

PN junction

- (I) Fundamentals (this lecture)
- (II) Fabrication (after midterm week)

Gary Chun Zhao, PhD

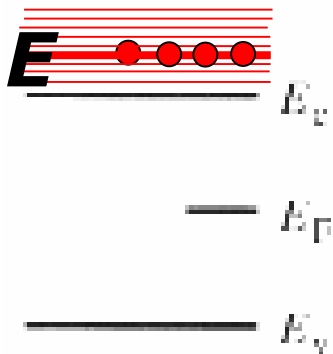
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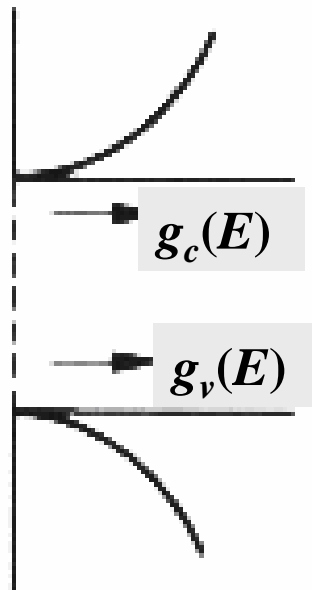
Last lecture: Distribution of **Electrons**

- Obtain **$n(E)$** by multiplying $g_c(E)$ and $f(E)$

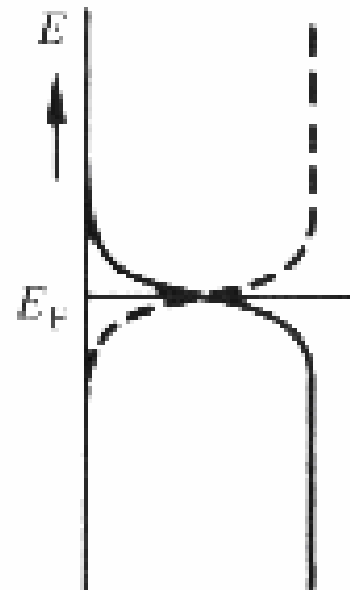
Energy band diagram



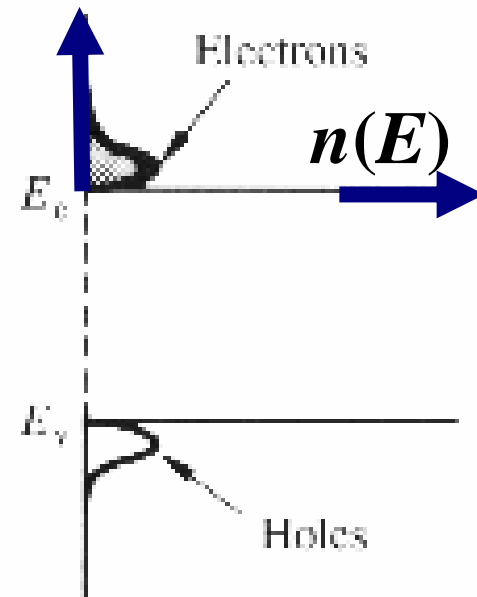
Density of States



Probability of occupancy



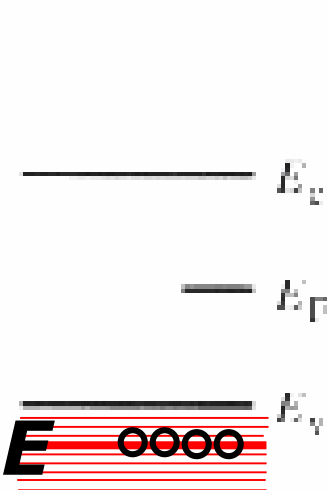
Carrier distribution



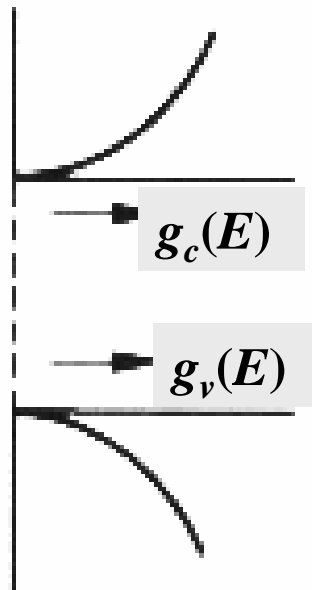
Last lecture: Distribution of Holes

- Obtain $p(E)$ by multiplying $g_v(E)$ and $1-f(E)$

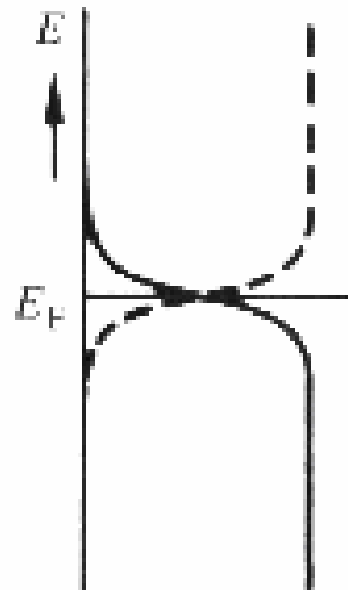
Energy band diagram



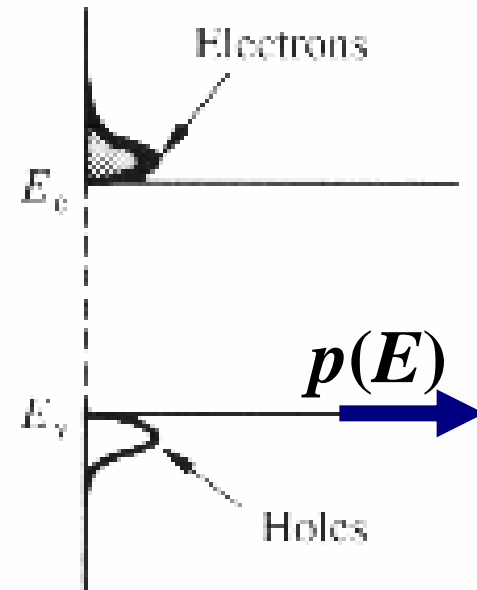
Density of States



Probability of occupancy



Carrier distribution



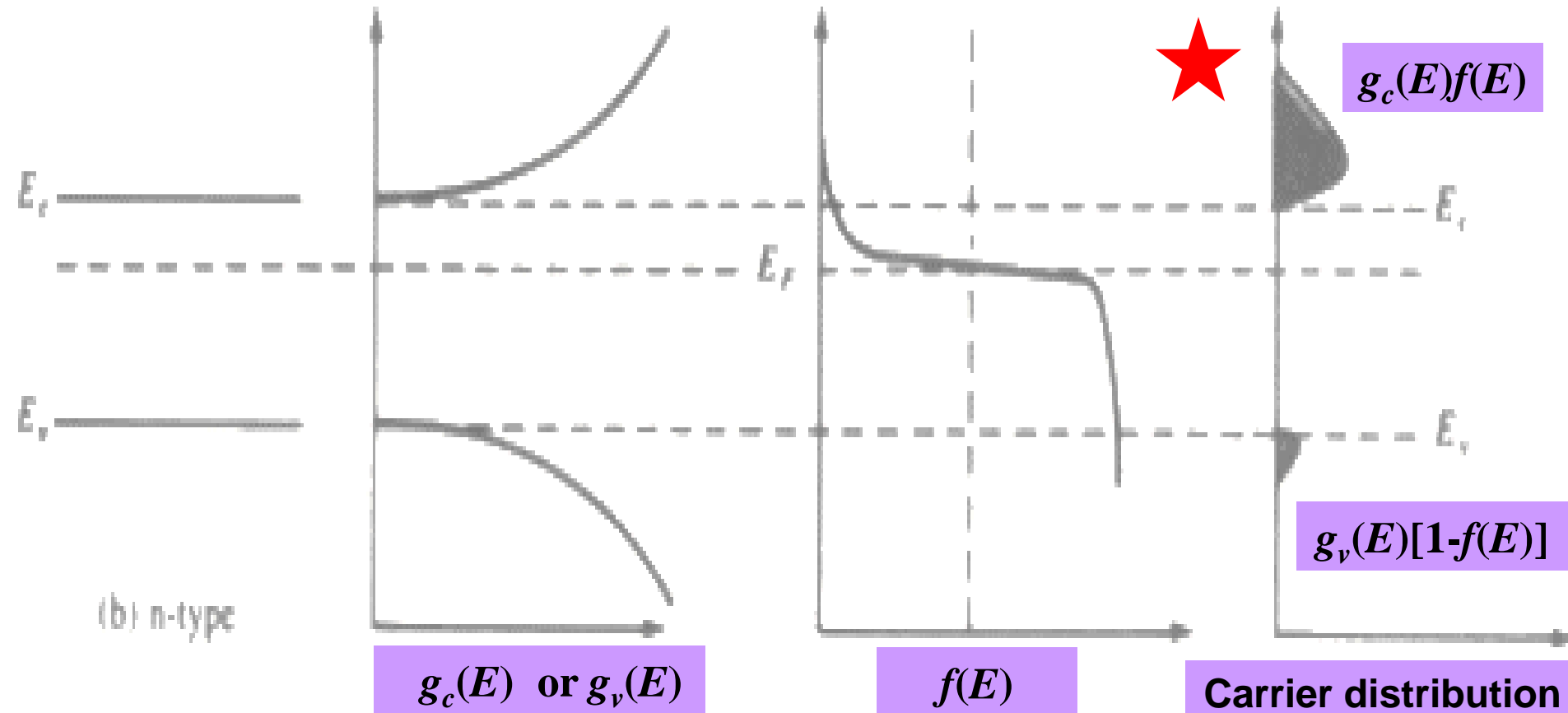
N-type Material

Energy band diagram

Density of States

Probability of occupancy

Carrier distribution



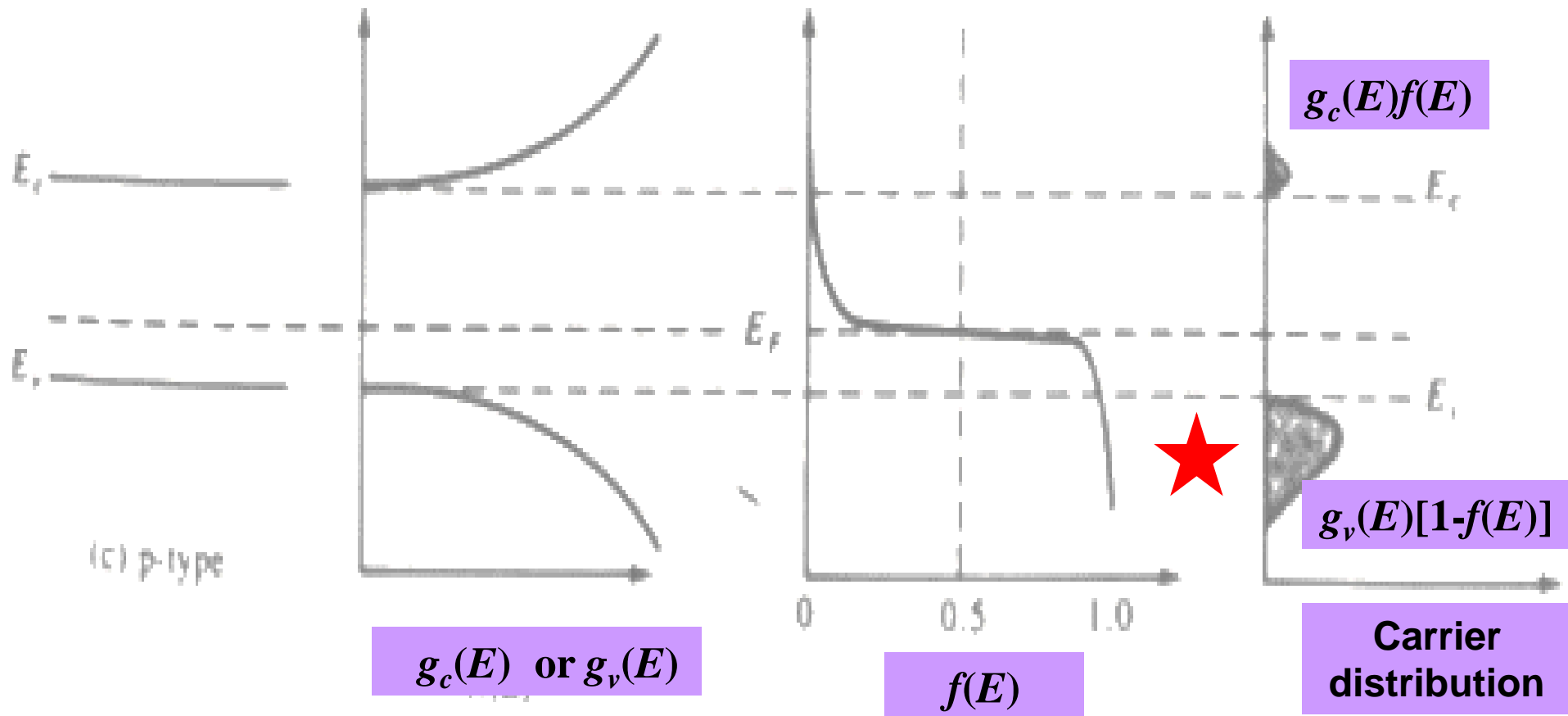
P-type Material

Energy band diagram

Density of States

Probability of occupancy

Carrier distribution



Last lecture: **total** current

- The **total** current flowing in a semiconductor is the **sum** of **drift current** and **diffusion current**:

$$J_{tot} = J_{p,drift} + J_{n,drift} + J_{p,diff} + J_{n,diff}$$



$$J_{p,drift} = qp\mu_p E, \quad J_{n,drift} = qn\mu_n E$$

$$J_{p,diff} = -qD_p \frac{dp}{dx}, \quad J_{n,diff} = qD_n \frac{dn}{dx}$$

Einstein Relation

- The characteristic constants for drift and diffusion are related:

$$D_n = \frac{kT}{q} \mu_n$$

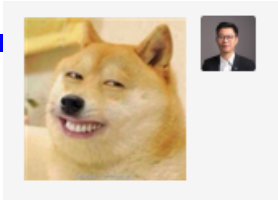
$$D_p = \frac{kT}{q} \mu_p$$

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}}$$

$$= 26 \text{ mV} \\ \text{at } T = 300 \text{ K}$$

- Note that $\frac{kT}{q} \cong 26 \text{ mV}$ at room temperature (**300K**)
 - This is often referred to as the “**thermal voltage**”.

PN junction – (I)



OUTLINE

- **Formation of depletion region (DR)**
- Built-in potential of DR
- Distribution of electric field and electric potential in DR
- Effect of applied voltage on DR
- Depletion capacitance of DR*

Reference Reading

- Chapter 3.1 (page 92-116)

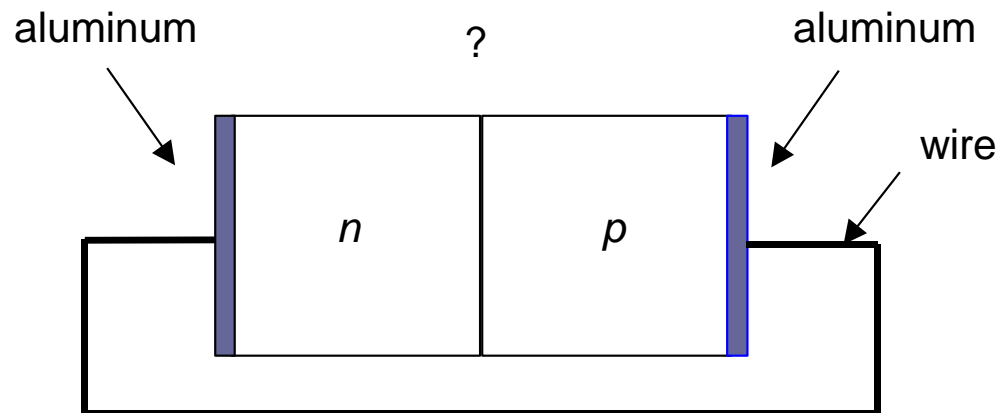
Junctions of n- and p-type Regions

p-n junctions form the **essential basis** of all semiconductor devices.

A silicon chip may have 10^8 to 10^9 p-n junctions today.

