EEE112
Integrated Electronics & Design: Exercise Problem

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Solution

<u>Contents</u>

A. Semiconductor fundamentals

Atomic Density

1. $$\frac{\text{Number of atoms in the unit cell}}{\text{Lattice constant cubed}} = \frac{4}{(5\text{x}10^{-8}\text{ cm})^3} = 3.2\text{x}10^{22}\text{ atoms cm}^{-3}$$

2. $18.7\text{x}10^{22}$ atoms cm^{-3}

Crystal Lattice Plane

1. The intercepts of the plane on the axes are 2, 3, 2 (in directions a, b, c)

   The reciprocals are therefore (1/2 1/3 1/2)

   Multiplying by the lowest common denominator (6) gives (323)

2. Axes intercepts are 6 2 3

3. The vector direction is described by the component sizes [2 4 1]

   The plane is also the (2 4 1) plane

   The intercepts of the plane are found as (1/2 1/4 1/1) multiplied by the lowest common denominator (4)

   Plane intercepts are (2 1 4)

4. Vector direction is described by component values [1 1 3]

   The plane is also (1 1 3)

   The plane has intercepts at (1/1 1/1 1/3) * 3 = (3 3 1)

Energy Levels

1. $E_1 = -13.6$ eV ($-2.17\text{x}10^{-18}$ J)
   $E_2 = -3.39$ eV
   $E_3 = -1.51$ eV

2. n=5

Energy Band

1. $\Delta E = (1/2)mv_2^2 - (1/2)mv_1^2$

   $v_2 = \Delta v + v_1$ so $v_2^2 = v_1^2 + (2v_1\Delta v) + \Delta v^2$ but $\Delta v \ll v_1$

   therefore $\Delta E = (1/2)m(2v_1\Delta v) = 5.7\text{x}10^{-9}$ eV

2. $\Delta v = 0.878$ cm/s

Density of States and Probability

1.

$$N = \int_0^{1eV} g(E)dE = \frac{4\pi(2m)^{3/2}}{h^3}\int_0^{1eV}\sqrt{E}dE$$

$$N = \frac{4\pi(2m)^{3/2}}{h^3}\frac{2}{3}E^{3/2}\Big|_0^{1eV} = 4.5\times10^{27}m^{-3} \equiv 4.5\times10^{21} states/cm^3$$

2.

$$N = \int_{1eV}^{2eV} g(E)dE = \frac{4\pi(2m)^{3/2}}{h^3}\int_{1eV}^{2eV}\sqrt{E}dE$$

$$N = \frac{4\pi(2m)^{3/2}}{h^3}\frac{2}{3}E^{3/2}\Big|_{1eV}^{2eV} = 8.29\times10^{27}m^{-3} \equiv 8.29\times10^{21} states/cm^3$$

3.

$$f_F(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+e^{(3kT)}} = 0.0474 or 4.74\%$$

4.

$$f_F(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+e^{\left(\frac{-(0.3+0.0259)}{0.0259}\right)}} = 3.43\times10^{-6}$$

$$f_F(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+e^{\left(\frac{-(0.3+2(0.0259))}{0.0259}\right)}} = 1.26\times10^{-6}$$

5.

$$1-f_F(E) = 1-\frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = 0.01 = 1-\frac{1}{1+e^{\left(\frac{5.95-6.25}{kT}\right)}} \therefore kT = 0.06529eV \text{ or } T = 756K$$

6.

$$f_F(E) = 1-\frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = 0.005 = \frac{1}{1+e^{\left(\frac{(5.7-5.5)}{kT}\right)}} \therefore T = 438K$$

7.

$$\frac{e^{\left(\frac{-(E-E_F)}{kT}\right)} - \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}}{\frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}} = 0.05 = e^{\left(\frac{-(E-E_F)}{kT}\right)} \therefore E-E_F \approx 3kT$$

8.

$$\frac{e^{\left(\frac{-(E-E_F)}{kT}\right)}-\frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}}{\frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}}=0.01=e^{\left(\frac{-(E-E_F)}{kT}\right)}\ \therefore E-E_F=4.6kT$$

9.

$$f_F(E_C+\mathrm{kT})=\frac{1}{1+e^{\left(\frac{E_C+kT-E_F}{kT}\right)}}\approx e^{\left(\frac{-(E_C+kT-E_F)}{kT}\right)}=e^{\left(\frac{-(0.2+0.0259)}{0.0259}\right)}=1.63\times10^{-4}$$

$$n_0=N_Ce^{\left(\frac{-(E_C-E_F)}{kT}\right)}=(2.8\times10^{19})e^{\left(\frac{-0.2}{0.0259}\right)}=1.24\times10^{16}cm^{-3}$$

10.

$$f_F(E_C+\mathrm{kT})=\frac{1}{1+e^{\left(\frac{E_C+kT-E_F}{kT}\right)}}\approx e^{\left(\frac{-(E_C+kT-E_F)}{kT}\right)}=e^{\left(\frac{-(0.25+0.0259)}{0.0259}\right)}=1.63\times10^{-4}$$

$$n_0=N_Ce^{\left(\frac{-(E_C-E_F)}{kT}\right)}=(2.8\times10^{19})e^{\left(\frac{-0.25}{0.0259}\right)}=1.8\times10^{15}cm^{-3}$$

11.

$$N_C=(1.04\times10^{19})\left(\frac{350}{300}\right)^{3/2}=1.31\times10^{19}cm^{-3}$$

$$kT=(0.0259)\left(\frac{350}{300}\right)=0.0302eV$$

Probability of free state at E

$$1-f_F(E_V-kT)=1-\frac{1}{1+e^{\left(\frac{E_V-kT-E_F}{kT}\right)}}\approx e^{\left(\frac{-(E_F-(E_V-kT))}{kT}\right)}=e^{\left(\frac{-(0.25+0.0302)}{0.0302}\right)}=9.34\times10^{-5}$$

hole concentration

$$p_0=N_Ve^{\left(\frac{-(E_F-E_V)}{kT}\right)}=(1.31\times10^{19})e^{\left(\frac{-0.25}{0.0302}\right)}=3.33\times10^{15}cm^{-3}$$

12.

$$p_0=N_Ve^{\frac{-(E_F-E_V)}{kT}}=(1.04\times10^{19})e^{\frac{-(0.2)}{0.0259}}=4.61\times10^{15}cm^{-3}$$

13.
At 350 K
kT = 0.032 eV

$$n_i^2 = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{350}{300}\right)^3 e^{\left(\frac{-1.12}{0.0302}\right)} = 3.62\times10^{22}$$

$$n_i = 1.9\times10^{11} cm^{-3}$$

At 400 K
kT = 0.0345 eV

$$n_i^2 = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{400}{300}\right)^3 e^{\left(\frac{-1.12}{0.0345}\right)} = 5.5\times10^{24}$$

$$n_i = 2.34\times10^{12} cm^{-3}$$

14.

