XI'AN JIAOTONG-LIVERPOOL UNIVERSITY
西交利物浦大学
COURSEWORK SUBMISSION COVER Page

Group Number	G40				
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Date: 2024.10.23...............

Part 1: Calculation Problems

➔ **Q1:**

The condition to determine whether a given vector field can represent a magnetic field is that its divergence must be zero. This follows from one of Maxwell's equations:

$$\nabla \cdot \mathbf{B} = 0 \tag{1}$$

Thus, if the divergence of the given vector field is zero, it could represent a magnetic field. If not, then it cannot be a magnetic field.

The given vector field is:

$$\mathbf{A}(\mathrm{r}, \phi, \mathrm{z}) = \frac{acos^2\phi}{r^2}\hat{r} \tag{2}$$

Now, we calculate the divergence of this field in cylindrical coordinates.

$$\nabla \cdot \boldsymbol{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z} \tag{3}$$

Since $A\phi = 0$ and $Az = 0$, we only need to calculate the term involving Ar.

The component Ar is given as:

$$A_r = \frac{a}{r^2}cos^2\varphi \tag{4}$$

Now, substitute this into the divergence formula and simplifies:

$$\nabla \cdot \vec{A} = \frac{1}{r}\left(-\frac{a\,cos^2\varphi}{r^2}\right) = -\frac{acos^2\varphi}{r^3} \tag{5}$$

Since the divergence is not zero:

$$\nabla \cdot \vec{A} = -\frac{a\,cos^2\varphi}{r^3} \neq 0 \tag{6}$$

Therefore, the given vector field does not meet the criteria for representing a magnetic field as its divergence is non-zero. Thus, this field cannot represent a magnetic field.

➔ **Q2:**

According to the differential form of Generalized Ampere's Law,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{1}$$

Since it's a source free region, we have:

$$\rho = 0, \boldsymbol{J} = 0 \tag{2}$$

The Generalized Ampere's Law (PHASOR FORM) can be simplified as:

$$\nabla \times \boldsymbol{B}(r,t) = jw\varepsilon\mu\boldsymbol{E}(r,t) \tag{3}$$

Since the expression of $\boldsymbol{B}$ **is**:

$$\boldsymbol{B}(x,y,z,t) = Acos(kz - \omega t)\hat{y} \tag{4}$$

Phasor form of $\boldsymbol{B}$ is**:**

$$\boldsymbol{B}(r,t) = Re\{\tilde{B}(r)e^{jwt}\}\, which\ \tilde{B}(r) = Ae^{-jkz}\hat{y} \tag{5}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & Re\{\tilde{B}_y(r)e^{jwt}\} & 0 \end{vmatrix} = jw\varepsilon\mu\boldsymbol{E}(r,t) \tag{6}$$

Therefore, we have:

$$jw\varepsilon\mu\boldsymbol{E}(r,t) = Re\left\{\left(-\frac{\partial\widetilde{B_y}(r)}{\partial z}\hat{x} + \frac{\partial\widetilde{B_y}(r)}{\partial x}\hat{z}\right)e^{jwt}\right\} = Re\{jAke^{-jkz}\hat{x}e^{jwt}\} \tag{7}$$

Divide both side with $jw\varepsilon\mu$ we have:

$$\boldsymbol{E}(r,t) = Re\left\{\frac{Ak}{w\varepsilon\mu}e^{-jkz}\hat{x}e^{jwt}\right\} = \frac{Ak}{w\varepsilon\mu}\cos(wt - kz)\hat{x} \tag{8}$$

The relationship of space frequency k and angular frequency is:

$$k = w\sqrt{\varepsilon\mu} \tag{9}$$

The $\boldsymbol{E}(r,t)$ can also be expressed as:

$$\boldsymbol{E}(r,t) = \frac{Aw}{k}\cos(wt - kz)\hat{x} \tag{10}$$

➔ **Q3:**

Considering Ampere-Maxwell law, the magnetic field:

$$\nabla \times \boldsymbol{B} = \mu_0 \,(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}) \tag{1}$$

Since there is no conduction current between parallel-plate capacitor, we can ignore $\boldsymbol{J}$, thus the equation can be simplified to

$$\nabla \times \boldsymbol{B} = \mu_0 \,\frac{\partial \boldsymbol{D}}{\partial t} \tag{2}$$

In cylindrical coordinate, and we know B is in φ direction, so the cross product simplifies to

$$\nabla \times \boldsymbol{B} = \frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi)\hat{z} \tag{3}$$

Substituting the given magnetic field,

$$\frac{\mathrm{wvcos(\omega t)}}{\mathrm{d}c^2}\hat{z} = \mu_0 \,\frac{\partial \boldsymbol{D}}{\partial t} \tag{4}$$

