CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 6 Passive Components-

Resistors & Capacitors

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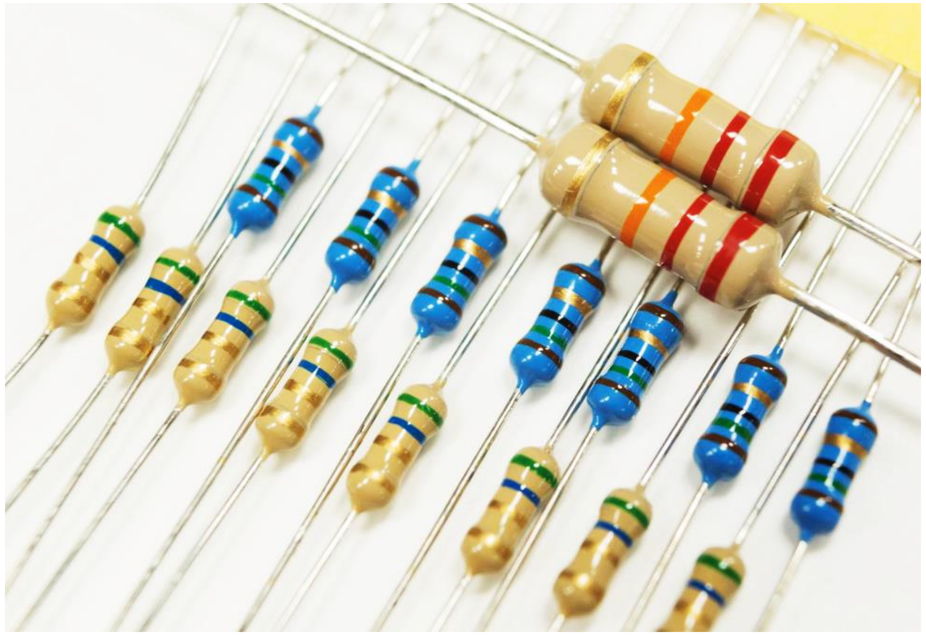
OUTLINE

- ➢ Resistor
  - ✓ Resistance vs Resistivity
- ➢ Capacitor
  - ✓ Calculation
  - ✓ Dielectric material filled

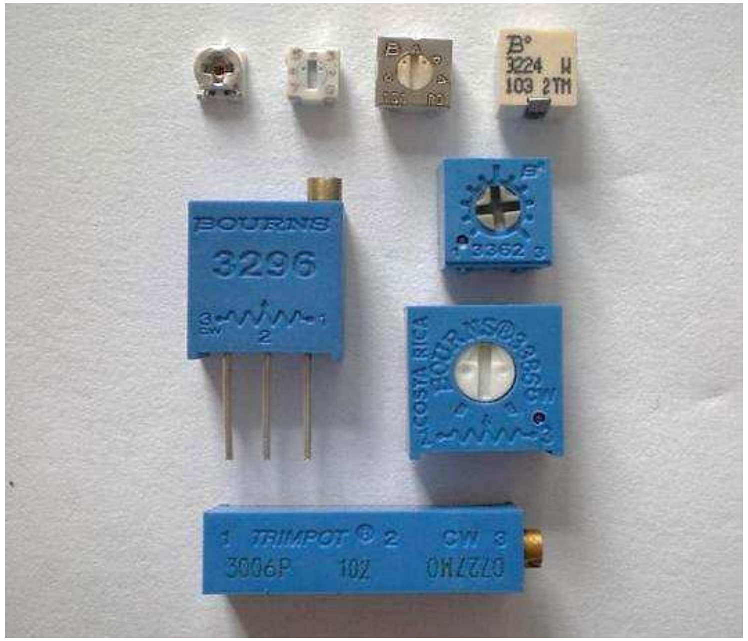
Passive Components:
Electronic components which can only receive energy

1.1 RESISTOR

A resistor is a passive two-terminal electrical component that implements electrical resistance as a circuit element.



