The Operational Amplifier

EEE103 ELECTRICAL CIRCUITS (Part 2)
Week 5
S1, 2023/24

Lecturer: Xuchen Wang
Xuchen.Wang@xjtlu.edu.cn

Overview of Part 2

- Part 2 introduces operational amplifier and discusses circuit impedance components and dc circuit transients.

  Week 5: The operational amplifier

  Week 6: Capacitor and Inductor

  Week 8: Basic RC and RL circuit

- In Class Assignment: Week 8 tutorial time. (Take in class)
- Homework Assignment 2: Released in Week 8 together with Part 3, due in week 12.
- Office hour: Thursday, 11:00am-1:00pm (Location: EE312)

Content

1. The characteristics of operational amplifier
2. The inverting amplifier
3. Other basic operational amplifier circuits:
    - The non-inverting amplifier
    - The voltage follower
    - The summing amplifier
    - Cascaded operational amplifier circuit
4. Comparators and the Schmitt triggers

Content

1. The characteristics of operational amplifier

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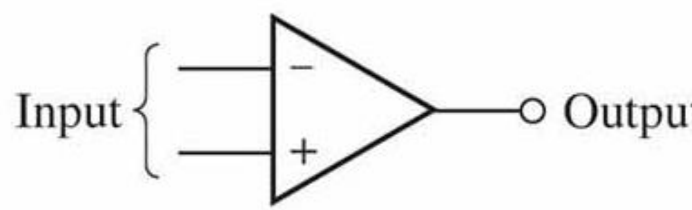
4. Comparators and the Schmitt triggers

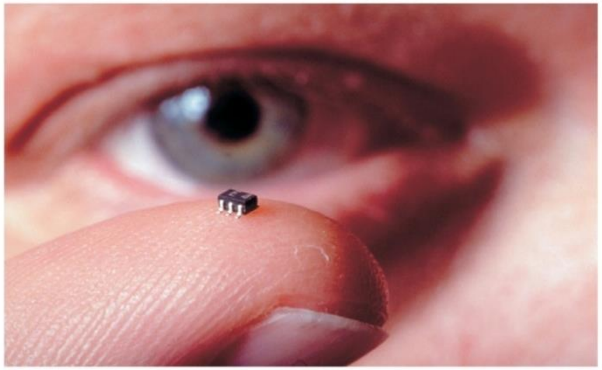
Introduction

- An operational amplifier (op amp) is an integrated circuit that amplifies the difference between input voltages and produces a single output.

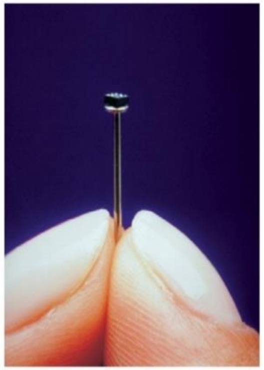


- The origin of operational amplifier is to perform mathematical operations such as addition, subtraction, multiplication, division, differentiation, and integration for analog computers. This is where the term "operational amplifier" comes from!
- The op amp finds daily usage in a large variety of electronic applications. Modern op amps are constructed using transistors in conjunction with resistors and capacitors to perform the desired characteristics.

A vacuum tube op amp



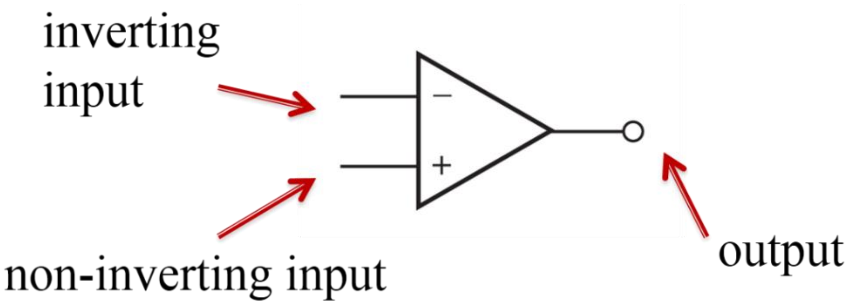
A LMV321 op amp



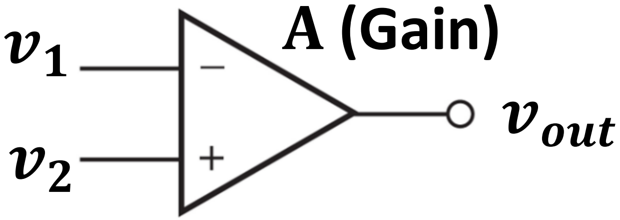
A LMC6035 op amp

Introduction

- The symbol of op amps have three principal terminals (two inputs and one output):



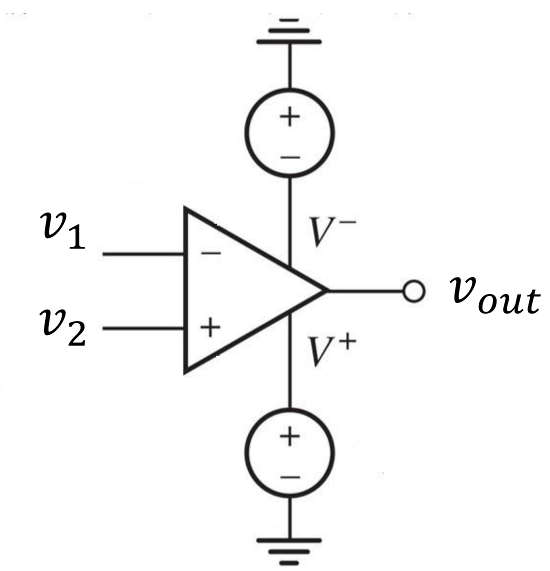
- The op amp senses the difference between two input signals and amplifies this difference to produce an output signal.

