



Xi'an Jiaotong-Liverpool University

西交利物浦大學

# MEC208 Instrumentation and Control System

*2024-25 Semester 2*

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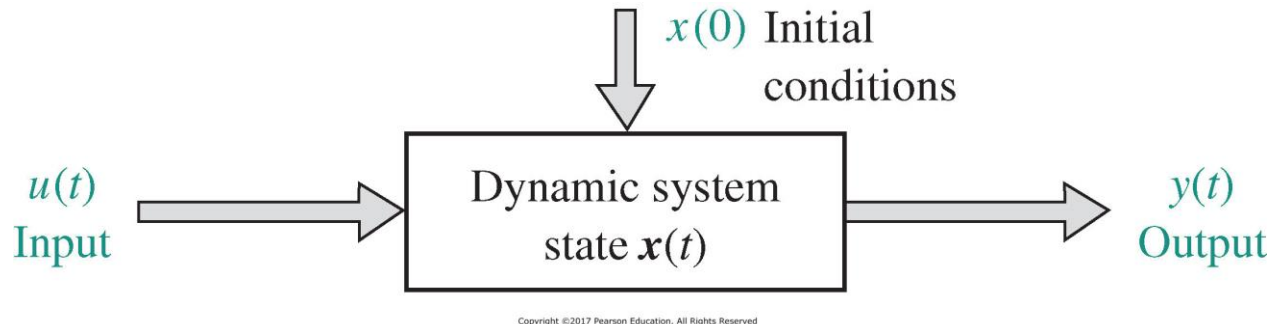
# State Variable Models

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- ☐ **State Variables**
- ☐ **State-space Modeling**
- ☐ **State Space Representation in Matrix Form**
- ☐ **Time-domain response (Solution of State-space Models)**
- ☐ **Conversion between State-space Model and Transfer Function**

# State of a System

The **State** of a system is a set of variables whose values, together with **the input signals** and the **equations describing the dynamics**, will provide the **future state** and **output** of the system;  
**It is the minimum information needed about the system in order to determine its future behavior.**



State variables:  $[x_1(t), x_2(t), x_3(t), \dots x_n(t)]$   $x(t)$

Input signals:  $[u_1(t), u_2(t), \dots u_m(t)]$   $u(t)$

Output signals:  $[y_1(t), y_2(t), \dots y_p(t)]$   $y(t)$

# State-space Modelling

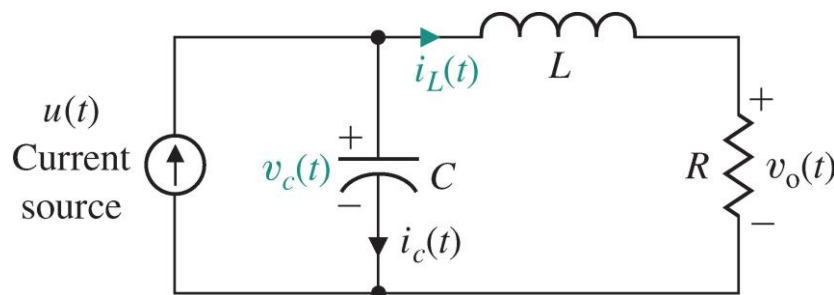
First-order linear differential equations:

$$\begin{cases} \dot{x}_1(t) = a_{11}x_1(t) + a_{12}x_2(t) + \cdots + a_{1n}x_n(t) + b_{11}u_1(t) + \cdots + b_{1m}u_m(t) \\ \dot{x}_2(t) = a_{21}x_1(t) + a_{22}x_2(t) + \cdots + a_{2n}x_n(t) + b_{21}u_1(t) + \cdots + b_{2m}u_m(t) \\ \vdots \\ \dot{x}_n(t) = a_{n1}x_1(t) + a_{n2}x_2(t) + \cdots + a_{nn}x_n(t) + b_{n1}u_1(t) + \cdots + b_{nm}u_m(t) \end{cases}$$

$$\begin{cases} y_1(t) = c_{11}x_1(t) + c_{12}x_2(t) + \cdots + c_{1n}x_n(t) + d_{11}u_1(t) + \cdots + d_{1m}u_m(t) \\ y_2(t) = c_{21}x_1(t) + c_{22}x_2(t) + \cdots + c_{2n}x_n(t) + d_{21}u_1(t) + \cdots + d_{2m}u_m(t) \\ \vdots \\ y_n(t) = c_{n1}x_1(t) + c_{n2}x_2(t) + \cdots + c_{nn}x_n(t) + d_{n1}u_1(t) + \cdots + d_{nm}u_m(t) \end{cases}$$

*\*note: boldface small letter: vector; boldface capital letter: matrix.*

# Example 9.3 revisit



input: source current  $u(t)$

output: resistor voltage  $v_o(t)$

Ordinary Differential Equations:

$$i_c(t) = C \frac{dv_c}{dt} = u(t) - i_L(t)$$

$$L \frac{di_L}{dt} = -Ri_L(t) + v_c(t)$$

Output:

$$v_o(t) = Ri_L(t)$$

- Choose variables, let  $x_1(t) = v_c(t)$ ,  $x_2(t) = i_L(t)$ ,  $y(t) = v_o(t)$

$$\frac{dx_1}{dt} = -\frac{1}{C}x_2(t) + \frac{1}{C}u(t)$$

$$\frac{dx_2}{dt} = \frac{1}{L}x_1(t) - \frac{R}{L}x_2(t)$$

$$y(t) = v_o(t) = Rx_2(t)$$



$$\dot{x}_1 = -\frac{1}{C}x_2 + \frac{1}{C}u$$

$$\dot{x}_2 = \frac{1}{L}x_1 - \frac{R}{L}x_2$$

$$y = Rx_2$$

# Process of State-space Modelling

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Model the system by linear ordinary differential equations (ODEs)

