Xi'an Jiaotong-Liverpool University
西交利物浦大学

MEC208 Instrumentation and Control System

2024-25 Semester 2

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Lecture 19

Outline

Root Locus Method

- ❑ The Root Locus Concept
- ❑ Root Locus Plotting Procedure
- ❑ Root Locus Using Matlab
- ❑ Parameter Design based on the Root Locus Method
- ❑ **PID Controllers**
  - **Concept**
  - **PID Tuning**
- ❑ **Design Examples**

Lesson Aims

- To learn PID design through Root Locus Method and other tuning methods.
- To understand control system synthesis methods in the s-domain.
- To conduct computer aided control system design and analysis

PID Controllers

One form of controller widely used in industrial process control is the three-term PID Controller, acting on the tracking error:

$$\boxed{\boldsymbol{G_c(s) = K_P + \frac{K_I}{s} + K_D s}}$$

Assume that $E(s) = R(s) - Y(s)$, the ***s***-domain output is

$$U(s) \; = K_P E(s) \; + \frac{K_I}{s} E(s) + K_D s E(S)$$

Apply inverse Laplace transform, the **time**-domain output is

$$u(t) = \underbrace{K_P}_{\textbf{Proportional}} e(t) + \underbrace{K_I}_{\textbf{Integral}} \int_0^t e(\tau)\, dt + \underbrace{K_D}_{\textbf{Derivative}} \frac{de(t)}{dt}$$

The three terms are **P**roportional, an **I**ntegral, and a **D**erivative terms, whose gains are represented by K_P, K_I, K_D respectively.

Table 7.4 Effect of Increasing the PID Gains K_p, K_D, and K_I on the Step Response

PID Gain	Percent Overshoot	Settling Time	Steady-State Error
Increasing K_P	Increases	Minimal impact	Decreases
Increasing K_I	Increases	Increases	Zero steady-state error
Increasing K_D	Decreases	Decreases	No impact

- **Proportional-integral (PI) controller** (set $K_D = 0$):

$$\boxed{G_c(s) = K_P + \frac{K_I}{s}}$$

- **Proportional-derivative (PD) controller** (set $K_I = 0$):

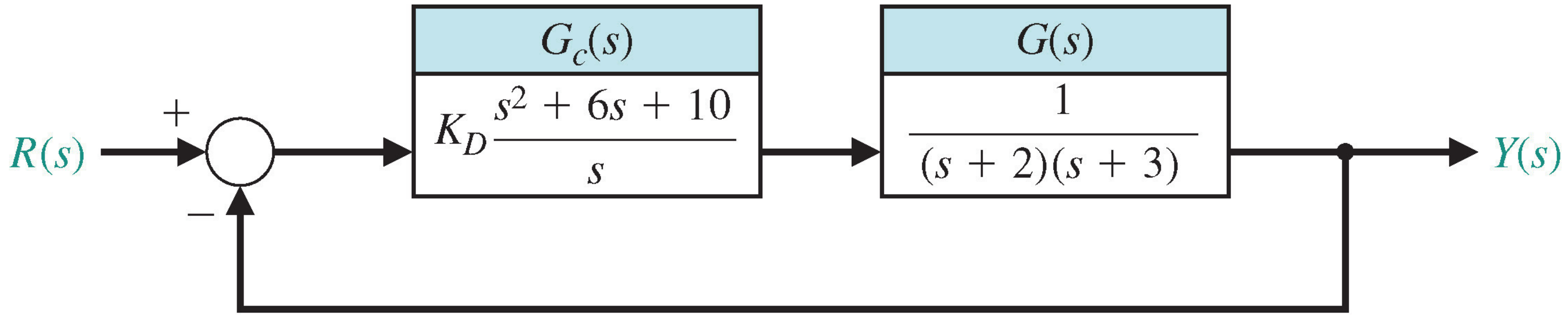
$$\boxed{G_c(s) = K_P + K_D s}$$

How a PID Controller Works

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s = \frac{K_D\left(s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right)}{s}$$

A PID controller adds two zeroes and one pole at the origin ($s = 0$) to the system under control.

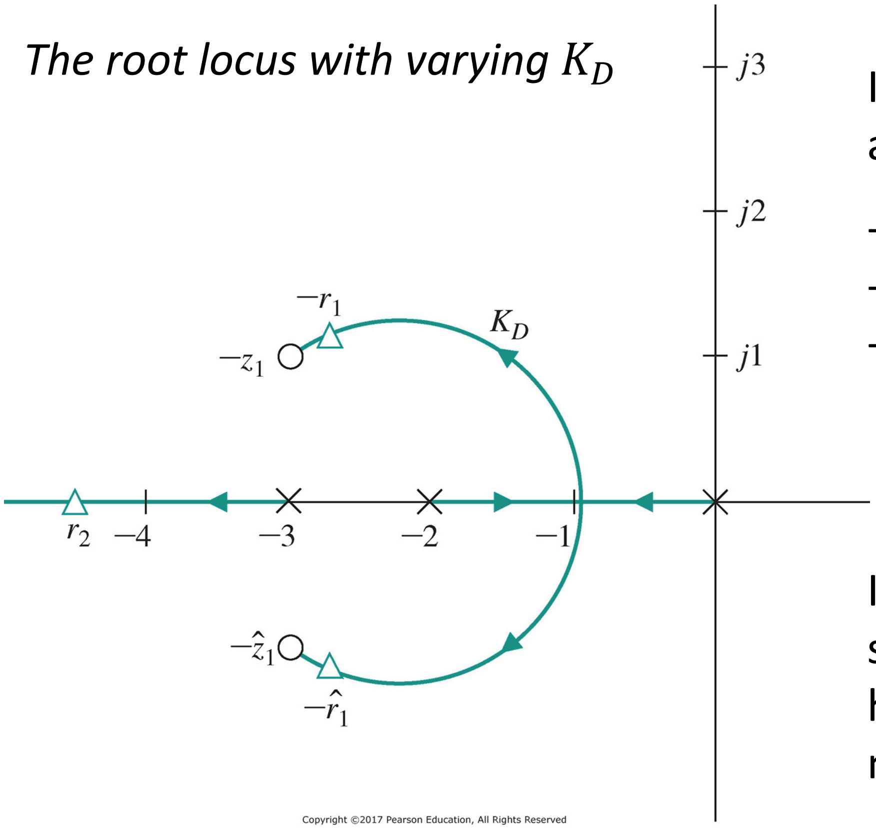
Example:



G_c in this CL system is a PID controller.

$$\Delta(s) = 1 + G(s)G_c(s) = 1 + \frac{K_D(s+z_1)(s+\hat{z}_1)}{s(s+2)(s+3)}$$

The root locus with varying K_D



If choose a value of K_D corresponding to $-r_1$ and -$\hat{r}_1$, then for a step input:

- the percent overshoot will be $P.O. \leq 2\%$,
- the steady-state error will be $e_{ss} = 0$;
- the settling time will be approximately $T_s = 1s$ $(\approx \frac{4}{3} = 1.333s)$.

If a shorter settling time is desired, then we select $-z_1$ and $-\hat{z}_1$ to lie further in the left-hand s-plane and set K_D to drive the roots near the complex zeroes.

PID Tuning

- Historically, before the advancement of embedded computing and PLC technology, PID was realized using analog circuitry (i.e., op-amps). It was well proven with the following advantages:
  - Functional simplicity – simple to operate
  - Good performance over a wide range of operating conditions

- To implement the PID controller, three parameters must be determined: proportional gain K_P, integral gain K_I and derivative gain K_D.

- The process of determining the gains is often called **PID tuning**. In the last two chapters, we have seen how a controller can be tuned analytically to meeting control design specification.

- Another common approach is <u>manual PID tuning methods</u>, whereby the PID control gains are obtained by trial-and-error with minimal analysis using step responses obtained via simulation, or in some cases, actual testing on the system. The gains based on observations and experience.

