Xi'an Jiaotong-Liverpool University
西交利物浦大学

MEC208 Instrumentation and Control System

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Dr. Bangxiang Chen

Email: Bangxiang.Chen@xjtlu.edu.cn

Office: SC554E

Department of Mechatronics and Robotics

School of Advanced Technology

Lecture 12

Today's outline

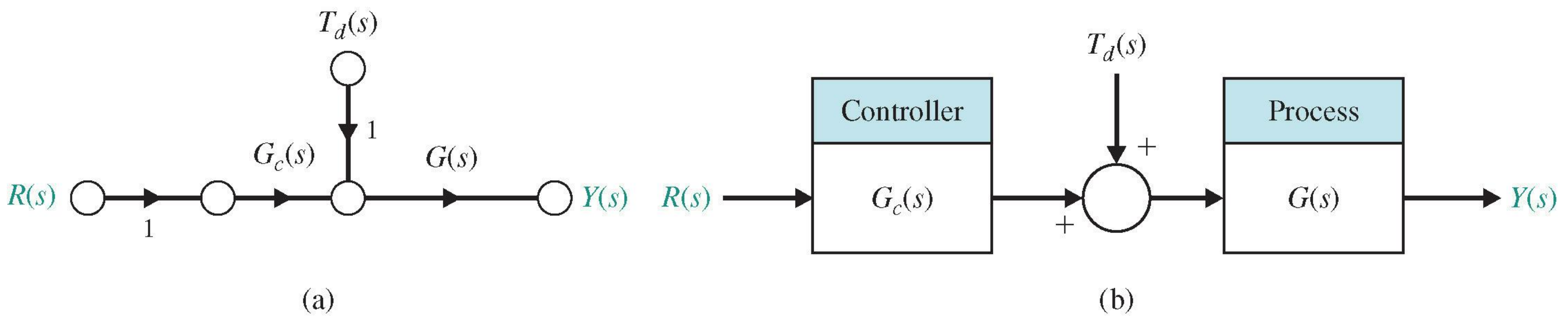
Feedback Control System Characteristics

- ❑ Error Signal Analysis
- ❑ Sensitivity of Control System to Parameter Variations
- ❑ Disturbance Rejection and Measurement Noise Attenuation
- ❑ Control of the Transient Response and Steady-state Error
- ❑ "Cost" of Feedback

Open-loop (OL) Control System

An open-loop control system operates without feedback and directly generates the output in response to an input signal.

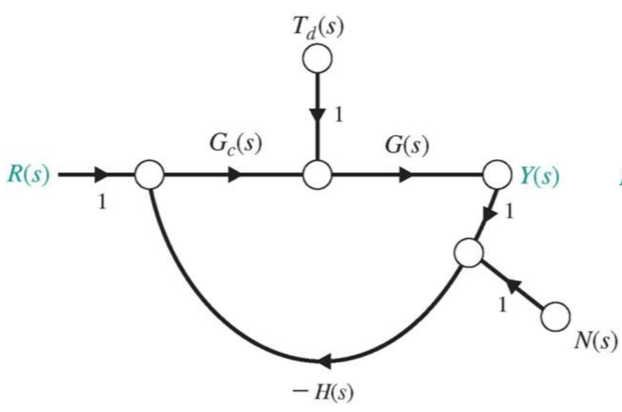
- Disturbance, $T_d(s)$, directly influences the output $Y(s)$. In the absence of feedback, the control system **may be** highly sensitive to disturbances and to accuracy and variations of $G(s)$ parameters.



Closed-loop (CL) Control System

A closed-loop control system uses a set of measurement from the system and compares them with the **desired output reference(s)** to generate an **error signal(s)** that is used by the controller to manipulate the plant input(s).

- The introduction of feedback to improve the control system may be necessary, depending on the application requirement;
- Feedback control exists beyond physical systems; it can be found in biological and physiological systems (i.e., heart rate control).