Define a set of state variables

Substitute the state variables into the ODEs and rewrite ODEs into

Rewrite ODEs into a set of first order differential equations

Don't forget about the output equations

**For a system, the number of state variables required is equal to the number of **independent energy-storage** elements.**

# State-space Representation in Matrix Form

Linear first-order differential equations:

$$\dot{x}_1(t) = a_{11}x_1(t) + a_{12}x_2(t) + \cdots + a_{1n}x_n(t) + b_{11}u_1(t) + \cdots + b_{1m}u_m(t)$$

$$\dot{x}_2(t) = a_{21}x_1(t) + a_{22}x_2(t) + \cdots + a_{2n}x_n(t) + b_{21}u_1(t) + \cdots + b_{2m}u_m(t)$$

$\vdots$

$$\dot{x}_n(t) = a_{n1}x_1(t) + a_{n2}x_2(t) + \cdots + a_{nn}x_n(t) + b_{n1}u_1(t) + \cdots + b_{nm}u_m(t)$$

State Space Equations:

System matrix

$$\dot{\mathbf{x}}(t) = \mathbf{Ax}(t) + \mathbf{Bu}(t)$$

Input matrix

Output matrix

$$\mathbf{y}(t) = \mathbf{Cx}(t) + \mathbf{Du}(t)$$

Feed-forward matrix

*Dimensions:*  $\mathbf{x}(n \times 1)$ ,  $\mathbf{u}(m \times 1)$ ,  $\mathbf{y}(p \times 1)$ ;  $\mathbf{A}(n \times n)$ ,  $\mathbf{B}(n \times m)$ ,  $\mathbf{C}(p \times n)$ ,  $\mathbf{D}(p \times m)$ .

*\*note: boldface small letter: vector; boldface capital letter: matrix.*



# Time-domain response

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s),$$

$$\mathbf{X}(s) = [s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{x}(0) + [s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B}\mathbf{U}(s)$$

State transition matrix

$$\Phi(s) = [s\mathbf{I} - \mathbf{A}]^{-1}$$

Then:

$$\mathbf{x}(t) = \Phi(t)\mathbf{x}(0) + \int_0^t \Phi(t - \tau)\mathbf{B}\mathbf{u}(\tau) d\tau$$

If  $\mathbf{u}(t) = 0$ , then

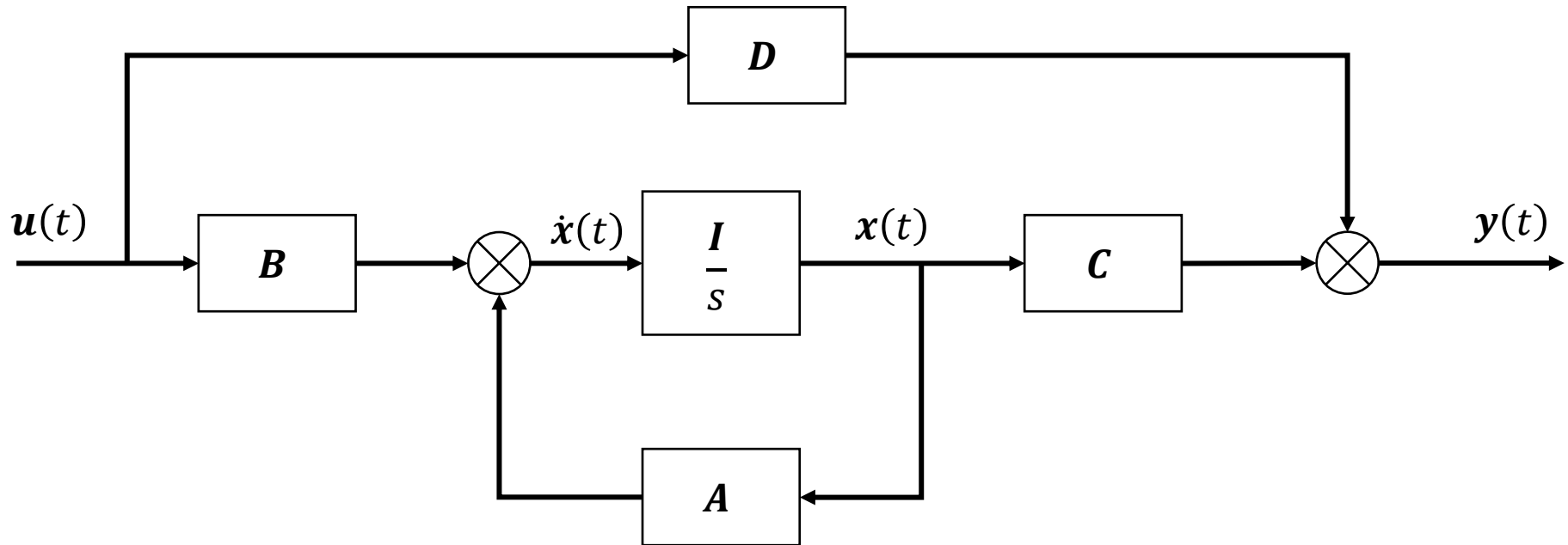
$$\text{unforced response} \quad \mathbf{x}(t) = \Phi(t)\mathbf{x}(0)$$

If  $\mathbf{u}(t) \neq 0$ , then

$$\text{forced response} \quad \mathbf{x}(t) = \Phi(t)\mathbf{x}(0) + \int_0^t \Phi(t - \tau)\mathbf{B}\mathbf{u}(\tau) d\tau \quad \text{Equivalent form}$$

