

# **EEE112**

## **Integrated Electronics & Design: Exercise Problem**

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**A. Semiconductor fundamentals**

**B. PN junction**

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### **Exercise problems**

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# Semiconductor Fundamentals

## Atomic Density

1. What is the volume density of atoms for a face-centered cubic structure with lattice constant  $a_0 = 5 \text{ \AA}$ ?
2. The lattice constant of a body-centered cubic structure is  $a_0 = 4.75 \text{ \AA}$ . What is the volume density of atoms?

## Crystal Lattice Plane

1. What are the Miller Indices of the plane shown in figure 1?
2. A plane is described as a (123) plane. What are the intercepts on a Cartesian axes?
3. Describe the direction in Figure 2 and the plane perpendicular to it
4. Describe the lattice direction shown in Figure 3 and the corresponding perpendicular plane

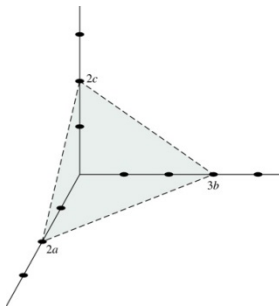


Figure 1

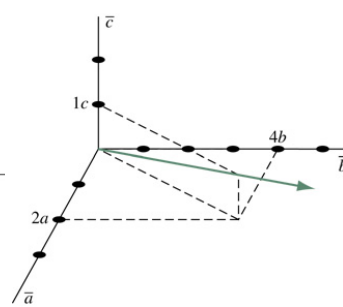


Figure 2

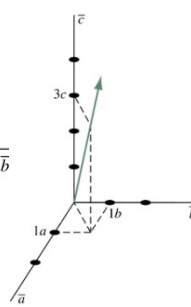


Figure 3

## Energy Levels

1. Determine the first 3 allowed electron energies in the hydrogen atom

$$E_n = -m_0 e^4 / (4\pi\epsilon_0)^2 2((h/2\pi)^2) n^2 \text{ where}$$

$m_0$  = mass of an electron =  $9.11 \times 10^{-31} \text{ kg}$

$e$  = electron charge =  $1.6 \times 10^{-19} \text{ C}$

$h$  = Planck Constant =  $6.625 \times 10^{-34} \text{ J-s}$

$\epsilon_0$  = Permittivity of free space =  $8.85 \times 10^{-12} \text{ F/m}$

$n$  is the energy level

2. For a one electron atom, determine the value of  $n$  such that  $E_{n+1} - E_n < 0.2 \text{ eV}$

## Energy Band

1. Calculate the change in Kinetic Energy of an electron when the velocity increases from  $10^7 \text{ cm/s}$  by  $1 \text{ cm/s}$

$$\text{Kinetic energy} = (1/2)mv^2$$

2. An electron travelling at  $v_0 = 2 \times 10^7$  cm/s increases in speed so that the change in Kinetic Energy is  $10^{-8}$  eV. Determine the increase in electron speed

### Density of States and Probability

1. Find the density of states for an electron per unit volume between 0 – 1 eV.

$$N = \int_{E_1}^{E_2} g(E) dE = \int_{E_1}^{E_2} \frac{4\pi(2m_p)^{3/2}}{h^3} \sqrt{E_v - E} dE$$

2. Find the density of states for an electron per unit volume between 1 eV – 2 eV.

3. Determine the probability that an electron occupies a state  $3kT$  above  $E_F$  at  $T=300$  K

4. If  $E_F = 0.3$  eV, determine the probability of a state being occupied by an electron at i)  $E = E_c + kT$  and ii)  $E = E_c + 2kT$  at  $T=300$  K

$$f_F(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}}$$

5. The Fermi Energy for a material is 6.25 eV and electrons follow the Fermi-Dirac distribution, Calculate the temperature for a 1% probability that a state 0.3 eV below the Fermi energy is empty.

6. The Fermi Energy for a material is 5.5 eV and electrons follow the Fermi-Dirac distribution, Calculate the temperature for a 0.5 % probability that a state 0.2 eV above the Fermi energy has an electron.

7. Calculate the energy in terms of  $kT$  and  $E_F$  at which the Boltzman approximation and the Fermi-Dirac function is 5% of the Fermi function

8. Repeat question 5 for a 1% variation

9. Calculate the probability that an energy state in the conduction band at  $E = E_c + kT$  is occupied by an electron and calculate the thermal equilibrium electron concentration in silicon at  $T=300$  K.  $E_F = E_c - 0.2$  eV, for silicon at  $T=300$  K  $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$

10. Repeat question 9 when  $E_F = E_c - 0.25$  eV

11. Calculate the probability that an energy state in the valence band at  $E = E_v - kT$  is empty of an electron and calculate the thermal equilibrium hole concentration in silicon at  $T=300$  K.  $E_F = E_v + 0.25$  eV, for silicon at  $T=350$  K  $N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$

12. Calculate the thermal equilibrium hole concentration in silicon at  $T=300\text{ K}$ , when  $E_F=E_V+0.2\text{ eV}$
13. Calculate the intrinsic carrier concentration in silicon at  $T=350\text{ K}$  and  $400\text{ K}$ .  $N_C$  and  $N_V$  vary with  $T^{3/2}$ . Assume that  $E_G=1.12\text{ eV}$
14. Calculate the intrinsic carrier concentration in GaAs at  $T=200\text{ K}$  and  $T=400\text{ K}$
15. Determine the position of the intrinsic Fermi level with respect to the centre of the bandgap in silicon at  $T=300\text{ K}$ .  $m_n^*=1.08m_0$  and  $m_p^*=0.56m_0$
16. Repeat question 15 for GaAs at  $T=300\text{ K}$