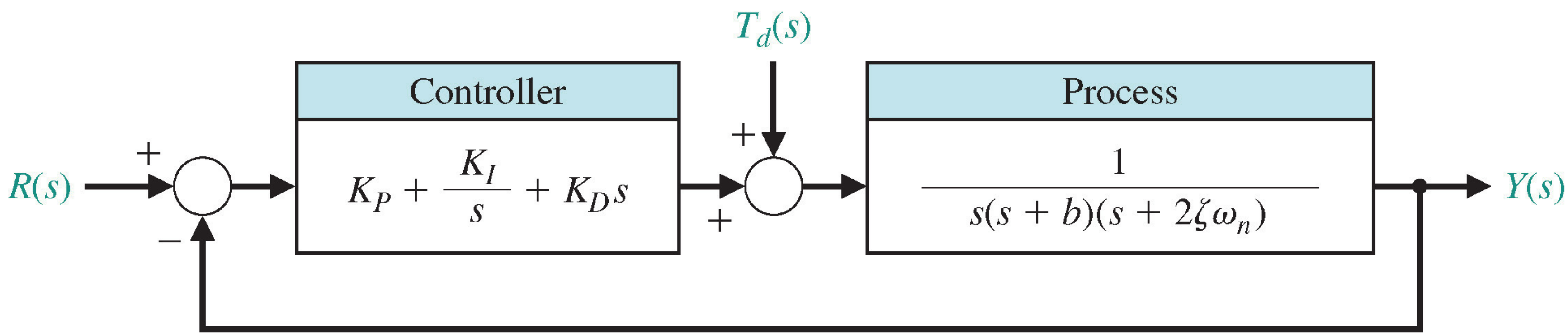
Manual Tuning Method

One approach to manually tuning is:

1. Set $K_I = 0$ and $K_\mathrm{D} = 0$;
2. Slowly increase the gain K_P until the output of the (practical) closed-loop system oscillates just on the edge of instability. This can be done either on the actual system or in simulation if it cannot be taken off-line;
3. Then, reduce the value of K_P to achieve what is known as the **quarter amplitude decay**, that is, the amplitude of the closed-loop response is reduced approximately to one-fourth of the maximum value in one oscillatory period. A rule-of-thumb is to start by halving the proportional gain K_P every time;
4. Then, increase K_I and/or K_D manually to achieve a desired response.

Example 19.1

For the following system, where $b = 10, \zeta = 0.707$ and $\omega_n = 4\, rad/s$. Design the PID controller so that the percent overshoot $P.O. \leq 15\%$ and settling time with 2% criterion $T_s < 3s$.

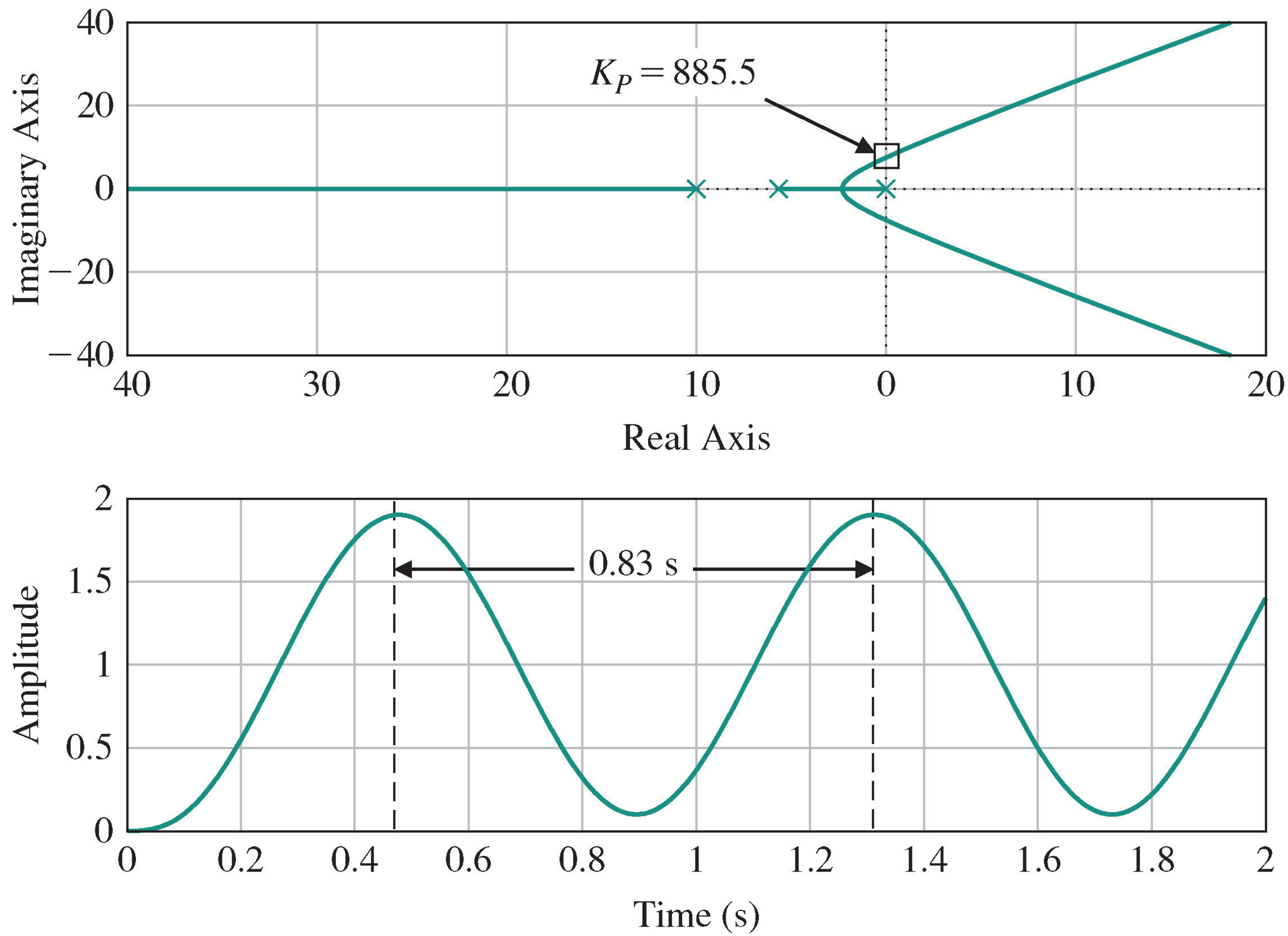


<u>Solutions:</u>

- Set $K_I = 0$ and $K_D = 0$; find the gain on the edge of instability using Routh-Hurwitz Criterion. The root locus with varying K_P can be also sketched.

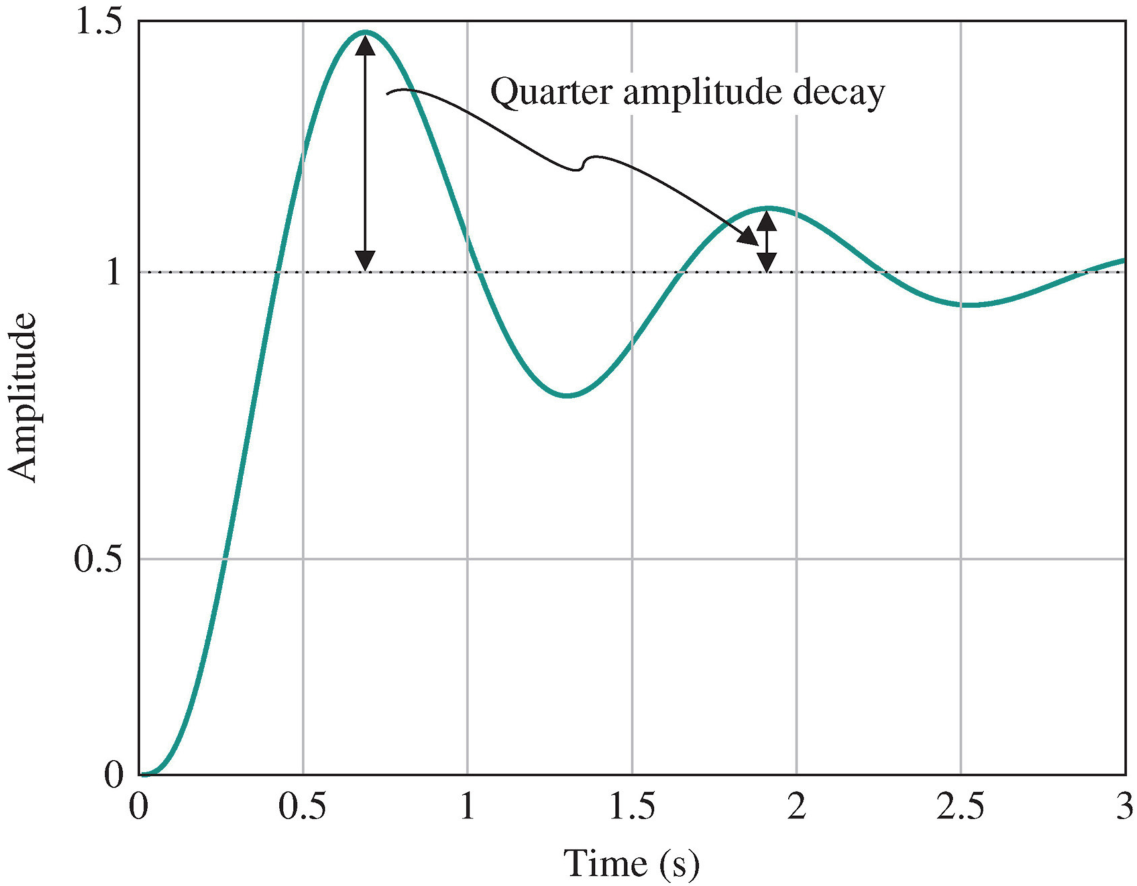
$$1 + K_P \left[\frac{1}{s(s+10)(s+5.66)} \right] = 0$$

Example 19.1 – near instability



Example 19.1 – Quart. amplitude decay through K_p

- Reduce $K_P = 885.5$ by half (start from $K_P = 442.75$) as a first step then keep reducing K_P to achieve a step response with approximately a quarter amplitude decay.

