CAN207 Continuous and Discrete Time Signals and Systems

Lecture-5

Introduction to Systems

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Room EE322

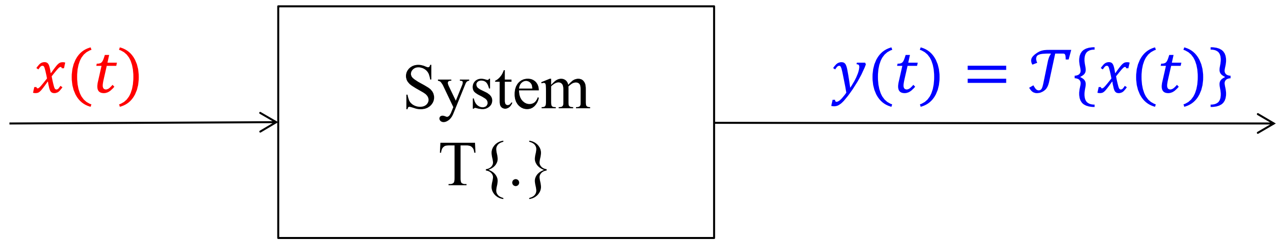
Content

- 1. Introduction
  - systems, some examples

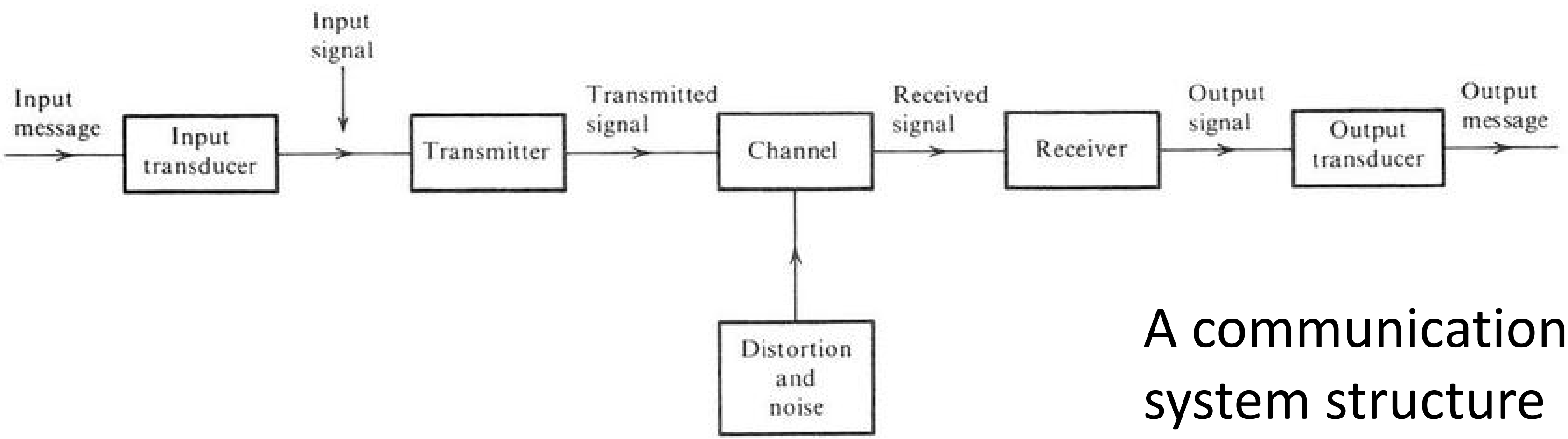
- 2. Systems classification (properties)
  - memory
  - invertibility
  - causality
  - stability
  - linearity
  - time-invariance

1.1 What is a system?

- A physical entity that operates on a set of primary signals (the inputs) to produce a corresponding set of signals (the outputs).
  - The operations, or processing, may take several forms: decomposition, filtering, extraction of parameters, combination, etc.