Integrating the equation,

$$\boldsymbol{D} = \frac{vsin(\omega \mathrm{t})}{\mu_0 \mathrm{d}c^2}\,\hat{z} \tag{5}$$

The relation between electric flux density and electric field intensity is

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} \tag{6}$$

And we know that $c^2 = \frac{1}{\epsilon_0 \mu_0}$, electric flux density can be simplified to:

$$\boldsymbol{D} = \frac{\epsilon_0 vsin(\omega \mathrm{t})}{\mathrm{d}}\hat{z} \tag{7}$$

After that, the relationship between displacement current density $\boldsymbol{J_d}$ and $\boldsymbol{D}$ is:

$$\boldsymbol{J_d} = \frac{\partial \boldsymbol{D}}{\partial t} \tag{8}$$

So, displacement current density $\boldsymbol{J_d}$ can be calculated as

$$\boldsymbol{J_d} = \frac{\epsilon_0 \omega vcos(\omega \mathrm{t})}{\mathrm{d}}\hat{z} \tag{9}$$

Overall,

$$\boldsymbol{E} = \frac{vsin(\omega \mathrm{t})}{d}\,\hat{z} \tag{10}$$

$$\boldsymbol{J_d} = \frac{\epsilon_0 \omega vcos(\omega \mathrm{t})}{\mathrm{d}}\hat{z} \tag{11}$$

- **Part 2: MATLAB Problems**

→ **A: Electric Field and Electric Potential**

1) A-1:

Handwriting electric field lines are shown in Fig 1.

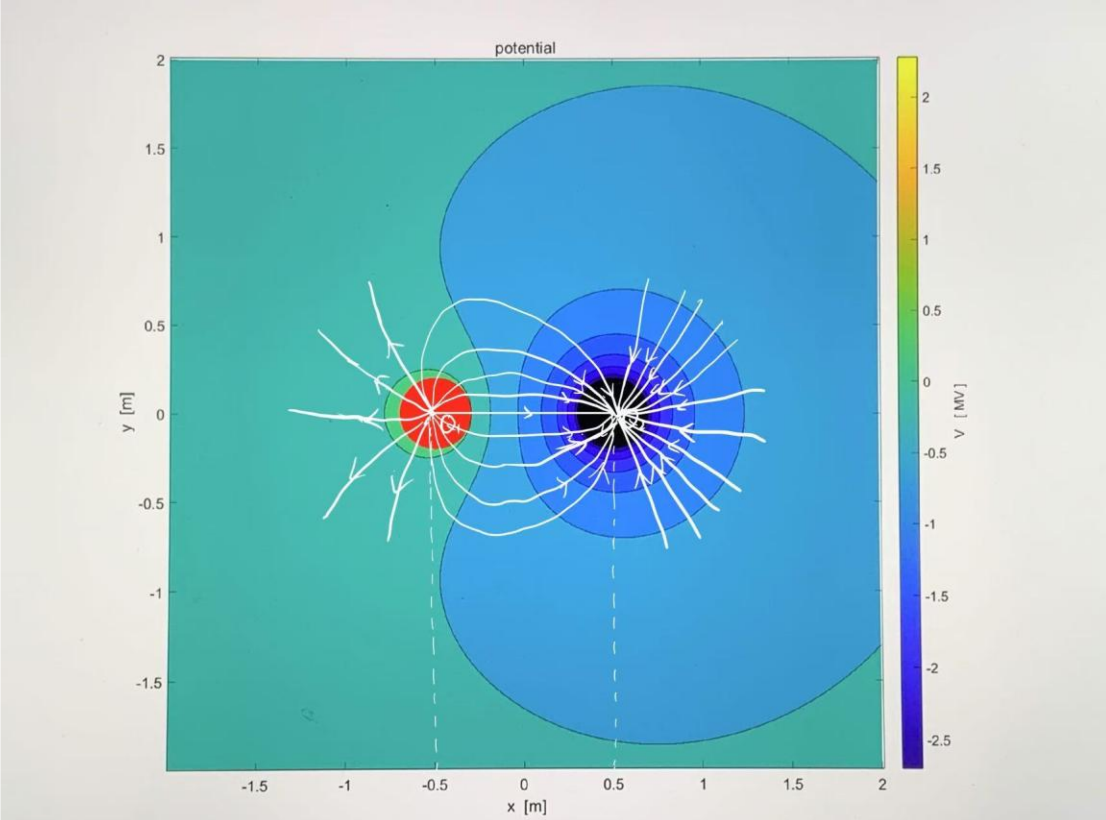

Fig 1. Handwritten Electric Potential Plot

2) A-2:

To visualize the electric field lines, we have to leverage "streamslice" function to generate field lines and modulate to an appropriate step to make the diagram clearly to understand the trend with plot showing in Fig 2. Additionally, the arrows point from the positive charge to the negative charge, as indicated in Fig 2 (*See detailed modified code in Appendix*). Please run the code in the real-time MATLAB script (.mlx file).

