Xi'an Jiaotong-Liverpool University
西交利物浦大学

MEC208 Instrumentation and Control System

2024-25 Semester 2

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Lecture 21-22

Outline

Frequency Response Methods – Bode Plot and Nyquist Plot

- ❑ Introduction of Bode Plot
- ❑ Frequency Response Plot – Bode Plot
- ❑ Frequency Response Measurements
- ❑ **Performance Specifications, Gain Margin, and Phase Margin**
  Compensator Structure and Design
- ❑ **Frequency Response Methods Using Matlab**
- ❑ **Another Frequency Response Plot – Nyquist/Polar Plot**

Lesson Aims

- To interpret and describe the concept of frequency response and its role in control system
- To sketch Bode plot and also how to obtain a computer-generated Bode plot
- To interpret Bode plot for performance specification and relative stability
- To design a controller to meet desired specification using frequency response methods
- To describe the main features of Nyquist plots and explain its relevance to stability and margin designs.

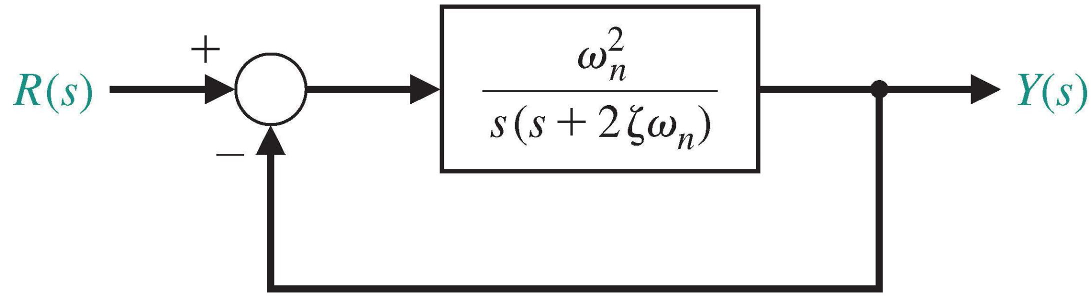
Frequency Response - Features

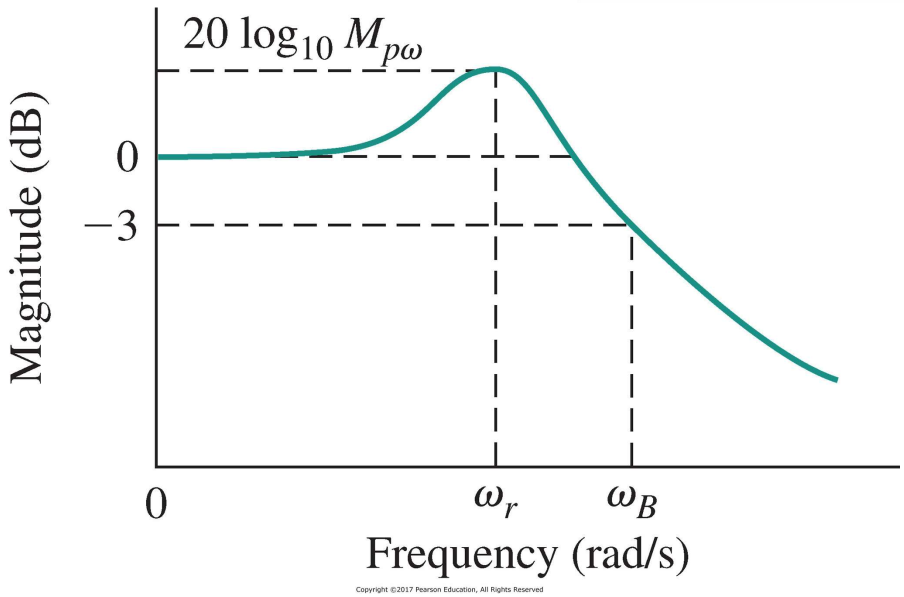
- A sine wave can be used to measure the open-loop frequency response of a system. In practice, a plot of amplitude versus frequency and phase versus frequency will be obtained. From these two plots, the transfer function (including both the loop TF and the closed-loop TF) of the system can be deduced.

- **Control bandwidth** of the system under control can be easily deduced from the plots. Behavior/response of the system towards measurement noise and disturbances can also be understood.

Performance Specifications in the Frequency Domain

- For a second-order system with a pair of complex poles:





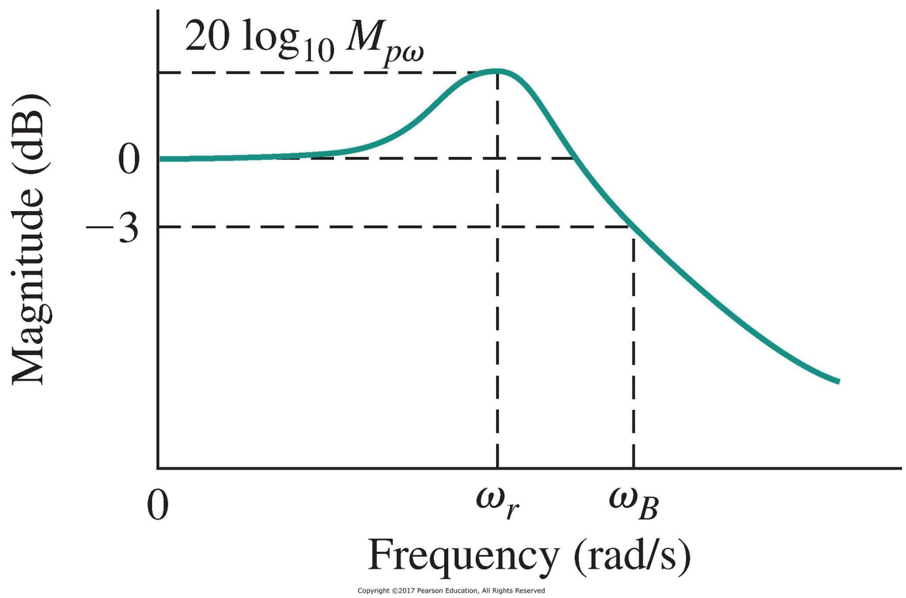
Two main frequency specifications:

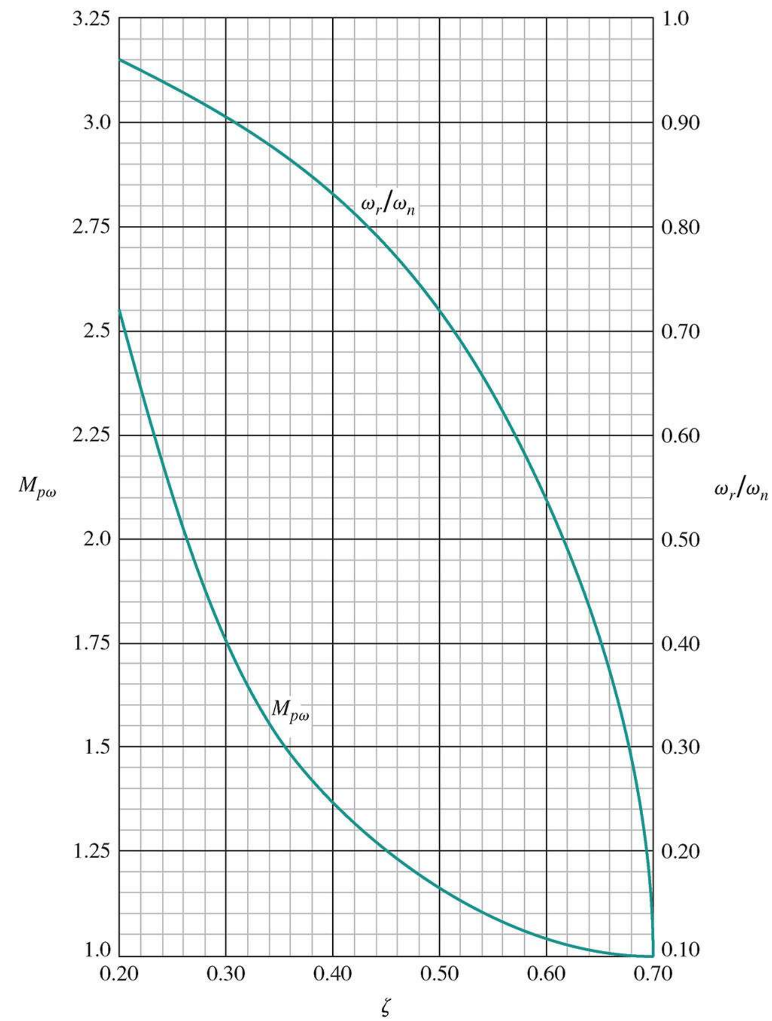
1. **Maximum magnitude - $M_{p\omega}$**
2. **Bandwidth - ω_B**

- For a higher-order system, if the frequency response is **dominated** by a pair of complex poles, the relationship between frequency response and time response will be valid.

Maximum Magnitude

At the resonant frequency $\boldsymbol{\omega_r}$ the maximum gain value $\boldsymbol{M_{p\omega}}$ of the frequency response is attained.





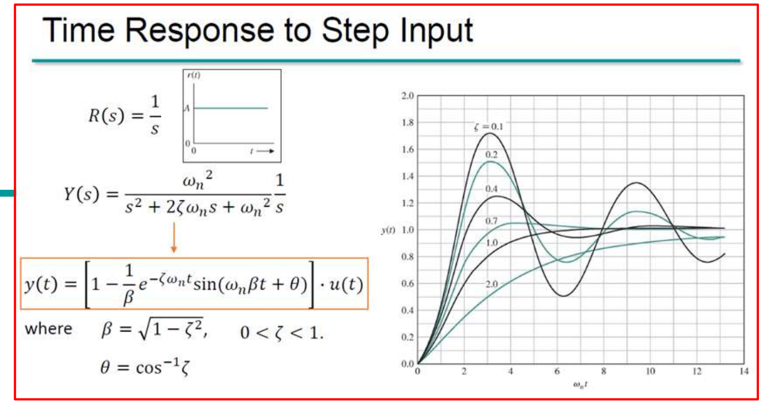
$$\omega_r = \omega_n\sqrt{1-2\zeta^2}, \zeta < 0.707$$

$$M_{p\omega} = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}, \zeta < 0.707$$

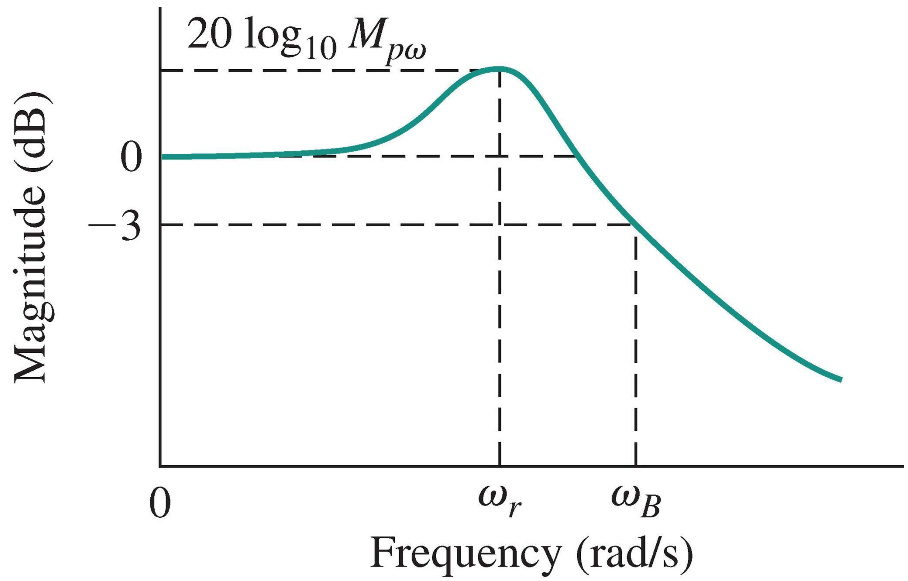
Maximum magnitude $M_{p\omega}$ ↑ → Damping ratio ζ ↓ → Percent Overshoot P.O. ↑

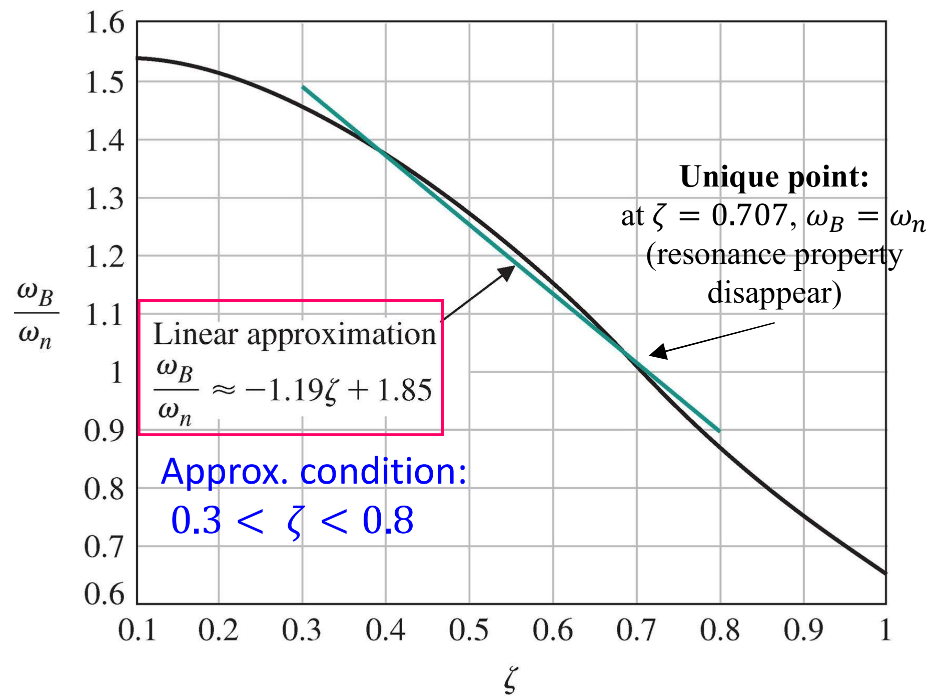
Bandwidth

From Lecture 13, about "settling time" and "natural freq.".



