

# Semiconductor Fundamentals – (III)

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2.5 Boltzmann approximation &  $E_F$ ,  $n$ ,  $p$

2.6 Carrier drift and diffusion

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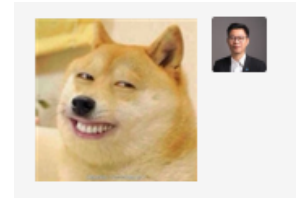
***Mar 2024***

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**2.5 Boltzmann approximation &  $E_F$ ,  $n$ ,  $p$**

2.6 Carrier drift and diffusion



# Last lecture:

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- **Negative charges:**
  - Conduction electrons (density =  $n$ ) **mobile**
  - Ionized acceptor atoms (density =  $N_A^-$ ) **immobile**
- **Positive charges:**
  - Holes (density =  $p$ ) **mobile**
  - Ionized donor atoms (density =  $N_D^+$ ) **immobile**
- The **net charge density** (C/cm<sup>3</sup>) in a semiconductor is
$$\rho = q(p - n + N_D^+ - N_A^-)$$
- Law of Mass Action:  $n \times p = n_i^2$

How to deduce the **relationship** between  $E_F$  and  $n/p$ ?

## 2.5 Boltzmann approximation & $E_F$ , $n$ , $p$

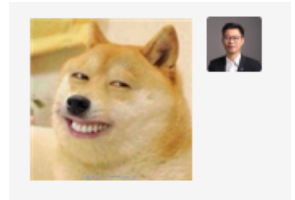
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- **Fermi function and Fermi level**

- Density of States

- Boltzmann Approximation

- Electron and Hole Concentrations



# Thermal Equilibrium

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- **No external forces** are applied:
  - electric field = 0, magnetic field = 0
  - mechanical stress = 0
  - no light
- **Dynamic situation** in which every process is **balanced** by its **inverse process**
  - Electron-hole pair (EHP) **generation rate** = EHP **recombination rate**
- **Thermal agitation** → electrons and holes exchange energy with the crystal lattice and each other
  - Every energy state in the conduction band and valence band has a certain probability of being occupied by an electron

# Statistical Thermodynamics: Fermi energy

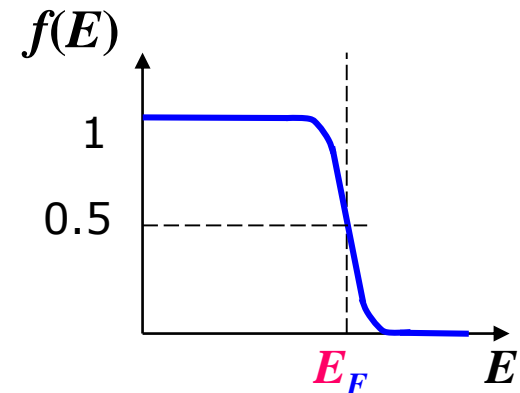
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- The Fermi energy,  $E_F$ , is the energy associated with a particle, which is in thermal equilibrium with the system of interest. The energy is strictly associated with the particle and does not consist even in part of heat or work. This same quantity is called the electrochemical potential,  $\mu$ , in most thermodynamics texts.
- <http://hyperphysics.phy-astr.gsu.edu/Hbase/solids/fermi.html#c2>
- <http://hyperphysics.phy-astr.gsu.edu/Hbase/solids/fermi.html#c1>

# Fermi function and Fermi level

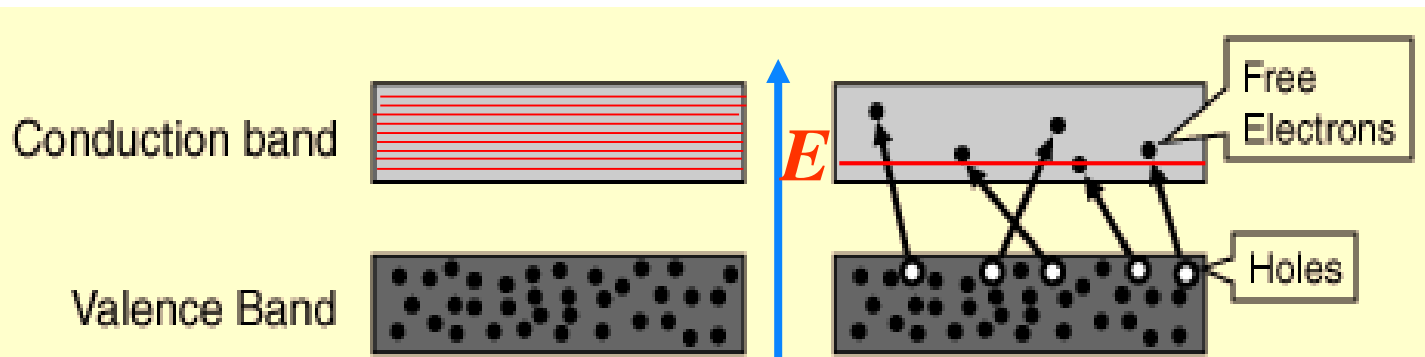
- **Probability** that a **state** at energy level,  $E$ , is occupied by one electron is,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$



- $f(E)$ : Fermi-Dirac function
- An increase in  $E$  will reduce  $f(E)$
- **$E_F$  --- Fermi-level**

➤ When  $E = E_F$ ,  $f(E=E_F) = \underline{0.5}$ .

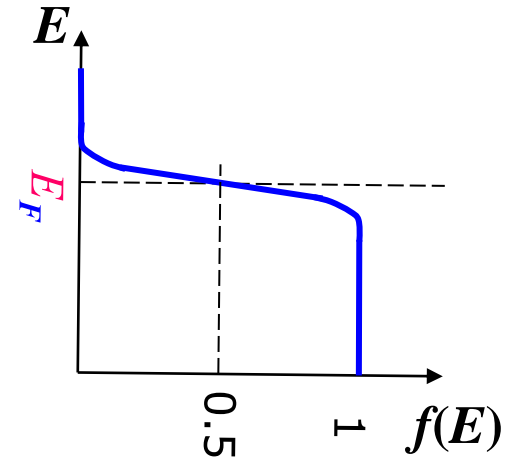
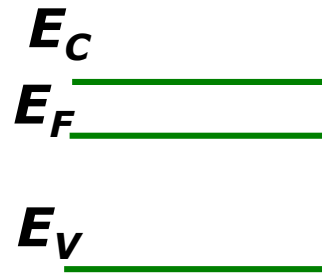


**textbook**  
**P.66**

# Fermi function and Fermi level

- Probability that a **state** at energy level,  $E$ , is occupied by one electron is,

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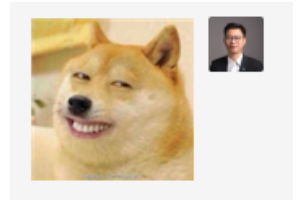
1. **Simplify** Fermi-Dirac function: **Boltzmann Approximation**
2. What is the **states' density**? (**Density of States**)



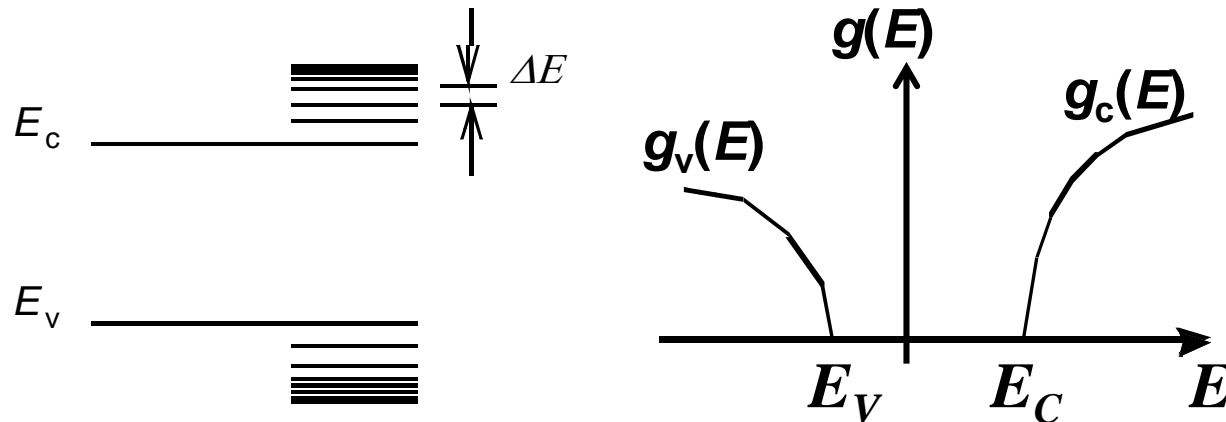
## 2.5 Boltzmann approximation & $E_F$ , $n$ , $p$

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- Fermi function and Fermi level
- **Density of States**
- Boltzmann Approximation
- Electron and Hole Concentrations



# Density of States



$g(E)\Delta E =$  **number of states** per  $\text{cm}^3$  in the energy range between  $E$  and  $E+\Delta E$

Near the band edges:

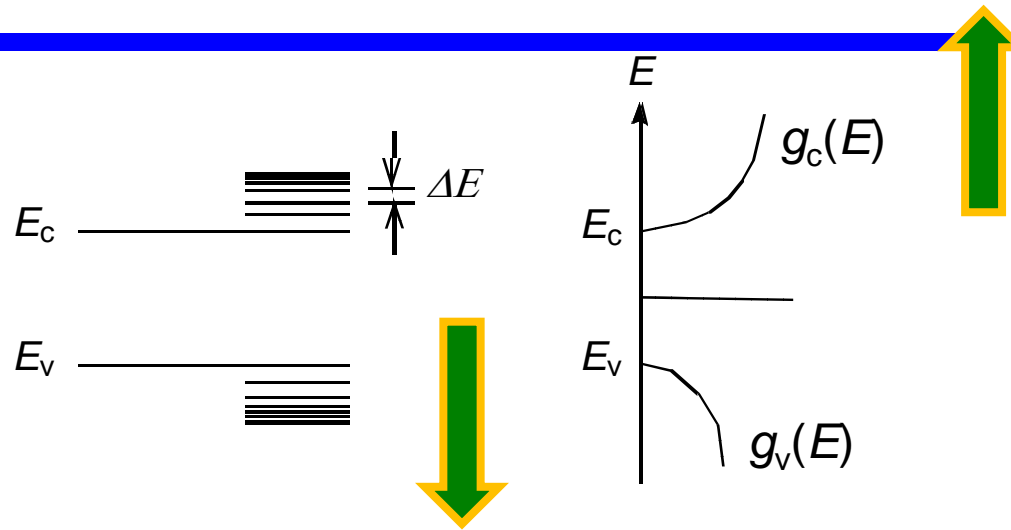
$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \quad \boxed{E \geq E_c}$$

density of states in the **conduction band**

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3} \quad \boxed{E \leq E_v}$$

density of states in the **valence band**

# Density of States



$g(E)dE =$  **number of states** per  $\text{cm}^3$  in the energy range between  **$E$**  and  **$E+dE$**

Near the band edges:

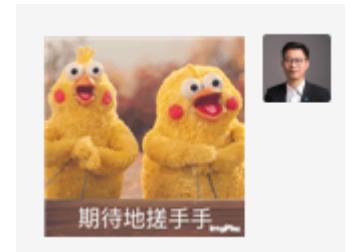
$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \quad E \geq E_c$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3} \quad E \leq E_v$$

## 2.5 Boltzmann approximation & $E_F$ , $n$ , $p$

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- Fermi function and Fermi level
- Density of States
- **Boltzmann Approximation**
- Electron and Hole Concentrations



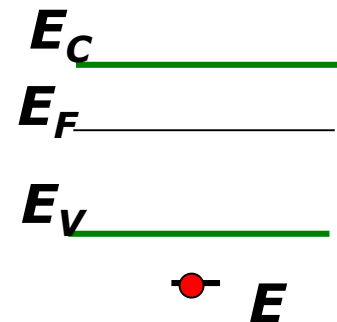
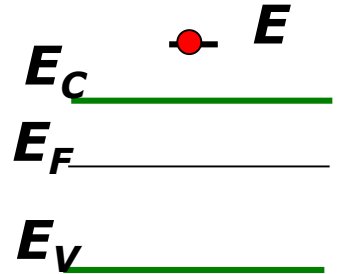
# Boltzmann Approximation