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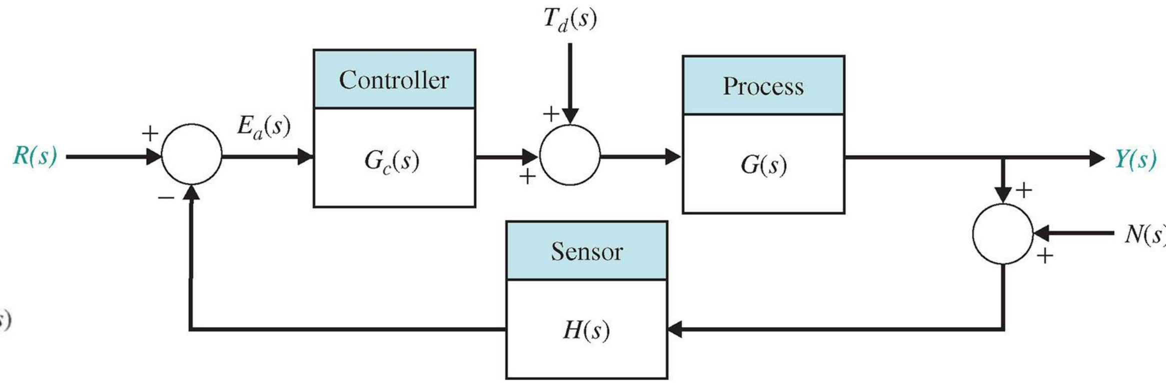
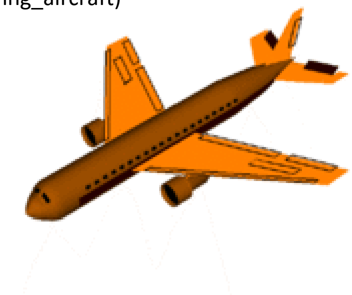


(b)

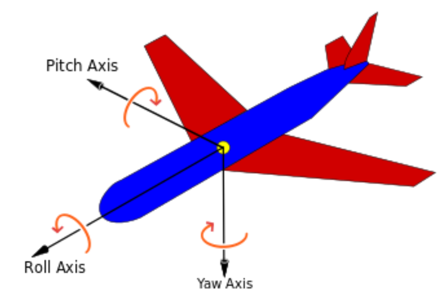
A **negative** feedback closed-loop control system

Why do we need closed-loop control: “Advantages”

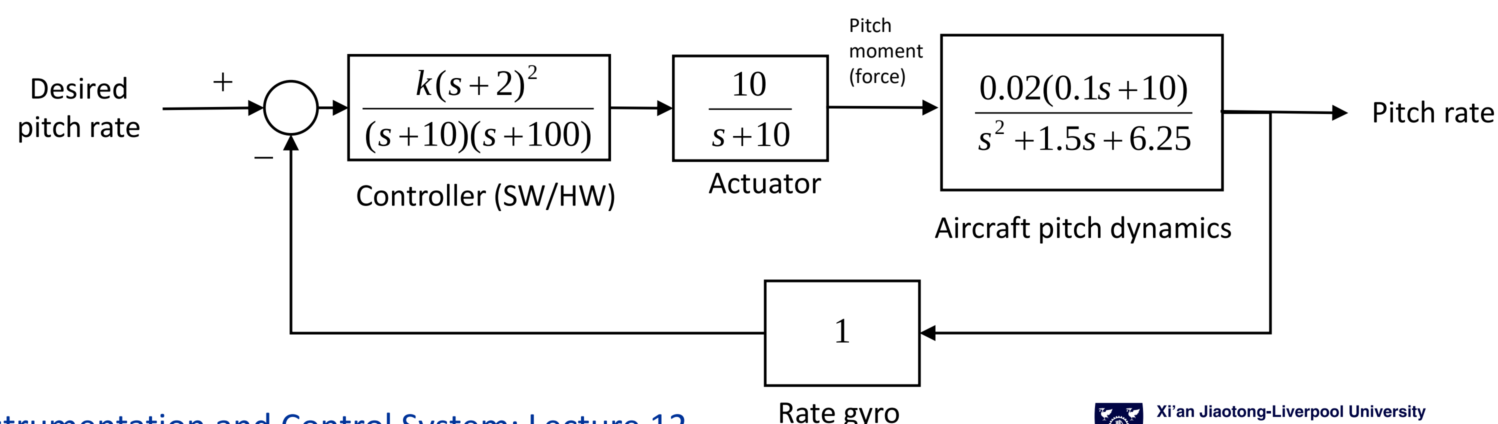
https://en.wikipedia.org/wiki/Flight_dynamics_(fixed-wing_aircraft)



- Decrease sensitivity of the system to parameter variations
- Improve disturbance rejection
- Improve measurement noise attenuation
- Reduce or eliminate steady-state error of the system
- Define/produce the desired transient response of the system



