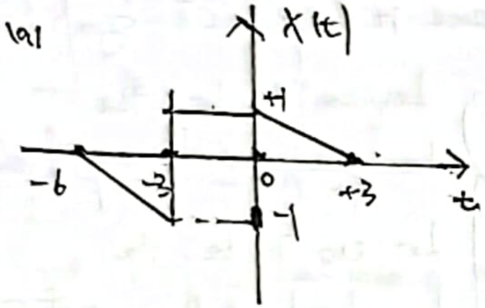
CAN207 — Assn 1.

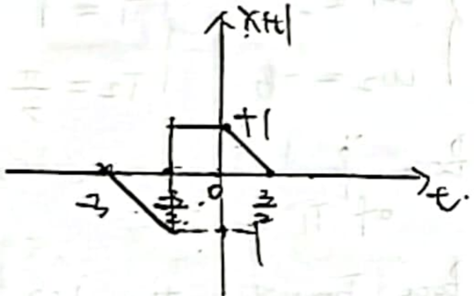
Q1

(a)



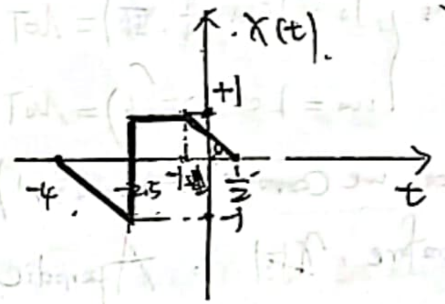
Step1 : Scale

$x(t) \rightarrow x(2t)$



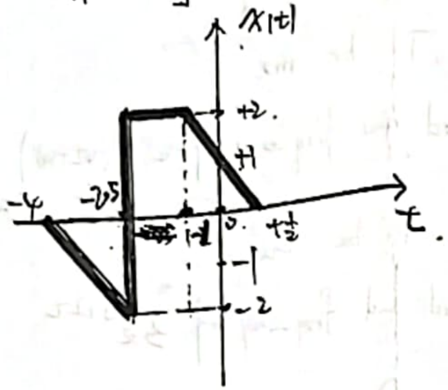
Step2 : Shift

$x(2t) \rightarrow x[2(t+1)]$

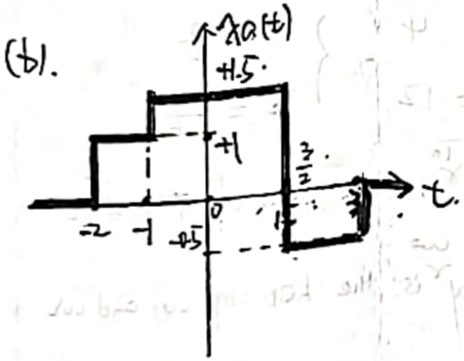


Step3 : Multiply.

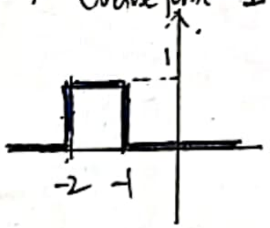
$x[2(t+1)] \rightarrow 2x[2(t+1)]$



(b).

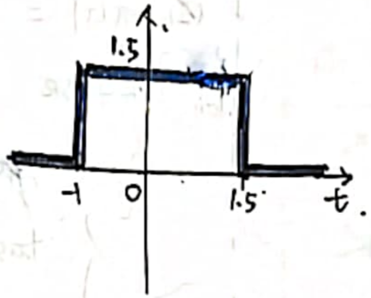


1° Waveform I



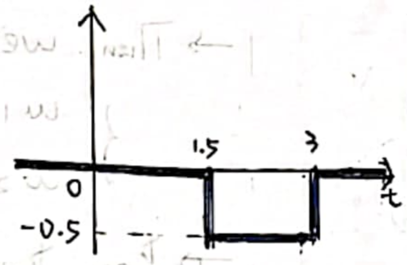
$x_1(t) = -u(t+1) + u(t+2)$

2° Waveform II



$x_2(t) = 1.5[u(t+1) - u(t-1.5)]$

3° Waveform III



$x_3(t) = (-0.5)[u(t-1.5) - u(t-3)]$

4° ~~Waveform IV~~ Sum up.

$x_a(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t)$

$= u(t+2) + 0.5u(t+1)$

$-2u(t-1.5) + 0.5\,u(t-3)$

(c). 问信号. Even, Odd or Neither

(I). $x(t) = \sin(3t - \frac{\pi}{2})$

→ Because it is a CT signal

→ So $x(-t) = \sin(-3t - \frac{\pi}{2})$ ①

$x(t) = \sin(3t - \frac{\pi}{2})$ ②

→ 1° Simplify ①, we can derive

→ $x(-t) = \sin(-3t)\cos(-\frac{\pi}{2}) + \cos(-3t) \cdot \sin(-\frac{\pi}{2})$

→ $x(-t) = -\cos(3t)$ ③

→ 2° Simplify ②, we can derive:

→ $x(t) = \sin(3t)\cos(\frac{\pi}{2}) + \cos(3t) \cdot \sin(-\frac{\pi}{2})$

→ $x(t) = -\cos(3t)$ ④

Obviously, ③ = ④

→ Thus, $x(t) = x(-t)$

In summary, $x(t)$ is an "Even" Signal.

