

Announcements:

➤ **Reading material:**

- **Microelectronics: Circuit Analysis and Design:**
 - Chapter 10: Sec. 10.1 – Bipolar Transistor Current Sources
 - Chapter 11: Sec. 11.4 – Differential Amplifier with Active Load

➤ **Practice exercises:**

- **Microelectronics: Circuit Analysis and Design, Chapter 10**
 - Chapter 10: Ex. 10.1-10.7
 - Chapter 11: Ex. 11. 10

Current Mirror Circuits

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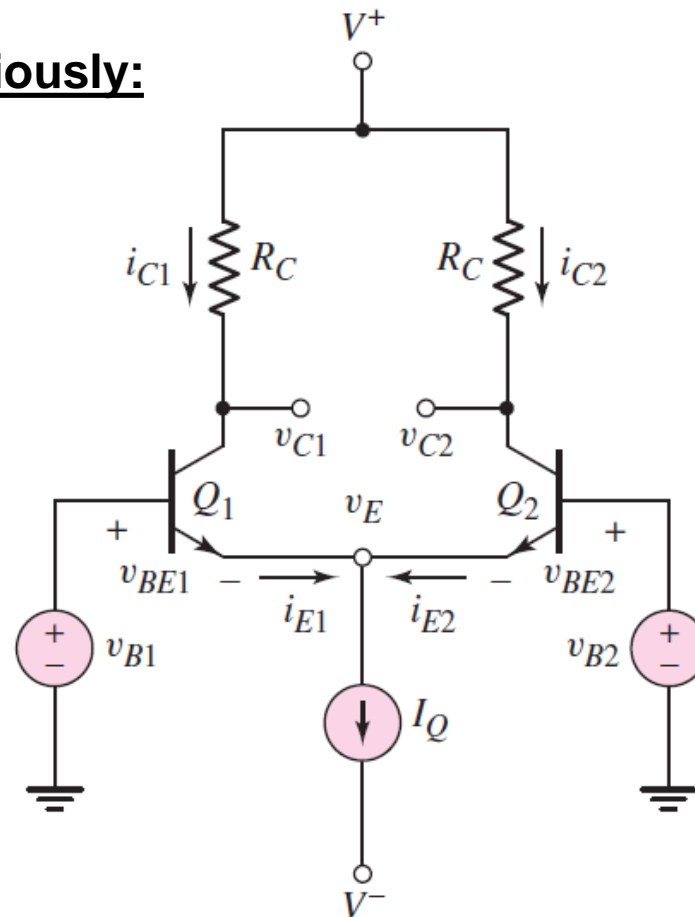
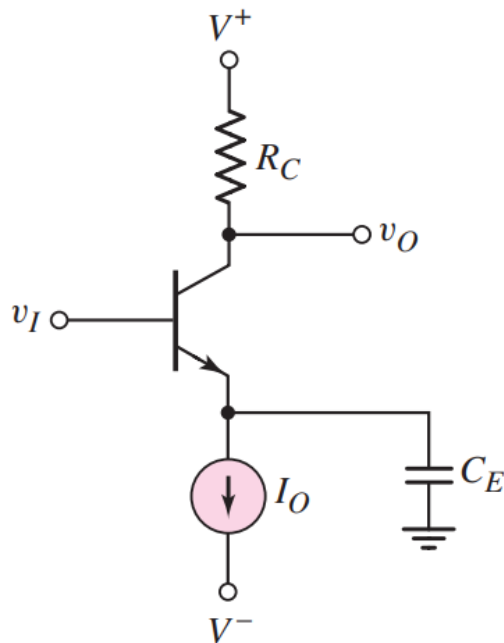
Outline

- **Part 1:** Two-Transistor and Multitransistor Current Mirrors:
 - Describe operational principle and analyze the characteristics of the basic current mirror circuits used to provide a constant output current.
- **Part 2:** Widlar Current Mirror:
 - Analyze and describe the characteristics of the Widlar current circuit used to provide a low-value constant output current.
- **Part 3:** Differential Amplifier with Active Loads:
 - Analyze operation and characteristics of BJT differential amplifiers with active loads.

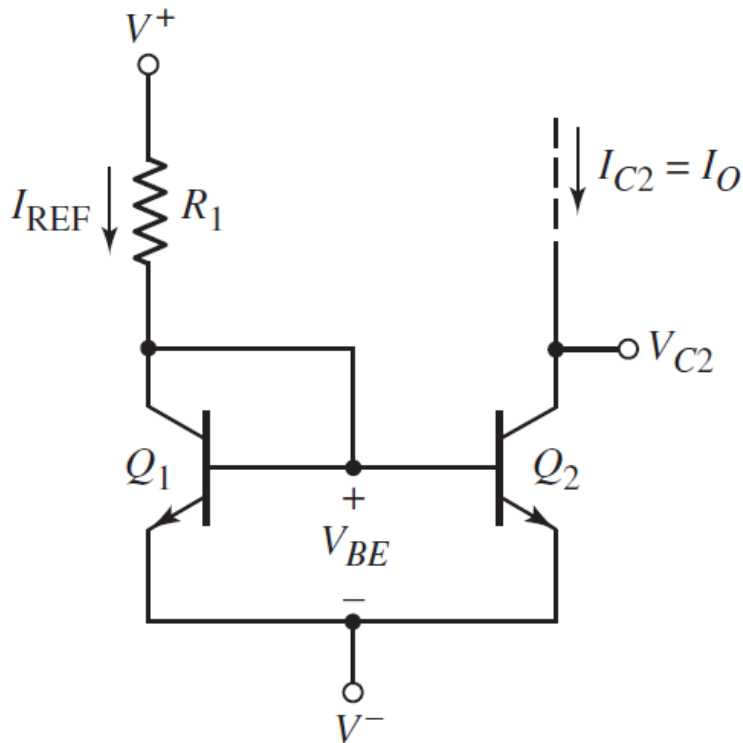
Part 1:

**Two-Transistor and
Multitransistor Current
Mirrors**

Linear amplifiers you studied previously:

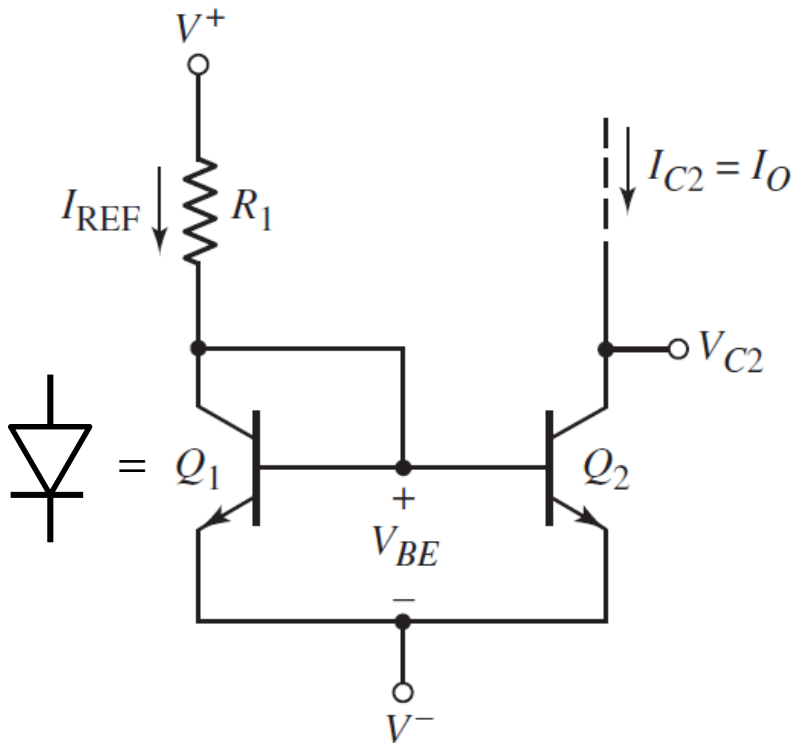


- In the previous lectures, when the bipolar transistors are used within a linear amplifying device, they must be biased in the forward-active mode.
- The bias may be a current source that establishes the quiescent collector current.
- We now need to consider the types of circuits that can be designed to establish the bias currents I_O and I_Q .