The bandwidth is the frequency at which the magnitude's frequency response declines to 3 dB (≈70%) from its low-frequency value.





$$M_{p\omega} = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}, \zeta < 0.707$$

$$\omega_B = \omega_n\sqrt{(1-2\zeta^2) + \sqrt{(2\zeta^2-1)^2 + 1}}$$

Bandwidth ω_B ↑ (with a constant ζ) → Natural frequency ω_n ↑ → Settling time T_s ↓

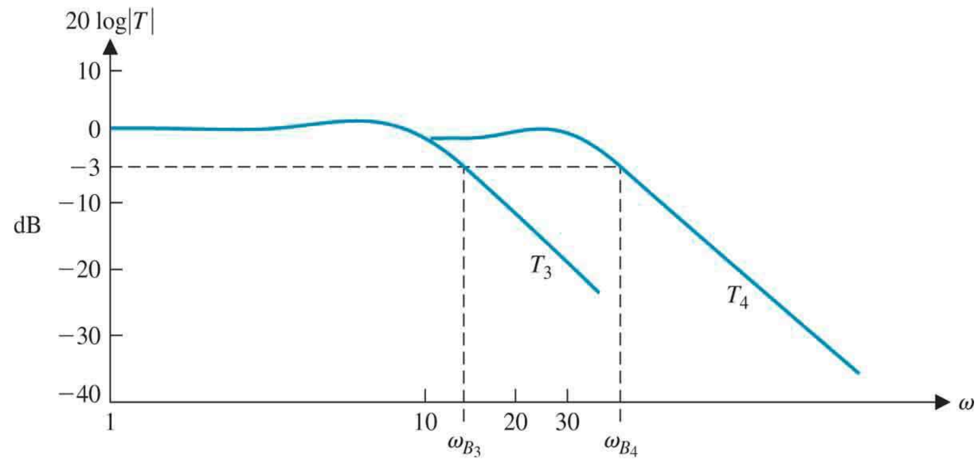
Bandwidth vs. Settling Time

For the same damping ratio ζ, bandwidth/natural frequency is increased by 3 times:

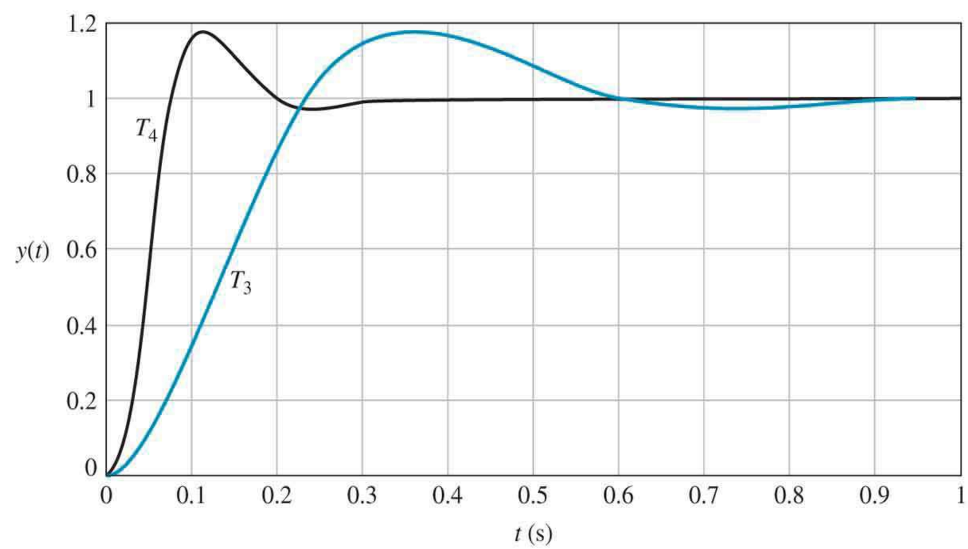
$$T_3(s) = \frac{100}{s^2 + 10s + 100}$$

$$T_4(s) = \frac{900}{s^2 + 30s + 900}$$

Larger bandwidth → smaller settling time



(a)



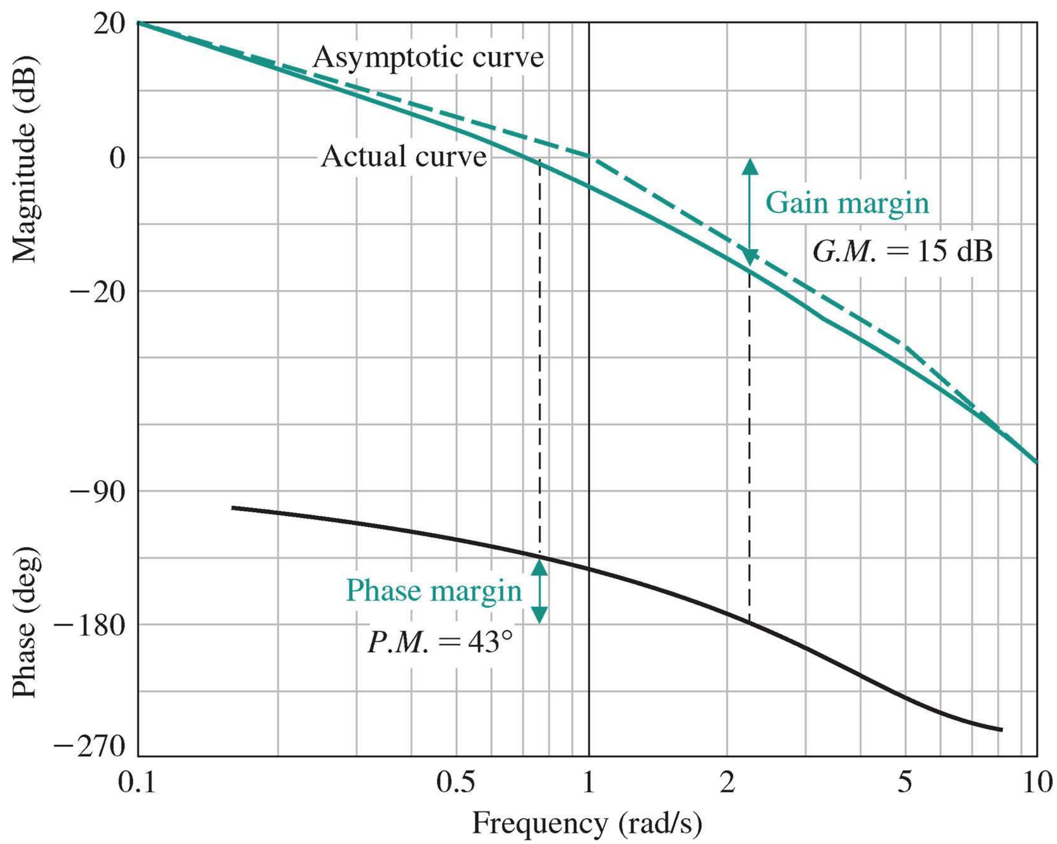
(b)

Gain Margin and Phase Margin

- For a closed-loop control system, with loop transfer function $L(j\omega)$, the characteristic equation is

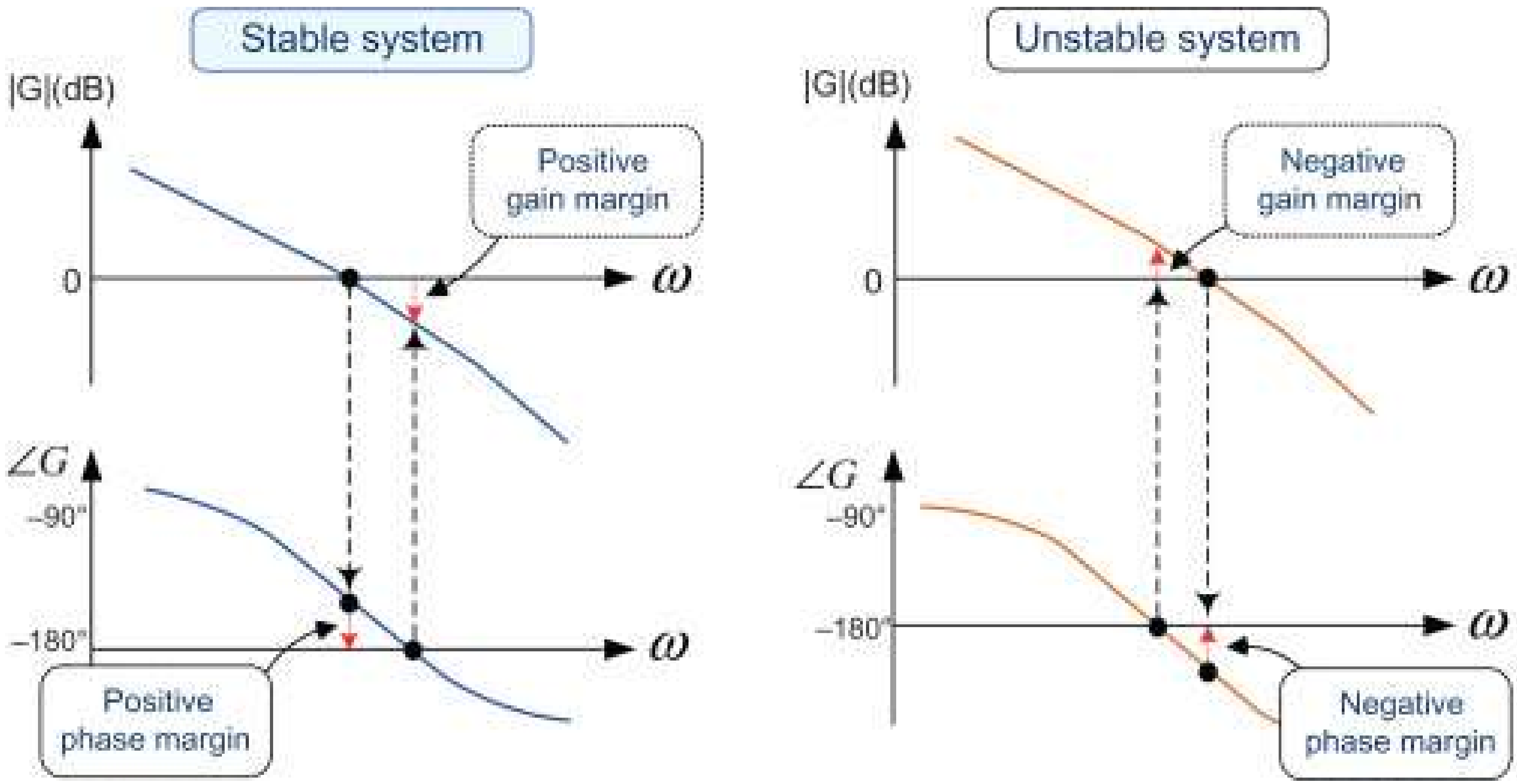
$$1 + L(j\omega) = 0 \rightarrow L(j\omega) = -1 + j0.$$

- It indicates that when $|L(j\omega)| = 1$ (or $20\log(|L(j\omega)|) = 0$ dB) and $\angle L(j\omega) = -180^o$, the closed-loop system is marginally stable.



- The gain margin GM is the increase in the gain of the $L(j\omega)$ when phase = -180^o that will result in a marginally stable system;
- The phase margin $\emptyset_{pm}$ is the amount of phase shift of the $L(j\omega)$ at unity magnitude (0 dB) that will result in a marginally stable system.