(II). $x(t) = u(t) - 0.5$

Because it is a CT signal

→ So, $\begin{cases} x(-t) = u(-t) - 0.5 & ① \\ x(t) = u(t) - 0.5 & ② \end{cases}$

And $-x(t) = 0.5 - u(t)$ ③

→ Obviously ① ≠ ②,

which indicates that $x(t)$ Can NOT be "Even" Signal.

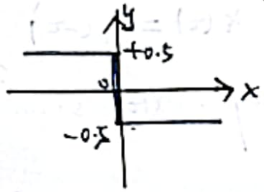
→ Then we try to compare ① and ③

Since $u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$

Therefore, ① $x(-t) = u(-t) - 0.5 = \begin{cases} 0.5, & t<0 \\ -0.5, & t>0 \end{cases}$

③ $-x(t) = 0.5 - u(t) = \begin{cases} 0.5, & t<0 \\ -0.5, & t>0 \end{cases}$

with the same diagram



⇒ In Summary, $x(t)$ is "Odd" Signal.

(d) base period + base frequency

(I) $x(t) = 4\cos(4t + 40°) + 3e^{-j12t}$.

→ Because it is a CT Signal.

→ So, Let ω_1, T_1 be the base period and frequency of $4\cos(4t+40°)$

Let ω_2, T_2 be the base period and frequency of $3e^{-j12t}$.

→ Then, we can find that

$\begin{cases} \Omega_1 = 4 \\ \Omega_2 = 12 \end{cases}$, $\begin{cases} T_1 = \frac{\pi}{2} \\ T_2 = \frac{\pi}{6} \end{cases}$

→ Base Period T_0 is the LCM of T_1 and T_2

Base frequency Ω_0 is the LCD of ω_1 and ω_2

→ Thus, $T_0 = \text{LCM}(\frac{\pi}{2}, \frac{\pi}{6}) = \frac{\pi}{2}$

$\Omega_0 = \text{LCD}(4, 12) = 4$.

→ $\boxed{\text{Missing Step for periodicity Verification}}$

For a CT signal, if it is periodic, then its frequency "ω" should be Any Real Number.

And in this question $\omega_0 = 4 \in R$, proving periodicity.

(II). $x(t) = \cos(2\pi t) + \sin 6t$.

Step 1: Verify Periodicity

Because it is a CT signal

→ So, Let ω_1, T_1 be the base period and frequency of $\cos(2\pi t)$

Let ω_2, T_2 be the base period and frequency of $\sin 6t$.

→ Then we can find that

$\begin{cases} \Omega_1 = 2\pi \\ \Omega_2 = 6 \end{cases}$ $\begin{cases} T_1 = 1 \\ T_2 = \frac{\pi}{3} \end{cases}$

→ Base Period T_0 is the LCM of T_1 and T_2.

Base Frequency Ω_0 is the LCD of ω_1 and ω_2

→ Thus, $T_0 = \text{LCM}(\frac{\pi}{3}, 1) = \text{NOT Find}$

$\Omega_0 = \text{LCD}(2\pi, 6) = \text{NOT Find}$

→ Since we cannot find $\text{LCM}(\frac{\pi}{3}, 1)$

→ Therefore, $x(t)$ is Aperiodic.

(2) Power Signal, energy signal, neither

(I). $x(t) = e^{-2t} u(t)$

Step 1: Signal Type

→ it is a CT signal

Step 2: Periodicity

→ $x(t)$ is non-periodic signal

Step 3: Diagram

→ $x(t)$ attenuates with time $t \uparrow$

Step 5: Power / Energy

$$\rightarrow E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$

$$= \int_0^{+\infty} e^{-4t} dt$$

$$= +\frac{1}{4} < \infty$$

$$\rightarrow P_x = 0$$

→ Thus, $x(t)$ is an energy signal.

(II) $x(t) = e^{j(2t+\frac{\pi}{4})}$

Step 1. Signal Type.

→ it is a CT signal

Step 2: Periodicity

→ Since $w=2 \in \mathbb{R}$,

$x(t)$ is periodic signal

Step 3: Power / Energy

1° → ~~Power~~ Energy

$$\rightarrow E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \infty$$

2° → Power

Since $x(t)$ is periodic CT signal

$$\rightarrow P_x = \frac{1}{T_0} \int_{<T_0>} |x(t)|^2 dt$$

$$= \frac{1}{\pi} \int_{<\pi>} \left| e^{j(2t+\frac{\pi}{4})} \right|^2 dt$$

$$= \frac{\pi}{\pi} = 1$$

⇒ Therefore, $x(t)$ is a power signal.

Q3 (a) Causal, stable, Linear, TI?

(I). $y[n] = x[3-2n]$.

1° Verify Causality.

Since $x[n]$ to $x[3-2n]$ suffer from flipping,

→ Thus, Non-Causality (NC).

2° Verify Stability.

· Apply "BIBO" Theorem,

If $x[n]$ is Bounded, $|x[n]| \le B_x$.

Thus $|y[n]| = |x[3-2n]| \le B_x = B_y$.

→ So, Stable (S).