Two-transistor current source (current mirror) circuit

Two-transistor current source circuit (also known as a **current mirror**) – the basic building block in the design of integrated circuit current sources. The circuit consists of:

- Positive and negative voltage supply V^+ and V^- ;
- Two matched or identical transistors (Q_1 and Q_2):
 - Connected collector and base terminals of Q_1
 - Connected base terminals of Q_1 & Q_2 ;
 - Connected emitter terminals of Q_1 & Q_2 ;
- Reference current resistor R_1 (connected to collector of Q_1).

Two-transistor current source analysis – Current Relationships

1. The reference current I_{REF} is established by resistor R_1 and transistor Q1 connected as a diode:

$$I_{REF} = \frac{V^+ - V_{BE} - V^-}{R_1}$$

2. As $V_{BE1} = V_{BE2} = V_{BE}$ then $I_{B1} = I_{B2}$ and $I_{C1} = I_{C2}$.
Therefore, KCL for collector node of Q1:

$$I_{REF} = I_{C1} + I_{B1} + I_{B2} = I_{C1} + 2I_{B2}$$

3. Replacing I_{C1} by I_{C2} and noting that $I_{B2} = I_{C2}/\beta$:

$$I_{REF} = I_{C2} + 2 \frac{I_{C2}}{\beta} = I_{C2} \left(1 + \frac{2}{\beta} \right)$$

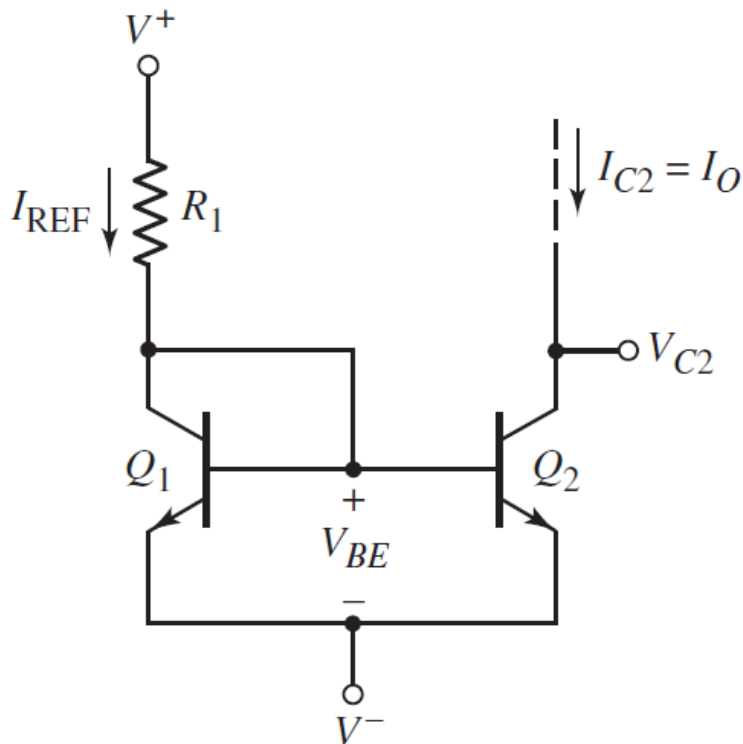
4. Output current is then:

$$I_O = I_{C2} = \frac{I_{REF}}{\left(1 + \frac{2}{\beta} \right)}$$

Note that the last equation gives the ideal output current for infinite Early voltage ($V_A = \infty$)

Example:

Task: Design a two-transistor current source to meet a set of specifications.



Circuit parameters:

$$V^+ = 5 \text{ V};$$

$$V^- = 0 \text{ V};$$

$$I_O = 200 \text{ } \mu\text{A};$$

Transistor parameters:

$$V_{BE(on)} = 0.6 \text{ V}$$

$$\beta = 100;$$

$$V_A = \infty;$$

Solution:

1. The reference current is:

$$I_{REF} = I_O \left(1 + \frac{2}{\beta} \right) = 200 \left(1 + \frac{2}{100} \right) = 204 \text{ } \mu\text{A}$$

2. The resistor R_1 is then found as:

$$R_1 = \frac{V^+ - V_{BE(on)} - V^-}{I_{REF}} = \frac{5 - 0.6}{204 \cdot 10^{-6}} = 21.6 \text{ k}\Omega$$

3. Here, $V_{BE(on)}$ is assumed to be 0.6 V.

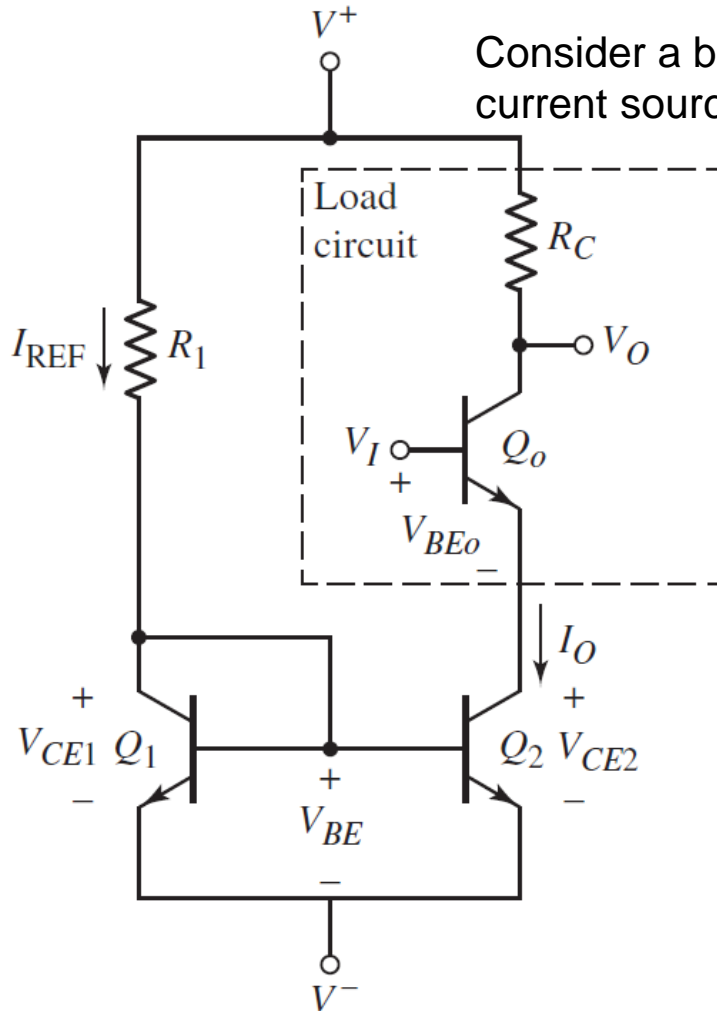
Calculate the output current if it is actually 0.7 V:

$$I_{REF} = \frac{V^+ - V_{BE(on)} - V^-}{R_1} = \frac{5 - 0.7}{21.6} = 199 \text{ } \mu\text{A};$$

Effectively, the output current can be approximated as $I_O \cong I_{REF}$

$$I_O = \frac{I_{REF}}{1 + 2/\beta} = \frac{199}{1 + 2/100} = 195 \text{ } \mu\text{A}.$$

