EEE210:
Energy Conversion and Power Systems

Basic principles in power system analysis-Part II

Weitao Yao

Email: Weitao.Yao@xjtlu.edu.cn

Office: SC348

XJTLU | ELECTRICAL & ELECTRONIC ENGINEERING

Xi'an Jiaotong-Liverpool University 西交利物浦大学

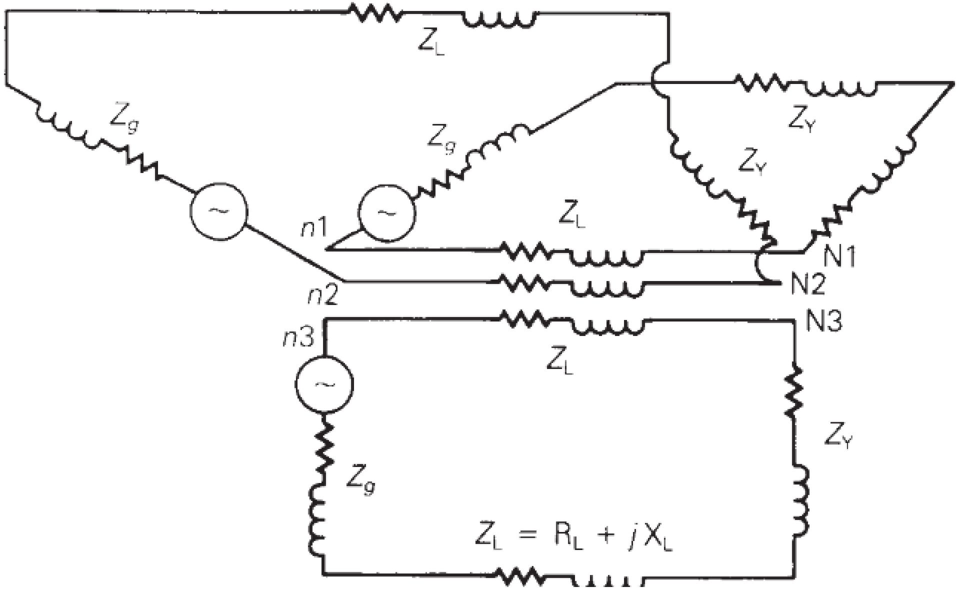
Highlights

- Phasors
- Complex power
- **Power in balanced three-phase circuits**

Balanced Three-Phase Circuits

What is the structure of the three-phase system?

Three single-phase systems?

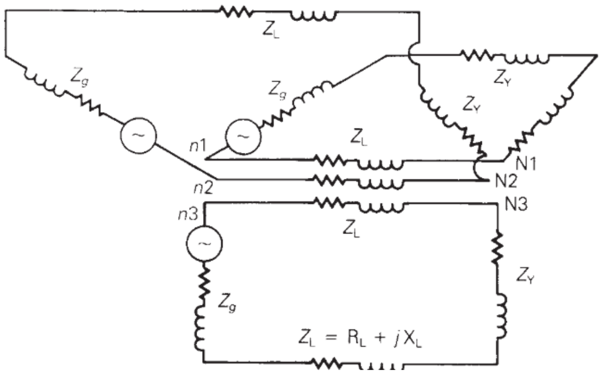


(1) a generator represented by a voltage source and a generator impedance Zg;
(2) a forward and return conductor represented by two series line impedances ZL;
(3) a load represented by an impedance ZY.

Balanced Three-Phase Circuits

What is the structure of the three-phase system?

Three single-phase systems? Feasible?

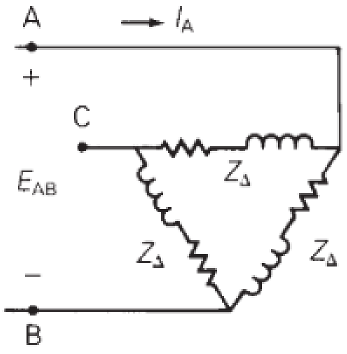


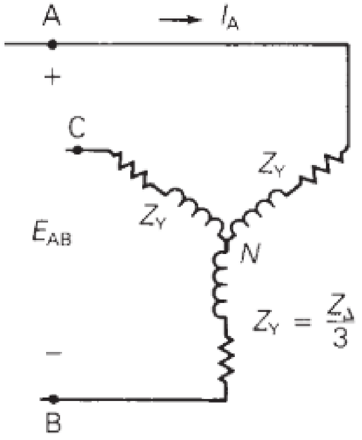
Each separate single-phase system requires:

Both the forward and return conductors have a current capacity (or ampacity) equal to or greater than the load current

First drawbacks

Any solution to prevent energy this assets waste?

