Announcements:

- Provide your feedback on Practical Lectures:
  - Please tap here to proceed…
- Reading material:
  - Microelectronics: Circuit Analysis and Design, Chapter 12:
    - Sec. 12.9 – Stability of the feedback circuit
    - Sec. 12.10 – Frequency compensation
- Practice exercises:
  - Microelectronics: Circuit Analysis and Design, Chapter 12:
    - Ex. 12.18-12.22
    - TYU 12.15-12.17
- Assignment 2:
  - Due date: 11:59PM Friday, 8 November (Week 8)
  - Cut-off date: 11:59PM Friday, 15 November (Week 9)
- Next lecture:
  - Week 9 (this week): New lecturer – Dr. Xiaoyang Chen
    - Group 1: Thu 16:00-18:00 @BSG02
    - Group 2: Fri 16:00-18:00 @EB138

Frequency Response and Stability of Amplifier Circuits with Feedback

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Outline

- Part 1: Frequency Response Analysis:
  - Review the principles of frequency response analysis: derive transfer functions and Bode plots of one- and two-pole amplifiers;
- Part 2: Stability of the Feedback Circuit:
  - Determine the stability criteria of feedback circuits;
- Part 3: Frequency Compensation
  - Consider frequency compensation techniques, methods by which unstable feedback circuits can be stabilized.

Part 1: Frequency Response Analysis (review)

Frequency response analysis (overview)

The circuit frequency response is usually determined (analytically) by using the **complex frequency $\boldsymbol{s}$**:

1. Each capacitor is represented by its complex impedance $\mathbf{1/sC}$, and each inductor is represented by $\boldsymbol{sL}$. For amplifiers, we only consider **the effect of capacitors** due to Miller effect;

2. Based on the complex impedances, we determine the **system gain (transfer function as a product of polynomials)**;

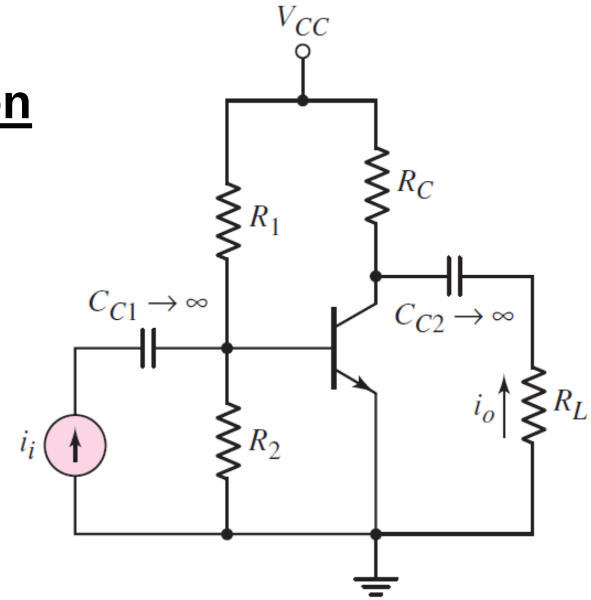
3. Once a transfer function is found, we set $s = j\omega = j2\pi f$. The system transfer function then reduces to a **complex function of frequency**;

4. The complex function of frequency can be reduced to **a magnitude and a phase**;

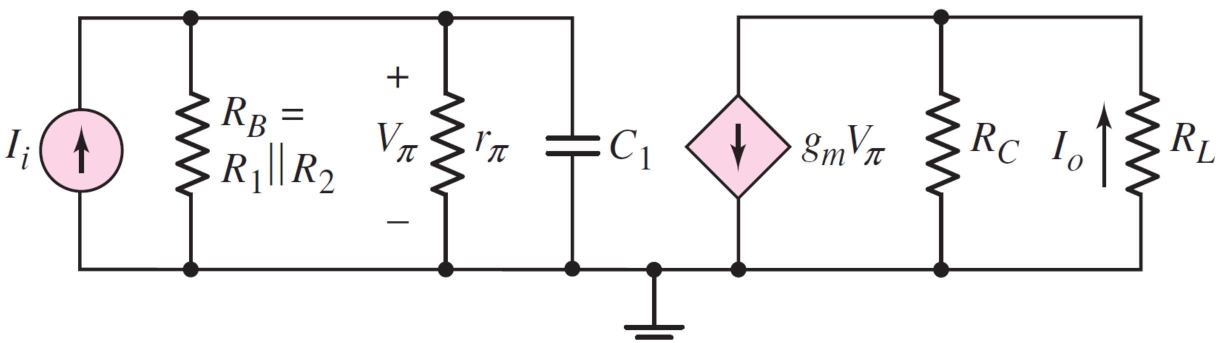
5. Finally, we can use analytical tools such as **Bode plots or Nyquist diagram** to analyze the system frequency response.

One-stage (pole) amplifier: transfer function

Consider a simple single-stage common-emitter current amplifier:



0. From Week 4 lecture on Frequency response, small-signal (high frequency) equivalent circuit have been obtained as follows:

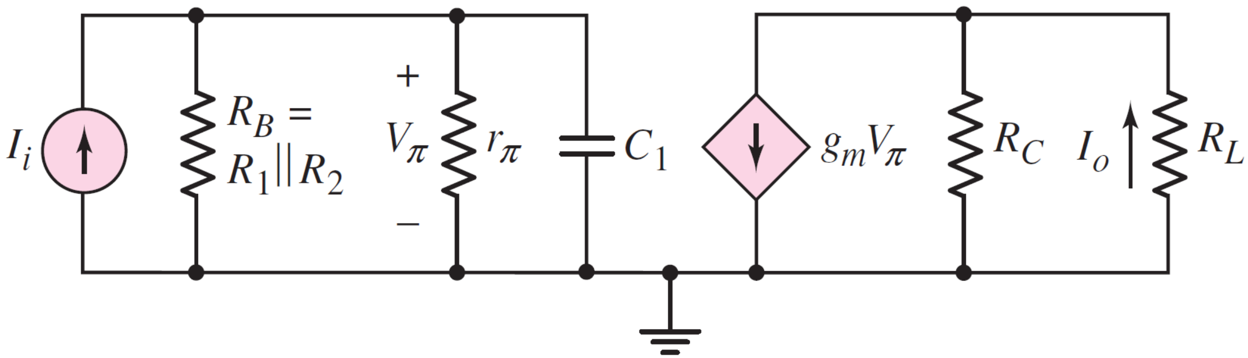


where C_1 includes:

- Forward-biased BE junction capacitance;
- Miller capacitance.