### Carrier Concentration

1. Find the thermal equilibrium carrier concentrations in silicon at  $T=300\text{ K}$ , assuming that  $E_F=E_V+0.25\text{ eV}$ . If  $E_G=1.12\text{ eV}$ , then  $E_F=E_C-0.87\text{ eV}$ .
2. Repeat question 1 when the Fermi level is  $0.2\text{ eV}$  below the conduction band
3. Determine the hole concentration in silicon at  $T=300\text{ K}$  if the electron concentration is  $n_0=1\times 10^{16}\text{ cm}^{-3}$
4. Repeat question 3 when  $n_0=1\times 10^5\text{ cm}^{-3}$ .
5. Determine the thermal equilibrium electron and hole concentrations in silicon doped with Phosphorus at a concentration of  $N_d=2\times 10^{16}\text{ cm}^{-3}$ , assuming  $N_a=0$
6. The concentration of majority carriers in n-type silicon at  $T=300\text{ K}$  is to be  $n_0=10^{15}\text{ cm}^{-3}$ . Determine the concentration of phosphorous atoms to be added and the concentration of minority carriers.
7. Calculate the thermal equilibrium electron and hole concentration in a Germanium sample at  $T=300\text{ K}$  in which  $N_d=5\times 10^{13}$  and  $N_a=0$ . Assume  $n_i=2.4\times 10^{13}\text{ cm}^{-3}$
8. Germanium is doped with donors at a concentration of  $N_d=10^{14}\text{ cm}^{-3}$ . Calculate the thermal equilibrium concentration of electrons and holes.
9. Consider a compensated p-type doped silicon semiconductor at  $T=300\text{ K}$  with doping concentrations of  $N_a=2\times 10^{16}\text{ cm}^{-3}$  and  $N_d=5\times 10^{15}\text{ cm}^{-3}$ . Find the thermal equilibrium carrier concentrations
10. Consider a compensated germanium semiconductor at  $T=300\text{ K}$  doped at  $N_a=5\times 10^{13}\text{ cm}^{-3}$ ,  $N_d=1\times 10^{13}\text{ cm}^{-3}$ . Find a thermal equilibrium electron and hole concentrations.

11. An n-type **silicon** device is to operate at  $T=450$  K. At this temperature the intrinsic carrier concentration must contribute no more than 3% of the total electron concentration. Determine the minimum doping concentration required.

12. An n-type germanium device is to operate at  $T=400$  K. At this temperature the intrinsic carrier concentration must not contribute more than 10% of the total electron concentration. Determine the minimum donor concentration required.

13. A silicon device operates at  $T=300$  K, it contains an acceptor impurity concentration of  $N_a=10^{16} \text{ cm}^{-3}$ . Determine the donor concentration that must be added so that the silicon is n-type and the Fermi energy is 0.2 eV **below the conduction band**.

14. Determine the position of the **Fermi level with respect to the valence band** in **p-type** GaAs at  $T=300$  K where the doping concentrations are  $N_a=5 \times 10^{16} \text{ cm}^{-3}$  and  $N_d=4 \times 10^{15} \text{ cm}^{-3}$ .

15. Consider p-type silicon at  $T=300$  K doped with Boron. Assuming the limit at which the Boltzman approximation is  $E_F - E_V=3kT$ . Determine the Fermi level and the maximum doping at which the Boltzman approximation is true.

