Xi'an Jiaotong-Liverpool University
西交利物浦大学

MEC208 Instrumentation and Control System

2024-25 Semester 2

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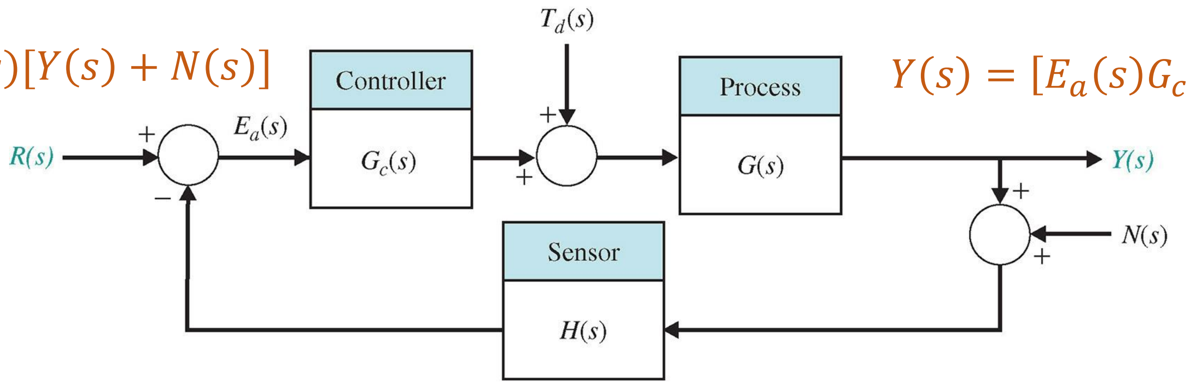
Correction: Definition of Error Signal

$E_a(s) = R(s) - H(s)[Y(s) + N(s)]$

$Y(s) = [E_a(s)G_c(s) + T_d(s)]G(s)$



(b)

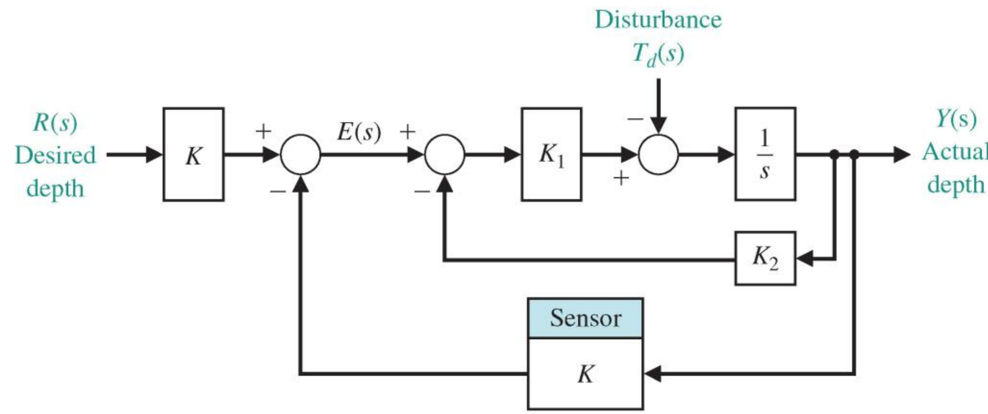
Tracking error definition: $E(s) = R(s) - Y(s)$

To facilitate our discussion, unity feedback system is assumed, i.e., $H(s) = 1$.

Correction：Example 12.5

Consider the following system:

1) Compute the transfer function $T(s) = \frac{Y(s)}{R(s)}$;
2) Determine the sensitivity $S_{K_1}^T$ and $S_{K_2}^T$;
3) Calculate the output's steady-state response due to unit-step input $R(s) = 1/s$;
4) Calculate the steady-state error, $\lim_{s\to0} sE$, due to unit-step disturbance $T_d(s) = 1/s$.



Thought process: (1) manipulate block diagram to obtain $\frac{Y}{R}$; (2) Calculate $S_{K_1}^T = \frac{\partial T/T}{\partial K_1/K_1}$ and $S_{K_2}^T = \frac{\partial T/T}{\partial K_2/K_2}$; (3) $y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to0} sY$; (4) derive $T_1 = \frac{Y}{T_d}$, calc. steady- state error $\lim_{s\to0} sE$

(Reminder: calculation steps must be shown/included in the exam)

Final answer: $T = \frac{KK_1}{s+K_1(K_2+K)}$, $S_{K_1}^T = \frac{s}{s+K_1(K_2+K)}$, $S_{K_2}^T = -\frac{K_1K_2}{s+K_1(K_2+K)}$, $y_{ss} = \frac{K}{K_2+K}$,

$T_1 = -\frac{\frac{1}{s}}{1+\frac{1}{s}\cdot K_1(K_2+K)}$, $\lim_{s\to0} sE = \lim_{s\to0} s\left(0 - T_1\frac{1}{s}\right) = \frac{1}{K_1(K_2+K)}$

[**Note**: this example has a –ve T_d input, different from +ve input in the figure in pg. 8; the sign of T_1 in this exercise should be derived correctly]

Lecture 14

Outline

Time-Domain Performance of Feedback Control Sys.

- ❑ Test Input Signals
- ❑ Performance of Second-Order System
- ❑ Effects of a Third Pole and a Zero on the Second-Order System Response
- ❑ **Pole location on the *s*-Plane and Transient Response**
- ❑ **Steady-State Error of Feedback Control Systems**
- ❑ **System Simulation Using Matlab**

Terminology revisited

- Example: transfer function of a plant is $T(s) = \frac{s+1}{s(s+2)}$. Define the following parameters:
  - Order of the characteristic equation = ?
  - Poles = ? (roots of the denominator's s-polynomial of the transfer func.)
  - Zeroes = ? (roots of the numerator's s-polynomial of the transfer func.)
  - Rank = ? (number of poles – number of zeroes)
  - Type = ? (number of poles at the origin)

Pole location on the *s*-Plane and Transient Response

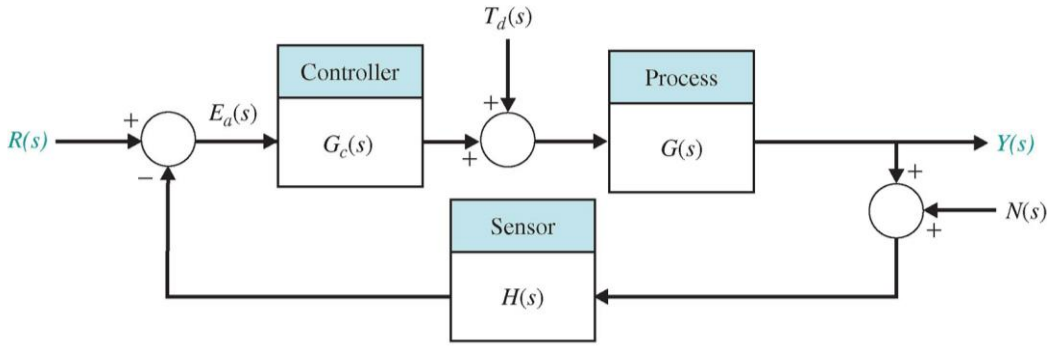
Transfer function for a closed-loop system can be written as:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\sum P_i(s)\Delta_i(s)}{\Delta(s)}$$

Characteristic equation of the system: $\Delta(s) = 0$

For a unit feedback control system: $\boldsymbol{\Delta(s) = 1 + G_c(s)G(s) = 0}$

Time response of a system depends on the poles and zeros of its transfer function $T(s)$; while for a closed-loop system, the poles are the roots of the characteristic equation: $\Delta(s)$.



Transfer function:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)}$$

Time Response of System: General Form

If the system (with DC gain = 1) has no repeated roots, its unit step response can be formulated as a partial fraction expansion as:

$$Y(s) = \frac{1}{s} + \sum_{i=1}^{M} \frac{A_i}{s+\sigma_i} + \sum_{K=1}^{N} \frac{B_k s + C_k}{s^2 + 2\alpha_k s + ({\alpha_k}^2 + {\omega_k}^2)}$$

where A_i, B_k and C_k are constants; the roots of the system must be either

$s = -\sigma_i$ or $s = -\alpha_k \pm j\omega_k$

The transient response expression can be obtained by inverse Laplace transform:

$$y(t) = 1 + \sum_{i=1}^{M} A_i e^{-\sigma_i t} + \sum_{k=1}^{N} D_k e^{-\alpha_k t} \sin(\omega_k t + \theta_k)$$