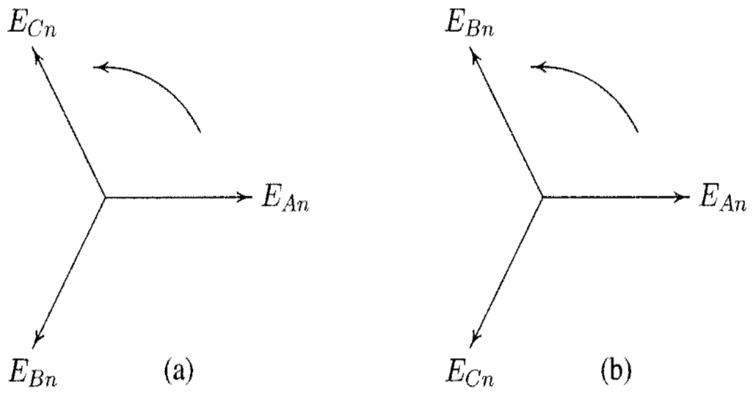




Balanced Three-Phase Circuits

Basics: balanced source

At the generation station, three sinusoidal voltages are generated having the same amplitude but displaced in phase by 120 degree.



Positive phase sequence   Negative phase sequence

For positive phase sequence:

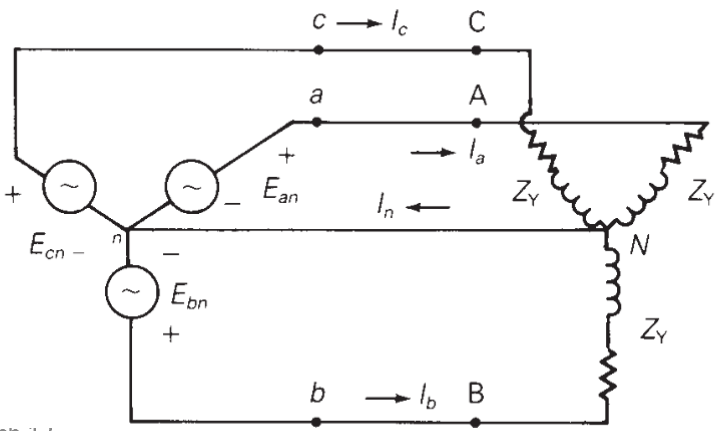
$$E_{An} = |E|\angle 0°$$
$$E_{Bn} = |E|\angle -120°$$
$$E_{Cn} = |E|\angle -240°$$

Please write the generated voltage for negative phase sequence after class.

Balanced Three-Phase Circuits

The star connected loads (balanced-Y connections)

- A three-phase Y-connected (or ‘‘wye-connected’’) voltage source feeding a balanced-Y-connected load.
- For a Y connection, the neutrals of each phase are connected.

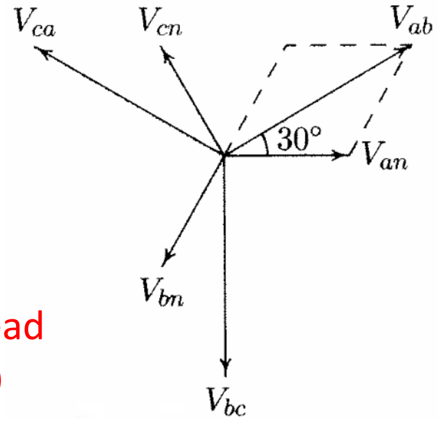


$$V_{an} = |V_P|\angle 0°$$
$$V_{bn} = |V_P|\angle -120°$$
$$V_{cn} = |V_P|\angle -240°$$

V_P is the magnitude of phase voltage.

Thus, the voltage between any two phases, for example, A and B (line voltage or phase-to phase voltage)

$$V_{ab} = V_{an} - V_{bn}$$
$$= |V_P|(1\angle 0° - 1\angle 120°)$$
$$= \sqrt{3}|V_P|\angle 30°$$



the line-to-line voltages are $\sqrt{3}$ times the phase voltages and lead by 30° (for positive sequence)

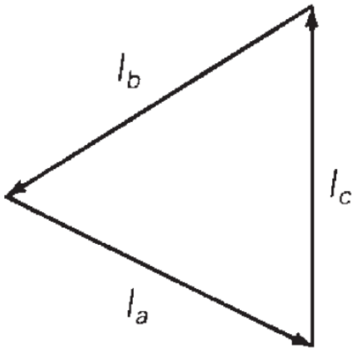
Balanced Three-Phase Circuits

The star connected loads (balanced-Y connections)

The currents in lines are also phase currents:

$$I_L = I_P$$
$$I_{\mathrm{n}} = I_c + I_b + I_c$$

*for balanced currents, I_{n} is always 0



$$V_{\mathrm{an}} = |V_P|\angle 0°$$
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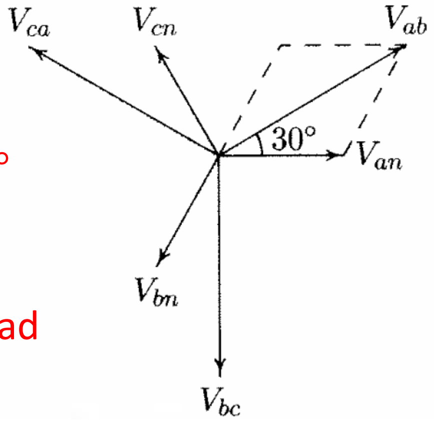
V_P is the magnitude of phase voltage.

