CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 3 Static Fields II

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OUTLINE

- Review of Coulomb's Law
- Visualisation of Electric Field
- Maxwell's Equations for Static Fields
  - Gauss's Law for Electric field
  - Gauss's Law for Magnetic field
  - **Electric** Field Loop Theorem
  - **Magnetic** Field Loop Theorem (Ampere's Law)
- Magnetic Forces

3.3 E-FIELD LOOP THEOREM

1. The Integral Form

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

a) Work done by moving a charge around a **closed** loop must be zero in an electrostatic field.

b) The static electric field is conservative (保守场).

3.3 WORK OF MOVING A CHARGE

To move a charge q a distance dl in an electric field, we must overcome the force arising from the electric field on q, which is $\vec{F}_E = q\vec{E}$.

Therefore, the force we used is

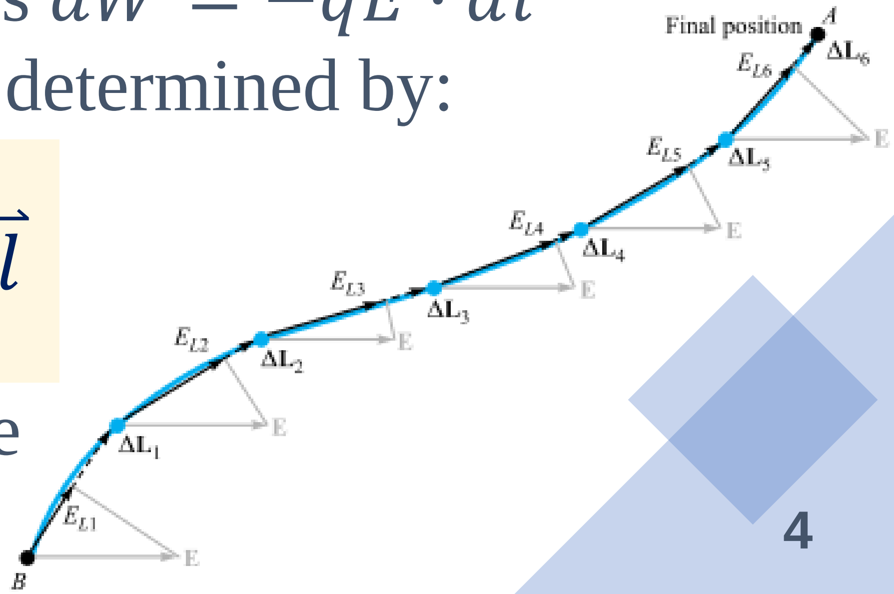
$$\vec{F}_{apply} \cdot \hat{l} = -q\vec{E} \cdot \hat{l}$$

where $\hat{l}$ is the unit vector in the direction of dl.

So, the differential work done by a moving charge q is $dW = -q\vec{E} \cdot d\vec{l}$

The work required to move the charge from B to A is determined by:

$$W_{AB} = \int \vec{F}_{apply} \cdot d\vec{l} = -q \int_B^A \vec{E} \cdot d\vec{l}$$



A negative sign is required since we are asking for the work required to move the charge against the field.

3.3 POTENTIAL DIFFERENCE

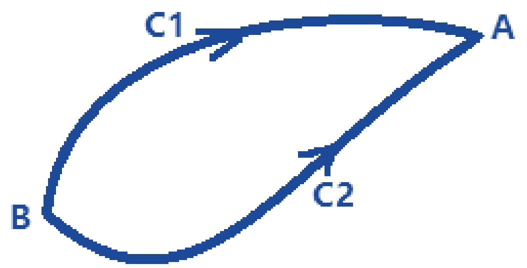
Define the ***potential difference*** or the ***voltage*** between points A and B as the work per unit of charge required to move the charge from A to B:

$$V_{AB} = \frac{W_{AB}}{q} = -\int_B^A \vec{E} \cdot d\vec{l} = V_A - V_B$$

The potential of a point (*e.g.*, point A), should be based on the potential of a **reference point**, usually the potential at infinite distance to the source, ($V_\infty = 0$):

$$V_A = V_A - V_\infty = -\int_\infty^A \vec{E} \cdot d\vec{l}$$

If we move a charge from B to A, then return to B (moving round the path below) :



$$\int_B^B \vec{E} \cdot d\vec{l} = \int_{c_1} \vec{E} \cdot d\vec{l} - \int_{c_2} \vec{E} \cdot d\vec{l}$$

$$= 0$$

* The net work done is zero.

* The circulation equals to zero means the static electric field is ***conservative*** (保守场).

The voltage between two points is only defined by the relative position of the two points regardless of the path taken.

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

Work done by moving a charge around a closed loop in static fields must be zero (this field is conservative).

QUIZ 3.4

A proton moves in a straight line from point A to point B inside a linear accelerator, and the distance from A to B is 0.5m. The electric field along this line is **uniform** with the magnitude 15 MV/m in the direction from A to B . Given that the charge of the proton is 1.602×10^{-19} C, determine:

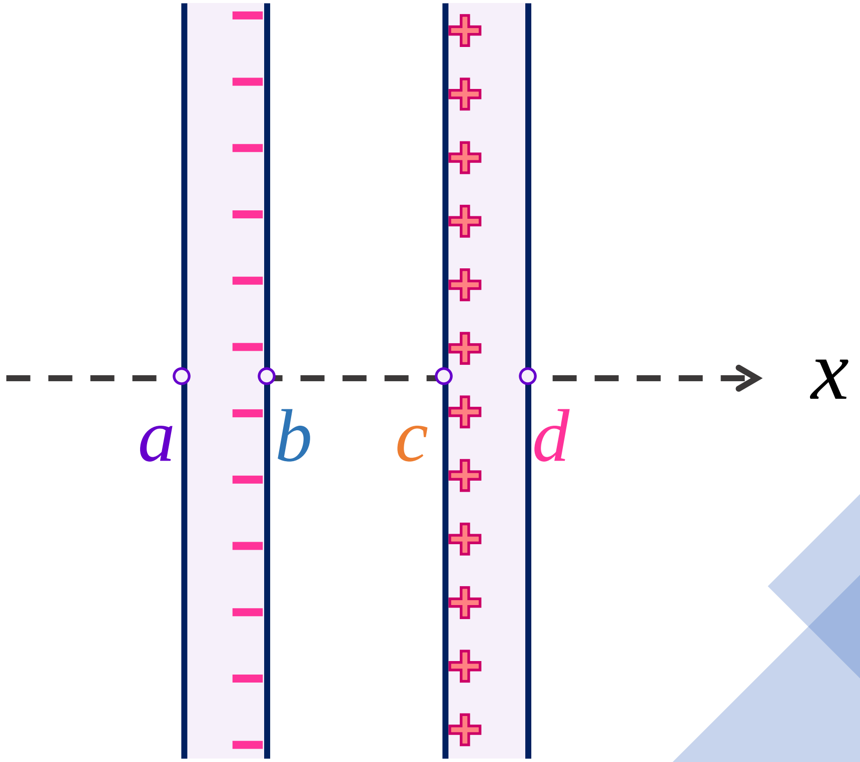
a) the force on the proton due to the electric field

b) the work done on the proton by the electric field

c) the potential difference V_{AB}

QUIZ 3.5

Two very large parallel metal plates carry charge densities of the same magnitude but opposite signs. Assume they are close enough together to be treated as **ideal** infinite plates. Taking the potential to be zero at the left surface of the negative plate, sketch a graph of the potential as a function of x within the regions from a to d.



3.3 E-FIELD LOOP THEOREM

2. The Differential Form

$$\nabla \times \vec{E} = 0$$

The static **electric fields** diverge away from points of positive charge and converge toward points of negative charge, such fields cannot circulate back on themselves.

3.3 E-FIELD LOOP THEOREM

Recall the Curl Theorem:

$$\iint_S (\nabla \times \vec{A}) \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{l}$$

$$\Downarrow$$

$$\oint_C \vec{E} \cdot d\vec{l} = \iint_S \nabla \times \vec{E} \cdot d\vec{s} = 0$$

$$\Downarrow$$

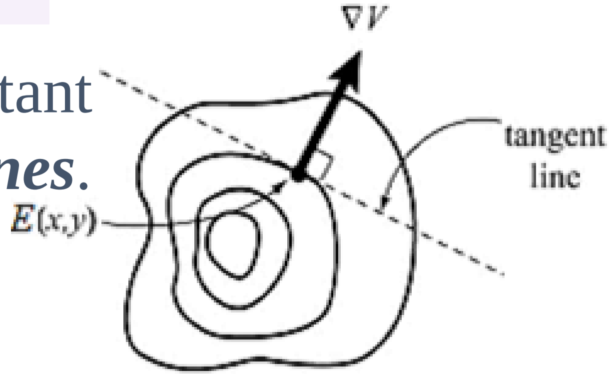
$$\nabla \times \vec{E} = 0$$

This static electric field is called ***irrotational*** field (无旋场).

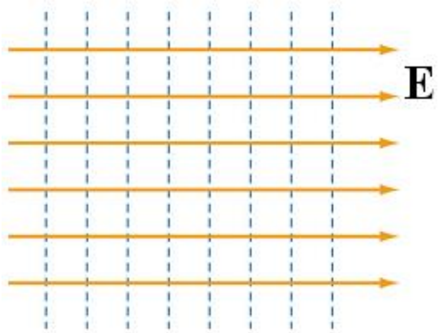
If a vector field is irrotational, this field can be represented in terms of the gradient of a scalar field:

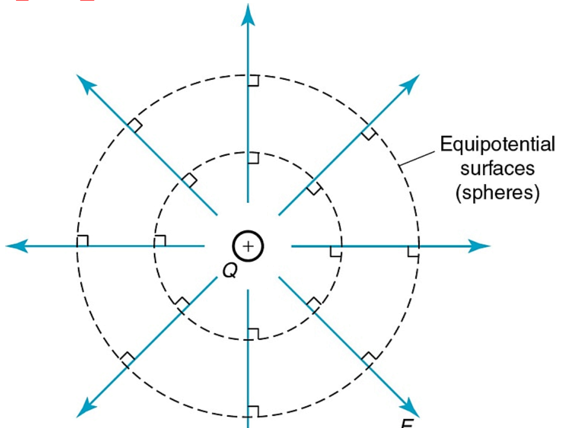
$$\vec{E} = -\nabla V$$

The curves characterised by constant $V(x, y, z)$ are called ***equipotential lines***.



Since $\vec{E} = -\nabla V$, we can show that the direction of $\vec{E}$ is always perpendicular to the equipotential lines through the point.





(a) A constant $\vec{E}$ field (b) A point charge

QUIZ 3.6

In a certain region of free space, the potential due to a point charge Q at the origin is:

$$V(x,y,z) = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+y^2+z^2}}$$

Determine the expression of the electric field intensity.

$$\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z} = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

QUIZ 3.7

In a certain region, a charge distribution exists that is spherically symmetric but non-uniform, which means the volume charge density is distance dependent from the centre of the distribution. The electric potential $V(r)$ due to the charge distribution is given by the following expression in free space. Determine the expression of the corresponding electric flux density.

$$V(r) = \begin{cases} \dfrac{\rho_0 a^2}{18\varepsilon_0}\left[1 - 3\left(\dfrac{r}{a}\right)^2 + 2\left(\dfrac{r}{a}\right)^3\right] & for\; r < a \\ 0 & for\; r \geq a \end{cases}$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r sin\theta}\frac{\partial V}{\partial \varphi}\hat{\varphi}$$

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3.4 AMPERE'S LAW

Ampere's circuital law (Ampere's Law):

The line integral of the magnetic field intensity $\vec{H}$ around a closed path equals the current enclosed.

$$\oint_C \vec{H} \cdot d\vec{l} = I_{encl}$$

Integral Form

"Enclosing" is done by the path C around which the magnetic field is integrated.

