CAN209 Advanced Electrical Circuits and Electromagnetics

Lecture 3 Dipoles & Nature of Materials

Dr. Zhenzhen Jiang

Assistant Professor, School Assessment Coordinator

Faculty: School of Advanced Technology

Email: zhenzhen.jiang02@xjtlu.edu.cn

Xi'an Jiaotong-Liverpool University
西交利物浦大学

OUTLINE

- Dipoles
  - Electric Dipole
  - Magnetic Dipole
- Nature of Materials
  - Conductors
  - Dielectrics and Permittivity
- Nature of Magnetic Materials
  - The Atomic Model
  - Classification (Paramagnetic, Diamagnetic, Ferromagnetic)
  - Permeability

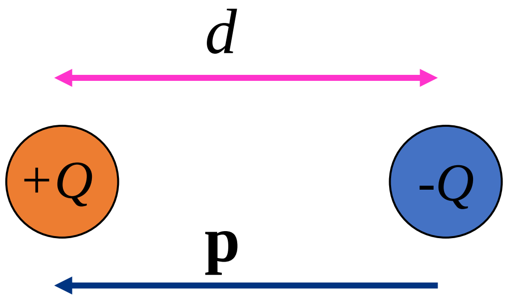
1.1 ELECTRIC DIPOLE

A pair of charges of equal magnitude but opposite sign is called an ***electric dipole***.

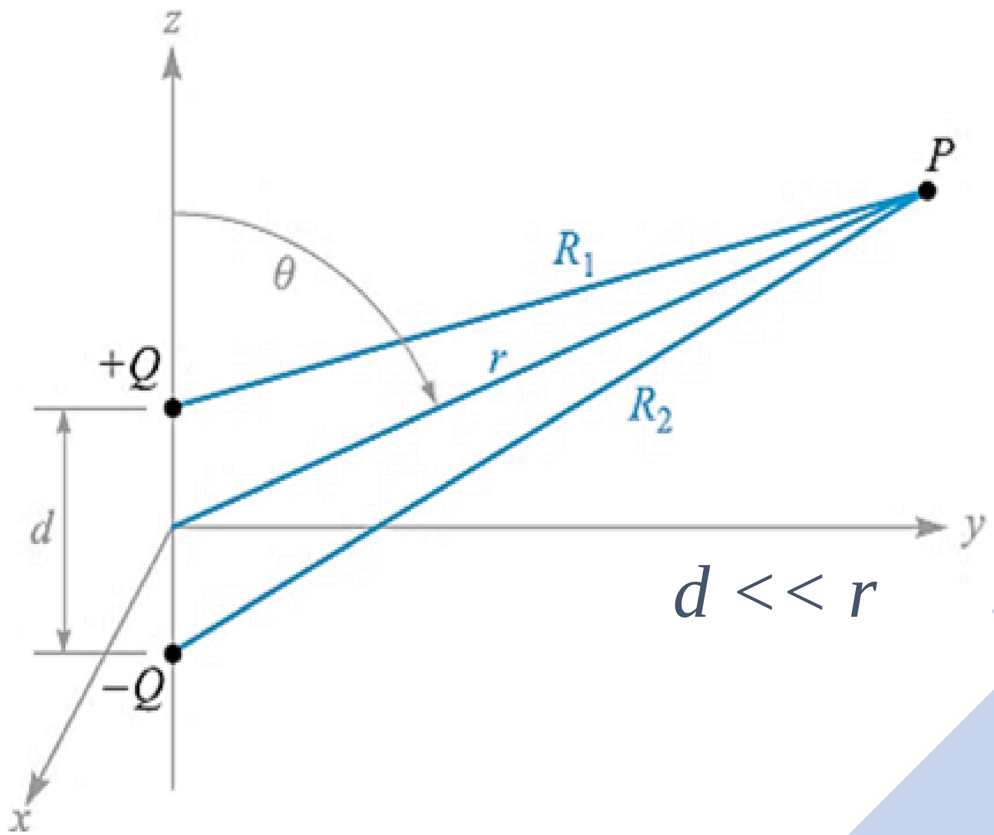
Define the ***dipole moment*** as $Q\vec{d}$ and assign it the symbol $\vec{p}$ (a measure of the separation of negative and positive charges in a system).

$$\vec{p} = Q\vec{d}$$

$\vec{d}$ is the vector length directed **from $-Q$ to $+Q$**.