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$\text{If } E - E_F > 3kT, f(E) \cong e^{-(E - E_F)/kT}$$

because of  $\exp[(E - E_F)/(kT)] \gg 1$

$$\text{If } E_F - E > 3kT, f(E) \cong 1 - e^{-(E_F - E)/kT}$$



Probability that a **state** is **empty**:

$$\underline{1 - f(E)} \cong e^{-(E_F - E)/kT} = e^{-(E_F - E)/kT}$$

Probability that a **state** is occupied by a **hole**

## 2.5 Boltzmann approximation & $E_F$ , $n$ , $p$

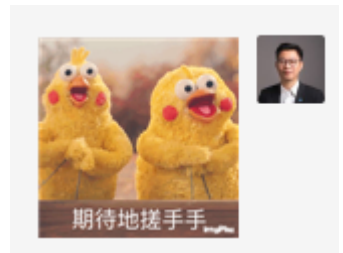
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➤ Fermi function and Fermi level

➤ Density of States

➤ Boltzmann Approximation

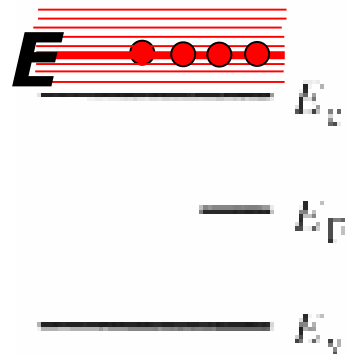
➤ **Electron and Hole Concentrations**



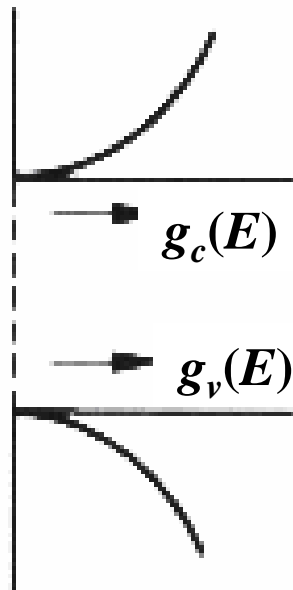
# Equilibrium Distribution of **Electrons**

- Obtain  **$n(E)$**  by multiplying  **$g_c(E)$**  and  **$f(E)$**

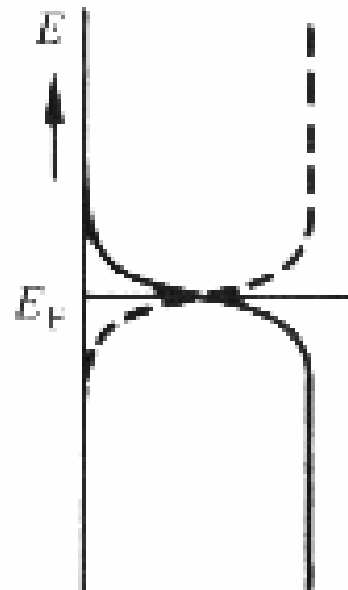
Energy band diagram



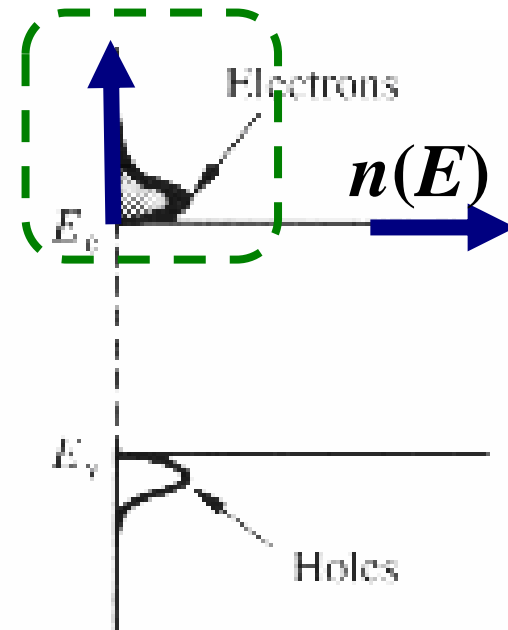
Density of States



Probability of occupancy



Carrier distribution



# Equilibrium **Electron Concentrations**

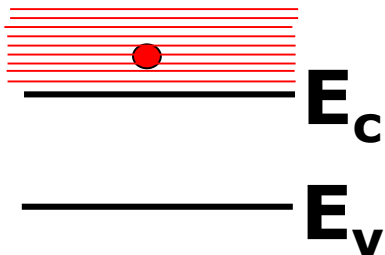
- **Integrate**  $n(E)$  over all the energies in the conduction band to obtain  $n$ :

$$n = \int_{E_c}^{\infty} g_c(E) f(E) dE$$

*(Note: In the original image, a red box highlights the infinity symbol, a red arrow points from the text 'top of conduction band' to the lower limit  $E_c$ , and the integrand  $g_c(E)f(E)dE$  is enclosed in a green dashed box.)*

- By using the **Boltzmann approximation**, and extending the integration limit to  $\infty$ , we obtain

$$n = N_c e^{-(E_c - E_F)/kT} \quad \text{where} \quad N_c = 2 \left( \frac{2\pi m_n^* kT}{h^2} \right)^{3/2}$$

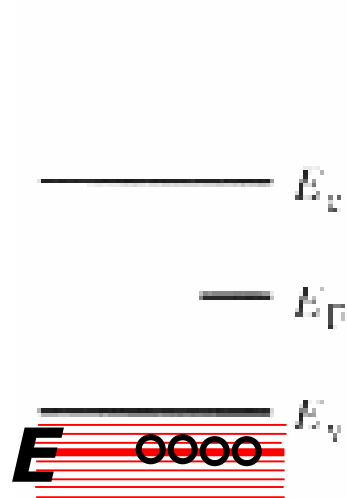




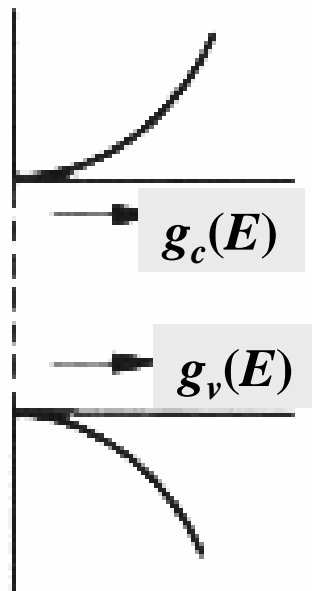
# Equilibrium Distribution of **Holes**

- Obtain  **$p(E)$**  by multiplying  $g_v(E)$  and  $1-f(E)$

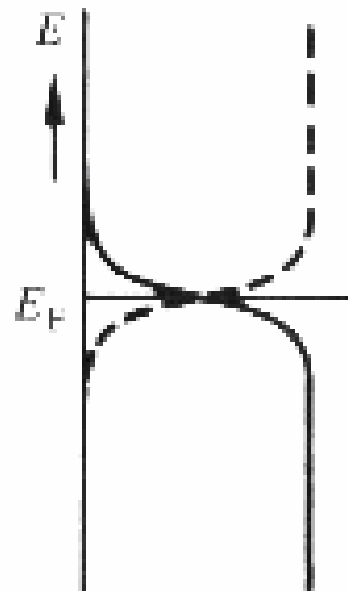
Energy band diagram



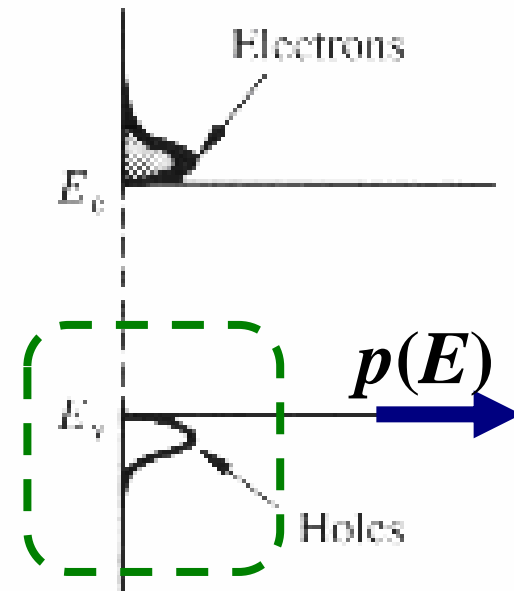
Density of States



Probability of occupancy



Carrier distribution



# Equilibrium Hole Concentrations

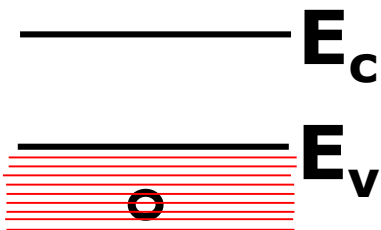
- **Integrate**  $p(E)$  over all the energies in the valence band to obtain  $p$ :

$$p = \int_{-\infty}^{E_v} g_v(E) [1 - f(E)] dE$$

of valenceband

- By using the **Boltzmann approximation**, and extending the integration limit to  $-\infty$ , we obtain

$$p = N_v e^{-(E_F - E_v)/kT} \quad \text{where} \quad N_v = 2 \left( \frac{2\pi m_p^* kT}{h^2} \right)^{3/2}$$



# Intrinsic Carrier Concentration

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$$\begin{aligned} np &= \left( N_c e^{-(E_c - E_F)/kT} \right) \left( N_v e^{-(E_F - E_v)/kT} \right) \\ &= N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT} \\ &= n_i^2 \end{aligned}$$

**Law of Mass Action**

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

# Electron and hole concentrations

$$n = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$p = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$n \cdot p = n_i^2$$

+

p.83,ref1,  $E_{Fi} \approx E_i$

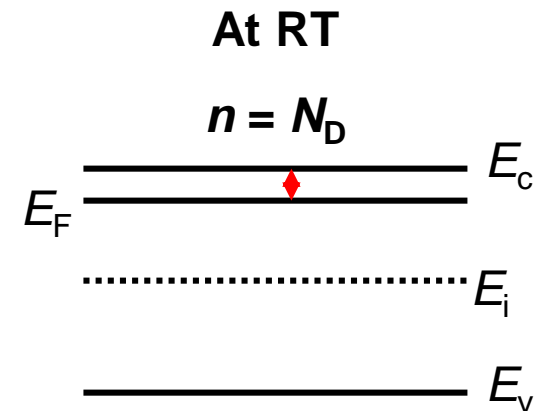
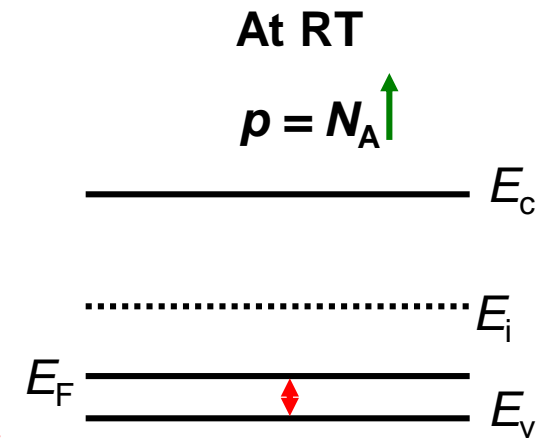
$$n_i = N_C \exp\left[\frac{-(E_C - E_i)}{kT}\right]$$

$$n_i = N_V \exp\left[\frac{-(E_i - E_V)}{kT}\right]$$



$$n = n_i \exp\left[\frac{(E_F - E_i)}{kT}\right]$$

$$p = n_i \exp\left[\frac{-(E_F - E_i)}{kT}\right]$$



## HW3: Energy-band diagram

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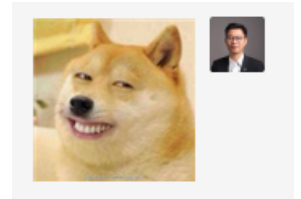
Question: Where is  $E_F$  for  $n = 10^{17} \text{ cm}^{-3}$  ?