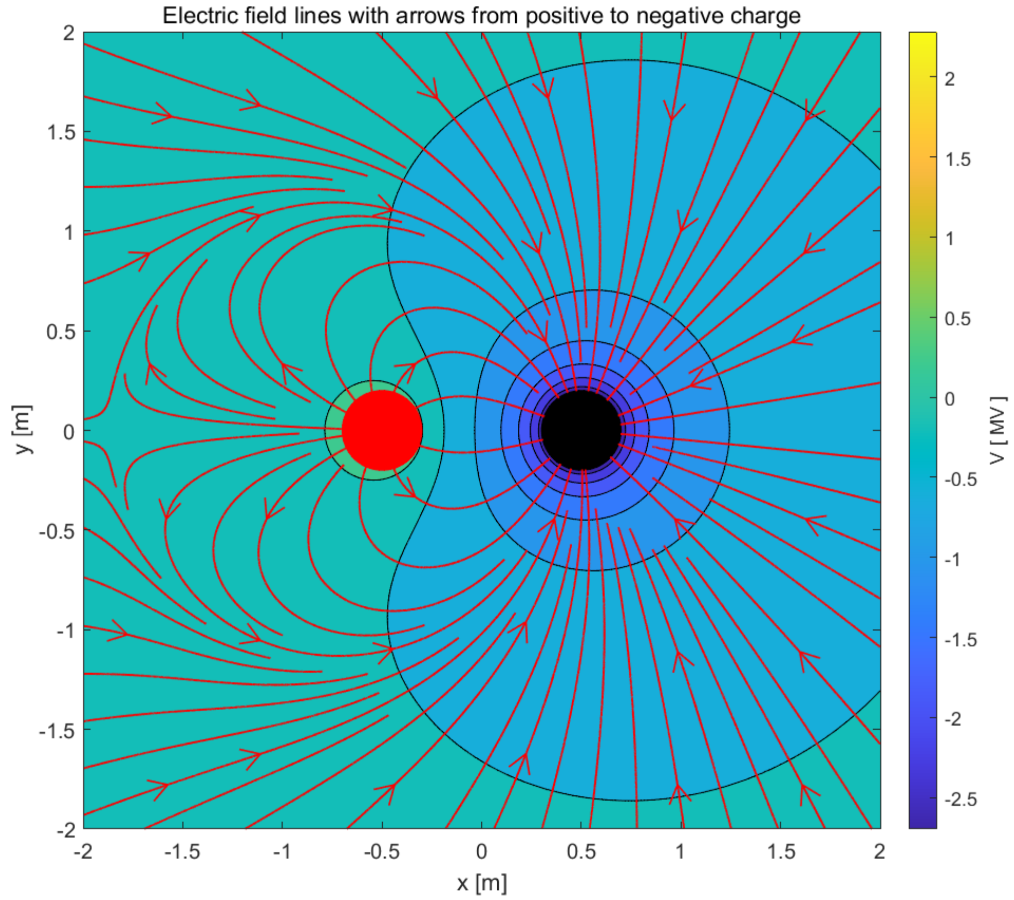

Fig 2. Electric Field Lines Visualization

➔ B: Parallel-plate Capacitor with Circular Plates

1) B-1:

✧ Diagram visualization

When $d/R = 0.5$, the equipotential contours and electric field lines around the center of the parallel-plate capacitor are nearly uniform, shown in Fig 3. Because edge effect is negligible in the center region of the capacitor. The electric field intensity is almost uniform between the plates, except near the edges where field lines start curving outward.

