Chapter 7: Frequency Response

Dr. Bing Han

Email: Bing.Han@xjtlu.edu.cn

Office: EE220

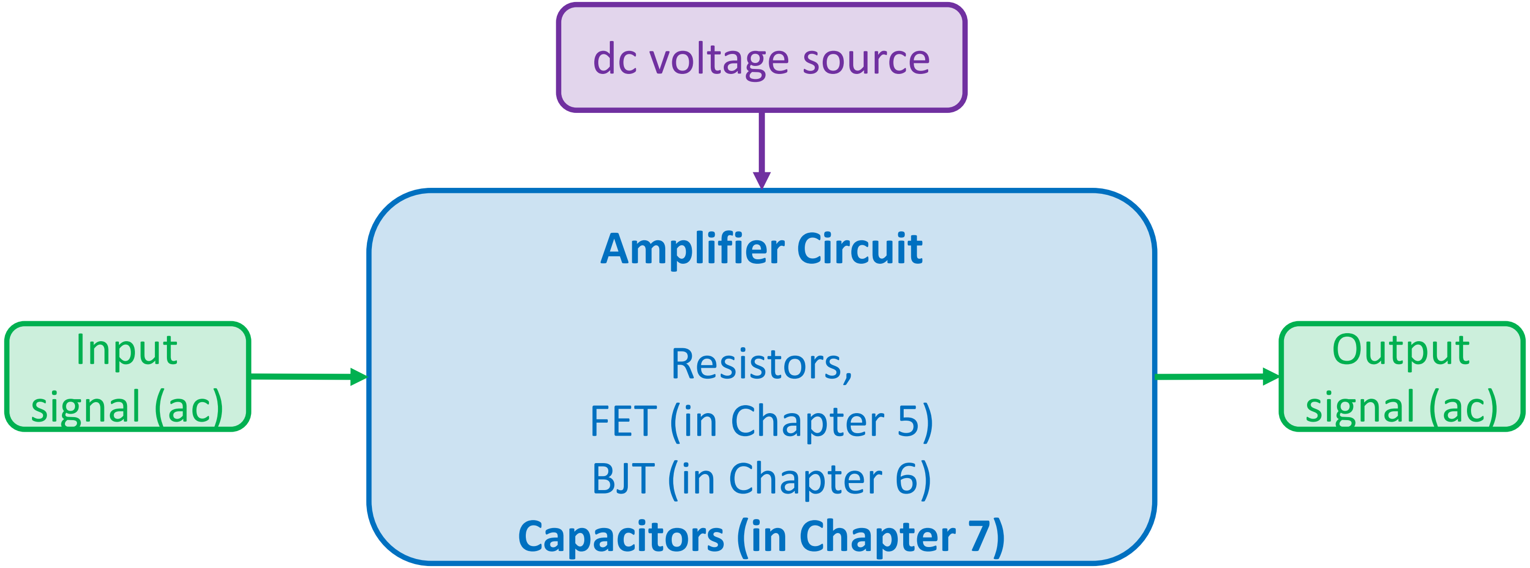
Preview

- Derive the system transfer functions of two simple RC circuits, develop the Bode plots for the magnitude of the transfer functions
- Discuss the general frequency response characteristics of amplifiers.
- Analyze the frequency response of transistor circuits with capacitors

- Optional: Determine the frequency response of the bipolar/MOS transistor, and determine the Miller effect and Miller capacitance.
- Optional: Determine the high-frequency response of basic transistor circuit configurations including the cascode circuit.

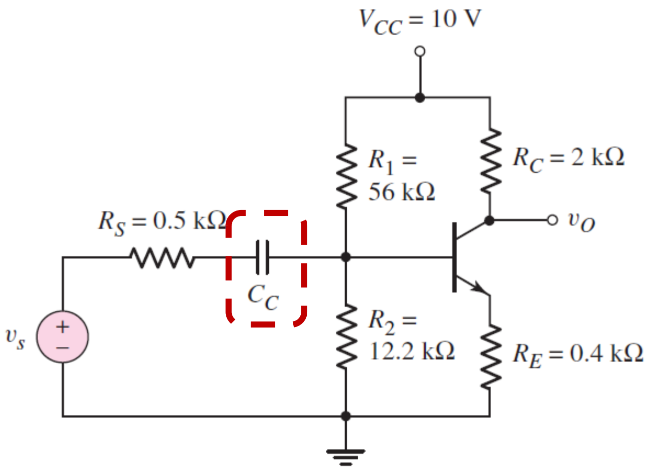
Linear Amplifier

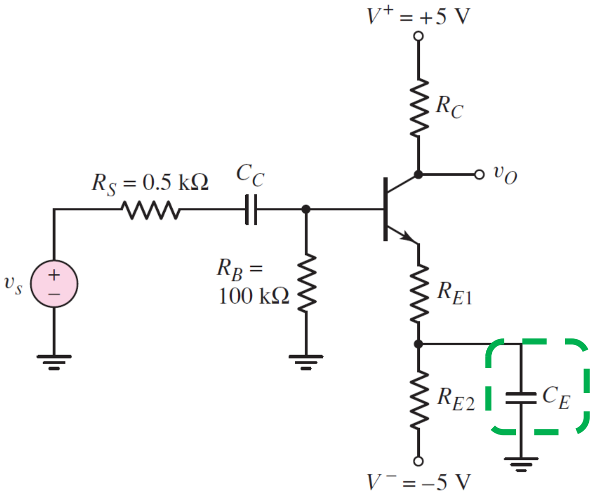
- FET and BJT can be used as the amplifying device



Amplifier Gain and Frequency

- In Chapter 5 and 6, we assume the frequency of the ac signal is high enough. So, the coupling and bypass capacitors are treated as short circuits in ac analysis





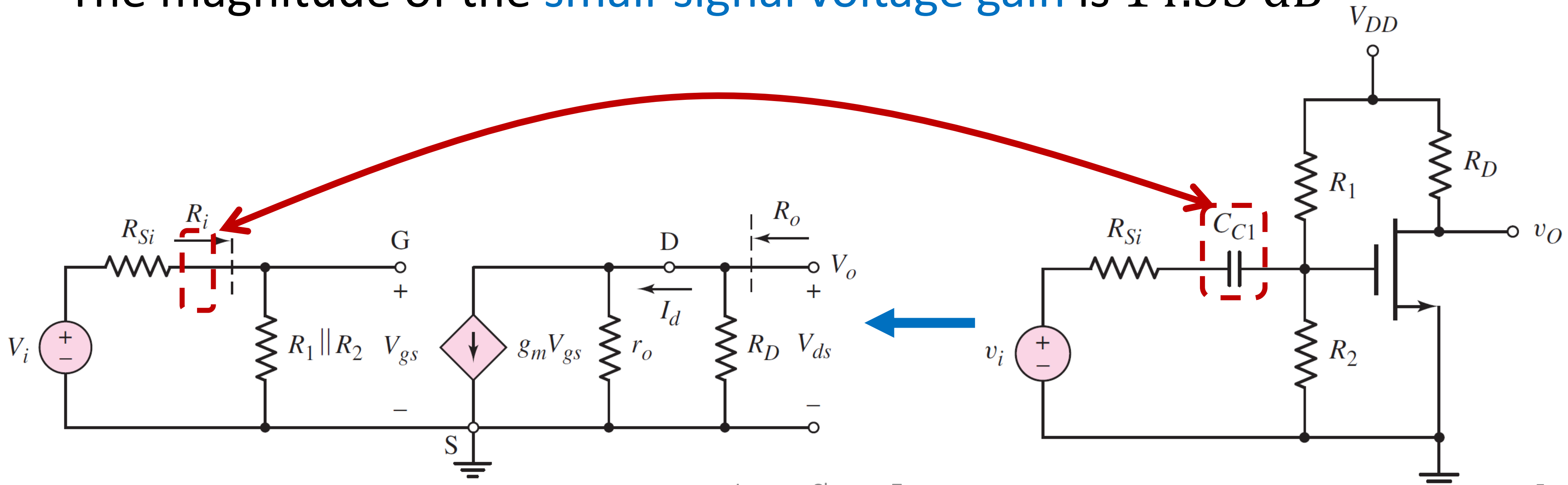
- All amplifier gain factors are functions of signal frequency

A Quick Question on Example 4.2

- We set coupling capacitor as short circuits, then we find the small-signal voltage gain of this common-source amplifier circuit

$$A_v = -5.21 \qquad A_{v\,\mathrm{dB}} = 20\log_{10}|A_v| = 20\log_{10}(5.21) = 14.33\ \mathrm{dB}$$

- The magnitude of the small-signal voltage gain is 14.33 dB



Two Basic RC Circuits

Derive the system transfer functions of two circuits, develop the Bode diagrams of the magnitude the transfer functions, and become familiar with sketching the Bode diagrams.

Complex Frequency

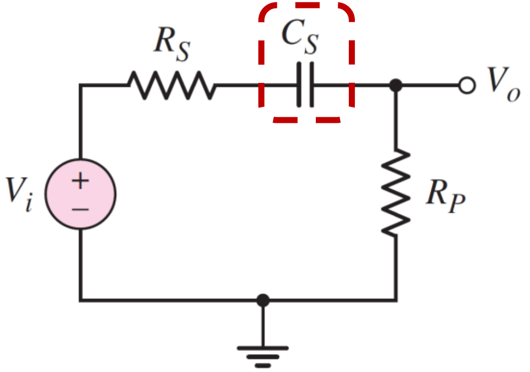
- The frequency response of a circuit is usually determined by using the complex frequency s
  - $s = j\omega = j2\pi f$
  - The complex impedance of capacitor is $\frac{1}{sC}$
  - The complex impedance of inductor is sL

Name of function	Expression
Voltage transfer function	$\boldsymbol{T(s) = V_o(s)/V_i(s)}$
Current transfer function	$T(s) = I_o(s)/I_i(s)$
Transresistance transfer function	$T(s) = V_o(s)/I_i(s)$
Transconductance transfer function	$T(s) = I_o(s)/V_i(s)$

RC Circuit 1

- RC circuit 1: series coupling capacitor circuit
  - The capacitor C_S behaves as a coupling capacitor
  - C_S and R_P are in series

- When V_i has zero frequency
  - The impedance of C_S is infinite → Open Circuit
  - $V_o = 0$
- When V_i has very high frequency
  - The impedance of C_S is very small → Short Circuit
  - $V_o = \frac{R_P}{R_S+R_P} V_i$



Series coupling capacitor circuit

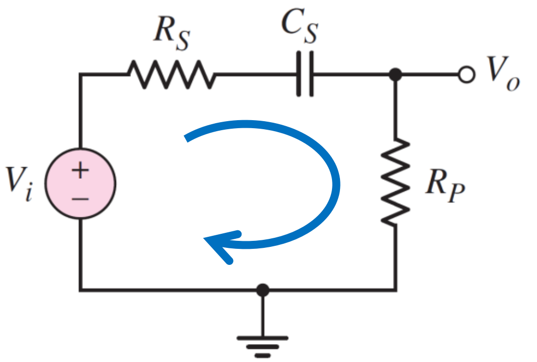
Voltage Transfer Function

- Write a KVL equation (or a voltage divider equation) in the loop
- The voltage transfer function is

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_P}{R_S + R_P + \frac{1}{sC_S}} = \frac{sR_PC_S}{1 + s(R_S + R_P)C_S} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{s(R_S + R_P)C_S}{1 + s(R_S + R_P)C_S}\right]$$

  - $\tau_S = (R_S + R_P)C_S$ is time constant

$$T(j\omega) = K\left(\frac{j\omega\tau_S}{1 + j\omega\tau_S}\right)$$



Series coupling capacitor circuit

Mathematical Derivation

$$T(j\omega) = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{j\omega\tau_S}{1 + j\omega\tau_S}\right]$$

- The magnitude of $T(j\omega)$ is

$$|T(j\omega)| = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{\omega\tau_S}{\sqrt{1 + \omega^2{\tau_S}^2}}\right] \qquad |T(jf)| = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{2\pi f\tau_S}{\sqrt{1 + (2\pi f\tau_S)^2}}\right]$$

- The magnitude of $T(j\omega)$ in **dB** is: $|T(jf)|_{\mathrm{dB}} = 20\log_{10}|T(jf)|$

$$|T(jf)|_{\mathrm{dB}} = 20\log_{10}\left[\left(\frac{R_P}{R_S + R_P}\right)\cdot\frac{2\pi f\tau_S}{\sqrt{1 + (2\pi f\tau_S)^2}}\right]$$

$$= 20\log_{10}\left(\frac{R_P}{R_S + R_P}\right) + 20\log_{10}(2\pi f\tau_S) - 20\log_{10}\sqrt{1 + (2\pi f\tau_S)^2}$$

Mathematical Derivation

$$|T(jf)|_{\mathrm{dB}} = 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) + 20\log_{10}(2\pi f\tau_S) - 20\log_{10}\sqrt{1+(2\pi f\tau_S)^2}$$

- In the first term
  - the dB value is always less than zero, as $\frac{R_P}{R_S+R_P} < 1$
- In the second term
  - When $f = \frac{1}{2\pi\tau_S}$, $20\log_{10}(1) = 0$
- In the third term
  - When $f = \frac{1}{2\pi\tau_S}$, the value is -3 dB $\rightarrow -20\log_{10}\sqrt{2} = -3\mathrm{dB}$
  - $f = \frac{1}{2\pi\tau_S}$ is the break-point frequency, corner frequency or 3 dB frequency

Bode Plot – High Frequency

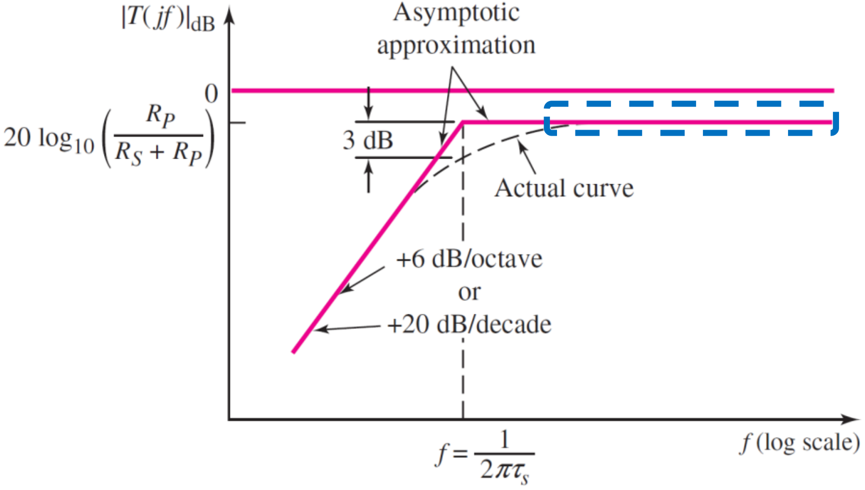
$$|T(jf)|_{\text{dB}} = 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) + 20\log_{10}(2\pi f\tau_S) - 20\log_{10}\sqrt{1+(2\pi f\tau_S)^2}$$

- When $f \gg \frac{1}{2\pi\tau_S}$ ← corner frequency
  - $2\pi f\tau_S \approx \sqrt{1+(2\pi f\tau_S)^2}$
  - The second and third terms cancel

- $|T(jf)|_{\text{dB}} \cong 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) < 0$

A constant value

Bode plot

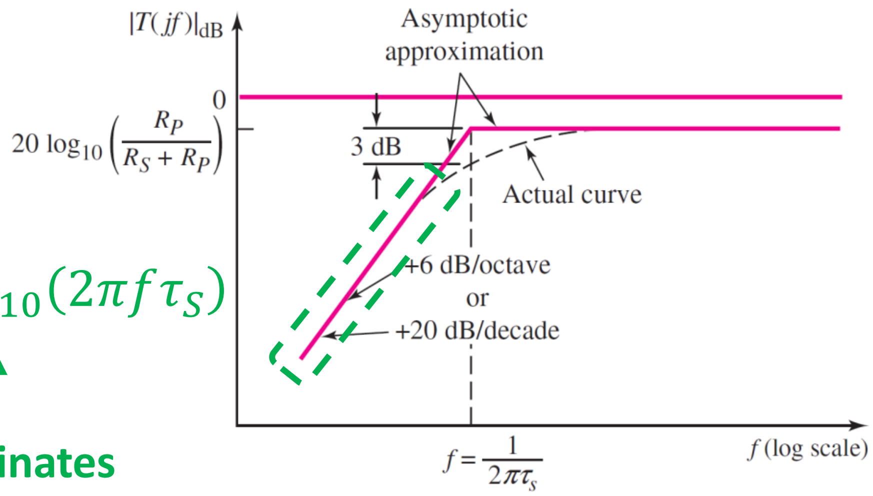


Bode Plot – Low Frequency

$$|T(jf)|_{\mathrm{dB}} = 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) + 20\log_{10}(2\pi f\tau_S) - 20\log_{10}\sqrt{1+(2\pi f\tau_S)^2}$$

- When $f \ll \frac{1}{2\pi\tau_S}$ ← corner frequency
  - $\sqrt{1+(2\pi f\tau_S)^2} \approx 1$
  - The third terms is essentially 0

Bode plot



- $|T(jf)|_{\mathrm{dB}} \cong 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) + 20\log_{10}(2\pi f\tau_S)$

The second term **dominates**

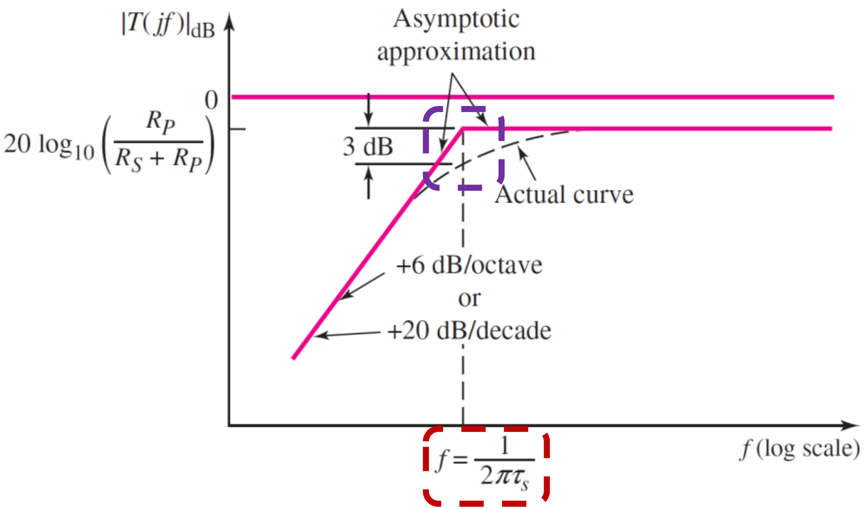
Bode Plot – Corner Frequency

$$|T(jf)|_{\text{dB}} = 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) + 20\log_{10}(2\pi f\tau_S) - 20\log_{10}\sqrt{1+(2\pi f\tau_S)^2}$$

- When $f = \dfrac{1}{2\pi\tau_S}$ ← corner frequency
  - The second terms is 0
    - $20\log_{10}(1) = 0$
  - The third term is -3 dB
    - $-20\log_{10}\sqrt{2} = -3\text{ dB}$

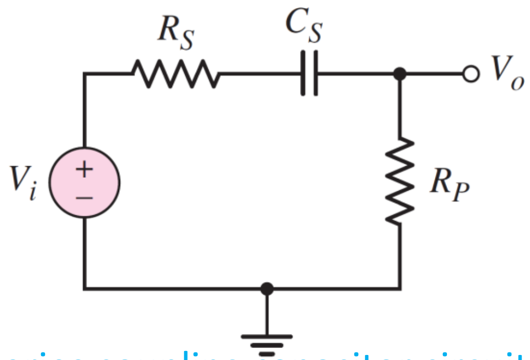
- $|T(jf)|_{\text{dB}} = 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) - 3\text{ dB}$