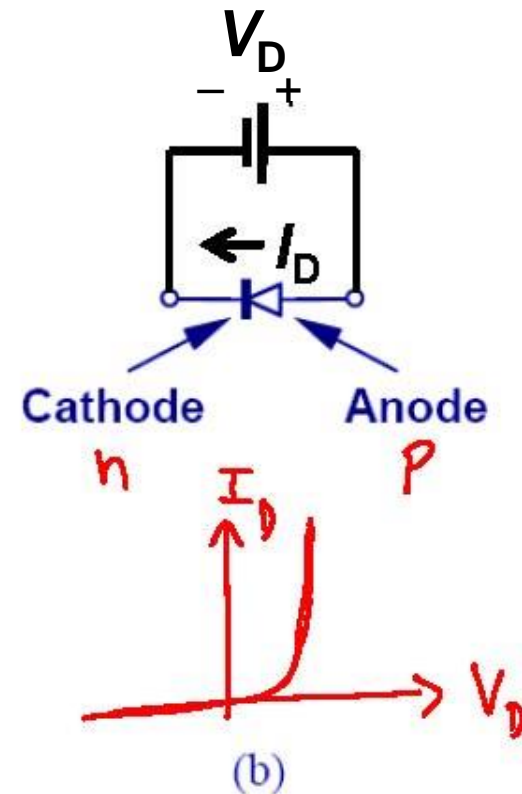
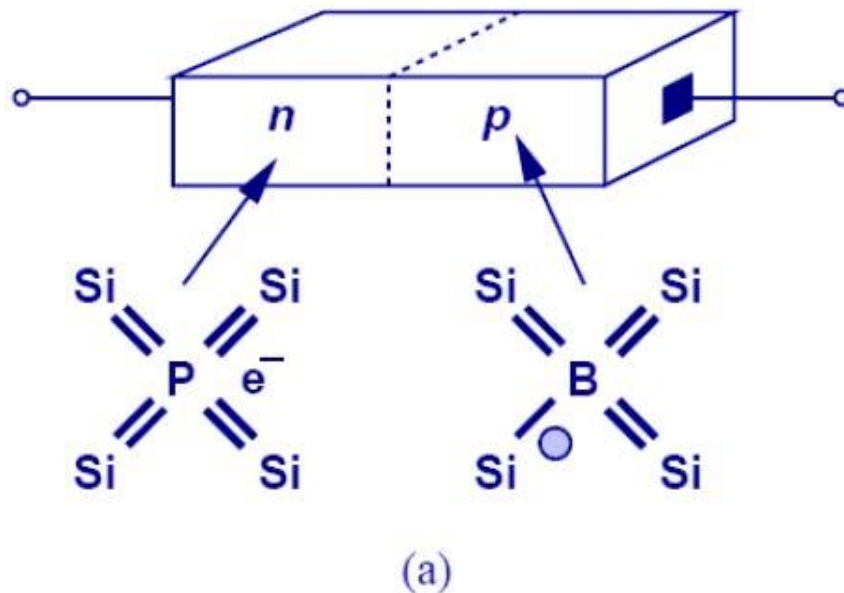
What happens to the electrons and holes if

n and p regions are brought into contact ?



PN Junction Diode

- When a **P-type** semiconductor region and an **N-type** semiconductor region are **in contact**, a **PN junction** diode is formed.

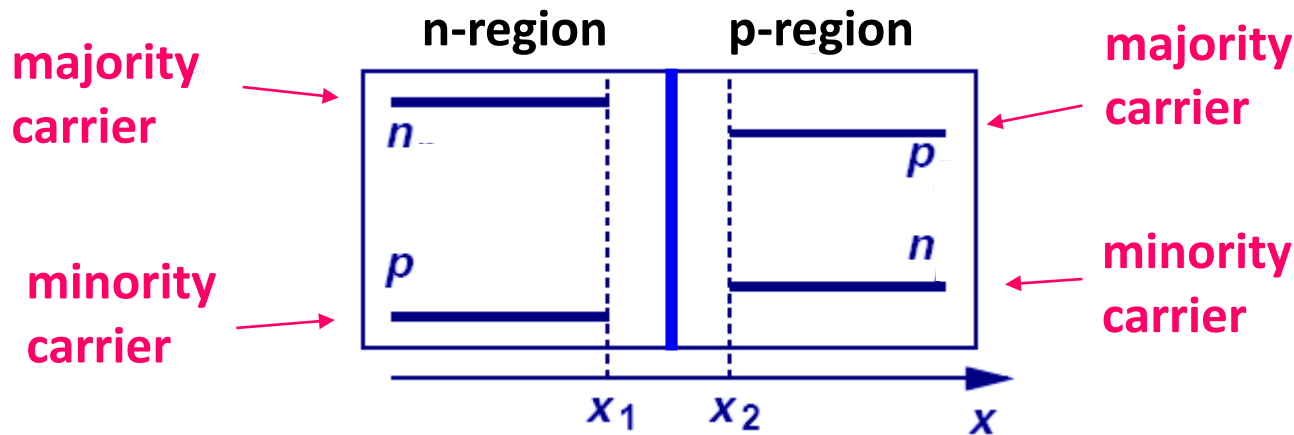


Carrier concentration distribution in thermal equilibrium

n-type

p-type

- Because of the **difference** in hole and electron **concentrations** on each side of the junction, carriers **diffuse** across the junction:



Notation:

$n_n \equiv$ **electron** concentration on **N-type side** (cm^{-3}) $\approx N_D$

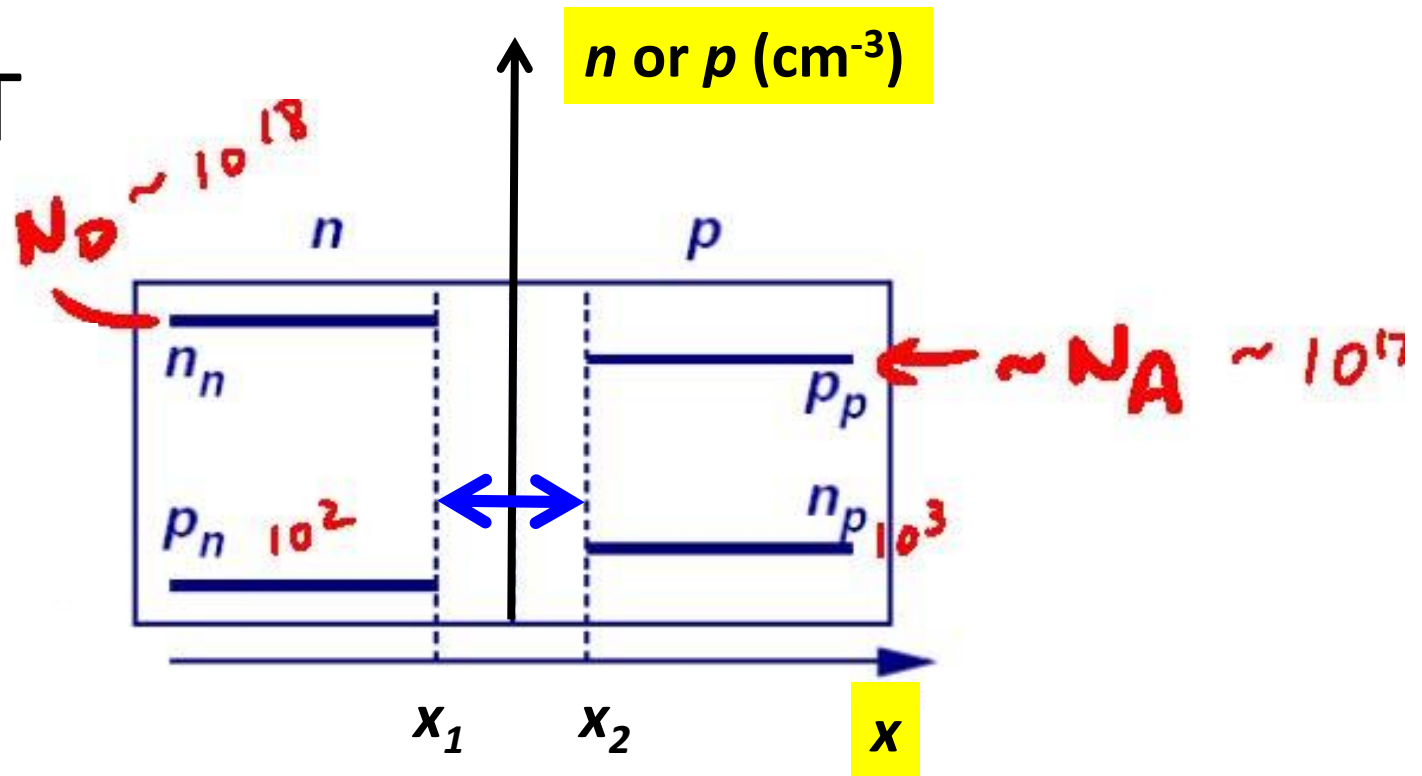
$p_n \equiv$ **hole** concentration on **N-type side** (cm^{-3}) $\approx n_i^2/N_D$

$p_p \equiv$ **hole** concentration on **P-type side** (cm^{-3}) $\approx N_A$

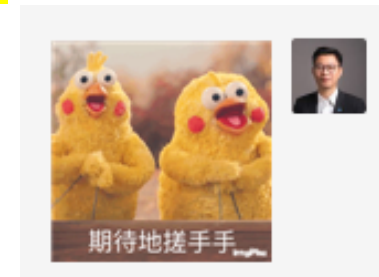
$n_p \equiv$ **electron** concentration on **P-type side** (cm^{-3}) $\approx n_i^2/N_A$

Log scale

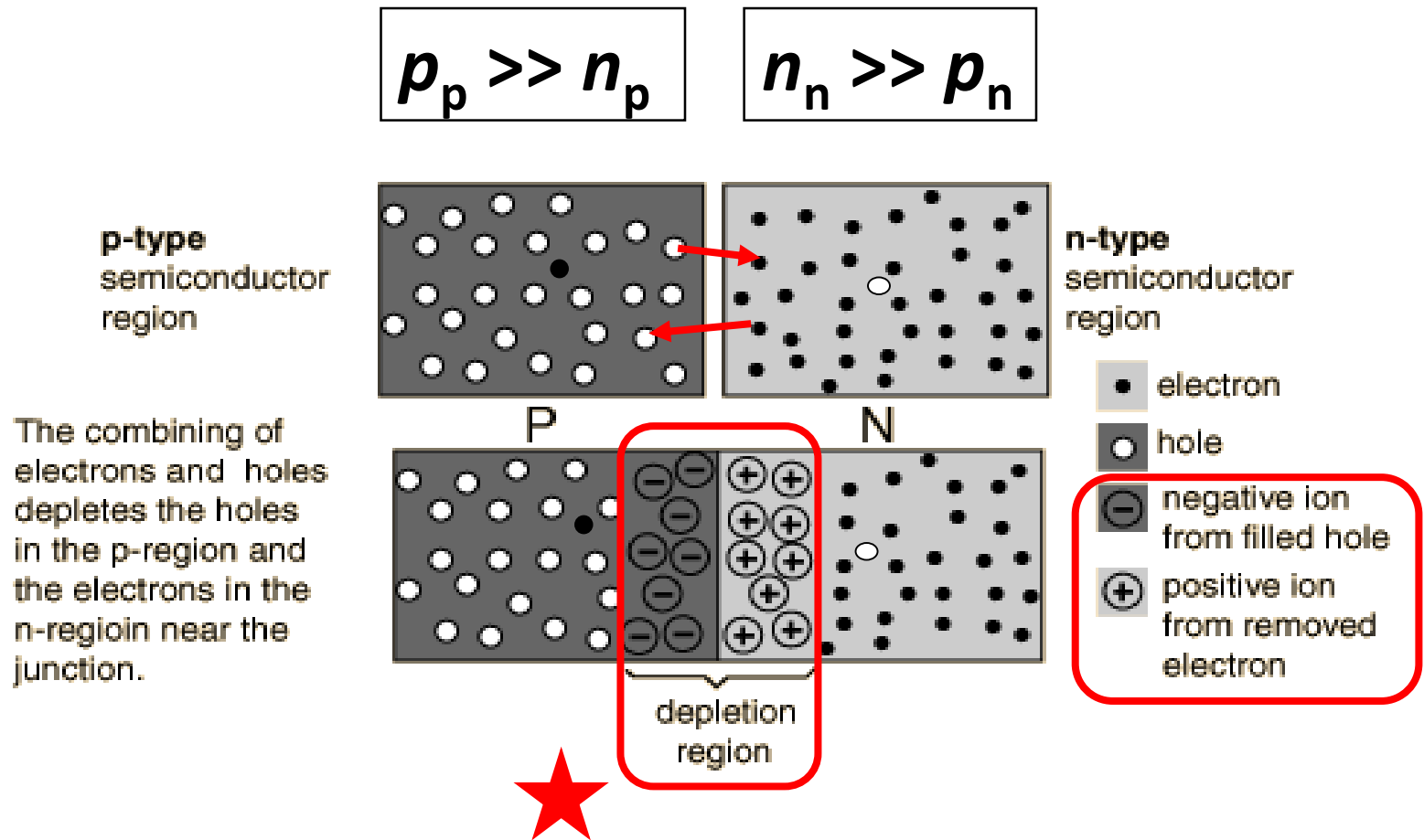
At RT



Carrier **Depletion Region**

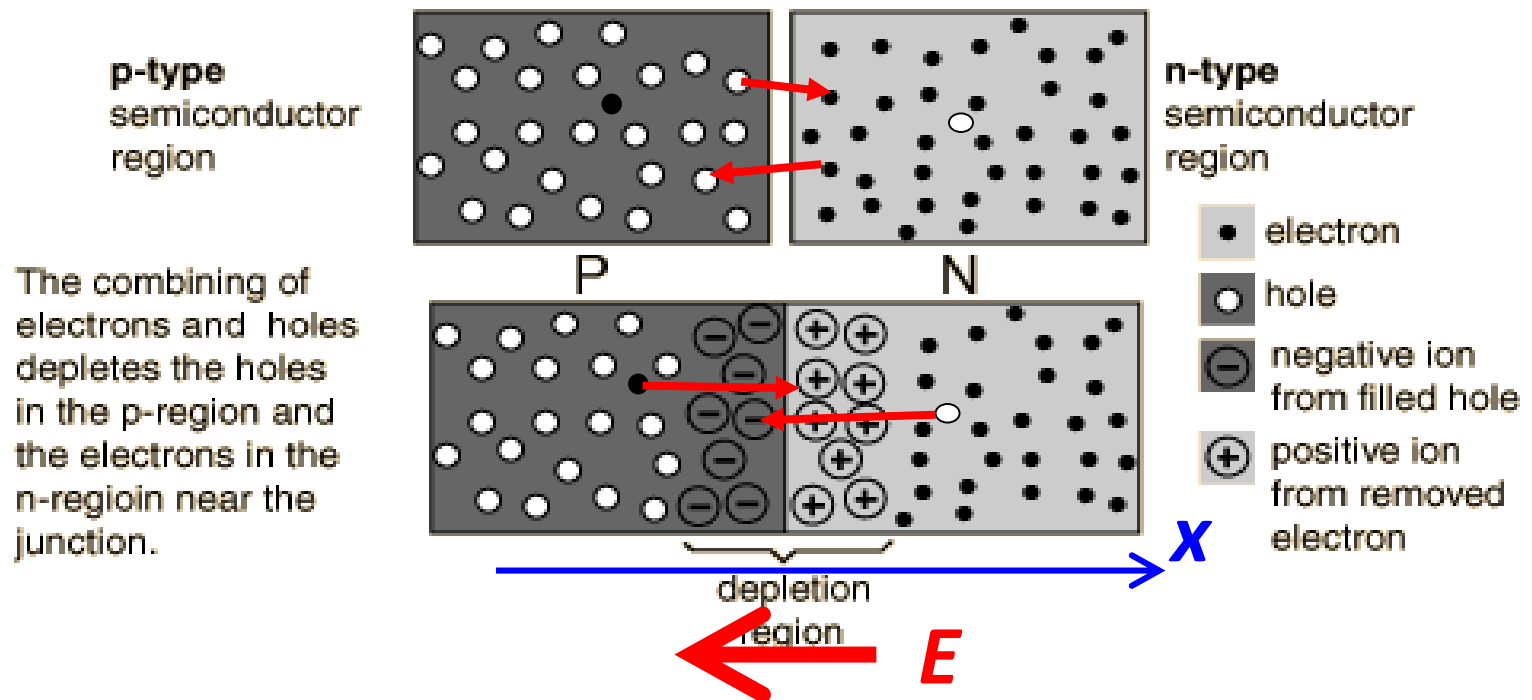


Carrier Diffusion across Junction



Carrier **Drift** across Junction

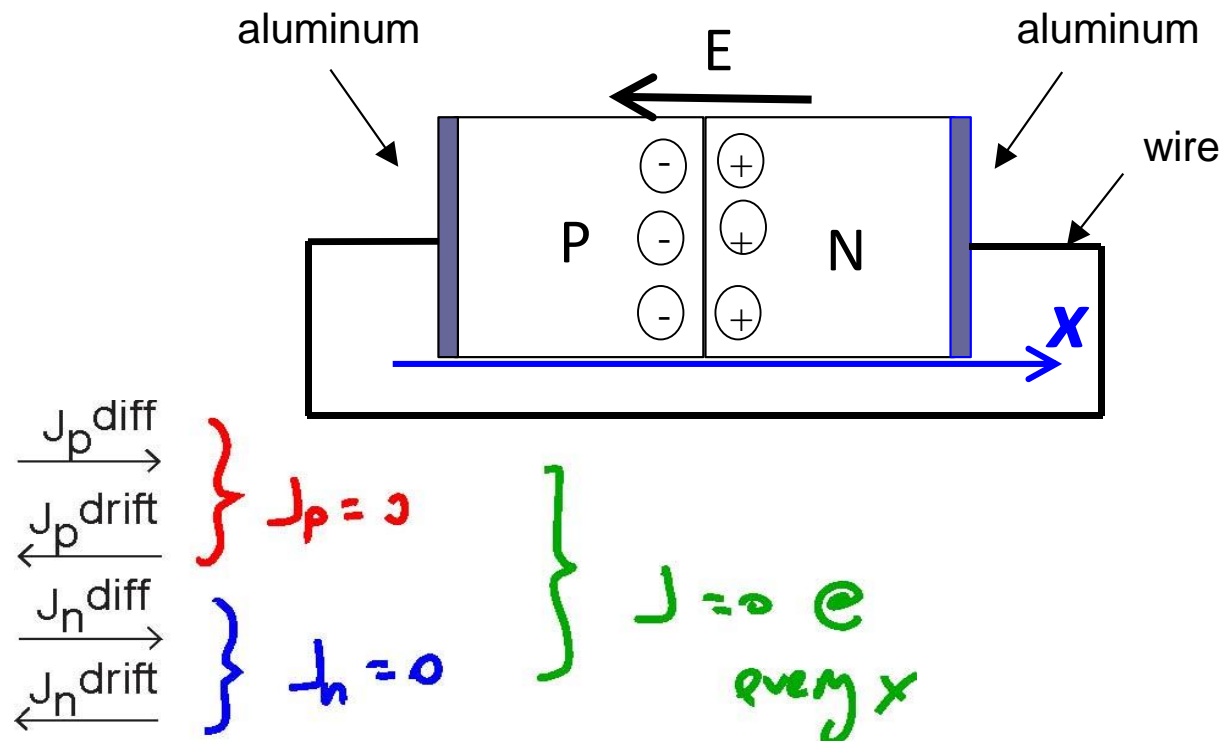
- Because charge density $\neq 0$ in the depletion region, an **electric field** exists, hence there is **drift current**.