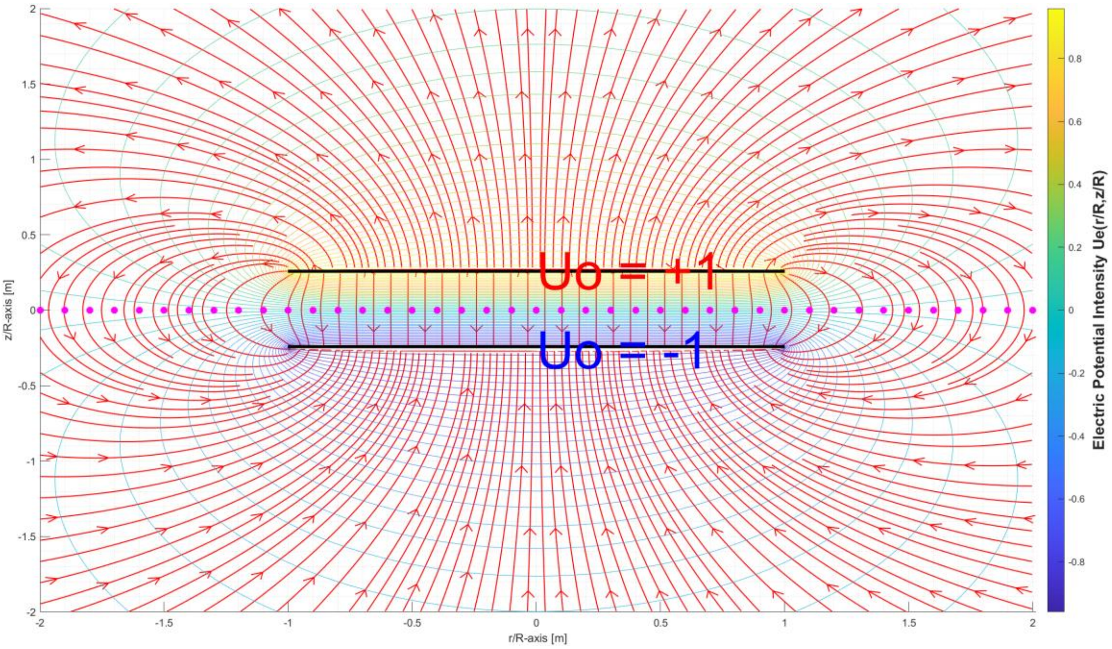

Fig 3. Electric Field in a Parallel-Plate Capacitor when $d/R = 0.5$

✧ Edge Effects Analysis

In the center region, far from the plate edges, the influence of edge effects (i.e., field curvature and distortion) can be ignored. Near the edges, however, the field lines begin to spread out due to the finite size of the plates. At the center, the field remains almost perfectly perpendicular to the plates, maintaining a nearly constant field intensity and direction, as the distance to the edges is much larger relative to the field extent.

✧ Electric Field Intensity between Capacitor

For a single plate, assuming the absence of nearby charged bodies, the electric field near the plate can be approximated as:

$$\boldsymbol{E} = \frac{-\rho}{2\epsilon_0}\,\hat{z} \tag{1}$$

The electric field arising between oppositely charged plates can only be obtained by superposition of the fields of both plates, since one is in the opposite direction. The total electric field between the plates is:

$$\boldsymbol{E} = \frac{-\rho}{\epsilon_0}\hat{z} \tag{2}$$

The field is constant in magnitude and direction in the central region of the plates, giving rise to the uniform distribution described above.

