EE211 — Assn 2.

Q1 : $V^+ = 15V$, $V^- = -5V$, $I_Q = 0.8mA$

$R_C = 25k\Omega$, $\beta = \infty$, $V_A = \infty$.

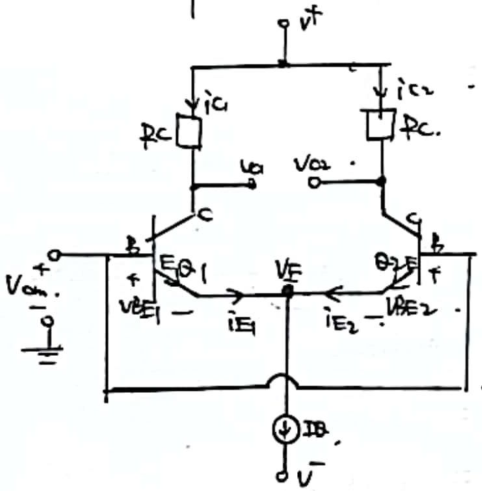
$V_{BE(on)} = 0.7V$, $V_T = 26mV$,

$V_{B1} = 1.001\,V$, $V_{B2} = 0.999\,V$

i. $V_{cm} = \frac{V_{B1}+V_{B2}}{2} = 1V$.

→ $V_E = V_{cm} - V_{BE(on)} = 0.3V$

→ Q-point Circuit shown as below.



→. Assume negligible base current,

→. $i_{C1} \approx i_{E1}$ and $i_{C2} \approx i_{E2}$.

→ Apply KCL at Node (V_E)

→ $i_{E1} + i_{E2} = I_Q$

→ $i_{CQ1} = i_{CQ2} = \frac{I_Q}{2}$.

→ Apply KVL.

→. $V_{CE1} = V^+ - i_{CQ1} R_C - V_E$

$V_{CE2} = V^+ - i_{CQ2} \cdot R_C - V_E$

$i_{CQ1} = i_{CQ2} = \frac{I_Q}{2}$, $V_E = 0.3V$

$I_Q = 0.8mA$, $R_C = 25k\Omega$, $V^+ = 5V$

→. $V_{CE1} = V_{CE2} = 15 - \frac{0.8m}{2} \times 25k - 0.3$

→ $V_{CE1} = V_{CE2} = 4.7V$

→ In Summary

$i_{CQ1} = 0.4mA$

$i_{CQ2} = 0.4mA$

$V_E = 0.3V$

$V_{CE1} = 4.7V$

$V_{CE2} = 4.7V$

ii. $i_{C1} = I_S \exp\left(\frac{V_{BE1}}{V_T}\right)$

$i_{C2} = I_S \cdot \exp\left(\frac{V_{BE2}}{V_T}\right)$

$I_Q = i_{C1} + i_{C2}$

→ $i_{C1} = \frac{I_Q}{1+\exp\left(\frac{-V_d}{V_T}\right)}$

$i_{C2} = \frac{I_Q}{1+\exp\left(\frac{+V_d}{V_T}\right)}$

Where $V_d = V_{BE1} - V_{BE2}$

Apply KVL

→. $V_d = V_{B1} - V_{B2} = V_{BE1} - V_{BE2}$

→ $V_d = 2mV$.

$V_T = 26mV$

$I_Q = 0.8mA$.

$i_{C1} = \frac{I_Q}{1+\exp\left(\frac{-V_d}{V_T}\right)}$

$i_{C2} = \frac{I_Q}{1+\exp\left(\frac{+V_d}{V_T}\right)}$

⇒ $i_{C1} = \frac{0.8m}{1+\exp\left(\frac{-2m}{26m}\right)} = 0.42mA$

$i_{C2} = \frac{0.8m}{1+\exp\left(\frac{2m}{26m}\right)} = 0.38mA$

→ Apply KVL

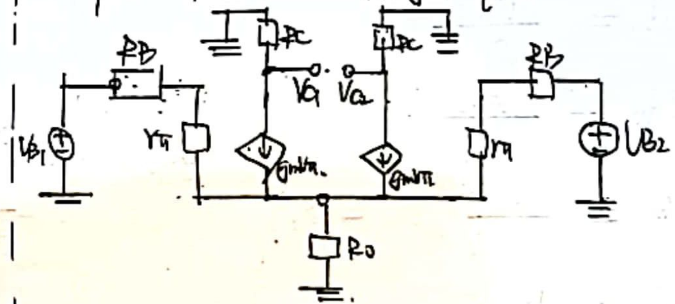
→. $V_{C1} = V^+ - i_{C1} \cdot R_C$; $V_{C2} = V^+ - i_{C2} \cdot R_C$.