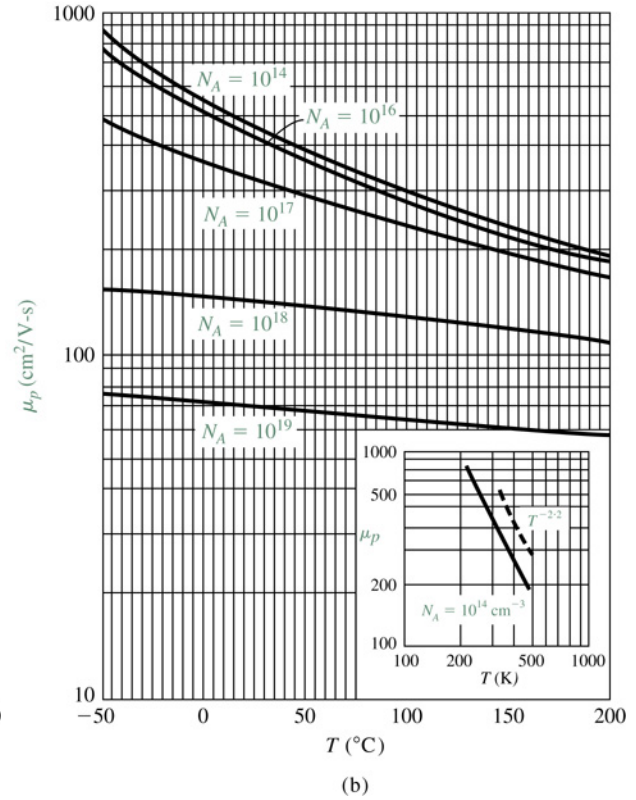
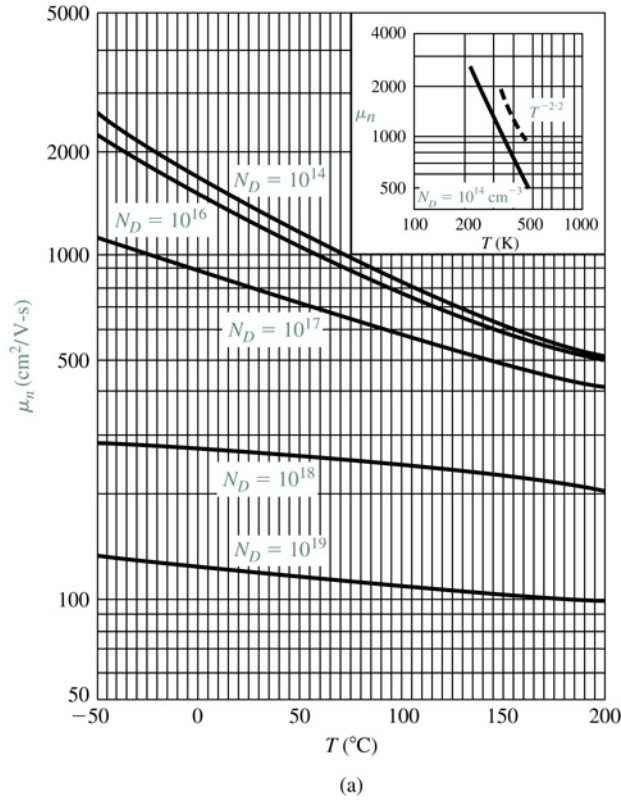
16. Consider n-type silicon at  $T=300$  K doped with phosphorous. Determine the doping concentration such that  $E_d-E_F=4.6kT$ .

### Drift and Diffusion of Carriers

	$\mu_n (\text{cm}^2/\text{V-s})$	$\mu_p (\text{cm}^2/\text{V-s})$
Silicon	1350	480
Galium Arsenide	8500	400
Germanium	3900	1900

1. Consider a silicon semiconductor at  $T=300$  K with an impurity doping concentration of  $N_d=10^{16} \text{ cm}^{-3}$ ,  $N_a=0$ . The electron and hole mobilities are given in the table. Calculate the drift current densities for an applied electric field of  $\mathcal{E}=35 \text{ V/m}$

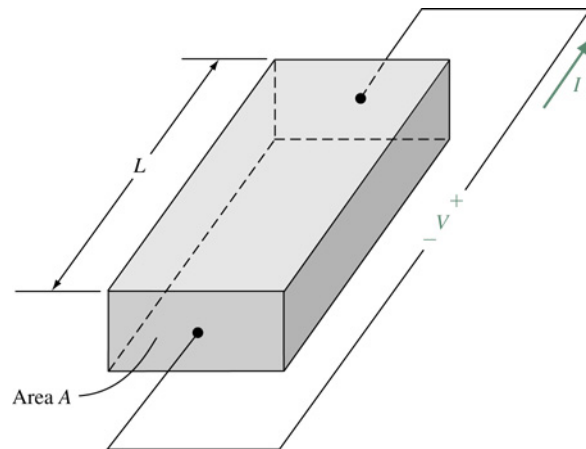
2. Consider a GaAs sample at  $T=300$  K with doping concentrations  $N_a=0$  and  $N_d=10^{16} \text{ cm}^{-3}$ . The electron hole mobilities are shown in the table. Calculate the drift current density for an applied electric field  $\mathcal{E}=10 \text{ V/m}$ .



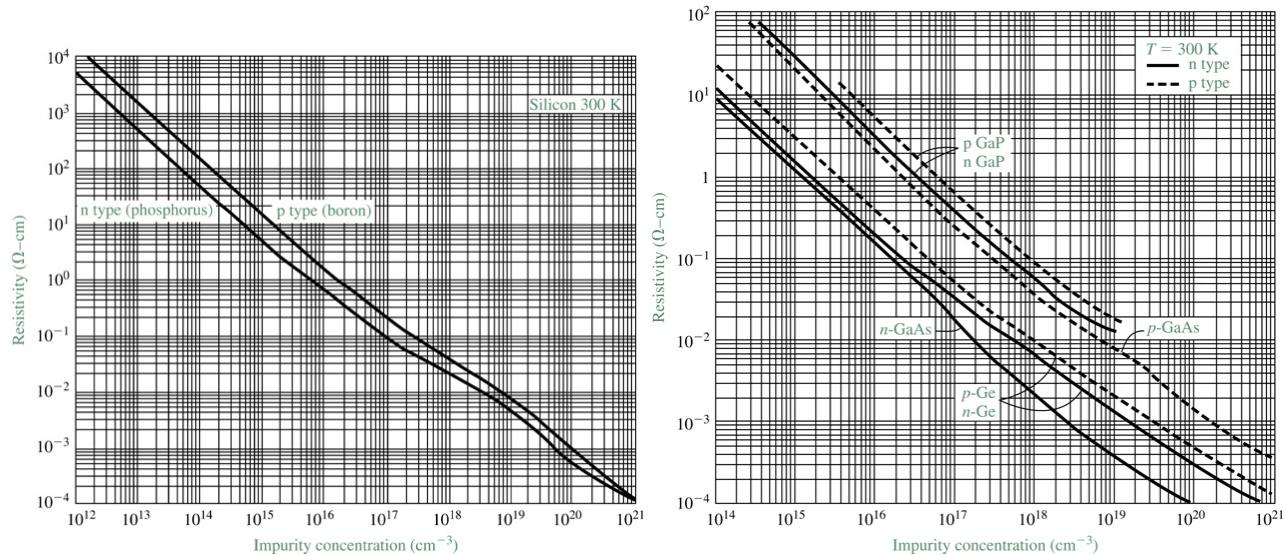
3. Find the electron mobility for i)  $N_d=10^{17} \text{ cm}^{-3}$ ,  $T=150 \text{ K}$  and for ii)  $N_d=10^{16} \text{ cm}^{-3}$ ,  $T=0$ . Find the hole mobility for i)  $N_a=10^{16} \text{ cm}^{-3}$ ,  $T=50 \text{ K}$  and for ii)  $N_d=10^{17} \text{ cm}^{-3}$ ,  $T=150 \text{ K}$ .