When the charges are symmetrically placed along the z axis, and the point of observation is very far away (d is much smaller compare with r), what will the potential and electric field be like?



1.1 ELECTRIC DIPOLE

Voltage

The voltage at observation point P is:

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

When P is far enough, $R_1 \parallel r \parallel R_2$

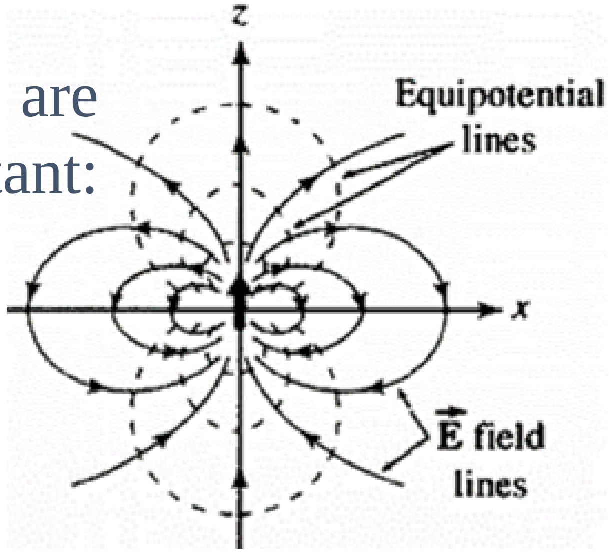
- $R_1 R_2 \approx r^2$
- $R_2 - R_1 = d\cos\theta$

The final result of voltage is:

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{d\cos\theta}{r^2}\right) = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

The equipotential surfaces are obtained by letting V be constant:

$$\frac{\cos\theta}{r^2} = constant$$



Electric-field Intensity

Since $\vec{E} = -\nabla V$, the electric field intensity is:

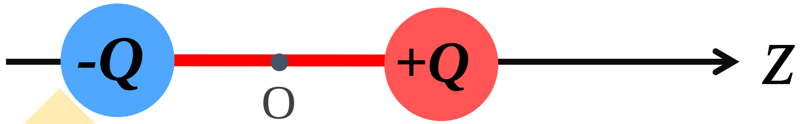
$$\vec{E} = -\nabla V = -\left(-\frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\hat{r} - \frac{Qd\sin\theta}{4\pi\varepsilon_0 r^3}\hat{\theta}\right)$$

$$= \frac{Qd}{4\pi\varepsilon_0 r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$$

The electric field depends on the product of the charge & the separation distance (dipole moment).

QUIZ 1.1

An electron and a proton separated by a distance of 10^{-11} m are symmetrically arranged along the + z axis with z = 0 as its bisecting plane in free space. Given that the amount of charge each of them carried is $1.6 \times 10^{-19}\mathrm{C}$, determine the electric dipole moment and the potential at point P (3, 4, 12) m in the Cartesian coordinate system.

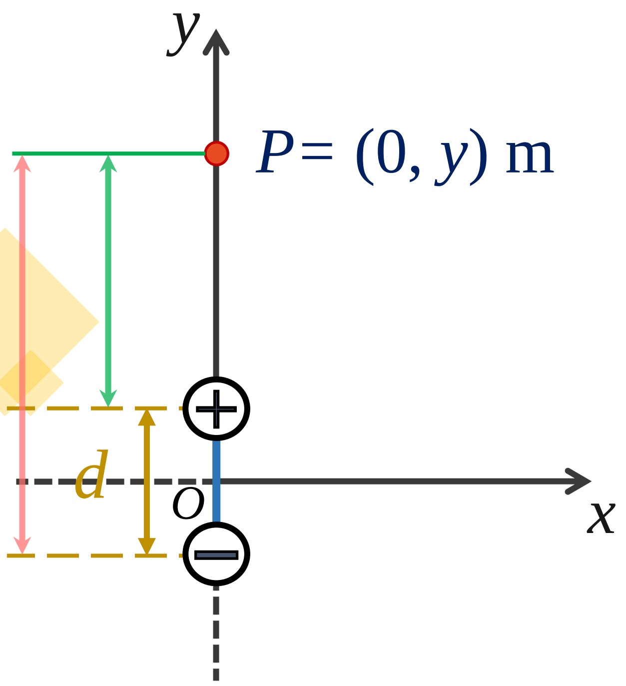


QUIZ 1.2

An electric dipole is centred at the origin with the dipole moment along the +*y* axis. Derive an approximate expression for the electric field at a point *P* on the *y* axis where *y* is much larger than *d*.