$$n_i^2 = N_C N_V e^{\left(\frac{-E_S}{kT}\right)} = (4.7\times10^{17})(7\times10^{15})\left(\frac{200}{300}\right)^3 e^{\left(\frac{-1.42}{0.0173}\right)}$$

$$n_i = 1.48 cm^{-3}$$

$$n_i^2 = N_C N_V e^{\left(\frac{-E_S}{kT}\right)} = (4.7\times10^{17})(7\times10^{15})\left(\frac{400}{300}\right)^3 e^{\left(\frac{-1.42}{0.0345}\right)}$$

$$n_i = 3.22\times10^{9} cm^{-3}$$

15.

$$E_{FI} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4}(0.0259)\ln\left(\frac{0.56}{1.08}\right) = -12.8 meV$$

16.

$$E_{FI} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4}(0.0259)\ln\left(\frac{0.48}{0.067}\right) = 38.2 meV$$

Carrier Concentration

1.

$$p_0 = (1.04\times10^{19})e^{\left(\frac{-0.25}{0.0259}\right)} = 6.68\times10^{14} cm^{-3}$$

$$n_0 = (2.8\times10^{19})e^{\left(\frac{-0.87}{0.0259}\right)} = 7.23\times10^{4} cm^{-3}$$

2.

$$p_0 = (1.04\times10^{19})e^{\left(\frac{-0.92}{0.0259}\right)} = 3.89\times10^{3} cm^{-3}$$

$$n_0 = (2.8\times10^{19})e^{\left(\frac{-0.2}{0.0259}\right)} = 1.24\times10^{16} cm^{-3}$$

3.

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4\,cm^{-3}$$

4.

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10})^2}{10^{5}} = 2.25\times10^{15}\,cm^{-3}$$

5.

$$n_0 = \frac{2\times10^{16}}{2} + \sqrt{\left(\frac{2\times10^{16}}{2}\right)^2 + (1.5\times10^{10})^2} \cong 2\times10^{16}\,cm^{-3}$$

$$p_0 = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^{16}} = 1.13\times10^4\,cm^{-3}$$

6.

Since $n_0 >> n_i$

$N_d \cong n_0 = 10^{15}\,cm^{-3}$

$$p_0 = \frac{n_i^2}{n_0} = \frac{\left(1.5\times10^{10}\right)^2}{10^{15}} = 2.25\times10^5\,cm^{-3}$$

7.

$$n_0 = \frac{5\times10^{13}}{2} + \sqrt{\left(\frac{5\times10^{13}}{2}\right)^2 + (2.4\times10^{13})^2} \cong 5.97\times10^{13}\,cm^{-3}$$

$$p_0 = \frac{\left(2.4\times10^{13}\right)^2}{5.97\times10^{13}} = 9.65\times10^{12}\,cm^{-3}$$

8.

$$n_0 = \frac{10^{14}}{2} + \sqrt{\left(\frac{10^{14}}{2}\right)^2 + (2.4\times10^{13})^2} \cong 1.055\times10^{14}\,cm^{-3}$$

$$p_0 = \frac{\left(2.4\times10^{13}\right)^2}{1.055\times10^{14}} = 5.46\times10^{12}\,cm^{-3}$$

9.

$$p_0 = \frac{2\times10^{16} - 5\times10^{15}}{2} + \sqrt{\left(\frac{2\times10^{16} - 5\times10^{15}}{2}\right)^2 + (1.5\times10^{10})^2} \cong 1.5\times10^{16}\,cm^{-3}$$

$$n_0 = \frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{16}} = 1.5\times10^4\,cm^{-3}$$

10.

$$N_a - N_d = 5\times10^{13} - 1\times10^{13} = 4\times10^{13}\,cm^{-3}$$

$$p_0 = \frac{4\times10^{13}}{2} + \sqrt{\left(\frac{4\times10^{13}}{2}\right)^2 + (2.4\times10^{13})^2} = 5.12\times10^{13}\,cm^{-3}$$

$$n_0 = \frac{\left(2.4\times10^{13}\right)^2}{5.124\times10^{13}} = 1.12\times10^{13}\,cm^{-3}$$

11.

$$n_i^2 = N_C N_V e^{\left(\frac{-E_G}{kT}\right)} = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{475}{300}\right)^2 e^{\left(\frac{-1.12}{0.0259}\frac{300}{475}\right)} = 1.59\times10^{27}$$

$$n_i = 3.99\times10^{13}\,cm^{-3}$$

$$n_0 = 1.03N_d = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + (3.99\times10^{13})^2}$$

$$N_d = 2.27\times10^{14}\,cm^{-3}$$

12.

$$n_i^2 = N_C N_V e^{\left(\frac{-E_G}{kT}\right)} = (6\times10^{18})(1.04\times10^{19})\left(\frac{400}{300}\right)^2 e^{\left(\frac{-0.66}{0.0259}\frac{300}{400}\right)}$$

$$n_i = 8.607\times10^{14}\,cm^{-3}$$

$$n_0 = 1.1N_d = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + (8.607\times10^{14})^2}$$

$$N_d = 2.6\times10^{15}\,cm^{-3}$$

13.

$$E_C - E_F = kT\ln\left(\frac{N_c}{N_d - N_a}\right) \therefore N_d - N_a = N_c e^{\left(\frac{-(E_c - E_F)}{kT}\right)}$$

$$N_d - N_a = (2.8\times10^{19})e^{\left(\frac{-0.2}{0.0259}\right)} = 1.24\times10^{16}\,cm^{-3}$$

$$N_d = 1.24\times10^{16} + N_a = 2.24\times10^{16}\,cm^{-3}$$

14.

$$p_0 = N_a - N_d = N_v e^{\frac{-(E_F - E_v)}{kT}} \quad \therefore E_F - E_v = kT \ln\left(\frac{N_v}{N_a - N_d}\right)$$

$$E_F - E_v = (0.0259)\ln\left(\frac{7\times10^{18}}{5\times10^{16} - 4\times10^{15}}\right) = 0.13eV$$

15.

$$E_{Fi} - E_F = \frac{E_G}{2} - (E_a - E_v) - (E_F - E_a) = kT\ln\left(\frac{N_a}{n_i}\right)$$

$$0.56 - 0.045 - 3(0.0259) = 0.437 = (0.0259)\ln\left(\frac{N_a}{n_i}\right)$$

$$\therefore N_a = n_i e^{\left(\frac{0.437}{0.0259}\right)} = 3.2\times10^{17}\,cm^{-3}$$

16.

