CAN102
Electromagnetism and Electromechanics

2023/24-S2

Lecture 15 Linear DC Machine

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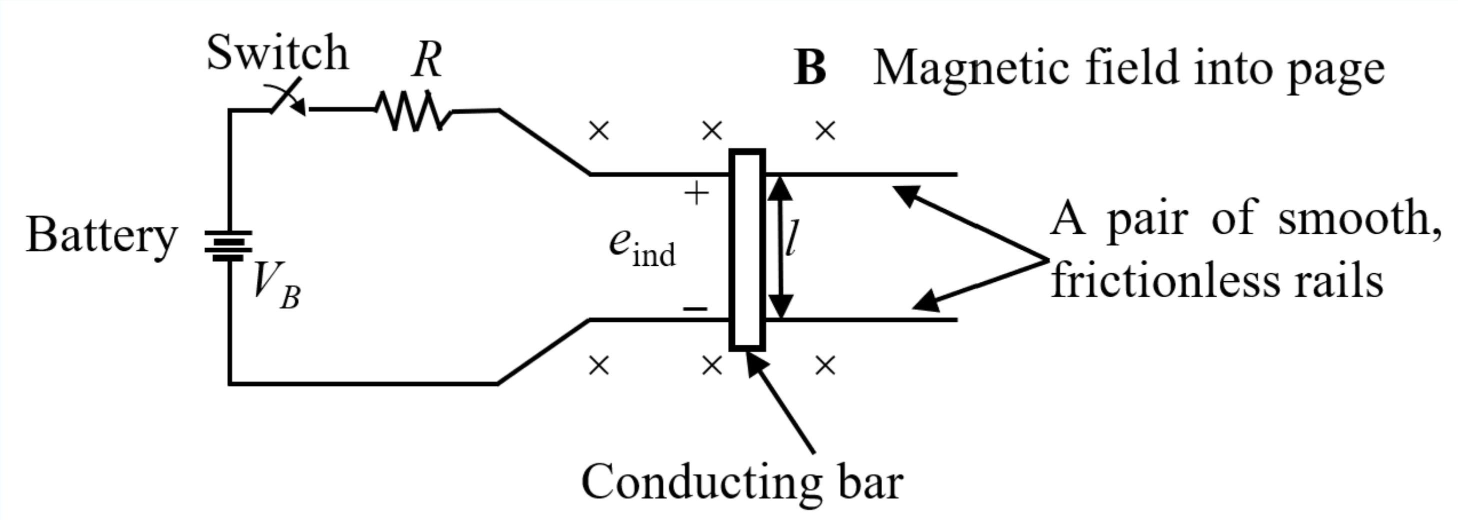
Linear DC machines

- Four basic equations
- Starting behavior
- The Linear DC Machine Starting Problems
- The Linear DC Machine as a Motor
- The Linear DC Machine as a Generator



Linear DC machines-Four Basic Equations

1. Force on a wire

- The force on a wire in the presence of a magnetic field:

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$$

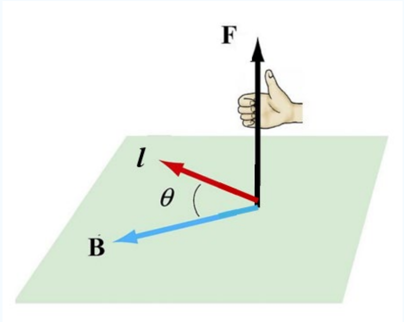
F: force on wire
i: magnitude of current in wire
l: length of wire, in current's direction
B: magnetic flux density vector

The magnitude of the force

$$F = ilB\sin\theta$$

θ is the angle between the wire and the flux density vector

Direction of the force:
Right-handed rule for cross product



Linear DC machines-Four Basic Equations

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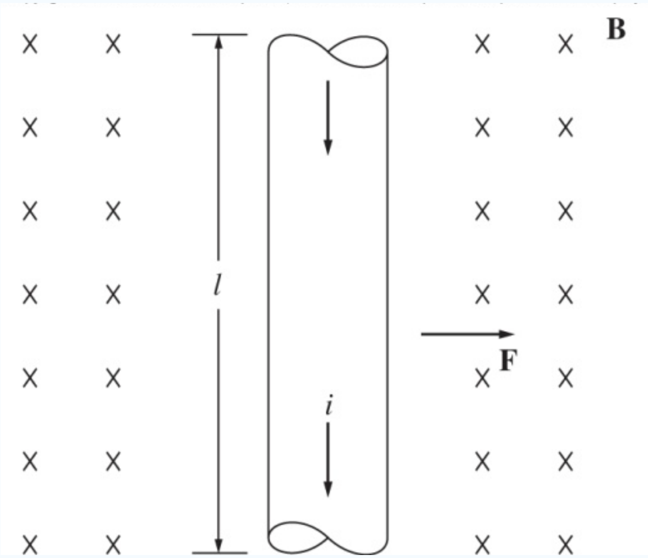
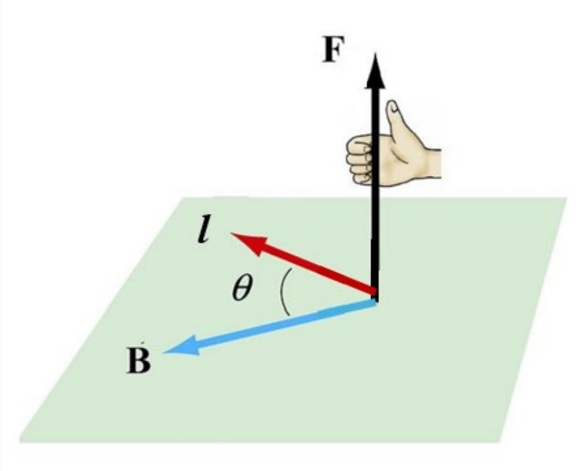




The induction of a force in a wire by a current in the presence of a magnetic field is the basis of motor action.

Almost every type of motor depends on this basic principle for the forces and torques which make it move

Linear DC machines-Four Basic Equations

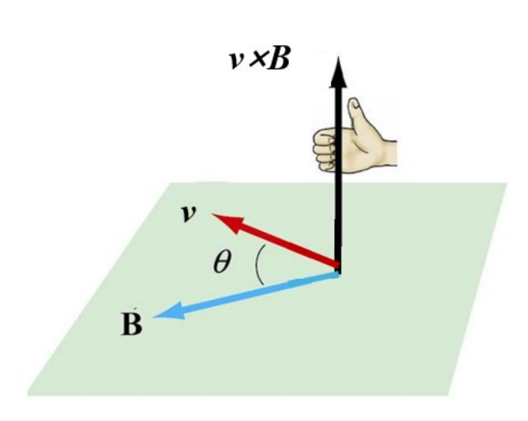
2. Induced voltage in a moving wire

- The voltage induced in a wire moving in a magnetic field

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \bullet \mathbf{l}$$

v: velocity of the wire
l: length of conductor in the magnetic field, points along the direction of the wire toward the end making the smallest angle with respect to the vector $\mathbf{v} \times \mathbf{B}$
B: magnetic flux density vector

The positive end of induced voltage in the wire is in the direction of the vector $\mathbf{v} \times \mathbf{B}$