One-stage (pole) amplifier: transfer function

Common-emitter high-freq. small-signal equivalent circuit:



Let us find the current gain of the amplifier ($A_i = I_o/I_i$):

1. The output current

$$I_o = g_m V_\pi \left(\frac{R_C}{R_C + R_L}\right)$$

2. The voltage V_π is:
where $R_\pi = r_\pi||R_1||R_2||$

$$V_\pi = I_i\left(r_\pi||R_1||R_2||\frac{1}{sC_1}\right) = \cdots = I_i\left(\frac{R_\pi}{1 + sR_\pi C_1}\right)$$

3. We can obtain the current gain of the amplifier as a function of ***s***:

$$A_i(s) = \frac{I_o}{I_i} = g_m R_\pi \left(\frac{R_C}{R_C + R_L}\right)\left(\frac{1}{1 + sR_\pi C_1}\right)$$

4. Now, we can substitute $s = j2\pi f$ and convert (3) into the form: $A_i(f) = \dfrac{A_{io}}{1 + j(f/f_1)}$

where $A_{io} = g_m R_\pi \left(\frac{R_C}{R_C+R_L}\right)$ is the low-frequency gain and $f_1 = \frac{1}{2\pi R_\pi C_1}$ is the corner frequency.

One-stage (pole) amplifier: Bode plots

5. To track gain amplitude and phase, the complex function in (4.) can be written in the polar form $A_i(jf) = |A_i(jf)|\angle\phi$:

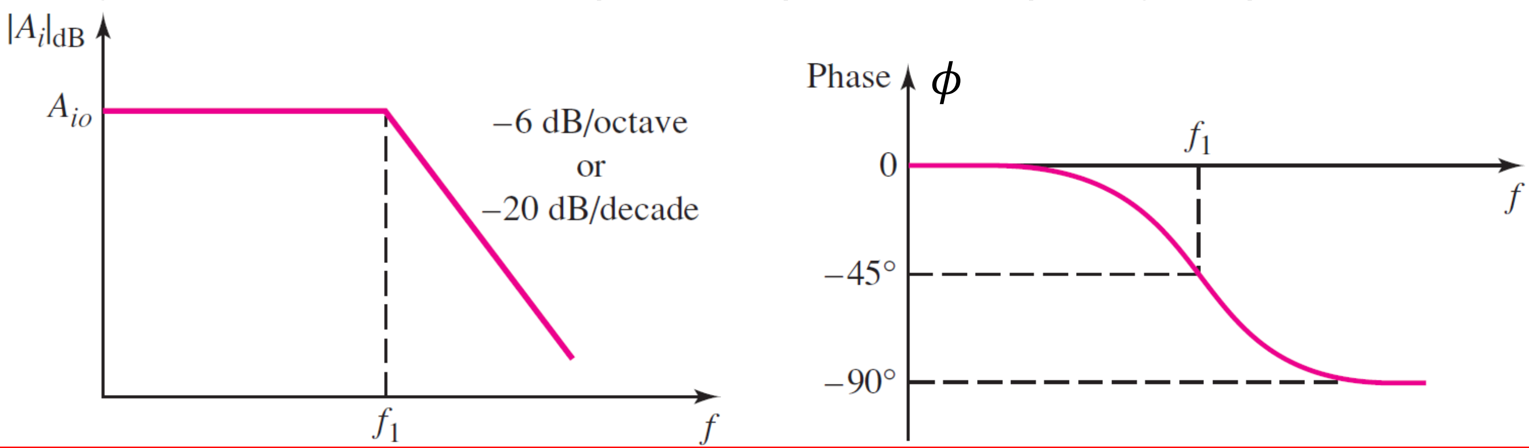
$$A_i(jf) = \frac{A_{io}}{\sqrt{1+\left(\frac{f}{f_1}\right)^2}} \angle - \tan^{-1}\left(\frac{f}{f_1}\right)$$

where
$\tan^{-1} x \equiv \arctan x$

6. From (4.) we can derive corner frequency f_1 and low-freq. gain A_{io}:

$$A_{io} = g_m R_\pi \left(\frac{R_C}{R_C + R_L}\right) \text{and } f_1 = \frac{1}{2\pi R_\pi C_1}$$

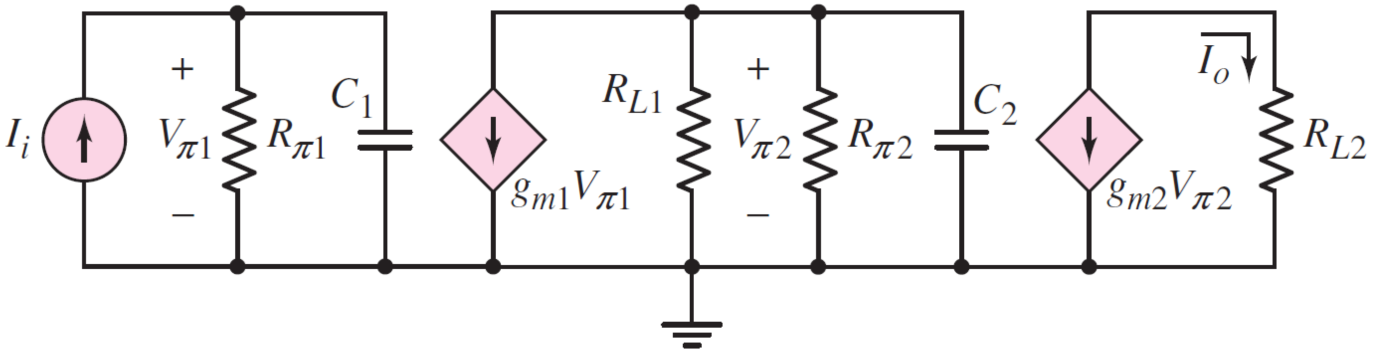
7. Finally, we can draw a Bode plot to represent frequency response of the amp:



The output current is in phase with the input current at low frequencies. At high frequencies, the output current becomes 90 degrees out of phase with respect to the input current.