$$E_F - E_{Fi} = \frac{E_G}{2} - (E_c - E_d) - (E_d - E_F) = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$0.56 - 0.045 - 4.6(0.0259) = 0.39586 = (0.0259)\ln\left(\frac{N_d}{1.5\times10^{10}}\right)$$

$$\therefore N_d = 6.52\times10^{16}\,cm^{-3}$$

Drift and Diffusion of Carriers

1.

$$n \approx N_d = 10^{16}\,cm^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^{4}\,cm^{-3}$$

$$n >> p \;\therefore J_{drf} = e(\mu_n n + \mu_p p)\,\mathcal{E}$$

$$J_{drf} = e(\mu_n n + \mu_p p)\,\mathcal{E} = e\mu_n n\,\mathcal{E}$$

$$J_{drf} = (1.6\times10^{19})(1350)(10^{16})(35) = 75.6\ \mathrm{A/cm^2}$$

2.

$$J_{drf} = e\mu_n n\,\mathcal{E}$$

$$J_{drf} = (1.6\times10^{-19})(8500)(10^{16})(10) = 136\ \mathrm{A/cm^2}$$

3.

a i) $\mu_n \approx 500\ cm^2/V\text{-}s$

ii) $\mu_n \approx 1500\ cm^2/V\text{-}s$

b i) $\mu_p \approx 380\ cm^2/V\text{-}s$

ii) $\mu_p \approx 200\ cm^2/V\text{-}s$

4.
a i) $\mu_n \approx 800$ cm^2/V-s
ii) $\mu_p \approx 300$ cm^2/V-s
b i) $\mu_n \approx 3800$ cm^2/V-s
ii) $\mu_p \approx 200$ cm^2/V-s

5.

$$I = J_{drf} A \to A = \frac{I}{J_{drf}} = \frac{2\times10^{-3}}{100} = 2\times10^{-5} cm^2$$

$$R = \frac{V}{I} = 2.5k\Omega = \frac{L}{e\mu_p N_a A}$$

Let $\mathrm{N}_a = 10^{16} cm^{-3}$

$\mu_p \approx 400 cm^2$/V-s from graph

$$L = \sigma AR = e\mu_p N_a AR = (1.6\times10^{-19})(400)(10^{16})(2\times10^{-5})(2.5\times10^3) = 3.2\times10^{-2} cm$$

6.

$$\sigma = e\mu_n N_d = \frac{1}{\rho} = \frac{1}{0.1} = 10(\Omega\text{-cm})^{-1}$$

From the Figure

$\mathrm{N}_d \cong 9\times10^{16} cm^{-3}$

$$\mu_n = \frac{10}{(1.6\times10^{19})(9\times10^{16})} \cong 695 cm^2/\text{V-s}$$

7.

$5V$ for a $10k\Omega$ resistor I=.5mA

For current density of 50A/cm^2

$$A = \frac{I}{J} = \frac{0.5\times10^{-3}}{50} = 10^{-5} cm^2$$

$$L = \frac{5}{100} = 5\times10^{-2} cm$$

$$\sigma = \frac{L}{RA} = \frac{5\times10^{-2}}{(10^4)(10^{-5})} = 0.5(\Omega - cm)^{-1}$$

$$\sigma \approx e\mu_p p = e\mu_p (N_a - N_d)$$

choosing $\mathrm{N}_a = 1.25\times10^{16} cm^{-3}$, then $\mathrm{N}_a + \mathrm{N}_d = 1.75\times10^{16} cm^{-3}$

hole mobility from graph $\mu_p = 410 cm^2$/V-s

$$\sigma = 0.492(\Omega\text{-cm})^{-1}$$

8.

$$\sigma = e\mu_n(N_d - N_a) = 16 = (1.6\times10^{19})\mu_n(N_d - 10^{17})$$

Using the graphs and some trial and error

$N_d \cong 3,4\times10^{17}\,cm^{-3}$

$\mu_n \cong 420 cm^2/\text{V-s}$

9.

$$J_{drf} = -eD_p\frac{dp}{dx} \approx -eD_p\left(\frac{p(0.01)-p(0)}{0.01-0}\right)$$

$$20 = -(1.6\times10^{-19})(10)\left(\frac{p(0.01)-4\times10^{17}}{0.01}\right) \Rightarrow p(0.01) = 2.75\times10^{17}\,cm^{-3}$$

10.

$$\left|J_{drf}\right| = eD_n\frac{\Delta n}{\Delta x} = (1.6\times10^{-19})(225)\left(\frac{10^{18}-7\times10^{17}}{0.1}\right) = 108A/\text{cm}^2$$

11.

Using the Einstien relation

$$D = \left(\frac{kT}{e}\right)\mu = (0.0259)(1200) = 31.1\text{cm}^2/\text{s}$$

12.

$$\frac{D}{\mu} = \frac{kT}{e} \Rightarrow \mu = \frac{210}{0.0259} = 8108cm^2/\text{V-s}$$

Solution 2: Doping

a) If a Si atom were to replace a Ga atom, **n-type**

If a Si atom were to replace an As atom, **p-type**

b) p-type

the majority carrier concentration $p = N_A = 10^{17}$ cm^{-3}.

the minority carrier concentration $n = 10^3$ cm^{-3}.

ii) At temperatures above **1000K,** the sample will be intrinsic.

Solution 3:

(a)
i) **n-type**
electron concentration n = 1x10^{16} cm^{-3}
hole concentration p = 1 x10^{4} cm^{-3}
ii)

(b)
i)
$p = 1.03\times10^{15} cm^{-3}$
n = 1 x10^{5} cm^{-3}

ii)
$[B] \approx N_A = 1.1\times10^{16} cm^{-3}$

Solution 4:

(a) n=p=0.

(b) n = 1x10^{15}cm^{-3}
p = 1 x10^{5} cm^{-3}

$E_F - E_i = 0.300eV$

(c) $n = 4.53\times10^{15} cm^{-3}$
p = 3.53 x10^{15} cm^{-3}
$E_F - E_i = 0.006eV$

(d) As T increases, the semiconductor becomes more intrinsic. Therefore, as T increases, E_f approaches E_i until they essentially become equal.

Solution 5: Drift & Resistivity

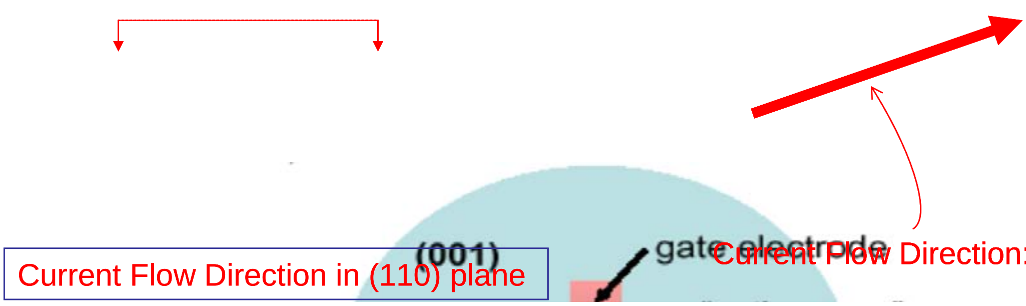
c)
$E_F - E_i = 0.45eV$

d)
$\rho \approx 0.04\Omega \cdot cm$

e)
$R = 533.3\Omega$

Solution 6: Miller indices

As shown in the figure below, the direction of current flow is parallel to the wafer flat, which is along the (110) crystal plane. Therefore, the Miller indices for the direction of current flow in the MOSFET are [$\bar{1}$10]. (Refer to the figure on the right, below.)