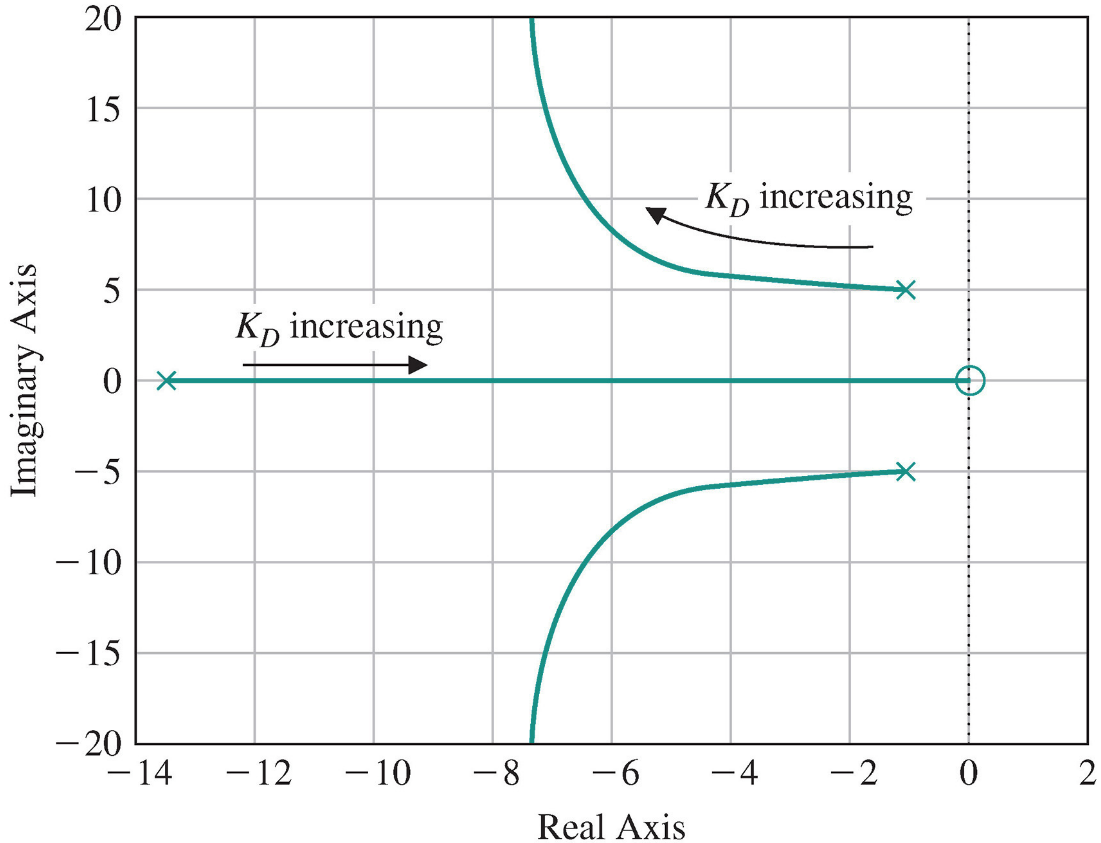


$K_P = 370$

Example 19.1 – Effect on RL by varying K_D

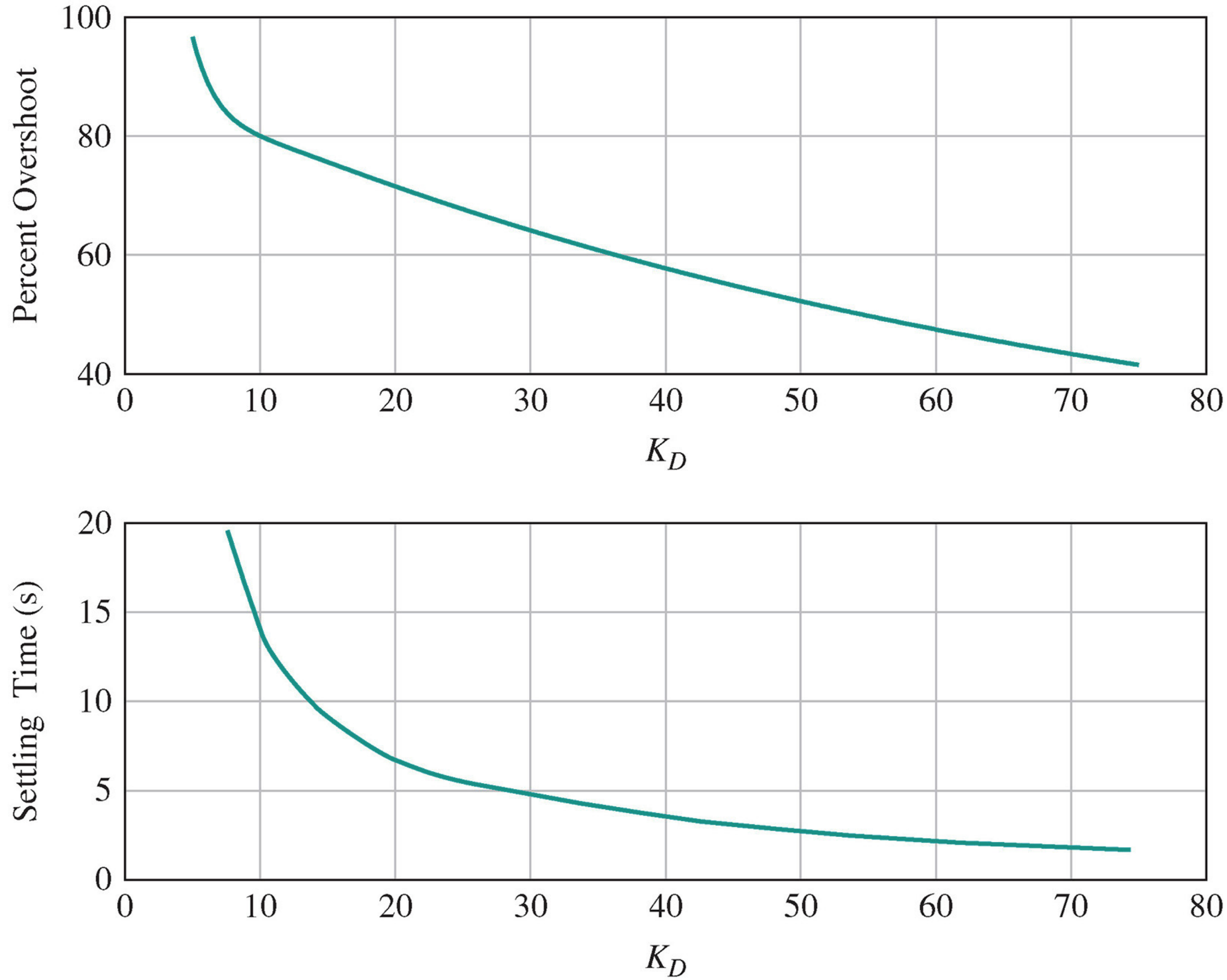
- Set $K_P = 370$ and $K_I = 0$, sketch the root locus with varying K_D.

$$1 + K_D\left[\frac{s}{s(s+10)(s+5.66)+K_P}\right] = 0$$



- In the beginning, the complex poles dominate the system behavior.
- As K_D increases (but small), associated damping ratio increases and thereby decreases P.O.; the complex poles to the left also increases $\zeta\omega_n$, thereby reducing the settling time;
- Until $K_D = 75$, when $K_D > 75$, the real root begins to dominate the response.