Bode plot



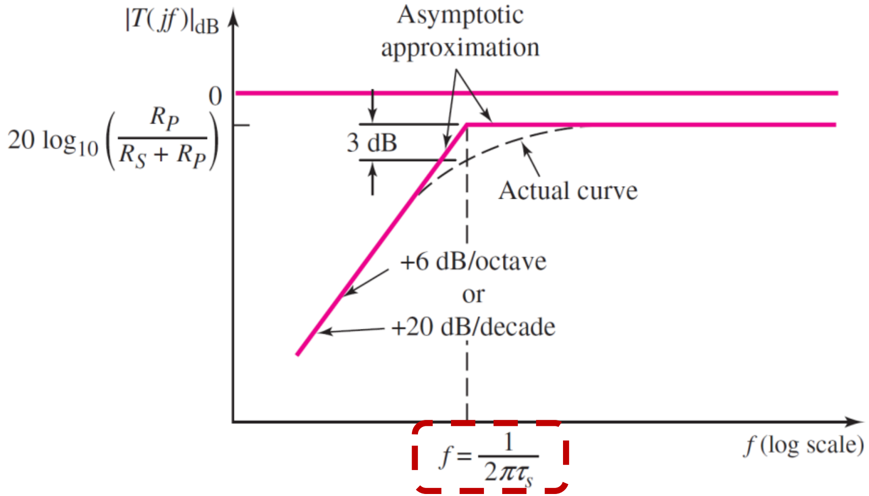
A Quick Summary: High-Pass Network

- This series coupling capacitor circuit is a high-pass network, since the high-frequency signals are passed through to the output
  - $f = \frac{1}{2\pi\tau_S}$ is the corner frequency
  - τ_S is the time constant



Series coupling capacitor circuit

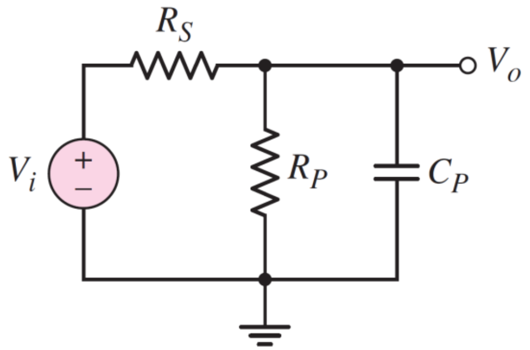
Bode plot



RC Circuit 2

- RC circuit 2: parallel load capacitor circuit
  - C_P and R_P are in parallel
  - The capacitor C_P behaves as a load capacitor

- When V_i has zero frequency
  - The impedance of C_P is infinite → Open Circuit
  - $V_o = \frac{R_P}{R_S + R_P} V_i$
- When V_i has very high frequency
  - The impedance of C_P is very small → Short Circuit
  - $V_o = 0$



Parallel load capacitor circuit

Voltage Transfer Function

- Write a KCL equation in the output node

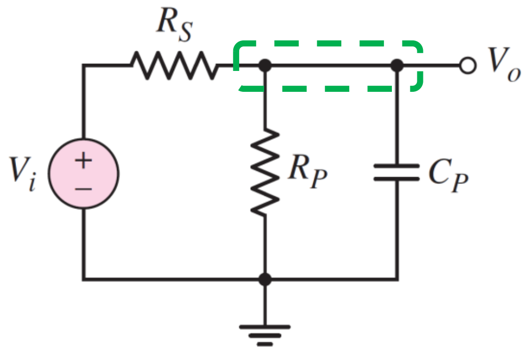
$$\frac{V_o - V_i}{R_S} + \frac{V_o}{R_P} + \frac{V_o}{(1/sC_P)} = 0$$

- The voltage transfer function is

$$T(s) = \frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + s\left(\frac{R_S R_P}{R_S + R_P}\right)C_P}\right]$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + s(R_S \parallel R_P)C_P}\right] = K\left(\frac{1}{1 + s\tau_P}\right)$$

  - τ_P is a time constant



Parallel load capacitor circuit

Bode Plot: Mathematical Derivation

$$T(s) = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + s\tau_P}\right] \qquad T(jf) = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + j2\pi f\tau_P}\right]$$

- The magnitude of $T(jf)$ is

$$|T(jf)| = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{\sqrt{1 + (2\pi f\tau_P)^2}}\right]$$

- The magnitude of $T(jf)$ in **dB** is: $|T(jf)|_{\text{dB}} = 20\log_{10}|T(jf)|$

$$|T(jf)|_{\text{dB}} = 20\log_{10}\left(\frac{R_P}{R_S + R_P}\right) - 20\log_{10}\sqrt{1 + (2\pi f\tau_P)^2}$$

Mathematical Derivation

$$|T(jf)|_{\mathrm{dB}} = 20\log_{10}\left(\frac{R_P}{R_S + R_P}\right) - 20\log_{10}\sqrt{1 + (2\pi f\tau_P)^2}$$

- In the first term
  - The dB value is always less than zero, as $\frac{R_P}{R_S+R_P} < 1$ (the same with RC circuit 1)
- In the second term
  - When $f = \frac{1}{2\pi\tau_P}$, $-20\log_{10}\sqrt{1+1} = -3\ \mathrm{dB}$
  - $f = \frac{1}{2\pi\tau_S}$ is the break-point frequency, corner frequency or 3 dB frequency

Bode Plot – Low Frequency

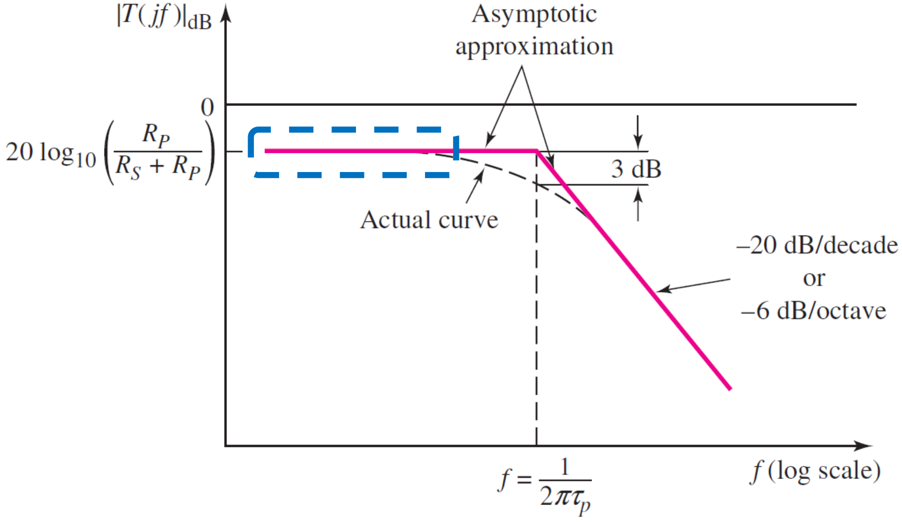
$$|T(jf)|_{\text{dB}} = 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) - 20\log_{10}\sqrt{1+(2\pi f\tau_P)^2}$$

- When $f \ll \frac{1}{2\pi\tau_P} \leftarrow$ corner frequency
  - The second term is essentially zero

- $|T(jf)|_{\text{dB}} \cong 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) < 0$

A constant value

Bode plot



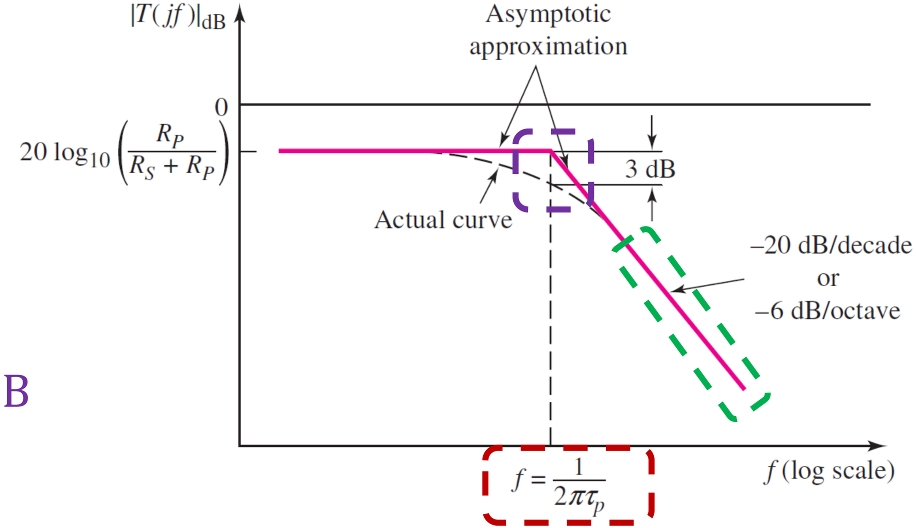
Bode Plot – High and Corner Frequencies

$$|T(jf)|_{\text{dB}} = 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) - 20\log_{10}\sqrt{1+(2\pi f\tau_P)^2}$$

- When $f \gg \dfrac{1}{2\pi\tau_P}$ ← corner frequency
  - The second term dominates
- When $f = \dfrac{1}{2\pi\tau_P}$
  - The second term is -3 dB

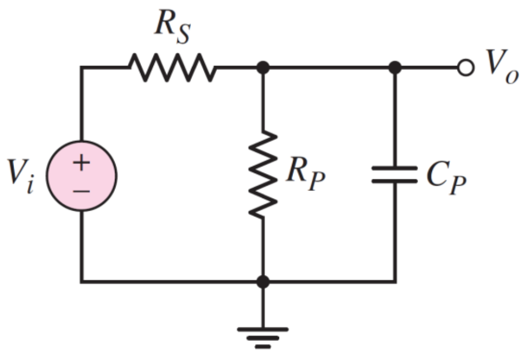
$$|T(jf)|_{\text{dB}} = 20\log_{10}\left(\frac{R_P}{R_S+R_P}\right) - 3\text{ dB}$$

Bode plot



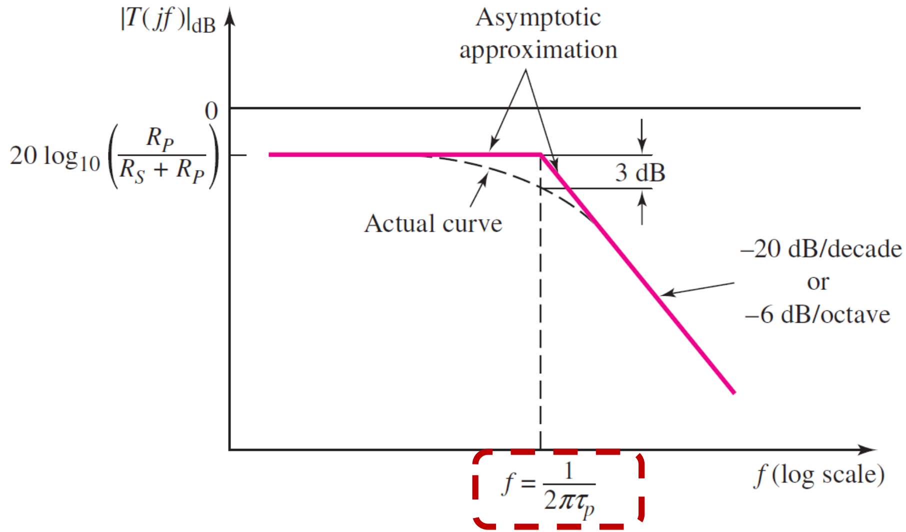
A Quick Summary: Low-Pass Network

- The parallel load capacitor circuit is called a low-pass network, since the low-frequency signals are passed through to the output
  - $f = \frac{1}{2\pi\tau_P}$ is the corner frequency
  - τ_P is the time constant



Parallel load capacitor circuit

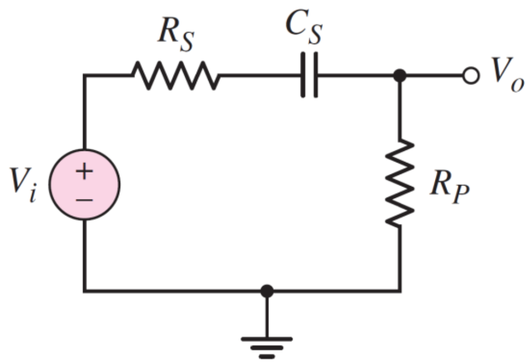
Bode plot



Example 7.1

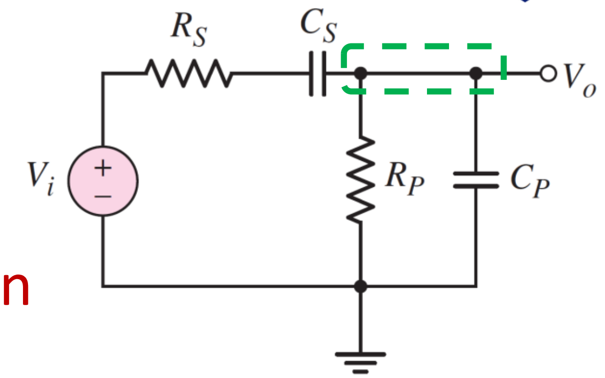
Determine the corner frequency and Bode plot for the circuit.

Assume $R_S = 1\ \text{k}\Omega, R_P = 10\ \text{k}\Omega, C_S = 1\ \mu\text{F}$.



Short-Circuit and Open-Circuit Time Constants

- Combine the RC circuit 1 and the RC circuit 2
  - The circuit with two capacitors
  - C_S is the coupling capacitor
  - C_P is the load capacitor
- We can determine the voltage transfer function by writing a KCL equation at the output node



$$\frac{V_o - V_i}{R_S + \frac{1}{sC_S}} + \frac{V_o}{R_P} + \frac{V_o}{\frac{1}{sC_P}} = 0$$

$$V_i\left(\frac{1}{R_S + \frac{1}{sC_S}}\right) = V_o\left(\frac{1}{R_S + \frac{1}{sC_S}} + \frac{1}{R_P} + \frac{1}{\frac{1}{sC_P}}\right)$$

Open- and Short-Circuit Time Constants

- The result is

$$T(s)=\frac{V_o(s)}{V_i(s)}=\frac{1}{R_S+\frac{1}{sC_S}\left(\frac{1}{R_S+\frac{1}{sC_S}}+\frac{1}{R_P}+\frac{1}{\frac{1}{sC_P}}\right)}=\frac{1}{\frac{R_S+R_P}{R_P}+\frac{1}{sC_SR_P}+sC_PR_S+\frac{C_P}{C_S}}$$

$$T(s)=\frac{V_o(s)}{V_i(s)}=\frac{R_P}{R_S+R_P}\times\frac{1}{1+\frac{1}{sC_SR_P}\frac{R_P}{R_S+R_P}+sC_PR_S\frac{R_P}{R_S+R_P}+\frac{R_P}{R_S+R_P}\frac{C_P}{C_S}}$$

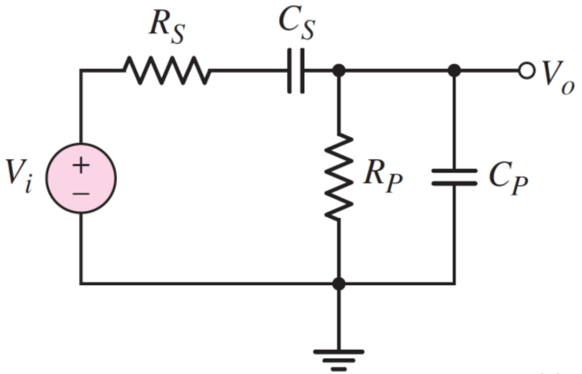
$$T(s)=\frac{V_o(s)}{V_i(s)}=\frac{R_P}{R_S+R_P}\times\frac{1}{1+\frac{1}{s\tau_S}+s\tau_P+\frac{R_P}{R_S+R_P}\frac{C_P}{C_S}}$$

Open- and Short-Circuit Time Constants

- The voltage transfer function of the circuit is

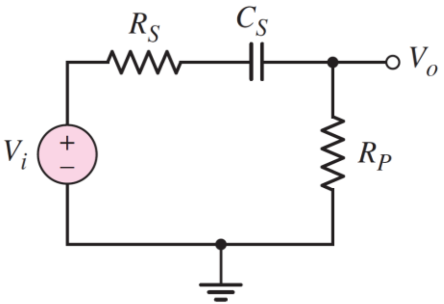
$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_P}{R_S + R_P} \times \frac{1}{1 + \frac{1}{s\tau_S} + s\tau_P + \frac{R_P}{R_S + R_P}\frac{C_P}{C_S}}$$

- Usually, $C_P \ll C_S$
  - At low frequencies, C_P was made an open circuit
    - τ_S is called an open-circuit time constant
    - $\tau_S = C_S(R_S + R_P)$, $R_S + R_P$ is the effective resistance seen by the capacitor C_S
  - At high frequencies, C_S was made a short circuit
    - τ_P is called a short-circuit time constant
    - $\tau_P = C_P(R_S \parallel R_P)$, $R_S \parallel R_P$ is the effective resistance seen by the capacitor C_P

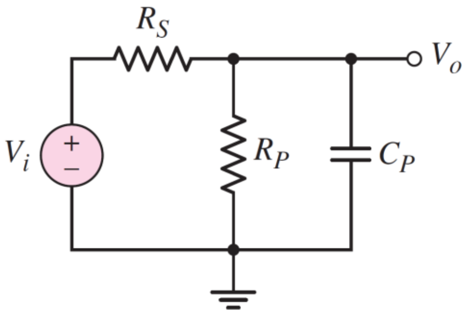


Bode Plot and Corner Frequencies

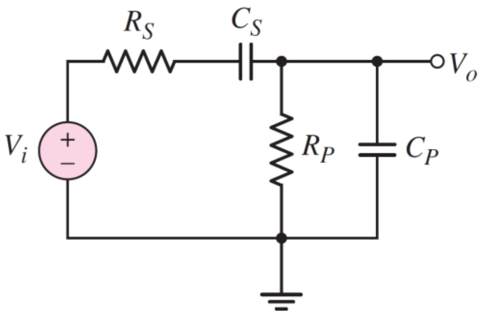
- Combine the high-pass and low-pass networks



+



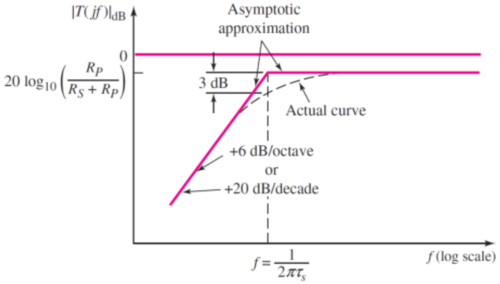
=



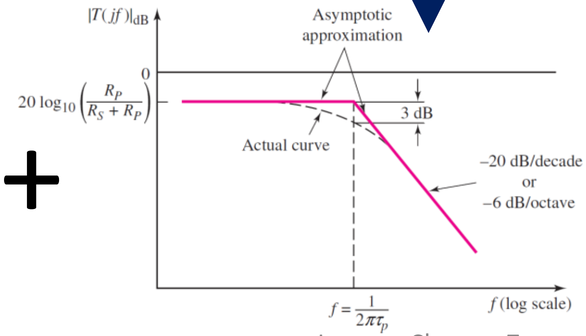
Series coupling capacitor circuit
(high-pass network)

Parallel load capacitor circuit
(low pass network)