Two-stage (pole) amplifier: transfer function

For the two-stage amplifier (i.e., common-emitter):



Let us find the current gain of the amplifier ($A_i = I_o/I_i$):

1. The output current is found as:

$$I_o = -g_{m2}V_{\pi 2},$$

where

$$V_{\pi 2} = -g_{m1}V_{\pi 1}\left(R_{L1}||R_{\pi 2}||\frac{1}{sC_2}\right) \text{ and } V_{\pi 1} = I_i\left(R_{\pi 1}||\frac{1}{sC_1}\right)$$

2. The resulting current gain expression will be:

$$A_i = \frac{I_o}{I_i} = (g_{m1}g_{m2})(R_{\pi 1})(R_{L1}||R_{L2})\left[\frac{1}{1 + sR_{\pi 1}C_1}\right]\left[\frac{1}{1 + s(R_{L1}||R_{\pi 2})C_2}\right]$$

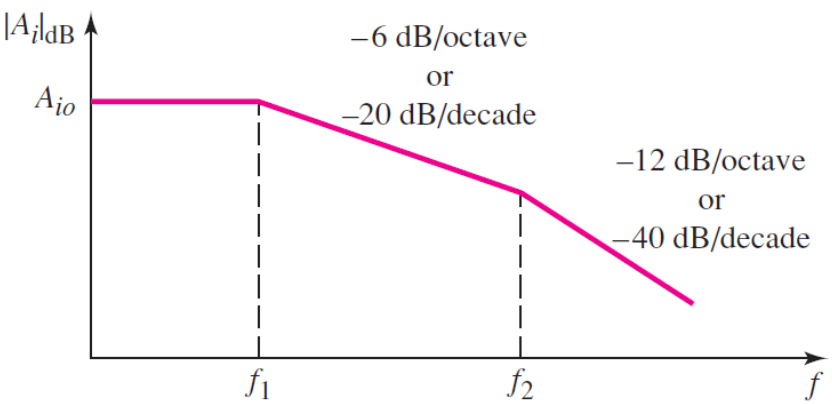
<u>Two-stage (pole) amplifier: Bode plots</u>

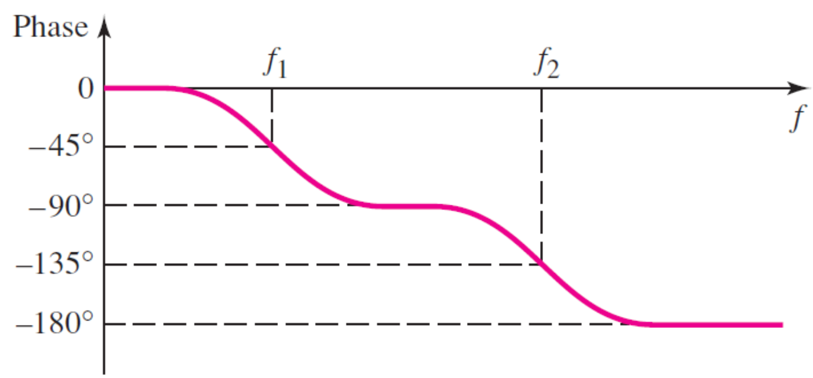
3. Setting $s = j(2\pi f)$, (2.) can be rewritten to the form: $A_i(f) = \dfrac{A_{io}}{\left(1 + j\dfrac{f}{f_1}\right)\left(1 + j\dfrac{f}{f_2}\right)}$

where $f_1 = 1/2\pi R_{\pi 1} C_1$ and $f_2 = 1/2\pi (R_{L1}||R_{\pi 2}) C_2$ are the corner frequencies of the first and the second pole.

4. Finally, represent the complex function in the form of its absolute value and phase (i.e., polar form), and draw the corresponding Bode plots:

$$A_i = \frac{A_{io}}{\sqrt{1 + (f/f_1)^2}\sqrt{1 + (f/f_1)^2}} \angle - \left[\tan^{-1}\left(\frac{f}{f_1}\right) + \tan^{-1}\left(\frac{f}{f_2}\right)\right]$$



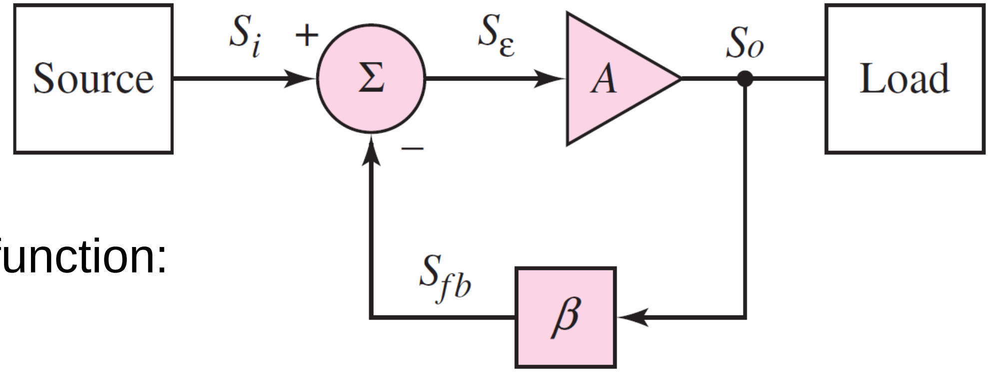


The output current is in phase with the input current at low frequency. At high frequencies, the output current becomes 180 degrees out of phase with respect to the input current.