- A system may contain many subsystems with their own inputs / outputs

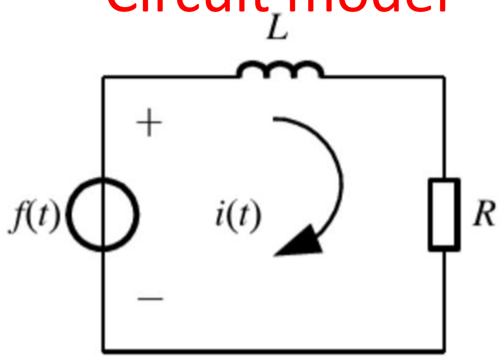


A communication system structure

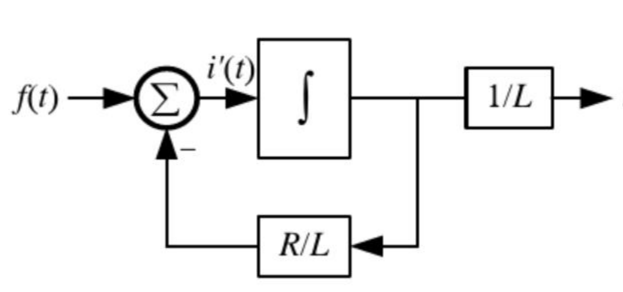
1.2 System Model

- The abstraction of a physical system, in the form of:

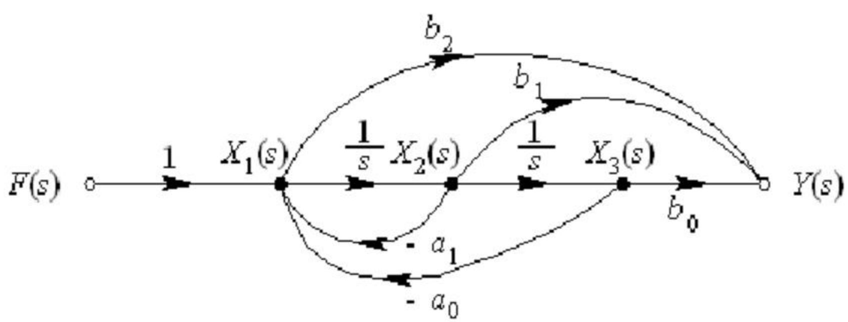
Circuit model



Block diagram



Signal flow graph

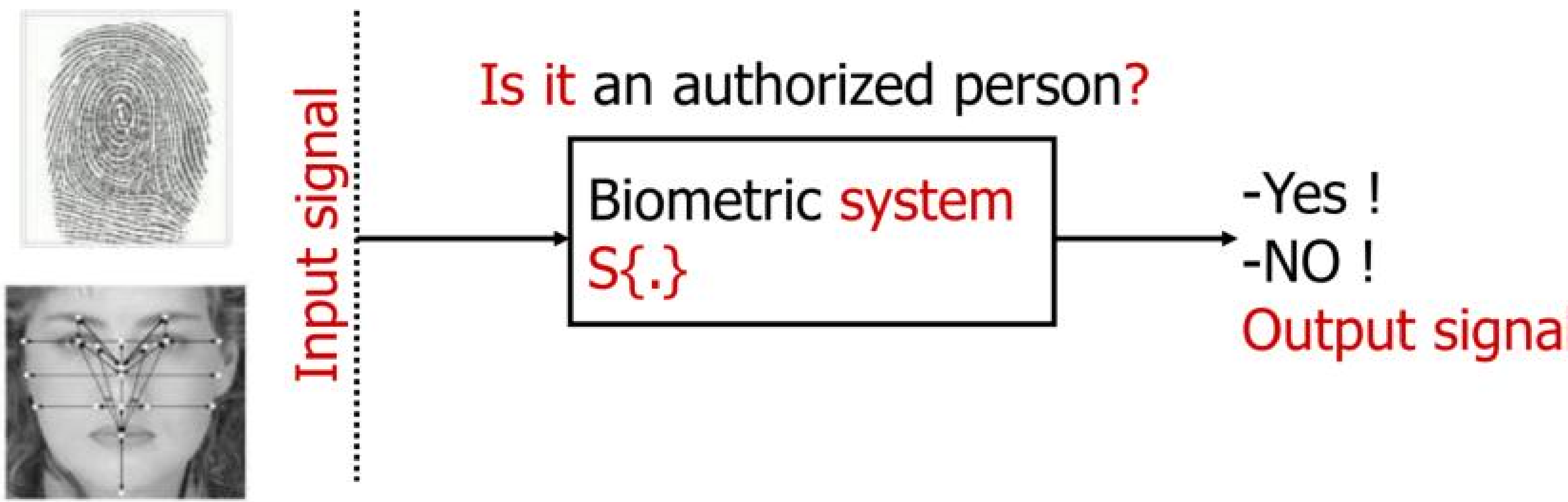


Mathematical expression

$$L\frac{\mathrm{d}i(t)}{\mathrm{d}t}+Ri(t)=f(t)$$

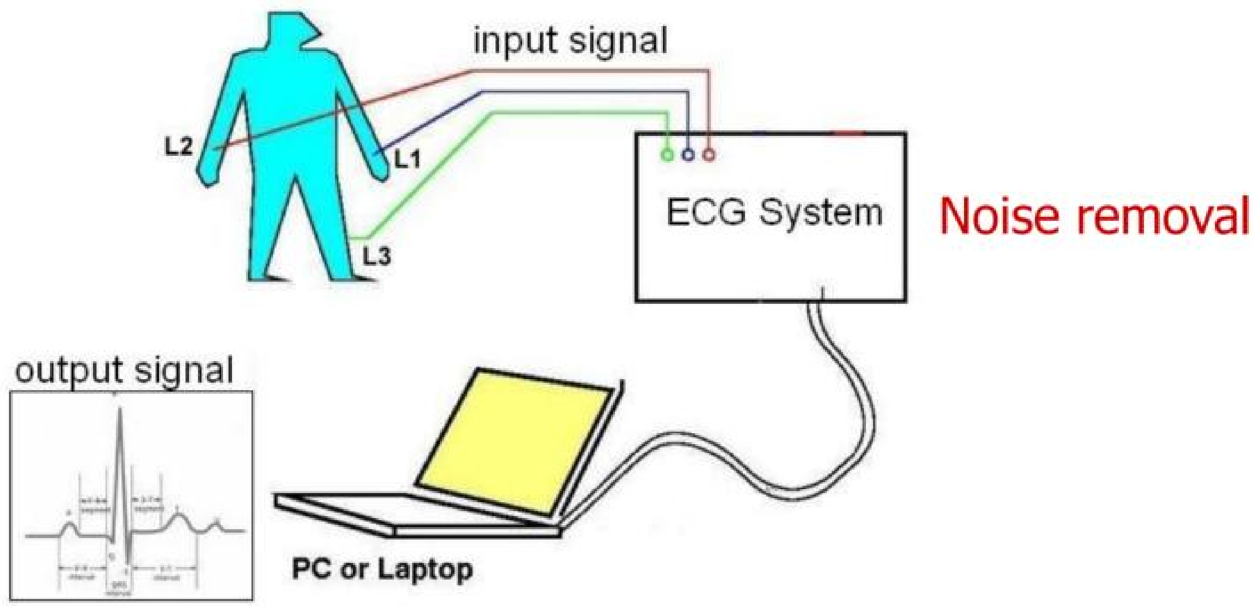
1.3 Some Examples

- Biometric systems:
  - Enables the identification, verification or authentication of an individual based on physiological, behavioral and molecular characteristics. Biometric techniques include recognizing faces, hands, voices, signatures, irises, fingerprints, DNA patterns, etc.



1.3 Some Examples

- Electrocardiogram (ECG):
  - An electrocardiogram (ECG) is a noninvasive graphic approach that records the electrical activity of the heart over time.



1.4 Input & Output of a system

Multiple Input Multiple Output

CT system

(a)

DT system

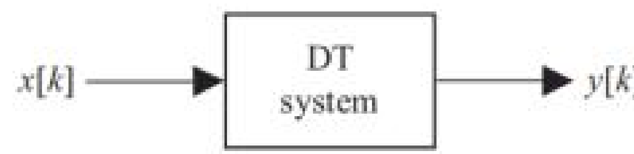
(a)

Single Input Single Output

CT system

(b)

DT system



(b)

2. System Properties (classifications)

- Systems can be classified according to some generic properties that the system satisfies.
  - For a system to possess a given property, the property must hold true for all possible input signals that can be applied to the system.
- Here listed 6 basic properties of systems:
  - 1. Memory (memory systems VS memoryless system);
  - 2. Invertibility (invertible sys. VS non-invertible sys.);
  - 3. Causality (causal sys. VS non-causal sys.);
  - 4. Stability (stable sys. VS unstable sys.);
  - 5. Linearity (linear sys. VS non-linear sys.);
  - 6. Time invariance (time-invariant sys. VS time-variant sys.).