# Covert State-space Model to Transfer Function

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned} \quad \Rightarrow \quad \begin{aligned}s\mathbf{X}(s) &= \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) \\ \mathbf{Y}(s) &= \mathbf{C}\mathbf{X}(s) + \mathbf{D}\mathbf{U}(s)\end{aligned} \quad \Rightarrow \quad \mathbf{U}(s) \rightarrow \boxed{\mathbf{G}(s)} \rightarrow \mathbf{Y}(s)$$



$$\mathbf{Y}(s) = (\mathbf{C}[s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B} + \mathbf{D})\mathbf{U}(s)$$

$$\mathbf{G}(s) = \mathbf{C}[s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B} + \mathbf{D}$$

# Covert Transfer Function to State-space Model

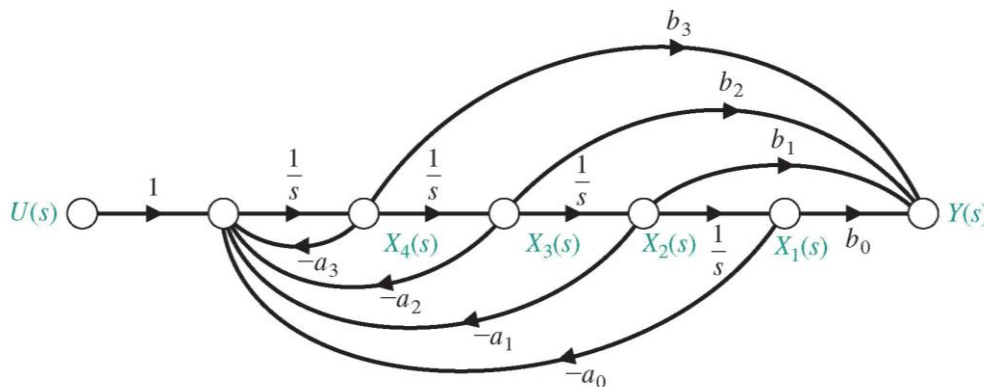
Method 1: to develop graphic model of the system and use this model to determine state variables.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} \quad n \geq m$$

Divided by  $s^n$

$$G(s) = \frac{b_m s^{-(n-m)} + b_{m-1} s^{-(n-m+1)} + \dots + b_1 s^{-(n-1)} + b_0 s^{-n}}{1 + a_{n-1} s^{-1} + \dots + a_1 s^{-(n-1)} + a_0 s^{-n}}$$

$$\begin{aligned} G(s) &= \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \\ &= \frac{b_3 s^{-1} + b_2 s^{-2} + b_1 s^{-3} + b_0 s^{-4}}{1 + a_3 s^{-1} + a_2 s^{-2} + a_1 s^{-3} + a_0 s^{-4}} \end{aligned}$$



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -a_0 x_1 - a_1 x_2 - a_2 x_3 - a_3 x_4 + u$$

$$y = b_0 x_1 + b_1 x_2 + b_2 x_3 + b_3 x_4$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [b_0 \quad b_1 \quad b_2 \quad b_3] \quad D = [0]$$

# Covert Transfer Function to State-space Model

Method 2: State-space Model can be also obtained by introducing an intermediate variable  $Z(s)$ .

For simplicity, assume  $n = 4$ :

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_3s^3 + b_2s^2 + b_1s + b_0}{s^4 + a_3s^3 + a_2s^2 + a_1s + a_0} \frac{Z(s)}{Z(s)}$$

$$Y(s) = (b_3s^3 + b_2s^2 + b_1s + b_0)Z(s)$$

$$U(s) = (s^4 + a_3s^3 + a_2s^2 + a_1s + a_0)Z(s)$$

Then taking inverse Laplace transform of both equations:

$$y = b_3 \frac{d^3z}{dt^3} + b_2 \frac{d^2z}{dt^2} + b_1 \frac{dz}{dt} + b_0z$$

$$u = \frac{d^4z}{dt^4} + a_3 \frac{d^3z}{dt^3} + a_2 \frac{d^2z}{dt^2} + a_1 \frac{dz}{dt} + a_0z$$

# Covert Transfer Function to State-space Model

$$y = b_3 \frac{d^3 z}{dt^3} + b_2 \frac{d^2 z}{dt^2} + b_1 \frac{dz}{dt} + b_0 z \quad u = \frac{d^4 z}{dt^4} + a_3 \frac{d^3 z}{dt^3} + a_2 \frac{d^2 z}{dt^2} + a_1 \frac{dz}{dt} + a_0 z.$$

Define the four state variables as follows:

$$\begin{aligned} x_1 &= z \\ x_2 &= \dot{x}_1 = \dot{z} \\ x_3 &= \dot{x}_2 = \ddot{z} \\ x_4 &= \dot{x}_3 = \dddot{z}. \end{aligned}$$

Then the differential equation can be written equivalently as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= x_4, \end{aligned}$$