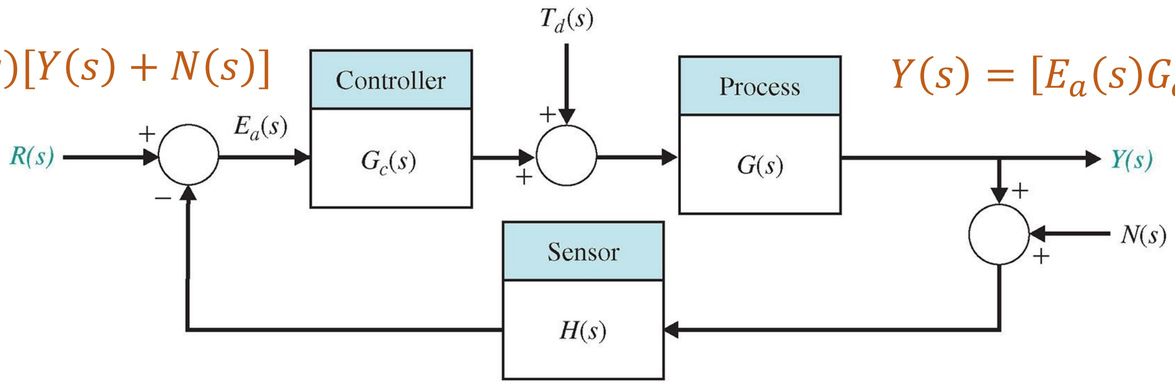
E.g.



Error Signal Analysis

$E_a(s) = R(s) - H(s)[Y(s) + N(s)]$

$Y(s) = [E_a(s)G_c(s) + T_d(s)]G(s)$



(b)

Tracking error definition: $E(s) = R(s) - Y(s)$

To facilitate our discussion, unity feedback system is assumed, i.e., $H(s) = 1$.

The output can be obtained from the block diagram:

$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}R(s) + \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) - \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s)$$

Therefore:

$$E(s) = R(s) - Y(s) = \frac{1}{1 + G_c(s)G(s)}R(s) - \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) + \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s)$$

$$E(s) = \frac{1}{1 + L(s)}R(s) - \frac{G(s)}{1 + L(s)}T_d(s) + \frac{L(s)}{1 + L(s)}N(s)$$

where **loop gain** $L(s) = G_c(s)G(s)H(s) = G_c(s)G(s)$

Sensitivity Functions

$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} R(s) + \frac{G(s)}{1 + G_c(s)G(s)} T_d(s) - \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} N(s)$$

$$E(s) = \frac{1}{1 + L(s)} R(s) - \frac{G(s)}{1 + L(s)} T_d(s) + \frac{L(s)}{1 + L(s)} N(s)$$

Define (Closed loop characteristic function): $F(s) = 1 + L(s)$

Sensitivity Function
(i.e., towards *R*)

$$\boldsymbol{S(s) = \frac{1}{F(s)} = \frac{1}{1 + L(s)}}$$

Complementary Sensitivity Function
(i.e., towards *N*)

$$\boldsymbol{C(s) = \frac{L(s)}{1 + L(s)}}$$

NOTE: $\boldsymbol{S(s) + C(s) = 1}$

$$E(s) = S(s)R(s) - S(s)G(s)T_d(s) + C(s)N(s)$$

Sensitivity towards Parameter Variations

A plant/process, represented by $G(s)$, is subject to a changing environment, aging, uncertainty in the exact values of the process parameters, and other factors that affect the process.

- In an OL control system, all these errors and changes result in a changing and inaccurate output;
- However, a CL system senses the change in the output due to the process changes and attempts to correct the output.

A primary advantage of a CL feedback control system is its ability to reduce the system's sensitivity to parameter variation.

How to Reduce Sensitivity towards Param. Var.?

To analyze influences of $G(s)$ param. variation on tracking error under CL control, we first assume $T_d(s) = N(s) = 0$

Suppose the process (or plant) undergoes a change such that the true plant model is $G(s) + \Delta G(s)$, we then consider the tracking error $E(s)$ due to $\Delta G(s)$.

$$E(s) + \Delta E(s) = \frac{1}{1 + G_c(s)[G(s) + \Delta G(s)]} R(s)$$

Then the change in the tracking error is:

$$\Delta E(s) = \frac{-G_c(s)\Delta G(s)}{[1 + G_c(s)G(s) + G_c(s)\Delta G(s)](1 + G_c(s)G(s)\,)} R(s)$$

Since usually $G_c(s)G(s) \gg G_c(s)\,\Delta G(s)$ for all complex frequencies of interest, we have

$$\Delta E(s) = \frac{-G_c(s)\Delta G(s)}{[1 + L(s)]^2} R(s)$$

Therefore, the change in tracking error is reduced by the factor $1 + L(s)$.