Thus, the voltage between any two phases, for example, A and B (line voltage or phase-to phase voltage)

$$V_{ab} = V_{an} - V_{bn}$$
$$= |V_P|(1\angle 0° - 1\angle 120°)$$
$$= \sqrt{3}|V_P|\angle 30° = \sqrt{3}V_{an}\angle 30°$$

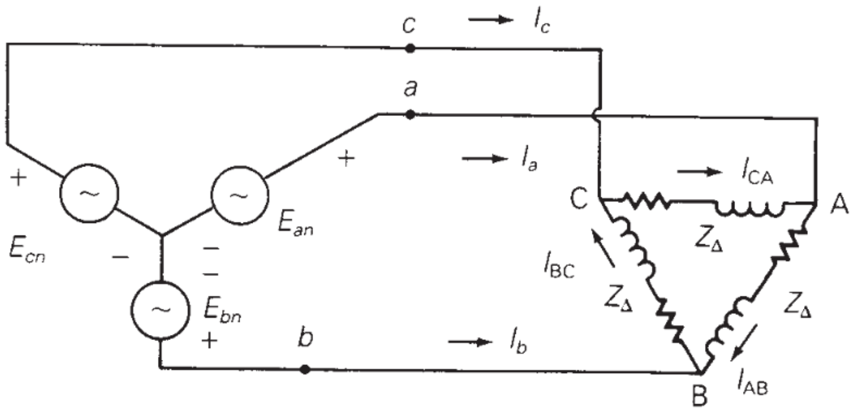


the line-to-line voltages are $\sqrt{3}$ times the phase voltages and lead by 30° (for positive sequence)

Balanced Three-Phase Circuits

The delta connected loads

- Eequal load impedances ZD are connected in a triangle whose vertices form the buses, labeled A, B, and C
- The D connection does not have a neutral bus.



Different from Y connections:

$$V_L = V_P$$

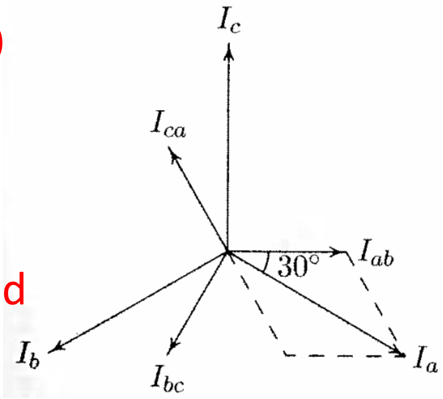
If

$$I_{ab} = |I_P|\angle 0°$$
$$I_{bc} = |I_P|\angle -120°$$
$$I_{ca} = |I_P|\angle -240°$$

The line currents is

$$I_a = I_{ab} - I_{ca}$$
$$= |I_P|(1\angle 0° - 1\angle -240°)$$
$$= \sqrt{3}|I_P|\angle -30°$$
$$= \sqrt{3}I_{ab}\angle -30°$$

The line currents are $\sqrt{3}$ times the phase currents and lag by $30°$ (for positive sequence)