Fixed resistors



Variable resistors

1.2 RESISTANCE

The resistance of a conductor of length *dl* can be obtained by

$$dR = \frac{dV}{I} = \frac{-\vec{E}\cdot d\vec{l}}{\iint_S \vec{J}\cdot d\vec{s}} \qquad (1)$$

If we assume that the potential at end *a* of the conductor is higher than that at end *b*, the total resistance of the conductor is:

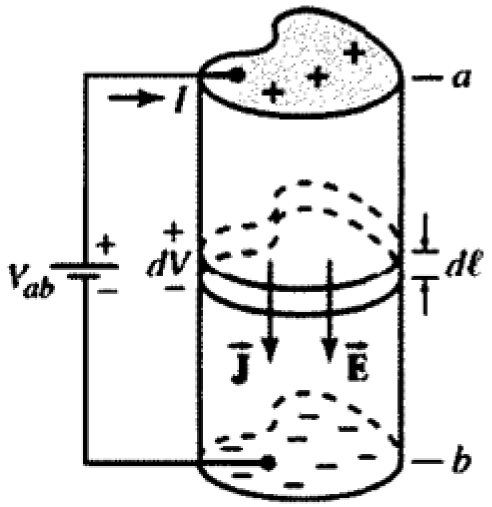
$$R = \int_b^a dR = \int_b^a \frac{-\vec{E}\cdot d\vec{l}}{\iint_S \vec{J}\cdot d\vec{s}} \qquad (2)$$

This is a general eq. to find the resistance of a conducting medium whose conductivity changes in the direction of the current.

For homogeneous medium having constant σ, it reduces to:

$$R = \int_b^a \frac{-\vec{E}\cdot d\vec{l}}{\iint_S \vec{J}\cdot d\vec{s}} = \frac{V_{ab}}{I} \qquad (3)$$

Simplified model:



A potential difference is maintained across the two ends of a conducting wire of length *l*. Define *A* is the cross-sectional area of the wire.

The electric field is:

$$V_{ab} = -\int_b^a \vec{E}\cdot d\vec{l} \qquad \Rightarrow E = \frac{V_{ab}}{l} \qquad (4)$$

If σ is the conductivity of the conducting material, the current density at any cross section of the wire is:

$$J = \sigma E = \frac{\sigma V_{ab}}{l} \qquad (5)$$

The current through the wire is:

$$I = \iint_S \vec{J}\cdot d\vec{s} = \frac{\sigma V_{ab} A}{l} = \frac{V_{ab}}{R} \qquad (6)$$

The resistance of the piece of the conducting material is:

$$R = \frac{V_{ab}}{I} = \frac{l}{\sigma A} = \frac{\rho l}{A}$$

1.2 RESISTANCE & RESISTIVITY

	Unit	Expression	Physical meaning
Resistance	Ω	$R = \frac{l}{\sigma A} = \frac{\rho l}{A}$	Resistance is property of an object, depends on geometry (shape and size) as well as resistivity.
Resistivity	Ω·m	$\rho = \frac{1}{\sigma} = \frac{\vec{E}}{\vec{J}}$	Resistivity is property of a substance. Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature, not on its shape or size.

QUIZ 1.1

A 3000 km long cable consists of seven copper wires, each of diameter a = 0.73 mm, bundled together and surrounded by an insulating sheath. Calculate the resistance of the cable if the resistivity of the copper is $3 \times 10^{-6}\,\Omega \cdot cm$.

Ans: $3.1 \times 10^{4}\ \Omega$

QUIZ 1.2

An 18-gauge copper wire (the size usually used for lamp cords) has a diameter of 1.02 mm and a cross-sectional area of 8.2×10^{-7} m^2. The resistivity of the copper is $1.72 \times 10^{-8}\ \Omega \cdot m$ and it carries a current of 1.67 A. Determine the following:

a) The magnitude of the electric field intensity in the wire.

b) The resistance of the wire given the length is 50m.

Ans: 0.035V/m; 1.05Ω

QUIZ 1.3

A long, round wire of radius a and conductivity σ is coated with a material of conductivity 0.1σ.

a) What should the thickness of the coating b be so that the resistance per unit length of the uncoated wire is reduced by 50%?

b) Assuming a total current I in the coated wire, find $\vec{J}$ and $\vec{E}$ both in the core and in the coating material.