For large $L(s)$, we have $1 + L(s) \approx L(s)$, then

$$\Delta E(s) \approx -\frac{1}{L(s)} \frac{\Delta G(s)}{G(s)} R(s)$$

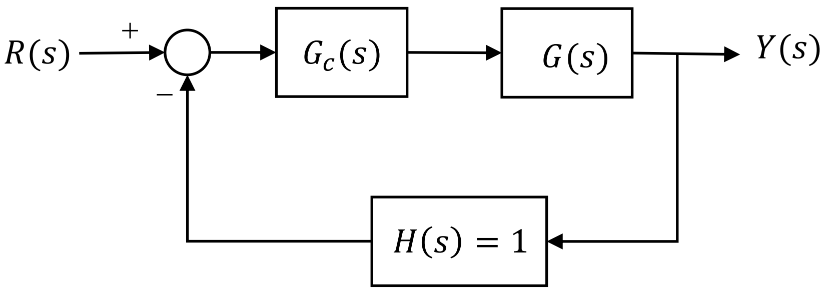
Large $L(s)$ implies smaller sensitivity.

Definition of System Sensitivity

By definition: $S = \dfrac{\partial T/T}{\partial G/G}$, where system transfer function $T(s) = \dfrac{Y(s)}{R(s)}$

In the limit, for small incremental changes:

$$S = \frac{\partial T/T}{\partial G/G} = \frac{\partial \ln T}{\partial \ln G}$$



System sensitivity is the ratio of the change in the system transfer function *T*(*s*) to the change of a process transfer function *G*(*s*) (or parameter) for a small incremental change.

Sensitivity for OL system: 1

Sensitivity for CL system: since $T(s) = \dfrac{G_c(s)G(s)}{1 + G_c(s)G(s)}$

$$S_G^T = \frac{\partial T \cdot G}{\partial G \cdot T} = \frac{G_c}{[1 + G_c G]^2} \cdot \frac{1 + G_c G}{G_c} \quad \rightarrow \quad S_G^T = \frac{1}{1 + G_c G}$$

To determine the influence of process parameter α (of $G(s)$), use chain rule:

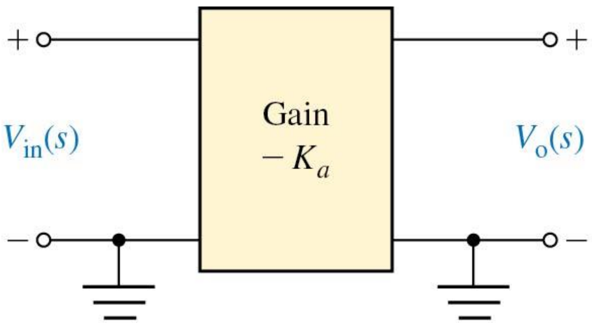
$$S_\alpha^T = S_G^T \, S_\alpha^G$$

Example 12.1: Feedback Amplifier

$$S = \frac{\partial T / T}{\partial G / G}$$

$$\beta = \frac{R_2}{R_1}$$

Open-loop Amplifier



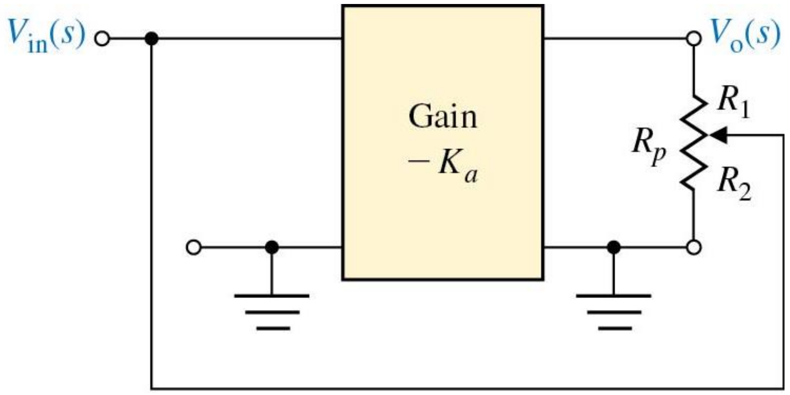
$$V_0 = -K_a V_{in}$$

$$T = \frac{V_0}{V_{in}} = -K_a$$

Sensitivity to the changes in the amplifier gain is:

$$S_{K_a}^{T} = 1$$

CL amplifier with Feedback



Assume that the transfer function is

$$T = \frac{V_0}{V_{in}} = \frac{-K_a}{1 + K_a \beta}$$

Through chain rule, we know that:

$$S_{K_a}^{T} = S_G^T \, S_{K_a}^{G} = \frac{1}{1+K_a\beta}$$

For $K_a = 10^4$ and $\beta = 0.1$:

$$S_{K_a}^{T} = \frac{1}{1 + 10^3} = 0.001$$

Disturbance Rejection

Feedback control reduces the negative effect of disturbance signals:

- A disturbance signal is an unwanted input signal that affects the output signal.
- Many control systems are subject to extraneous disturbance signals that cause the system to provide an inaccurate output.
  - ➢ Electronic amplifiers have inherent noise generated within the integrated circuits or transistors;
  - ➢ Radar antennas are subject to wind gusts;
  - ➢ Many systems generate unwanted distortion signals due to nonlinear elements.
- The benefit of feedback systems is that the effect of distortion, noise, and unwanted disturbances can be effectively reduced.

To analyze rejection of disturbance, assume $R(s) = N(s) = 0$.

$$E(s) = -S(s)G(s)T_d(s) = -\frac{G(s)}{1 + L(s)}T_d(s)$$

For a fixed $G(s)$ and a given $T_d(s)$, as the loop gain $L(s)$ increases, the effect of $T_d(s)$ on the tracking error decreases. For good disturbance rejection, we require a large loop gain over the frequencies of interest associated with the expected disturbance signals.

Measurement Noise Attenuation

A noise signal that is prevalent in many systems is the noise generated by the **measurement sensor.**

To analyze attenuation of measurement noise, assume $R(s) = T_d(s) = 0$.

$$E(s) = C(s)N(s) = \frac{L(s)}{1 + L(s)} N(s)$$

As the loop gain $L(s)$ decreases, the effect of $N(s)$ on the tracking error decreases. For effective measurement noise attenuation, we need a small loop gain over the frequencies associated with the expected noise signals.

How to realize disturbance rejection and measurement noise attenuation at the same time?

- In practice, disturbances are often at low frequencies, while measurement noise signals are often at high frequencies.
- Therefore, the controller should be of high gain at low frequencies and low gain at high frequencies.

Control of Transient Response

One of the most important characteristics of control systems is their **transient response**, which is a function of time.

Another purpose of control systems is to provide a desired satisfactory transient response:

- If an OL control system does not provide a satisfactory transient response, then the process, $G(s)$, may need to be replaced with a more suitable process;
- By contrast, a CL system can often be adjusted to yield the desired response by adjusting the feedback loop parameters (e.g., controller and feedback path parameters).

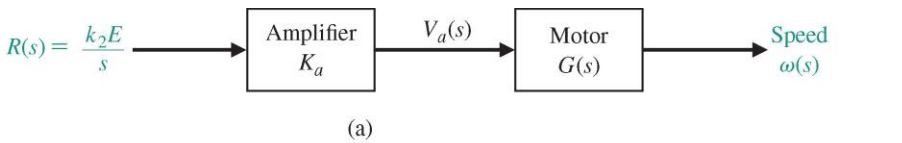
A feedback control system is valuable because it provides the engineer with the ability to **adjust/manipulate** the transient response.

Example 12.2: Speed Control System

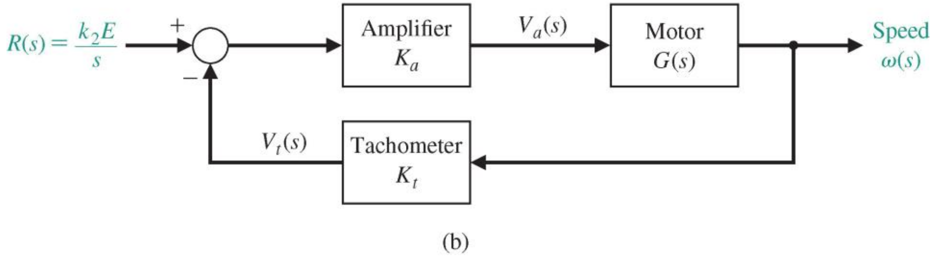
A speed control system, is often used in industrial processes to move materials and products, in one of the two modes:

(a) OL control system;

(b) CL control with feedback.