Two-transistor current source analysis – Output resistance



1. The change in load current with a change in voltage $V_O = V_{C2}$ determines the output conductance:

$$\frac{dI_O}{dV_{C2}} = \frac{1}{r_o}$$

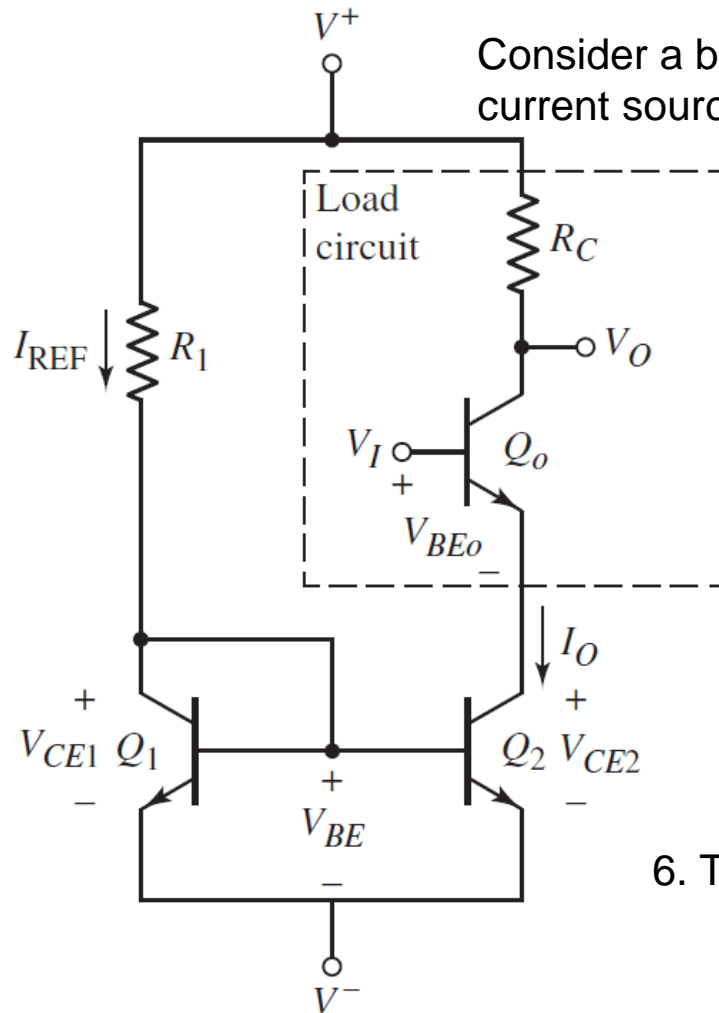
2. The output (collector) voltage is determined by:

$$V_{C2} = V_{CE2} - V^- \Rightarrow dV_{C2} = dV_{CE2}$$

3. Taking the Early voltage into account, the output-reference currents relationship is established by:

$$I_O = \frac{I_{REF}}{\left(1 + \frac{2}{\beta}\right)} \times \frac{1 + \frac{V_{CE2}}{V_A}}{1 + \frac{V_{CE1}}{V_A}}$$

Two-transistor current source analysis – Output resistance



Consider a basic transistor amplifier biased with the two-transistor current source. The stability of a load current generated in a constant-current source is a function of the output resistance looking back into the output transistor Q2:

4. From the circuit configuration $V_{CE1} = V_{BE}$ and the differential change in I_O to V_{CE2} can be found as:

$$\frac{dI_O}{dV_{CE2}} = \frac{I_{REF}}{1 + 2/\beta} \times \frac{\frac{1}{V_A}}{1 + \frac{V_{BE}}{V_A}}$$

5. Assuming that $V_{BE} \ll V_A$: $\frac{dI_O}{dV_{CE2}} \cong \frac{I_O}{V_A} = \frac{1}{r_o}$

6. The small signal output resistance is then: $r_o = \frac{V_A}{I_O}$

The output resistance of a current source determines not only the stability of the load current, but also the performance of the amplifier it is connected to (remember diff-amps)

Exercise:

Task: Determine the change in load current produced by a change in collector–emitter voltage in a two-transistor current source.

Circuit parameters:

$$V^+ = 5 \text{ V};$$

$$V^- = -5 \text{ V};$$

$$R_1 = 9.3 \text{ k}\Omega.$$

Transistor parameters:

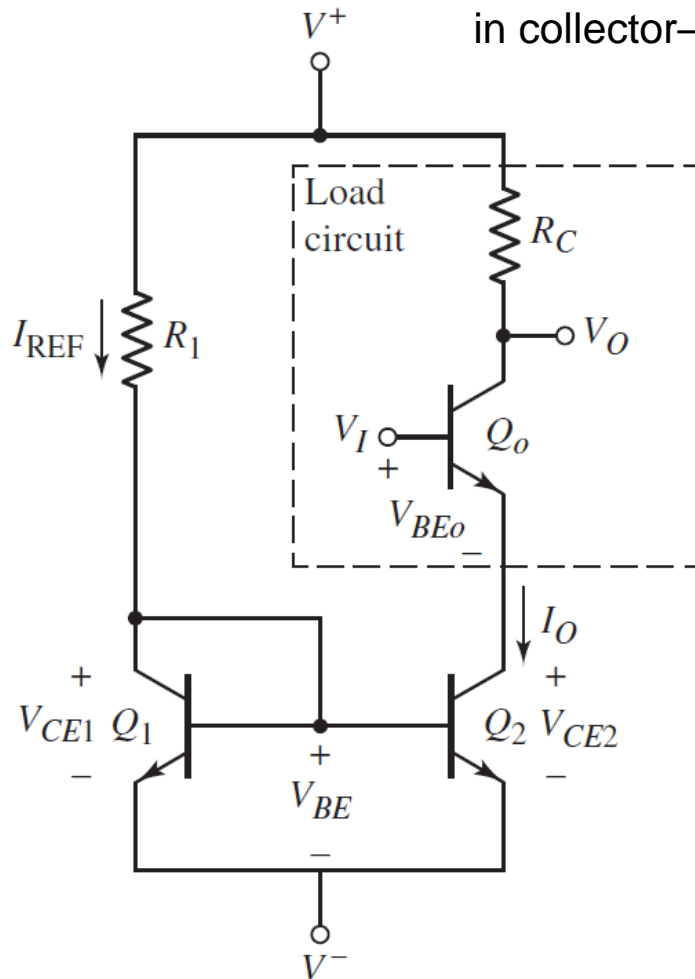
$$V_{BE(on)} = 0.7 \text{ V}$$

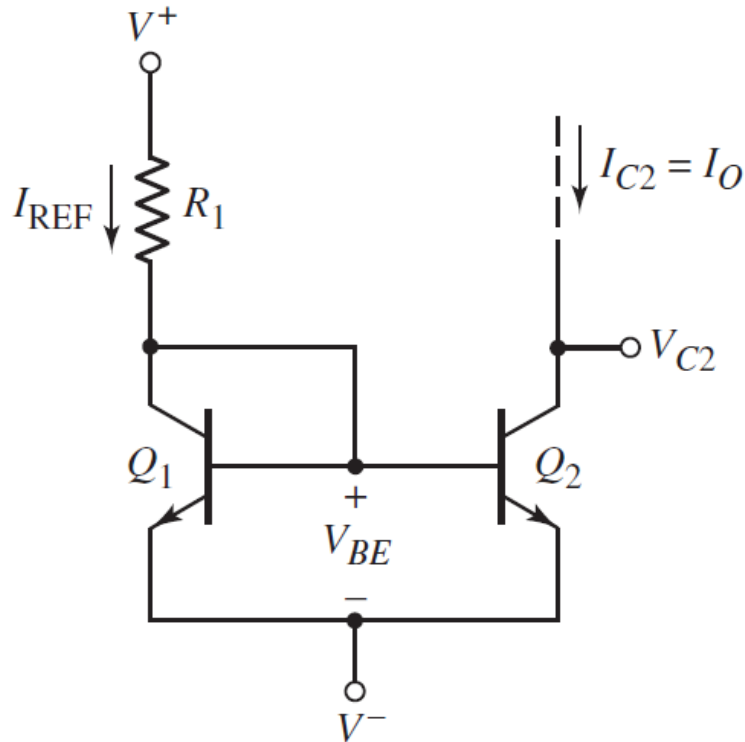
$$\beta = 50;$$

$$V_A = 80 \text{ V}.$$

Find percentage of I_O change when V_{CE2} varies from 0.7 to 5V:

Solution:



Two-transistor current source analysis – Mismatched Transistors

1. Consider exponential collector current-voltage relationships (neglecting the Early effect and base currents):

$$I_{REF} \cong I_{C1} = I_{S1} e^{V_{BE1}/V_T}$$

and

$$I_O = I_{C2} = I_{S2} e^{V_{BE2}/V_T}$$

For not identical transistors $I_{S1} \neq I_{S2}$; however, due to the circuit configuration $V_{BE1} = V_{BE2} = V_{BE}$.