Binomial expansion for the case $-1 < x < 1$:

$$(1+x)^n \cong 1 + nx + n(n-1)\frac{x^2}{2} + \cdots$$



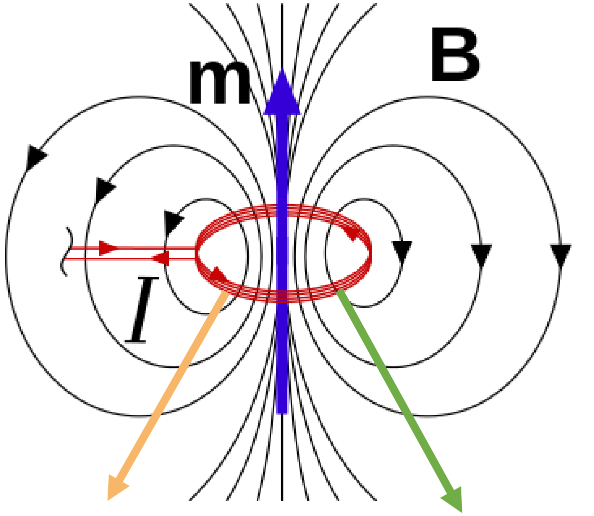
OUTLINE

- Dipoles
  - Electric Dipole
  - Magnetic Dipole
- Nature of Materials
  - Conductors
  - Dielectrics and Permittivity
- Nature of Magnetic Materials
  - The Atomic Model
  - Classification (Paramagnetic, Diamagnetic, Ferromagnetic)
  - Permeability

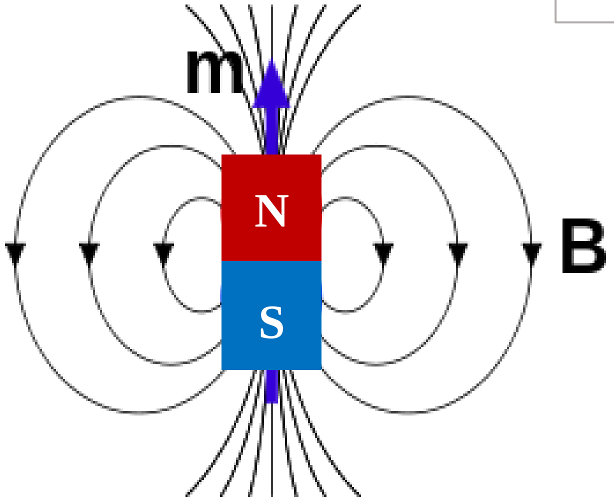
1.2 MAGNETIC DIPOLE

The magnetic fields are generated by:

direction of the dipole moment?



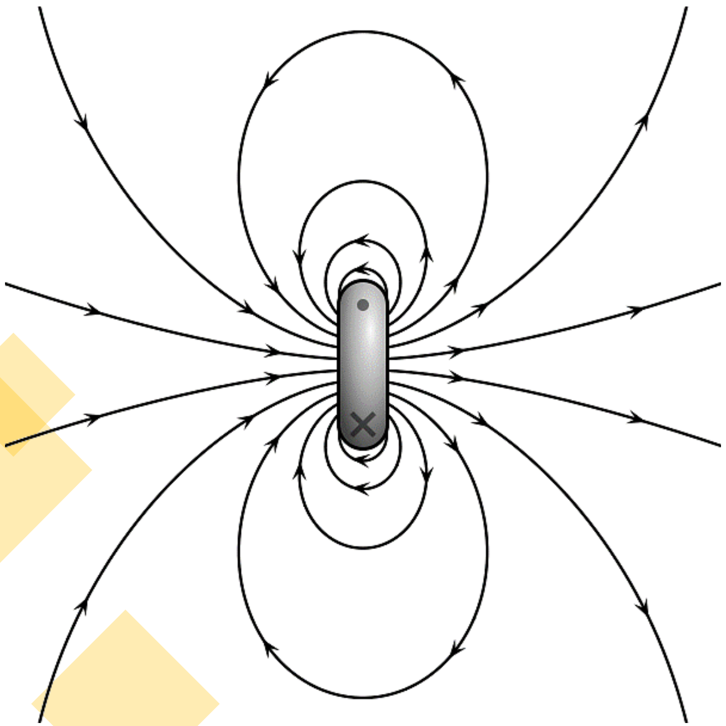
Area: $\vec{S}$ Turns: N



$\vec{B}$ is the <u>magnetic field flux density</u>, describing the strength of the magnetic intensity generated by the dipole;

$\vec{m}$ is the <u>magnetic dipole moment</u>, describing the property of the dipole.