Linear DC machines-Four Basic Equations

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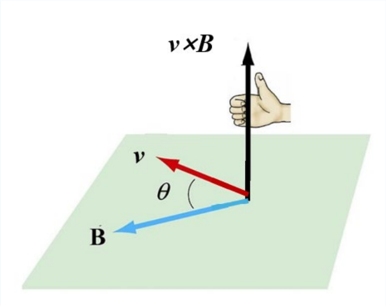

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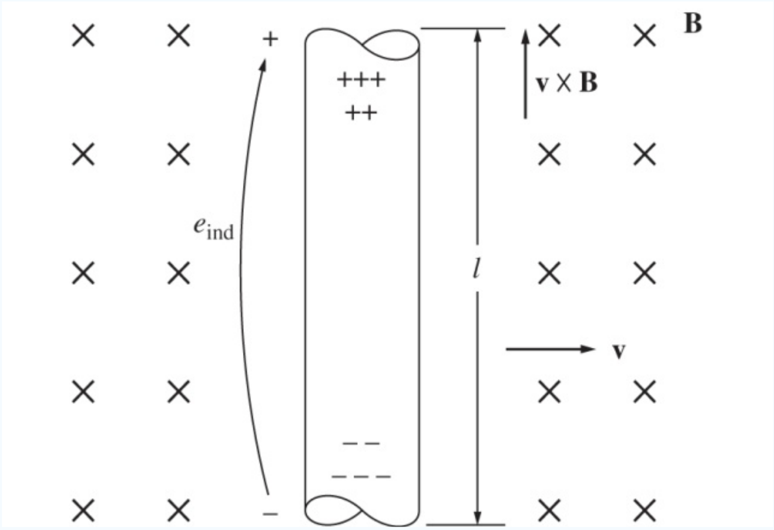
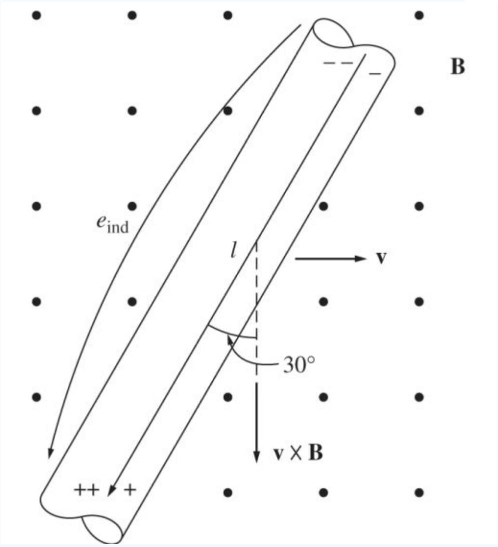




Generator action

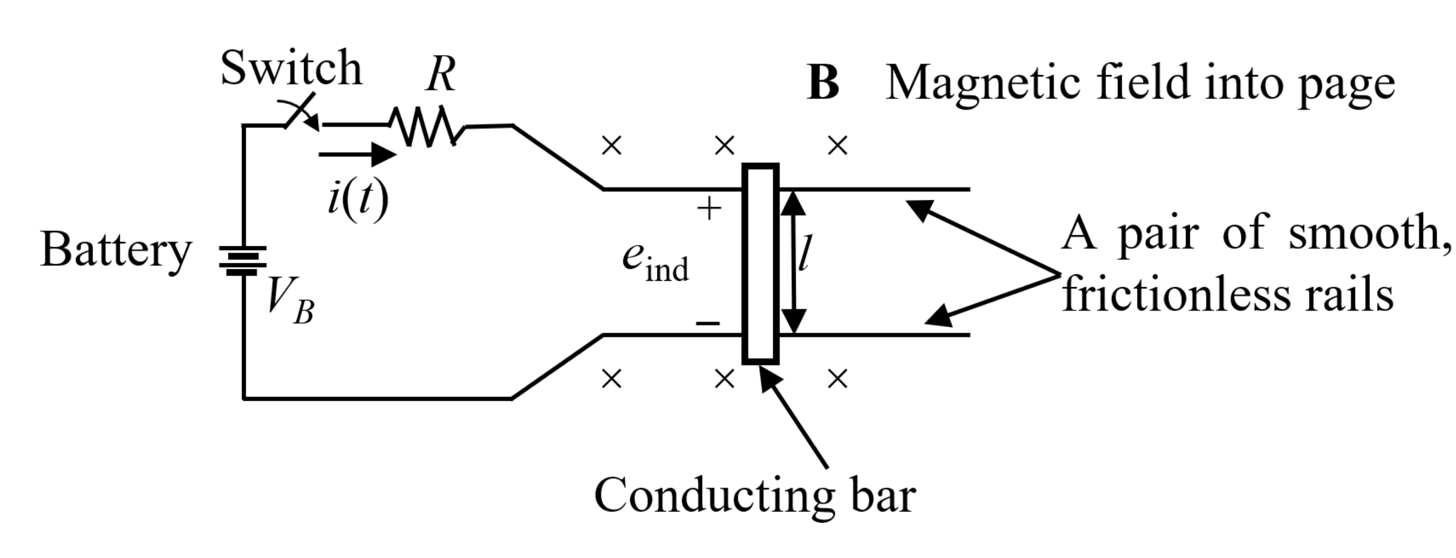
Linear DC machines-Four Basic Equations

3. Kirchhoff's voltage law:

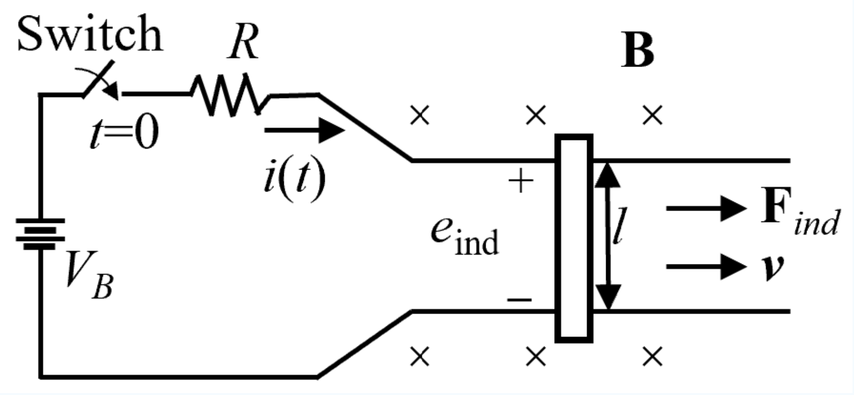
$$V_B = e_{ind} + iR$$

4. Newton's law:

For the bar: $F_{net} = ma$



Starting the Linear DC machine



To start-close the switch

⇓

A current flows in the bar

Initially, $e_{ind} = 0$, $i = V_B/R$

⇓

The force is induced on the bar $\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$