Note: In modern MOSFETs, the transistor layout orientation is very important because carrier mobilities are generally anisotropic (not the same in every direction). For example, electron mobility (μ_n) is highest in the <100> direction, whereas hole mobility (μ_p) is largest in the <110> direction.

Solution 7: Diamond lattice crystal structure

$$\text{Atomic density} = 4.44 \times 10^{22}\, atoms / cm^3$$

<u>Solution 8</u>: Semiconductor doping

(a) Boron (B) is preferred over indium (In) as the dopant species to achieve highly conductive p-type silicon because of its low ionization energy (IE). The IE of boron is only 45 meV, whereas that of indium is much higher (160 meV). Consequently, it is much easier to activate boron dopants (and thus achieve highly conductive p-type silicon) than indium dopants. The addition of indium dopants to silicon will not result in a significant increase in the number of holes at room temperature.

(b) From the plot of intrinsic carrier concentration (n_i) *vs.* temperature (T), given in Lecture #2, n_i for silicon reaches 10^{18} cm^{-3} at a temperature of ~1000K (727 ^{o}C). Therefore, at very high temperatures (e.g. >1000^{o}C), the intrinsic carrier concentration will be much higher than the net dopant concentration, and thus the sample becomes intrinsic ($n \approx p \approx n_i$).

<u>Solution 9</u>: Carrier concentrations

(a) $n = p = n_i = 1\times10^{10}$ cm^{-3}.

(b) p-type

$$p = 1\text{x}10^{16}\ \text{cm}^{-3}$$
$$n = 1.0\times10^{4}\,cm^{-3}$$

(c) n-type.

$$n = 9\times10^{16}\,cm^{-3}$$
$$p = 1.11\times10^{3}\,cm^{-3}$$

Solution 10: Energy Band Diagram

$$E_F = E_i + 480\,mV$$
$$E_C - E_F = 80\,mV\ .$$

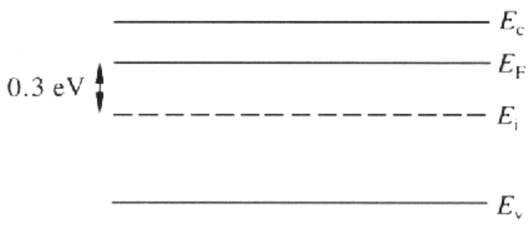
Solution 11: Carrier Mobility and Drift Velocity

a) $\mu_n = 800\,cm^2\,V^{-1}\,s^{-1}$

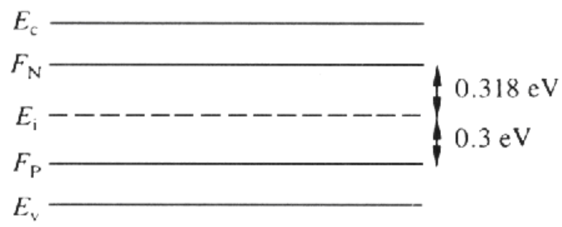
b) use graph in textbook or notes

$$\rho = 0.19\,\Omega - cm$$

Solution 12: Fermi Levels



(a) Before



(b) After

(a) Under equilibrium conditions (energy band diagram (a)):

$\boldsymbol{n_0} = \mathbf{1.0\times10^{15}cm^{-3}}$
$\boldsymbol{p_0} = \mathbf{1.0\times10^{5}cm^{-3}}$

B. PN junction

1.

$$V_{bi} = V_t \ln\left(\frac{N_a N_a}{n_i^2}\right) = (0.0259)\ln\frac{(2\times10^{16})(5\times10^{15})}{(1.5\times10^{10})^2} = 0.695V$$

2.

$$V_{bi} = V_t \ln\left(\frac{N_a N_a}{n_i^2}\right) = (0.0259)\ln\frac{(5\times10^{17})(10^{16})}{(1.5\times10^{10})^2} = 0.796V$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_a}{n_i^2}\right) = (0.0259)\ln\frac{(10^{15})(2\times10^{16})}{(1.5\times10^{10})^2} = 0.653V$$

3.

$$x_n = \left[\frac{2\varepsilon_s V_{bi}}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{0.5}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.695)}{1.6\times10^{-19}} \left(\frac{2\times10^{16}}{5\times10^{15}} \right) \left(\frac{1}{2\times10^{16} + 5\times10^{15}} \right) \right]^{0.5} = 0.379 \mu m$$

$$x_p = \left[\frac{2\varepsilon_s V_{bi}}{e} \left(\frac{N_d}{N_a} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{0.5}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.695)}{1.6\times10^{-19}} \left(\frac{5\times10^{15}}{2\times10^{16}} \right) \left(\frac{1}{2\times10^{16} + 5\times10^{15}} \right) \right]^{0.5} = 0.0948 \mu m$$

$$W = \left[\frac{2\varepsilon_s V_{bi}}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{0.5} = \left[\frac{2(11.7)(8.85\times10^{-14})(0.695)}{1.6\times10^{-19}} \left(\frac{2\times10^{16} + 5\times10^{15}}{2\times10^{16} 5\times10^{15}} \right) \right]^{0.5}$$

$$= 0.474 \mu m \Rightarrow W = x_p + x_n = 0.379 + 0.0948 = 0.474 \mu m$$

$$|\mathcal{E}_{\max}| = \frac{eN_d x_n}{\varepsilon_s} = \frac{(1.6\times10^{-19})(5\times10^{15})(0.379\times10^{-4})}{(11.7)(8.85\times10^{-14})} = 2.93\times10^{4}\ \text{V/cm}$$

4.

$$V_{bi} = (0.0259) \ln \frac{(2\times10^{16})(5\times10^{15})}{(1.5\times10^{10})^2} = 0.718\text{V}$$

$$x_n = \left[\frac{2\varepsilon_s V_{bi}}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right]^{0.5}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.718)}{1.6\times10^{-19}} \left(\frac{5\times10^{15}}{5\times10^{16}} \right) \left(\frac{1}{5\times10^{16} + 5\times10^{15}} \right) \right]^{0.5} = 0.0411 \mu m$$

$$x_p = \left[\frac{2(11.7)(8.85\times10^{-14})(0.695)}{1.6\times10^{-19}} \left(\frac{5\times10^{16}}{5\times10^{15}} \right) \left(\frac{1}{5\times10^{16} + 5\times10^{15}} \right) \right]^{0.5} = 0.0452 \mu m$$

$$|\mathcal{E}_{\max}| = \frac{eN_d x_n}{\varepsilon_s} = \frac{(1.6\times10^{-19})(5\times10^{16})(4.11\times10^{-6})}{(11.7)(8.85\times10^{-14})} = 3.18\times10^{4}\ \text{V/cm}$$