1.2 MAGNETIC DIPOLE



A current loop that goes into the page at the x and comes out at the dot produces a magnetic field (lines).
The north pole is to the right and the south to the left.

Amperian loop model:

- Since all electric currents attract and repel each other like magnets, it was natural to hypothesize that all magnetic fields are due to electric current loops.
- In this model developed by Ampère, the elementary magnetic dipole that makes up all magnets is a sufficiently small amperian loop of current I.
- The dipole moment of this loop is

$$\vec{m} = NI\vec{S}$$

- where $\vec{S}$ is the area of the loop.

The direction of the magnetic moment is in a direction normal to the area enclosed by the current consistent with the direction of the current using the right-hand rule.

OUTLINE

- ➢ Dipoles
  - ✓ Electric Dipole
  - ✓ Magnetic Dipole
- ➢ Nature of Materials
  - ✓ Conductors
  - ✓ Dielectrics and Permittivity
- ➢ Nature of Magnetic Materials
  - ✓ The Atomic Model
  - ✓ Classification (Paramagnetic, Diamagnetic, Ferromagnetic)
  - ✓ Permeability

2.1 CLASSIFICATION

free electrons

According to their electrical properties, classify materials into:

- ***Conductors***: a material that possesses a relatively large number of **free electrons**. The electrons can drift freely in the conductor. Its ability of conducting electric current is described by the conductivity. (Au, Ag, Cu, Al,…..) Large conductivity σ
- ***Dielectrics*** (Insulators): a material **without** free electrons in its lattice structure. In an ideal dielectric, positive and negative charges are so sternly bound that they are inseparable. For perfect dielectrics, it has zero conductivity (Rubber, plastic, glass,….) Tiny conductivity σ
- ***Semiconductors***: In some special materials such as silicon and germanium, a small fraction of the total number of valence electrons are free to move about randomly with the space lattice. NOT interested in this module.

2.2 CONDUCTORS

Basic Properties

1. The **static** electric field intensity inside a conductor is **zero**.

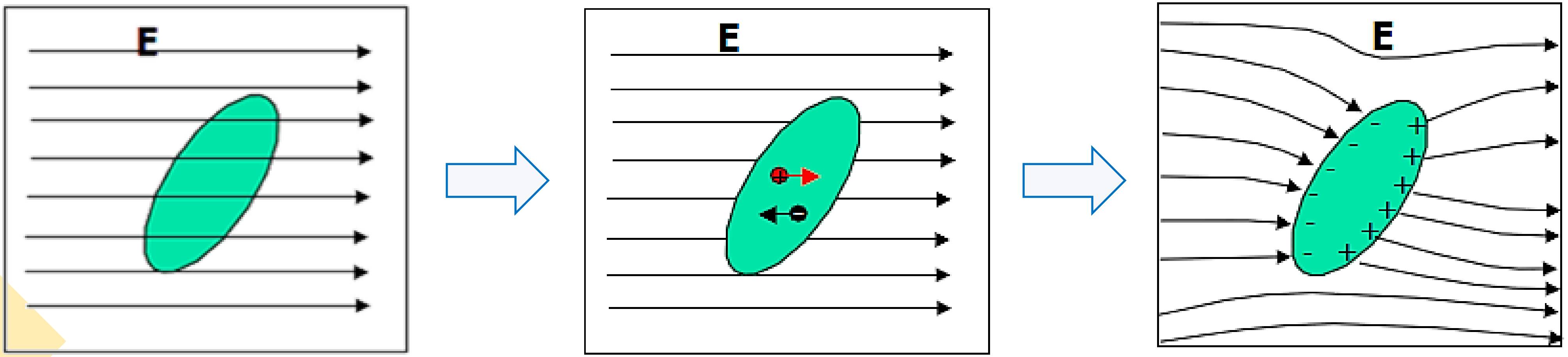
2. The **static** electric field intensity at the surface of a conductor is everywhere directed **normal** to that surface.