(a)



(b)

Open-loop:
$$\frac{\omega(s)}{V_a(s)} = G(s) = \frac{K_1}{\tau_1 s + 1}$$
$$V_a(s) = \frac{k_2 E}{s} K_a$$

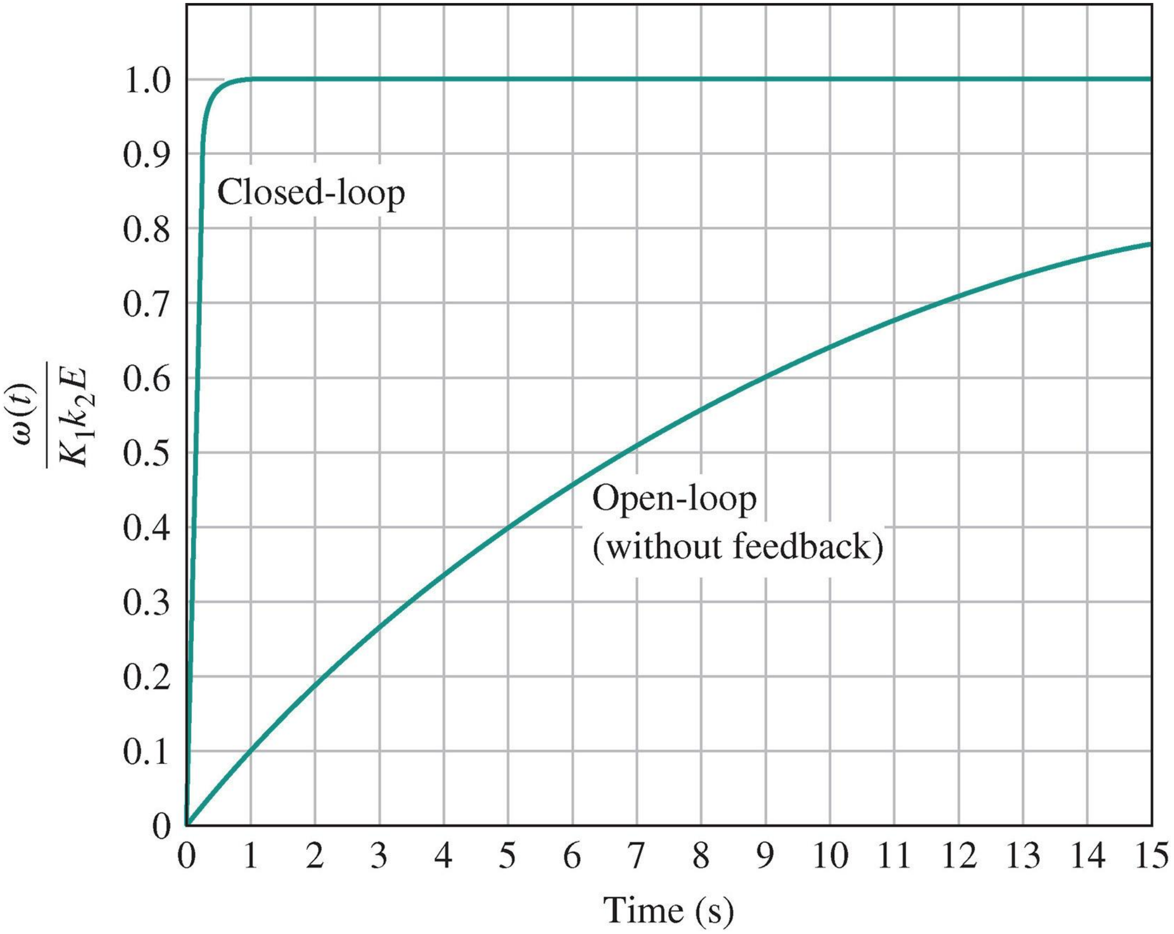
$$\omega(s) = G(s) V_a(s) \longrightarrow \omega(t) = K_1 (k_2 E)(1 - e^{-t/\tau_1}) K_a$$

Closed-loop:
$$\frac{\omega(s)}{R(s)} = \frac{K_a G(s)}{1 + K_a K_t G(s)} = \frac{K_a K_1}{\tau_1 s + 1 + K_a K_t K_1} = \frac{K_a K_1 / \tau_1}{s + (1 + K_a K_t K_1)/\tau_1}$$

$$\omega(t) = \frac{K_a K_1}{1 + K_a K_t K_1} (k_2 E)(1 - e^{-pt})$$
$$p = (1 + K_a K_t K_1)/\tau_1$$

Transient Response

The response of the open-loop and closed-loop speed control system when $\tau = 10$ and $K_1 K_a K_t = 100$. The time to reach 98% of the final value for the open-loop and closed-loop system is 40 seconds and 0.4 seconds, respectively.



Steady-state Error (and Output)

The **steady-state error (output)** is the error (output) value after the transient response has decayed, leaving only the continuous response.