3° Verify Linearity

$y[ax_1[n] + bx_2[n]] = ay_1[n] + by_2[n]$

→ Hence, Linear (L).

4° Verify Time-Invariant.

Since $x[n] \rightarrow x[3-2n]$ contains scale and flip,

→ therefore, Time-Variant (TV)

In Summary. $y[n] = x[3-2n]$

is a system with
- Non-Causality
- Stability
- Linear
- Time-Variant

$\boxed{\text{or briefly : NC, S, L, TV}}$

(II). $y(t) = \cos(\pi t)\, x(t)$

1° Verify Causality.

Since $y(t)$ only depends on time "t"

Inputs signal

→ Thus, Causality (C)

2° Verify Stability

Apply "BIBO" Theorem

If $x(t)$ is bounded: $|x(t)| \le B_x < \infty$.

Given that $\cos(\pi t) \in [-1, 1]$

→ So, $|y(t)| \le |+1| B_x = B_y < \infty$.

→ So, Stability (S)

3° Verify Linearity

Step 1: L → S.

→ $a x_1(t) + b x_2(t)$

→ $y(t) = \cos(\pi t) \cdot [a x_1(t) + b x_2(t)]$

→ $y(t) = a \cos(\pi t) \cdot x_1(t) + b \cos(\pi t) \cdot x_2(t)$ (1)

Step 2: S → L.

→ $y(t) = \cos(\pi t)\, x(t)$

→ $a y_1(t) + b y_2(t)$

$y(t) = a \cos(\pi t)\, x_1(t) + b \cos(\pi t)\, x_2(t)$. (2)

Step 3: Conclusion.

Because the Eq(1) is the same as Eq(2)

→ Hence, Linearity (L).

4° Verify Time-Invariant

Step 1: D → S.

→ $x(t)$ to $x(t-t_0)$

→ $y(t) = \cos(\pi t)\, x(t-t_0)$ (3)

Step 2: S → D.

→ $y(t) = \cos(\pi t)\, x(t)$

→ $y(t) = \cos(\pi t - \pi t_0) \cdot x(t-t_0)$ (4)

Step 3: Conclusion

Because Eq(3) ≠ Eq(4)

→ therefore, Time-Variant (TV)

In Summary, $y(t) = \cos(\pi t)\, x(t)$

is a system satisfies with.
- Causality
- Stability
- Linearity
- Time-Variant.

$\boxed{\text{Or briefly : C, S, L, TV}}$

(III). $h(t) = u(t+3) - u(t-3)$

1° Causality.

Because it is a CT signal..

→ So we just need to check

$h(t) = 0$, if $t<0$

→ $\boxed{\text{Thus, Non-Causality (NC)}}$

2°. Stability

By checking whether $h(t)$ is absolutely Integrable, we can determine its Stability

→ $\int_{-\infty}^{+\infty} |h(t)|\, dt. = 6 < \infty$

→ $\boxed{\text{Thus, Stability (S)}}$

3° Linearity.

→ Impulse Response suggests Linearity

→ $\boxed{\text{Thus, Linearity (L)}}$

4° Time - Invariant

→ LTI System has impulse response

→ $\boxed{\text{Thus, Time - Invariant (TI)}}$

(IV). $h[n] = 5^n u[-n]$

1° Causuality

Because it is a DT signal

⇒ So we just need to check

$h[n] = 0$, if $n<0$

$\boxed{\text{→ Thus, non-Causality (NC)}}$

2° Stability

By checking whether $|h[n]|$ is absolutely Integrable, we can determine its Stability

→ $\cancel{\int_{-\infty}^{+\infty} |h(t)|\, dt}$ → $\sum_{k=-\infty}^{+\infty} |5^k u[-k]| \cancel{<\infty}$.

$= \sum_{k=-\infty}^{0} 5^k = \cancel{k=0} \sum_{k=0}^{+\infty} (\frac{1}{5})^k = \frac{5}{4}$.

$\boxed{\text{→ Thus, } \cancel{\text{Non-}}\text{Stability (} \overset{S}{\cancel{NS}} \text{)}}$

3° Linearity, Time - Invariant

→ LTI system has impulse response

→ Thus, $\begin{cases} \text{Linearity (L)} \\ \text{Time - Invariant (TI)} \end{cases}$

(b). Find $h(t)$

I). $y(t) = x(t-7)$

Let $\begin{cases} x(t) = \delta(t) \\ y(t) = h(t) \end{cases}$

Hence, $h(t) = \delta(t-7)$

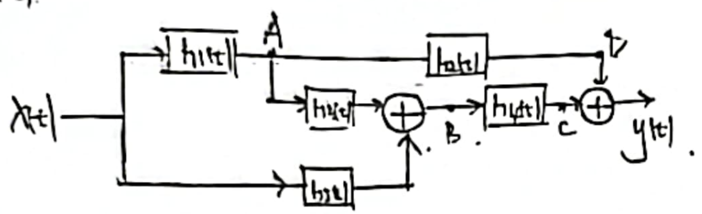
II). $y(t) = \int_{-\infty}^{t} x(\tau - 7)\, d\tau$.

Let $\begin{cases} x(t) = \delta(t) \\ y(t) = h(t) \end{cases}$

Hence, $h(t) = \int_{-\infty}^{t} \delta(\tau - 7)\, d\tau$.