and

$$\dot{x}_4 = -a_0 x_1 - a_1 x_2 - a_2 x_3 - a_3 x_4 + u,$$

and the corresponding output equation is

$$y = b_0 x_1 + b_1 x_2 + b_2 x_3 + b_3 x_4.$$

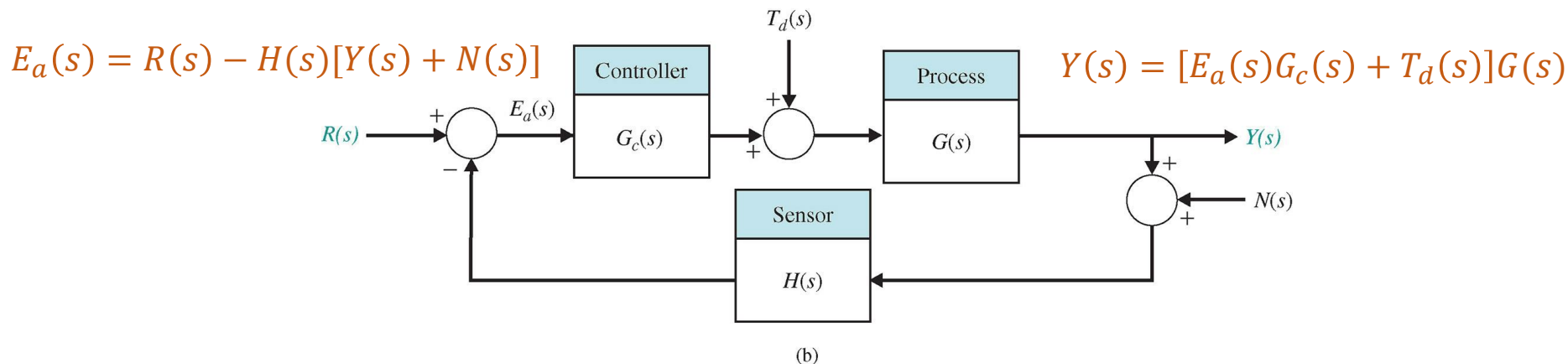
Method 3: Select state variable with physical meanings.

# Feedback Control System Characteristics

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- ☐ **Error Signal Analysis**
- ☐ **Sensitivity of Control System to Parameter Variations**
- ☐ **Disturbance Rejection and Measurement Noise Attenuation**
- ☐ **Control of the Transient Response and Steady-state Error**
- ☐ **“Cost” of Feedback**

# Error Signal Analysis



**Tracking error definition:**  $E(s) = R(s) - Y(s)$

To facilitate our discussion, unity feedback system is assumed, i.e.,  $H(s) = 1$ .

The output can be obtained from the block diagram:

$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}R(s) + \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) - \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s)$$

Therefore:  $E(s) = R(s) - Y(s) = \frac{1}{1 + G_c(s)G(s)}R(s) - \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) + \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s)$

$$E(s) = \frac{1}{1 + L(s)}R(s) - \frac{G(s)}{1 + L(s)}T_d(s) + \frac{L(s)}{1 + L(s)}N(s)$$

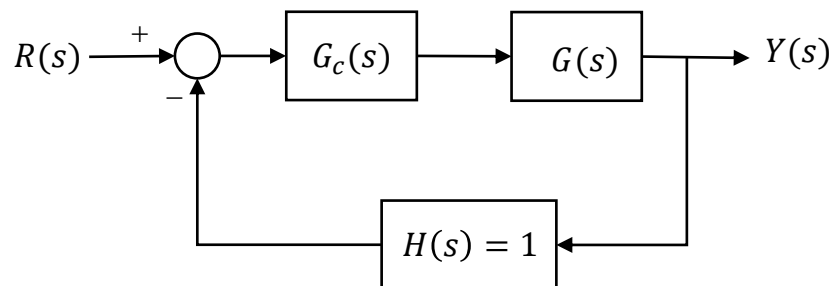
where **loop gain**  $L(s) = G_c(s)G(s)H(s) = G_c(s)G(s)$

# Definition of System Sensitivity

By definition:  $S = \frac{\partial T/T}{\partial G/G}$ , where system transfer function  $T(s) = \frac{Y(s)}{R(s)}$

In the limit, for small incremental changes:

$$S = \frac{\partial T/T}{\partial G/G} = \frac{\partial \ln T}{\partial \ln G}$$



**System sensitivity** is the ratio of the change in the system transfer function  $T(s)$  to the change of a process transfer function  $G(s)$  (or parameter) for a small incremental change.

Sensitivity for OL system: 1

Sensitivity for CL system: since  $T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}$

$$S_G^T = \frac{\partial T \cdot G}{\partial G \cdot T} = \frac{G_c}{[1 + G_c G]^2} \cdot \frac{1 + G_c G}{G_c} \rightarrow S_G^T = \frac{1}{1 + G_c G}$$

To determine the influence of process **parameter  $\alpha$  (of  $G(s)$ )**, use chain rule:

$$S_\alpha^T = S_G^T S_\alpha^G$$



# Disturbance Rejection

Feedback control reduces the negative effect of disturbance signals:

- A disturbance signal is an unwanted input signal that affects the output signal.
- Many control systems are subject to extraneous disturbance signals that cause the system to provide an inaccurate output.
  - Electronic amplifiers have inherent noise generated within the integrated circuits or transistors;
  - Radar antennas are subject to wind gusts;
  - Many systems generate unwanted distortion signals due to nonlinear elements.
- The benefit of feedback systems is that the effect of distortion, noise, and unwanted disturbances can be effectively reduced.

To analyze rejection of disturbance, assume  $R(s) = N(s) = 0$ .

$$E(s) = -S(s)G(s)T_d(s) = -\frac{G(s)}{1 + L(s)}T_d(s)$$

For a fixed  $G(s)$  and a given  $T_d(s)$ , as the loop gain  $L(s)$  increases, the effect of  $T_d(s)$  on the tracking error decreases. **For good disturbance rejection, we require a large loop gain over the frequencies of interest associated with the expected disturbance signals.**

# Measurement Noise Attenuation

A noise signal that is prevalent in many systems is the noise generated by the **measurement sensor**.

To analyze attenuation of measurement noise, assume  $R(s) = T_d(s) = 0$ .

$$E(s) = C(s)N(s) = \frac{L(s)}{1 + L(s)} N(s)$$

As the loop gain  $L(s)$  decreases, the effect of  $N(s)$  on the tracking error decreases. **For effective measurement noise attenuation, we need a small loop gain over the frequencies associated with the expected noise signals.**

## How to realize disturbance rejection and measurement noise attenuation at the same time?

- In practice, disturbances are often at low frequencies, while measurement noise signals are often at high frequencies.
- **Therefore, the controller should be of high gain at low frequencies and low gain at high frequencies.**

# Control of Transient Response

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One of the most important characteristics of control systems is their **transient response**, which is a function of time.