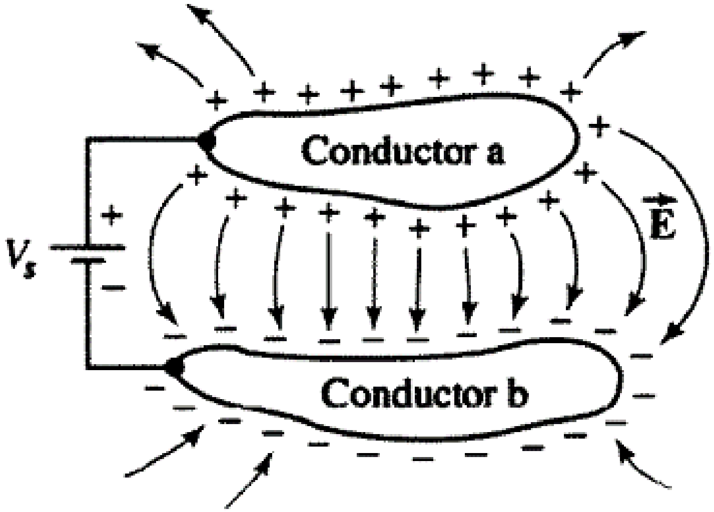
OUTLINE

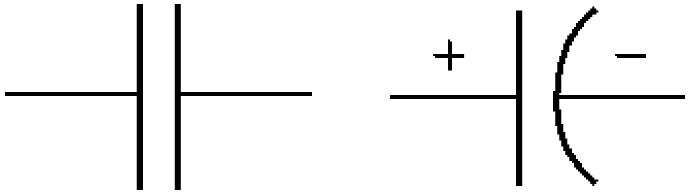
- Resistor
  - ✓ Resistance vs Resistivity
- Capacitor
  - ✓ Calculation
  - ✓ Dielectric material filled

2.1 CAPACITOR

A capacitor is a two-terminal passive device which stores electric charge.

2.1 CAPACITOR





Unit: 1 F (farad) = 1 C/V

Capacitor

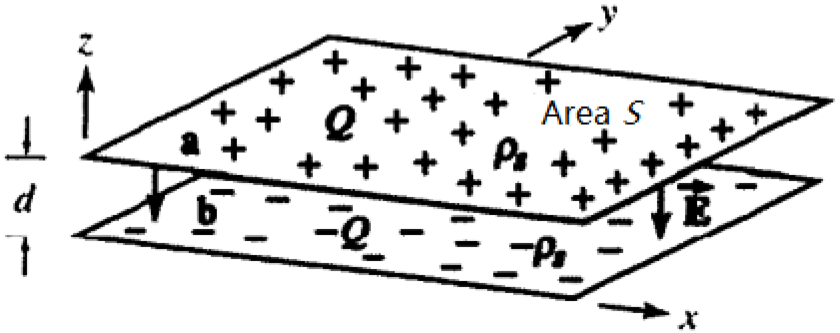
- A capacitor is a device which stores electric charge.
- Its basic configuration is two conductors carrying equal but opposite charges