5.

$$W = \left[\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{0.5}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.695 + 5)}{1.6\times10^{-19}} \left(\frac{2\times10^{16} + 5\times10^{15}}{2\times10^{16} 5\times10^{15}} \right) \right]^{0.5} = 1.36 \mu\text{m}$$

6.

$$V_{bi} = (0.0259)\ln\frac{(5\times10^{16})(5\times10^{15})}{(1.5\times10^{10})^2} = 0.718\text{V}$$

$$\text{i)}x_n = \left[\frac{2(11.7)(8.85\times10^{-14})(0.718+8)}{1.6\times10^{-19}}\left(\frac{5\times10^{15}}{5\times10^{16}}\right)\left(\frac{1}{5\times10^{16}+5\times10^{15}}\right)\right]^{0.5} = 0.143\mu m$$

$$x_p = \left[\frac{2(11.7)(8.85\times10^{-14})(0.718+8)}{1.6\times10^{-19}}\left(\frac{5\times10^{16}}{5\times10^{15}}\right)\left(\frac{1}{5\times10^{16}+5\times10^{15}}\right)\right]^{0.5} = 1.43\mu m$$

$$W = x_n + x_p = 1.57\mu m$$

$$\text{ii)}x_n = \left[\frac{2(11.7)(8.85\times10^{-14})(0.718+12)}{1.6\times10^{-19}}\left(\frac{5\times10^{15}}{5\times10^{16}}\right)\left(\frac{1}{5\times10^{16}+5\times10^{15}}\right)\right]^{0.5} = 0.173\mu m$$

$$x_p = \left[\frac{2(11.7)(8.85\times10^{-14})(0.718+8)}{1.6\times10^{-19}}\left(\frac{5\times10^{16}}{5\times10^{15}}\right)\left(\frac{1}{5\times10^{16}+5\times10^{15}}\right)\right]^{0.5} = 1.73\mu m$$

$$W = x_n + x_p = 1.9\mu m$$

7.

$$|\mathcal{E}_{\max}| = \left[\frac{2e(V_{bi}+V_R)}{\varepsilon_s}\left(\frac{N_a N_d}{N_a+N_d}\right)\right]^{0.5}$$

assume V_{bi} = 0.75 V (Need to choose a value)

$$10^5 \approx \left[\frac{2(1.6\times10^{-19})(0.695+5)}{(11.7)(8.85\times10^{-14})}\left(\frac{10^{18}N_d}{10^{18}+N_d}\right)\right]^{0.5} \Rightarrow N_d = 3.02\times10^{15}cm^{-3}$$

$$V_{bi} = (0.0259)\ln\frac{(10^{18})(3.02\times10^{15})}{(1.5\times10^{10})^2} = 0.783\text{V close to chosen value}$$

8.

i) For V=8 V, W=1.57 μm

$$|\mathcal{E}_{\max}| = \frac{2(V_{bi}+V_R)}{W} = \frac{2(0.718+8)}{1.57\times10^{-4}} = 1.11\times10^5 \text{ V/cm}$$

ii) For V=12 V, W=1.9 μm

$$|\mathcal{E}_{\max}| = \frac{2(V_{bi}+V_R)}{W} = \frac{2(0.718+12)}{1.9\times10^{-4}} = 1.34\times10^5 \text{ V/cm}$$

9.

$$C' = \left[\frac{e\varepsilon_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{0.5} = \left[\frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(2\times10^{16})(5\times10^{15})}{2(0.695+5)(2\times10^{16}+5\times10^{15})}\right]^{0.5}$$

$$= 7.63\times10^{-9}\ \text{F/m}$$

$$C + AC' = (10^{-4})(7.63\times10^{-9}) = 0.763\,pF$$

10.

$$V_{bi} = (0.0259)\ln\frac{(10^{15})(2\times10^{16})}{(1.5\times10^{10})^2} = 1.12\text{V}$$

i) $V_R = 0$

$$C = AC' = A\left[\frac{e\varepsilon_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{0.5} = (10^{-4})\left[\frac{(1.6\times10^{-19})(13.1)(8.85\times10^{-14})(10^{15})(2\times10^{16})}{2(1.12)(10^{15}+2\times10^{16})}\right]^{0.5}$$

$$C = 0.888\,pF$$

ii) $V_R = 5V$

$$C = (10^{-4})\left[\frac{(1.6\times10^{-19})(13.1)(8.85\times10^{-14})(10^{15})(2\times10^{16})}{2(1.12+5)(10^{15}+2\times10^{16})}\right]^{0.5} = 0.38\,pF$$

11.

$$N_d = \frac{2}{e\varepsilon_s(slope)} = \frac{2}{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(3.92\times10^{15})} = 3.08\times10^{15}\,cm^{-3}$$

Built in potential is then:

$$V_{bi} = V_t\ln\left(\frac{N_a N_d}{n_i^2}\right) \therefore N_a = \frac{n_i^2}{N_d}e^{\frac{V_{bi}}{V_t}} = \frac{(1.5\times10^{10})^2}{3.08\times10^{15}}e^{\left(\frac{0.742}{0.0259}\right)} = 2.02\times10^{17}\,cm^{-3}$$

12.

$$C' = \frac{C}{A} = \frac{1.1\times10^{-8}}{10^{-4}} = 1.1\times10^{-8}\,\text{F/m}^2$$

$$Slope = \frac{\left(1/C'\right)^2}{V_{bi}+V_R} = \frac{\left(1/1.1\times10^{-8}\right)^2}{0.782+4} = 1.728\times10^{15}$$

$$\frac{2}{e\varepsilon_s(slope)} = \frac{2}{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(1.728\times10^{15})} = 7\times10^{15}\,cm^{-3}$$

$$V_{bi} = V_t\ln\left(\frac{N_a N_d}{n_i^2}\right) \therefore N_d = \frac{n_i^2}{N_a}e^{\frac{V_{bi}}{V_t}} = \frac{(1.5\times10^{10})^2}{7\times10^{15}}e^{\left(\frac{0.782}{0.0259}\right)} = 4.17\times10^{17}\,cm^{-3}$$