exponential terms

Steady-state output

Damped sinusoidal terms

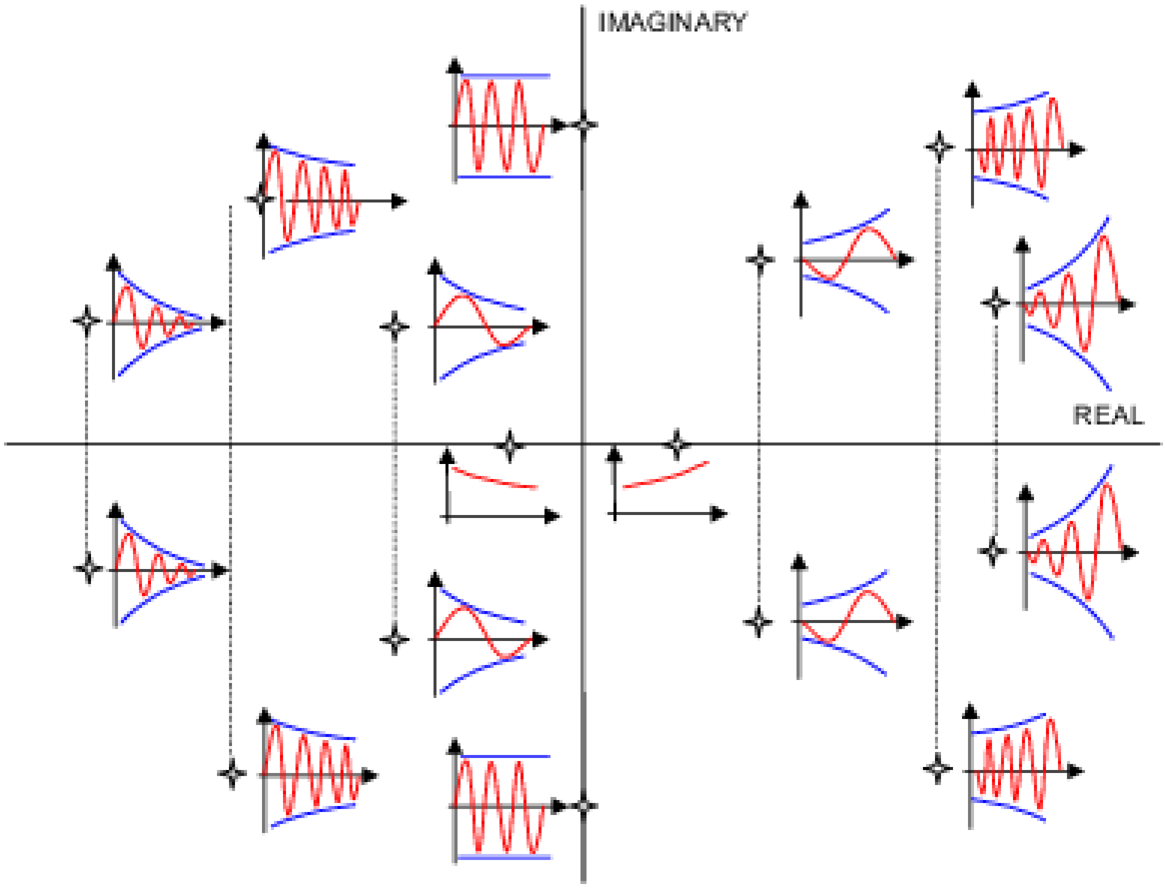
where D_k is a constant depends on B_k, C_k, α_k and ω_k.

For the response to be stable (bounded for a step input, a.k.a. BIBO stable) – **the poles must be in the left-hand side of the *s*-plane (i.e., real parts are negative)**.

Step Response for Various Root Locations in the s-Plane

$$\boldsymbol{y(t) = 1 + \sum_{i=1}^{M} A_i e^{-\sigma_i t} + \sum_{k=1}^{N} D_k e^{-\alpha_k t} \sin(\omega_k t + \theta_k)}$$

$(s = -\sigma_i)$ $(s = -\alpha_k \pm j\omega_k)$



Root Location and System Design

- It is important for the control system designer to understand the complete relationship of the **frequency domain** representation of a linear system, the **poles and zeros** of its transfer function, and its **time-domain response** to step and other inputs;

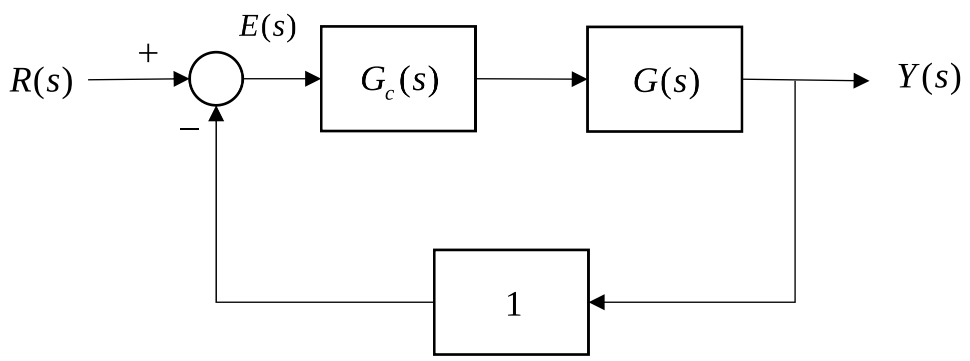
- In such areas as signal processing and control, many analysis and design calculations are done in the s-plane, where a system model is represented in terms of the poles and zeros of its transfer function;

- The control system designer will envision the effects of the step and impulse response of adding, deleting, or moving poles and zeros of $T(s)$ in the s-plane;

- A control designer should be familiar with the effects of pole-zero locations on system response. For example, moving a zero closer to a specific pole will reduce its relative contribution to the output response.

Steady-State Error of Feedback Control System

- One of the basic reasons for using feedback, despite its cost and increased complexity, is on the reduction of steady-state tracking error.
- Consider a unity negative feedback system:



- The standard form the loop transfer function is:

$$G_cG(s)=\frac{K\left(1+\tau'_1 s\right)\left(1+\tau'_2 s\right).....\left(1+\tau'_m s\right)}{s^k\left(1+\tau_1 s\right)\left(1+\tau_2 s\right).....\left(1+\tau_n s\right)}$$

Poles at origin, s=0

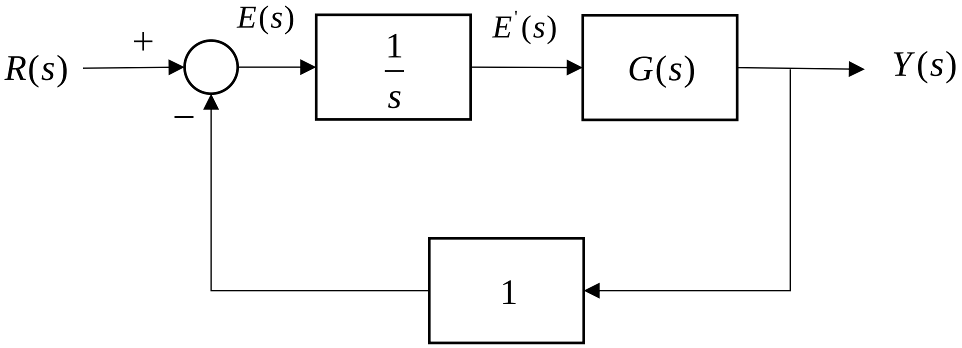
- System type is given by the number of poles at origin, s=0.

$$\text{system type}=k,(\text{type } 0 \text{ for } k=0, \text{ type } 1 \text{ for } k=1, \text{ etc...})$$

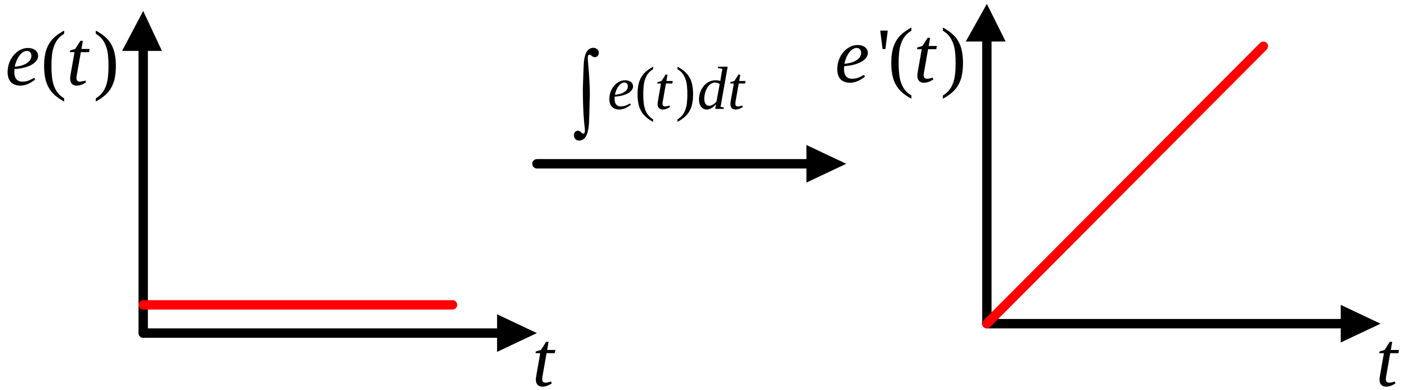
Steady-State Error of Feedback Control System

- Pole at origin, s=0 often purposely included in a closed loop system to reduce steady-state error.