Thermal equilibrium: balance **between drift and diffusion**

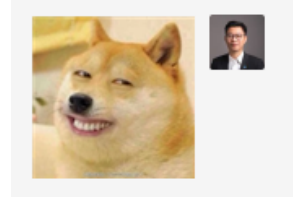
Carrier **Drift** across the Junction

Thermal equilibrium: balance **between drift and diffusion**



PN junction – (I)

OUTLINE



- The formation of depletion region
- **Built-in potential (two methods for V_{bi})**
- Distribution of electric field and electric potential
- Effect of Applied Voltage
- Depletion capacitance

Reference Reading

- Chapter 3.1 (Page 92-116)

PN Junction in Equilibrium

- In equilibrium, the **drift** and **diffusion** components of current are **balanced**; therefore the **net current** flowing across the junction is **zero**.

$$J_{p,drift} + J_{p,diff} = 0$$



$$J_{n,drift} + J_{n,diff} = 0$$

$$J_{tot} = J_{p,drift} + J_{n,drift} + J_{p,diff} + J_{n,diff} = 0$$

$$J_{p,drift} = qp\mu_p E,$$

$$J_{n,drift} = qn\mu_n E$$

$$J_{p,diff} = -qD_p \frac{dp}{dx},$$

$$J_{n,diff} = qD_n \frac{dn}{dx}$$

Built-in Potential, V_{bi}

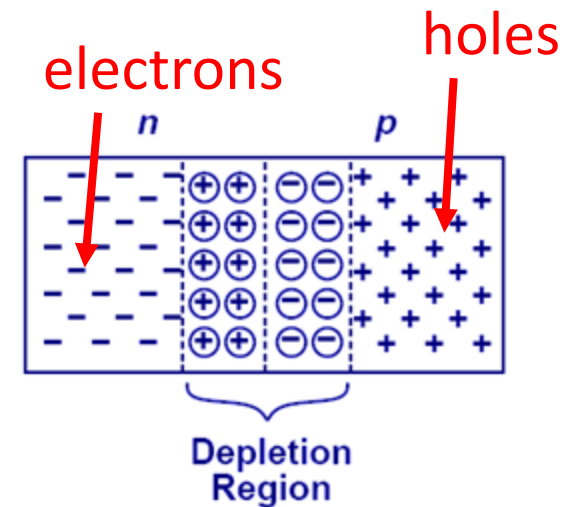
- Because of the **electric field** in the depletion region, there exists a **potential drop** across the junction:

$$qp\mu_p E = qD_p \frac{dp}{dx} \Rightarrow p\mu_p \left(-\frac{dV}{dx} \right) = D_p \frac{dp}{dx}$$

$$\Rightarrow -\mu_p \int_{x_1}^{x_2} dV = D_p \int_{p_n}^{p_p} \frac{dp}{p}$$

$$E = -\frac{dV}{dx}$$

$$\Rightarrow V(x_1) - V(x_2) = \frac{D_p}{\mu_p} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \left(\frac{N_A}{n_i^2 / N_D} \right)$$



$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = V_{bi}$$

(at RT)

(Unit: Volts)

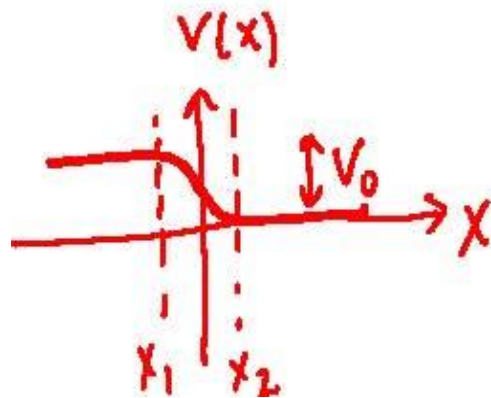
drift

diffusion

$$qp\mu_p E = qD_p \frac{dp}{dx} \Rightarrow p\mu_p \left(-\frac{dV}{dx} \right) = D_p \frac{dp}{dx}$$

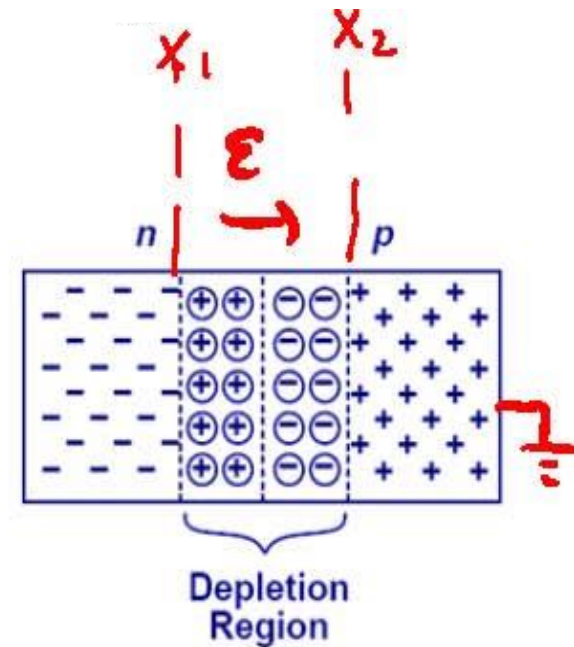
$$\Rightarrow -\mu_p \int_{x_1}^{x_2} dV = D_p \int_{p_n}^{p_p} \frac{dp}{p}$$

$$\Rightarrow V(x_1) - V(x_2) = \frac{D_p}{\mu_p} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \left(\frac{N_A}{n_i^2 / N_D} \right)$$



$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

26mV



(Unit: Volts)

Built-In Potential Example

HW6

- Estimate the **built-in potential** for PN junction below.
 - Note that

N	P
$N_D = 10^{18} \text{ cm}^{-3}$	$N_A = 10^{15} \text{ cm}^{-3}$

$V_0 \lesssim 1 \text{ V}$ for a
Si PN junction

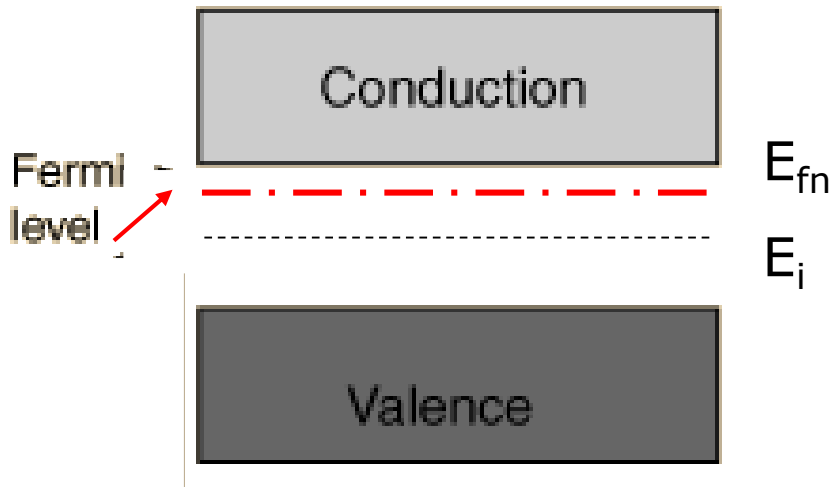
$$V_0 = \frac{kT}{q} \ln \left(\frac{N_D N_A}{n_i^2} \right) = \frac{kT}{q} \ln \left(\frac{10^{18} 10^{15}}{10^{20}} \right) = \frac{kT}{q} \ln (10^{13})$$
$$= 13 \cdot \frac{kT}{q} \ln (10) = 13 \cdot 0.06 \text{ V} = 0.78 \text{ V}$$

$$\frac{kT}{q} \ln(10) \cong 26 \text{ mV} \times 2.3 \cong 60 \text{ mV}, \quad \text{at RT}$$

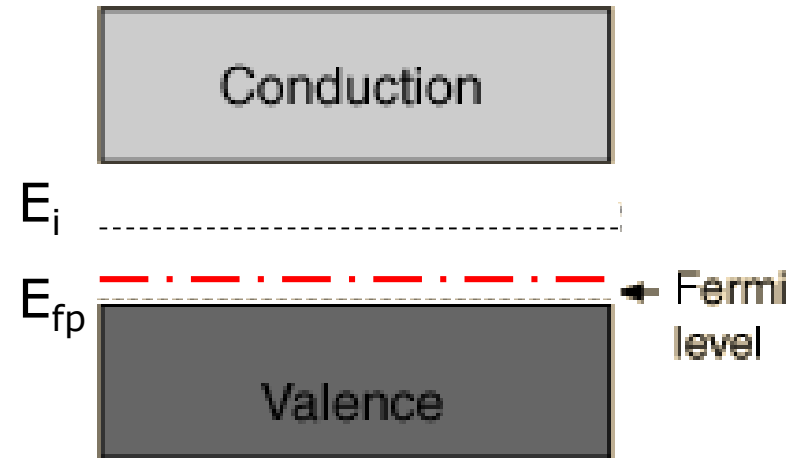
$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

Energy bands of n- and p- type

n-type

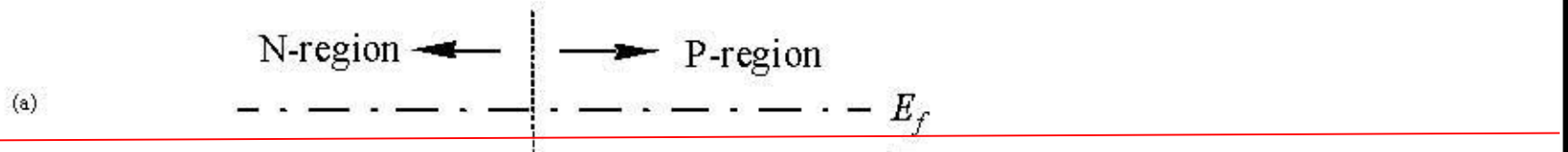


p-type

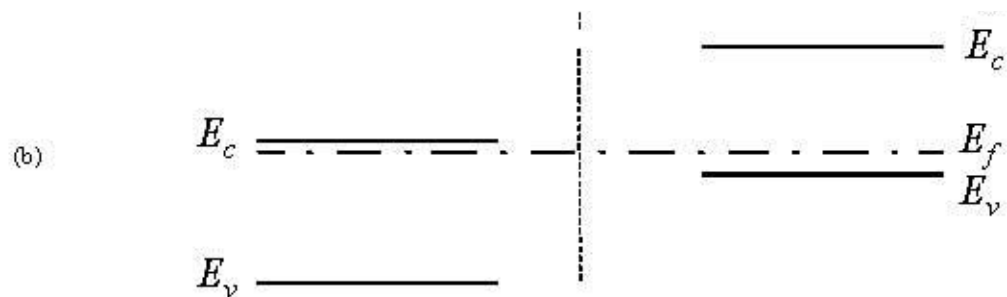


- If n-type and p-type are in the same thermal equilibrium system, **they have the same Fermi level.**

Energy Band Diagram and Depletion Layer

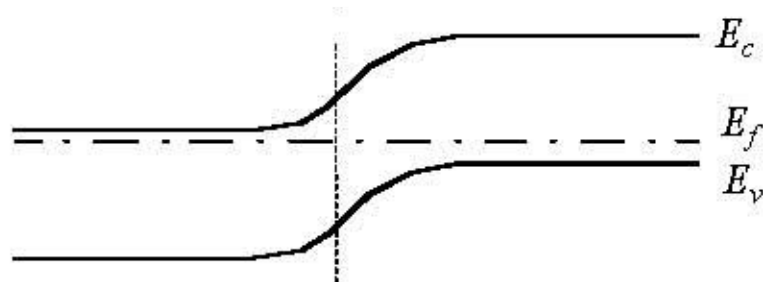


Energy Band Diagram and Depletion Layer



Energy Band Diagram and Depletion Layer

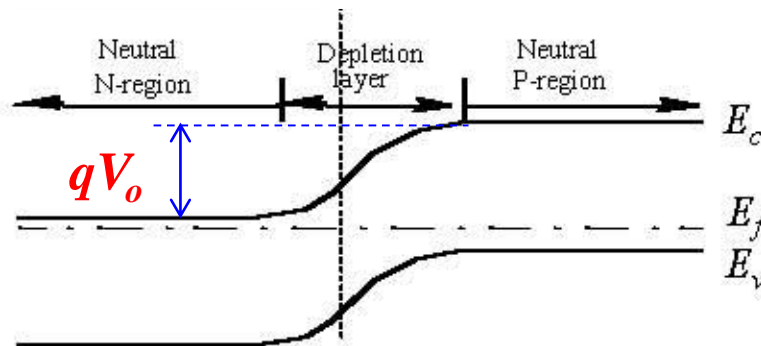
(c)



Energy Band Diagram and Depletion Layer

$$n = N_D = N_C \exp\left(\frac{E_{fn} - E_{Cn}}{kT}\right)$$

$$p = N_A = N_V \exp\left(\frac{E_{vp} - E_{fp}}{kT}\right)$$



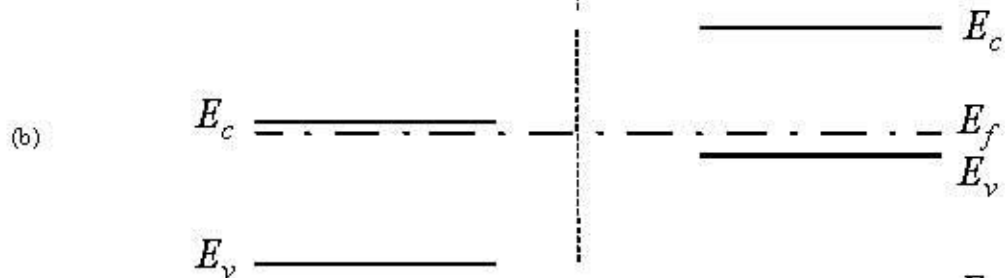
(d)

$$V_o = \frac{kT}{q} \ln \frac{N_D N_A}{n_i^2}$$

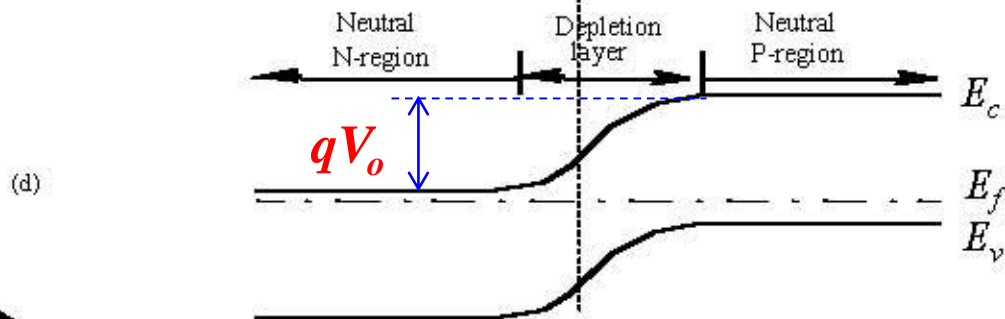
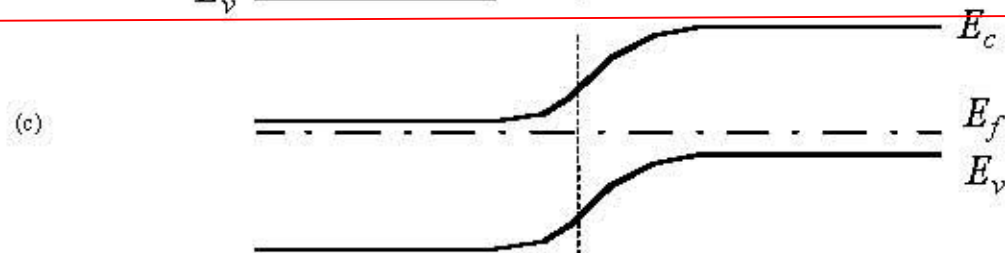
Energy Band Diagram and Depletion Layer



$$n = N_D = N_C \exp\left(\frac{E_{fn} - E_C}{kT}\right)$$



$$p = N_A = N_V \exp\left(\frac{E_V - E_{fp}}{kT}\right)$$



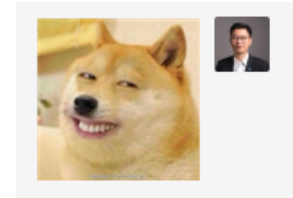
$$V_o = \frac{kT}{q} \ln \frac{N_D N_A}{n_i^2}$$