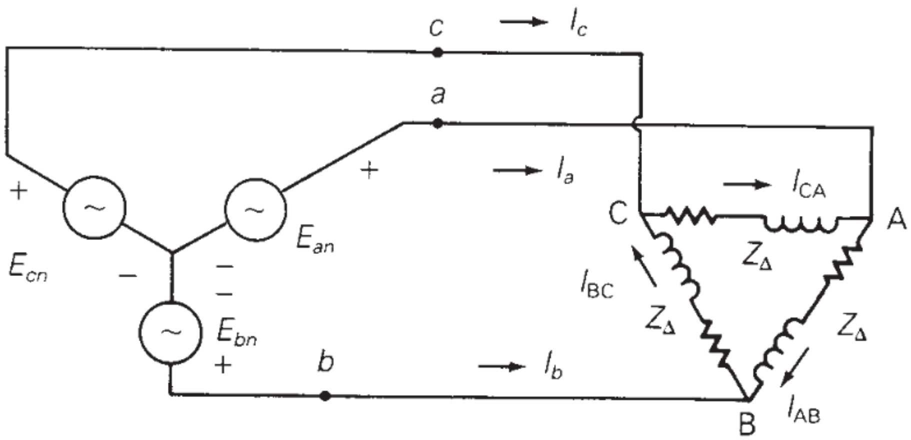


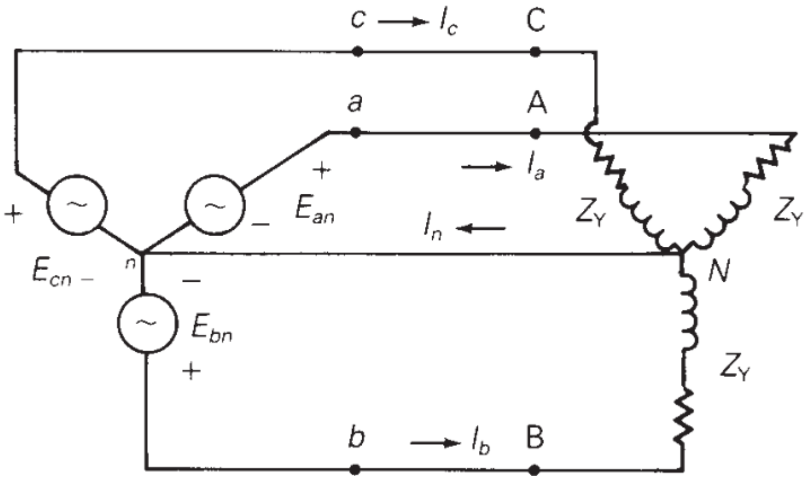
Balanced Three-Phase Circuits

Simplified analysis

into single phase

Could we simplify the analysis of a balanced three phase circuit?





Balanced Three-Phase Circuits

Simplified analysis– Delta to Star conversion

For delta connection:

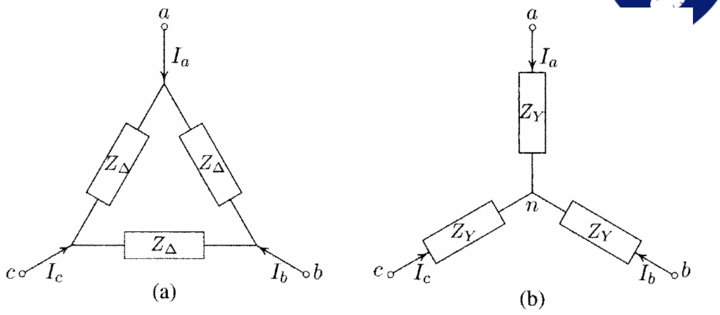
$$I_a = \sqrt{3} I_{ab} \angle -30° = \frac{\sqrt{3} V_{ab} \angle -30°}{Z_\Delta}$$

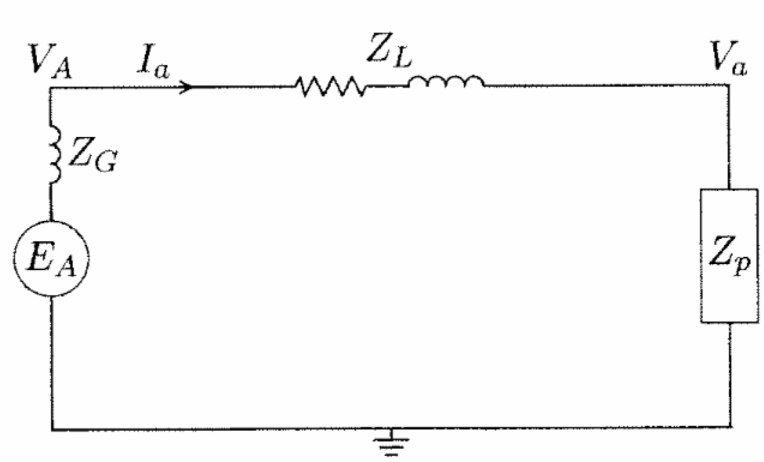
For Y connection:

$$I_a = \frac{V_{an}}{Z_Y}; (V_{ab} = \sqrt{3} V_{an} \angle 30°)$$

$$I_a = \frac{V_{ab} \angle -30°}{\sqrt{3} Z_Y}$$

$$Z_Y = \frac{Z_\Delta}{3}$$





Balanced Three-Phase Circuits

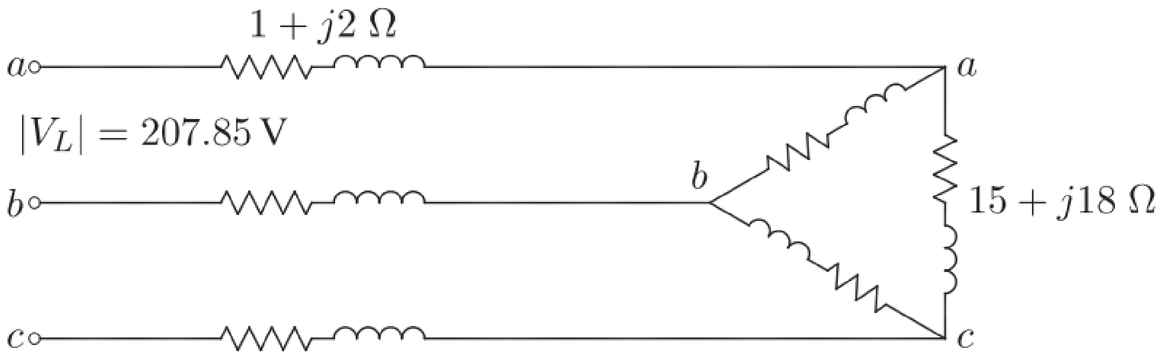
Example questions:

A balanced delta connected load of15 + j18 Ω per phase is connected at the end of a three-phase line. The line impedance is 1+j2 Ω per phase. The line is supplied from a three-phase source with a line-to-line voltage of 207.85 V rms. Taking Van as reference, determine the following:

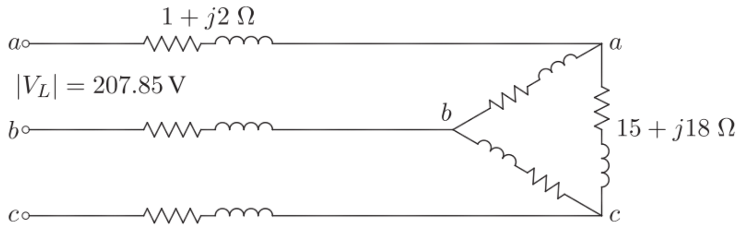
(a) Current in phase a.

