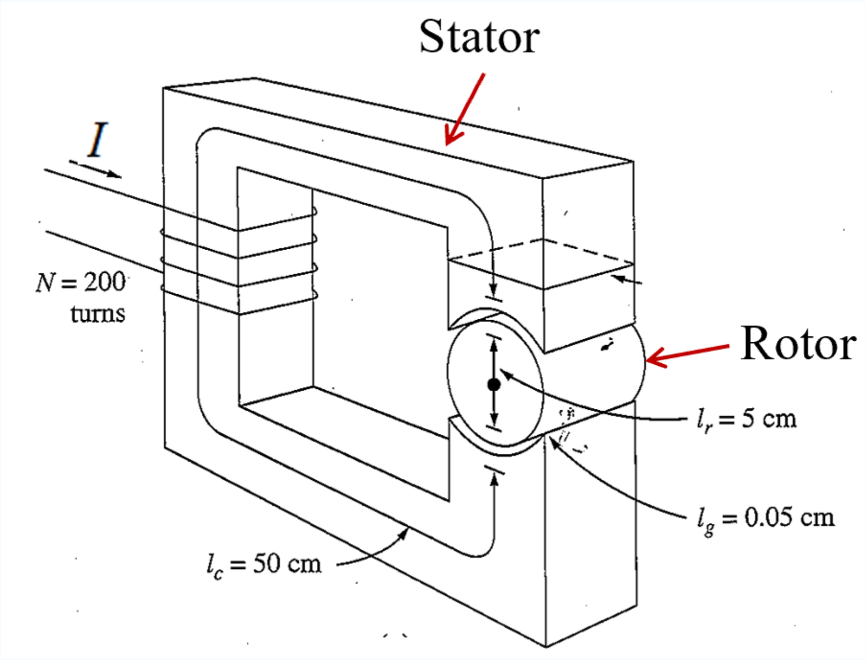
Series magnetic circuit

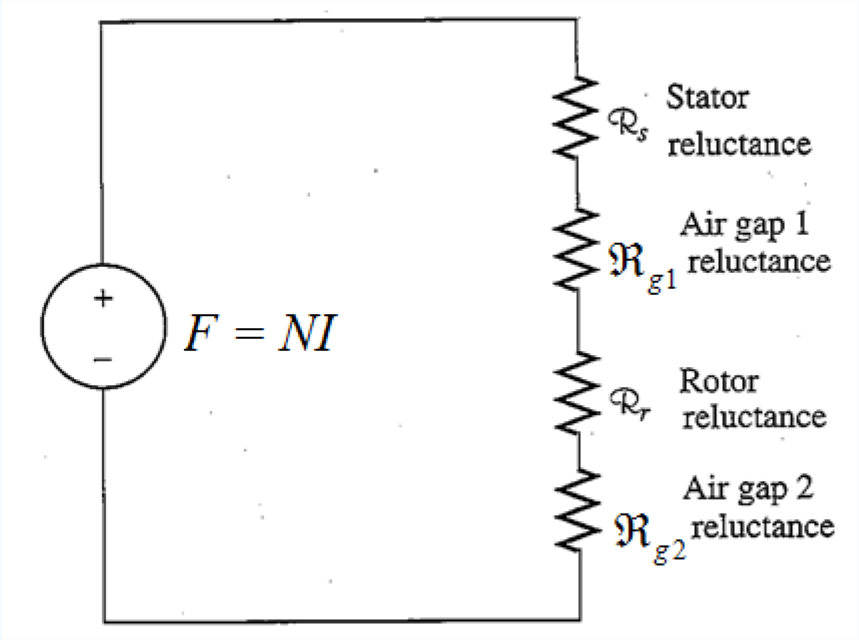
Magnetic Circuital Laws

Reluctance in Series Example 2

The cross-sectional area of each air gap (including fringing is 14 cm^2) and the iron of the core has a relative permeability of 2000.

If the current in the wire is adjusted to be 1 A, what will the resulting flux density in the air gaps be?