HW7

PN junction – (I)

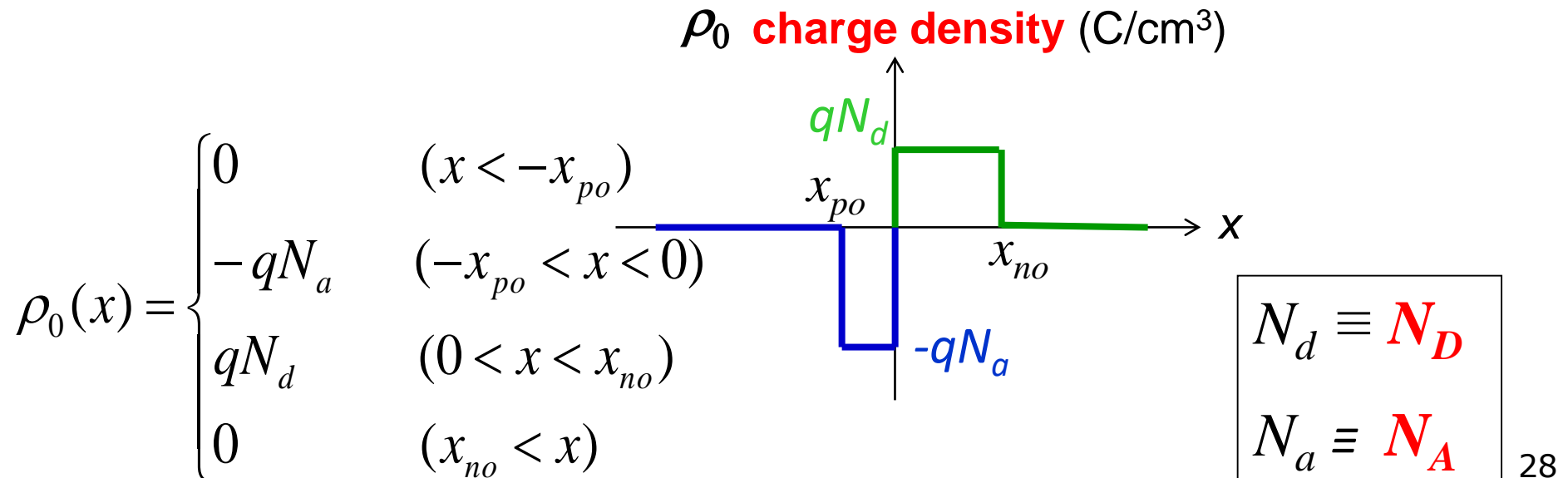
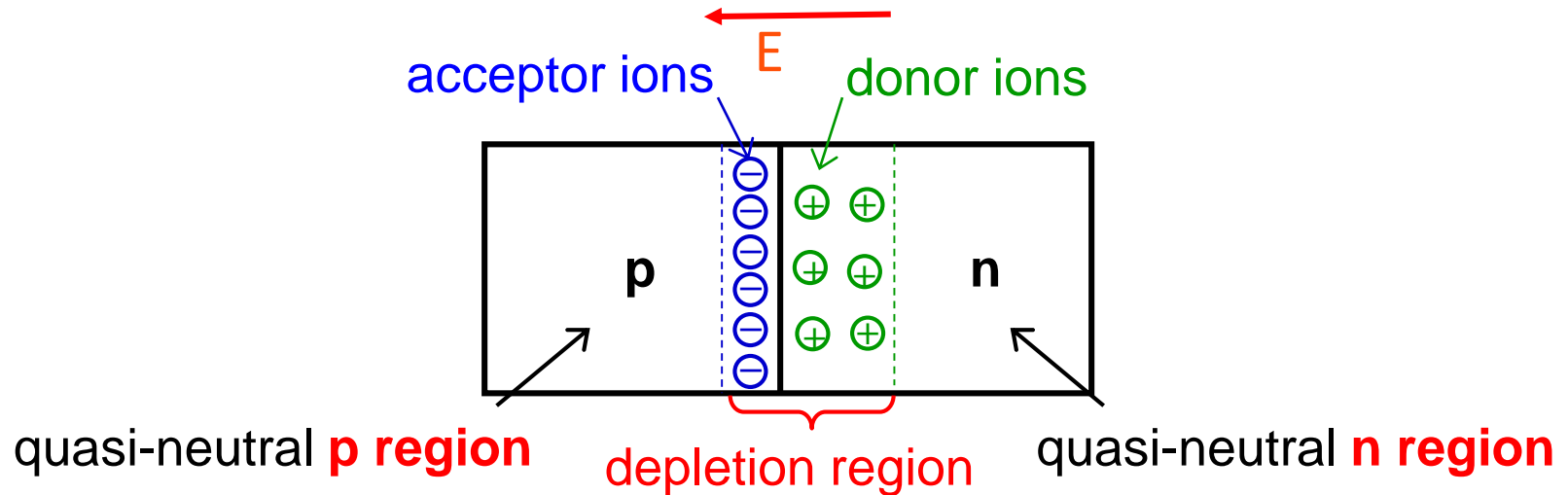
OUTLINE

- The formation of depletion region
- Built-in potential
- **Distribution of electric field and electric potential**
- Effect of Applied Voltage
- Depletion capacitance



Depletion Approximation

Charge is stored in the depletion region.



Two Governing Laws

$$E = -\frac{dV}{dx} \quad \text{or} \quad E = -\frac{d\phi}{dx}$$

Gauss's Law describes the relationship of **charge (density)** and **electric field**.

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon} \int_V \rho dV = \frac{Q_{encl}}{\epsilon}$$

$$\epsilon = \epsilon_S$$

$$\frac{dE}{dx} = \frac{\rho}{\epsilon}$$

$$E(x) - E(x_0) = \frac{1}{\epsilon} \int_{x_0}^x \rho(x) dx$$

Poisson's Equation describes the relationship between **electric field distribution** and **electric potential**

$$\frac{d^2 \phi(x)}{dx^2} = -\frac{dE(x)}{dx} = -\frac{\rho(x)}{\epsilon}$$

$$\phi(x) - \phi(x_0) = \int_{x_0}^x -E(x) dx$$

Depletion Approximation 1 (Electric field)

$$E_0(x) - E_0(x_0) = \frac{1}{\epsilon_{Si}} \int_{x_0}^x \rho_0(x) dx$$

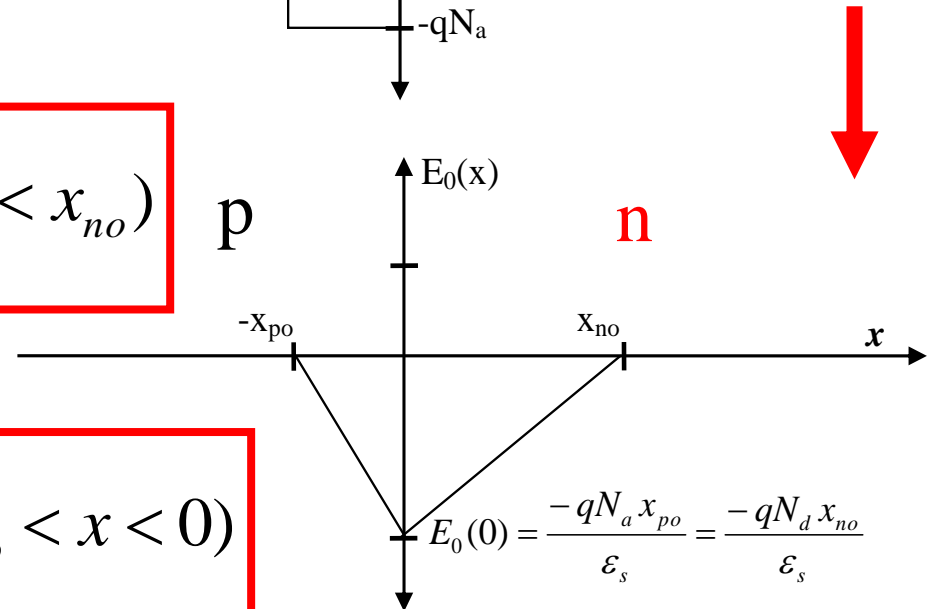
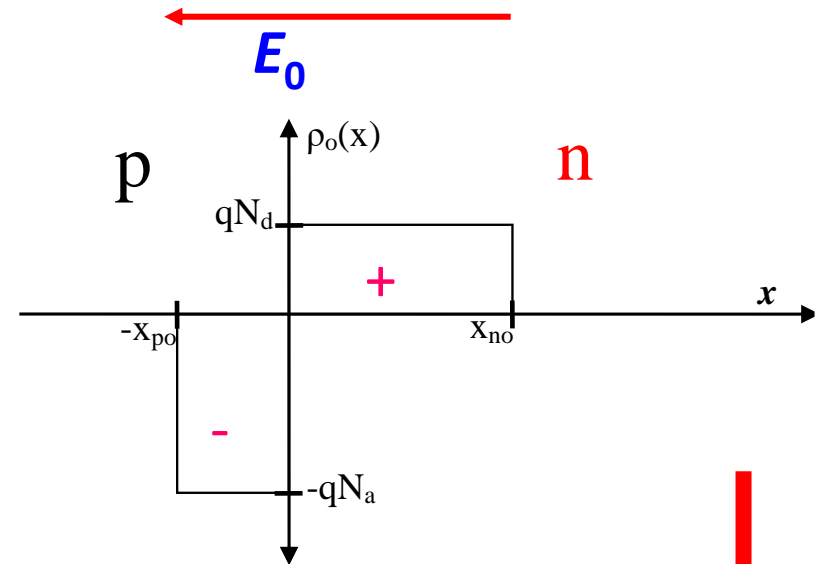
n Side:

$$E_0(x) - E_0(x_{no}) = \frac{1}{\epsilon_{Si}} \int_{x_{no}}^x qN_d dx$$

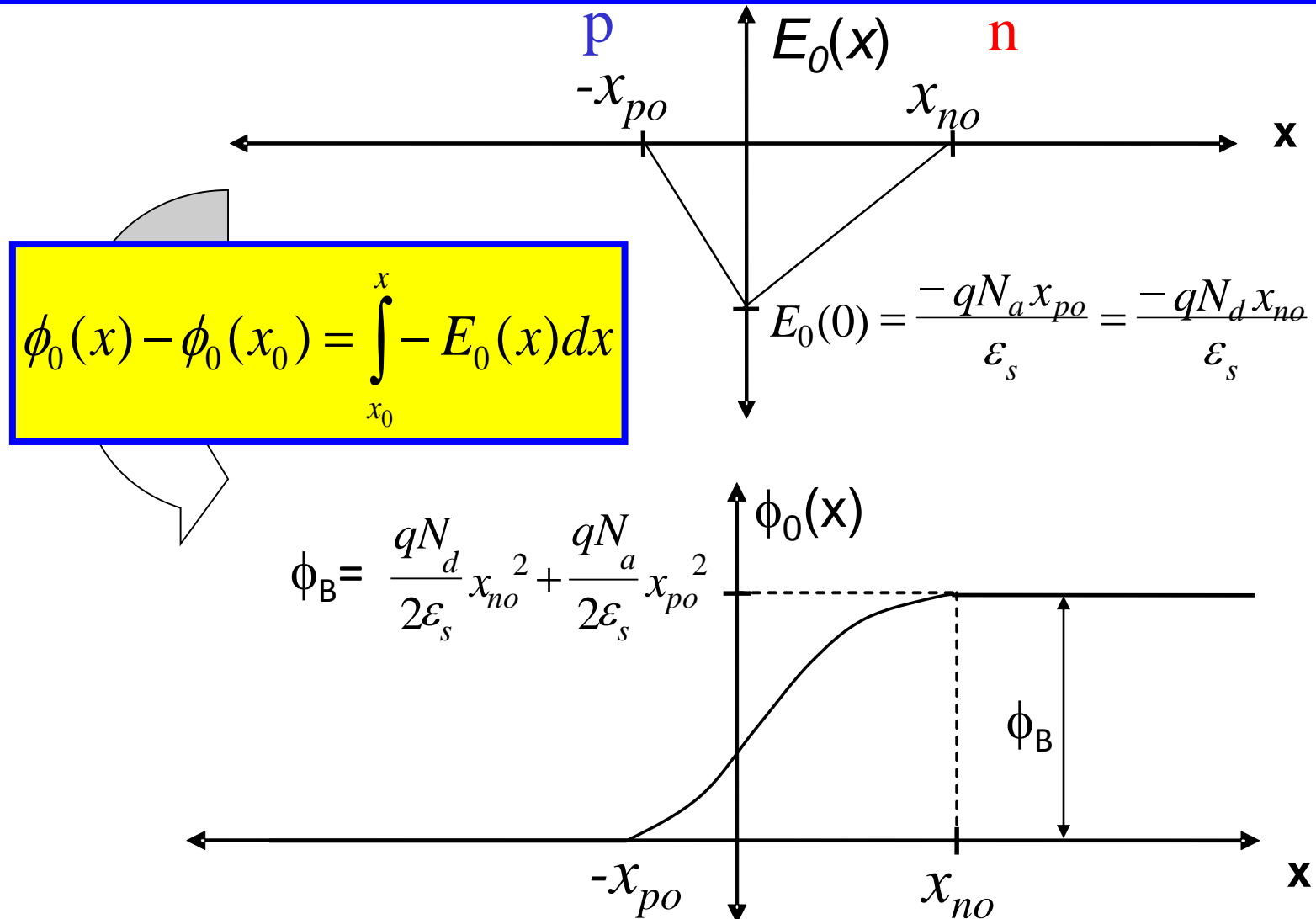
$$E_0(x) = \frac{qN_d}{\epsilon_{Si}} (x - x_{no}) \quad (0 < x < x_{no})$$

p Side:

$$E_0(x) = \frac{-qN_a}{\epsilon_s} (x + x_{po}) \quad (-x_{po} < x < 0)$$



Depletion Approximation 2 (Electrostatic potential)



Depletion Approximation 3

$$\phi_0(x) = \int_{-x_{po}}^x -E_0(x)dx + \phi_0(-x_{po}) = \int_{-x_{po}}^x \frac{qN_a}{\epsilon_s} (x + x_{po})dx + 0$$



$$\phi_0(x) = \frac{qN_a}{2\epsilon_s} (x + x_{po})^2 \quad (-x_{po} < x < 0)$$

$$\phi_0(x) = \int_0^x -E_0(x)dx + \phi_0(0) = \int_0^x -\frac{qN_d}{\epsilon_s} (x - x_{no})dx + \frac{qN_a}{2\epsilon_s} (0 + x_{po})^2$$



$$\phi_0(x) = \frac{qN_d}{2\epsilon_s} x(2x_{no} - x)^2 + \frac{qN_a}{2\epsilon_s} x_{po}^2 \quad (0 < x < x_{no})$$

Built-in Potential, ϕ_B

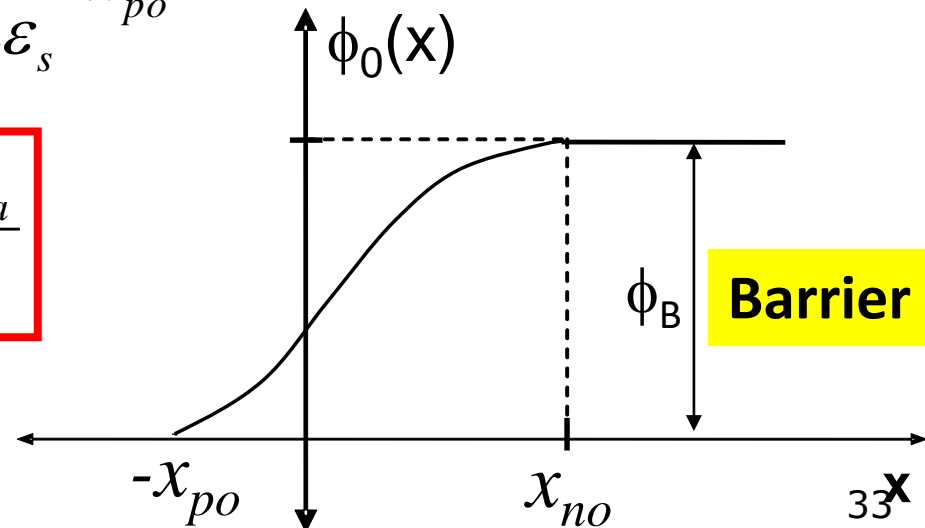
$$\phi_0(x) = \frac{qN_a}{2\epsilon_s} (x + x_{po})^2 \quad (-x_{po} < x < 0)$$

$$\phi_0(x) = \frac{qN_d}{2\epsilon_s} x(2x_{no} - x)^2 + \frac{qN_a}{2\epsilon_s} x_{po}^2 \quad (0 < x < x_{no})$$