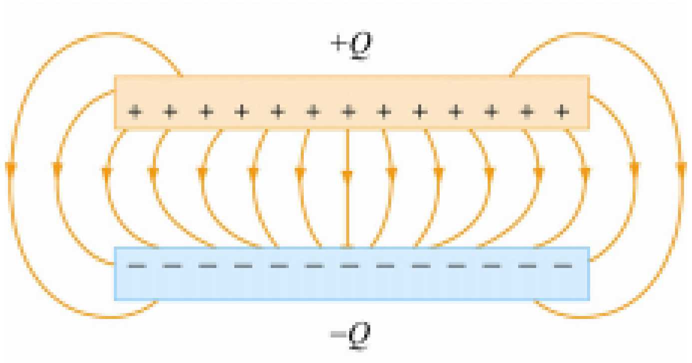
Capacitance

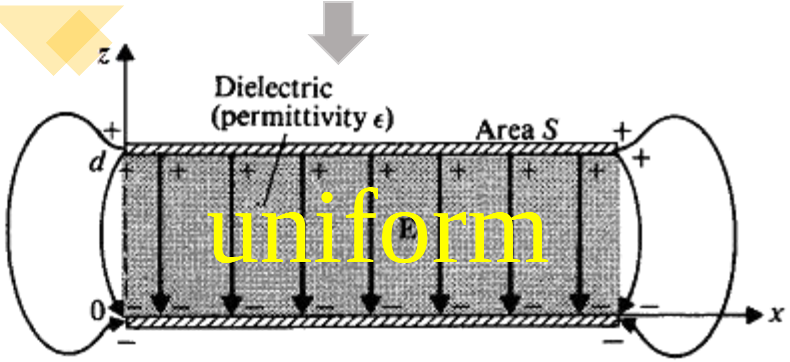
- measures the capability of energy storage in electrical devices.
- the amount of charge Q stored in a capacitor is linearly proportional to the electric potential difference V between the two conductors:

$$\frac{Q}{V} = constant = C$$

CASE 1: PARALLEL PLATES







$d \ll \sqrt{S}(side\ length)$

Two parallel conducting plates with area *S* in vacuum are separated by a distance *d*, and they form a parallel-plate capacitor. The total charge on the top plate is +*Q* and that on the other plate is –*Q*.

- Edge effects: The e-field lines at the edge of the plates are not straight lines, and the field is not contained entirely between the plates.
- Fringing fields: The non-uniform fields near the edge.

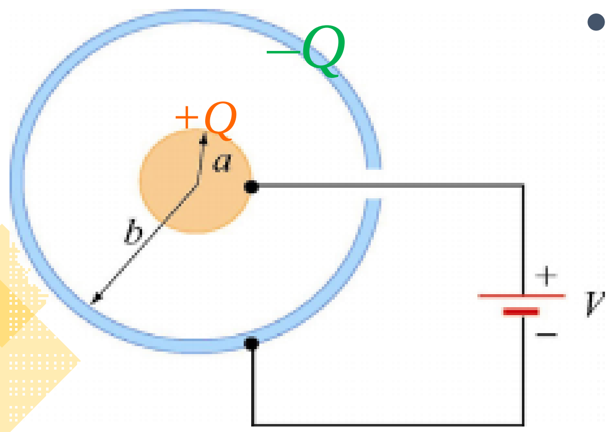
$$\oiint_{S'} \vec{E}\cdot d\vec{s} = \frac{Q_{encl}}{\varepsilon_0} = \frac{\rho_s S}{\varepsilon_0} \qquad \therefore \vec{E} = -\frac{\rho_s}{\varepsilon_0}\hat{z} = -\frac{Q}{\varepsilon_0 S}\hat{z}$$

The potential *V* is: $V = \int_d^0 \vec{E}\cdot d\vec{l} = Ed = \frac{Q}{\varepsilon_0 S}d$

So, the capacitance of a parallel–plate is:

$$C = \frac{Q}{V} = \frac{\varepsilon_0 S}{d}$$

CASE 2: SPHERICAL CAPACITOR



- Two concentric spherical conducting shells of radii a and b
- The inner shell has a charge $+Q$ uniformly distributed over its surface, & the outer shell has an equal but opposite charge $-Q$.

The electric field is non-vanishing only in the region $a < r < b$.

Using Gauss's law:

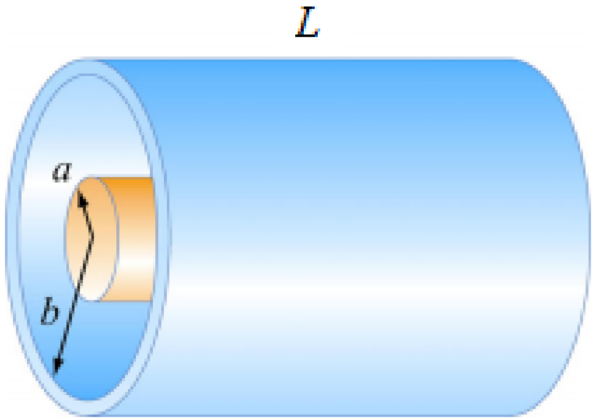
$$\oiint_{S'} \vec{E} \cdot d\vec{s} = E_r(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0} \qquad \therefore E_r = \frac{Q}{4\pi\varepsilon_0 r^2}$$

