Tutorial – 1

(Uncontrolled and Controlled Rectifiers)

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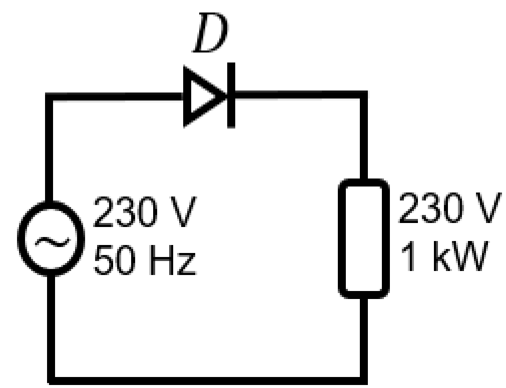
Problem 2.1

Q1) A single-phase 230 V, 1 kW heater is connected across single-phase 230 V, 50 Hz supply through a diode. Calculate the power delivered to the heater element. Also find the peak diode current and input power factor.



Solution

Heater resistance, $R = \frac{{V_s}^2}{P} = \frac{230^2}{1000} = 52.9\ \Omega$

RMS output voltage, $V_{RMS} = \frac{V_m}{2} = \frac{\sqrt{2} \times V_s}{2} = 162.63\ V$

Power absorbed by heater element $= \frac{{V_{RMS}}^2}{R} = \frac{162.63^2}{52.9} = 500\ W$

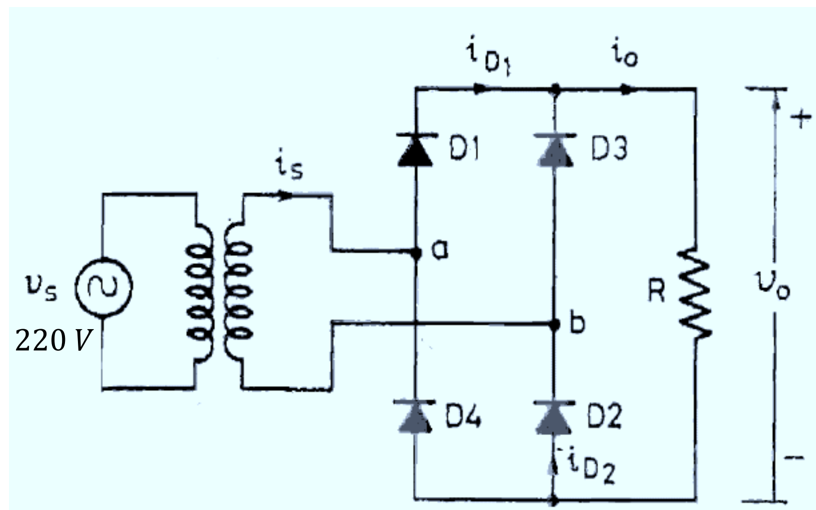
Peak value of diode current $= \frac{V_m}{R} = 6.1478\ A$

Input power factor $= \frac{\text{Power delivered to load}}{\text{Input VA}} = \frac{V_{RMS} \times I_{RMS}}{V_S \times I_{RMS}} = 0.707$

Problem 2.2

Q1) A single-phase bridge rectifier is required to supply an average voltage of 110V to a resistive load $R = 10\Omega$. The mains supply is 220V RMS. Determine

- The rms value of the output voltage for the rectifier and then the turns ratio of the transformer;
- The current ratings of the diodes;
- The power consumed by the load;



Problem 2.2

Solution

1) Given output voltage, $V_o = 110\ V = \frac{2V_m}{\pi} \Rightarrow V_m = 172.78V$