Another purpose of control systems is to provide a desired satisfactory transient response:

- If an OL control system does not provide a satisfactory transient response, then the process,  $G(s)$ , may need to be replaced with a more suitable process;
- By contrast, a CL system can often be adjusted to yield the desired response by adjusting the **feedback loop parameters (e.g., controller and feedback path parameters)**.

A feedback control system is valuable because it provides the engineer with the ability to **adjust/manipulate** the transient response.

# Steady-state Error (and Output)

The **steady-state error (output)** is the error (output) value after the transient response has decayed, leaving only the continuous response.

**Final Value Theorem (only for stable system):**

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s)$$

Assume a unit step input as a comparable input ( $r(t) = 1, t > 0; R(s) = \frac{1}{s}$ ):

Open-loop:  $E_{OL}(s) = R(s) - Y(s) = (1 - G_c(s)G(s))R(s)$

$$e_{OL}(\infty) = \lim_{s \rightarrow 0} s(1 - G_c(s)G(s)) \left( \frac{1}{s} \right) = 1 - G_c(0)G(0)$$

To calculate **steady-state output** towards input  $R$ , simply use:

Closed-loop (assume  $T_d(s) = N(s) = 0$ , and  $H(s) = 1$ ):

$$E_{CL}(s) = \frac{1}{1 + G_c(s)G(s)} R(s)$$

$$e_{CL}(\infty) = \lim_{s \rightarrow 0} s \frac{1}{1 + G_c(s)G(s)} \frac{1}{s} = \frac{1}{1 + G_c(0)G(0)}$$

$$\begin{aligned} \lim_{t \rightarrow \infty} y(t) &= \lim_{s \rightarrow 0} sY(s) \\ &= \lim_{s \rightarrow 0} sT_{CL}R(s) \end{aligned}$$

Large  $L(0) = G_c(0)G(0)$  will lead to small steady-state error.

# “Cost” of Feedback Control

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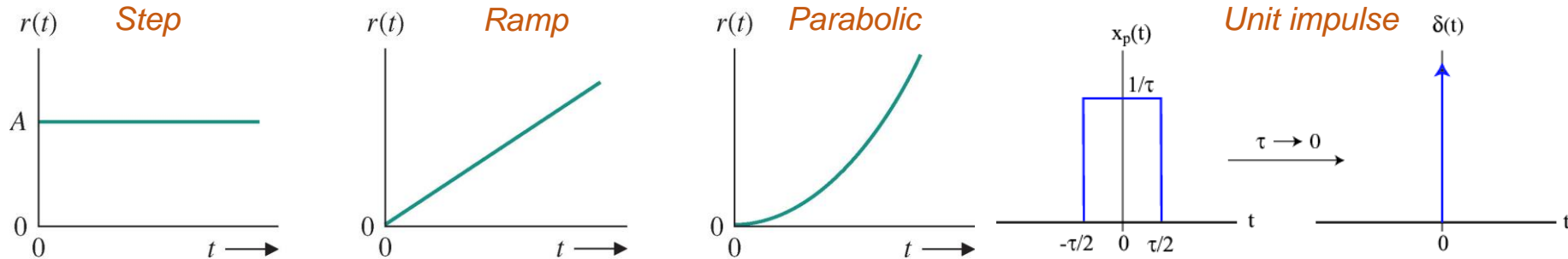
- **Increased number of components and complexity in the system.**
  - To add feedback, it is necessary to consider several “physical” feedback components, e.g. sensors. The sensor is often the most expensive component in a control system, which may introduce noise, inaccuracy, and robustness issues.
- **Loss of Gain.**
  - Loop gain:  $G_c(s)G(s)$
  - Closed-loop gain:  $\frac{G_c(s)G(s)}{1+G_c(s)G(s)}$
- **Introduction of the possibility of instability.**
  - Even if an open-loop system is stable, the closed-loop system may not be always stable (will be discussed in later chapters).

# Time-Domain Performance of Feedback Control System

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- ☐ Test Input Signals
- ☐ Performance of First-Order and Second-Order System
- ☐ Effects of a Third Pole and a Zero on the Second-Order System Response
- ☐ Pole Location on the s-plane and the Transient Response
- ☐ Steady-State Error of Feedback Control Systems

# Test Input Signal in Time- and s-Domain



**Table 5.1 Test Signal Inputs**

Test Signal	$r(t)$	$R(s)$
Step	$r(t) = A, t > 0$ $= 0, t < 0$	$R(s) = A/s$
Ramp	$r(t) = At, t > 0$ $= 0, t < 0$	$R(s) = A/s^2$
Parabolic	$r(t) = At^2, t > 0$ $= 0, t < 0$	$R(s) = 2A/s^3$

$$r(t) = \delta(t)$$

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ +\infty, & t = 0 \end{cases}$$