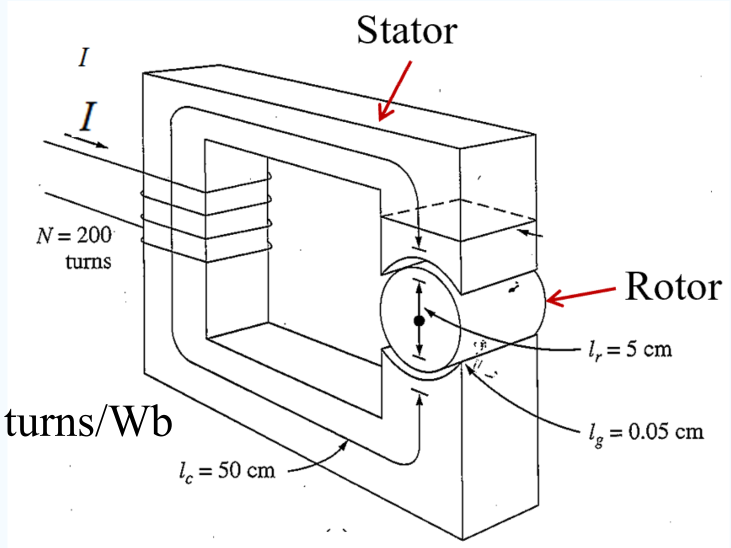


Magnetic Circuital Laws

Reluctance in Series Example 2

$B_g = \Phi / A_g$ $\qquad$ $A_g = 14\ \text{cm}^2$

$$\Phi = F / \Re$$



The reluctance of the stator:

$$\Re_s = \frac{l_s}{\mu_r \mu_0 A_s} = \frac{0.5}{(2000)(4\pi \times 10^{-7})(0.0012)} = 166000 \quad \text{A} \cdot \text{turns/Wb}$$

The reluctance of the rotor:

$$\Re_r = \frac{l_r}{\mu_r \mu_0 A_r} = \frac{0.05}{(2000)(4\pi \times 10^{-7})(0.0012)} = 16600 \quad \text{A} \cdot \text{turns/Wb}$$

The reluctance of the air gap:

$$\Re_g = \frac{l_g}{\mu_r \mu_0 A_g} = \frac{0.0005}{(1)(4\pi \times 10^{-7})(0.0014)} = 284000 \quad \text{A} \cdot \text{turns/Wb}$$

Proper units

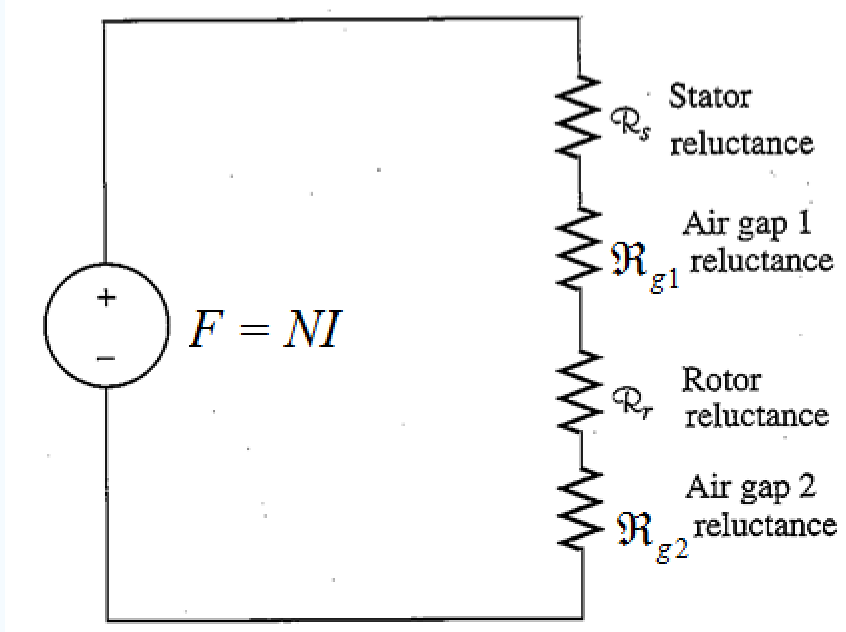
The total reluctance of the motor:

$$\Re_{total} = \Re_s + \Re_{g1} + \Re_r + \Re_{g2} = 166000 + 284000 + 16600 + 284000 = 751000 \quad \text{A} \cdot \text{turns/Wb}$$

