

# Controlled Rectifiers (Three-phase)

**Dr. Suneel Kommuri**

Email: [Suneel.Kommuri@xjtlu.edu.cn](mailto:Suneel.Kommuri@xjtlu.edu.cn)

Dept. Electrical & Electronic Engineering

# Outline

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## – Review of last week lecture

### 1. Three-phase Half-wave controlled rectifiers

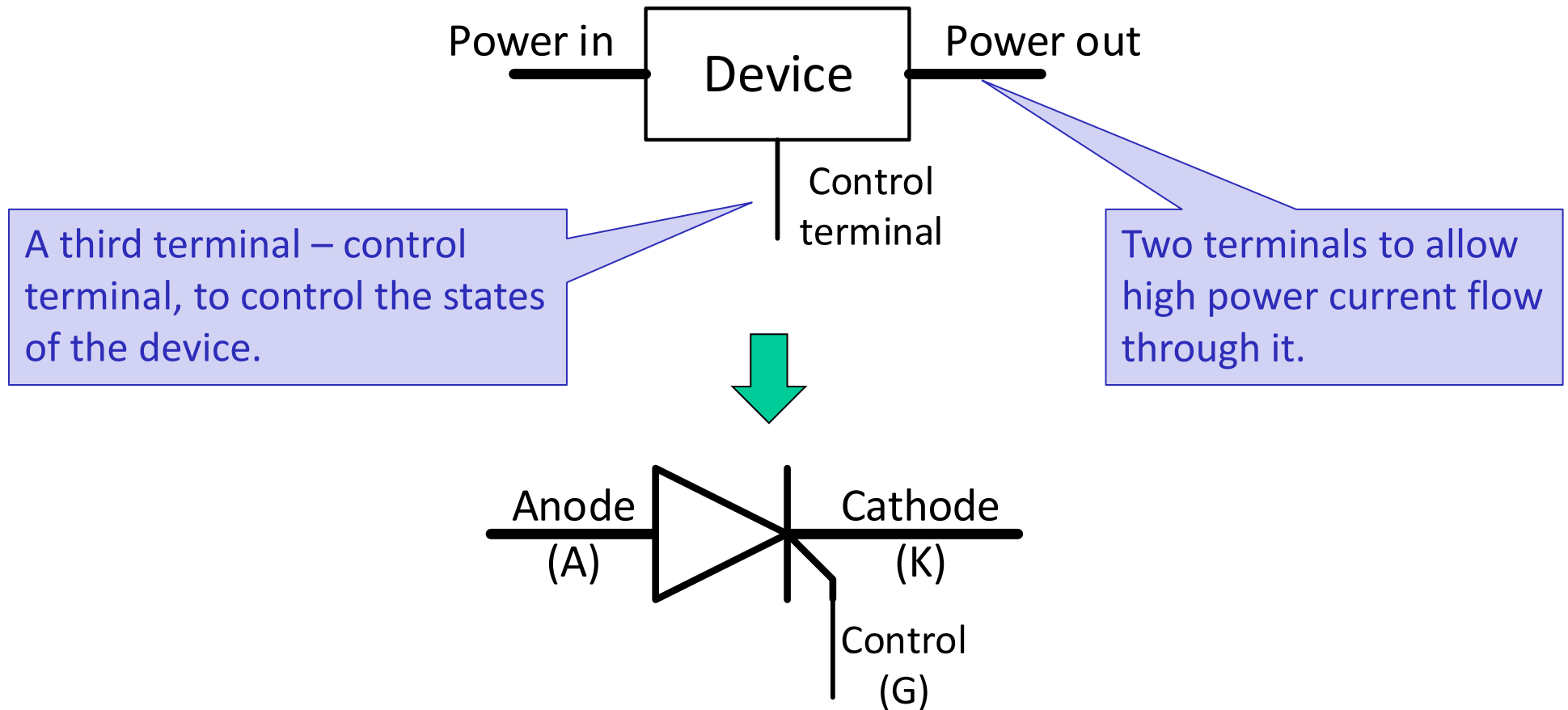
1. Firing/triggering angle  $\alpha$ ;
2. Resistive loading;
3. Inductive loading;

### 2. Full-wave (bridge)

1. Resistive loading;
2. Inductive loading.

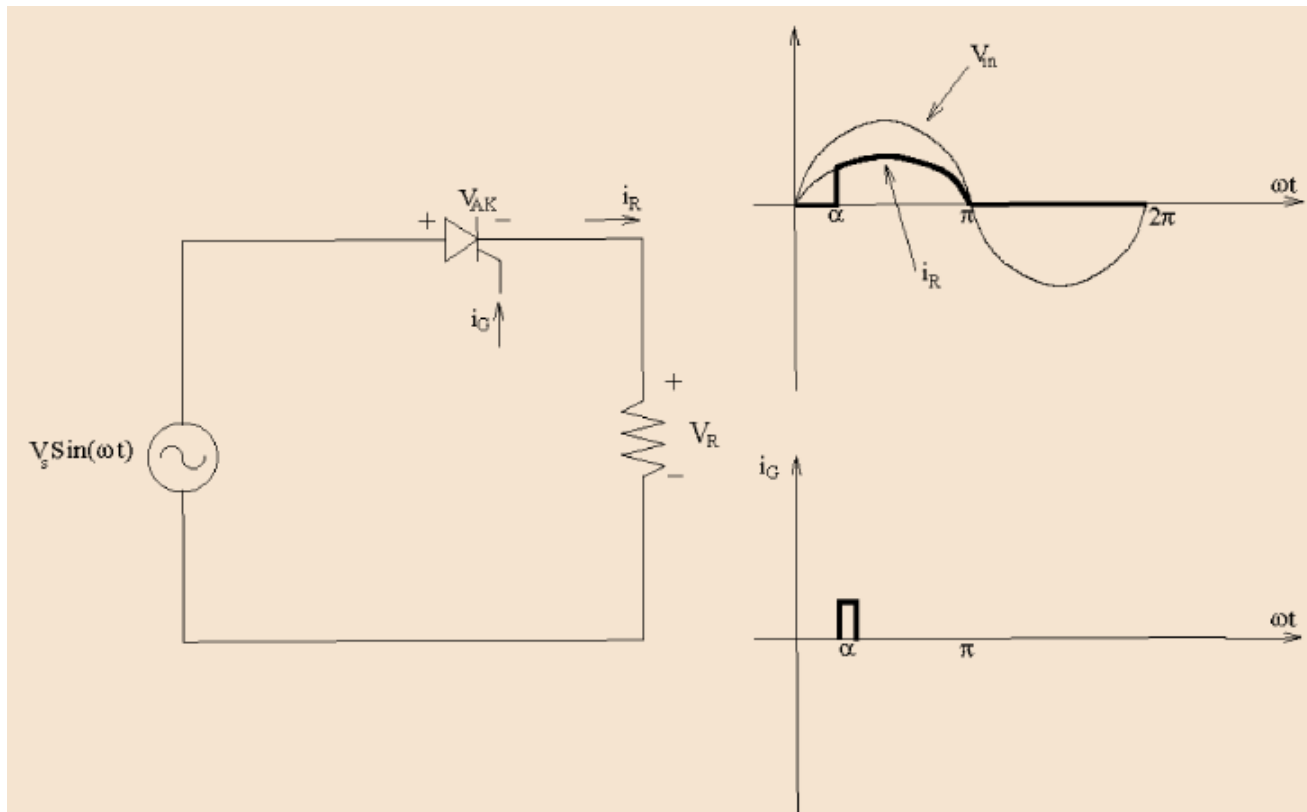
# Power semiconductor devices – Review

- Terminals of a controllable power electronic device



# SCR in a rectification circuit – Review

- SCR/Thyristor: Acts like a diode where you can select when conduction will start, but not when it stops.
  - Semi-controlled: we control the turn on point, but only turns off when circuit conditions force it to.



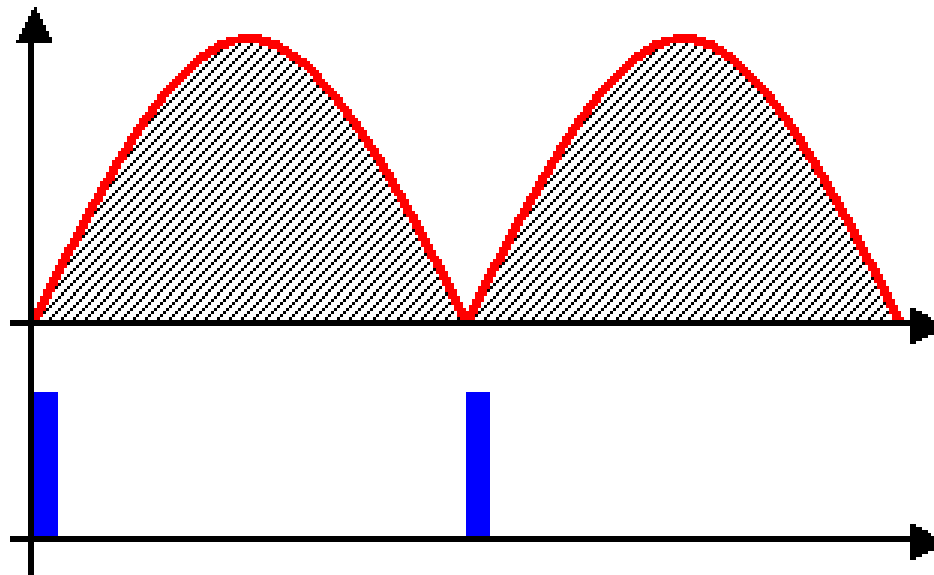
# SCR Turn-On Analysis – Review

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- Two conditions must be met before the SCR can conduct:
  1. The SCR must be forward biased ( $v_{AK} > 0$ ).
  2. A current must be applied to the gate of SCR.
- A *SCR is turned ON* by increasing the anode current. This can be accomplished in one of the following ways:
  1. Forward voltage triggering
  2. Gate triggering
  3. The  $dv/dt$  triggering
  4. Temperature triggering
  5. Light triggering