Part 2: Stability of the Feedback Circuits

The Stability Problem

At some frequencies, the subtraction may become addition – the negative feedback becomes positive, resulting in instability.



Recall, the ideal closed-loop transfer function:

$$A_f = \frac{S_o}{S_i} = \frac{A}{1 + \beta A}$$

In practice, the open-loop gain is a function of the individual transistor parameters, including capacitance; therefore, it is also a function of frequency:

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)} = \frac{A(s)}{1 + T(s)},$$

where $T(s) = \beta A(s)$ is the loop gain and $s = j\omega$ is the complex frequency.

Now, the closed-loop gain function can be presented as:

$$A_f(j\omega) = \frac{A(j\omega)}{1 + |T(j\omega)|\angle\phi}$$

What will happen to the closed-loop gain if $|T(j\omega)| = 1$ and $\phi = 180°$?: $A_f \to \infty$

To study the stability of feedback circuits, we must analyze the frequency response of the loop gain factor $T(j\omega)$.

Two-stage (pole) feedback amplifier: Nyquist plot

Let us consider the two-stage amplifier studied previously with a feedback loop:

1. Its loop gain can be found as follows:

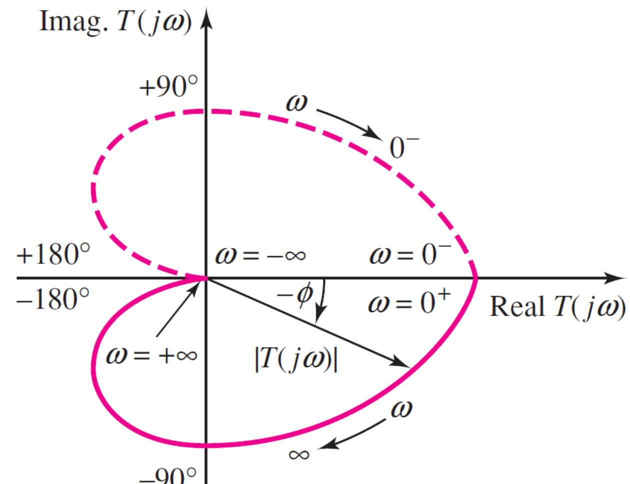
$$T(j\omega) = \beta A_i = \frac{\beta A_{io}}{\left(1 + j\dfrac{\omega}{\omega_1}\right)\left(1 + j\dfrac{\omega}{\omega_2}\right)}$$

2. In polar form notation it will look:

$$T(j\omega) = \frac{\beta A_{io}}{\sqrt{1 + \left(\dfrac{\omega}{\omega_1}\right)^2}\sqrt{1 + \left(\dfrac{\omega}{\omega_2}\right)^2}} \angle - \left[\tan^{-1}\left(\frac{\omega}{\omega_1}\right) + \tan^{-1}\left(\frac{\omega}{\omega_2}\right)\right]$$

3. Now, the Nyquist plot (polar plot of the loop gain factor $T(j\omega)$) can be drawn:

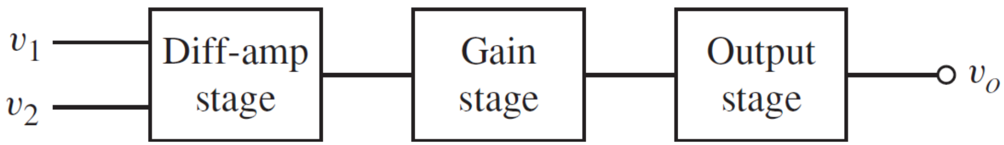
- The Nyquist diagram is a plot of the real and imaginary components of *T (jω)* as the frequency *ω* varies from minus infinity to plus infinity.



The loop gain never becomes -1; hence, the considered two-stage feedback amplifier is conceptually stable.

Nyquist Stability Criterion

In practice, op-amps are three-stage (poles) with a feedback loop.



1. Its loop gain will take the form:

$$T(j\omega) = \beta A_i = \frac{\beta A_{io}}{\left(1 + j\dfrac{\omega}{\omega_1}\right)\left(1 + j\dfrac{\omega}{\omega_2}\right)\left(1 + j\dfrac{\omega}{\omega_3}\right)}$$

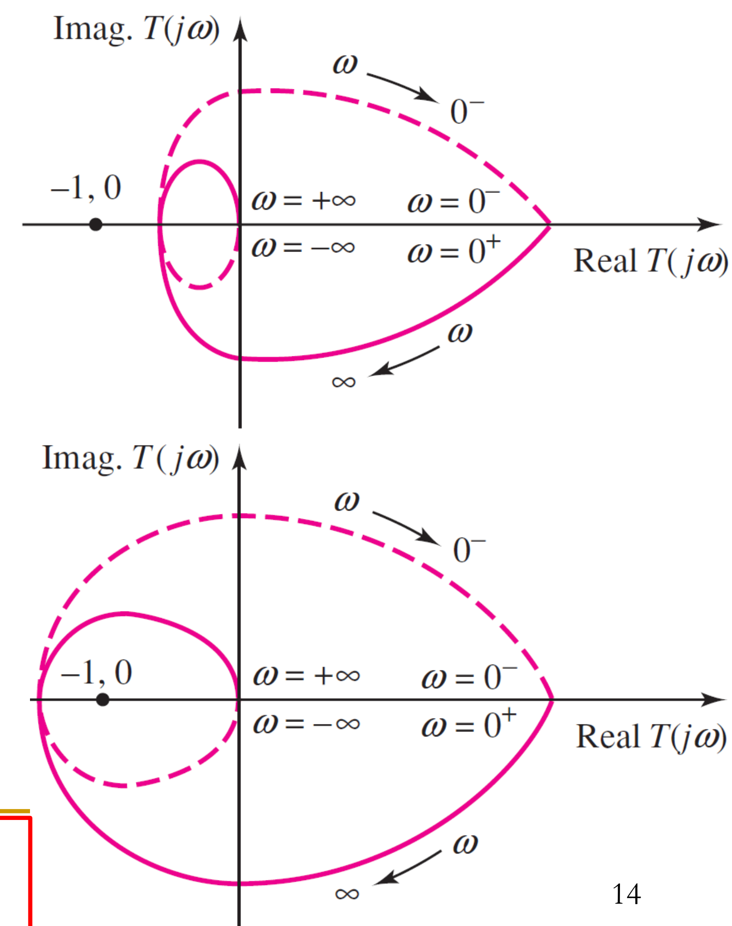
2. In polar form it will look:

$$T(j\omega) = \frac{\beta A_{io}}{\sqrt{1 + \left(\dfrac{\omega}{\omega_1}\right)^2}\sqrt{1 + \left(\dfrac{\omega}{\omega_2}\right)^2}\sqrt{1 + \left(\dfrac{\omega}{\omega_3}\right)^2}} \angle\phi,$$

where

$$\phi = -\left[\tan^{-1}\left(\frac{\omega}{\omega_1}\right) + \tan^{-1}\left(\frac{\omega}{\omega_2}\right) + \tan^{-1}\left(\frac{\omega}{\omega_3}\right)\right]$$

3. Draw the Nyquist plot (two distinct realizations):

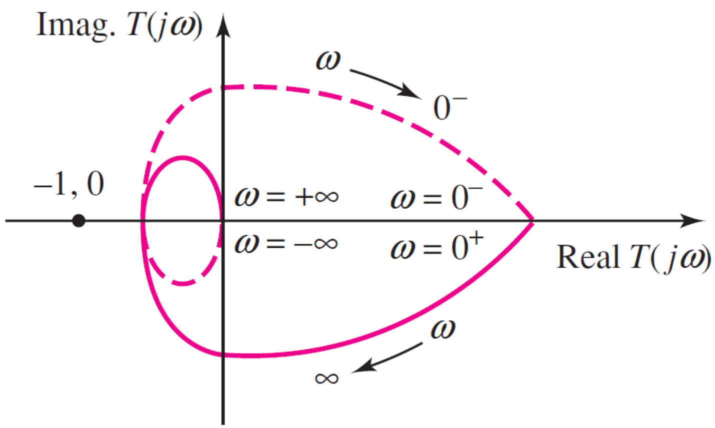


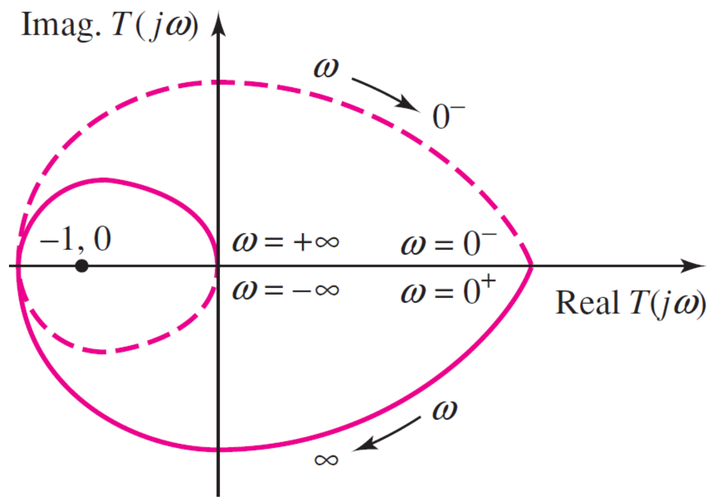
If the Nyquist plot encircles or goes through the point (−1, 0), the amplifier is unstable.

Nyquist Stability Criterion

Test for stability (without a need to draw Nyquist plot):

- If $|T\ (j\omega)| \geq 1$ at the frequency at which the phase is −180 degrees, then the amplifier is unstable.





This simple test allows us to use the Bode plots (considered previously), instead of explicitly constructing the Nyquist diagram.

Example

Given the loop gain function: $T(jf) = \dfrac{\beta(100)}{\left(1 + j\dfrac{f}{10^5}\right)^3}$

Determine the stability of an amplifier for $\beta = 0.2$ and $\beta = 0.02$.

Solution:

1. Rewrite the loop gain in the polar form $T(jf) = |T(jf)|\angle\phi$:

$$T(jf) = \frac{\beta(100)}{\left[\sqrt{1 + \left(\frac{f}{10^5}\right)^2}\right]^3} \angle -3\tan^{-1}\left(\frac{f}{10^5}\right)$$

2. Find the frequency f_{180} at which the phase $\phi = -180° = -\pi$:

$$-3\tan^{-1}\left(\frac{f_{180}}{10^5}\right) = -\pi \quad \Rightarrow \quad f_{180} = 10^5 \cdot \tan\left(\frac{\pi}{3}\right) = 1.73 \cdot 10^5 \text{ Hz}$$

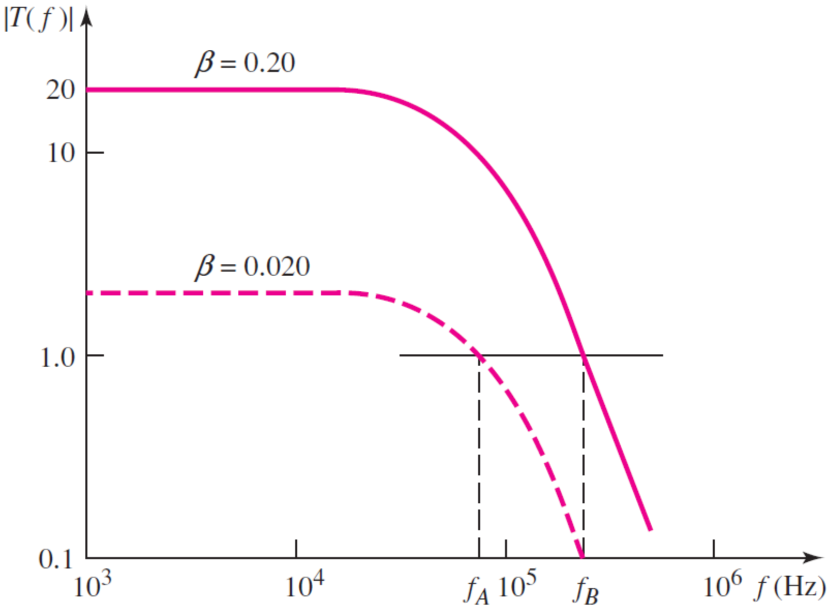
3. Now, the magnitude of the loop gain at the phase of $-180°$ can be found:

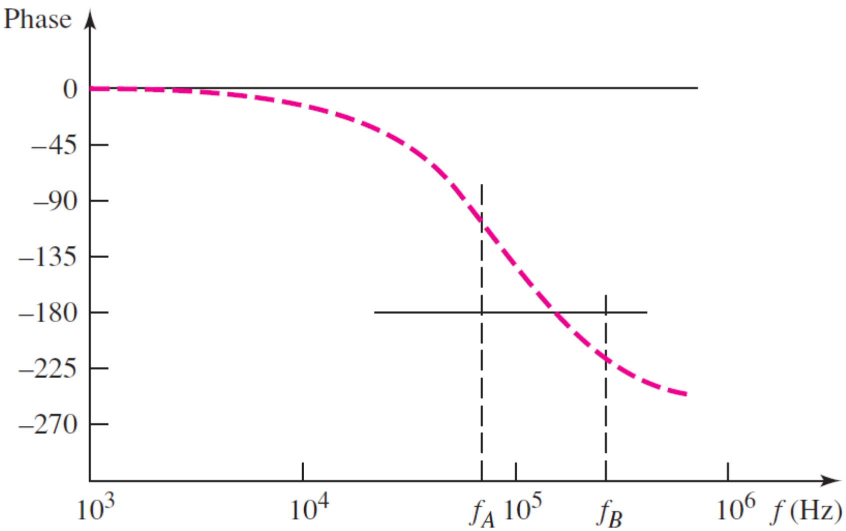
$$|T(f_{180})|_{\beta=0.2} = \frac{0.2(100)}{\left[\sqrt{1 + (1.73)^2}\right]^3} = 2.5 \quad \text{and} \quad |T(f_{180})|_{\beta=0.02} = 0.25$$

$\boxed{\text{The system is unstable for } \beta = 0.20 \text{ and stable for } \beta = 0.02.}$

Bode plot stability analysis

Effectively, Bode plots represent the same data as a Nyquist plot, but in a different way; therefore, it can also be used for stability analysis.
Consider the Bode plots of the loop gain from the previous Exercise:

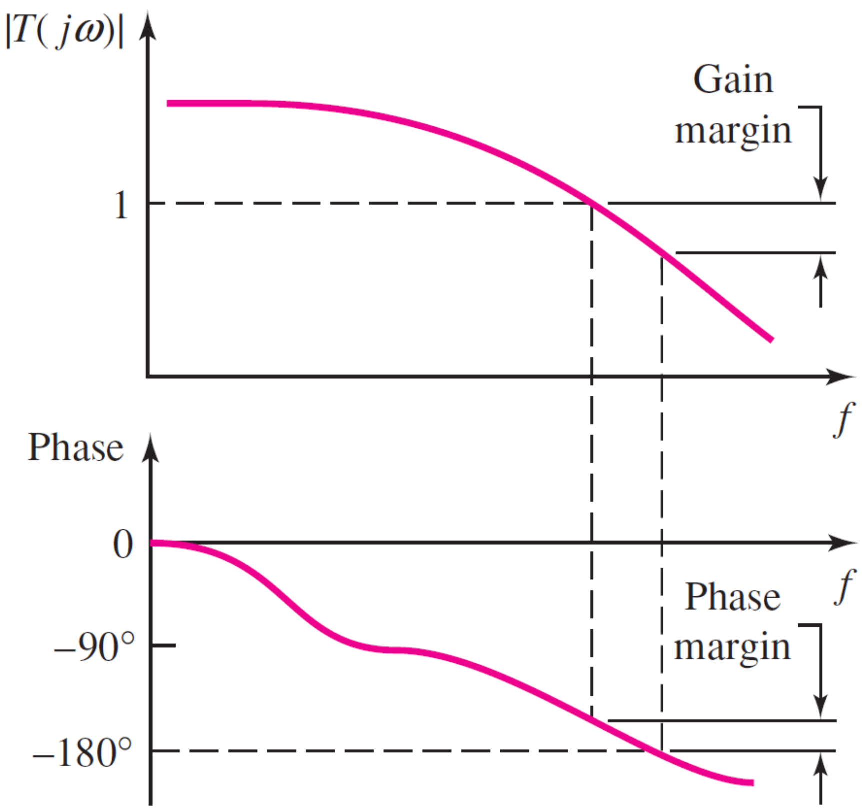




- For $\beta = 0.02$, we see that $|\phi| < 180$ at the frequency when $|T(f)| = 1$ – stable;
- For $\beta = 0.2$, we see that $|\phi| > 180$ at the frequency when $|T(f)| = 1$ – unstable;

Phase and Gain (Stability) Margins

Using Bode plots or Nyquist diagram we can also determine the degree of stability of a feedback amplifier:



So far we know:
If $|\phi(f)| < 180$ when $|T(f)| = 1$ the system is stable. In this case, the stability margin is determined by:

- **Phase margin**: the difference between the phase angle at $f_{|T(f)|=1}$ and -180 degrees:
  $$PM = \phi_{|T(f)|=1} - (-180)$$

- **Gain margin:** the difference between the gain magnitude at -180 degrees and one (typically in dB) :
  $$GM = -20\log|T(f_{180})|$$

The phase margin indicates how much the loop gain can increase and still maintain stability. A typical desired phase margin is in the range of 45 to 60°

Example

Given the loop gain function: $T(f) = \dfrac{\beta(1000)}{\left(1 + j\dfrac{f}{10^3}\right)\left(1 + j\dfrac{f}{5 \cdot 10^4}\right)\left(1 + j\dfrac{f}{10^6}\right)}$

Determine the value of β that yields a phase margin of 45 degrees and the corresponding low-frequency closed-loop gain.