4. Use the graphs above to find the electron and hole mobilities in silicon for i)  $N_d=10^{17} \text{ cm}^{-3}$ ,  $N_a=5 \times 10^{16} \text{ cm}^{-3}$  and ii) in GaAs for  $N_d=N_a=10^{17} \text{ cm}^{-3}$ ,  $T=150 \text{ K}$

5. Consider a bar of silicon at  $T=300 \text{ K}$  uniformly doped with acceptors and geometry as shown. For an applied voltage of  $5 \text{ V}$ , a current of  $2 \text{ mA}$  is required. Current density must be no larger than  $J_{\text{drf}}=100 \text{ A/cm}^2$ . Find the required cross-sectional area, length and doping concentration.



6. For a semiconductor device at  $T=300$  K, the required material is to be n-type with a resistivity  $\rho=0.1 \Omega\text{-cm}$ . Determine the required impurity doping concentrations and the resulting electron mobility. Use the figures below.



7. A silicon semiconductor at  $T=300$  K and with the geometry shown above is initially doped with donors at a concentration of  $N_d=5 \times 10^{15} \text{ cm}^{-3}$ . Acceptors are added to form a p-type material. The resistance should be  $R=10 \text{ k}\Omega$  and handle a current density of  $J_{\text{drf}}=50 \text{ A/cm}^2$  when  $5 \text{ V}$  is applied and have an  $\mathcal{E}$  field of less than  $\mathcal{E}=100 \text{ V/m}$ . Design the semiconductor.

8. Consider compensated n-type silicon at  $T=300$  K with a conductivity of  $\sigma=16 (\Omega\text{-cm})^{-1}$  and an acceptor doping concentration of  $N_a=10^{17} \text{ cm}^{-3}$ .

9. Determine the carrier density gradient to produce a given diffusion current density. The hole concentration in silicon at  $T=300$  K varies linearly from  $x=0$  to  $x=0.01 \text{ cm}$ . The hole diffusion coefficient is  $D_p=10 \text{ cm}^2/\text{s}$ , the hole diffusion current density is  $J_{\text{drf}}=20 \text{ A/cm}^2$  and the hole concentration at  $x=0$  is  $p=4 \times 10^{17} \text{ cm}^{-3}$ . Determine the hole concentration at  $x=0.01 \text{ cm}$

10. Assume that, in an n-type GaAs semiconductor at  $T=300$  K, the electron concentration varies linearly from  $1 \times 10^{18}$  to  $7 \times 10^{17} \text{ cm}^{-3}$  over a distance of  $0.1 \text{ cm}$ . Calculate the magnitude of the diffusion current density if the diffusion coefficient is  $D_n=225 \text{ cm}^2/\text{s}$

11. Determine the diffusion coefficient for a carrier mobility of  $\mu=1200 \text{ cm}^2/\text{V-s}$  at  $T=300$  K

12. For a semiconductor with a diffusion coefficient of  $D=210 \text{ cm}^2/\text{s}$ , determine the carrier mobility.

### **Problem 1: Intrinsic semiconductor**

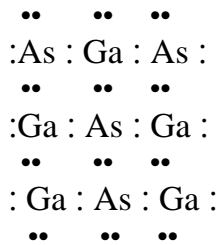
Refer to the plot of intrinsic carrier concentration ( $n_i$ ) vs. absolute temperature ( $T$ ) in the Lecture Notes:

a) Explain qualitatively the differences in intrinsic carrier concentrations for Ge, Si and GaAs. (Why is  $n_i$  highest for Ge? Why is it lowest for GaAs?)

b) Explain qualitatively why  $n_i$  increases with increasing temperature.

### **Problem 2: Doping**

a) Consider the two-dimensional representation of the semiconductor GaAs shown below:



If Si atoms are inserted as dopants and exclusively replace Ga atoms in the lattice, will the Si doped GaAs material be n-type or p-type? What if the Si atoms exclusively replace As atoms?

b) Consider a Si sample under equilibrium conditions, doped with boron to a concentration  $10^{17} \text{ cm}^{-3}$ .

i) At  $T = 300\text{K}$ , is this material n-type or p-type? What are the majority and minority carrier concentrations?

ii) As the temperature of this sample is increased,  $n_i$  will eventually increase to be higher than the dopant concentration, and the sample will become intrinsic ( $n \approx p \approx n_i$ ). Estimate the temperature at which this occurs, by finding the temperature at which  $n_i$  be much greater (at least  $10\times$  higher) than  $n$  and  $p$ . (You can use the formula for  $n_i$  given in the Lecture Notes, or simply use the plot of  $n_i$  vs.  $T$ .)