# Semiconductor Fundamentals – (III)

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2.5 Boltzmann approximation &  $E_F$ ,  $n$ ,  $p$

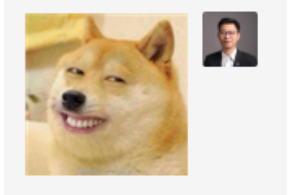
**2.6 Carrier drift and diffusion**



# 2.6 Carrier drift and diffusion

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- **Carrier scattering**

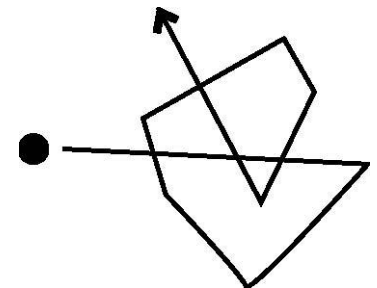
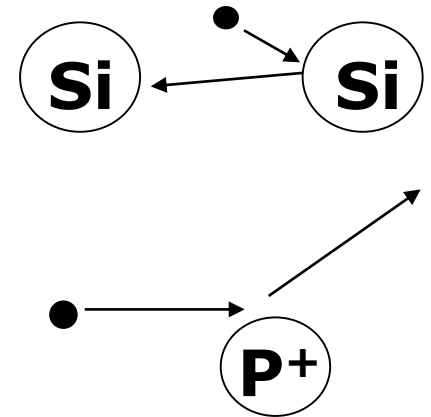


- Carrier drift:
  - *Carrier mobility*
  - *Conductivity & Resistivity*
  - *Energy band model*
- Carrier diffusion

**Reading: Chapter 2.6**

# Thermal Motion

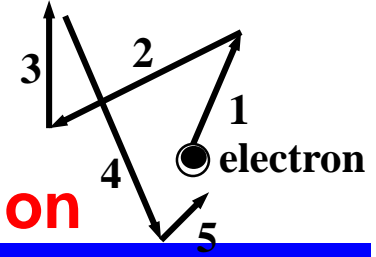
- In thermal equilibrium, **carriers** are **not sitting still**:
  - undergo **collisions** with **vibrating Si atoms** (Brownian motion)
  - electrostatically **interact** with **charged dopants** and with each other
- Characteristic **time constant** of thermal motion
  - mean free time between collisions:
  - $\tau_c \equiv$  collision time [s]
  - In between collisions, carriers acquire high velocity:  $v_{th} \equiv$  thermal velocity [cm/s]
  - ...**but get nowhere!** (on average)
- Characteristic **length** of thermal motion:
  - $\lambda \equiv$  mean free path [cm],  $\lambda = v_{th} \tau_c$



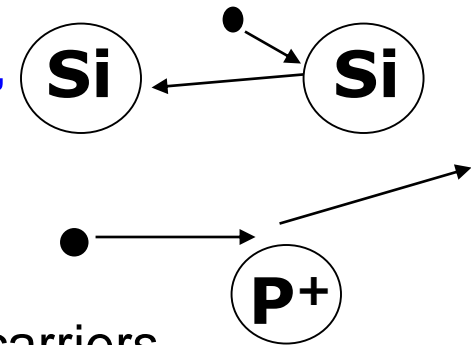


# Carrier Scattering

random motion



- **Mobile electrons** and atoms in the Si lattice are always in **random thermal motion**.
  - **Average velocity** of thermal motion for electrons in Si:  $\sim 10^7$  cm/s @ 300K
  - Electrons make frequent “**collisions**” with the **vibrating atoms**
    - “Lattice Scattering” or “Phonon Scattering”
  - Other scattering mechanisms:
    - deflection by **ionized impurity atoms**
    - deflection due to Coulombic force between carriers
- **The average current in any direction is zero, if no electric field is applied.**



# Effective Mass

- Under an **externally applied force,  $F_{\text{ext}}$** , the movement of electrons (or holes) is influenced by the positively charged protons and by negatively charged electron in the lattice. So, the movement in the crystal is different from that in vacuum.
- The total force  $F_{\text{total}}$

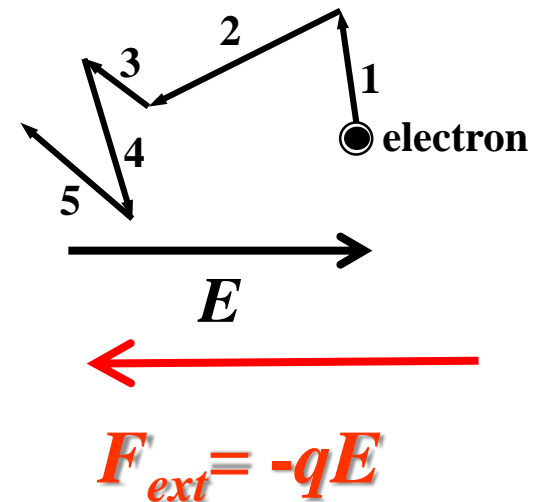
$$F_{\text{total}} = F_{\text{ext}} + F_{\text{int}} = ma$$

where  **$a$**  is the **acceleration**,  $F_{\text{int}}$  is the internal force. We can write

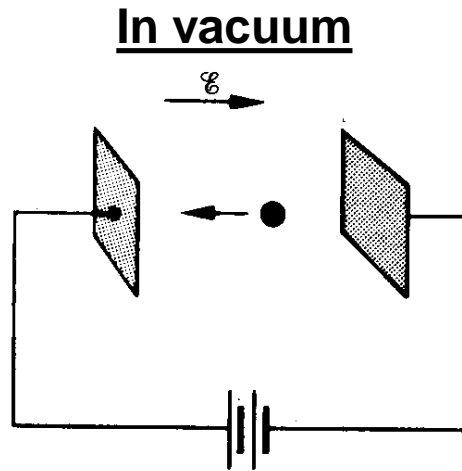
$$F_{\text{ext}} = m^* a$$

where  **$m^*$**  is called **effective mass**.

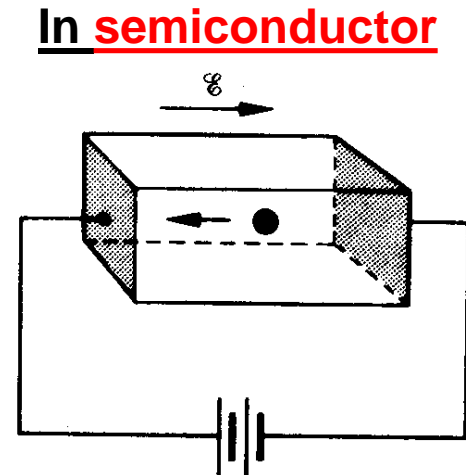
Notation:  $m_n^*$  for **electrons**,  $m_p^*$  for **holes**,



# Electrons as Moving Particles



$$F = (-q)E = m_o a$$



$$F_{\text{ext}} = (-q)E = m_n^* a$$

where  $m_n^*$  is the **electron effective mass**.

If  $\tau_{cn}$  is **electron mean free time** between collisions,

$$|a| = dv/dt \approx v_e / \tau_{cn}$$

$$|a| = qE / m_n^*$$

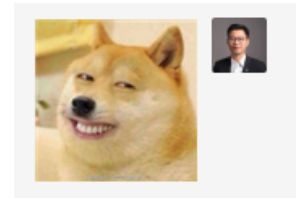
$$\Rightarrow v_e = \frac{q \tau_{cn} E}{m_n^*}, \quad v_h = \frac{q \tau_{cp} E}{m_p^*}$$

**average drift velocity**

## 2.6 Carrier drift and diffusion

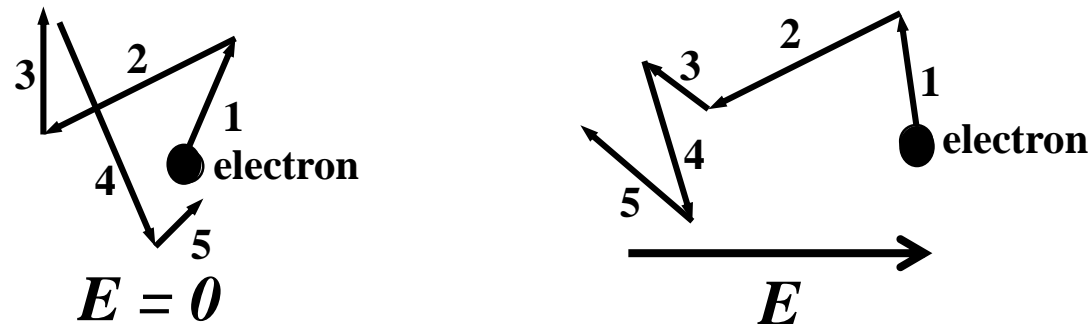
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- Carrier scattering
- **Carrier drift:**
  - *Carrier mobility*
  - *Conductivity & Resistivity*
  - *Energy band model*
- Carrier diffusion



# Carrier Drift

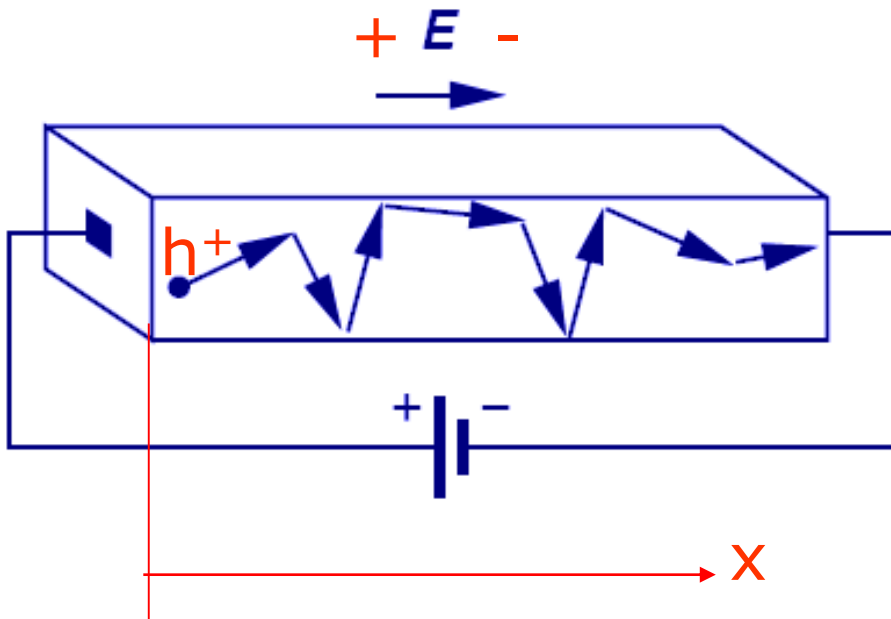
- When an **electric field** (e.g., due to an externally applied voltage) is applied to a **semiconductor**, **mobile charge-carriers** will be accelerated by the **electrostatic force**. This force superimposes on the random motion of electrons:



- **Electrons drift** in the direction **opposite** to the  **$E$ -field**  
→ Current flows
- ❖ Because of scattering, electrons in a semiconductor **do not achieve constant acceleration**. However, they can be viewed as classical particles moving at a constant average drift velocity.

# Carrier Drift

- The process in which **charged particles move** because of an **electric field** is called **drift**.
- Charged particles within a semiconductor move with an average **velocity proportional** to the **electric field**.
  - The **proportionality constant** is the carrier **mobility**.