Gain/Phase Margins in a stable/unstable system

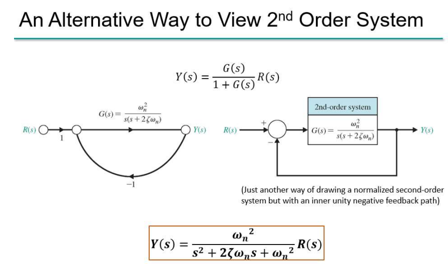


To relate **Phase Margin** *and* **Damping Ratio** *(exact relationship)*

$$T_{CL}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega s + \omega_n^2}$$

$$L(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\zeta\omega_n)}$$

Lecture 13



- To determine the gain crossover frequency ω_c, we let

$$\frac{\omega_n^2}{\omega_c\sqrt{\omega_c^2 + 4\zeta^2\omega_n^2}} = 1$$

$$\left(\omega_c^2\right)^2 + 4\zeta^2\omega_n^2\left(\omega_c^2\right) - \omega_n^4 = 0$$

$$\frac{\omega_c^2}{\omega_n^2} = \sqrt{4\zeta^4 + 1} - 2\zeta^2 \quad \Rightarrow \quad \frac{\omega_c}{\omega_n} = \left(\sqrt{4\zeta^4 + 1} - 2\zeta^2\right)^{1/2}$$

$$\phi_{PM} = 180^o - 90^o - \tan^{-1}\frac{\omega_c}{2\zeta\omega_n} = 90^o - \tan^{-1}\frac{1}{2\zeta}\left(\sqrt{4\zeta^4 + 1} - 2\zeta^2\right)^{1/2}$$

$$\phi_{PM} = \tan^{-1}\frac{2}{\left(\sqrt{4 + 1/\zeta^4} - 2\right)^{1/2}}$$

Approximation that relates **Phase Margin** *and* **Damping Ratio**

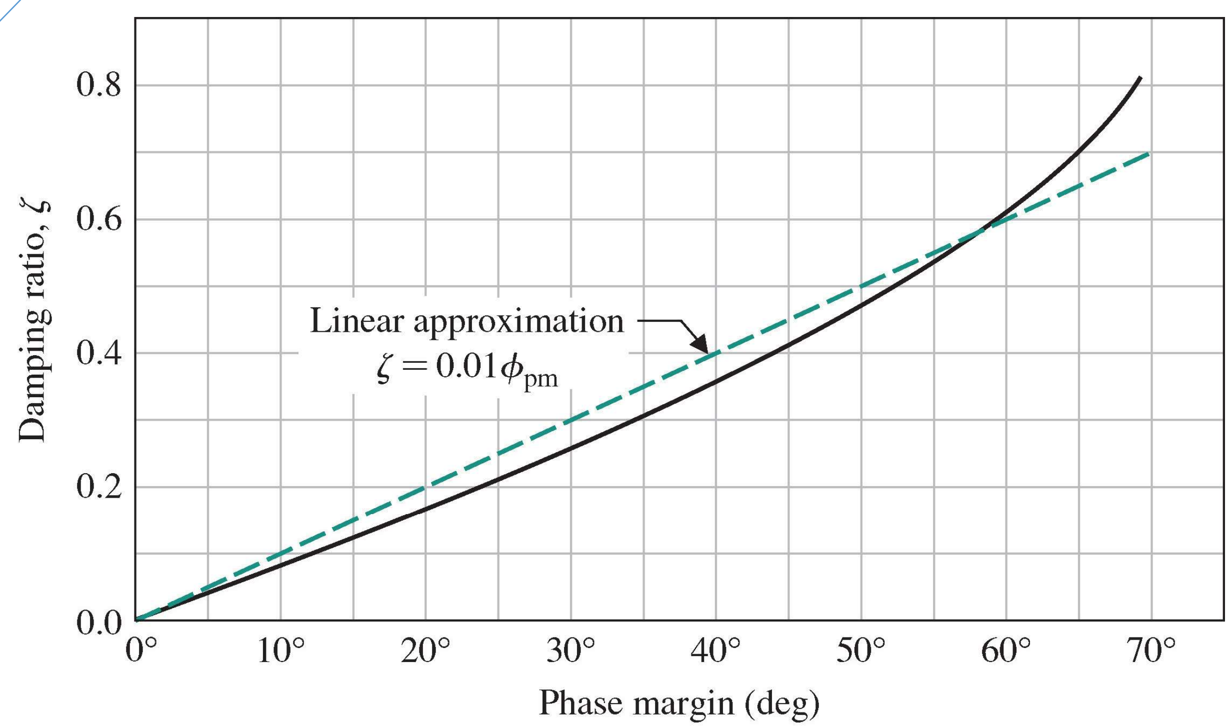
- The relationship between ϕ_{PM} and ζ is not linear.
- For $\zeta < 0.707$, through empirical means, it has been established that for most **2nd order systems** (or higher-order system with dominant underdamped 2nd order behavior), the relationship can be approximated to a linear form:

$$\boxed{\zeta \approx 0.01\emptyset_{PM}}$$

Unit $\emptyset_{PM}$ in deg.

Approximation of

$$\Phi_{PM} = \tan^{-1} \frac{2}{\left(\sqrt{4 + 1/\zeta^4} - 2\right)^{1/2}}$$



Example 21.1

- For the following two loop transfer functions, estimate their damping factor using phase margin:

i) $L(j\omega) = G_c\,(j\omega)\,G(j\omega) = \dfrac{1}{j\omega(j\omega+1)}$

$$\frac{1}{\omega_c\sqrt{\omega_c^2+1}} = 1$$

$$\omega_c = 0.786\,rad/s$$

$$\angle L(j\omega_c) = -90^o - \tan^{-1}\frac{0.786}{1} = -128.2^o$$

$$\emptyset_{PM} = -128.2^o - (-180^o) = 51.8^o$$

$$\zeta \approx 0.01 \times 51.8^o = 0.518$$

Based on s-domain analysis: $\zeta = 0.5$

ii) $L(j\omega) = G_c\,(j\omega)\,G(j\omega) = \dfrac{1}{j\omega(j\omega+1)(0.2j\omega+1)}$

$$\frac{1}{\omega_c\sqrt{(\omega_c^2+1)(0.04\omega_c^2+1)}} = 1$$

$$\omega_c = 0.779\,rad/s$$

$$\angle L(j\omega_c) = -90^o - \tan^{-1}\frac{0.779}{1} - \tan^{-1}\frac{0.2(0.779)}{1}$$
$$= -136.8^o$$

$$\emptyset_{PM} = -136.8^o - (-180^o) = 43.2^o$$

$$\zeta \approx 0.01 \times 43.2^o = 0.432$$

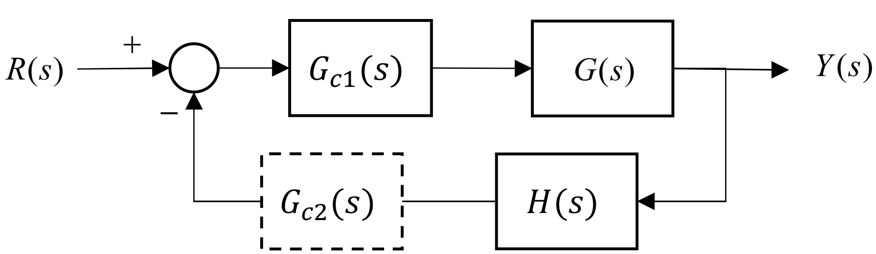
Based on s-domain analysis: $\zeta = 0.39$

Compensator Structure and Design

- A compensator is an additional component that is inserted into a control system to compensate for deficient performance.
- Compensator $G_c(s)$ can be chosen to alter either the shape of the root locus or the frequency response.
- Compensator $G_c(s)$ is used with a process $G(s)$ so that the overall loop gain can be set to satisfy the steady state error requirement, and then $G_c(s)$ is used to adjust the system dynamics favorably without affecting the steady state error.

$$G_c(s) = \frac{K\prod_{i=1}^{M}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}$$

Example



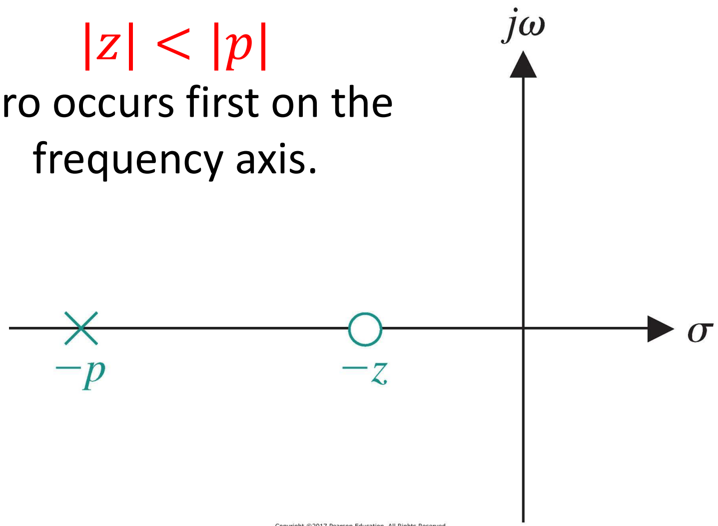
- E.g., A first-order compensator has the following form:

$$G_c(s) = \frac{K(s+z)}{(s+p)}$$

Phase – Lead Compensator

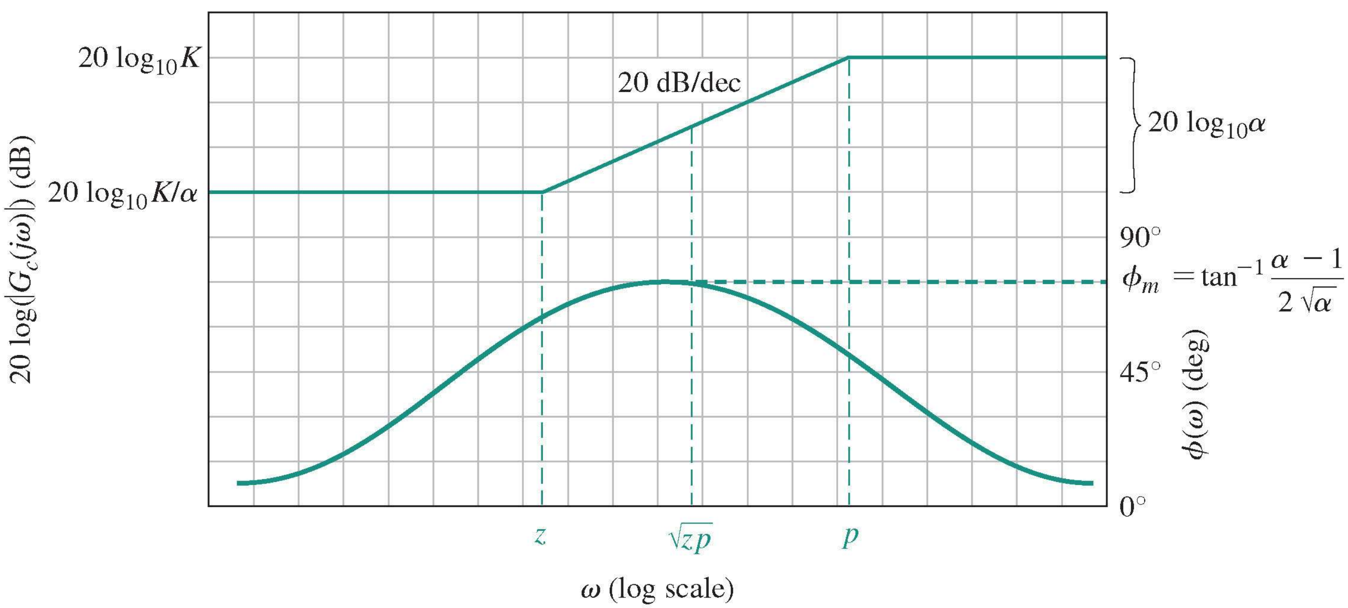
$|z| < |p|$

Zero occurs first on the frequency axis.



$$G_c(j\omega) = \frac{K(j\omega + z)}{(j\omega + p)} = \frac{K(1 + j\omega\alpha\tau)}{\alpha(1 + j\omega\tau)}$$

$$\tau = \frac{1}{p} \text{ and } \alpha = p/z > 1.$$



The maximum phase lead $\emptyset_m$ occurs at ω_m.