An electric current through a surface produces a circulating magnetic field around any path that bounds that surface.

Recall the "Curl Theorem":

$$\iint_S (\nabla \times \vec{A}) \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{l}$$

$$\Downarrow$$

$$\oint_C \vec{H} \cdot d\vec{l} = \iint_S \nabla \times \vec{H} \cdot d\vec{s} = \iint_S \vec{J} \cdot d\vec{s}$$

$\nabla \times \vec{H} = \vec{J}$ or $\nabla \times \vec{B} = \mu_0 \vec{J}$

Differential Form

A circulating magnetic field is produced by an electric current.

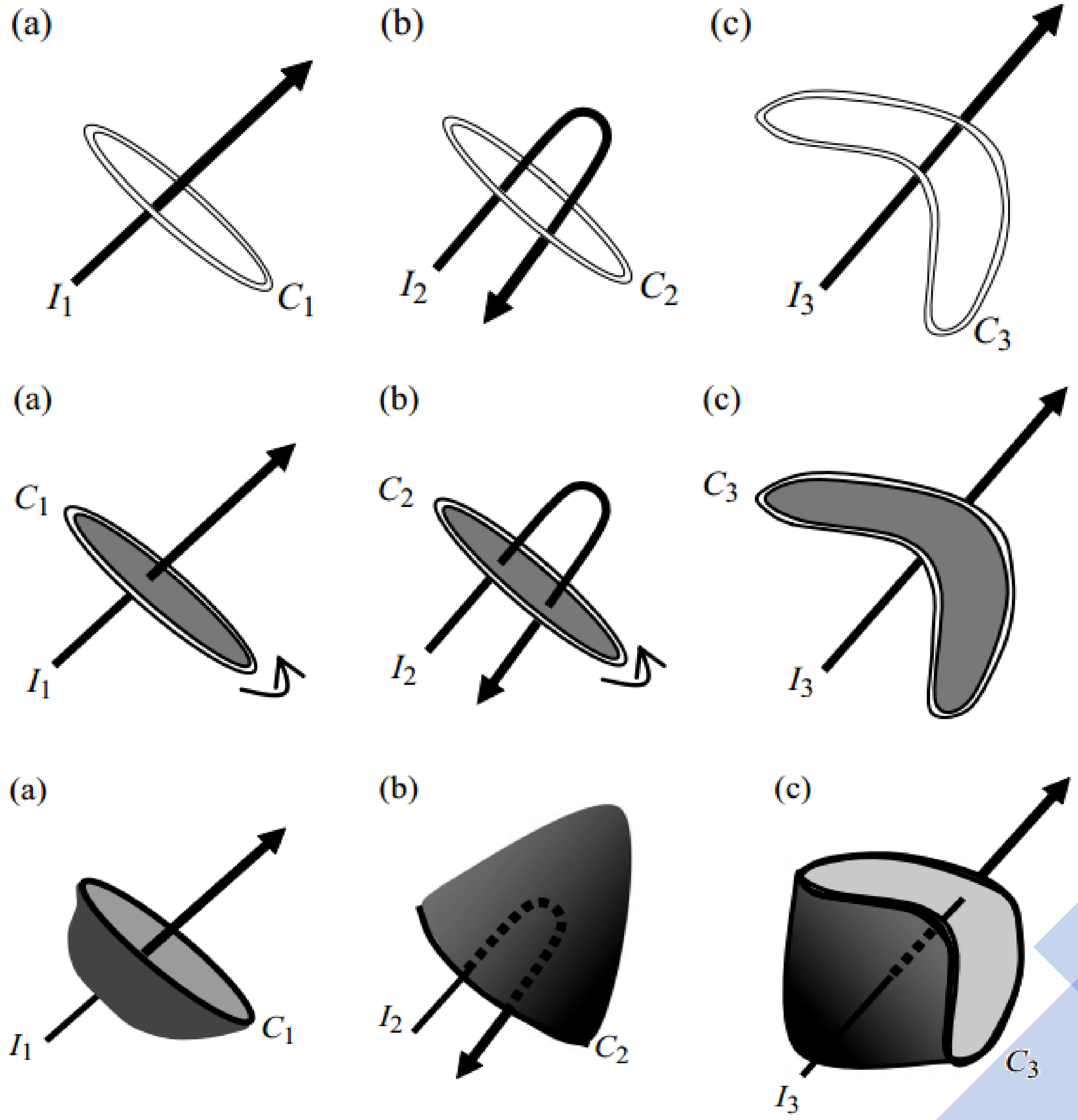
3.4 ENCLOSED CURRENT

‘Enclosing’ is done by the path C around which the magnetic field is integrated.

Which of the currents are enclosed by the path?