Reluctance in Series Example 2

$$B_g = \Phi / A_g \qquad A_g = 14 \text{ cm}^2$$

$$\Phi = F / \Re$$



(b)

The total reluctance:

$$\Re_{total} = 751000 \quad \text{A} \cdot \text{turns/Wb}$$

The net magnetomotive force applied to the core:

$$F = NI = (200)(1.0) = 200 \quad (\text{A} \cdot \text{turns})$$

The total flux:

$$\Phi = \frac{F}{\Re_{total}} = \frac{200}{751000} = 0.00266 \quad \text{Wb}$$

Then the magnetic flux density in the gap: $B = \dfrac{\Phi}{A_g} = \dfrac{0.00266}{0.0014} = 0.19 \quad \text{T} \quad (\text{Wb/m}^2)$

Reluctance in Series Example 2

$$\frac{\Re_s}{\Re_g} = \frac{166000}{284000} = 0.5845$$

$$\Rightarrow \quad \Re_s \; (\Re_r) \text{ not } << \Re_g$$

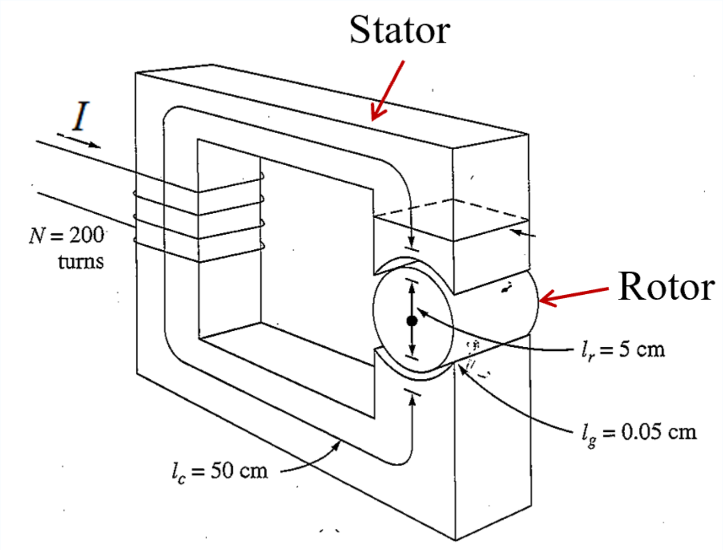
$$\frac{\Re_r}{\Re_g} = \frac{16600}{284000} = 0.05845$$

especially $\Re_s$

The core has a relative permeability of 2000.

Two reasons:

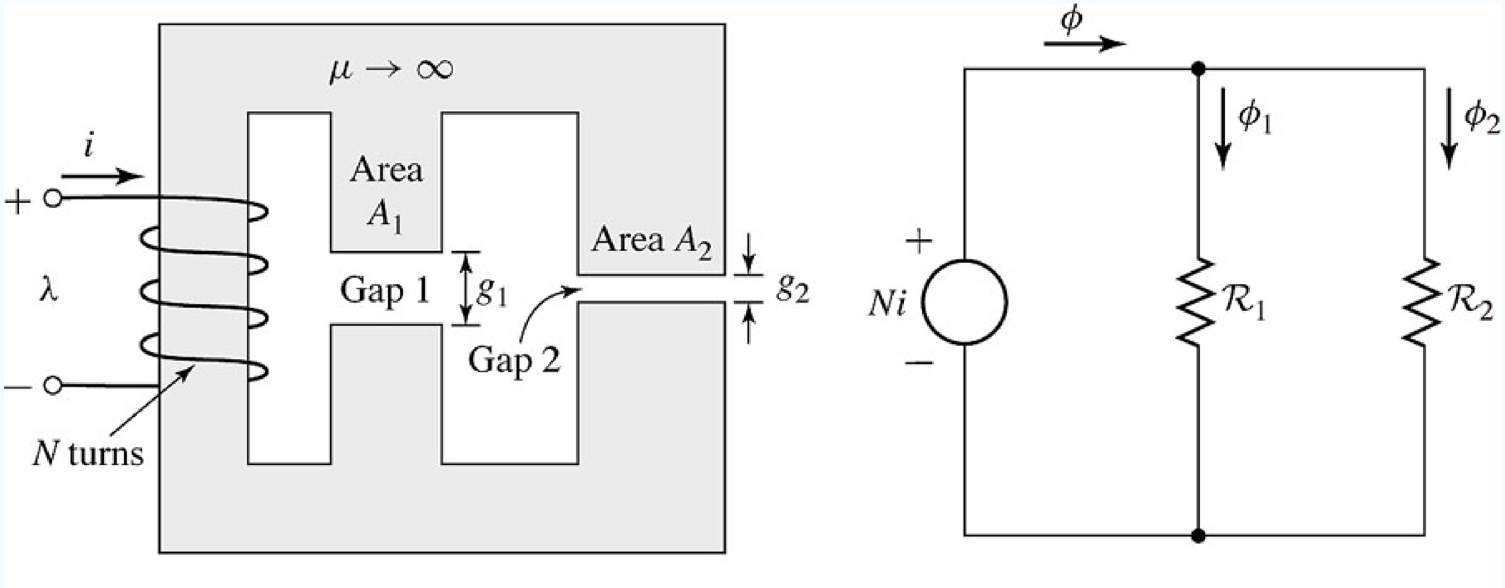
1. μ_r not big enough
2. Length of core is much longer than that of gap



Magnetic Circuital Laws - Reluctance in Parallel

The magnetic circuits with two or more flux paths, neglecting leakage flux, are classified as parallel magnetic circuits.

