Assignment 1: CT Signals and Systems

Deadline: Nov. 11th, 9:00 a.m.

Submission: Submit the electronic version to Learning Mall.

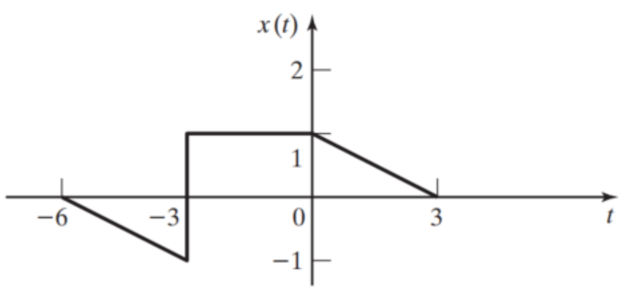
Information: This assignment takes 15% in the total mark.

Late submission: 5% each day, less than 1 day is counted as 1 day.

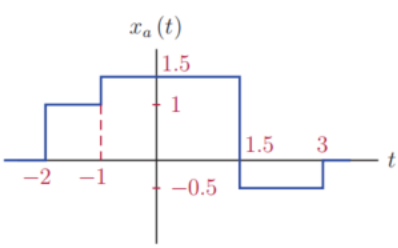
Submissions later than 5 working days won't be accepted.

Question 1 (L3-4) 20 marks

(a) For the signal $x(t)$ shown below, plot $2x(2t+2)$ 4



(b) Express the signal shown below using scaled and time shifted unit step function $u(t)$. 4



(c) For each of the following signals, determine whether they are even, odd or neither. 4

I) $x(t) = \sin(3t - \frac{\pi}{2})$

II) $x(t) = u(t) - 0.5$

(d) For the given signals, if the signal is periodic, find its fundamental period and its fundamental frequency; otherwise, prove that the signal is not periodic. 4

I) $x(t) = 4\cos(4t + 40°) + 3e^{-j12t}$

II) $x(t) = \cos(2\pi t) + \sin(6t)$

(e) Determine whether the following signals are power signal, energy signal or neither: 4

I) $x(t) = e^{-2t}u(t)$

II) $x(t) = e^{j(2t+\pi/4)}$

Question 2 (L5-6) 20 marks

(a) For the systems given below, decide whether they are causal, stable, linear and time-invariant? Conclusions only. 4

I) Input-output relationship: $y[n] = x[3 - 2n]$;

II) Input-output relationship: $y(t) = cos(\pi t)x(t)$;

III) Impulse response: $h(t) = u(t+3) - u(t-3)$;

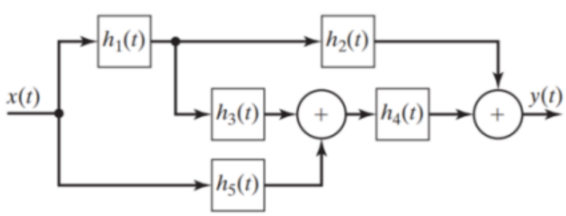
IV) Impulse response: $h[n] = 5^n u[-n]$.

(b) Suppose the following systems take $x(t)$ as the input and $y(t)$ as the corresponding output. Find the impulse response $h(t)$. 4

I) $y(t) = x(t-7)$;

II) $y(t) = \int_{-\infty}^{t} x(\tau - 7) d\tau$.

(c) Consider the LTI system shown as below: 4



Express the system impulse response as a function of the impulse responses of the subsystems.

(d) Suppose the systems with impulse response $h(t)$ take $x(t)$ as the input. Find the output $y(t)$. 4

$x(t) = u(1-t)$ and $h(t) = e^{-t}u(t-2)$;

(e) For the convolution between two time-domain signals f(t) and g(t), the ***differentiation property*** is: 4

$$\frac{d}{dt}(f(t) * g(t)) = \frac{df(t)}{dt} * g(t) = f(t) * \frac{dg(t)}{dt}$$

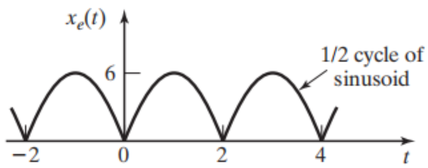
Calculate $\frac{d}{dt}(e^{-t} * u(t))$.

Question 3 (L7-9) 20 marks

(a) Find the Fourier coefficients of the exponential form: 4

$$x(t) = 2\sin^2 4t + \cos 4t \text{ and}$$

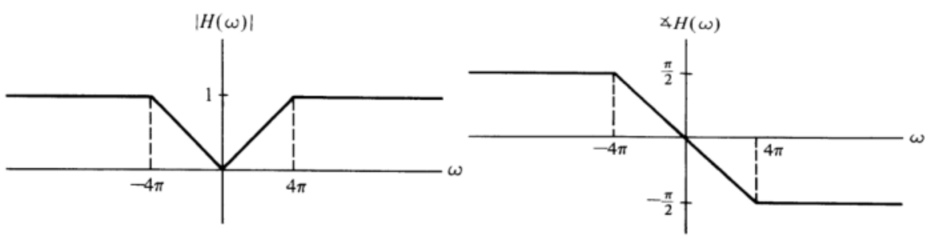
(b) Calculate the Fourier coefficients for each signal: 8



(c) A signal $x(t)$ has a Fourier transform $X(\omega)$. 4

Calculate the Fourier transform of $x(at)cos(\omega_0 t)$, with $0 < a < 1$.

(d) The magnitude and phase spectrum of a LTI system are plotted below: 4



If input signal is $x(t) = 1 + 2cos(2\pi t)$, find the corresponding output.

Question 4 (L10-11) 20 marks

(a) A stable system is characterized by the transfer function: 10

$$H(s) = \frac{15s(s+1)}{(s+3)(s-1)(s-2)}$$

I) Draw the zero-pole plot of the system;

II) Determine the ROC of the system;

III) Find the impulse response of the system;

IV) Decide whether the system's magnitude response is lowpass, highpass, bandpass or bandstop.

(b) The characteristic equation of a continuous-time causal system is given: 5

$$D(s) = s^2 + 2s + a$$

For the system to be stable, decide the range of the real value a in the equation.

(c) Given the relationships: 5

$$y(t) = x(t) * h(t) \text{ and } g(t) = x(3t) * h(3t)$$

$$x(t) \xleftrightarrow{\mathcal{F}} X(\omega) \text{ and } h(t) \xleftrightarrow{\mathcal{F}} H(\omega)$$

Use Fourier transform properties to show that g(t) has the form like: $g(t) = Ay(Bt)$, and determine the values of A and B.

Question 5 (L12-13) 20 marks

(a) The following differential equation is used to model a RLC circuit whose input is $x(t) = e^{-t}u(t)$: 10

$$y''(t) + 5y'(t) + 6y(t) = x(t)$$

With the initial conditions:

$$\mathrm{y}(0^{+}) = 1 \quad \text{and} \quad \mathrm{y}'(0^{+}) = 0$$

Solve the differential equation in ***time domain*** to get:

i) Zero-input response;

ii) Zero-state response;

iii) Overall response.

(b) Solve sub-question c) in ***frequency domain***. 10