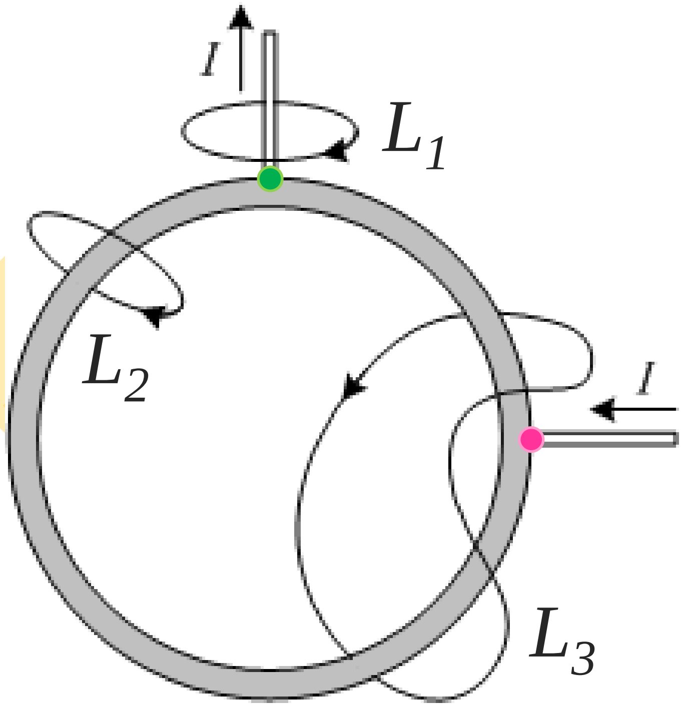
The easiest way to answer that question is to imagine a soap film (皂膜) stretched across the path. The enclosed current is the **net** current that penetrates the soap film:

- if you wrap the fingers of your right hand around the path in the direction of integration, your thumb points in the direction of positive current.
- the enclosed current is exactly the same irrespective of the shape of the surface you choose.



QUIZ 3.8

A uniform **resistance** wire (the grey part in the figure below) has two lead wires connecting to a dc current source supplying constantly. It can be known that $\oint_{L_1} \vec{B} \cdot d\vec{l} = -\mu_0 I$. Determine $\oint_{L_2} \vec{B} \cdot d\vec{l}$ and $\oint_{L_3} \vec{B} \cdot d\vec{l}$ represented by μ_0 and I.



3.4 AMPERE’S LAW

Ampere’s law in magnetism is analogous to Gauss’s law in electrostatics.

In order to apply them, the system must possess certain symmetry.

Biot-Savart Law	$d\vec{H} = \frac{1}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$	General current source *e.g.*: finite wire
Ampere’s Law	$\oint_C \vec{H} \cdot d\vec{l} = I_{enc}$	Current source has certain symmetry *e.g.*: infinite wire

Ampere’s law is applicable to the following current configurations:

1. Infinitely long straight wires carrying a steady current I
2. Infinitely large sheet of thickness b with a current density J
3. Infinite solenoid (螺线管)
4. Toroid (螺线圈)

COMPARISON

$$\mathbf{E} \leftrightarrow \mathbf{H}$$

$$\mathbf{D} \leftrightarrow \mathbf{B}$$

$$\mathbf{D} = \varepsilon\mathbf{E} \leftrightarrow \mathbf{B} = \mu\mathbf{H}$$

- **E**
  - Electric Field Intensity
  - Unit: V/m
- **D**
  - Electric Flux Density
  - Unit: C/m^2

$$\oiint \vec{D} \cdot d\vec{s} = \varepsilon \oiint \vec{E} \cdot d\vec{s} = Q_{encl}$$

- **H**
  - Magnetic Field Intensity
  - Unit: A/m
- **B**
  - Magnetic Flux Density
  - Unit: Tesla = Wb/m^2

$$\oint_C \vec{B} \cdot d\vec{l} = \mu \oint_C \vec{H} \cdot d\vec{l} = \mu I_{encl}$$

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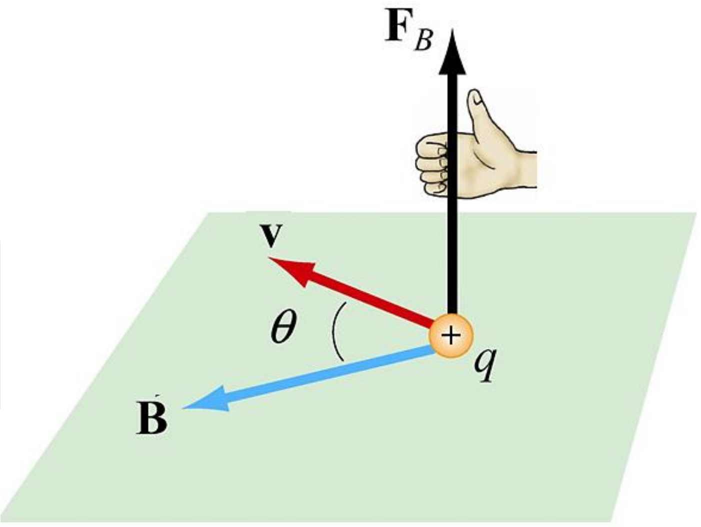
4.1 MAGNETIC FORCE

Magnetic forces on a moving charge:

- Experiment shows:
  - $\vec{F}_B$ is perpendicular to $\vec{v}$ and $\vec{B}$;
  - F_B is proportional to q, v and B;
- This is the magnetic force on the charged particle q

$$\Rightarrow \vec{F}_B = q(\vec{v} \times \vec{B})$$



Right-hand rule for the direction of magnetic force on a **positive** charge moving in a magnetic field:

1.Place the vectors tail to tail

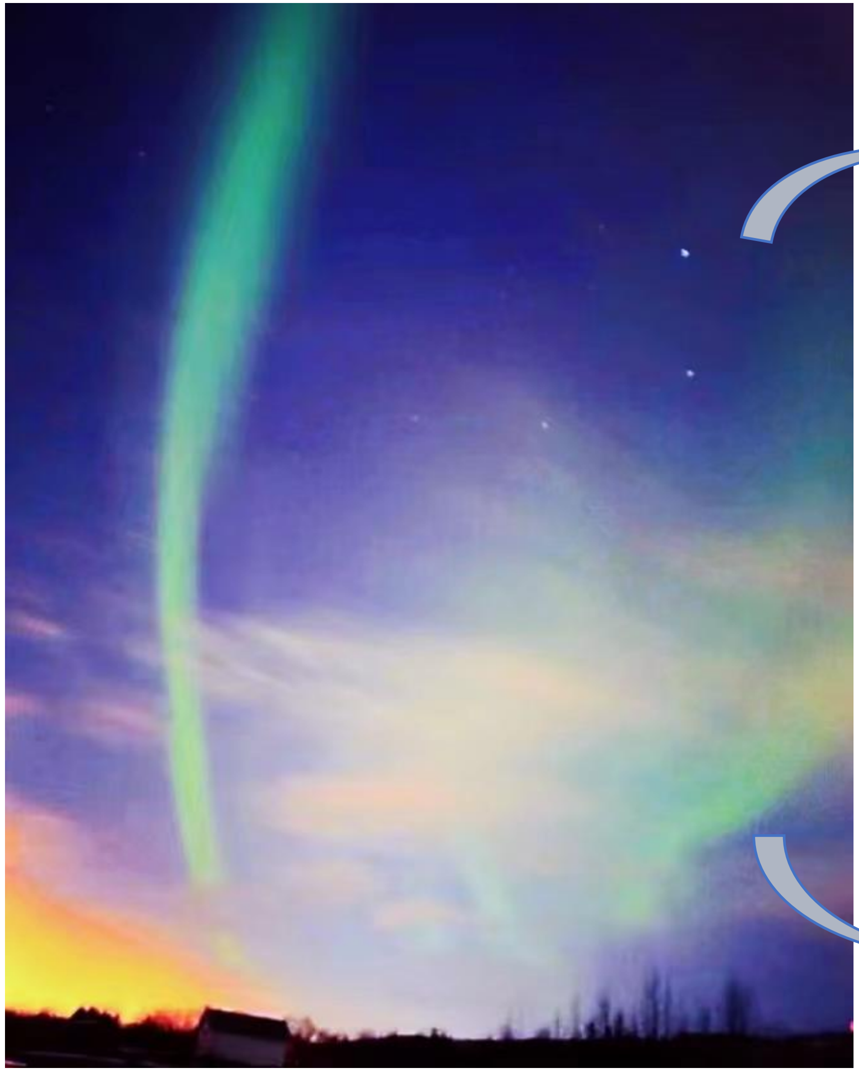
2.Imagine turning toward in the plane (through the smaller angle)

3.The force acts along a line perpendicular to the plane. Do right-hand as shown. The thumb points the direction of the force.

4.1 MAGNETIC FORCE

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

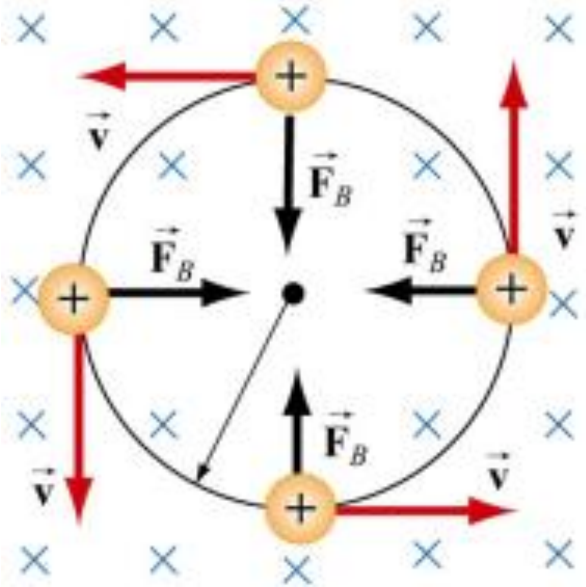
Natural view: Aurora 极光



at poles?

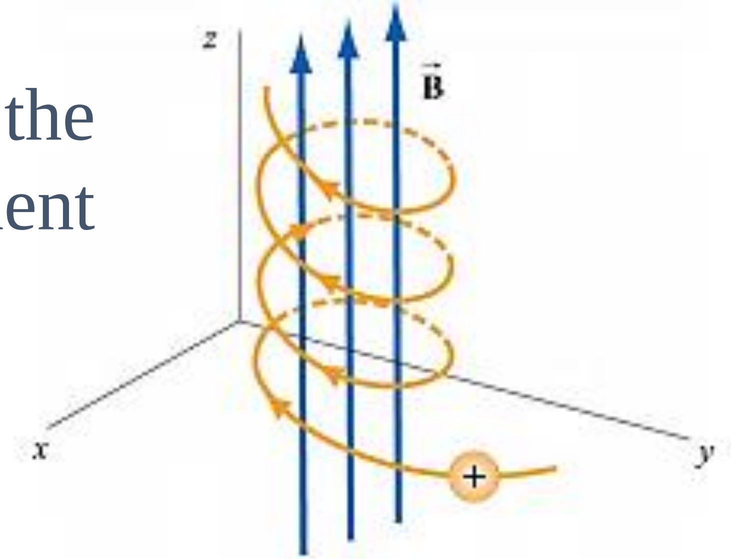
QUIZ 4.1

1. What would happen if a charged particle moves through a uniform magnetic field with its initial velocity $\boldsymbol{v}$ at a right angle to $\mathbf{B}$?