→ $V_{C1} = 15 - 0.42m \times 25k = \cancel{4.5}$ 4.62 V

$V_{C2} = 15 - 0.38m \times 25k = \cancel{5.5}$ 5.38 V.

iii.

Step 1 : Draw the Small-signal equivalent circuit.



Note : Replace Current Source with R_O

Voltage Source with V_B and a Resistor R_B.

Step 2: Analysis.

Apply KVL

→ $V_{C1} = -g_m V_{\pi1} \cdot R_C$

$V_{C2} = -g_m V_{\pi2} \cdot R_C$

→ $V_o = V_{C2} - V_{C1}$

→ $V_o = g_m R_C (V_{\pi1} - V_{\pi2}) = \frac{\beta}{r_\pi} R_C (V_{\pi1} - V_{\pi2})$

Apply KCL, KVL and Ohm's Law.

We can find 5 unknowns: $I_{b1}, I_{b2}, V_{\pi1}, V_{\pi2}, V_e$

with 5 Equations

$$\begin{cases} I_{b1} = \frac{V_{B1} - V_e - V_{\pi1}}{R_B} = \frac{V_{B1} - V_e}{R_B + r_\pi} \\ I_{b2} = \frac{V_{B2} - V_e - V_{\pi2}}{R_B} = \frac{V_{B2} - V_e}{R_B + r_\pi} \\ I_{b1} + I_{b2} + g_m V_{\pi1} + g_m V_{\pi2} = \frac{V_e}{R_o} \\ I_{b1} = \frac{V_{\pi1}}{r_\pi} \\ I_{b2} = \frac{V_{\pi2}}{r_\pi} \end{cases}$$

→ $V_o = \frac{\beta R_C}{r_\pi + R_B} (V_{B1} - V_{B2})$;

$V_d = V_{B1} - V_{B2}$

→ $V_o = \frac{\beta R_C}{r_\pi + R_B} \cdot V_d$.

Since $R_B = 0\Omega$, $r_\pi = \frac{\beta V_T}{I_{CQ}}$, $I_{CQ} = \frac{I_Q}{2}$.

→ Thus, $V_o = \frac{I_Q \cdot R_C}{2V_T} \cdot V_d$.

Differential-mode gain

→ Hence, $A_d = \frac{I_Q \cdot R_C}{2V_T} = \frac{0.8m \cdot 25k}{2 \cdot 26m} = 384.62$

We know that CMRR is defined as.

$CMRR = \left| \frac{A_d}{A_{cm}} \right|$ in absolute value

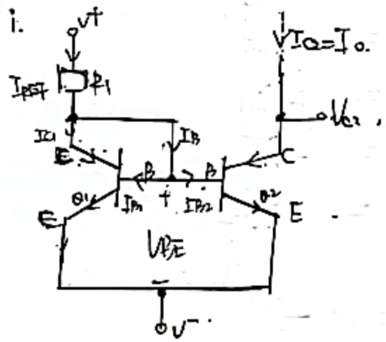
→ then $CMRR = \left| \frac{384.62}{-0.01} \right| = 38462$.

→ and $CMRR = 20 \log_{10} \left| \frac{A_d}{A_{cm}} \right| = 91.70\ dB$

→ In Summary:
- Differential-mode gain $A_d = 384.62$.
- CMRR in absolute value $= 38462$.
- CMRR in decibels $= 91.70\ dB$.

Q2.

(a) i.



Step1: KVL around the left Q1.

→ $V^+ - V^- = I_{REF} \cdot R_1 + V_{BE(on)}$

→ $I_{REF} = \frac{V^+ - V^- - V_{BE(on)}}{R_1} = \frac{10 - 0 - 0.7}{50k} = 0.186mA$

Step2: KCL of BJT Q1 collector

→ $I_{REF} = I_{C1} + I_B = I_{C1} + I_{B1} + I_{B2}$

Step 3: Current relationship

Since $V_{BE1} = V_{BE2}$

→ then $I_{B1} = I_{B2}$, $I_{C1} = I_{C2}$.