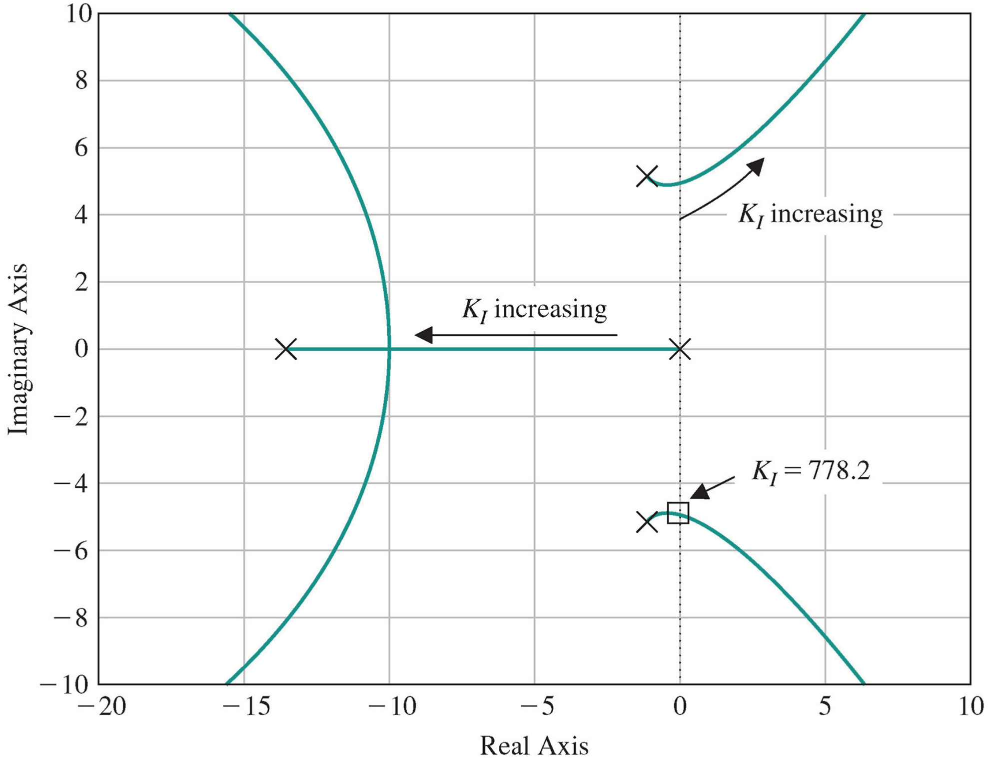
Percent overshoot and settling time with $K_P = 370$, $K_I = 0$, and $5 \le K_D < 75$.



Example 19.1 – Effect on RL by varying K_I

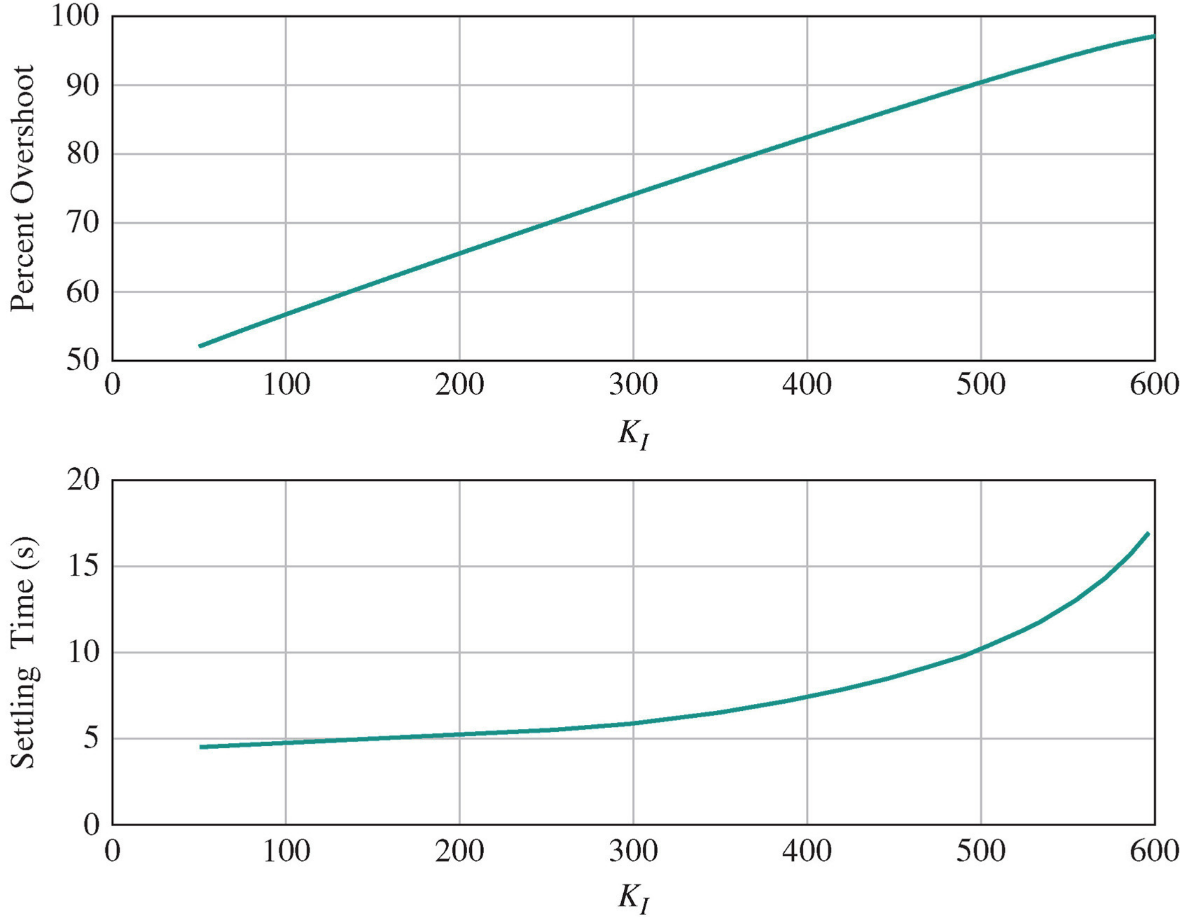
- Set $K_P = 370$ and $K_D = 0$, sketch the root locus with varying K_I.

$$1 + K_I \left[\frac{1}{s^2(s+10)(s+5.66) + K_P s} \right] = 0$$



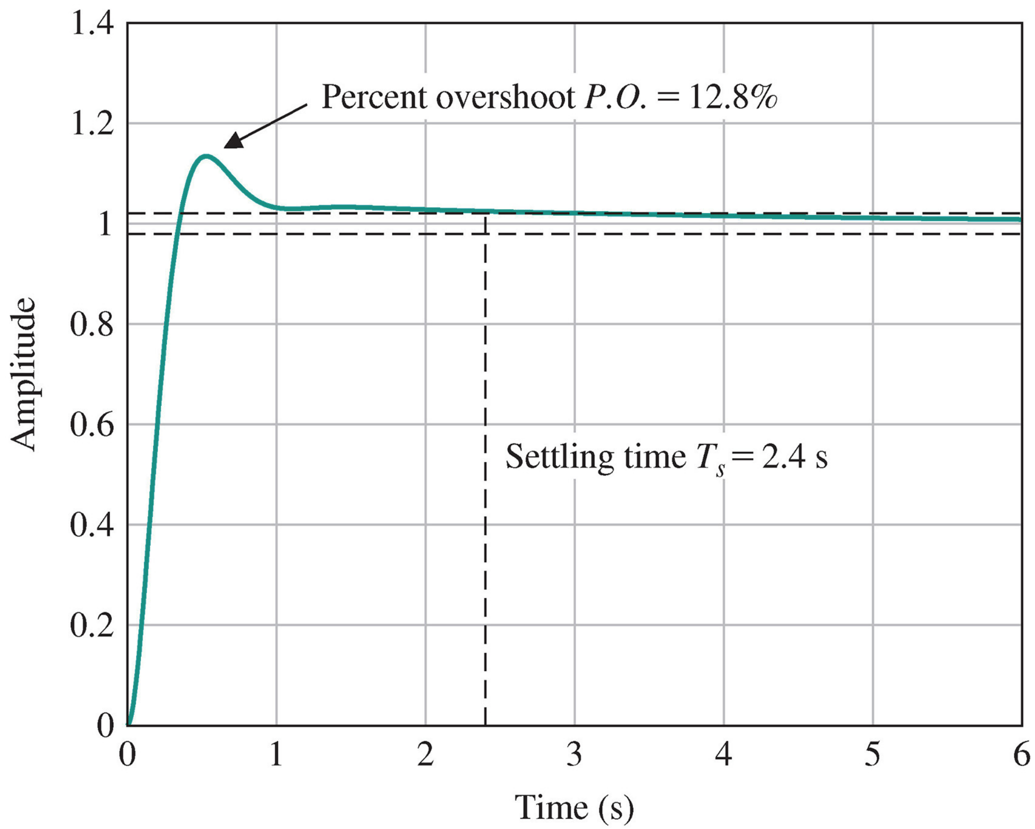
- As K_I beyond 778.2, the system will become unstable (marginally stable at $K_I = 778.2$)
- With a stable system, the pair of complex poles dominate the system behavior. As K_I increases, the complex pair poles move right and therefore increases the settling time.