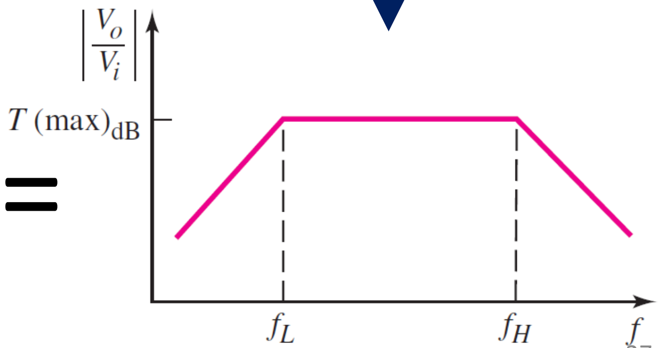
Bode plot



+

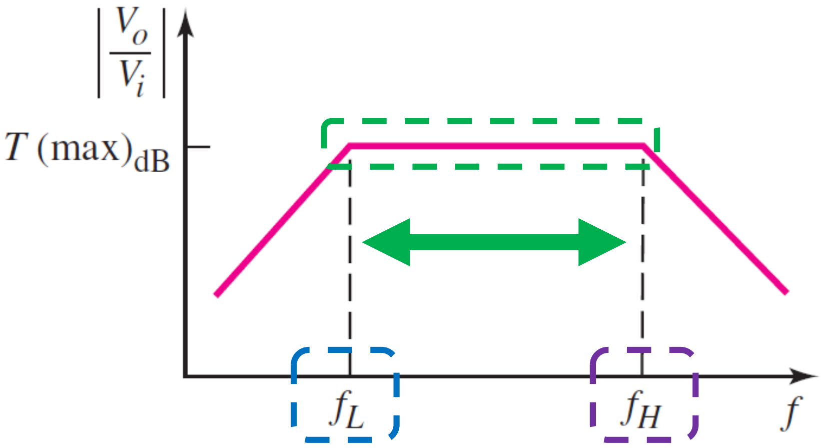


=



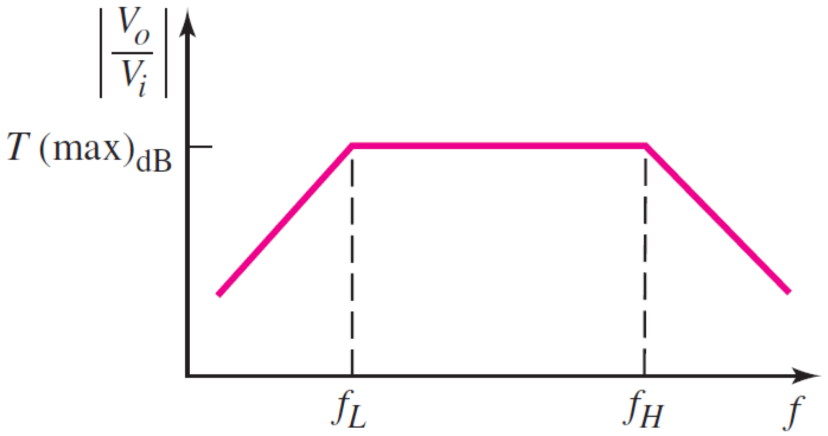
Bode Plot and Corner Frequencies

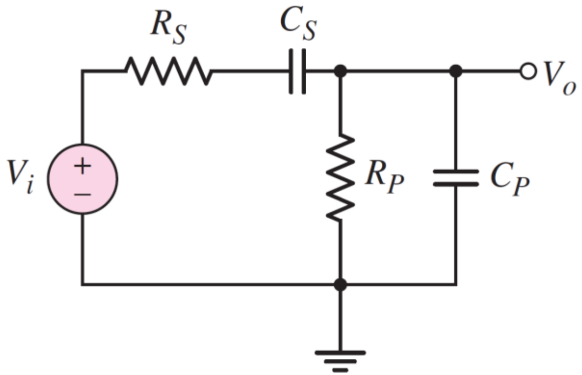
- The corner frequencies of the Bode plot
  - Lower corner frequency, $f_L = \dfrac{1}{2\pi\tau_S}$
  - Upper corner frequency, $f_H = \dfrac{1}{2\pi\tau_P}$
- The mid-band range, or the bandwidth is $f_{\mathrm{BW}} = f_H - f_L \cong f_H$



Bode Plot and Corner Frequencies

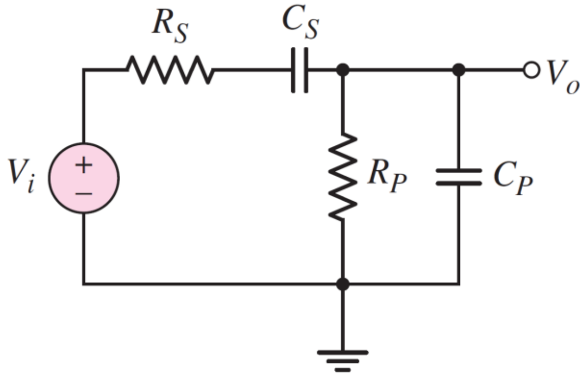
- At the high end of the frequency spectrum
  - The voltage transfer function magnitude decreases because the load capacitor does not act as perfect open circuit
- At the low end of the frequency spectrum
  - The voltage transfer function magnitude decreases because coupling capacitor does not act as perfect short circuit





Example 7.2

Determine the corner frequencies and bandwidth of a passive circuit containing two capacitors. Assume $R_S = 1\ \mathrm{k\Omega}, R_P = 10\ \mathrm{k\Omega}, C_S = 1\ \mu\mathrm{F}, C_P = 3\ \mathrm{pF}$.

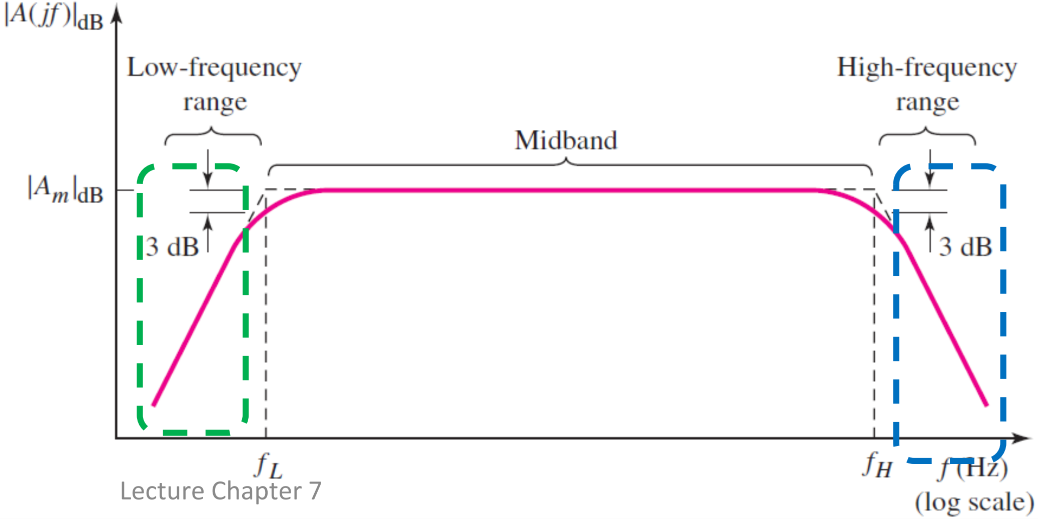


Frequency Response: Transistor Amplifiers with Circuit Capacitors

Analyze the frequency response of transistor circuits with capacitors.

Amplifier Gain and Frequency

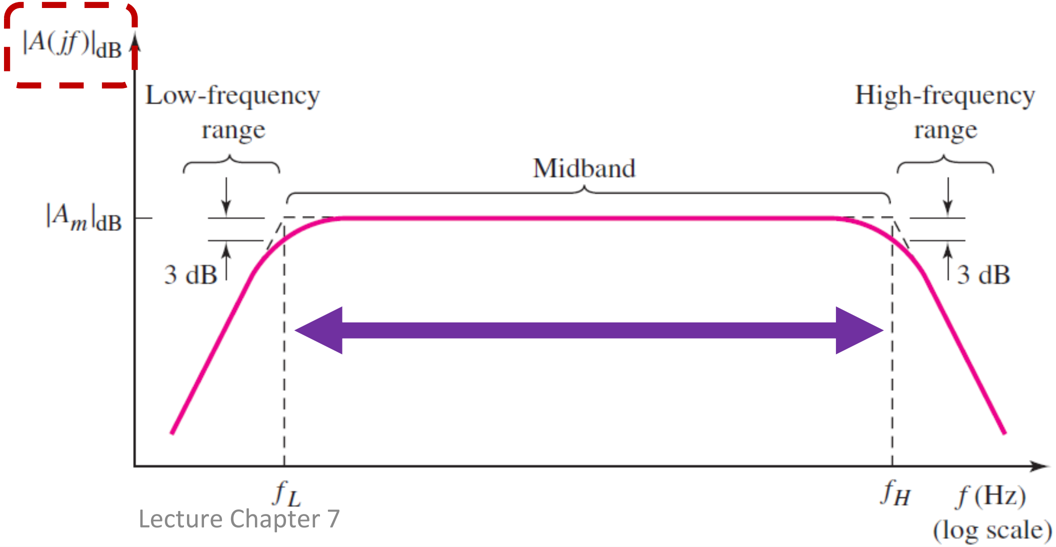
- The Bode plots for transistor amplifiers are similar to RC circuits
- When the source frequency is low: $f < f_L$
  - Coupling and bypass capacitor effects ()
- When the source frequency is high: $f > f_H$
  - Stray capacitance and transistor capacitance effects



Amplifier Gain and Frequency

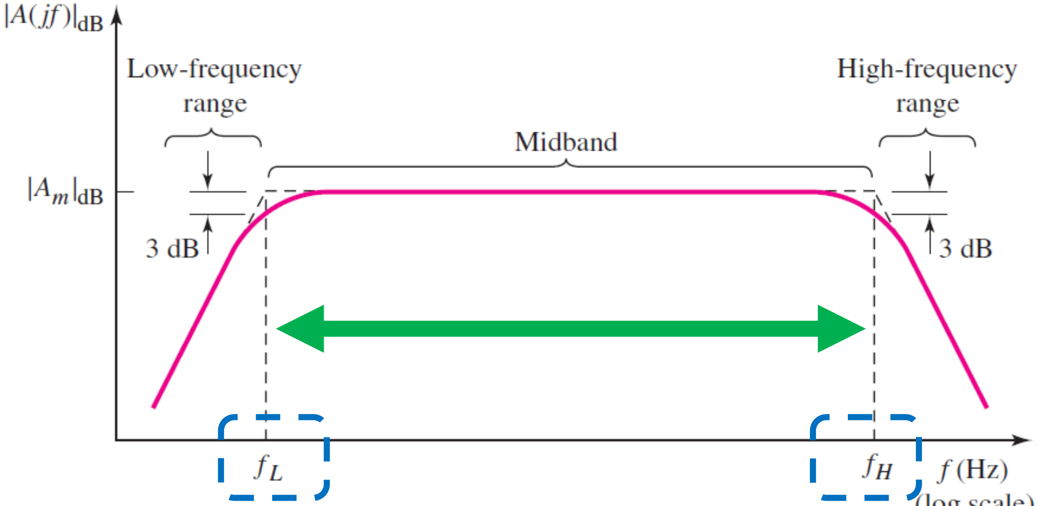
- In the mid-band range: $f_L < f < f_H$
  - Coupling and bypass capacitors act as short circuits
  - Load capacitor, stray and transistor capacitances act as open circuits
- The small-signal voltage gain we determined in Chapter 5 and 6 is the mid-band gain
- The amplifier gain is also plotted in terms of decibels (dB)

$$|A|_{\mathrm{dB}} = 20\log_{10}|A|$$



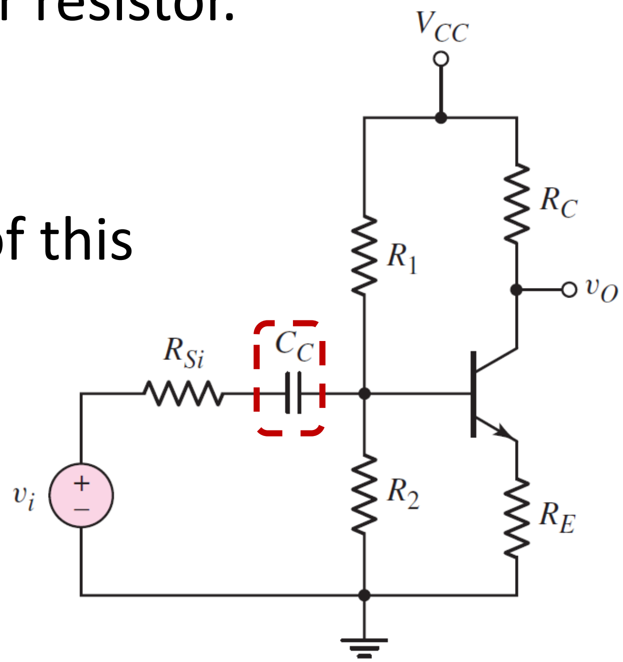
Amplifier Gain and Frequency

- Corner frequency
  - The gain at $f = f_L$ and $f = f_H$ is 3 dB less than the mid-band gain ($|A|_{\mathrm{dB}}$)
- The bandwidth of the amplifier is defined as
  - $f_{\mathrm{BW}} = f_H - f_L$



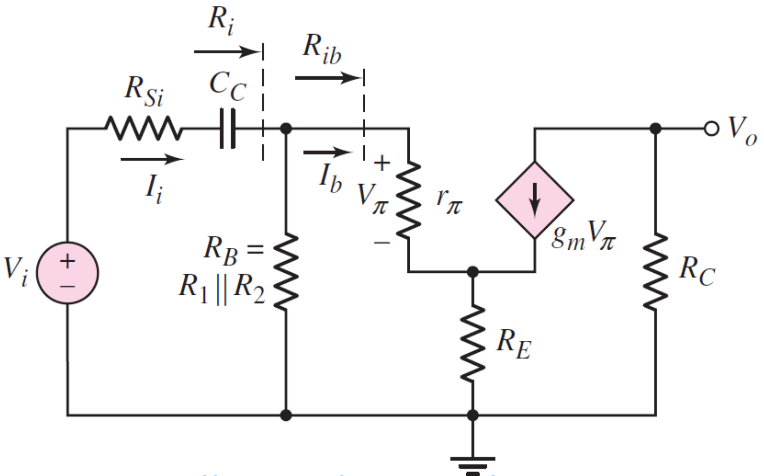
Transistor Amplifiers with Capacitors

- Three types of capacitors will be considered in this Chapter
  - Coupling capacitor, load capacitor, and bypass capacitor
- This is a common-emitter circuit with emitter resistor. C_C is the input coupling capacitor
- What is the general shape of the Bode plot of this amplifier circuit?
  - High pass network or low pass network?

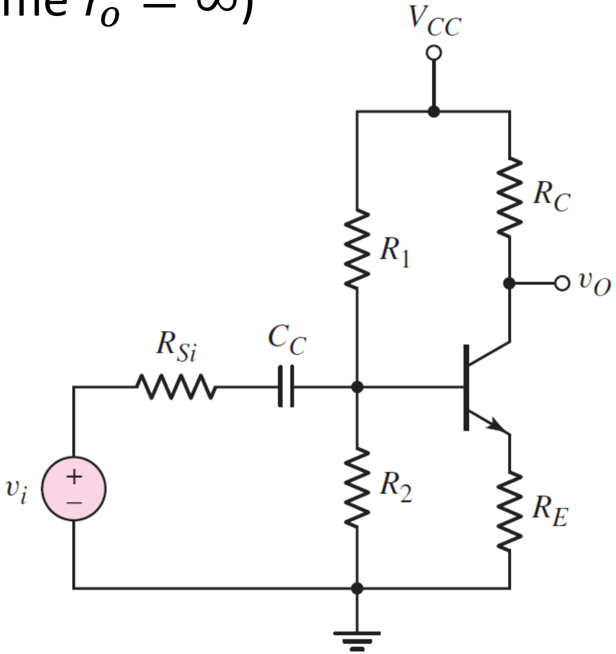


Common-Emitter Circuit with Coupling Capacitor

- Draw the small-signal equivalent circuit
  - Set all dc sources to zero
  - Replace the BJT by the hybrid-π model (assume $r_o = \infty$)
  - Keep the coupling capacitor in the circuit

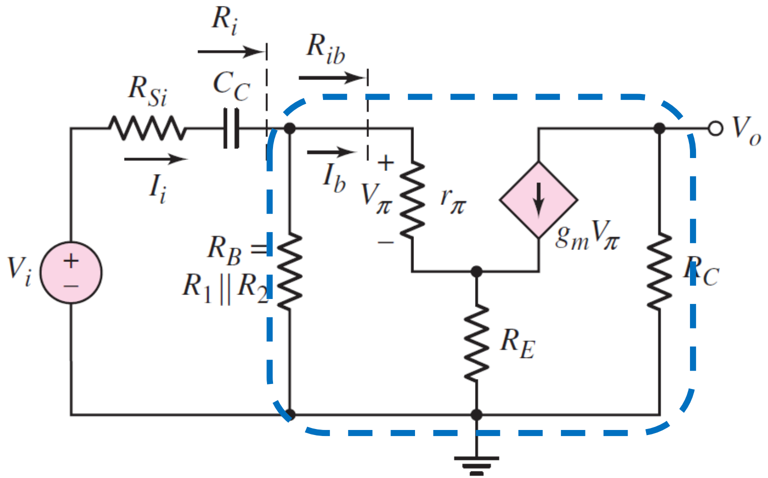


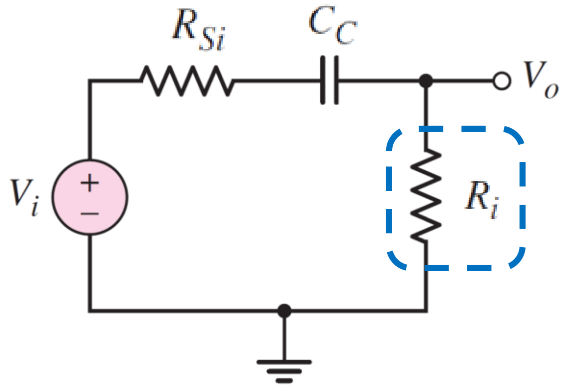
small-signal equivalent circuit



Common-Emitter Circuit with Coupling Capacitor

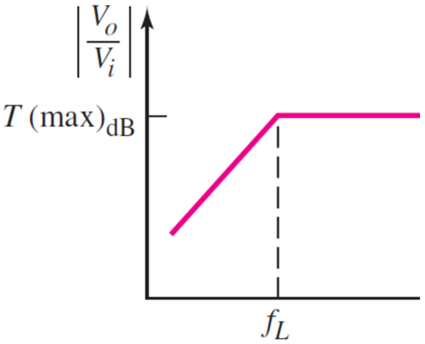
- The input resistance of the circuit is
  - $R_i = R_1 \parallel R_2 \parallel R_{ib}$
  - $R_{ib} = r_\pi + (1+\beta)R_E$
  - See Chapter 6 contents for more details…

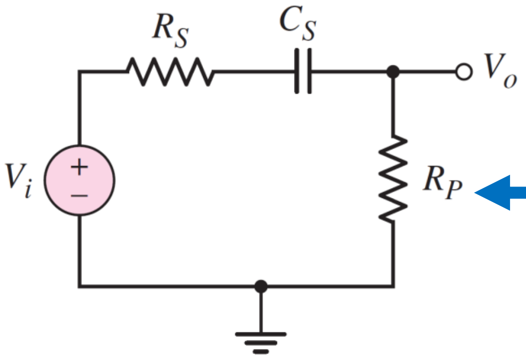


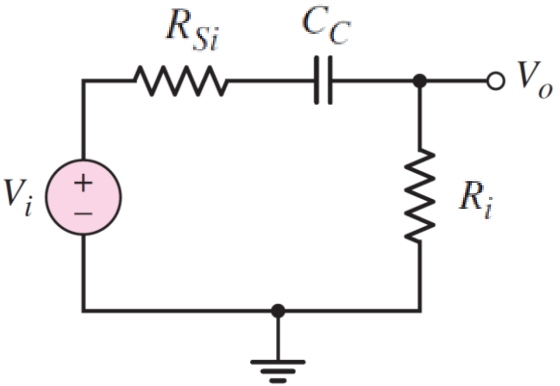


Common-Emitter Circuit with Coupling Capacitor

- This common-emitter circuit is equivalent to a series coupling capacitor circuit (RC circuit 1)
- We note that this common-emitter circuit is a high-pass network







Series coupling capacitor circuit

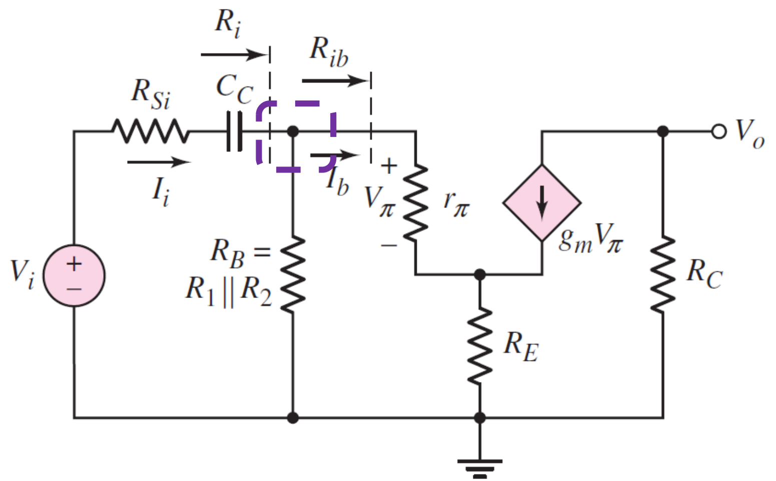
Bode Plot of a Common-Emitter Circuit

- To find the small-signal voltage gain, considering coupling capacitor
  - $A_v(s) = \dfrac{V_o(s)}{V_i(s)}$
  - $V_o \leftrightarrow V_\pi \leftrightarrow I_b \leftrightarrow I_i \leftrightarrow V_i$
- The output voltage is

$$V_o = -g_m V_\pi R_C = -g_m (I_b r_\pi) R_C$$

- Write a current divider equation in the base node

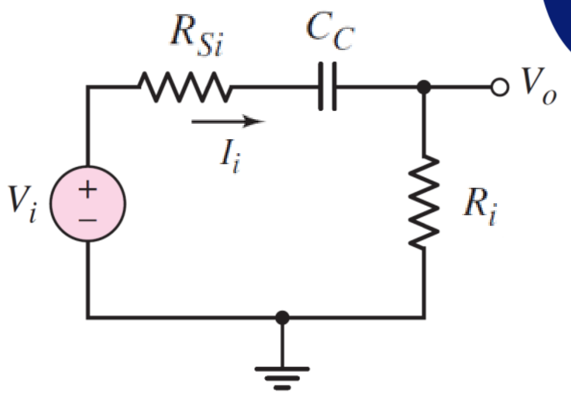
$$I_b = \left(\frac{R_B}{R_B + R_{ib}}\right) I_i$$