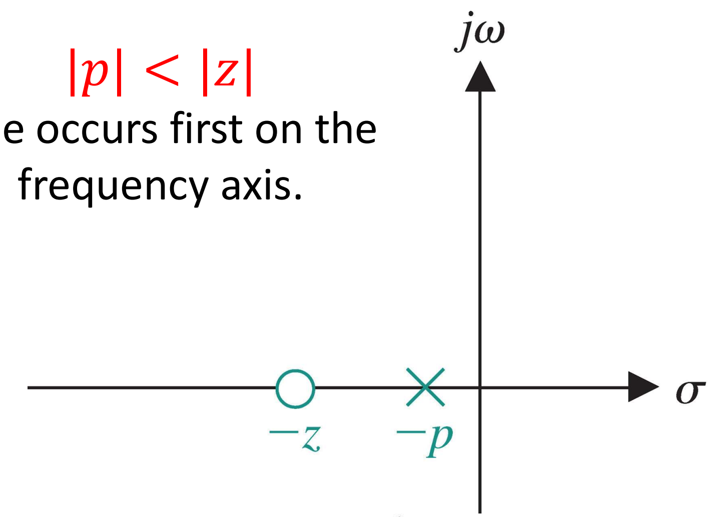
$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}$$

$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1}$$

Phase - Lag Compensator

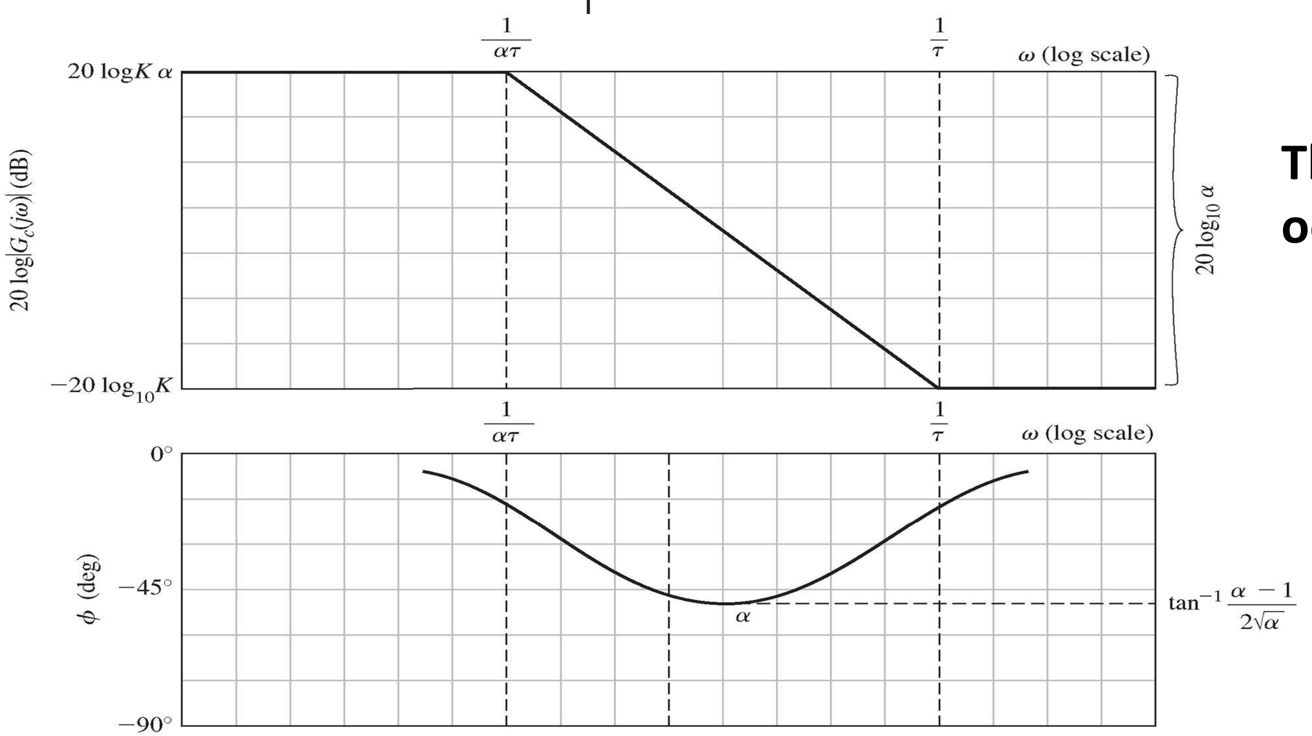
$|p| < |z|$

Pole occurs first on the frequency axis.



$$G_c(j\omega) = \frac{K(j\omega + z)}{(j\omega + p)} = \frac{K\alpha(1 + j\omega\tau)}{1 + j\omega\alpha\tau}$$

$$\tau = \frac{1}{z} \text{ and } \alpha = z/p > 1.$$



The maximum phase lag $\emptyset_m$ occurs at ω_m.

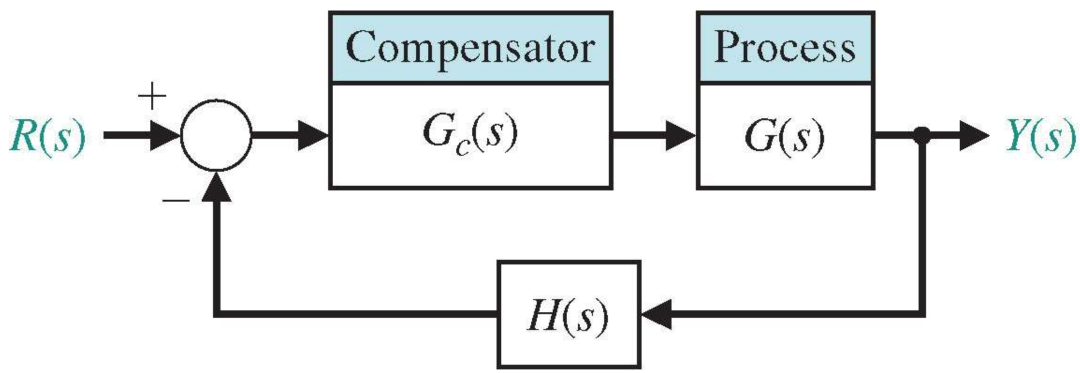
$$\boxed{\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}}$$

$$\boxed{\sin\phi_m = \frac{\alpha - 1}{\alpha + 1}}$$

Example 21.2

Let's consider the below system, we assume $G(s) = \frac{10}{s^2}$, and $H(s) = 1$. It is clear that without compensator, the closed-loop system is marginally stable. Design the compensator to satisfy the following specifications using frequency response methods:

1. Settling time (with 2% criterion) $T_s \leq 4s$;
2. System damping ratio $\zeta \geq 0.45$.



Solutions:

Step 1.

$$T_s = \frac{4}{\zeta\omega_n} = 4s$$

$$\omega_n = \frac{1}{\zeta} = \frac{1}{0.45} = 2.22\ rad/s$$

$$\omega_B = (-1.19\zeta + 1.85)\omega_n = 3.00$$

→ The bandwidth of the closed-loop system should be larger than 3 rad/s.

Step 2.

The phase margin of the system is required to be approximately

$$\phi_{pm} = \frac{\zeta}{0.01} = \frac{0.45}{0.01} = 45^o$$

The phase margin of the uncompensated system ($G(s)$) is 0^o → The compensator needs to provide a phase-lead angle of at least 45^o to the loop transfer function.

$$\frac{\alpha - 1}{\alpha + 1} = \sin\phi_m = \sin 45^o \longrightarrow \alpha = 5.8$$

Phase – Lead Compensator

$$G_c(j\omega) = \frac{K(j\omega + z)}{(j\omega + p)} = \frac{K(1 + j\omega\alpha\tau)}{\alpha(1 + j\omega\tau)}$$

$$\tau = \frac{1}{p} \text{ and } \alpha = p/z > 1.$$

The maximum phase lead $\emptyset_m$ occurs at ω_m.

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}$$

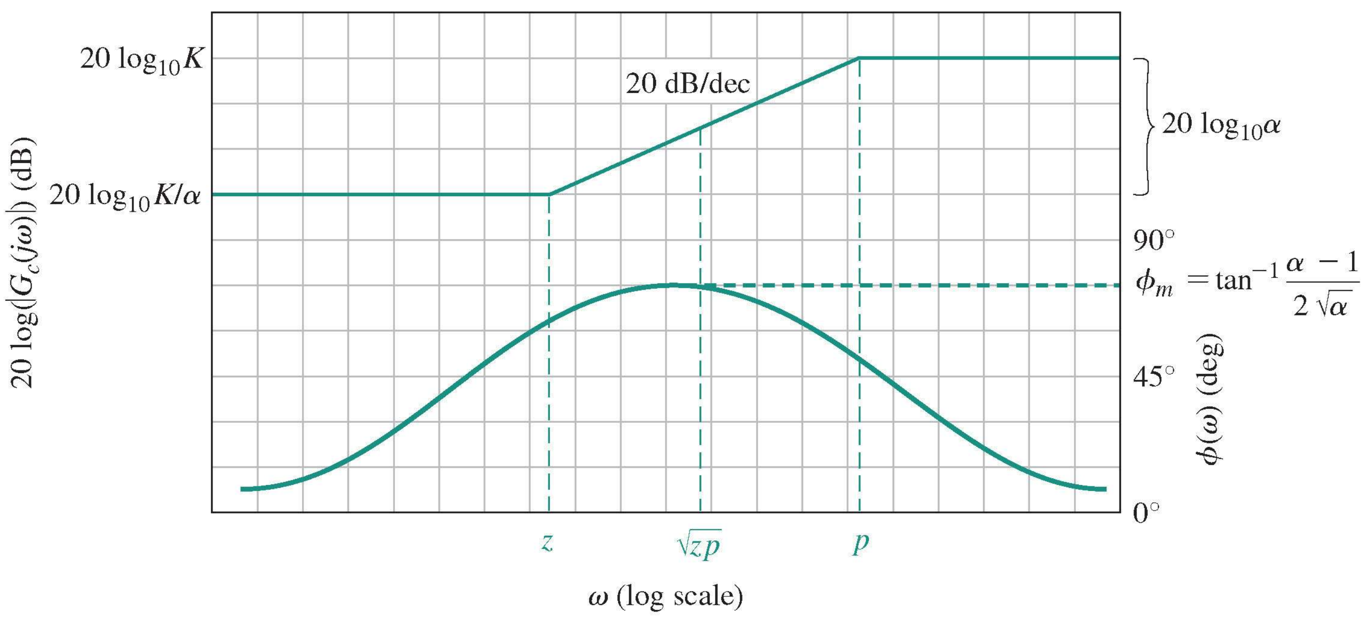
$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1}$$

We use $\alpha = 6$. The gain added to the system at the frequency ω_m is

$$10 \log \alpha = 7.78 \text{ dB}$$

We want to have ω_m equal to the compensated slope crossing the 0 dB axis, thus, the compensated crossover frequency is located by evaluating the frequency where the uncompensated magnitude curve is equal to -7.78 dB. So,

$$\omega_m = 4.95$$



Hint: "Placing" the ω_m at the freq. at which the uncompensated system's magnitude is -7.78dB, so that the compensated system will have 0 dB at this frequency point, so that the phase margin is as designed/desired, about 45°.

$$20 \log \frac{10}{\omega^2} = -7.78dB$$

$$\frac{10}{\omega^2} = 0.4083$$

$$\omega = 4.95 \text{ rad/s}$$

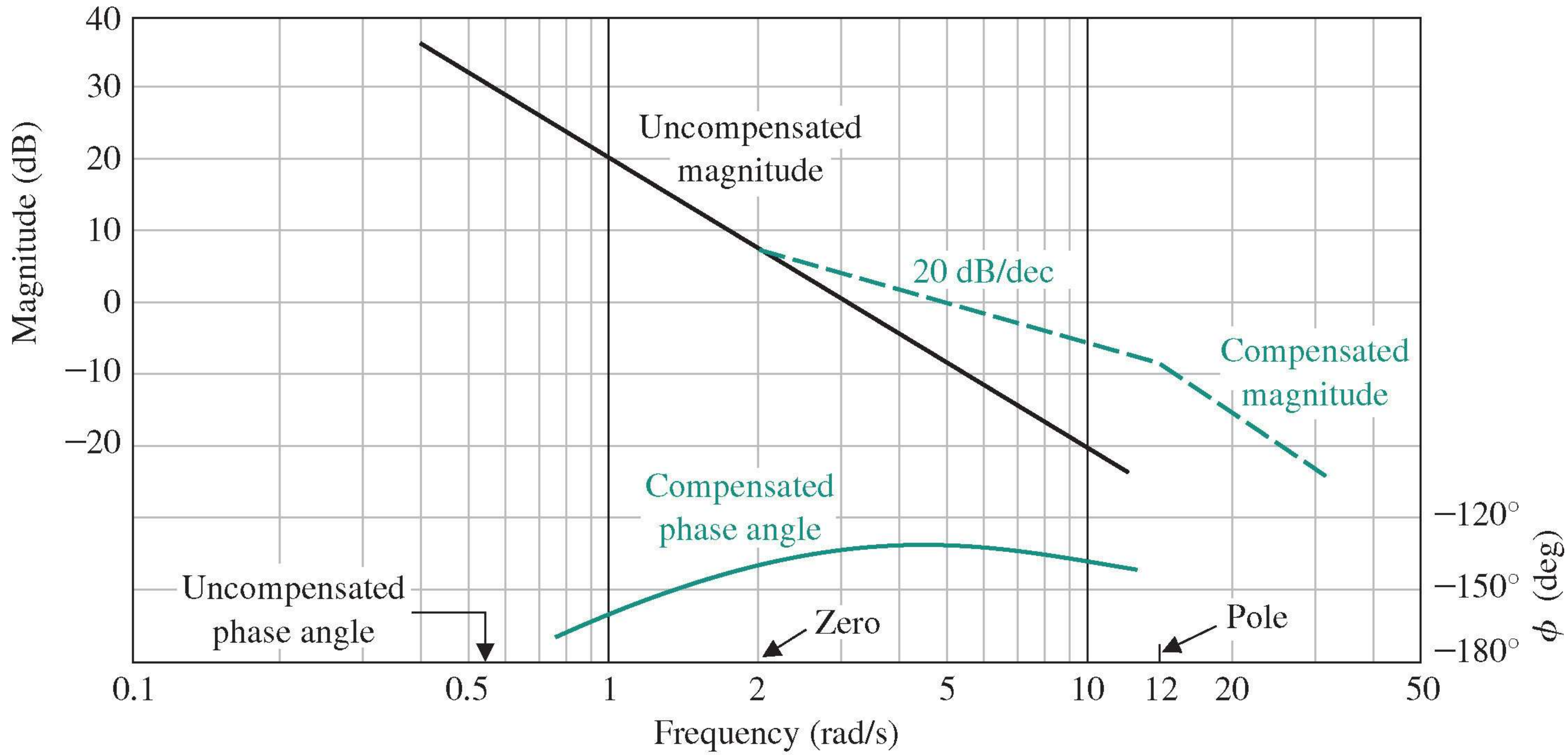
Step 3.