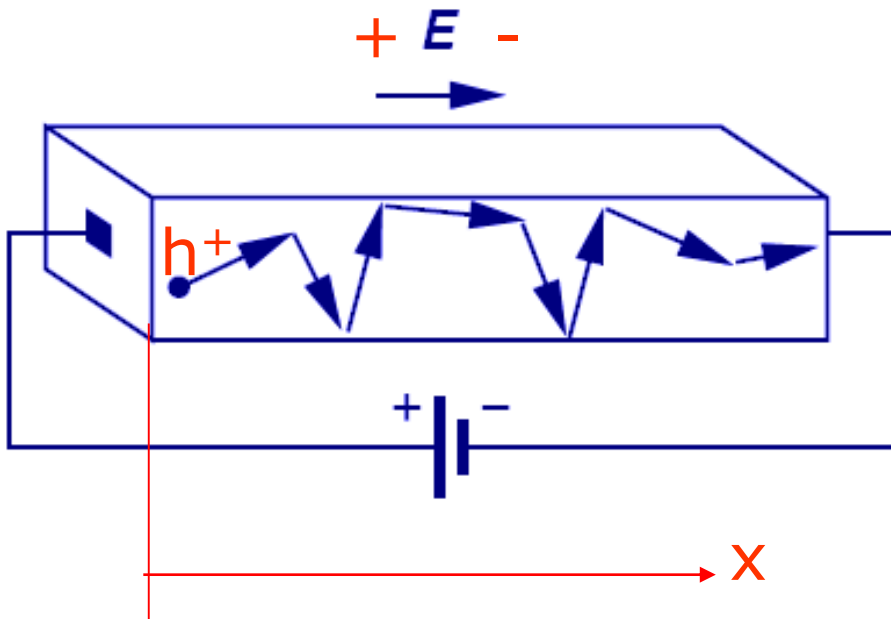
Hole velocity  $\vec{v}_h = \mu_p \vec{E}$

Electron velocity  $\vec{v}_e = -\mu_n \vec{E}$

# Carrier Drift

$$v_e = \frac{q\tau_{cn}E}{m_n^*}, \quad v_h = \frac{q\tau_{cp}E}{m_p^*} \Rightarrow$$

$$\mu_n = \frac{q\tau_{cn}}{m_n^*}, \quad \mu_p = \frac{q\tau_{cp}}{m_p^*}$$



Hole velocity  $\vec{v}_h = \mu_p \vec{E}$

Electron velocity  $\vec{v}_e = -\mu_n \vec{E}$

## Notation:

$\mu_p \equiv$  hole mobility ( $\text{cm}^2/\text{V}\cdot\text{s}$ )

$\mu_n \equiv$  electron mobility ( $\text{cm}^2/\text{V}\cdot\text{s}$ )

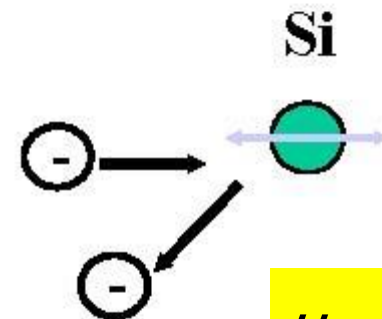
## $1/\mu = 1/\mu_L + 1/\mu_I$ *Carrier Mobility*

- Mobile carriers are always in random thermal motion. If no electric field is applied, the average current in any direction is zero.

- Mobility is reduced by**

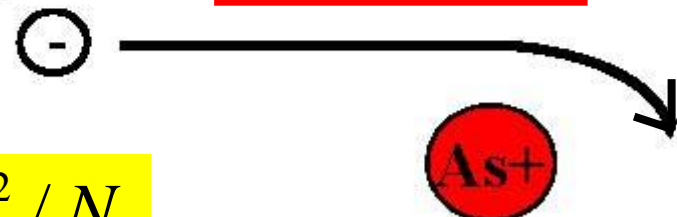
1) collisions with the vibrating atoms

“phonon scattering”



$$\mu_L \propto T^{-3/2}$$

2) deflection by ionized impurity atoms “Coulombic scattering”



$$\mu_I \propto T^{+3/2} / N_I$$

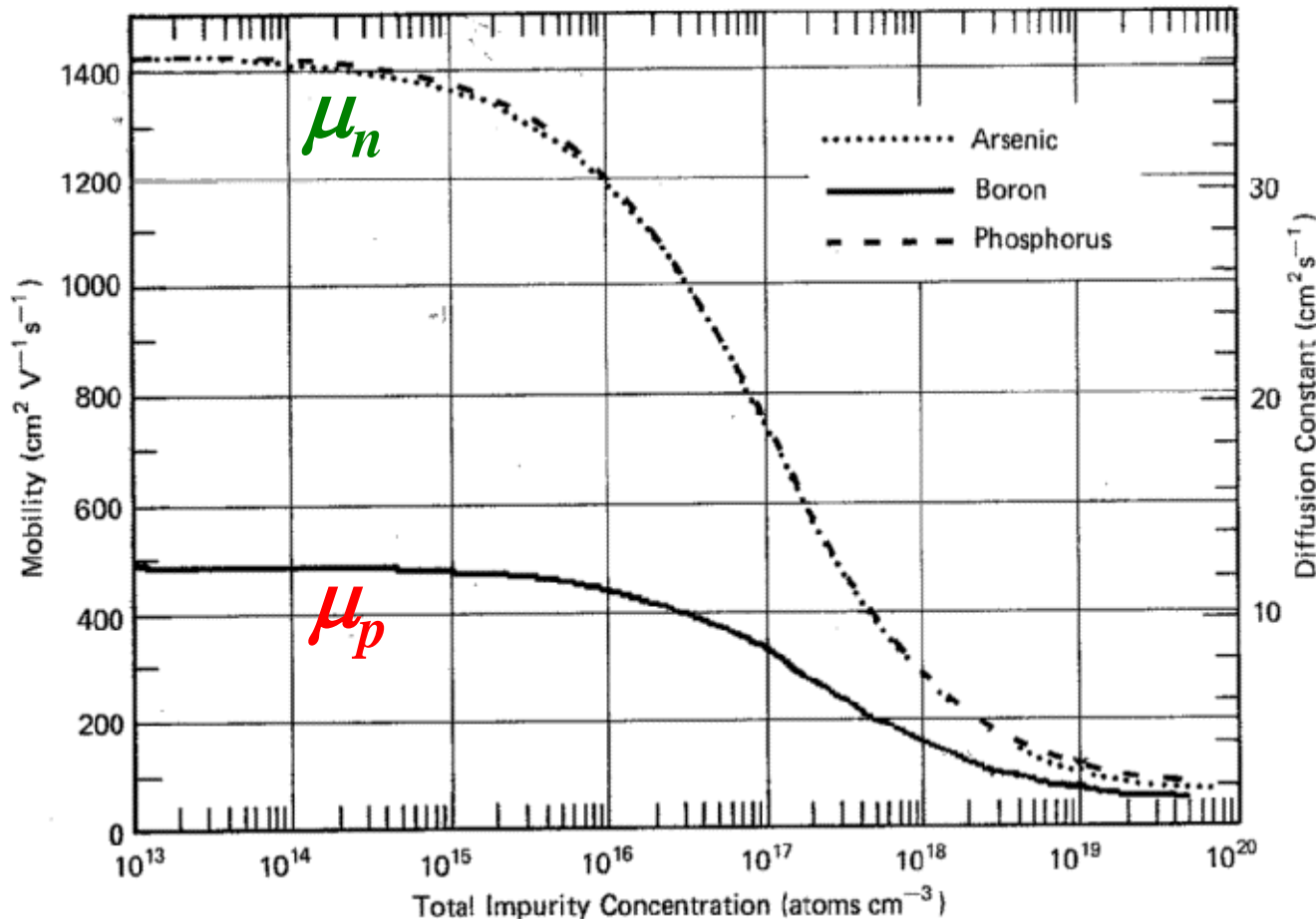


# Drift Velocity and Carrier Mobility

Mobile charge-carrier **drift velocity** is **proportional** to applied **E-field**:

$$|v| = \mu E$$

$\mu$  is the **mobility** (Units:  $\text{cm}^2/\text{V}\cdot\text{s}$ )

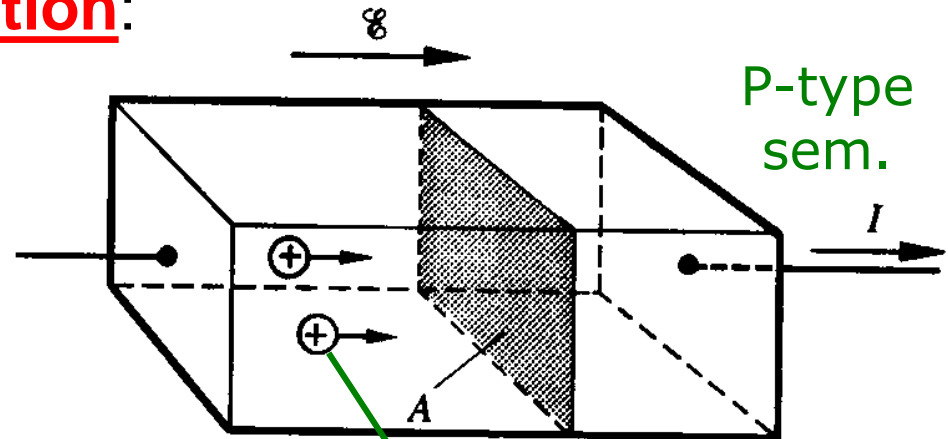


Note: Carrier **mobility** depends on **total dopant concentration** ( $N_D + N_A$ ) !

# Drift Current

- **Drift current** is **proportional** to the **carrier velocity** and **carrier concentration**:

- 1)  $p$  --- hole density
- 2)  $q = 1.6 \times 10^{-19} \text{ C}$   
--- One electron charge
- 3) Charges passing through 'A' per second  
--- The definition of **current**.



$v_h t A$  = volume from which **all holes** cross plane in **time  $t$**

$p v_h t A$  = **# of holes** crossing plane in **time  $t$**

$q p v_h t A$  = **charge** crossing plane in **time  $t$**

$q p v_h A$  = **charge** crossing plane **per unit time** = **hole current**

➔ **Hole current per unit area (i.e. current density)  $J_{p,drift} = q p v_h$**

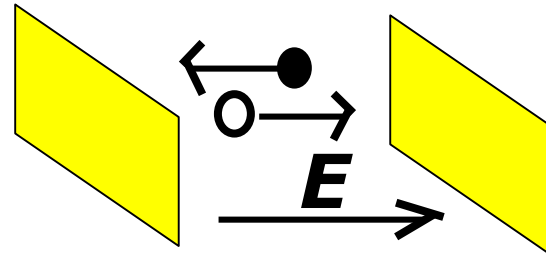
# Electrical Conductivity $\sigma$

Negatively charged electron  
Direction of electron drift

When an electric field is applied, current flows due to drift of mobile electrons and holes:

electron current density:  $J_n = (-q)nv_e = qn\mu_n E$

hole current density:  $J_p = (+q)pv_h = qp\mu_p E$



total current density:  $J = J_n + J_p = (qn\mu_n + qp\mu_p)E$

$$J = \sigma E$$

★ conductivity

$$\sigma \equiv qn\mu_n + qp\mu_p$$

Units:  $(\Omega \cdot \text{cm})^{-1}$

# Electrical Resistivity $\rho$

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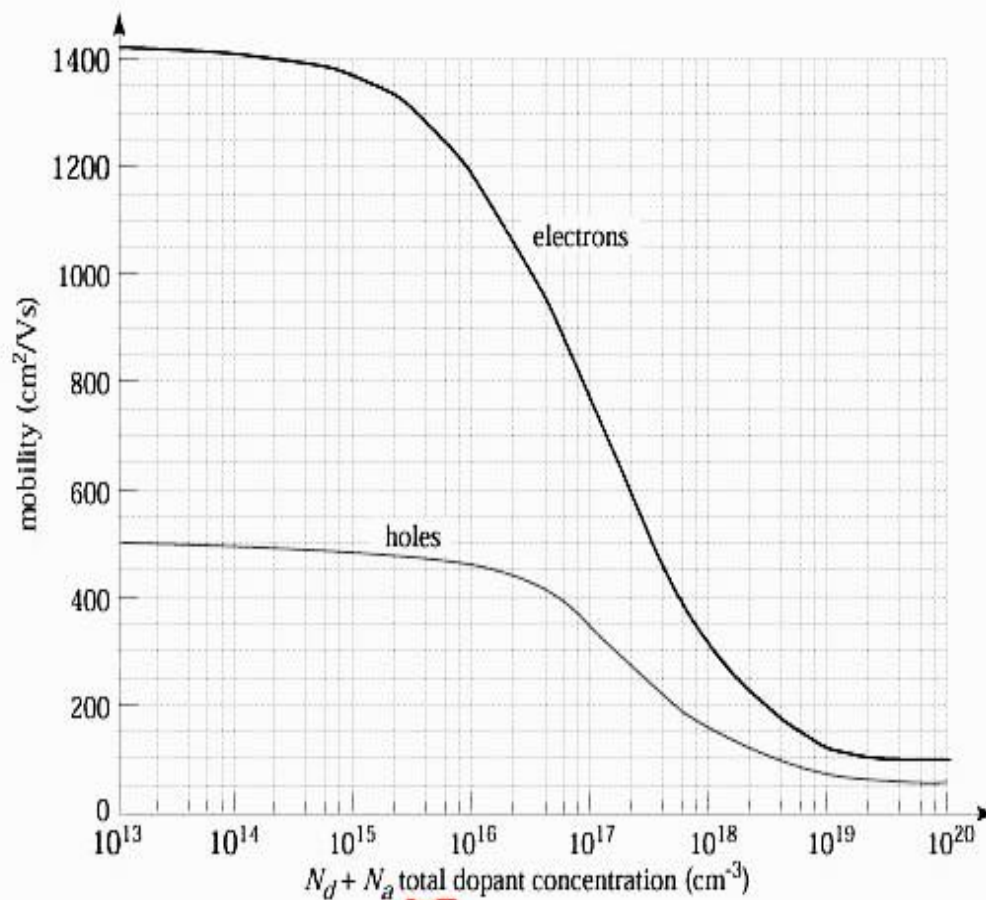
$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$$

$$\rho \cong \frac{1}{qn\mu_n} \text{ for n-type material}$$

$$\rho \cong \frac{1}{qp\mu_p} \text{ for p-type material}$$

(Units: ohm•cm)

# HW4



- Estimate the **resistivity** of a **Si sample** doped with **phosphorus** to a concentration of  **$10^{15} \text{ cm}^{-3}$**  and **boron** to a concentration of  **$10^{17} \text{ cm}^{-3}$** .



$$\rho \approx ?$$

- The **electron mobility** and **hole mobility** are  **$700 \text{ cm}^2/\text{Vs}$**  and  **$350 \text{ cm}^2/\text{Vs}$** , respectively. (Why??)