Percent overshoot and settling time with $K_P = 370$, $K_D = 0$, and $50 \le K_I < 600$.



Example 19.1 – Overall

Finally, we choose $K_P = 370, K_D = 60, K_I = 100$, the step response of $P.O. = 12.8\%$ and $T_s = 2.4s$ is achieved.



Ziegler-Nichols PID Tuning Method

- Two important PID controller gain tuning methods were published in 1942 by John G. Ziegler and Nathaniel B. Nichols **intend to achieve a fast closed-loop step response without excessive oscillations and excellent disturbance rejection.** The two approaches are classified under the general heading of Ziegler-Nichols tuning methods.

- The Ziegler-Nichols tuning methods are based on assumed forms of the models of the process, but the models do not have to be precisely known. This makes the tuning approach very practical in process control applications.

- It is suggested to consider the Ziegler-Nichols rules to obtain initial controller designs followed by design iteration and refinement.

- Remember that the Ziegler-Nichols rules will not work with all plants or processes.

The Closed-loop Ziegler-Nichols Tuning Method (3 steps) considers the closed-loop system response to a step input (or step disturbance) with the PID controller in the loop:

1. Set $K_I = 0$ and $K_\mathrm{D} = 0$;

2. Increase K_P (in simulation or on the actual system) until the closed-loop system reaches the boundary of instability. The gain on the border of instability, denoted by $\boldsymbol{K_U}$, is called the **ultimate gain**. The period of the sustained oscillations, denoted by $\boldsymbol{T_U}$, is called the **ultimate period**.

3. Once K_U and T_U are determined, the PID gains are computed using the following relationship.

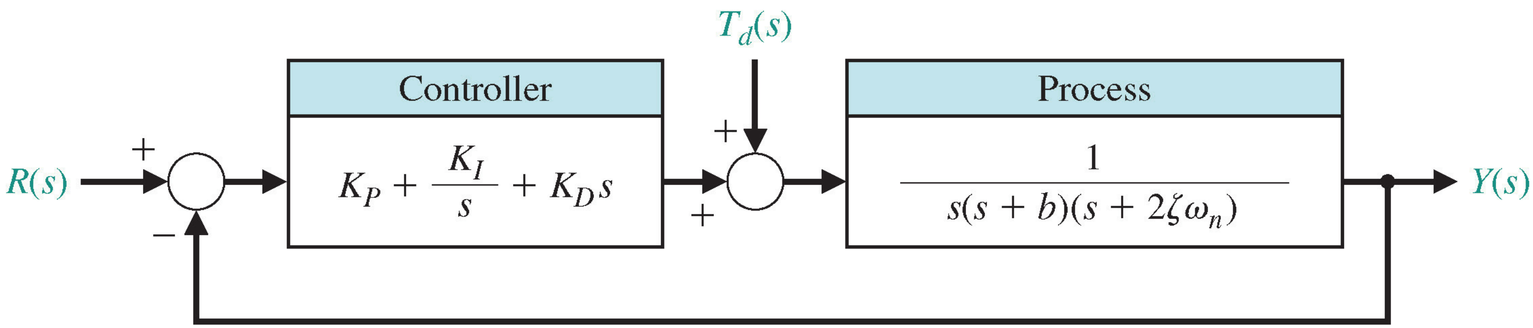
Table 7.7 Ziegler-Nichols PID Tuning Using Ultimate Gain, K_U, and Oscillation Period, P_U

Ziegler-Nichols PID Controller Gain Tuning Using Closed-loop Concepts

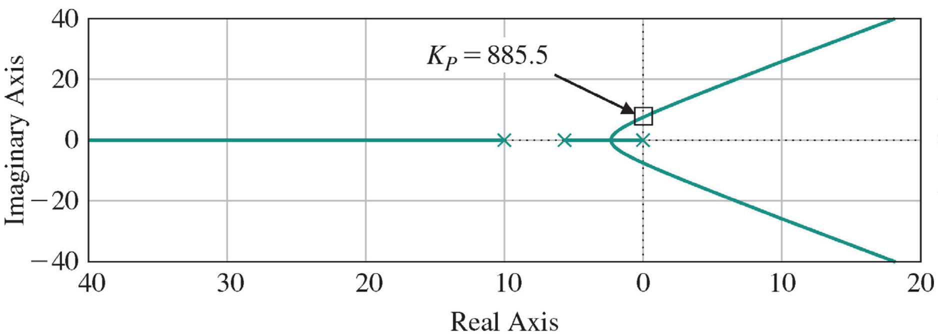
Controller Type	K_P	K_I	K_D
Proportional (P) $G_c(s) = K_P$	$0.5K_U$	–	–
Proportional-plus-integral (PI) $G_c(s) = K_P + \frac{K_I}{s}$	$0.45K_U$	$\frac{0.54K_U}{T_U}$	–
Proportional-plus-integral-plus-derivative (PID) $G_c(s) = K_P + \frac{K_I}{s} + K_D s$	$0.6K_U$	$\frac{1.2K_U}{T_U}$	$\frac{0.6K_U T_U}{8}$

Example 19.2

Re-consider the previous example.