3. Conductor's surface is an **equipotential** surface (等势面). The tangential component of the static electric field intensity on the surface is zero.

4. Net charge can **only** reside on the surface when reaches electrostatic equilibrium.

2.2 CONDUCTORS

When tangential component of electric field intensity $\vec{E}_{tan} \neq 0$, the charges will move from where the potential is higher to where the potential is lower. The moving will stop only when $\vec{E}_{tan} = 0$.



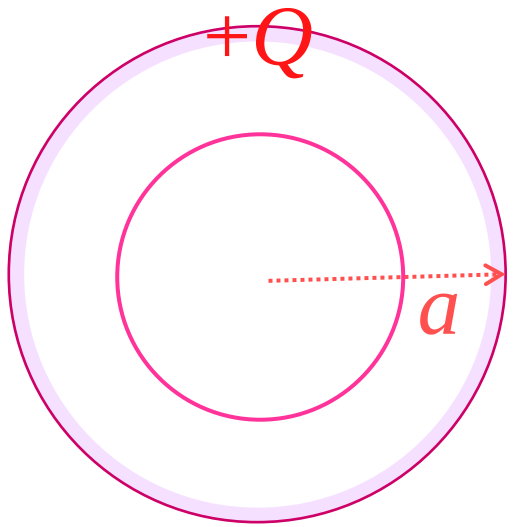
If there are two points inside the conductor, the potential difference is:

$$V_{AB} = -\int_{B}^{A} \vec{E}_{in} \cdot d\vec{l} = 0$$

QUIZ 2.1

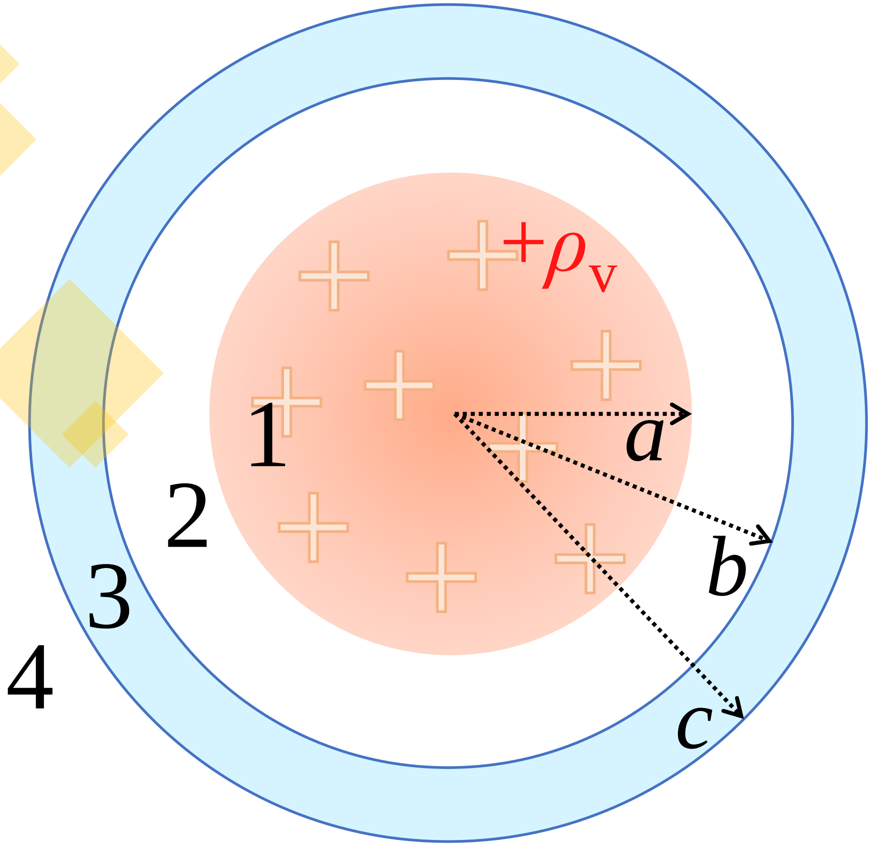
Consider a metallic spherical shell of radius a and charge $+Q$. Find the electric flux density and the electric potential everywhere in free space.