⇓

By Newton's law, the bar will accelerate to the right $F_{net} = ma$

⇓

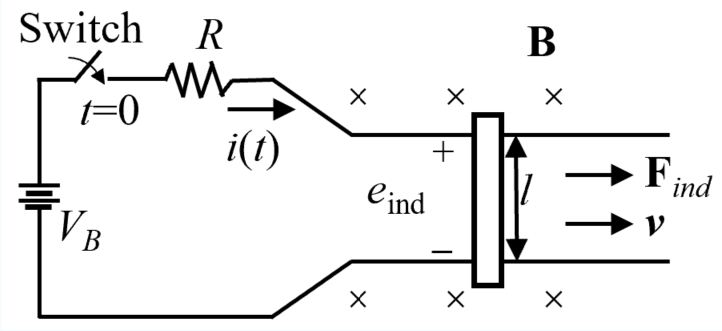
When the velocity of the bar begins to increase, the voltage is induced across the bar $e_{ind} = vBl$

⇓

The voltage now reduces the current flowing in the bar, since by Kirchhoff's voltage law: $i \downarrow = \frac{V_B - e_{ind} \uparrow}{R}$ $V_B = e_{ind} + iR$

Starting the Linear DC machine-Steady state

$$i \downarrow = \frac{V_B - e_{ind} \uparrow}{R}$$



As e_{ind} increases, the current i decreases

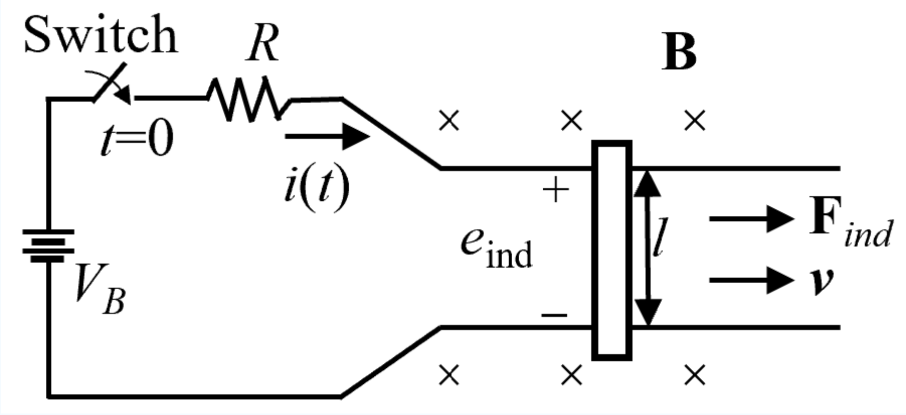
The induced force is thus decreased $F\downarrow = i\downarrow lB$

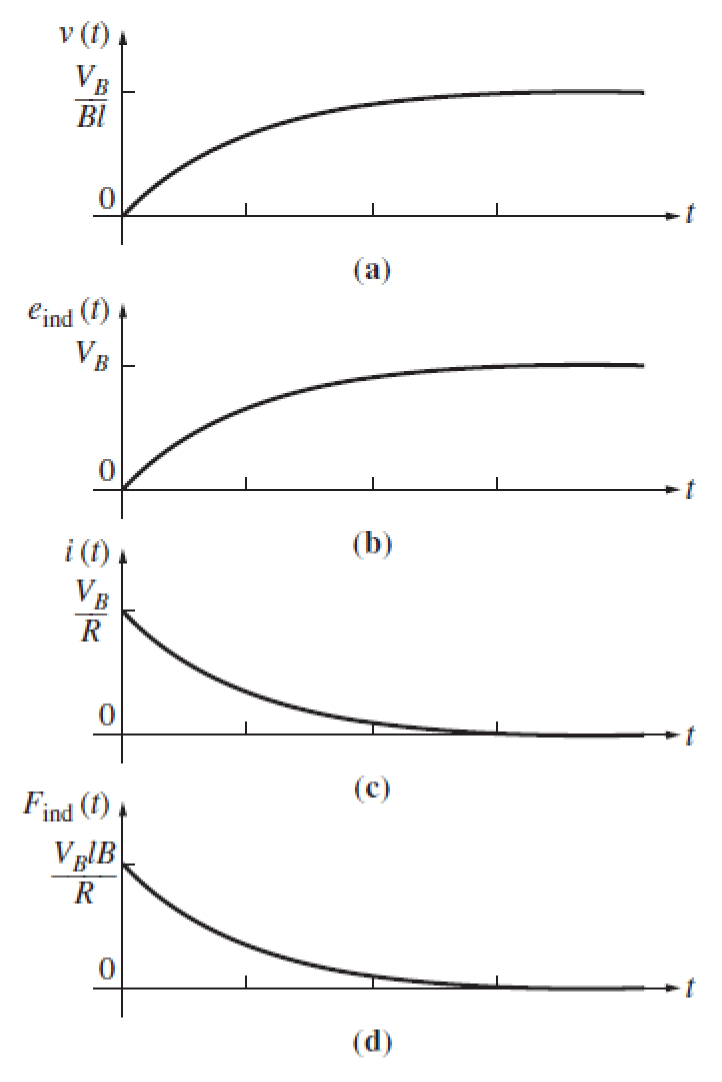
until eventually $F_{net} = F_{ind} = 0$, at that point, $i=0$, $e_{ind} = V_B$, The bar moves at a constant no-load speed $v_{nl} = e_{ind} / Bl = V_B / Bl$

The bar will continue at this speed v_{nl} unless some external force disturbs it.

Starting the Linear DC machine-No Load

The linear DC machine on starting
(a) velocity $v\ (t)$ as a function of time
(b) induced voltage $e_{ind}(t)$
(c) current $I\ (t)$
(d) induced force $F_{ind}(t)$





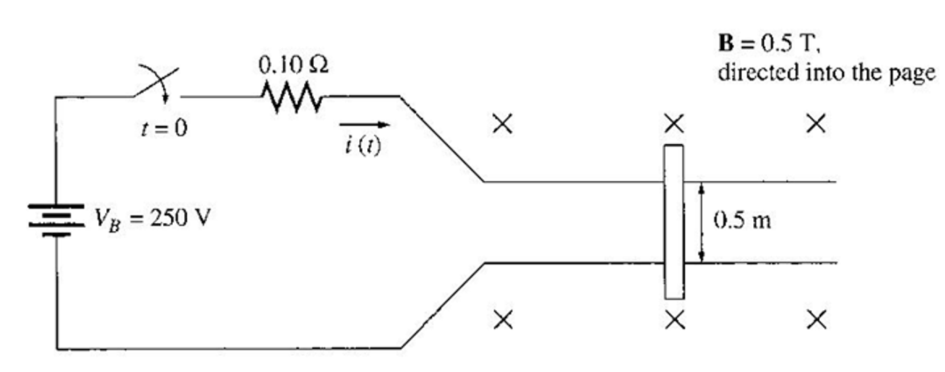
This is precisely the behavior observed in real motors on starting

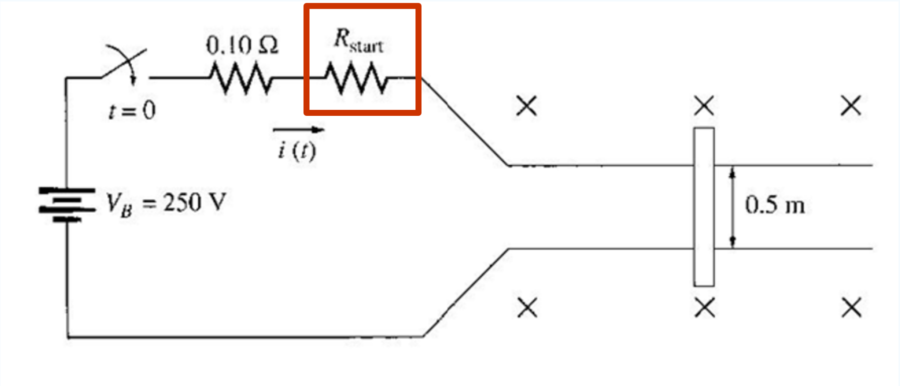
The Linear DC machine –Starting problems

At starting conditions, the speed of the bar is zero, so $e_{ind} = 0$. the current flow at starting is: $i_{start} = V_B/R = 250/0.1 = 2500$ A.