2.1 Memory

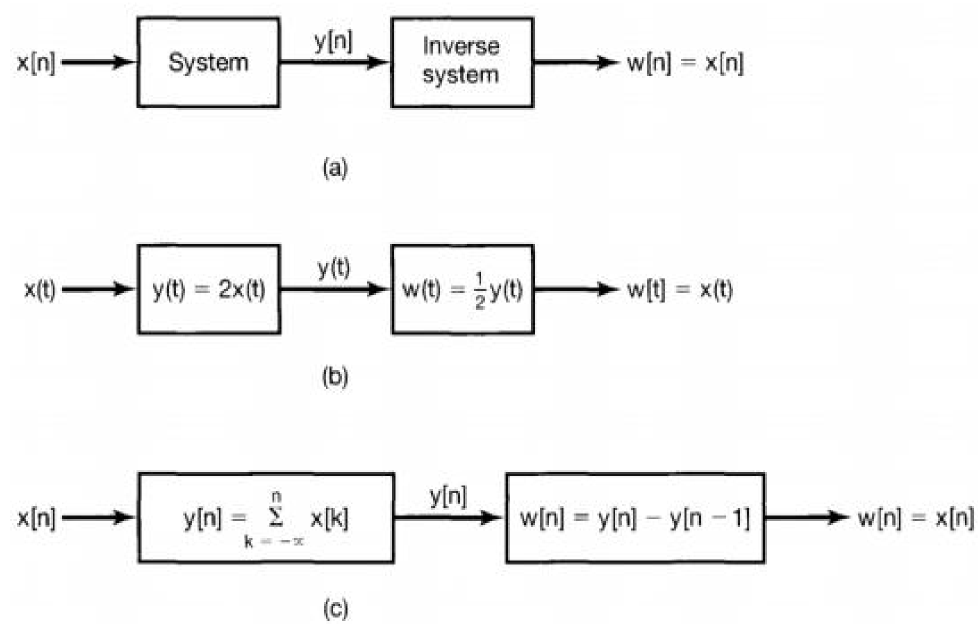
- A system is said to be ***memoryless*** if its output for each value of the independent variable at a given time is dependent only on the input at that same time.
  – Roughly speaking, the concept of *memory* in a system corresponds to the presence of a mechanism in the system that **retains** or **stores** information about input values at times other than the current time.
    - For example, the **delay** must retain or store the preceding value of the input.
    - Similarly, the **accumulator** must "remember" or store information about past inputs. In particular, the accumulator computes the running sum of all inputs up to the current time, and thus, at each instant of time, the accumulator must add the current input value to the preceding value of the running sum.

2.1 Memory

- Examples:
  - a) $y[n] = (2x[n] - x^2[n])^2$;
  - b) the ***identity system***: $y(t) = x(t)$;
  - c) a resistor system, considering the voltage on it as the output, and take the current flowing through the resistor as the input;
  - d) a capacitor system, i.e. the ***integrator***: $y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$;
  - e) the ***differentiator***: $y(t) = x'(t) = \frac{dx(t)}{dt}$;
  - f) the unit delay: $y[n] = x[n-1]$.

2.2 Invertiblity

- A system is said to be invertible if distinct inputs lead to distinct outputs.
  - To be invertible, two different inputs cannot produce the same output;
  - if a system is invertible, then an inverse system exists that, when cascaded with the original system, yields an output w[n] equal to the input x[n] to the first system.