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & r \geq a \\ 0 & r < a \end{cases}$$



QUIZ 2.2

As shown in the Figure, the charge is uniformly distributed in the free space within a spherical region of radius a. An isolated conducting spherical shell is placed concentrically with inner radius b and outer radius c. Determine the electric field intensity in the regions 1, 2, 3, 4.



2.3 DIELECTRICS

$$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} = 8.854 \times 10^{-12}\ \mathrm{F/m}$$

Dielectrics/Insulators: there are not significant numbers of free charges present within them, quartz, Teflon, rubber, glass, …

Polarisation charges: electric charges confined to atoms or molecules.

Under the influence of an electric force, the molecules of a dielectric material experience distortion => being polarised.

For any medium

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

where ε_0 is the permittivity in vacuum (constant; Unit: F/m)

ε_r is the relative permittivity (unitless)

ε is the permittivity of the medium (F/m)

OUTLINE

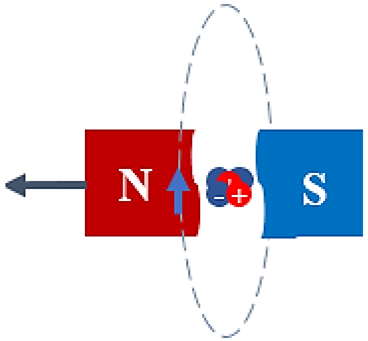
- Dipoles
  - ✓ Electric Dipole
  - ✓ Magnetic Dipole
- Nature of Materials
  - ✓ Conductors
  - ✓ Dielectrics and Permittivity
- Nature of Magnetic Materials
  - ✓ The Atomic Model
  - ✓ Classification (Paramagnetic, Diamagnetic, Ferromagnetic)
  - ✓ Permeability

3.1 ATOMIC MODEL

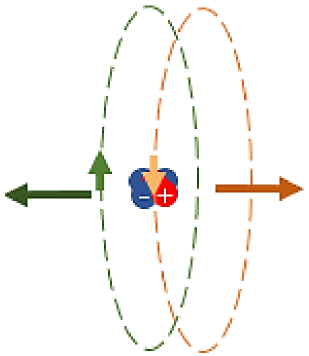
How do magnetic materials affect magnetic field?

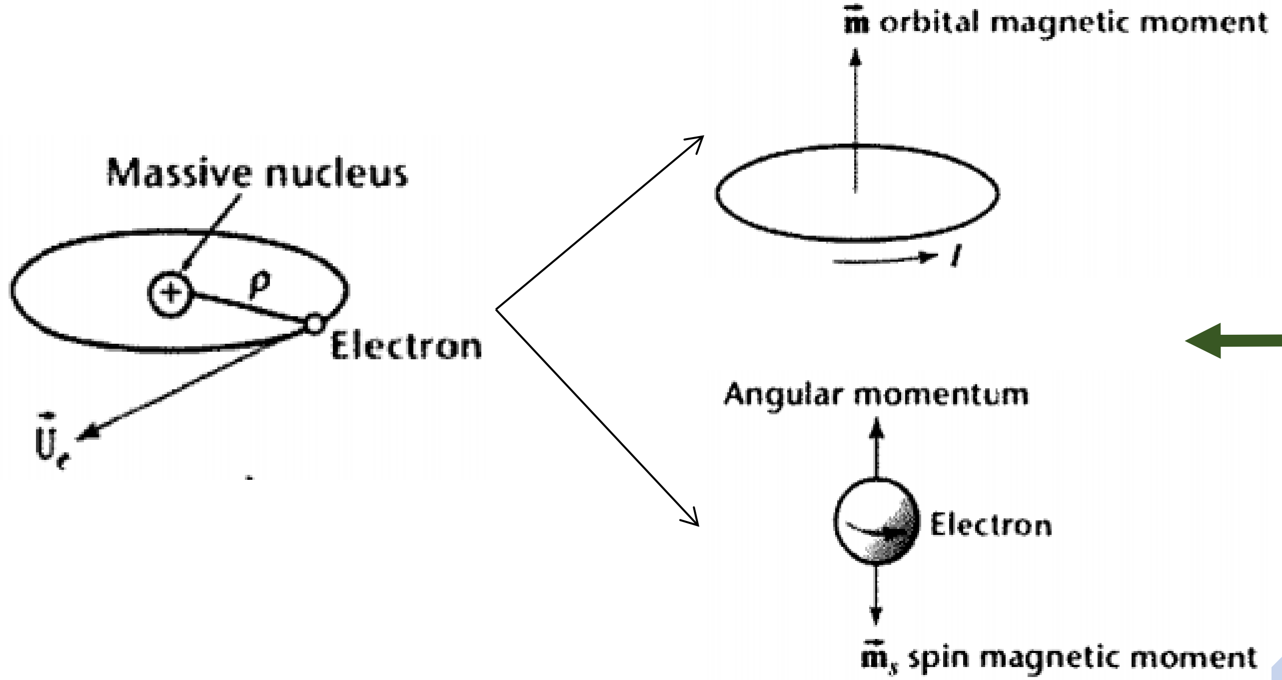
Although accurate quantitative results can only be predicted using quantum theory, the simple atomic model yields reasonable qualitative results and provides a satisfactory theory. The simple atomic model assumes that there is a central positive nucleus surrounded by electrons in various circular orbits.