- Consider a system with an Integral-controller:



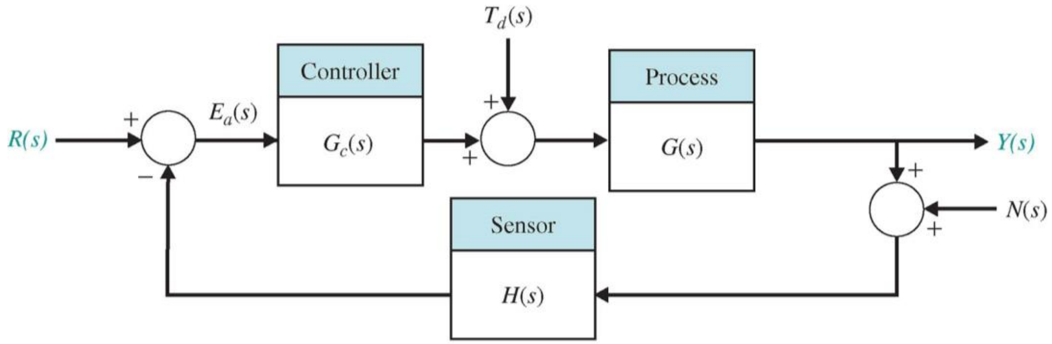
$$E'(s) = \frac{1}{s}E(s) \xrightarrow{\mathcal{L}^{-1}} e'(t) = \int e(t)dt$$



Steady-state error is very small & the controller action is not sufficiently strong

Steady-state error is "accumulated" so that it is more significant to obtain stronger control action

Steady-State Error of Feedback Control System



- Consider a unit negative feedback system ($H(s) = 1$), in the absence of external disturbances ($T_d(s) = 0$) and measurement noise ($N(s) = 0$), tracking error is:

$$E(s) = R(s) - Y(s) = \frac{1}{1 + G_c(s)G(s)} R(s)$$

Using the final value theorem (FVT), the **steady-state error** is:

$$\boxed{\lim_{t \to \infty} e(t) = e_{ss} = \lim_{s \to 0} s \frac{1}{1 + G_c(s)G(s)} R(s)}$$

Steady-State Error to Step Inputs – Generalized

- **Step Input** of magnitude A:

$$e_{ss} = \lim_{s\to 0} s \frac{A/s}{1 + G_c(s)G(s)} = \frac{A}{1 + \lim_{s\to 0} G_c(s)G(s)}$$

The loop transfer function can be written in general form as

$$G_c(s)G(s) = \frac{K \prod_{i=1}^{M}(s + z_i)}{s^N \prod_{k=1}^{Q}(s + p_k)}$$ where $z_i \neq 0, p_k \neq 0$.

The number of integration indicates a system with **type number** that is equal to ***N***, which determines the steady-state error of the system.

Denote the **position error constant**:

$$K_p = \lim_{s\to 0} G_c(s)G(s) = K \frac{\prod z_i}{\prod p_k}$$

- For a type-0 system ($N = 0$):

$$e_{ss} = \frac{A}{1 + K_p}$$

- For a type-N system with $N \geq 1$:

$$e_{ss} = \lim_{s\to 0} \frac{A}{1 + K \frac{\prod z_i}{(s^N \prod p_k)}} = \lim_{s\to 0} \frac{As^N}{s^N + K \frac{\prod z_i}{\prod p_k}} = \lim_{s\to 0} \frac{As^N}{s^N + K_p} = 0.$$

Steady-State Error to Step Inputs – Numerical example for k = 0, 1, and 2

- Alternatively, we can visualize the derivation in another way.
- Re-express the pole-zero terms in the form of $(1+\tau s)$, we can see that:

$$\boxed{e_{ss}(\infty)=\lim_{s\to 0}\frac{A}{1+G_cG(s)}=\lim_{s\to 0}\frac{A}{1+\dfrac{K_p(1+\tau'_1 s)(1+\tau'_2 s).....(1+\tau'_m s)}{s^k(1+\tau_1 s)(1+\tau_2 s).....(1+\tau_n s)}}}$$

$k=0$

$$e(\infty)=$$

$$\lim_{s\to 0}\frac{A}{1+\dfrac{K_p(1+\tau'_1 s)(1+\tau'_2 s)...}{1(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=\frac{A}{1+K_p}}$$

$k=1$

$$e(\infty)=$$

$$\lim_{s\to 0}\frac{A}{1+\dfrac{K_p(1+\tau'_1 s)(1+\tau'_2 s)...}{s(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=0}$$

$k=2$

$$e(\infty)=$$

$$\lim_{s\to 0}\frac{A}{1+\dfrac{K_p(1+\tau'_1 s)(1+\tau'_2 s)...}{s^2(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=0}$$

Steady-State Error to Ramp Inputs

- **Ramp Input** with a slope A:

$$e_{ss} = \lim_{s\to 0} s \frac{A/s^2}{1 + G_c(s)G(s)} = \frac{A}{s + \lim_{s\to 0} sG_c(s)G(s)} = \frac{A}{\lim_{s\to 0} sG_c(s)G(s)}$$

$$G_c(s)G(s) = \frac{K\prod_{i=1}^{M}(s+z_i)}{s^N \prod_{k=1}^{Q}(s+p_k)}$$ where $z_i \neq 0, p_k \neq 0$.

- For a type-0 system ($N = 0$): $e_{ss} = \infty$

- For a type-1 system ($N = 1$):

$$e_{ss} = \lim_{s\to 0} \frac{A}{sK\frac{\prod(s+z_i)}{s\prod(s+p_k)}} = \frac{A}{K\frac{\prod z_i}{\prod p_k}} = \frac{A}{K_v}$$

Denote the **velocity error constant**:

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = K\frac{\prod z_i}{\prod p_k}$$

- For a type-N system with $N > 1$:

$$e_{ss} = \lim_{s\to 0} \frac{A}{sK\frac{\prod(s+z_i)}{s^N\prod(s+p_k)}} = \frac{As^{N-1}}{K\frac{\prod z_i}{\prod p_k}} = \frac{As^{N-1}}{K_v} = 0$$

Steady-State Error to Ramp Inputs – Numerical example for k = 0, 1, and 2

- Alternatively:

$$\boxed{e_{ss}(\infty)=\lim_{s\to 0}\frac{A}{s+sG_cG(s)}=\lim_{s\to 0}\frac{1}{s+s\dfrac{K_v(1+\tau'_1 s)(1+\tau'_2 s).....(1+\tau'_m s)}{s^k(1+\tau_1 s)(1+\tau_2 s).....(1+\tau_n s)}}}$$

$k = 0$

$$e_{ss}(\infty)=$$

$$\lim_{s\to 0}\frac{A}{s+s\dfrac{K_v(1+\tau'_1 s)(1+\tau'_2 s)...}{1(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=\infty}$$

$k = 1$

$$e_{ss}(\infty)=$$

$$\lim_{s\to 0}\frac{A}{s+\dfrac{K_v(1+\tau'_1 s)(1+\tau'_2 s)...}{(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=\frac{A}{K_v}}$$

$k = 2$

$$e_{ss}(\infty)=$$

$$\lim_{s\to 0}\frac{A}{s+\dfrac{K_v(1+\tau'_1 s)(1+\tau'_2 s)...}{s^1(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=0}$$

Steady-State Error to Acceleration Inputs

- **Acceleration Input** $R(s) = A/s^3$ ($r(t) = At^2/2$):

$$e_{ss} = \lim_{s\to 0} s \frac{A/_{s^3}}{1 + G_c(s)G(s)} = \frac{A}{s^2 + \lim_{s\to 0} s^2 G_c(s)G(s)} = \frac{A}{\lim_{s\to 0} s^2 G_c(s)G(s)}$$

$$G_c(s)G(s) = \frac{K\prod_{i=1}^{M}(s+z_i)}{s^{N}\prod_{k=1}^{Q}(s+p_k)} \qquad \text{where } z_i \neq 0, p_k \neq 0.$$

- For a type-N system with $N < 2$: $\quad e_{ss} = \infty$

- For a type-2 system ($N = 2$):

$$e_{ss} = \lim_{s\to 0} \frac{A}{s^2 K \frac{\prod(s+z_i)}{s^2\prod(s+p_k)}} = \frac{A}{K\frac{\prod z_i}{\prod p_k}} = \frac{A}{K_a}$$

Denote the **acceleration error constant**:

$$\boxed{K_a = \lim_{s\to 0} s^2 G_c(s)G(s) = K\frac{\prod z_i}{\prod p_k}}$$

- For a type-N system with $N > 2$:

$$e_{ss} = \lim_{s\to 0} \frac{A}{s^2 K \frac{\prod(s+z_i)}{s^{N}\prod(s+p_k)}} = \frac{As^{N-2}}{K\frac{\prod z_i}{\prod p_k}} = \frac{As^{N-2}}{K_a} = 0$$

Steady-State Error to Acceleration Inputs – Numerical example for k = 0, 1, and 2

- Alternatively:

$$\boxed{e(\infty)=\lim_{s\to 0}\frac{A}{s^2+s^2G(s)}=\lim_{s\to 0}\frac{A}{s^2+^2\dfrac{K_a(1+\tau'_1 s)(1+\tau'_2 s).....(1+\tau'_m s)}{s^k(1+\tau_1 s)(1+\tau_2 s).....(1+\tau_n s)}}}$$

$k = 0$

$e_{ss}(\infty)=$

$$\lim_{s\to 0}\frac{A}{s^2+s^2\dfrac{K_a(1+\tau'_1 s)(1+\tau'_2 s)...}{1(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=\infty}$$