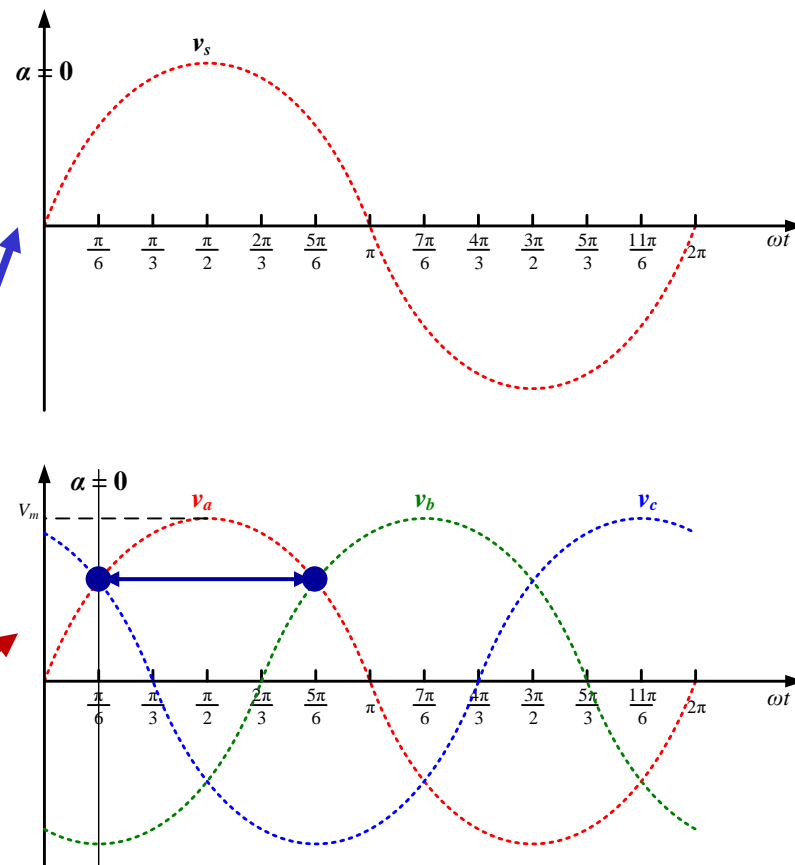
# Phase Control – Review

- In this circuit, the control of the output DC voltage is realized by modifying the triggering pulse phase or firing angle, this is called *Phase Control*.
- Change the firing angle  $\alpha$  from 0 to  $\pi$ :



# 1.1 Firing angle (triggering angle) – Recall

- In controlled rectifier, controllability of the circuit is realized by triggering the thyristors at different phases, which is called the **firing/triggering angle**. It is usually represented by “ $\alpha$ ”;
  - This trigger signal is a current pulse at the “gate” terminal of thyristors;
  - For **single phase circuit**,  $\alpha = 0$  means trigger signal is sent at  $\omega t = 0$ ;
  - For **three-phase circuit**,  $\alpha = 0$  means trigger signal is sent at  $\omega t = \pi/6$ , which is the first **natural commutation (phase changing) point**.

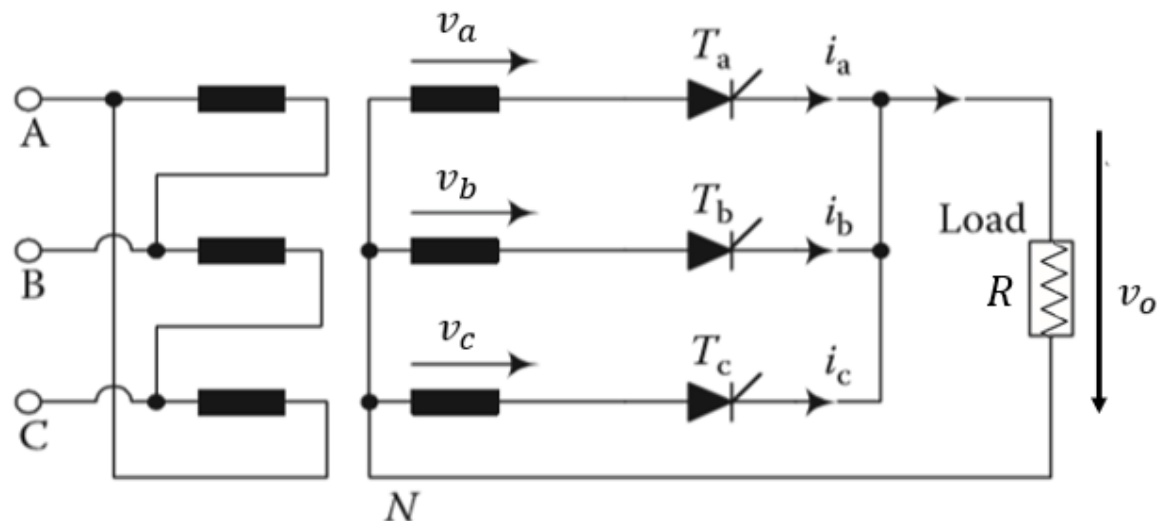


# 1.1 Three-phase, half-wave, R-load

- Resistive load

- ❖ Three phase supply – primary in delta and secondary in star connection.

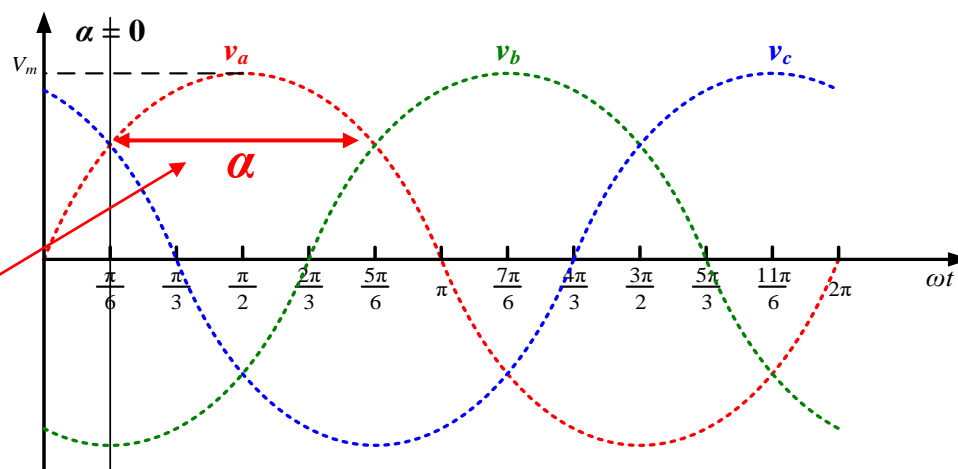
- ❖ Common-cathode connection.



- Natural commutation (phase-changing) point

- It is considered as the starting point for thyristor triggering angle  $\alpha$ , i.e.  $\alpha = 0^\circ$ .

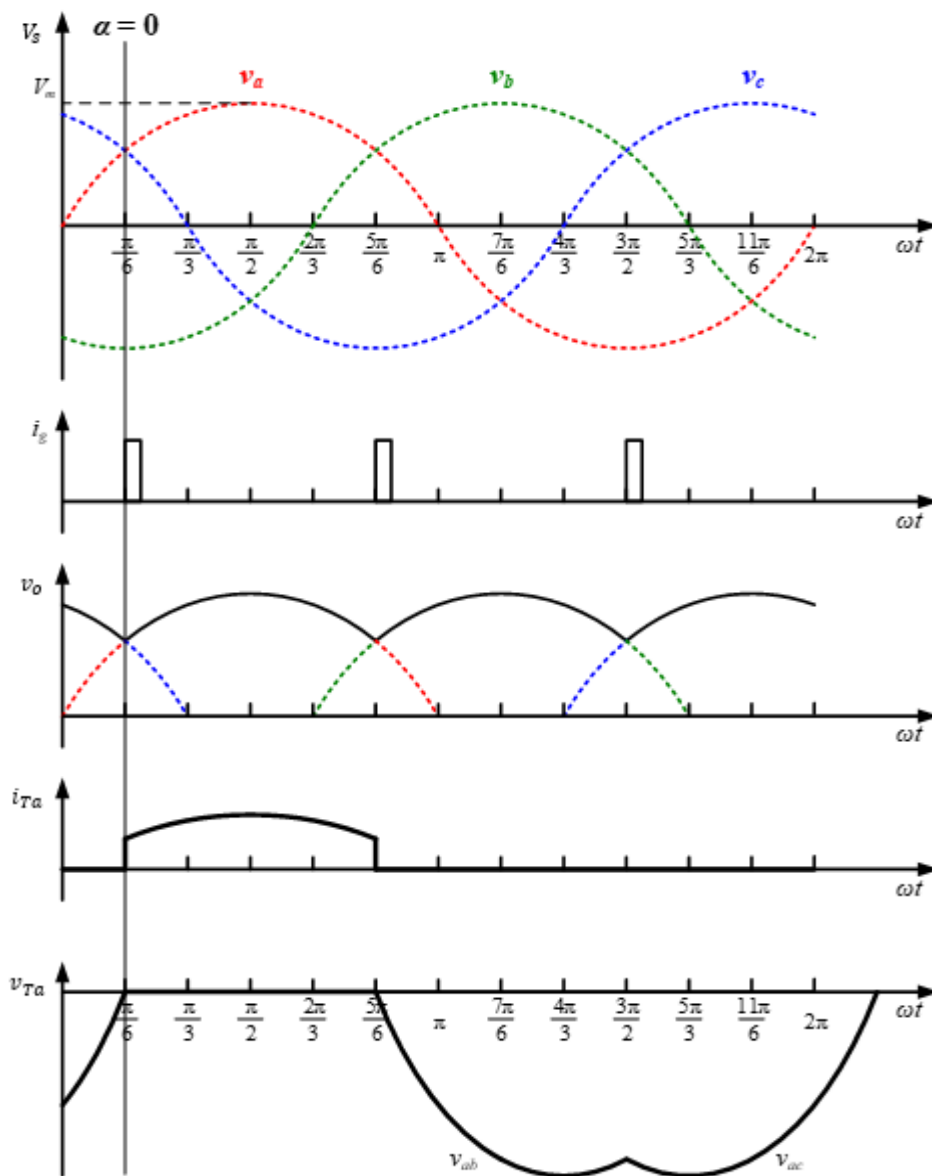
- Phase-shift range:  $\alpha \leq 120^\circ$ .