We can now determine the pole and zero of the compensator.

$$p = \omega_m \sqrt{\alpha} = 12.0 \qquad z = \frac{p}{\alpha} = 2.0$$

The compensator is $$G_c(s) = \frac{K}{6} \frac{(1 + s/2.0)}{(1 + s/12.0)}$$

Choose $K = 6$ to keep the loop gain as 1.



Step 4.

Validate, and re-design if the specifications are not satisfied.

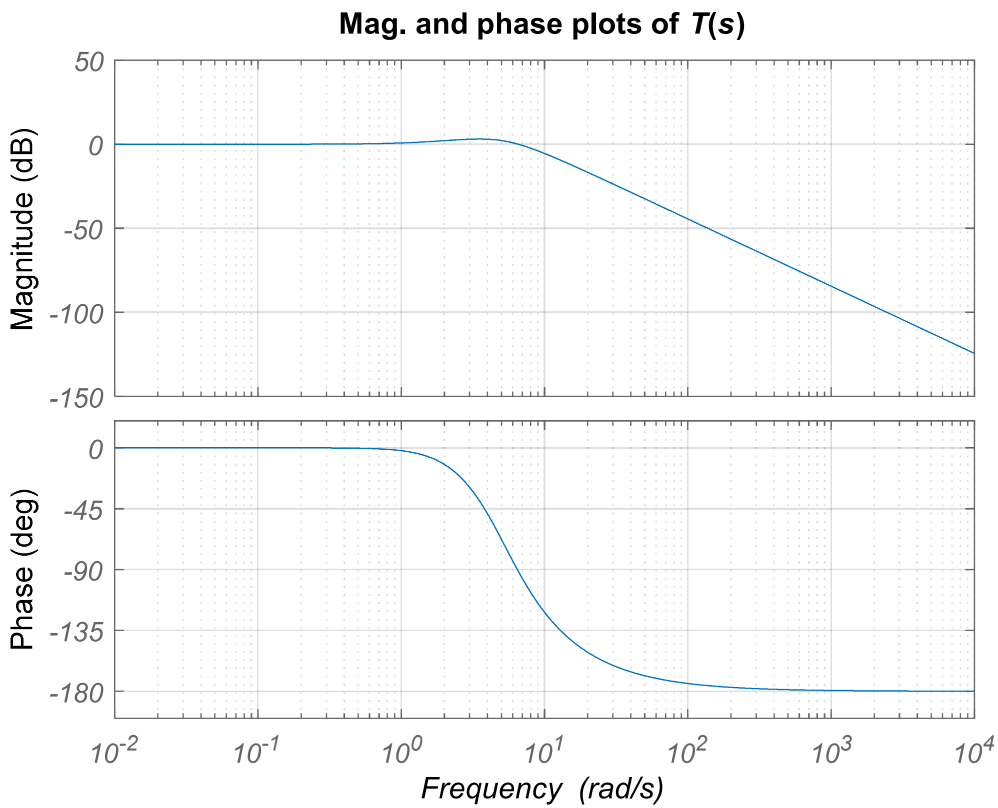
The total loop transfer function is

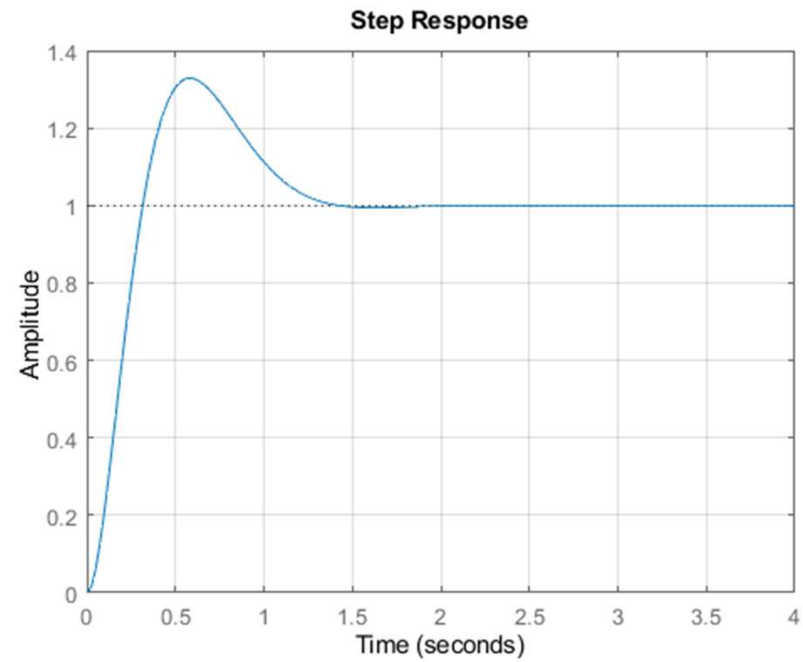
$$L(s) = \frac{10(1 + s/2)}{s^2(1 + s/12)} = \frac{60(s + 2)}{s^2(2 + 12)}$$

The closed-loop transfer function is

$$T(s) = \frac{60(s + 2)}{s^3 + 12s^2 + 60s + 120}$$

Obtain the bode plots for *T*(*s*) and step response of the system.

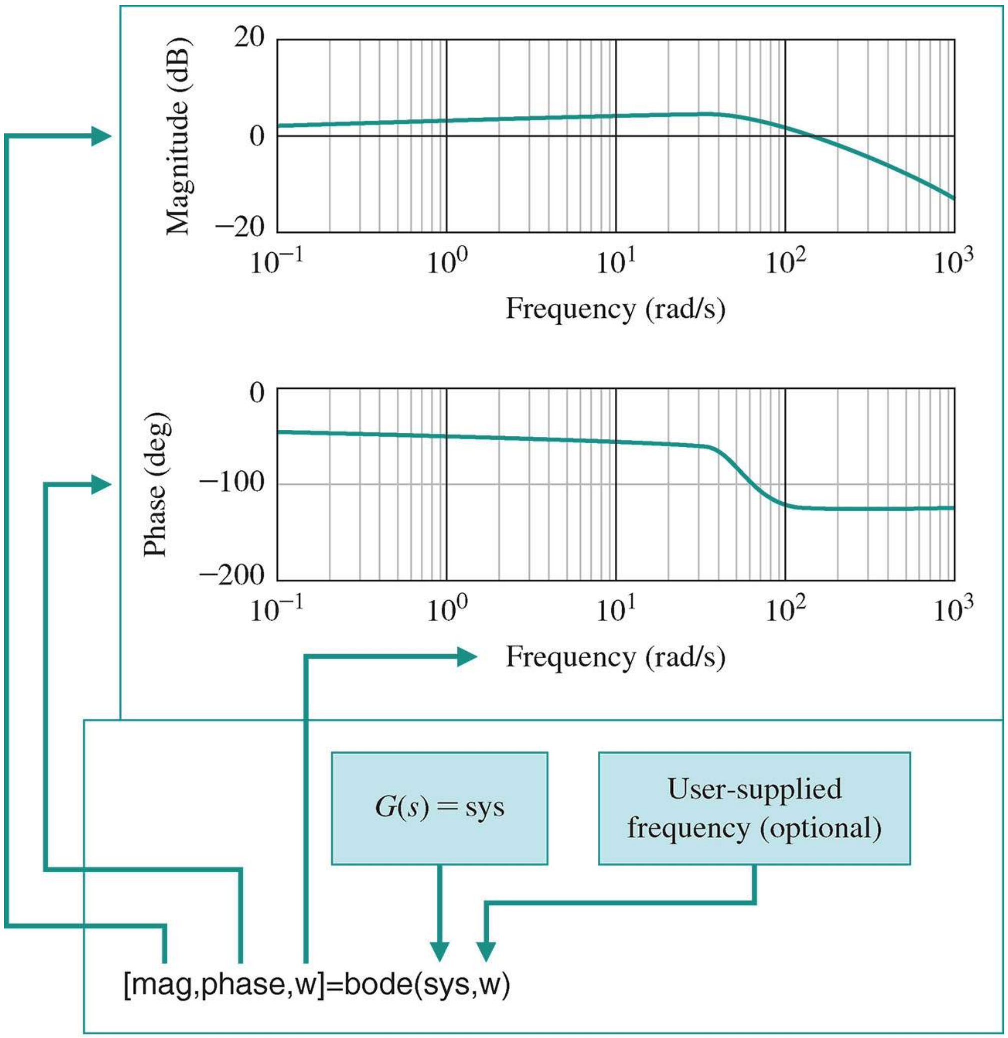


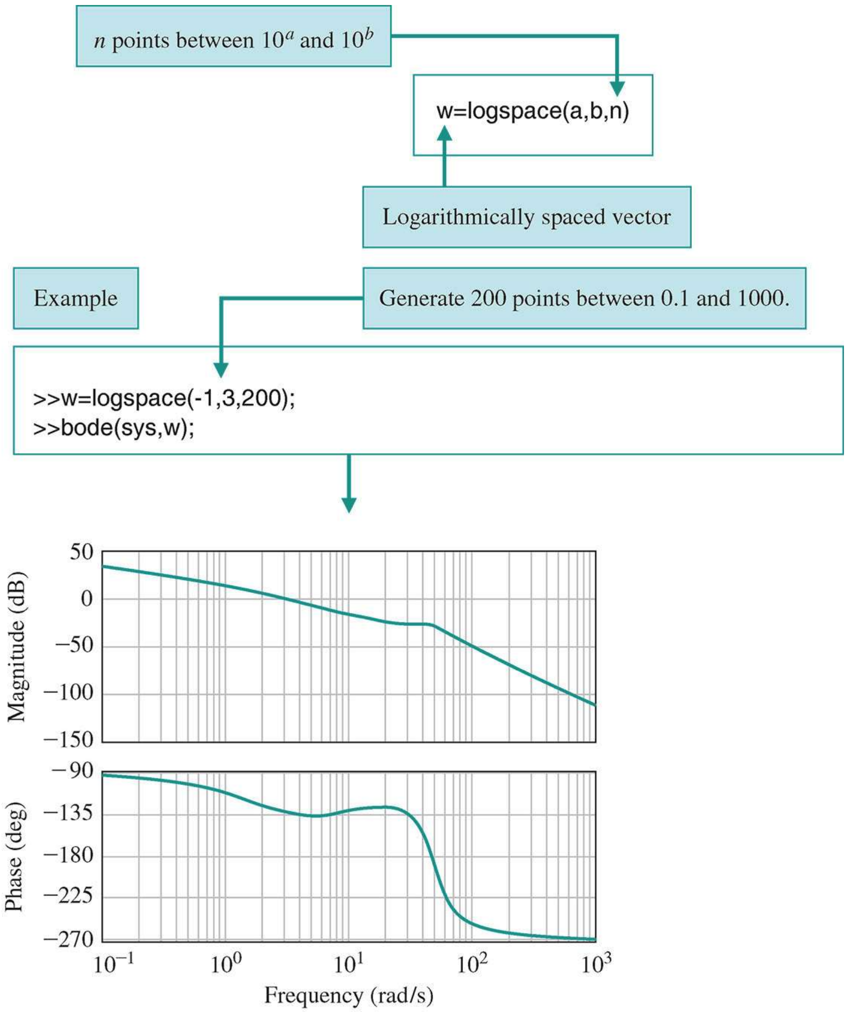


$$T_S = 1.3\ s; P.O. = 34\%;$$
$$\omega_B = 8.4\ rad/s$$

→ The design specifications are satisfied.

Frequency Response using Matlab

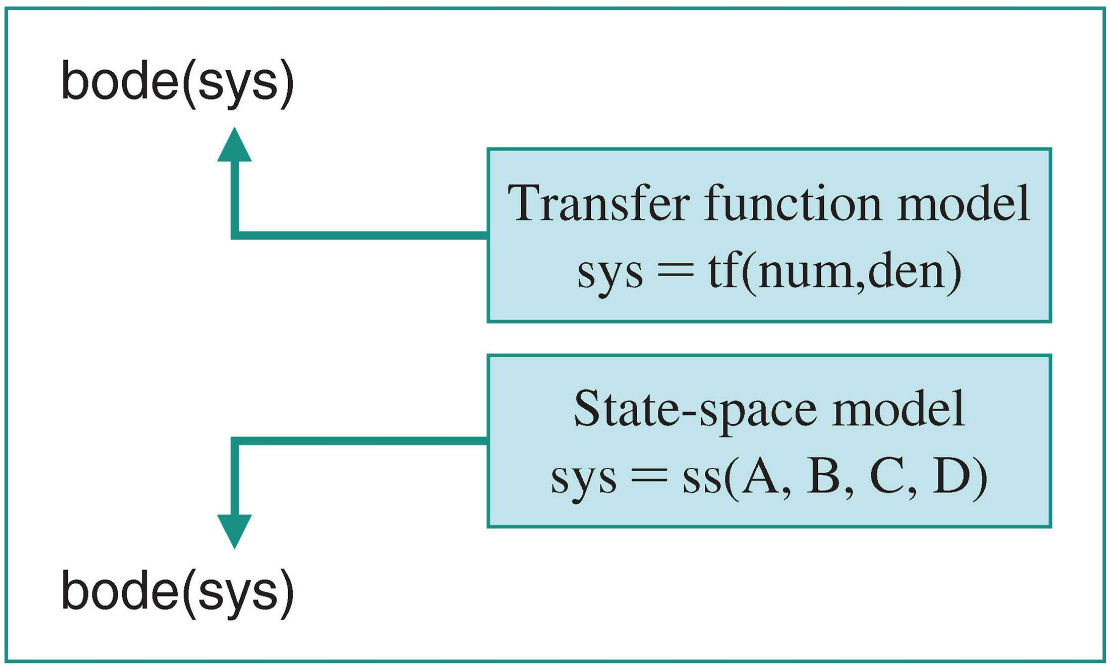




```
% Bode plot script for Figure 8.39
%
num=5*[0.1 1];
f1=[1 0]; f2=[0.5 1]; f3=[1/2500 .6/50 1];
den=conv(f1,conv(f2,f3));
%
sys=tf(num,den);
bode(sys)
```

Compute

$$s(1+0.5s)\left(1+\frac{0.6}{50}s+\frac{1}{50^2}s^2\right)$$



Example 21.2 (in class)

A unity feedback system has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K}{(s+2)^2}$$

(a) Assume $K = 40$, calculate the system phase margin;

(b) Select a gain K so that the phase P.M. $\geq 55^o$.