$k = 1$

$e_{ss}(\infty)=$

$$\lim_{s\to 0}\frac{A}{s^2+s\dfrac{K_a(1+\tau'_1 s)(1+\tau'_2 s)...}{(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=\infty}$$

$k = 2$

$e_{ss}(\infty)=$

$$\lim_{s\to 0}\frac{A}{s^2+\dfrac{K_a(1+\tau'_1 s)(1+\tau'_2 s)...}{(1+\tau_1 s)(1+\tau_2 s)...}}$$

$$\boxed{e_{ss}(\infty)=\frac{A}{K_a}}$$

Summary Table

Table 5.2 Summary of Steady-State Errors

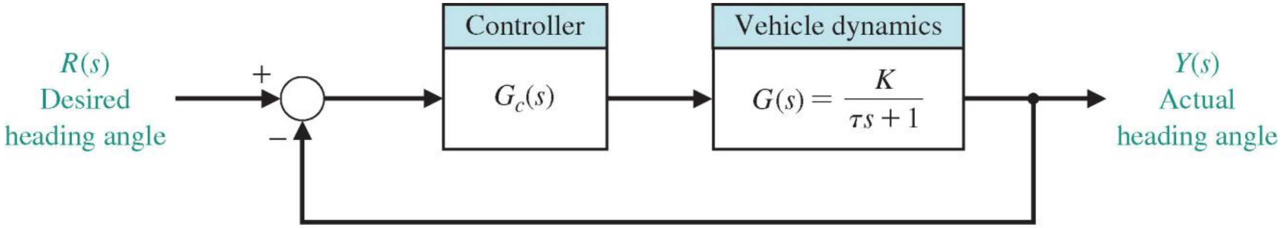
Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Input: Ramp, $r(t) = At$, $R(s) = A/s^2$	Input: Parabola, $r(t) = At^2/2$, $R(s) = A/s^3$
0	$e_{ss} = \frac{A}{1 + K_p}$	∞	∞
1	$e_{ss} = 0$	$\frac{A}{K_v}$	∞
2	$e_{ss} = 0$	0	$\frac{A}{K_a}$

❖ The control system **error constants** K_p, K_v and K_a , describe the ability of a system to reduce or eliminate the steady-state error. Therefore, they are utilized as numerical measure of the steady-state performance. The designer determines the error constants for a given system and attempts to determine methods of increasing the error constants while maintaining an acceptable transient response.

Example 14.1: Mobile Robot Steering Control

Consider the following system of mobile robot. Transfer function of controller is

$$G_c(s) = K_1 + K_2/s$$



Loop transfer function: $G_c(s)G(s) = \frac{K(K_1 s + K_2)}{\tau s^2 + s}$

- When $K_2 = 0$ -> $G_c(s)G(s) = \frac{KK_1}{\tau s + 1}$ -> type-0 system:

  For step input: $e_{ss} = \frac{A}{1 + K_p}$ where $\boxed{K_p = \lim_{s \to 0} G_c(s)G(s) = KK_1}$

  For ramp input: $e_{ss} = \infty$

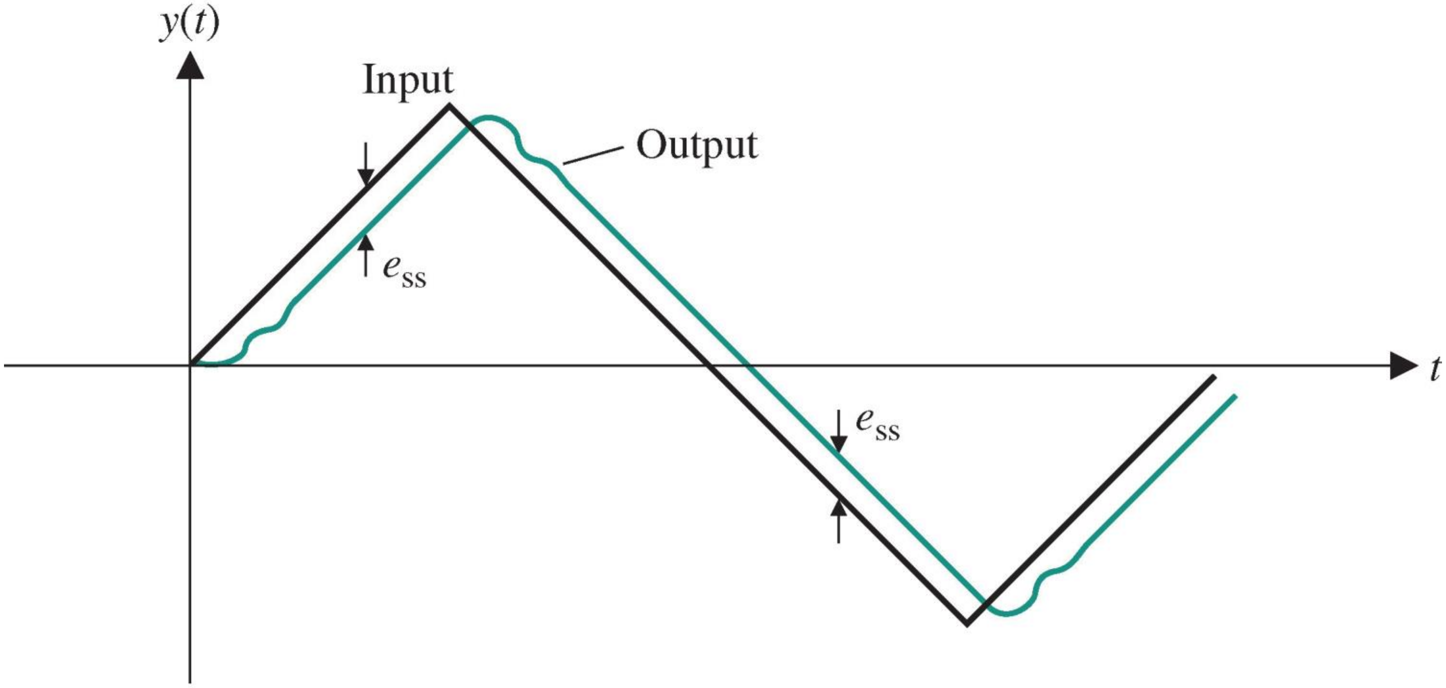
- When $K_2 > 0$ -> $G_c(s)G(s) = \frac{K(K_1 s + K_2)}{s(\tau s + 1)}$ -> type-1 system:

  For step input: $e_{ss} = 0$

  For ramp input: $e_{ss} = \frac{A}{K_v}$ where $\boxed{K_v = \lim_{s \to 0} sG_c(s)G(s) = KK_2}$

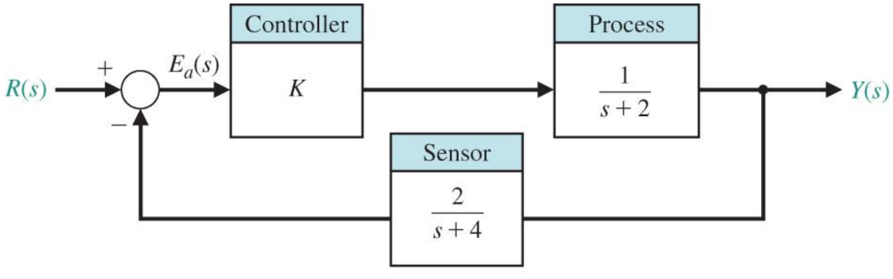
Example 14.1: Mobile Robot Steering Control

Transient and steady-state responses of type-1 system (in the previous example when $K_2 > 0$) subjected to a triangular wave (or seesaw) input



Example 14.2: Steady-State Error for CL System with Non-unity Feedback Path

Consider the following system, determine K so that the e_{ss} for a unit step input is minimized.



Solutions:

$$E(s) = R(s) - Y(s) = [1 - T(s)]R(s)$$

$$T(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)H(s)} = \frac{K(s+4)}{(s+2)(s+4)+2K}$$

For a unit step input: $\quad e_{ss} = \lim_{s\to 0} sE(s) = 1 - T(0)$

To minimize ESS, it requires: $\quad T(0) = \dfrac{4K}{8+2K} = 1$

Therefore: $\quad K = 4$ yields zero steady-state error

Performance Index

A performance index is a quantitative measure of the performance of a system and is chosen so that emphasis is given to the important system specifications.

- A system is considered an optimum control system when the system parameters are adjusted so that the index reaches an extremum, which are typically a minimum.