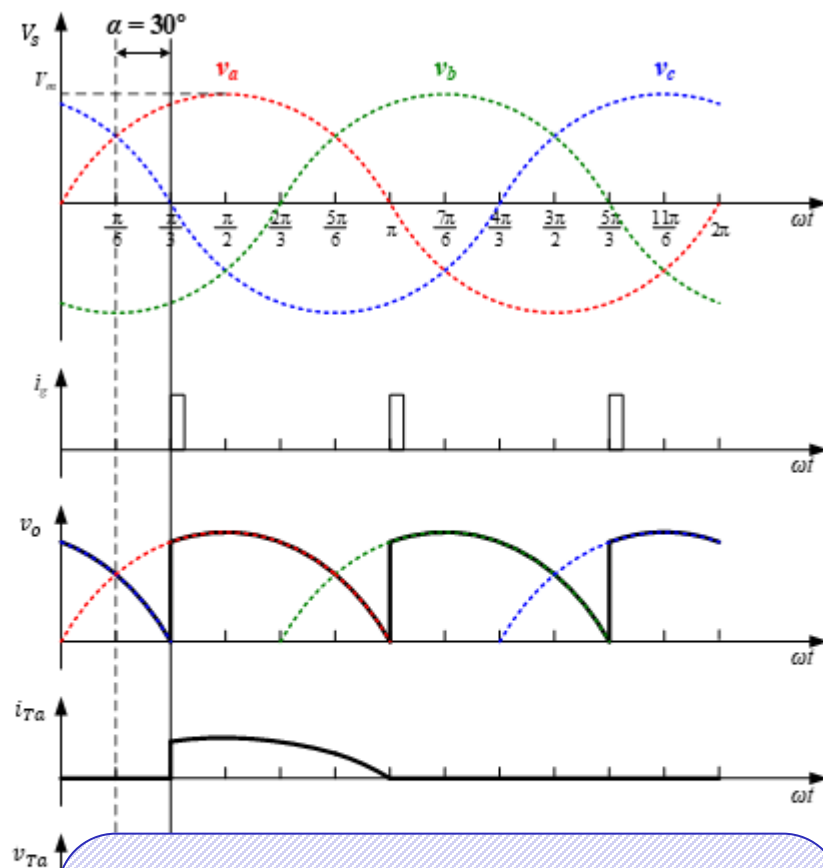
# 1.1 Three-phase, half-wave, R-load

- When  $\alpha = 0^\circ$  (Same as the **uncontrolled, 3-phase, half-wave**)
  - Example: At  $\alpha = 0$  ( $\omega t = \pi/6$ ), as soon as  $T_a$  is forward biased (red line  $v_a$  becomes the largest one), a trigger signal is provided to  $T_a$ , so  $T_a$  starts to conduct;
  - At  $\omega t = 5\pi/6$ , when  $v_b$  becomes the largest one, another trigger signal is provided to  $T_b$ , so  $T_b$  starts to conduct;
  - At  $\omega t = 3\pi/2$ , when  $v_c$  becomes the largest one, the trigger signal to  $T_c$  is provided, so  $T_c$  starts to conduct.



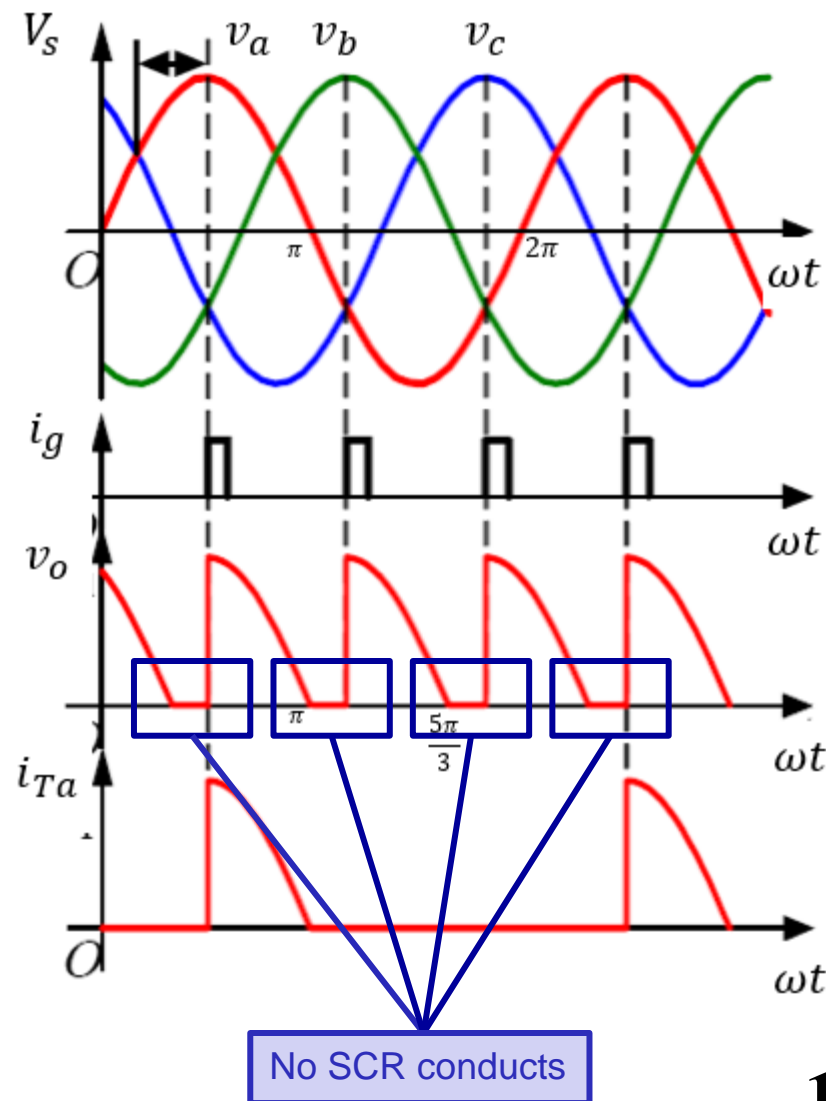
# 1.1 Three-phase, half-wave, R-load

- When  $\alpha = 30^\circ$ 
  - From  $\omega t = \pi/6$  to  $\pi/3$ , although  $T_a$  is forward biased (red line  $v_a$  is the largest one), no trigger signal is provided to  $T_a$ , so  $T_a$  cannot conduct;
  - At  $\alpha = 30^\circ$  ( $\omega t = \pi/3$ ), a trigger signal is provided to  $T_a$ , so  $T_a$  starts to conduct;
  - At  $\omega t = 5\pi/6$ , when  $v_b$  becomes the largest one, since no trigger signal is provided to  $T_b$ , it will not conduct until  $\omega t = \pi$ . It will conduct only when the trigger signal provided.



# 1.1 Three-phase, half-wave, R-load

- When  $\alpha = 60^\circ$ 
  - From  $\omega t = \pi/6$  to  $\pi/2$ , although  $T_a$  is forward biased (red line  $v_a$  is the largest one), no trigger signal is provided to  $T_a$ , so it cannot conduct;
  - At  $\alpha = 60^\circ$  ( $\omega t = \pi/2$ ), a trigger signal is provided to  $T_a$ , therefore it starts to conduct;
  - From  $\omega t = \pi$  to  $7\pi/6$ ,  $v_a$  is no longer the largest one,  $T_a$  stops; since no trigger signal is provided to  $T_b$ , it will not conduct either. **In this region, no SCR will conduct.**
  - At  $\omega t = 7\pi/6$ ,  $T_b$  is triggered and forward biased, therefore it conducts.