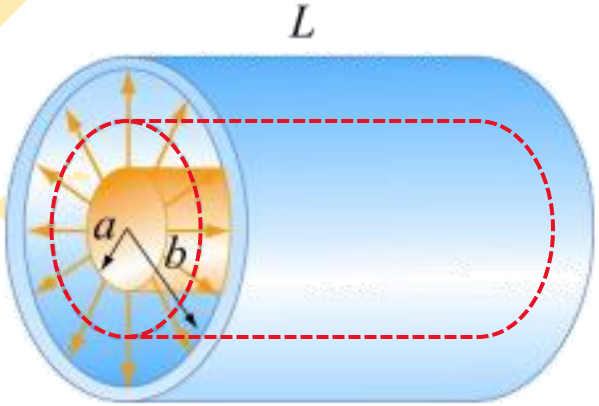
The potential difference between the two conducting shells:

$$V_{ab} = \int_a^b E_r dr = \frac{Q}{4\pi\varepsilon_0}\frac{b-a}{ab}$$

$$\boxed{C = \frac{Q}{V_{ab}} = 4\pi\varepsilon_0 \frac{ab}{b-a}}$$

CASE 3: CYLINDRICAL CAPACITOR





- Inner radius a surrounded by a coaxial cylindrical shell of inner radius b. Filled with dielectrics with ε. The length of both cylinders is L.
- The capacitor is charged so that the inner cylinder has charge $+Q$ while the outer shell has a charge $-Q$.

Assume that L is much larger than $b-a$, the separation of the cylinders, so that edge effects can be neglected.

Using Gauss's law:

$$\oiint_{S'} \vec{E}\cdot d\vec{s} = E_r(2\pi rL) = \frac{Q_{encl}}{\varepsilon} \qquad \therefore E_r = \frac{Q}{2\pi\varepsilon rL}$$

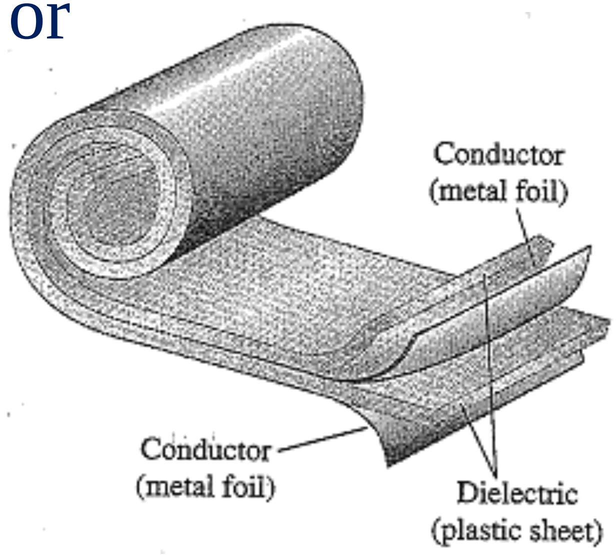
$$V_{ab} = \int_a^b \frac{Q}{2\pi\varepsilon rL}\,dr = \frac{Q}{2\pi\varepsilon L}\ln\left(\frac{b}{a}\right)$$

Gaussian Surface: **Cylinder**

$$C_l = \frac{Q_l}{V_{ab}} = \frac{2\pi\varepsilon\cancel{L}}{\ln(b/a)}$$

2.2 WITH DIELECTRICS

Most capacitors have an insulating material, such as paper or plastic, between their conducting plates.