Common performance index:

Integral of squared error $ISE = \int_0^T e^2(t) \cdot dt$

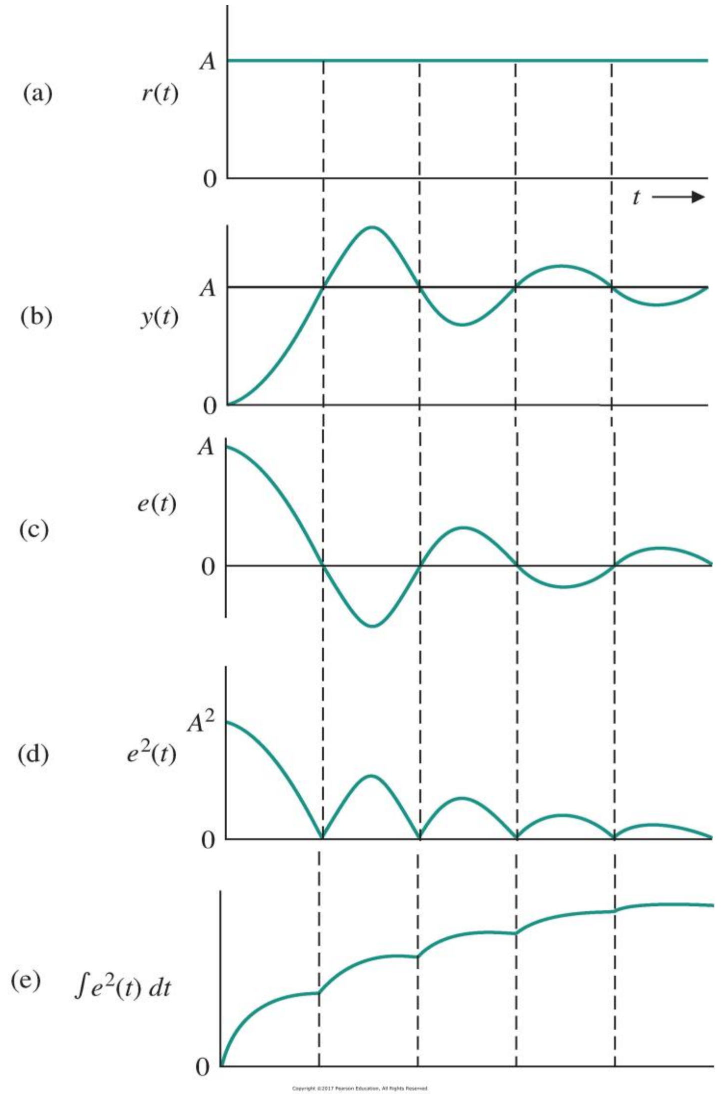
Integral of absolute error $IAE = \int_0^T |e(t)| \cdot dt$

Integral of time multiplied by absolute error $ITAE = \int_0^T t|e(t)| \cdot dt$

Integral of time multiplied by squared error $ITSE = \int_0^T te^2(t) \cdot dt$

General form: $$I = \int_0^T f(e(t), r(t), y(t), t) \cdot dt$$

Integral of errors - visual



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Optimum Coefficients for General $T(s)$ based on ITAE Criterion

Given **unit-step/-ramp input** and $$T(s) = \frac{b_0}{s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots + b_1 s + b_0}$$

Table 5.3 The Optimum Coefficients of $T(s)$ Based on the ITAE Criterion for a Step Input

$s + \omega_n$

$s^2 + 1.4\omega_n s + \omega_n^2$

$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$

$s^4 + 2.1\omega_n s^3 + 3.4\,\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$

$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$

$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.60\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$

Table 5.4 The Optimum Coefficients of $T(s)$ Based on the ITAE Criterion for a Ramp Input

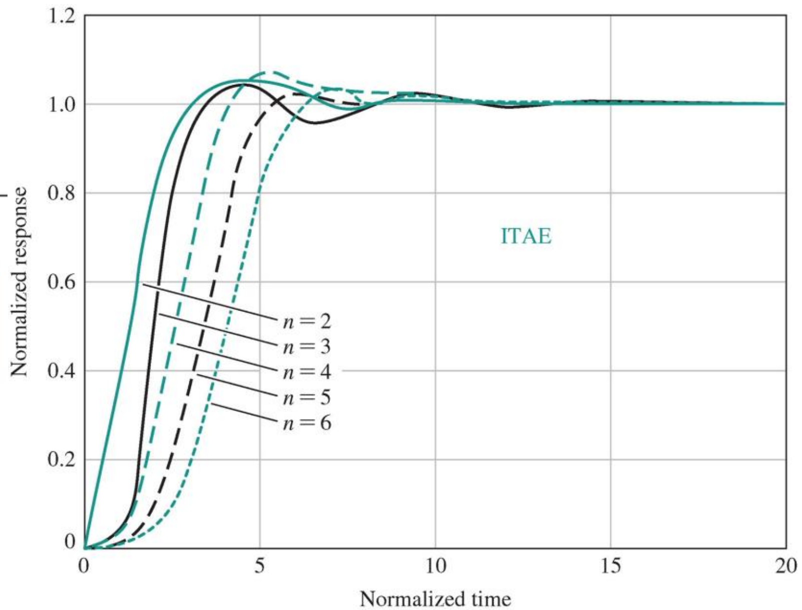
$s^2 + 3.2\omega_n s + \omega_n^2$

$s^3 + 1.75\omega_n s^2 + 3.25\omega_n^2 s + \omega_n^3$

$s^4 + 2.41\omega_n s^3 + 4.93\omega_n^2 s^2 + 5.14\omega_n^3 s + \omega_n^4$

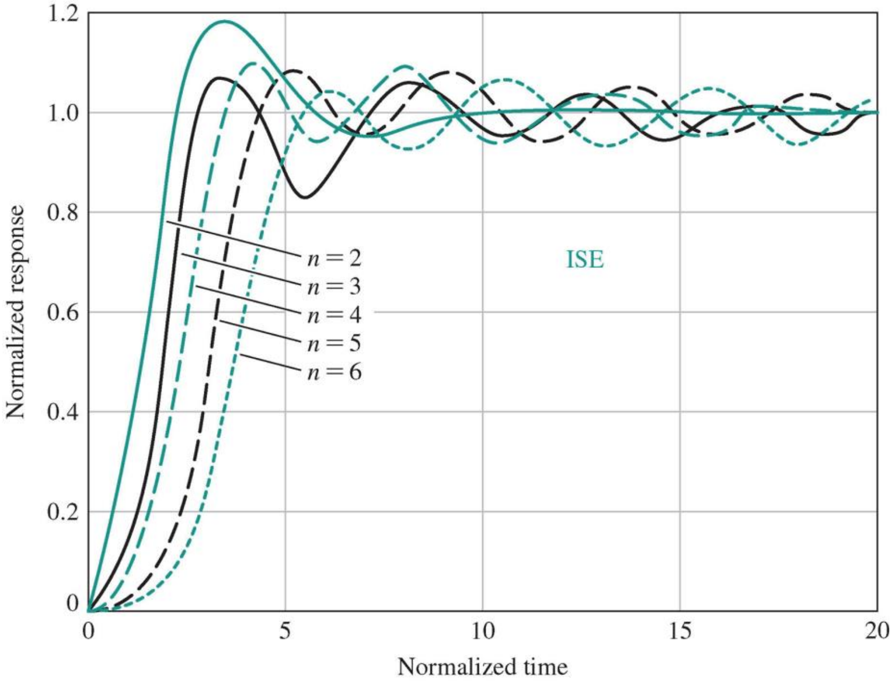
$s^5 + 2.19\omega_n s^4 + 6.50\omega_n^2 s^3 + 6.30\omega_n^3 s^2 + 5.24\omega_n^4 s + \omega_n^5$

Time-domain output → integral of "errors" → find optimum wrt different damping

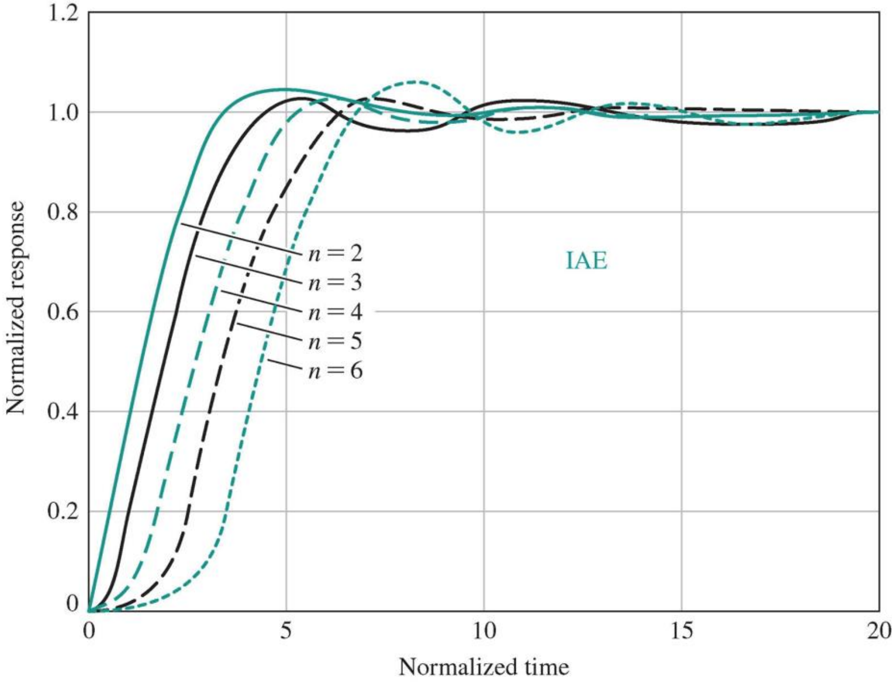


(c)

ISE, IAE (normalized time $\omega_n t$)



(a)