→ $h(t) = u(t-7)$

(c).



Point A : $x(t) * h_1(t)$.

Point B : $[x(t) * h_1(t)] * h_3(t) + x(t) * h_5(t)$.

Point C : $\{ [x(t) * h_1(t)] * h_3(t) + x(t) * h_5(t) \} * h_4(t)$ ①

Point D : $[x(t) * h_1(t)] * h_2(t)$. ②

Apply Commutative、Associative、Distribute Property

Then Eq① and Eq② can be simplified to

$$\begin{cases} x(t) * \left(h_1(t) * h_3(t) * h_4(t) + h_5(t) * h_4(t)\right) & ③ \\ x(t) * \left(h_1(t) * h_2(t)\right) & ④ \end{cases}$$

We know $y(t) = \text{Point C} + \text{Point D}$.

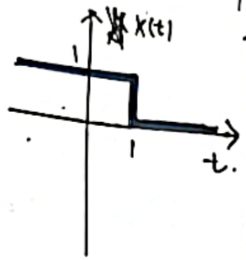
$\rightarrow y(t) = x(t) * \left[h_1(t) * \left(h_2(t) + h_3(t) * h_4(t)\right) + h_4(t) * h_5(t) \right]$

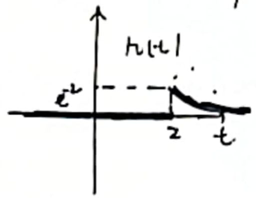
Therefore, the total System Impulse Response $h(t)$

$\rightarrow h(t) = h_1(t) * \left[h_2(t) + \left(h_3(t) * h_4(t)\right)\right] + \left(h_4(t) * h_5(t)\right)$

Step 1: Draw the diagram of $x(t)$、$h(t)$

1° $x(t) = u(1-t)$ 2° $h(t) = e^{-t} u(t-2)$





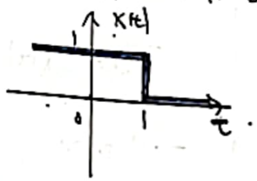
Step 2: Property of Convolution

Since $x(t) * h(t) = h(t) * x(t)$,

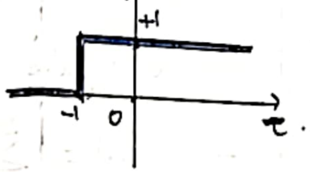
thus, fix $h(t)$, shift $x(t)$.

Step 3: Shift $x(t)$.

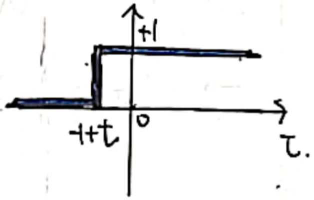
1° $x(\tau) = u(1-\tau)$



2° $x(-\tau)$



3° $x(-\tau+t)$



Step 4: Discussion.

1° Case 1 $-1+t \leq 2$.

$\rightarrow h(t) * x(t)$

$= \int_{-\infty}^{+\infty} h(\tau) \cdot x(t-\tau)\, d\tau$

$= \int_{2}^{+\infty} e^{-\tau}\, d\tau$

$= +e^{-2}$.

2° Case 2 : $-1+t > 2$.

$\rightarrow h(t) * x(t)$

$= \int_{-\infty}^{+\infty} h(\tau) \cdot x(t-\tau)\, d\tau$

$= \int_{t-1}^{+\infty} e^{-\tau}\, d\tau$

$= +e^{1-t}$.

Step 5: Conclusion

$$y(t) = h(t) * x(t) = \begin{cases} +e^{-2}, & \text{if } t \leq +3 \\ +e^{1-t}, & \text{if } t > +3 \end{cases}$$

(e).

From Question, we know that

$$\frac{d}{dt}\left(f(t) * g(t)\right) = \frac{df(t)}{dt} * g(t) = \frac{dg(t)}{dt} * f(t)$$

So, if we want to find the value of

$$\frac{d}{dt}\left(e^{-t} * u(t)\right).$$

We are able to let. $f(t) = u(t)$, $g(t) = e^{-\tau}$

→ Hence, $\frac{df(t)}{dt} = \delta(t)$, $g(t) = e^{-t}$.

$$\rightarrow \frac{d}{dt}\left(f(t) * g(t)\right) = \delta(t) * e^{-t}.$$

$$= \int_{-\infty}^{+\infty} e^{-\tau} \cdot \delta(t-\tau)\, d\tau.$$

$$= e^{-t}.$$

Q3.