$$R(s) = 1$$

**Note:** this is a continuous-time impulse.

# First-order System: Unit-step Time Response

Normalized form  $\frac{Y(s)}{R(s)} = \frac{1}{Ts + 1}$

T - time constant

Unit-step time response

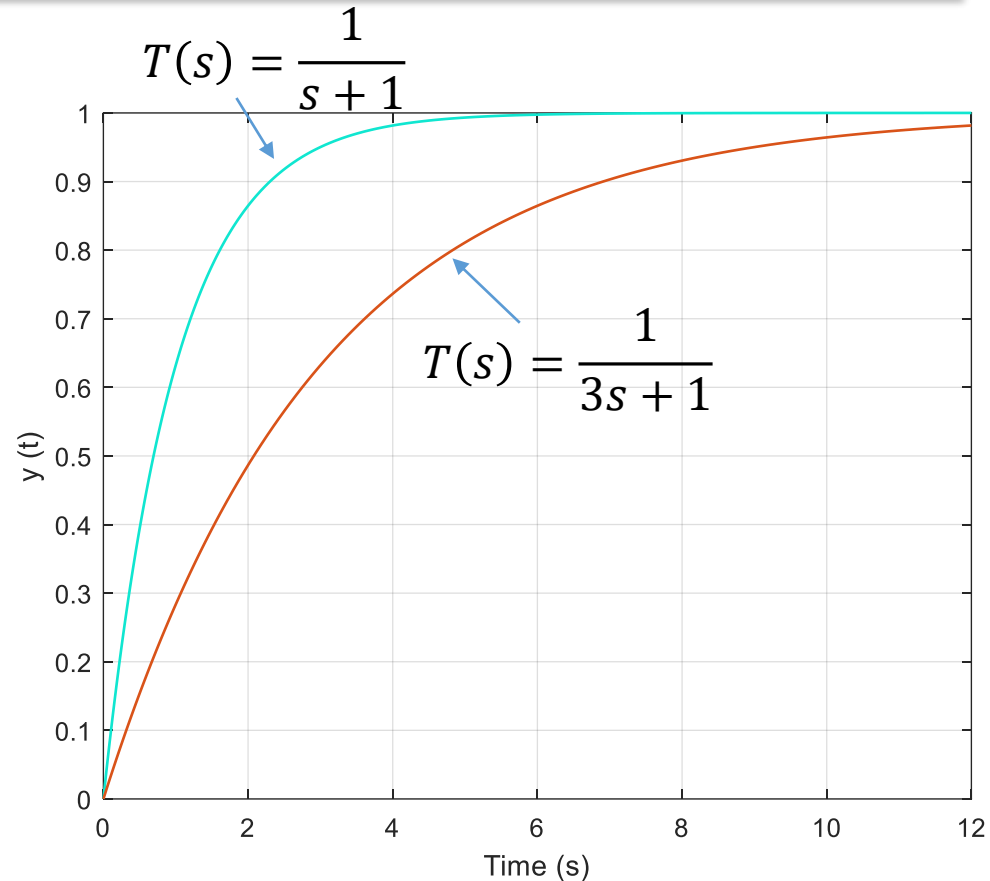
$$y(t) = 1 - e^{-t/T}$$

Time points of interest

$$y(T) = 0.632$$

$$y(3T) = 0.95$$

$$y(4T) = 0.982$$





# Second-Order system

The **normalized form** is :

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n$  – Natural frequency  
 $\zeta$  – Damping ratio/coefficient

$\zeta > 1$       *overdamped*

$\zeta = 1$       *critical damped*

$\zeta < 1$       *under damped*

Unit-step time response:

$$\zeta > 1 \qquad y(t) = 1 - Ae^{-\frac{1}{T_1}t} - Be^{-\frac{1}{T_2}t}$$

One component can be ignored if  $T_1$  and  $T_2$  are far away enough to each other

$$\zeta < 1 \qquad y(t) = 1 - e^{-\zeta\omega_n t} \left( \cos \sqrt{1 - \zeta^2} \omega_n t + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \sqrt{1 - \zeta^2} \omega_n t \right)$$

# Second-Order system - under damped

$$\zeta < 1$$

$$y(t) = 1 - e^{-\zeta\omega_n t} \left( \cos \sqrt{1 - \zeta^2} \omega_n t + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \sqrt{1 - \zeta^2} \omega_n t \right)$$

Peak Time:

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

Rise Time:

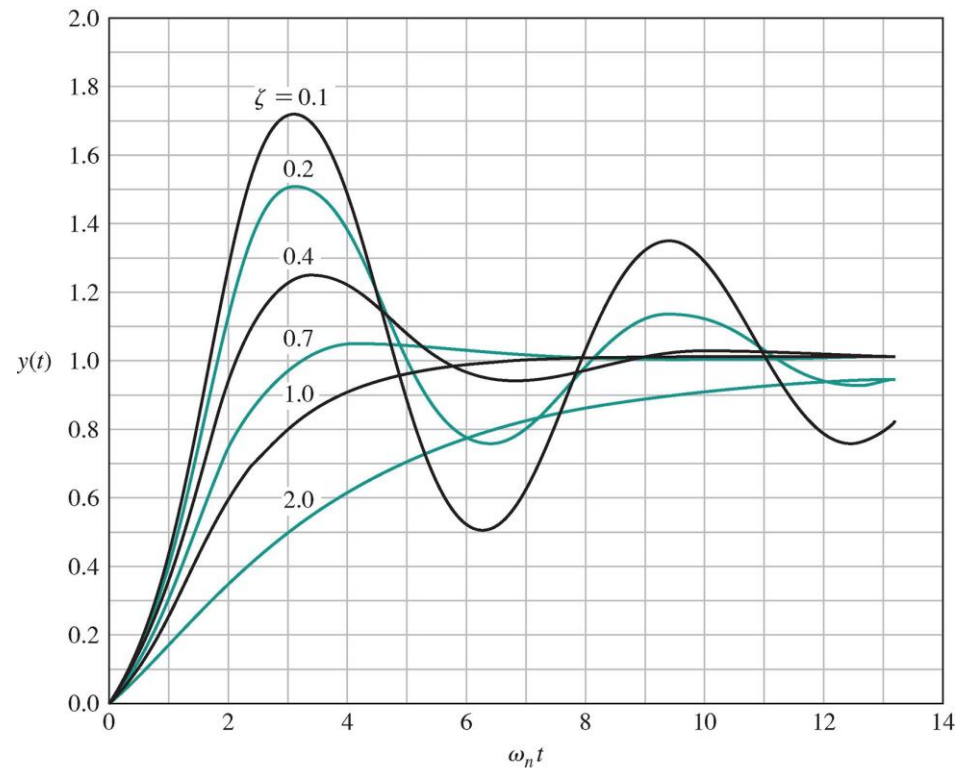
$$T_{r1} = \frac{2.16\zeta + 0.60}{\omega_n} \quad (0.3 < \zeta < 0.8)$$