At $x = x_{no}$

$$\phi_0 = \phi_B = \frac{qN_d}{2\epsilon_s} x_{no}^2 + \frac{qN_a}{2\epsilon_s} x_{po}^2$$

$$\phi_B = V_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}$$



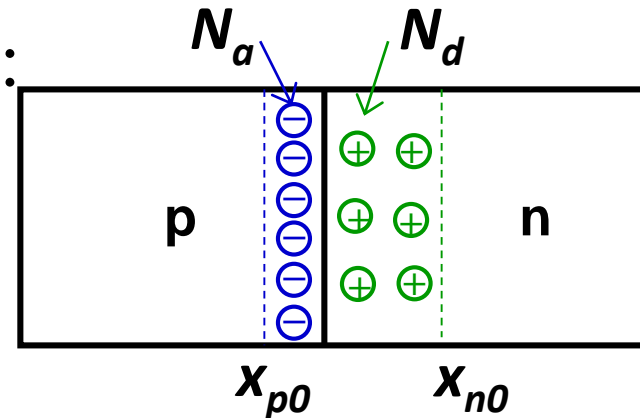
Still don't know x_{n0} and x_{p0}

1. Require overall **charge neutrality**:

$$qN_a x_{p0} = qN_d x_{n0}$$

2. Require $\phi(x)$ **continuous** at $x = 0$:

$$\phi_B = \frac{qN_d}{2\epsilon_s} x_{n0}^2 + \frac{qN_a}{2\epsilon_s} x_{p0}^2$$



Two equations with two unknowns. Solution:

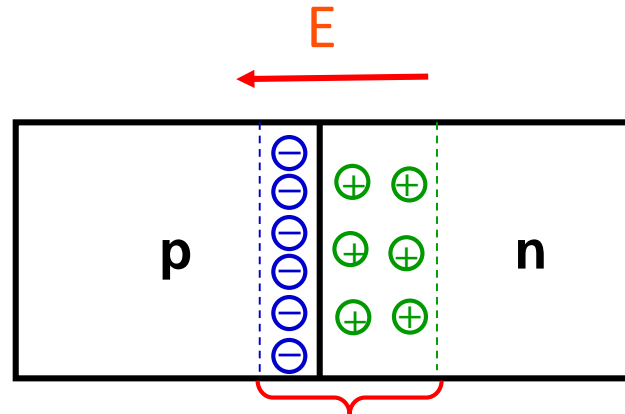
★

$$x_{n0} = \sqrt{\frac{2\epsilon_s \phi_B N_a}{q(N_a + N_d)N_d}}$$

★

$$x_{p0} = \sqrt{\frac{2\epsilon_s \phi_B N_d}{q(N_a + N_d)N_a}}$$

Depletion Region Width W_{dep}



$$\phi_B = \frac{kT}{q} \ln \frac{N_D N_A}{n_i^2}$$

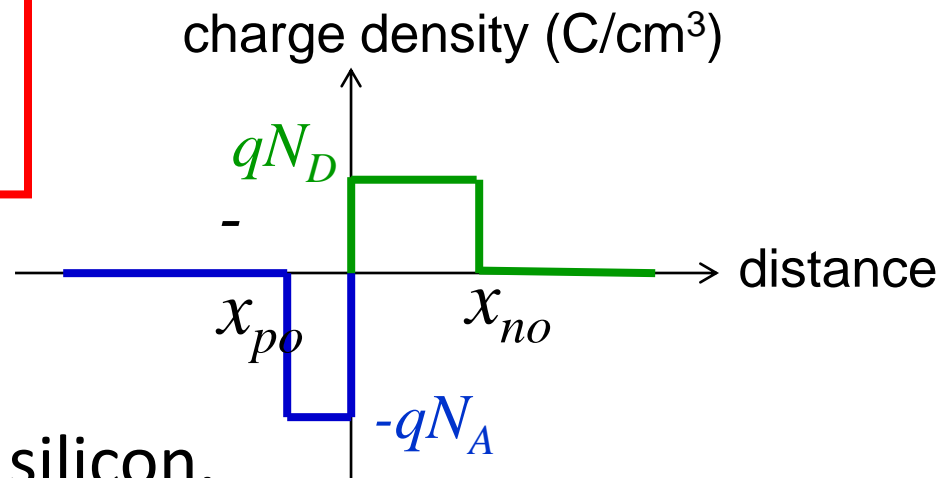
$$\epsilon_{Si} = \epsilon_{r,Si} \epsilon_0$$

depletion region width W_{dep}

$$W_{dep} = x_{po} + x_{no} = \sqrt{\frac{2\epsilon_{Si}}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) \phi_B}$$

$$\epsilon_{Si} \approx 10^{-12} \text{ F/cm}$$

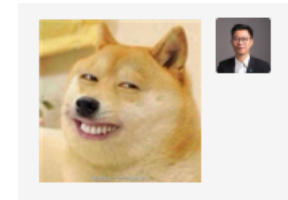
is the permittivity of silicon.



PN junction – (I)

OUTLINE

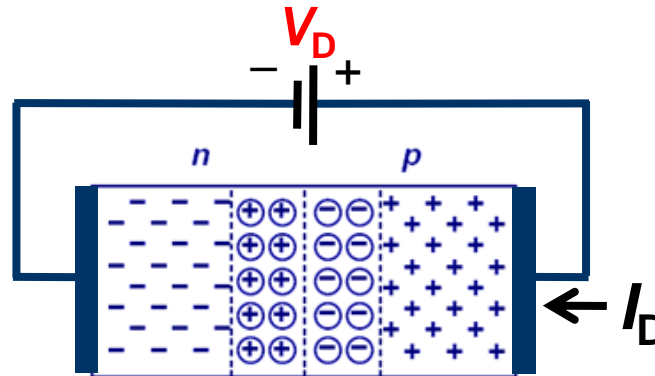
- The formation of depletion region
- Built-in potential
- Distribution of electric field and electric potential
- **Effect of Applied Voltage**
- Depletion capacitance



Effect of Applied Voltage

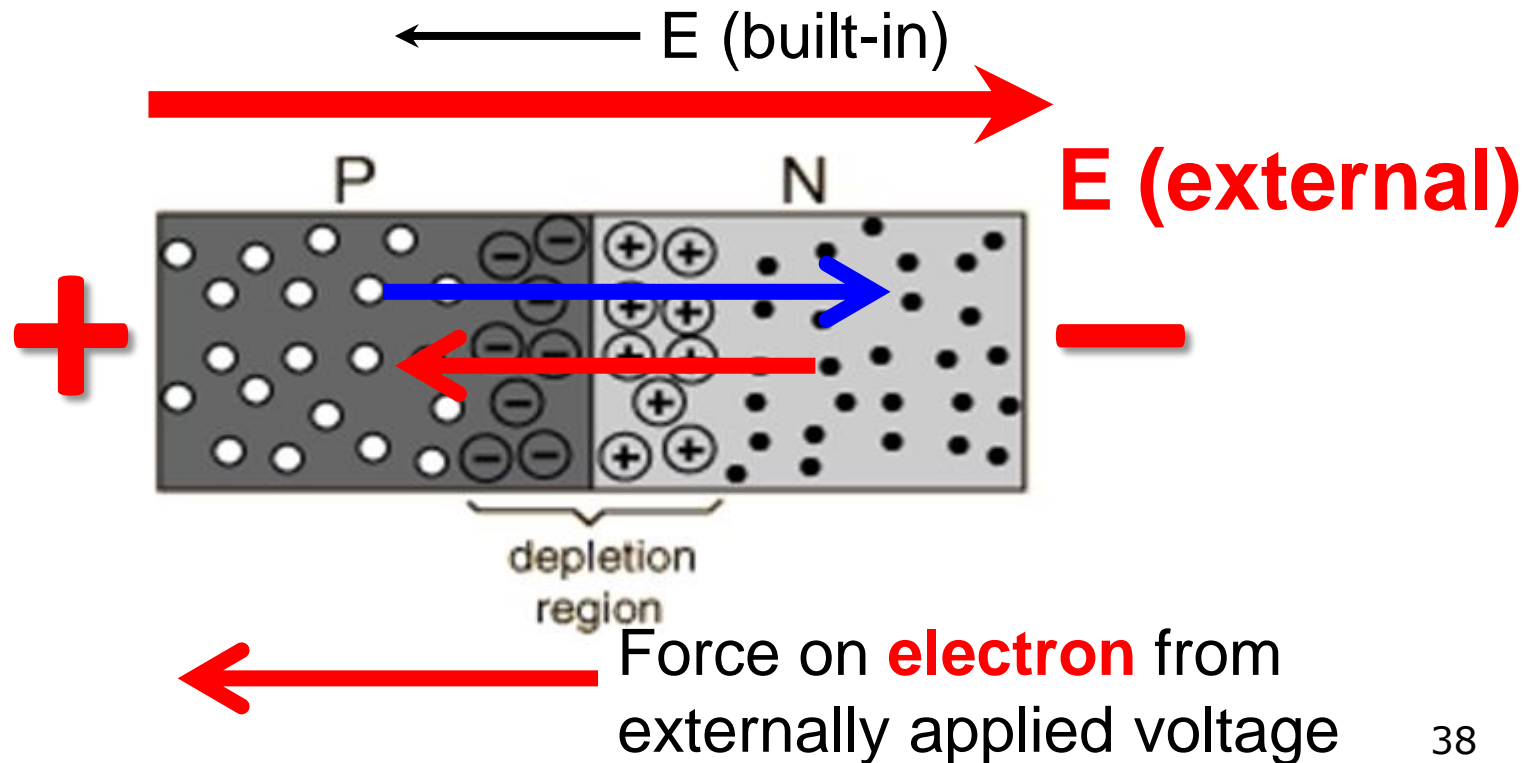
- The quasi-neutral N-type and P-type regions have low resistivity, whereas the depletion region has high resistivity.
 - Thus, when an **external voltage V_D** is applied across the diode, almost all of this voltage **is dropped across the depletion region**. (Think of a voltage divider circuit.)
- If $V_D < 0$ (**reverse bias, or V_R**), the **potential barrier** to carrier diffusion is increased by the applied voltage.
- If $V_D > 0$ (**forward bias, or V_F**), the **potential barrier** to carrier diffusion is reduced by the applied voltage.

$$V_D = \begin{cases} V_R & (V_D < 0) \\ V_F & (V_D > 0) \end{cases}$$



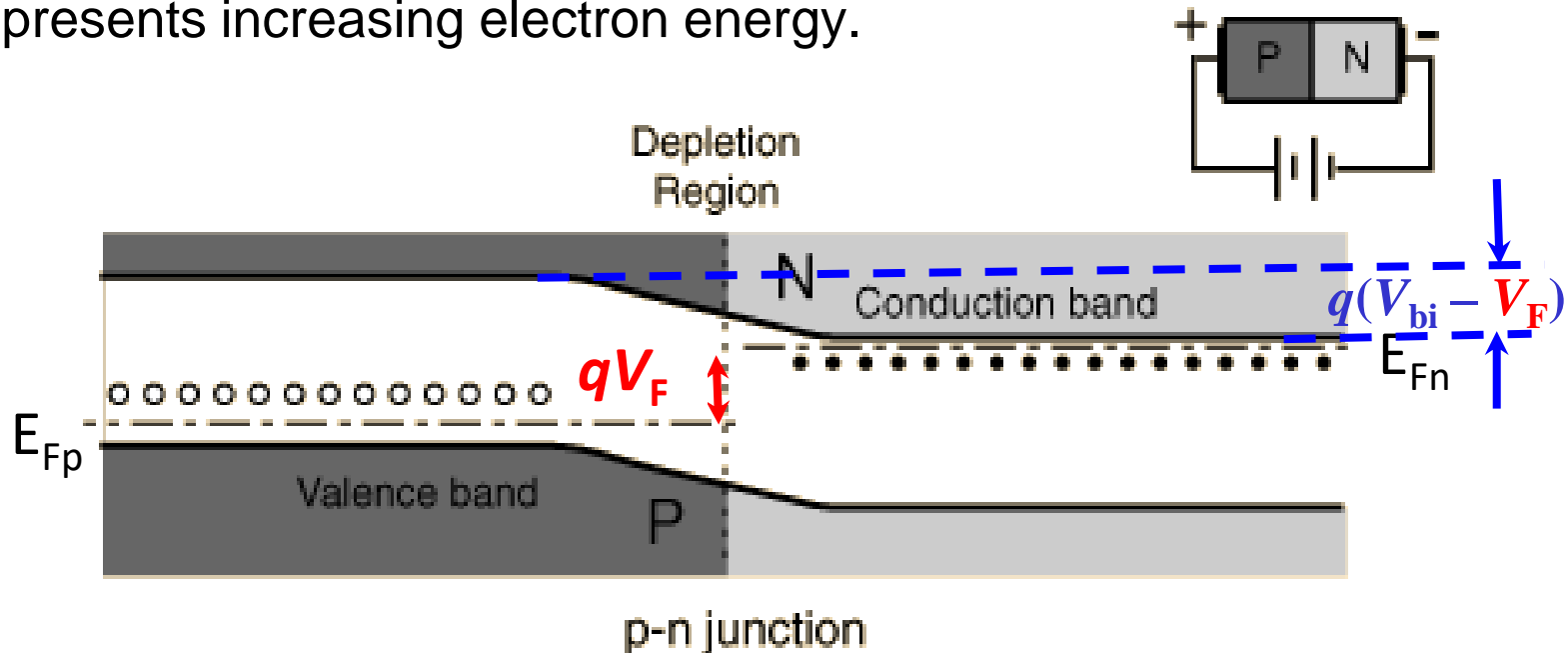
+Bias effect on electrons in depletion zone

- Forward bias
- An applied voltage in the **forward direction** as indicated assists **electrons** in **overcoming** the **coulomb barrier** of the space charge in depletion region. Electrons will flow with very small resistance in the forward direction.



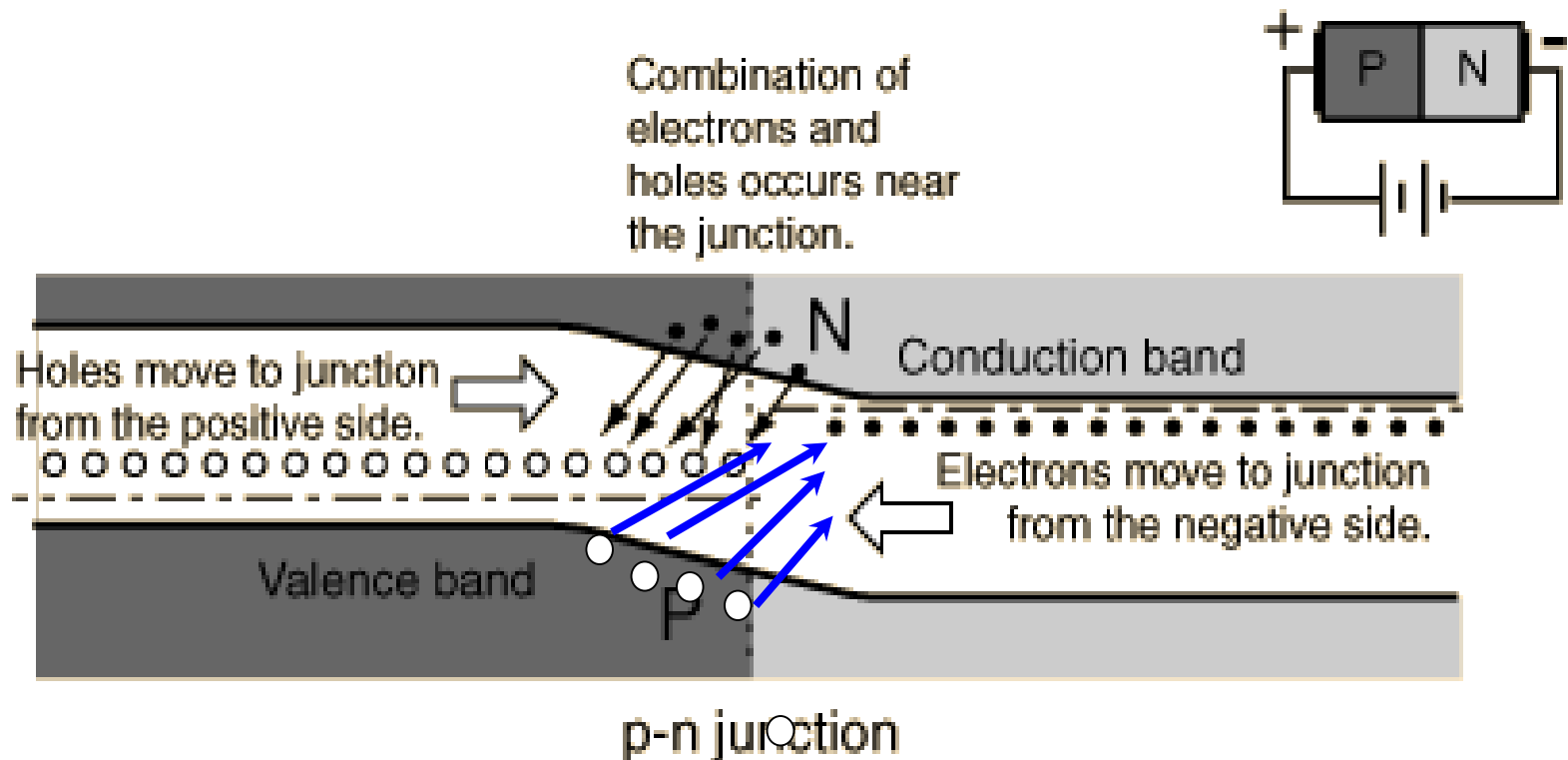
+Bias effect on electrons in depletion zone

To forward bias the p-n junction, the p side is made more positive, so that it is "downhill" for electron motion across the junction. An electron can move across the junction and fill a vacancy or "hole" near the junction. It can then move from vacancy to vacancy leftward toward the positive terminal, which could be described as the hole moving right. The conduction direction for electrons in the diagram is right to left, and the upward direction represents increasing electron energy.