This starting is normally 10 times the rated current of the machine. Such current can cause severe damage to a motor.

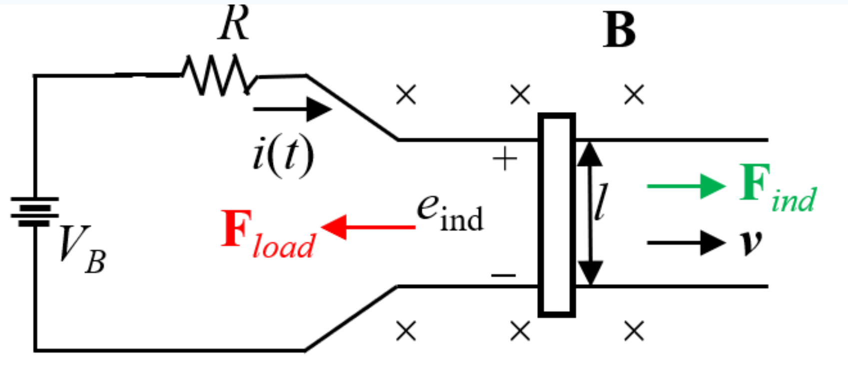
The easiest method for this simple linear machine is to insert an extra resistance into the circuit during starting to limit the current flow until e_{ind} build up enough to limit it.





The Linear DC machine as a Motor (Apply Load)

If the linear machine is initially running at the no-load steady-state conditions, an external load force F_{load} is applied to it, then



$F_{net}=F_{load}-F_{ind}=F_{load}>0$, opposite the direction of the motion

The bar begins to slow down, the induced voltage on the bar drops, $e_{ind}\downarrow=v\downarrow Bl$

As the induced voltage decreases, the current flow in the bar rises $i\uparrow=\dfrac{V_B-e_{ind}\downarrow}{R}$

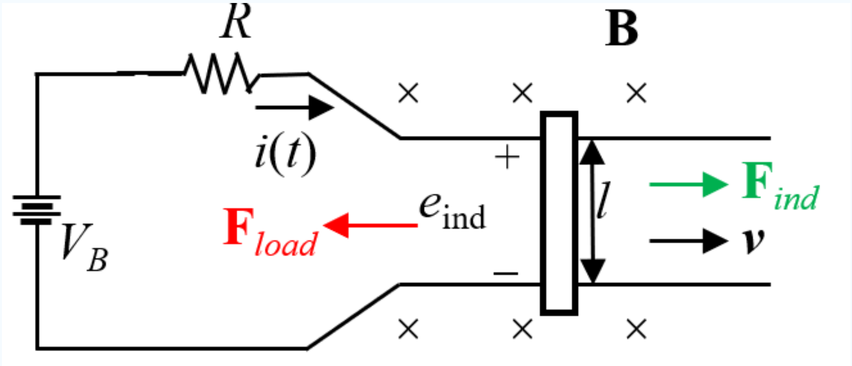
The induced force rises $F_{ind}\uparrow=i\uparrow lB$

The Linear DC machine as a Motor (Apply Load)

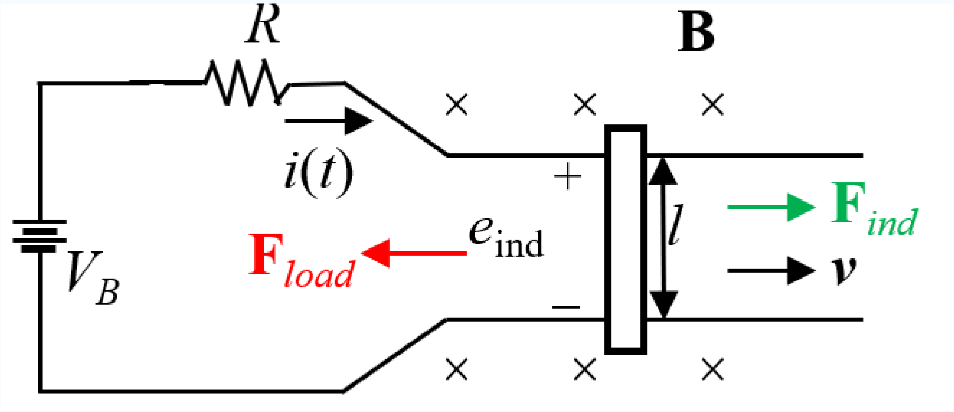
The induced force rises $F_{ind}\uparrow = i\uparrow\, lB$

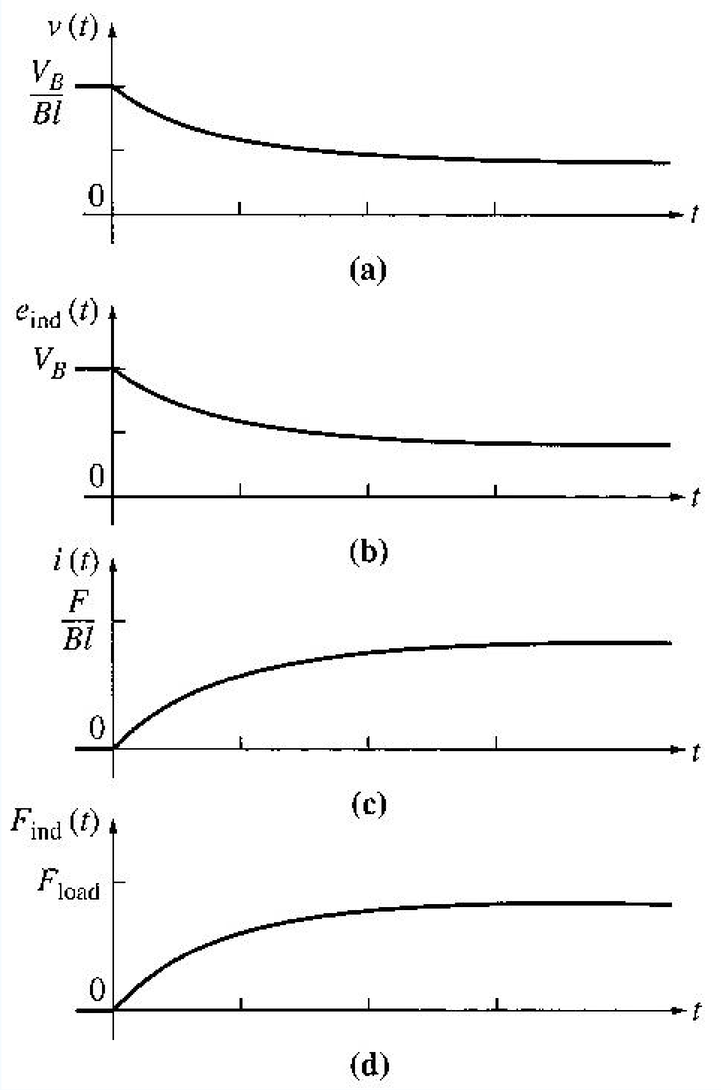
The induced force rises until it is equal and opposite to the load force. $\boldsymbol{F}_{net} = \boldsymbol{F}_{load} - \boldsymbol{F}_{ind} = 0$

Finally, the bar again travels at a lower speed v_{load}.