2% Settling Time:

$$T_s = \frac{4}{\zeta\omega_n}$$

Percent Overshoot:

$$P.O. = 100e^{-\zeta\pi / \sqrt{1 - \zeta^2}}$$



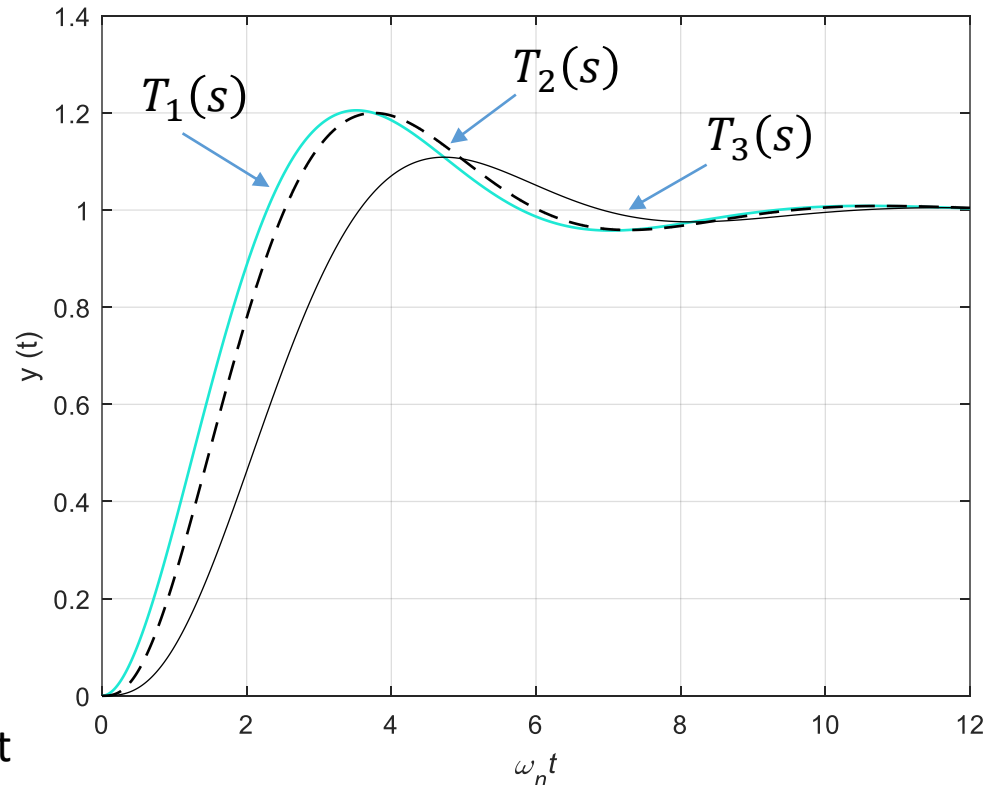
# Additional pole – Underdamped Second-Order system