Step 4: Assume Both BJT are biased in forward active region

→ then,
$$\begin{cases} I_{C1} = \beta I_{B1} \\ I_{C2} = \beta I_{B2} \\ I_{C1} = I_{C2} = I_O \\ I_{REF} = I_{C1} + I_{B1} + I_{B2} \end{cases}$$

→ $I_{REF} = I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_{C1}}{\beta} = (1 + \frac{2}{\beta}) I_{C1}$

→ $I_O = I_{C2} = I_{C1} = \frac{I_{REF}}{(1 + \frac{2}{\beta})} = \frac{0.186m}{1 + \frac{2}{80}} = 0.181mA$

→ $I_{B1} = \frac{I_{C1}}{\beta} = I_{B2} = \frac{I_{C2}}{\beta} = \frac{0.181m}{80} = 2.268\mu A$

Step 5: Conclusion

$I_{REF} = 0.186\ mA$

$I_O = 0.181\ mA$

$I_{B1} = 2.268\ \mu A$

$I_{B2} = 2.268\ \mu A$

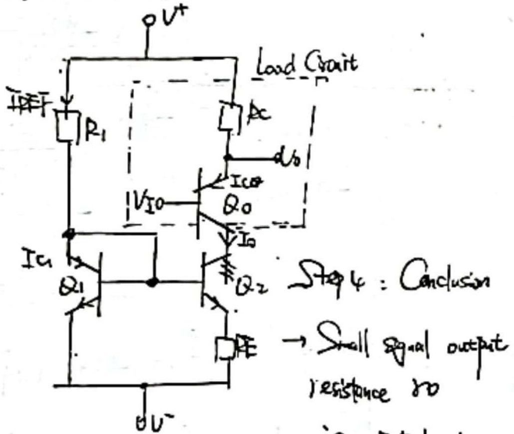
ii.

Step 1 : Find ~~VCE2 and~~ VCE1

→ From the configuration, $V_{CE1} = V_{BE(on)} = 0.7V$

Step 2 : Apply a load Circuit to find r_o

→



Step 4 : Conclusion

→ Small signal output resistance r_o is 551 kΩ.

Step 3 : Deduction

From the diagram

$$\rightarrow \frac{dI_o}{dV_{C2}} = \frac{1}{r_o} \quad ①$$

Apply ohm's law.

$$\rightarrow V_{C2} = V_{CE2} - V^-$$

$$\rightarrow dV_{C2} = dV_{CE2} \quad ②$$

Given the relation that

$$\rightarrow I_o = \frac{I_{REF}}{(1+\frac{2}{\beta})} \times \frac{1+\frac{V_{CE2}}{V_A}}{1+\frac{V_{CE1}}{V_A}} \quad ③$$

Combine ①.②.③

$$\rightarrow \frac{dI_o}{dV_{CE2}} = \frac{I_{REF}}{1+\frac{2}{\beta}} \times \frac{\cancel{V_A}\,1/V_A}{1+\frac{V_{CE1}}{V_A}} = \frac{dI_o}{dV_{C2}} = \frac{1}{r_o}.$$

Since $V_{BE(on)} = V_{CE1}$

$$\rightarrow \frac{dI_o}{dV_{C2}} = \frac{I_{REF}}{1+\frac{2}{\beta}} \times \frac{1}{V_A + V_{BE(on)}}$$

Considering that $V_A = 100 >> V_{BE(on)}$

Hence

$$\rightarrow \frac{dI_o}{dV_{C2}} = \frac{I_{REF}}{1+\frac{2}{\beta}} \times \frac{1}{V_A} \quad ④$$

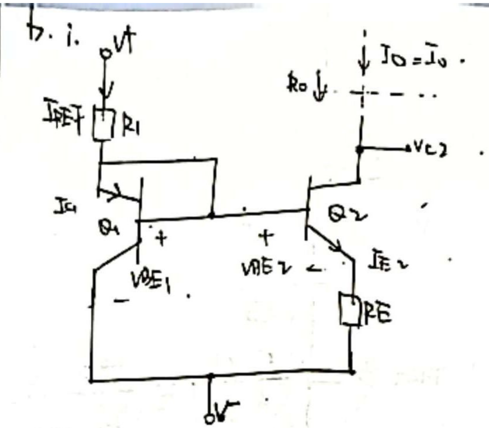
$I_{REF} = 0.186\,mA$

$\beta = 80$

$$\rightarrow \frac{1}{r_o} = \frac{0.186m}{1+\frac{2}{80}} \times \frac{1}{100}$$

$\Rightarrow r_o = 551k\Omega$.

b. i.