(a) Find Fourier Coefficients.

$$x(t) = 2\sin^2 4t + \cos 4t$$

Step 1 : Simplify the representation

$$\rightarrow x(t) = 1 - \cos 8t + \cos 4t.$$

Since $\cos\theta = \frac{e^{j\theta} + e^{j(-\theta)}}{2}$

$$\rightarrow x(t) = 1 - \frac{e^{j8t} + e^{j(-8t)}}{2} + \frac{e^{j4t} + e^{j(-4t)}}{2}$$

$$\rightarrow x(t) = 1 + \tfrac{1}{2} e^{j(4t)} + \tfrac{1}{2} e^{j(-4t)} - \tfrac{1}{2} e^{j(8t)} - \tfrac{1}{2} e^{j(-8t)}$$

$$\rightarrow x(t) = \left(-\tfrac{1}{2}\right) \cdot e^{j(-8t)} + \tfrac{1}{2} e^{j(-4t)} + 1 + \tfrac{1}{2} e^{j(4t)} - \tfrac{1}{2} e^{j(8t)}$$

Step 2 : find the Base Angular Frequency

Since $x(t)$ is a CT signal

→ Hence, the common Angular Frequency

$$\Omega_0 = GCD(8, 4) = 4$$

→ We also know that. the Synthesis Equation

$$\rightarrow x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\Omega_0 t}.$$

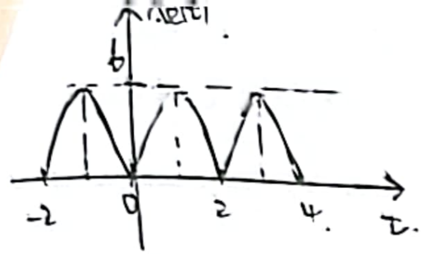
Step 3 : Write the Fourier Series.

$$\rightarrow x(t) = \left(-\tfrac{1}{2}\right) \cdot e^{j(-2)\Omega_0 t} + \tfrac{1}{2} e^{j(-1)\Omega_0 t} + 1 + \tfrac{1}{2} e^{j\Omega_0 t} - \tfrac{1}{2} e^{j2\Omega_0 t}$$

Step 4 : Coefficie

In Summary

$$\begin{cases} a_{-2} = -\frac{1}{2} \\ a_{-1} = +\frac{1}{2} \\ a_0 = +1 \\ a_1 = +\frac{1}{2} \\ a_2 = -\frac{1}{2} \end{cases}$$



Step1 : $x_e(t)$

From Diagram, we can find that

if $x_e(t) = A\,|\sin(\omega t+\varphi)|$

→ then
$$\begin{cases} A = +6 \\ \omega = \frac{2\pi}{4} = \frac{\pi}{2} \\ \varphi = 0 \end{cases}$$

→ Thus, $x_e(t) = 6\,|\sin\frac{\pi}{2}t|$, with $T_1 = 2$

Step 2 : Analysis Equation

→ $a_n = \frac{1}{T}\int_T x(t)\cdot e^{-jn\Omega_0 t}\,dt$.

→ $a_n = \frac{1}{2}\int_0^2 6\sin\frac{\pi}{2}t\cdot e^{-jn\Omega_0 t}\,dt$

$$= \frac{3}{2j}\int_0^2 \left[e^{j\frac{\pi}{2}t} - e^{j(-\frac{\pi}{2}t)}\right]\cdot e^{-jn\pi t}\,dt$$

$$= \frac{3}{2j}\int_0^2 e^{j(\frac{\pi}{2}-n\pi)t} - e^{j(-\frac{\pi}{2}-n\pi)t}\,dt$$

$$= \frac{3}{2j}\int_0^2 e^{j(\frac{\pi}{2}-n\pi)t}\,dt - \frac{3}{2j}\int_0^2 e^{j(-\frac{\pi}{2}-n\pi)t}\,dt$$

1° Part 1 : $\frac{3}{2j}\int_0^2 e^{j(\frac{\pi}{2}-n\pi)t}\,dt$.

$$= \frac{3}{2j}\cdot\frac{1}{j(\frac{\pi}{2}-n\pi)}\cdot\left[e^{j(\frac{\pi}{2}-n\pi)t}\right]_0^2.$$

$$= \frac{3}{\pi-2n\pi}\left[e^{j(1-2n)\pi}-1\right]\cdot(-1)$$

$$= \frac{6}{\pi-2n\pi}$$

2° Part 2 : $\frac{3}{2j}\int_0^2 e^{j(-\frac{\pi}{2}-n\pi)t}\,dt$

$$= \frac{3}{2j}\cdot\frac{1}{j(-\frac{\pi}{2}-n\pi)}\left[e^{j(-\frac{\pi}{2}-n\pi)t}\right]_0^2.$$

$$= \frac{3}{\pi+2n\pi}\cdot\left[e^{j(-1-2n)\pi}-1\right]$$

$$= \frac{-6}{\pi+2n\pi}$$

3° a_n = Part 1° − Part 2°.

$$\Rightarrow a_n = \frac{6}{\pi-2n\pi} - \frac{-6}{\pi+2n\pi}$$

$$\Rightarrow a_n = 6\left(\frac{1}{\pi-2n\pi}+\frac{1}{\pi+2n\pi}\right)$$

(c).

Step 1 : Simplification

Let $x_1(t) = x(at)$

Since $\cos(\omega_0 t) = \frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}$

→ $x(at)\cdot\cos(\omega_0 t) = \frac{1}{2}x_1(t)\cdot e^{j\omega_0 t} + \frac{1}{2}x_1(t)\,e^{-j\omega_0 t}$

Step 2 : Property

We know that if $x(t) \xrightarrow{F} X(j\omega)$

then $x(t)\cdot e^{j\omega_0 t} \xrightarrow{F} X(j(\omega-\omega_0))$.