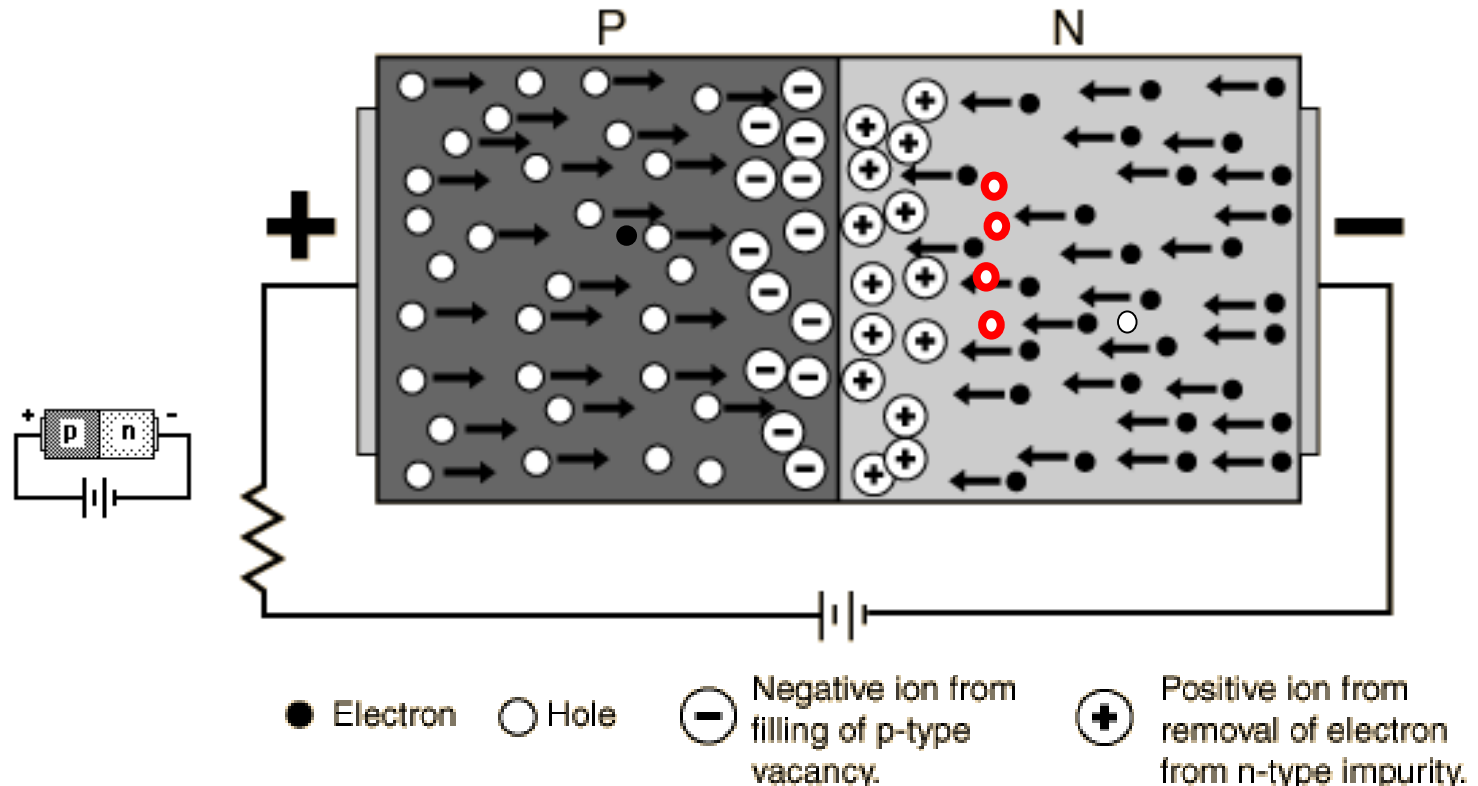
Forward Biased Conduction

When the p-n junction is forward biased, the **electrons** in the n-type material which have been elevated to the conduction band and which have diffused across the junction find themselves at a higher energy than the holes in the p-type material. They readily **combine with those holes**, making possible a **continuous forward** current through the junction.



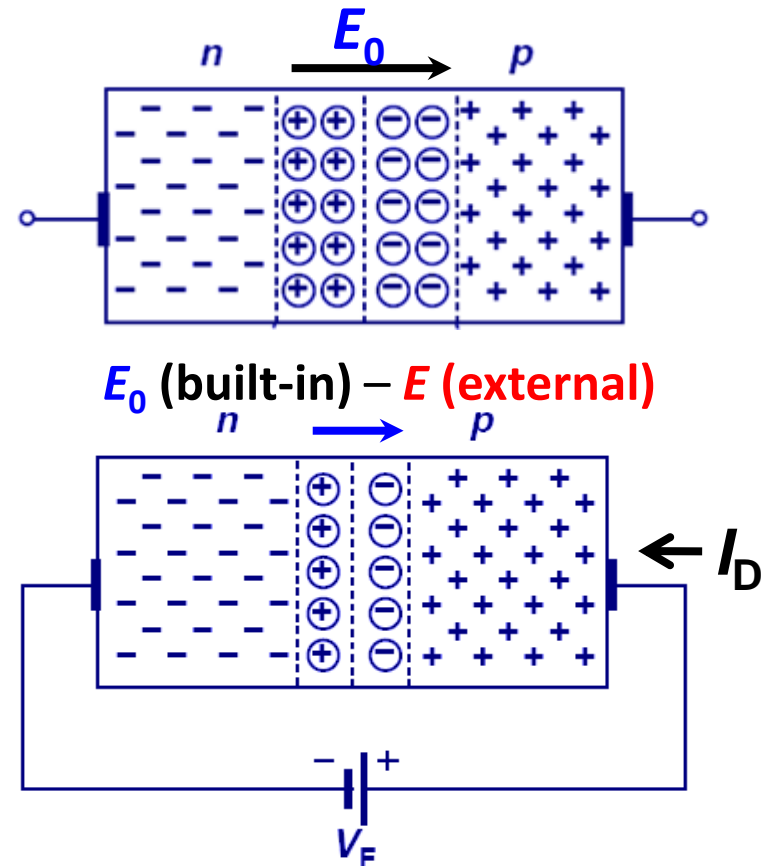
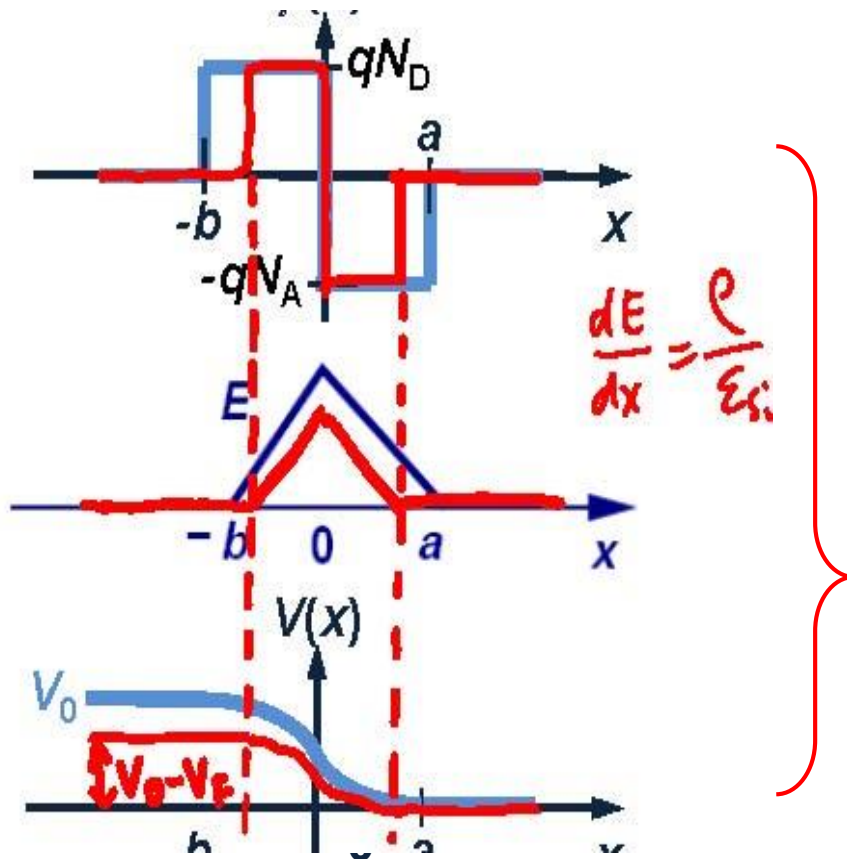
Forward Biased Conduction

The forward current in a p-n junction when it is forward-biased (illustrated below) involves **electrons** from the n-type material moving leftward **across the junction** and **combining with holes** in the p-type material. Electrons can then **proceed further leftward** by jumping from hole to hole, so the holes can be said to be moving to the right in this process.

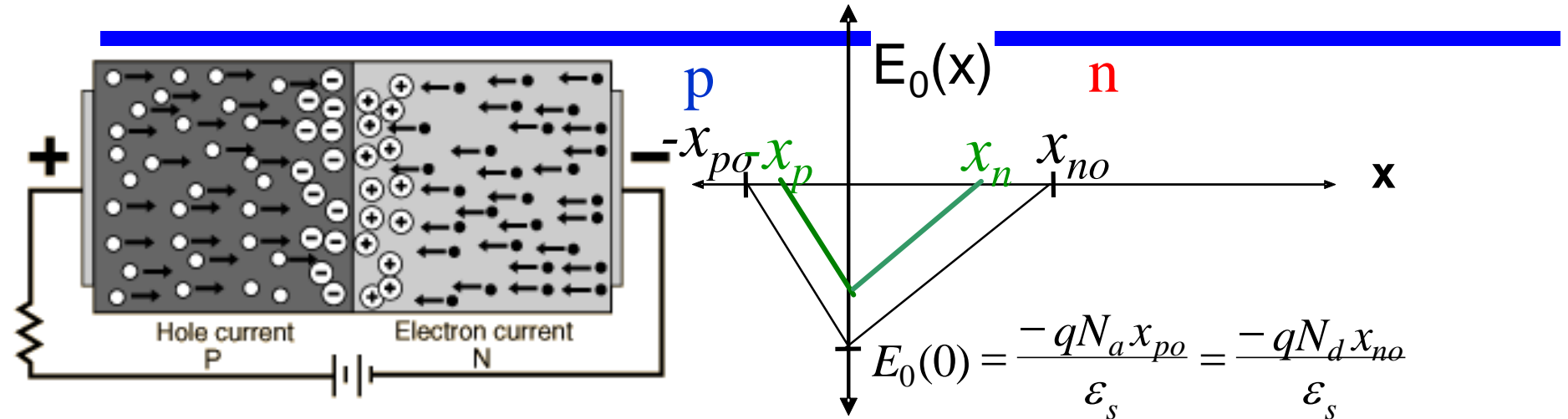


PN Junction under Forward Bias

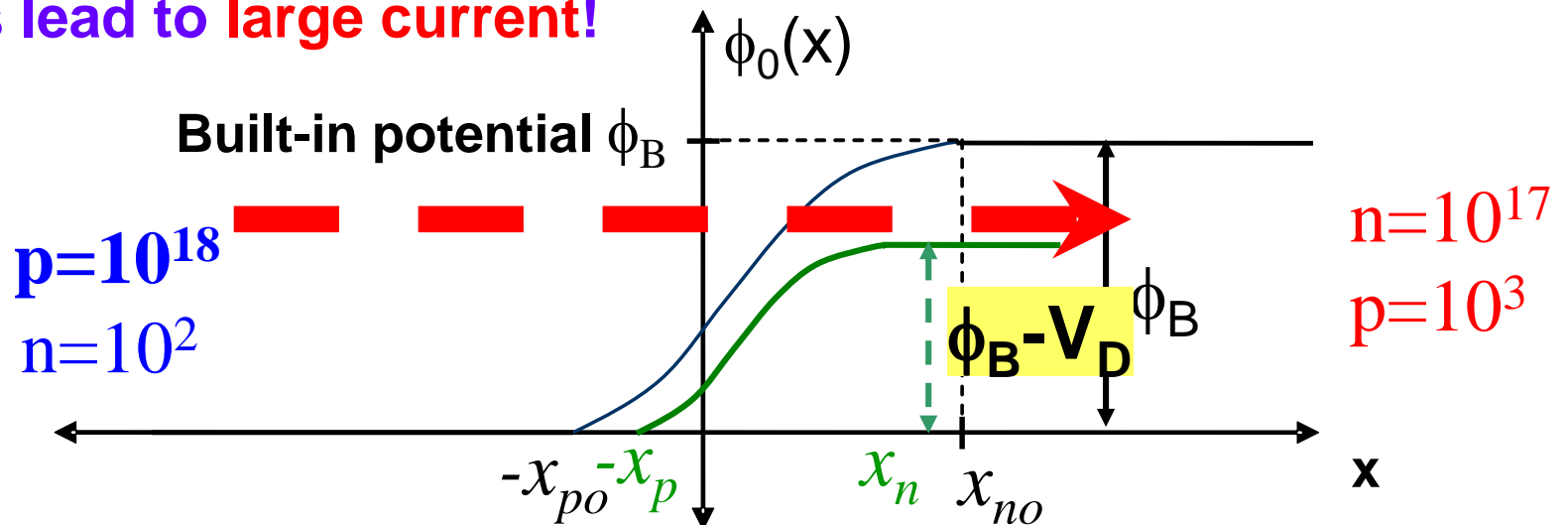
- A **forward** bias **decreases** the **potential drop** across the junction. As a result, the magnitude of the **electric field** **decreases**, and the **width** of the depletion region **narrows**.



Depletion Approx. – with $V_D > 0$ forward bias



Lower barrier and large hole (electron) density at the right places lead to large current!



Depletion Region Width W_{dep}

At $V_D=0$

$$W_{dep} = x_{po} + x_{no} = \sqrt{\frac{2\epsilon_{Si}}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) \phi_B}$$

At $V_D>0$

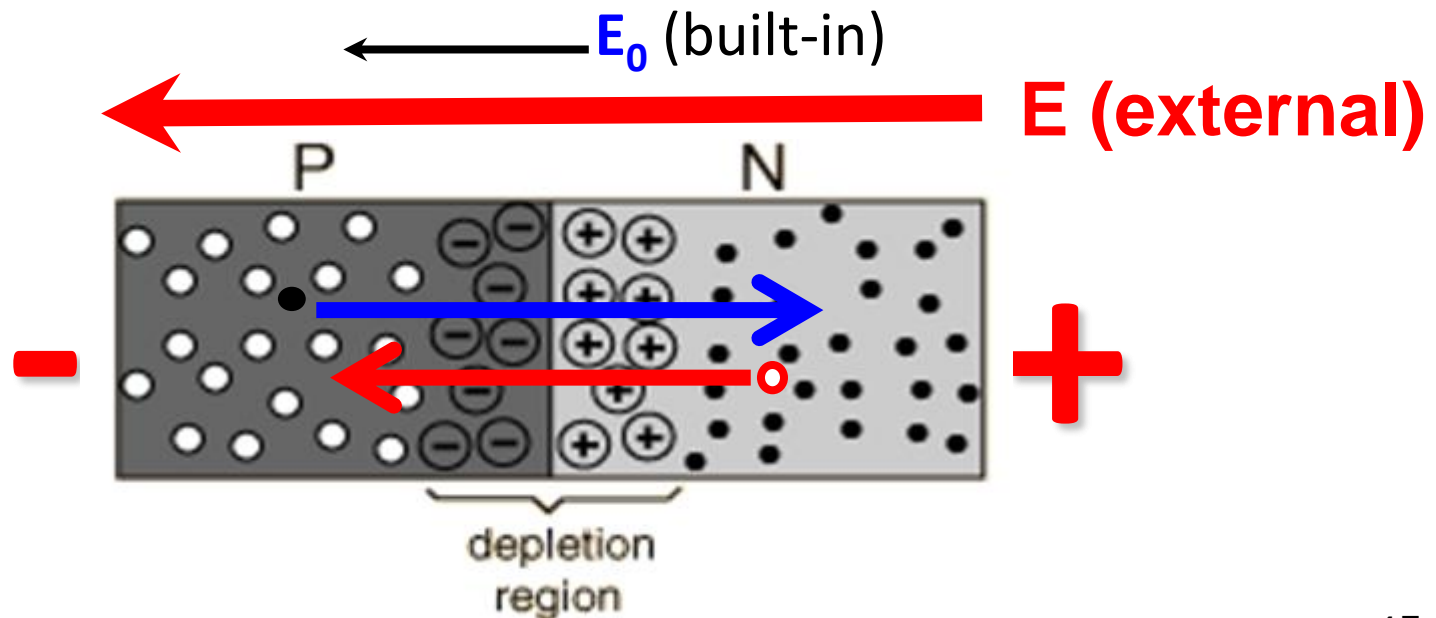
$$W_{dep} = x_p + x_n = \sqrt{\frac{2\epsilon_{Si}}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) (\phi_B - V_D)}$$

- The **width of the depletion region** is a function of the **bias voltage** and is dependent on N_A and N_D .