Step 1 : Find R1

Apply kvl around transistor Q1

$$\rightarrow V^+ - V^- - V_{BE(on)} = I_{REF} \cdot R_1$$

$$\rightarrow R_1 = \frac{V^+ - V^- - V_{BE(on)}}{I_{REF}} = \frac{3-(-3)-0.7}{2m} = 2.65k\Omega$$

Step 2 : Find RE

Apply kvl around the bottom,

$$\rightarrow V_{BE1} = \cancel{V_{BE(on)}} = V_{BE2} + I_{E2}R_E \quad ①$$

Since $I_o = 10\mu A$, and we ignore Base current

→ Then $I_{C2} = 10\mu A = I_{E2}$. ②

Apply KCL to the Collector Node of Q_1.

$$\rightarrow I_{REF} = I_{C1} + I_B = I_{C1}$$

$$\rightarrow I_{C1} = 2mA \quad ③$$

Equation ① can be simplified to.

$$V_T \cdot \ln\left(\frac{I_{C1}}{I_S}\right) - V_T \cdot \ln\left(\frac{I_{C2}}{I_S}\right) = I_{E2} \cdot R_E$$

$$\rightarrow R_E = \frac{V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right)}{I_{E2}} \quad ④$$

Combine ②、③、④.

$$\rightarrow R_E = \frac{26m \cdot \ln\left(\frac{2m}{10\mu}\right)}{10\mu} = 13.78k\Omega$$

ii.

Apply kvl to the ground loop.

$$\rightarrow V_{BE1} = V_{BE2} + I_{E2}R_E \quad ①$$

Since we know that

$$\begin{cases} I_{E2} = I_o = 10\mu A \\ R_E = 13.78\,k\Omega \\ V_{BE1} = V_{BE(on)} = 0.7V \end{cases}$$

$$\rightarrow V_{BE2} = V_{BE1} - I_{E2}R_E$$

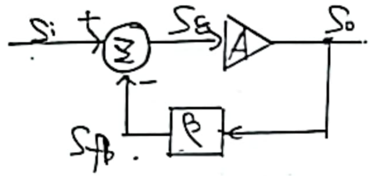
$$= 0.7 - 10\mu \cdot 13.78k$$

$$= 0.56\,V.$$

Q3: $S_i = 1\,mV$, $S_o = 99.9\,mV$, $S_{fb} = 0.999\,mV$.

i. Step1 : Deduction.

From the Configuration shown as below.



$$\Rightarrow \begin{cases} S_i - \beta S_o = S_\varepsilon \\ S_\varepsilon \cdot A = S_o \end{cases}$$

$$\Rightarrow \frac{S_o}{S_i} = \frac{A}{1+A\beta}$$

Step2 : Find A_f, β, A

1° closed-loop gain A_f

$$\Rightarrow A_f = \frac{S_o}{S_i} = \frac{A}{1+A\beta} = \frac{99.9m}{1m} = 99.9$$

2° feedback transfer function β.

$$\Rightarrow \beta = \frac{S_{fb}}{S_o} = \frac{0.999m}{99.9m} = \frac{1}{100}.$$

3° open-loop gain A

$$\Rightarrow \cancel{A} \cancel{=} \cancel{\frac{1}{\beta}} = \frac{\cancel{S_{fb}}\, S_o}{S_\varepsilon} = \frac{\cancel{S_{fb}}\, S_o}{S_i - \cancel{\beta} S_{fb}}$$

$$\Rightarrow A = \frac{\cancel{0.999m}\ 99.9m}{1m - 0.999m} = \cancel{999000}\ 99900$$

ii. From i, we are able to derive

$$\rightarrow A = 99900$$

Since we know that.

$$A_f = \frac{A}{1+A\beta}.$$

$$\rightarrow \text{then } \beta = \left(\frac{A \cdot A_f}{A - A_f}\right)^{-1} = \frac{99900 - 200}{99900 \times 200} = 4.99m$$

$\rightarrow$ Therefore, transfer-function $\beta = 4.99 \times 10^{-3}$.

iii. From i and ii, we know that

$$\rightarrow A_f = \frac{A}{1+A\beta}.$$

$$\rightarrow \frac{dA_f}{dA} = \frac{1+A\beta - A\beta}{(1+A\beta)^2}$$

$$\rightarrow dA_f = \frac{dA}{(1+A\beta)^2}$$

Then Divide A_f in Both Sides.

$$\rightarrow \frac{dA_f}{A_f} = \frac{dA}{(1+A\beta)^2} \cdot \frac{1+A\beta}{A} = \frac{1}{(1+A\beta)}\left(\frac{dA}{A}\right)$$

Next, Substitute $\beta = 0.005$, $A = 10^5$, $dA/A = \pm 50\%$

$$\cancel{\rightarrow \frac{dA_f}{A_f} = \frac{\pm 50\%}{2.002 \times 10^{-3}} =}$$

$\rightarrow$ Case 1 : A increases by 50%.