Final Value Theorem (only for stable system):

$$\boxed{\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)}$$

Assume a unit step input as a comparable input ($r(t) = 1, t > 0; R(s) = \frac{1}{s}$):

Open-loop:

$$E_{OL}(s) = R(s) - Y(s) = \big(1 - G_c(s)G(s)\big)R(s)$$

$$e_{OL}(\infty) = \lim_{s\to 0} s\big(1 - G_c(s)G(s)\big)\left(\frac{1}{s}\right) = 1 - G_c(0)G(0)$$

To calculate **steady-state output** towards input *R*, simply use:

$$\boxed{\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sT_{CL}R(s)}$$

Closed-loop (assume $T_d(s) = N(s) = 0$, and $H(s) = 1$):

$$E_{CL}(s) = \frac{1}{1 + G_c(s)G(s)}R(s)$$

$$e_{CL}(\infty) = \lim_{s\to 0} s\frac{1}{1+G_c(s)G(s)}\frac{1}{s} = \frac{1}{1+G_c(0)G(0)}$$

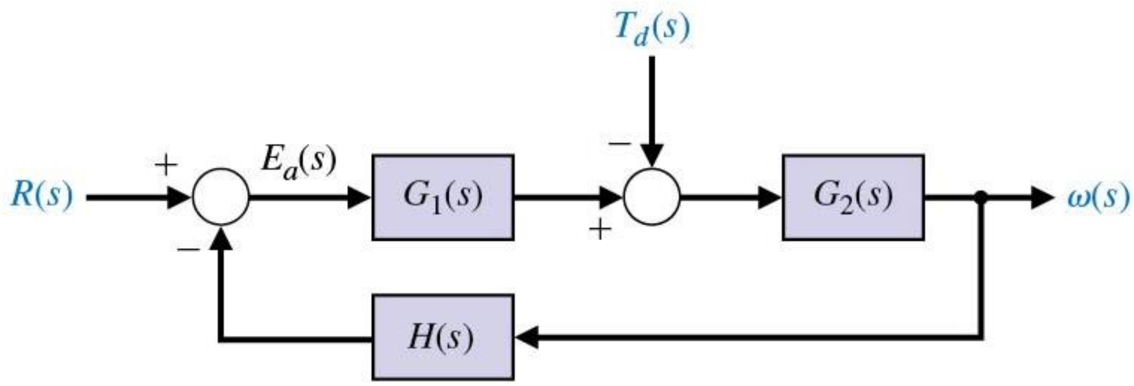
Large $L(0) = G_c(0)G(0)$ will lead to small steady-state error.

"Cost" of Feedback Control

- **Increased number of components and complexity in the system.**
  - To add feedback, it is necessary to consider several "physical" feedback components, e.g. sensors. The sensor is often the most expensive component in a control system, which may introduce noise, inaccuracy, and robustness issues.
- **Loss of Gain.**
  - Loop gain: $G_c(s)G(s)$
  - Closed-loop gain: $\frac{G_c(s)G(s)}{1+G_c(s)G(s)}$
- **Introduction of the possibility of instability.**
  - Even if an open-loop system is stable, the closed-loop system may not be always stable (will be discussed in later chapters).

Example 12.3 (in-class)

Consider the following system, where $G_1(s) = \frac{5}{s}$, $G_2(s) = \frac{1}{s(s+p)}$, and $H(s) = \frac{1}{10}$.
Assuming zero disturbance, calculate the sensitivity of the CL system $T(s) = \frac{\omega}{R}$ to changes in the G_2's parameter p.



Thought process: (1) Question asking for S_p^T (suggest to use $\frac{\partial T/T}{\partial p/p}$); (2) derive T_{CL}, $\frac{\partial T}{\partial p}$; (3) find S_p^T.

(Reminder: calculation steps must be shown/included in the exam)

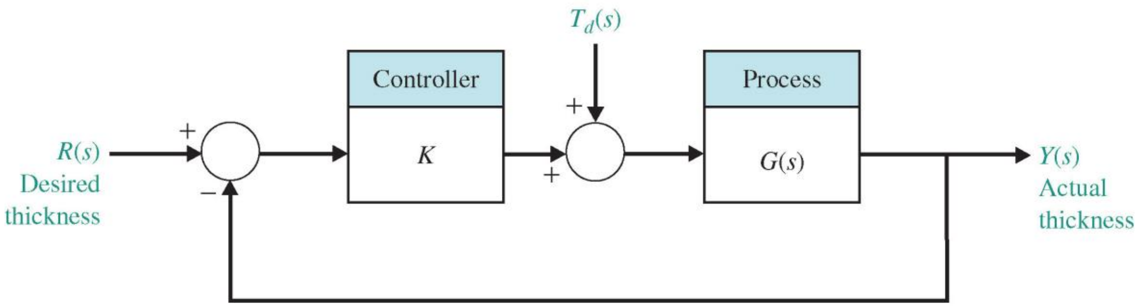
Final answer: $S_p^T = -\frac{2ps^2}{2s^3+2ps^2+1}$

Example 12.4 (in-class)

Consider the following system, where $G(s) = \frac{1}{s(s+50)}$,

(a) calculate the sensitivity of the CL system (with zero T_d) towards the changes in the controller gain K;

(b) with $K = 900$, estimate the output's steady-state error towards a unit-step input $R(s) = \frac{1}{s}$.