The Linear DC machine as a Motor (Apply Load)





The linear DC machine operating at no-load conditions and then loaded as a motor

(a) velocity $v\ (t)$ as a function of time
(b) induced voltage $e_{ind}\ (t)$
(c) current $i\ (t)$
(d) induced force $F_{ind}\ (t)$

The Linear DC machine as a Motor (Apply Load)

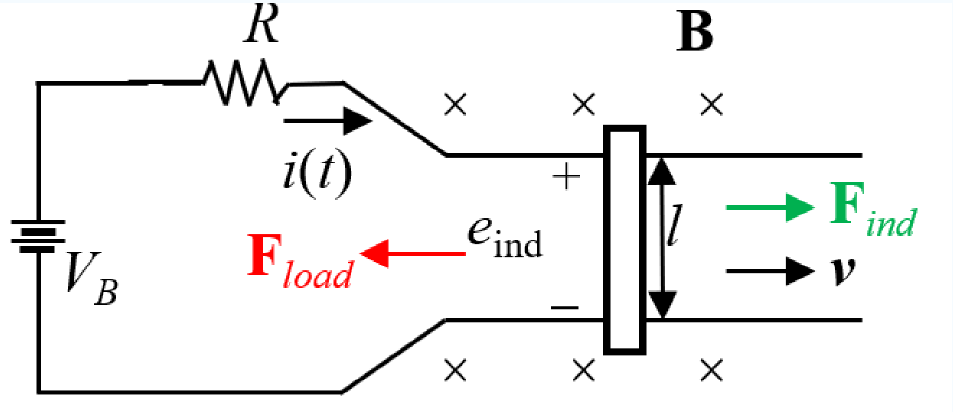
Now an induced force in the direction of motion of the bar, power is being converted *from electrical form to mechanical form* to keep the bar moving

Electric Power $P_e = e_{ind} i = \left(V_B - \frac{F_{ind}}{Bl} R \right) \frac{F_{ind}}{Bl} = \left(V_B - \frac{F_{load}}{Bl} R \right) \frac{F_{load}}{Bl}$

Mechanical Power $P_m = F_{ind} v_{load} = F_{load} \left(\frac{e_{ind}}{Bl} \right) = F_{load} \left(\frac{V_B - iR}{Bl} \right) = \frac{F_{load}}{Bl} \left(V_B - \frac{F_{load}}{Bl} R \right)$

The machine is operating as a motor

$$P_{conv} = P_e = P_m$$
$$P_{conv} = e_{ind} i = F_{ind} v_{load}$$



Rotational Motion

Angular velocity ω

is defined as the rate of change of the **angular displacement** φ with respect to time: $\omega = d\varphi/dt$

SI units: radians per second.

f_m angular velocity expressed in revolutions per second
n_m angular velocity expressed in revolutions per minute

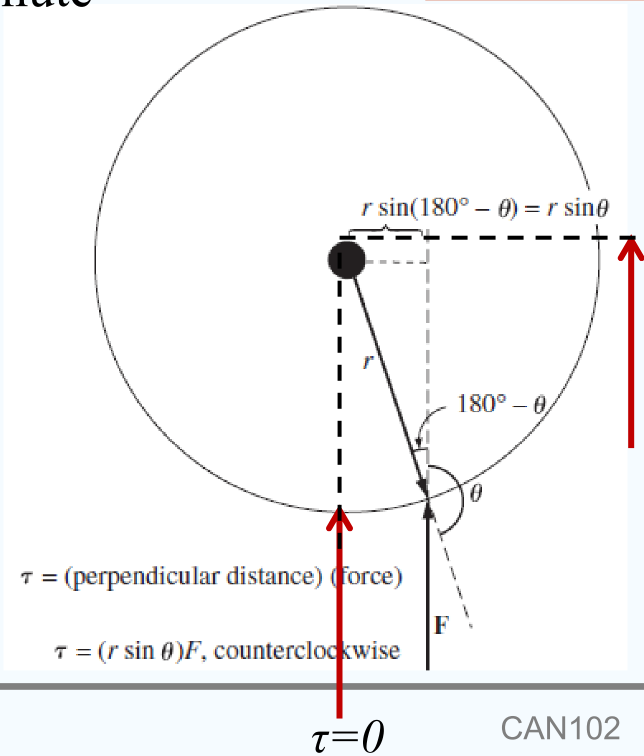
$$f_m = \frac{\omega_m}{2\pi}$$
$$n_m = 60 f_m$$
$$\omega_m = \frac{2\pi n_m}{60}$$

Torque τ

is defined as the product of the force applied to the object and the smallest distance between the line of action of the force and the object's axis of rotation:

τ = (Force applied) (perpendicular distance)

SI units: Newton-meters



Rotational Motion

Work W

For rotational motion, work is the application of a torque through an angle:

$$W = \int \tau d\varphi$$

If the torque is constant :

$$W = \tau\varphi$$

SI units: Joules

Power P

Power is the rate of doing work:

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\varphi)$$

Assuming constant torque :

$$P = \tau\frac{d\varphi}{dt} = \tau\omega$$

SI units: Joules per second

(watts, W)

The Linear DC machine –Real DC Motor Behaves

$$P_{conv} = e_{ind} i = F_{ind} v_{load}$$

A real DC motor behaves in a precisely analogous fashion when it is loaded:

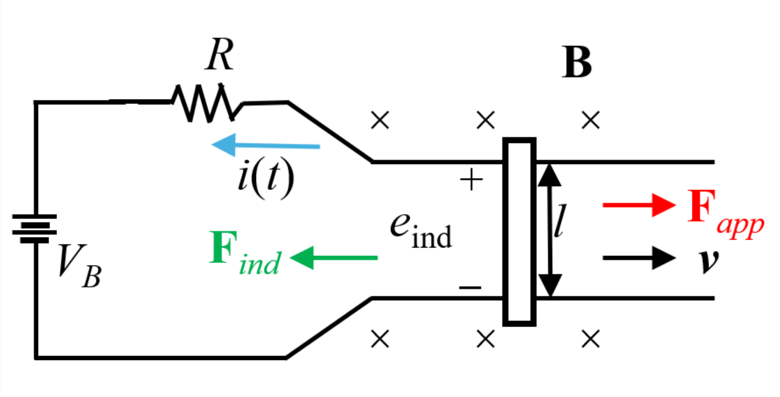
As a load is added to its shaft, the motor begins to slow down, which reduces its internal voltage, increasing its current flow. The increased current flow increases its induced torque, and the induced torque will equal the load torque of the motor at a new, slower speed.