Therefore, the relationship between the output (bias) and reference currents is found as:

$$I_O = I_{REF} \left(\frac{I_{S2}}{I_{S1}} \right).$$

The reverse-saturation currents I_{S1} and I_{S2} of transistors are functions of the cross-sectional area of the B–E junctions; therefore, it can be used to our advantage. For example, by using different sizes of transistors, we can design the circuit such that $I_O \neq I_{REF}$

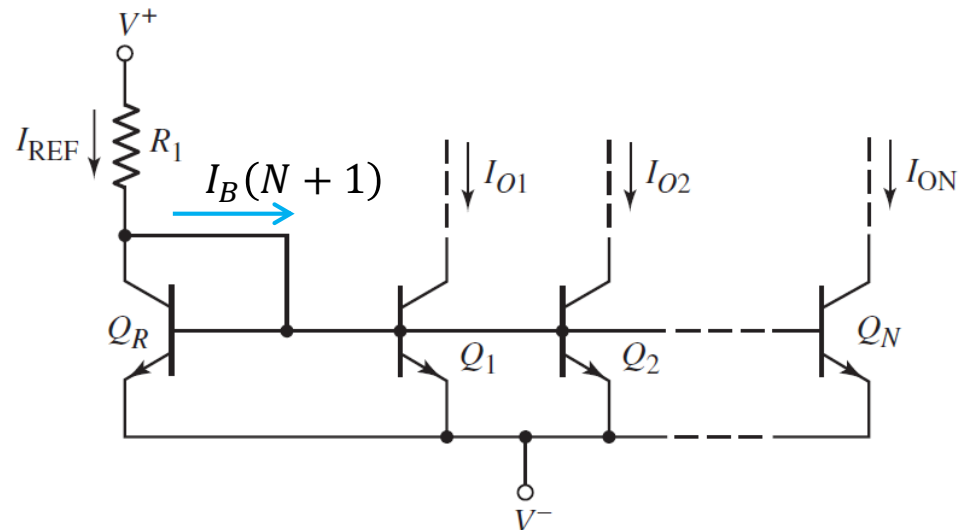
Multitransistor Current Mirrors

To generate multiple load currents, the reference transistor is placed alongside a series of transistors with their base-emitter junctions connected in parallel. The reference current is still found as:

$$I_{REF} = \frac{V^+ - V_{BE} - V^-}{R_1}$$

In this circuit, the reference current also feeds all the transistor base currents.

Performing the same analysis as for the two-transistor current mirror, the reference and output currents relationship is:



$$I_{O1} = I_{O2} = \dots = I_{ON} = \frac{I_{REF}}{1 + (1 + N)/\beta}$$

Practice on your own!
(based on slide 7)

A limitation of this circuit is that the same current is produced in each load, but usually **different** bias currents are required. It is possible to scale the currents - within limits - by scaling up the B-E area of transistors (as discussed previously)

Part 2:

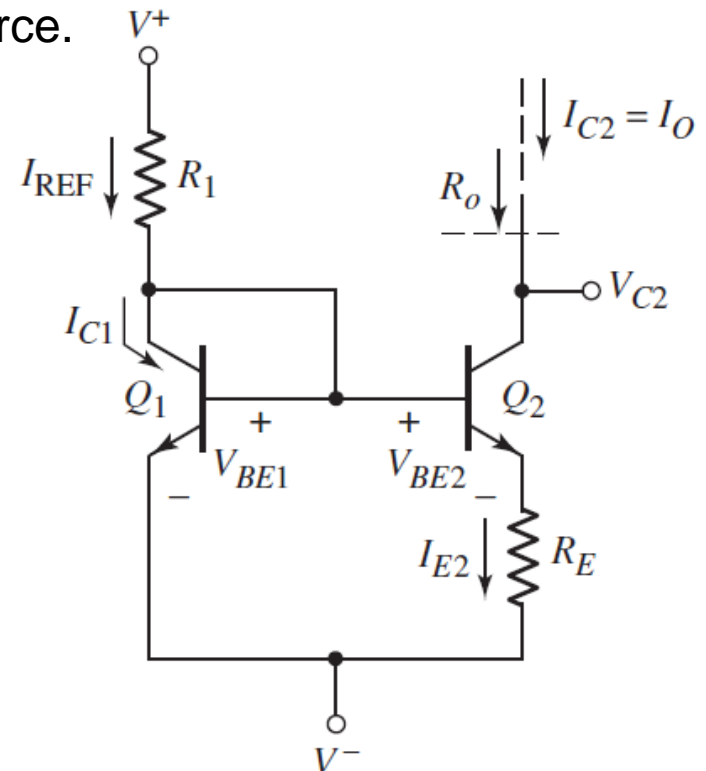
Widlar Current Mirror

Widlar current source

Another way to establish output current being significantly different from the reference current is to use the Widlar current source.

It is particularly useful in internal integrated circuit biasing arrangements because:

1. A range of currents can be produced by changing the value of R_E rather than the size of the output transistor,
2. The resistance values required are much smaller (see the exercise on the next slide).
3. Both of the above features lead to a much more compact design.



Exercise: counter-example of importance of the Widlar current source

Task: Design a two-transistor current source to meet a set of specifications (find R_1).

Two-transistor current source
(this is not Widlar current source):

Circuit parameters:

$$V^+ = 5 \text{ V};$$

$$V^- = -5 \text{ V};$$

$$I_O = 10 \text{ }\mu\text{A};$$

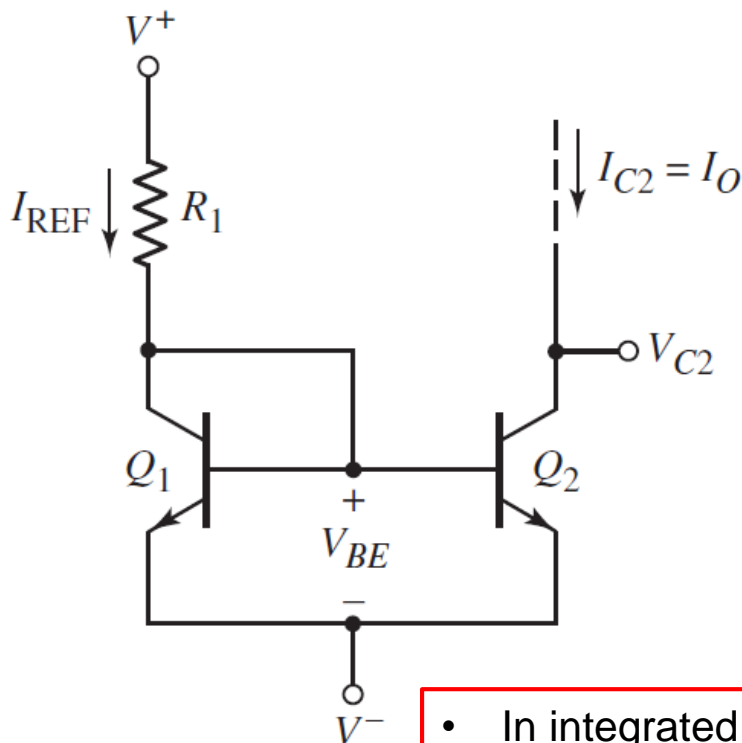
Transistor parameters:

$$V_{BE(on)} = 0.7 \text{ V}$$

$$\beta = 100;$$

$$V_A = \infty;$$

Solution:

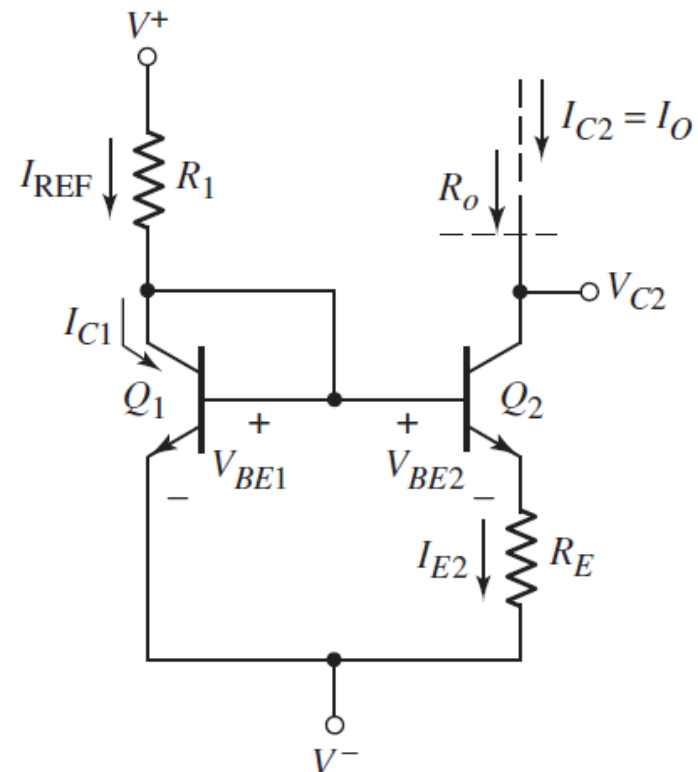


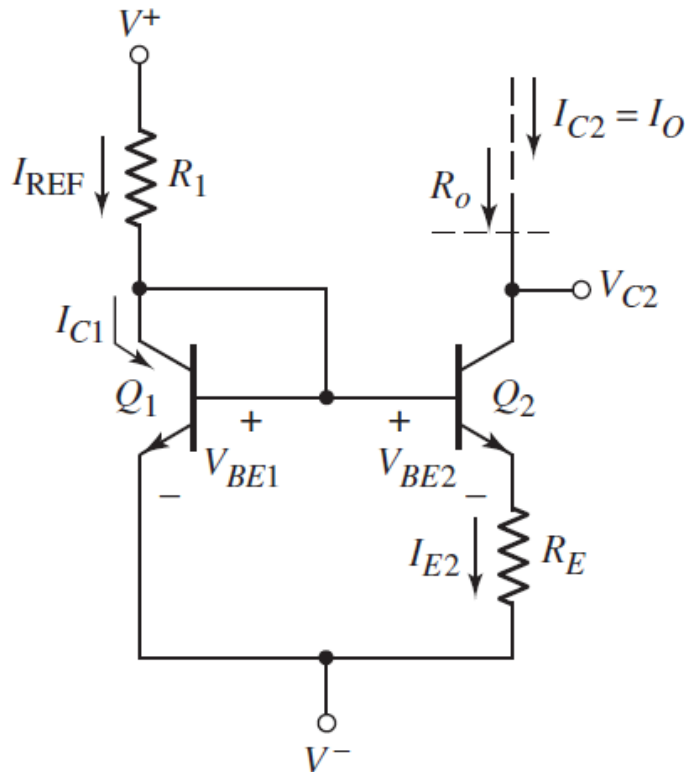
- In integrated circuits (IC), resistors on the order of $1 \text{ M}\Omega$ require large areas and are difficult to fabricate accurately.
- Therefore, it is required to **limit** IC resistor values.

The Widlar current source: Design and operational principle

Compared to the two-transistor current mirror, the Widlar current source circuit includes **resistor R_E** to the emitter of Q2:

- A voltage difference is produced across resistor R_E , so that now $V_{BE2} < V_{BE1}$.
- A smaller B-E voltage of Q2 produces a smaller collector current, meaning that the load current I_O is significantly less than the reference current I_{REF} .
- As a result, the Widlar current source circuit allows to use few kOhm range resistors for both R_1 and R_E (will be shown in the next exercise).



The Widlar current source – Current relationship

1. If Q1 and Q2 are identical and $\beta \gg 1$:

$$I_{REF} \cong I_{C1} = I_S \exp\left(\frac{V_{BE1}}{V_T}\right)$$

and

$$I_O = I_{C2} = I_S \exp\left(\frac{V_{BE2}}{V_T}\right)$$

2. Solving for the B-E voltages:

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) \text{ and } V_{BE2} = V_T \ln\left(\frac{I_O}{I_S}\right)$$

3. Their difference yields:

$$V_{BE1} - V_{BE2} = V_T \ln\left(\frac{I_{REF}}{I_O}\right)$$

4. Applying KVL to the lower loop we see:

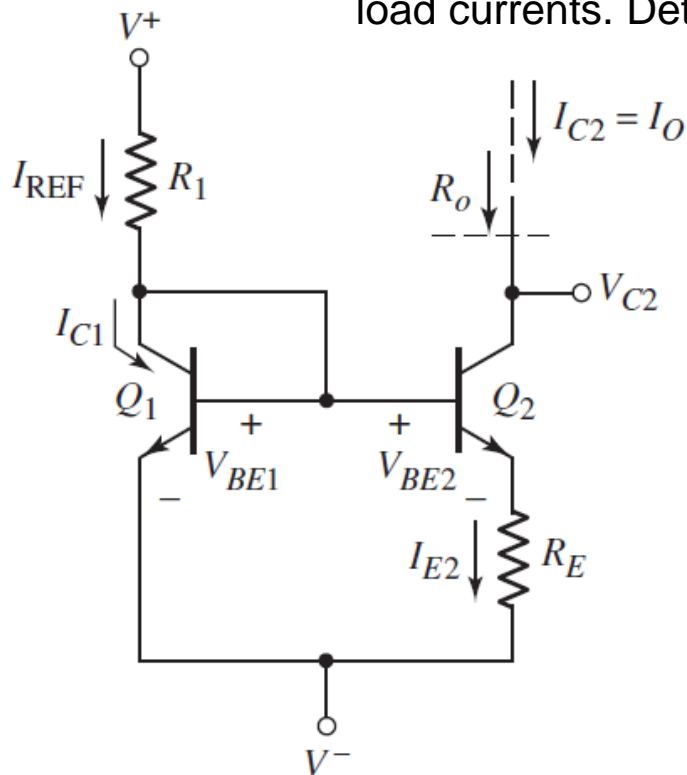
$$V_{BE1} - V_{BE2} = I_{E2} R_E \cong I_O R_E$$

5. Combining (3.) and (4.) will give the currents' relationship: $I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right)$

This transcendental equation cannot be solved directly for I_O . However, a numerical solution or a trial and error approach can be applied.

Example:

Task: Design a Widlar current source to achieve specified reference and load currents. Determine the difference between the two B-E voltages.