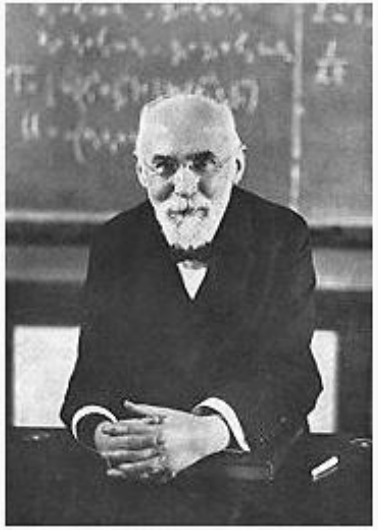
$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

2. What if the initial velocity of the charged particle has a component parallel to the magnetic field $\mathbf{B}$?



4.1 MAGNETIC FORCE

When a charged particle moves in a **mixed** electric field and magnetic field, it experiences a **total** force *Lorentz Force* $\vec{F}_L$ exerted by both fields:



$$\vec{F}_L = \vec{F}_E + \vec{F}_B = q(\vec{E} + \vec{v} \times \vec{B})$$

Electric force
- always in the direction of the E-field
- acts on a charged particle no matter it is moving or not
- expends energy in displacing a charged particle

Magnetic force
- always perpendicular to the magnetic field.
- acts on it only when it is in motion.
- does no work when a particle is displaced

4.1 MAGNETIC FORCE

Magnetic forces on a current-carrying wire:

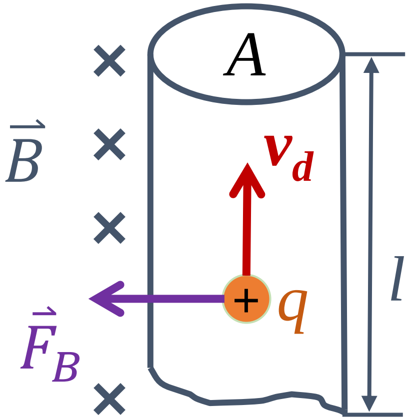
The total amount of charge in the segment:

$$Q_{tot} = q(nAl)$$

n is the number of charges per unit volume.

Then

$$\vec{F}_B = Q_{tot}\vec{v}_d \times \vec{B}$$
$$= qnAl(\vec{v}_d \times \vec{B})$$
$$= I(\vec{l} \times \vec{B})$$



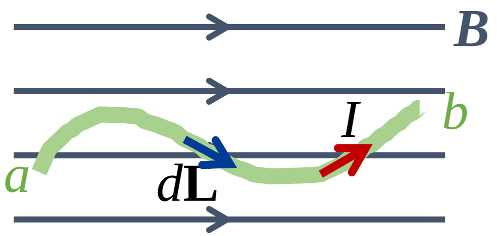
$$= q(\vec{v} \times \vec{B})$$

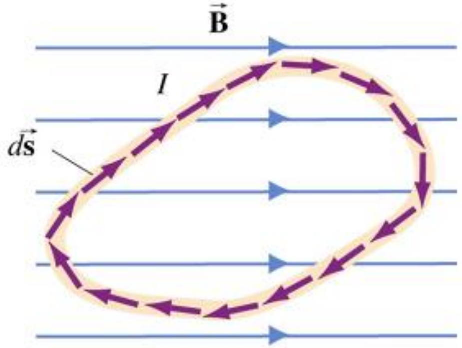
where $\vec{l}$ is the length vector with a magnitude l and directed along the direction of the current.

Current: $I = nqv_dA$

When a current-carrying conductor is placed in an external magnetic field, the magnetic force exerted on it is:

$$\vec{F}_B = I\left(\int_a^b d\vec{l}\right) \times \vec{B} = I\vec{l} \times \vec{B}$$





If the wire forms a **closed** loop of arbitrary shape, in a **uniform** field, the force on the loop becomes:

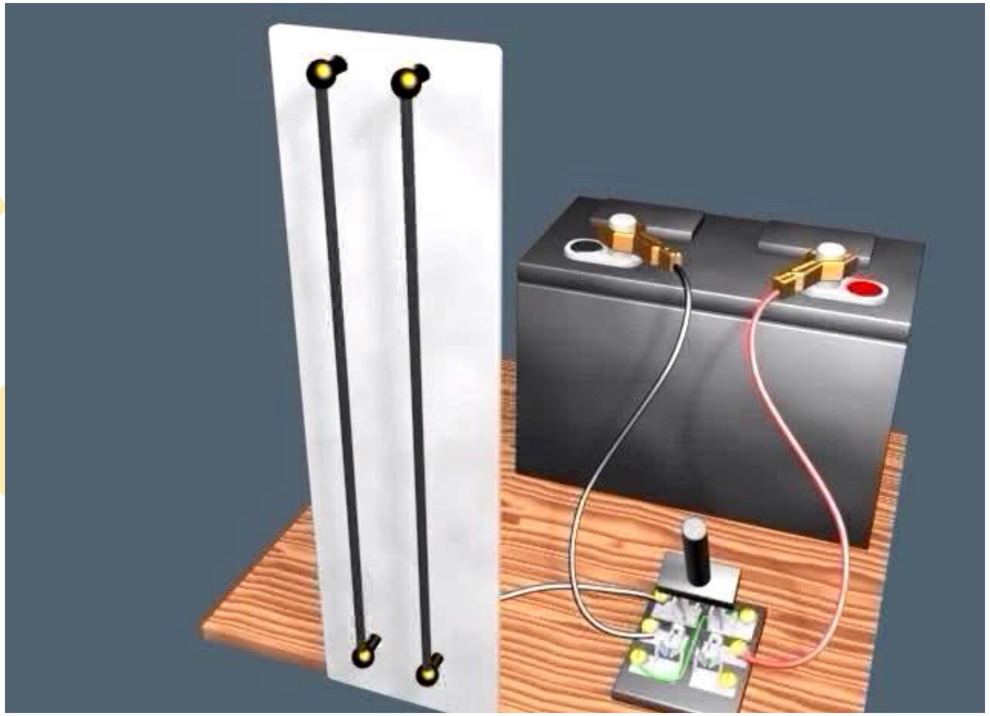
$$\vec{F}_B = I\left(\oint d\vec{l}\right) \times \vec{B} = 0$$