<u>Solution 1</u>: pn Junction Electrostatics

a) We have an abrupt pn junction with N_d=1x10^{16} cm^{-3}, and N_a=5x10^{16} cm^{-3}. Since the doping concentrations on both sides are comparable, we use the standard full expressions:

i)
$V_{bi} = 0.76V$

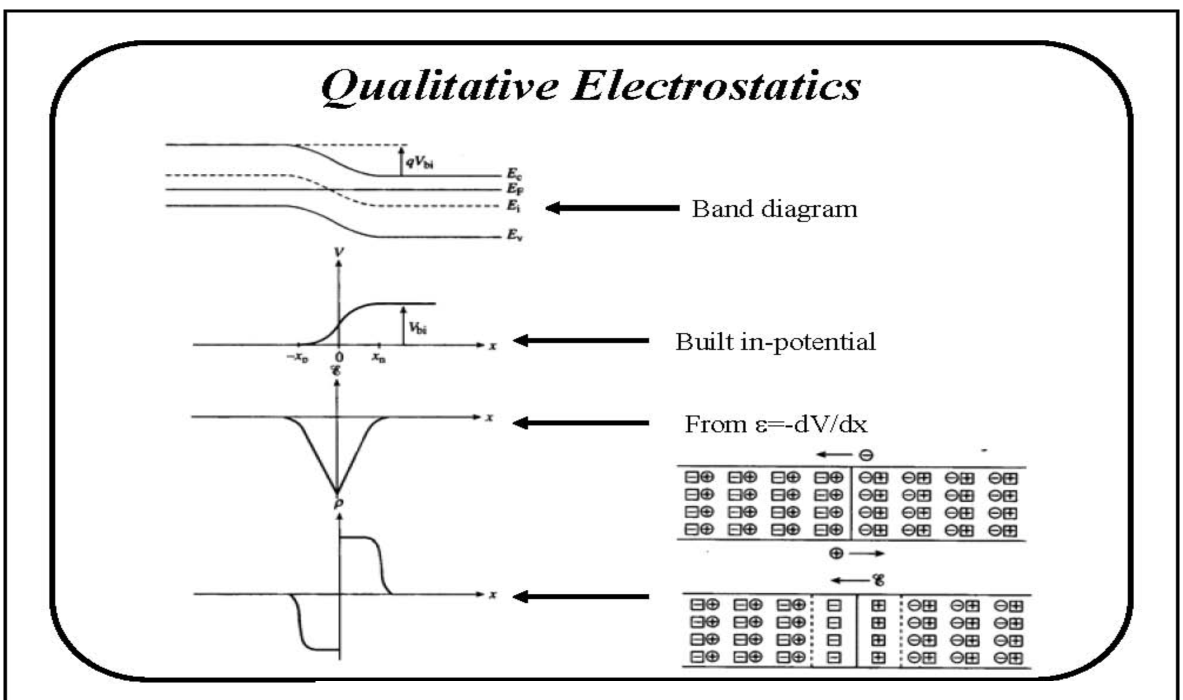
ii)
$W = 3.45\times10^{-5}cm$

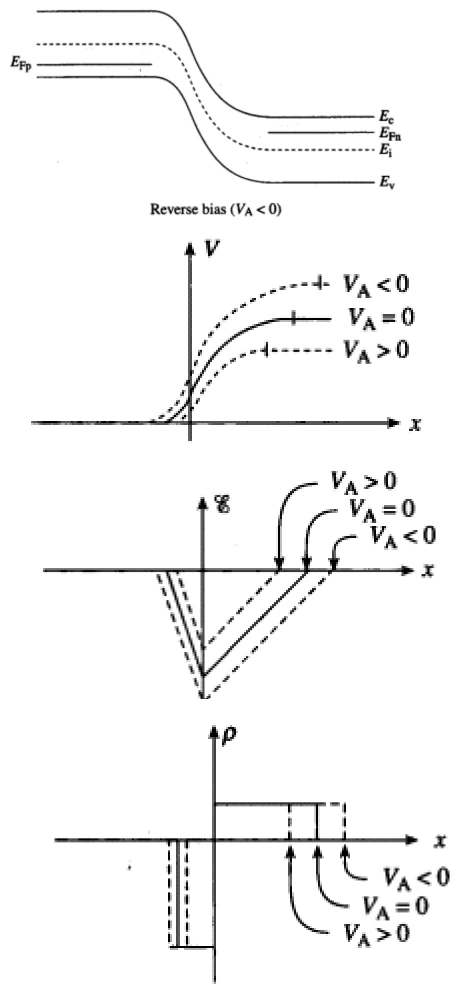
iii)

$$|\varepsilon_{\max}| = \frac{2V_{bi}}{W} = 4.4\times10^{4} V/cm$$

iv)
$|\varepsilon_{\max}'| = 1.3\times10^{5} V/cm$
where:
$W' = 8.7\times10^{-5}cm$

b) The requested plots are given on the next page.





<u>Solution 2</u>: pn Junction Electrostatics

(a)

$$V_{bi} = 0.66V$$

(b)