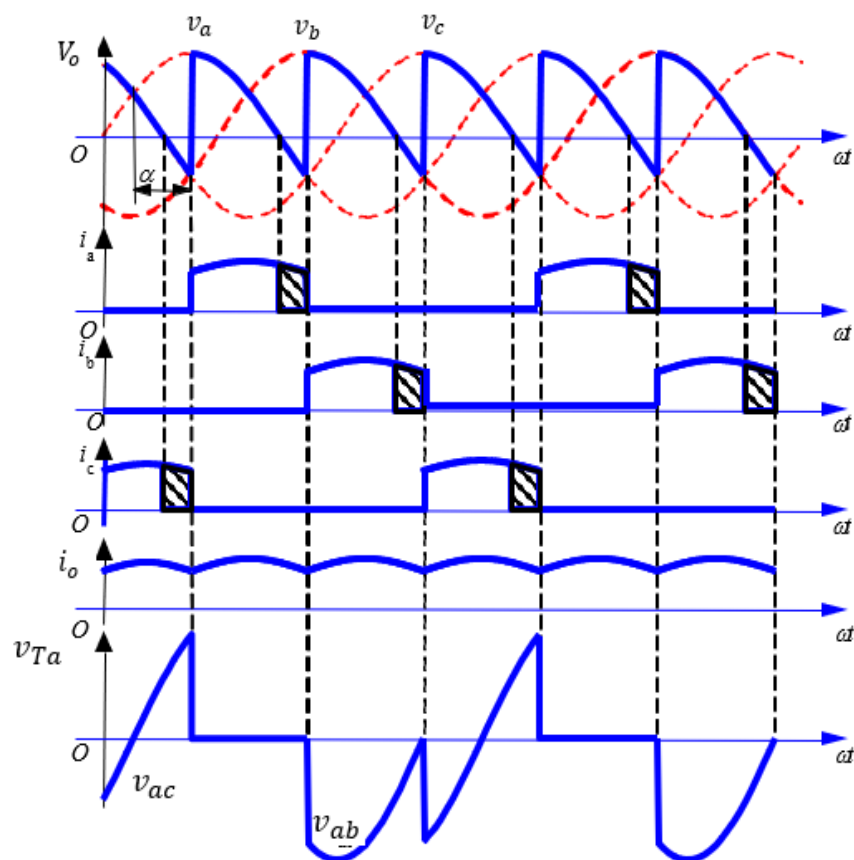
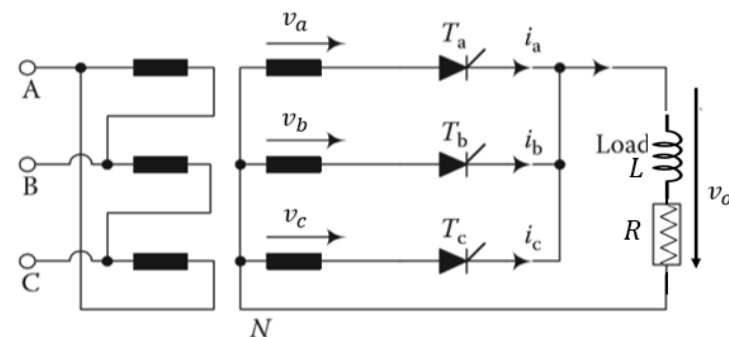
# Resistive load, quantitative analysis

- When  $\alpha \leq 30^\circ$ ,
  - Average value of output voltage,  $V_o = \frac{1}{2\pi/3} \int_{\alpha+\pi/6}^{\alpha+5\pi/6} V_m \sin \omega t d(\omega t) = \frac{3\sqrt{3}}{2\pi} V_m \cos \alpha$
  - RMS voltage,  $V_{RMS} = \left[ \frac{1}{2\pi/3} \int_{\alpha+\pi/6}^{\alpha+5\pi/6} V_m^2 \sin^2 \omega t d(\omega t) \right]^{1/2} = \sqrt{3} V_m \left[ \frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{1/2}$
  - Average load current,  $I_o = \frac{V_o}{R} = \frac{3\sqrt{3}V_m}{2\pi R} \cos \alpha$
  - RMS load current,  $I_{RMS} = \frac{V_{RMS}}{R} = \frac{\sqrt{3}V_m}{R} \left[ \frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{1/2}$
- When  $\alpha > 30^\circ$ ,
  - Average output voltage,  $V_o = \frac{1}{2\pi/3} \int_{\alpha+\pi/6}^{\pi} V_m \sin \omega t d(\omega t) = \frac{3V_m}{2\pi} [1 + \cos(\alpha + 30^\circ)]$
  - RMS voltage,  $V_{RMS} = \left[ \frac{1}{2\pi/3} \int_{\alpha+\pi/6}^{\pi} V_m^2 \sin^2 \omega t d(\omega t) \right]^{1/2} = \frac{\sqrt{3}V_m}{2\sqrt{\pi}} \left[ \left( \frac{5\pi}{6} - \alpha \right) + \frac{1}{2} \sin(2\alpha + \pi/3) \right]^{1/2}$



# 1.2 Three-phase, half-wave, RL-load

- Inductive load (R-L)
  - The load inductance  $L$  is large enough, the output current  $i_o$  is continuous and almost flat;
  - When  $\alpha \leq 30^\circ$ , the rectified voltage waveform is similar to resistive load;
  - When  $\alpha > 30^\circ$  (eg.  $\alpha = 60^\circ$  ):
    - At  $\omega t = \pi$ ,  $v_a$  is zero but  $i_a$  is not zero due to RL load. So,  $T_a$  keeps conducting beyond  $\pi$ .
    - $v_o$  goes negative beyond  $\omega t = \pi$ .
  - When  $T_b$  is turned on, load current shifts from  $T_a$  to  $T_b$ .



# Inductive load, quantitative analysis

- Load current  $i_o$  is always continuous, and

$$\text{Average value of output voltage, } V_o = \frac{1}{2\pi/3} \int_{\alpha+\pi/6}^{\alpha+5\pi/6} V_m \sin \omega t d(\omega t) = \frac{3\sqrt{3}}{2\pi} V_m \cos \alpha$$

$$\text{RMS voltage, } V_{RMS} = \left[ \frac{1}{2\pi/3} \int_{\alpha+\pi/6}^{\alpha+5\pi/6} V_m^2 \sin^2 \omega t d(\omega t) \right]^{1/2} = \sqrt{3} V_m \left[ \frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{1/2}$$

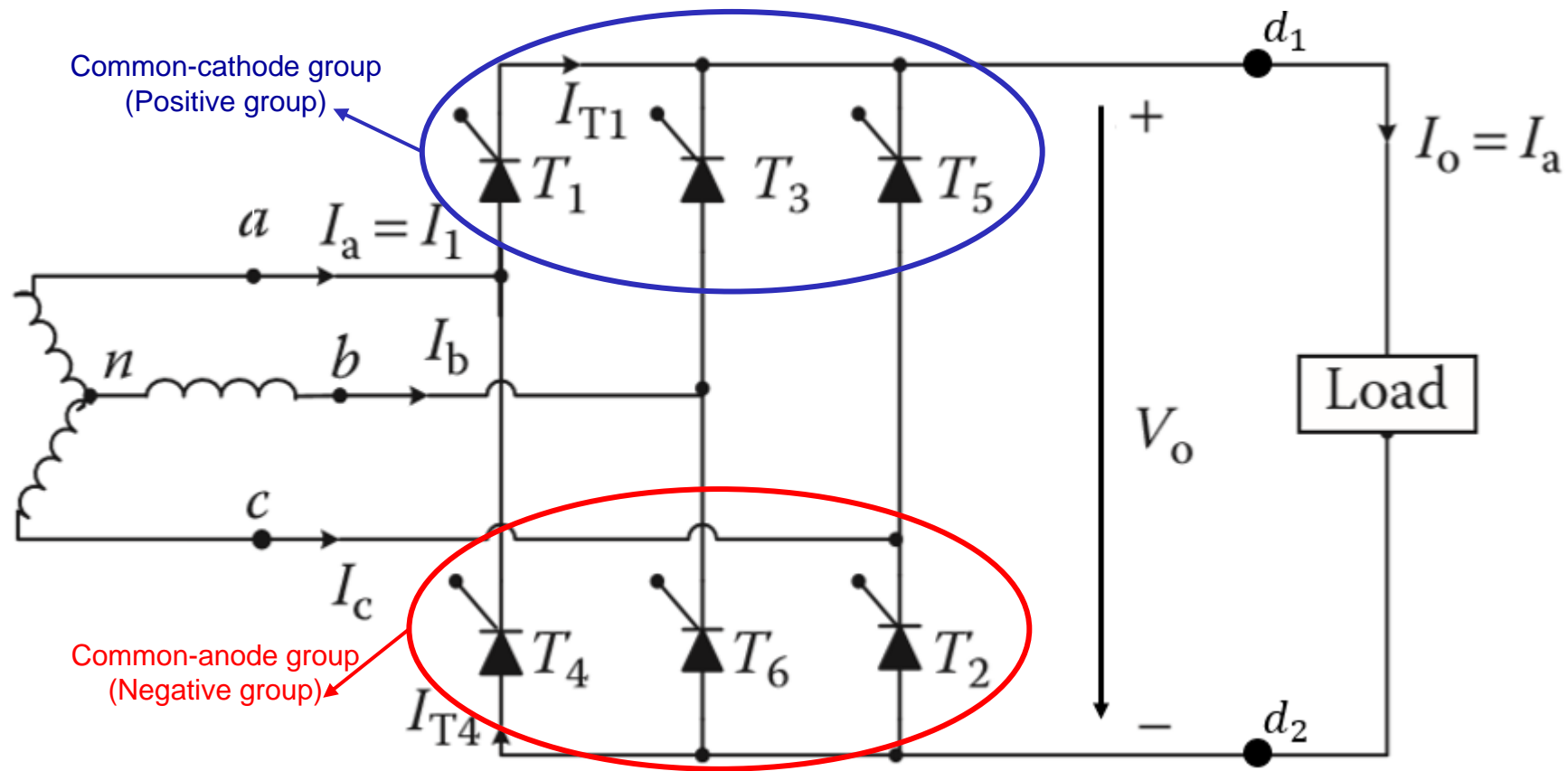
- The average value of thyristor current = Average value of source current,  $I_{SA} = \frac{(I_o \times 120)}{360}$
- RMS value of thyristor current:

$$I_{T,RMS} = \left[ \frac{I_o^2 \times 120}{360} \right]^{1/2} = \frac{I_o}{\sqrt{3}}$$

How?



## 2 Three-phase bridge fully-controlled rectifier



- Numbering of the 6 thyristors indicates the trigger sequence:

$$T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4 \rightarrow T_5 \rightarrow T_6$$

Commutation occurs every  $60^\circ$  interval, alternatively in upper and lower group of SCRs.