Solution:

1. $PM = 45°$ implies that the phase at $|T(f)| = 1$ is at: $\phi = -135°$.

2. If you recall Bode phase plot of the two-pole amplifier you will see that $\phi = -135°$ corresponds to: $f = 5 \cdot 10^4$ Hz.

3. From the loop gain magnitude function find β for $|T(f)| = 1$:

$$\beta = \frac{|T(f)|}{A_{io}} \sqrt{1 + \left(\frac{f}{f_1}\right)^2} \sqrt{1 + \left(\frac{f}{f_2}\right)^2} \sqrt{1 + \left(\frac{f}{f_3}\right)^2} = 0.0708$$

4. The corresponding closed-loop gain: $A_{f0} = \dfrac{A_o}{1 + \beta A_o} = 13.9$

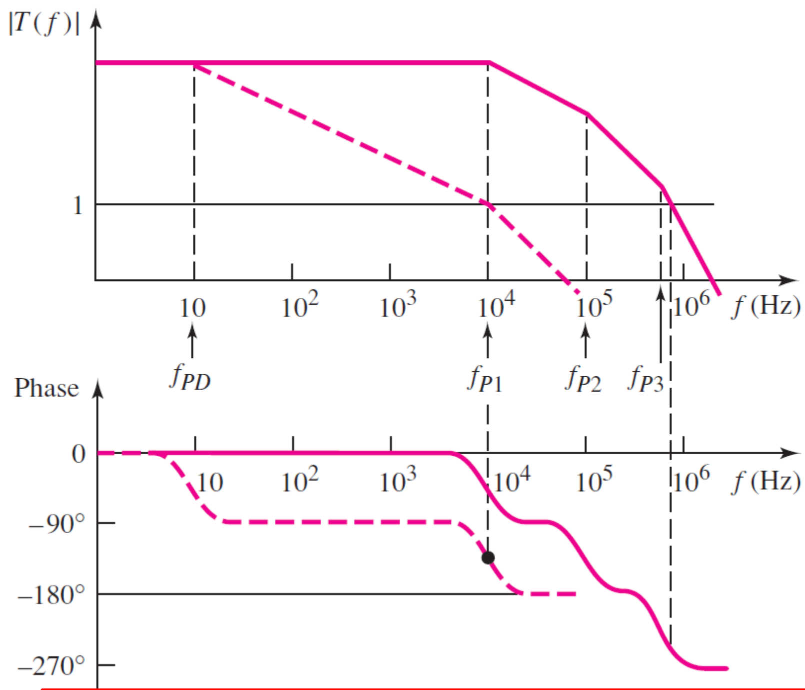
For $\beta \approx 0.07$ the loop gain magnitude is less than one at $f > 5 \cdot 10^4$ Hz. If $f < 5 \cdot 10^4$ Hz, the phase of the loop gain is $|\phi| < 135°$. These imply that the system is stable.

Part 3: Frequency Compensation

Frequency compensation with dominant pole

The simplest method to stabilize a system is to introduce a new pole in the loop gain function for which the loop gain $|T(f)| = 1$ occurs when $|\phi| < 180°$.

Consider the Bode plots of a three-pole loop gain magnitude and phase:



- Original system frequency response is represented by the solid line:
  - The magnitude of the loop gain $|T(f)| = 1$ when $\phi \approx -270°$ **– the system is unstable.**
- After the introduction of a new pole with a very low corner frequency f_{PD} the new frequency response will be shifted left as shown by dashed line:
  - The magnitude of the loop gain $|T(f)| = 1$ when $|\phi| < 180°$ **– the system is stable.**

Since the pole is introduced at a low frequency and since it dominates the frequency response, it is called a dominant pole.

Example

Given the loop gain function: $$T(f) = \frac{1000}{\left(1 + j\frac{f}{10^4}\right)\left(1 + j\frac{f}{5\cdot 10^6}\right)\left(1 + j\frac{f}{10^8}\right)}$$

Determine the dominant pole required to stabilize the feedback system, such that the phase margin is at least 45 degrees.

Solution:

1. By inserting a dominant pole, the loop gain function will look:

$$T(f) = \frac{1000}{\left(1 + j\frac{f}{f_{PD}}\right)\left(1 + j\frac{f}{10^4}\right)\left(1 + j\frac{f}{5\cdot 10^6}\right)\left(1 + j\frac{f}{10^8}\right)}$$

2. As per the phase Bode plot of the four-pole system (dashed line on the previous slide), a phase $\phi = -135°$ will be determine by the second pole corner frequency $f \approx 10^4$ Hz (given that $f_{PD} \ll 10^4$ Hz):

$$|T(f_{135})| = 1 = \frac{1000}{\sqrt{1 + \left(\frac{10^4}{f_{PD}}\right)^2}\sqrt{1 + \left(\frac{10^4}{10^4}\right)^2}\sqrt{1 + \left(\frac{10^4}{10^6}\right)^2}\sqrt{1 + \left(\frac{10^4}{10^8}\right)^2}}$$

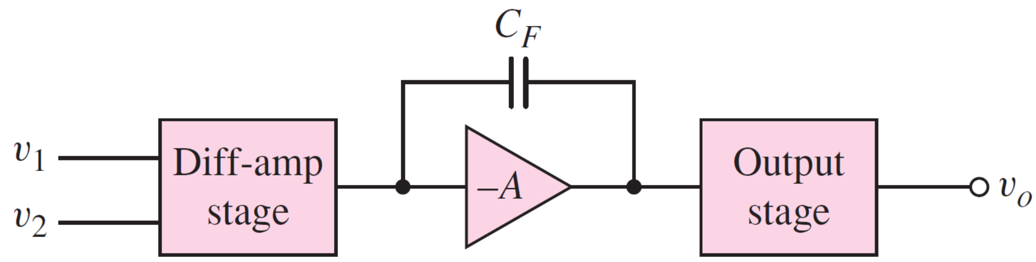
3. Solving for the dominant pole frequency: $f_{PD} = 14.14\ \mathrm{Hz}$

With high-gain amplifiers, the dominant pole must be at a very low frequency to ensure stability of the feedback circuit.