$$W = 9.7\times10^{-5}\,cm = 0.97\,\mu m$$

$$x_{po} = 0.09\,\mu m$$

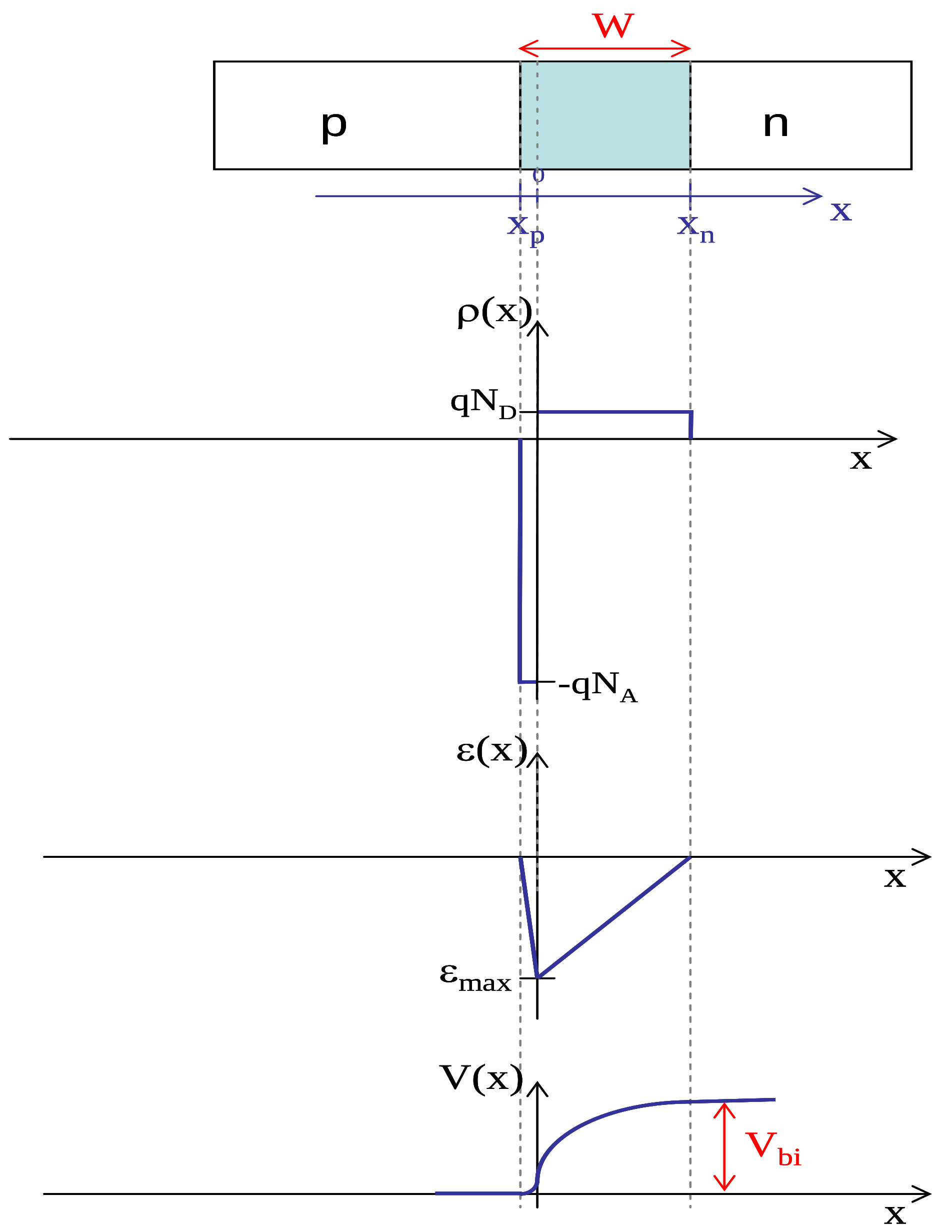
$$x_{no} = 0.88\,\mu m$$

(c)

$$V(0) = 0.06V$$

$$\mathcal{E}(0) = -1.4\times10^{4}\,V/cm$$

(d) The sketches are:


W
p
n
0
x
x_p
x_n
$\rho(x)$
qN_D
x
$-qN_A$
$\varepsilon(x)$
x
ε_{max}
V(x)
V_{bi}
x

C. MOS capacitors

1.

$$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right) = -(0.0259)\ln\left(\frac{N_a}{1.5\times10^{10}}\right)$$

For $\mathrm{N}_a = 10^{15}\, cm^{-3}$

$$\phi_{Fp} = -0.288V$$

For $\mathrm{N}_a = 10^{17}\, cm^{-3}$.

$$\phi_{Fp} = -0.407V$$

2.

$$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right)$$

$$N_a = (1.5\times10^{10})\ln\left(\frac{0.34}{0.0259}\right) = 7.54\times10^{15}\, cm^{-3}$$

3.

$$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right) = -(0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = -0.347V$$

$$x_{dT} = \left(\frac{4\varepsilon_s\left|\phi_{Fp}\right|}{eN_a}\right)^{0.5} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.347)}{(1.6\times10^{-19})(10^{16})}\right]^{0.5} = 0.3\mu m$$

4.

a) $$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right) = -(0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = -0.376V$$

$$x_{dT} = \left(\frac{4\varepsilon_s\left|\phi_{Fp}\right|}{eN_a}\right)^{0.5} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.376)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{0.5} = 0.18\mu m$$

b) $$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right) = -(0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right) = -0.288V$$

$$x_{dT} = \left(\frac{4\varepsilon_s\left|\phi_{Fp}\right|}{eN_a}\right)^{0.5} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.288)}{(1.6\times10^{-19})(10^{15})}\right]^{0.5} = 0.863\mu m$$

5.

$$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right) = -(0.0259)\ln\left(\frac{10^{14}}{1.5\times10^{10}}\right) = -0.228V$$

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_G}{2e} + \left|\phi_{Fp}\right|\right) = 3.2 - (3.25 + 0.56 + 0.288) = -0.838V$$

6.

$$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right) = -(0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = -0.347V$$

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_G}{2e} + \left|\phi_{Fp}\right|\right) = 3.2 - (3.25 + 0.56 + 0.47) = -0.957V$$

7.

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{550\times10^{-8}} = 6.28\times10^{-8}\,\mathrm{F/cm^2}$$

$$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right) = -(0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = -0.347V$$

$$x_{dT} = \left(\frac{4\varepsilon_s\left|\phi_{Fp}\right|}{eN_a}\right)^{0.5} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.347)}{(1.6\times10^{-19})(10^{16})}\right]^{0.5} = 30\mu m$$

$$C'_{\min} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\varepsilon_s}\right)x_{dT}} = \frac{(3.9)(8.85\times10^{-14})}{550\times10^{-8} + \frac{3.9}{11.7}0.3\times10^{-4}} = 2.23\times10^{-8}\,\mathrm{F/cm^2}$$

$$\frac{C'_{\min}}{C_{ox}} = \frac{2.23\times10^{-8}}{6.28\times10^{-8}} = 0.355$$

$$C'_{FB} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\varepsilon_s}\right)\sqrt{\frac{kT}{e}\left(\frac{\varepsilon_s}{eN_a}\right)}} = \frac{(3.9)(8.85\times10^{-14})}{550\times10^{-8} + \frac{3.9}{11.7}\sqrt{0.0259\left(\frac{(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(10^{16})}\right)}} = 5.03\times10^{-8}\,\mathrm{F/cm^2}$$

$$\frac{C'_{FB}}{C_{ox}} = \frac{5.03\times10^{-8}}{6.28\times10^{-8}} = 0.8$$

8.

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{250\times10^{-8}} = 1.38\times10^{-7}\ \text{F/cm}^2$$

$$\phi_{Fp} = -V_t \ln\left(\frac{N_a}{n_i}\right) = -(0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = -0.376V$$

$$x_{dT} = \left(\frac{4\varepsilon_s\left|\phi_{Fp}\right|}{eN_a}\right)^{0.5} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.376)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{0.5} = 0.18\mu m$$

$$C'_{\min} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\varepsilon_s}\right)x_{dT}} = \frac{(3.9)(8.85\times10^{-14})}{250\times10^{-8} + \frac{3.9}{11.7}0.18\times10^{-4}} = 4.06+\times10^{-8}\ \text{F/cm}^2$$

$$\frac{C'_{\min}}{C_{ox}} = 0.294$$

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\varepsilon_s}\right)\sqrt{\frac{kT}{e}\left(\frac{\varepsilon_s}{eN_a}\right)}} = \frac{(3.9)(8.85\times10^{-14})}{250\times10^{-8} + \frac{3.9}{11.7}\sqrt{0.0259\left(\frac{(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(3\times10^{16})}\right)}} = 1.05\times10^{-7}\ \text{F/cm}^2$$