Bode Plot of a Common-Emitter Circuit

- The input current is

$$I_i = \frac{V_i}{R_{Si} + \dfrac{1}{sC_C} + R_i}$$

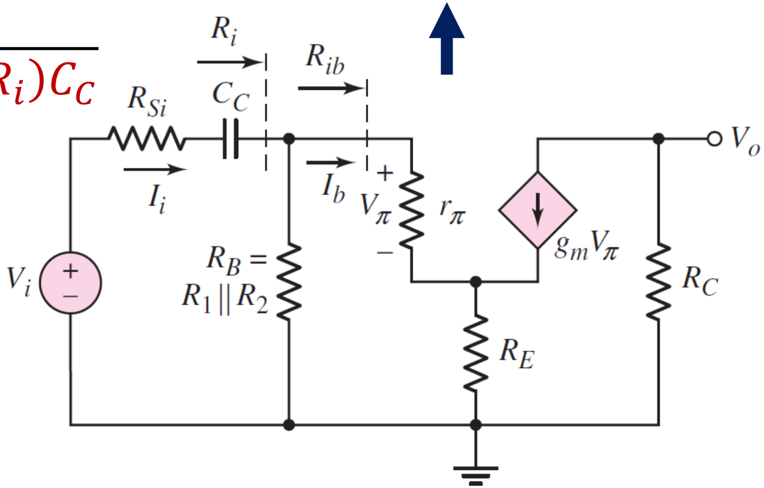


- The small-signal voltage gain is

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}}\right) \frac{sC_C}{1 + s(R_{Si} + R_i)C_C}$$

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \boxed{\frac{-g_m r_\pi R_C}{(R_{Si} + R_i)}\left(\frac{R_B}{R_B + R_{ib}}\right)} \frac{s\tau_s}{1 + s\tau_s}$$

We have got this result in Chapter 6

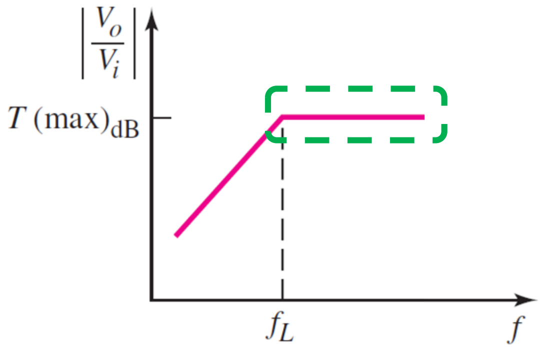


Bode Plot of a Common-Emitter Circuit

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{-g_m r_\pi R_C}{(R_{Si} + R_i)}\left(\frac{R_B}{R_B + R_{ib}}\right)\frac{s\tau_s}{1 + s\tau_s}$$

- The maximum gain can be achieved when the signal has very high frequency
- The maximum magnitude of the small-signal voltage gain, in decibels, is

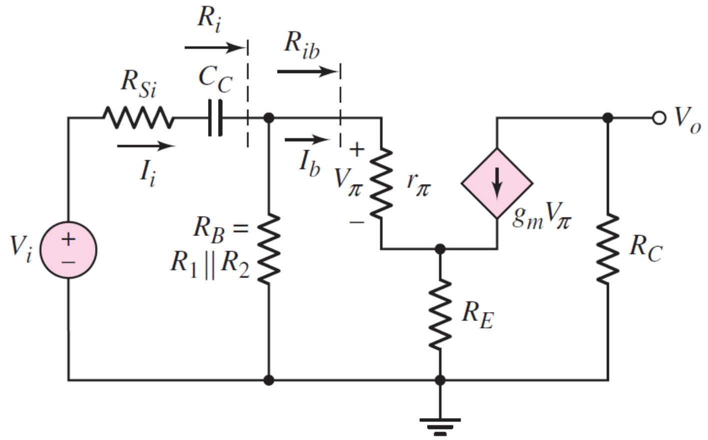
$$|A_v(\max)|_{\text{dB}} = 20\log_{10}|A_v(\max)|$$
$$= 20\log_{10}\frac{g_m r_\pi R_C}{(R_{Si} + R_i)}\left(\frac{R_B}{R_B + R_{ib}}\right)$$

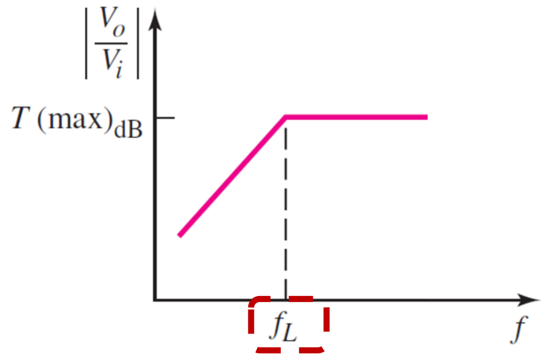


Bode Plot of a Common-Emitter Circuit

- The effective resistance seen by the coupling capacitor is $R_{Si} + R_i$
- The lower corner frequency is

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(R_{Si} + R_i)C_C}$$



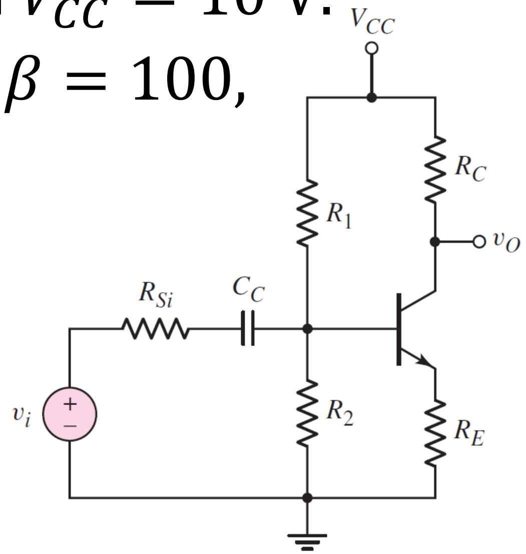


Example 7.3

Calculate the corner frequency and maximum gain of a bipolar common-emitter circuit with a coupling capacitor.

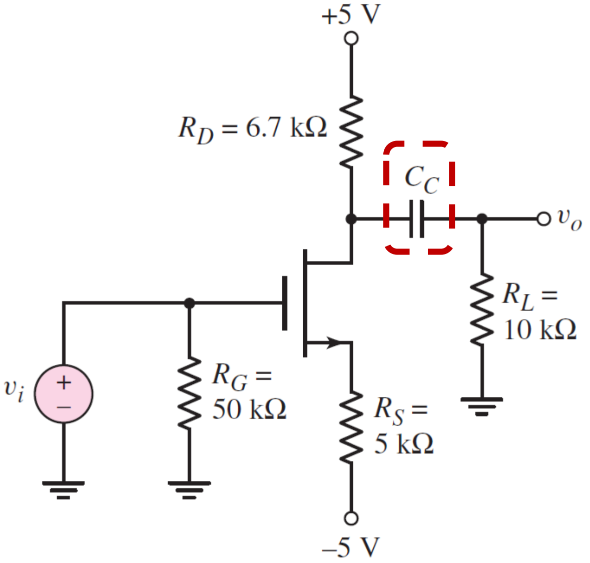
The parameters are: $R_1 = 51.2\ \text{k}\Omega, R_2 = 9.6\ \text{k}\Omega, R_C = 2\ \text{k}\Omega, R_E = 0.4\ \text{k}\Omega, R_{Si} = 0.1\ \text{k}\Omega, C_C = 1\ \mu\text{F}$, and $V_{CC} = 10\ \text{V}$.

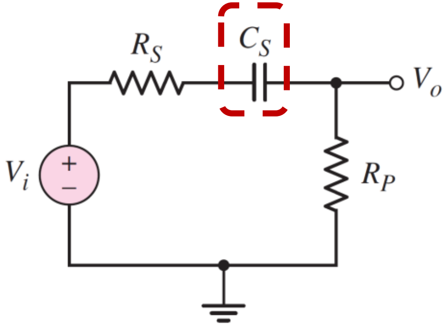
The transistor parameters are: $V_{BE}(\text{on}) = 0.7\ \text{V}, \beta = 100$, and $V_A = \infty$.



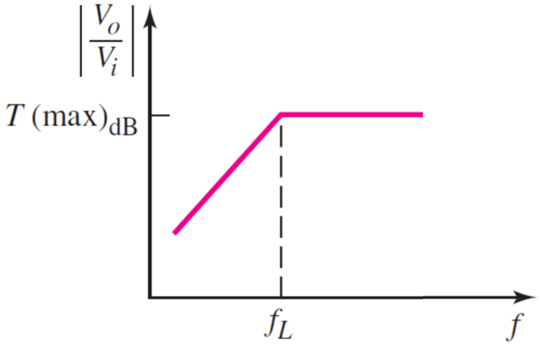
Common-Source Circuit with Coupling Capacitor

- The output signal is connected to the load through a coupling capacitor
- This common-source MOSFET amplifier is equivalent to a parallel load capacitor circuit (RC circuit 2)



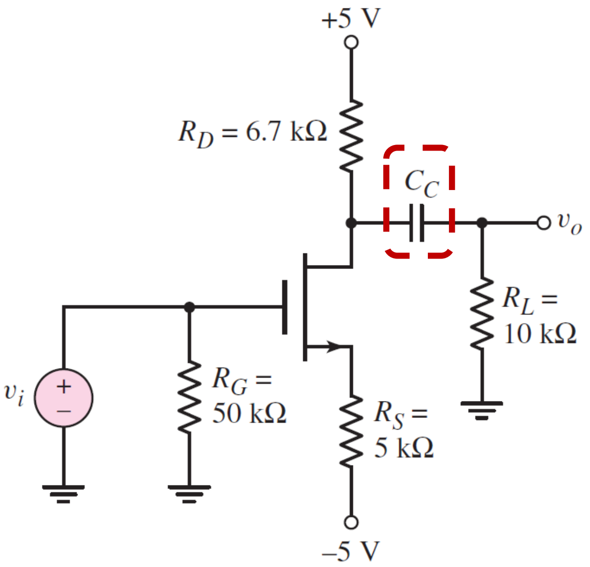


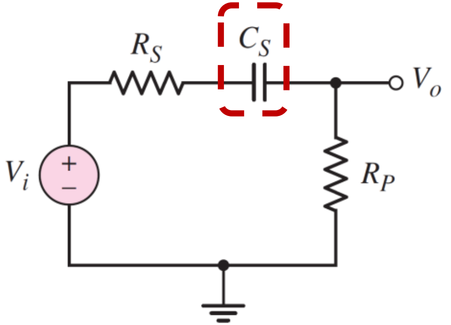
parallel load capacitor circuit



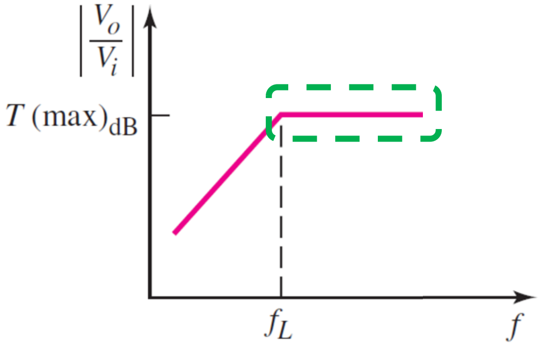
Common-Source Circuit with Coupling Capacitor

- This circuit is a low-pass network
  - The maximum small-signal voltage gain (in dB)←we found this value in Chapter 5
  - The lower corner frequency f_L



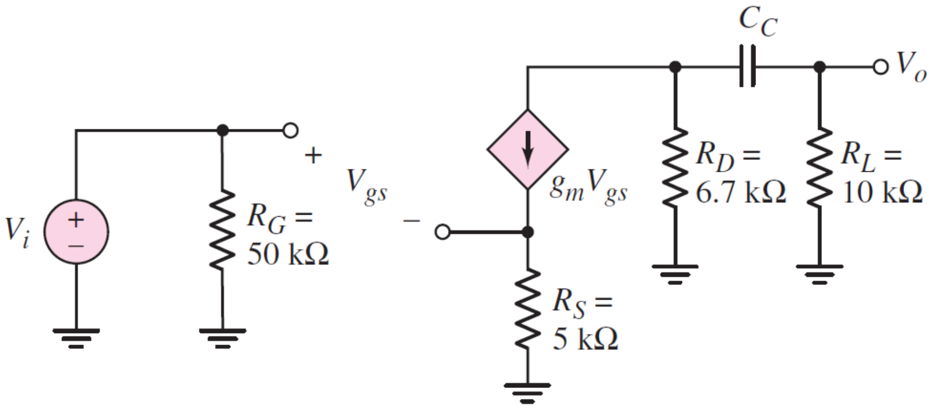


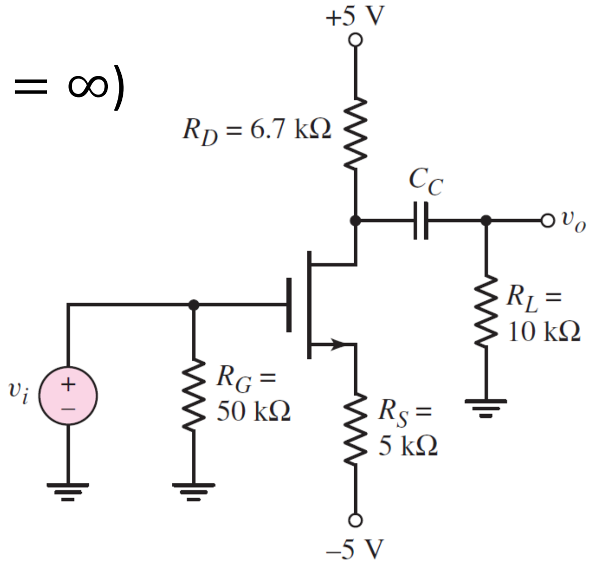
parallel load capacitor circuit



Common-Source Circuit with Coupling Capacitor

- To find the maximum small-signal voltage gain, we need to draw the small-signal equivalent circuit (step 2 ac analysis)
  - Set all dc sources to zero
  - Assume the frequency is high enough, so C_C can be treated as a short circuit
  - Replace the FET by small-signal model (assume $r_o = \infty$)





Common-Source Circuit with Coupling Capacitor

- The output voltage is

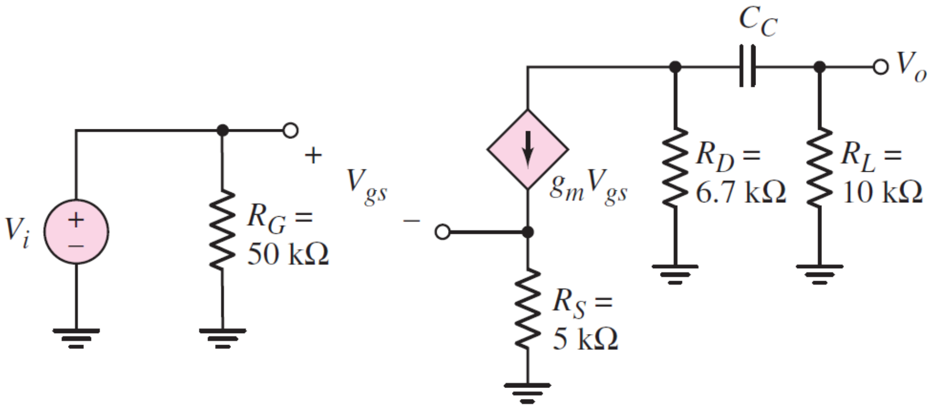
$$V_o = -g_m V_{gs}(R_D \parallel R_L)$$

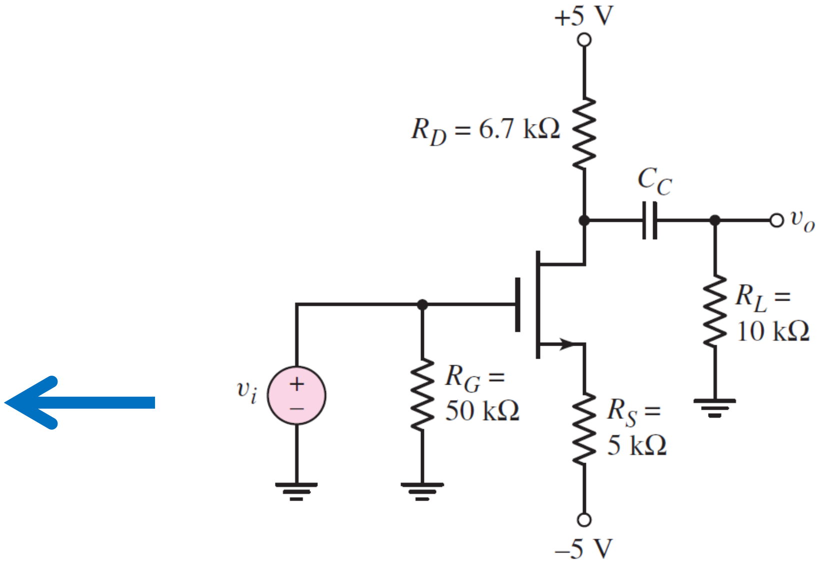
- The input voltage is

$$V_i = V_{gs} + g_m R_S V_{gs}$$

- The maximum small-signal voltage gain is

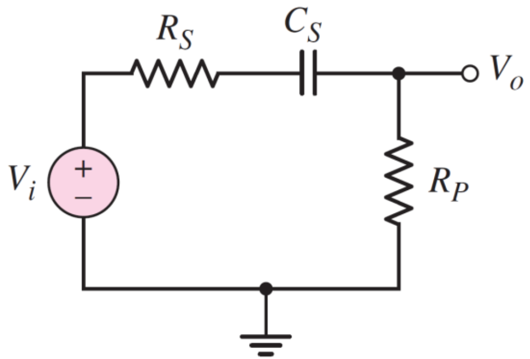
$$|A_v(\max)| = \frac{g_m(R_D \parallel R_L)}{1 + g_m R_S}$$





Output Coupling Capacitor: Common-Source Circuit

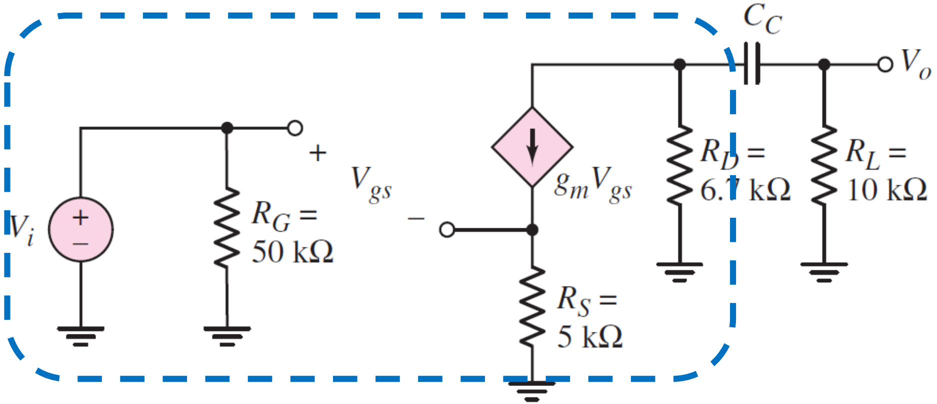
- The time constant is a function of the effective resistance seen by capacitor C_c
  - Set all independent sources equal to zero $V_i = 0$



$$\tau_S = (R_o + R_L)C_C = (R_D + R_L)C_C$$

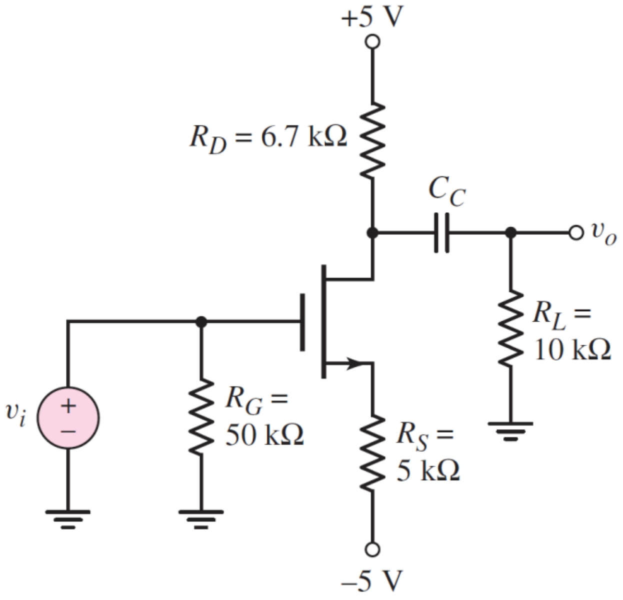
- The lower corner frequency is

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(R_D + R_L)C_C}$$



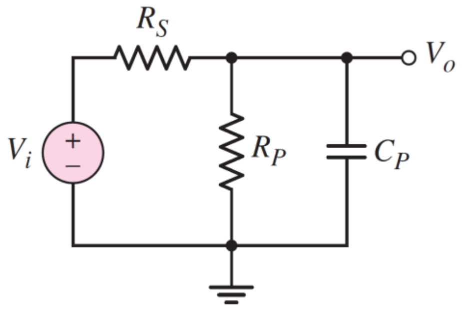
Example 7.4

This circuit is to be used as a simple audio amplifier. Design the circuit such that the lower corner frequency is $f_L = 20\ \text{Hz}$.