In that case

$$\rightarrow \begin{cases} x_1(t)\cdot e^{j\omega_0 t} \xrightarrow{F} X_1(\omega-\omega_0) \\ x_1(t)\cdot e^{-j\omega_0 t} \xrightarrow{F} X_1(\omega+\omega_0) \end{cases}.$$

Step 3. : Find $X_1(\omega)$.

Because $x_1(t) = x(at)$

According to
$$\begin{cases} \text{if } x(t) \xrightarrow{F} X(\omega) \\ \text{then } x(at) \xrightarrow{F} \frac{1}{|a|}X\left(\frac{\omega}{|a|}\right) \end{cases}$$

Considering $a \in (0,1)$

→ $x_1(t) \xrightarrow{F} \frac{1}{a}X\left(\frac{\omega}{a}\right)$

→ $X_1(\omega) = \frac{1}{a}X\left(\frac{\omega}{a}\right)$

$$\rightarrow \begin{cases} X_1(\omega-\omega_0) = \frac{1}{a}X\left(\frac{\omega-\omega_0}{a}\right) \\ X_1(\omega+\omega_0) = \frac{1}{a}X\left(\frac{\omega+\omega_0}{a}\right) \end{cases}$$

Step 4: Fourier Transform

$$x(at)\cdot\cos(\omega_0 t) = \frac{1}{2}x_1(t)\cdot e^{j\omega_0 t} + \frac{1}{2}x_1(t)\cdot e^{-j\omega_0 t}$$

$$\rightarrow F\{x(at)\cdot\cos(\omega_0 t)\} = \frac{1}{2a}X\left(\frac{\omega-\omega_0}{a}\right) + \frac{1}{2a}X\left(\frac{\omega+\omega_0}{a}\right)$$

(d) Step 1 : $X(\omega)$

Since $x(t) = 1+2\cos(2\pi t)$

$\rightarrow$ then $X(\omega) = 2\pi\delta(\omega)+2\pi\delta(\omega-2\pi)+2\pi\delta(\omega+2\pi)$

Step 2 : $H(\omega)\cdot X(\omega)$

Since $Y(\omega) = H(\omega)\cdot X(\omega)$

$\rightarrow$ then $Y(\omega) = 2\pi H(\omega)\cdot\delta(\omega)+2\pi H(\omega)\cdot\delta(\omega-2\pi)+2\pi H(\omega)\cdot\delta(\omega+2\pi)$

$\rightarrow Y(\omega) = 2\pi H(0)\cdot\delta(\omega)+2\pi H(2\pi)\cdot\delta(\omega-2\pi)+2\pi H(-2\pi)\cdot\delta(\omega+2\pi)$

From the given graphs, we can derive that

$\rightarrow H(0) = 0$; $H(2\pi) = \frac{1}{2}\cdot e^{j(-\frac{\pi}{4})}$; $H(-2\pi) = \frac{1}{2}\cdot e^{j(+\frac{\pi}{4})}$

$\rightarrow Y(\omega) = 0+2\pi\cdot\frac{1}{2}\cdot e^{j(-\frac{\pi}{4})}\cdot\delta(\omega-2\pi)+2\pi\cdot\frac{1}{2}\cdot e^{j(+\frac{\pi}{4})}\cdot\delta(\omega+2\pi)$

$\rightarrow Y(\omega) = \frac{1}{2}\cdot[2\pi\delta(\omega-2\pi)]\cdot e^{j(-\frac{\pi}{4})}+\frac{1}{2}[2\pi\delta(\omega+2\pi)]\cdot e^{j(+\frac{\pi}{4})}$

$\rightarrow y(t) = \frac{1}{2}\cdot\left[e^{j(2\pi t-\frac{\pi}{4})}+e^{j(-2\pi t+\frac{\pi}{4})}\right]$

$\rightarrow y(t) = \cos\left(2\pi t-\frac{\pi}{4}\right)$

Q4.

(a). $H(s) = \frac{15s(s+1)}{(s+3)(s-2)(s-1)}$.

I) 1° From the Representation, it is obvious that

zeroes : 0, −1

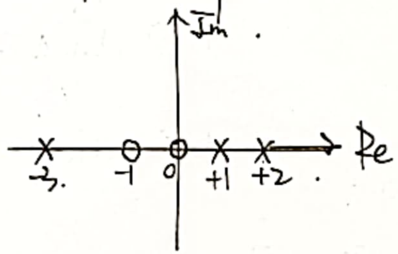
poles : −3, +1, +2

2° However, we need to check the Infinity.

Since the denominator has a higher order

$\rightarrow$ Infinity is belong to zeroes.

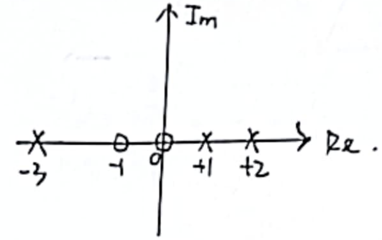
3° Finally, draw the zero-pole plot of this system



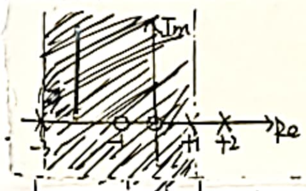
(Infinity does not show in this picture)

(II).