Circuit parameters:

$$V^+ = 5 \text{ V};$$

$$V^- = -5 \text{ V};$$

Solution:

1. Resistance R_1 is:

$$R_1 = \frac{V^+ - V_{BE1}(on) - V^-}{I_{REF}} = 9.3 \text{ k}\Omega;$$

2. Resistance R_E is:

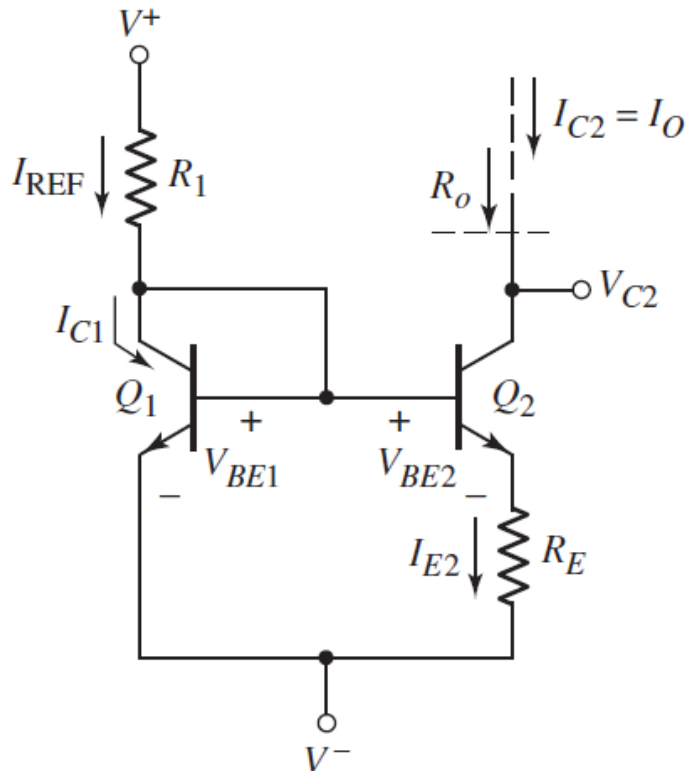
$$R_E = \frac{V_T}{I_O} \ln \left(\frac{I_{REF}}{I_O} \right) = 9.58 \text{ k}\Omega;$$

3. The difference between the two B-E voltages:

$$V_{BE1} - V_{BE2} = I_O R_E = 0.115 \text{ V}.$$

- 115mV difference in the B–E voltages of Q1 and Q2 produces approximately two orders of magnitude difference between the reference and load currents;
- As a result, we can produce a very low bias current using resistors in the low kOhm range;
- Moreover, additional R_E resistor gives the designer additional flexibility in adjusting the output to reference current ratio.

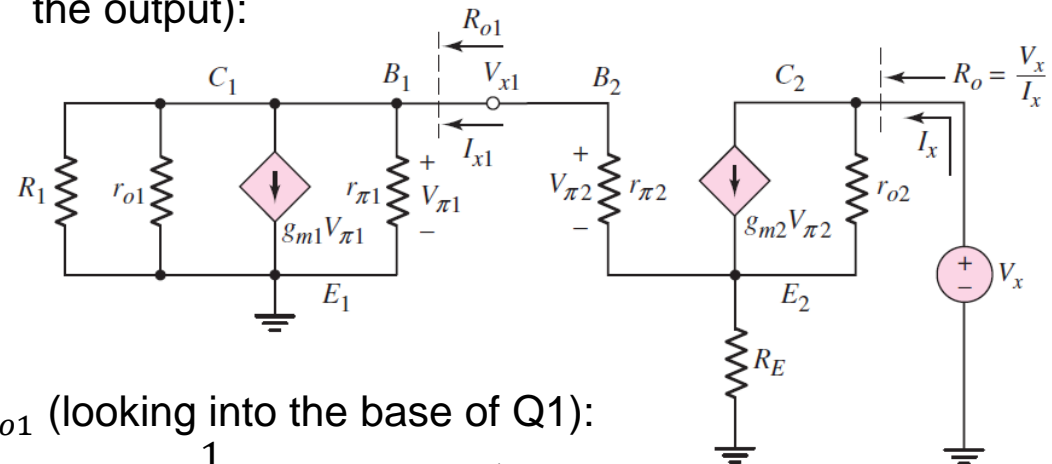
The Widlar current source – Output resistance



Applying test voltage to the output, we need to find output current to derive output resistance as:

$$R_O = \frac{V_x}{I_x}$$

1. This output resistance can be determined using the small-signal equivalent circuit (applying test voltage to the output):



2. First, let us determine resistance R_{O1} (looking into the base of Q1):

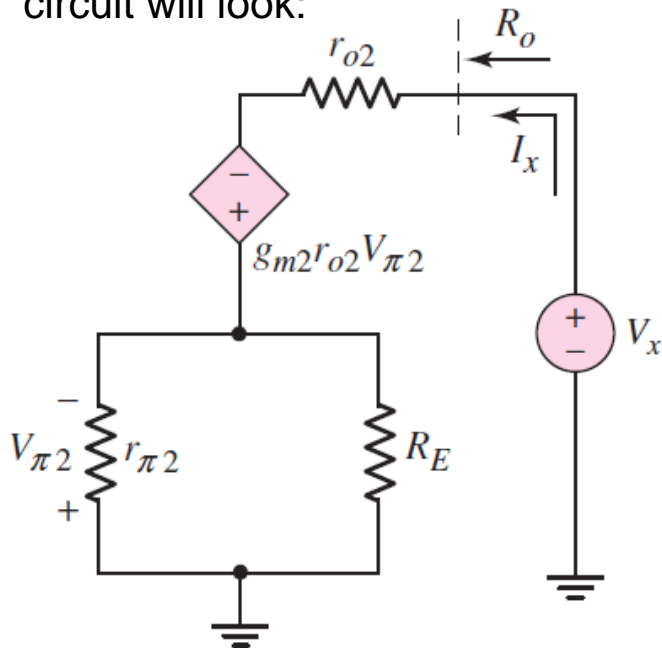
$$R_{O1} = r_{\pi 1} \parallel \frac{1}{g_{m1}} \parallel r_{o1} \parallel R_1 \cong \frac{1}{g_{m1}} \quad \text{because } \frac{1}{g_{m1}} \ll r_{\pi 1}, r_{o1}, \text{ and } R_1$$

For instance, based on the previous example for $I_{REF} = 1 \text{ mA}$: $\frac{1}{g_{m1}} = \frac{V_T}{I_{C1}} = 26 \text{ Ohm}$

3. Given the fact that $r_{\pi 2} \gg R_{O1}$ we can ignore R_{O1} in the further analysis.

The Widlar current source – Output resistance

4. Following the Norton-Thevenin equivalence*, the equivalent circuit will look:



5. Let us define $R'_E = r_{\pi 2} || R_E$ and write KVL equation:

$$V_x = I_x r_{o2} - g_{m2} r_{o2} V_{\pi 2} + I_x R'_E$$

6. Substituting for $V_{\pi 2} = -I_x R'_E$:

$$V_x = I_x (r_{o2} + g_{m2} r_{o2} R'_E + R'_E)$$

7. Therefore, the output resistance is:

$$\frac{V_x}{I_x} = R_O = r_{o2} \left(1 + R'_E \left(g_{m2} + \frac{1}{r_{o2}} \right) \right)$$

8. As $g_{m2} \gg \frac{1}{r_{o2}}$, the output resistance:

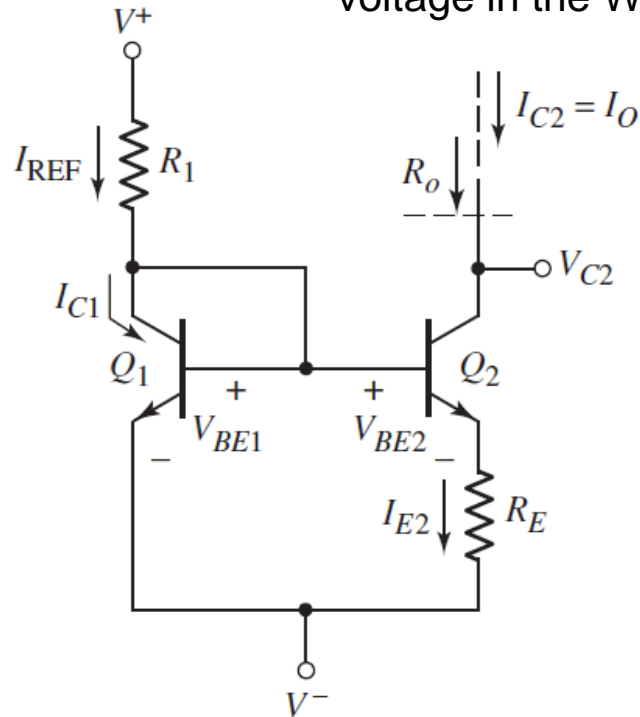
$$R_O \cong r_{o2} (1 + g_{m2} R'_E)$$

The output resistance of the Widlar current source is $(1 + g_{m2} R'_E)$ times larger than that of the simple two-transistor current source. Therefore, the Widlar current source is more stable.