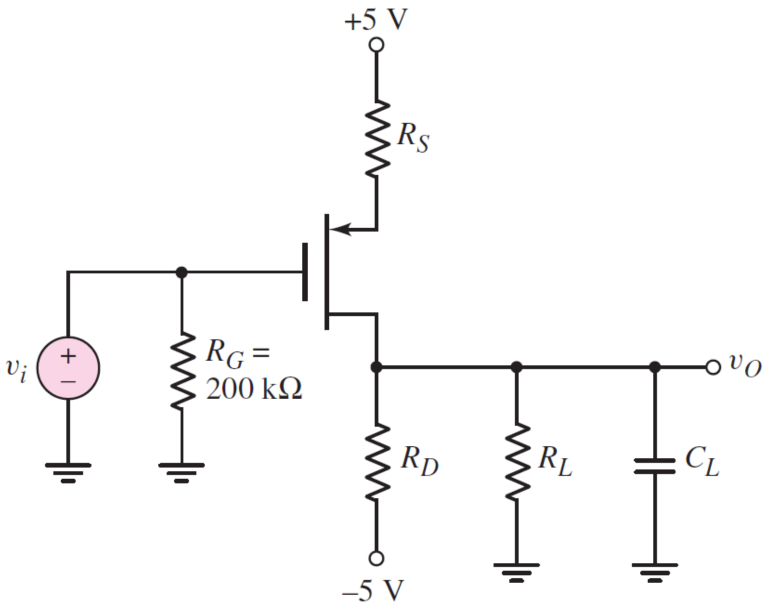


Transistor Amplifiers with Capacitors

- Three types of capacitors will be considered
  - Coupling capacitor, load capacitor, and bypass capacitor
- What is the Bode plot of this amplifier circuit?

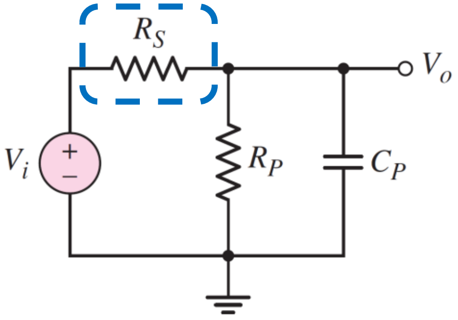


Parallel load capacitor circuit
(low-pass network)

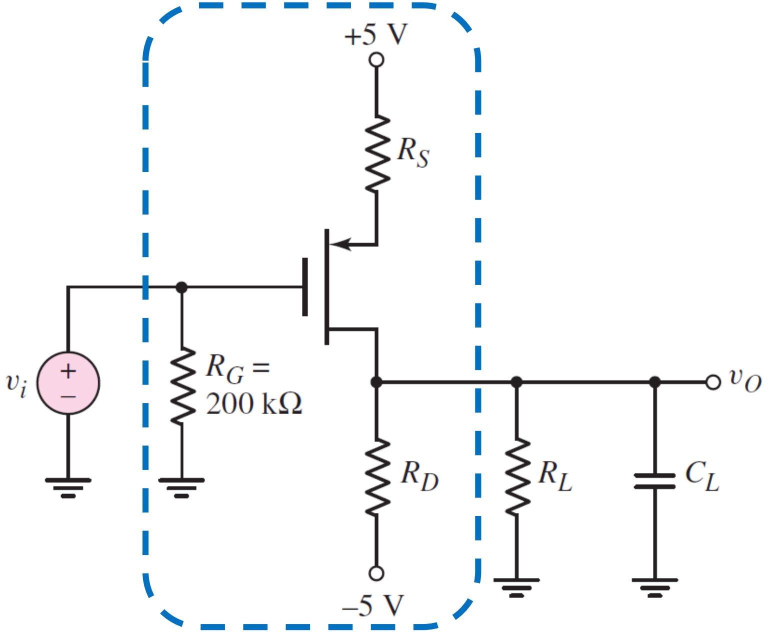


Load Capacitor Effects

- This is a low-pass network
- Draw the general shape of the Bode plot
  - Upper corner frequency
  - The maximum small-signal voltage gain

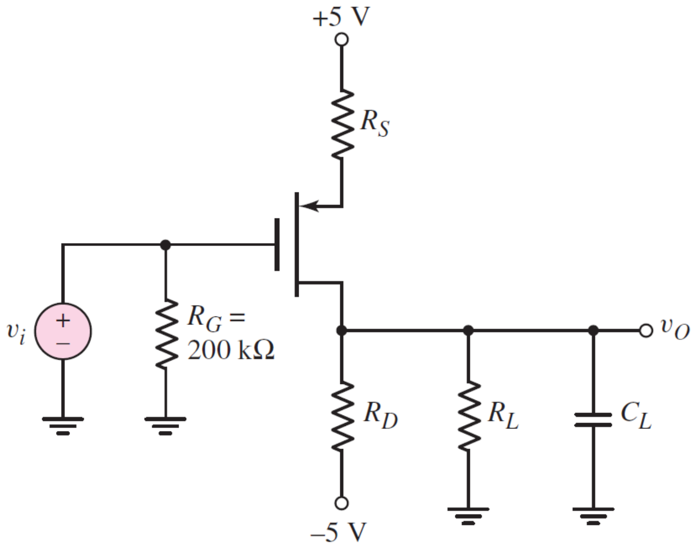


Parallel load capacitor circuit
(low-pass network)



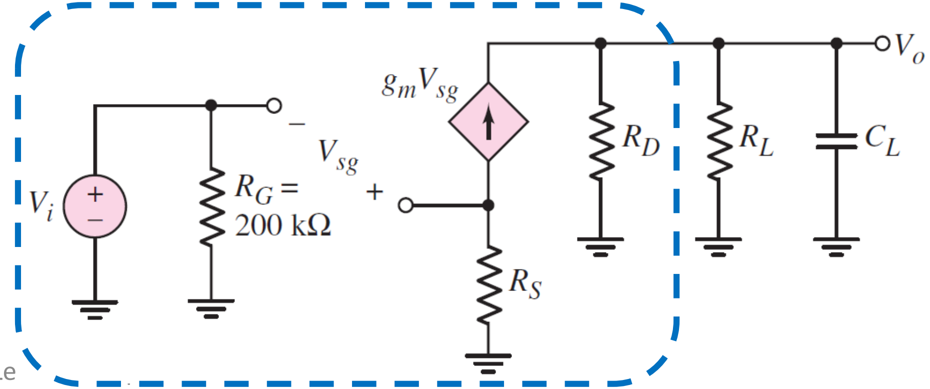
Load Capacitor Effects

- The effective resistance seen by the load capacitor is
  - $R_{eq} = R_o \parallel R_L$
- The time constant is
  - $\tau_P = R_{eq} C_L$



- The upper corner frequency is

$$f_H = \frac{1}{2\pi\tau_P}$$



Le

Load Capacitor Effects

- The maximum small-signal voltage gain is

$$|A_v|_{\max} = \frac{g_m(R_D \parallel R_L)}{1 + g_m R_S}$$

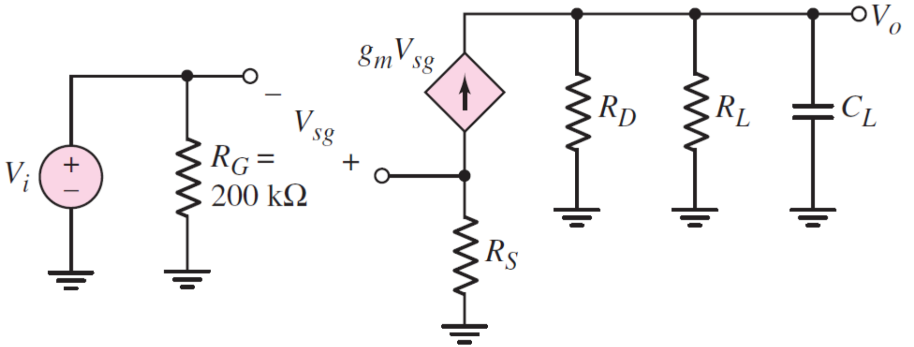
  - C_L is treated as an open circuit

- The output voltage is

$$V_o = g_m V_{sg}(R_D \parallel R_L)$$

- The input voltage is

$$V_i = -(1 + g_m R_S)V_{sg}$$

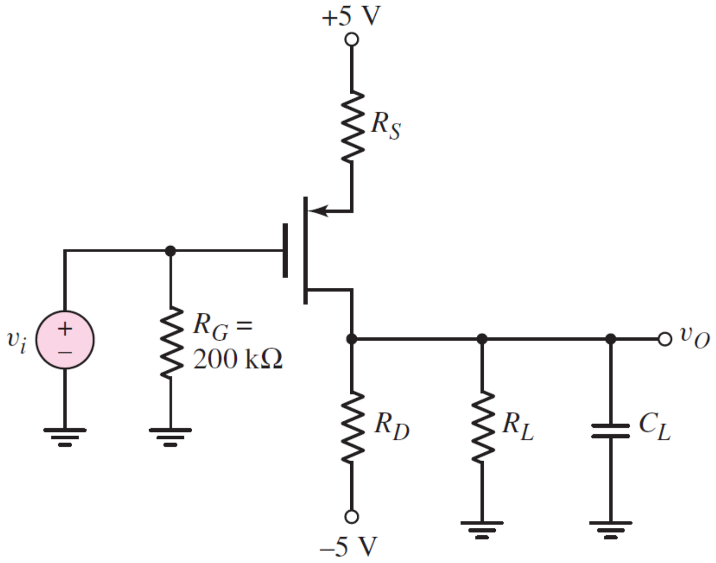


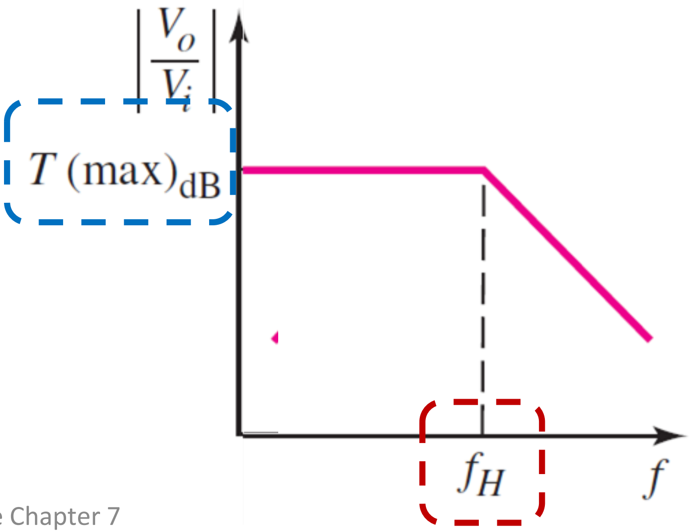
Load Capacitor Effects

- The Bode plot of the common-source with source resistor circuit is

$$|A_v|_{\max} = \frac{g_m(R_D \parallel R_L)}{1 + g_m R_S}$$

$$f_H = \frac{1}{2\pi(R_o \parallel R_L)C_L}$$



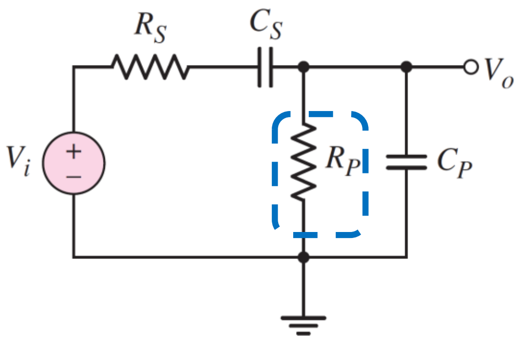


Problem-Solving Technique: Bode Plot

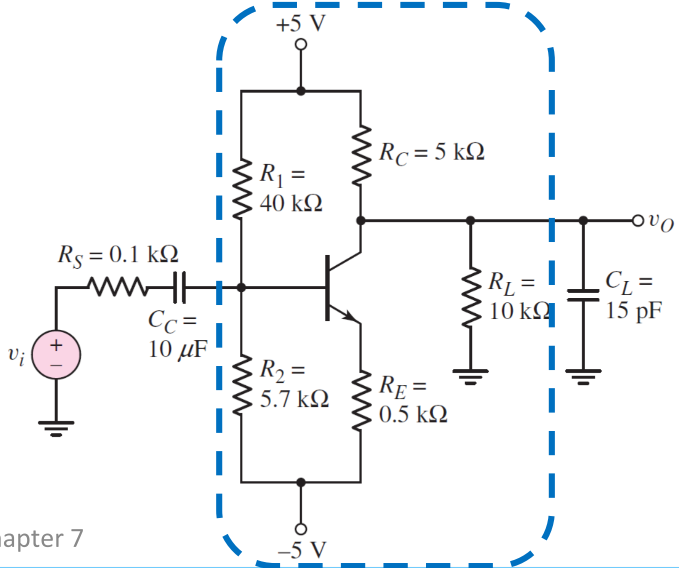
- For a particular capacitor:
  - 1. Determine whether the capacitor is producing a low-pass or high-pass circuit
    - Sketch the general shape of the Bode plot
  - 2. Find the corner frequency from $f = 1/2\pi\tau$
    - The time constant is $\tau = R_{eq}C$
    - The R_{eq} is the effective resistance seen by the capacitor
  - 3. Find maximum gain magnitude (mid-band gain)
    - Coupling and bypass capacitors act as short circuits
    - Load capacitors act as open circuits
    - See more details in Chapter 5 and 6

Coupling and Load Capacitors

- A circuit with both a coupling capacitor and load capacitor
- The values of these two capacitor differ by orders of magnitude
  - $C_C \gg C_L$
- The corner frequencies are far apart and can be treated separately



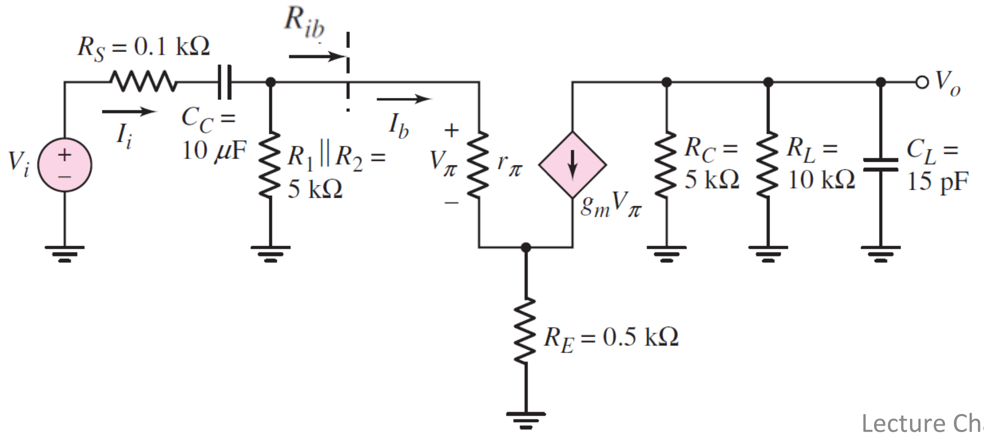
Combination of RC circuit 1 and 2

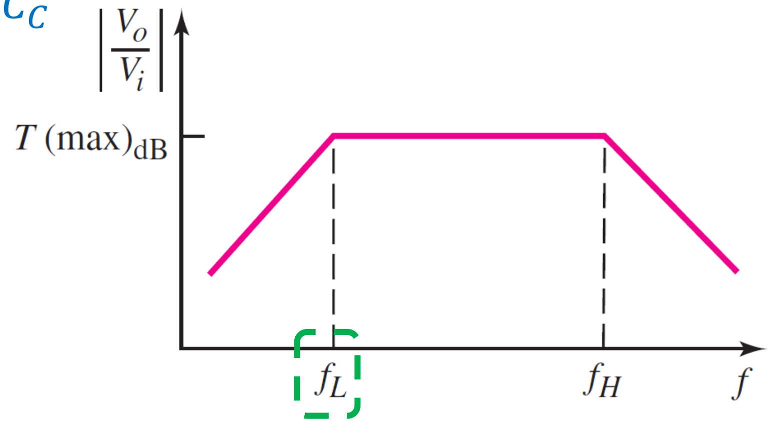


Coupling and Load Capacitors

- The effective resistance seen by the coupling capacitor C_C is
  - $R_{eq} = R_S + (R_1 \parallel R_2 \parallel R_{ib})$
  - $R_{ib} = r_\pi + (1+\beta)R_E$
- The lower corner frequency is

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi R_{eq} C_C} = \frac{1}{2\pi[R_S + (R_1 \parallel R_2 \parallel R_{ib})]C_C}$$

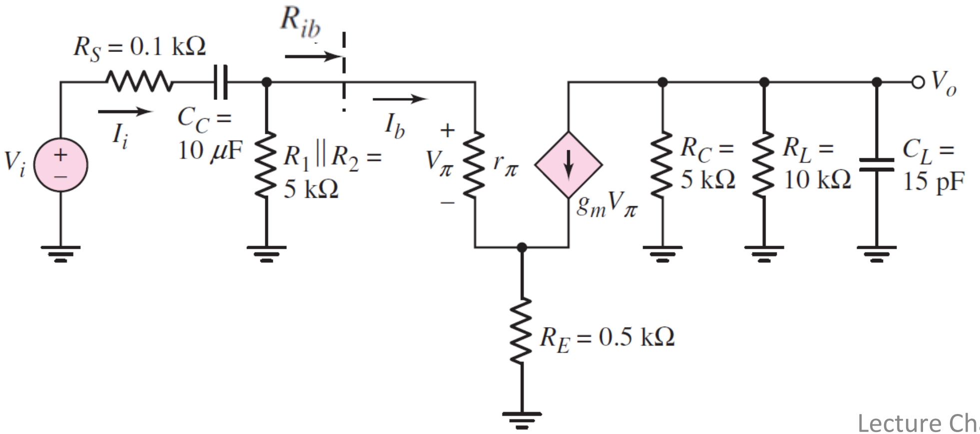


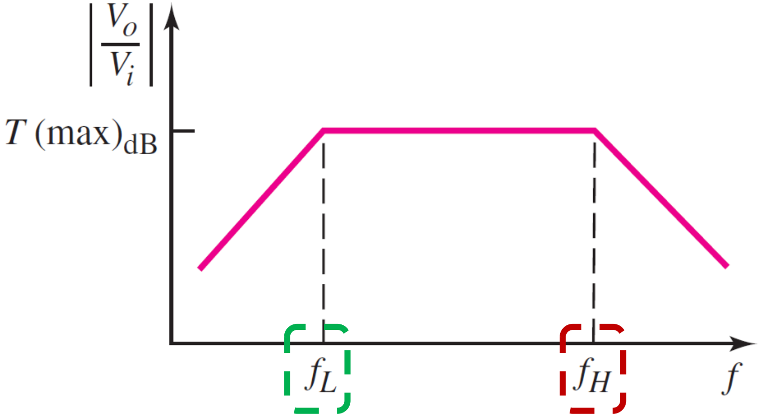


Coupling and Load Capacitors

- The effective resistance seen by the load capacitor C_L is
  - $R_{eq} = R_C \parallel R_L$
- The upper corner frequency is

