MTH102 Engineering Mathematics II

Lesson 11: Random process

Term: 2024

Outline

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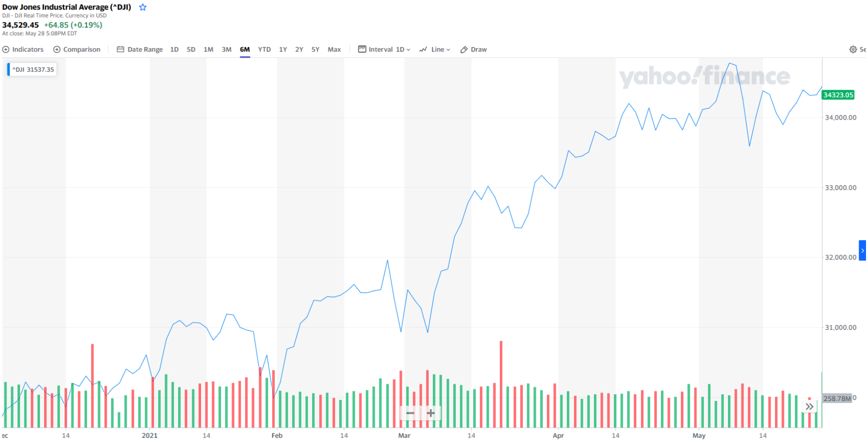
Definition

- A random process is a collection of random variables defined on a set of indices T as
$$\{X(t),\ t \in T\}.$$
$X(t)$ and T can be either discrete or continuous.
- The random variables can be indexed with respect to time. For example, the temperature during the day.
- The random variables can be indexed with respect to space. For example, the temperature of the water in the ocean.
- The distribution of the random process is defined by the collection of joint cumulative distribution functions
$$F_{X(1),X(2),\ldots,X(k)}(x_1, x_2, \ldots, x_k) = P(X(1) \le x_1, X(2) \le x_2, \ldots, X(k) \le x_k).$$

Example: Dow Jones Industrial Average



Outline

Definition

We are interested in some "events" which are occurring at random points of time, and let $X(t)$ be the number of events that occur in the time interval $[0, t]$. The collection of random variables $\{X(t), t \geq 0\}$ is said to be a **Poisson process with rate** λ, $\lambda > 0$, if

(i) $X(0) = 0$.

(ii) The numbers of events that occur in disjoint time intervals are independent, i.e. for any $s_1 < t_1 < s_2 < t_2$,

$$(X(t_1) - X(s_1)) \text{ and } (X(t_2) - X(s_2)) \text{ are independent.}$$

(iii) The distribution of the number of events that occur in an interval depend only on the length of the interval and not on its location, i.e. for any $0 \leq s \leq t$ and $k \in \mathbb{N}$,

$$P(X(t) - X(s) = k) = P(X(t - s) = k).$$

(iv) For any $t \geq 0$, $X(t)$ has a Poisson distribution with mean λt, i.e.

$$P(X(t) = k) = e^{-\lambda t}(\lambda t)^k / k!,\ \ k = 0, 1, 2, \ldots.$$

Example

Defects occur along an undersea cable according to a Poisson process of rate $\lambda = 1$ per mile.

(a) What is the probability that 1 defect appears in the first mile of cable?

(b) What is the probability that no defects appear in the first two miles of cable?

(c) Given that there are 2 defects in the first 5 miles of cable, what is the conditional probability that there is 1 defect appear in the first 3 miles of cable?

Example

(a) $P(X(1)=1)=e^{-1}$.

(b) $P(X(2)=0)=e^{-2}$.

(c)

$$
\begin{aligned}
P(X(3)=1|X(5)=2) &= \frac{P(X(3)=1, X(5)=2)}{P(X(5)=2)} \\
&= \frac{P(X(3)=1, X(5)-X(3)=1)}{P(X(5)=2)} \\
&= \frac{P(X(3)=1)P(X(5)-X(3)=1)}{P(X(5)=2)} \\
&= \frac{P(X(3)=1)P(X(2)=1)}{P(X(5)=2)} \\
&= \frac{3e^{-3}\cdot 2e^{-2}}{\frac{5^2}{2!}e^{-5}} \\
&= 12/25.
\end{aligned}
$$