Answer:

(a) 36.9°. Flow of thoughts: form $L(j\omega) = |mag|\angle angle$; solve for ω_1 when $L(j\omega) = 1 = 0\text{dB}$; calculate $\angle L(j\omega_1)$, then P.M.

(b) e.g., for P.M. = 60° , $K = 16$. Flow of thoughts: set the desired P.M., solve for ω_2 meeting the required $\angle L(j\omega_2)$, then calculate through $|L(j\omega_2)| = 1 = 0dB$ the required gain K.

Lecture 22

Nyquist/Polar plot

- Another well-known Frequency Response method is through Nyquist/Polar plot.
- It utilizes "Function plane", in contrast to "s-plane" or "bode diagram".
- Manual plotting procedure is available.
    - Nyquist plot can be used to assess absolute and relative stability (and therefore also to design for meeting control system performance specification).

Frequency Response Plot – Polar(Nyquist) Plot

Transfer function of a system, $G(s)$, can be described in the frequency domain by the relation

$$\boldsymbol{G}(j\omega) = G(s)\Big|_{s=j\omega} = R(\omega) + jX(\omega)$$

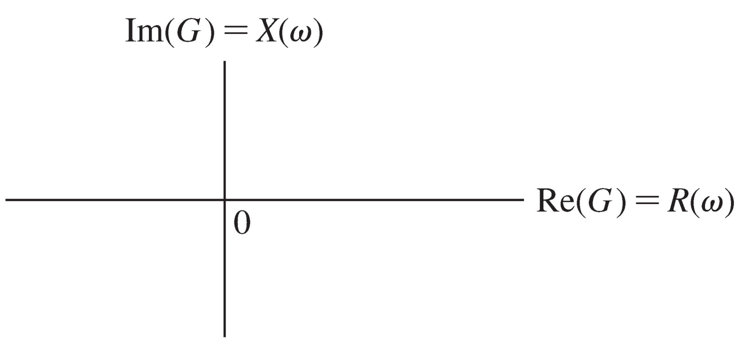
$$R(\omega) = Re[G(j\omega)]$$

$$X(\omega) = Im[G(j\omega)]$$

Alternatively, the transfer function can be represented by magnitude and phase as

$$\boldsymbol{G}(j\omega) = |\boldsymbol{G}(j\omega)|e^{j\phi(\omega)} = |\boldsymbol{G}(j\omega)|\angle\phi(\omega)$$

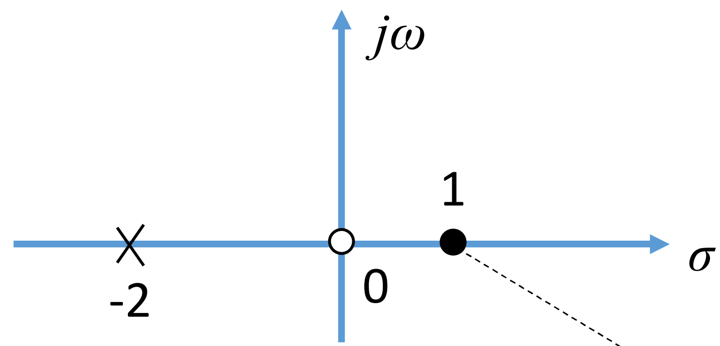
$$\phi(\omega) = \tan^{-1}\frac{X(\omega)}{R(\omega)} \; and \; |\boldsymbol{G}(j\omega)|^2 = [R(\omega)]^2 + [X(\omega)]^2$$



The polar plot representation of the frequency response is obtained by using the above equations.

s-plane and Function Plane

- Given any transfer function, e.g., $G(s) = \frac{s}{s+2}$:
  - The *s*-plane plot of this transfer function's poles and zeros is:

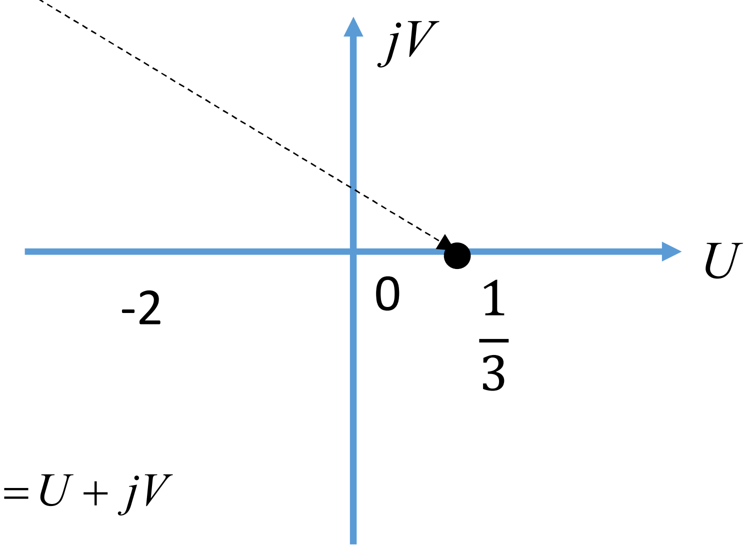


  - The function-plane of this transfer function is based on the following derivation:

$$G(s) = \frac{s}{s+2}$$

Let $s = \sigma + j\omega$,

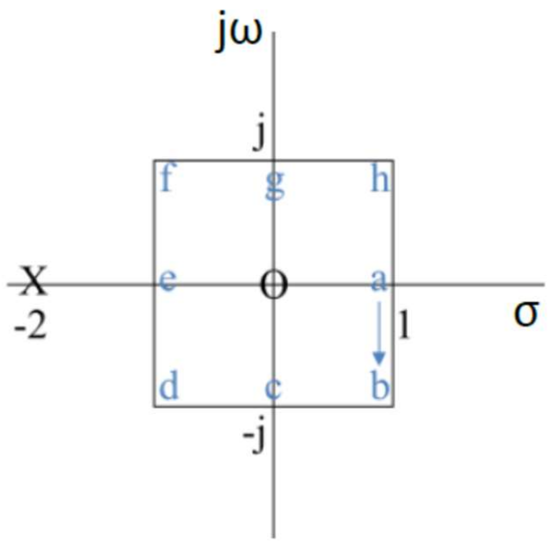
$$\begin{aligned}
G(s) &= \frac{\sigma + j\omega}{\sigma + j\omega + 2} \\
&= \frac{(\sigma + j\omega)(\sigma + 2 - j\omega)}{(\sigma + 2 + j\omega)(\sigma + 2 - j\omega)} \\
&= \frac{\sigma(\sigma+2) + \omega^2 + 2j\omega}{(\sigma+2)^2 + \omega^2} \\
&= \frac{\sigma(\sigma+2) + \omega^2}{(\sigma+2)^2 + \omega^2} + j\left[\frac{2\omega}{(\sigma+2)^2 + \omega^2}\right] = U + jV
\end{aligned}$$



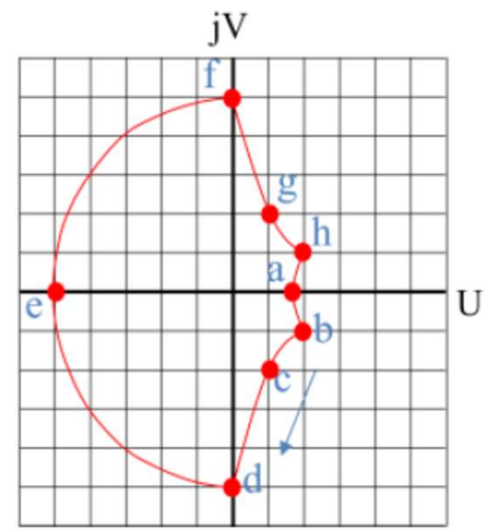
Contour Mapping from the s-Plane to the Function Plane

$$G(s) = \frac{s}{s+2}$$

$$G(s) = \frac{\sigma(\sigma+2)+\omega^2}{(\sigma+2)^2+\omega^2} + j\left[\frac{2\omega}{(\sigma+2)^2+\omega^2}\right] = U + jV$$



	σ	ω	U	V
a	1	0	0.33	0
b	1	-1	0.4	-0.2
c	0	-1	0.2	-0.4
d	-1	-1	0	-1
e	-1	0	-1	0
f	-1	1	0	1
g	0	1	0.2	0.4
h	1	1	0.4	0.2



- What is being varied at the *s*-plane is an arbitrary chosen point along the square drawn on the s-plane with its major points highlighted in the table.
- A closed contour in the *s*-plane gives a closed contour in the function plane.

Basic Definition

- As usual, we consider the characteristic equation of a closed-loop system:

$$F(s) = 1 + L(s) = \frac{K\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)} = 0$$

where $L(s) = K(s)G(s)H(s)$.

- For the CL system to be stable, all the poles of the CL transfer function, i.e., the zeros of $F(s)$ must lie in the LHS of the s-plane.

- Base on the contour mapping technique, if we choose a contour Γ_s in the s-plane that encloses the entire RHS, and we can determine whether any zeros of $F(s)$ lie within Γ_s by utilizing Cauchy's theorem, or more accurately the Principle of the Argument.

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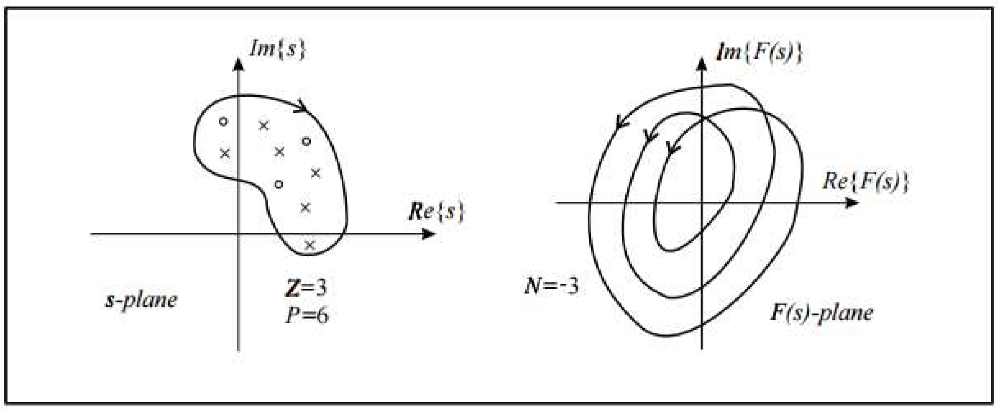
Cauchy's Theorem - Principle of the Argument

- Cauchy's theorem:
  - If a contour Γ_s in the s-plane encircles Z zeros and P poles of the system's s-plane and does not pass through any poles (or zeros) and the traversal is in the clockwise direction along the contour, the corresponding contour Γ_F in the $F(s)$-plane encircles the origin of the $F(s)$-plane $N = (Z - P)$ times in the clockwise direction.
  - Further interpretation: Recall that $F(s) = 1 + L(s)$. The number of zeros of $F(s)$ in the s-plane, i.e., Z, is the sum of the number of clockwise encirclements about the origin at the $F(s)$-function plane (N) and the number of poles of $F(s)$ in the s-plane (P). Mathematically, P can be known from $L(s)$ as the poles of $F(s)$ corresponds exactly to the poles of $L(s)$.