## 2.1 Resistive load

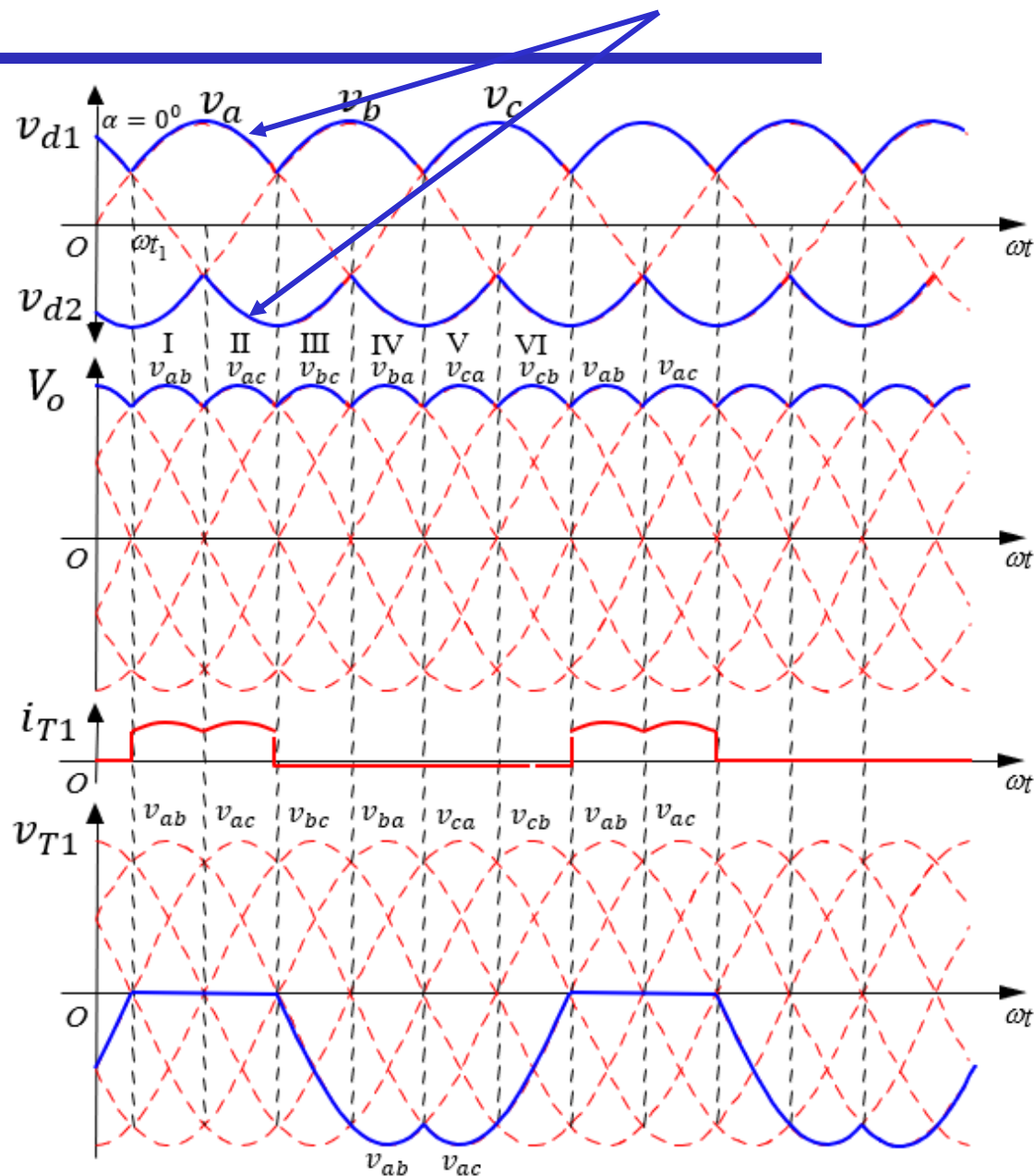
Positive and negative groups of SCRs are fired at an interval of  $120^\circ$

- Resistive load
  - $\alpha = 0^\circ$
  - Thyristors behave like diodes
  - $T_1$  is triggered at  $\omega t = \pi/6$ ,  $T_2$  starts at  $\pi/2$ ...
  - Load (output) voltage is similar to uncontrolled case

$$V_{an} = V_m \sin \omega t$$

$$V_{bn} = V_m \sin(\omega t - 120^\circ)$$

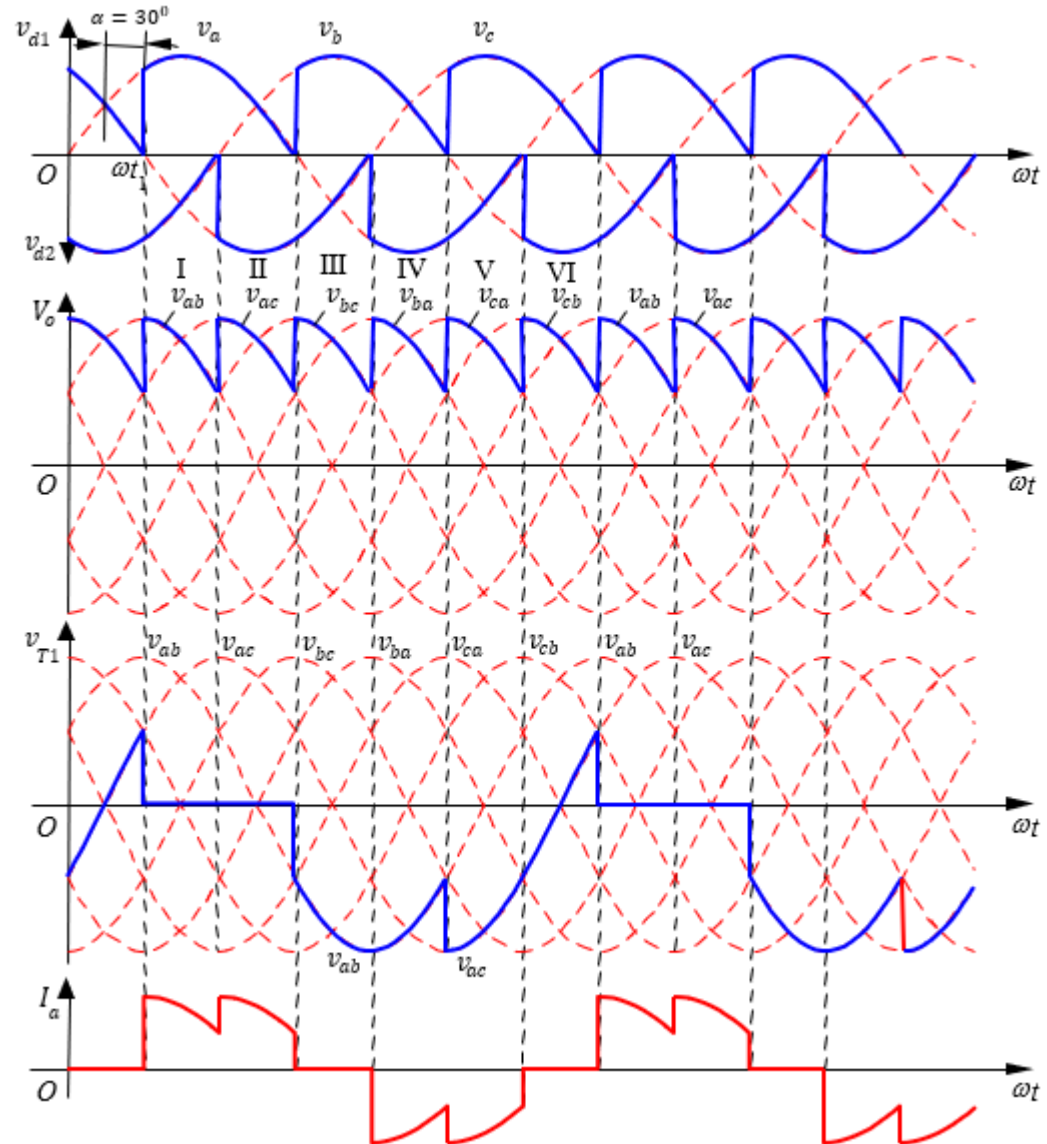
$$V_{cn} = V_m \sin(\omega t - 240^\circ)$$