4.1 MAGNETIC FORCE

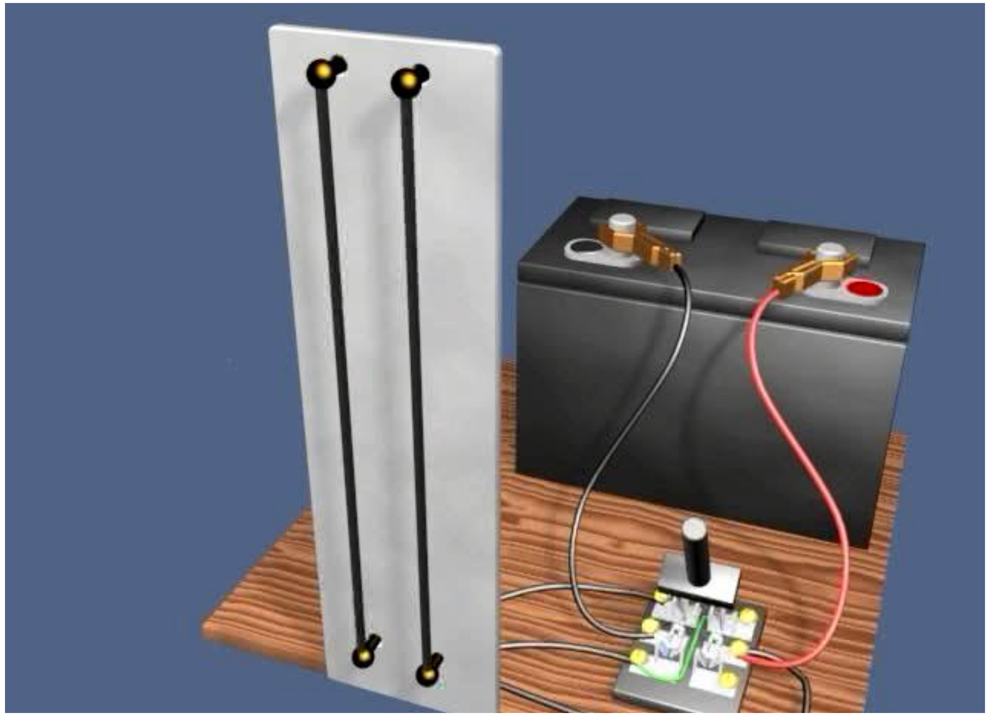
A current-carrying wire produces a magnetic field.

In a magnetic field, a wire carrying a current experiences a net force.

Two current-carrying wires to exert force on each other.



Parallel – current in **same** direction

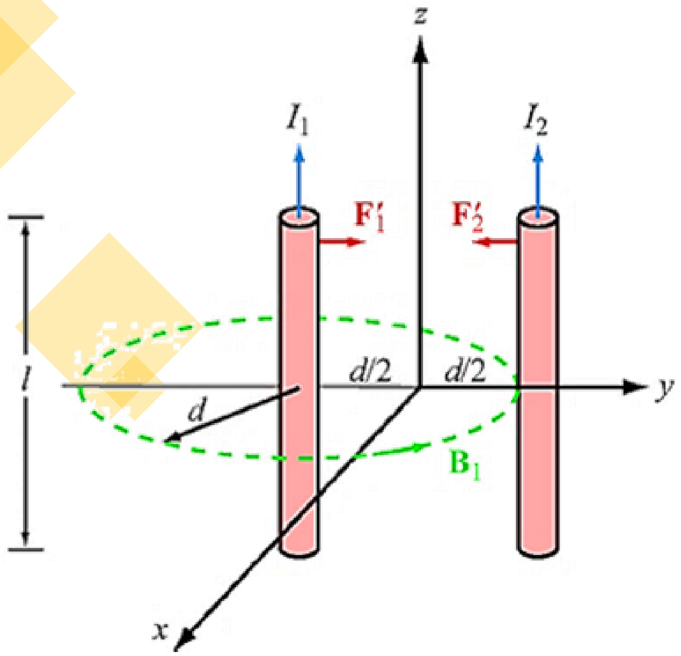


Series – current in **different** direction

4.1 MAGNETIC FORCE

The magnetic force $\vec{F}_{21}$, <u>exerted on wire 2 by wire 1</u> is generated by the magnetic field lines due to I_1.

At an arbitrary point P on wire 2, we have $\vec{B}_1 = -(\frac{\mu_0 I_1}{2\pi d})\hat{x}$, which points in the direction perpendicular to wire 2.