✧ Numerical and Graphical Verification

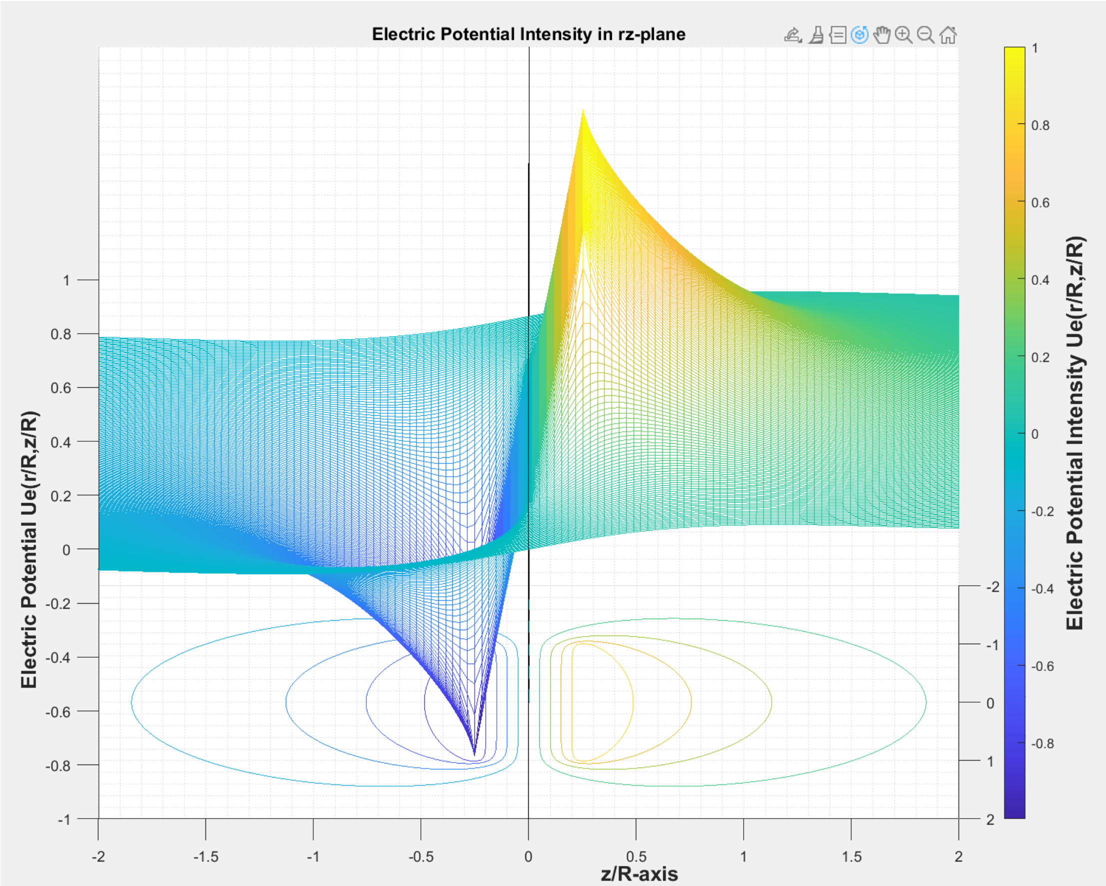

Fig 4. Electric Potential in Three Dimensions

In the contour map shown in Fig 4 for $d/R = 0.5$, we observe that:

- For $r < 0.65$, the equipotential lines are equidistant and nearly parallel—this indicates a uniform field.
- The electric field lines are perpendicular to the plates, and their spacing is roughly constant in the middle.

2) B-2:

✧ Charge Accumulation at the Edges

Observing Fig 3, if $d/R = 0.5$, there is an enormous increase in the electric field around the edges of a parallel-plate capacitor. This results from the charge accumulation that takes place at the sharp or curved boundaries of the plates. According to the edge effect, charges tend to accumulate heavily at the edges of the plates, leading to a significant local increase in electric field intensity near these regions.

✧ Application of Gauss's Law and Edge Field Enhancement

Gauss's Law explains the enhancement of electric fields at the edges. The electric flux through a closed surface enclosing the edge is given by:

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{Q_{encl}}{\varepsilon_0} \qquad (1)$$

This increase in electric field intensity is related to the accumulation of charge in the edge region within a small area. The field lines become concentrated along the boundary, resulting in higher field intensity.

3) B-3:

✧ Geometric Symmetry of Plates and Electric Field

At $z = 0$, the point lies right in the middle between the two plates, which are separated by a distance d. The potential difference between the plates is $U_0 = 1V$, with one plate at $+U_0$, and the other at $-U_0$. From symmetry, we conclude that the electric field between the plates depends linearly on z along the z-axis:

$$\boldsymbol{E} = -\frac{dV}{dz}\,\hat{z} \qquad (1)$$

Because the electric field has a constant magnitude between the plates and is symmetrically distributed, the potential at $z = 0$ is midway between the two plates.

✧ Potential Difference Calculation

Using the results from B-1, we know that the electric field along the zzz-axis is uniformly distributed. The potential difference between $z = -d/2$ and $z = +d/2$ is:

$$V_{z=\frac{d}{2}} - V_{z=0} = \int_{\frac{d}{2}}^{0} \vec{E}d\vec{l} = \int_{0}^{-\frac{d}{2}} \vec{E}d\vec{l} = \int_{\frac{d}{2}}^{0} \vec{E}d\vec{l} \tag{2}$$

$$V_{z=\frac{d}{2}} - V_{z=-\frac{d}{2}} = \int_{\frac{d}{2}}^{-\frac{d}{2}} \vec{E}d\vec{l} = \int_{0}^{-\frac{d}{2}} \vec{E}d\vec{l} + \int_{\frac{d}{2}}^{0} \vec{E}d\vec{l} = 2V \tag{3}$$

Since the electric field intensity is constant and $z = 0$ is equidistant from the plates, the integrals along the path from $z = 0$ to each plate are the same, such that the total potential difference is 2V.

Therefore, we conclude that:

$$V_{z=0} = 0V \tag{4}$$

✧ Explanation Based on Field Symmetry

Since $z = 0$ is equidistant from the two plates and the electric field is uniform, the potential at $z = 0$ must be zero. Indeed, as seen from the contour plot, along the center line, the potential is independent of radius r and remains constant at 0V. This equipotential plane symmetrically divides the capacitor between the positive and negative potentials of the plates.