Thought process: (1) Question asking for $S_K^{T_{cl}}$(use either $\frac{\partial T_{CL}/T_{CL}}{\partial K/K}$ or chain rule $S_G^{T_{CL}} \times S_K^G$); (2) derive T_{CL}, $\frac{\partial T_{CL}}{\partial K}$; (3) find $S_K^{T_{cl}}$; (4) derive and calc. steady- state error $\lim_{s \to 0} sE$ with $R(s) = \frac{1}{s}$.

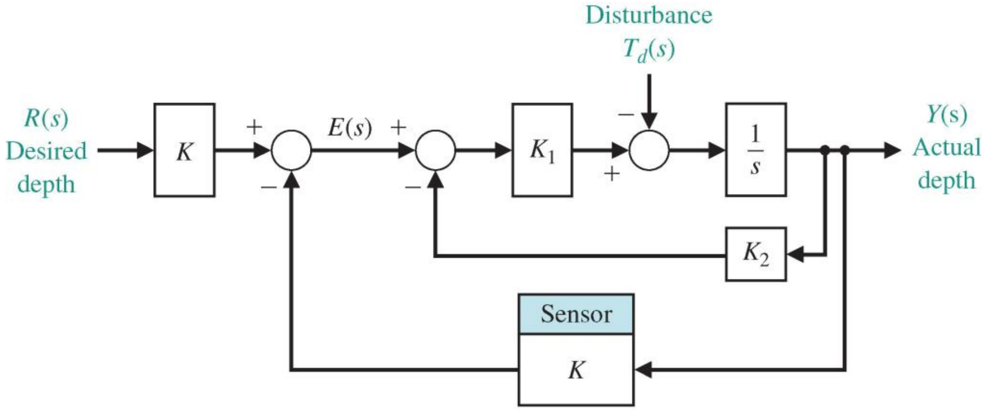
(Reminder: calculation steps must be shown/included in the exam)

Final answer: (a) $S_K^{T_{cl}} = \frac{s(s+50)}{s^2+50s+K}$, (b) 0.

Example 12.5 (in-class)

Consider the following system:

1) Compute the transfer function $T(s) = \frac{Y(s)}{R(s)}$;
2) Determine the sensitivity $S_{K_1}^T$ and $S_{K_2}^T$;
3) Calculate the output's steady-state response due to unit-step input $R(s) = 1/s$;
4) Calculate the steady-state error, $\lim_{s\to 0} sE$, due to unit-step disturbance $T_d(s) = 1/s$.



Thought process: (1) manipulate block diagram to obtain $\frac{Y}{R}$; (2) Calculate $S_{K_1}^T = \frac{\partial T/T}{\partial K_1/K_1}$and $S_{K_2}^T = \frac{\partial T/T}{\partial K_2/K_2}$; (3) $y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY$; (3) derive $T_1 = \frac{Y}{T_d}$, calc. steady- state error $\lim_{s\to 0} sE$

(Reminder: calculation steps must be shown/included in the exam)

Final answer: $T = \frac{KK_1}{s+K_1(K_2+K)}$, $S_{K_1}^T = \frac{s}{s+K_1(K_2+K)}$, $S_{K_2}^T = -\frac{K_1K_2}{s+K_1(K_2+K)}$, $y_{ss} = \frac{K}{K_2+K}$,

$T_1 = -\frac{\frac{1}{s}}{1+\frac{1}{s}\cdot K_1(K_2+K)}$, $\lim_{s\to 0} sE = \lim_{s\to 0} s\left(0 - T_1\frac{1}{s}\right) = \frac{1}{K_1(K_2+K)}$

[**Note**: this example has a –ve T_d input, different from +ve input in the figure in pg. 8; the sign of T_1 in this exercise should be derived correctly]

Concluding Remarks

- **What has been covered:** "Feedback Control System Characteristic"
  - Feedback control system
  - Error signal analysis
  - Parameter variation
  - Disturbance
  - Measurement noise
  - Transients, steady-state error
  - Cost of feedback