The power converted from electrical form to mechanical form in the real rotating motor:

$$\boxed{P_{conv} = \tau_{ind} \omega}$$

The Linear DC machine as a Generator (Apply Driving)

If the linear machine is initially running at the no-load steady-state conditions, an external driving force F_{app} is applied to it, then



$F_{net}=F_{app}-F_{ind}=F_{app}>0$, in the direction of the motion

⇓

The bar begins to accelerate, the induced voltage on the bar increase and will be larger than the battery voltage V_B, $e_{ind}\uparrow=v\uparrow Bl>V_B$

⇓

As the induced voltage increases, the current flow in the bar rises

$$i\uparrow=\frac{e_{ind}\uparrow-V_B}{R}$$

⇓

The induced force rises $F_{ind}\uparrow=i\uparrow lB$, to the left

The Linear DC machine as a Generator (Apply Driving)

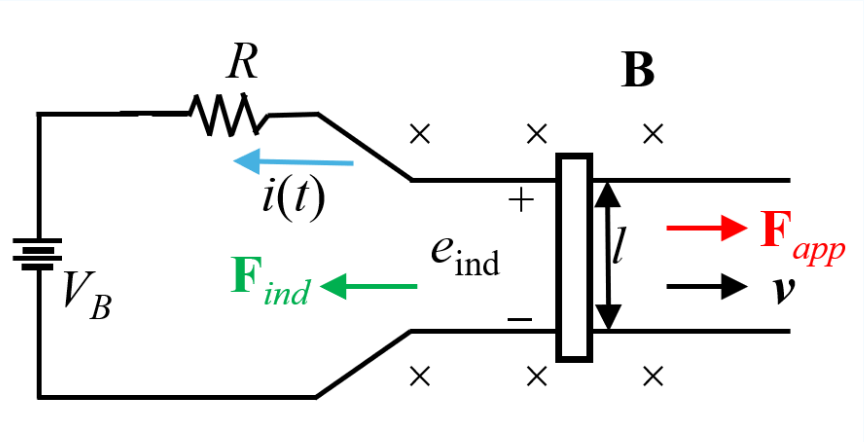
The induced force rises $F_{ind}\uparrow = i\uparrow\ lB$

⇓

The induced force rises until it is equal and opposite to the applied driving force. $\boldsymbol{F}_{net}=\boldsymbol{F}_{app}-\boldsymbol{F}_{ind}=0$

⇓

Finally, the bar again travels at a higher speed v_{app}.



The Linear DC machine as a Generator (Apply Driving)

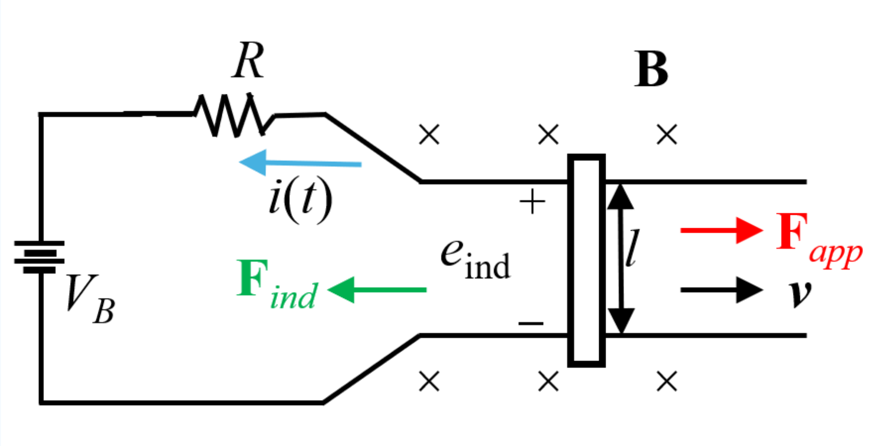
Now an induced force is opposite to the direction of motion of the bar, power is being converted *from mechanical form to electrical form* to charge the batter

Electric Power $$P_e = e_{ind} i = \left(V_B + \frac{F_{ind}}{Bl} R\right)\frac{F_{ind}}{Bl} = \left(V_B + \frac{F_{app}}{Bl} R\right)\frac{F_{app}}{Bl}$$

Mechanical Power $$P_m = F_{ind} v_{app} = F_{app}\left(\frac{e_{ind}}{Bl}\right) = F_{app}\left(\frac{V_B + iR}{Bl}\right) = \frac{F_{app}}{Bl}\left(V_B + \frac{F_{app}}{Bl} R\right)$$

This machine is operating as a generator

$$\boxed{\begin{aligned} P_{conv} &= P_e = P_m \\ P_{conv} &= e_{ind} i = F_{ind} v_{app} \end{aligned}}$$



The Linear DC machine –Real DC Generator Behaves

A real DC generator behaves in precisely this manner:

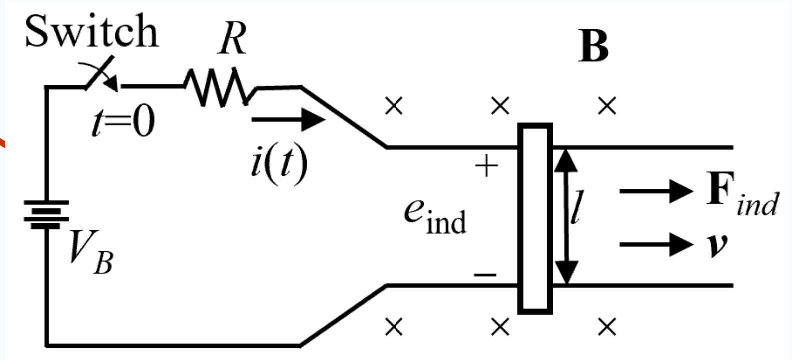
A torque is applied to the shaft in the direction of motion, the speed of the shaft increase, the internal voltage increases, current flows out of the generator to the loads.

The amount of mechanical power converted to electric form in the real rotating generator:

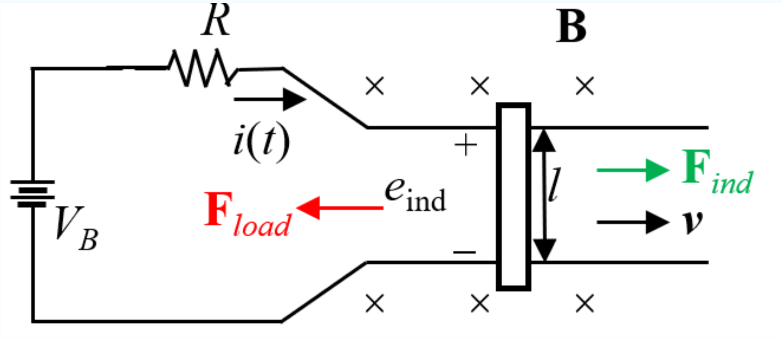
$$P_{conv} = \tau_{ind}\omega$$

The Linear DC machine –Summary

$F = 0$, $e_{ind} = V_B$, $i = 0$, and $v_{nl} = V_B/Bl$.