(b) Total complex power supplied from the source.

(c) Magnitude of the line-to-line voltage at the load terminal



Balanced Three-Phase Circu

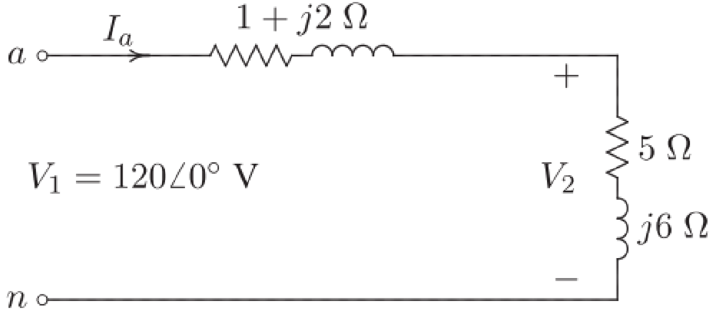


Example questions:

(a) Current in phase a.

1. transforming the delta connected load to an equivalent Y-connected load:

$$Z_P = \frac{z_\Delta}{3} = \frac{15+j18}{3} = 5 + j6\ \Omega$$



Assume the voltage on phase a as the base (with 0 phase angle)

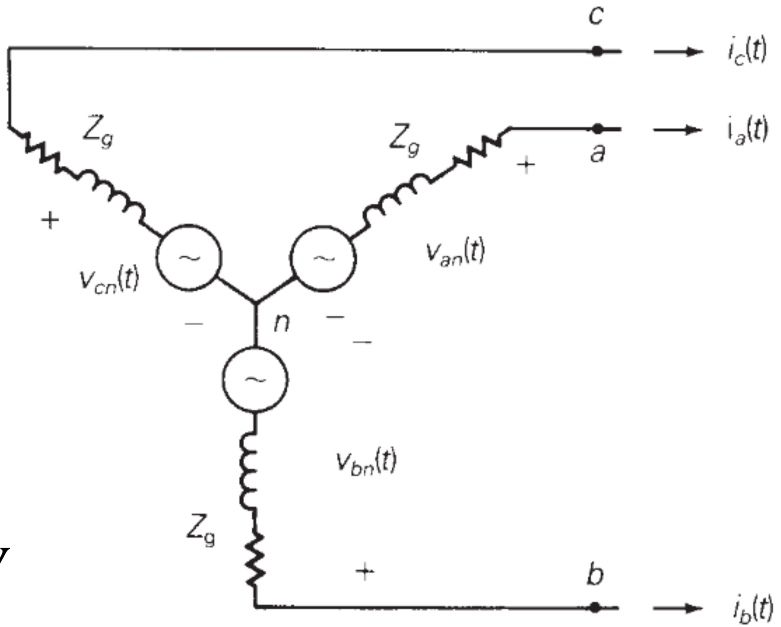
$$|V_{an}| = \frac{|V_{LL}|}{\sqrt{3}} = \frac{207.85}{\sqrt{3}} = 120\ V; Assume\ V_{an}\ has\ a\ phase\ angle\ of\ 0\ degree: V_{an} = 120\angle 0° V$$

$$I_a = \frac{V_{an}}{Z_P} = \frac{120\angle 0°}{6 + j8} = 12\angle -53.13°\ \ A$$

Balanced Three-Phase Circuits

Instantaneous power in balanced three-phase circuits -- generators

Taking phase a as the example: assume that the generator is operating under balanced steady-state conditions:



$$v_{an}(t) = \sqrt{2}V_{LN}\cos(wt + \varphi) \quad V$$
$$i_a(t) = \sqrt{2}I_L\cos(wt + \beta) \quad A$$

$$p_a(t) = v_{an}(t)i_a(t)$$
$$= 2V_{LN}I_L\cos(wt + \varphi)\cos(wt + \beta)$$
$$= V_{LN}I_L\cos(\varphi - \beta) + V_{LN}I_L\cos(2wt + \varphi + \beta) \quad W$$

Second drawbacks

Double frequency harmonics for single phase

Balanced Three-Phase Circuits

Complex power in balanced three-phase circuits -- generators

Phase a: $p_a(t) = V_{LN}I_L\cos(\varphi-\beta) + V_{LN}I_L\cos(2wt+\varphi+\beta)\quad W$