* – Parallel current source and resistor become series voltage source and the same resistance; Thevenin voltage is equal to the Norton current times the Norton resistance ²¹

Exercise:

Task: Determine the change in load current with a change in collector voltage in the Widlar current source.



Transistor parameters:

$$V_{BE1}(\text{on}) = 0.7 \text{ V}$$

$$\beta = 100;$$

$$V_A = 80 \text{ V}.$$

Solution:

Circuit parameters:

$$V^+ = 5 \text{ V};$$

$$V^- = -5 \text{ V};$$

$$R_1 = 9.3 \text{ k}\Omega;$$

$$R_E = 9.58 \text{ k}\Omega;$$

$$dV_{C2} = 4.3 \text{ V}.$$

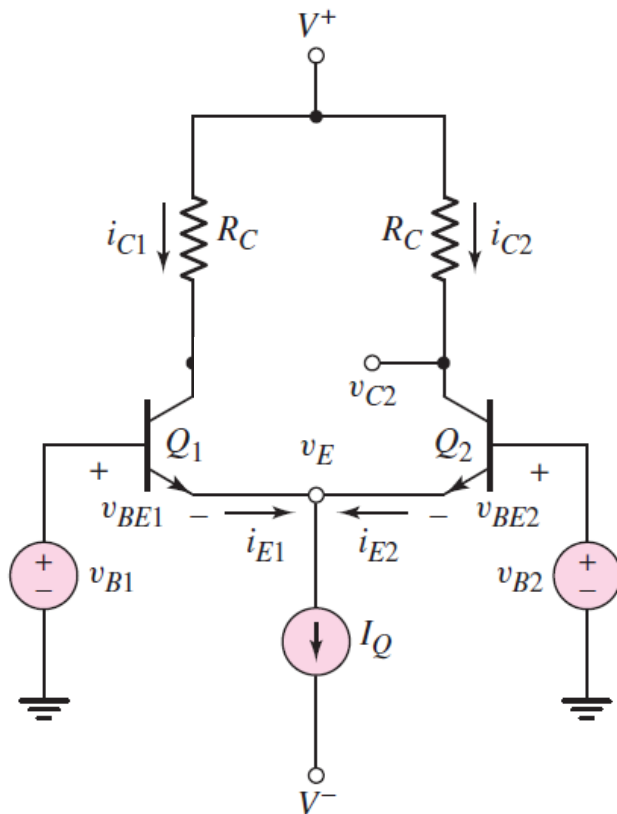
The stability of the load current, as a function of a change in output voltage, is improved in the Widlar current source, compared to the simple two-transistor current source.

Part 3:

Differential Amplifier with Active Loads

Differential amplifier with active loads

Diff-amp circuit:



From the previous lecture we know that:

Common mode

Input resistance: $R_{icm} \approx \beta R_0$

Voltage gain: $A_{cm} \approx -\frac{R_C}{2R_0}$

Differential mode

Input resistance: $R_{id} = \frac{4\beta V_T}{I_Q}$

Voltage gain: $A_d = \frac{\beta R_C}{2(r_\pi + R_B)}$

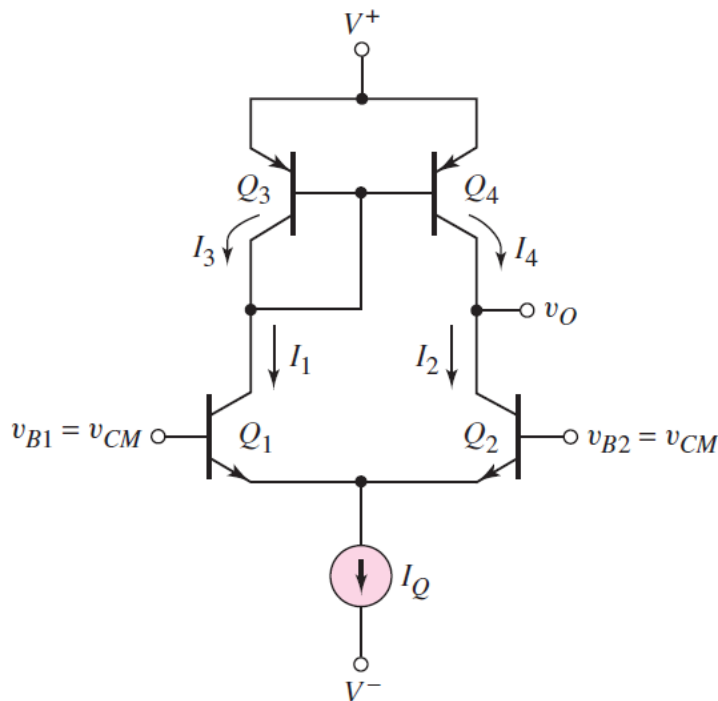
To design a high-spec diff-amp, we need to have:

1. High **common-mode input impedance**, e.g., by making R_0 large – the Widlar CS provides it;
2. High **differential-mode input impedance**, e.g., by making I_Q small – the Widlar CS provides it;
3. Low **common-mode voltage gain**, e.g., by making R_0 large and R_C small – trade-off with A_d ;
4. High **differential mode voltage gain**, e.g., by making R_C large – limited by V^+ and IC itself.

To overcome limitation from (4.) and to increase the differential-mode gain, **active loads** are used in place of R_C of a diff-amp circuit.

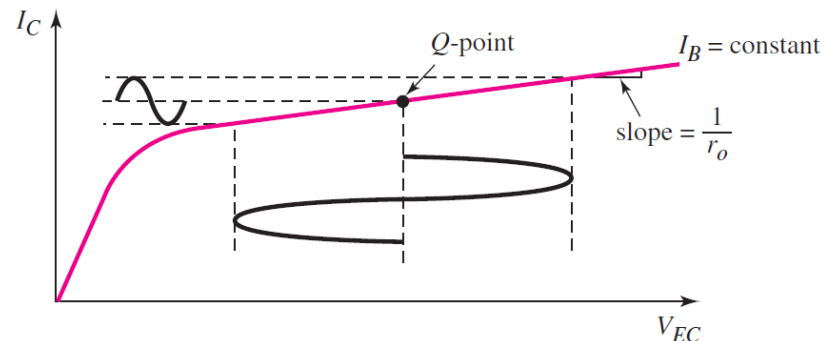
Differential amplifier with active loads – Operational principle

Diff-amp with active load:



Active load is a transistor current source used in place of resistive loads (note, the figure shows pnp transistor current mirror).