That means $\frac{dA_f}{A_f} = \frac{1}{1+1.5A\beta} \cdot +50\% = +0.067\%$.

$\rightarrow$ Case 2. : A decreases by 50%

That means $\frac{dA_f}{A_f} = \frac{1}{1+0.5A\beta} \cdot (-50\%)$

$$\frac{dA_f}{A_f} = -0.199\%$$

$\Rightarrow$ In Summary, the variation of closed-loop gain A_f in percentage Value is from ~~−0.199%~~ −0.199% to +0.067%

iv. Step1 : Deduction

$$\rightarrow A_f(s) = \frac{A(s)}{1+A(s)\cdot\beta}$$

$$= \frac{\frac{A_o}{1+s/\omega_H}}{1+\beta\frac{A_o}{1+s/\omega_H}}$$

$$= \frac{A_o}{1+\beta A_o} \cdot \frac{1}{1+\frac{s}{\omega_H \cdot (1+\beta A_o)}}$$

where $\begin{cases} A_{fo} = \frac{A_o}{1+\beta A_o} \\ \omega_{fH} = \omega_H \cdot (1+\beta A_o) \end{cases}$

Step2 : Find A_{fo}, ω_{fH}

$$\rightarrow A_{fo} = \frac{A_o}{1+\beta A_o} = \frac{10^5}{1+10^2} = 990.10$$

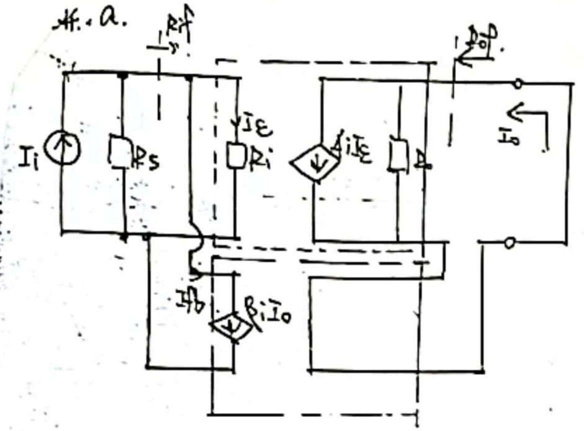
$$\rightarrow \omega_{fH} = \omega_H \cdot (1+\beta A_o)$$

$$= 10 \cdot (1+10^{-2} \times 10^5)$$

$$= 1010 \text{ rad/s}.$$

$\rightarrow$ In Summary, $\begin{cases} A_f(s) = \frac{A_o}{1+\beta A_o} \cdot \frac{1}{1+\frac{s}{\omega_H \cdot (1+\beta) A_o}} \\ \text{closed-loop low frequency gain } A_{fo} = 990.10 \\ \text{closed-loop corner frequency } \omega_{fH} = 1010\text{ rad/s} \end{cases}$

4. a.



i. Type: Shunt-Series Current Amplifier

Reason: Input: feedback affects input signal in terms of current.

Output: Short-Circuit the output load, it is current-sensing.

ii. Since $R_S = \infty$, and Apply KCL on Input terminal.

$\rightarrow I_i - I_{fb} = I_\varepsilon.$

$\rightarrow I_\varepsilon = 1mA - 0.99mA = 0.01\ mA$

Apply ohm's law at output node

$\rightarrow I_o = A_i I_\varepsilon.$

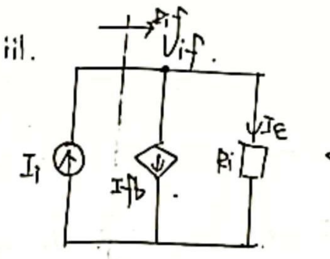
$\rightarrow A_i = \frac{I_o}{I_\varepsilon}$

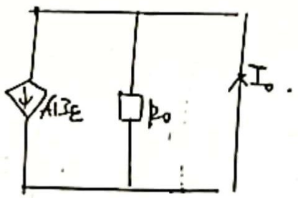
$I_o = 500mA$

$I_\varepsilon = 0.01mA$

$\Rightarrow$ Open-loop Gain $A_i = \frac{500\ mA}{0.01\ mA} = 5\times10^4$

iii.