$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi R_{eq} C_L} = \frac{1}{2\pi (R_C \parallel R_L) C_C}$$



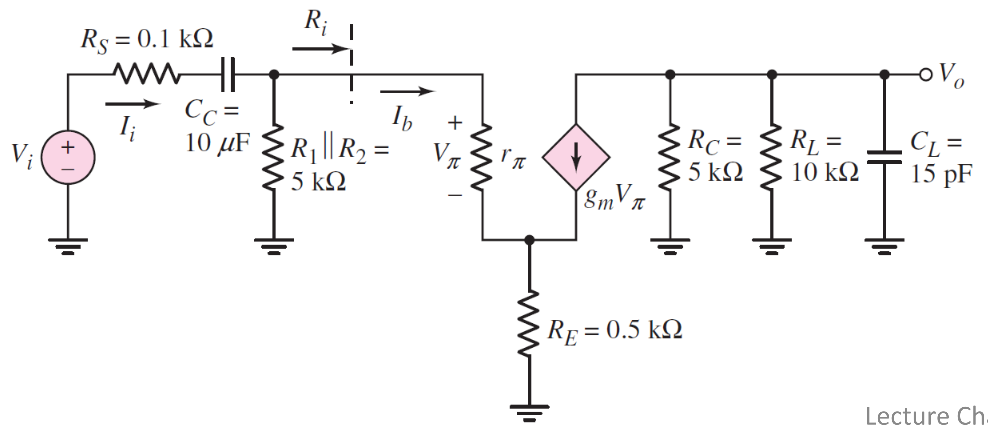


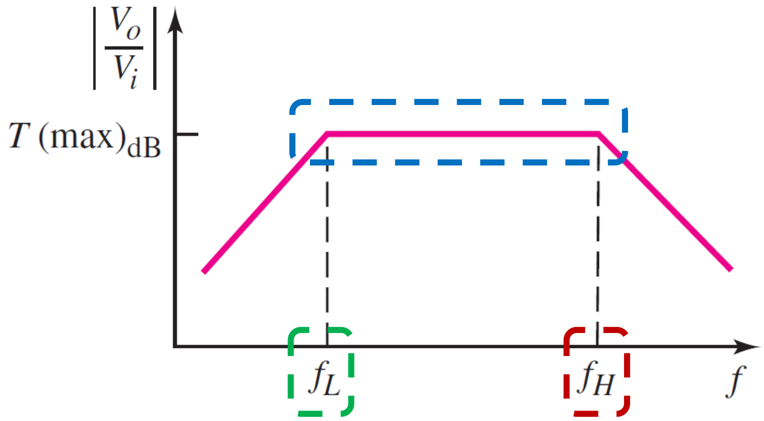
Coupling and Load Capacitors

- The magnitude of the mid-band gain (the maximum gain) is

$$|A_v|_{\max} = g_m r_\pi (R_C \parallel R_L)\left(\frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + R_i}\right)\left(\frac{1}{R_S + (R_1 \parallel R_2 \parallel R_i)}\right)$$

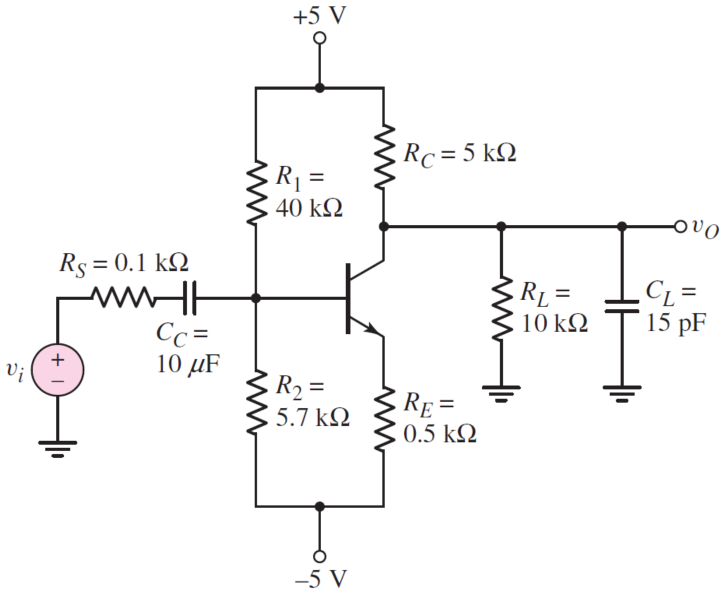
  - The frequency is high enough ($f \gg f_L$), so C_C acts as a short circuit
  - The frequency is low enough ($f \ll f_H$), , so C_L acts as an open circuit





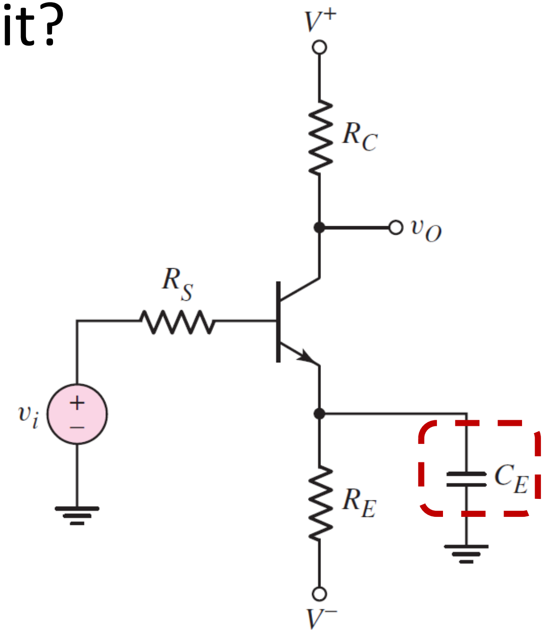
Example 7.5

Determine the mid-band gain, corner frequencies, and bandwidth of a circuit containing both a coupling capacitor and a load capacitor. The transistor parameters are $V_{BE}(\text{on}) = 0.7\text{ V}, \beta = 100, V_A = \infty$.



Transistor Amplifiers with Capacitors

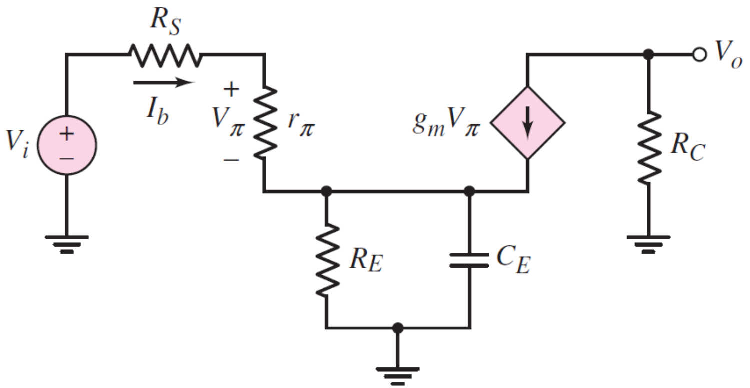
- Three types of capacitors will be considered
  - Coupling capacitor, load capacitor, and bypass capacitor
- This is a common-emitter circuit with an emitter bypass capacitor
- What is the Bode plot of this amplifier circuit?



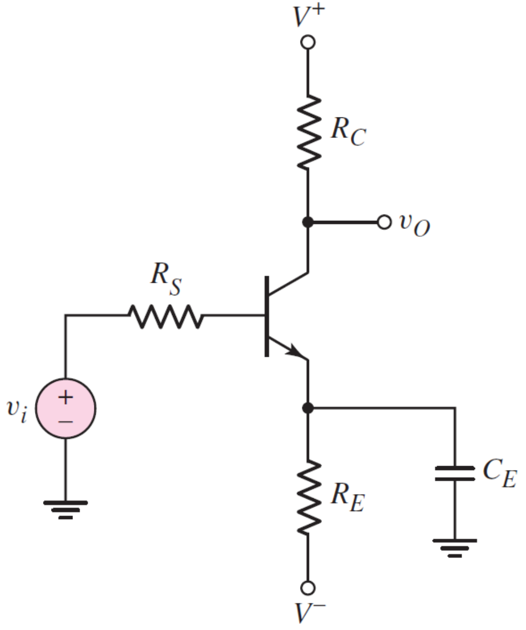
Bypass Capacitor Effects

- Draw the small-signal equivalent circuit
- The output voltage is

$$V_o = -g_m V_\pi R_C = -g_m (I_b r_\pi) R_C$$



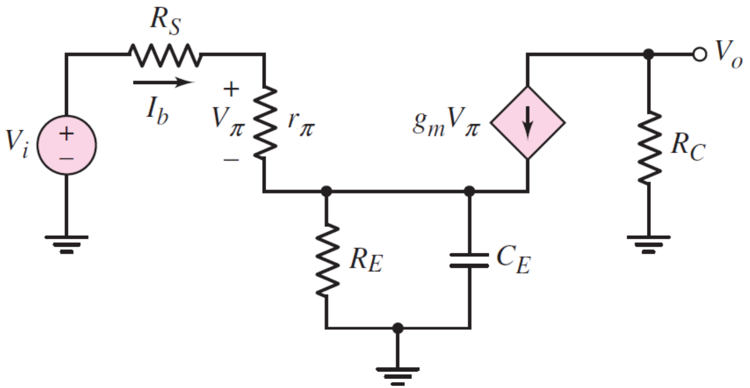
small-signal equivalent circuit



Bypass Capacitor Effects

- The base current is the input current

$$I_b = I_b = \frac{V_i}{R_S + R_{ib}} = \frac{V_i}{R_S + \left[r_\pi + (1+\beta)\left(R_E \parallel \frac{1}{sC_E}\right)\right]}$$



small-signal equivalent circuit

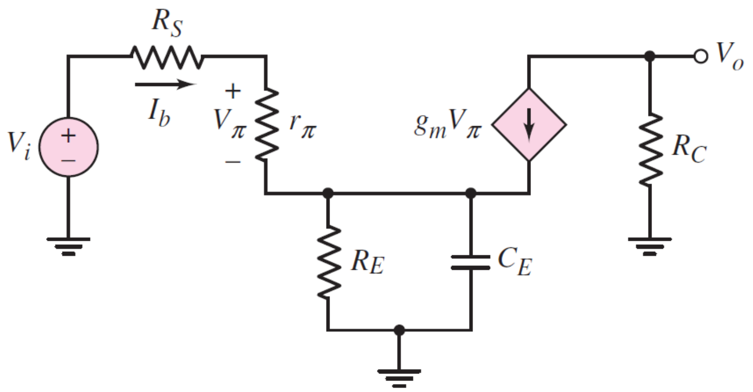
Bypass Capacitor Effects

- Combining previous two equations, we can get

$$V_o = -g_m(I_b r_\pi)R_C = -g_m \frac{V_i}{R_S + r_\pi + (1+\beta)\left(R_E \parallel \frac{1}{sC_E}\right)} r_\pi R_C$$

- The small-signal voltage gain is

$$A_v(s) = -\frac{g_m r_\pi R_C}{R_S + r_\pi + (1+\beta)\left(R_E \parallel \frac{1}{sC_E}\right)}$$



Bypass Capacitor Effects

$$A_v(s) = -\frac{g_m r_\pi R_C}{R_S + r_\pi + (1+\beta)\left(R_E \parallel \frac{1}{sC_E}\right)}$$

- Expanding the parallel combination of R_E and $\frac{1}{sC_E}$ rearranging terms, we find

$$A_v(s) = -\frac{g_m r_\pi R_C}{R_S + r_\pi + (1+\beta)R_E} \times \frac{1 + sR_E C_E}{1 + s\frac{R_E(R_S + r_\pi)C_E}{R_S + r_\pi + (1+\beta)R_E}}$$

- We can write the voltage gain in terms of time constants as

$$A_v(s) = -\frac{g_m r_\pi R_C}{R_S + r_\pi + (1+\beta)R_E} \times \frac{1 + s\tau_A}{1 + s\tau_B}$$

$$\tau_A = R_E C_E$$

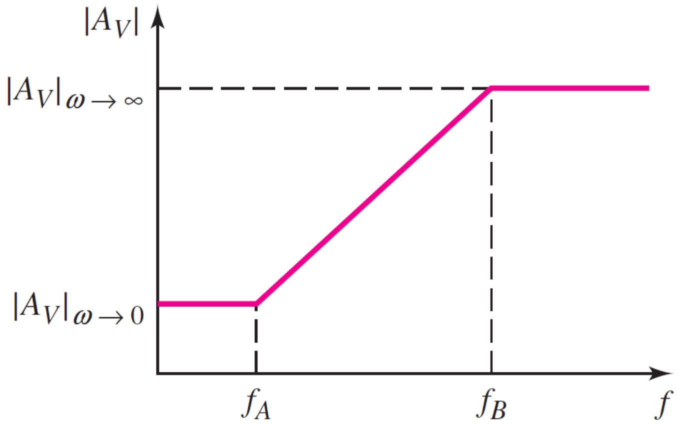
$$\tau_B = \frac{R_E(R_S + r_\pi)C_E}{R_S + r_\pi + (1+\beta)R_E}$$

Bypass Capacitor Effects

$$A_v(s) = -\frac{g_m r_\pi R_C}{R_S + r_\pi + (1+\beta)R_E} \times \frac{1 + s\tau_A}{1 + s\tau_B}$$

$$|A_v|_{\omega \to 0} = \frac{g_m r_\pi R_C}{R_S + r_\pi + (1+\beta)R_E} \qquad |A_v|_{\omega \to \infty} = \frac{g_m r_\pi R_C}{R_S + r_\pi}$$

- The Bode plot of the voltage gain magnitude is
  - $f_B = \frac{1}{2\pi\tau_B}$
  - $f_A = \frac{1}{2\pi\tau_A}$
  - τ_A and τ_B are time constants



Summary

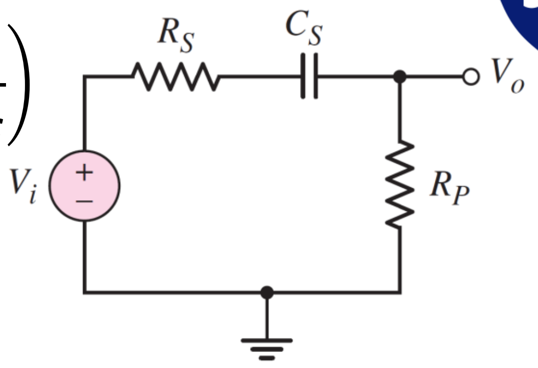
- Construct the Bode plots of the gain magnitude from a transfer function written in terms of the complex frequency s
  - Series coupling capacitor circuit and parallel load capacitor circuit
- Construct the Bode plots of the gain magnitude of amplifier circuits, taking into account circuit capacitors
  - Coupling capacitor, load capacitor and bypass capacitor

- Optional: Determine the Miller capacitance of a BJT circuit using the expanded hybrid-π equivalent circuit
- Optional: Determine the Miller capacitance of an FET circuit using the expanded small-signal equivalent circuit
- Optional: Describe the relative frequency responses of amplifier circuits

Thank You

Optional: Time Response

$$T(s) = \frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{s(R_S + R_P)C_S}{1 + s(R_S + R_P)C_S}\right] = K\left(\frac{s\tau_S}{1 + s\tau_S}\right)$$



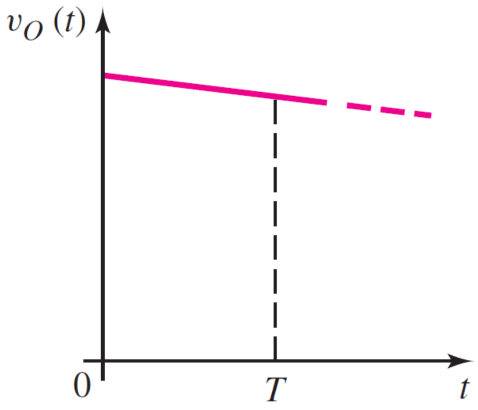
Series coupling capacitor circuit

- If the input voltage is a step function
  - $V_i(s) = \frac{1}{s}$
- The output voltage is

$$V_o(s) = T(s)V_i(s) = K\left(\frac{\tau_S}{1 + s\tau_S}\right) = K\left(\frac{1}{s + \frac{1}{\tau_S}}\right)$$

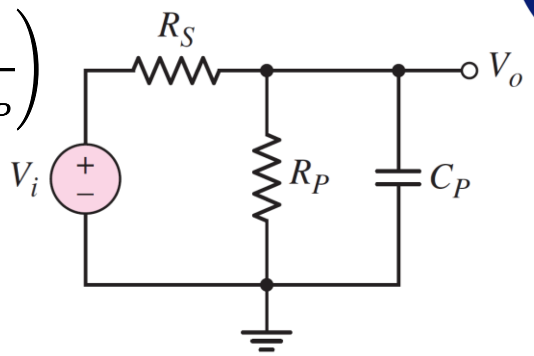
- Taking the inverse Laplace transform

$$v_O(t) = Ke^{-\frac{t}{\tau_S}}$$



Optional: Time Response

$$T(s) = \frac{V_o(s)}{V_i(s)} = \left(\frac{R_P}{R_S + R_P}\right)\left[\frac{1}{1 + s(R_S \parallel R_P)C_P}\right] = K\left(\frac{1}{1 + s\tau_P}\right)$$



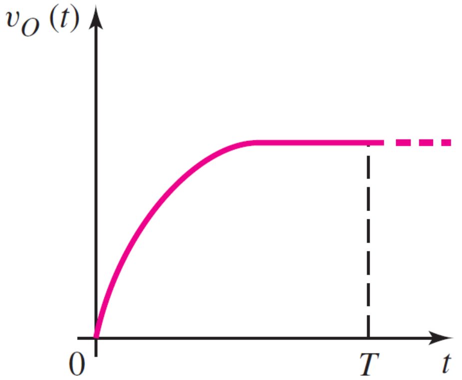
Parallel load capacitor circuit

- If the input voltage is a step function
  - $V_i(s) = \frac{1}{s}$
- The output voltage is

$$V_o(s) = T(s)V_i(s) = \frac{K}{s}\left(\frac{1}{1 + s\tau_P}\right) = \frac{K}{s}\left(\frac{1/\tau_P}{s + 1/\tau_P}\right)$$

- Taking the inverse Laplace transform

$$v_o(t) = K\left(1 - e^{-\frac{t}{\tau_P}}\right)$$



Frequency Response: Bipolar Transistor (Optional)

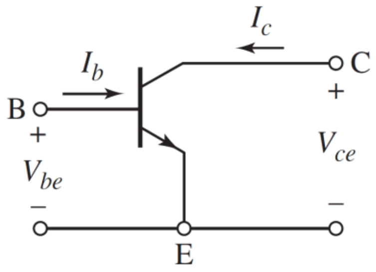
Determine the frequency response of the bipolar transistor, and determine the Miller effect and Miller capacitance.