The transistors in the active load circuit are biased at a Q-point in the forward-active mode as shown below:

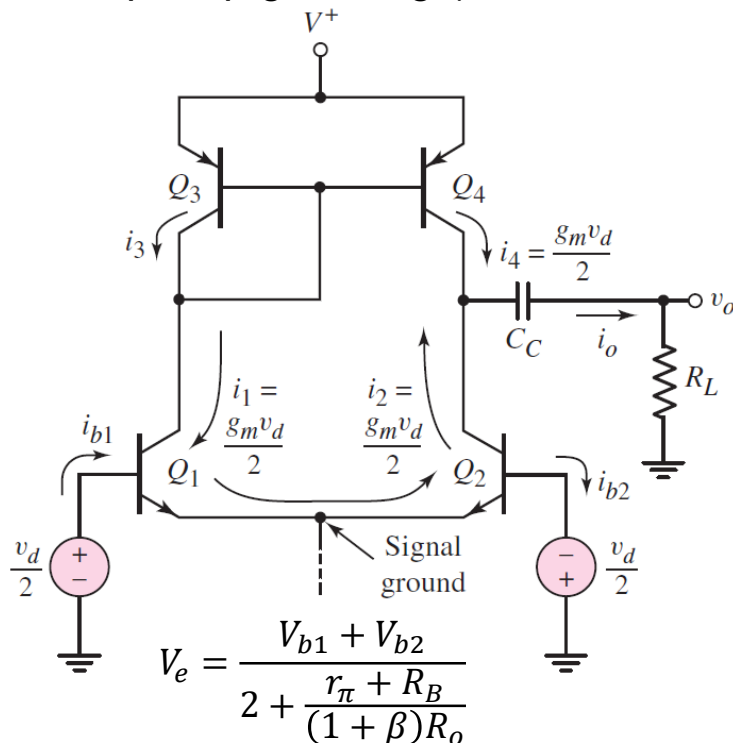


The relation between the change in current and change in voltage is proportional to the small-signal output resistance r_o of the transistor.

Within the same space of IC, the value of r_o can be much larger than that of a discrete resistive load, so the diff-mode voltage gain will be larger with the active load.

Differential amplifier with active loads – Small-signal qualitative analysis

Diff-amp with active load, i.e., three-transistor current source (resistor R_L is the small-signal resistance of the op-amp gain stage):



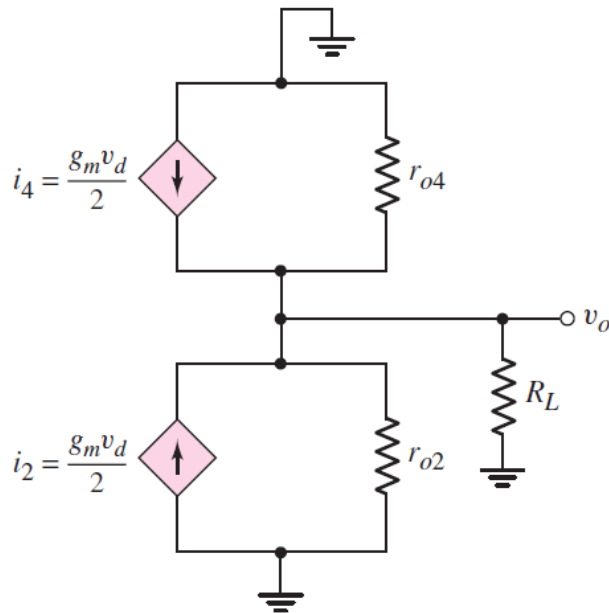
When purely diff-mode input voltage is applied ($v_{cm} = 0$):

0. Remember that common-emitter node is at signal ground ($v_e = 0$);
1. The signal voltage at the base of Q1 produces a signal collector current $i_1 = (g_m v_d)/2$;
2. Neglecting the base currents, Q3 collector current $i_3 = i_1$;
3. Since Q3 and Q4 share B-E voltage, the current mirror produces Q4 collector current $i_4 = i_3$;
4. The opposite signal voltage at the base of Q2 produces a signal collector current $i_2 = (g_m v_d)/2$ (with the direction shown);
5. Finally, both currents, i_2 and i_4 , add to produce a signal current i_o in the load resistance R_L , producing voltage output v_o .

This analysis is a simple evaluation of the circuit operation principle.

Differential amplifier with active loads – Small-signal analysis

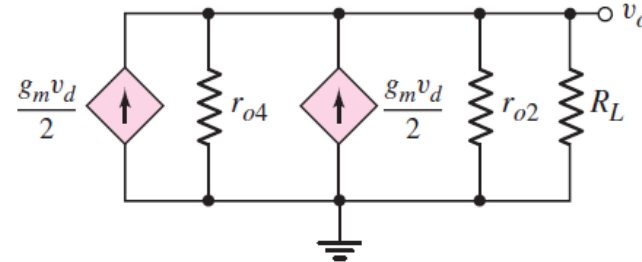
Small-signal equivalent circuit at the collector nodes of Q2 and Q4:



4. Finally, the output resistance of the diff-amp is nothing but:

$$R_O = r_{o2} || r_{o4}$$

1. Or, it can be rearranged as follows:



2. From the figure above, the output voltage is:

$$v_o = 2 \left(\frac{g_m v_d}{2} \right) (r_{o2} || r_{o4} || R_L)$$

3. Therefore, the small-signal diff-mode voltage gain is:

$$A_d = \frac{v_o}{v_d} = g_m (r_{o2} || r_{o4} || R_L)$$

alternatively

$$A_d = \frac{g_m}{g_{o2} + g_{o4} + G_L},$$

where g_{o2} , g_{o4} and G_L are the corresponding conductances.

To minimize loading effects, we need $R_L > R_O$. However, since R_O is generally large for active loads, we may not be able to satisfy this condition. And this is the challenge that circuit designers face. 27

Exercise:

Task: Determine the differential-mode gain of a diff-amp with active load, with and without loading effect.

Diff-amp with active load:

Transistor parameters:

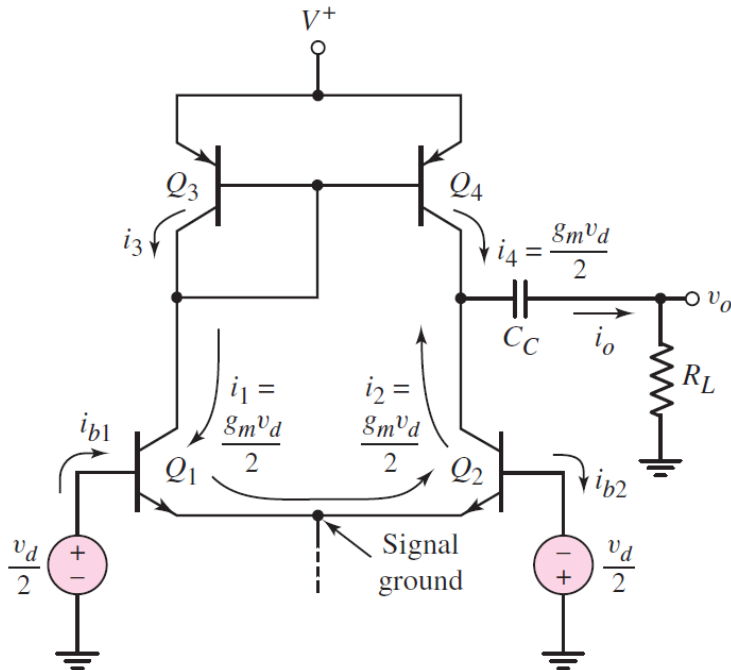
$$V_A = 100 \text{ V.}$$

Circuit parameters:

$$I_Q = 0.2 \text{ mA;}$$

$$R_L = 100 \text{ k}\Omega.$$

Solution:



A 100 k Ω load caused almost an order of magnitude decrease in the gain.

In this lecture we have seen:

- How the current mirror circuit is able to create current source and how the basic circuit can be easily adapted to provide a number of separate sources with a range of output currents.
- How the range of currents produced by the basic current repeater circuit can be extended with the Widlar circuit without the need for large value resistors, making it particularly suitable for use in integrated circuits.
- How an active load can be used to produce a high gain differential amplifier.

Announcements:

➤ **Reading material:**

- **Microelectronics: Circuit Analysis and Design:**
 - Chapter 10: Sec. 10.1 – Bipolar Transistor Current Sources
 - Chapter 11: Sec. 11.4 – Differential Amplifier with Active Load

➤ **Practice exercises:**

- **Microelectronics: Circuit Analysis and Design, Chapter 10**
 - Chapter 10: Ex. 10.1-10.7
 - Chapter 11: Ex. 11. 10

Thank you for your attention.....

The End