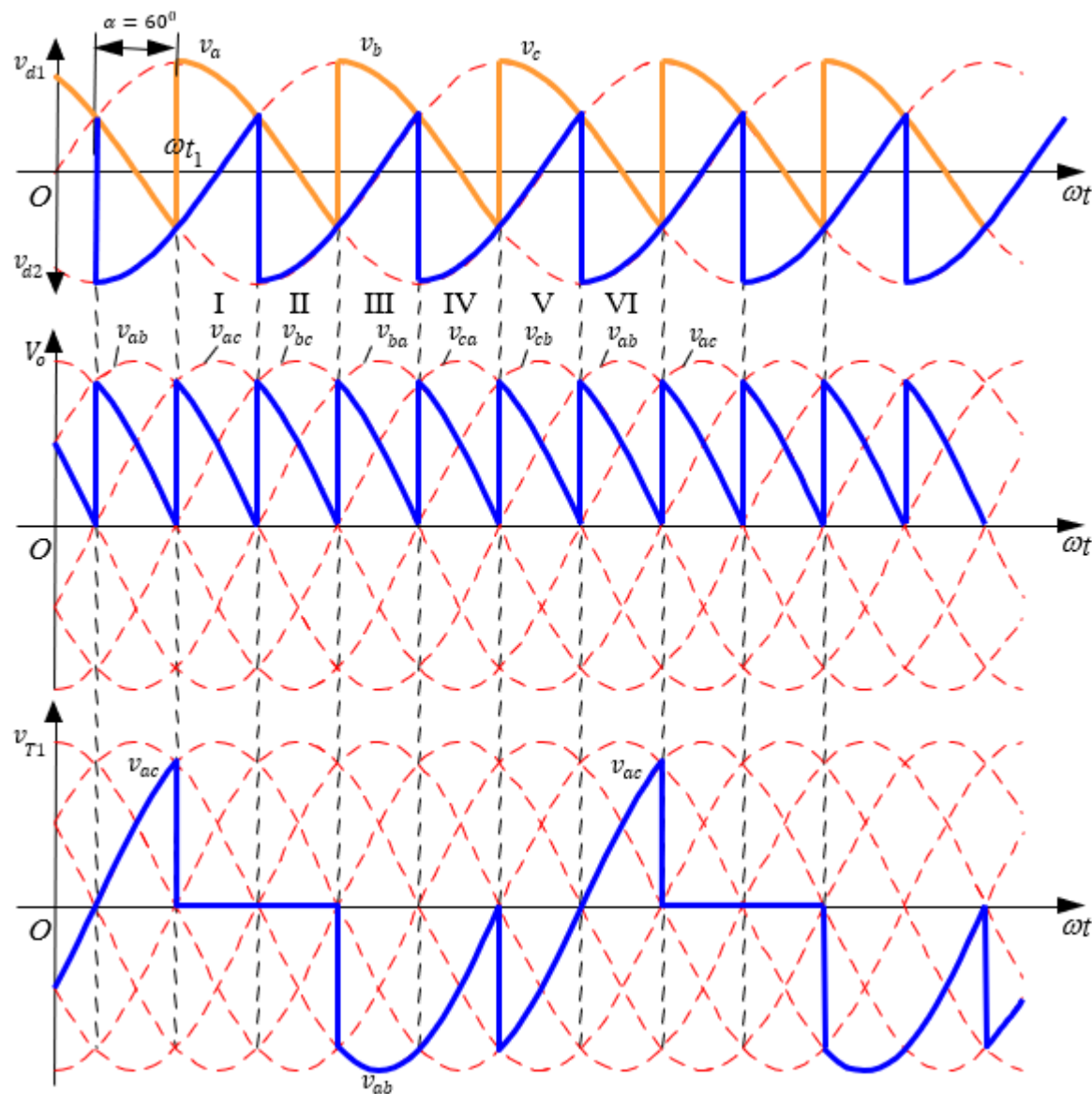
## 2.1 Resistive load

- Resistive load
  - $\alpha = 30^\circ$
  - $T_1$  starts conducting at  $\omega t = 60^\circ$ ,  $T_2$  conducts at  $120^\circ$ ,  $T_3$  conducts  $180^\circ$  and so on....
  - At  $\omega t = 60^\circ$ ,  $T_1$  connected to line “a” from positive group and  $T_6$  connected to line “b” would conduct, therefore output voltage follows the line voltage  $v_{ab}$ .
  - Output voltage is continuous



## 2.1 Resistive load

- Resistive load
  - $\alpha = 60^\circ$
  - $T_1$  starts conducting at  $\omega t = 90^\circ$ ,  $T_2$  conducts at  $150^\circ$ ,  $T_3$  conducts  $210^\circ$  and so on....
  - Output voltage is continuous



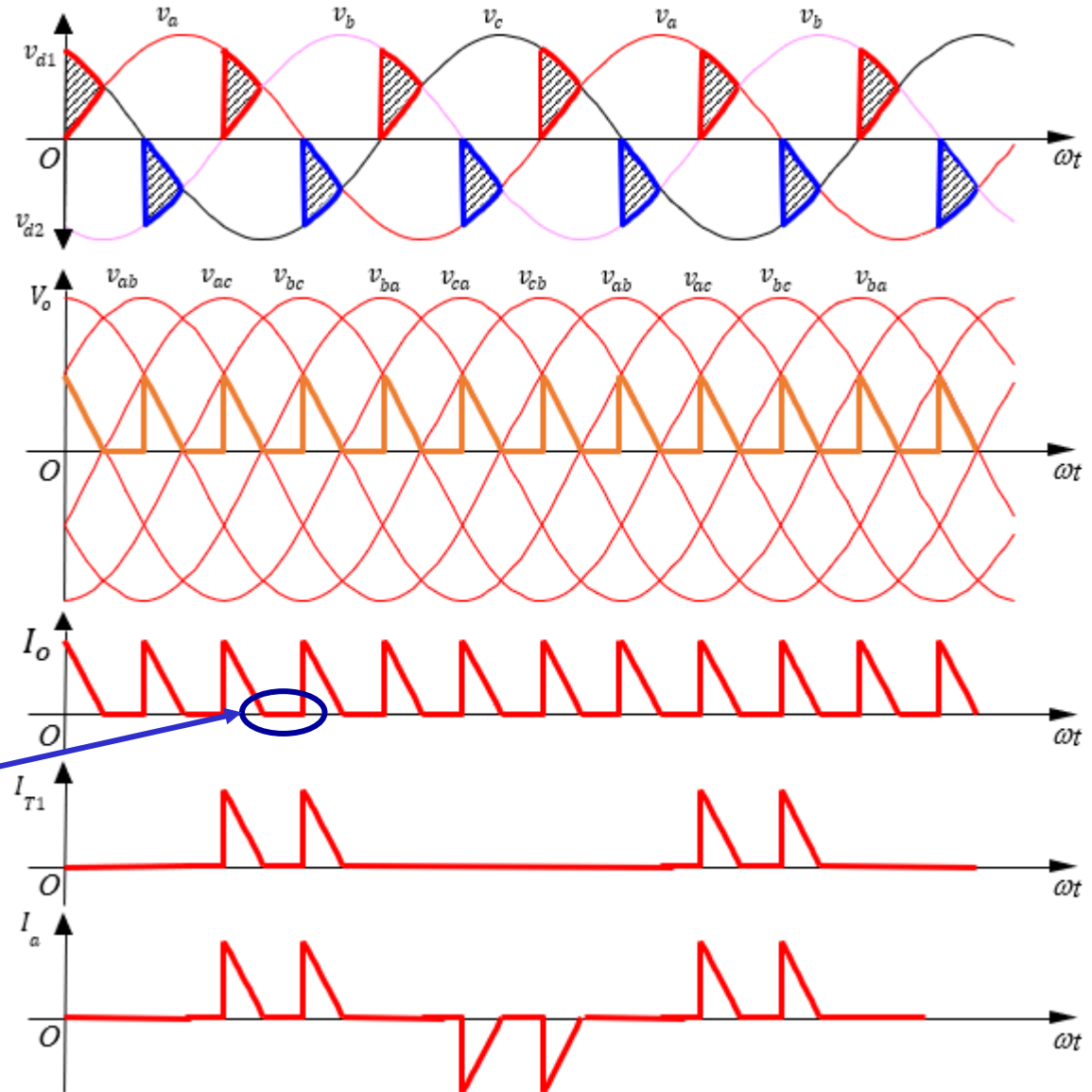
## 2.1 Resistive load

- Resistive load
  - $\alpha = 90^\circ$
  - T1 starts conducting at  $\omega t = 120^\circ$
  - Output is discontinuous due to resistive load
  - At  $\omega t = 150^\circ$ , output voltage,