# Example 1

Consider a Si sample doped with  $10^{16}/\text{cm}^3$  Boron.  
What is its **resistivity**?

Answer:

$$N_A = 10^{16}/\text{cm}^3, N_D = 0 \quad (N_A \gg N_D \rightarrow \text{p-type})$$

$$\rightarrow p \approx 10^{16}/\text{cm}^3 \quad \text{and} \quad n \approx 10^4/\text{cm}^3$$

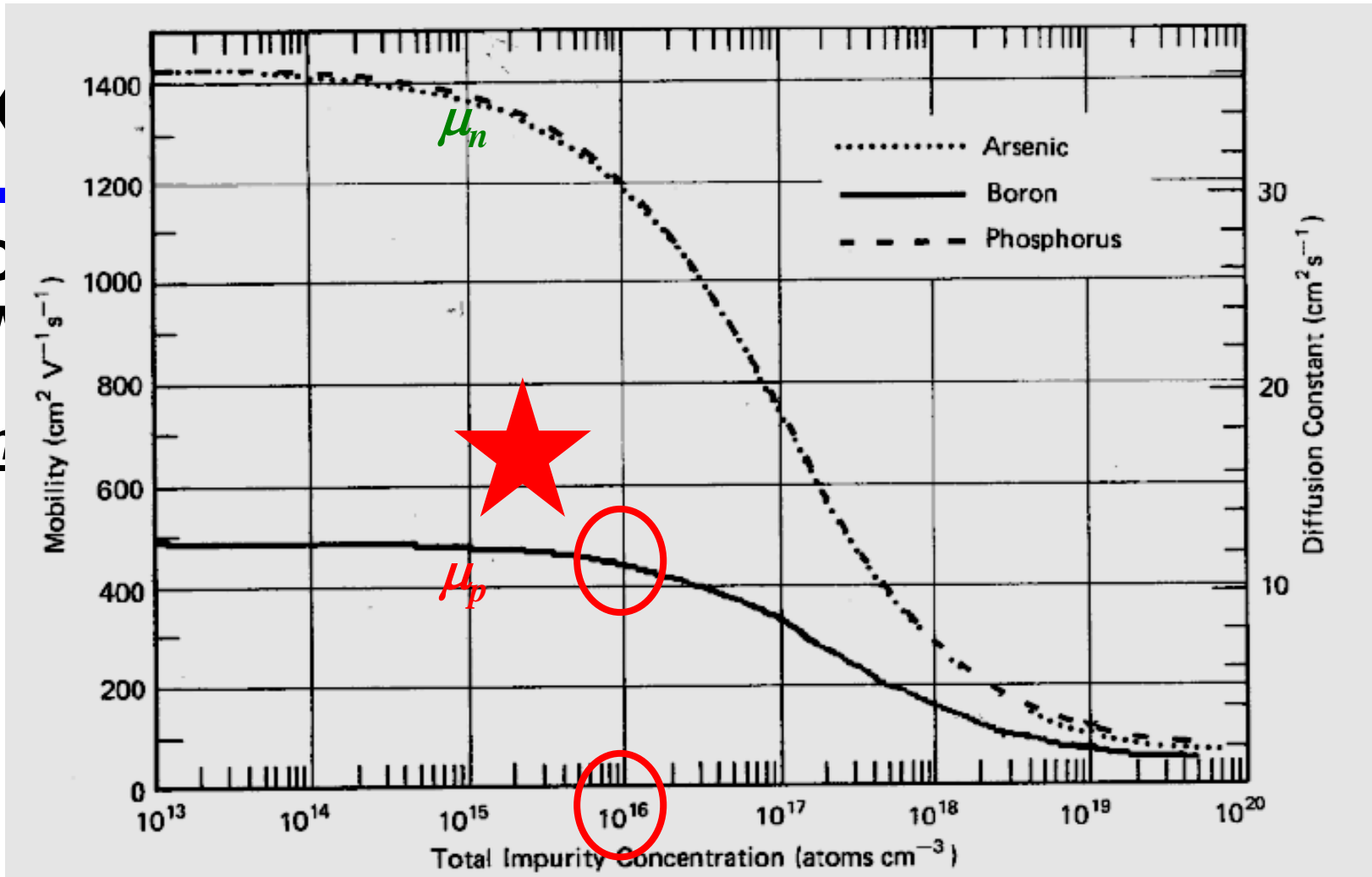
$$\begin{aligned} \rho &= \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qp\mu_p} \\ &= \left[ (1.6 \times 10^{-19})(10^{16})(450) \right]^{-1} = 1.4 \, \Omega \cdot \text{cm} \end{aligned}$$

From  $\mu$  vs.  $(N_A + N_D)$  plot 

Ex

C  
V

Ar



$$= \left[ (1.6 \times 10^{-19}) (10^{16}) (450) \right]^{-1} = 1.4 \, \Omega \cdot \text{cm}$$

From  $\mu$  vs.  $(N_A + N_D)$  plot

# Example 2

Consider the same Si sample, **doped additionally with  $10^{17}/\text{cm}^3$  Arsenic**. What is its **resistivity**?

Answer:

$$N_A = 10^{16}/\text{cm}^3, N_D = 10^{17}/\text{cm}^3 \quad (N_D \gg N_A \rightarrow \text{n-type})$$

$$\rightarrow n \approx 9 \times 10^{16}/\text{cm}^3 \quad \text{and} \quad p \approx 1.1 \times 10^3/\text{cm}^3$$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qn\mu_n}$$

$$= \left[ (1.6 \times 10^{-19})(9 \times 10^{16})(700) \right]^{-1} = 0.10 \, \Omega \cdot \text{cm}$$

The sample is converted to **n-type material** by adding **more donors** than **acceptors**, and is said to be “**compensated**”.