### **Problem 3: Carrier Concentrations and Energy Band Diagram**

a) Consider a Si sample maintained at  $T = 300\text{K}$  under equilibrium conditions, doped with phosphorus to a concentration  $1 \times 10^{16} \text{ cm}^{-3}$ .

i). What are the electron and hole concentrations ( $n$  and  $p$ ) in this sample? Is the material n type or p-type?

ii). Draw the energy band diagram for this sample. Indicate  $(E_c - E_F)$  and  $(E_F - E_i)$  to within 0.01 eV. (Numeric answer required.)

b) Suppose the sample type is converted to the opposite type by counter-doping it with boron, such that  $E_i - E_F = 0.30 \text{ eV}$ .

i). What are  $n$  and  $p$  in the counter-doped sample?

ii). What is the concentration of boron?

### **Problem 4: Temperature Dependence of carrier densities and $E_f$**

Consider a Si sample doped with arsenic to a concentration  $1 \times 10^{15} \text{ cm}^{-3}$ .

a) At  $T = 0\text{K}$ , what are the equilibrium hole and electron concentrations?



- b) At  $T = 300\text{K}$ , what are the equilibrium hole and electron concentrations? Where is the Fermi level located? (Numeric answer required.)
- c) At  $T = 600\text{K}$ , what are the equilibrium hole and electron concentrations? (Use Figure in the Lecture Nodes to determine  $n_i$  at  $600\text{K}$ .) Where is the Fermi level located? (Numeric answer required.)
- d) Explain qualitatively the trend in  $E_F - E_i$  with increasing temperature.

### **Problem 5: Drift**

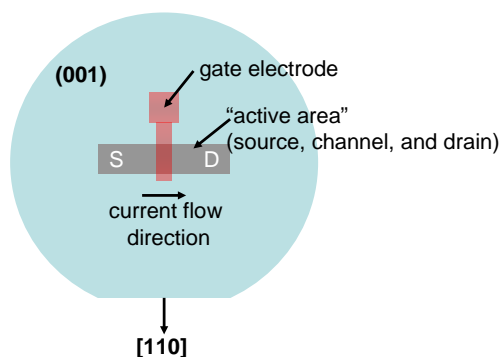
A n-type silicon sample is maintained at room temperature. When an electric field with a strength of  $1000\text{V/cm}$  is applied to the sample, the hole velocity is  $2 \times 10^5 \text{ cm/sec}$ .

- a) What is the mobility of an electron in this sample?
- b) Calculate the electron and hole concentrations.
- c) Draw the energy band diagram, showing the position of  $E_F$  with respect to  $E_c$  and  $E_v$ .
- d) What is the resistivity of this sample?
- e) Suppose this sample is used to make an integrated-circuit resistor. The width and height of the sample are  $10 \mu\text{m}$  and  $1.5 \mu\text{m}$ , respectively, and the length of the sample is  $20 \mu\text{m}$ . Calculate the resistance of the sample.

### **Problem 6: Miller indices**

The performance of a MOSFET depends upon the crystalline orientation of the channel surface, as well as the crystalline orientation of the current flow direction. These parameters must be precisely controlled in an integrated-circuit manufacturing process; therefore, silicon wafer substrates are carefully prepared by slicing along (001) planes, and are marked by a “flat” or notch at the edge of the wafer to indicate the [110] direction.

If a MOSFET is laid out on the surface of a (001) silicon wafer as shown in the figure below, what are the Miller indices for the direction of current flow?



Plan view of a MOSFET on a Si wafer

### **Problem 7: Diamond lattice crystal structure**

Germanium (Ge) is a semiconductor material of renewed interest, because it can potentially yield higher MOSFET performance as compared with silicon (Si). Ge atoms are arranged in a diamond lattice structure, as are Si atoms. Given that the lattice constant of Ge is  $5.65\text{\AA}$  at room temperature (300K), calculate the atomic density of Ge (atoms/cm<sup>3</sup>).

### **Problem 8: Semiconductor doping**

(a) Explain why boron (B) is preferred over indium (In) as the dopant species to achieve highly conductive p-type silicon. (hints: The low ionization energy (IE) of boron is 45 meV, whereas that of indium is 160 meV)

(b) At very high temperatures (*e.g.*  $>1000^{\circ}\text{C}$ ), the conductivity of silicon is not significantly affected by moderate doping ( $N_A$  or  $N_D$  less than  $10^{18}/\text{cm}^3$ ), *i.e.* it is an intrinsic semiconductor. Explain why this is the case.

### **Problem 9: Carrier concentrations**

Consider a silicon sample maintained at 300K. Calculate the electron density ( $n$ ) and hole density ( $p$ ) for the following cases:

(a) the silicon is undoped

(b) the silicon is doped with  $10^{16}/\text{cm}^3$  boron atoms (all ionized)

(c) the silicon is doped with  $10^{16}/\text{cm}^3$  boron atoms (all ionized) and  $10^{17}/\text{cm}^3$  arsenic atoms (all ionized)

### **Problem 10: Energy Band Diagram**

Consider a Si sample in thermal equilibrium at  $T = 300\text{K}$ , with electron concentration  $n = 1 \times 10^{18} \text{ cm}^{-3}$ . Draw the energy band diagram for this sample. Indicate  $(E_c - E_F)$  and  $(E_F - E_i)$  to within 0.01 eV.