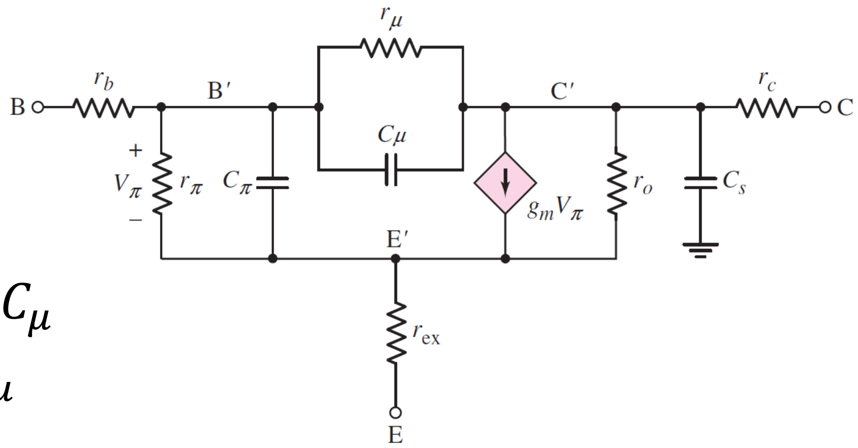
Frequency Response: Bipolar Transistor

- The frequency response of circuits as a function of external resistors and capacitors, and we have assumed the transistor to be ideal.
- However, both bipolar transistors and FETs have internal capacitances that influence the high-frequency response of circuits.
- Develop an expanded small-signal hybrid-π model of the bipolar transistor
- Analyze the frequency characteristics of the bipolar transistor.

Expanded Small-Signal Hybrid-π Model

- Common-emitter npn bipolar transistor
- Base to emitter
  - Base series resistance r_b
  - Forward biased junction capacitance C_π
  - Emitter series resistance r_{ex} (usually very small)
- Collector to emitter
  - Collector series resistance r_c
  - Junction capacitance C_s
- Base to collector
  - Reverse-biased junction capacitance C_μ
  - Reverse-biased diffusion resistance r_μ

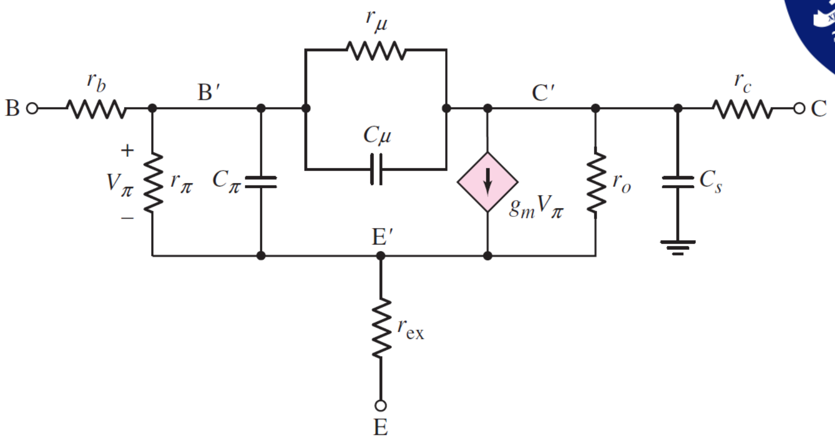


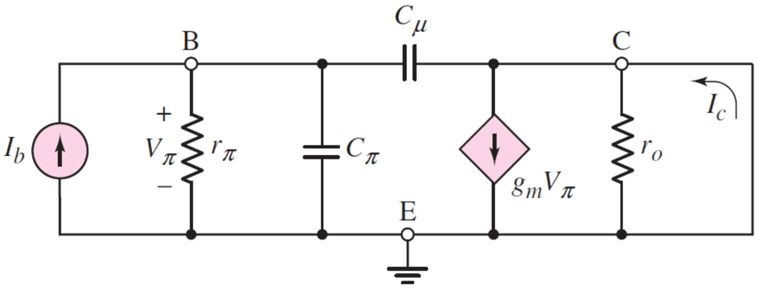


Short-Circuit Current Gain

- Simplified hybrid-π equivalent circuit
  - r_b, r_c, r_{ex}, r_μ, and C_s are neglected
  - The transistor must still be biased in the forward-active region
- Writing KCL equation

$$I_b = \frac{V_\pi}{r_\pi} + \frac{V_\pi}{\frac{1}{j\omega C_\pi}} + \frac{V_\pi}{\frac{1}{j\omega C_\mu}}$$



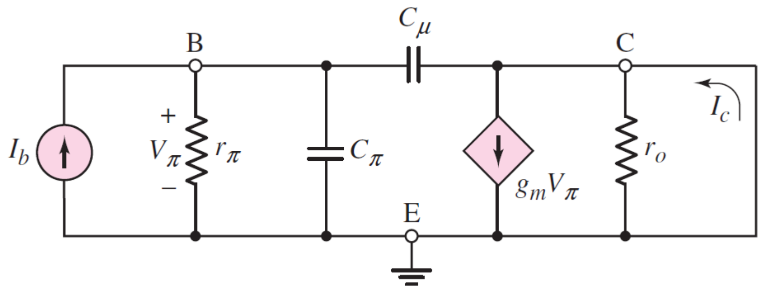


Short-Circuit Current Gain

- Writing KCL equation at the output node

$$\frac{V_\pi}{\frac{1}{j\omega C_\mu}} + I_c = g_m V_\pi \qquad I_c = V_\pi\left(g_m - j\omega C_\mu\right)$$

$$I_b = \frac{I_c}{\left(g_m - j\omega C_\mu\right)}\left[\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)\right]$$



- The small-signal current gain is

$$A_i = \frac{I_c}{I_b} = h_{fe} = \frac{g_m - j\omega C_\mu}{\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)} \cong \frac{g_m r_\pi}{1 + j\omega r_\pi\left(C_\pi + C_\mu\right)}$$

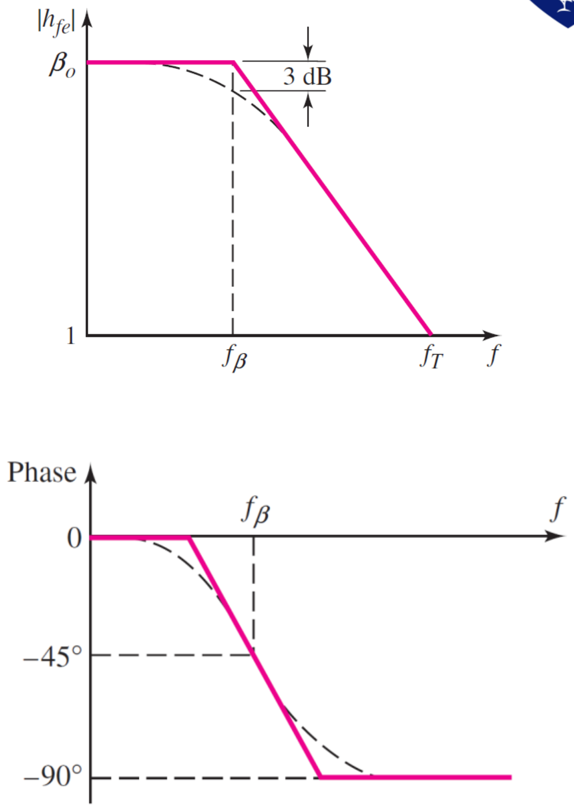
Short-Circuit Current Gain

- The low frequency current gain is just β

$$h_{fe} \cong \frac{g_m r_\pi}{1 + j\omega r_\pi(C_\pi + C_\mu)} = \frac{\beta}{1 + j\omega r_\pi(C_\pi + C_\mu)}$$

- Beta cutoff frequency (corner frequency) is

$$f_\beta = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$$



Cutoff Frequency

- At the cutoff frequency f_T, the gain goes to 1

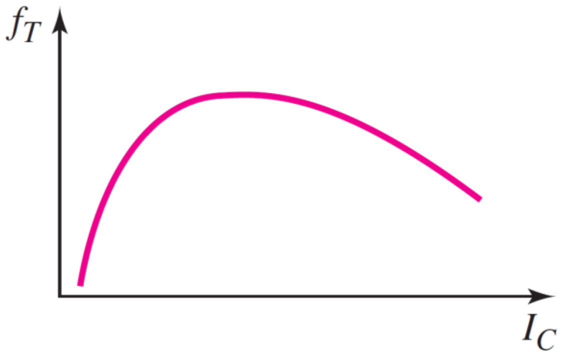
$$h_{fe} = \frac{\beta}{1 + j\omega r_\pi (C_\pi + C_\mu)} = \frac{\beta}{1 + j\left(\frac{f}{f_\beta}\right)}$$

$$1 = \frac{\beta}{\sqrt{1 + \left(\frac{f_T}{f_\beta}\right)^2}} \cong \frac{\beta}{\sqrt{\left(\frac{f_T}{f_\beta}\right)^2}} = \frac{\beta f_\beta}{f_T}$$

- Unity-gain bandwidth

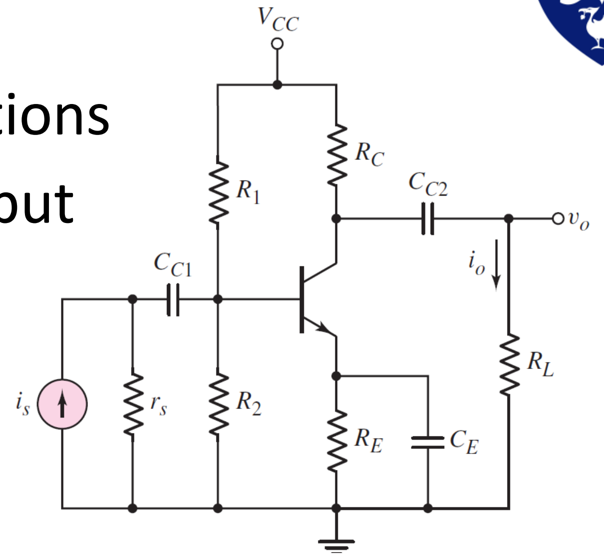
$$f_T = \beta f_\beta = \beta \frac{1}{2\pi r_\pi (C_\pi + C_\mu)} = \frac{g_m}{2\pi (C_\pi + C_\mu)}$$

- The cutoff frequency is also a function of the dc collector current I_C

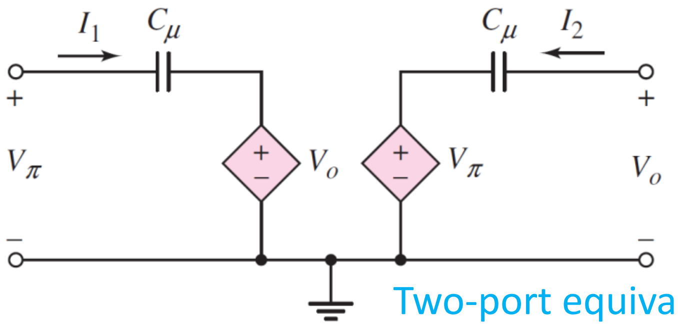


Miller Effect and Miller Capacitance

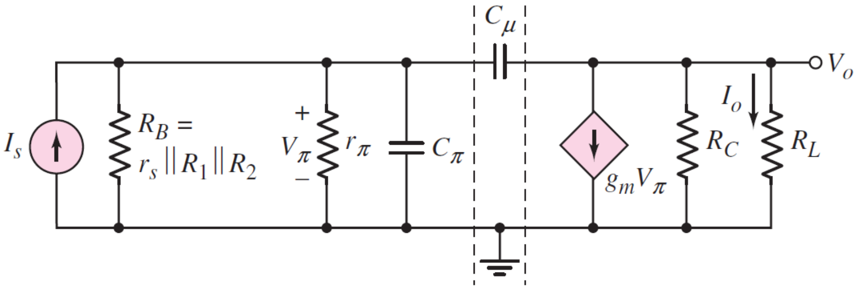
- The Miller effect, or feedback effect, is a multiplication effect of C_μ in circuit applications
- Writing KVL equations at the input and output terminals



$$V_\pi = I_1\left(\frac{1}{j\omega C_\mu}\right) + V_o \qquad V_o = I_2\left(\frac{1}{j\omega C_\mu}\right) + V_\pi$$

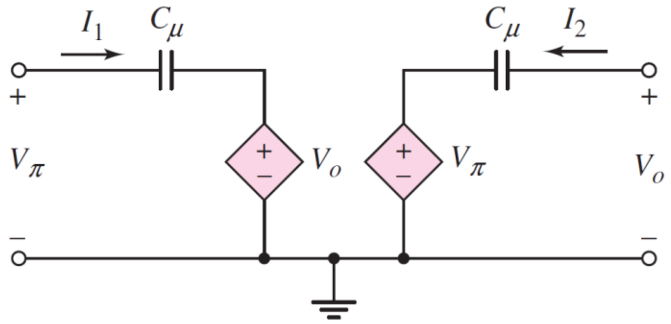


Two-port equivalent circuit of C_μ

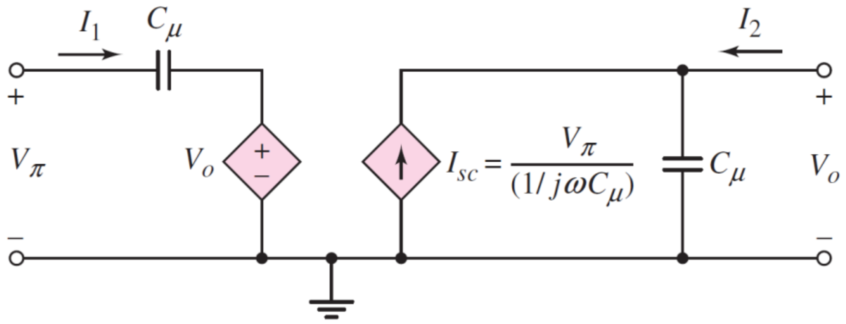


Miller Effect and Miller Capacitance

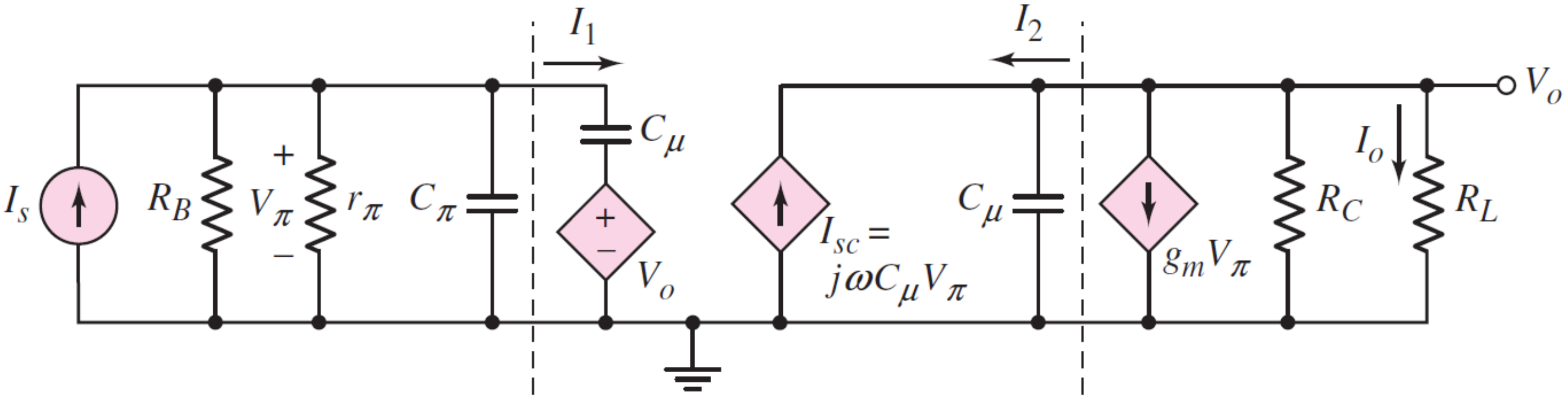
- Convert the Thevenin equivalent circuit to a Norton equivalent circuit



Thevenin equivalent circuit

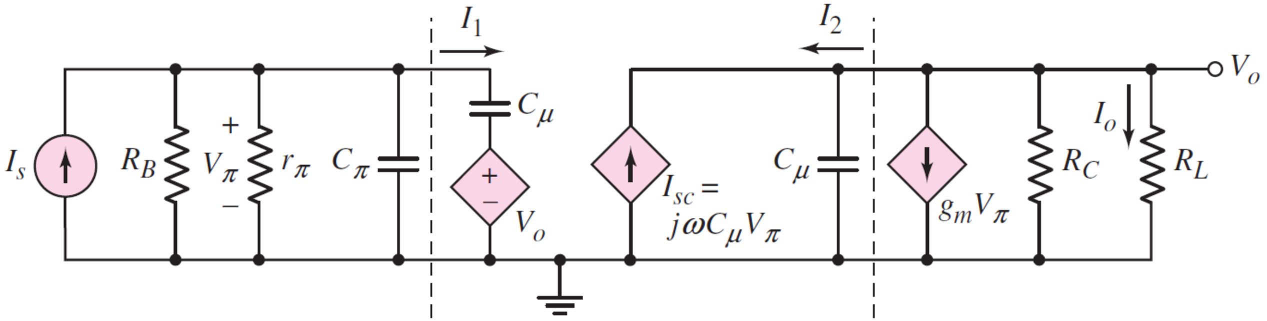


Norton equivalent circuit

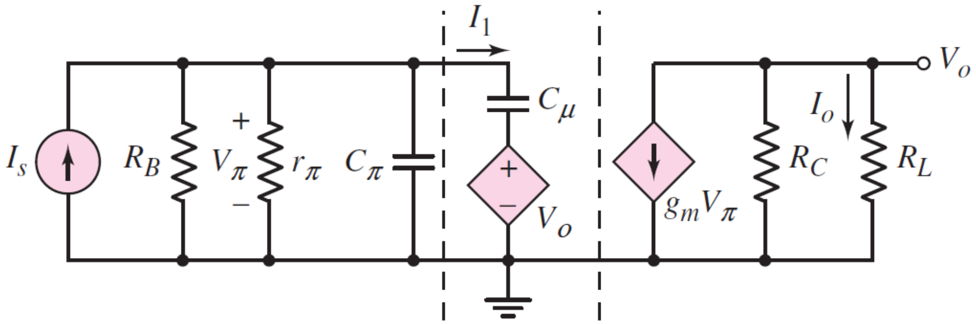


Miller Effect and Miller Capacitance

- Usually the impedance of C_μ is much greater than $R_C \parallel R_L$
- I_{SC} is negligible compared to the $g_m V_\pi$ source

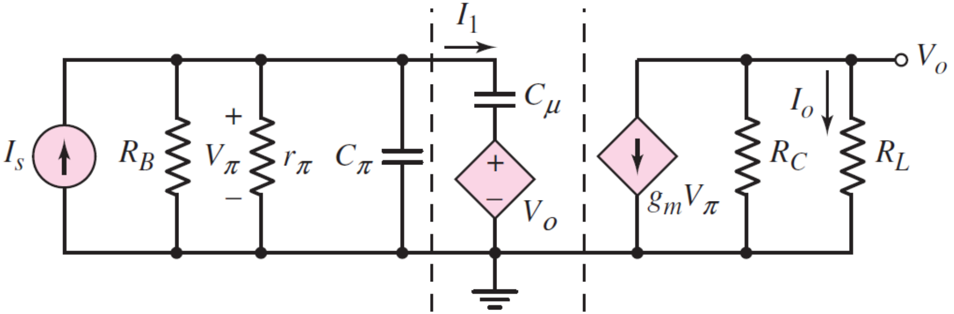


$$I_1 = \frac{V_\pi - V_o}{\frac{1}{j\omega C_\mu}} = j\omega C_\mu (V_\pi - V_o)$$



Miller Effect and Miller Capacitance

- The output voltage is



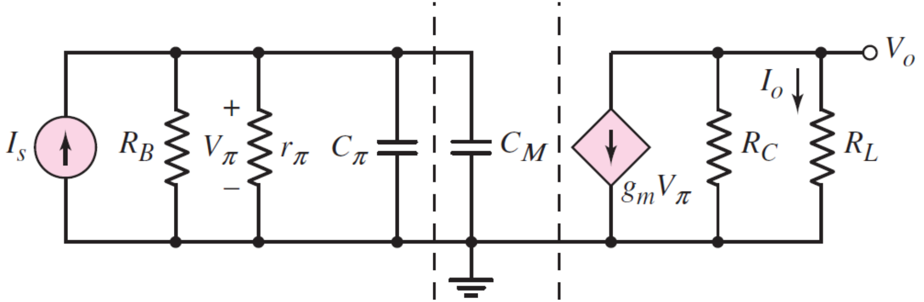
$$V_o = -g_m V_\pi (R_C \parallel R_L)$$

$$I_1 = j\omega C_\mu (V_\pi - V_o) = j\omega C_\mu [1 + g_m (R_C \parallel R_L)] V_\pi = j\omega C_M V_\pi$$

- C_M is the Miller capacitance

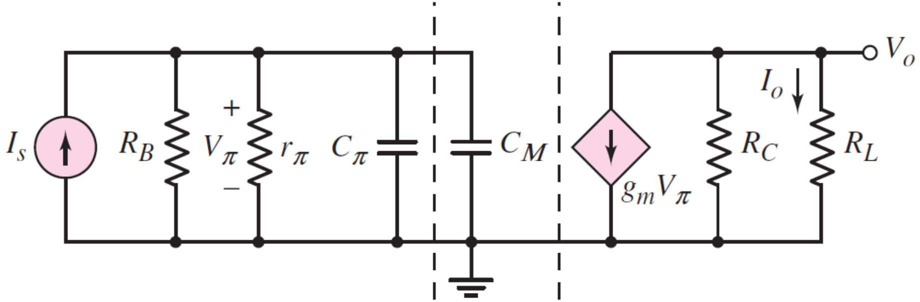
$$\boxed{C_M = C_\mu [1 + g_m (R_C \parallel R_L)]}$$

- The multiplication effect of C_μ is the Miller effect



Example 7.6

Determine the 3 dB frequency of the current gain for the circuit with Miller effect. The circuit parameters are $R_C = R_L = 4\ \mathrm{k\Omega}, r_\pi = 2.6\ \mathrm{k\Omega}, R_B = 200\ \mathrm{k\Omega}, C_\pi = 0.8\ \mathrm{pF}, C_\mu = 0.05\ \mathrm{pF}$, and $g_m = 38.5\ \mathrm{mA/V}$

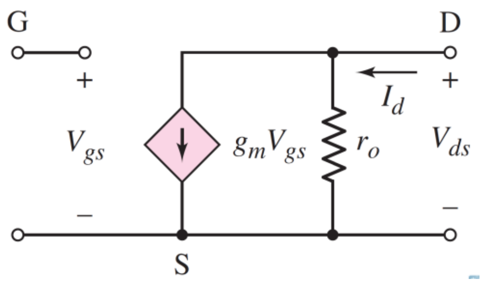


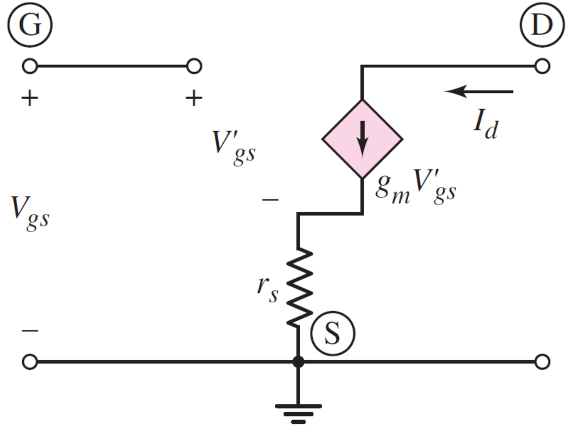
Frequency Response: The FET (Optional)

Determine the frequency response of the MOS transistor, and determine the Miller effect and Miller capacitance.