From  $\mu$  vs.  $(N_A + N_D)$  plot

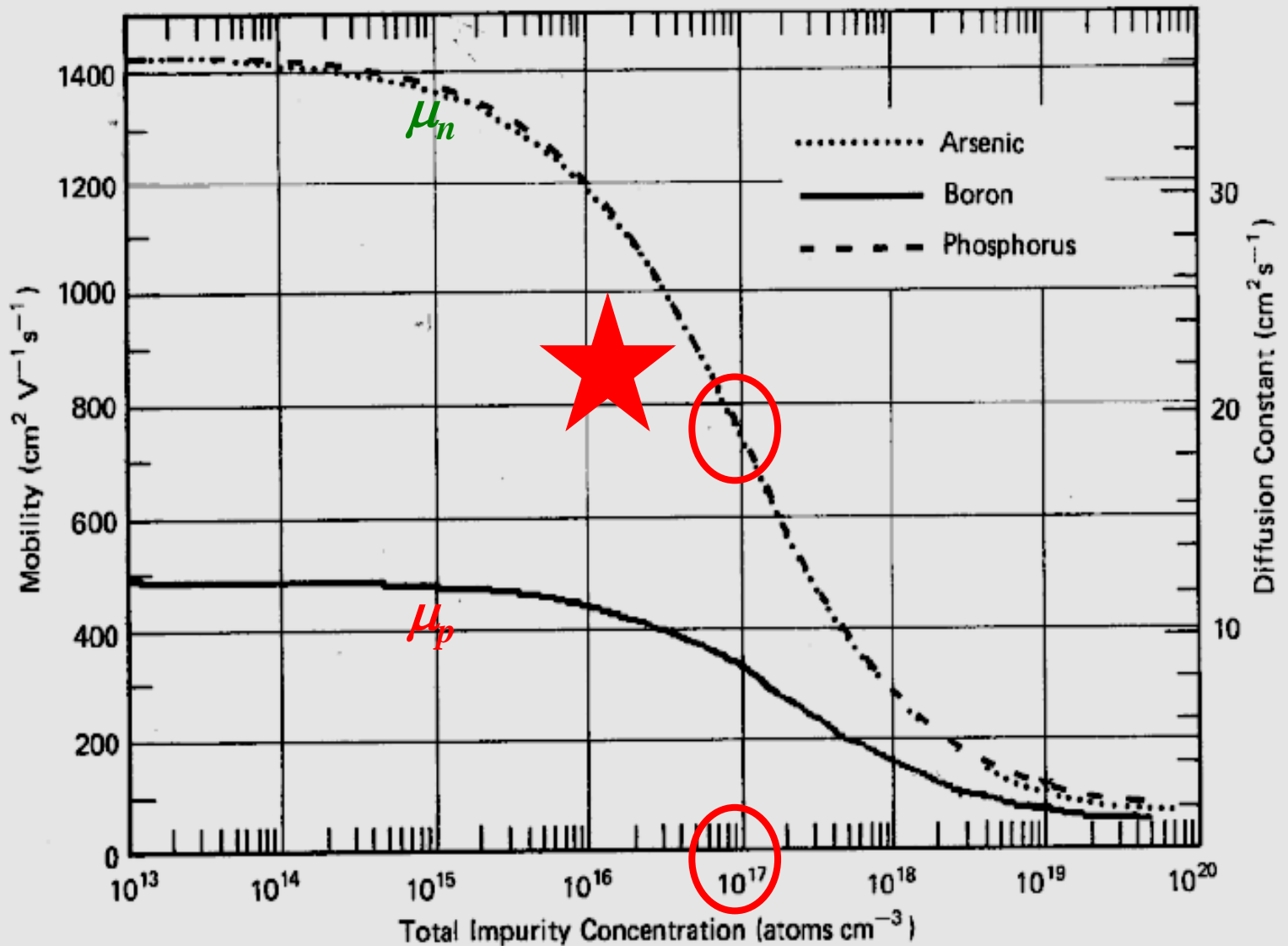


E

C

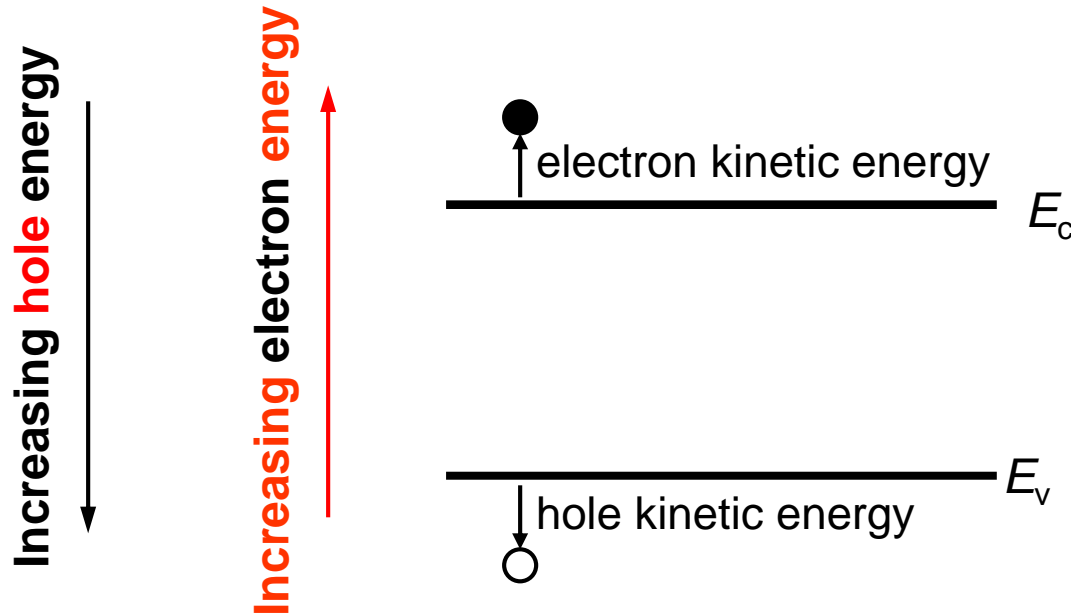
W

A



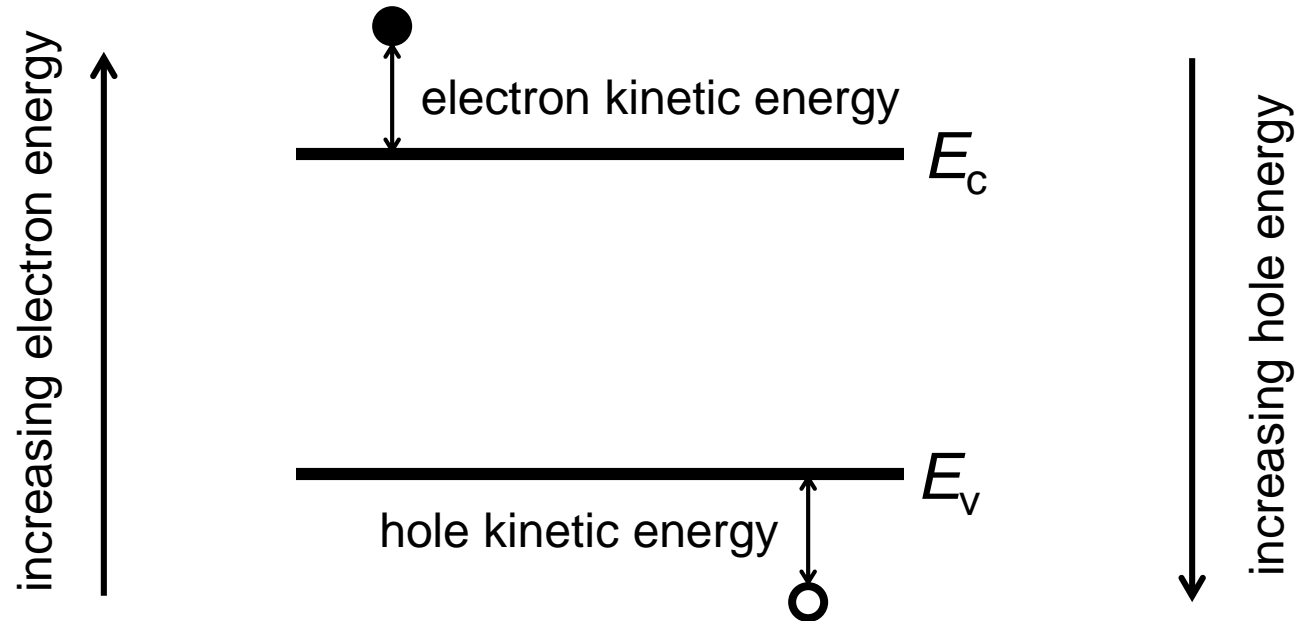
The sample is converted to **n-type material** by adding **more donors** than **acceptors**, and is said to be “**compensated**”.

# Electrons and Holes (Band Model)



- **Electrons and holes** tend to seek **lowest-energy** positions
  - **Electrons** tend to **fall**
  - **Holes** tend to **float up** (like bubbles in water)

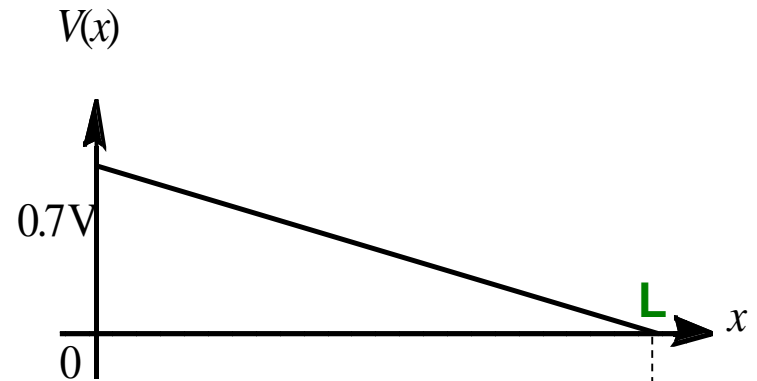
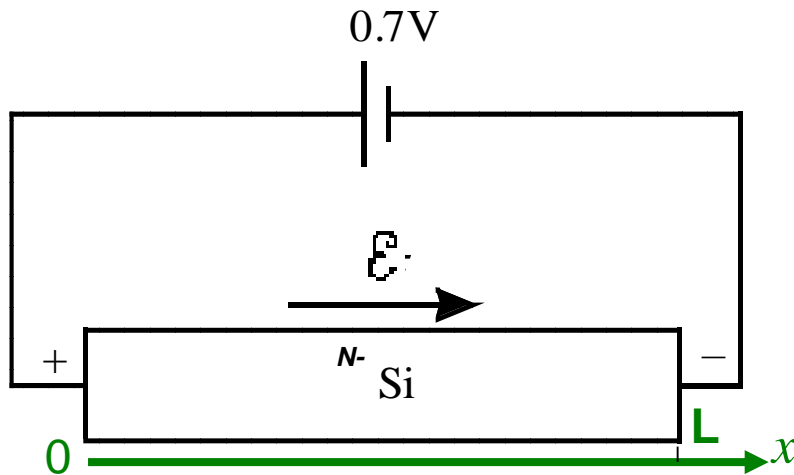
# Potential vs. Kinetic Energy



$E_c$  represents the electron **potential energy**:

$$\text{P.E.} = E_c - E_{\text{reference}}$$

# Electrostatic Potential, $V$

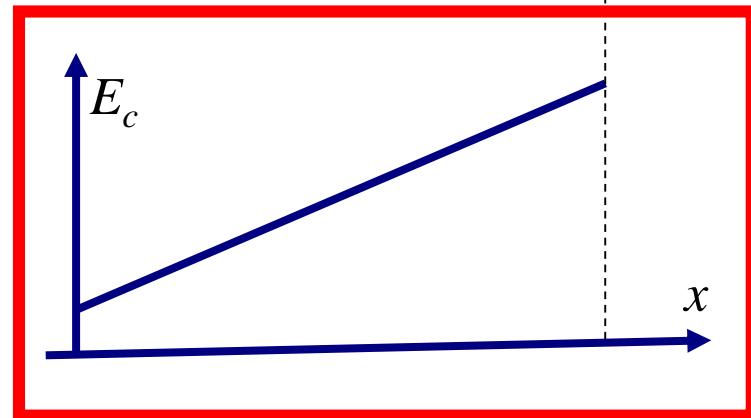


- The **potential energy** of a particle with charge  $-q$  is related to the electrostatic **potential  $V(x)$** :

$$\text{P.E.} = -qV$$

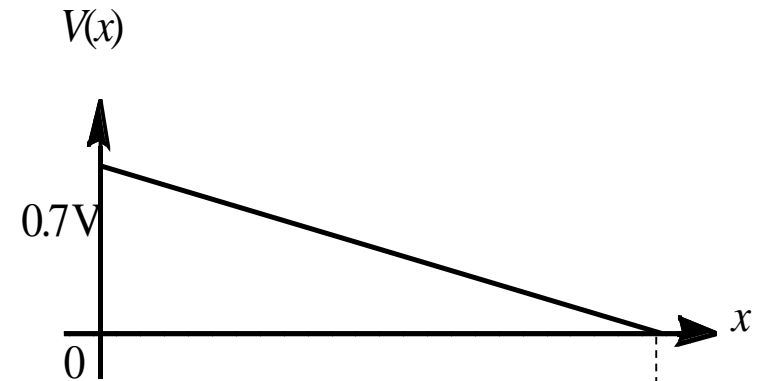
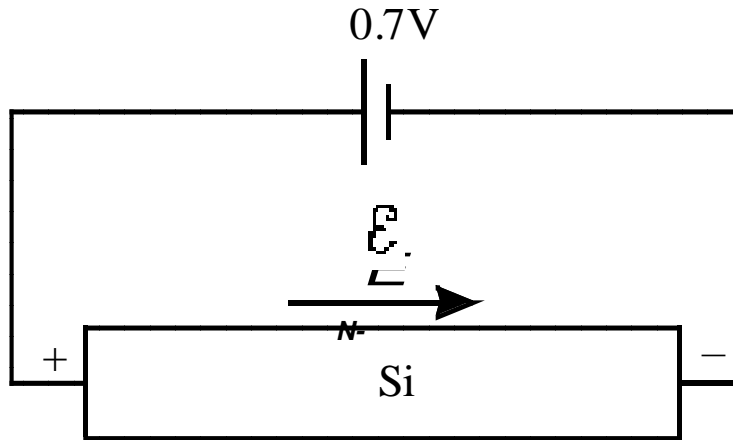
$$V = \frac{1}{q} (E_{\text{reference}} - E_c)$$

$E_{\text{reference}}$  is constant



# Electric Field, $\mathcal{E}$

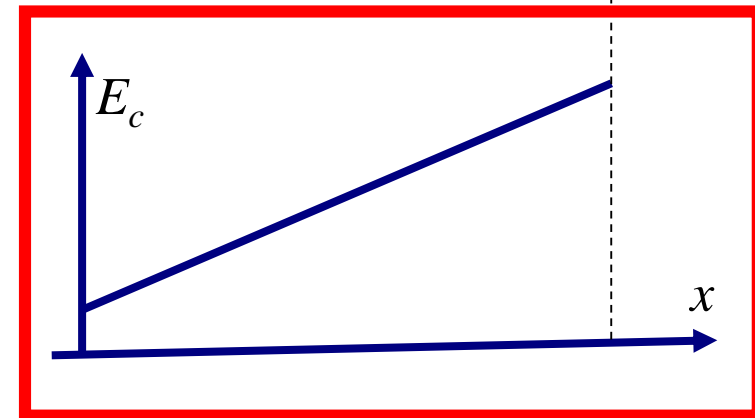
$$\mathcal{E} = -\frac{dV}{dx}$$



$$V = \frac{1}{q} (E_{\text{reference}} - E_c)$$



$$\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx}$$

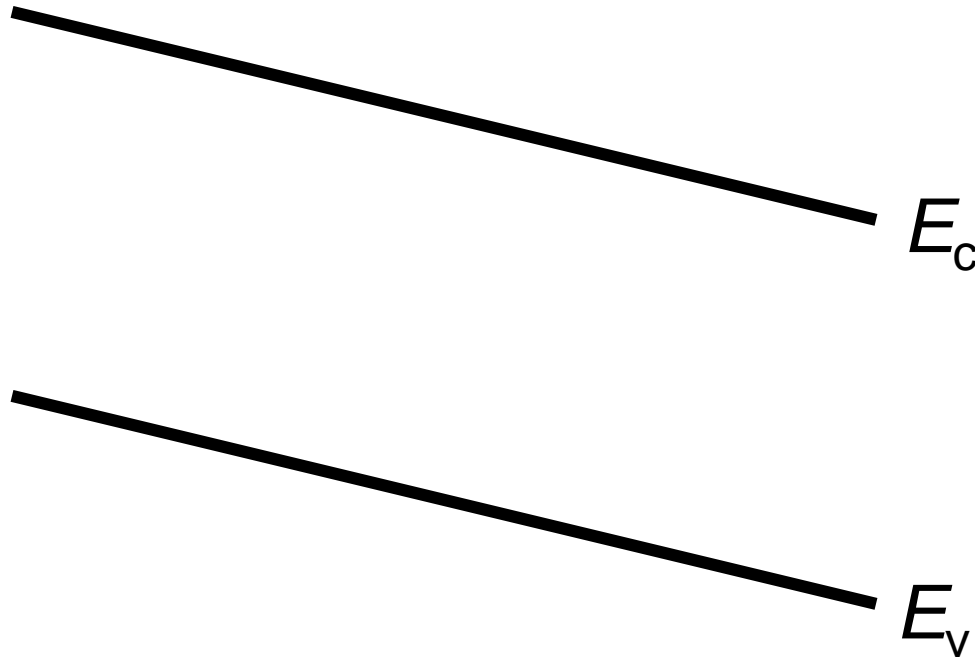


- Variation of  $E_c$  with position is called “**band bending**.”