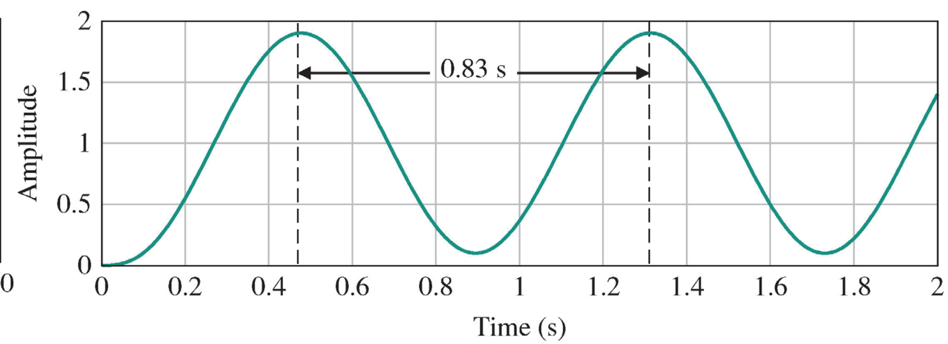


Root Locus with varying K_P
$(K_D = K_I = 0)$



$K_U = 885.5$

Step Response

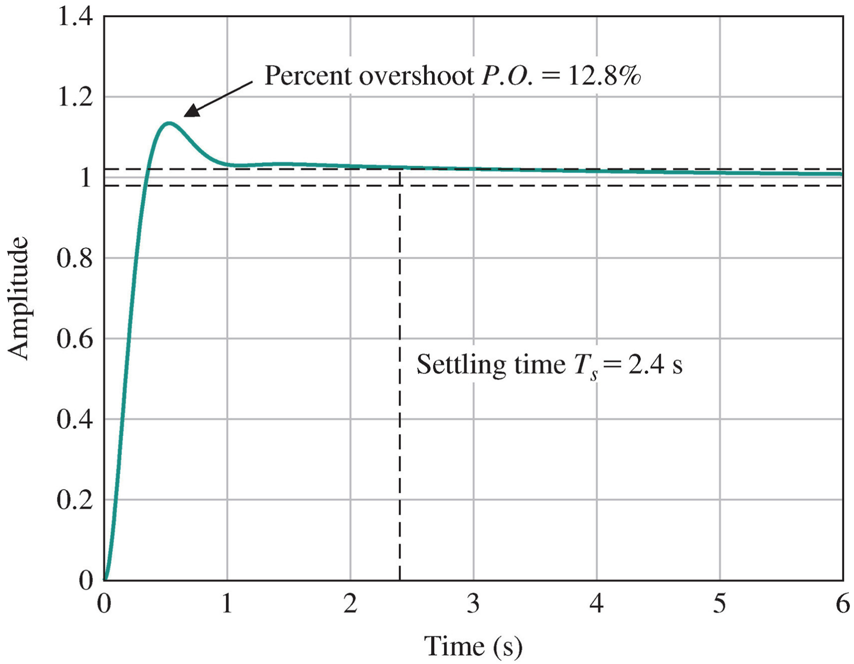


$T_U = 0.83s$

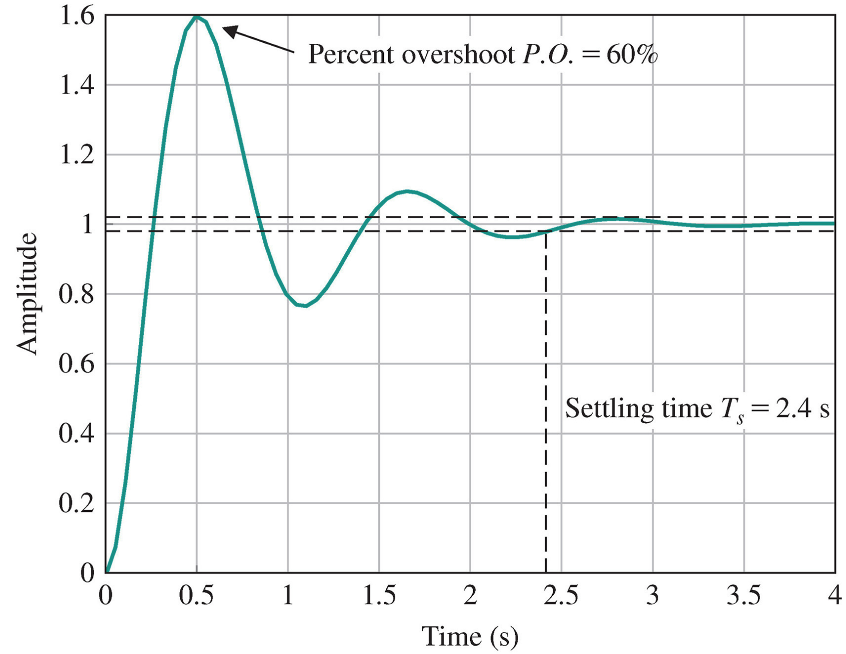
By using the Ziegler-Nichols formulas we obtain

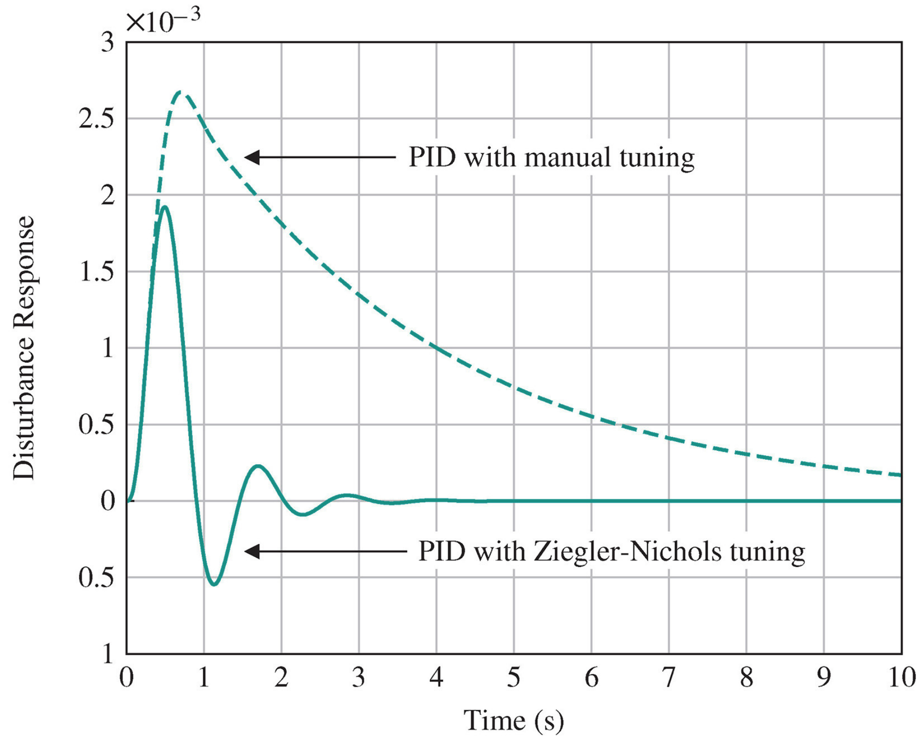
$$K_P = 0.6K_U = 531.3, \qquad K_I = \frac{1.2K_U}{T_U} = 1280.2, \qquad \text{and } K_D = \frac{0.6K_U T_U}{8} = 55.1$$

Time response for Manual Tuning



Time response for the Ziegler–Nichols Tuning



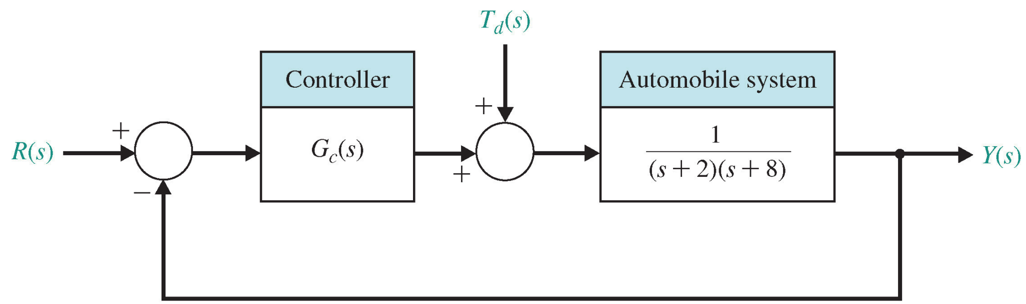


Disturbance response for the Ziegler–Nichols PID tuning versus the manual tuning.

- We see that the step disturbance performance of the Ziegler-Nichols PID controller is indeed better than the manually tuned controller;
- While Ziegler-Nichols approach provides a structured procedure for obtaining the PID controller gains, the appropriateness of the Ziegler-Nichols tuning depends on the requirements of the problem under investigation.

Example 19.3: Design for Automobile Velocity Control (through the analytical method)

A velocity control system for maintain the velocity between the two automobiles are shown as follows:



Control Goal:

- Maintain the prescribed velocity between the two vehicles and maneuver the active vehicle as command.

Variable to Be Controlled:

- The relative velocity between vehicles, denoted by $y(t)$.

Design specification (DS):

- DS1. Zero steady-state error to a step input.
- DS2. Steady-state error due to a ramp input of $e_{ss} \leq 25\%$ of the input slope/magnitude.
- DS3. Percent overshoot of $P.O. \leq 5\%$ to a step input.
- DS4. Settling time of $T_s \leq 1.5\ s$ to a step input (using 2% criterion).

Solutions:

Step 1.

$G(s)$ is a type 0 system. To guarantee a zero steady-state error to a step input (DS1), the controller needs to increase the system type by at least 1. That is, a controller with one **integrator**.

We consider the **PI** controller $G_c = K_P + \frac{K_I}{s} = K_P \frac{s+\frac{K_I}{K_P}}{s}$.

To meet DS2, it requires that

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{1 + G_c(s)G(s)} \cdot \frac{1}{s^2} = \lim_{s \to 0} \frac{(s+2)(s+8)}{K_P(s + \frac{K_I}{K_P})} = \frac{16}{K_I} \leq 25\%$$

Therefore, the integral gain must satisfy

$$K_I \geq 64$$

Step 2.

Consider values of K_P and K_I to make the system stable. The closed-loop characteristic equation is

$$\Delta(s) = s^3 + 10s^2 + (16 + K_P)s + K_I$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 16 + K_P \\ s^2 & 10 & K_I \\ s^1 & \frac{10(K_P+16)-K_I}{10} & 0 \\ s^0 & K_I & 0 \end{array}$$