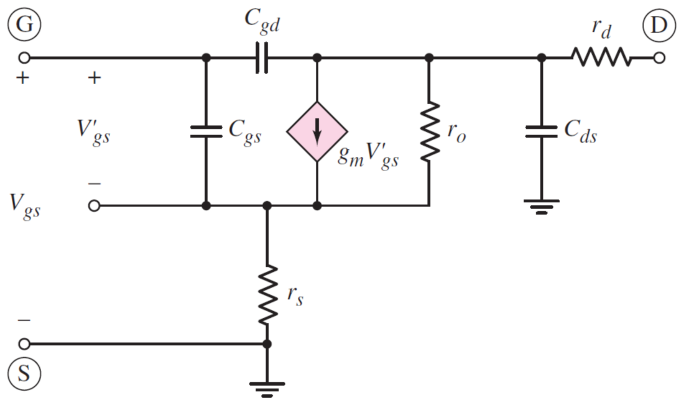
High-Frequency Equivalent Circuit

- V'_{gs} is the internal gate-source voltage
- A simplified low-frequency equivalent circuit





Simplified low-frequency equivalent circuit



Equivalent circuit of the n-channel common-source MOSFET

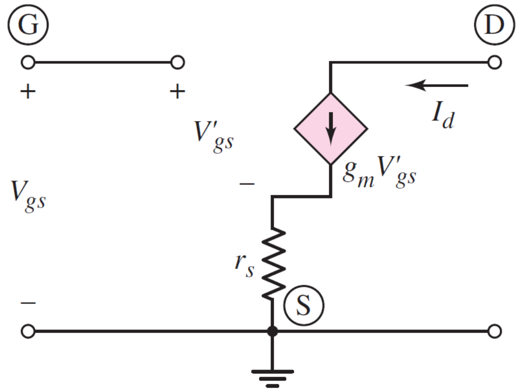
Simplified Low-Frequency Equivalent Circuit

- For this circuit, the drain current is

$$I_d = g_m V'_{gs}$$

$$V_{gs} = V'_{gs} + \left(g_m V'_{gs}\right) r_s = (1 + g_m r_s) V'_{gs}$$

$$I_d = \left(\frac{g_m}{1 + g_m r_s}\right) V_{gs} = g'_m V_{gs}$$

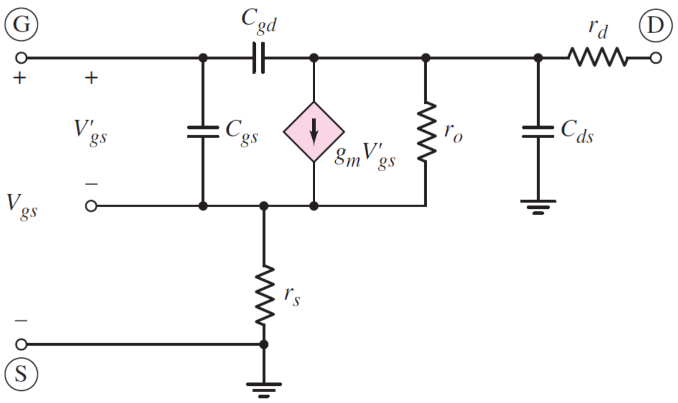


Simplified low-frequency equivalent circuit of the n-channel common-source MOSFET

- The p-channel MOSET is exactly the same as that of an n-channel devices
    - All voltage polarities and current directions are reversed

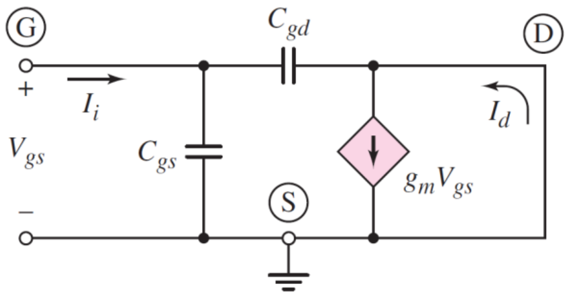
Short-Circuit Current Gain

- Neglect r_s, r_d, r_o and C_{ds}
- Short-circuit current gain
- Apply KCL equation



$$I_i = \frac{V_{gs}}{\frac{1}{j\omega C_{gs}}} + \frac{V_{gs}}{\frac{1}{j\omega C_{gd}}} = j\omega\left(C_{gs} + C_{gd}\right)V_{gs}$$

$$\frac{V_{gs}}{\frac{1}{j\omega C_{gd}}} + I_d = g_m V_{gs} \qquad I_d = V_{gs}\left(g_m - j\omega C_{gd}\right)$$



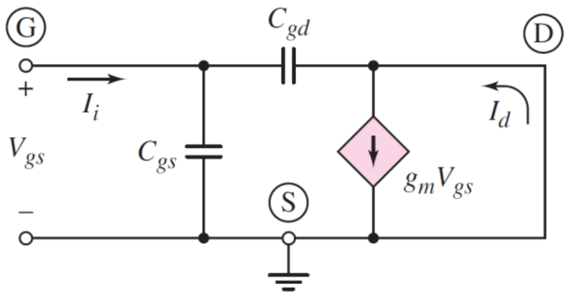
$$A_i = \frac{I_d}{I_i} = \frac{g_m - j\omega C_{gd}}{j\omega\left(C_{gs} + C_{gd}\right)} \cong \frac{g_m}{j\omega\left(C_{gs} + C_{gd}\right)}$$

Unity-Gain Frequency

- The unity-gain frequency f_T is defined as the frequency at which the magnitude of the short-circuit current gain goes to 1.

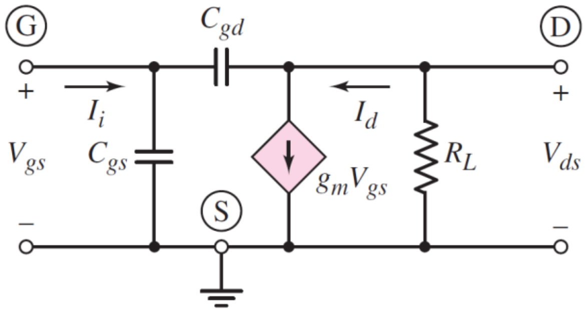
$$A_i \cong \frac{g_m}{j\omega\left(C_{gs} + C_{gd}\right)} = 1$$

$$f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)}$$



Miller Effect and Miller Capacitance

- Miller effect and Miller capacitances are factors under high frequency
- Find the Miller capacitance
- Writing a KCL equation



$$I_i = \frac{V_{gs}}{\frac{1}{j\omega C_{gs}}} + \frac{V_{gs} - V_{ds}}{\frac{1}{j\omega C_{gd}}} = j\omega C_{gs} V_{gs} + j\omega C_{gd}(V_{gs} - V_{ds})$$

$$\frac{V_{gs} - V_{ds}}{\frac{1}{j\omega C_{gd}}} = g_m V_{gs} + \frac{V_{ds}}{R_L}$$

$$\frac{V_{ds}}{R_L} + g_m V_{gs} + j\omega C_{gd}(V_{ds} - V_{gs}) = 0$$

Miller Effect and Miller Capacitance

$$I_i = j\omega C_{gs} V_{gs} + j\omega C_{gd}(V_{gs} - V_{ds})$$

$$\frac{V_{ds}}{R_L} + g_m V_{gs} + j\omega C_{gd}(V_{ds} - V_{gs}) = 0$$

$$\frac{V_{ds}}{R_L} + g_m V_{gs} + j\omega C_{gd}(V_{ds} - V_{gs}) = 0$$

- Combine two equations to eliminate V_{ds}

$$I_i = j\omega\left[C_{gs} + C_{gd}\left(\frac{1 + g_m R_L}{1 + j\omega R_L C_{gd}}\right)\right]V_{gs} \cong j\omega[C_{gs} + C_{gd}(1 + g_m R_L)]V_{gs}$$

  - Normally, $\omega R_L C_{gd}$ is much less than 1

Miller Effect and Miller Capacitance

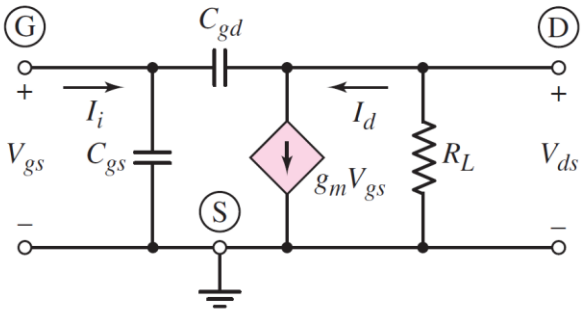
$$I_i \cong j\omega\left[C_{gs} + C_{gd}(1 + g_m R_L)\right]V_{gs} = j\omega\left(C_{gs} + C_M\right)V_{gs}$$

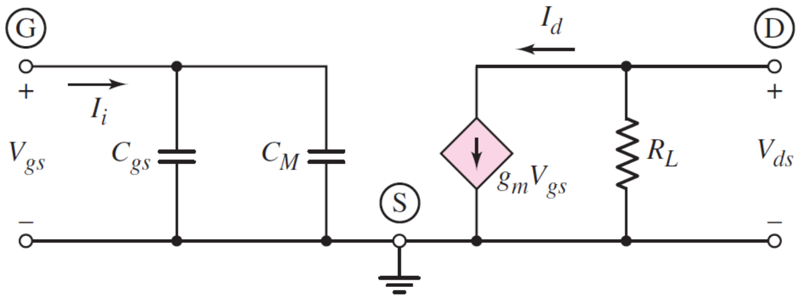
- C_M is the Miller capacitance

$$\boxed{C_M = C_{gd}(1 + g_m R_L)}$$

- C_G is the equivalent input gate capacitance

$$C_G = C_{gs} + C_M$$





High Frequency Response of Transistor Circuits

Determine the high-frequency response of basic transistor circuit configurations including the cascode circuit.

High Frequency Response of Transistor Circuits

- The high-frequency equivalent circuits for BJT and FET
  - Miller effect of BJT: $C_M = C_\mu[1 + g_m(R_C \parallel R_L)]$
  - Miller effect of FET: $C_M = C_{gd}(1 + g_m R_L)$

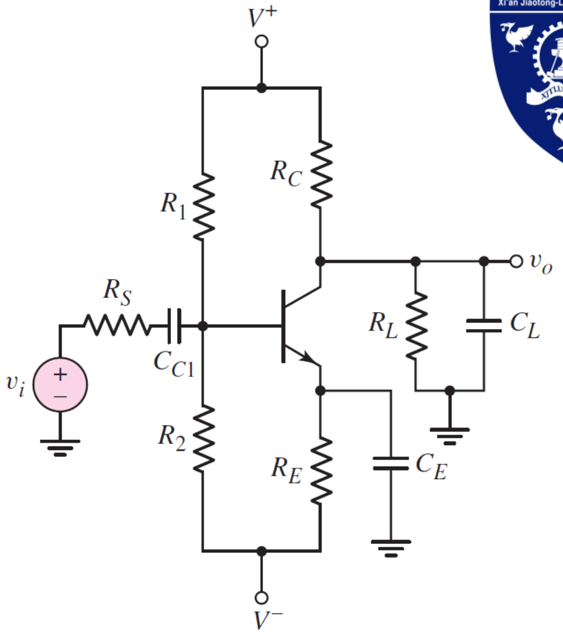
- Analysis the high-frequency characteristics of transistor circuits
  - Common-emitter configuration
  - Common-source configuration

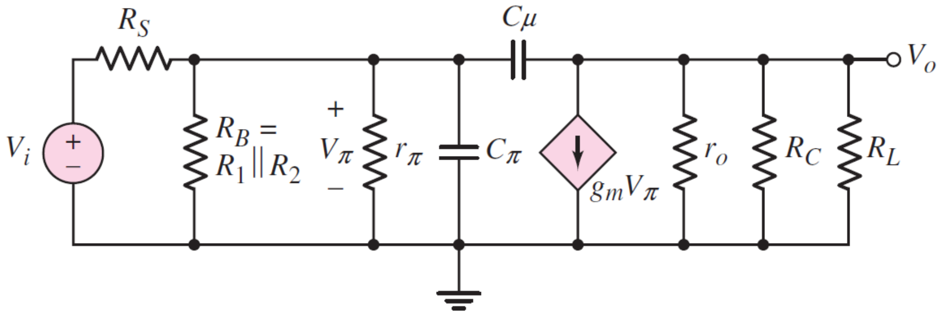
Common-Emitter Circuit

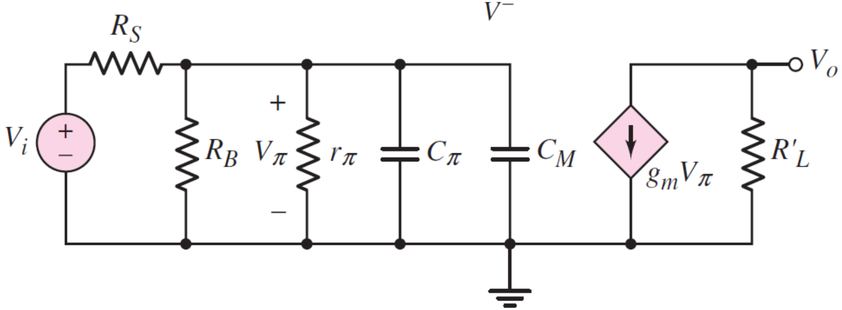
- Assume C_{C1} and C_E are short circuits and C_L is an open circuit
- Considering the Miller effect

$$C_M = C_\mu(1 + g_m R_L')$$

$$R_L' = r_o \parallel R_C \parallel R_L$$





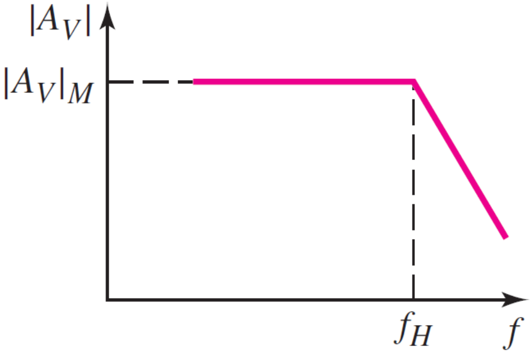


Common-Emitter Circuit

- The upper 3 dB frequency is

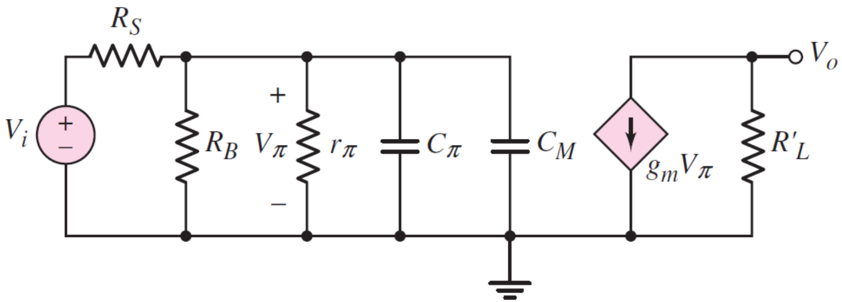
$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi\left(R_{eq}C_{eq}\right)}$$

  - $R_{eq} = r_\pi \parallel R_B \parallel R_S$
  - $C_{eq} = C_\pi + C_M$



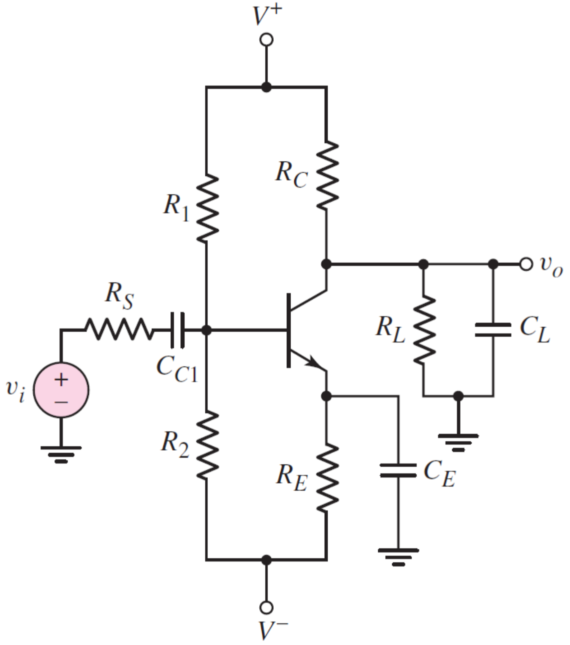
- The midband voltage gain magnitude is
  - Assume C_π and C_M are open circuits

$$|A_v|_M = \left|\frac{V_o}{V_i}\right|_M = \frac{g_m R_L'}{\frac{r_\pi \parallel R_B + R_S}{r_\pi \parallel R_B}}$$



Example 7.7

Determine the upper corner frequency and midband gain of a common-emitter circuit. The circuit parameters are $V^+ = 5\text{ V}, V^- = -5\text{ V}$, $R_S = 0.1\text{ k}\Omega, R_1 = 40\text{ k}\Omega, R_2 = 5.72\text{ k}\Omega, R_E = 0.5\text{ k}\Omega, R_C = 5\text{ k}\Omega$ and $R_L = 10\text{ k}\Omega$. The transistor parameters are $V_{BE}(\text{on}) = 0.7\text{ V}, \beta = 150, V_A = \infty, C_\pi = 35\text{ pF}$, and $C_\mu = 4\text{ pF}$.



Common-Source Circuit

- The high-frequency response of the common-source circuit is similar to that of the common-emitter circuit
  - C_{π} is replaced by C_{gs}
  - C_{μ} is replaced by C_{gd}
- The high-frequency small-signal equivalent circuit of the FET is then essentially identical to that of the bipolar transistor.