$$T_1(s) = \frac{1}{s^2 + 0.9s + 1}$$

$$T_2(s) = \frac{1}{(s^2 + 0.9s + 1)(0.22s + 1)}$$

$$T_3(s) = \frac{1}{(s^2 + 0.9s + 1)(s + 1)}$$

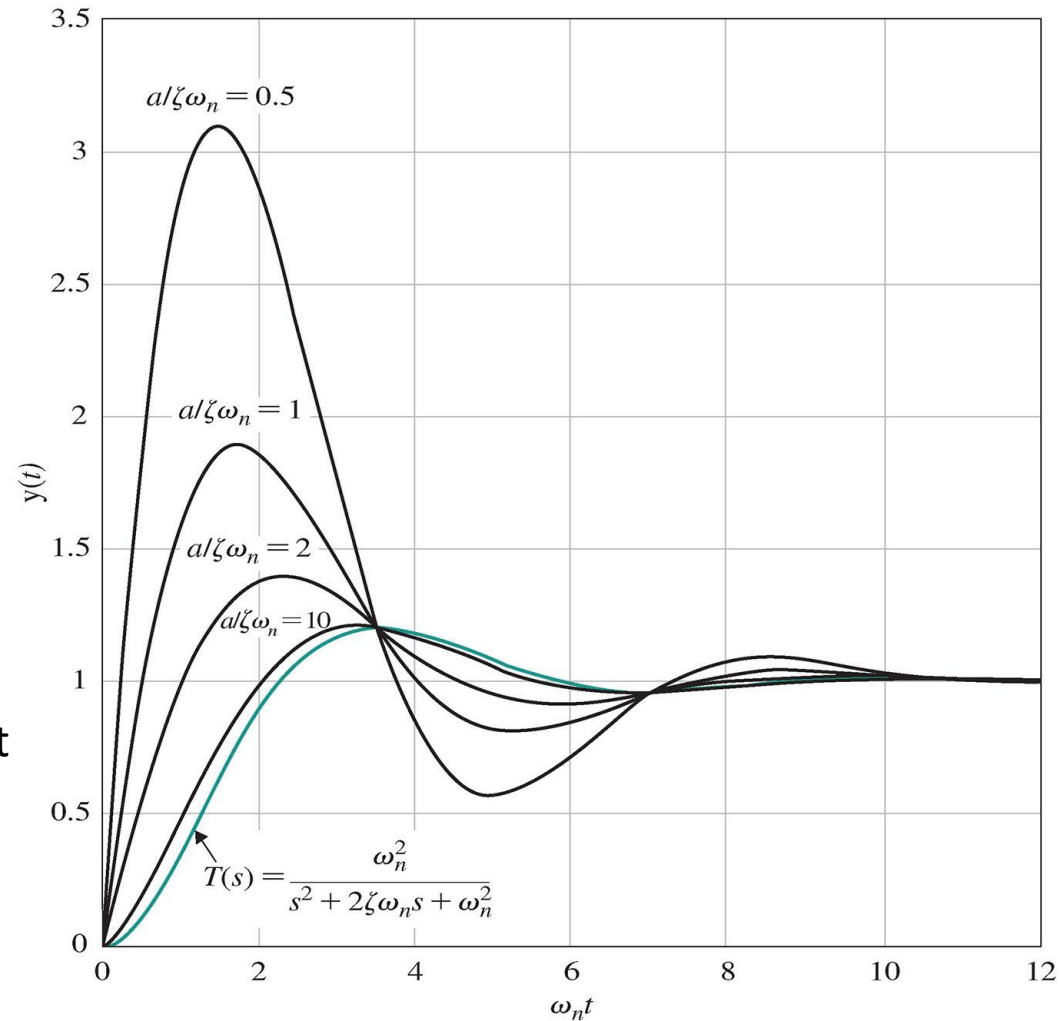
- Additional pole to the second-order system results in small overshoot and larger peak time
- If the pole's real part is faraway from the real part of original poles, the effect is minor and can be neglected



# Additional zero – Underdamped Second-Order system

$$T(s) = \frac{\frac{\omega_n^2}{a}(s + a)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Additional zero to the second-order system results in larger overshoot and smaller peak time
- If the zero's real part is faraway from the real part of original poles, the effect is minor and can be neglected



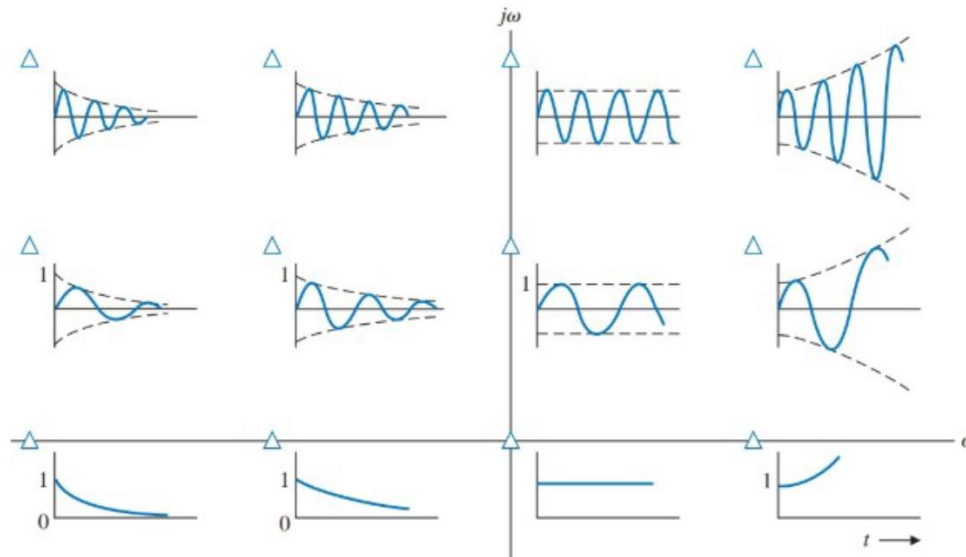
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# Higher-order system – Unit-step Response

If the system (with DC gain = 1) has no repeated roots, its unit step response can be formulated as a partial fraction expansion as:

$$Y(s) = \frac{1}{s} + \sum_{i=1}^M \frac{A_i}{s + \sigma_i} + \sum_{K=1}^N \frac{B_k s + C_k}{s^2 + 2\alpha_k s + (\alpha_k^2 + \omega_k^2)}$$

$$y(t) = 1 + \sum_{i=1}^M A_i e^{-\sigma_i t} + \sum_{k=1}^N D_k e^{-\alpha_k t} \sin(\omega_k t + \theta_k)$$



# Steady-State Error

$$e_{ss} = \lim_{s \rightarrow 0} s \frac{A/s}{1 + G_c(s)G(s)} = \frac{A}{1 + \lim_{s \rightarrow 0} G_c(s)G(s)}$$

$$G_c(s)G(s) = \frac{K \prod_{i=1}^M (s + z_i)}{s^N \prod_{k=1}^Q (s + p_k)}$$

The number of integration indicates a system with **type number** that is equal to **N**, which determines the steady-state error of the system.

**Table 5.2 Summary of Steady-State Errors**

Number of Integrations in $G_c(s)G(s)$ , Type Number	Input		
	Step, $r(t) = A$ , $R(s) = A/s$	Ramp, $r(t) = At$ , $R(s) = A/s^2$	Parabola, $r(t) = At^2/2$ , $R(s) = A/s^3$
0	$e_{ss} = \frac{A}{1 + K_p}$	$\infty$	$\infty$
1	$e_{ss} = 0$	$\frac{A}{K_v}$	$\infty$
2	$e_{ss} = 0$	0	$\frac{A}{K_a}$

# Good luck with your exam

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Office hour: 2-4 pm Thursday SC554E