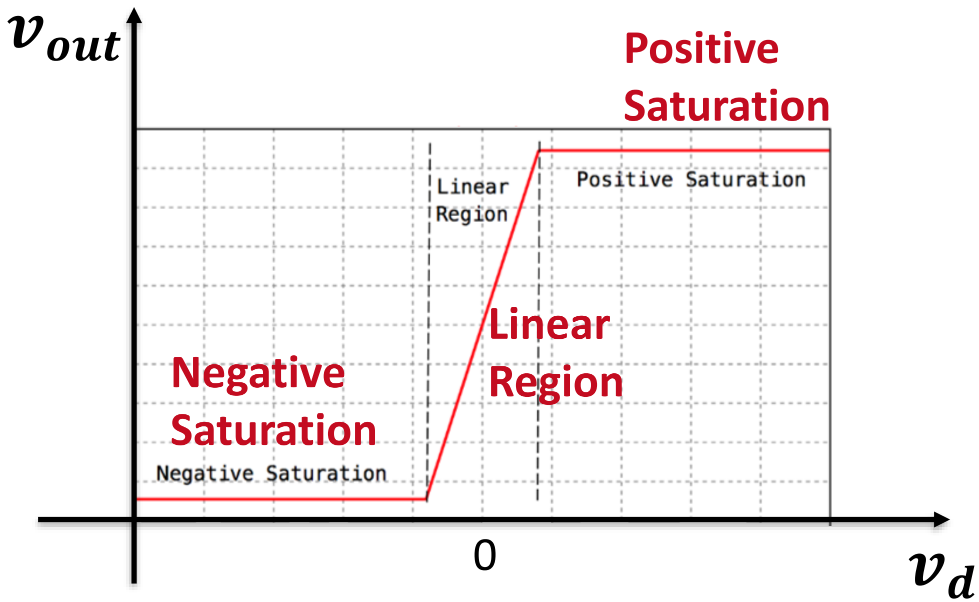


$$v_{out} = \mathrm{A}v_d = \mathrm{A}(v_2 - v_1)$$

Power supply and saturation

- An op amp requires power supplies.
- Usually, equal and opposite voltages are connect to V+ and V−
- The output of op amp is a linear response bounded by the positive and negative saturation regions





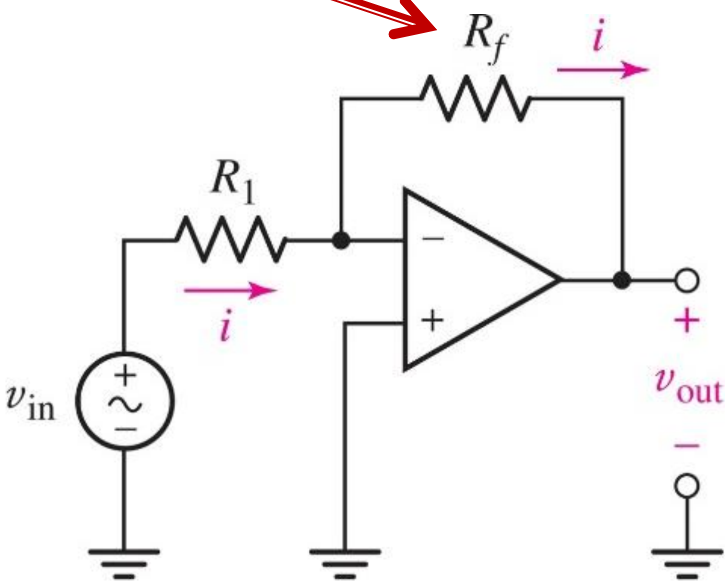
- Typical values: 5 to 24 volts.
- Power supply ground must be the same as the signal ground

Content

Feedback amplifier

- The enormous but unpredictable gain of the op amp is made usable through negative feedback.

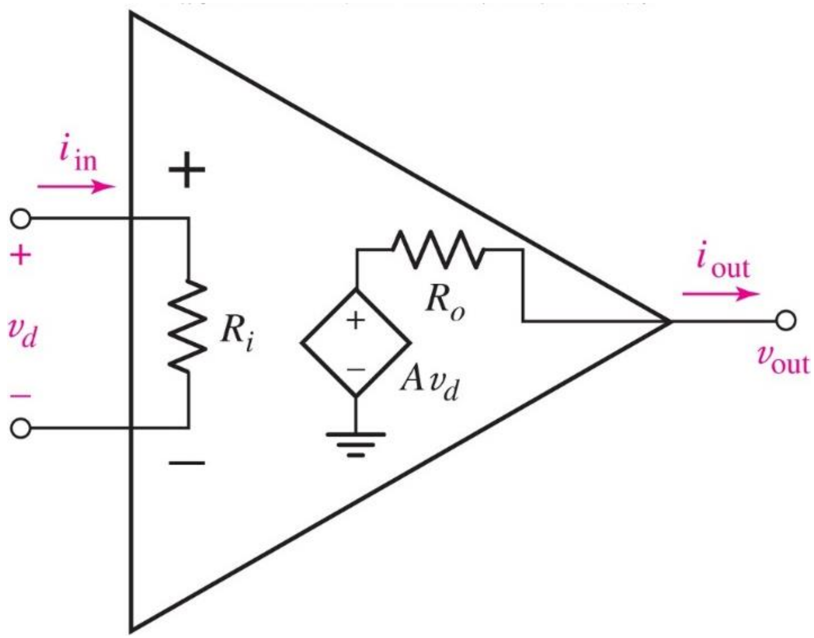
this “feedback” resistor allows the output to affect the input terminal.



Inverting amplifier

The Ideal Op Amp

- The internal differential gain A is considered to be infinite.
- The differential input voltage v_d is assumed to be zero. If A is very large and if the output v_{out} is finite, then the two input voltages must be nearly equal.
- The effective input resistance to the op-amp is assumed to be infinite, so the input current, i_{in} , is essentially zero.



- The output resistance R_o is assumed to be zero in the ideal case, so the output voltage is connected directly to the dependent voltage source, and the output voltage is independent of any load connected to the output.

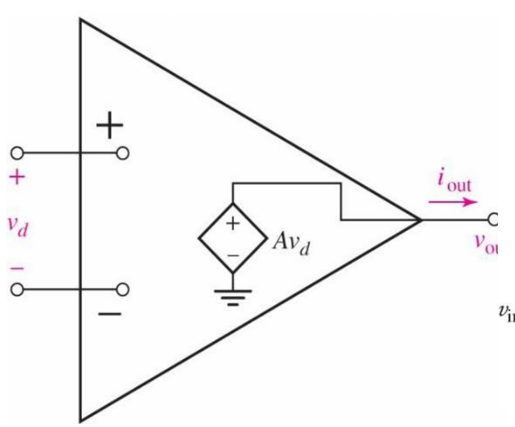
In summary: $A = \infty$, $R_o = 0\ \Omega$, and $R_i = \infty\ \Omega$, the op amp behaves according to the ideal op amp rules. ($v_d = 0$ *and* $i_{in} = 0$)

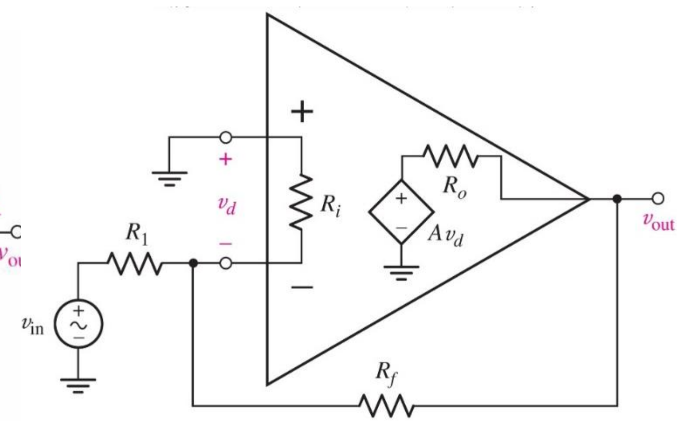
Two Ideal Op Amp Rules

- **Rule 1:** No current ever flows into either input terminal. (Current can flow at output terminal!)
- **Rule 2:** There is no voltage difference between the two input terminals.

$$v_{out} = Av_d \neq 0$$

Note the two rules here are for the op amp with external circuit analysis!