$$V_{ab} = V_{an} - V_{bn}$$

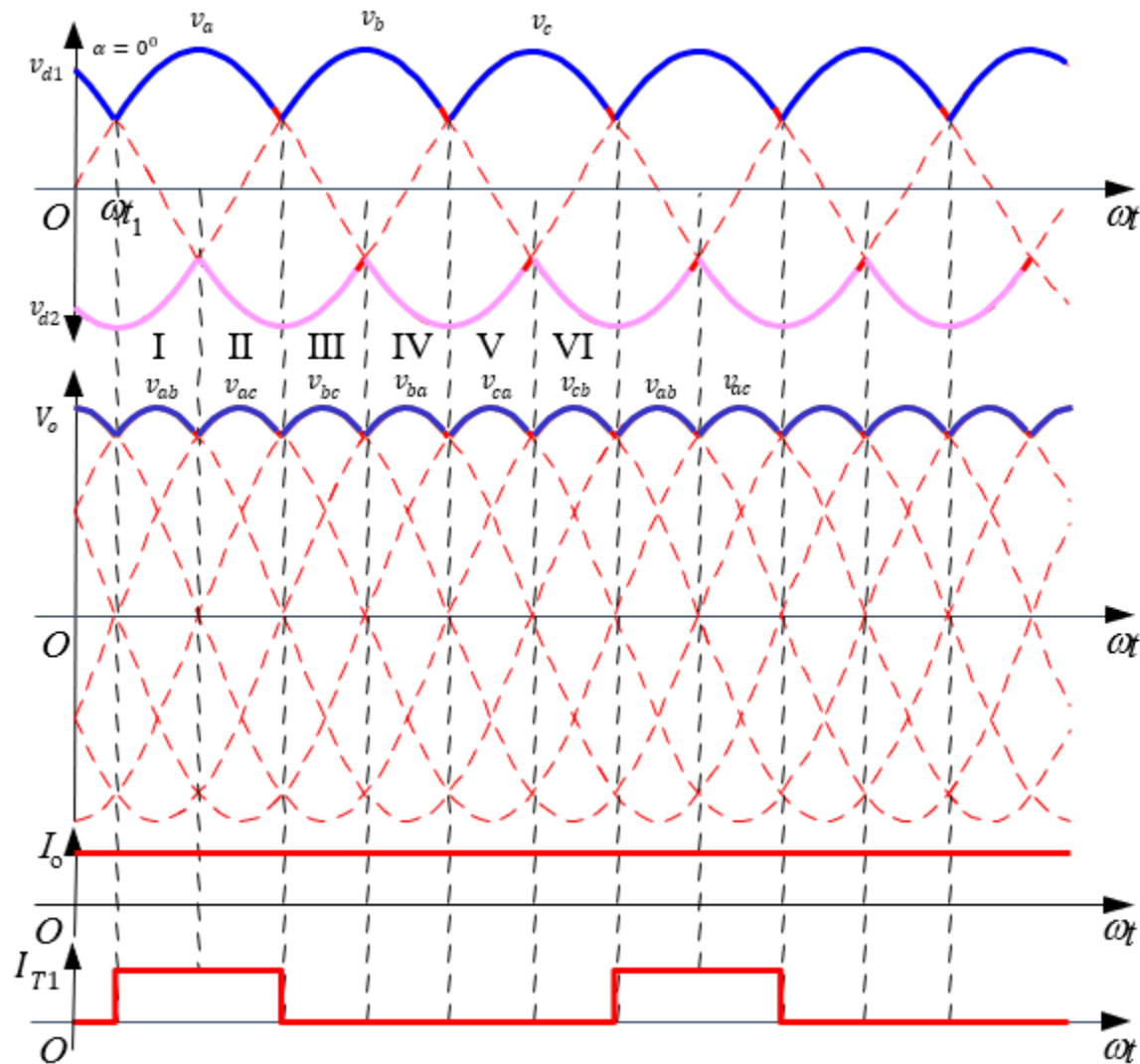
$$= V_m \sin 150^\circ$$

$$= V_m \sin(150^\circ - 120^\circ) = 0$$



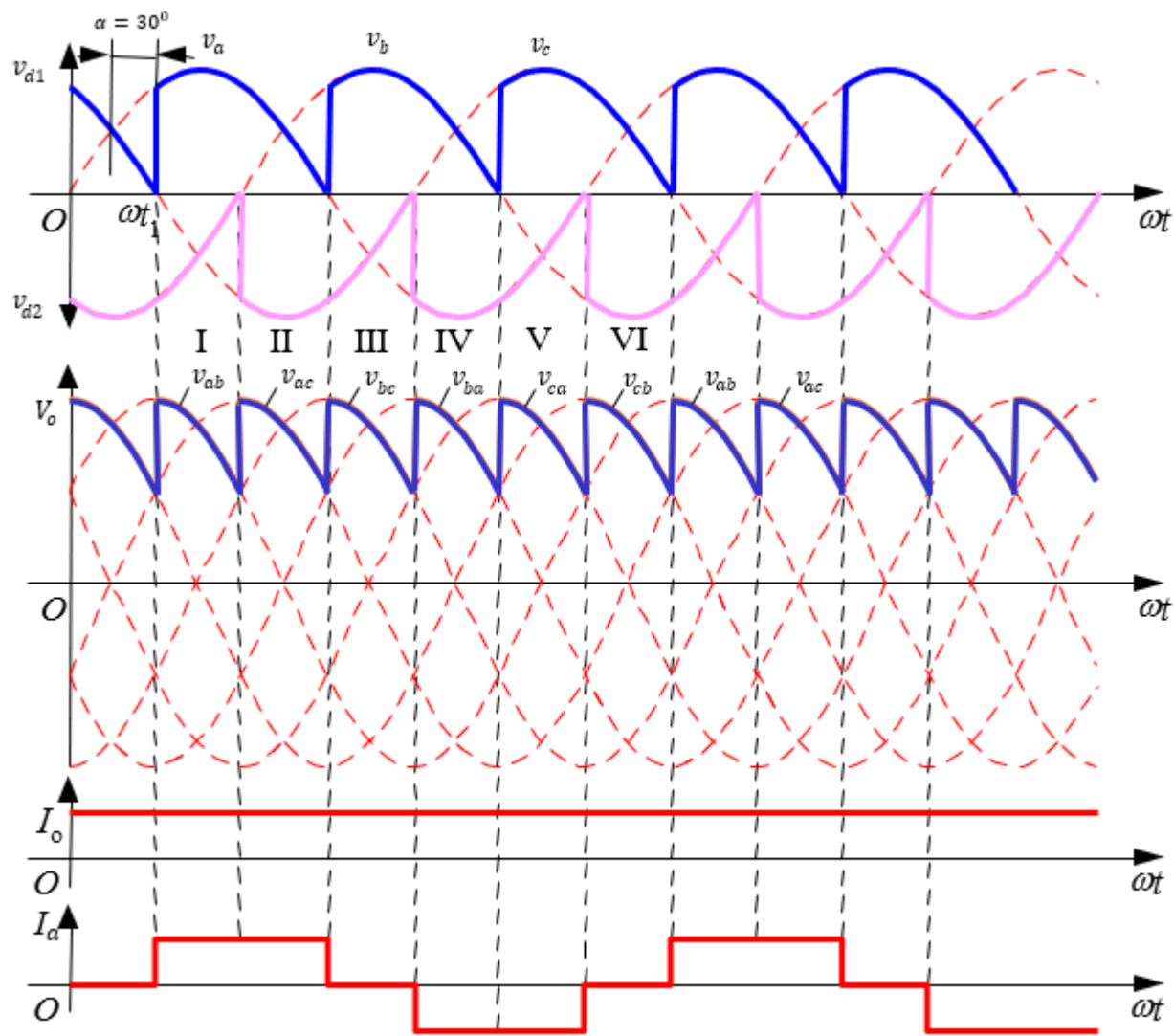
## 2.2 Inductive load

- Inductive load  
(R-L load)
  - $\alpha = 0^\circ$
  - Note that load inductance  $L$  is large so that the load current is continuous and constant at magnitude.
  - $T_1$  is triggered at  $\omega t = \pi/6$
  - Output voltage is continuous



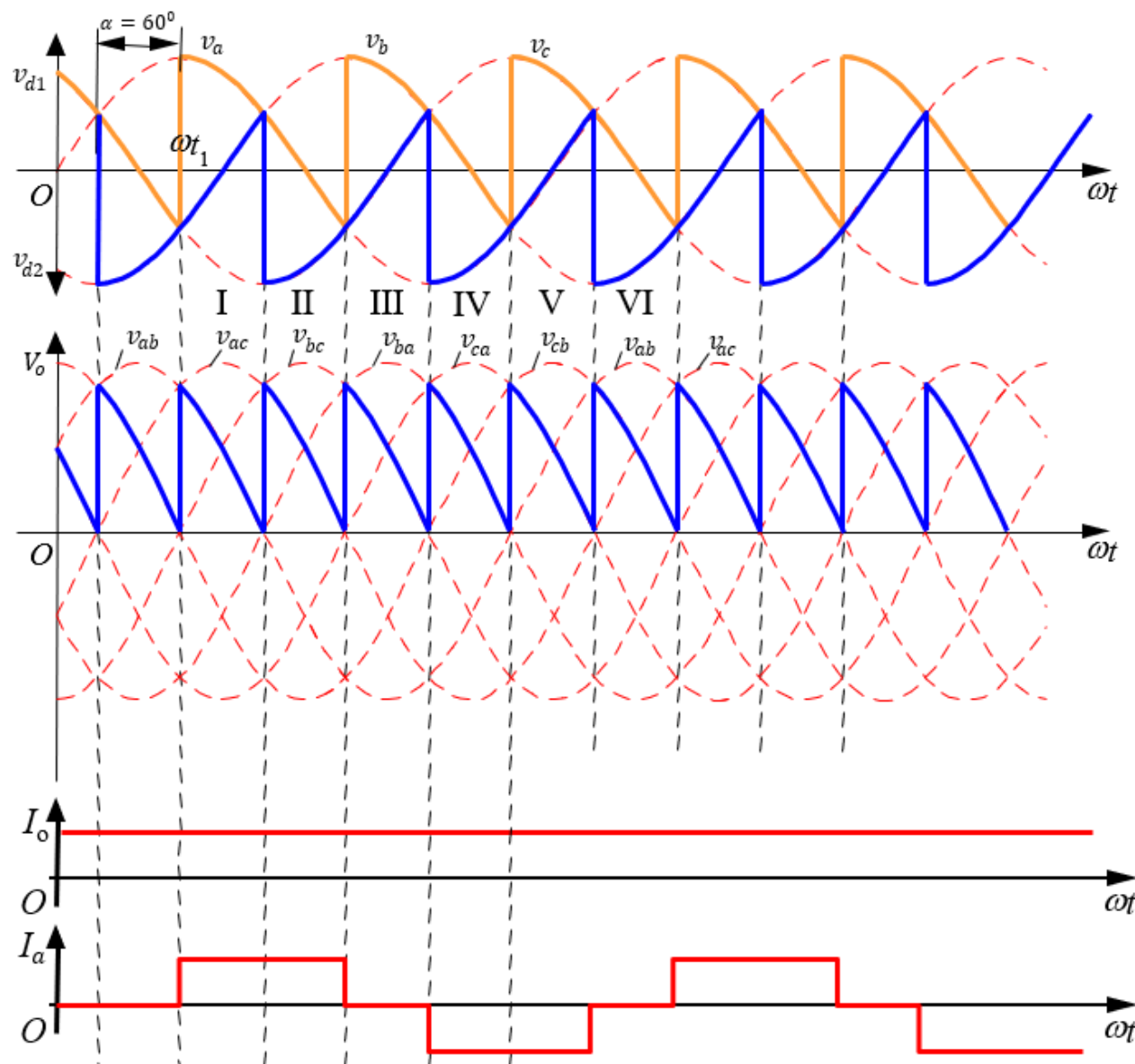
## 2.2 Inductive load

- Inductive load  
(R-L load)
  - $\alpha = 30^\circ$
  - $T_1$  is triggered at  $\omega t = \pi/3$
  - Output voltage is continuous