-Bias effect on electrons in depletion zone

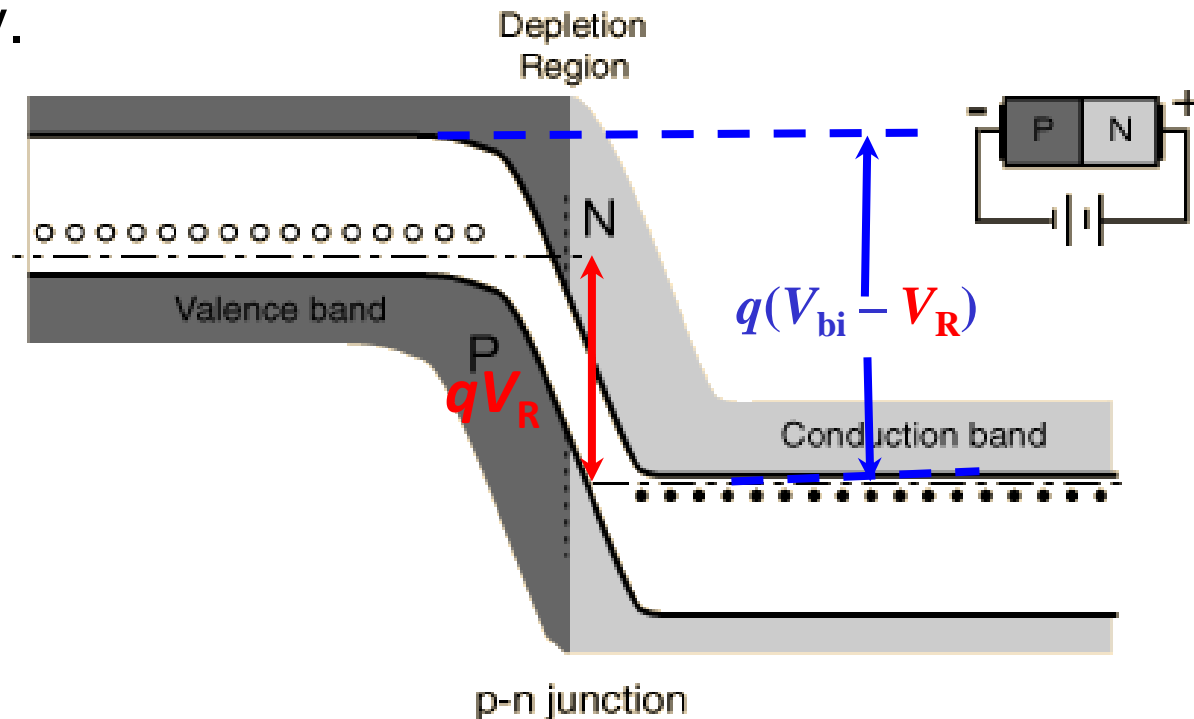
Reverse bias

An applied voltage with the indicated polarity further impedes the flow of electrons across the junction. For conduction in the device, electrons from the N region must move to the junction and combine with holes in the P region. A **reverse voltage** drives the **electrons** away from the junction, preventing conduction.



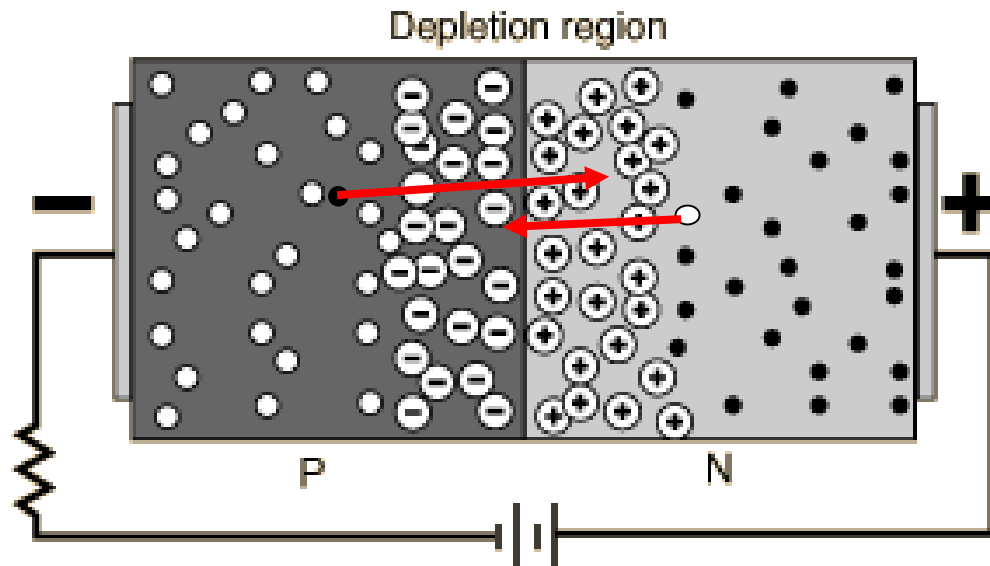
Bias effect on electrons in depletion zone

- To reverse-bias the p-n junction, the p side is made more negative, making it "uphill" for electrons moving across the junction. The conduction direction for electrons in the diagram is right to left, and the upward direction represents increasing electron energy.



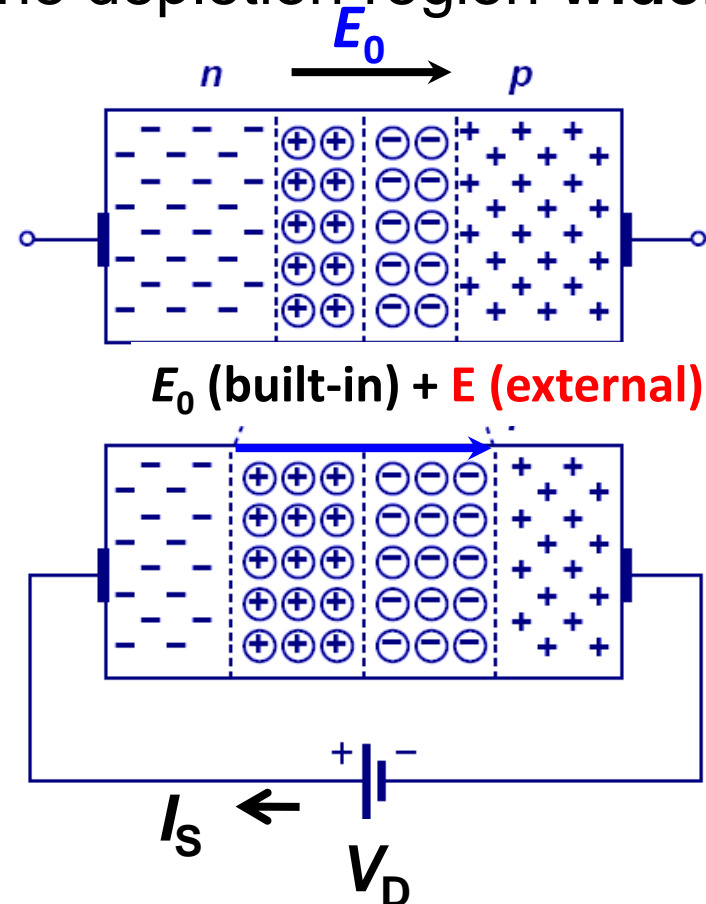
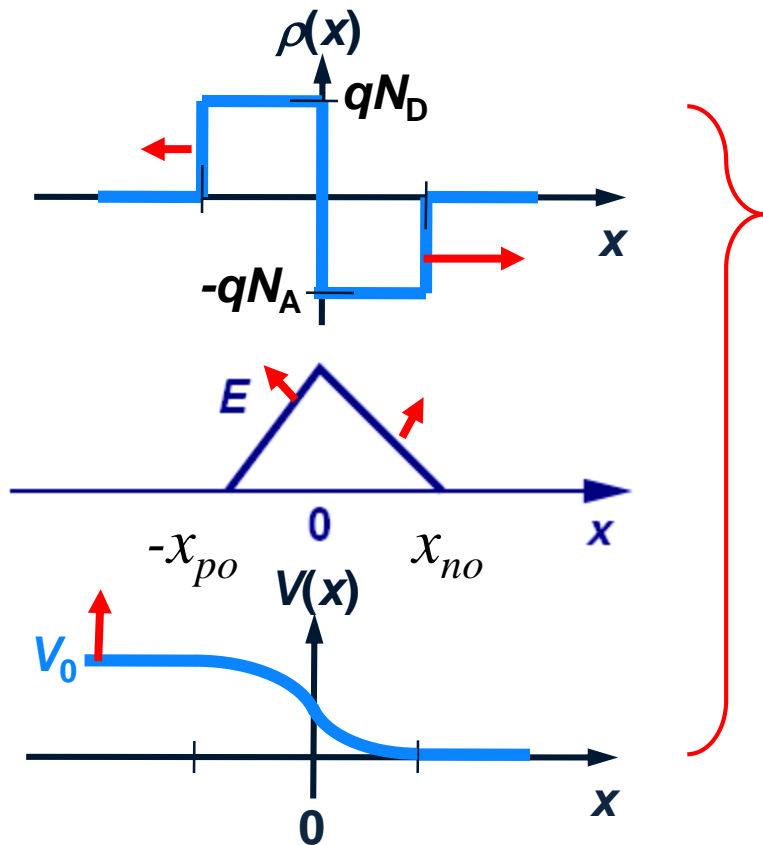
Reverse Biased P-N Junction

The application of a reverse voltage to the p-n junction will cause a transient current to flow as both electrons and holes are pulled away from the junction. When the potential formed by the widened depletion layer equals the applied voltage, the **current** will **cease** except for the small thermal current.

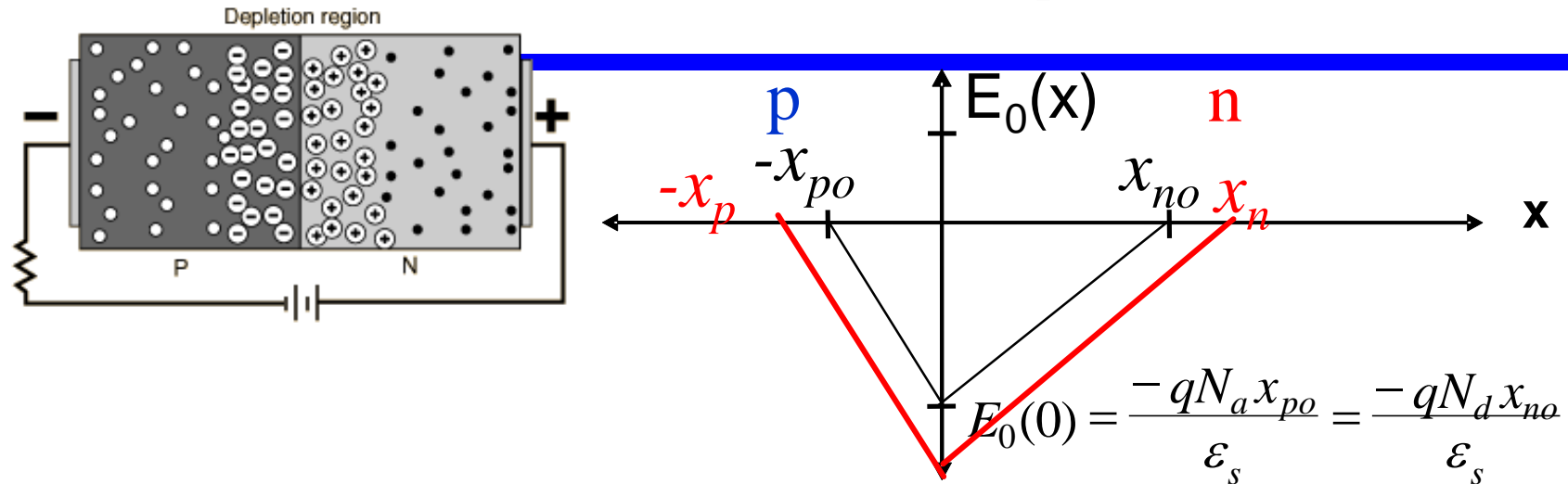


PN Junction under Reverse Bias

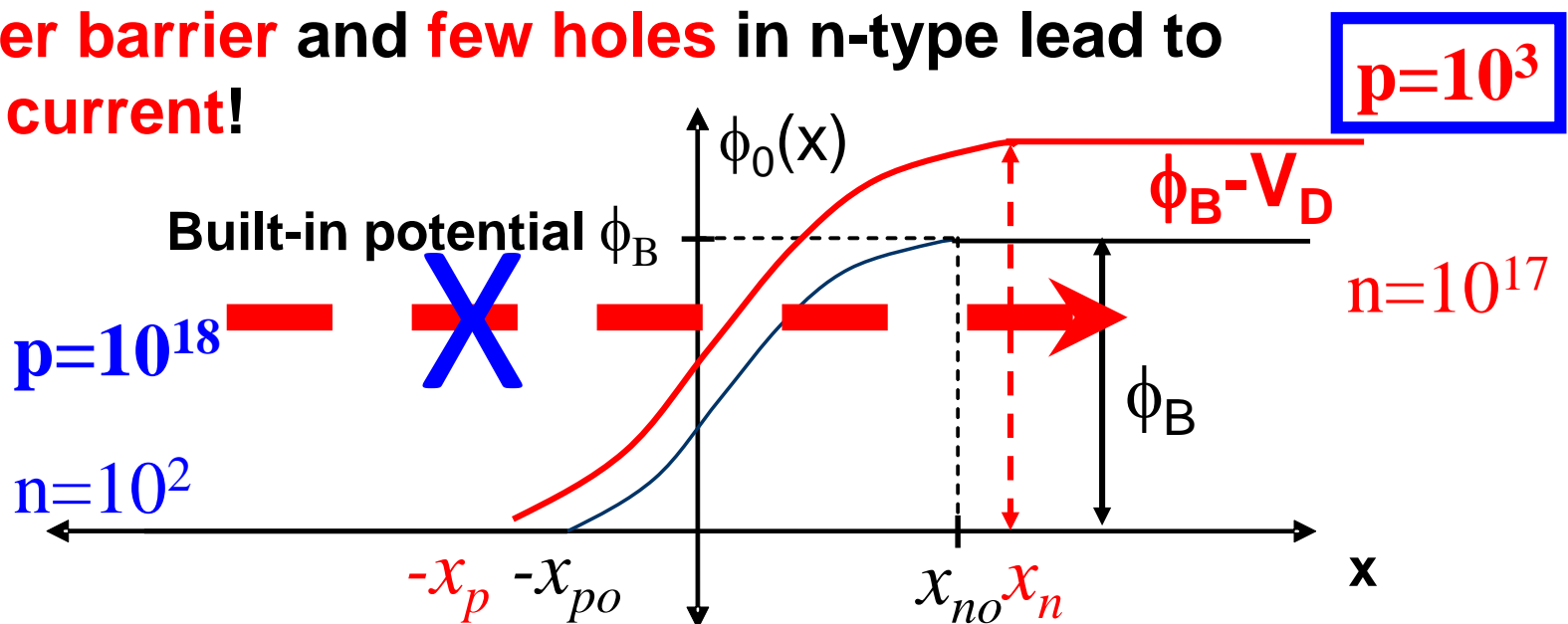
- A **reverse bias** increases the **potential drop** across the junction. As a result, the magnitude of the **electric field** **increases**, and the **width** of the depletion region **widens**.



Depletion Approx. – with $V_D < 0$ reverse bias



Higher barrier and few holes in n-type lead to little current!



Depletion Region Width W_{dep}

At $V_D=0$

$$W_{dep} = x_{po} + x_{no} = \sqrt{\frac{2\epsilon_{Si}}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) \phi_B}$$

At $V_D < 0$

$$W_{dep} = x_p + x_n = \sqrt{\frac{2\epsilon_{Si}}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) (\phi_B - V_D)}$$

- The **width of the depletion region** is a function of the **bias voltage** and is dependent on N_A and N_D .
- If one side is much more heavily doped than the other (which is commonly the case), this can be simplified then:

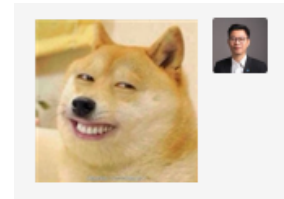
$$W_{dep} \cong \sqrt{\frac{2\epsilon_{Si}}{qN} (\phi_B - V_D)}$$

where **N** is the doping concentration on the **more lightly doped side**.