From (I), we can derive



Since the system is stable, the ROC needs to include $j\omega$-axis and be bounded by poles



ROC $\in(-3,+1)$ Re(s)

(III). $H(s) = \frac{15s(s+1)}{(s+3)(s-1)(s-2)}$.

Applying Partial Fraction Expansion.

$\rightarrow H(s) = \frac{A}{s+3}+\frac{B}{s-1}+\frac{C}{s-2}$

$\rightarrow \begin{cases} A+B+C = 15 \\ -3A+B+2C = 15 \\ 2A-6B-3C = 0 \end{cases} \rightarrow \begin{cases} A = \frac{9}{2} \\ B = \frac{-15}{2} \\ C = +18 \end{cases}$

$\rightarrow H(s) = \frac{\frac{9}{2}}{s+3}+\frac{\frac{-15}{2}}{s-1}+\frac{18}{s-2}$

$\rightarrow h(t) = \mathcal{L}^{-1}\{H(s)\}$

Finally, Impulse Response

$h(t) = \frac{9}{2}e^{-3t}u(t)+\frac{15}{2}e^{t}u(-t)-18e^{2t}u(-t)$

(IV). From III., we know that

$h(t) = \frac{9}{2}e^{-3t}u(t) + \frac{15}{2}e^{t}u(-t) - 18e^{2t}u(t)$

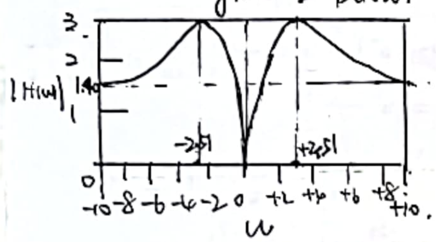
Apply Fourier Transform

$\rightarrow H(\omega) = \frac{9}{2}\cdot\frac{1}{3+j\omega} - \frac{15}{2}\cdot\frac{1}{-1+j\omega} + 18\cdot\frac{1}{-2+j\omega}$

$= \frac{\frac{9}{2}(-1+j\omega)(-2+j\omega) + (-\frac{15}{2})\cdot(3+j\omega)(-2+j\omega) + 18(3+j\omega)(-1+j\omega)}{(3+j\omega)(-1+j\omega)(-2+j\omega)}$

$= \frac{15(j\omega)^2 + 15(j\omega)}{(3+j\omega)(-1+j\omega)(-2+j\omega)}$

To determine the kind of filter, we have to plot $H(\omega)$ in MATLAB, and then we can derive the diagram as below.



with $\begin{cases} y\text{-axis}: \text{Magnitude} \\ x\text{-axis}: \omega. \end{cases}$

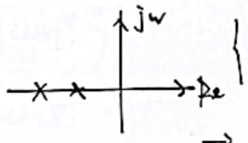
$\Rightarrow$ In Summary, this is Band-Pass Filter.

(b). $D(s) = s^2+2s+a$.

We know that if a system is stable, then its ROC contains $j\omega$ axis.

Since characteristic function implies the denominator of Transfer function

$\rightarrow$ So, we have to guarantee that the biggest root of $D(s)$ is no larger than 0



$\begin{cases} S_{max} = -1+\sqrt{1-a} \\ S_{max} < 0 \end{cases}$

$\rightarrow 0 < a \leq 1$

(c). $\begin{cases} y(t) = x(t) * h(t). \\ g(t) = x(3t) * h(3t). \end{cases}$

$\begin{cases} x(t) \xleftrightarrow{F} X(\omega). \\ h(t) \xleftrightarrow{F} H(\omega). \end{cases}$

1° From Scaling Property, we know

if $x(t) \xleftrightarrow{F} X(\omega)$

then $x(at) \xleftrightarrow{F} \frac{1}{|a|}X(\frac{\omega}{a})$

$\rightarrow$ thus $x(3t) \xleftrightarrow{F} \frac{1}{3}X(\frac{\omega}{3})$

2° From time-domain Convolution Property

$\rightarrow$ if $x(t) \xleftrightarrow{F} X(\omega)$, $h(t) \xleftrightarrow{F} H(\omega)$

$\rightarrow$ then $F\{x(t)*h(t)\} = X(\omega)\cdot H(\omega)$

3° Combine these two properties

$\rightarrow$ it is clear that

$\rightarrow Y(\omega) = H(\omega)\cdot X(\omega)$.

$\rightarrow G(\omega) = \frac{1}{3}X(\frac{\omega}{3})\cdot H(\frac{\omega}{3})\cdot\frac{1}{3}$

$\rightarrow G(\omega) = \frac{1}{9}X(\frac{\omega}{3})\cdot H(\frac{\omega}{3})$.