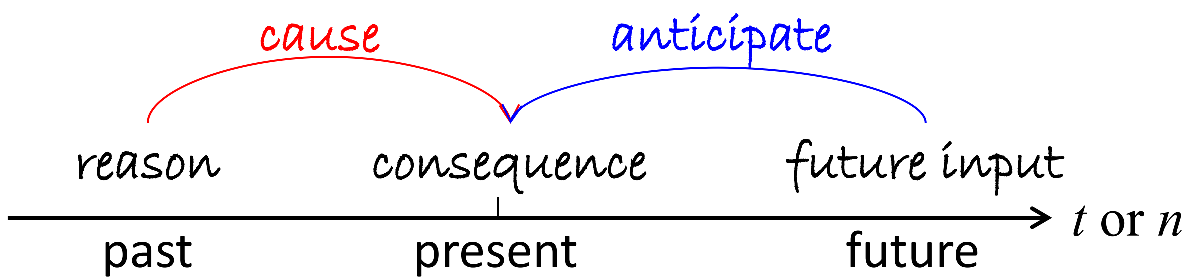
  - Example: $y(t) = 2x(t)$
  - Inverse system: $w(t) = \frac{1}{2}y(t)$

2.2 Invertiblity

- Examples:
  - a) Incrementally linear system: $y(t) = 3x(t) + 5$;
  - b) Cosine system: $y(t) = \cos[x(t)]$;
  - c) Integrating system: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$;
  - d) Differentiator: $y(t) = x'(t) = \frac{dx(t)}{dt}$;
  - e) Exponential output: $y[k] = e^{x[k]}$;
  - f) Differencer: $y[k] = x[k] - x[k-1]$.

2.3 Causality

- A system is causal if the output at any time depends only on values of the input at the present time and in the past.
  - Also referred to as being ***non-anticipative***;
  - All memoryless systems are causal systems.

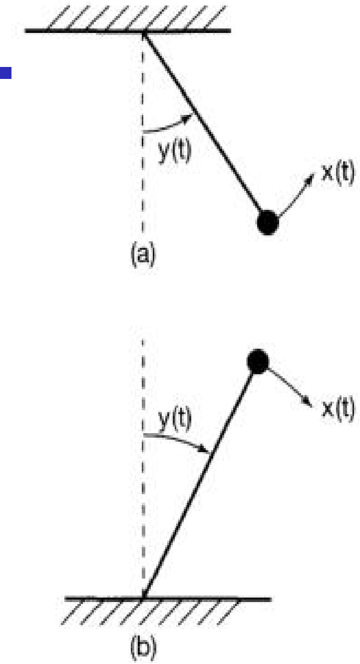


- Tips: 1. All implementable systems => causal;
- 2. Scaling (and flipping) systems => non-causal;
- 3. Shifting: delay => causal; advances => non-causal.

2.3 Causality

- Examples:
  - a) Time delay system: $y(t) = x(t-2)$;
  - b) Time advance system: $y[k] = x[k+2]$;
  - c) Expansion system: $y(t) = x(t/2)$;
  - d) Decimation system: $y[k] = x[2k]$;
  - e) Integrating system: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$;
  - f) the ***differentiator***: $y(t) = x'(t) = \frac{dx(t)}{dt}$.

2.4 Stability



- Informally, a stable system is one in which small inputs lead to responses that do not diverge.
- Mathematically, one definition of stable is based on the concept of "***bounded***":
  - A signal is said to be ***bounded in magnitude*** if:

$$|x(t)| \leq B_x < \infty, \text{for } t \in (-\infty, \infty)$$

  - For a system, it can be referred to as ***bounded-input, bounded-output (BIBO) stable*** if an arbitrary bounded-input signal always produces a bounded-output signal.
    - In other words, if the bounded input defined above is applied to a stable system, it is always possible to find a finite number B_y such that:

$$|y(t)| \leq B_y < \infty, \text{for } t \in (-\infty, \infty)$$

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2.4 Stability

- Methods:
- 1. By definition.
  - Set $|x(t)| \leq B_x$, substitue in

$$y(t) = f\{B_x\} = B_y$$

    To see whether B_y is a finite value.

- 2. Find counter example.
  - If you can find one example with finite input but infinite output, the system is unstable.
  - This doesn't need to prove.

2.4 Stability

- Examples:
  - a) Incrementally linear system: $y(t) = 3x(t) + 5$;
  - b) Cosine system: $y(t) = \cos[x(t)]$;
  - c) Integrating system: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$;
  - d) the ***differentiator***: $y(t) = x'(t) = \frac{dx(t)}{dt}$;
  - e) Sinusoidal system: $y[k] = 5\sin(x[k]) + 1$;
  - f) Exponential system: $y[k] = e^{x[k]}$.