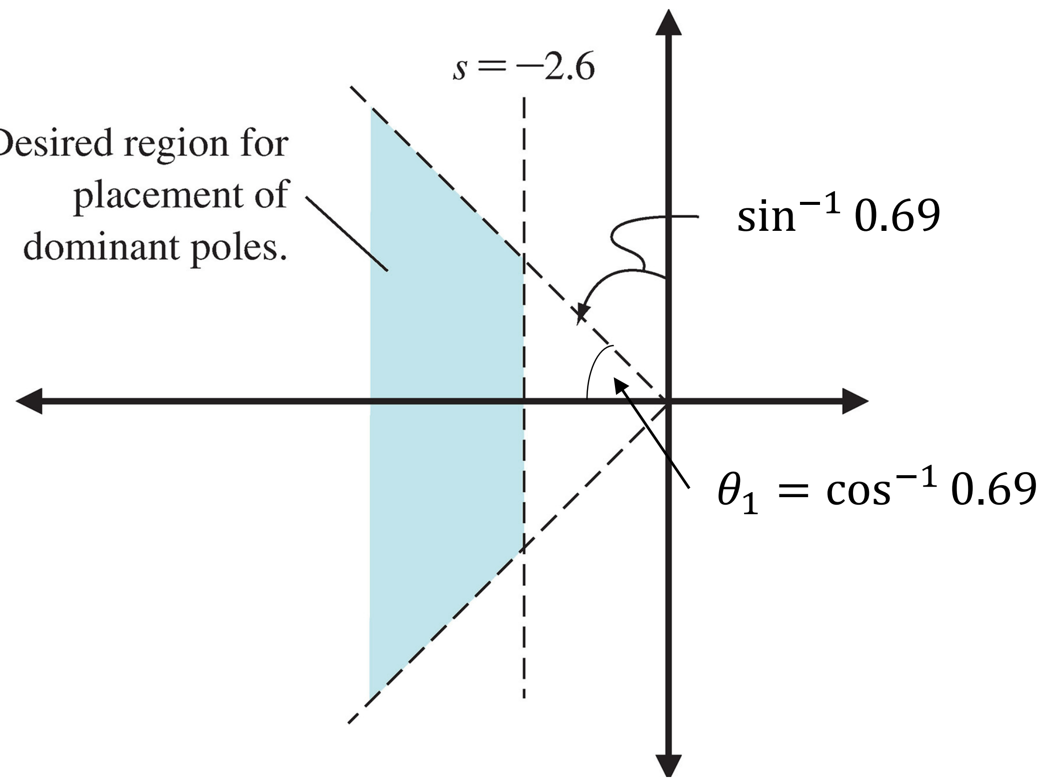
Therefore, we need

$$K_I > 0 \quad \text{and} \quad K_P > \frac{K_I}{10} - 16$$

Step 3.

Consider DS3 and DS4, we can obtain the desired region in the s-plane for locating the dominant system poles.

$$T_s = \frac{4}{\zeta\omega_n} < 1.5$$
$$\zeta\omega_n > 2.67\ (\approx 2.6)$$
$$Re(s) = -\zeta\omega_n < -2.6$$



Percent Overshoot:

$$\boxed{P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

$$P.O. \leq 5\% \longrightarrow \zeta \geq 0.69$$

$$\theta < \cos^{-1}\zeta$$
$$\theta < \cos^{-1} 0.69\ (i.e., \theta_1)$$

Step 4.

Consider the root locus. Here, we want to have the dominant poles to the left of the $s = -2.6$ line. We know from our experience sketching the root locus that since we have three poles (at $s = 0, s = -2$ and $s = -8$) and one zero (at $s = -K_I/K_P$), we expect two branches of the loci to go to infinity along two asymptotes at $\emptyset = \pm 90^o$ centered at

$$\sigma_A = \frac{\sum(-p_i) - \sum(-z_i)}{n - M}$$

where $n = 3$ and $M = 1$. In this case

$$\sigma_A = \frac{-2 - 8 - (-\frac{K_I}{K_P})}{2} = -5 + \frac{1}{2}\frac{K_I}{K_P}$$

We want to have $\sigma_A < -2.6$ so that the two branches will enter/bend into the desired regions.

$$\sigma_A = -5 + \frac{1}{2}\frac{K_I}{K_P} < -2.6 \longrightarrow \frac{K_I}{K_P} < 4.7$$

Step 5.

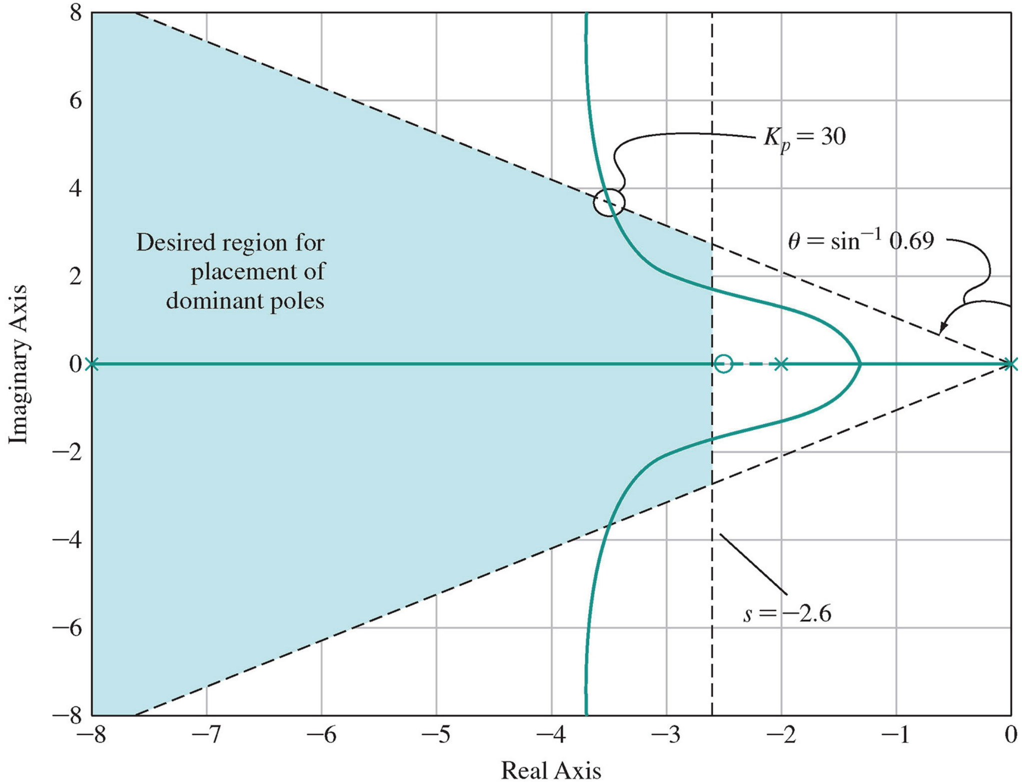
As the first design choice, we select K_P and K_I such that

$$K_I \geq 64, \qquad K_P > \frac{K_I}{10} - 16, \qquad \text{and} \qquad \frac{K_I}{K_P} < 4.7$$

Suppose we choose $\frac{K_I}{K_P} = 2.5$. Then the closed-loop characteristic equation is

$$1 + K_p \frac{s + 2.5}{s(s+2)(s+8)} = 0$$

Sketch the root locus for the above equation.



- The crossing of the locus with the DS3's dotted line can be determined by substituting $s = -0.691\omega_n + j0.724\omega_n$ into $1 + G_c GH = 0$.

Solution:

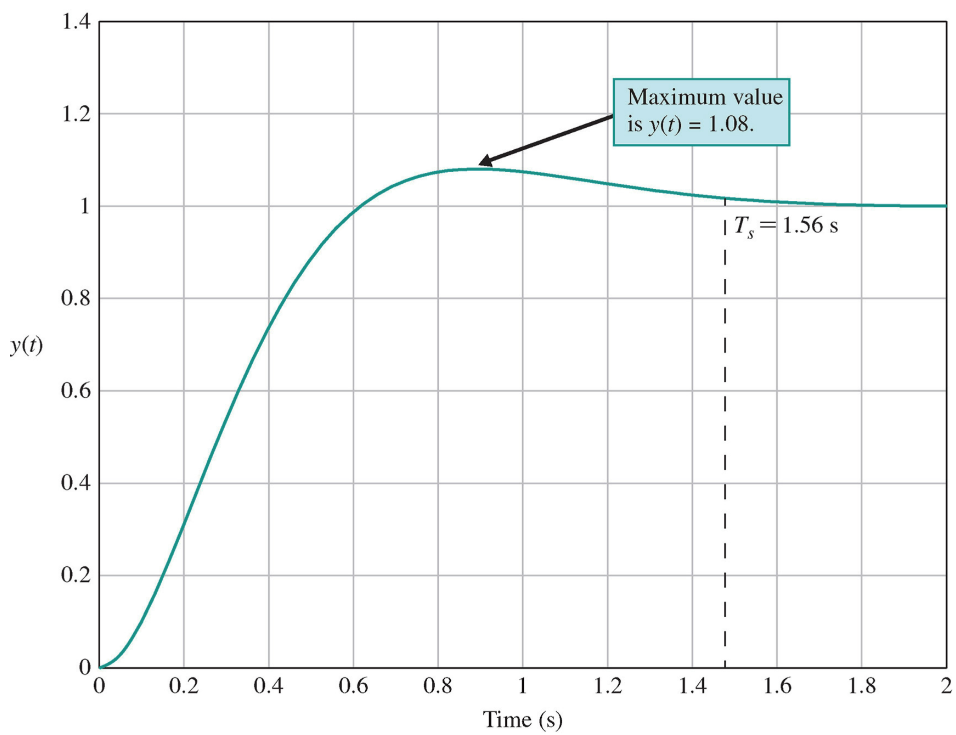
- We need to select $K_P < 30$ to meet the DS3 ($P.O. \leq 5\%$, i.e. $\zeta < 0.69$) requirement.