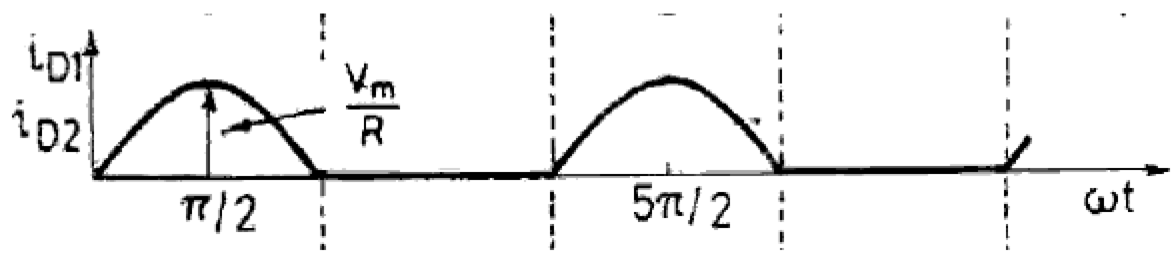
Let us find RMS value of output voltage, $V_{RMS} = V_s = \frac{V_m}{\sqrt{2}} = 122.17V$

Now, the turns ratio (primary to secondary) $= \frac{220}{122.17} = 1.8$

2) Peak value of diode current, $I_m = \frac{V_m}{R} = \frac{172.78}{10} = 17.27A$

Avg value of diode current, $I_{DA} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\omega t\,.\,d(\omega t) = \frac{I_m}{\pi} = 5.5A$

3) Power to the load, $P_{load} = \frac{V_o^2}{R} \Rightarrow 1210\text{W}$



Problem 3.1

Q1) A three-phase full-bridge rectifier charges a 240 V battery. Input voltage to rectifier is 3-phase, 230 V, 50 Hz. Current limiting resistance in series with battery is 8Ω and an inductor makes the load current almost ripple free. Determine

a) Power delivered to battery and the load;
b) Input displacement factor;
c) Current distortion factor;
d) Input power factor;
e) Input HF or THD;

Problem 3.1

Solution

Line voltage

a) Here $V_{ml} = \sqrt{2} \times V_l = \sqrt{2} \times 230V$

Average output voltage, $V_o = \frac{3V_{ml}}{\pi} = \frac{3\sqrt{2}\times 230}{\pi} = 310.56V$

But, $V_o = E + I_0 R$

Average value of battery charging current, $I_o = \frac{V_o - E}{R} = \frac{310.56-240}{8} = 8.82A$

Power delivered to battery $= EI_o = 240 \times 8.82 = 2116.8W$

Power delivered to load, $P_d = EI_o + I_o^2 R$

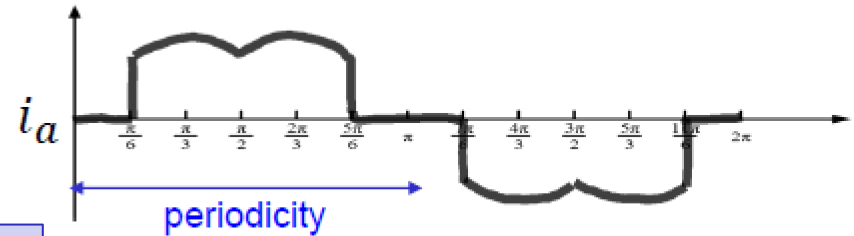
So, $P_d = 240 \times 8.82 + 8.82^2 \times 8 = 2739.16W$

Problem 3.1

Solution

b) For ripple free load current, phase-a current i_a, or transformer secondary current i_s would be constant at $I_o = 8.82A$ from $\omega t = 30°$ to $150°$ and $-I_o$ from $210°$ to $330°$ and so on. As positive and negative half cycles are identical, average value of $i_s = 0$, i.e. $I_{s,avg} = 0$.