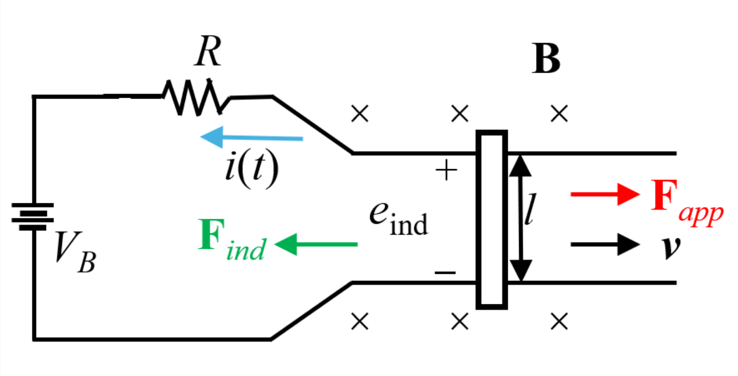
Starting a linear DC machine



The linear DC machine as a motor

$F_{ind} = F_{load}$ (with opposite directions) at a lower speed v_{load}.

Electric power $e_{ind}i$ is being converted to mechanical power $F_{ind}v_{load}$



The linear DC machine as a generator

$F_{ind} = F_{app}$ (with opposite directions) at a higher speed v_{app}.

Mechanical power $F_{ind}v_{app}$ is being converted to electric power $e_{ind}i$

The Linear DC machine –Summary

It is interesting that the same machine can act as both motors and generators:

- The only difference between the two is whether the externally applied forces are in the direction of motion (generator) or opposite to the direction of motion (motor)

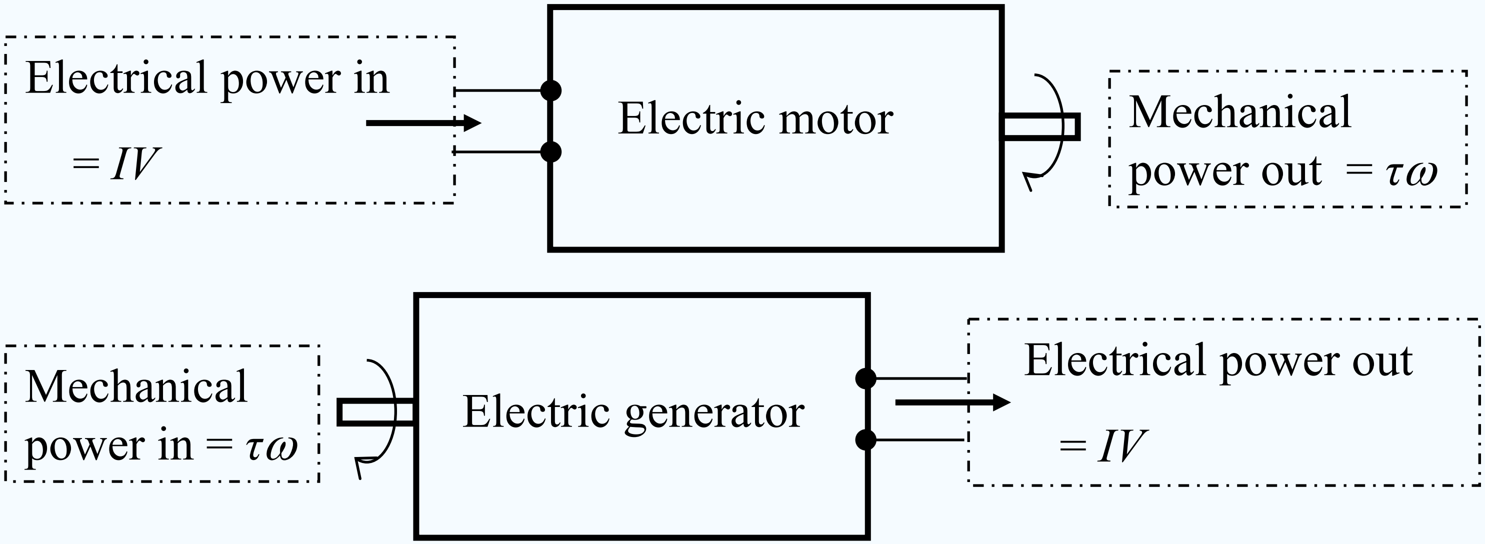
- Electrically, when $e_{ind} > V_B$, the machine acts as a generator, and when $e_{ind} < V_B$, the machine acts as a motor

- Whether the machine is a motor or a generator, both induced force (motor action), and induced voltage (generator action) are present at all times

- The machine is a generator when it moved rapidly ($v_{app}>v_{nl}$) and a motor when it moved more slowly ($v_{load}<v_{nl}$) , but whether it was a motor or a generator, **it always moved in the same direction**

- The induced force is always opposite to the applied force to oppose its change.

Power Conversion

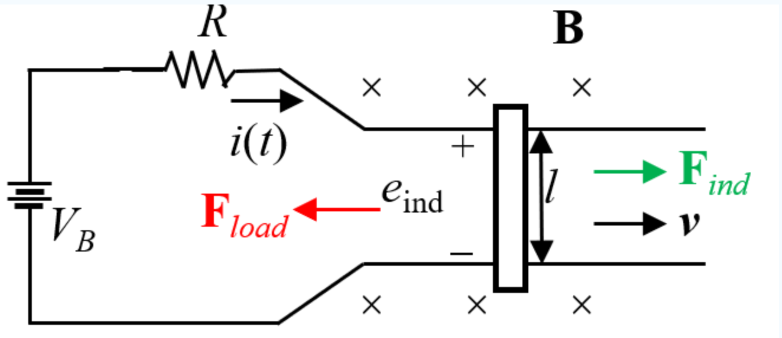


•In general, a three-dimensional vector is required to completely describe the rotation of an object in space.

•In CAN102, a counterclockwise (CCW) angle of rotation is taken as positive, and a clockwise (CW) one is negative.

•Now, all the concepts of the rotation motion reduces to scalars.

Example



A linear machine has the following characteristics:

$B = 0.33$ T into page $\quad R = 0.50\ \Omega$

$l = 0.5$ m $\quad V_B = 120$ V

(a) If this bar has a load of 10 N attached to it opposite to the direction of motion, what is the steady-state speed of the bar?

(b) If the bar runs off into a region where the flux density falls to 0.30 T, what happens to the bar? What is its final steady-state speed?

(c) Suppose V_B is now decreased to 80 V with everything else remaining as in part (b). What is the new steady-state speed of the bar?

(d) From the results for parts (b) and (c), what are two methods of controlling the speed of a linear machine (or a real DC motor)?

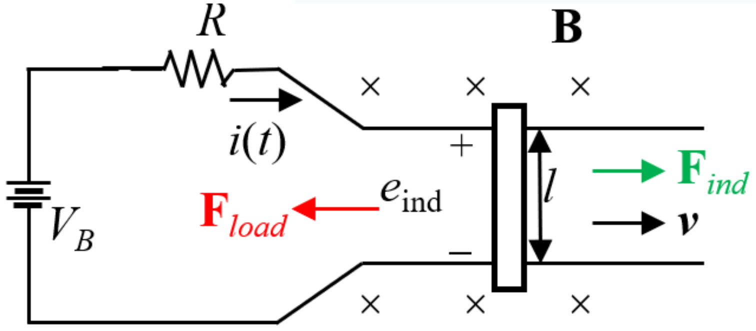
Example

$B = 0.33$ T into page $R = 0.50\ \Omega$
$l = 0.5$ m $V_B = 120$ V

(a) With a load of 10 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_{\text{load}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{load}}}{Bl} = \frac{10\ \text{N}}{(0.33\ \text{T})(0.5\ \text{m})} = 60.5\ \text{A}$$



The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 120\ \text{V} - (60.5\ \text{A})(0.50\ \Omega) = 89.75\ \text{V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{89.75\ \text{V}}{(0.33\ \text{T})(0.5\ \text{m})} = 544\ \text{m/s}$$

Example

(b) If the flux density drops to 0.30 T while the load on the bar remains the same, there will be a speed transient until $F_{\text{load}} = F_{\text{ind}} = 10\text{ N}$ again.