Phase b: $p_b(t) = V_{LN}I_L\cos(\varphi-\beta) + V_{LN}I_L\cos(2wt+\varphi+\beta-240)\quad W$

Phase c: $p_c(t) = V_{LN}I_L\cos(\varphi-\beta) + V_{LN}I_L\cos(2wt+\varphi+\beta+240)\quad W$

The total power:

Zero

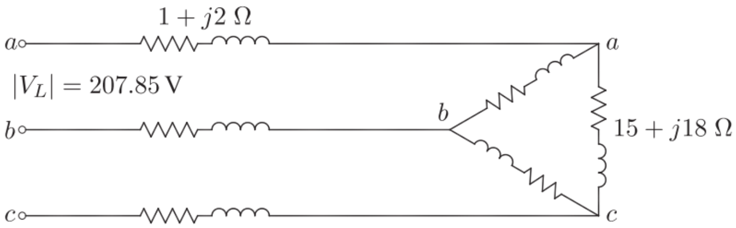
$$P_{3\emptyset}(t) = p_a(t) + p_b(t) + p_c(t)$$
$$= 3V_{LN}I_L\cos(\varphi-\beta) + V_{LN}I_L[\cos(2wt+\varphi+\beta) + \cos(2wt+\varphi+\beta-240)$$
$$+ \cos(2wt+\varphi+\beta+240)\,]$$

Considering $V_{LN} = V_{LL}/\sqrt{3}$

A constant value

$$P_{3\emptyset} = \sqrt{3}V_{LL}I_L\cos(\varphi-\beta)\quad W$$
$$Q_{3\emptyset} = \sqrt{3}V_{LL}I_L\sin(\varphi-\beta)\quad \text{var}$$
$$S_{3\emptyset} = P_{3\emptyset} + jQ_{3\emptyset} = 3V_{LN}I_L'(phasor\ representations)$$

Balanced Three-Phase Circu



Example questions:

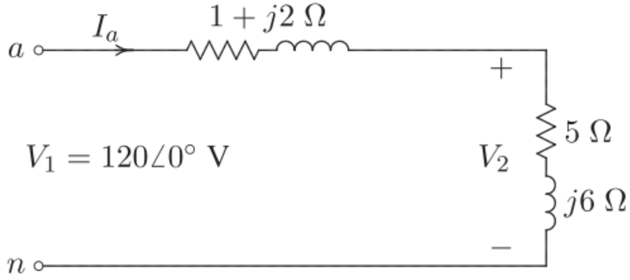
(b) Total complex power supplied from the source

$$S = 3V_{an} * I_a' = 3 * 120\angle 0° * 12\angle 53.13° = 4320\angle 53.13°\ \ VA = 2592\,W + j3456\,Var$$

(c) *Magnitude of the line-to-line voltage at the load terminal*

$$V_2 = V_1 - V_{drop} = V_1 - I_a * Z_{line,per-phase} = 120\angle 0° - 12\angle 53.13° * (1 + j2)$$
$$= 93.72\angle - 2.93°\ V$$

Thus, the magnitude of the line-to-line voltage at the load terminal:



$$|V_{LL}| = \sqrt{3}|V_2| = \sqrt{3} * 93.72 = 162.3\ V$$

Balanced Three-Phase Circuits

Choice between star and delta connections:

The choice between star and delta connections depends on several factors such as the type of load, power requirements, and cost.

- Star connections are commonly used for low and medium power loads, while delta connections are preferred for high-power loads. Star connections provide a neutral point, which is essential for single-phase loads and can also provide some degree of protection against ground faults.

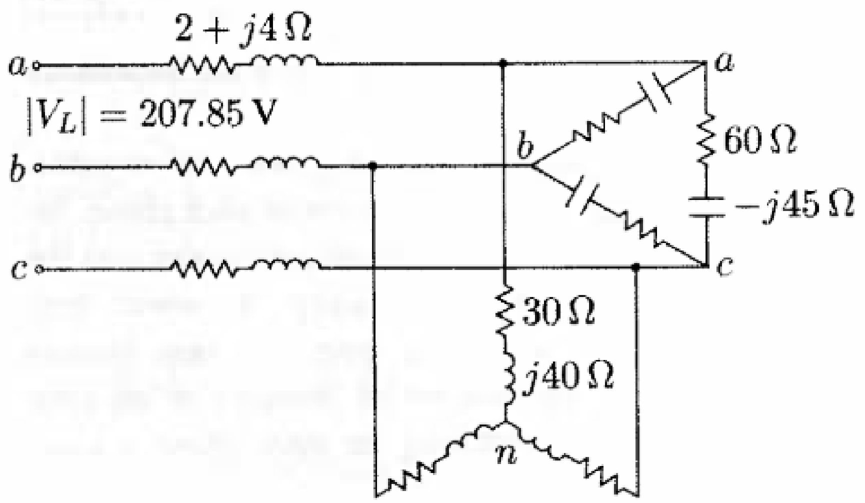
- Delta connections are more suitable for balanced loads and can handle higher currents with lower line voltages.