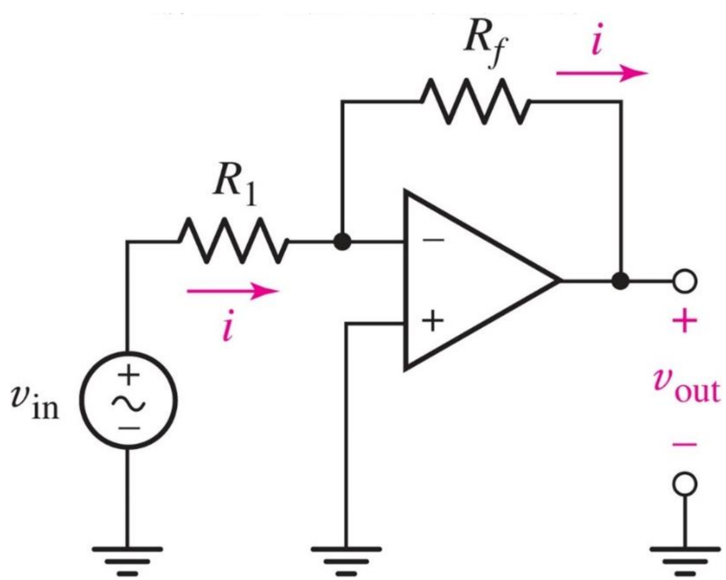




A ideal Op Amp model   Negative feedback amplifier

The Inverting Amplifier

Apply KVL and KCL, Ohm's law, and the ideal op amp rules to find v_{out}



- According to op amp rule 1, apply KVL:

$$-v_{in} + R_1 i + R_f i + v_{out} = 0 \qquad [1]$$

- According to op amp rule 2, apply KCL:

$$i = \frac{v_{in} - 0}{R_1} \qquad [2]$$

- Combing Eq.[1] and [2], we obtain v_{out}:

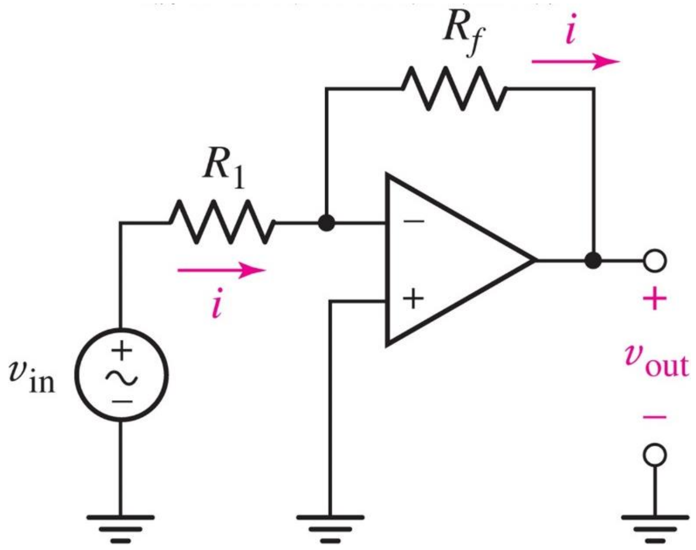
$$v_{out} = -\frac{R_f}{R_1} v_{in} \qquad [3]$$

The resulting answer shows that the output v_{out} is proportional to the input v_{in} by the factor of $-\frac{R_f}{R_1}$, or amplifies the input by a negative constant defined by resistor values.

Example 1

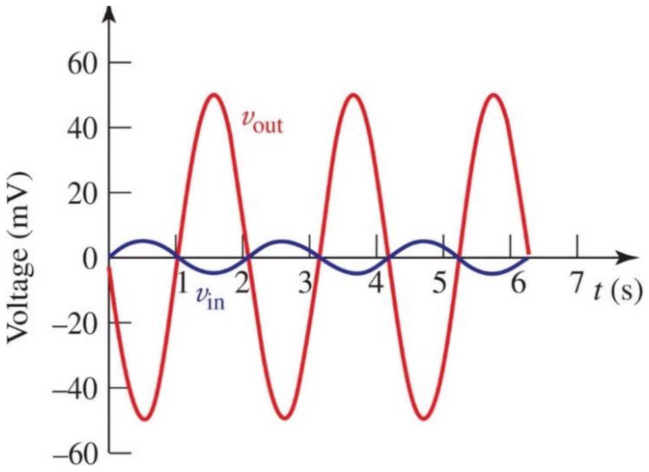
Derive an expression for inverting amplifier v_{out} in terms of v_{in} the circuit show below.

$$v_{in}(t) = 5sin3t\ mV, R_f = 47k\Omega, R_1 = 4.7k\Omega$$



Solution:

$$v_{out}(t) = -\frac{R_f}{R_1} v_{in} = -50sin3t\ mV$$

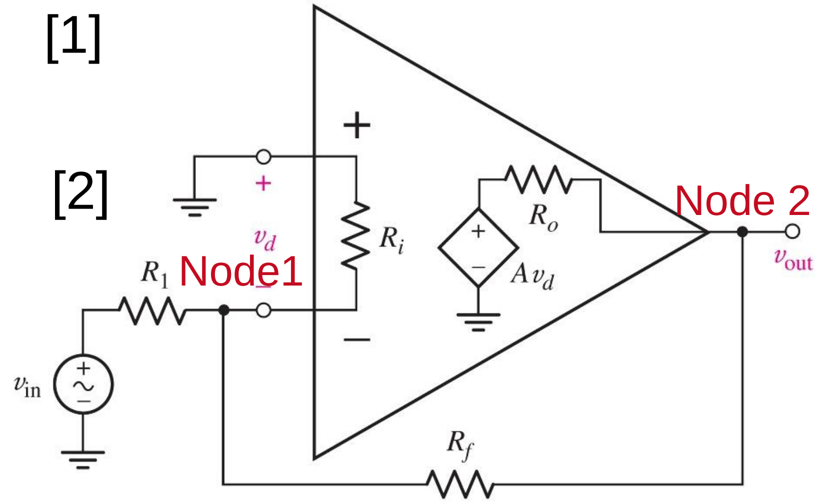


Inverting Amplifier with a Real Op Amp

- Note we are no longer using ideal op amp rules for real op amp model, thus we write two nodal equations:

At node 1: $\frac{-v_d-v_{in}}{R_1}+\frac{-v_d-v_{out}}{R_f}+\frac{-v_d}{R_i}=0$ [1]

At node 2: $\frac{v_{out}+v_d}{R_f}+\frac{v_{out}-Av_d}{R_0}=0$ [2]



- According to Eq.[1] and [2], and eliminate v_d, we obtain v_{out}:

$$v_{out}=[\frac{R_0+R_f}{R_0-AR_f}\left(\frac{1}{R_1}+\frac{1}{R_f}+\frac{1}{R_i}\right)-\frac{1}{R_f}]^{-1}\frac{v_{in}}{R_1} \quad [3]$$

- Now if we allow A = ∞, R_o = 0 Ω, and R_i = ∞ Ω, Eq.[3] reduces to

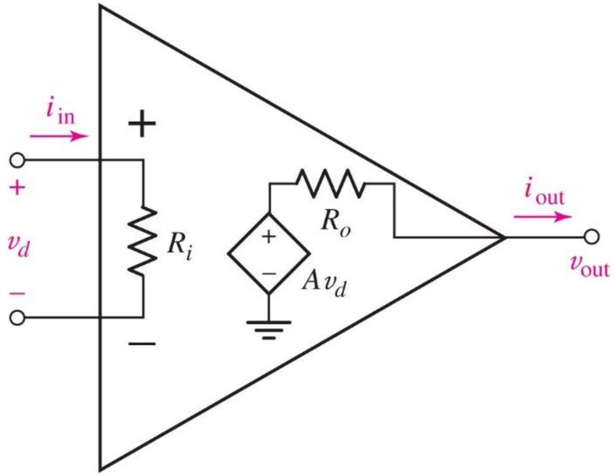
$$v_{out}=-\frac{R_f}{R_1}v_{in} \quad [4]$$

which is exactly what we derived for the inverting amplifier when the op amp is ideal.

A more detailed Op Amp model

The practical characteristics of op amp:

- The open loop gain A is the range of 10^5 to 10^6.
- The effective input resistance R_i is the range of MΩ to TΩ.
- The output resistance R_o is in the range of several to dozens ohms.



Part Number	μA741	LM324	LT1001	LF411	AD549K
Description	General purpose	Low-power quad	Precision	Low-offset, low-drift JFET input	Ultralow input bias current
Open-loop gain *A*	2×10^5 V/V	10^5 V/V	8×10^5 V/V	2×10^5 V/V	10^6 V/V
Input resistance	2 MΩ	*	100 MΩ	1 TΩ	10 TΩ
Output resistance	75 Ω	*	*	~1 Ω	~15 Ω
Input bias current	80 nA	45 nA	0.5 nA	50 pA	75 fA
Input offset voltage	1.0 mV	2.0 mV	7 μV	0.8 mV	0.150 mV
CMRR	90 dB	85 dB	110 dB	100 dB	100 dB

* Not provided by manufacturer.

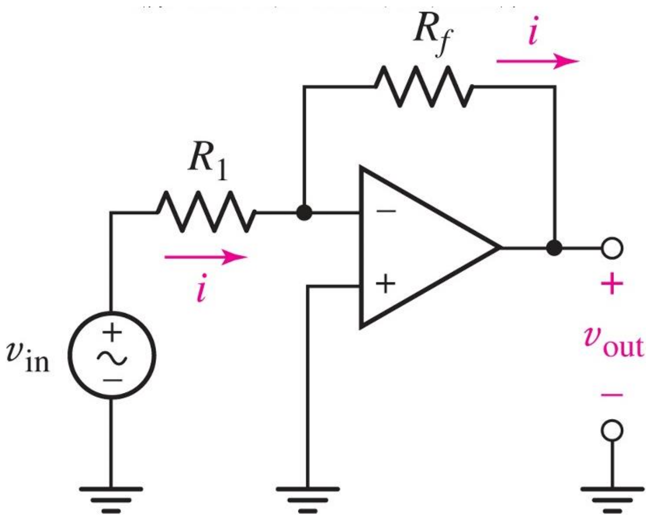
A more detailed Op Amp Model

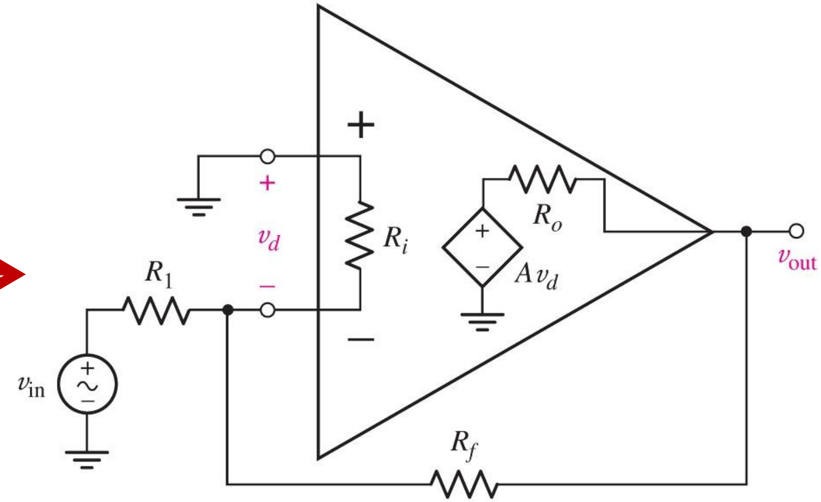
Example 2

Using the following values for *u*A741 op amp, reanalyze the inverting amplifier circuit and find v_{out}.

$$v_{in}(t) = 5sin3t\, mV, R_f = 47k\Omega, R_1 = 4.7k\Omega$$

For a 741 op amp: $A = 2 \times 10^5$, $R_i = 2\text{M}\Omega$, $R_o = 75\Omega$





Solution: $v_{out}(t) = -49.9977sin3t\, mV$

(Ideal op amp: $v_{out}(t) = -50sin3t\, mV$)

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The Non-inverting Amplifier

Solve V_{out}, use KVL, KCL, and op amp rules.