Miller compensation

Instead of adding an extra dominant pole to obtain a stable system, Miller compensation implies moving the first pole f_{P1} (whatever stage it belongs to) to a low frequency.

Consider the three-stage amplifier with a **compensation capacitor** in the 2nd stage:



Miller compensation uses Miller effect for a benefit:

- The effective Miller input capacitance to a transistor amplifier is a feedback capacitance multiplied by the magnitude of the gain of the amplifier stage plus one:

$$C_M = C_F(1 + A)$$

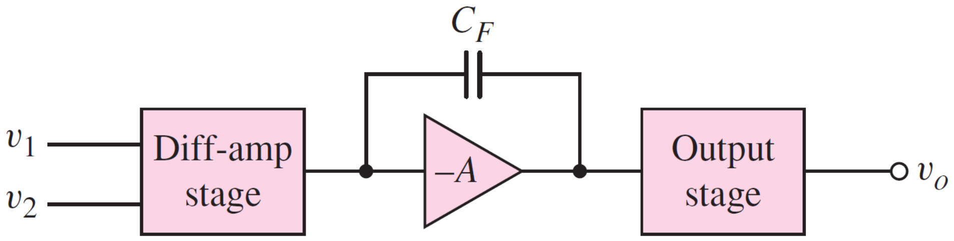
- Knowing this, we can adjust the corner frequency as we wish:

$$f_{P1} = \frac{1}{2\pi R_2 C_M},$$

where R_2 is the effective resistance between the amplifier input node and the ground ($R_2 = R_{1o}||R_{2i}$).

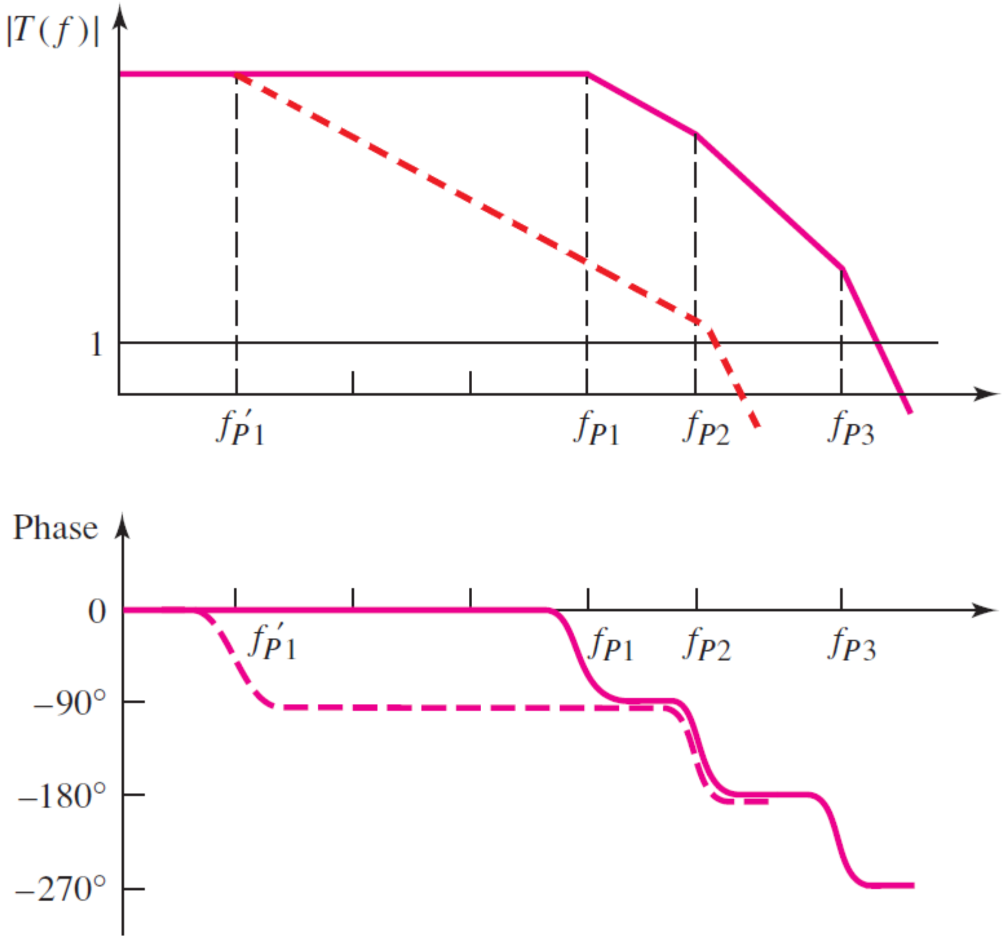
Exercise

Consider a gain stage with an amplification 10^3 and input-ground resistance $5 \cdot 10^5$ Ohm. Determine the required feedback capacitor to move the dominant-pole frequency to 10 Hz.



Solution:

Miller compensation (concluding remarks)



Previously, we assumed that moving the pole f_{P1} does not affect other poles; however, it is not true:

- A detailed analysis shows that pole $\boldsymbol{f_{P2}}$ does not remain constant; it increases. This is called **pole-splitting**.

What does this fact mean to the stability of a system?

In this lecture we learned:

- Frequency response analysis involves using analytical tool (i.e., Bode plots, Nyquist diagram) to study a system response in terms of gain magnitude and phase to periodic signals of various frequencies;

- Feedback systems are prone to instability, which occurs when a loop gain magnitude does not drop below one while phase is grater or equal to 180 degree (ie, Nyquist stability criterion);

- A degree to which a system is stable can be determined from the system frequency response characteristics (ie, phase and gain margins);

- A system stability can be improved or even achieved via frequency compensation techniques (ie, dominant pole and Miller compensation), which usually occurs at a cost of reduced gain.

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The End