PN junction – (I)

OUTLINE

- The formation of depletion region
- Build-in potential
- Distribution of electric field and electric potential
- Effect of Applied Voltage
- **Depletion capacitance**



parallel-plate capacitor:

- Capacitance per unit area:

Apply *small signal* on top of bias:

$$C = \epsilon_s / t_{\text{ins}}$$

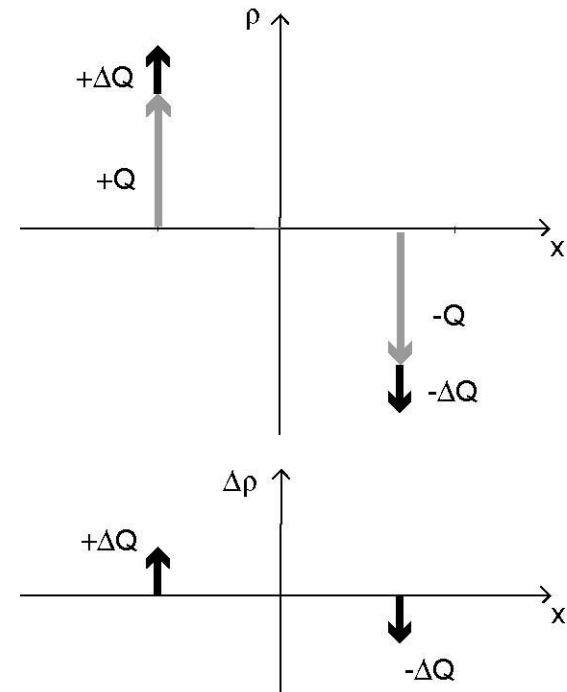
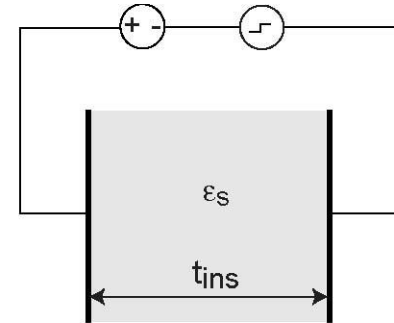
$$\epsilon_s = \epsilon_{r,s} \epsilon_0$$

$$C = Q/V$$

$\epsilon_{r,s}$ is the **relative dielectric constant** of insulators.

ϵ_0 is the permittivity of free space.

V ΔV



Depletion capacitance

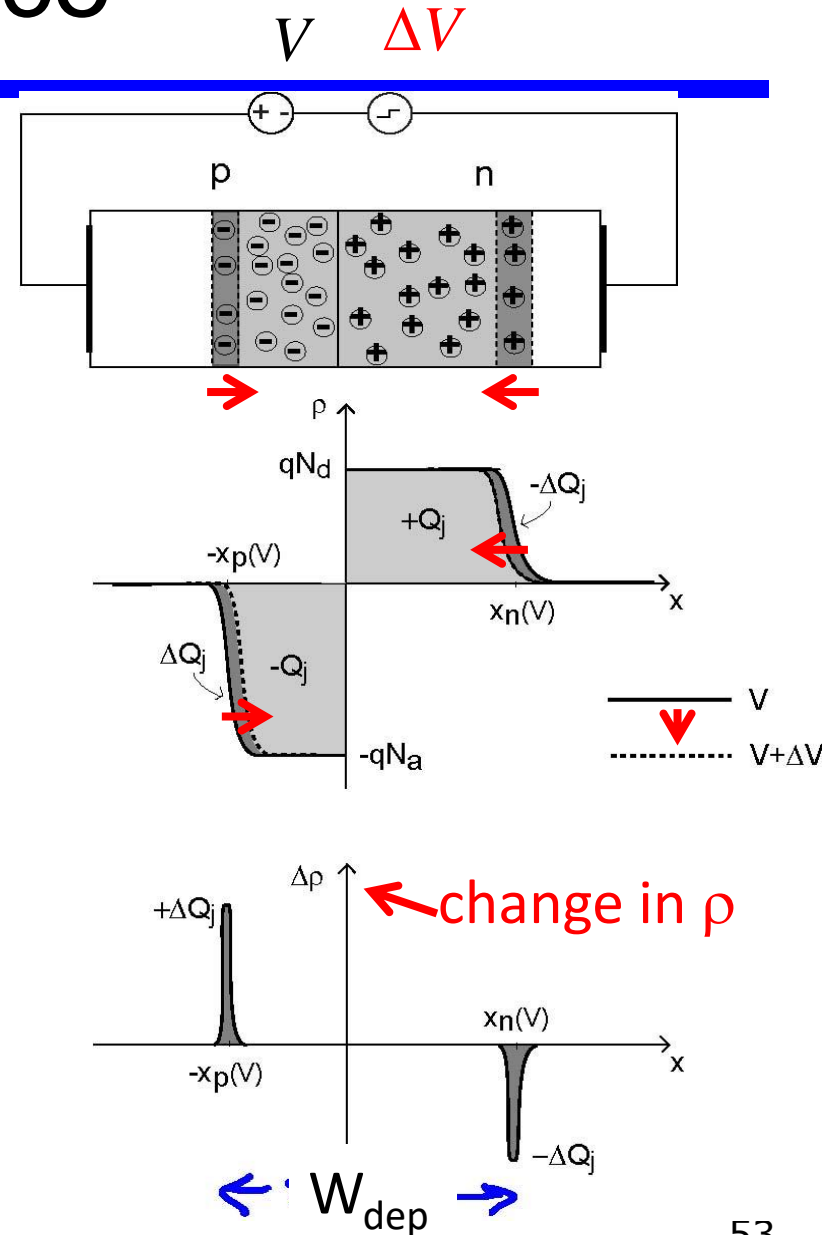
Apply *small signal* on top of bias:

Change in ΔV across diode causes:

change of ΔQ_j at $-x_p$
change of $-\Delta Q_j$ at x_n

$$V \gg |\Delta V|$$

$$W_{\text{dep}} \gg \Delta W_{\text{dep}}$$



Depletion capacitance per unit area (depletion approx.)

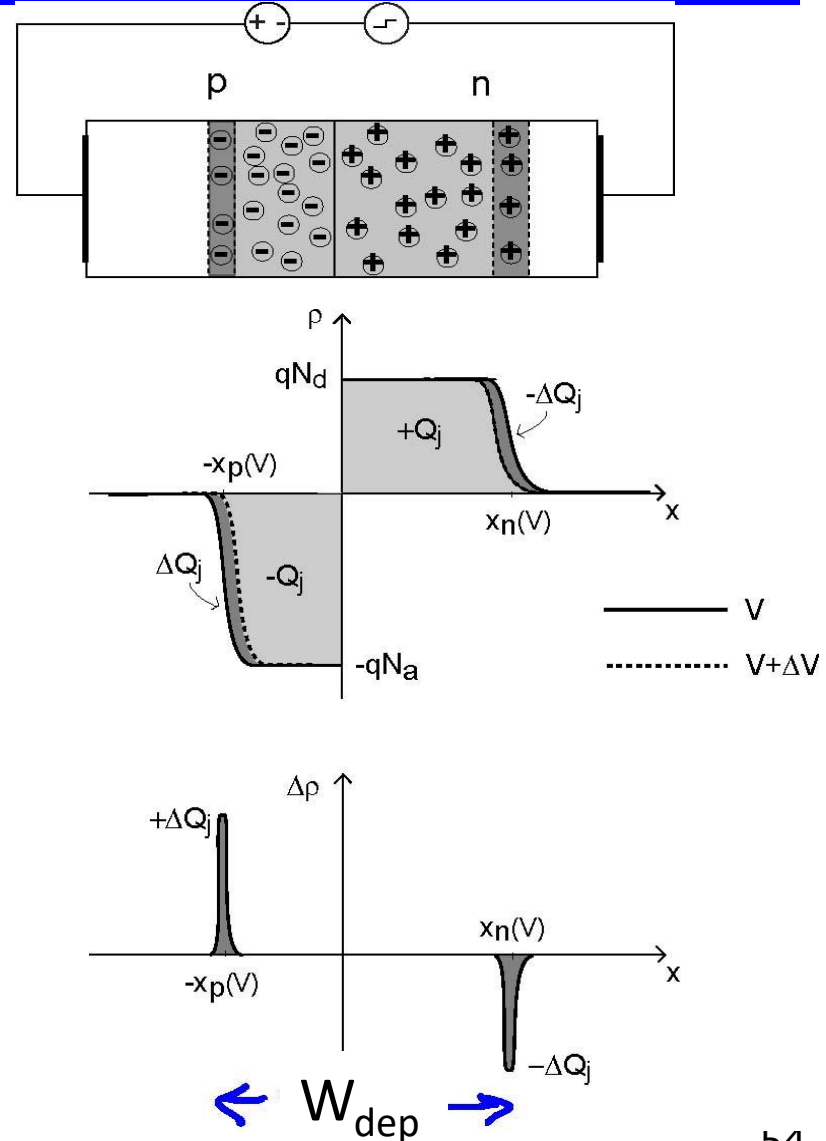
- In analogy, in pn junction:

$$C_j(V) = \frac{\epsilon_s}{W_{dep}(V)}$$

$$C_j(V) = \frac{\epsilon_s}{W_{dep}(V)} =$$

$$\sqrt{\frac{q\epsilon_s N_a N_d}{2(\phi_B - V)(N_a + N_d)}} = \frac{C_{j0}}{\sqrt{1 - V / \phi_B}}$$

$$C_{j0} = \sqrt{\frac{\epsilon_{si} q}{2} \frac{N_a N_d}{N_a + N_d} \frac{1}{\phi_B}}$$



Alternative view of capacitance: depletion charge

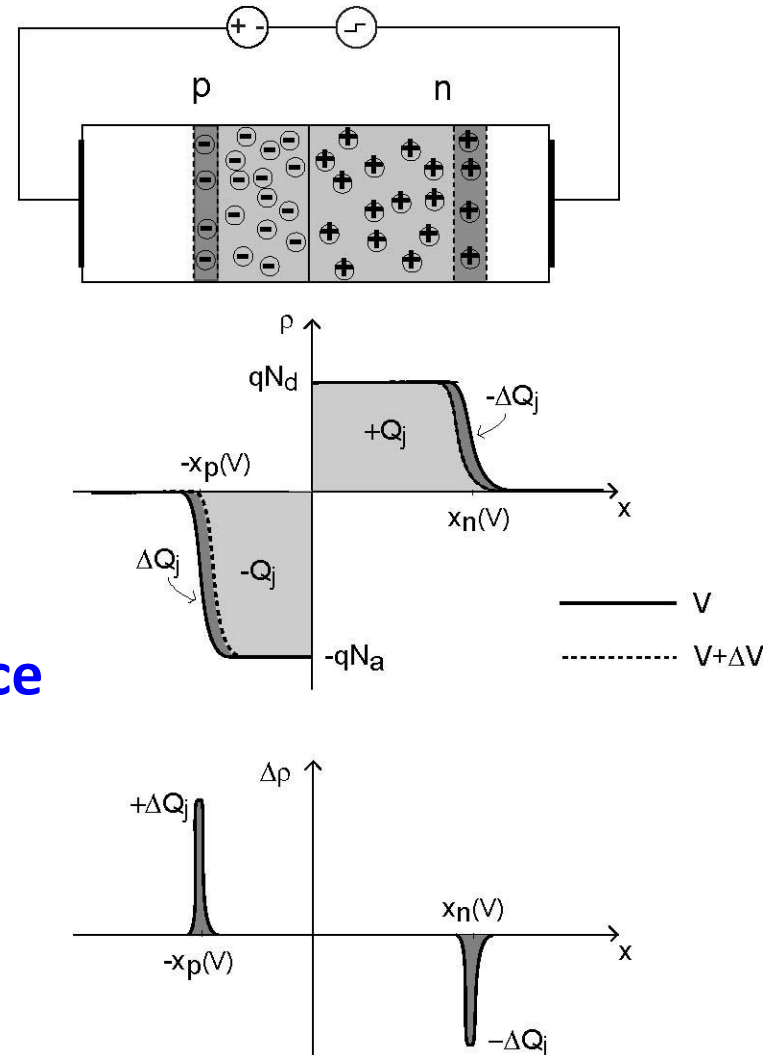
- Within depletion approximation:
- C_j is slope of Q_j vs. V characteristics:

$$C_j(V) = \sqrt{\frac{q\epsilon_s N_a N_d}{2(N_a + N_d)(\phi_B - V)}} = C_{jo} / \sqrt{1 - \frac{V}{\phi_B}}$$

$$C_j = \frac{dQ_j}{dV}$$


Differential capacitance

~~$$C_j = \frac{Q_j}{V}$$~~



Summary-1

- A depletion region (in which n and p are each much smaller than the net dopant concentration) is formed at the junction between p- and n-type regions
 - A **built-in potential barrier (voltage drop)** exists across the depletion region, opposing carrier diffusion (due to a concentration gradient) across the junction:


$$\phi_0 = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$


- At equilibrium ($V_D=0$), no net current flows across the junction

- **Width of depletion region**

$$W_j \cong \sqrt{\frac{2\epsilon_{Si}}{qN} (\phi_0 - V_D)}$$

- decreases with increasing forward bias (p-type region biased at higher potential than n-type region)
- increases with increasing reverse bias (n-type region biased at higher potential than p-type region)

- Charge stored in depletion region → **capacitance**


$$C_j = \frac{A_D \epsilon_{Si}}{W_j}$$

Summary-2

Current flowing in a semiconductor is comprised of drift and diffusion components: $J_{tot} = qp\mu_p E + qn\mu_n E + qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$

A region depleted of mobile charge exists at the junction between P-type and N-type materials.

- A built-in potential drop (V_0) across this region is established by the charge density profile; it opposes diffusion of carriers across the junction. A reverse bias voltage serves to enhance the potential drop across the depletion region, resulting in very little (drift) current flowing across the junction.
- The **width of the depletion region (W_{dep})** is a function of the bias voltage (V_D).



$$W_{dep} = \sqrt{\frac{2\epsilon_{si}}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 - V_D)}$$

$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

Electron and hole concentrations

$$n = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$p = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$n \cdot p = n_i^2$$

+

$$n_i = N_C \exp\left[\frac{-(E_C - E_i)}{kT}\right]$$

$$n_i = N_V \exp\left[\frac{-(E_i - E_V)}{kT}\right]$$

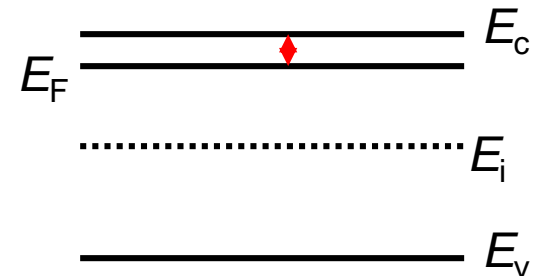


$$n = n_i \exp\left[\frac{(E_F - E_i)}{kT}\right]$$

$$p = n_i \exp\left[\frac{-(E_F - E_i)}{kT}\right]$$

At RT

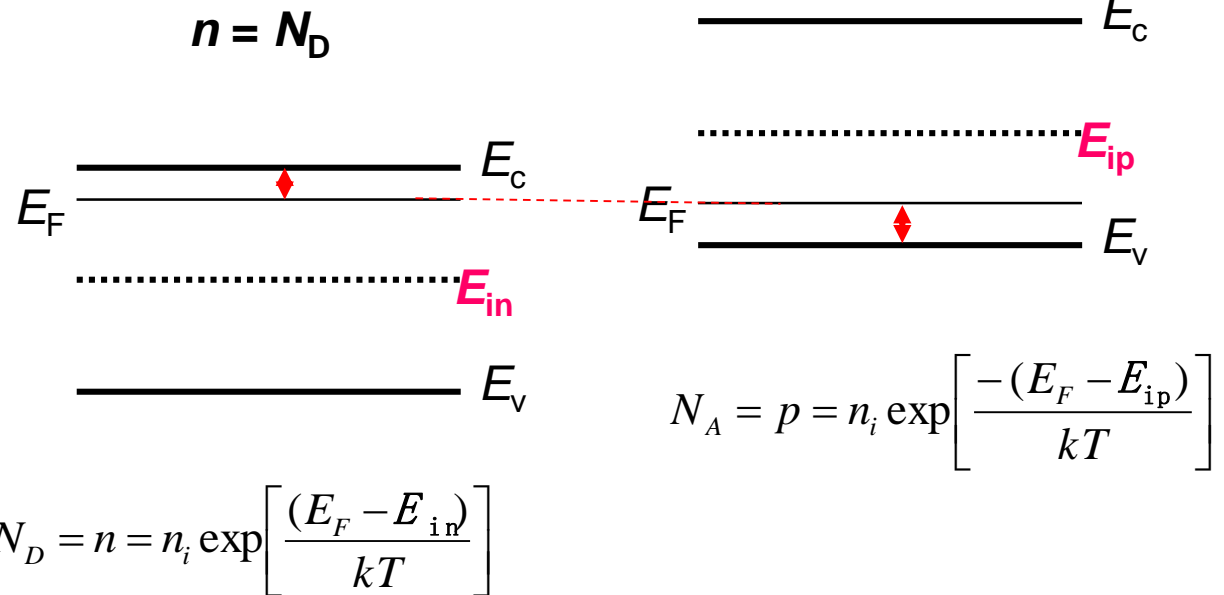
$$n = N_D$$



HW7

Electron and hole concentrations

$$n = n_i \exp\left[\frac{(E_F - E_i)}{kT}\right], \quad p = n_i \exp\left[\frac{-(E_F - E_i)}{kT}\right]$$



$$qV_0 = E_{ip} - E_{in}$$

$$\Rightarrow V_0 = \frac{kT}{q} \ln \frac{N_D N_A}{n_i^2}$$