- According to op amp rule 1:

at node *a*: $0 = \frac{v_a}{R_1} + \frac{v_a - v_{out}}{R_f}$ [1]

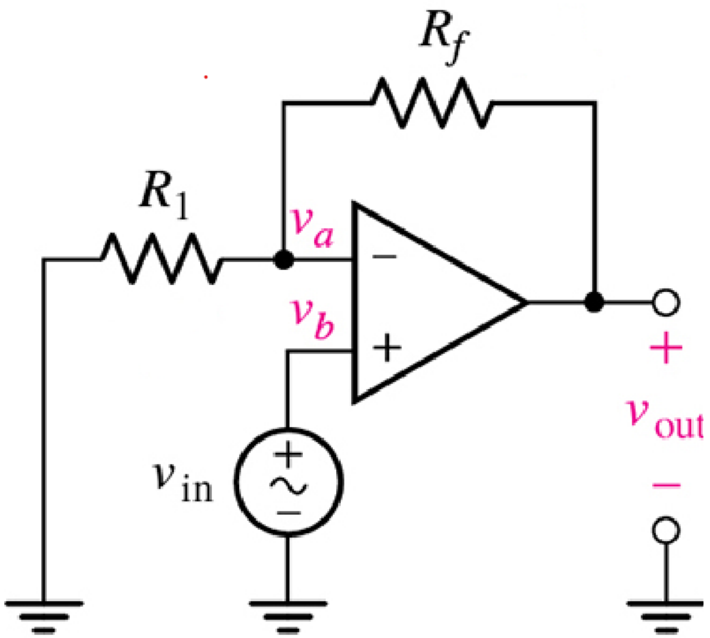
at node *b*: $v_b = v_{in}$ [2]

- According to op amp rule 2, since $v_a = v_b = v_{in}$,

$$0 = \frac{v_{in}}{R_1} + \frac{v_{in} - v_{out}}{R_f} \quad [3]$$

- Rearranging Eq.[3], We obtain v_{out}

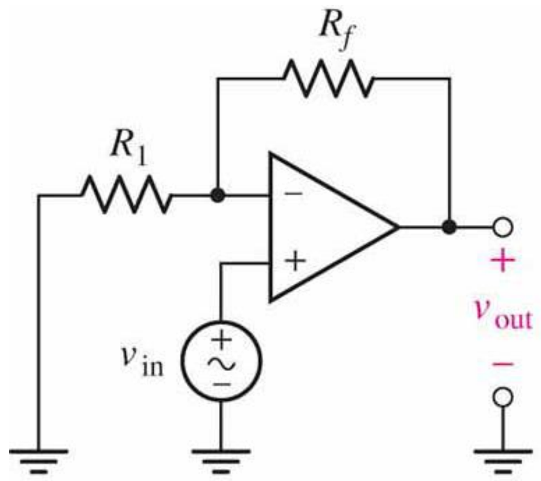
$$v_{out} = (1 + \frac{R_f}{R_1})v_{in} \quad [4]$$



Example 3

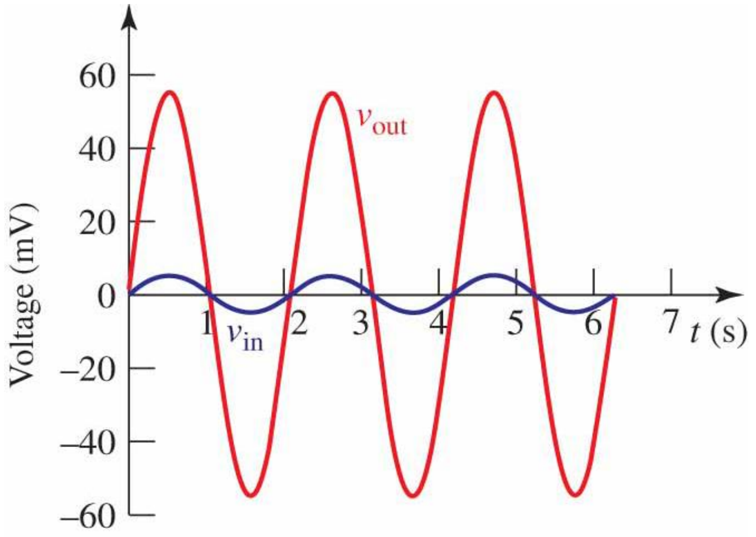
Derive an expression for non-inverting amplifier V_{out} in terms of V_{in} the circuit show below.

$$v_{in}(t) = 5sin3t\ mV, R_f = 47k\Omega, R_1 = 4.7k\Omega$$



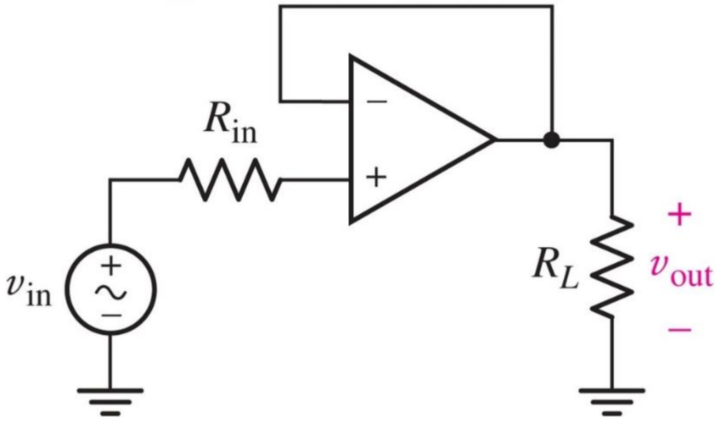
Solution:

$$v_{out}(t) = (1 + \frac{R_f}{R_1})v_{in} = 55sin3t\ mV$$



The Voltage Follower

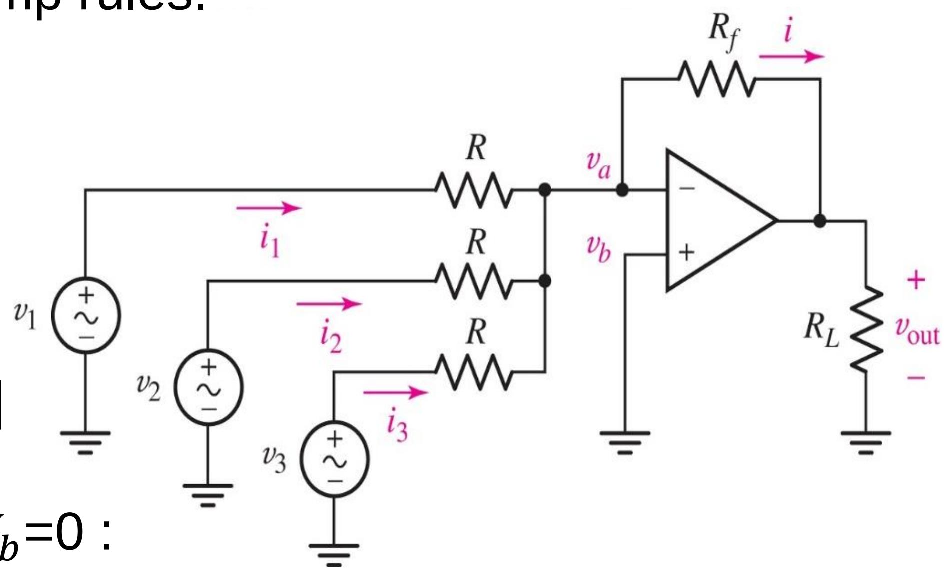
- The voltage follower is a noninverting amplifier
- R_1 set to ∞ and R_f set to zero.
- Output is identical to input in both sign and magnitude.
- Voltage follower draws no current from the input.
- Act as buffer between v_{in} and v_{out}



$$v_{out} = v_{in}$$

The Summing amplifier

Solve V_{out}, use KVL, KCL, and op amp rules.