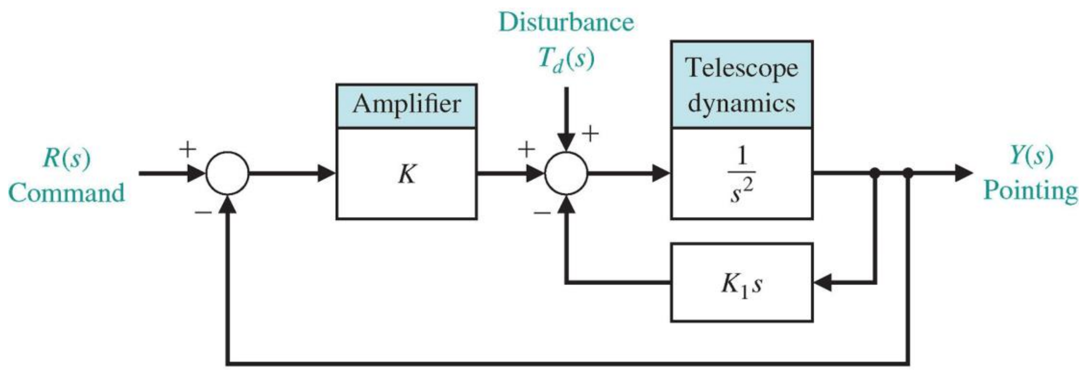


(b)

Example 14.3 (Design for P.O. and steady-state error specifications): Hubble Space Telescope Control

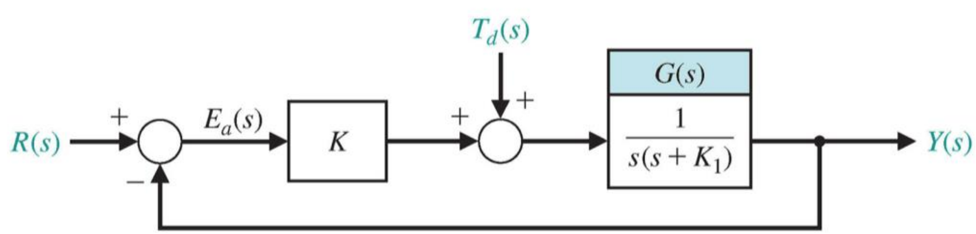
For the following control system, choose K_1 and K, to satisfy:

(1) Percent overshoot ($P.O.$) of the output to a step command $r(t)$ is $\leq 10\%$
(2) Steady-state error to a ramp command is minimized;
(3) Effect of a step disturbance is reduced.



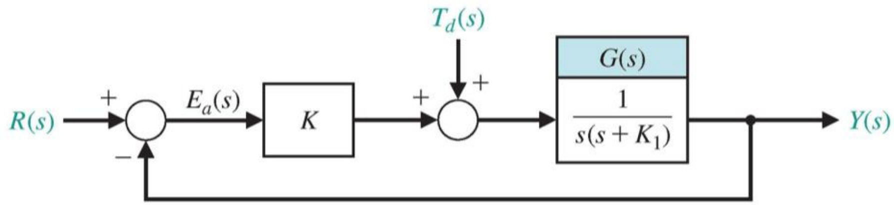
(a)

Step 1. Re-arrange the block diagram to achieve a standard form.



(b)

Example 14.3



Step 2. Obtain Y(s) and E(s) in terms of R(s), T_d(s) and system parameters.

$$Y(s) = \frac{KG(s)}{1+KG(s)}R(s) + \frac{G(s)}{1+KG(s)}T_d(s)$$

$$E(s) = R(s) - Y(s) = \frac{1}{1+KG(s)}R(s) - \frac{G(s)}{1+KG(s)}T_d(s)$$

Step 3. Consider requirement (1): $P.O. \le 10\%$ *for a step input.*

Characteristic equation of the system is

$$1 + KG(s) = 0 \longrightarrow s^2 + K_1 s + K = 0$$

Standard form: $s^2 + 2\zeta\omega_n s + \omega_n{}^2$

$$2\zeta\omega_n = K_1, \qquad \omega_n{}^2 = K.$$

For $P.O. \le 10\%$, it must be satisfied that $\zeta \ge 0.6$. we choose ζ=0.6, therefore

$$\boxed{P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

$$\frac{K_1}{1.2} = \sqrt{K}$$

Example 14.3

$$E(s) = R(s) - Y(s) = \frac{1}{1 + KG(s)} R(s) - \frac{G(s)}{1 + KG(s)} T_d(s)$$

Step 4. Consider requirement (2): minimize e_{ss} to a ramp input: assume $T_d(s) = 0$

$$E(s) = \frac{1}{1 + KG(s)} R(s) = \frac{s^2 + K_1 s}{s^2 + K_1 s + K} \frac{A}{s^2}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{A}{K/K_1}$$

To minimize ESS, we need large value of K/K_1.

Step 5. Consider requirement (3): minimize e_{ss} to a step disturbance: assume $R(s) = 0$

$$E(s) = -\frac{G(s)}{1 + KG(s)} T_d(s) = -\frac{1}{s^2 + K_1 s + K} \frac{B}{s}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = -\frac{B}{K}$$

To minimize ESS, we need large value of K.

Example 14.3

Step 6. Choose suitable values.

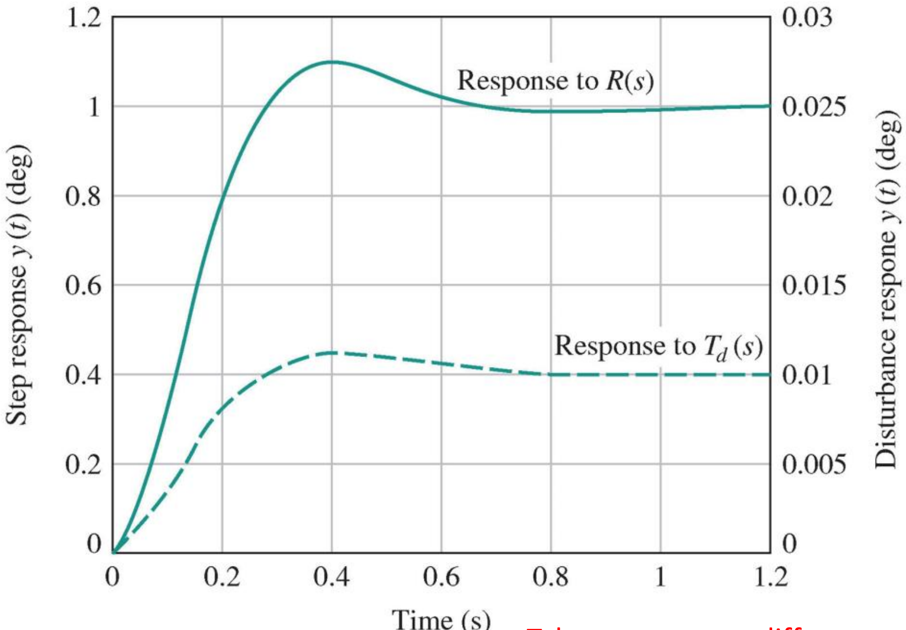
$\frac{K_1}{1.2} = \sqrt{K}$;

To minimize 1st e_{ss}, we need large value of K/K_1.

To minimize 2nd e_{ss}, we need large value of K.

Can choose $K = 100$, then according to $\frac{K_1}{1.2} = \sqrt{K}$, $K_1 = 12$, and $K/K_1 = 8.33$.

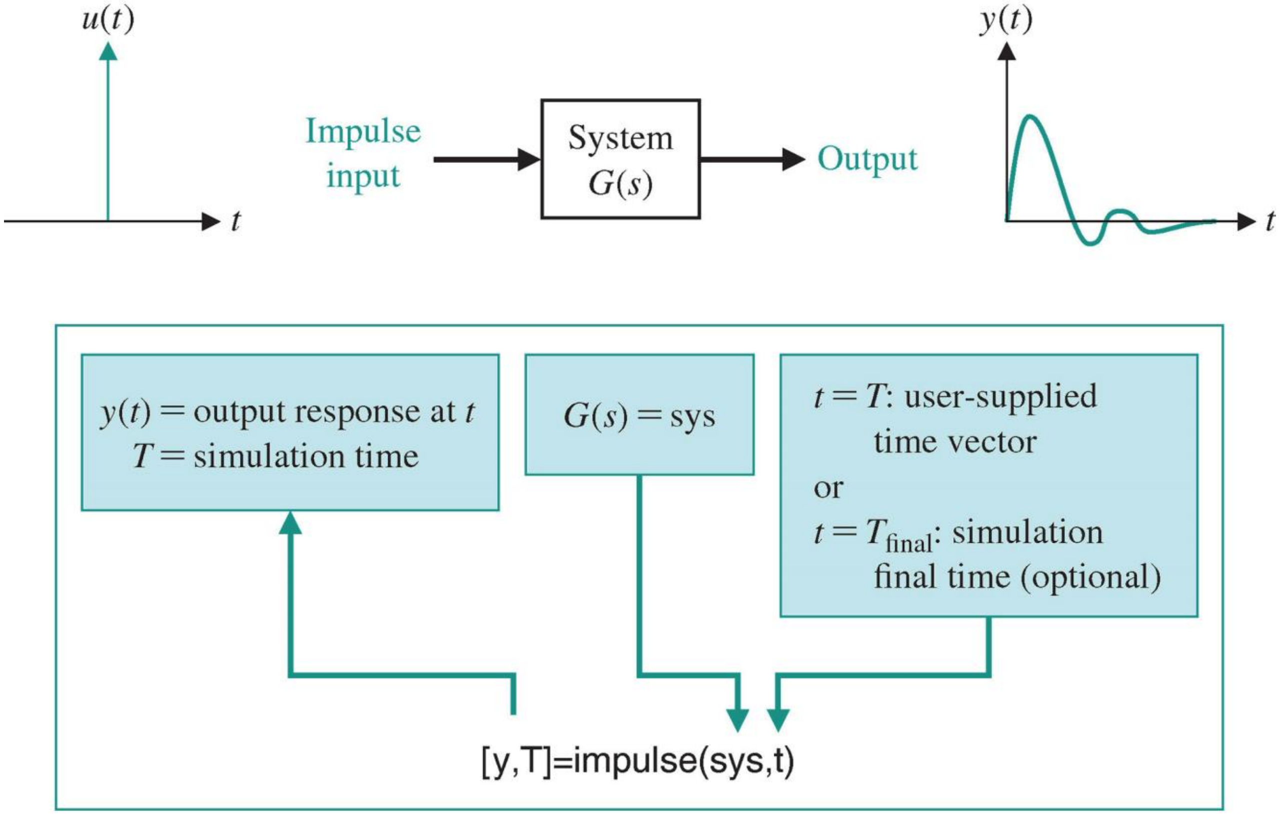
Therefore,
e_{ss} for a ramp input is $\frac{A}{8.33} \approx 0.12A$, e_{ss} for a step disturbance is $-\frac{B}{100} = -0.01B$.
All the requirements are deemed satisfied.