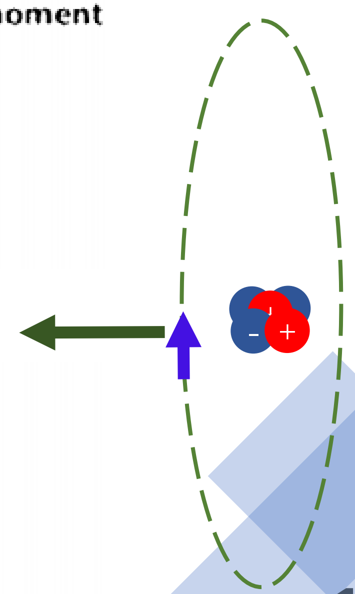
Un-paired



All paired







3.2 CLASSIFICATION

Each atom contains many different component moments, and their combination determines the magnetic characteristics of the material and provides its general **magnetic classification**:

1. Diamagnetic material (抗磁性)
   - produce a magnetisation that opposes the magnetic field.
   - $\vec{B}_{inner} < \vec{B}_{external}$.
2. Paramagnetic material (顺磁性)
   - produce a magnetisation in the same direction as the applied magnetic field.
   - $\vec{B}_{inner} > \vec{B}_{external}$.
3. Ferromagnetic material (铁磁性)
   - can have a magnetisation independent of an applied B-field with a complex relationship between the two fields.
   - $\vec{B}_{inner} >> \vec{B}_{external}$.

3.3 PERMEABILITY

Examples:

Paramagnetism	**Diamagnetism**
$\mu_r > 1$	$\mu_r < 1$
Aluminum, Calcium, Magnesium, Tungsten	Copper, Diamond, Gold, Silver, Lead

For diamagnetic materials and paramagnetic materials: $\mu_r \approx 1$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

where μ_0 is the permeability in vacuum (constant; Unit: H/m)

μ_r is the **relative permeability** (unitless)

μ is the **permeability** of the medium (constant; Unit: H/m)

DIAMAGNETIC MATERIALS

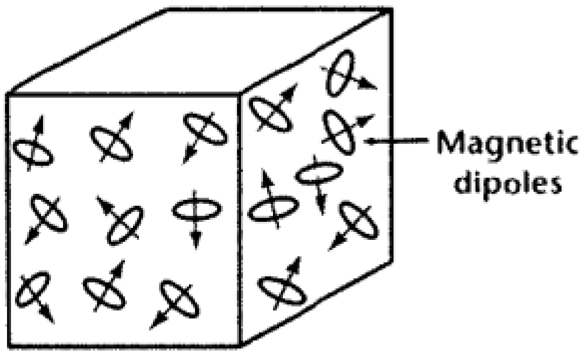
For an orbiting electron whose moment **m** is in **the same direction** as the applied field $\mathbf{B_0}$.

- The magnetic field produces an outward force on the orbiting electron. Since the orbital radius is quantized and cannot change, the inward Coulomb force of attraction is also unchanged. The force unbalance created by the outward magnetic force must therefore be compensated for by a reduced orbital velocity. Hence, the orbital moment decreases, and a smaller internal field results.
- If we had selected an atom for which **m** and $\mathbf{B_0}$ were opposed, the magnetic force would be inward, the velocity would increase, the orbital moment would increase, and greater cancellation of $\mathbf{B_0}$ would occur. Again, a smaller internal field would result.