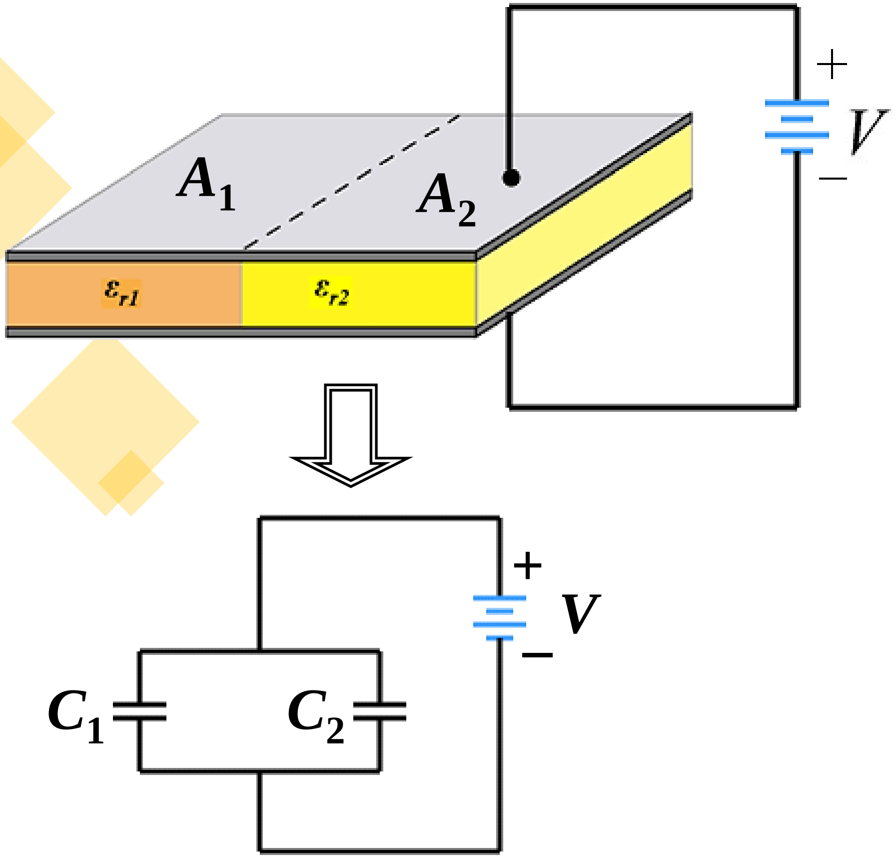
Reasons:

- To maintain a physical separation of the plates;
- Increase the maximum possible potential difference between the conducting plates;
- Capacitance increases when the space between the conductors is filled with dielectrics.

$$C = \frac{Q}{V} = \frac{\varepsilon S}{d}$$

QUIZ 2.1

Two dielectrics with dielectric constants ε_{r1} and ε_{r2} each fill half the space between the plates of a parallel-plate capacitor. Each plate has an area, and the plates are separated by a distance *d*. Find the capacitance of the system.



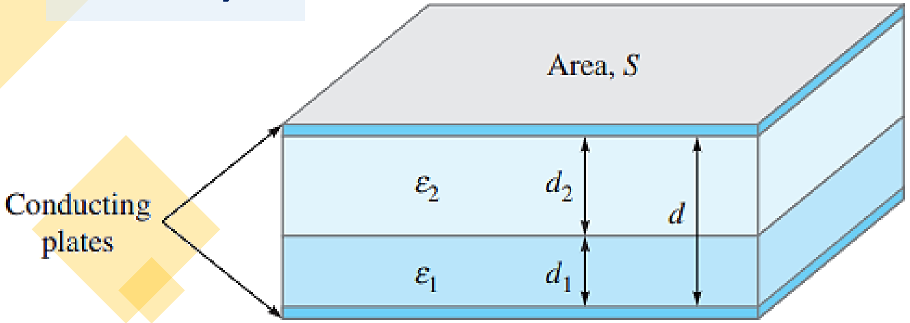
The potential difference on each half of the capacitor is the same, so the system can be treated as being composed of two capacitors connected in parallel.

Thus, the capacitance of the system is $C = C_1 + C_2$

QUIZ 2.2

A parallel-plate capacitor containing two dielectrics with the dielectric interface parallel to the plates. By analysing the *boundary conditions* at the dielectric interface, determine the capacitance.

$$C = \frac{Q}{V}$$



QUIZ 2.3

Suppose the parallel plates each have an area of 2000 cm^2 and are separated by 1 cm apart. The capacitor is charged to a potential difference V_0=3000 V. After that, a sheet of insulating plastic is inserted and filling the space between the plates completely. Now the potential difference decreases to 1000 V with the same charge on each plate.

Determine the following:

(a) The original capacitance C_0.

(b) The magnitude of charge Q on each plate.

(c) The capacitance after the dielectric inserted.

(d) The relative permittivity of the insulator.

NEXT…

- ➢ Time-varying Fields
- ➢ Inductors