4) B-4:

✧ Graphical Comparison

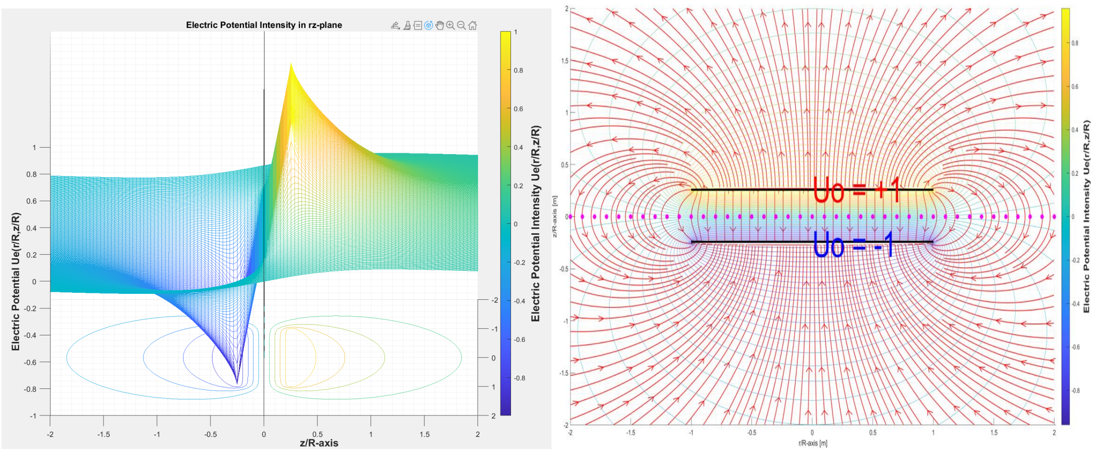

Fig 5. 3D image of the potential distribution and Electric Field in a Parallel-Plate Capacitor when $d/R = 0.5$

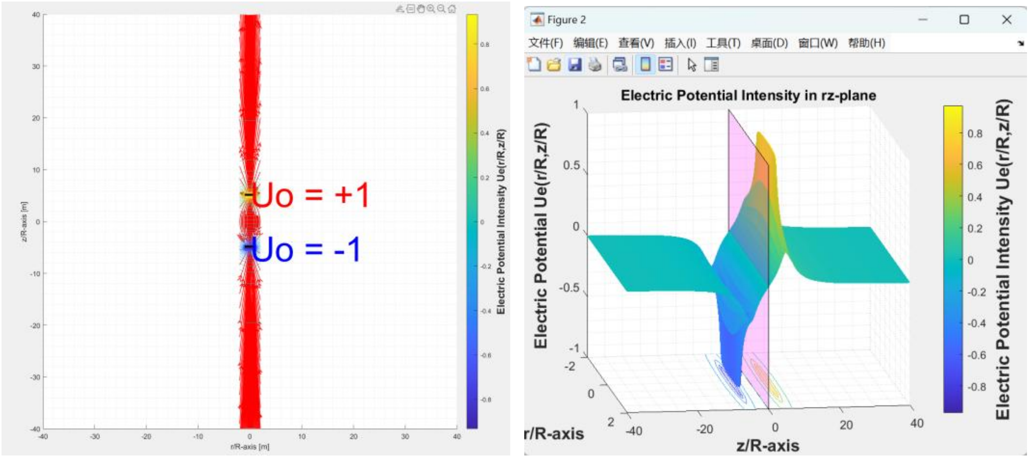

Fig 5. 3D image of the potential distribution and Electric Field in a Parallel-Plate Capacitor when $d/R = 10$

When we increase the ratio d/R, we explain the observed changes from three aspects:

1. Changes in Electric Field Lines

✧ **Observation:**

Compared with the results obtained from $d/R = 0.5$. When increasing the ratio d/R, electric field lines become sparse and no longer so uniform.

✧ **Explanation:**

The reason is that the interaction between charges weakens, and the electric field intensity decreases accordingly. For a parallel-plate capacitor, according to the formula of a parallel-plate capacitor:

$$C = \frac{\varepsilon_0 S}{d} \tag{1}$$

where

S stands for the area of the plate

d means the distance between the two plates

When the ratio d/R increases, capacitance C decreases.

Based on the relationship $C = \frac{Q}{V}$, when capacitance C decreases and potential differences between the two plates keep. The total charges on the plate will decrease.

According to Gauss's law, the electric field intensity decrease. Hence, the electric field lines become sparse. When the ratio d/R increases, the edge effect becomes stronger, which make the electric field lines no longer so uniform.

2. Changes in Electric Potential Distribution

✧ **Observation:**

Compared with the results obtained from $d/R = 0.5$, when increasing the ratio d/R, the spacing between equipotential lines increases. The equipotential lines become sparser and nonuniform.

✧ **Explanations:**

Since it is known that the electric field is just the negative value of the potential gradient, a decrease in the intensity of the electric field would amount to a decrease in the gradient of the potentials. Potential gradient reflects the rate of change of electric potential between two electrode plates. A smaller potential gradient will increase the distance between equipotential lines and make the potential lines sparser. Due to the edge effect, increasing the ratio d/R results in an increase in the non-uniformity of the electric field distribution. This affects electric potential distribution, too. Consequently, there is non-uniformity regarding equipotential lines.