Balanced Three-Phase Circuits

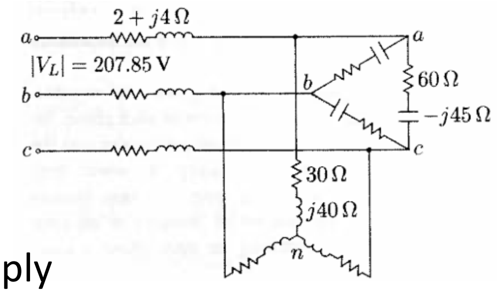
Practice:

A three-phase line has an impedance of 2+j4 Ω. The power supply is Y connected.
The line feeds two balanced three-phase loads that are connected in parallel. The first load is Y-connected and has an impedance of 30+j40 Ω per phase. The second load is a delta connected and has an impedance of 60-j45 Ω between two phases. The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85V. Taking the phase voltage Va as reference, determine:

- The current, real power and reactive power drawn from the supply
- The line voltage at the combined loads
- The current per phase in each load
- The total real and reactive powers in each load and the line



Balanced Three-Phase Circuits



Practice:

1. Determine the current, real power and reactive power drawn from the supply

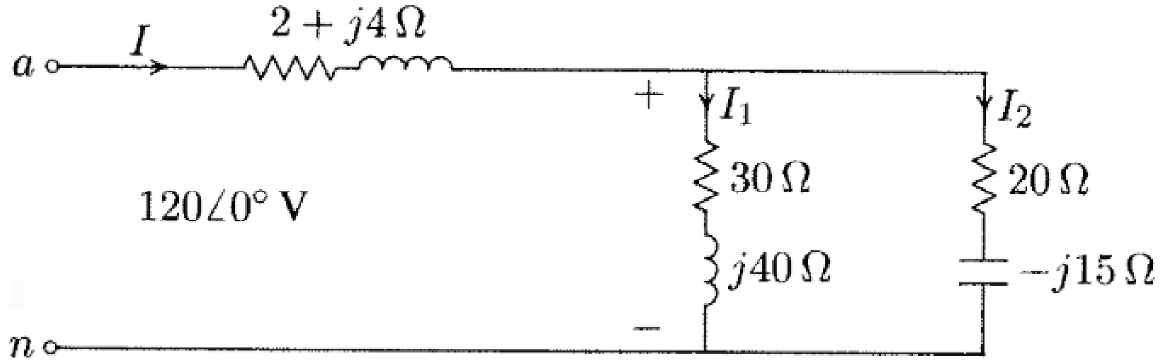
- Convert the delta connected load into Y connected load:

$$Z_{2Y} = \frac{Z_{2\text{d}}}{3} = \frac{60 - j45}{3} = 20 - j15 \;\; \Omega$$

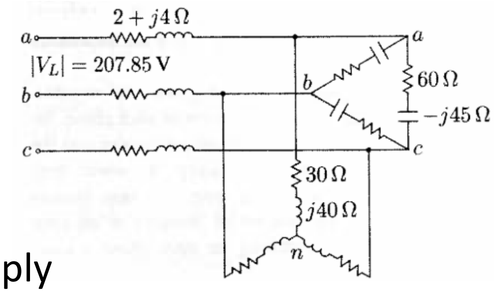
- The phase (line to neutral) voltage at the supply side is

$$V_{LN,S} = \frac{V_{LL,S}}{\sqrt{3}} = \frac{207.85}{\sqrt{3}} = 120\angle 0° \, V$$

- Assume the angle of phase voltage at the supply side is 0, the single-phase equivalent circuit is shown as follows:



Balanced Three-Phase Circuits



Practice:

1. Determine the current, real power and reactive power drawn from the supply

- The total impedance is:

$$Z_{total} = Z_{line} + Z_1 // Z_{2Y} = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)} = 2 + j4 + 22 - j4 = 24\,\Omega$$

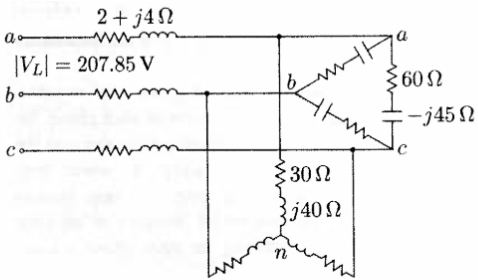
- The current in phase a is:

$$I_{LN,S} == \frac{V_{LN,S}}{Z_{total}} = \frac{120\angle 0°}{24} = 5\angle 0° A$$

The three-phase power supplied is

$$S_{3\emptyset} = 3V_{LN,S} I'_{LN,s} = 3 * 120\angle 0° * 5\angle 0° = 1800\, W$$

Balanced Three-Phase Circuits



Practice:

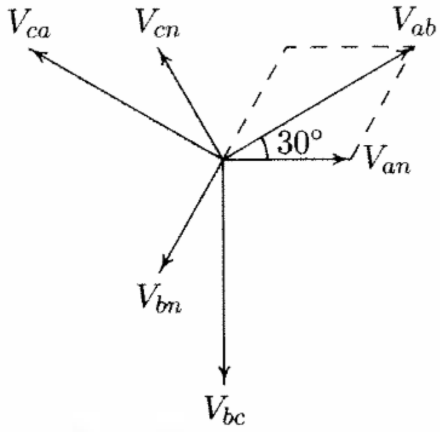
2. Determine the line voltage at the combined loads

- The voltage at phase a at the load terminal is

$$V_{LN,L} = V_{LN,1} - I_{LN,s}Z_{line} = 120\angle 0° - (2 + j4) * 5\angle 0°$$
$$= 110 - j20 = 111.8\angle - 10.3° \ V$$

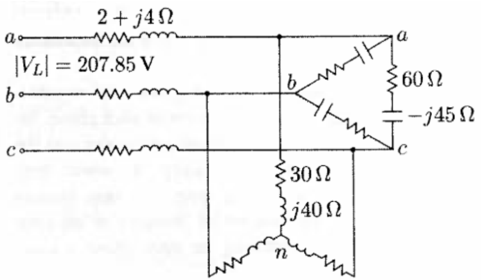
- The line voltage at the load terminal is:

$$V_{ab,L} = \sqrt{3}\angle 30° V_{LN,L} = \sqrt{3}\angle 30° * 111.8\angle - 10.3°$$
$$= 193.64\angle 19.7° \ V$$



So $V_{bc,L}$ & $V_{ca,L}$???

Balanced Three-Phase Circuits



Practice:

3. Determine the current per phase in each load

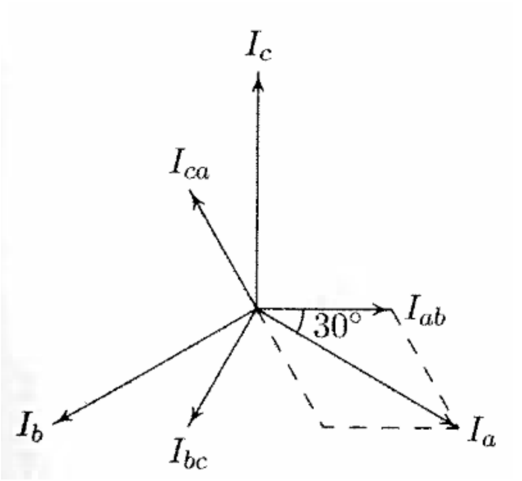
- The current per phase in the Z1 and the equivalent Y of Z2 is

$$I_{LN,1} = \frac{V_{LN,L}}{Z_1} = \frac{110 - j20}{30 + j40} = 1 - j2 = 2.236\angle -63.4° \quad A$$

$$I_{LN,2} = \frac{V_{LN,L}}{Z_2} = \frac{110 - j20}{20 - j15} = 4 + j2 = 4.472\angle 26.56° \quad A$$

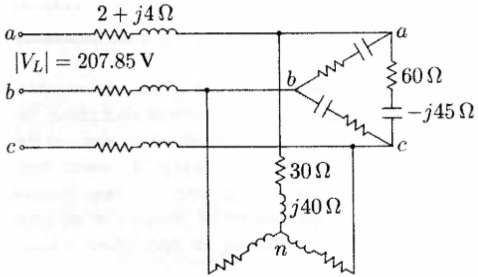
The phase current in the original delta-connected load is given by:

$$I_{ab,2} = \frac{I_{LN,2}}{\sqrt{3}\angle -30°} = \frac{4.472\angle 26.56°}{\sqrt{3}\angle -30°} = 2.582\angle 56.56° \ A$$



So $I_{b,1}$, $I_{c,1}$ and $I_{bc,2}$, $I_{ca,2}$????

Balanced Three-Phase Circuits



Practice:

- 4. Determine the total real and reactive powers in each load and the line

- The three phase power absorbed by each load is:

$$S_1 = 3V_{LN,L}I'_{LN,1} = 3 * (111.8\angle - 10.3°) * (2.236\angle 63.4°) = 450\,W + j600\,var$$

$$S_2 = 3V_{LN,L}I'_{LN,2} = 3 * (111.8\angle - 10.3°) * (4.472\angle - 26.56°) = 1200\,W - j900\,var$$

- The three phase power absorbed by the line is:

$$S_L = 3Z_{line}\left|I_{LN,S}\right|^2 = 3 * (2 + j4) * 5^2 = 150\,W + j300\,var$$

****Power balance

$$S_{3\emptyset} = 1800\,W = S_1 + S_2 + S_L = 450 + j600 + 1200 - j900 + 150 + j300$$

Next Week (Week 6)

In-class Quiz and Essay Writing

Time: 11:10-12:30

Content:

- Part 1: Two Questions (W4-W5 content)
- Part 2: Essay Writing: Smart Grid-Related Title. (Within 400 words)

Form: Open-Book

Submission: Via Learning Mall Before **12:50, 26th March**. Late submissions will not be accepted.

Requirement: Electronic devices are allowed, but communication, WeChat, or social apps are forbidden.

Thanks for your attendance!