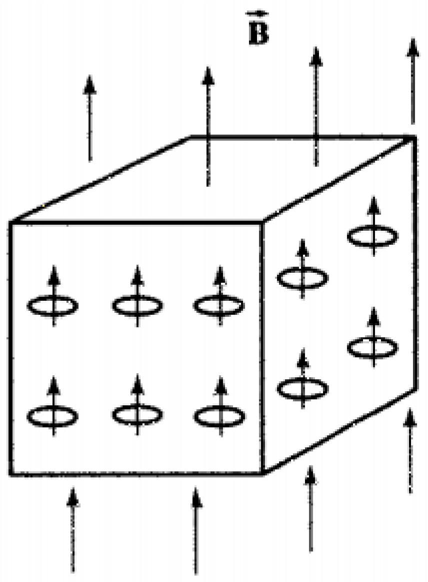
Diamagnetic effect is present in all materials, since it arises from an interaction of the external magnetic field with every orbiting electron.

However, it is overshadowed by other effects in many materials.

PARAMAGNETIC MATERIALS



a)



b)

When an external field is applied, there is a small torque on each atomic moment, and these moments tend to become aligned with the external field.

The orbital magnetic moment ***m***, is always aligned with the applied field, so it will increase ***B*** inside the material

the diamagnetic effect is still operating on the orbiting electrons and may counteract the increase.

If the net result is a **decrease** in B, the material is still called ***diamagnetic***. However, if there is an **increase** in B, the material is termed ***paramagnetic***.

FERROMAGNETIC MATERIALS

Not required

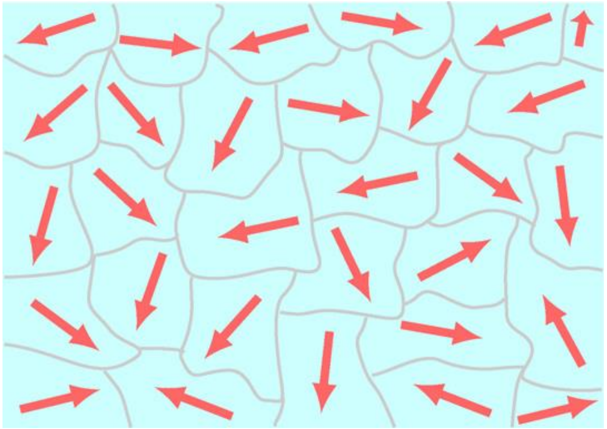
In <u>ferromagnetic</u> materials, each atom has a relatively large dipole moment, Interatomic forces cause these moments to line up over regions containing a large number of atoms.

- These regions are called **magnetic domains**.
- With an external magnetic field applied, those domains which have moments in the direction of the applied field increase their size: $B_{int} >> B_{appl}$
- Ferromagnetic materials: iron, cobalt, nickel...

$(\mu_r \gg 1, \chi_m \gg 1)$

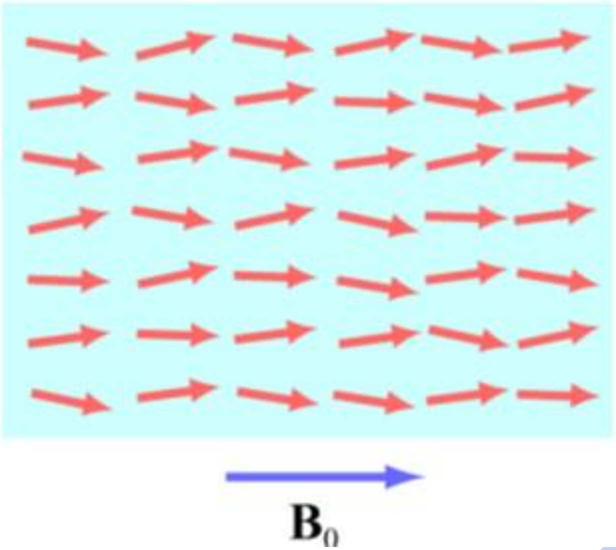
Without an external B field, the domains take on random orientations

→ no net magnetisation



Unmagnetised domains

Magnetised domains



NEXT….

- Boundary Conditions
- Electric currents