The new steady state current will be

$$F_{\text{load}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{load}}}{Bl} = \frac{10\text{ N}}{(0.30\text{ T})(0.5\text{ m})} = 66.7\text{ A}$$

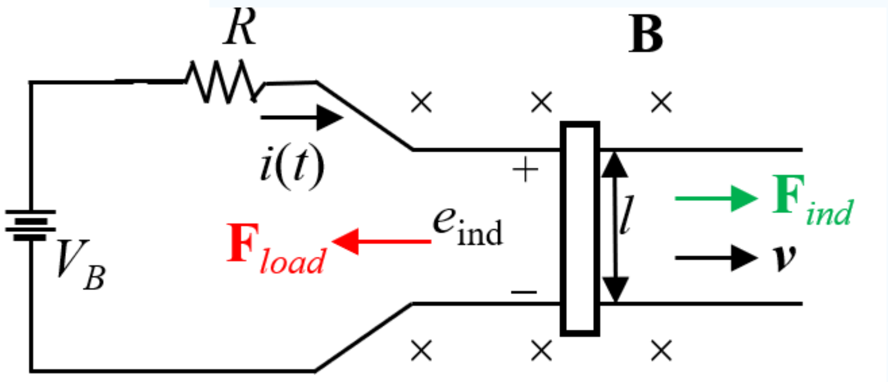
The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 120\text{ V} - (66.7\text{ A})(0.50\ \Omega) = 86.65\text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{86.65\text{ V}}{(0.30\text{ T})(0.5\text{ m})} = 577\text{ m/s}$$

$B = 0.33$ T into page $R = 0.50\ \Omega$
$l = 0.5$ m $V_B = 120$ V



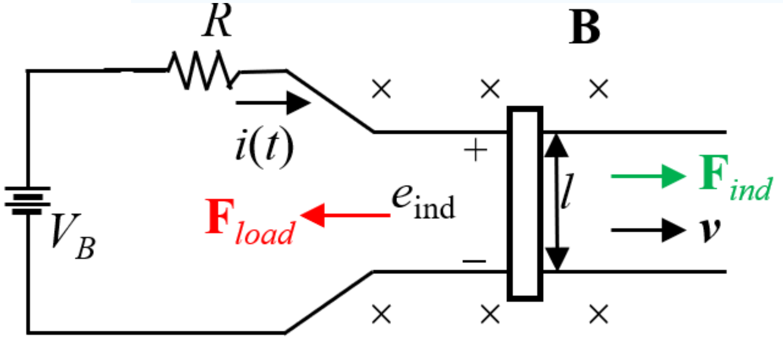
0.33T	60.5 A	89.75 V	544 m/s
0.30 T	66.7 A	86.65 V	577 m/s

$B \downarrow \Rightarrow v \uparrow$

Example

(c) If the battery voltage is decreased to 80 V while the load on the bar remains the same, there will be a speed transient until $F_{\text{load}} = F_{\text{ind}} = 10\text{ N}$ again.

$B = 0.33\text{ T}$ into page $R = 0.50\ \Omega$
$l = 0.5\text{ m}$ $V_B = 120\text{ V}$



The new steady state current will be

$$F_{\text{load}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{load}}}{Bl} = \frac{10\text{ N}}{(0.30\text{ T})(0.5\text{ m})} = 66.7\text{ A}$$

120 V	66.7 A	86.65 V	577 m/s
80 V	66.7 A	46.65 V	311 m/s

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 80\text{ V} - (66.7\text{ A})(0.50\ \Omega) = 46.65\text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{46.65\text{ V}}{(0.30\text{ T})(0.5\text{ m})} = 311\text{ m/s}$$

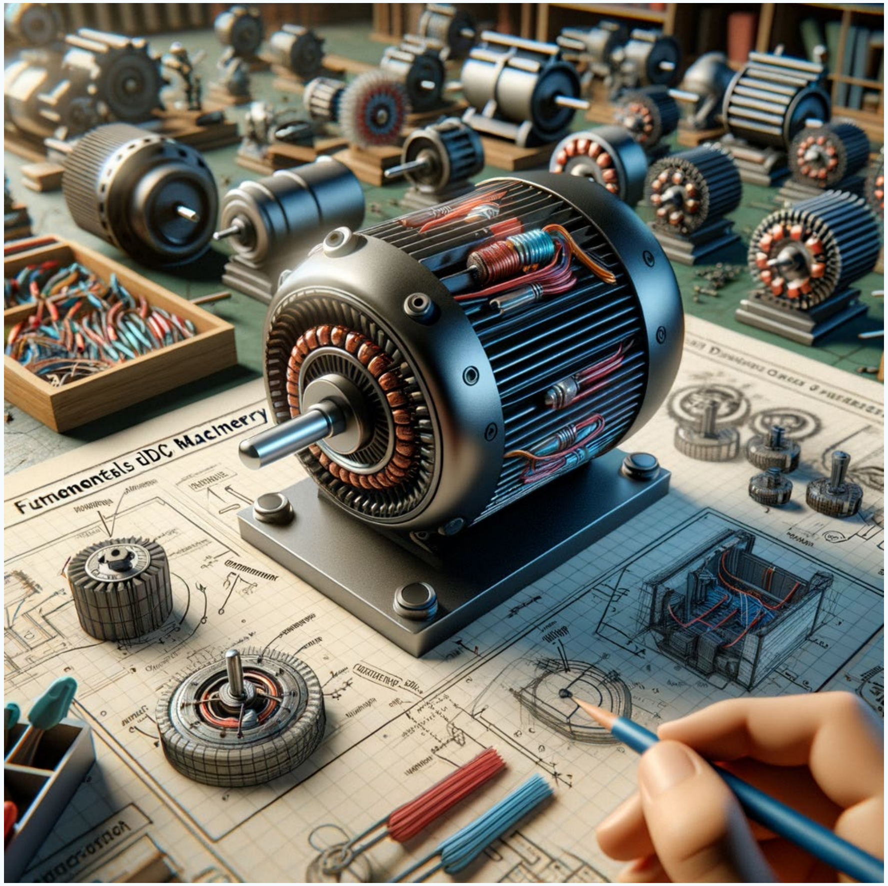
$$V_B \downarrow \Rightarrow v \downarrow$$

Example

(d) From the results of the two previous parts, we can see that there are two ways to control the speed of a linear DC machine:

- *reducing* the flux density B of the machine *increases* the steady-state speed, and
- *reducing* the battery voltage V_B *decreases* the stead-state speed of the machine.
- Both of these speed control methods work for real DC machines as well as for linear machines.

Next



DC Machinery
Fundamentals

Thanks for your attention