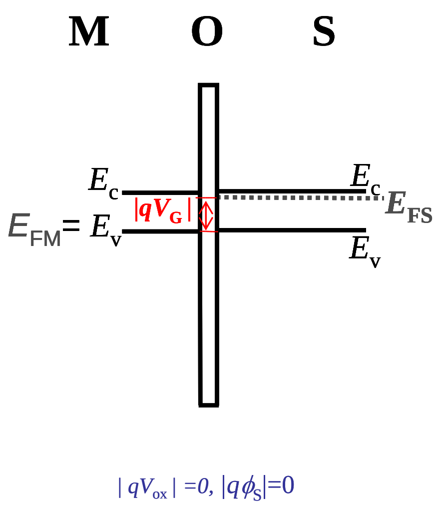
$$\frac{C'_{FB}}{C_{ox}} = 0.761$$

<u>Solution 1</u>: MOS Fundamentals

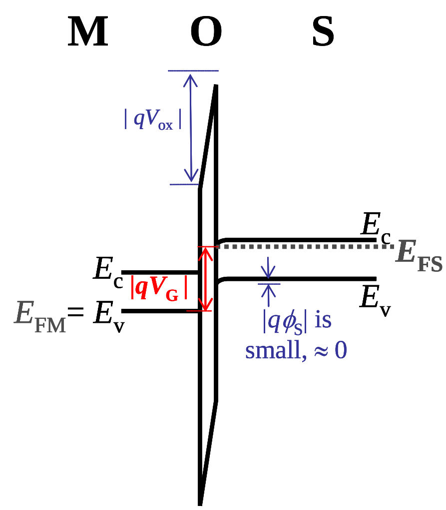
a) The flat-band voltage (V_{FB}) of this PMOS capacitor is determined as follows:

$$V_{FB} = 1.04V$$

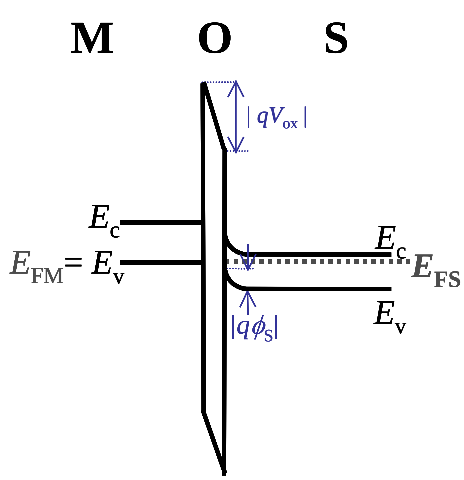
b) The sketches of the energy-band diagrams (for each condition) are shown below:



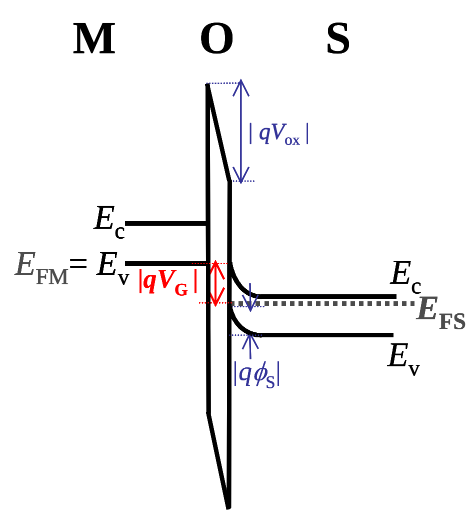
i) Flat-band
($V_G = V_{FB}$)



ii) Accumulation
($V_G > V_{FB}$)



iii) Equilibrium
($V_G = 0V$)

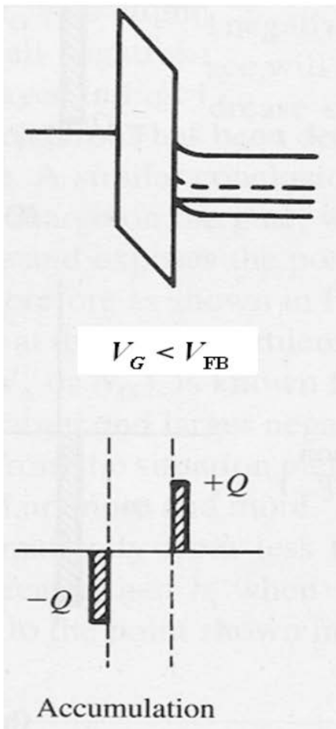


iv) Strong inversion
($V_G < V_T$)

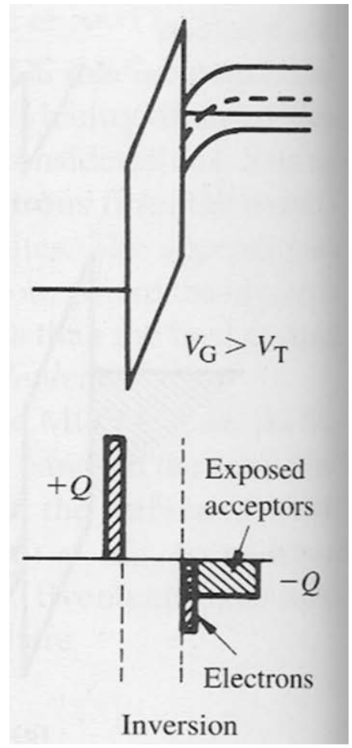
Solution 2: MOS Capacitance

(a) p-type

(b) The charge density diagrams are shown below:



(a) Point 1



(b) Point 2

(c) From point 1 on the *C-V* curve,

$$x_o = 3.45 \times 10^{-7} cm = 3.45 nm$$

(d) From point 2 on the *C-V* curve,
Solving by iteration, we get: $\mathbf{N_A \sim 7.5x10^{17}\ cm^{-3}}$.

(e) high frequency

Solution 3: MOS Capacitance

x_d = $\mathbf{5*10^{-6}m}$

D. MOSFETs and ICs

1.
$W = 11.8\mu m$

2.
$C_{ox} = 1.73\times10^{-7}\,\mathrm{F/cm^2}$
$\mathrm{V}_{GS} = 1V, I_D = 1.01mA$
$\mathrm{V}_{GS} = 2V, I_D = 7.19mA$
$\mathrm{V}_{GS} = 3V, I_D = 19.0mA$

3.
$\mu_n = 773\mathrm{cm^2/V\text{-}s}$

4.
$V_{TN} = 0.625V$

5.
$\beta = 0.104\mathrm{mA/V^2}$
$I_D = 0.574\mathrm{mA}$

6.
$\beta = 0.154\mathrm{mA/V^2}$
$\frac{W}{L} = 14.9$