- According to op amp rule 1:

$$i_1 + i_2 + i_3 = i \qquad [1]$$

- Therefore, at the node labeled V_a:

$$\frac{v_1 - v_a}{R} + \frac{v_2 - v_a}{R} + \frac{v_3 - v_a}{R} = \frac{v_a - v_{out}}{R_f} \qquad [2]$$

- According to op amp rule 2, $V_a = V_b = 0$:

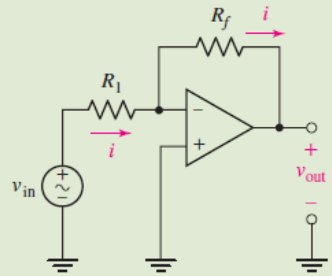
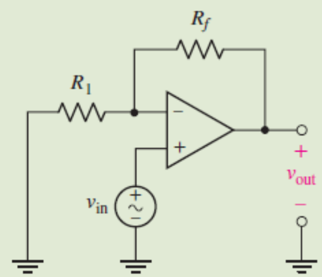
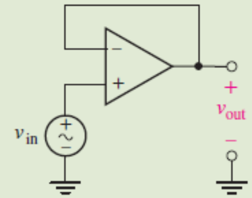
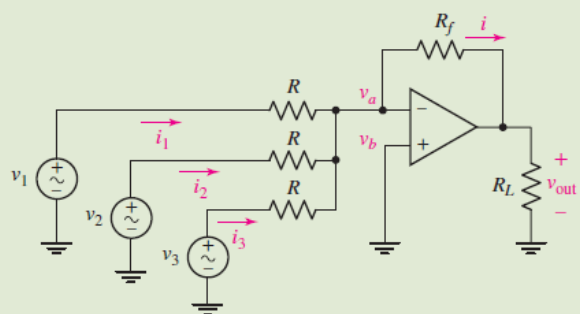
$$\frac{v_1}{R} + \frac{v_2}{R} + \frac{v_3}{R} = -\frac{v_{out}}{R_f} \qquad [3]$$

- Rearranging Eq.[3], We obtain V_{out}

$$v_{out} = -\frac{R_f}{R}(v_1 + v_2 + v_3) \qquad [4]$$

This amplifier performs the operation of adding. It also introduces a gain of $-\frac{R_f}{R}$ (same with inverting amplifier).

Summary of basic Op Amp Circuits

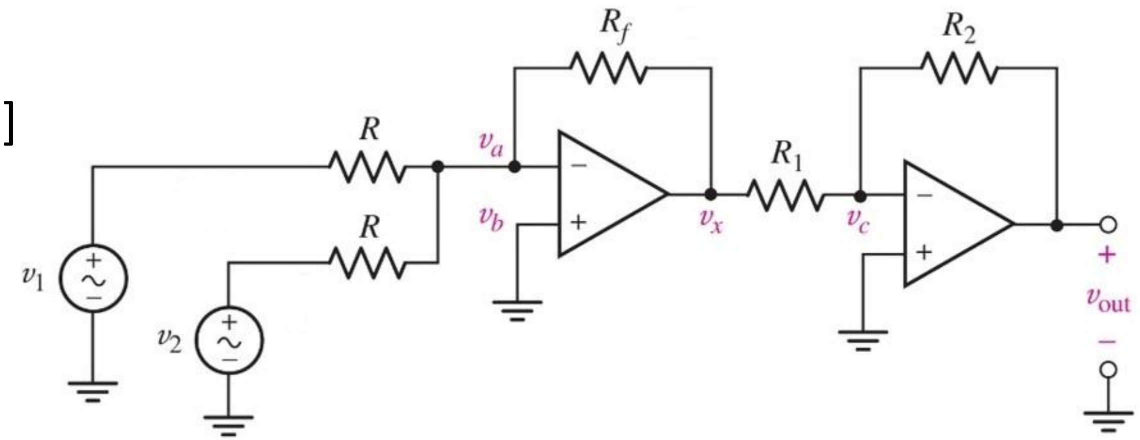
Name	Circuit Schematic	Input-Output Relation
Inverting Amplifier		$v_{out} = -\frac{R_f}{R_1} v_{in}$
Noninverting Amplifier		$v_{out} = \left(1 + \frac{R_f}{R_1}\right) v_{in}$
Voltage Follower (also known as a Unity Gain Amplifier)		$v_{out} = v_{in}$
Summing Amplifier		$v_{out} = -\frac{R_f}{R}(v_1 + v_2 + v_3)$

Cascaded stages

Solve V_{out}, use KVL, KCL, and op amp rules.

- According to op amp rule 1, at the node labeled v_c

$$\frac{v_c - v_x}{R_1} + \frac{v_c - v_{out}}{R_2} = 0 \qquad [1]$$



- According to op amp rule 2, v_c= 0:

$$\frac{v_x}{R_1} + \frac{v_{out}}{R_2} = 0 \quad [2]$$

- According to op amp rule 1, at the node labeled v_a:

$$\frac{v_a - v_x}{R_f} + \frac{v_a - v_1}{R} + \frac{v_a - v_2}{R} = 0 \qquad [3]$$

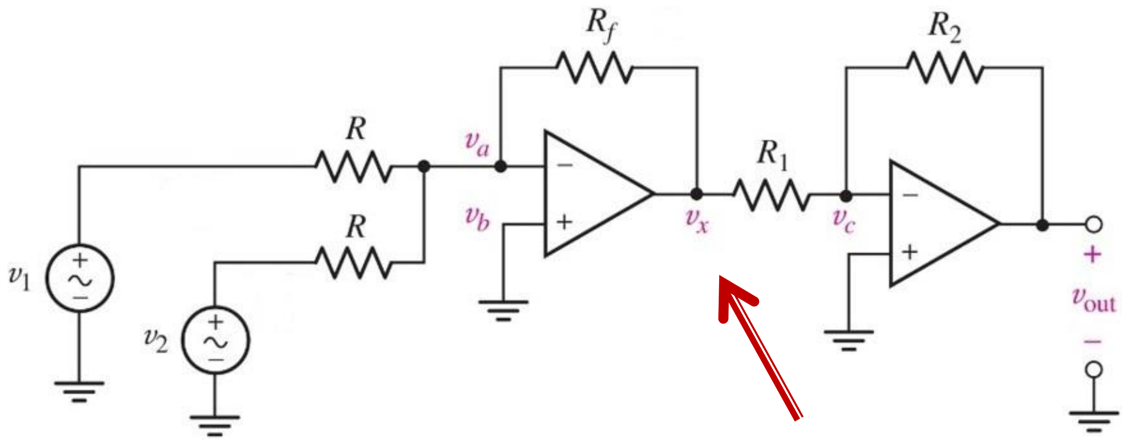
- According to op amp rule 2, v_a = 0, Eq.[3] becomes:

$$\frac{v_x}{R_f} + \frac{v_1}{R} + \frac{v_2}{R} = 0 \qquad [4]$$

Cascaded stages

- Combing Eq.[2] and [4], We obtain v_{out}

$$v_{out} = \frac{R_2}{R_1}\frac{R_f}{R}(v_1 + v_2) \qquad [5]$$



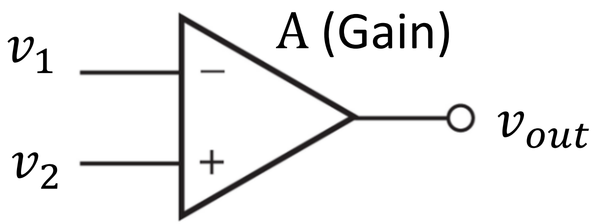
This voltage is not affected by the circuit on the right.

- Op amps can be combined in stages to create the desired relationship between the outputs and the inputs .
- The cascaded circuit act as a summing amplifier, but without inverting the sign between the input and output.

Common Mode Rejection

- When $v_1 = v_2$,the output should be zero, but real op amps produce a small “common mode” voltage.
- Defining $v_{o\mathrm{CM}}$ as the output obtained when both inputs are equal ($v_1 = v_2 = v_{CM}$), we can determine the common-mode gain of the op amp A_{CM}:

$$A_{\mathrm{CM}} = \left|\frac{v_{\mathrm{oCM}}}{v_{\mathrm{CM}}}\right|$$



$$v_{out} = Av_d + A_{\mathrm{CM}}v_{\mathrm{CM}}$$

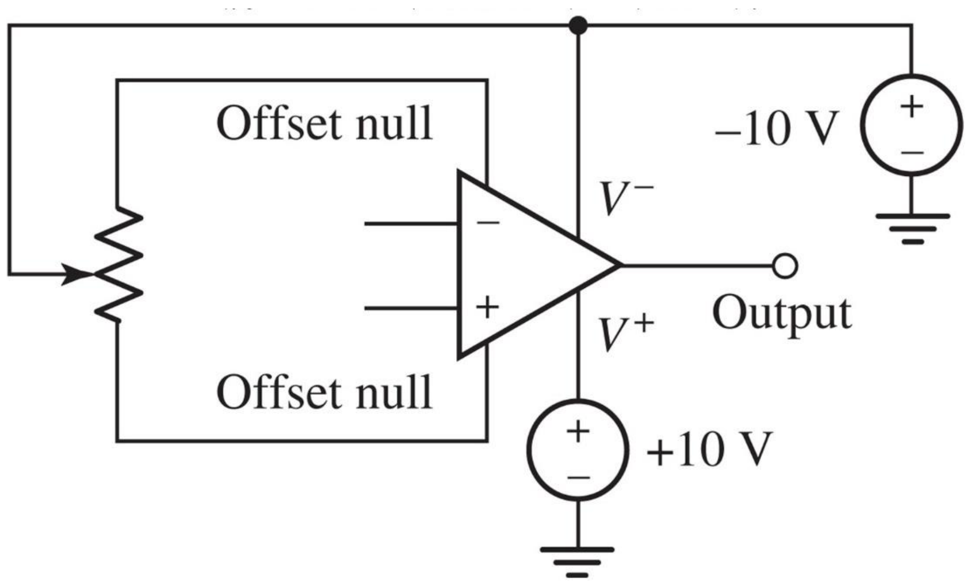
This voltage should be minimized !

- In practice, we would like to minimize the common-mode gain. We define the common-mode rejection ratio, or CMRR as the ability of the op amp to reject the common mode voltage. CMRR is the ratio of differential-mode gain A to the common-mode gain A_{CM}, or

$$\mathrm{CMRR} \equiv \left|\frac{A}{A_{\mathrm{CM}}}\right|$$

Input Offset Voltage

- Real op amps have a nonzero output even when the two input terminals are shorted together. The value of the output voltage under this condition is called offset voltage.
- Non-zero output “offsets” can be removed by following circuit:



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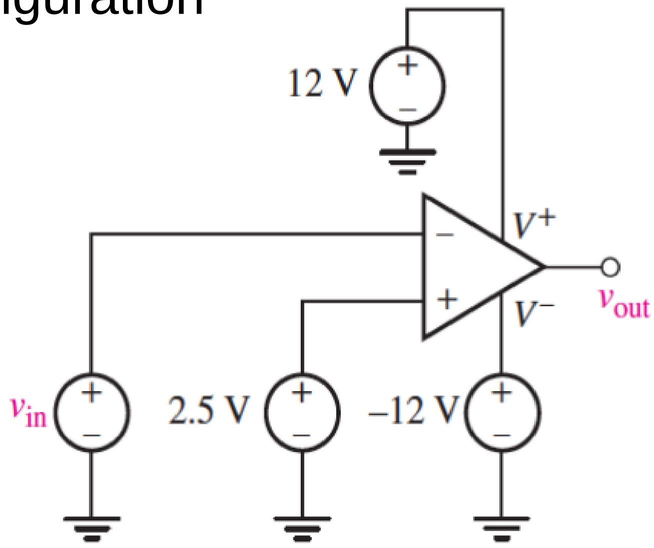
Comparator

- Closed loop is the preferred method of using op amp as an op amp (all the previous op amp circuits)
- Op-amp can also be used an open-loop configuration with different design: comparators!
- Since the op amp has a very large open-loop gain A, it can force the op amp into saturation with small difference.
- Thus, the output depends on how input compares to the reference voltage.
- For this example, reference is 2.5 V, output is at op amp saturation at ±12 V