### **Problem 11: Carrier Mobility and Drift Velocity**

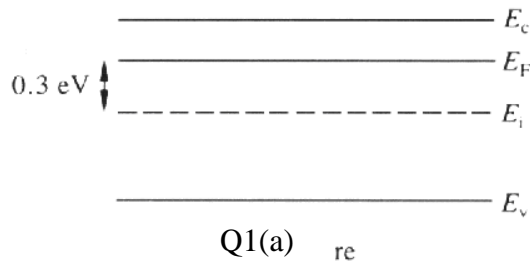
A p-type (uncompensated) silicon sample is maintained at 300K. When an electric field with strength  $5 \times 10^3 \text{ V/cm}$  is applied to the sample, the electron drift velocity is  $4 \times 10^6 \text{ cm/sec}$ .

a) What is the mobility of this sample?

b) What is the resistivity of this sample?

### **Problem 12: Fermi Levels**

The equilibrium and steady state conditions before and after illumination of a silicon sample maintained at  $T = 300\text{K}$  are characterized by the energy band diagrams shown below.

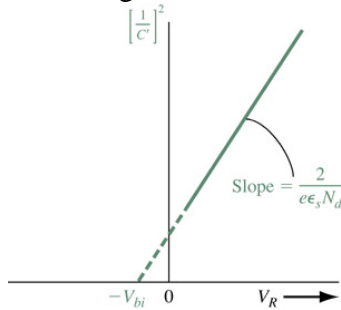


a) What are the equilibrium carrier concentrations  $n_0$  and  $p_0$ ?

### **PN Junction**

1. Calculate the built in potential barrier of a silicon pn junction at  $T=300$  K with doping concentrations of  $N_a=2 \times 10^{16} \text{ cm}^{-3}$  and  $N_d=5 \times 10^{15} \text{ cm}^{-3}$
2. Repeat question 1 for carrier concentrations of
  - i)  $N_a=5 \times 10^{17} \text{ cm}^{-3}$  and  $N_d=10^{16} \text{ cm}^{-3}$
  - ii)  $N_a=10^{15} \text{ cm}^{-3}$  and  $N_d=2 \times 10^{16} \text{ cm}^{-3}$
3. Calculate the space charge widths and peak electric field in a silicon pn junction at  $T=300$  K with uniform doping concentrations of  $N_a=2 \times 10^{16} \text{ cm}^{-3}$  and  $N_d=5 \times 10^{15} \text{ cm}^{-3}$ , find  $x_n$ ,  $x_p$ ,  $W$  and  $|\mathcal{E}_{\max}|$
4. A silicon pn junction at  $T=300$  K with zero applied bias has doping concentrations of  $N_a=5 \times 10^{16} \text{ cm}^{-3}$  and  $N_d=5 \times 10^{15} \text{ cm}^{-3}$ , find  $x_n$ ,  $x_p$ ,  $W$  and  $|\mathcal{E}_{\max}|$ .
5. Calculate the width of the space charge region in a silicon pn junction with uniform doping concentrations  $N_a=2 \times 10^{16} \text{ cm}^{-3}$  and  $N_d=5 \times 10^{15} \text{ cm}^{-3}$  at  $T=300$  K when a reverse-bias voltage of  $V_R=5$  V is applied
6. A silicon pn junction at  $T=300$  K is reverse biased at  $V_R=8$  V and has doping concentrations of  $N_a=5 \times 10^{15} \text{ cm}^{-3}$  and  $N_d=5 \times 10^{16} \text{ cm}^{-3}$ , find  $x_n$ ,  $x_p$ ,  $W$ , repeat for  $V_R=12$  V
7. Determine the n-type doping required in a silicon pn junction to operate at  $T=300$  K with a p-type doping concentration of  $N_a=10^{18} \text{ cm}^{-3}$  such that the maximum electric field in the space charge region is  $|\mathcal{E}_{\max}|=10^5 \text{ V/cm}$  at a reverse bias voltage of  $V_R=10$  V.
8. Determine the maximum electric field in a pn silicon with doping concentrations of  $N_a=5 \times 10^{15} \text{ cm}^{-3}$  and  $N_d=5 \times 10^{16} \text{ cm}^{-3}$ , for reverse bias voltages of  $V_R=8$  V and  $V_R=12$  V.
9. Calculate the junction capacitance of a pn junction at  $V_R=5$  V assuming the cross sectional area of the pn junction is  $A=10^{-4} \text{ cm}^2$ .
10. Consider a GaAs pn junction at  $T=300$  K doped at concentrations of  $N_a=1 \times 10^{15} \text{ cm}^{-3}$  and  $N_d=2 \times 10^{16} \text{ cm}^{-3}$  and with a junction area of  $A=10^{-4} \text{ cm}^2$  when the reverse bias voltage is  $V_R=0$  V and  $V_R=5$  V.

11. Determine the impurity concentrations in a  $p^+n$  junction at  $T=300$  K, assuming the intercept on the voltage axis of the curve below gives  $v_{bi}=0.742$  V and that the slope is  $3.92 \times 10^{15} \text{ (F/cm}^2\text{)}^{-2}/\text{V}$



12. The experimentally measured junction capacitance of a one sided  $p^+n$  silicon junction biased at  $V_R=4$  V at  $T=300$  K is  $C=1.10$  pF. The built in potential barrier is found to be  $v_{bi}=0.782$  V The cross-sectional area is  $A=10^{-4} \text{ cm}^2$ . Find the doping concentrations.