1) If the permeability of material is infinite

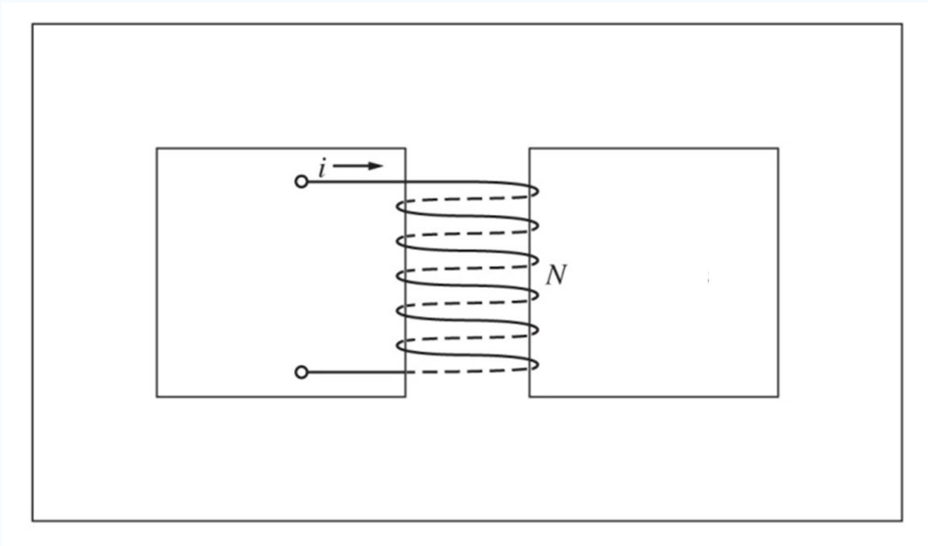


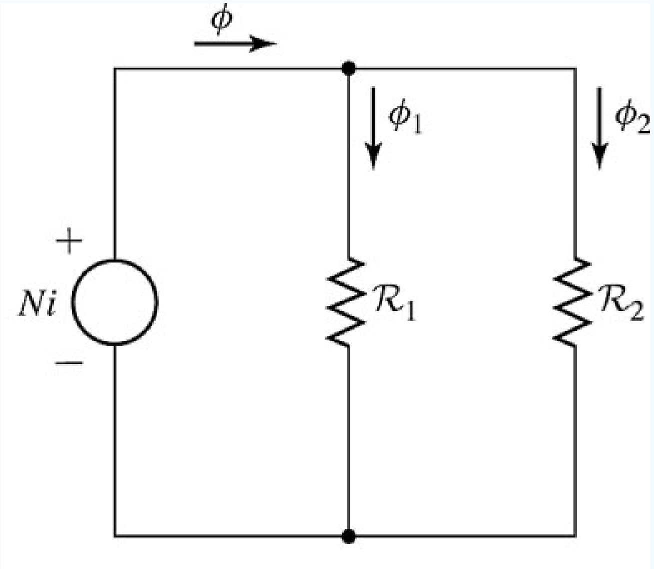
(a) (b)

$$\frac{1}{\Re_{eq}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \ldots + \frac{1}{\Re_n} = \sum_{i=1}^{n} \frac{1}{\Re_i}$$

Magnetic Circuital Laws - Reluctance in Parallel

2) If the permeability of material is not infinite, the reluctance of core is not considered as negligible.

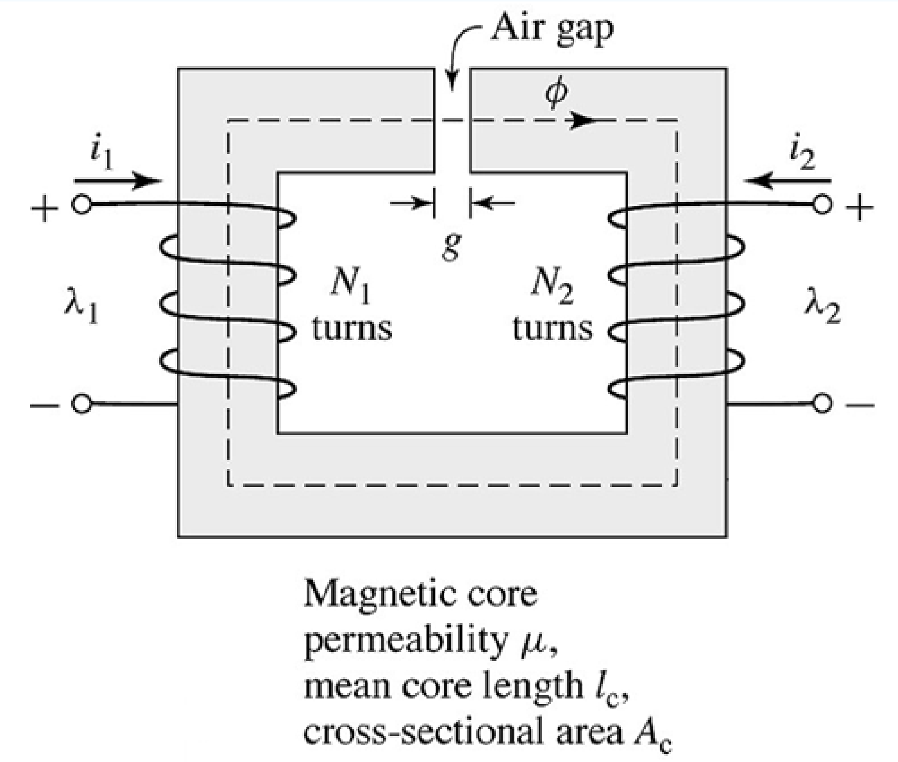




$$\Phi = \Phi_1 + \Phi_2$$

$$\frac{1}{\Re_{eq}} = \frac{1}{\Re_1} + \frac{1}{\Re_2}$$

Magnetic Circuit with Two Windings



In this case, the MMF acting on the magnetic circuit is given by the total ampere-turns acting on the circuit. For the current directions shown by the Fig, the flux produced by the two windings is in the same direction. The total MMF:

$$F = N_1 i_1 + N_2 i_2$$

The total resultant core flux produced by the total magnetomotive force of the two windings with assumption of $A_c = A_g$:

$$\Phi = (N_1 i_1 + N_2 i_2)\frac{\mu_0 A_c}{g} \quad \text{if } \mu_r \text{ is large}$$