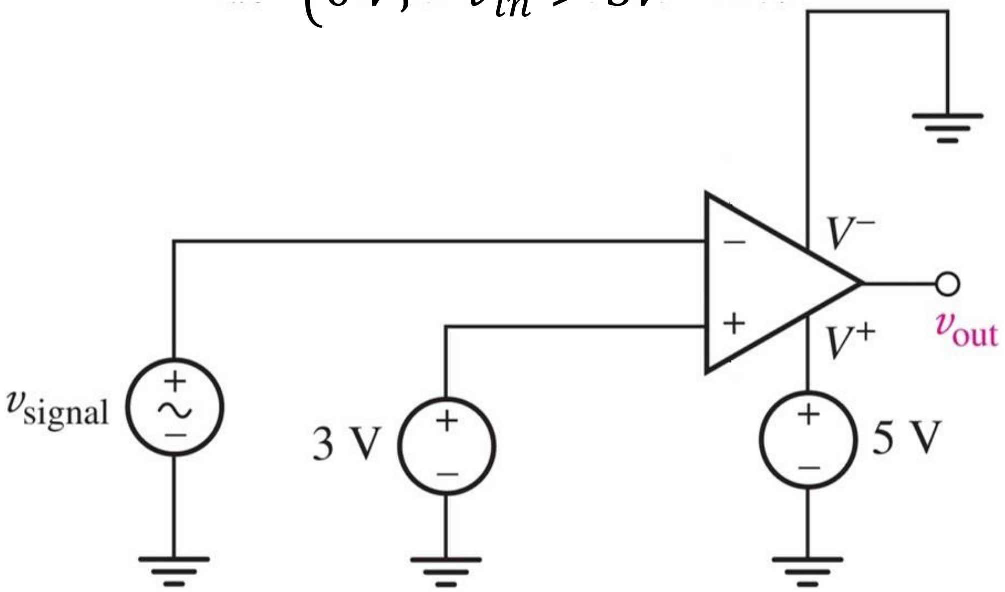
$$v_{out} = \begin{cases} 12V, & v_{in} < 2.5V \\ -12V, & v_{in} > 2.5V \end{cases}$$

Example 4

- Design a circuit that provides a "logic 1" 5 V output if a certain voltage signal drops below 3 V, and zero volts otherwise.

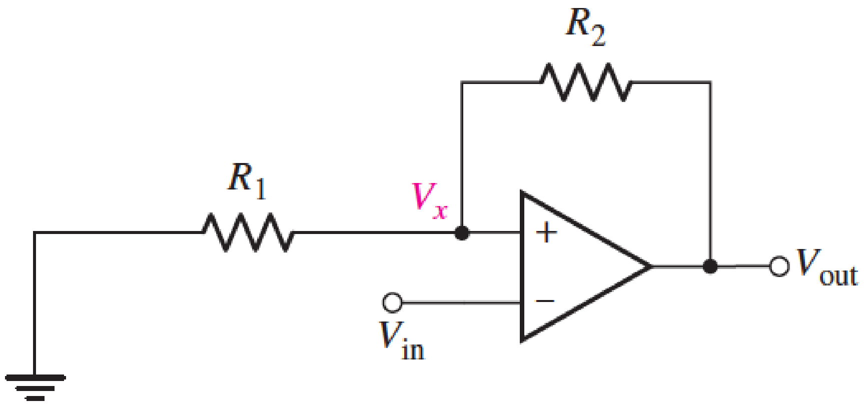
Solution:

$$v_{out} = \begin{cases} 5\ V, & v_{in} < 3V \\ 0\ V, & v_{in} > 3V \end{cases}$$



Schmitt Trigger

- Comparator circuits can also be used in a positive feedback configuration, such as Schmitt trigger circuit:



- The output voltage will be forced to the supply voltage V^+ and V^-,depending on the sign of the voltage difference between V_x and V_{in}.

$$v_{out} = \begin{cases} V^-, & v_{in} > V_x \\ V^+, & v_{in} < V_x \end{cases} \qquad [1]$$

Schmitt Trigger

- The voltage at the non-inverting terminal V_x is:

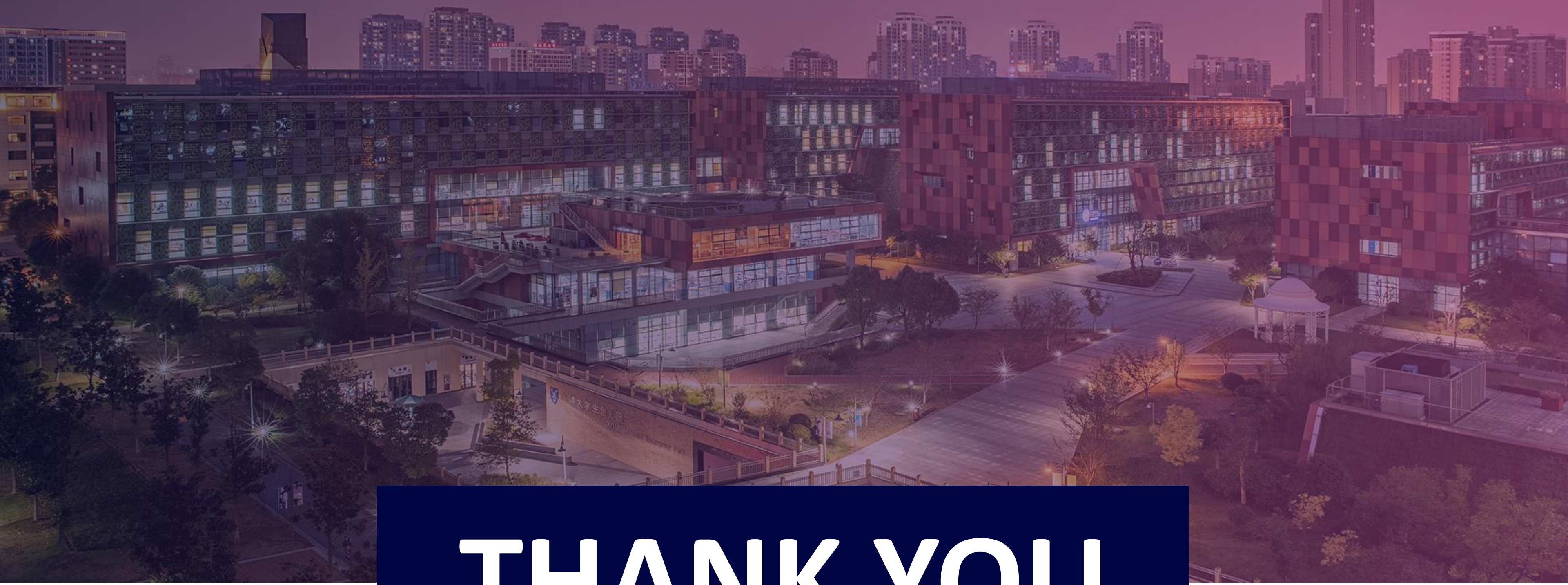
$$V_x = \frac{R_1}{R_1+R_2} V_{out} \qquad [2]$$

- Assuming that $V_{out} = V^+$ at initial state, the output voltage will remain in high state until V_{in} exceeds V_x . The upper threshold voltage is given by:

$$V_{in} = V_T^{upper} = \frac{R_1}{R_1 + R_2} V^+ \qquad [3]$$

- According to Eq [1], when V_{in} exceeds V_x , V_{out} will be forced to be V^-. The output voltage will remain in low state until V_{in} is less than V_x . The lower threshold voltage is given by:

$$V_{in} = V_T^{lower} = \frac{R_1}{R_1 + R_2} V^- \qquad [4]$$



THANK YOU