### **Problem 1: pn Junction Electrostatics**

a) A pn step junction is made in silicon with  $N_A = 5 \times 10^{16} \text{ cm}^{-3}$  and  $N_D = 1 \times 10^{16} \text{ cm}^{-3}$ . Assume  $T = 300\text{K}$ . Calculate the following:

- i) the built-in potential  $V_{bi}$
- ii) the depletion layer width  $W$  at zero bias
- iii) the maximum electric field at zero bias
- iv) the maximum electric field at 5V reverse bias

b) Sketch the energy band diagram, charge density distribution, electric field distribution, and potential distribution as a function of position  $x$  for both zero bias and 5V reverse bias.

### **Problem 2: pn Junction Electrostatics**

A silicon step junction maintained at  $T = 300\text{K}$  under equilibrium conditions has a p-side doping of  $N_A = 1 \times 10^{16} \text{ cm}^{-3}$  and n-side doping of  $N_D = 1 \times 10^{15} \text{ cm}^{-3}$ .

a) Calculate the built-in voltage,  $V_{bi}$  (sometime it is called built-in potential  $V_0$  or  $\phi_B$ ).

b) Calculate the depletion width,  $W$ . What are  $x_{po}$  and  $x_{no}$ ?

$$\text{Note that } x_{po} = \frac{N_D}{N_A + N_D} W \quad \text{and} \quad x_{no} = \frac{N_A}{N_A + N_D} W$$

c) What is the electrostatic potential at  $x = 0$ , i.e. how much voltage is dropped across the p-side?

$$\text{Note that } V(0) = \frac{qN_A}{2\epsilon_s} x_{po}^2 = V_{bi} - \frac{qN_D}{2\epsilon_s} x_{no}^2$$

d) Calculate the peak electric field (at  $x = 0$ ):

$$E(0) = -\frac{2V_{bi}}{W}$$

e) Sketch (roughly to scale) the charge density distribution  $\rho(x)$ , electric field distribution  $\mathcal{E}(x)$ , and electrostatic potential  $V(x)$ .

## **MOS Fundamentals**

1. Determine the potential  $\phi_{Fp}$  in silicon at  $T=300$  K for a)  $N_a=10^{15} \text{ cm}^{-3}$  and b)  $N_a=10^{17} \text{ cm}^{-3}$
2. Consider p-type silicon at  $T=300$  K. Determine the doping concentration if  $\phi_{Fp}=-0.34$  V
3. Calculate the maximum space charge width for silicon at  $T=300$  K doped to  $N_a=10^{16} \text{ cm}^{-3}$
4. Consider an oxide-to-p-type silicon junction at  $T=300$  K. The impurity doping concentration in the silicon is  $N_a=3 \times 10^{16} \text{ cm}^{-3}$ , calculate the maximum space charge width in the silicon. Repeat for an impurity concentration of  $N_a=10^{15} \text{ cm}^{-3}$ .
5. Calculate the metal semiconductor work function difference  $\phi_{ms}$ . For an aluminium-silicon dioxide junction  $\phi'_m = 3.2$  V and for a silicon-silicon dioxide junction  $\chi'=3.25$  V. Assume that  $E_G=1.12$  eV and let the p-type doping be  $N_a=10^{14} \text{ cm}^{-3}$
6. Calculate the metal-semiconductor work function difference  $\phi_{ms}$  for an aluminium-silicon dioxide-silicon device if the silicon is doped to a concentration of  $N_a=10^{16} \text{ cm}^{-3}$ . Assume  $T=300$  K
7. Calculate  $C_{ox}$ ,  $C'_{min}$ ,  $C'_{FB}$  for an MOS capacitor. Consider a p-type silicon substrate at  $T=300$  K doped to  $N_a=10^{16} \text{ cm}^{-3}$ . The oxide is silicon dioxide with a thickness of  $550 \text{ \AA}$  and the gate is aluminium.
8. Consider an MOS device with the following parameters. Aluminium gate, p-type substrate,  $N_a=3 \times 10^{16} \text{ cm}^{-3}$ ,  $t_{ox}=250 \text{ \AA}$  and  $Q'_{ss}=10^{11} \text{ cm}^{-2}$ . Determine  $C'_{min}/C_{ox}$  and  $C'_{FB}/C_{ox}$ .