Summary

magnetomotive force 磁动势 $F = NI$

$$F = Hl = \frac{B}{\mu_r \mu_0} l = \frac{\Phi}{A \mu_r \mu_0} l$$

Flux density **B**
磁通密度
Flux Φ
磁通量
Magnetic field intensity **H**
磁场强度
Magnetic reluctance $\Re$
磁阻

relative permeability μ_r
相对磁导率
Mean path length of the core/air gap l
平均路径长度
Cross-sectional area A
横截面积

$$F = \Phi \Re = BA\Re = \mu_r \mu_0 H \Re$$

$$\Re = \frac{l}{\mu_r \mu_0 A}$$

Summary

Assumptions :

- Constant magnetic permeability: linear relationship between B-H– gives results of acceptable engineering accuracy
- No leakage
- Air gap is small and the fringing can be ignored: $A_c = A_g$

Under the conditions provided, the magnetic circuit model introduced is true.

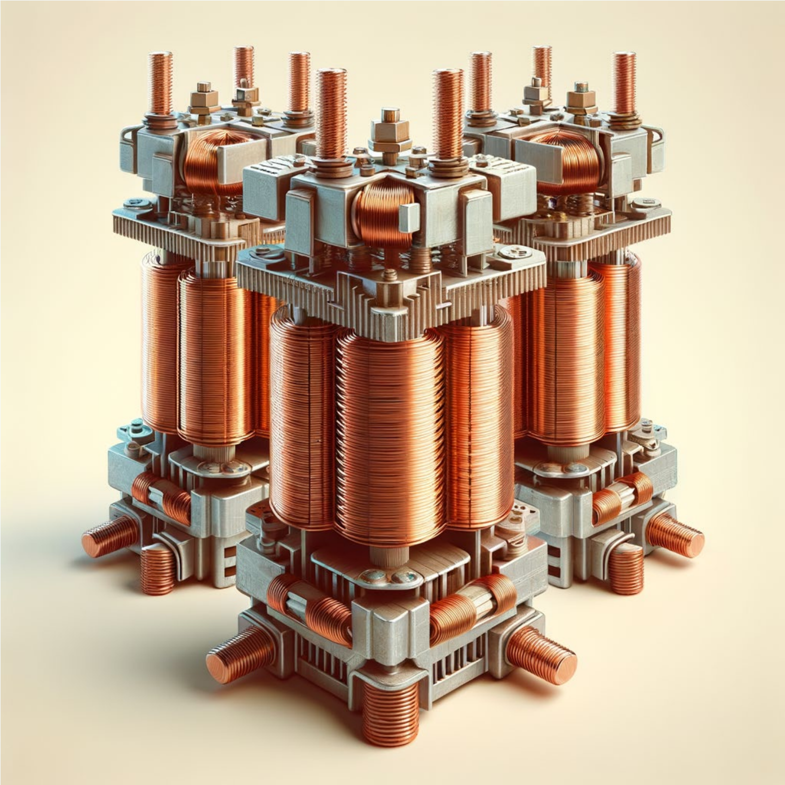
Reluctance in Series

$$\Re_{eq} = \Re_1 + \Re_2 + \ldots + \Re_n = \sum_{i=1}^{n} \Re_i$$

Reluctance in Parallel

$$\frac{1}{\Re_{eq}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \ldots + \frac{1}{\Re_n} = \sum_{i=1}^{n} \frac{1}{\Re_i}$$

Next



Transformers

Thanks for your attention