# HW 5: Carrier Drift (Band Diagram Visualization)

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$$\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx}$$



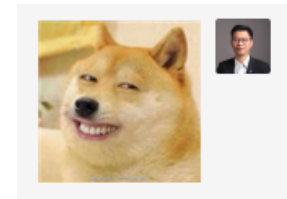
Q1: what is the **direction** of **electric field**?

Q2: what is the **direction** of **carriers' drift**?

## 2.6 Carrier drift and diffusion

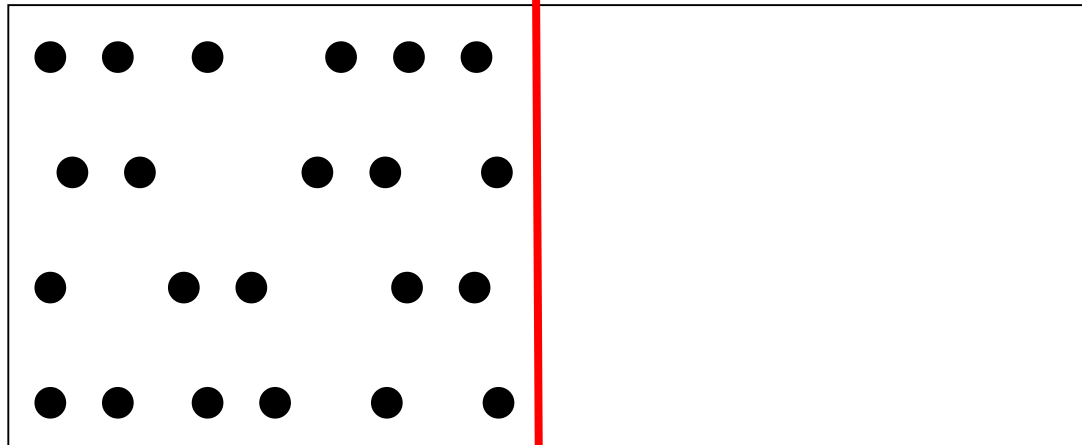
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- Carrier scattering
- Carrier drift:
  - *Carrier mobility*
  - *Conductivity & Resistivity*
  - *Energy band model*
- **Carrier diffusion**



# Diffusion

- **Diffusion** occurs when there exists a **concentration gradient**
- In the figure below, imagine that we fill the **left chamber** with a gas at temperature  $T$
- If we suddenly remove the **divider**, what happens?
- The gas will fill the **entire volume** of the new chamber.
- **How does this occur?**

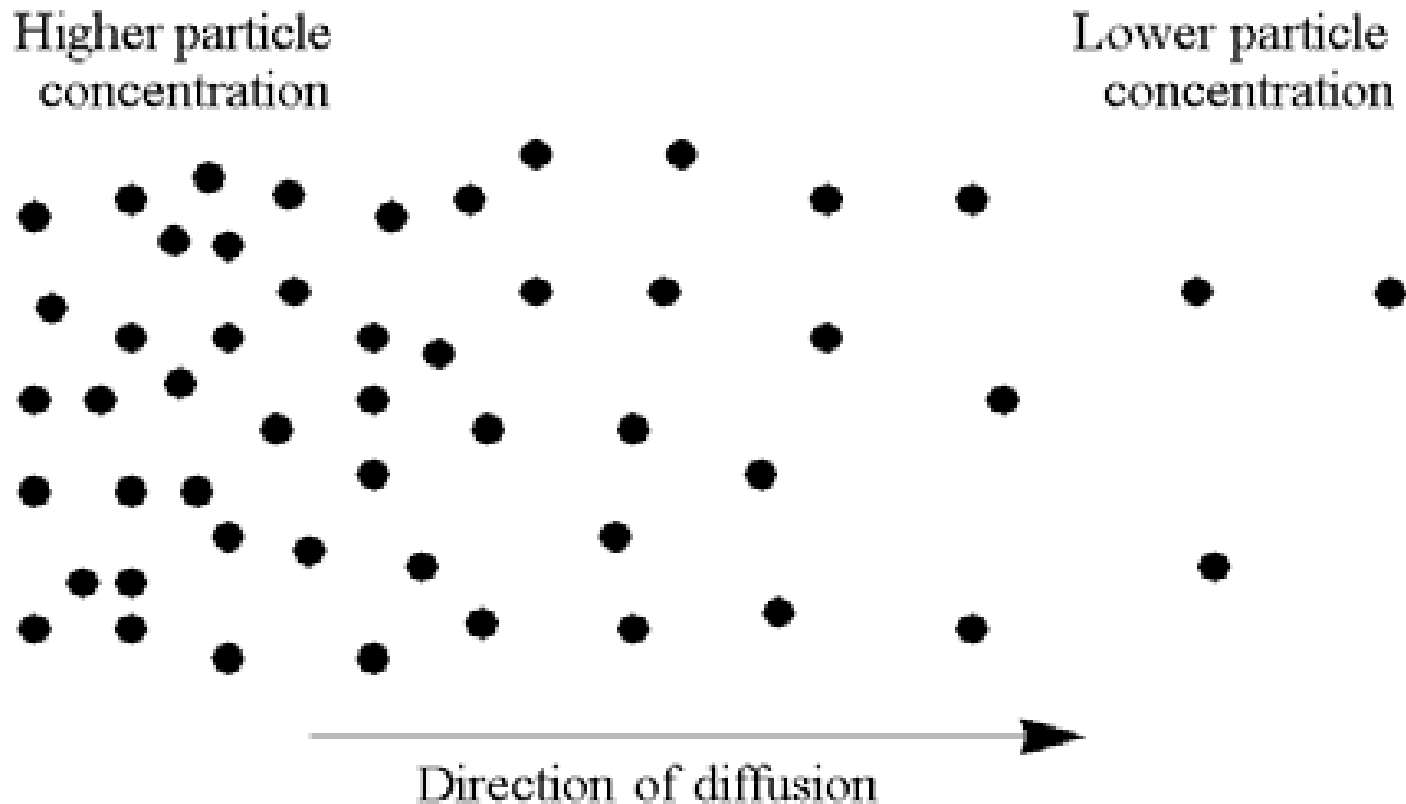




# Diffusion

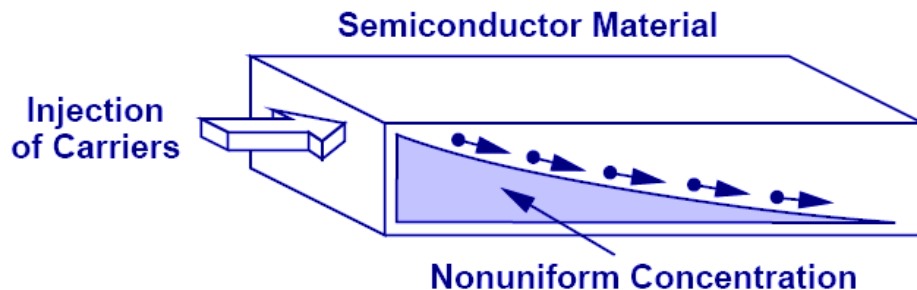
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- Particles **diffuse** from **higher concentration** to **lower concentration** locations.



# Carrier Diffusion

- Due to thermally induced random motion, mobile particles tend to move from a region of **high concentration** to a region of **low concentration**.
  - Analogy: ink droplet in water
- Current flow due to **mobile charge diffusion** is proportional to the **carrier concentration gradient**.
  - The **proportionality constant** is the **diffusion constant**.



$$J_p = -qD_p \frac{dp}{dx}$$

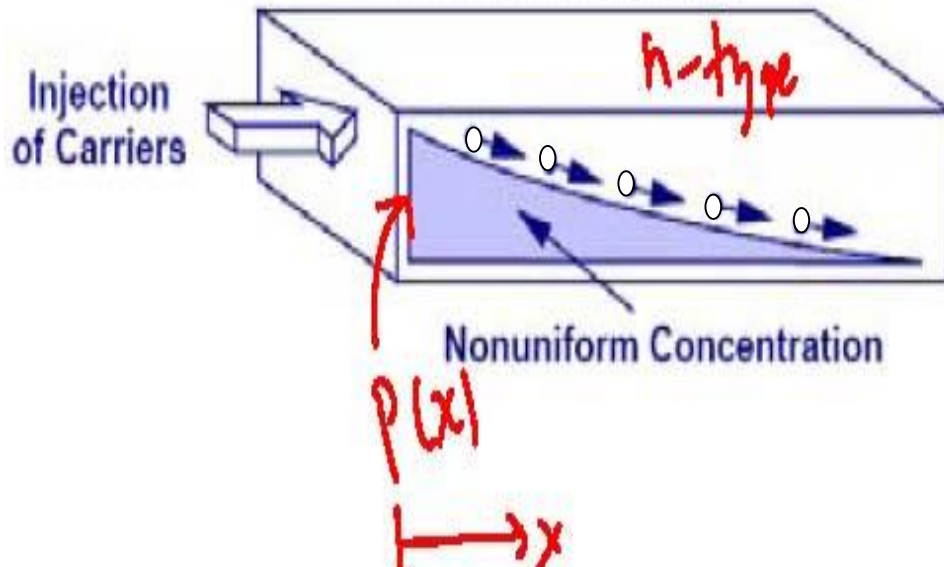
**Notation:**

$D_p$   $\equiv$  hole diffusion constant (cm<sup>2</sup>/s)

$D_n$   $\equiv$  electron diffusion constant (cm<sup>2</sup>/s)

# Carrier Diffusion

- Current flow due to **mobile charge diffusion** is proportional to the carrier concentration gradient.
  - The **proportionality constant** is the **diffusion constant**.



$$J_p = -qD_p \frac{dp}{dx}$$

## Notation:

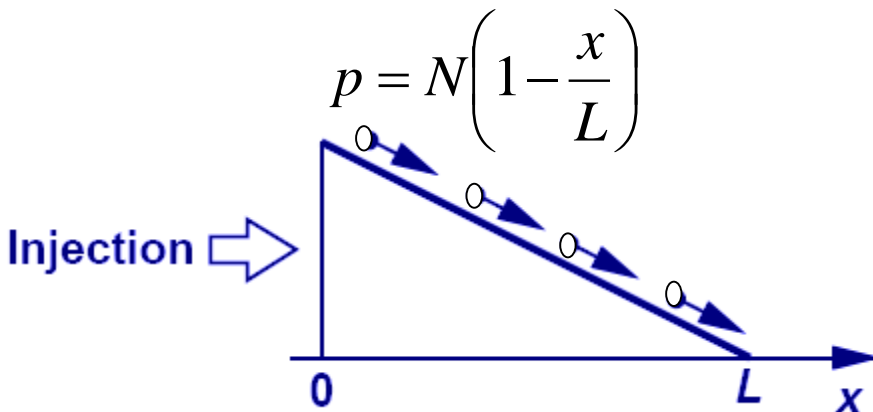
$D_p \equiv$  hole diffusion constant ( $\text{cm}^2/\text{s}$ )

$D_n \equiv$  electron diffusion constant ( $\text{cm}^2/\text{s}$ )

# Diffusion Examples

Linear **concentration profile**

→ **constant** diffusion current

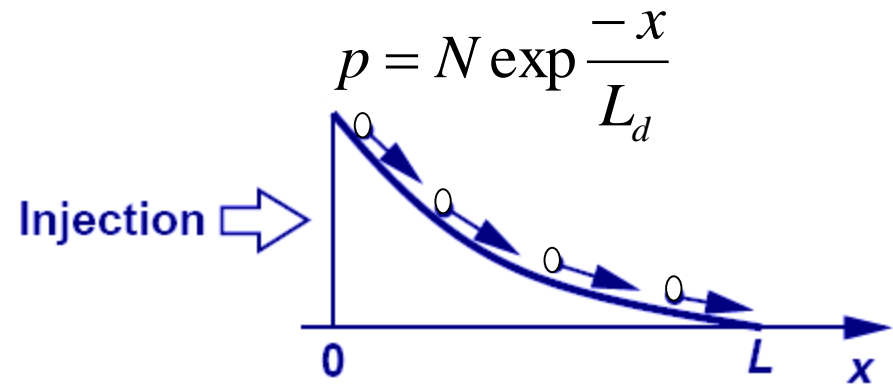


$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= qD_p \frac{N}{L}$$

Non-linear **concentration profile**

→ **varying** diffusion current



$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= \frac{qD_p N}{L_d} \exp \frac{-x}{L_d}$$