$$i_s(t) = I_{s,avg} + \sum_{n=1,3,5}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$



$$a_n = \frac{2}{2\pi}\left[\int_{\pi/6}^{5\pi/6} I_0(t) \cos n\omega t \, d\omega t + \int_{7\pi/6}^{11\pi/6} -I_0(t) \cos n\omega t \, d\omega t\right]$$

$$b_n = \frac{2}{2\pi}\left[\int_{\pi/6}^{5\pi/6} I_0(t) \sin n\omega t \, d\omega t + \int_{7\pi/6}^{11\pi/6} -I_0(t) \sin n\omega t \, d\omega t\right]$$

$$\Rightarrow \quad a_1 = 0, \qquad b_1 = \frac{2\sqrt{3}}{\pi} I_0$$

$$i_s(t) = i_a(t) = \frac{2\sqrt{3}}{\pi} I_0 \left(\sin \omega t - \frac{1}{5}\sin 5\omega t - \frac{1}{7}\sin 7\omega t + \frac{1}{11}\sin 11\omega t + \cdots \right)$$

See Page.33 (Week 3)

Problem 3.1

Solution

From the original Fourier series expression, fundamental component of source current is given by

$i_{s1} = \frac{2\sqrt{3}}{\pi} I_o \sin \omega t$ and $\phi_1 = tan^{-1}[\frac{0}{b_1}] = 0°$

Input displacement factor, $DF = \cos \phi_1 = 1.$

c) RMS value of fundamental source current, $I_{s1} = \frac{2\sqrt{3}}{\pi} \times \frac{I_o}{\sqrt{2}}$

RMS value of source current, $I_s = \sqrt{\frac{2}{2\pi} \int_{\pi/6}^{5\pi/6} {I_0}^2 \, d\omega t} = \sqrt{\frac{2}{3}} I_o$

Current distortion factor, $CDF = \frac{I_{s1}}{I_s} = \frac{2\sqrt{3} \times I_o}{\pi \times \sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{2} I_o} = \frac{3}{\pi} = 0.955$

Problem 3.1

Solution

d) Input PF $= CDF \times DF = 0.955 \times 1 = 0.955$

e) $HF = THD = \left[\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right]^{1/2} = \left[\left(\frac{1}{0.955}\right)^2 - 1\right]^{1/2} = 0.3106$

Problem 4.1

Q1) A single-phase half-wave controlled rectifier with R load is supplied from a 230 V, 50 Hz AC source. When average dc output voltage is 50% of maximum possible average dc output voltage, determine

1) Firing angle of SCR.
2) Average dc output voltage.
3) RMS output voltage.
4) Average and RMS output currents.
5) Average and RMS currents of SCR.

[Assume $R = 20\ \boldsymbol{\Omega}$]

Exercise problem (Week 4)

Problem 4.1

Solution

1) When $\alpha = 0^0$, the maximum possible average output voltage is available across load.

$$V_{o,max} = \frac{V_m}{2\pi}(1 + \cos\alpha) \rightarrow V_{o,max} = \frac{V_m}{2\pi}(1 + \cos 0) = \frac{V_m}{\pi}$$

Consider that $V_o = 50\% \times V_{o,max} = 0.5 \times \frac{V_m}{\pi} \rightarrow \frac{V_m}{2\pi}(1 + \cos\alpha) = 0.5 \times \frac{V_m}{\pi}$

Therefore, the firing angle of SCR is obtained as $\alpha = 90^0$.

2) Average dc output voltage, $V_o = \frac{V_m}{2\pi}(1 + \cos\alpha) = \frac{V_m}{2\pi}(1 + \cos 90) = 51.7V$

3) RMS output voltage, $V_{RMS} = \frac{V_m}{2}\sqrt{\frac{2(\pi - \alpha) + \sin 2\alpha}{2\pi}} = 115.01V$

4) Average output current, $I_o = \frac{V_o}{R} = \frac{51.7}{20} = 2.58\ A$;