2.5 Linearity

- A linear system is a system that possesses the important property of ***superposition***:

If an input consists of the weighted sum of several signals, then the output is the superposition of the responses of the system to each of those signals.

- Mathematically: a CT system with the following set of inputs and outputs: $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$. The superposition principle includes:
  - Additive property: $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$;
  - Homogeneity property: $\alpha x_1(t) \rightarrow \alpha y_1(t)$;
  - Overall: $\alpha x_1(t) + \beta x_2(t) \rightarrow \alpha y_1(t) + \beta y_2(t)$.
- A consequence of the linearity property is the special case when the input is zero, then the output y(t) must also be zero for all time t.

2.5 Linearity

- Method 1 - using definition
  - Step 1: Identify the system's function;
  - Step 2: Linear combination first, then going through the system;
  - Step 3: System first, then linear;
  - Step 4: Compare the results from steps 2 and 3.

- Method 2 - several useful tips
  - 1. $x(t) + C$: NL
  - 2. Higher order (or nonlinear operation) of $x(t)$ or $y(t)$: NL
  - 3. Containing the absolute value of $x(t)$ or $y(t)$: NL
  - 4. Constant coefficient differential and integral equation: L

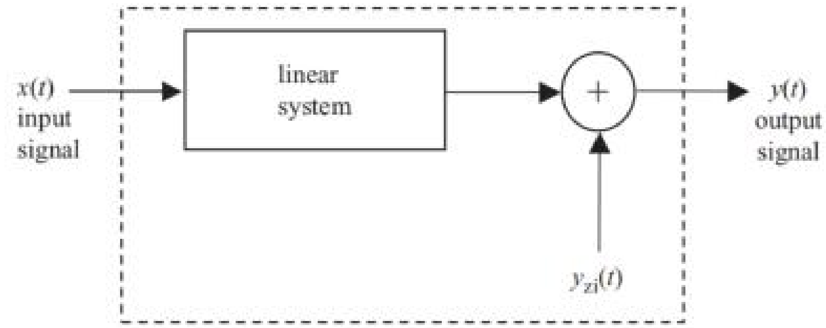
2.5 Linearity

- Examples:
  - a) amplifier with additive bias: $y(t) = 3x(t) + 5$;
  - b) exponential amplifier: $y(t) = e^{x(t)}$;
  - c) sinusoidal system: $y[k] = |x[k]|$.
  - d) amplifier: $y(t) = 3x(t)$;
  - e) differencing system: $y[k] = 3x[k] - 2x[k-2]$;
  - f) differentiator: $y(t) = \frac{dx(t)}{dt}$;
  - g) integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$;

2.5 Linear - Incrementally linear system

- System $y(t) = 3x(t) + 5$ satisfies a different type of linearity, called ***incrementally linear***.
- Consider two different inputs $x_1(t)$ and $x_2(t)$, the respective outputs of system $y(t) = 3x(t) + 5$ are given by
  - input $x_1(t)$ is $y_1(t) = 3x_1(t) + 5$ and input $x_2(t)$ is $y_2(t) = 3x_2(t) + 5$.
- Calculating the difference on both sides of the above equations yield
  - $y_2(t) - y_1(t) = 3[x_2(t) - x_1(t)]$
  - or $\Delta y(t) = 3\Delta x(t)$.

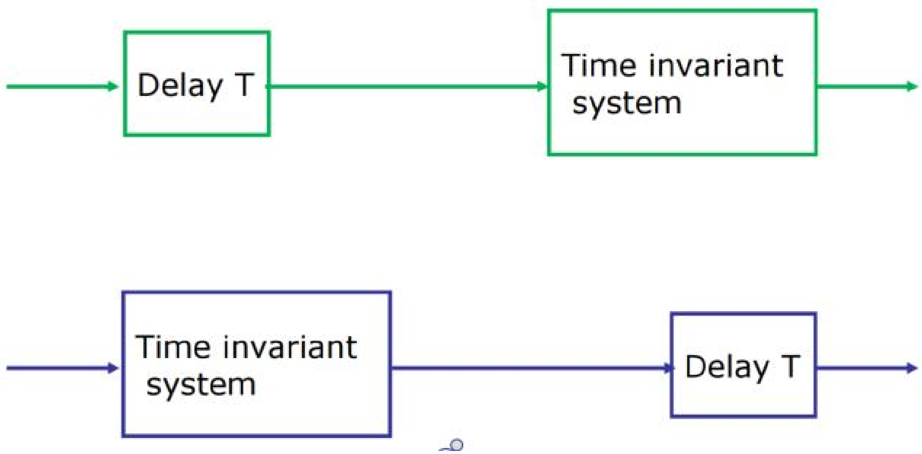
Incrementally linear system expressed as a linear system with an additive offset.