3. Changes in 3D Potential Distribution Images

✧ **Observation:**

Compared with the results obtained from $d/R = 0.5$, when increasing the ratio d/R, the potential gradient in 3D images becomes less steep, and the variation of potential values between the two electrodes is smoother.

✧ **Explanations:**

Due to the more dispersed distribution of electric field lines, the change in potential between the two plates becomes smoother. Therefore, in 3D images, the potential gradient becomes less steep. This is due to the decrease in electric field intensity leading to a decrease in potential gradient which makes it much smoother.

4. Electric Field Distribution Resembling Point Charge

✧ **Observation:**

We can see that as the ratio of d/R starting from 0.5 and going up to 10, the distribution of electric field lines will gradually take the shape of two equal and opposite point charges.

✧ **Explanations:**

As d/R increases, all the charge on the plates starts gathering towards the center of the plates. Such concentration makes the electric field lines increasingly resemble the pattern formed between two equal and opposite point charges.

5) B-5:

✧ **General formula for Parallel-Plate Capacitors**

The capacitance between two parallel plates, ignoring edge effects and before the insertion of a conductive circular plate, is given by the following expression:

$$C = \frac{\varepsilon_0 s}{d} = \frac{\varepsilon_0 \pi R^2}{d} \tag{1}$$

✧ **Capacitance After Adding of a Conductive Circular Plate**

When an infinite conductive circular plate of thickness t is inserted between the two plates, at a distance d_1 from the plate at $+U_0$ and d_2 from the plate at $-U_0$, the system becomes two capacitors in series. These two sections can be treated as two capacitors in series, with distances d_1 and d_2, plus the thickness t of the inserted plate.

The parallel total capacitance is given by:

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{2}$$

where $C_1 = \frac{\varepsilon_0 \pi R^2}{d1}$ and $C_2 = \frac{\varepsilon_0 \pi R^2}{d2}$

Thus, the total capacitance becomes:

$$C_{tot} = \frac{\varepsilon_0 \pi R^2}{d_1 + d_2 + t} \tag{3}$$

✧ Electric Field and Voltage Distribution

The presence of the conductive plate splits the electric field into two regions, each with a constant electric field intensity E. Since the electric field intensity for a parallel-plate capacitor does not depend on the distance between the plates, we have:

$$E_1 = E_2 = \frac{\sigma}{\varepsilon_0} \tag{4}$$

where σ is the surface charge density on the plates. The voltage drop across each section of the capacitor can be computed as:

$$V_1 = E_1 d_1,\ V_2 = E_2 d_2 \tag{5}$$

Thus, the total voltage across the two sections is:

$$V_{total} = V_1 + V_{inner} + V_2 = E_1 d_1 + E_2 d_{21} + V_{inner} = U_0 \tag{6}$$

Since the electric field inside the conductive plate is zero,

$$V_{inner} = 0V \tag{7}$$

✧ Final Expression for the Total Capacitance

By substituting the expressions for E_1 and E_2 and using the relationship between charge, voltage, and capacitance, the final expression for the total capacitance becomes:

$$C_{tot} = \frac{\varepsilon_0 \pi R^2}{d_1 + d_2} \tag{8}$$

Alternatively, if the thickness of the conductive plate t is negligible, the capacitance simplifies to:

$$C_{tot} = \frac{\varepsilon_0 \pi R^2}{d - t} \tag{9}$$

Appendix A-2:

```
% Number of grid point [N = 10001]
 N = 1001;
% Charges
 Q = [+20, -60, 0, 0, 0] .* 1e-6;
% Radius of circular charged conductor;
 a = 0.2;
% X & Y components of position of charges [0, 0, 0, 0, 0]
 xC = [-0.5, 0.5, 0.5, -0.5, 0];
 yC = [ 0, 0, 0, 0, 0];
% constants
 eps0 = 8.854e-12;
 kC = 1/(4*pi*eps0);
% Dimensions of region / saturation levels
% [dimensions of region -2 to 2 / minR = 1e-6 / Esat = 1e6 / Vsat = 1e6]
 minX = -2;
 maxX = 2;
 minY = -2;
 maxY = 2;
 minR = 1e-6;
 minRx = 1e-6;
 minRy = 1e-6;
 Vsat = kC * max(abs(Q)) / a;
 Esat = kC * max(abs(Q)) / a^2;
% SETUP ==============================================================
 % fields
 V = zeros(N,N);
 Ex = zeros(N,N); Ey = zeros(N,N);
 % [2D] region
 x = linspace(minX,maxX,N);
 y = linspace(minY, maxY,N);
 % color of charged object + red / - black
 col1 = [1 0 0];
 col2 = [0 0 0];
 %if Q(1) < 0; col1 = [0 0 0]; end;
 % grid positions
 [xG, yG] = meshgrid(x,y);
% CALCULATION: POTENTIAL & ELECTRIC FIELD
for n = 1 : 2
 Rx = xG - xC(n);
 Ry = yG - yC(n);

 index = find(abs(Rx)+ abs(Ry) == 0);
 Rx(index) = minRx; Ry(index) = minRy;
```

```
  R = sqrt(Rx.^2 + Ry.^2);
  R(R==0) = minR;
  V = V + kC .* Q(n) ./ (R);

  R3 = R.^3;
  Ex = Ex + kC .* Q(n) .* Rx ./ R3;
  Ey = Ey + kC .* Q(n) .* Ry ./ R3;
end
  if max(max(V)) >= Vsat; V(V > Vsat) = Vsat; end
  if min(min(V)) <= -Vsat; V(V < -Vsat) = -Vsat; end
  E = sqrt(Ex.^2 + Ey.^2);
  if max(max(E)) >= Esat; E(E > Esat) = Esat; end
  if min(min(E)) <= -Esat; E(E < -Esat) = -Esat; end

  if max(max(Ex)) >= Esat; Ex(Ex > Esat) = Esat; end
  if min(min(Ex)) <= -Esat; Ex(Ex < -Esat) = -Esat; end

  if max(max(Ey)) >= Esat; Ey(Ey > Esat) = Esat; end
  if min(min(Ey)) <= -Esat; Ey(Ey < -Esat) = -Esat; end
% make all the field lines start from the out surface of the positive conductor
for n = 1:2
    % Calculate the distance from each point to the center of the conductor
    distance_from_center = sqrt((xG - xC(n)).^2 + (yG - yC(n)).^2);

    % Find the points where the distance is less than the conductor's radius (i.e., inside the conductor)
    inside_conductor = distance_from_center < a;

    % Set the electric field Ex, Ey to 0 inside the conductor
    Ex(inside_conductor) = 0;
    Ey(inside_conductor) = 0;

    % Set the electric field magnitude E to 0 inside the conductor as well
    E(inside_conductor) = 0;
end
% GRAPHICS
figure(1)
  set(gcf,'units','normalized','position',[0.01 0.52 0.23 0.32]);
  surf(xG,yG,V./1e9);
  xlabel('x [m]'); ylabel('y [m]'); zlabel('V [ V ]');
  title('potential','fontweight','normal');

  rotate3d
  view(76,32);
```

```
  set(gca,'fontsize',12)
  set(gca,'xLim',[-2, 2]); set(gca,'yLim',[-2, 2]);

  shading interp;
h = colorbar;
  h.Label.String = 'V [ MV ]';
  colormap(parula);
  axis square
  box on

%%
figure(2)
set(gcf,'units','normalized','position',[0.25 0.1 0.23 0.32]);
zP = V./1e6;
contourf(xG,yG,zP,12);
%set(gca,'xLim',[-5,5]); set(gca,'yLim', [-5, 5]);
%set(gca,'xTick',-5:5); set(gca,'yTick', -5:5);

hold on;

% charged conductors
col = col1;
if Q(1) < 0; col = col2; end
pos1 = [-a+xC(1), -a, 2*a, 2*a];
h = rectangle('Position',pos1,'Curvature',[1,1]);
set(h,'FaceColor',col,'EdgeColor',col);

col = col1;
if Q(2) < 0; col = col2; end
pos2 = [-a-xC(1), -a, 2*a, 2*a];
h = rectangle('Position',pos2,'Curvature',[1,1]);
set(h,'FaceColor',col,'EdgeColor',col);

xlabel('x [m]'); ylabel('y [m]');
title('Electric field lines with arrows from positive to negative charge','fontweight','normal');
shading interp;
h = colorbar;
h.Label.String = 'V [ MV ]';
colormap(parula);
set(gca,'fontsize',12);
axis square;
```

```
% use streamslice to draw field lines
hLines = streamslice(xG, yG, Ex, Ey, 0.75);
set(hLines, 'Color', 'r', 'LineWidth', 1);
box on;
hold off
%
figure(3)

set(gcf,'units','normalized','position',[0.01 0.1 0.23 0.32]);
surf(xG,yG,E./1e6);
xlabel('x [m]'); ylabel('y [m]'); zlabel('E [ V / m ]');
title('electric field | E |','fontweight','normal');
colorbar
shading interp
h = colorbar;
h.Label.String = '| E | [ MV/m ]';
rotate3d
view(30,50);
axis square
set(gca,'fontsize',12)
box on
%%
toc
```