Take note on two different y-axes

System Performance Simulation Using Matlab

The **impulse** and **step** function.

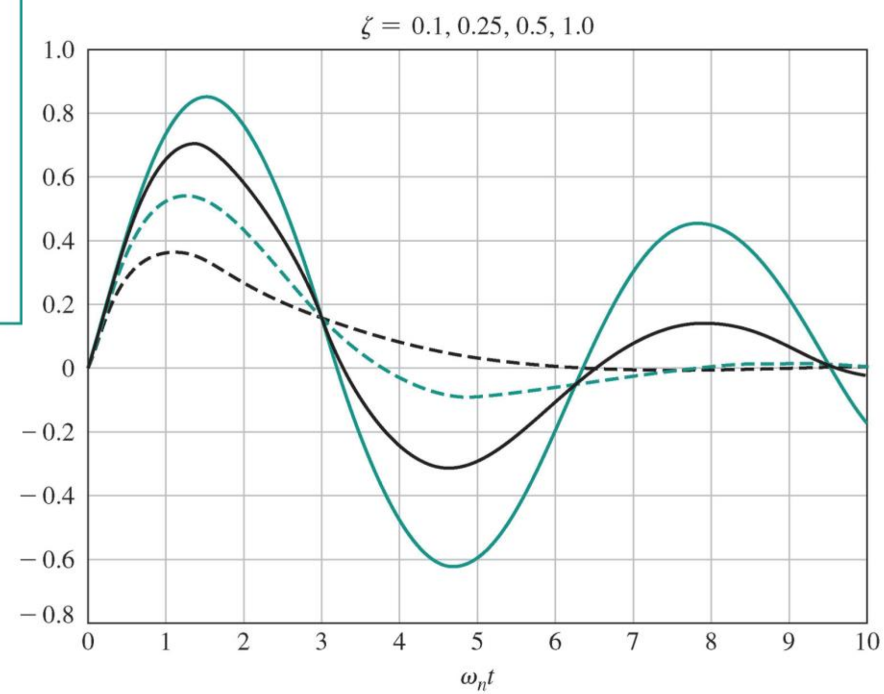


```
%Compute impulse response for a second-order system
%Duplicate Figure 5.5
%
t=[0:0.1:10]; num=[1];
zeta1=0.1; den1=[1 2*zeta1 1]; sys1=tf(num,den1);
zeta2=0.25; den2=[1 2*zeta2 1]; sys2=tf(num,den2);
zeta3=0.5; den3=[1 2*zeta3 1]; sys3=tf(num,den3);
zeta4=1.0; den4=[1 2*zeta4 1]; sys4=tf(num,den4);
%
[y1,T1]=impulse(sys1,t);
[y2,T2]=impulse(sys2,t);
[y3,T3]=impulse(sys3,t);
[y4,T4]=impulse(sys4,t);
%
plot(t,y1,t,y2,t,y3,t,y4)
xlabel(' \omega _nt'), ylabel('y(t)/\omega_n')
title('\zeta = 0.1, 0.25, 0.5, 1.0'), grid
```

Compute impulse response.

Generate plot and labels.

(b)



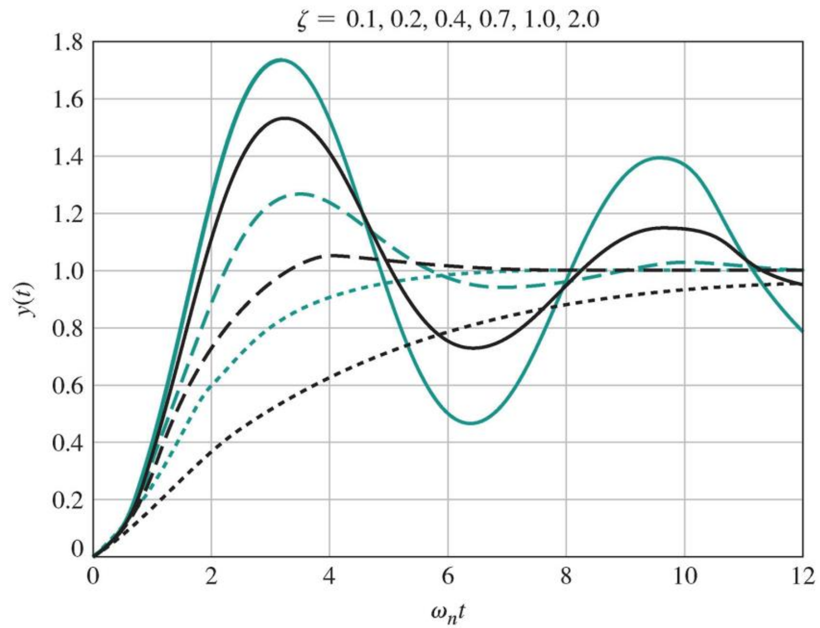
(a)

```
%Compute step response for a second-order system
%Duplicate Figure 5.4
%
t=[0:0.1:12]; num=[1];
zeta1=0.1; den1=[1 2*zeta1 1]; sys1=tf(num,den1);
zeta2=0.2; den2=[1 2*zeta2 1]; sys2=tf(num,den2);
zeta3=0.4; den3=[1 2*zeta3 1]; sys3=tf(num,den3);
zeta4=0.7; den4=[1 2*zeta4 1]; sys4=tf(num,den4);
zeta5=1.0; den5=[1 2*zeta5 1]; sys5=tf(num,den5);
zeta6=2.0; den6=[1 2*zeta6 1]; sys6=tf(num,den6);
%
[y1,T1]=step(sys1,t); [y2,T2]=step(sys2,t);
[y3,T3]=step(sys3,t); [y4,T4]=step(sys4,t);
[y5,T5]=step(sys5,t); [y6,T6]=step(sys6,t);
%
plot(T1,y1,T2,y2,T3,y3,T4,y4,T5,y5,T6,y6)
xlabel(' \omega_n t'), ylabel('y(t)')
title('\zeta = 0.1, 0.2, 0.4, 0.7, 1.0, 2.0'), grid
```

Compute step response.