N=3-6=-3 (hence, 3 counterclockwise encirclement)

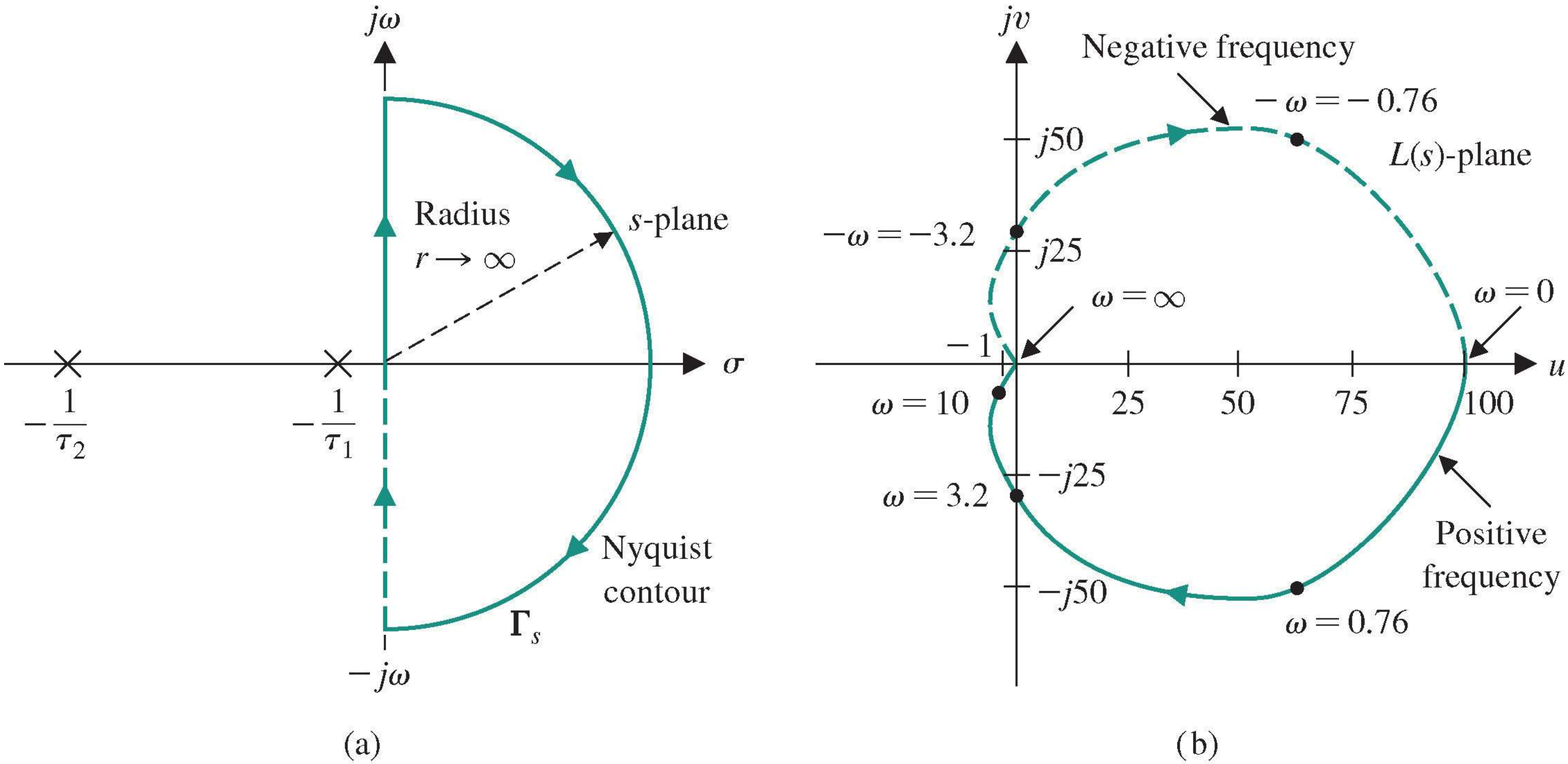
Nyquist Stability Criterion (Practical)

- A direct application of the Cauchy's theorem is rare because a factored form of $F(s)$ is usually not available, but instead, the one for $L(s)$ is usually the first thing available.

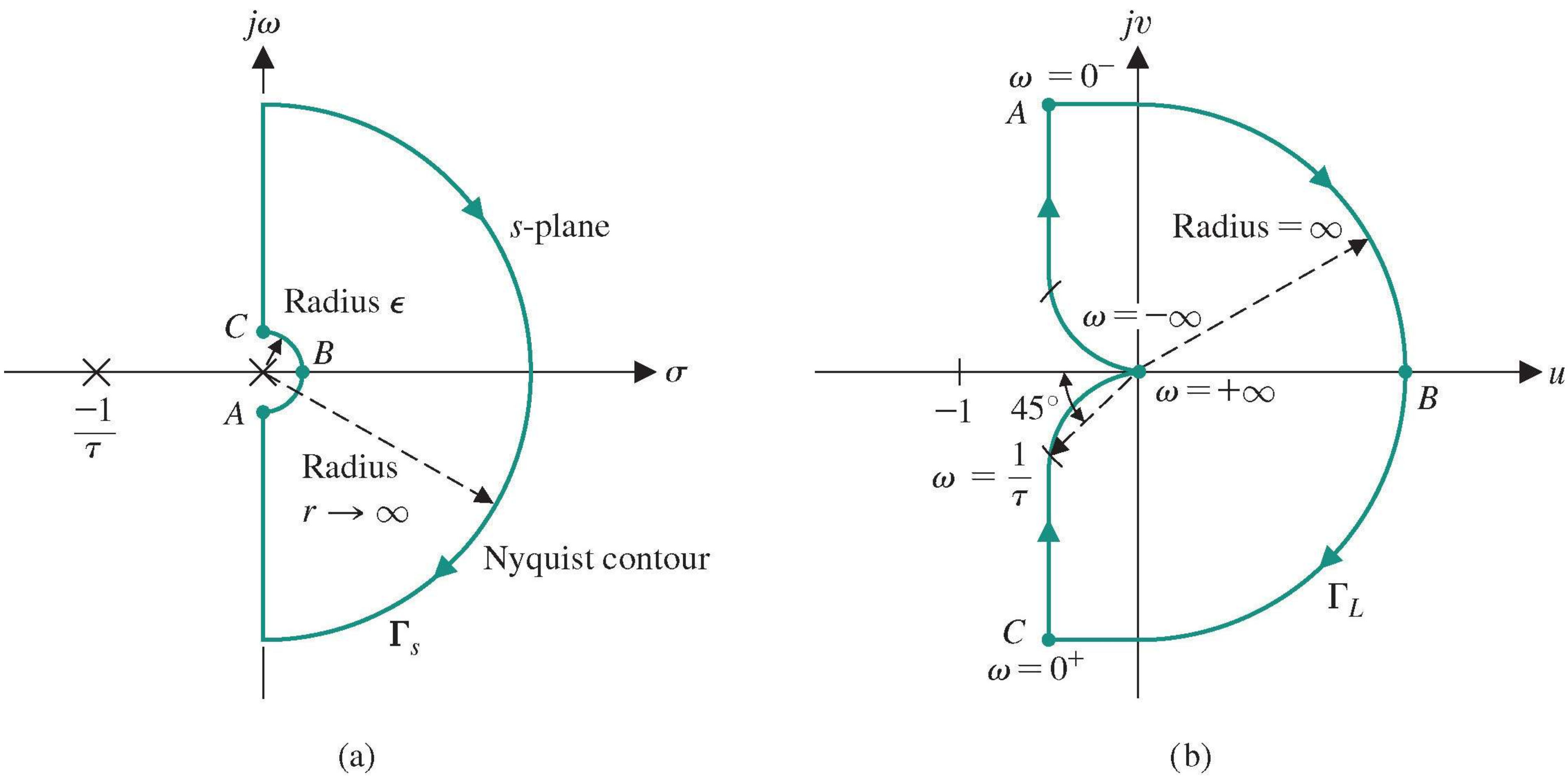
$$F(s) = 1 + L(s)$$
$$F'(s) = F(s) - 1 = L(s)$$

- Let contour Γ_L be the contour on the $L(s)$ function plane that is mapped from the whole RHS of the s-plane ($0 < \sigma < \infty$ and $-\infty < \omega < +\infty$, and not passing through any pole or zero), NSC states that (two distinctive scenarios):
  - For $L(s)$ without any pole in the RHS of the s-plane: The feedback system is stable if an only if the contour Γ_L in the $L(s)$-plane does not encircle the (-1,0) point.
    - Mathematically, with $Z = N + P$, since $P = 0$, we need $N = 0$ to make $Z = 0$.
  - For $L(s)$ with pole(s) in the RHS of the s-plane: The feedback system is stable if and only if the contour Γ_L in the $L(s)$-plane encircles in a counterclockwise direction the (-1,0) point the same of times as the number of poles of $F(s)$ with positive real parts (i.e., number of poles of $F(s)$ in the RHS of the s-plane).
    - Mathematically, with $Z = N + P$, since P is a positive integer, we need N to be the same negative integer (this explains the "counterclockwise" requirement) to make $Z = 0$.

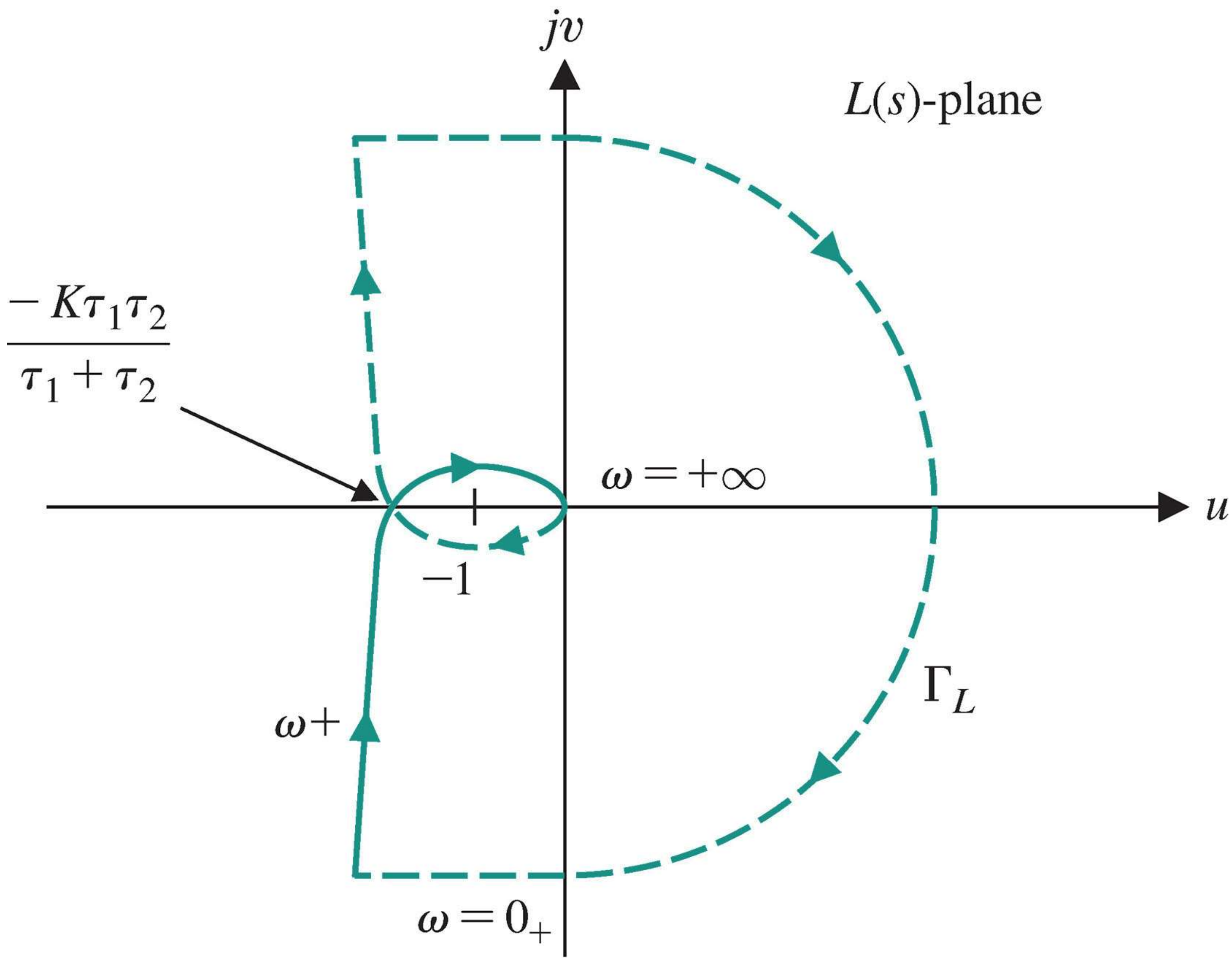
Nyquist contour and mapping for $L(s) = 100/(s + 1)(s/10 + 1)$