### **Problem 1: MOS Fundamentals**

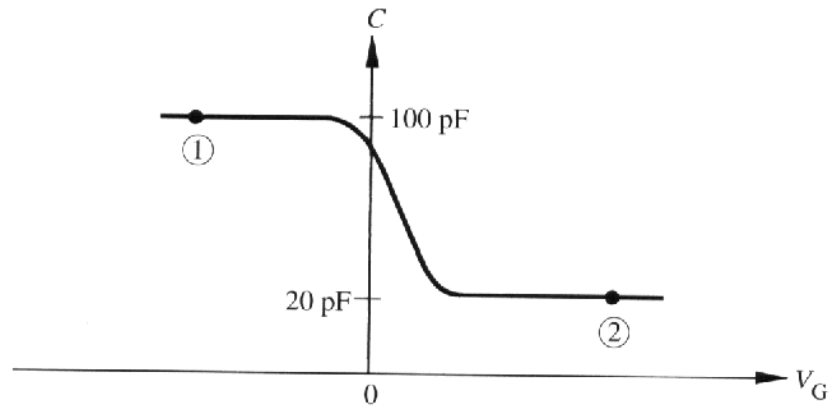
Consider an ideal MOS capacitor maintained at  $T = 300\text{K}$  with the following characteristics:

- Gate material is p+ polycrystalline-silicon (work function  $\Phi_M = 5.15\text{eV}$ )
- Substrate is n-type Si, with doping concentration  $10^{18} \text{ cm}^{-3}$
- Oxide thickness  $x_o = 2 \text{ nm}$

- a) What is the flat-band voltage,  $V_{FB}$ , of this capacitor?
- b) Sketch the energy-band diagrams, labeling  $qV_G$ ,  $q\phi_s$ ,  $qV_{ox}$  (no numerical values required), for the following bias conditions:
  - i) flat-band
  - ii) accumulation
  - iii) equilibrium
  - iv) strong inversion

### **Problem 2: MOS Capacitance**

The capacitance vs. gate voltage characteristic of a simple MOS capacitor of area  $1 \times 10^{-4} \text{ cm}^2$  is as shown:



- a) Is the semiconductor (silicon) substrate doped n-type or p-type? Explain briefly.
- b) Sketch the block charge density diagrams corresponding to points (1) and (2) on the  $C$ - $V$  characteristic.
- c) What is the thickness of the gate oxide ( $\text{SiO}_2$ ),  $x_o$ ?
- d) What is the semiconductor doping concentration?
- e) Is the measurement frequency low or high? Explain briefly.
- f) Indicate qualitatively how the  $C$ - $V$  characteristic would change if the semiconductor doping concentration were to be increased.

### **Problem 3: MOS Capacitance**

An MOS capacitor is made on uniformly doped p type material. With -20V on the gate with respect to the substrate it has a capacitance of 20pF. With +20V on the gate it has a capacitance of 10pF. What is the thickness of the depletion layer and the also the oxide if the capacitor has an area of  $10^{-6} \text{ m}^2$ .

## **MOSFETs and IC's**

1. Design the width of a MOSFET such that a specified current is induced for a given bias. Consider an ideal n-channel MOSFET with parameters  $L=1.25\text{ }\mu\text{m}$ ,  $\mu_n=650\text{ cm}^2/\text{V}\cdot\text{s}$ ,  $C_{ox}=6.9\times 10^{-8}\text{ F/cm}^2$  and  $V_T=0.65\text{ V}$ . Design the channel  $W$  such that  $I_D(\text{sat})=4\text{ mA}$  for  $V_{GS}=5\text{ V}$
2. The parameters of an n-channel MOSFET are  $\mu_n=650\text{ cm}^2/\text{V}\cdot\text{s}$ ,  $t_{ox}=200\text{ }\text{\AA}$ ,  $W/L=50$  and  $V_T=0.4\text{ V}$ . If the transistor is biased in the saturation region find the drain current for  $V_{GS}=1, 2, 3\text{ V}$
3. Consider an n-channel MOSFET with  $W=15\text{ }\mu\text{m}$ ,  $L=2\text{ }\mu\text{m}$ ,  $C_{ox}=6.9\times 10^{-8}\text{ F/cm}^2$ . Assume a drain current in the nonsaturation region for  $V_{DS}=0.1\text{ V}$  is  $I_D=35\text{ }\mu\text{A}$  at  $V_{GS}=1.5\text{ V}$  and  $I_D=75\text{ }\mu\text{A}$  at  $V_{GS}=2.5\text{ V}$ . Determine the inversion carrier mobility from experimental results.
4. Consider the n-channel MOSFET in Question 3. Using the results in the example, determine the threshold voltage of the MOSFET.
5. Consider a p-channel MOSFET with parameters  $\mu_p=300\text{ cm}^2/\text{V}\cdot\text{s}$ ,  $C_{ox}=6.9\times 10^{-8}\text{ F/cm}^2$ ,  $W/L=10$  and  $V_{TP}=-0.65\text{ V}$ . Determine the conduction parameter  $\beta$  and find the maximum current at  $V_{SG}=3\text{ V}$
6. The maximum current in a p-channel MOSFET must be  $I_D=0.85\text{ mA}$  at  $V_{SG}=3\text{ V}$ . If the transistor has the same electrical parameters as in question 5. Determine the required width to length ratio.

### **Problem 1: The MOSFET as a Resistor**

Calculate the resistance of a load MOST with an aspect ratio of 1 when the mobility of the electrons is  $1000\text{ cm}^2\text{V}^{-1}\text{sec}^{-1}$  and the gate capacitance per unit area is  $10^{-2}\text{ Fm}^{-2}$ . The drain voltage is  $V_{DD}=5\text{ V}$  and the threshold voltage  $V_T=0.5\text{ V}$ . (Hints: In this case the gate and the drain are connected together so that  $V_G=V_D$ .)

### **Problem 2: The nMOS logics**

Calculate  $W/L$  with the given specification (see two examples on lecture “nMOS logic IC design”).

### **Problem 3: The micro fabrication**

Describe the fabrication of an IC resistor, a pn junction, and an nMOSFET using planar technology.