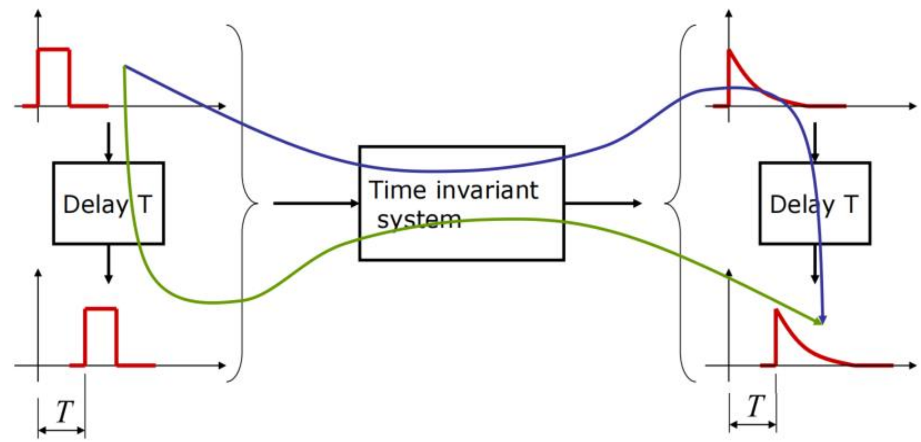


- Therefore, the change in the output is linearly related to the change in the input.
  - An incrementally linear system can be expressed as a combination of a linear system adds an offset $y_{zi}(t)$.

2.6 Time-invariance

- Conceptually, a system is time invariant if the behavior and characteristics of the system are fixed over time.
- Mathematically: A CT system with $x(t) \rightarrow y(t)$ is time-invariant iff $x(t - t_0) \rightarrow y(t - t_0)$





In a time invariant system delaying the input or the output leads to the same result.

2.6 Time-invariance

- Method 1 - using definition
  - Step 1: Identify the system's function;
  - Step 2: Delay the input, then going through the system;
  - Step 3: System first, then delay;
  - Step 4: Compare the results from steps 2 and 3.

- Method 2 - several useful tips
  - 1. $x(t)$ multiply or add a function of t: NTI (TV)
  - 2. Scaling of $x(t)$ or $y(t)$: TV
  - 3. Constant coefficient differential and integral equation: TI

2.6 Time-invariance

- Examples:
  - a) $y(t) = \sin(x(t))$;
  - b) $y(t) = t\sin(x(t))$;
  - c) $y[n] = x[n] + 3n$;
  - d) $y(t) = x(2t)$;
  - e) $y[k] = 3x[k] - 2x[2-k]$;
  - f) $y[n] = \sum_{k=-\infty}^{n} x[k]$;
  - g) integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$;
  - h) differentiator: $y(t) = \frac{dx(t)}{dt}$.

2.7 LTI systems

- Linear Time-Invariant (LTI) System: A system that satisfies both the ***linearity*** and the ***time-invariance*** properties
  - LTI systems are mathematically easy to analyse and characterise, and consequently, easy to design;
  - The systems do not change with time, which is a reasonably good approximation of most systems;
  - Many useful signal processing algorithms have been developed utilizing LTI systems.

Quiz 1

- Evaluate the following systems in terms of the 4 properties:

	Causal	Stable	Linear	Time-invariant
$y[n] = x[n] - x[n-3]$				
$y[n] = 5x[3n-2]$				
$y[n] = 2^{x[n]}$				
$y[n] = \sum_{k=-\infty}^{n} x[k]$				

Next ...

- LTI System
  - Properties of LTI systems
  - Convolution integral
  - Convolution sum