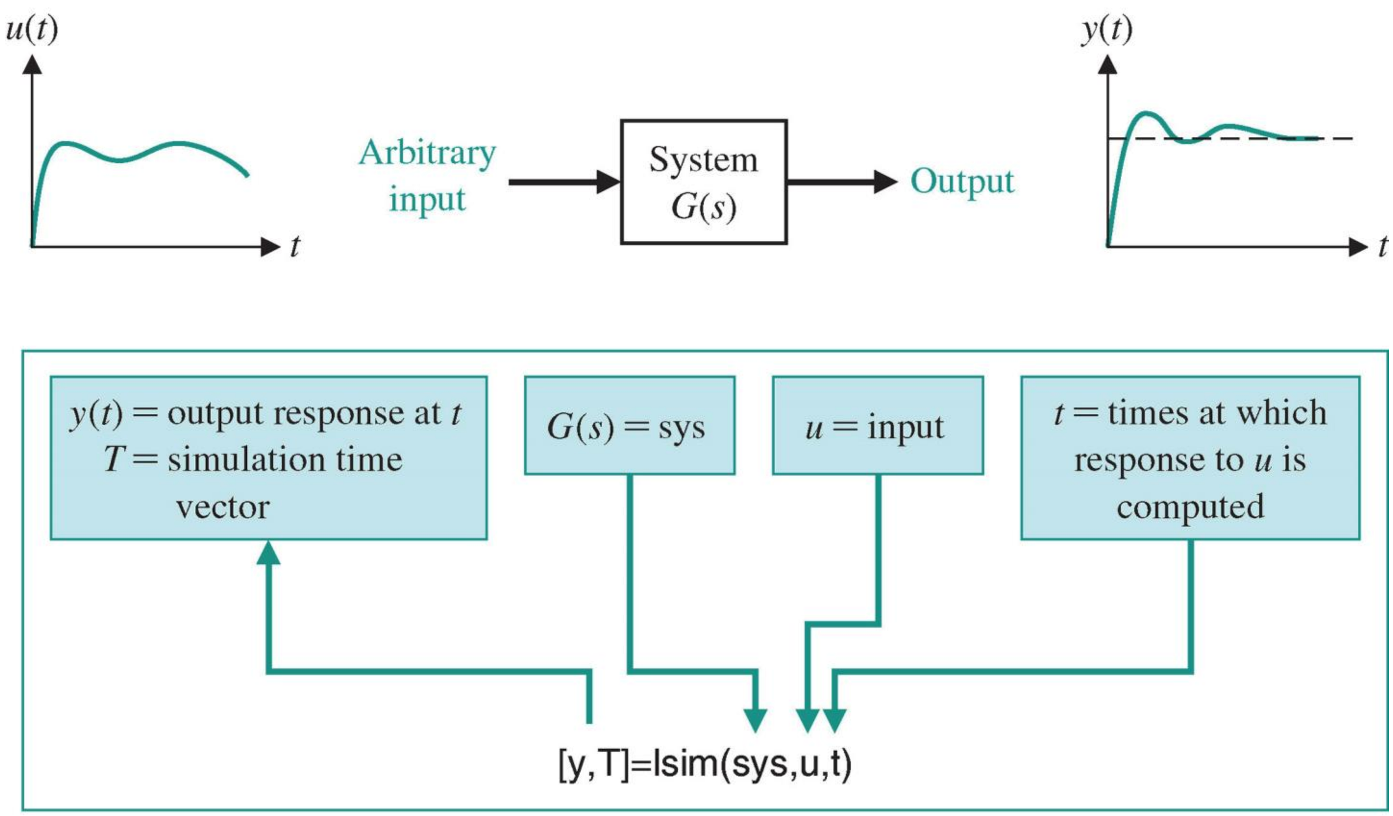
Generate plot and labels.

(b)



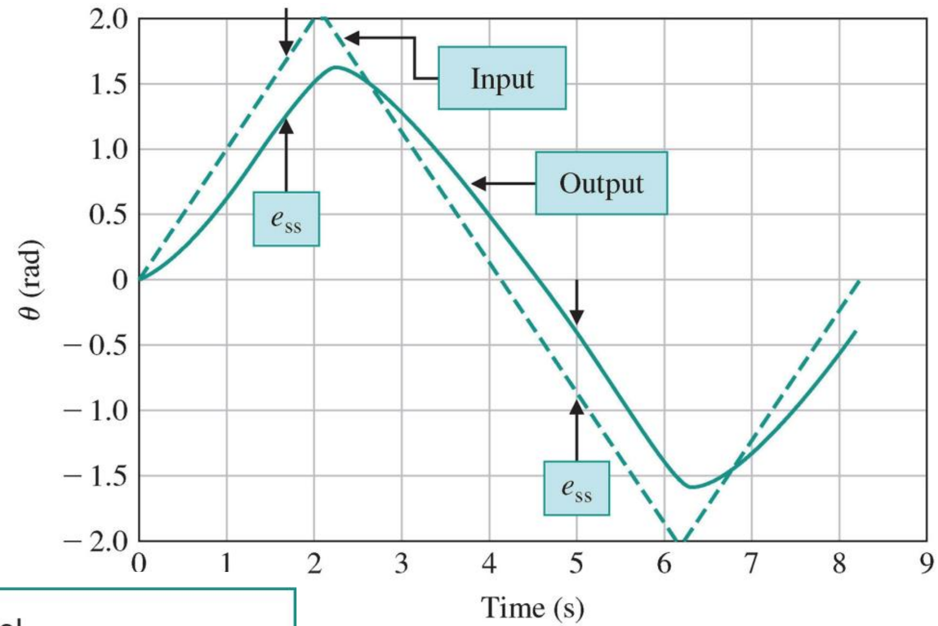
(a)

The **lsim** function.



Note:

- Apart from the presented numerical tools, **there are many other classical control design tools (*ilaplace*, *tf2ss*, *feedback*, etc.)** in MATLAB.
- You will need to self-learn some of the numerical tools/functions, through MATLAB's Help file and other resources, to complete your Computer Lab Work.



(a)


```
%Compute the response of the Mobile Robot Control
%System to a triangular wave input
%
numg=[10 20]; deng=[1 10 0]; sysg=tf(numg,deng);
[sys]=feedback(sysg, [1]);
t=[0:0.1:8.2]';
v1=[0:0.1:2]';v2=[2:-0.1:-2]';v3=[-2:0.1:0]';
u=[v1;v2;v3];
[y,T]=lsim(sys,u,t);
plot(T,y,t,u,'--'),
xlabel('Time (s)'), ylabel('\theta (rad)'), grid
```

$G(s)G_c(s)$

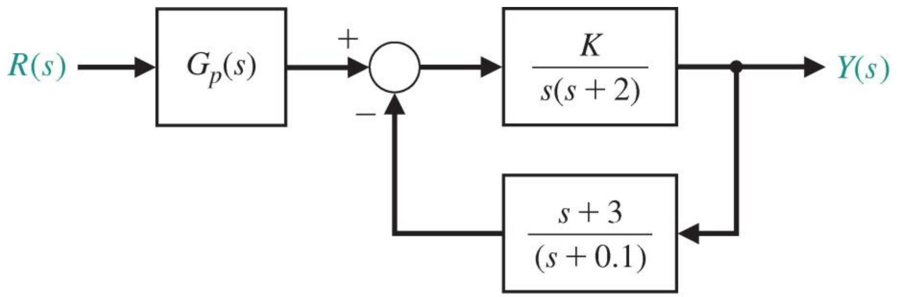
Compute triangular wave input.

Linear simulation.

(b)

Example 14.4

Given that $K = 0.4$ and $G_p(s) = k_1$, choose a suitable k_1 value to minimize e_{ss} to zero when subject to a unit step input.



Flow of thoughts: (1) Minimizing e_{ss} means minimizing $\lim_{s \to 0} sE$; (2) form $E = R - Y = (1 - T)R$ expression; (3) need T and R; (4) solve k_1 for $e_{ss} = 0$.

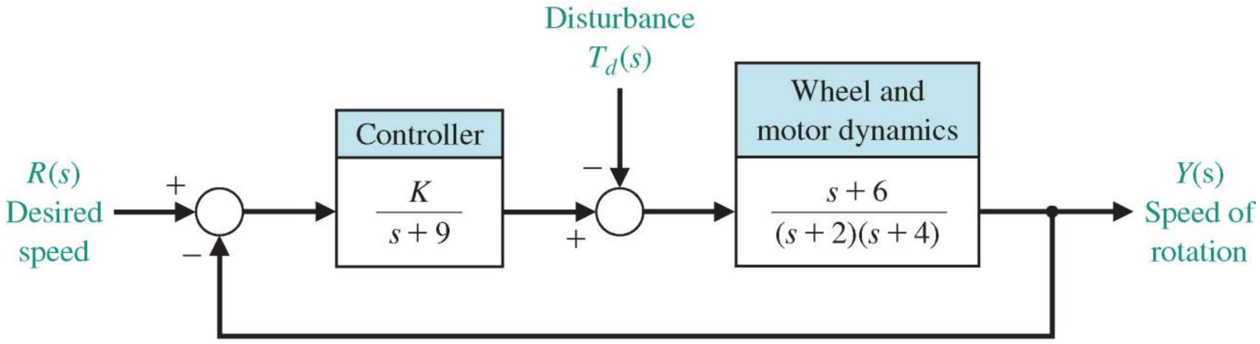
Answer: $k_1 = 30$.

Example 14.5

Consider the following system,

(1) Determine K to satisfy: e_{ss} to a unit step input < 0.05;

(2) Select a suitable K value through part (1), calculate e_{ss} due to the unit step disturbance.



Flow of thoughts for part (i): (1) $e_{ss} < 0.05$ means $\lim_{s\to 0} sE < 0.05$; (2) form $E = R - Y = (1-T)R$; (3) recall $T = \frac{G_cG}{1+G_cG}$ and $R = \frac{1}{s}$; (4) solve for K range.

Flow of thoughts for part (ii): (1) form "$E = R - Y = (1-T)R + \frac{G}{1+G_cG}T_d$" (standard form for positive T_d "+" input; need to adjust the sign for T_d "-" input; try practice yourself); (2) select K value, recall $T_d = \frac{1}{s}$ and $R = 0$; (3) solve e_{ss} value for the select K.

Answer: (i) $K > 228$; (ii) e.g., if $K = 250$, then $e_{ss} = 0.034$.

Concluding Remarks

- **What have been covered:** <u>"Time-domain Performance of Feedback Control Systems "</u>
  - Standard inputs, performance specification
  - Effects of a third pole or a zero; effects of pole location
  - Steady-state errors due to different inputs and system types
  - Performance index (about error)

Office hour: 2-4 pm Thursdays