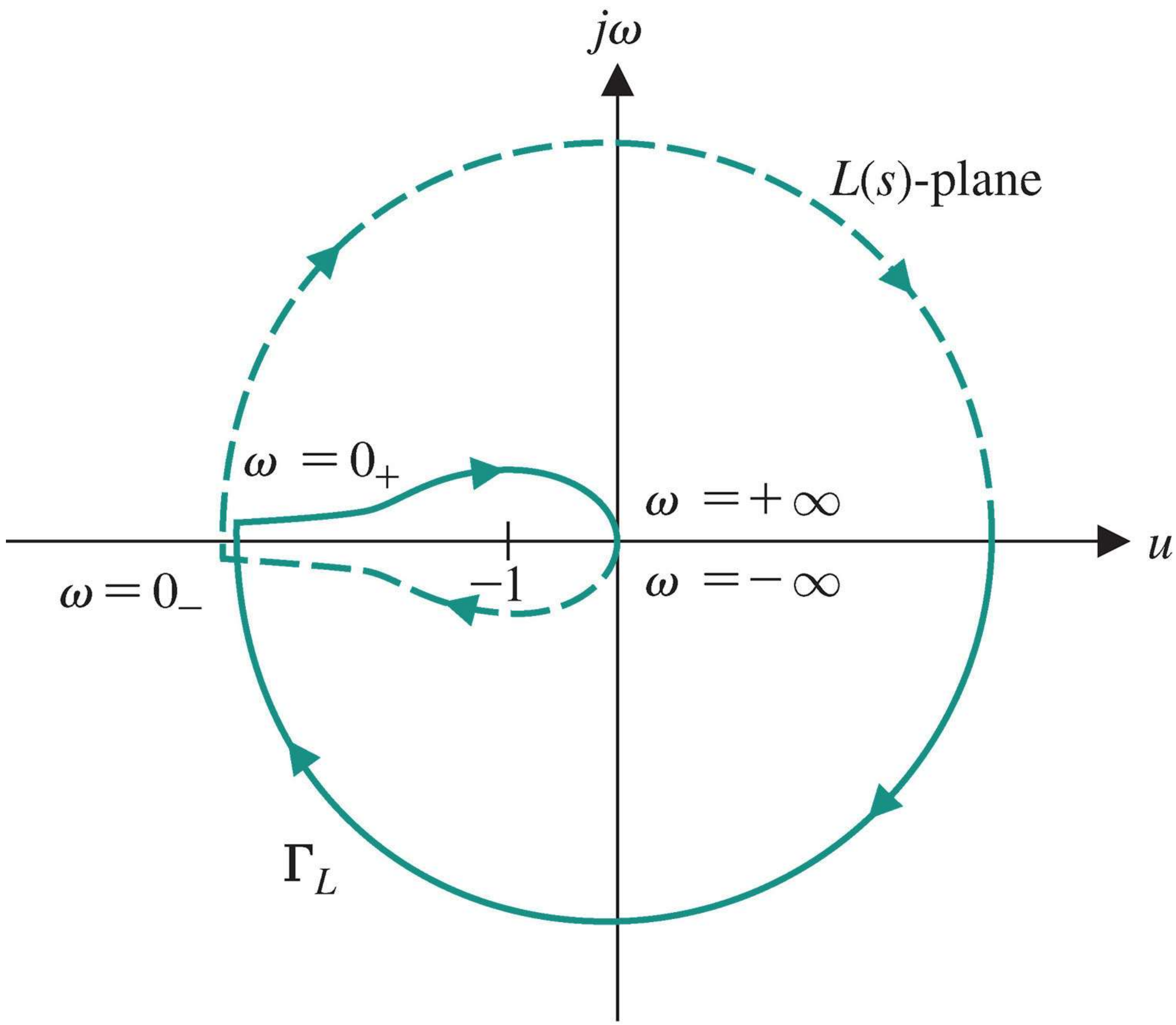
Nyquist contour and mapping for $L(s) = K/[s(\tau s + 1)]$



Nyquist plot for $L(s) = K/[s(\tau_1 s + 1)(\tau_2 s + 1)]$. The tic mark shown to the left of the origin is the −1 point.



Nyquist contour plot for $L(s) = K/[s^2(\tau s + 1)]$

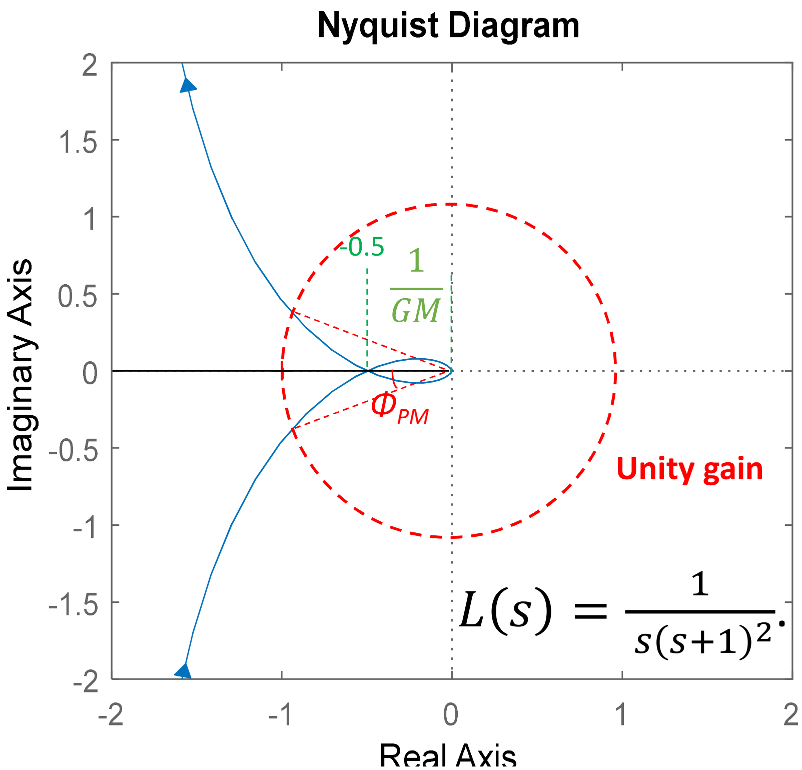


Relative Stability

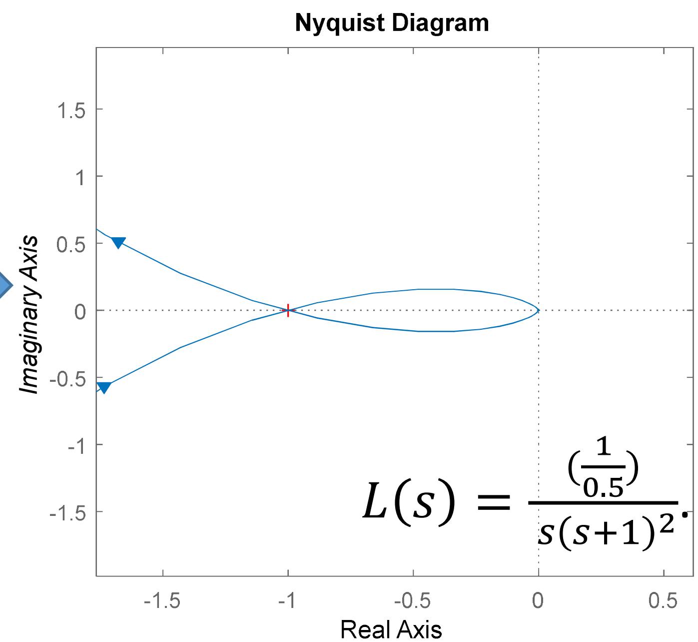
- Changing the gain of the controller will affect the stability of the CL system (we have seen this in RL and bode plots!).
- Whenever applicable, there exists margins (specifically, gain margin and phase margin) within which the CL system is stable. Nyquist (and Nichols) plots can be used to assess this relative stability.
- Definition 1: Gain margin is defined as the reciprocal of the gain at a particular frequency, i.e., the *phase crossover frequency*, during which the phase angle is (-)180^{o}.
  - Alternatively, Gain margin is defined as the change in open-loop gain required at 180^{o} phase shift to make the closed-loop system unstable.
- Definition 2: Phase margin is the difference between (-)180^{o} and the phase angle at a particular frequency, i.e., the *gain crossover frequency*, at which the system gain is unity.
  - Alternatively, Phase margin is the change in open-loop phase shift required at unity gain to make the closed-loop system unstable.

Nyquist Plot: Relative Stability, Stability Margin

- Example, for $L(s) = \frac{1}{s(s+1)^2}$.



Apply the gain k = *GM* to the forward path of the CL sys, we expect to get a marginally stable CL system.



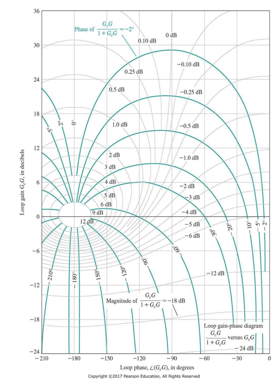
- If one were to adjust k to beyond *GM*, the (-1,0) will be clockwise-encircled 2 times (not 1, recall that there is another clockwise, infinite circle). Since $Z = N+P=2+0=2$, there are two zeros for the CL characteristic function $1+L(s)$, i.e., two poles of the CL transfer function are located at the RHS of the s-plane.

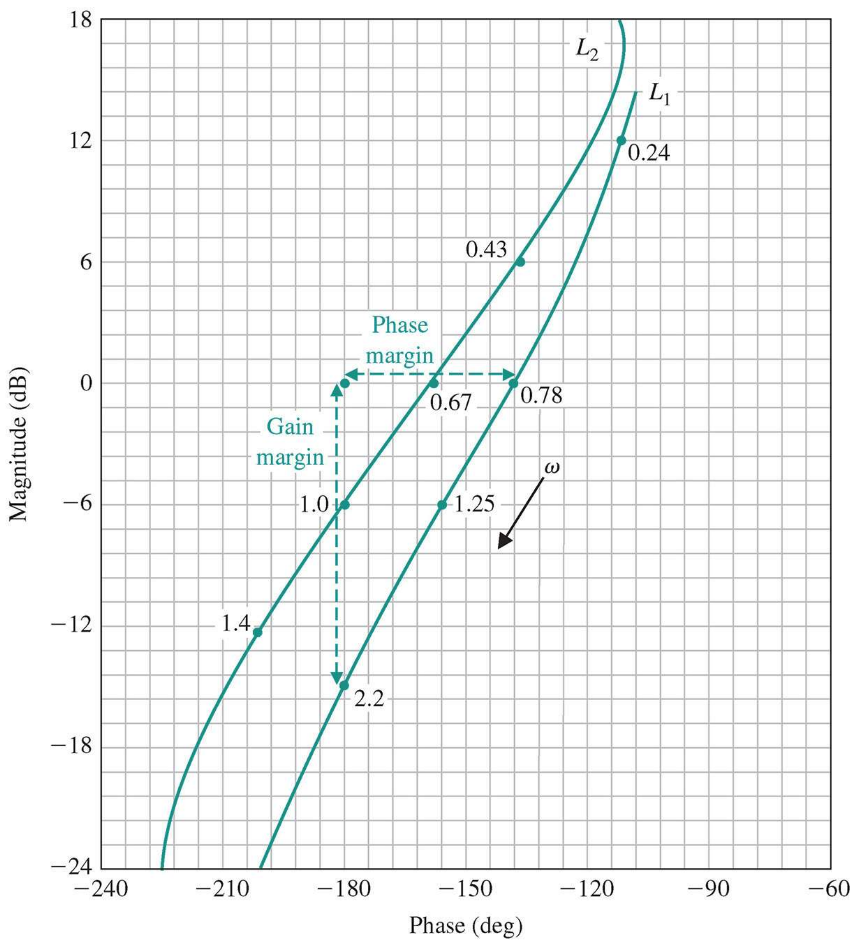
Nichols Chart: Relative Stability, Stability Margin

Nichols chart are basically the magnitude-phase plots. Examples:

(1) $L_1(j\omega) = \dfrac{1}{j\omega(j\omega+1)(0.2j\omega+1)}$

(2) $L_2(j\omega) = \dfrac{1}{j\omega(j\omega+1)^2}$





- If we intend to assess their relative stability, we have:

(1) phase margin is 43°, gain margin is 15 dB

(2) phase margin is 20°, gain margin is 5.7 dB

- Gain margin or phase margin is a measure of **relative stability**.
- We can conclude that system 1 is MORE stable than system 2:
  - ✓ Higher the (positive) margin indicates a more stable system.

Concluding Remarks

- **What have been covered:** Frequency Response Method "Bode plot and Nyquist plot"
  - Frequency performance specification through Bode plot: $M_{p\omega}$, ω_B
  - Stability margin from Bode plot: GM, $\emptyset_{pm}$
  - Applications: lead/lag/lead-lag compensator design
  - Numerical tool example: Matlab
  - Polar/Nyquist plot
  - Contour mapping, and its relationship with Nyquist stability Criterion
  - Stability margin from Nyquist plot: GM, $\emptyset_{pm}$
- **How to get in touch**: through LMO Module's "*General question and answer forum*" section or during my weekly consultation hour(s).

Additional Exercises (self-check)

Textbook ("Modern Control Systems" by R. C. Dorf & R. H. Bishop, 14th edition, Chapter 8 and Chapter 9's sub-section 9.4, 9.6):

- Skills Check - Can be found from Textbook pg. 596-600 (answer in pg. 620) or LMO MEC280's "Quizzes/Tutorials" section.
- Some other Skills Check in Chapter 9, especially those related to Bode plot's Gain/Phase margins and crossover frequencies, Textbook pg. 698-701 (answer in pg. 727).

Additional:
E8.1, E8.3, E8.8 (140 rad/s; 0 dB/dec and -20 dB/dec), **E8.11** (ω = 9.85 rad/s)
E9.2, E9.3, E9.6 (inf, 63.8°), **9.10, 9.12** (20; 19°, 14.6 dB; 12.4 rad/s, 9.82, 7.92 rad/s), **E9.28** (14.4°; 0.144, 63.3%; check actual/accurate using MATLAB, 67%)