RMS output current, $I_{RMS} = \frac{V_{RMS}}{R} = \frac{115.01}{20} = 5.755A$

Problem 4.1

Solution

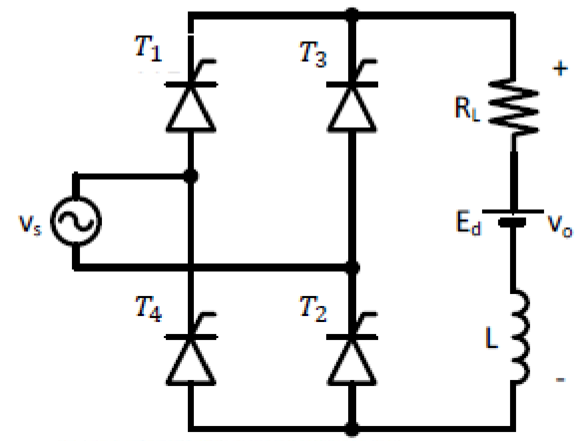
5) Since the SCR current waveform is same as output current waveform, average current of SCR is $I_o = 2.58\ A$
RMS current of SCR is $I_{RMS} = 5.755\ A$

Problem 4.2

Q2) A single-phase fully controlled bridge converter with RLE load is supplied from 230 V, 50 Hz ac supply. The average load current is 5 A which is constant over the working range. Determine the firing angle for

a) $E = 100\,V$

b) $E = -100\,V$



[Assume $R = 4\Omega$ and $L = 4$ mH]

Problem 4.2

The average output voltage with RLE load at firing angle α is,

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \; d\omega t = \frac{2V_m}{\pi}\cos\alpha$$

Check??

a) When $E = 100\,V$, the average current is $I_o = \frac{V_o - E}{R} \rightarrow 5 = \frac{V_o - 100}{4} \rightarrow V_o = 120V$

Consider, $V_o = \frac{2V_m}{\pi}\cos\alpha \rightarrow 120 = \frac{2\times\sqrt{2}\times 230}{\pi}\cos\alpha \rightarrow \alpha = 54.59^0$

b) When $E = -100\,V$, $V_o = I_oR + E = 5 \times 4 - 100 = -80\,V$

Therefore, $V_o = \frac{2V_m}{\pi}\cos\alpha \rightarrow -80 = \frac{2\times\sqrt{2}\times 230}{\pi}\cos\alpha \rightarrow \alpha = 112.7^0$

(Useful formulae for 3-phase Half-wave rectifier)

The average output voltage, $V_o = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$

The maximum average output voltage at $\alpha = 0^0, V_{o,max} = \frac{3\sqrt{3}V_m}{2\pi}$

The rms output voltage, $V_{RMS} = \left[\frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t \; d(\omega t)\right]^{1/2}$

For resistive load, $\alpha \geq 30^0$

$$V_o = \frac{3V_m}{2\pi}\left[1 + \cos\left(\frac{\pi}{6} + \alpha\right)\right]$$

$$V_{RMS} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m^2 \sin^2 \omega t \; d(\omega t)\right]^{1/2}$$

Problem 5.1

Q1. A three-phase half-wave controlled converter is operated from 3-phase, 230 V, 50 Hz supply with load resistance $R = 10\Omega$. An average output voltage of 50% of the maximum possible output voltage is required. Determine

a) the firing angle,
b) average and rms values of load current,
c) rectification efficiency

Exercise problem (Week 5)

Problem 5.1

Solution

a) For R load, voltage is continuous when $\alpha \leq 30^0$ and average output voltage is given by

$$V_o = \frac{3\sqrt{3}V_m}{2\pi} \cos\alpha$$

Its maximum possible value is, $V_{o,max} = \frac{3\sqrt{3}V_m}{2\pi} = \frac{3\sqrt{3}\times 187.7}{2\pi} = 155.3\ V$

Required average output voltage, $V_o = \frac{3\sqrt{3}V_m}{2\pi} \cos\alpha = 0.5 \times V_{o,max}$

$$\therefore\ \alpha = 60^0$$

Phase voltage $= \frac{\text{Line Voltage}}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.8 \Longrightarrow V_m = \sqrt{2} \times 132.8 = 187.7$

Problem 5.1

Solution

Since the firing angle $\alpha > 30^0$, the output voltage cannot be computed from the above expressions. *Note that the voltage as well as load current will be discontinuous.*