# Total Diffusion Current

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- Due to the non-uniform distribution of carriers

$$J_n = qD_n \frac{dn}{dx}$$

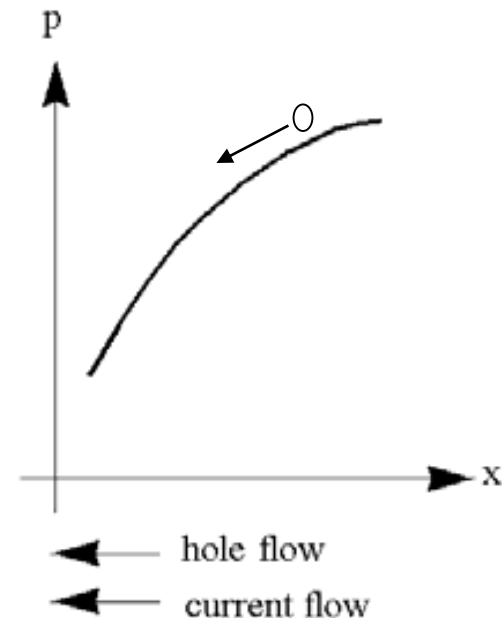
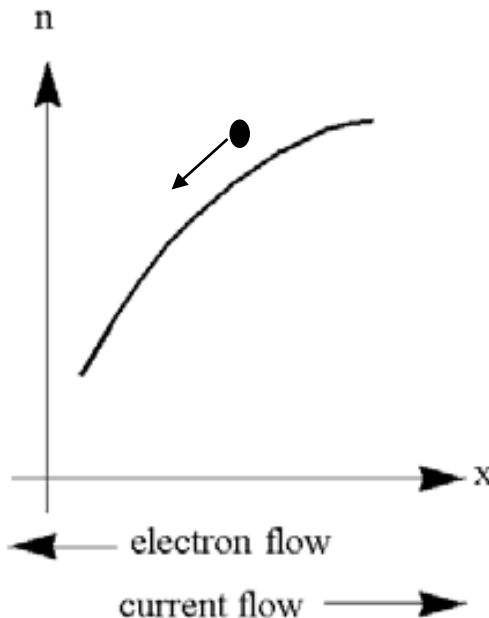
- $D_n$  --- electron diffusion constant
- Driving force: thermal energy, not electric field
- $dn/dx$  --- density gradient
- **Total** diffusion current
  - $J = J_n + J_p$

# Total Diffusion Current

- Diffusion current within a semiconductor consists of **hole** and **electron** components:

$$J_{p,diff} = -qD_p \frac{dp}{dx} \quad J_{n,diff} = qD_n \frac{dn}{dx}$$

$$J_{tot,diff} = q\left(D_n \frac{dn}{dx} - D_p \frac{dp}{dx}\right)$$



# Total current

- The **total current** flowing in a semiconductor is the sum of **drift current** and **diffusion current**:

$$J_{tot} = J_{p,drift} + J_{n,drift} + J_{p,diff} + J_{n,diff}$$

$$J_{p,drift} = qp\mu_p E, \quad J_{n,drift} = qn\mu_n E$$

$$J_{p,diff} = -qD_p \frac{dp}{dx}, \quad J_{n,diff} = qD_n \frac{dn}{dx}$$

# Einstein Relation



- The **characteristic constants** for **drift** and **diffusion** are related:

$$D_n = \frac{kT}{q} \mu_n$$

$$D_p = \frac{kT}{q} \mu_p$$

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}}$$

$$= 26 \text{ mV} \\ \text{at } T = 300 \text{ K}$$

- Note that  $\frac{kT}{q} \cong 26 \text{ mV}$  at **room temperature** (300K)
  - This is often referred to as the “**thermal voltage**”.



# Important Constants

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- Electronic charge,  $q = 1.6 \times 10^{-19} \text{ C}$
- Permittivity of free space,  $\epsilon_0 = 8.854 \times 10^{-14} \text{ F/cm}$
- Boltzmann constant,  $k = 8.62 \times 10^{-5} \text{ eV/K}$
- Planck constant,  $h = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
- Free electron mass,  $m_0 = 9.1 \times 10^{-31} \text{ kg}$
- Thermal voltage  $kT/q = 26 \text{ mV}$ , at  $T=300\text{K}$

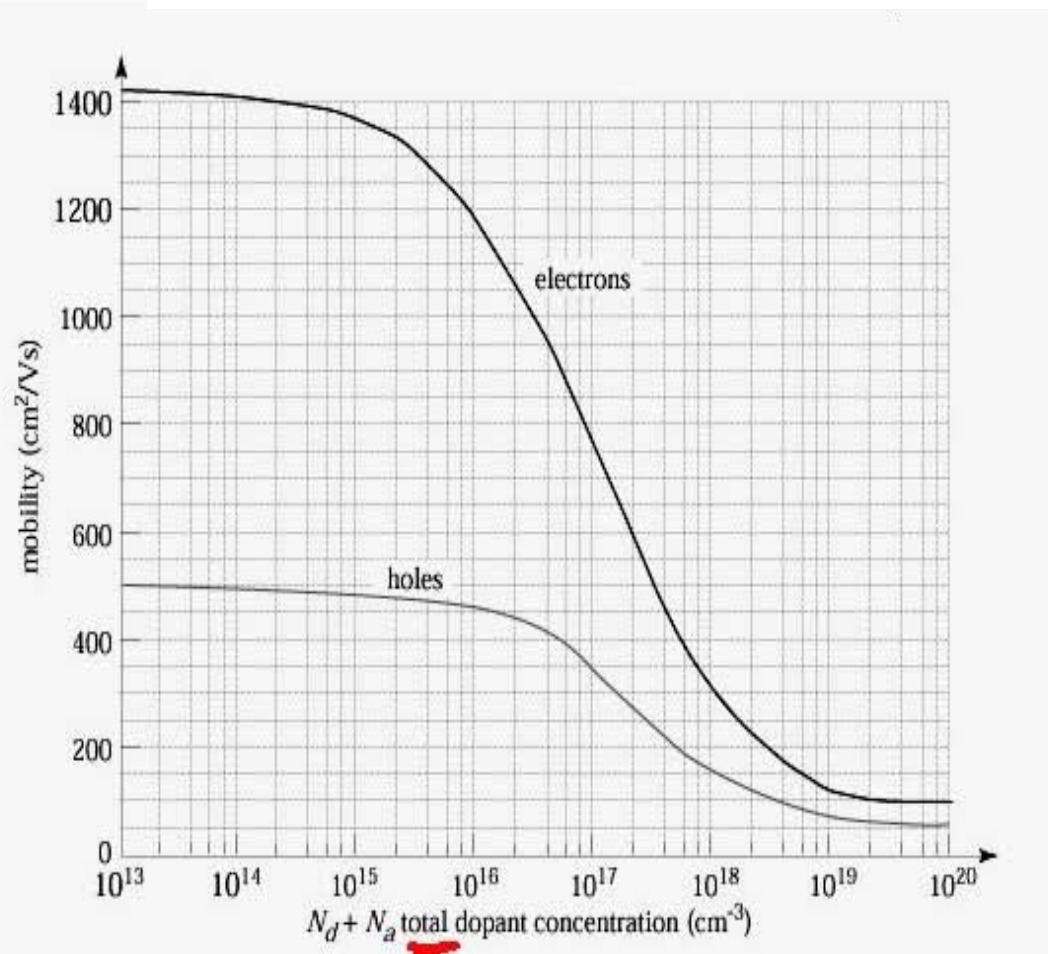
# HW3: Energy-band diagram

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Question: Where is  $E_F$  for  $n = 10^{17} \text{ cm}^{-3}$  ?

$$n = n_i \exp\left[\frac{(E_F - E_i)}{kT}\right]$$

# HW4



$$N_D = 10^{15} \text{ cm}^{-3}, N_A = 10^{17} \text{ cm}^{-3}$$

$$N_A - N_D \approx 10^{17} \text{ cm}^{-3}$$

$$p = 10^{17} \text{ cm}^{-3}, n \approx 10^3 \text{ cm}^{-3}$$

$$\sigma = \cancel{q n n} + q \mu_p p$$

↑ negligible

$$\sigma = 1.6 \times 10^{-19} (350) (10^{17})$$

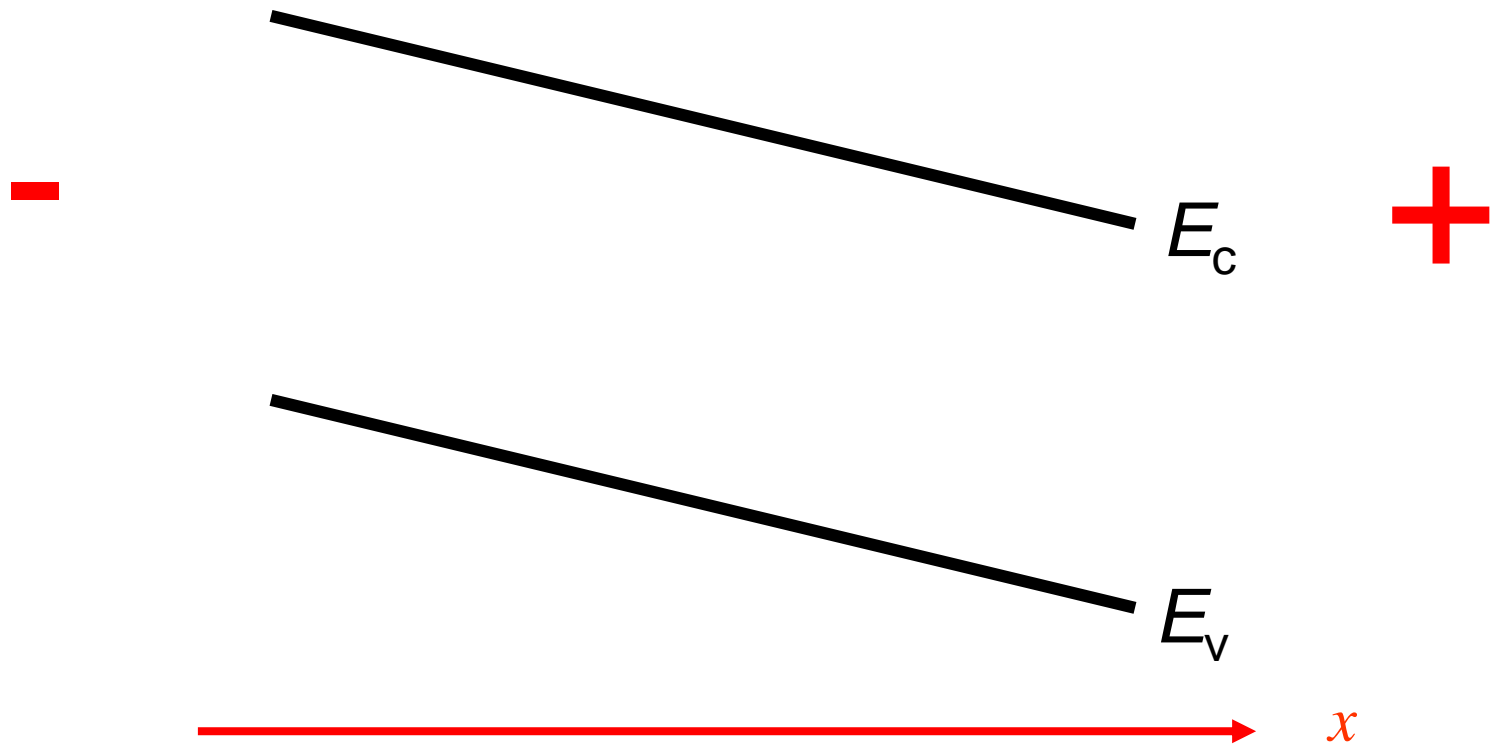
$$\approx 5 \times 10^0$$

$$\rho \approx 0.2 \Omega\text{-cm}$$

- The electron mobility and hole mobility are 700 cm<sup>2</sup>/Vs and 350 cm<sup>2</sup>/Vs, respectively.

# HW5: Carrier Drift (Band Diagram Visualization)

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Q1: what is the direction of electric field?

Q2: what is the direction of carriers' drift?