## 2.2 Inductive load

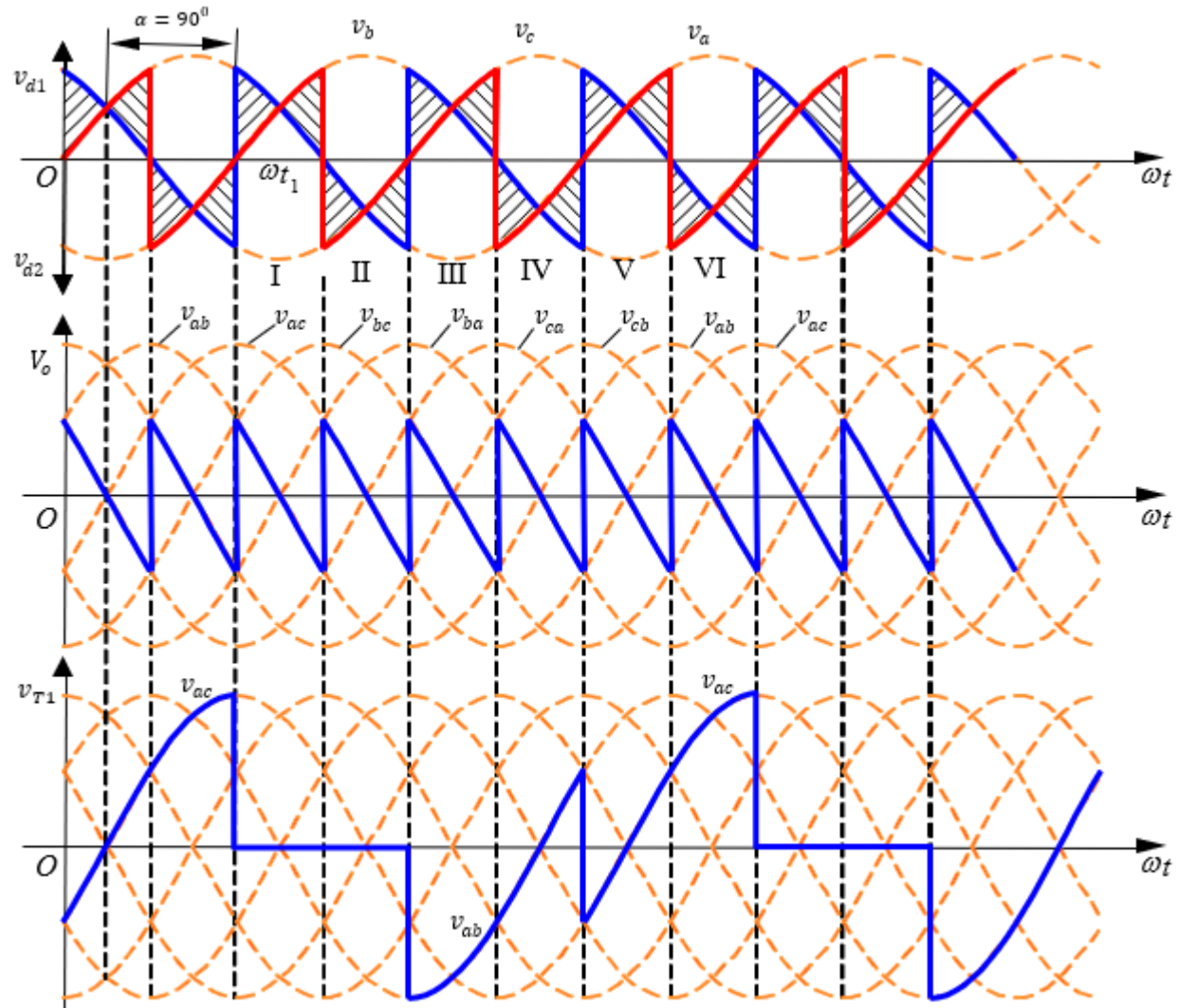
- Inductive load  
(R-L load)
  - $\alpha = 60^\circ$
  - $T_1$  is triggered at  $\omega t = \pi/2$
  - Output voltage is continuous





## 2.2 Inductive load

- Inductive load  
(R-L load)
  - $\alpha = 90^\circ$
  - $T_1$  is triggered at  $\omega t = 120^\circ$
  - Output voltage is continuous and goes negative due to RL load



# Quantitative analysis

- Average output voltage (R-load, for  $\alpha \leq 60^\circ$  and any  $\alpha$  for RL load)

$$\begin{aligned} V_{dc} &= \frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi/2+\alpha} v_{ab} d(\omega t) = \frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi/2+\alpha} \sqrt{3} V_m \sin \left( \omega t + \frac{\pi}{6} \right) d(\omega t) \\ &= \frac{3 \sqrt{3} V_m}{\pi} \cos \alpha \end{aligned}$$

- For resistive load, when  $\alpha > 60^\circ$ , load current  $I_o$  is discontinuous.

$$V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}} \sqrt{3} \sin \left( \omega t + \frac{\pi}{6} \right) d(\omega t) = \frac{3\sqrt{3}V_m}{\pi} \left[ 1 + \cos \left( \frac{\pi}{3} + \alpha \right) \right]$$

- RMS output voltage

$$\begin{aligned} V_{rms} &= \left[ \frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi/2+\alpha} 3V_m^2 \sin^2 \left( \omega t + \frac{\pi}{6} \right) d(\omega t) \right]^{1/2} \\ &= \sqrt{6} V_m \left( \frac{1}{4} + \frac{3 \sqrt{3}}{8\pi} \cos 2\alpha \right)^{1/2} \end{aligned}$$





# Quantitative analysis – Notes

- $V_m$  is the peak value of the phase voltage.

$$v_{an} = V_m \sin \omega t$$

$$v_{bn} = V_m \sin(\omega t - 120^\circ)$$

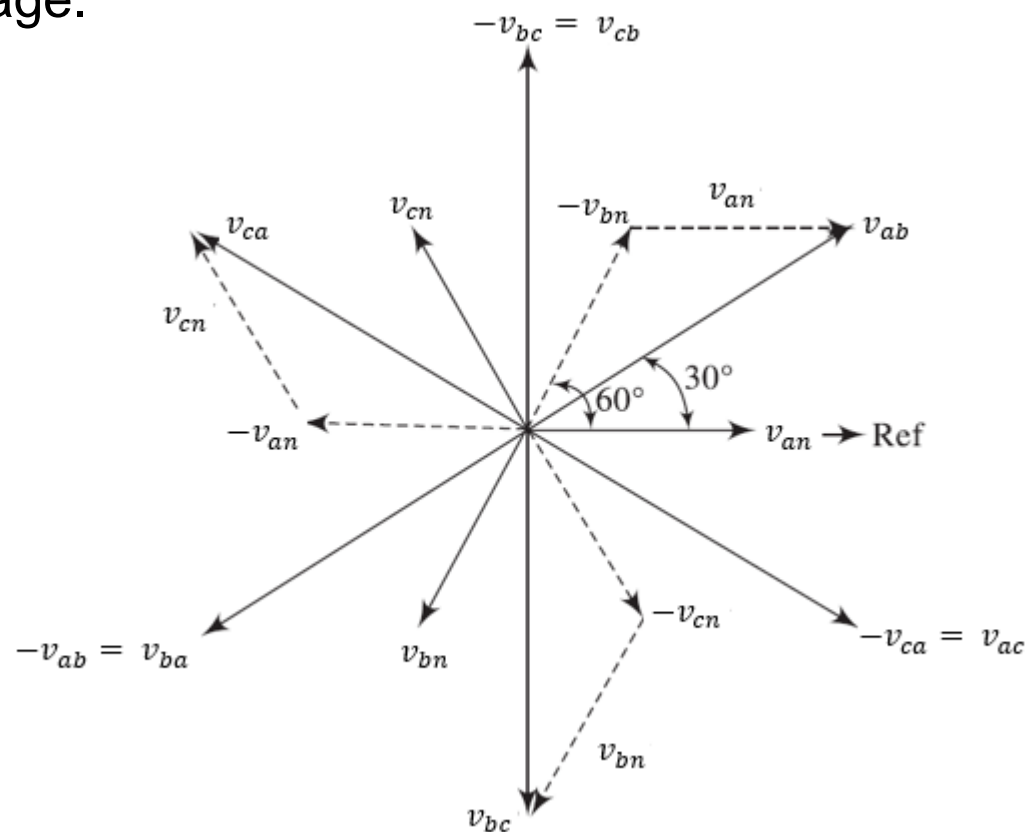
$$v_{cn} = V_m \sin(\omega t - 240^\circ)$$

- Line to line voltages lead the phase voltage by  $30^\circ$ .

$$v_{ab} = \sqrt{3}V_m \sin(\omega t + 30^\circ)$$

$$v_{bc} = \sqrt{3}V_m \sin(\omega t - 90^\circ)$$

$$v_{ca} = \sqrt{3}V_m \sin(\omega t - 210^\circ) \\ \cong V_{ml} \sin(\omega t - 210^\circ)$$



# Quantitative analysis

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- Average output current (load current) for resistive load

$$I_o = \frac{V_o}{R}$$

- Thyristor voltage and current
  - Same as three-phase half-wave rectifier
- For EMF load,  $L$  is large enough
  - All the same as inductive load except the calculation of average output current  $I_o = \frac{V_o - E}{R}$

## *Exercise – Try to solve!*

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- A three-phase half-wave controlled converter is operated from 3-phase, 230 V, 50 Hz supply with load resistance  $R = 10\Omega$ . An average output voltage of 50% of the maximum possible output voltage is required. Determine
  - a) the firing angle,
  - b) average and rms values of load current,
  - c) rectification efficiency

# Summary

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- **3-phase half-wave rectifier with R-load:** 1) Continuous conduction mode when  $\alpha < 30^\circ$ , 2) Discontinuous mode when  $\alpha > 30^\circ$ .
- **3-phase full-wave rectifier:** Triggering pulses should be
  - According the sequence of  $V_{T1} \rightarrow V_{T2} \rightarrow V_{T3} \rightarrow V_{T4} \rightarrow V_{T5} \rightarrow V_{T6}$
  - With  $60^\circ$  phase difference
  - 6 pulses in one period;
  - **Continuous conduction mode** for  $\alpha \leq 60^\circ$  in R-load and for any  $\alpha$  in RL load, otherwise, *discontinuous mode*.
- The output voltage waveforms for  $\alpha = 0^\circ, 30^\circ, 60^\circ$  of 3-phase fully controlled bridge rectifier with RL load will be same as the waveforms for  $\alpha = 0^\circ, 30^\circ, 60^\circ$  of 3-phase fully controlled bridge rectifier with R load.

***See you in the next class (March 24<sup>th</sup>)***

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**»»» Tutorial in the next lecture**

**The End**