Therefore, average value of output voltage is obtained as

$$V_o = \frac{3V_m}{2\pi}[1 + \cos(\alpha + 30^0)] = 0.5 \times V_{o,max}$$

$$\therefore \alpha = 67.7^0$$

Problem 5.1

Solution

b) Average load current, $I_o = \frac{V_o}{R} = \frac{77.65}{10} = 7.765\ A$

RMS voltage is $V_{RMS} = \frac{\sqrt{3}V_m}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6} - \alpha\right) + \frac{1}{2}\sin(2\alpha + \pi/3)\right]^{1/2}$

$$= \frac{\sqrt{3}\times 187.7}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6} - \frac{67.7\times\pi}{180}\right) + \frac{1}{2}\sin(2\times 67.7 + \pi/3)\right] = 104.76V$$

$$I_{RMS} = \frac{104.765}{10} = 10.477A$$

c) Rectifier efficiency $= \frac{V_o I_o}{V_{RMS} I_{RMS}} = \frac{77.65\times 7.765}{104.765\times 10.477} = 0.5493 = 54.93\%$

Problem 5.2

Q2. A three-phase half-wave controlled rectifier is fed from 3-phase, 440 V, 50-Hz ac supply and it is connected with the load resistance $R = 10\Omega$. When the firing angle of thyristor is 20^0, calculate

a) the dc output voltage,
b) the rms value of output voltage,
c) the form factor,
d) the ripple factor,
e) the average output current,
f) the rms load current,
g) dc output power
h) ac power supplied, and
i) rectification efficiency.

Problem 5.2

Solution

Phase voltage $= \frac{\text{Line Voltage}}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.04V \Longrightarrow V_m = \sqrt{2} \times 254.04 = 359.2\text{ V}$

Since firing angle is less than 30^0, the output voltage as well as load current will be continuous.

a) $\mathrm{V_o} = \frac{3\sqrt{3}\mathrm{V_m}}{2\pi} \cos\alpha = 279.15\text{ V}$

b) $\mathrm{V_{RMS}} = \left[\frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \mathrm{V_m^2} \sin^2 \omega t \; d(\omega t)\right]^{1/2} = 291.48\text{ V}$

c) Form factor $= \frac{\mathrm{V_{RMS}}}{\mathrm{V_o}} = 1.0441$

d) Ripple factor $= \sqrt{FF^2 - 1} = 0.30$

Problem 5.2

Solution

e) Average load current, $I_o = \frac{V_o}{R} = \frac{279.15}{10} = 27.91\ A$

f) RMS load current, $I_{RMS} = \frac{V_{RMS}}{R} = \frac{291.48}{10} = 29.14\ A$

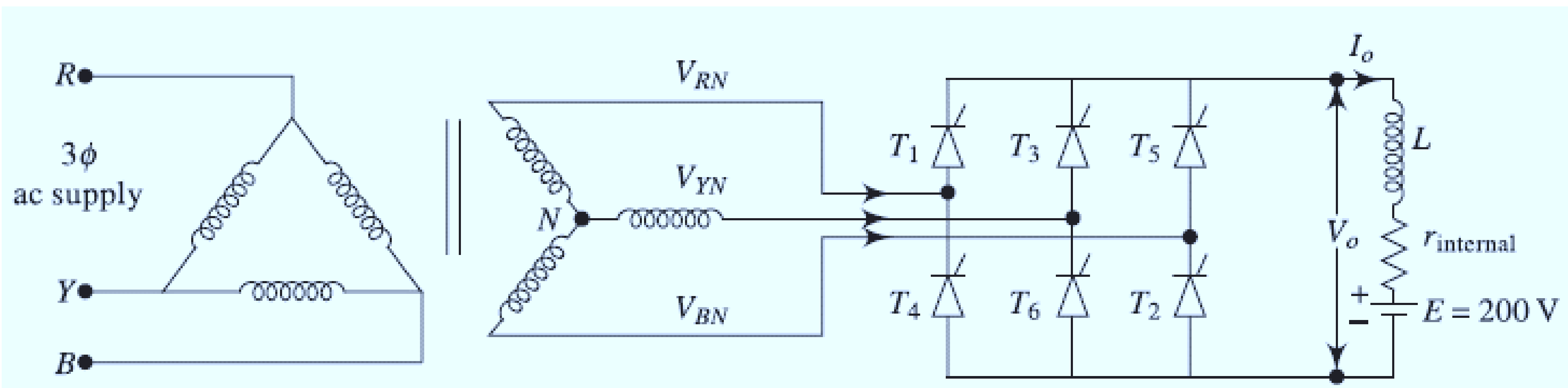
g) The dc output power, $P_o = V_o I_o = 7792.47\ W$