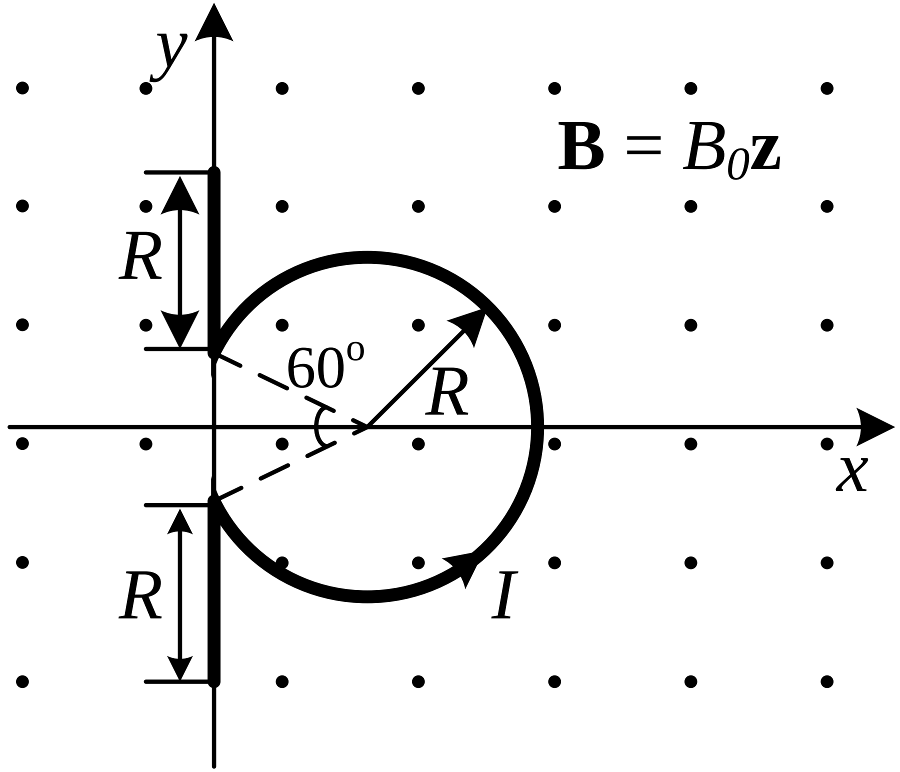
So,

$$\vec{F}_{21} = I_2 \vec{l} \times \vec{B}_1 = I_2 l\hat{z} \times \left(-\frac{\mu_0 I_1}{2\pi d}\hat{x}\right) = -\frac{\mu_0 I_1 I_2 l}{2\pi d}\hat{y}$$

- Two parallel wires carrying currents in the same direction will attract each other.
- If the currents flow in opposite directions, the resultant force will be repulsive.

QUIZ 4.2

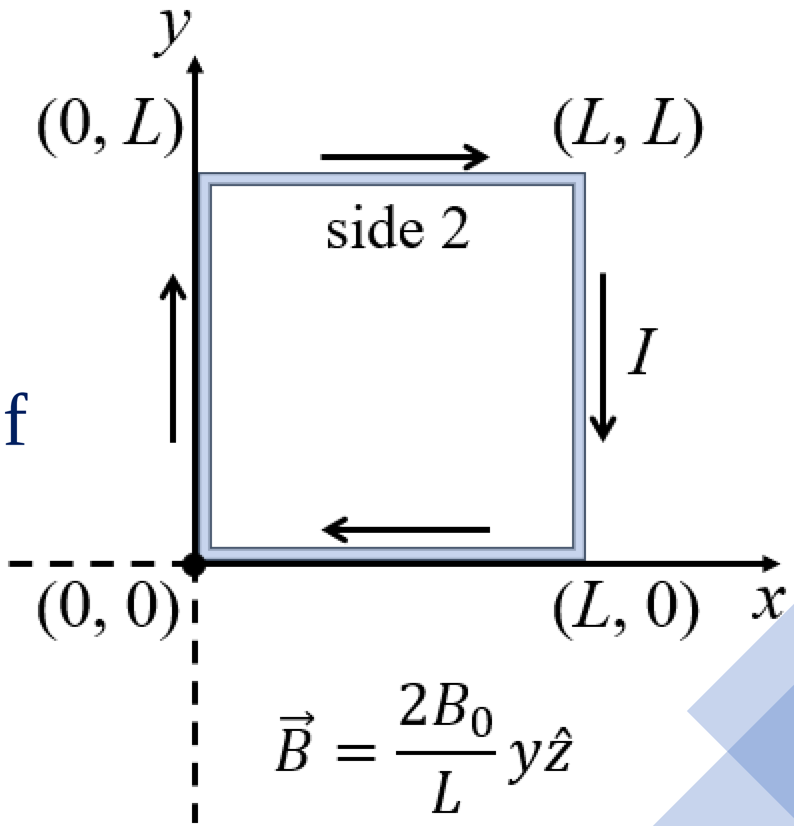
A wire lies in the xy plane and carries a current I. If the magnetic flux density in the region is $\vec{B} = B_0\,\hat{z}$, where B_0 is a positive constant. determine an expression for the magnetic force acting on the wire.



QUIZ 4.3

A square wire loop lies in the xy-plane and the four corners of the wire are located at (0,0), (0, L), (L, L), and (L, 0) respectively. The wire carries a clockwise current I and the magnetic field is given in the following equation:

$$\vec{B} = \frac{2B_0 y}{L}\hat{z}\ \mathrm{T}$$



where B_0 is a positive constant.

Calculate the magnetic force exerted on side 2 of the square loop.

MAXWELL'S EQUATIONS – STATIC FIELDS

Law	Integral	Differential	Physical meaning
Gauss's law for **E-field**	$\oiint_S \vec{E} \cdot \mathrm{d}\vec{s} = \frac{Q}{\varepsilon}$	$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$	Electric flux through a closed surface is proportional to the charges enclosed
E-field Loop Theorem	$\oint_C \vec{E} \cdot \mathrm{d}\vec{l} = 0$	$\nabla \times \vec{E} = 0$	Work done by moving a charge in the E-field along a closed loop is 0
Gauss's law for **H-field**	$\oiint_S \vec{B} \cdot \mathrm{d}\vec{s} = 0$	$\nabla \cdot \vec{B} = 0$	The total magnetic flux through a closed surface is 0
H-field Loop Theorem	$\oint_C \vec{H} \cdot \mathrm{d}\vec{l} = \mathrm{I}$	$\nabla \times \vec{H} = \vec{J}$	The H-field produced by an electric current is proportional to the current

NEXT….

- Dipoles
- Nature of Materials
- Boundary Conditions