(Assume $y(t) \xleftrightarrow{F} Y(\omega)$, $g(t) \xleftrightarrow{F} G(\omega)$)

Since $Y(\omega) = H(\omega)\cdot X(\omega)$

$\rightarrow Y(\frac{\omega}{3}) = H(\frac{\omega}{3})\cdot X(\frac{\omega}{3})$

$\rightarrow G(\omega) = \frac{1}{3}\left[\frac{1}{3}X(\frac{\omega}{3})\cdot H(\frac{\omega}{3})\right]$ ①

Then Apply Scaling Property of CTFT

$\rightarrow y(3t) \xrightarrow{F} \frac{1}{3}Y(\frac{\omega}{3})$

Next, Apply Inverse Fourier Transform to Eq.①

$\rightarrow g(t) = \frac{1}{3}y(3t)$

Finally, $\begin{cases} A = \frac{1}{3} \\ B = 3 \end{cases}$

Q5. $\begin{cases} y''(t)+5y'(t)+6y(t)=e^{-t}u(t). \\ y(0^+)=1 \\ y'(0^+)=0 \end{cases}$

(a) Time Domain

1° Since it is a RLC Circuit, thus we are able to apply "Switching Theorem"

→ $y(0^-)=y(0^+)=1$

→ $y'(0^-)=y'(0^+)=0$

2° Zero-Input Case.

Set $x(t)=0$ and Combine $\begin{cases} y(0^-)=1 \\ y'(0^-)=0 \end{cases}$

$y(t)=Ae^{st}$.

→ $Ae^{st}\cdot(s^2+5s+6)=0$

→ $s^2+5s+6=0$

→ $(s+2)(s+3)=0$

→ $s_1=-2$ and $s_2=-3$.

→ $y_{zi}(t)=A_1\cdot e^{-2t}+A_2\cdot e^{-3t}$.

$\begin{cases} y(0)=+1 \\ y'(0)=0 \end{cases}$ ⇒ $\begin{cases} A_1+A_2=+1 \\ -3A_2-2A_1=0 \end{cases}$ ⇒ $\begin{cases} A_1=+3 \\ A_2=-2. \end{cases}$

→ $y_{zi}(t)=\left(3e^{-2t}-2e^{-3t}\right)\cdot u(t)$

3°. Zero-State

→ $y_{zs}(t)=y_{zs}^{(p)}(t)+y_{zs}^{(h)}(t)$

→ $y_{zs}(t)=k\cdot e^{-t}+c_1\cdot e^{-2t}+c_2\cdot e^{-3t}$.

$\begin{cases} y_{zs}^{(p)}(t)=k\cdot e^{-t}. \\ y''(t)+5y'(t)+6y(t)=e^{-t}\cdot u(t) \end{cases}$

→ $k-5k+6k=+1$

→ $k=+\frac{1}{2}$

→ $y_{zs}^{(p)}(t)=\frac{1}{2}e^{-t}u(t)$

$\begin{cases} y'(0^+)=0,\ y(0^+)=1 \\ y_{zs}(t)=\frac{1}{2}\underset{u(t)}{e^{-t}}+c_1\cdot\underset{u(t)}{e^{-2t}}+c_2\cdot\underset{u(t)}{e^{-3t}}. \end{cases}$

⇒ $\begin{cases} 2c_1+3c_2=-\frac{1}{2} \\ c_1+c_2=-\frac{1}{2}. \end{cases}$

⇒ $\begin{cases} c_1=\cancel{+2}-1 \\ c_2=\cancel{\frac{3}{2}}+\frac{1}{2} \end{cases}$

$y_{zs}(t)=\frac{1}{2}\underset{u(t)}{e^{-t}}-e^{-2t}u(t)+\frac{1}{2}e^{-3t}u(t)$

4° Overall

$y(t)=y_{zi}(t)+y_{zs}(t)$

$=3e^{-2t}u(t)-2e^{-3t}u(t)+\frac{1}{2}e^{-t}u(t)-e^{-2t}u(t)+\frac{1}{2}e^{-3t}u(t)$

$=\frac{1}{2}e^{-t}u(t)+2e^{-2t}u(t)-\frac{3}{2}e^{-3t}u(t)$

(b) Frequency Domain.

Step 1 : Switch Theorem.

Since this is a RLC Circuit, So we can apply Switch Theorem

→ $y(0^+)=y(0^-)$

⇒ $y(0^-)=1$; $y'(0^-)=0$

Step 2 : ULT

Apply ULT on both sides of the equation

→ $\begin{cases} \text{LHS}: s^2Y(s)-sy(0^-)-y'(0^-)+5[sY(s)-y(0^-)] \\ \quad +6Y(s) \\ \text{RHS}: \frac{1}{s+1} \end{cases}$

→ $(s^2+5s+6)\cdot Y(s)=\frac{s}{s+1}+\frac{s^2+5s+6}{s+1}$

→ $Y(s)=\frac{1}{s+1}+\frac{s}{(s+1)(s+2)(s+3)}$

Apply Partial Fraction Expansion.

→ $Y(s)=\frac{1}{s+1}+\frac{-\frac{1}{2}}{s+1}+\frac{+2}{s+2}+\frac{-\frac{3}{2}}{s+3}$.

→ $Y(s)=\frac{+\frac{1}{2}}{s+1}+\frac{+2}{s+2}+\frac{-\frac{3}{2}}{s+3}$

→ $y(t)=\frac{1}{2}\cdot e^{-t}u(t)+2e^{-2t}u(t)-\frac{3}{2}e^{-3t}u(t)$

In Summary, the overall response

→ $y(t)=\frac{1}{2}\cdot e^{-t}u(t)+2e^{-2t}u(t)-\frac{3}{2}e^{-3t}u(t)$