h) AC power supplied, $P_{AC} = I_{RMS}{}^2 R = 8696.05\ W$

i) Rectification efficiency, $\eta = \frac{P_o}{P_{AC}} = 89.6\ \%$

Problem 5.3

Q3. A three-phase full-wave controlled rectifier is used to charge 200 V battery from a 3-phase, 220 V, 50-Hz ac supply. Assume the internal resistance of battery is 0.5Ω and a inductance is connected in series with a battery so that the 10 A constant charging current flows. A) Determine the firing angle of converter and the input power factor. B) If power flows from battery to ac side, what will be the firing angle of converter?



Problem 5.3

Solution

Phase voltage $= \frac{\text{Line Voltage}}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127.02$ $\quad V_m = \sqrt{2} \times 127 = 179.63$

a) The output voltage of the rectifier when it is charging battery is

$$V_o = E + I_o r_{internal} = 200 + 10 \times 0.5 = 205\, V$$

At firing angle α, the average dc output voltage is

$$V_o = 205 = \frac{3\sqrt{3}V_m}{\pi} \cos\alpha = 297.25 \cos\alpha \;\rightarrow\; \alpha = 46.39^0$$

RMS supply current, $I_s = \sqrt{\frac{2}{3}} I_o = 8.164\, A$

Problem 5.3

Solution

a) Power delivered to load, $P_o = EI_o + {I_o}^2 r_{internal} = 2050\ W$

We know that, $\sqrt{3}VI\cos\varphi = P_o \rightarrow \cos\varphi = \frac{P_o}{\sqrt{3}VI} = \frac{2050}{\sqrt{3}\times 220\times 8.16} = 0.65$

b) If the battery is used to deliver power to ac side, the output voltage

$V_o = E - I_o r_{internal} = 200 - 10\times 0.5 = 195\ V$

The output voltage will be negative.

At firing angle α, the average dc output voltage is

$V_o = -195 = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha = 297.25\cos\alpha \rightarrow \alpha = 130.99^0$

See you in next class (Week 8–April 07th)

- **No lecture next week** – Week 7 is a non-teaching week
- All the Best for Lab session – Lab materials will be soon uploaded in the module page!
- Lab – 2 students/group
- **Lab groups** – Allocate randomly or you choose??
- Lab schedule

Week8	Lab1	Monday	2:00-4:30	Group 1-8
			4:30-7:00	Group 9-16
		Wednesday	2:00-4:30	Group 17-24
			4:30-7:00	Group 25-32
		Thursday	2:00-4:30	Group 33-40
			4:30-7:00	Group 41-48
		Friday	2:00-4:30	Group 49-56
			4:30-7:00	Group 57-64
Week9	Lab2	Monday	2:00-4:30	Group 1-8
			4:30-7:00	Group 9-16
		Wednesday	2:00-4:30	Group 17-24
			4:30-7:00	Group 25-32
		Thursday	2:00-4:30	Group 33-40
			4:30-7:00	Group 41-48
		Friday	2:00-4:30	Group 49-56
			4:30-7:00	Group 57-64

The End