We can select $K_P = 26$ as a first try. Since we have $\frac{K_I}{K_P} = 2.5$, then $K_I = 65$. This satisfies the DS2 since $K_I > 64$. The resulting PID controller is

$$\boxed{G_c = 26 + \frac{65}{s}}$$

Step 6.

Check the step response (use Matlab).



- $P.O. = 8\%$, $T_s = 1.56$ s. DS3 and DS4 are not precisely satisfied; but the controller represents a very good first design.

"Reason of discrepancy"

- The formula used for P.O. and settling time are based on the characteristic (normalized) "2nd order system without any zero", whereas the system is a "3rd order system with a zero".

- We can then iterate the design process to refine it (e.g. choose different value in Step 5 and try again).

Example 19.4 (in-class)

Two unity feedback control systems have the loop transfer functions, sketch the root locus.

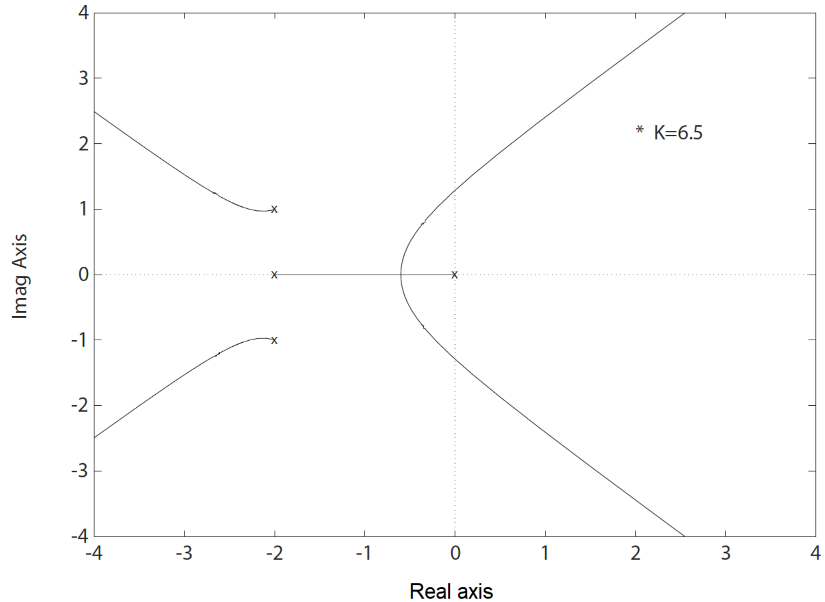
1) $L(s) = G_c(s)G(s) = \frac{K}{s(s+2)(s^2+4s+5)}$

2) $L(s) = G_c(s)G(s) = \frac{K(s^2+4s+8)}{s^2(s+4)}$

Example 19.4 (in-class)

Two unity feedback control systems have the loop transfer functions, sketch the root locus.

1) $L(s) = G_c(s)G(s) = \dfrac{K}{s(s+2)(s^2+4s+5)}$



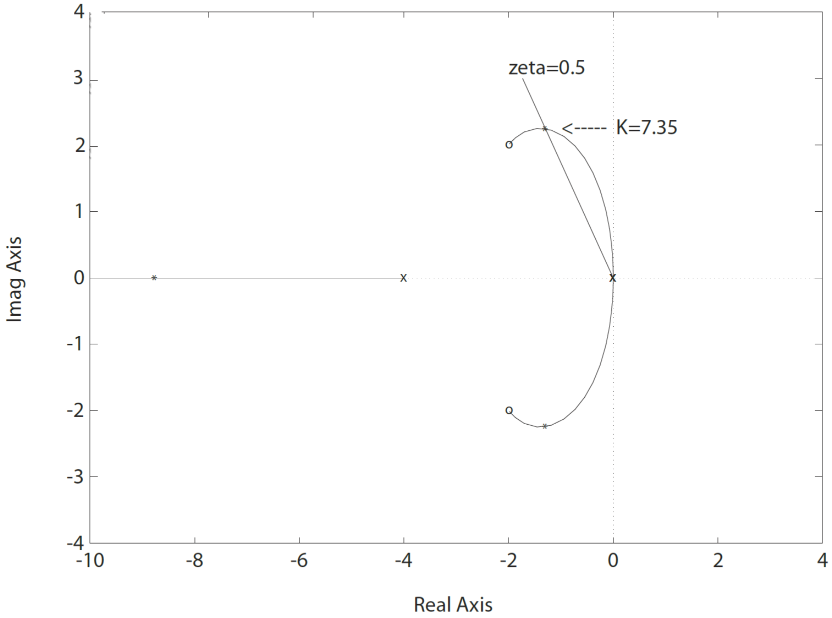
Answer: OL zeroes: none; OL poles: 0, -2, -2±j
S – symmetrical, N = 4, R = 4 – 0 = 4
O – sketch and recognize this feature in the plot
A – P.O.I. @real axis = -1.5; Angles of asymptotes: 45°, 135°, 225°, 315°
B – Apply the rule, B. I. B. O. points obtained are -0.58 (on locus, accept), -1.51±1.08j (ignore complex values)
A – ~~not relevant here~~ Apply this rule, get -153.4°,153.4°
+1 rule – Apply. Conclude that crossing occurs only at k = 18.89.

Example 19.4 (in-class)

Two unity feedback control systems have the loop transfer functions, sketch the root locus.

2) $L(s) = G_c(s)G(s) = \frac{K(s^2+4s+8)}{s^2(s+4)}$

Answer: OL zeroes: -2±*j*2; OL poles: 0, 0, -4
S – symmetrical, N = 3, R = 3 – 2 = 1
O – sketch and recognize this feature in the plot
A – P.O.I. @real axis = 0; Angles of asymptotes: 180°
B – Apply the rule, B. I. B. O. points obtained are 0, -2.41(only "0" on locus, accept; reject -2.41), -2.8±4.3*j* (ignore complex values)
A – angle of arrival. 45° (upper), -45°(lower)
+1 rule – Apply. Conclude that crossing occurs only at *k* = 0.



Additional Exercises (self-check)

Textbook ("Modern Control Systems" by R. C. Dorf & R. H. Bishop, 14th edition), Chapter 7:

- Skills Check - Can be found from Textbook pg. 514-518 (answer in pg. 543) or LMO MEC280's "Quizzes/Tutorials" section.

Additional:

E7.4, E7.6, E7.8, E7.9, E7.10, E7.15, E7.18, E7.20, E7.21 (answers are provided in the textbook, except the following two):

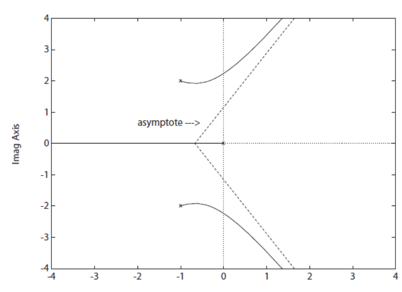
Partial answers to E7.9:

7.9(a): R = 3, Asymptotes angle +60°, +180°, +300°, POI@real = -2/3

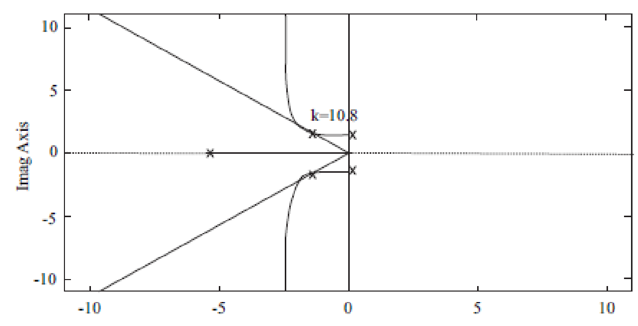
7.9(b): Departure angles: $\pm 26.6^o$

7.9(c): K = 10

7.9(d):



Partial answers E7.21: K = 10.8. [Solution steps: find out ω_n of the dominant oscillatory poles; calculate the exact poles with ζ=0.66; solve for K through $K|L(s)|$ = 1]



Concluding Remarks and Next Lectures

- **What have been covered:** "Root Locus Method"
  - Basic concept
  - Root Locus Plotting Procedure, and Matlab
  - Analytical design of gains/parameters using RLM
  - Recognize the basic forms of PID
  - Manual/empirical tuning (vs the analytical tuning method)
  - Interpretation of Root Locus

- **In our next lectures:** we will learn another Frequency Response Methods: "Bode plot" and "Nyquist plot".
- **What you can do from now till the next lecture:** revise the material, further reading, group study, and skill checks (LMO tutorial, or Textbook chapter 7).
- **How to get in touch**: through LMO Module's "General question and answer forum" section or during my weekly consultation hour(s).