$\rightarrow R_{if} = \frac{V_{if}}{I_i}$ and $I_o = A_i I_\varepsilon$.

Apply KCL and ohm's Law at input terminal

$\rightarrow I_i = I_{fb} + \frac{V_{if}}{R_i}$

$\rightarrow I_i = \beta_i I_o + \frac{V_{if}}{R_i}$

$I_o = A_i I_\varepsilon = A_i \frac{V_{if}}{R_i}$ $\rightarrow R_{if} = \frac{R_i}{1+A_i\beta_i}$

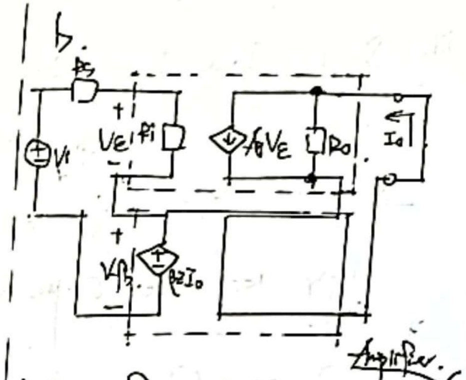
From ii., we know that $A_i = 5\times10^4$

$\beta_i = \frac{I_{fb}}{I_o} = 1.98\times10$

Since $R_{if} = \frac{R_i}{1+A_i\beta_i}$

$\Rightarrow$ Thus, $R_{if} = \frac{5k}{1+99} = 50\ \Omega$.

b.



i. Type: Series-Series Transconductance

Reason: Input: → feedback affects input signal in terms of voltage.

Output: → Short-Circuit the output, feedback signal is current sensing

ii. Apply KVL at Input node

$\rightarrow V_i = V_\varepsilon + V_{fb}.$

$\rightarrow V_i = V_\varepsilon + \beta_Z I_o$ ①

Apply KCL at Output node

$\rightarrow I_o = A_g V_\varepsilon$ ②

We know that

$\rightarrow A_{gf} = \frac{I_o}{V_i}$ ③

Then, Combine ①, ②, ③

$\rightarrow A_{gf} = \frac{A_g}{1+\beta_Z A_g}$ #

$\Rightarrow$ Therefore,

Closed-loop gain $A_{gf} = \frac{A_g}{1+\beta_Z A_g}$

iii. Step 1 : Input Resistance R_{if}

Apply KVL at Input node

→ $V_i = I_i R_i + R_Z I_o$ ①

Apply KCL at Output node

→ $I_o = A_g V_\varepsilon$. ②

Apply Ohm's law at Input node

→ $V_\varepsilon = I_i \cdot R_i$ ③

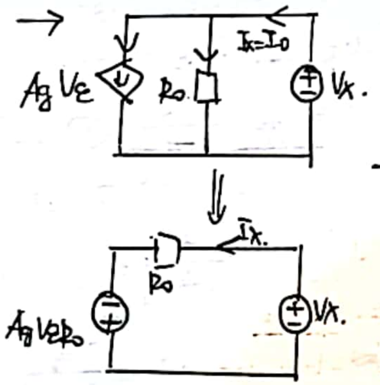
Combine ①, ②, ③

→ $V_i = I_i R_i + R_Z A_g R_i \cdot I_i$

→ $\frac{V_i}{I_i} = R_i(1 + R_Z A_g) = R_{if}$

Step 2 : Output Resistance R_{of}.

Apply a test voltage source at Output node

→ 

→ Apply KVL and Thevenin Equivalent Circuit

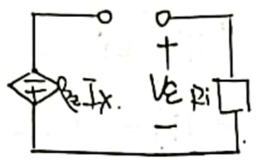
→ $V_x = I_x R_o - A_g V_\varepsilon R_o$ ④

If we would like to apply test source to find R_{of}

→ we have to Disable the voltage supply V_i

→ That means V_i has to be treated as Open-loop

→ And the circuit will be.



→ $V_\varepsilon = -R_Z I_x$. ⑤

Combine ④ and ⑤

→ $V_x = I_x(1 + A_g R_Z) R_o$

→ $\frac{V_x}{I_x} = R_o(1 + A_g R_Z) = R_{of}$

Step 3 : Conclusion.

In Summary

Input resistance $R_{if} = R_i(1 + R_Z A_g)$

Output resistance $R_{of} = R_o(1 + R_Z A_g)$